Lab 3.

Q1. $M = \dfrac{20(S-1)}{(S+2)(S^2+4)}$

↳ 2 imaginary poles
↳ 1 LHP.
$= (-2)$ pole

∴ M(S) is marginally stable.

Q2. $2S^4 + S^3 + 3S^2 + 5S + 10 = 0$.

using rh criterion:

$-\begin{vmatrix} 2 & 3 \\ 1 & 5 \end{vmatrix} = -7$   $-\begin{vmatrix} 2 & 10 \\ 1 & 0 \end{vmatrix} = 10$

| $S^4$ | 2 | 3 | 10 |
|---|---|---|---|
| $S^3$ | 1 | 5 | 0 |
| $S^2$ | -7 | 10 | 0 |
| $S$ | 45/7 | 0 | 0 |
| $S^0$ | 10 | 0 | 0 |

$-\dfrac{1}{-7}\begin{vmatrix} 1 & 5 \\ -7 & 10 \end{vmatrix} = \dfrac{45}{7}$

→ as there is a sign change

∴ System UNSTABLE

Q3. $S^4 + S^3 + 2S^2 + 2S + 3 = 0$.

| $S^4$ | 1 | 2 | 3 |
|---|---|---|---|
| $S^3$ | 1 | 2 | 0 |
| $S^2$ | 0 | 3 | 0 |
| $S^1$ | -3 | 0 | 0 |
| $S^0$ | 0 | 0 | 0 |

∴ UNSTABLE.

Q4. $S^5 + 4S^4 + 8S^3 + 2S + 4 = 0$.

| $S^5$ | 1 | 8 | 7 | 0 |
|---|---|---|---|---|
| $S^4$ | 4 | 8 | 4 | 0 |
| $S^3$ | 6 | 6 | 0 | 0 |
| $S^2$ | 4 | 4 | 0 | 0 |
| $S^1$ | 0 | 0 | 0 | 0 |
| $S^0$ | 4 | 0 | 0 | 0 |

No sign change
∴ System is STABLE.

**Q5.**

i(t) → + ⊙ → K → $\dfrac{s(3s+1)}{s^3+2s+4}$ → y(t)

     ↑ −y(t)            G(s)
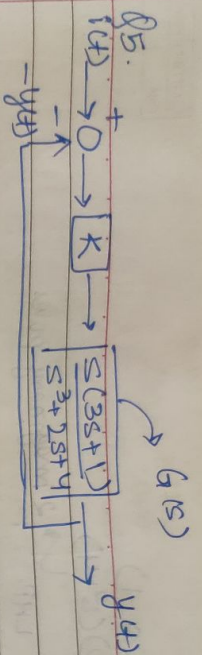
K → gain of proportional controller     $1 + KG(s) = 0$

$K(G(s))\,(i(t) - y(t)) = y(t)$.

$$1 + \frac{s^3+2s+4}{K\,s(3s+1)} = 0.$$

$$s^3 + 2s + 4 + Ks(3s+1) = 0$$

$$s^3 + (2+K)s + 4 + 3ks^2 = 0$$

$\underline{K>0}$       on table.

| | | |
|---|---|---|
| $s^3$ | 1 | $2+K$ |
| $s^2$ | $3k$ | 4 |
| $s^1$ | $\dfrac{-(4-3k^2-6k)}{3k}$   [k+2] | 0 |
| $s^0$ | 4 | |

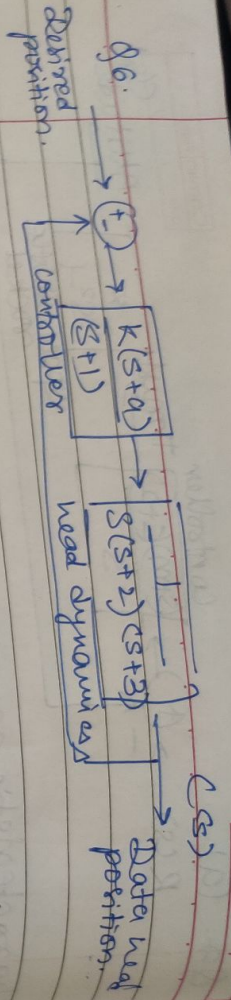$\Rightarrow \dfrac{-1}{3k}(4-3k^2-6k)$.

As $k > 0$

$$3k^2 + 6k - 4 > 0$$

$$k > -1 + \sqrt{\tfrac{7}{3}} \quad \text{ч } 0.53$$

when $k = -1+\sqrt{\tfrac{7}{3}}$ both elements of 3$^{rd}$ row is 0.

stable (marginally).

$$\boxed{k > -1 + \sqrt{\tfrac{7}{3}}}$$

System stable for →

Q6.



Desired position

Block: → (+/−) → K(s+a) [controller] → (s+1) → 1/[s(s+2)(s+3)] → C(s)
Data read dynamic position.

**Characteristic equation:**

$$1 + \frac{K(s+a)}{s(s+1)(s+2)(s+3)} = 0$$

$$S^4 + 6S^3 + 11S^2 + (6+K)S + Ka = 0$$

as K > 0  ∴ a > 0.

| | | | |
|---|---|---|---|
| $S^4$ | 1 | 11 | Ka |
| $S^3$ | 6 | 6+K | 0 |
| $S^2$ | $-\frac{1}{6}\vert 6$  $\frac{6+K}{-\frac{1}{6}\vert 6}$  Ka | 0 | |
| $S^1$ | $\frac{6Ka-(6+K)(10-K/6)}{10-K/6}$ | 0 | |
| $S^0$ | Ka | 0 | 0 |

→ Ka
→ 10 − K/6.

$$\therefore \ 10 - \frac{K}{6} > 0. \qquad 0 < K < 60.$$

at K = 60, a = 0
system → marginally stable

$$0 \le a \le \frac{10+3-K}{...} \ge \frac{3-K}{36}$$

∴ Stable region:  K ∈ (0, 60],  a ∈ [0, 10].

**Q7. (a)**

R(s) → ⊗ → [Controller $\frac{(s+m)(s+2)}{s}$] → thrust → [Spacecraft model $\frac{k}{s^2-1}$] → Attitude C(s)
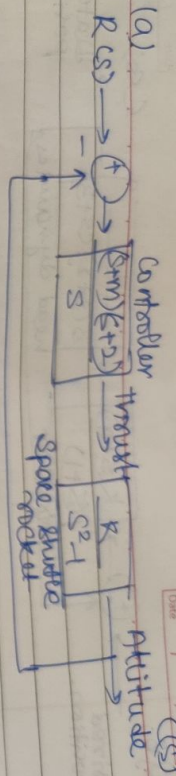
Characteristic equation:

$$1 + \frac{k(s+m)(s+2)}{s(s^2-1)} = 0$$

$$s^3 - s + ks^2 + k(m+2)s + 2km = 0$$

$$s^3 + ks^2 + (k(m+2)-1)s + 2km = 0$$

For stability:

$k > 0$

$2km \geq 0 \quad \therefore m \geq 0$

$k(m+2)-1 > 0 \longrightarrow m > \frac{1}{k} - 2$

| | | |
|---|---|---|
| $s^3$ | 1 | $k(m+2)-1$ |
| $s^2$ | k | $2km$ |
| $s^1$ | $\frac{1}{k}\begin{vmatrix} k & k(m+2)-1 \\ k & 2km \end{vmatrix}$ | 0 |
| $s^0$ | $(k-2)m + 2k-1$ | |

$$\begin{cases} >0 & \forall\ m \geq 0 \quad \text{if } k \geq 2 \\ <0 & \forall\ m \leq 0 \quad \text{if } k < 2, \quad k\in[1/2,2) \\ \geq 0 & \forall\ m \leq \frac{2k-1}{2-k}, \quad k\in[1/2,2) \end{cases}$$

System is $\left\{ m = \frac{2k-1}{2-k}, \quad k\in[1/2,2) \right\}$

∴ System stable   marginally stable

∴ f  k>2, m≥0.

or  k>2, m>0.

$$k\in[1/2,2), \quad 0 \leq m \leq \frac{2k-1}{2-k}$$

Q2.
(b) $G_1(s) = \frac{(s+m)(s+2)}{s}$

$G_2(s) = \frac{k}{s^2-1}$

steady state error $\longrightarrow$ $e = \dfrac{1}{\phantom{xxx}}$ when system stable

ramp response $\lim_{s\to 0} s\,G_1(s)\,G_2(s)$

$= \boxed{-\dfrac{1}{2km}}$ given error $\pm 0.1$

$k, m$ are both +ve

so; $\dfrac{-1}{2km} \;\eta = -0.1$

$\boxed{m \geq 5/k}$

given error $\pm 0.1$

Q8. $G_1 = 1/s(s+2)(s+4)$

(a,b)



$R(s) \rightarrow \bigotimes \xrightarrow{+} \boxed{K_1} \rightarrow \bigotimes \rightarrow \boxed{G_1(s)} \rightarrow C(s)$

$\boxed{k_2(s)}$
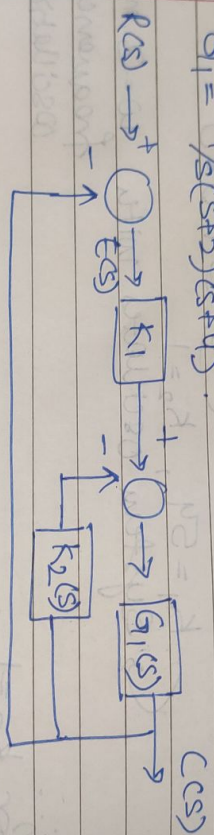
$1 + k_2\, s\, G_1(s) = 0.$

$G$ will have 2 equal $-ve$ roots.

$(s+2)(s+4) + k_2 = 0.$

$s^2 + 6s + 4 = 0 \;\begin{cases} -3 \\ -3 \end{cases}$ 2 equal roots

$k_2 = 1$

$G_1(s) = \dfrac{1}{s(1+(s+2)(s+4))}$

$$R(s) \longrightarrow \bigoplus_{(+-)} \xrightarrow{E(s)} \boxed{k_1} \longrightarrow \boxed{G(s)} \xrightarrow{C(s)}$$

$$1 + k_1 \, G(s) = 0$$

$$1 + \frac{k_1}{s(s+2)(s+4)+s} = 0.$$
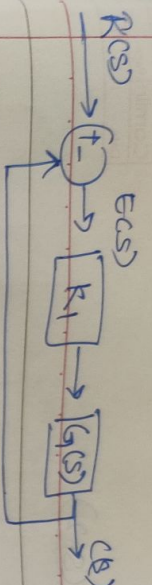
$$S^3 + 6S^2 + 9s + k_1 = 0 \qquad k_1 > 0.$$

$$\begin{array}{c|cc}
S^3 & 1 & 9 \\
S^2 & 6 & k_1 \\
S & \dfrac{54-k_1}{6} & 0 \\
6 & & 
\end{array}$$

$$\underbrace{0 < k_1 < 54}_{\text{for stable system}}$$

$$\begin{array}{l}
k_1 = 54, \quad k_2 = 1 \\
\hookrightarrow \text{System oscillates} \quad S^3 + 6S^2 + 9s + 54 = 0 \\
\qquad (S+6)(S^2+9) = 0 \qquad \omega = 3 \text{ rad/sec} \\
\qquad\qquad\qquad\qquad \hookrightarrow \text{frequency of oscillation.}
\end{array}$$

$$\text{for } k_2 = 1 \\
\quad \hookrightarrow k_1 \in (0.54)$$

$$\therefore$$

at $k_1 = 54 \hookrightarrow$ system is marginally stable.
(oscillates).

if $k_2$ not specified

$$\omega = \sqrt{\frac{8+k_2}{}} \text{ rad/sec}$$

when $k_1 = 6(8 + k_2)$.

$$(4)(c+2)+1)2$$

Q8 (c) as closed loop pole at will make system unstable, we won't able to approximate Step response as underdamped.

$$s^3 + 6s^2 + (8+K_2)s + K_1 = 0$$

$s = 5 \rightarrow$ pole

$$K_1 = -5(6S + K_2).$$

ego

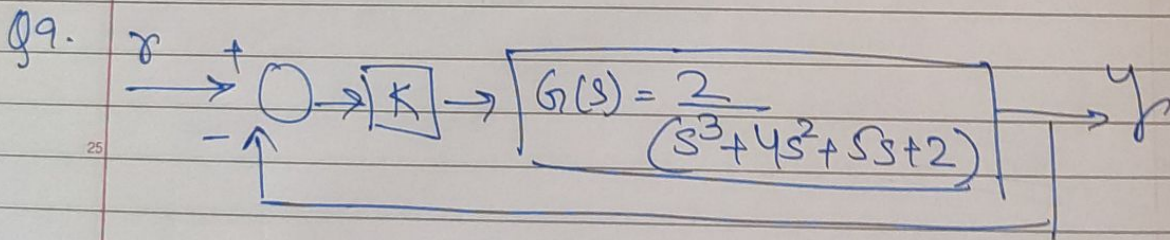$$K_1 = 5(K_2 + 3)$$

$$(8+5)(s^2 + s + K_2 + 3) = 0.$$

1 pole at $s = -5$
both complex poles have real $= -\frac{1}{2}$
real pole is 10 times away from img axis.

Dominant pole approximation
$K_2 = 1 \quad K_1 = 20.$

$$G(s) = \frac{20}{s^3 + 6s^2 + 9s + 20.}$$

$$G_{app}(s) = \frac{4}{s^2 + s + 4.}$$

Q9.



$$y(s) = \frac{K G(s)}{1 + K G(s)} r(s).$$

$$E(s) = \frac{R(s)}{1 + K(G(s)).}$$

$$R(s) = \frac{1}{s}.$$

$$e = \frac{1}{1 + K \lim_{s \to 0} G(s)}.$$

$$e = \frac{1}{1+k}.$$

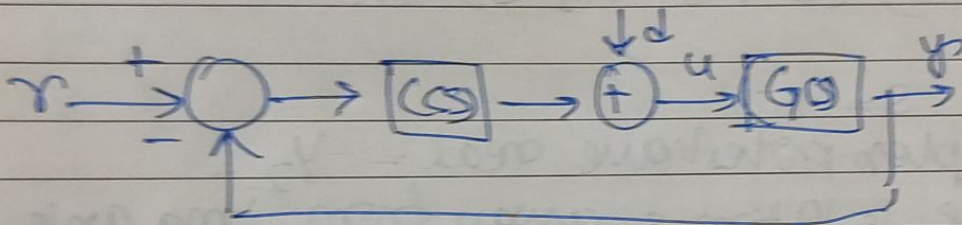$$G(s) = (-2, -1, -1) \text{ in LHP.}$$

$$\frac{1}{1+k} \leq 0.15$$

$$k \geq 17/3$$

Characteristic equ$^n$     $1 + k \, G(s) = 0$
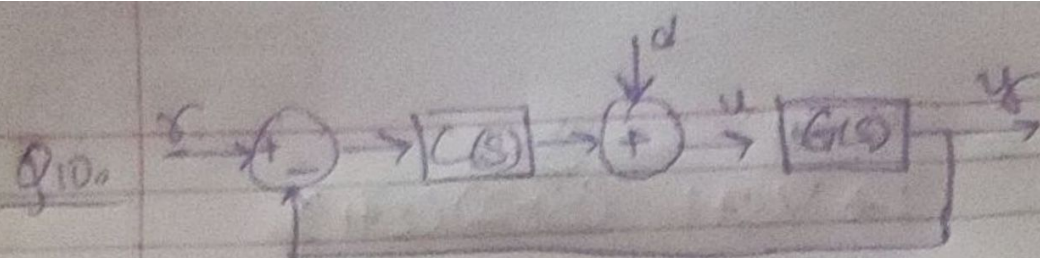
$$s^3 + 4s^2 + 5s + 2 + 2k = 0.$$

Q10



$$G(s) = 1/s-1$$

$$C(s) = \frac{s-1}{s+1}.$$

case 1.

Q10.



$$R(s) = D(s) \cdot \frac{1}{s}$$

$$G(s)\left(C(s)(R(s) - Y(s)) + D(s)\right) = Y(s).$$

$$Y(s) = U(s)G(s).$$

$r \rightarrow y$

$$G(s)C(s)R(s) = Y(s)\left(1 + G(s)C(s)\right)$$

$$\boxed{\frac{Y(s)}{R(s)} = \frac{GC}{1 + GC}}$$

$d \rightarrow y$ $\quad GD = Y(1 + GC)$

$$\boxed{\frac{Y}{D} = \frac{G}{1 + GC}}$$

$r \rightarrow u$ $\quad \dfrac{UG}{R} = \dfrac{GC}{1 + GC}$ $\qquad \boxed{\dfrac{U}{R} = \dfrac{C}{1 + GC}}$

$d \rightarrow u.$ $\quad \dfrac{GU}{D} = \dfrac{G}{1 + GC}$ $\qquad \boxed{\dfrac{U}{D} = \dfrac{1}{1 + GC}}$

Case ① $\quad C = \dfrac{S-1}{S+1} \qquad G = \dfrac{1}{S-1}$

$r \rightarrow y$ - $\dfrac{Y}{R} = \dfrac{1/S+1}{1 + 1/S+1} = \dfrac{1}{S+2}.$ $\qquad Y = \dfrac{1}{S+2} \cdot \dfrac{1}{S}$

$$Y = \dfrac{1}{2}\left(\dfrac{1}{S} - \dfrac{1}{S+2}\right)$$

$$\boxed{y(t) = \dfrac{1}{2}\left(u(t) - e^{-2t} u(t)\right)}$$

$d \to y$

$$\frac{Y}{D} = \frac{1/s+1}{1+1/s+1} \Rightarrow \frac{s+1}{(s+2)(s+1)}$$

$$Y = \frac{s+1}{(s+2)(s+1)(s)} \qquad \rightarrow \quad Y = \frac{1}{2s} + \frac{2}{3(s-1)} - \frac{1}{6(s+2)}$$

$$y(t) = \left(\frac{1}{2} + \frac{2}{3}e^{t} - \frac{1}{6}e^{-2t}\right)u(t).$$

$d \to u$.

$$\frac{U}{D} = \frac{1}{1+1/s+1} = \frac{s+1}{s+2}. \qquad U = \frac{s+1}{s(s+2)}.$$

$$U = \frac{1}{2}\left(\frac{1}{s+2} + \frac{1}{s}\right) \qquad u(t) = \left(\frac{1}{2} + \frac{1}{2}e^{-2t}\right)u(t)$$

$r \to u$.

$$\frac{U}{R} = \frac{s \cdot 1/s+1}{1+1/s+1} = \frac{s}{s+2}.$$

$$U = \frac{s}{(s+2)s} = \frac{3}{2}\frac{1}{s+2} - \frac{1}{2s} \qquad u(t) = \left(\frac{3}{2}e^{-2t} - \frac{1}{2}\right)u(t)$$

case ② $G(s) = \frac{s+1}{s+1}$    $C(s) = \frac{1}{s+1}$

$d \to u$

$$\frac{U}{D} = \frac{1}{1+GC} \Rightarrow \quad u(t) = \left(\frac{1}{2} + \frac{1}{2}e^{-2t}\right)u(t)$$

remains the same.

$r \to y$    $\frac{Y}{R} = \frac{GC}{1+GC} \rightsquigarrow \quad y(t) = \left(\frac{1}{2} - \frac{1}{2}e^{-2t}\right)u(t)$

remains the same

$r \to u$    $\frac{U}{R} = \frac{1/s+1}{1+1/s+1} \rightsquigarrow$    Same as case ①
$d \to y$.

$$u(t) = \left(-\frac{1}{2} + \frac{2}{3}e^{t} - \frac{e^{-2t}}{6}\right)u(t)$$

$d \to y$.    $\frac{Y}{D} = \frac{s+1/s+1}{1+s+1/s+1} \rightsquigarrow$    same as case ①
$r \to u$.

$$y(t) = \left(\frac{3}{2}e^{-2t} - \frac{1}{2}\right)u(t)$$