# Control & Instrumentation Lab, Autumn 2020-21

## Session 3: Model representation and time-domain analysis

1. (A) Consider a series R-L circuit with zero initial condition (R=10 Ohm, L=10 mH). At t = 0, an ideal DC voltage source 1V is applied. The circuit is represented in SIMULINK as shown in Fig. 1.

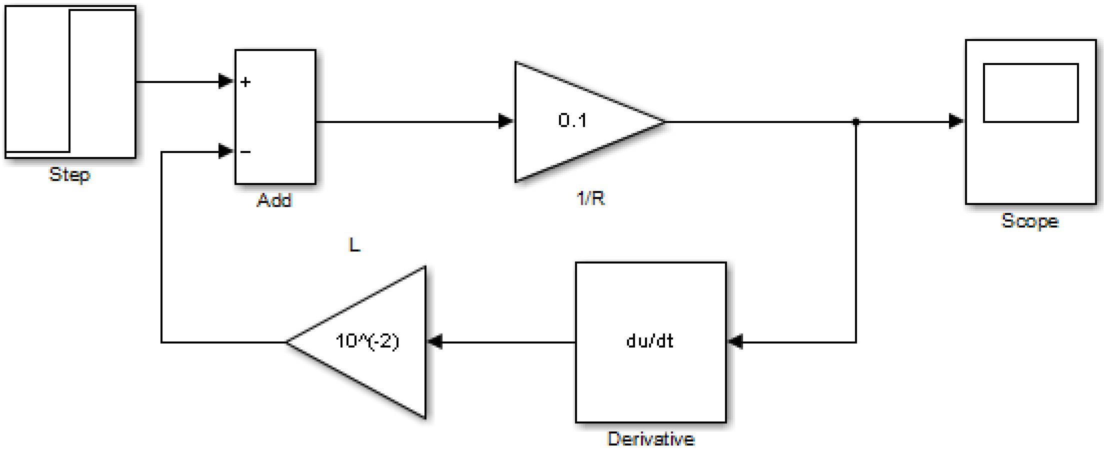


Fig. 1: Series R-L circuit represented in SIMULINK

Calculate the time constant of the circuit and choose the simulation run time accordingly. Is the simulation correct? Justify. If not, correct it and show responses (current and voltage across the inductor).

(B) Using the commands “tf” and “zpk”, enter the following transfer functions in MATLAB.

$$\text{(i)}\ G(s)=\frac{(s+1)}{s(s+2)(s+3)},\quad \text{(ii)}\ G(s)=\frac{-s-1}{s^2+5s+6}.$$

(C) Find the state-space matrices of the above transfer functions (in (B)) using “tf2ss” command.

2.

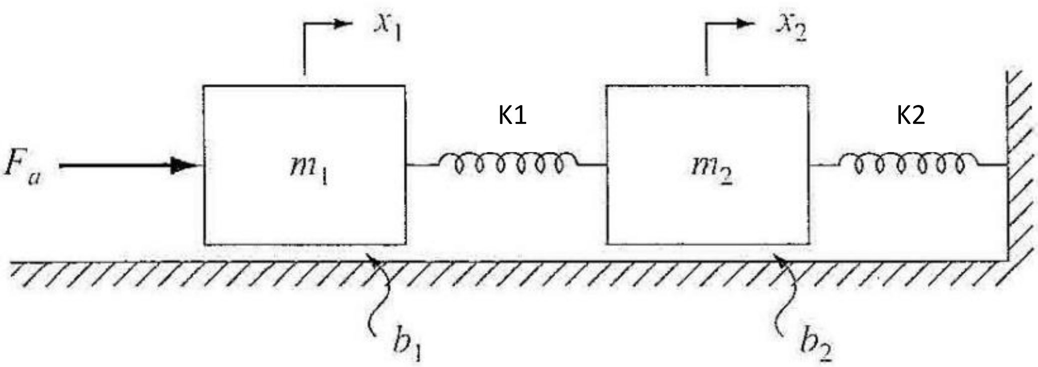


Fig. 2: Spring-mass-damper system

Write the equations of motion, and represent the dynamical system in state-space form. Using MATLAB command “ss2tf”, find the transfer function from $F_a$ to $x_2$. Plot unit step and unit impulse response by using MATLAB commands “step” and “impulse”, respectively. Also plot position $x_2$ when a sinusoidal force input $F_a$ is applied. Simulate the dynamical system in SIMULINK and verify the above responses. $m_1 = 1kg, k_1 = 1N/m, b_1 = 0.1Ns/m, m_2 = 2kg, k_2 = 1.5N/m, b_2 = 0.2Ns/m$

3. (A) The step response of a system is observed as shown in Fig. 3.

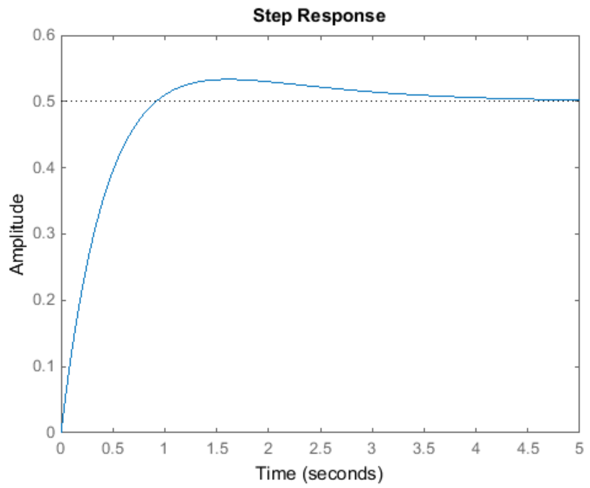


Figure 3: Step response of a system

What form do you think the model has? Give an example to justify your answer.

(B) Can you observe overshoot for an over-damped second-order system? If your answer is affirmative, give an example and show response.

4. (A) Plot the step response of the following transfer functions:

(i) $G(s)=\dfrac{-(s-1)}{(s+1)^2}$, (ii) $G(s)=\dfrac{(s-1)}{(s+1)^2}$ (iii) $G(s)=\dfrac{(s-1)^2}{(s+1)^2}$ (iv) $G(s)=\dfrac{(s^2-10s+27)}{(s+3)^3}$,

(v) $G(s)=\dfrac{(2s^2-s+1)}{(s+1)^3}$, (vi) $G(s)=\dfrac{(s^2-s+4)}{(s+1)^3}$.

In above, note down: properness (i.e. strictly proper or proper) of the transfer functions; existence of initial undershoot, zero crossing and overshoot; the number of positive zeros of $G(s)$, $G(s)-G(\infty)$ and $G(s)-G(0)$. With these observations (you may take some more examples), fill-in the following table.

| Properness | Initial undershoot | Zero crossing | Overshoot |
|---|---|---|---|
| Strictly proper | If and only if G(s) has an odd number of positive zeros. | | If $G(s)-G(0)$ has at least one positive zero. |
| Proper | | If G(s) has at least one positive zero. | |

(B) Give an example of a linear dynamical system that exhibits bounded response to an unbounded input. Plot the time response.

5. (A) With different values of $\delta$ (damping ratio) in the range 0.2 to 1.5 and $\omega_n = 16$ rad/s (natural frequency), plot the output response C(s) of the closed-loop system shown in Fig. 4 when a unit step input $R(s)$ is applied.

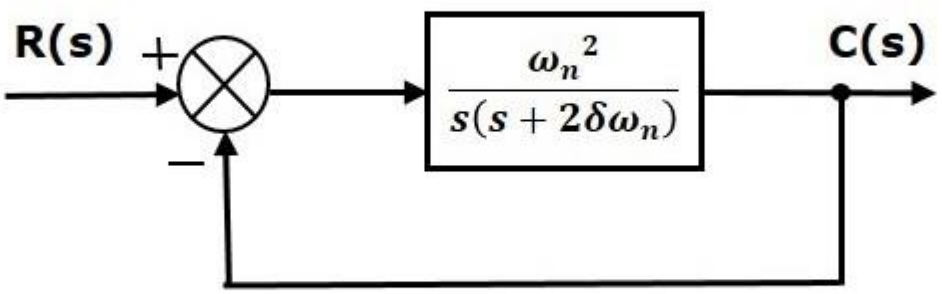


Fig. 4: Closed-loop system

(B) Write a program in MATLAB to calculate the rise time $(T_r)$, peak time $(T_p)$, percentage of overshoot $(\%OS)$, settling time $(T_s)$, damping ratio $(\delta)$ and natural frequency of a second-order dynamical system from its unit step response data (a sample data set is shown in Fig. 6). Note that damping frequency = $\omega_n\sqrt{1-\delta^2}$.

$y_{ss}$ … Steady state value.

$T_p$ … Time to reach first peak (undamped or underdamped only).

$\%OS$ … % of $y_{step}(T_p)$ in excess of $y_{ss}$.

$T_s$ … Time to reach and stay within 2% of $y_{ss}$.

$T_r$ … Time to rise from 10% to 90% of $y_{ss}$.

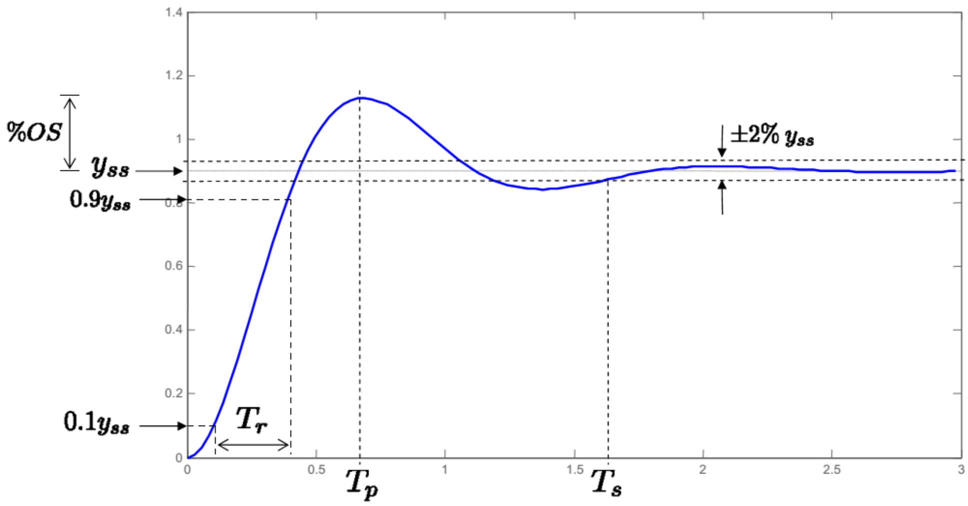


Fig. 5: Step response of an under-damped second-order system

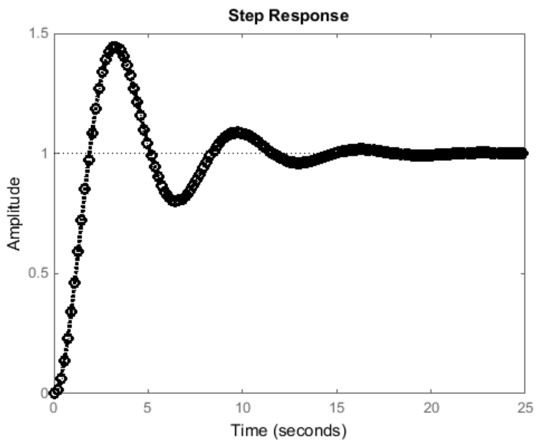


Fig. 6: Sample unit step-response data (to be provided by TA)