

Lab 3.

Q1. $M = \frac{20(s-1)}{(s+2)(s^2+4)}$

\hookrightarrow 1 LHP.
 \hookrightarrow 2 imaginary poles
 $= (-2)$ pole

$\therefore M(s)$ is marginally stable.

Q2. $2s^4 + s^3 + 3s^2 + 5s + 10 = 0$.

Using R-H criterion:

s^4	2	3	10
s^3	1	5	0
s^2	$\frac{-2}{-7}$	10	0
s	457	0	0
s^0	10	0	0

$$-\left| \begin{array}{cc} 2 & 3 \\ 1 & 5 \end{array} \right| = -7$$

$$-\left| \begin{array}{cc} 2 & 10 \\ 1 & 0 \end{array} \right| = 10$$

$$-\frac{1}{-7} \left| \begin{array}{cc} 1 & 5 \\ -7 & 10 \end{array} \right| = \frac{45}{7}$$

as there is a sign change
 \therefore System UNSTABLE

Q3. $s^4 + s^3 + 2s^2 + 2s + 3 = 0$.

s^4	1	2	3
s^3	1	2	0
s^2	0	3	0
s^1	$\frac{-3}{-3}$	0	0
s^0	0	0	0

UNSTABLE.

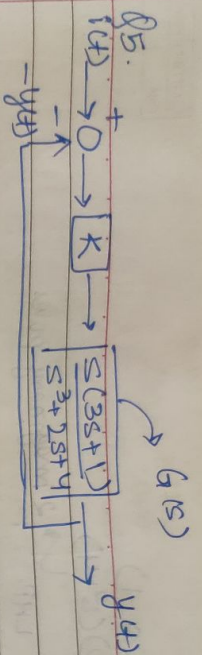
Q4. $s^5 + 4s^4 + 8s^3 + 7s + 4 = 0$.

s^5	1	8	7	0
s^4	4	8	4	0
s^3	6	6	0	0
s^2	4	4	0	0
s^1	0	0	0	0
s^0	4	0	0	0

No sign change

\therefore System is

STABLE



K → gain of proportional controller

$$1 + K G(s) = 0$$

$$K(G(s) - (1/s) - y(t)) = y(t)$$

$$1 + \frac{s^3 + 2s + 4}{K s(3s + 1)} = 0$$

$$s^3 + 2s + 4 + K s(3s + 1) = 0$$

$$s^3 + (2 + 3K)s + 4 + Ks^2 = 0$$

$$K > 0$$

or table

$$\begin{array}{c|ccc} s^3 & 1 & 2+K & 0 \\ s^2 & 3K & 4 & 0 \\ s^1 & -\frac{1}{3K} \frac{K+2}{3K} & 1 & 0 \\ s^0 & 4 & & \end{array} \rightarrow -\frac{1}{3K} (4 - 3K^2 - 6K)$$

$$\text{as } K > 0$$

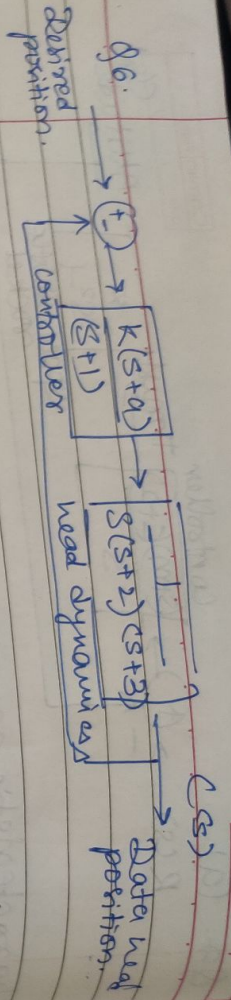
$$3K^2 + 6K - 4 > 0$$

$$K > -1 + \sqrt{\frac{7}{3}} \quad \vee \quad 0.53$$

when $K = -1 + \sqrt{\frac{7}{3}}$ both elements of 3rd row is 0.
stable (marginally).

$$\begin{array}{c|ccc} s^3 & 1 & 2+K & 0 \\ s^2 & 3K & 4 & 0 \\ s^1 & 6K & 0 & 0 \\ s^0 & 4 & 0 & 0 \end{array} \therefore K > -1 + \sqrt{\frac{7}{3}}$$

system stable for



characteristic equation:

$$1 + K(s+a) = 0$$

$$s(s+1)(s+2)(s+3) + K(s+a) = 0$$

$$s^4 + 6s^3 + 11s^2 + (6+K)s + Ka = 0$$

as $K > 0 \therefore a > 0$

s^4	1	11	Ka
s^3	6	$6+K$	0
s^2	6	$6+K$	0
s^1	$6Ka - (6+K)(6+K)$	$10-K/6$	Ka
s^0	Ka	0	0

$$\therefore 10 - K > 0 \therefore K < 10$$

$$0 < K < 10$$

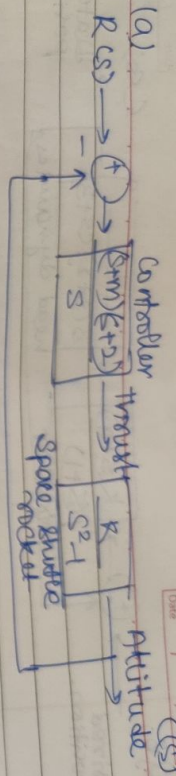
$$\text{at } K=10 \quad a=0$$

$$0 \leq a \leq \frac{10+3-K}{36}$$

system \rightarrow marginally stable

Stable region $\therefore K \in [0, 10]$
 $a \in [0, 10]$

Q7. (a)



characteristic equation:

$$1 + \frac{k(s+m)}{s(s^2-1)}(s+2) = 0$$

$$s^3 - s + ks^2 + k(m+2)s + 2km = 0$$

$$s^3 + ks^2 + (k(m+2)-1)s + 2km = 0$$

for stability: $k(m+2)-1 > 0 \implies m > \frac{1}{k}-2$

$$2km > 0 \implies m > 0$$

$$\begin{array}{c|c|c} s^3 & 1 & k(m+2)-1 \\ s^2 & k & 2km \\ s^1 & \frac{-1}{k} & \frac{k(m+2)-1}{2km} \\ s^0 & 1 & 0 \end{array}$$

$$\begin{cases} (k-2)(m+2k-1) > 0 & \forall m \geq 0 \text{ if } k \geq 2 \\ < 0 & \forall m \leq 0 \text{ if } k < \frac{1}{2} \\ \geq 0 & \forall m \leq \frac{2k-1}{2-k} \end{cases}$$

System is marginally stable if $m = \frac{2k-1}{2-k}$ $k \in [\frac{1}{2}, 2)$

System stable if $k > 2, m > 0$

$$k \in [\frac{1}{2}, 2), 0 \leq m \leq \frac{2k-1}{2-k}$$

Q 9.
(b) $G_1(s) = \frac{(sm)(s+2)}{s}$

$G_2(s) = \frac{k}{s^2-1}$

steady state error $\rightarrow e = \frac{1}{\lim_{s \rightarrow 0} s G_1(s) G_2(s)}$ when system stable

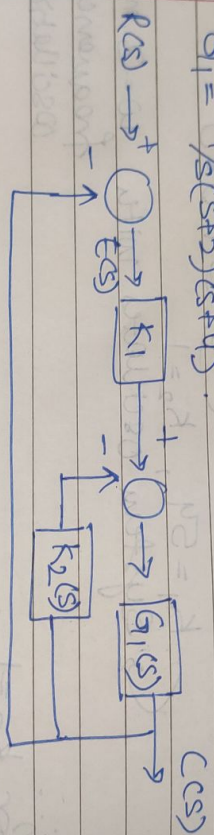
group response $s \rightarrow 0$
 $= (-1/2km)$

given $\text{error} \pm 0.1$

km are both +ve
so;
 $\frac{-1}{2km} \geq -0.1$
 $|m| \geq 5/k$

for
k

Q 8. (a,b) $G_1 = 1/s(s+2)(s+4)$



$1 + K_2 s G_1(s) = 0$

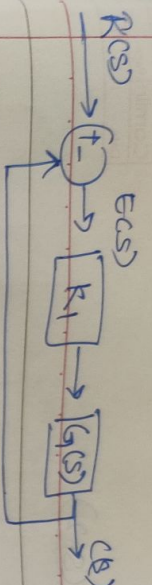
\rightarrow will have 2 equal -ve roots.

$(s+2)(s+4) + K_2 = 0$

$G_1(s) = \frac{1}{s(1+(s+2)(s+4))}$

$K_2 = 1$

$s^2 + 6s + 9 = 0$
 $\begin{cases} -3 \\ -3 \end{cases}$ 2 equal roots.



$$1 + K_1 G(s) = 0$$

$$1 + K_1 = 0$$

$$s(s+2)(s+4) + 5$$

$$s^3 + 6s^2 + 9s + K_1 = 0 \quad K_1 > 0$$

$$\begin{array}{c|ccc} s^3 & 1 & 9 & \\ s^2 & 6 & K_1 & \\ s & 54 & 0 & \\ & 6 & & \end{array}$$

$0 < K_1 < 54$
for stable system.

$$s^3 + 6s^2 + 9s + 54 = 0$$

$$(s+6)(s^2+9) = 0$$

$K_1 = 54, K_2 = 1$
System oscillates with frequency of oscillation.

for $K_2 = 1$
 $\Rightarrow K_1 \in (0, 54)$

at $K_1 = 54$, system is marginally stable (oscillates).

if K_2 not specified
 $\omega = \sqrt{8 + K_2}$ rad/sec
when $K_1 = 6(8 + K_2)$.

Q8 (c) as closed loop pole at will make system unstable, we won't be able to approximate step response as underdamped.

$$s^3 + 6s^2 + (8+k_2)s + k_1 = 0$$

$$s = -5 \rightarrow \text{pole}$$

$$k_1 = -s(6s + k_2)$$

eqn

$$k_1 = 5(k_2 + 3)$$

$$(8+5)(s^2 + s + k_2 + 3) = 0$$

1 pole at $s = -5$
both complex poles have real = $-1/2$
real pole is 10 times away from img axis.

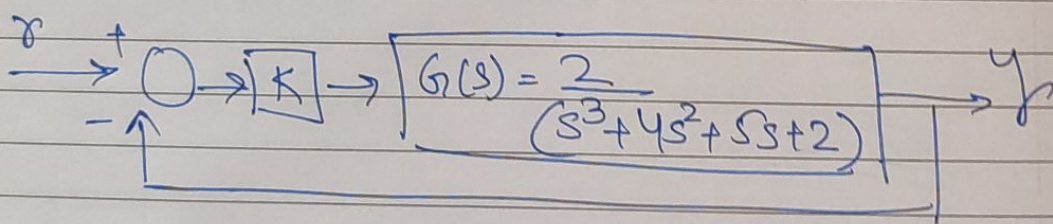
Dominant pole approximation

$$k_2 = 1 \quad k_1 = 20$$

$$G(s) = \frac{20}{s^3 + 6s^2 + 9s + 20}$$

$$G_{app}(s) = \frac{4}{s^2 + s + 4}$$

Q9.



$$Y(s) = \frac{K G(s)}{1 + K G(s)} X(s)$$

$$E(s) = \frac{R(s)}{1 + K G(s)}$$

$$R(s) = 1/s$$

$$e = \frac{1}{1 + K \lim_{s \rightarrow 0} G(s)}$$

$$e = \frac{1}{1+K}$$

$$G(s) = (-2, -1, -1) \text{ in LHP.}$$

$$\frac{1}{1+K} \leq 0.15$$

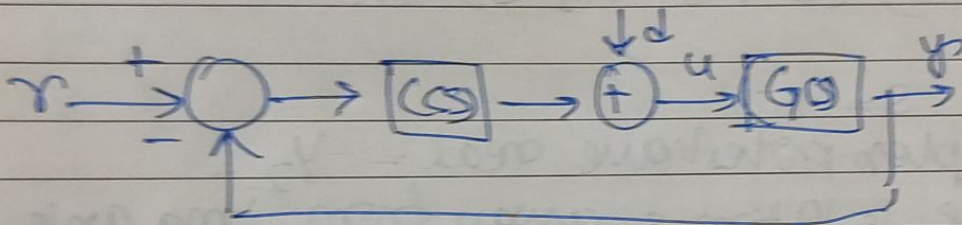
$$K \geq 17/3$$

Characteristic eqnⁿ

$$1+K G(s)=0$$

$$s^3 + 4s^2 + 5s + 2 + 2K = 0$$

Q10



$$G(s) = 1/s+1$$

$$C(s) = \frac{s+1}{s+1}$$

Case 1