# Causality in Earth Science

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Machine Learning for Earth System Science

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# Causality between two variables

- Consider two variables X and Y (may be spatio-temporal)
- "X causes Y" = Value of X influences value of Y
- Eg. i) Smoking causes cancer

  ii) Clouds cause rainfall
- Spatial causality: X(s) causes Y(s') where s, s' may be same
- Temporal causality: X(t) causes Y(t') where t'>=t
- Controlled process: if we can externally change the value of X, value of Y will change accordingly.

# Bi-directional Causality

- Bi-directional causality: “X causes Y” and “Y causes X”!
- Self-replenishing or self-destructive
- i) High temperature (X) causes water evaporation
  ii) Water evaporation creates clouds
  iii) Clouds cause rainfall (Y)
  iv) Rainfall (Y) brings down temperature! (X)

- i) High temperature (X) -> people use air conditioners
  ii) Air conditioners release $CO_2$
  iii) $CO_2$ (Y) causes higher temperature (X)!!!!

# Correlation and Causation

| X | Y |
|---|---|
| 12 | 105 |
| 25 | 176 |
| 13 | 109 |
| 19 | 140 |
| 23 | 168 |
| 37 | 225 |
| 16 | 115 |

| X | Y |
|---|---|
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| X | Y |
|---|---|
| 12 | 153 |
| 25 | 105 |
| 13 | 176 |
| 19 | 109 |
| 15 | 125 |
| 17 | 120 |
| 16 | 135 |

Whenever X increases, Y increases too.
Whenever X decreases, Y decreases too.

Whenever X increases, Y increases in next step.
Whenever X decreases, Y decreases in next step

Whenever X in/decreases, Y de/increases.
Whenever Y increases, X increases in next step!

# Correlation and Causation

| X | Y |
|---|---|
| 12 | 105 |
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High Correlation

| X | Y |
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High lagged Correlation

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| 13 | 176 |
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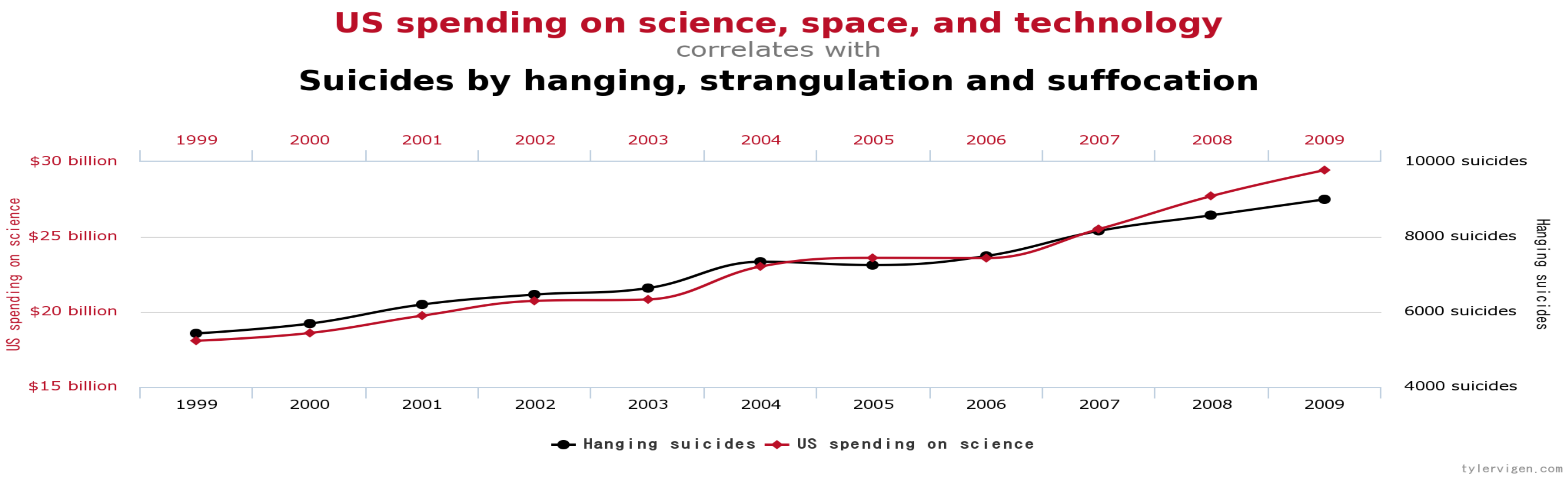
Whenever X in/decreases, Y de/increases.
Whenever Y increases, X increases in next step!

High lagged Correlation, high anti-correlation!

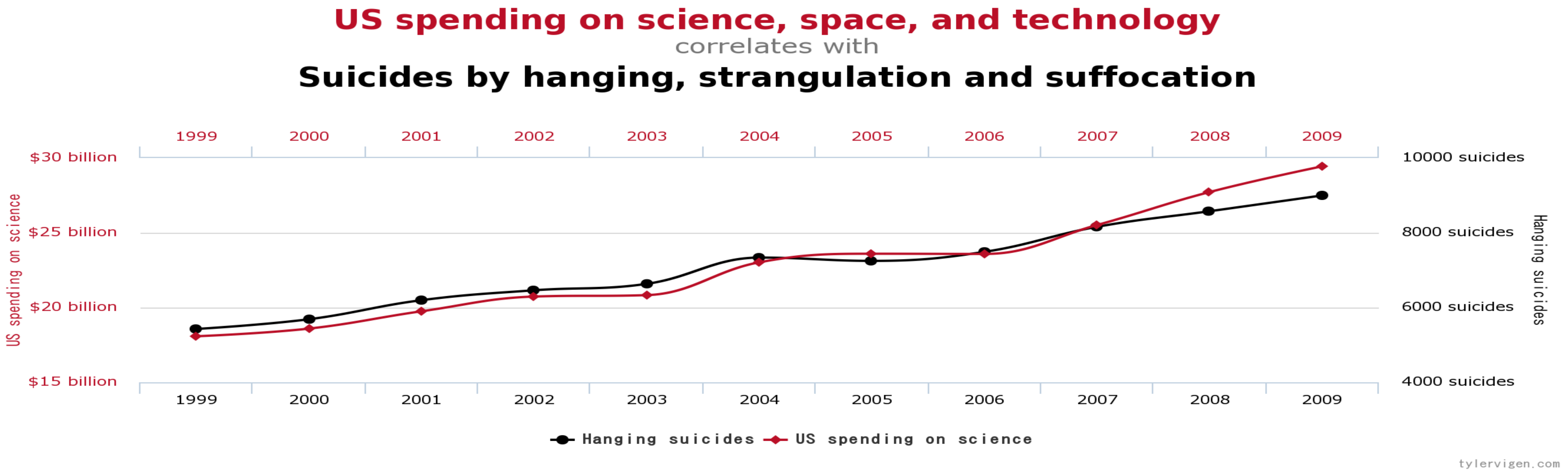
# Correlation and Causation

- High correlation need not indicate causal relationship
- "Spurious correlation": artificially enforced high correlation



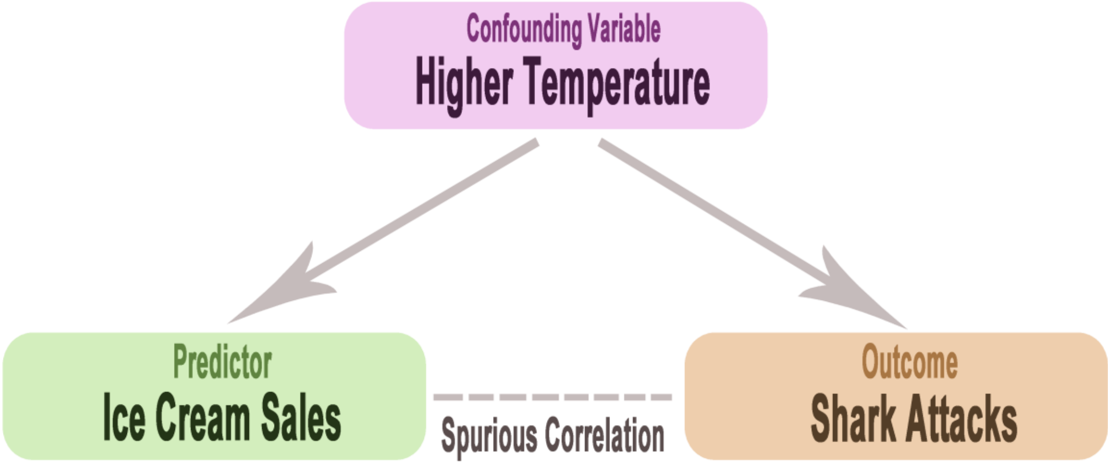
# Correlation and Causation

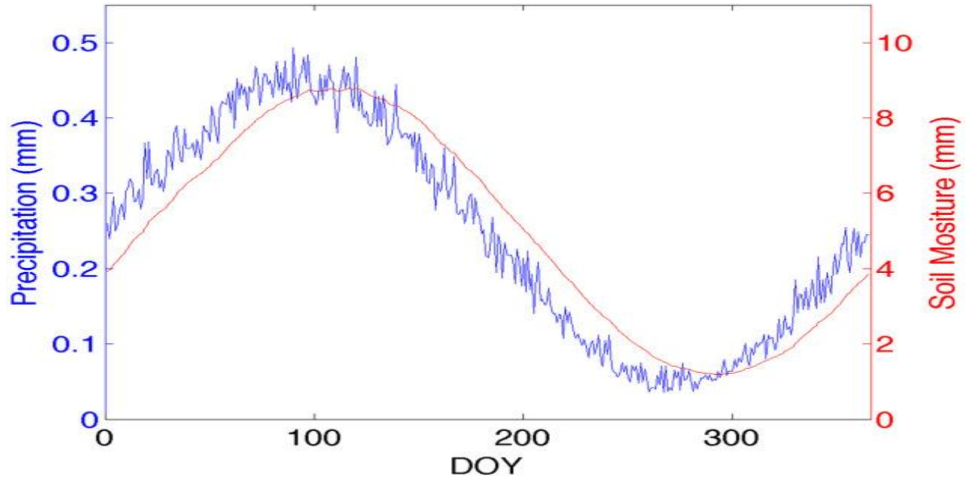
- High correlation need not indicate causal relationship
- "Spurious correlation": artificially enforced high correlation
- System involves more "confounding" variables which are not observed

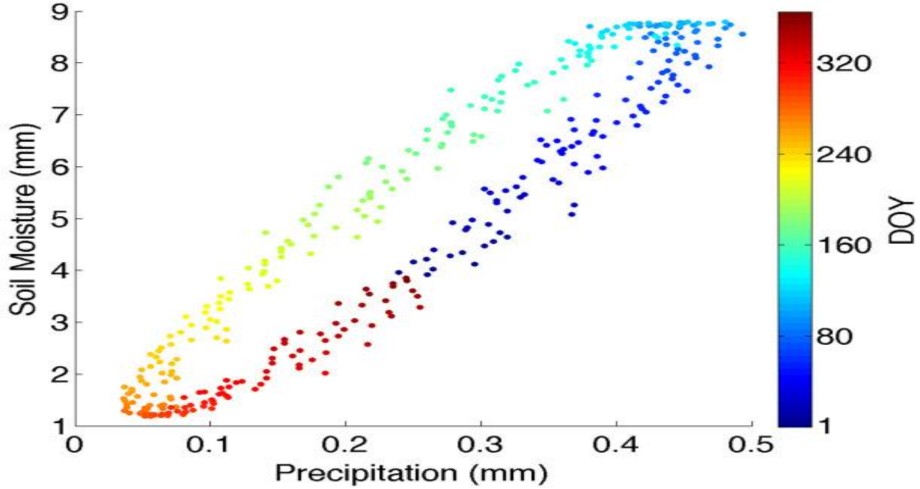


# Correlation and Causation

- "Explaining away": identify new variable Z which has causal relationship with both X and Y!



# Granger Causality in Time-series

- Express X as a linear function of past values of itself
- $X(t) \sim a_1X(t-1) + a_2X(t-2) + a_3X(t-3) + \ldots\ldots$
- Does the estimate improve if we include past values of Y?
- $X(t) \sim a_1X(t-1) + a_2X(t-2) + a_3X(t-3) + \ldots\ldots + b_1Y(t-1) + b_2Y(t-2) + b_3Y(t-3) + \ldots\ldots$
- If yes, then Y Granger-causes X.

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- If yes, then Y Granger-causes X.
- Does X Granger-cause Y?
- $Y(t) \sim c_1Y(t-1) + c_2Y(t-2) + c_3Y(t-3) + \ldots\ldots$
- $Y(t) \sim c_1Y(t-1) + c_2Y(t-2) + c_3Y(t-3) + \ldots\ldots + b_1X(t-1) + b_2X(t-2) + b_3X(t-3) + \ldots\ldots$
- If both yes, then bidirectional causality!

# Pearl Causality

- Smoking (X) may cause cancer (Y)
- $P(Y=1 \mid X=1) = 0.4$ (relation between cancer and smoking)

- Cancer (Y) can be caused by smoking (X), or many other factors (Z)
- $Prob(Y=1) = Prob(X=1,Y=1) + Prob(Y=1,Z=1) + Prob(Y=1,Z=2) + ....$

- $Prob(Y=1|X=1)$ helps us to estimate probability of cancer $Prob(Y=1)$
- But does it tell us anything about probability of smoking?

# Pearl Causality

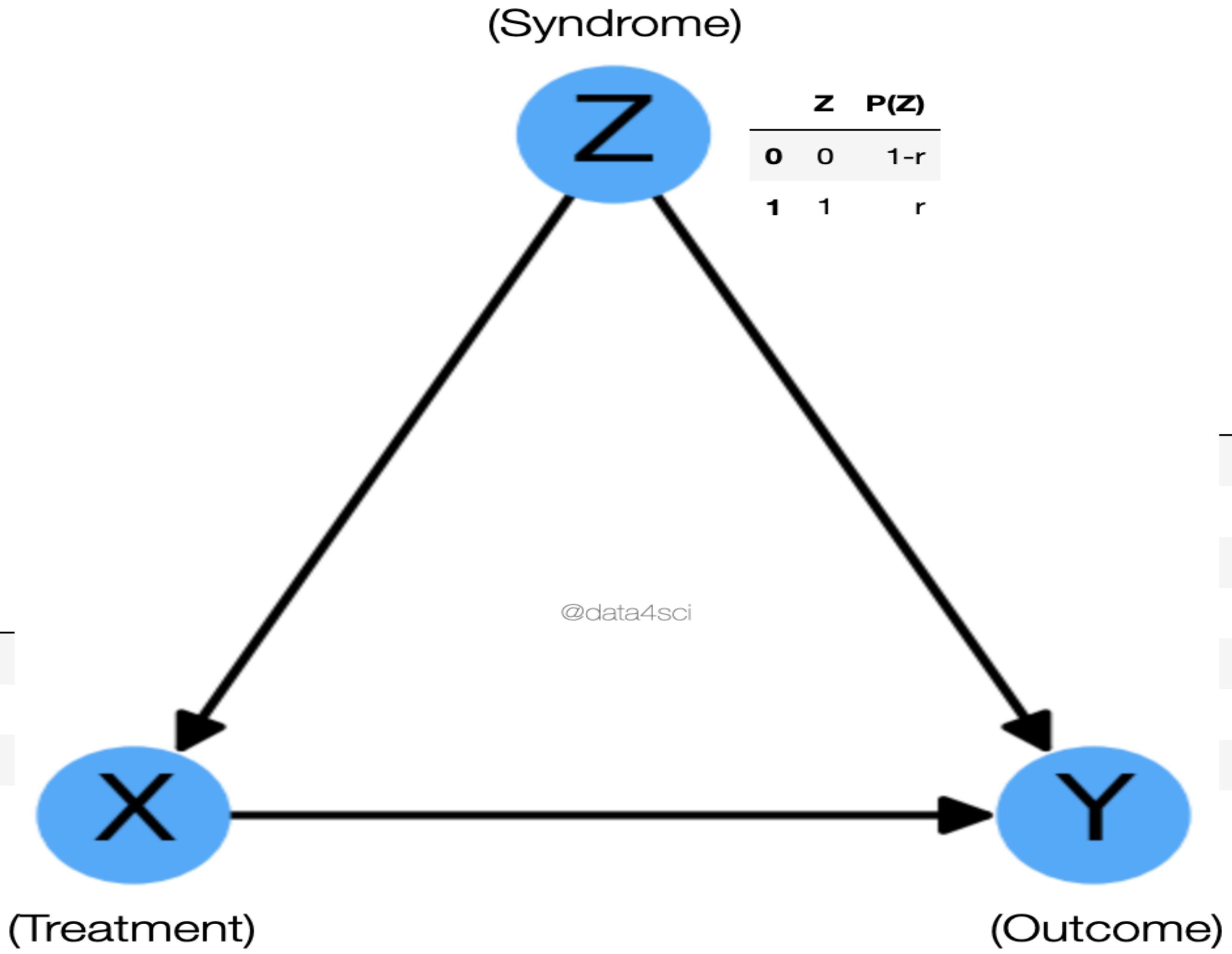
- Based on the notion of conditional independence
- X, Y are conditionally dependent on each other
- X->Y: Y may take certain values **if** X takes certain values

- Can we estimate Prob(Y) using Prob(X|Y)?
- Can we estimate Prob(X) using Prob(Y|X)?

- Prob(X|Y) helps us to estimate Prob(Y) but Prob(Y|X) doesn't help us to estimate Prob(X): X -> Y

- Prob(Y | X=x) is different from Prob(Y | do(X=x))

# Structural Causal Model




| | Z | P(Z) |
|---|---|---|
| **0** | 0 | 1-r |
| **1** | 1 | r |

| | X | Z | P(X \| Z) |
|---|---|---|---|
| **0** | 1 | 0 | $q_0$ |
| **1** | 1 | 1 | $q_1$ |
| **2** | 0 | 0 | $1-q_0$ |
| **3** | 0 | 1 | $1-q_1$ |

| | Y | X | Z | P(Y \| X,Z) |
|---|---|---|---|---|
| **0** | 1 | 1 | 0 | $p_{10}$ |
| **1** | 1 | 0 | 0 | $p_{00}$ |
| **2** | 1 | 0 | 1 | $p_{01}$ |
| **3** | 1 | 1 | 1 | $p_{11}$ |
| **4** | 0 | 1 | 0 | $1-p_{10}$ |
| **5** | 0 | 0 | 0 | $1-p_{00}$ |
| **6** | 0 | 0 | 1 | $1-p_{01}$ |
| **7** | 0 | 1 | 1 | $1-p_{11}$ |