Assignment (Poisson Process)

Customers arrive at a facility according to a Poisson process with the rate $\lambda = 2$ customers per minute. Suppose we begin observing the facility at some point in time.

1. What is the probability that eight customers arrive during a 5-minute interval? [0.1125]
2. On average, how many customers will arrive during a 3.2-minute interval? [6.4]
3. What is the probability that more than two customers arrive during a 1-minute interval? [0.3233]
4. What is the probability that four customers arrive during the interval that begins 3.3 minutes after we start observing and ends 6.7 minutes after we start observing? [0.09922]
5. On average, how many customers will arrive during the interval that begins 16 minutes after we start observing and ends 17.8 minutes after we start observing? [3.6]
6. What is the probability that seven customers arrive during the first 12.2 minutes we observe, given that five customers arrive during the first 8 minutes? [0.0079]
7. If three customers arrive during the first 1.2 minutes of our observation period, then, on average, how many customers will arrive during the first 3.7 minutes? [8]
8. If one customer arrives during the first 6 seconds of our observation, then what is the probability that two customers arrive during the interval that begins 12 seconds after we start observing and ends 30 seconds after we start observing? [0.0987]
9. If five customers arrive during the first 30 seconds of our observations, then, on average, how many customers will arrive during the interval that begins 1 minute after we start observing and ends 3 minutes after we start observing? [4]
10. If three customers arrive during the interval that begins 1 minute after we start observing and ends 2.2 minutes after we start observing, then, on average, how many customers will arrive during the first 3.7 minutes? [8]
11. What is the probability that the third customer arrives more than 0.2 minutes but less than 1.3 minutes after the second customer? [0.5960]
12. What is the expected value of the time between the first and second customer arrivals? [0.5]
13. If the time between the first and second customer arrivals is 0.6 minute, then what is the probability that it is more than 1.7 minutes between the third and fourth customer arrivals? [0.0334]
14. Suppose it has been 1.3 minutes since the tenth customer arrived. What is the probability that it is less than 0.8 minute until the 11th customer arrives? [0.7981]
15. Suppose it has been 0.7 minute since the sixteenth customer arrived. What is the expected value of the seventeenth customer's inter-arrival time? [1.2]
16. Two people, Kevin and Bob, leave for work at the same time. Kevin's commute requires an amount of time that is exponentially distributed with a mean of 10 minutes. Bob's commute is independent of Kelvin's and requires an exponentially distributed amount of time with a mean of 7 minutes. What is the probability that Bob arrives at work before Kelvin? [0.5882]
17. A store takes orders from three independent types of customers. The time until an order of type i is received is an exponential random variable with mean $1/\alpha_i$ hours where $\alpha_1 = 0.2$, $\alpha_2 = 0.5$, and $\alpha_3 = 0.05$. What is the probability that it is more than 1 hour until an order is received? [0.4724]