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Assignment.

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|Rat|

1. |0|1|2|3|4|5|

Prob. of moving right = 3/4.
Prob. of moving left = 1/4.

$i=2$, $N=5$, $P=3/4$, $q=1/4$. $p/q \neq 1$.

$$P_5 = \frac{1-(q/p)^i}{1-(q/p)^N} = \frac{1-(1/3)^2}{1-(1/3)^5} = 0.892.$$

Prob. of rat finding food before getting shocked = $\underline{0.892}$.

2. For markov chain, $TPM = \begin{bmatrix} 1 & 0 & 0 \\ 0.4 & 0 & 0.6 \\ 0 & 0 & 1 \end{bmatrix}$.

$P_i = P(X_n \text{ ends in state } 0 \mid x_0 = i)$

from TPM,

$P_0 = 1$ (initial state 0, final state 0)

$P_2 = 0$ (initial state 2, final state is never 0)

$$P_1 = \sum_{k=0}^{2} P(X_n \text{ ends at } 0, X_1 = k \mid x_0 = 1)$$

$$= \sum_{k=0}^{2} P(X_n \text{ ends at } 0 \mid x_1 = k) \cdot P(X_1 = k \mid x_0 = 0)$$

$$= ~~P_0 \times 0.4~~ \; P_0 \times (0.4) + P_1 \times (0) + P_2 \times (0.6)$$

Probability $\boxed{P_1 = 0.4}$ ← that markov chain ends at 0, if started from 1

3. Prob. of thrower win = 0.49 = q
find prob. that player A's amt. goes down to 5 before going to 10.

Prob. of player loosing = 0.51

$$P = \frac{1-(q/p)^{10}}{1-(q/p)^{15}} = \boxed{0.73}$$

$p \neq q$.

4. In Gambler ruin problem, gambler start with i and on success gamble, he either wins or loss 1, independent of current amount.
Objective :- Reach N before getting ruined.

$P_i \rightarrow$ Prob. gambler win.

$P_0 = 0$, $P_N = 1$

$$P_i = p P_{i+1} + q P_{i-1}$$

$$P_{i+1} - P_i = \frac{q}{p}(P_i - P_{i-1})$$

$$P_2 - P_1 = \frac{q}{p}(P_1 - P_0) = \frac{q}{p} P_1$$

$$P_3 - P_2 = \frac{q}{p}(P_2 - P_1) = \left(\frac{q}{p}\right)^2 P_1$$

$$\underbrace{P_{i+1} - P_i}= (q/p)^i P_1 .$$

$$P_{i+1} = P_1 + P_1 \sum_{k=1}^{i} (q/p)^k$$

$$= \sum_{k=1}^{i} \left(\frac{q}{p}\right)^k P_1 .$$

$$P_{i+1} = \begin{cases} P_1 \times \dfrac{1-(q/p)^{i+1}}{1-(q/p)} & p \neq q \\ P_1 (i+1) & p = q \end{cases}$$

$$1 = P_N = \begin{cases} P_1 \times \dfrac{\left(1-(q/p)^N\right)}{1-(q/p)} & p \neq q \\ P_1 N & p = q . \end{cases}$$

$$P_1 = \begin{cases} \dfrac{1 - q/p}{1-(q/p)^N} & p \neq q \\ 1/N & p = q \end{cases}$$

$$P_i = \begin{cases} \dfrac{1-(q/p)^i}{1-(q/p)^N} & p \neq q \\ i/N & p = q \end{cases}$$

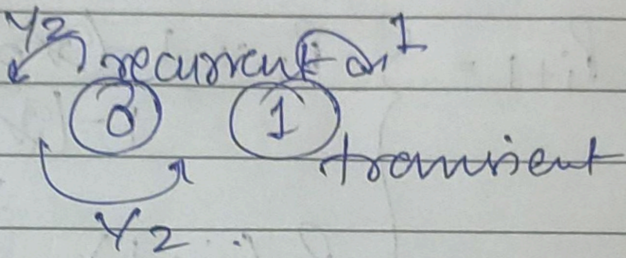
5(i) $\begin{bmatrix} 1/2 & 1/2 \\ 1/4 & 3/4 \end{bmatrix}$ it is regular as all terms are nonzero

$\pi_1 + \pi_2 = 1$

$\pi_1 = \frac{1}{2}\pi_1 + \frac{1}{4}\pi_2 \longrightarrow \pi_1 = 1/3, \quad \pi_2 = 2/3.$

$\lim_{x_n \to \infty} P(X_n = 1) = 1/3.$

(ii) $\begin{bmatrix} 1/2 & 1/2 \\ 0 & 1 \end{bmatrix}$



$\begin{bmatrix} + & + \\ 0 & + \end{bmatrix} \cdot \begin{bmatrix} + & + \\ 0 & + \end{bmatrix} = \begin{bmatrix} + & + \\ 0 & + \end{bmatrix}$

Never get all elements +ve.

$\therefore$ No reguler.

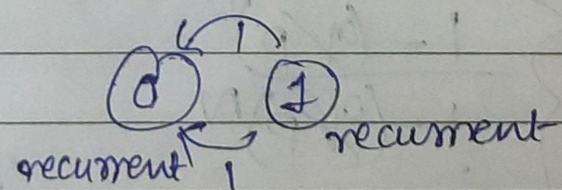
$\left.\begin{aligned} \pi_1 + \pi_2 &= 1 \\ \pi_1 &= \tfrac{1}{2}\pi_1 \end{aligned}\right\} \longrightarrow \begin{aligned} \pi_1 &= 0 \\ \pi_2 &= 1 \end{aligned}$

Stationary dist. is (0,1)

Matrix is non regular

this is not limitation distribution

(iii) $P = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.



State 1 & State 2 are communicable,

it is irreducible.

Period = len(2, 4, 6 ...) = 2

Hence, this is non-regular - DTMC.

$$\left.\begin{array}{l} \pi_1 + \pi_2 = 1 \\ \pi_1 = \pi_2 \end{array}\right\} \rightarrow \pi_1 = \pi_2 = \tfrac{1}{2}$$

But this is not limiting distribution.

6). Gambler starts with N dollars.

(i) Prob to win extra M before getting ruined.

$i = N \qquad N' = N + M.$

$$P = \begin{cases} \dfrac{1-(q/p)^N}{1-(q/p)^{N+M}} & p \neq q \\ N/(M+N) & p = q. \end{cases}$$

(ii) $p = q = \tfrac{1}{2}, \quad M = 10, \quad N = 50$

$$P = \frac{50}{60} = \underline{0.833}.$$

7) Given prob of 1 claim in year = 0.2

prob of 2 or more claims = 0.1

(i) TPM =

$$\begin{array}{c|c} & \begin{array}{ccc} E_0 & E_1 & E_2 \end{array} \\ \begin{array}{c} E_0 \\ E_1 \\ E_2 \end{array} & \begin{bmatrix} 0.3 & 0.7 & 0 \\ 0.3 & 0 & 0.7 \\ 0.1 & 0.2 & 0.7 \end{bmatrix} \end{array}$$

(ii) let π_0 = proportion of time for E_0.

π_1 = — " — E_1

π_2 = — " — E_2.

$$\left.\begin{aligned} \pi_0 &= 0.3\pi_0 + 0.3\pi_1 + 0.1\pi_2 \\ \pi_1 &= 0.7\pi_0 + 0.2\pi_2 \\ \pi_2 &= 0.1\pi_0 + 0.2\pi_1 + 0.7\pi_2 \\ \pi_1 + \pi_2 + \pi_0 &= 1 \end{aligned}\right\}$$

Solving $\rightarrow \left\{\begin{aligned} \pi_0 &= 0.186 \\ \pi_1 &= 0.2442 \\ \pi_2 &= 0.5678 \end{aligned}\right\}$

(iii) avg annual premium paid

$= 100\pi_0 + 80\pi_1 + 60\pi_2$

$= \underline{\underline{72.324}}$

8) $P = 0.4 \quad q = 0.6$

$$P_{10} = \frac{1-(0.6/0.4)^{10}}{1-(0.6/0.4)^{20}} = \underline{\underline{0.0174}}$$