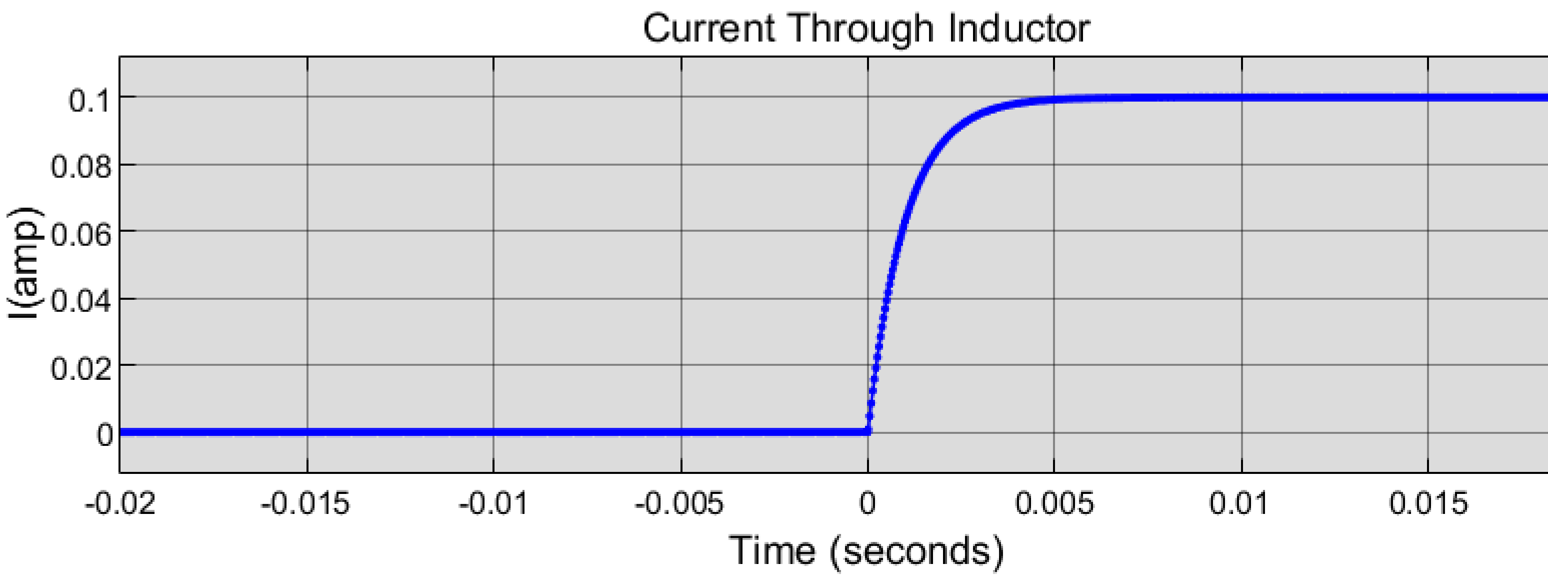
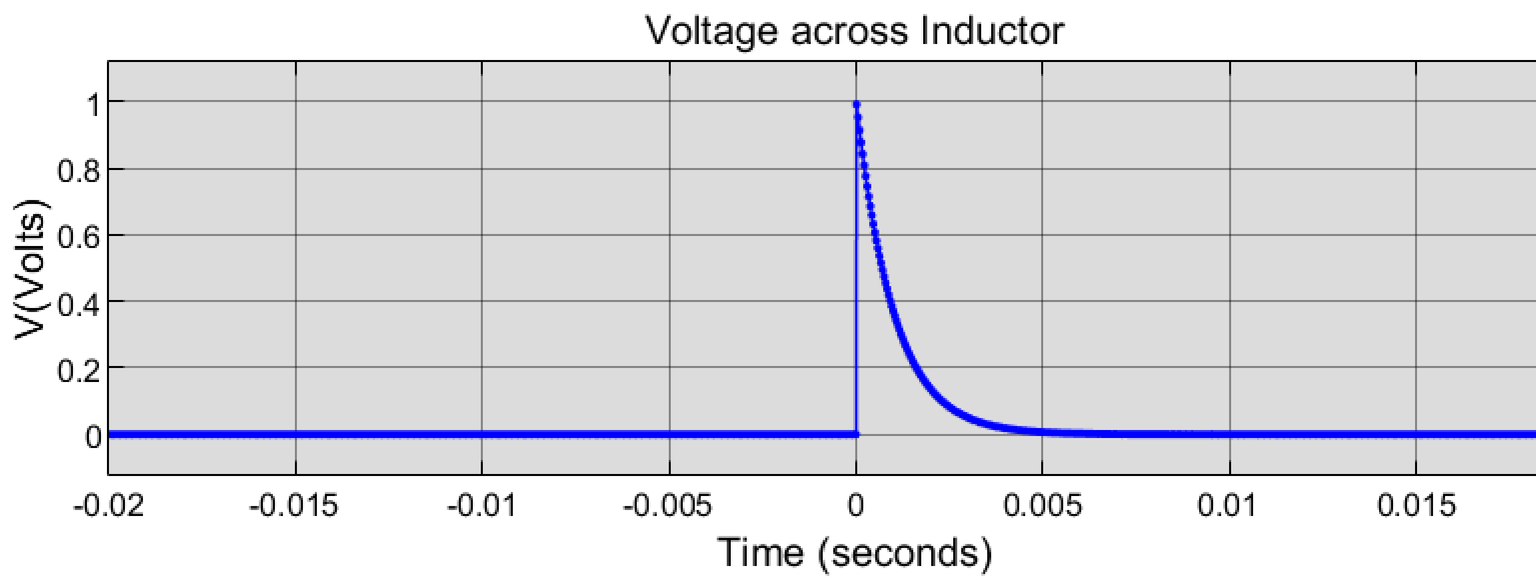
Q1





Time constant of the circuit is equal to $\frac{L}{R} = \frac{1mH}{10\Omega} = 10^{-3}s$

```
% a
% stimulation done on part1.slx
% The plot shows the current and voltage across the inductor vs time.
```

$$\text{(i)} \quad G(s) = \frac{(s+1)}{s(s+2)(s+3)}$$

```
% b (i)
% zeros = [-1]
% poles = [0,-2,-3]
% gain = 1

tf1=tf([1,1],[1,5,6,0])
```

```
tf1 =

        s + 1
  -----------------
  s^3 + 5 s^2 + 6 s

Continuous-time transfer function.
```

```
zpk1=zpk(-1,[0,-2,-3],1)
```

```
zpk1 =

      (s+1)
  -------------
  s (s+2) (s+3)

Continuous-time zero/pole/gain model.
```

(ii) $G(s) = \dfrac{-s-1}{s^2+5s+6}$

```
tf2=tf([-1,-1],[1,5,6])
```

```
tf2 =

     -s - 1
  -------------
  s^2 + 5 s + 6

Continuous-time transfer function.
```

```
zpk2=zpk(-1,[-2,-3],-1)
```

```
zpk2 =

    - (s+1)
  -----------
  (s+2) (s+3)

Continuous-time zero/pole/gain model.
```

```
% c
[A1, B1, C1, D1]=tf2ss([1,1],[1,5,6,0])
```

```
A1 = 3×3
    -5    -6     0
     1     0     0
     0     1     0
B1 = 3×1
     1
     0
     0
C1 = 1×3
     0     1     1
D1 = 0
```

```
[A2,B2,C2,D2]=tf2ss([-1,-1],[1,5,6])
```

```
A2 = 2×2
    -5    -6
     1     0
B2 = 2×1
     1
     0
C2 = 1×2
    -1    -1
D2 = 0
```

```
% time constant =L/R = 0.01/0.1 = 0.001 = 1ms
% set time is taken 10*time constant =0.01 s
```

Q2

Balancing forces in m_1 we get,

$$F_a - b_1\dot{x_1} - k_1(x_1 - x_2) = m_1\ddot{x_1}$$

and balancing forces in m_2 we get,

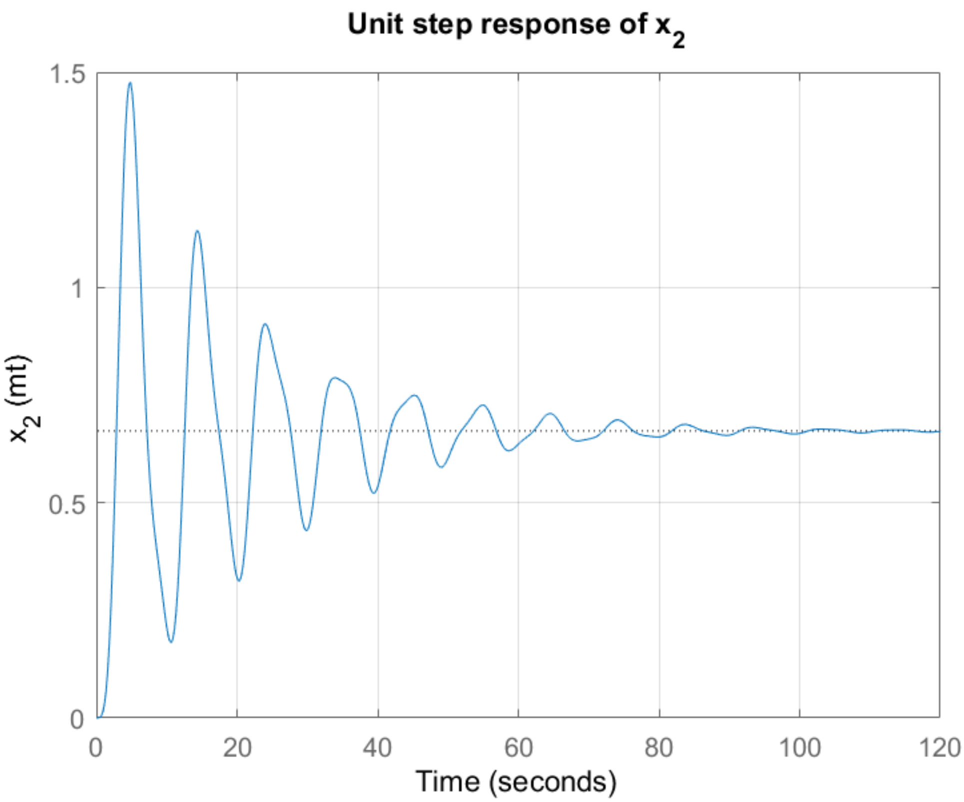
$$k_1(x_1 - x_2) - b_2\dot{x_2} - k_2x_2 = m_2\ddot{x_2}$$

```
m1=1;
k1=1;
b1=0.1;
m2=2;
k2=1.5;
b2=0.2;

A=[0,1,0,0; -k1/m1,-b1/m1,k1/m1,0; 0,0,0,1; k1/m2,0,-(k1+k2)/m2,-b2/m2];
B=[0; 1/m1; 0; 0];
C=[0,0,1,0];
D=0;

% transfer function
[b,a]=ss2tf(A,B,C,D);
tf1=tf(b,a)
```

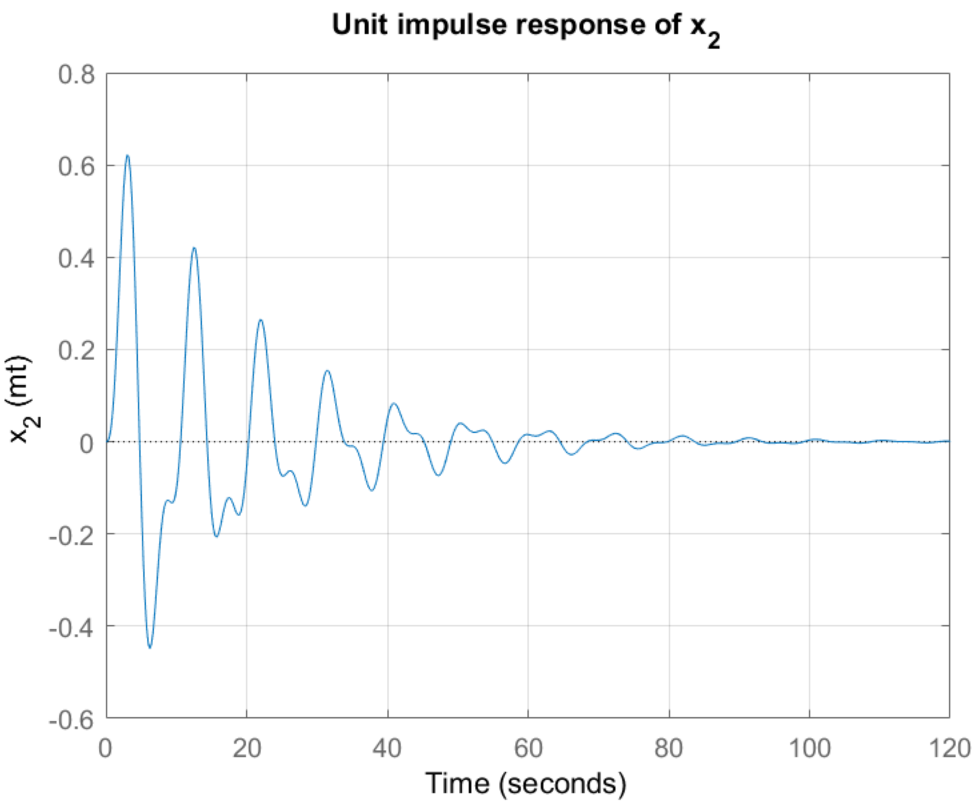
```
tf1 =

                          0.5
    ----------------------------------------
```

```
  s^4 + 0.2 s^3 + 2.26 s^2 + 0.225 s + 0.75

Continuous-time transfer function.
```

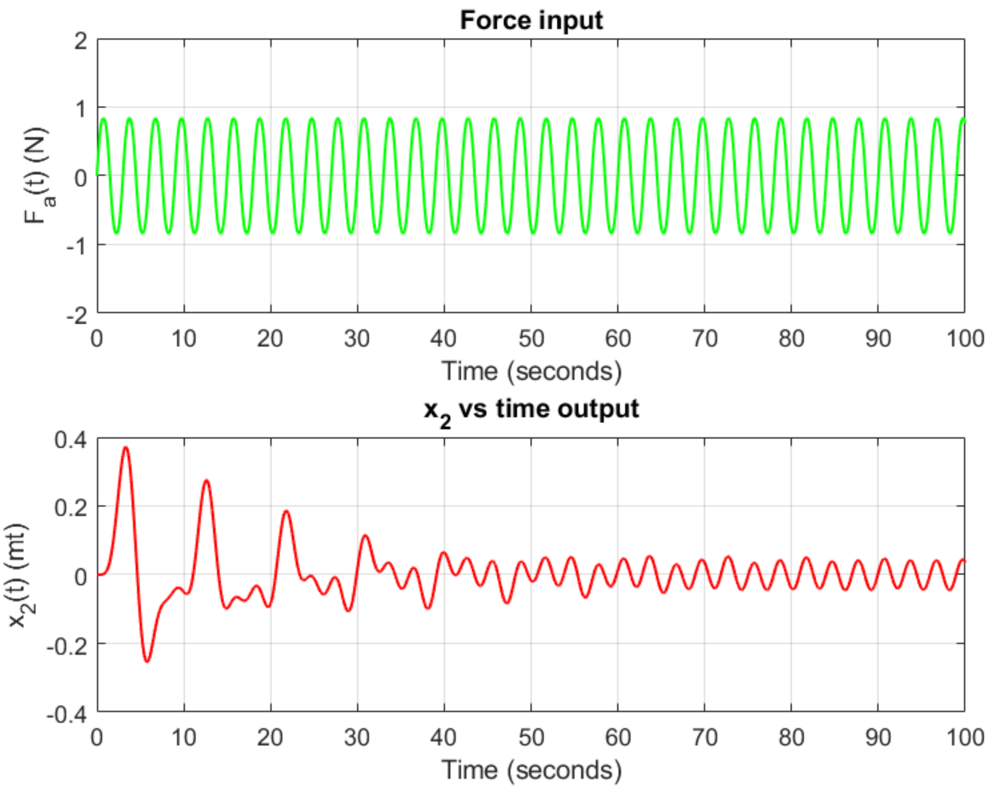
```
figure
step(tf1)
title("Unit step response of x_2")
grid on
ylabel("x_2 (mt)")
```




```
figure
impulse(tf1)
title("Unit impulse response of x_2")
grid on
ylabel("x_2 (mt)")
```

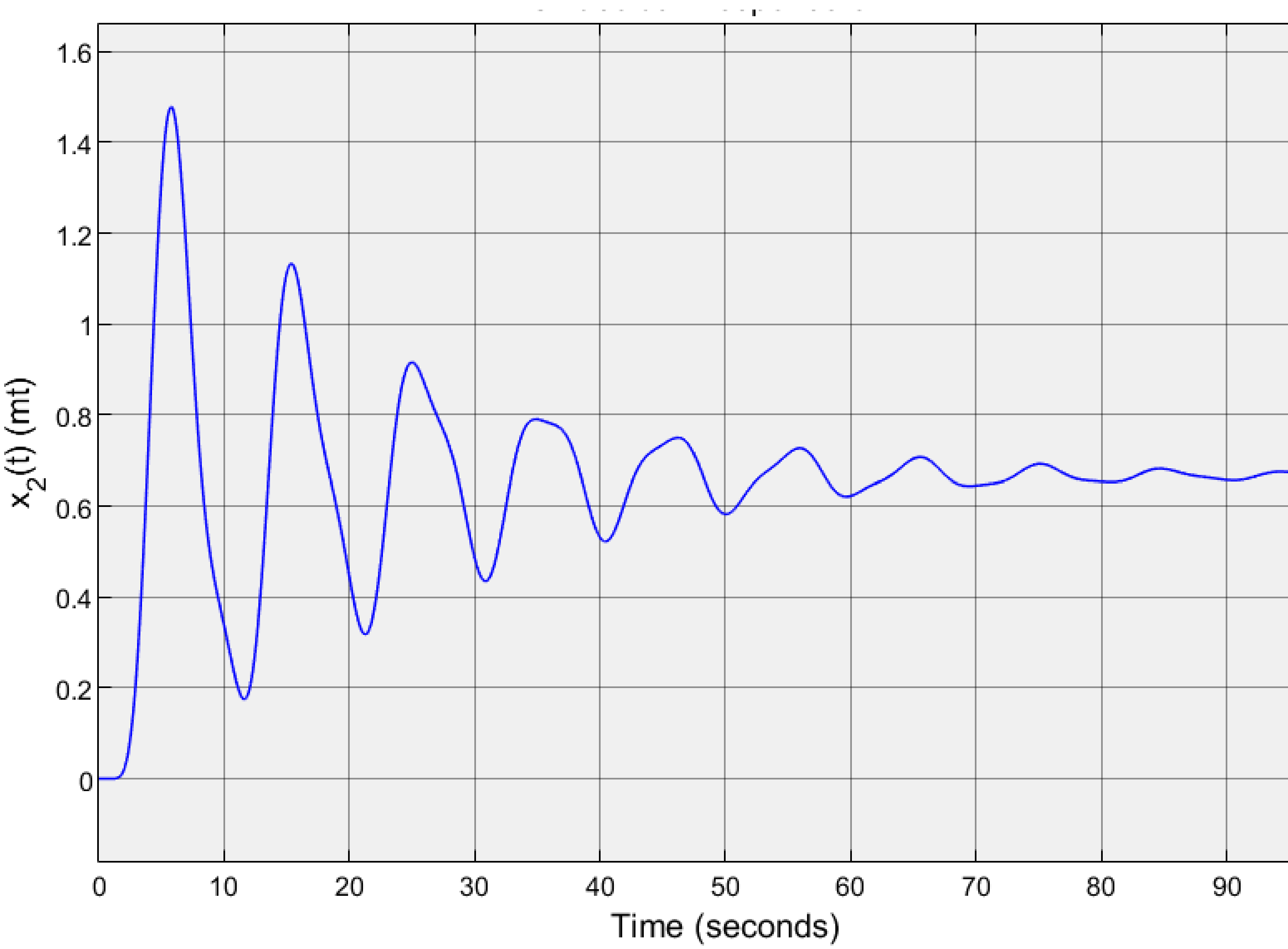



```
figure
t=0:1e-3:100;
u=sin(sin(2*pi*t/3));
[x2_out,tout]=lsim(tf1,u,t);
subplot(2,1,1)
plot(t,u,"LineWidth",1,"Color","g");
title("Force input");
ylabel("F_a(t) (N)");
xlabel("Time (seconds)");
ylim([-2,2]);
grid on
subplot(2,1,2)
plot(tout,x2_out,"LineWidth",1,"Color","r")
title("x_2 vs time output");
ylabel("x_2(t) (mt)");
xlabel("Time (seconds)");
grid on
```

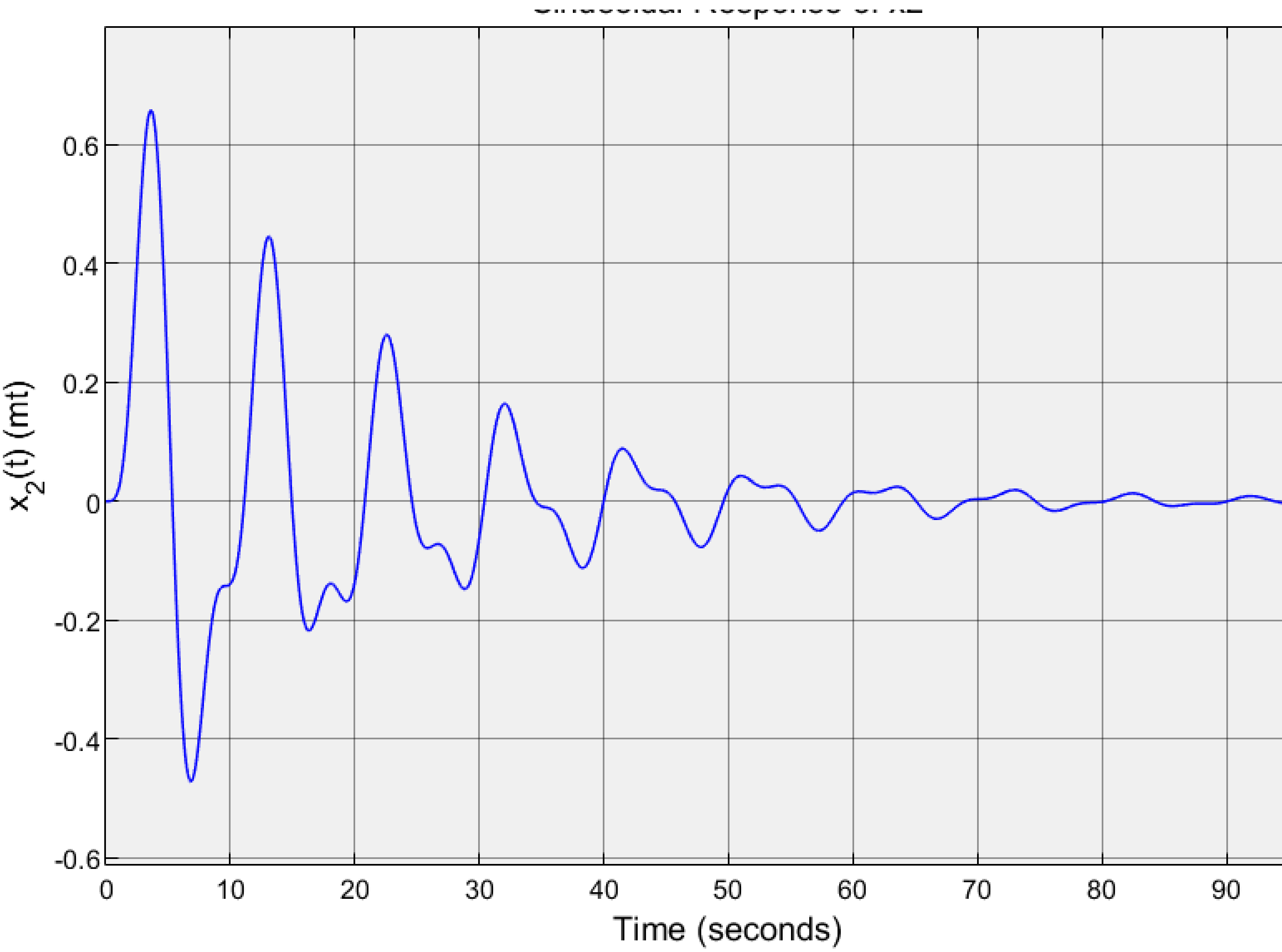


Stimulation done in part2.slx

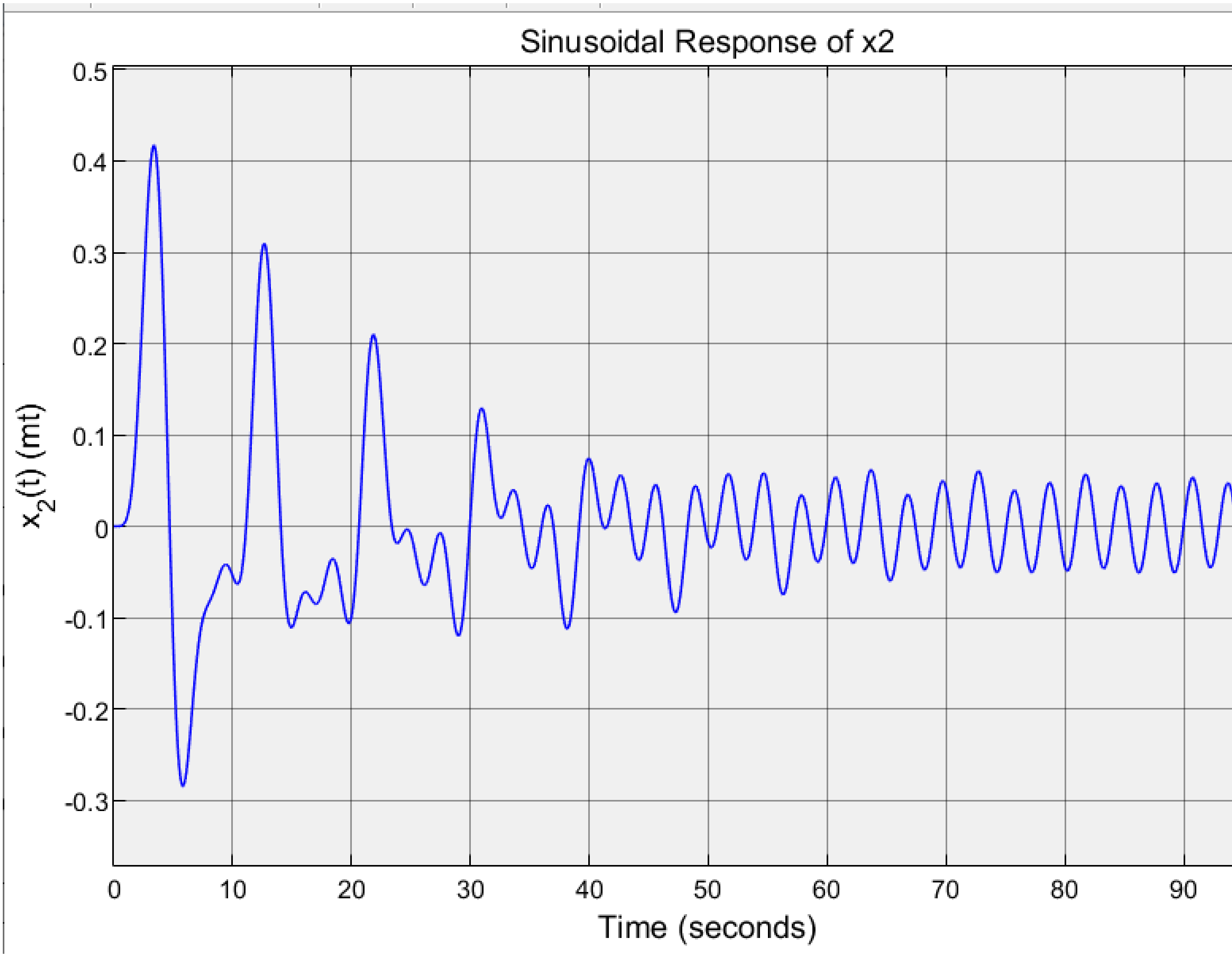
```
% all the plots are same for the stimulation results as well

% Applying an unit step output
```




```
% Applying an unit impulse output
```




```
% Applying a sinusoidal output
```


Sinusoidal Response of x2
$x_2(t)$ (mt)
Time (seconds)
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
0
10
20
30
40
50
60
70
80
90

Q3

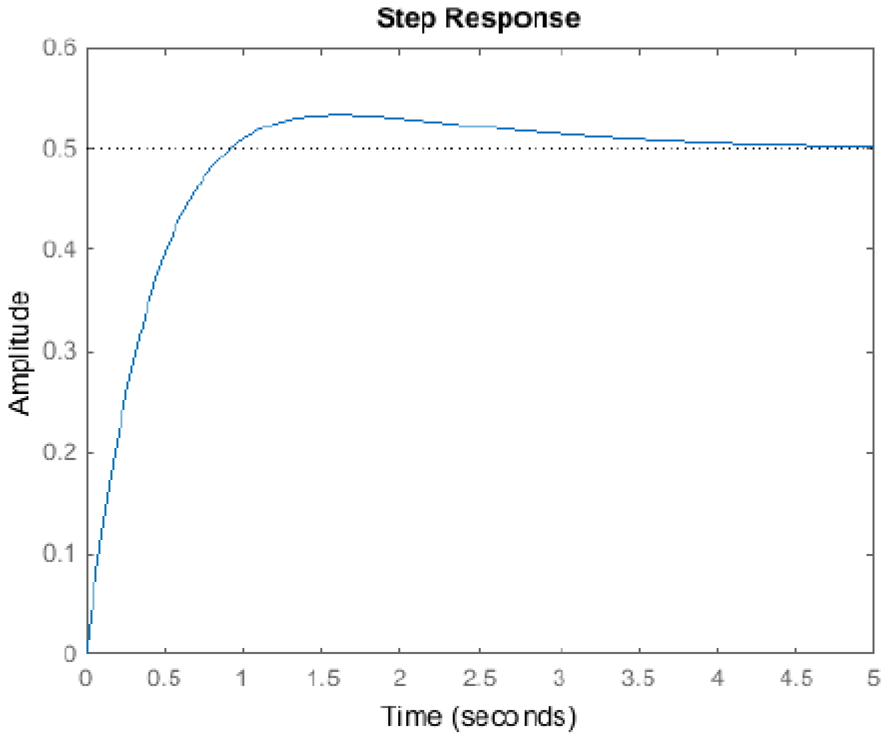
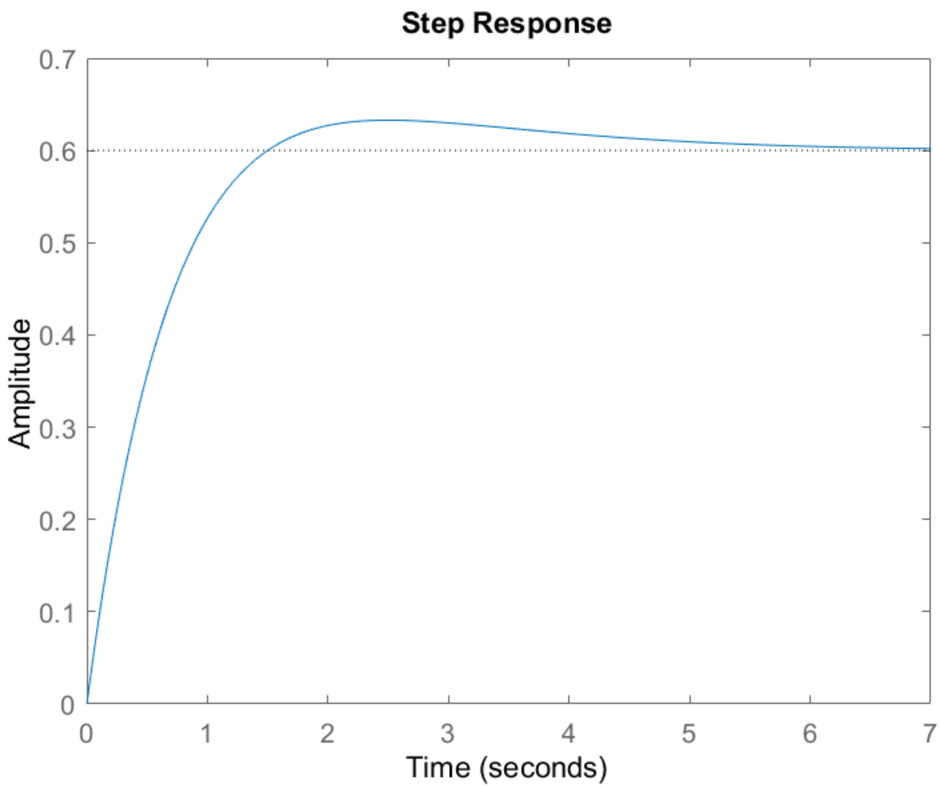
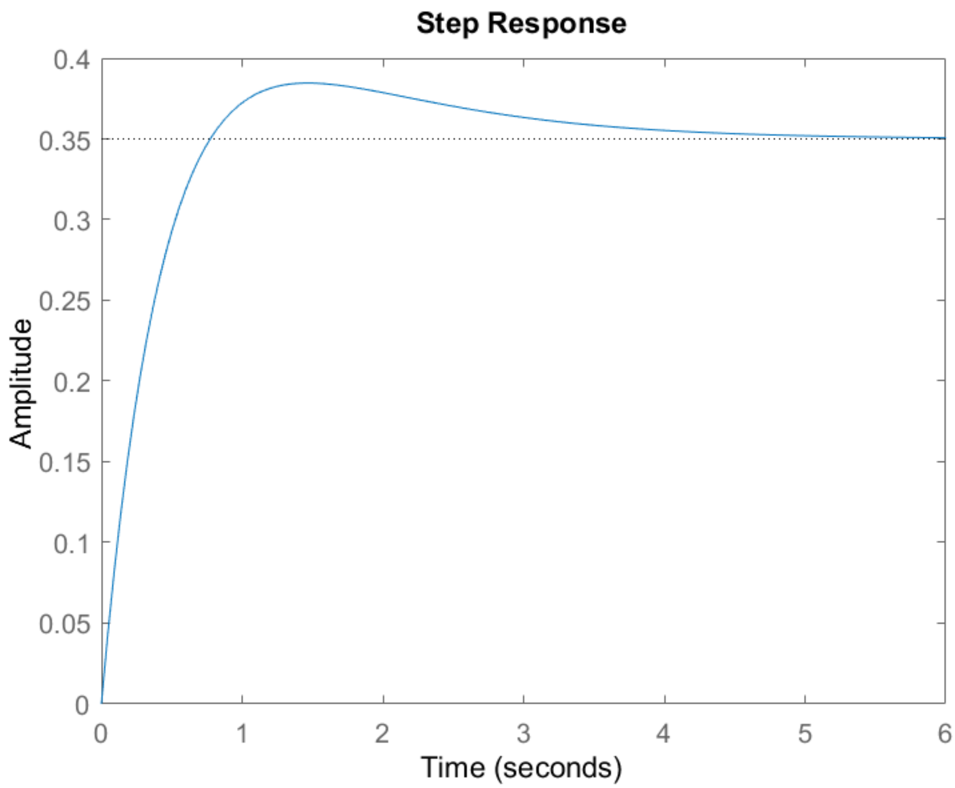


Figure 3: Step response of a system

```
% a
% the response has no oscillatory behaviour but it has an overshoot, thus
% the system is overdamped or critically damped. Overshoot signifies the
% presence of negative zero in system, as it amplifies the initial response.

%example: Critically damped system with poles [-1,-1] and zero -0.6
figure
step([1,0.6],[1,2,1])
```




```
% b
% Yes.
% we observe an overshoot in over-damped 2nd order system, if system has
% negative zero.

%example: Overdamped with poles [-1,-2] and zero -0.7
figure
step([1,0.7],[1,3,2])
```



Q4

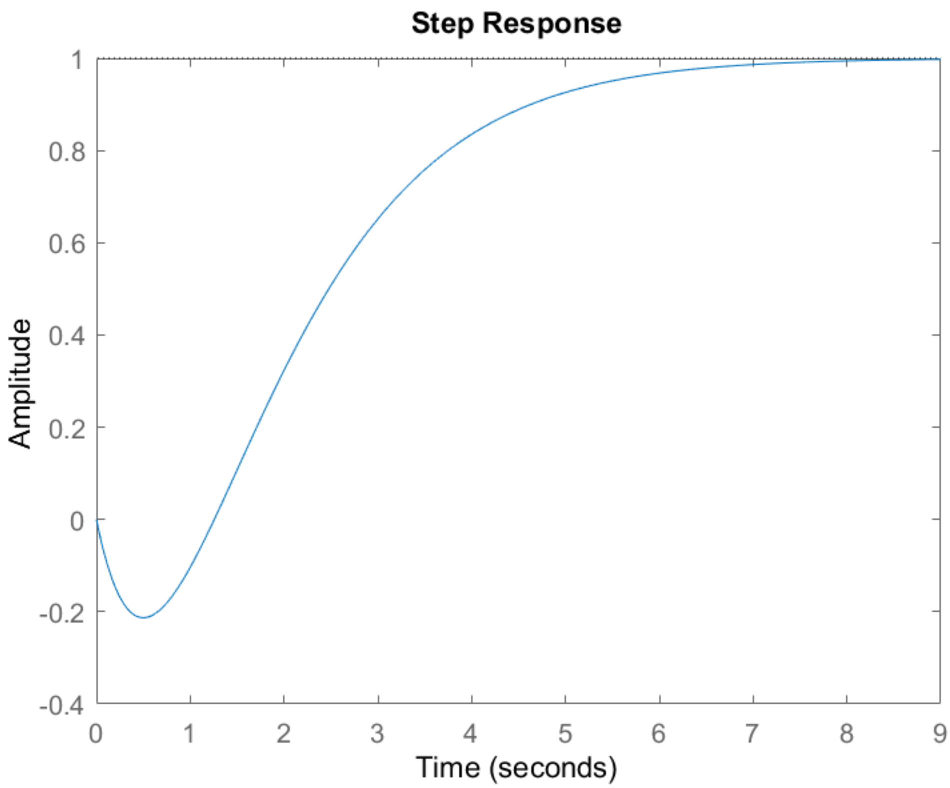
```
% a
% i
```

$$\text{(i)}\ G(s) = \frac{-(s-1)}{(s+1)^2}$$

```
figure
tf1=zpk(1,[-1,-1],-1)
```

```
tf1 =

  - (s-1)
  -------
  (s+1)^2

Continuous-time zero/pole/gain model.
```

```
step(tf1)
```



1. Strictly proper
2. Initial undershoot
3. zero crossing
4. no overshoot
5. 1 positive zero of $G(s)$
6. 1 positive zero of $G(s) - G(\infty)$
7. 0 positive zero of $G(s) - G(0)$

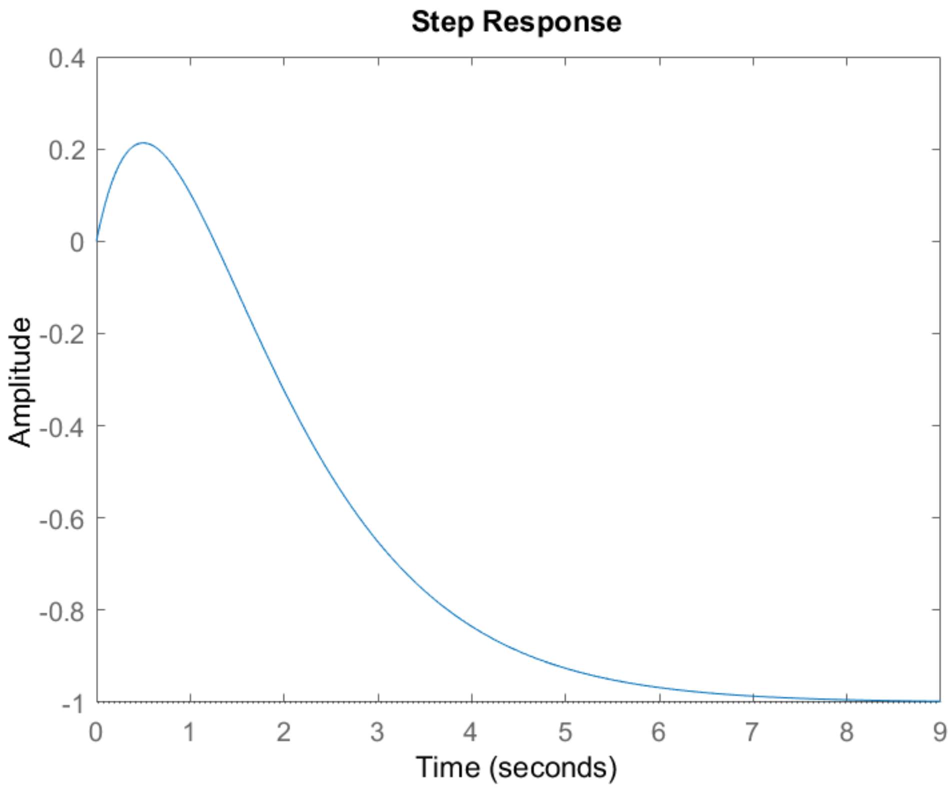
```
% ii
```

, (ii) $G(s) = \dfrac{(s-1)}{(s+1)^2}$

```
figure
tf2=zpk(1,[-1,-1],1)
```

```
tf2 =

   (s-1)
  -------
  (s+1)^2

Continuous-time zero/pole/gain model.
```

```
step(tf2)
```



1. Strictly proper
2. Initial undershoot
3. zero crossing
4. no negative overshoot
5. 1 positive zero of $G(s)$
6. 1 positive zero of $G(s) - G(\infty)$
7. 0 positive zero of $G(s) - G(0)$

```
% iii
```

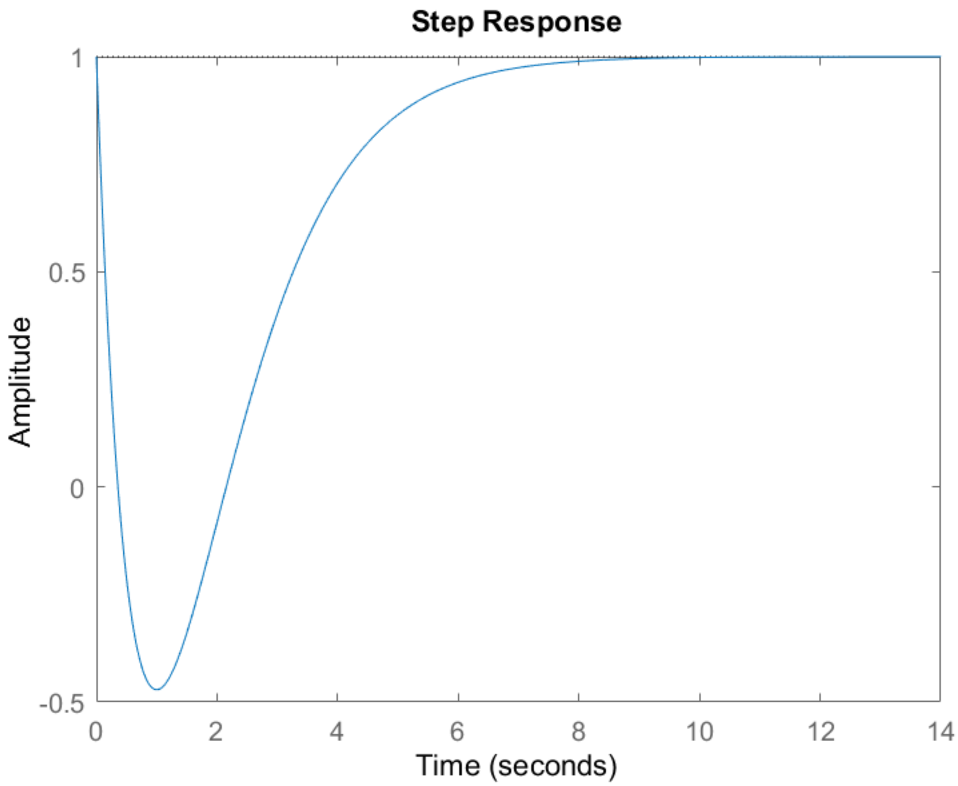
$$\text{(iii)}\ \ G(s) = \frac{(s-1)^2}{(s+1)^2}$$

```
figure
tf3=zpk([1,1],[-1,-1],1)
```

```
tf3 =

  (s-1)^2
  -------
  (s+1)^2
```

```
Continuous-time zero/pole/gain model.
```

```
step(tf3)
```



1. Proper
2. Initial undershoot
3. zero crossing
4. no overshoot
5. 2 positive zeros of $G(s)$
6. 0 positive zero of $G(s) - G(\infty)$
7. 0 positive zero of $G(s) - G(0)$

```
% iv
```

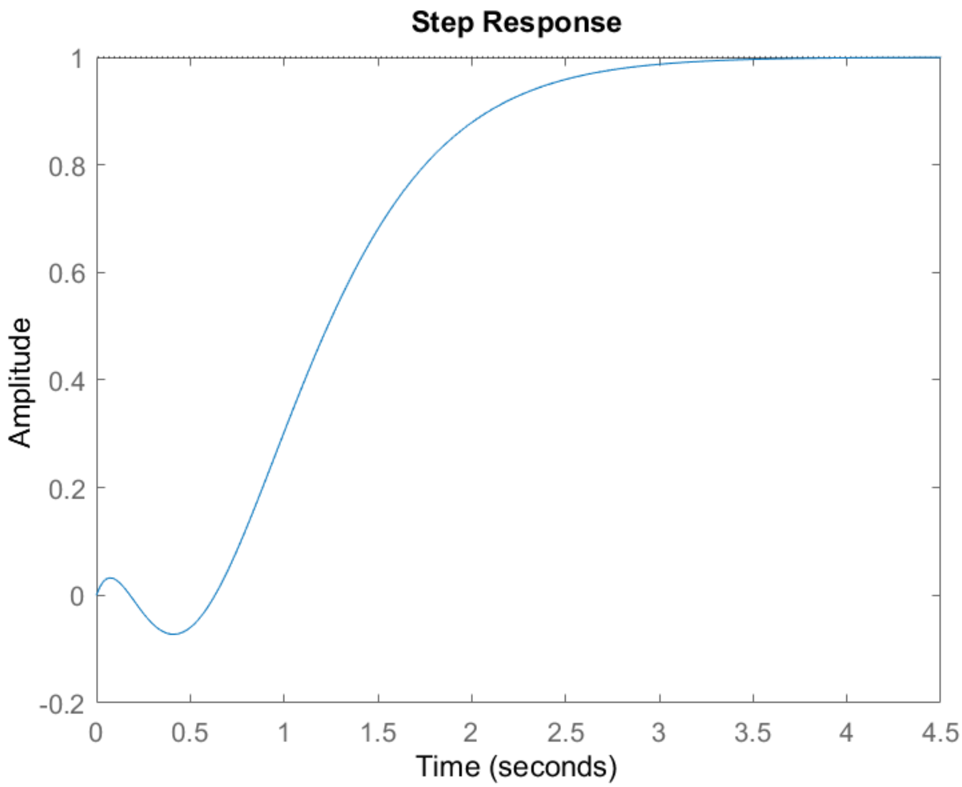
$$\text{(iv)} \quad G(s) = \frac{(s^2 - 10s + 27)}{(s+3)^3}$$

```
figure
tf4=tf([0,1,-10,27],[1,9,27,27])
```

```
tf4 =

      s^2 - 10 s + 27
```

```
  ----------------------
  s^3 + 9 s^2 + 27 s + 27

Continuous-time transfer function.
```

```
step(tf4)
```



1. Strictly proper
2. Initial undershoot
3. zero crossing
4. no overshoot
5. 0 positive zero of $G(s)$, 2 complex zeros
6. 0 positive zero of $G(s) - G(\infty)$,2 complex zeros
7. 0 positive zero of $G(s) - G(0)$, 2 complex zeros

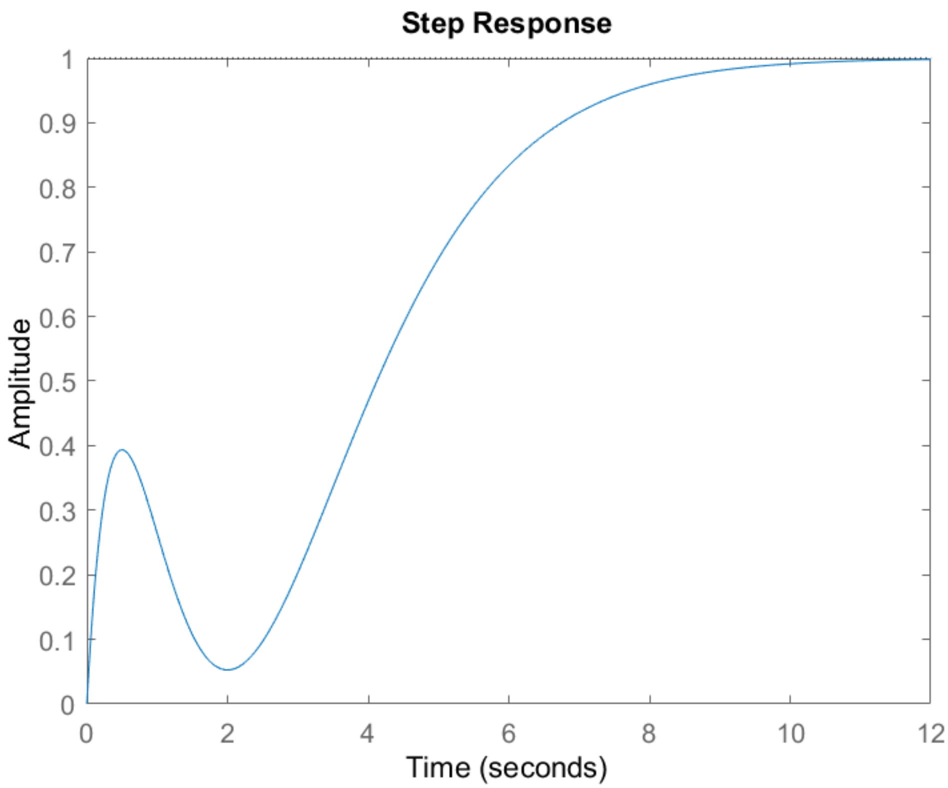
```
% v
```

$$\text{(v)}\quad G(s) = \frac{(2s^2 - s + 1)}{(s+1)^3}$$

```
figure
tf5=tf([2,-1,1],[1,3,3,1])
```

```
tf5 =

      2 s^2 - s + 1
  ---------------------
  s^3 + 3 s^2 + 3 s + 1

Continuous-time transfer function.
```

```
step(tf5)
```



1. Strictly proper
2. Initial undershoot
3. no zero crossing
4. no overshoot
5. 0 positive zero of $G(s)$, 2 complex zeros
6. 0 positive zero of $G(s)-G(\infty)$, 2 complex zeros
7. 0 positive zero of $G(s)-G(0)$, 2 complex zeros

```
% vi
```

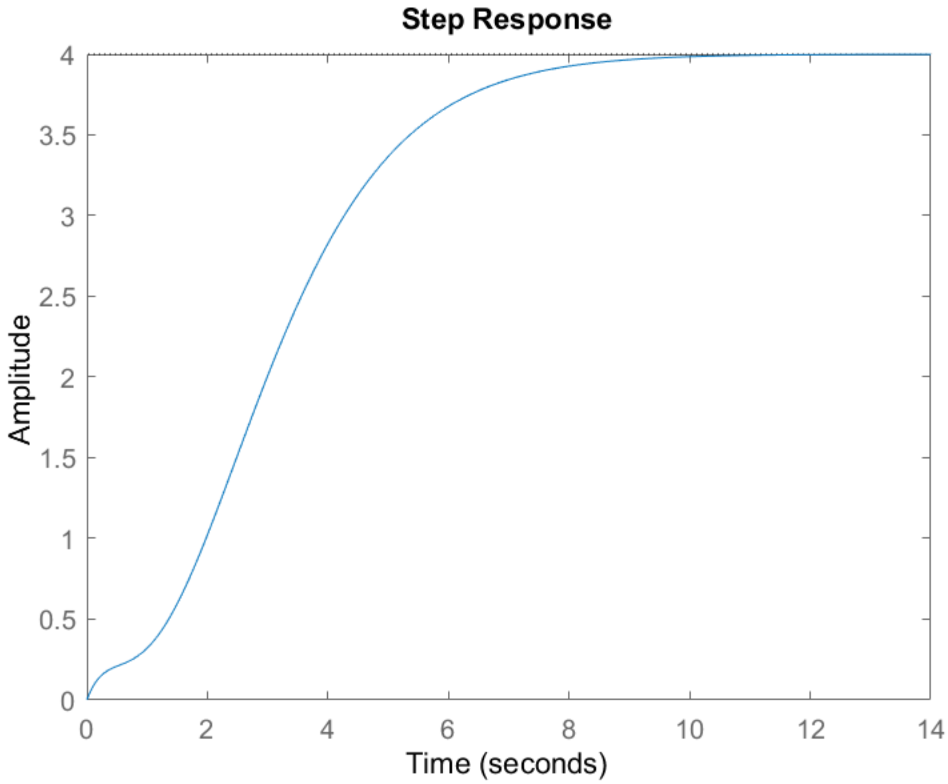
$$\text{(vi)}\;\; G(s)=\frac{(s^2-s+4)}{(s+1)^3}$$

```
figure
```

```
tf6=tf([1,-1,4],[1,3,3,1])
```

```
tf6 =

        s^2 - s + 4
  ---------------------
  s^3 + 3 s^2 + 3 s + 1

Continuous-time transfer function.
```

```
step(tf6)
```



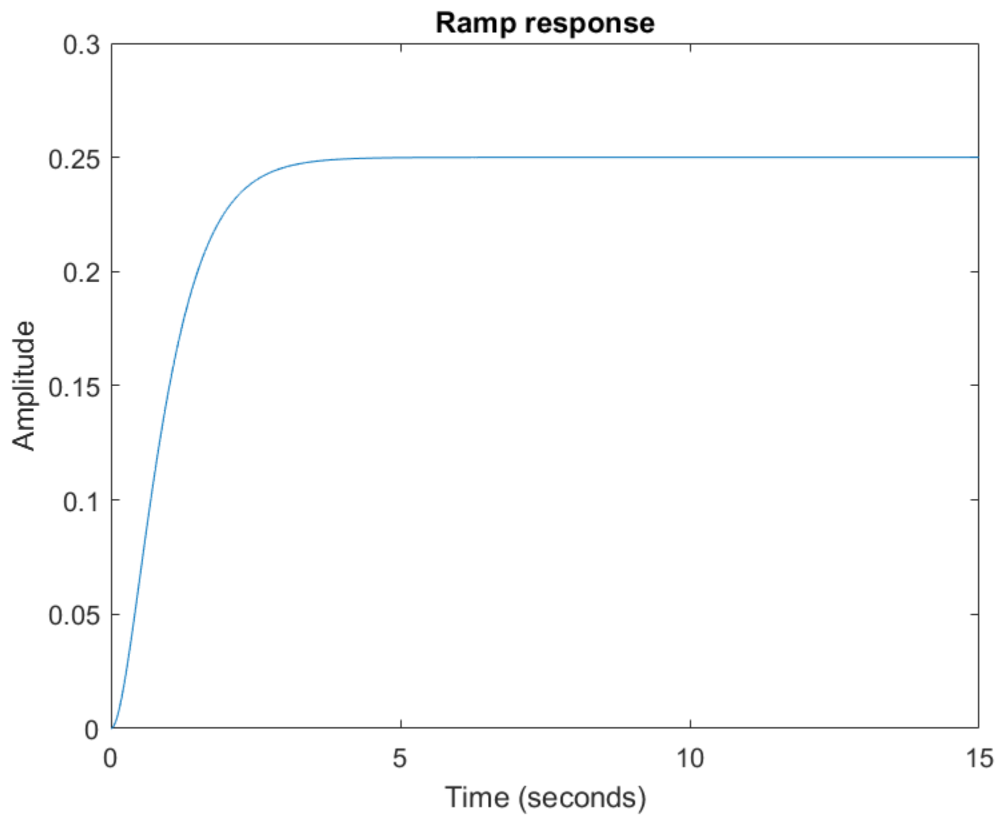
1. Strictly proper
2. no initial undershoot
3. no zero crossing
4. no overshoot
5. 0 positive zero of $G(s)$, 2 complex zeros
6. 0 positive zero of $G(s) - G(\infty)$, 2 complex zeros
7. 0 positive zero of $G(s) - G(0)$, 2 complex zeros

```
% b
% Yes, a linear dynamical system can give bounded response for unbounded input
% example: for a ramp function we'll get a bounded output

% numerator=s, denominator= (s^2+4s+4)
figure
```

```
tf7=tf([1,0],[1,4,4])
```

```
tf7 =

        s
  -------------
  s^2 + 4 s + 4

Continuous-time transfer function.
```

```
t=0:1e-4:15;
[Y,T]=lsim(tf7,t,t);
plot(T,Y)
title("Ramp response");
ylabel("Amplitude");
xlabel("Time (seconds)");
```




```
% unbounded ramp input --> bounded response, from a linear dynamical system
```

Q5

```
% a
% stimulation done in part3.slx
% output response C(s) for 14 different values of delta from 0.2 to 1.5 keeping
% natural frequency at 16 rad/sec
% where, R(s) provided by the waveform generator is unit step input at t=0
```

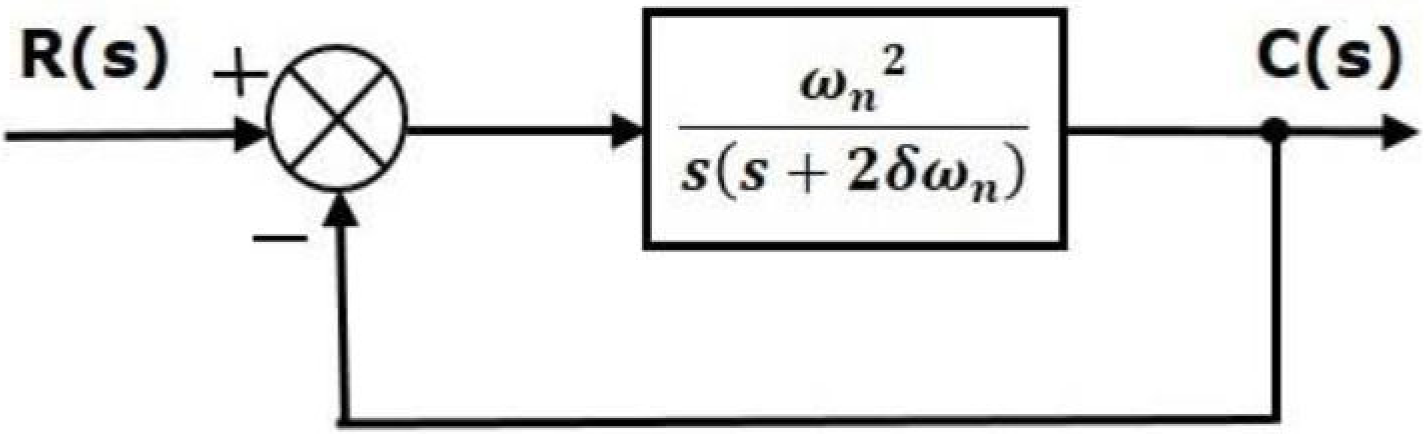
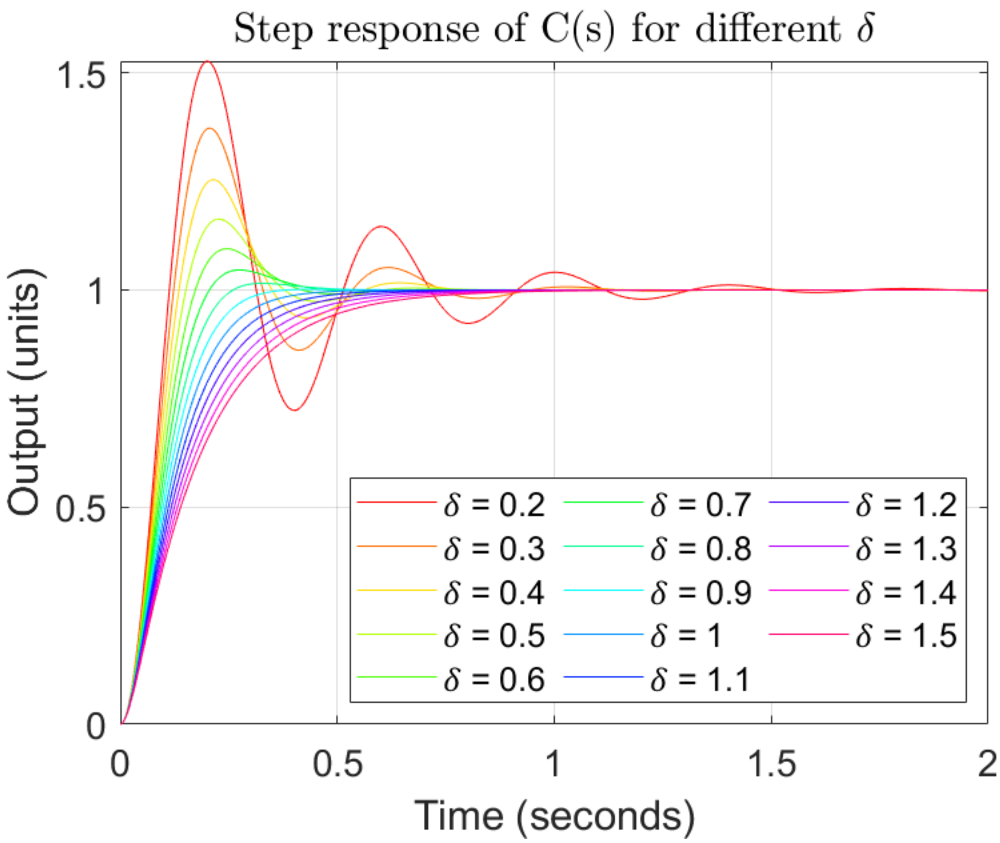


Fig. 4: Closed-loop system

```
figure;
w=16;
time=0.2:0.1:1.5;
color=hsv(14);
title("Step response of C(s) for different $\delta$",'Interpreter',"latex");
grid on;
xlabel("Time (seconds)");
ylabel("Output (units)");
for i=1:14
    delta=time(i);
    a=sim("part3.slx");
    plot(a.c,'color',color(i,:),'DisplayName',strcat(" = ",num2str(delta)));
    hold on
end
hold off;
title("Step response of C(s) for different $\delta$",'Interpreter',"latex");
grid on;
xlabel("Time (seconds)");
ylabel("Output (units)");
legend('NumColumns',3, 'Location',"southeast");
set(gca,"FontSize",14);
```




```
% With increase in damping ratio (delta), damping frequency of
% the output decreases.
% When (delta=1) the system is critically damped, As we increase delta
% system becomes overdamped and takes longer time to reach the final value.
```

```
% b
% data provided by TA
```

y_{ss} … Steady state value.

T_p … Time to reach first peak (undamped or underdamped only).

$\%OS$ … % of $y_{step}(T_p)$ in excess of y_{ss} .

T_s … Time to reach and stay within 2% of y_{ss} .

T_r … Time to rise from 10% to 90% of y_{ss} .

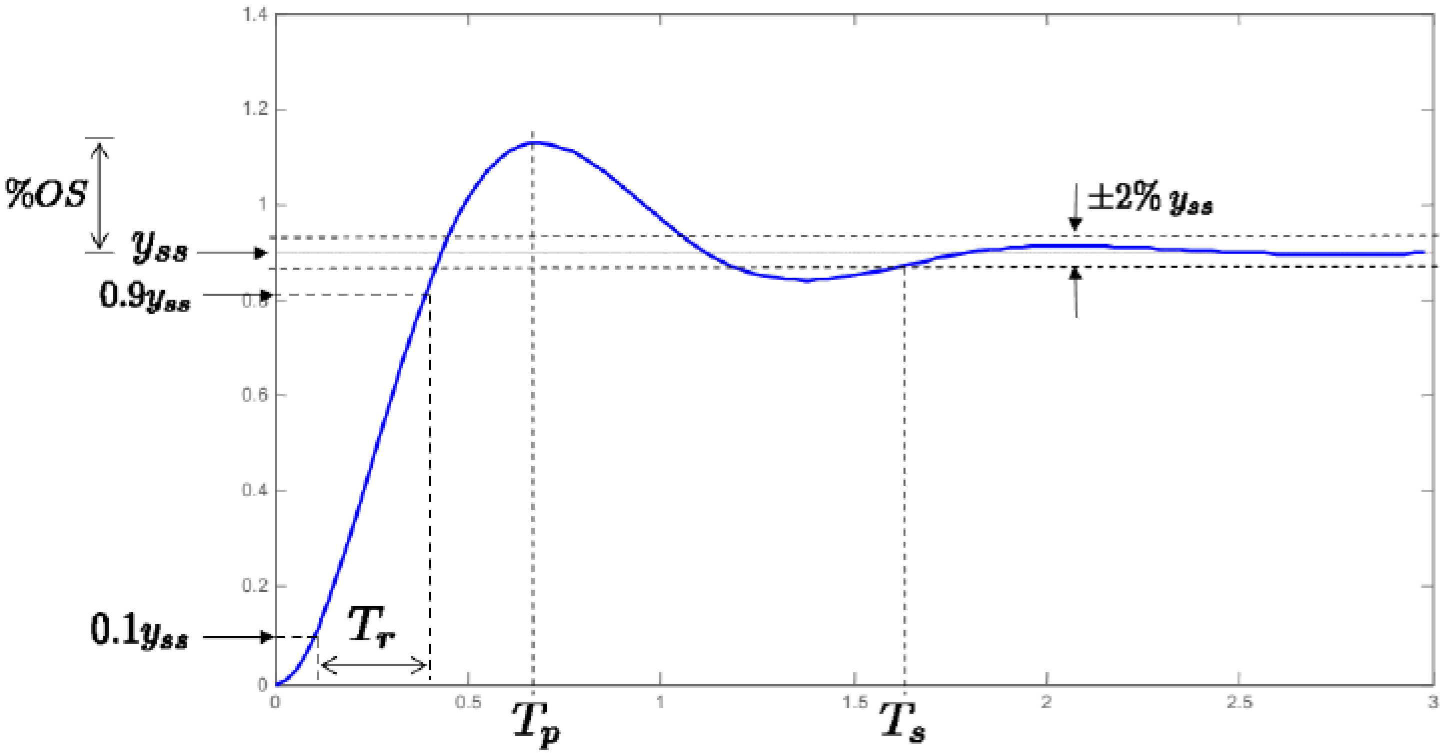


Fig. 5: Step response of an under-damped second-order systen

```
% transfer function

% step signal
t=0:0.01:20;
delta=0.6; %damping ratio
natural_freq=10;

th=atan(sqrt(1-delta^2)/delta);
damping_freq = natural_freq*(sqrt(1-delta^2))
```

```
damping_freq = 8
```

```
rise_time = (pi - th)/damping_freq
```

```
rise_time = 0.2768
```

```
peak_time = pi/damping_freq
```

```
peak_time = 0.3927
```

```
max_peak_overshoot = exp(-pi*delta/sqrt(1-delta^2))
```

```
max_peak_overshoot = 0.0948
```

```
settling_time = 4/(delta*natural_freq)
```

```
settling_time = 0.6667
pole = 1.0060
sys =
 
      0.6
  ---------
  z - 1.006
 
Sample time: 0.01 seconds
Discrete-time transfer function.

   Pole       Magnitude     Damping       Frequency      Time Constant
                                        (rad/seconds)      (seconds)

  1.01e+00     1.01e+00    -1.00e+00       6.00e-01        -1.67e+00
```