Spatio-temporal Extreme Events

AI60002

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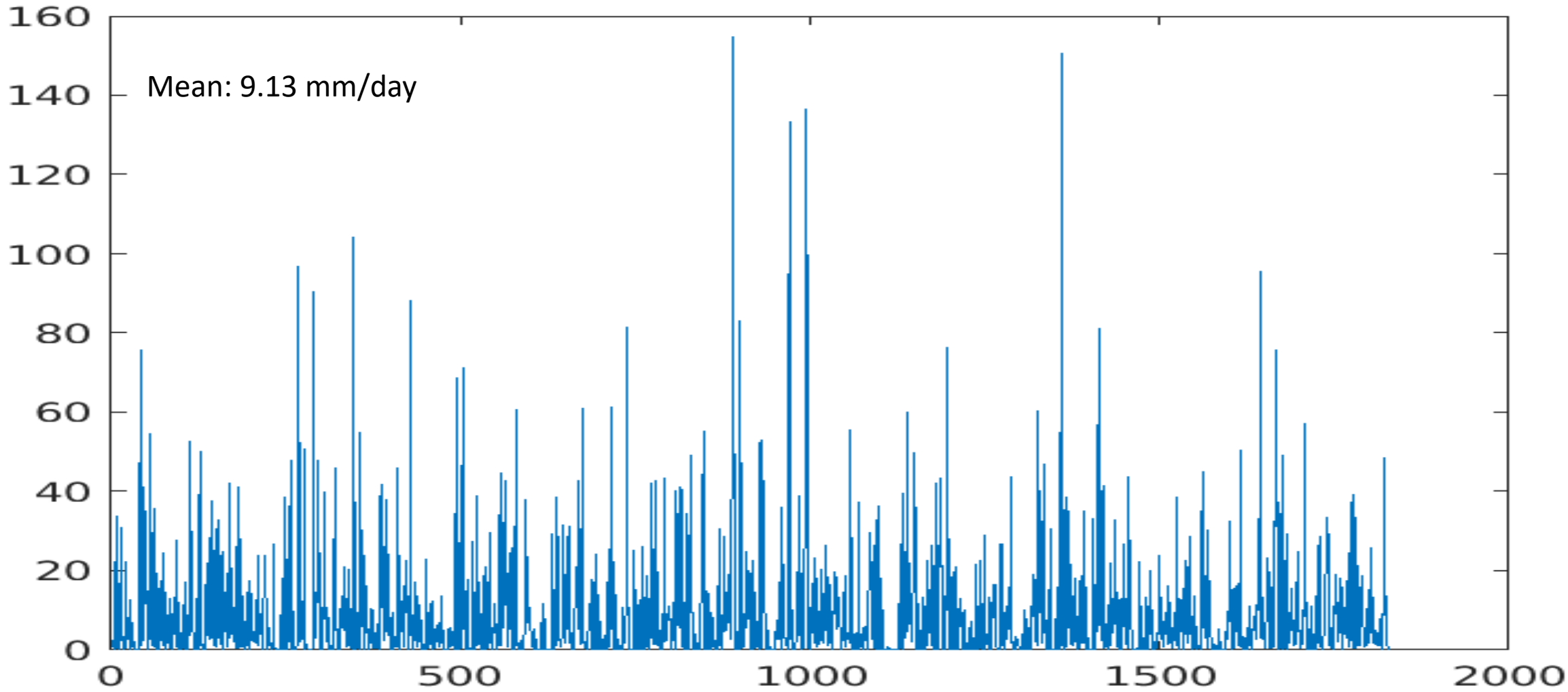
Anomaly

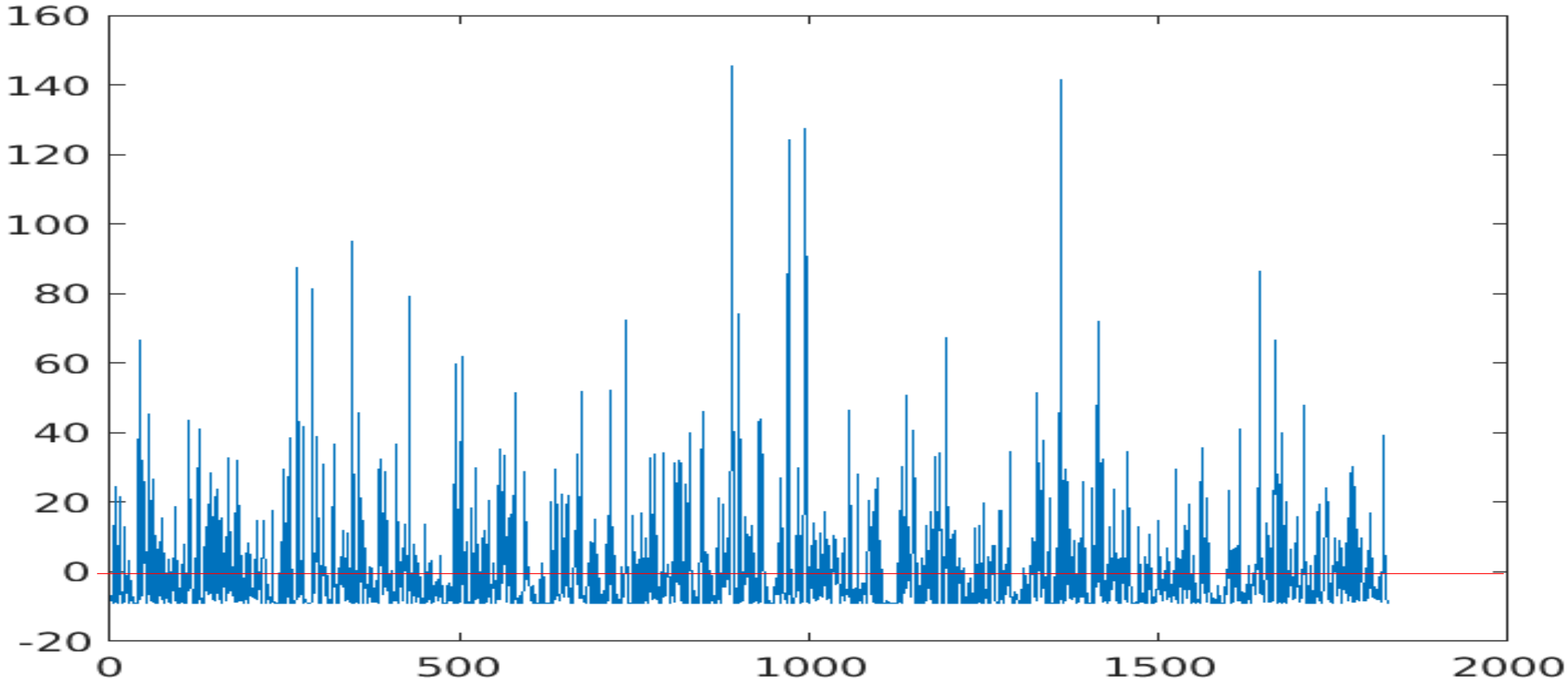
- Anomaly = Observation – expected value
- $Y(s,t) = X(s,t) - \mu(s,t)$
- $Y(s,t) > 0$: positive anomaly, $Y(s,t) < 0$: negative anomaly
- $Y(s,t) > \eta_U$: positive extreme event
- $Y(s,t) < -\eta_L$: negative extreme event

- η_U, η_L are usually double standard deviation of observations
- In some situations, one of the extreme events may not make sense

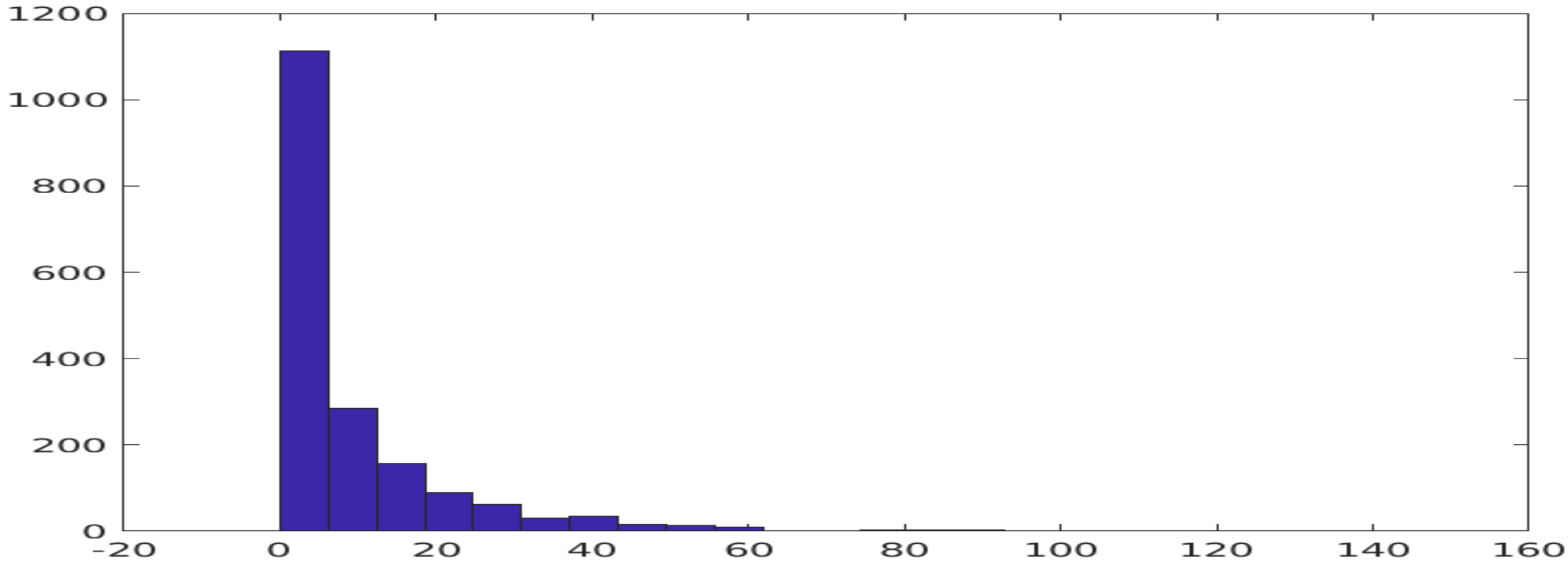
Daily Rainfall in Kharagpur (2000-2014, Jun-Sep)



Daily Rainfall Anomaly in Kharagpur (2000-2014, Jun-Sep)



Histogram of Daily Rainfall in Kharagpur



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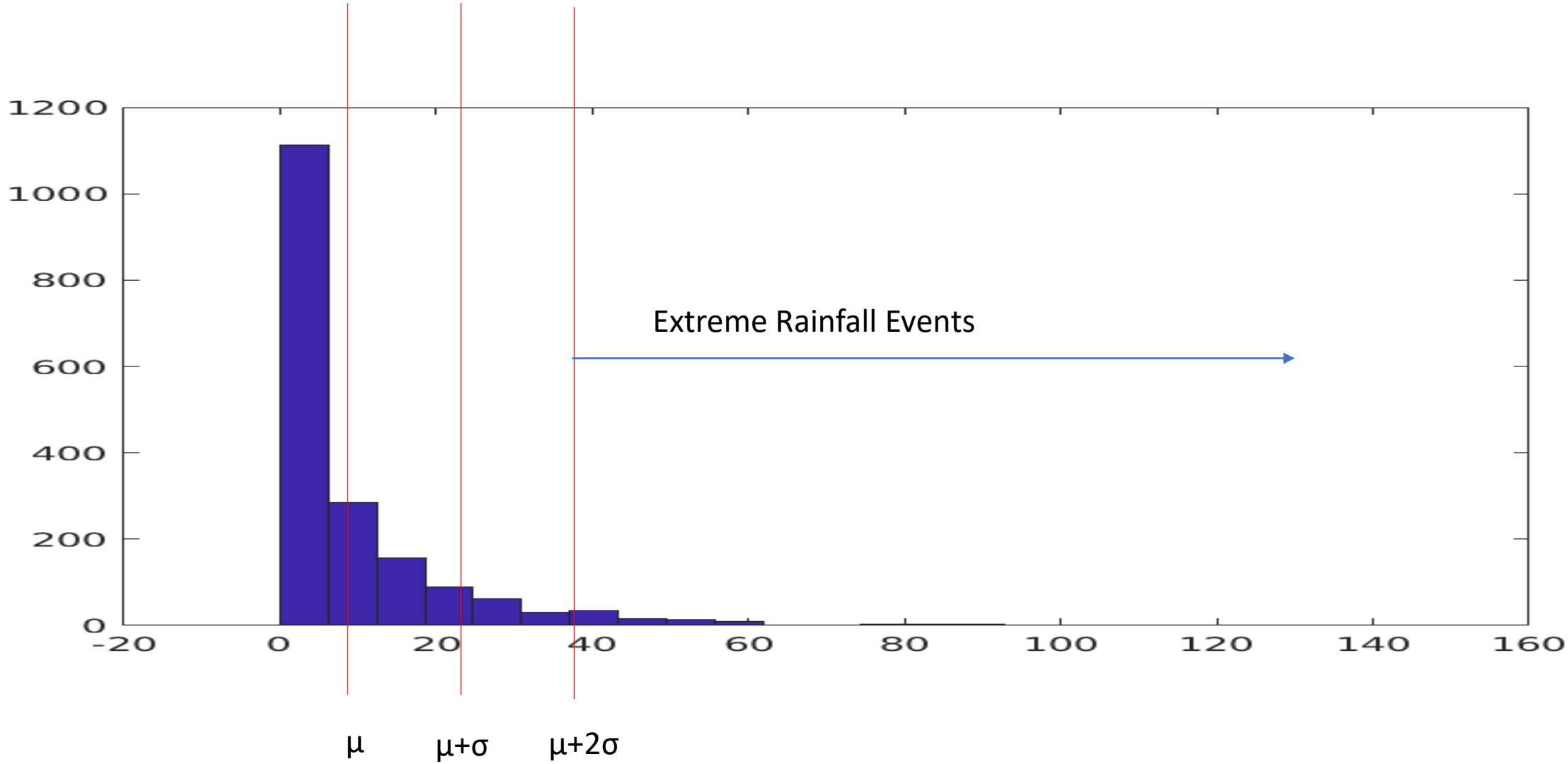


Percentiles

- Maximum rainfall in Kharagpur: 5th July 2007: 154 mm rainfall!
- It was caused by a deep depression in Bay of Bengal
- Quantile: cut-off points in probability distribution
- p-th Percentile = x: “p” percent of times, observation < x!

p	P-th Percentile in KGP	Frequency in 2000-2014
0.99	69 mm/day	18 days
0.9	25 mm/day	183 days
0.75	12 mm/day	457 days
0.5	3.4 mm/day	915 days

Skewed Distribution

- Mean daily rainfall at Kharagpur: 9.13 mm/day
- Median daily rainfall at Kharagpur: 3.4 mm/day!
- Median < Mean: On most of the days, KGP receives less rainfall than mean!
- 573/1830 days: more rainfall than the mean!
- Negative anomaly more frequent than Positive anomaly!
- Most days are “dry”, rainfall concentrated in a few “wet” days!
- Skewed distribution!

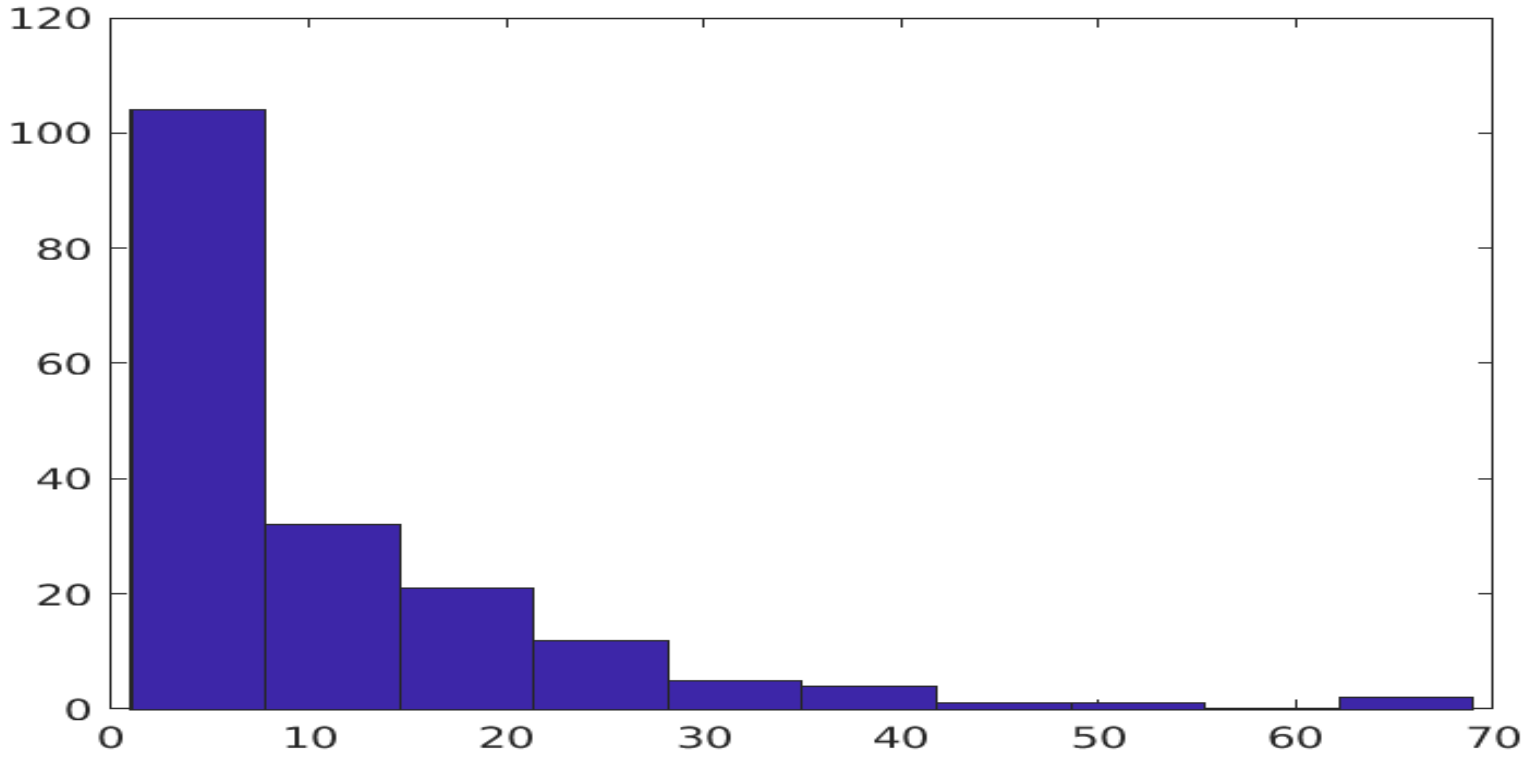
Return Periods

- Suppose any event happens today, when can you expect it to happen next?
- Common events may happen soon afterwards, rare events may happen much later!
- Return period = 1/p, where p is event probability
- Follows from Geometric Distribution
- 90% Percentile event: p=1-0.9 = 0.1, return period: 10 days
- i.e. expected difference between such events:10 days!

But actually …..

It seems that such events are “clustered” in time

Differences between two “90th-Percentile events” most likely to be <10 days!



Temporal Coherence of Anomaly

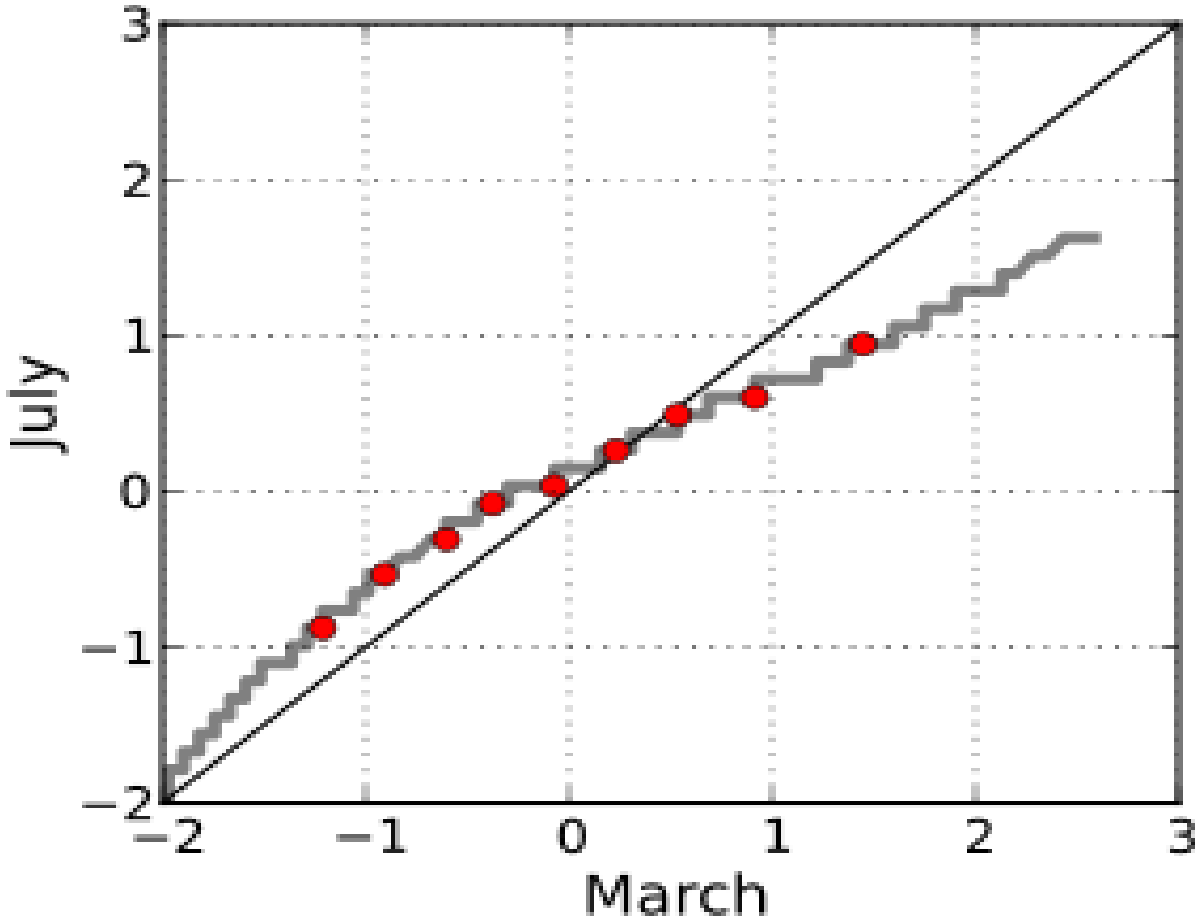
- Today positive anomaly => tomorrow positive anomaly (probably)
- KGP: prob(tomorrow positive anomaly) = 0.31
- KGP: prob(tomorrow positive anomaly | today positive anomaly) = 0.46!

- KGP: prob(tomorrow 90%-quantile event) = 0.1
- KGP: prob(tomorrow 90%-quantile event | today 90%-quantile event) = 0.25!

Spatial Coherence of Anomaly

- Anomalies are usually spatially coherent
- If one location has a positive anomaly, usually its surrounding locations also have it
- Whenever KGP has a positive rainfall anomaly during monsoon, on 60% occasions its surrounding locations also have positive rainfall anomaly!
- 90^{th} -Percentile rainfall in KGP => 90^{th}-Percentile in neighboring regions in 45% cases!
- 90^{th} -Percentile rainfall in KGP => 80^{th}-Percentile in neighboring regions in 60% cases!

Q-Q (Quantile-quantile plot)



A Q–Q plot comparing the distributions of standardized daily maximum temperatures at 25 stations in the US state of Ohio in March and in July. The curved pattern suggests that the central quantiles are more closely spaced in July than in March, and that the July distribution is skewed to the left compared to the March distribution. The data cover the period 1893–2001.

Wikipedia

Extreme Event Definitions

- PoT (Peak over Threshold) - An observation with high magnitude of anomaly
- Should also be a “peak” in the time-series (higher than neighbors)
- Threshold may be i) $\mu+2\sigma$

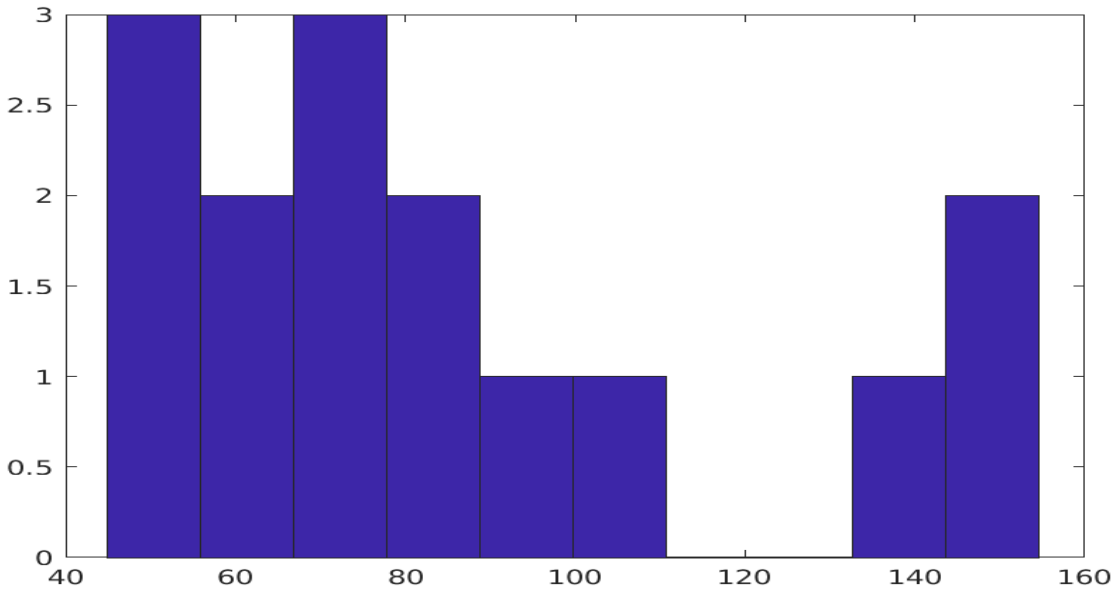
  ii) quantile (often 90^{th}, 95^{th}, 99^{th})
- Second definition: Block-maxima/minima
- Take a “set” of observations and calculate their maxima/minima
- The “set” can be spatial or temporal

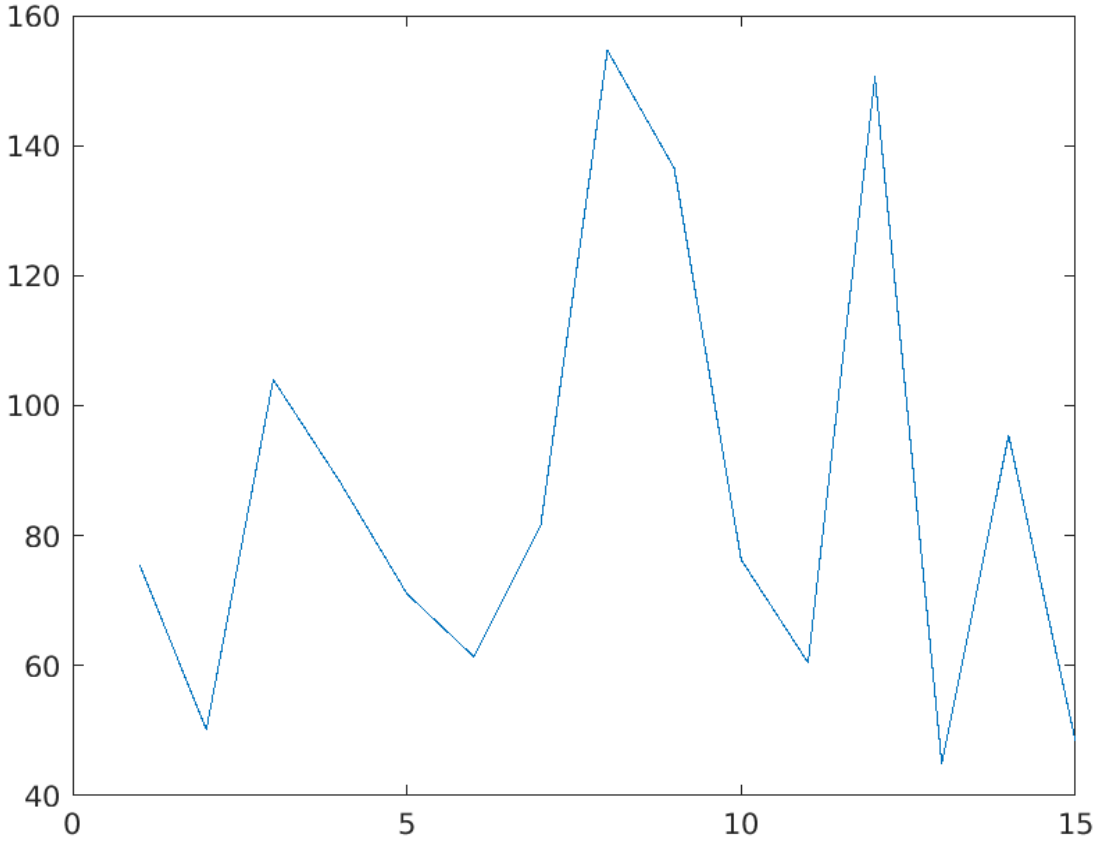
Block-wise Extremes

- Seasonal maximum precipitation over KGP
- Instead of dealing with all observations, we now deal with only the max. values

  Advantage: less variance

  Problem: less data





Extreme-Value Theory

- We focus on only the values at the “tail” of the distribution!
- Very different from the original distribution
- Original distribution more left-skewed than extreme distribution
- Needs a new distribution: Extreme-Value Distribution!