

Assignment 5

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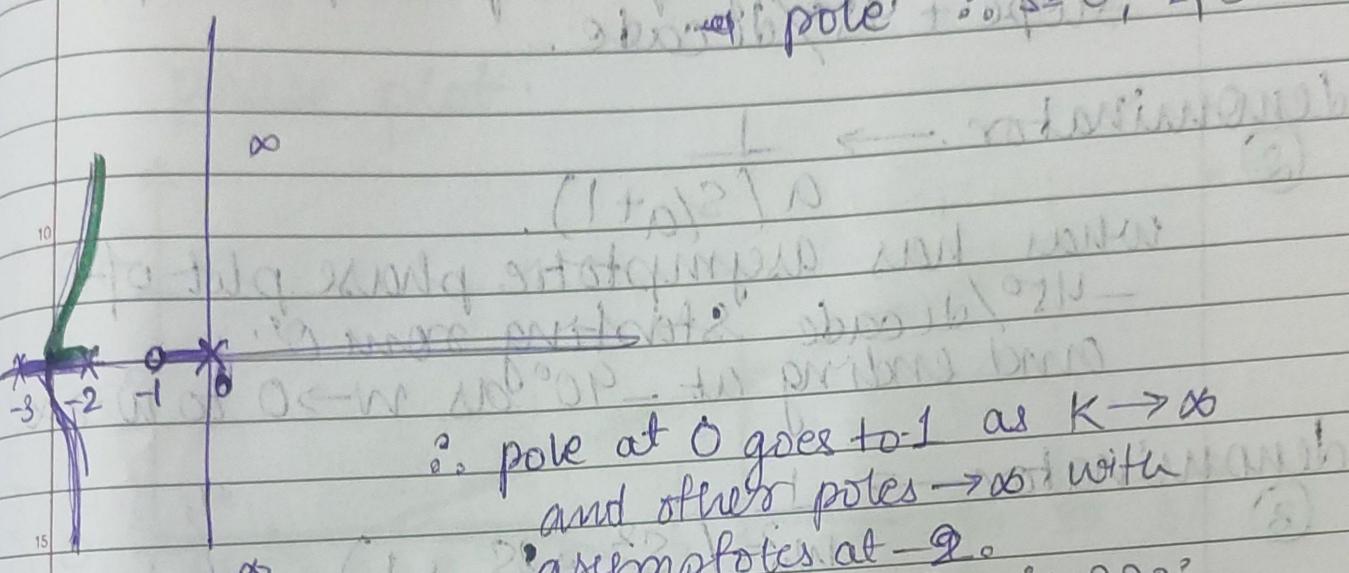
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Q1. characteristic equation

$$S(S+2)(S+3) + K(S+1) = 0$$

$$\text{Transfer funcn. } K = -(S+1) \rightarrow 0 \quad \text{pole zero} \quad \therefore S = -1$$

to following state
as ~~zero~~
pole $\therefore S = 0, -2, -3$



\therefore pole at 0 goes to -1 as $K \rightarrow \infty$
and other poles $\rightarrow \infty$ with
asymptotes at -2.
with angles of 90° & 270°

Q2. characteristic equation

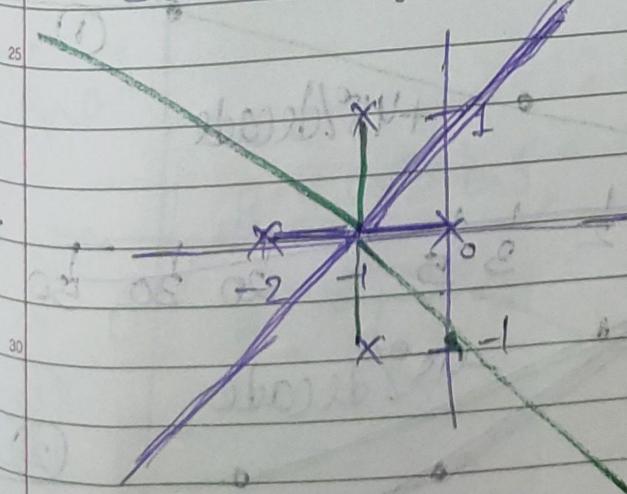
$$S(S+2)(S+1+j)(S+1-j) + K = 0$$

Transfer funcn.

$$S(S+2)(S+1+j)(S+1-j)$$

pole $\rightarrow 0, -2, -1-j, -1+j$

zero X



\therefore can be seen from
the plot there is
symmetry about -1
point.

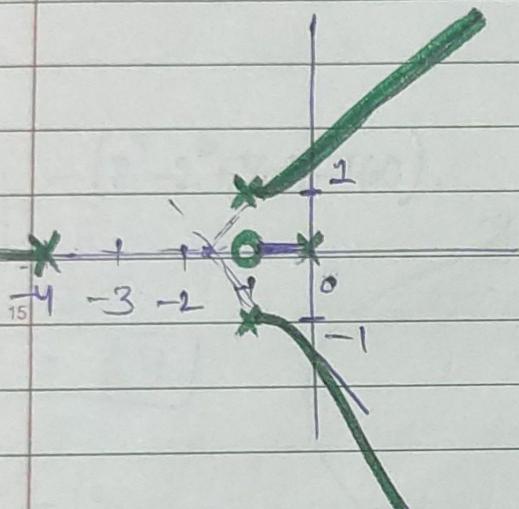
Q3. characteristic eqn⁴

$$s(s+4)(s^2 + 2s + 2) + K(s+1) = 0,$$

transfer funcⁿ $\rightarrow \frac{s+1}{s(s+4)(s^2 + 2s + 2)}$

5 centroid = $\frac{0 - 4 - 1 - 1 + 1}{4 - 1} = \frac{-5}{3}$ \rightarrow intersect of asymptotes

asymptote angles, $\theta = \frac{(2g+1)}{4-1} 180^\circ = 60^\circ, 180^\circ, 300^\circ$



8 asymptotes
intersect at centroid.

Q4. characteristic eqn⁴

$$s(s+3)(s^2 + 2s + 2) + K = 0$$

transfer funcⁿ

$$= \frac{8}{s(s+3)(s^2 + 2s + 2)}$$

zero \rightarrow
pole $0, -3, -1+j, -1-j$.

If odd number of open loop poles and zeros exists to the left side of point on real axis then point is root locus branch.

So on real axis root loci will be present betⁿ

2 poles $0, -3$.

for departure angle of pole $-1+j$.

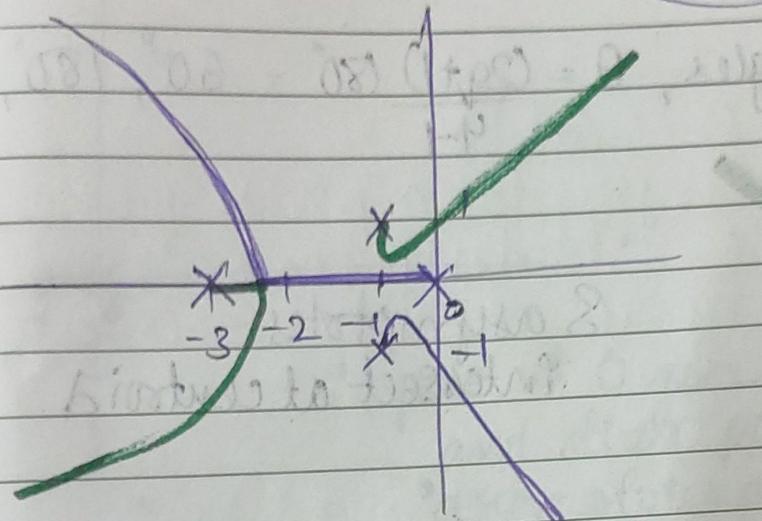
(S)

pole at 0 makes 135° with s_1 .

pole at -3 makes $\tan^{-1}(1/2)$ with s_1

pole at $-1-j$ makes 90° with s_1 .

$$\text{Q. angle of departure} = 180^\circ - 135^\circ - 90^\circ - \tan^{-1}(1/2) = -71.56^\circ$$



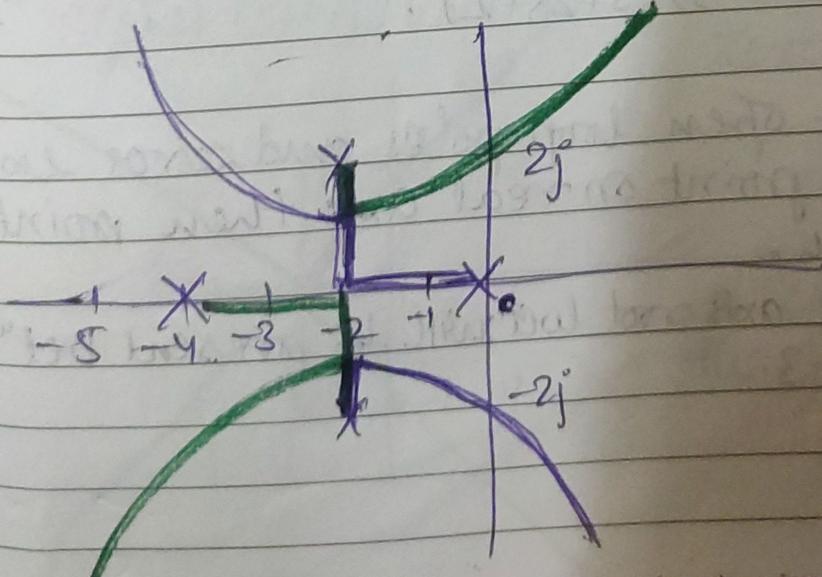
Q5. characteristic eqn⁴

$$S(S+4)(S^2+4S+20)+K=0$$

transfer func^a

$$\frac{1}{S(S+4)(S^2+4S+20)}$$

\rightarrow pole
0, -4,
 $-2-2j$,
 $-2+2j$.



intersection with
imaginary axis
using RH table.

$$K=2 \text{ for now is } 0.$$

∴ at, $K=260$, we have 2 poles
on imaginary axis $(0, \pm j)$.

for $K=260$. intersection with
real & imaginary axis at $\rightarrow \pm j10j$

$$K = -(2^7 - 2^6 + 36 \times 4 - 160).$$

$$K = -(S^4 + 8S^3 + 36S^2 + 80S)$$

$$S_{25} = 2$$

$$\boxed{K = 64}$$

$$\text{at } K = 0, \text{ i.e. } 260$$

$$4S^3 + 24S^2 + 60S + 80 = 0$$

$$2S^3 + 12S^2 + 36S + 20 = 0$$

$$S^3 + 6S^2 + 18S + 20 = 0.$$

∴ -2 is

breakaway
point on real
axis.

Breakaway
point:

in minimum gain

at which gain is zero

Q6. Characteristic eqn.

$$s(s+5)(s+6)(s^2+2s+2) + k(s+3) = 0$$

Transfer func. = $s+3$
 $s(s+5)(s+6)(s^2+2s+2)$.

pole

$$0, -5, -6, -1-j, -1+j$$

Zeros

$$-3$$

5 root locus
& 4 of them will go
to ∞ as k increases

If odd number of
open loop poles and
zeros exists to the left
side of a point on
real axis, then the point is
on root locus branch.

So on real axis, root loci will be
present betw $0, -3$ and $-5, -6$.

Centroid $\alpha = 2.5$

angle of asymptotes = $45^\circ, 135^\circ, 225^\circ, 315^\circ$.

Breakaway point betw -5 and -6 .

Angle of departure for $-1+j$ is

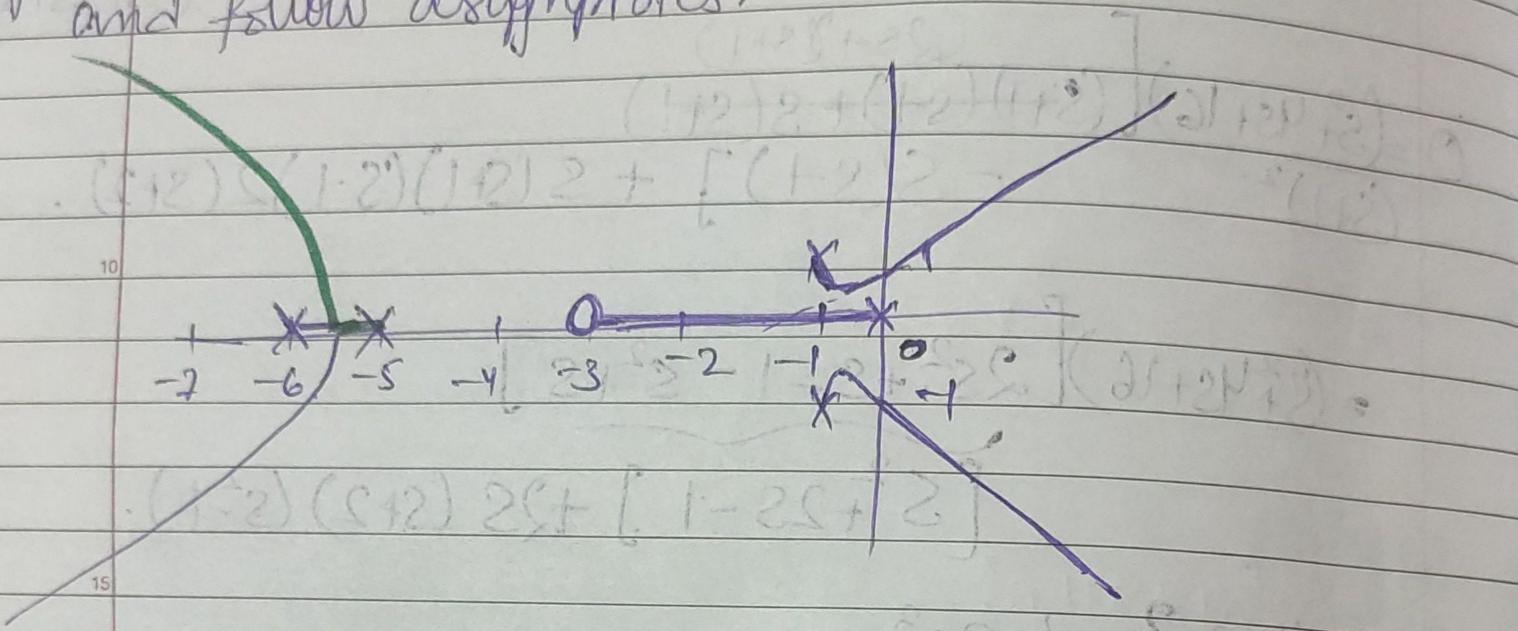
$$\begin{aligned} & 180^\circ - (180 - \tan^{-1} 1) \\ & - 90^\circ - \tan^{-1} y_4 \\ & - \tan^{-1} y_3 \\ & + \tan^{-1} y_2 \\ & = -43.78^\circ \end{aligned}$$

So root locus of pole
at 0 will end at -3

pole at $-1+j$ will start with -43.78°
and follow 45° asymptote from centroid.

pole at $-1-j$ will reflect $-1+j$ locus wst real axis,

poles at $-5, -6$ will breakaway
and follow asymptotes.



$$(Q7) G(s)H(s) = \frac{K(s+a)}{s(s-b)(s^2 + 2\zeta\omega_n s + \omega_n^2)}$$

$$= \frac{K(s+1)}{(s+1)(s^2 + 4s + 16)}$$

Zeros

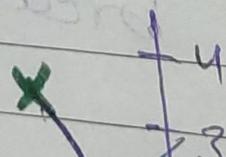
-1.

gain for stability?

poles

$$0, 1, -2 + 2\sqrt{3}j, -2 - 2\sqrt{3}j$$

0.47 ?
-2.263 breakaway points.



$$K = \frac{S(S+1)(S^2 + 4S + 16)}{(S+1)}$$

$$\frac{dK}{ds} = \frac{(S+1)}{(S+1)^2} \left[(S+1)(S^2 + 4S + 16) + S(S^2 + 4S + 16) \right] - \frac{S}{(S+1)} (S+1)(2S+4)$$

$$0 = \frac{(S^2 + 4S + 16)}{(S+1)^2} \left[(S+1)(S+1) + S(S+1) - S(S+1) \right] + S(S+1)(S-1)2(S+2).$$

$$= (S^2 + 4S + 16) \left[\underbrace{2S^2 + S - 1 - S^2 + S}_{[S^2 + 2S - 1]} + 2S(S+2)(S^2 - 1) \right].$$

$$= S^4 + 8S^3 - S^2 + 2S^3 + 4S^3 - 4S + 16S^2 + 32S - 16 + 2S(S^3 + 2S^2 - S - 2).$$

$$= S^4 + 6S^3 + 2S^2 + 28S - 16 + 2S^4 + 4S^3 - 2S^2 - S$$

$$= 3S^4 + 10S^3 + 21S^2 + 24S - 16.$$

$$\hookrightarrow S = -2.263, 0.448$$

breakaway points

continued.

B7

characteristic eqn

$$1 + G(s)(H(s)) = 0.$$

$$s(s+1)(s^2 + 4s + 16) + K(s+1) = 0.$$

open loop poles $\rightarrow 0, -1, -2 \pm 2\sqrt{3}j$

Zeros $\rightarrow 1$

$$\text{centroid} = \frac{0+1-2-2\sqrt{3}j}{3} = -2 + 2\sqrt{3}j + 1 \\ = -2/3.$$

angle of asymptotes $\frac{(m+1)\pi}{3} = 60^\circ, 180^\circ, 300^\circ$

Breakaway points

$$s = 0.448, -2.26.$$

Routh table.

long axis crosses over

| s^4 | 1 | 12 | K | $s^4 + 3s^3 + 12s^2 + s(K-16)$ |
|-------|---|----|-----|--------------------------------|
| | | | | $+K=0$ |

| s^3 | 3 | $K-16$ | $(2+28F^2)20N$ |
|-------|---|--------|----------------|
|-------|---|--------|----------------|

| s^2 | $\frac{s^2-K}{3}$ | K | $\rightarrow A(s)$ |
|-------|-------------------|-----|--------------------|
|-------|-------------------|-----|--------------------|

| s | $\frac{-K^2+59K-832}{s^2-K}$ | \rightarrow should be 0. |
|-----|------------------------------|----------------------------|
|-----|------------------------------|----------------------------|

K

$$K = \frac{59}{2} \pm \frac{3\sqrt{17}}{2}$$

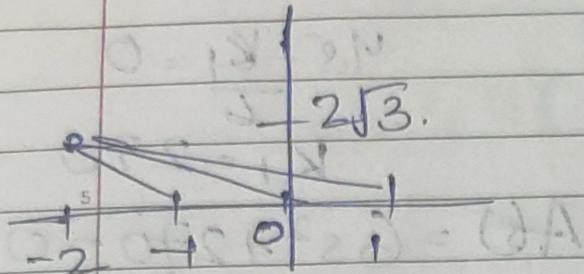
$$A(s) = \underline{s^2 - K} s^2 + K = 0$$

$$K = 35.68, \text{ or } 23.32.$$

$$s = \sqrt{\frac{3K}{K-52}}$$

$$s = \pm 2.86j, \text{ or } \pm 1.02j$$

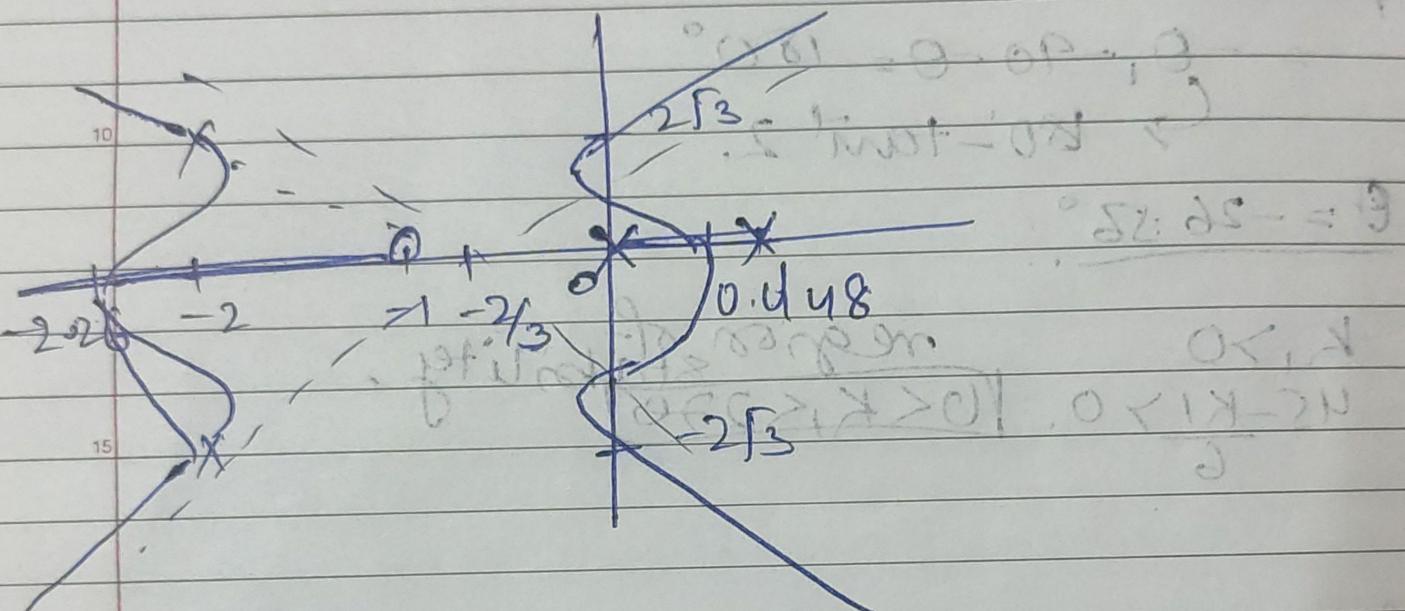
angle of departure.



$$180 - \tan^{-1}(2\sqrt{3}) = 180 + \tan^{-1}\sqrt{3}$$

$$-180 + \tan^{-1}(2\sqrt{3}) = 90 - 0^\circ$$

$$\theta_M = -54.8^\circ$$



$$k - 16 > 0$$

$$k > 0$$

$$\frac{52 - k}{3} > 0$$

$$16 < k < 52$$

$$k^2 - 59k + 832 > 0$$

$$(2)(1) k - 52$$

$$k^2 - 59k + 832 < 0$$

range for k for stability

$$23.32 < k < 35.68$$

$$k > 23.32 \quad \text{or} \quad k < 35.68$$

Q8.

$$G_1(s) = \frac{1}{s^3 + 6s^2 + 45s} = \frac{1}{s(s^2 + 6s + 45)}$$

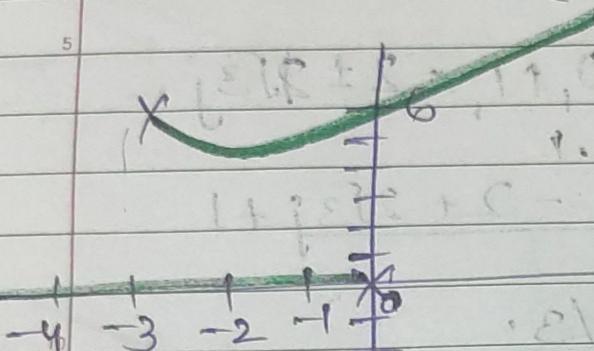
$$= \frac{1}{s((s+3)^2 + 6^2)}$$

pole

$$0, -3 - 6j, -3 + 6j.$$

$$\text{centroid} = -2$$

$$\text{angle of asymptotes} = 60^\circ, 180^\circ, 300^\circ$$



breaking pt.

$$K_1 = -(s^3 + 6s^2 + 45s)$$

$$\frac{dK_1}{ds} = 0$$

$$s = -2 \pm 3.32j$$

: no breaking pt.

Q

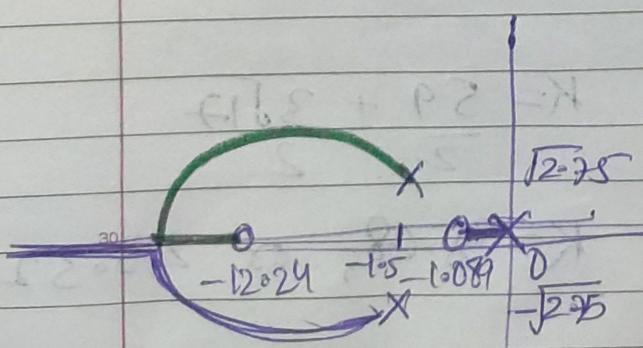
$$G_2(s) = \frac{0.025s^2 + 0.5s + 1}{s^3 + 3s^2 + 5s} = \frac{0.025(s^2 + 5s + 1)}{s(s^2 + 3s + 5)}$$

$$= \frac{3s^2 + 40s + 40}{40s(s^2 + 3s + 5)}$$

poles

$$0, -1.5 + \sqrt{2.75}j, -1.5 - \sqrt{2.75}j$$

$$s = -1.089, -12.244$$



$$\text{centroid} = 10.3$$

$$\text{angle of asympt} = (2n+1)\pi$$

$$= 180^\circ, 360^\circ$$

breaking away pt

$$K_2 = \frac{(s^3 + 3s^2 + 5s)}{(0.025s^2 + 5s + 1)}$$

$$\frac{dK_2}{ds} = 0 \Rightarrow 0.46 \pm 0.06j, \frac{23.63}{-2.75}, \frac{-2.75}{-2.75}$$

continued.

Q8

Routh Table 1.

| | | |
|-------|------------|------------------------|
| s^3 | 1 | 45 |
| s^2 | 6 | $K_1 \rightarrow A(s)$ |
| s | $45 - K_1$ | $\rightarrow 0$ |
| 1 | K_1 | |

$$45 - K_1 = 0$$

$$K_1 = 270$$

$$A(s) = 6s^2 + 270 = 0$$

$$s = \pm 6.91 j$$

angle of departure

$$= -0, -90 - 0 = 180^\circ$$

$\hookrightarrow 180^\circ - \tan^{-1} 2$.

$$\theta = -26.56^\circ$$

$$K_1 > 0$$

$$45 - K_1 > 0. \quad [0 < K_1 < 270]$$

region of stability.

Routh Table 2.

| | | |
|-------|------------------------------|------------------------|
| s^3 | 1 | $5 + K_2$ |
| s^2 | $3 + 0.075K_2$ | $K_2 \rightarrow A(s)$ |
| s | $0.075K_2^2 + 2.375K_2 + 15$ | $\rightarrow 0$ |
| 1 | $3 + 0.075K_2$ | |

$$A(s) = 3s^2 + 0.075K_2 s^2 + K_2 = 0$$

NO img. axis cross over

$$3 + 0.075K_2 > 0$$

$$3 > 0$$

$$K_2 > 0$$

$$0.075K_2^2 + 2.375K_2 + 15 > 0$$

$$3 + 0.075K_2 > 0$$

$\left. \begin{array}{l} 3 + 0.075K_2 > 0 \\ 3 > 0 \\ K_2 > 0 \end{array} \right\} \rightarrow K_2 > 0$

stability region.

II) value of k for identical character

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$$1 + K G_1(s) = 1 + K G_2(s).$$

$$s^3 + 6s^2 + 45s + K = s^3 + (3 + 0.075K)s^2 + (5 + K)s + K.$$

$$0.075K + 3 = 6$$

$$K = 40$$

$$K + 5 = 45$$

$$K = 40.$$

Q9.

Unity feedback system

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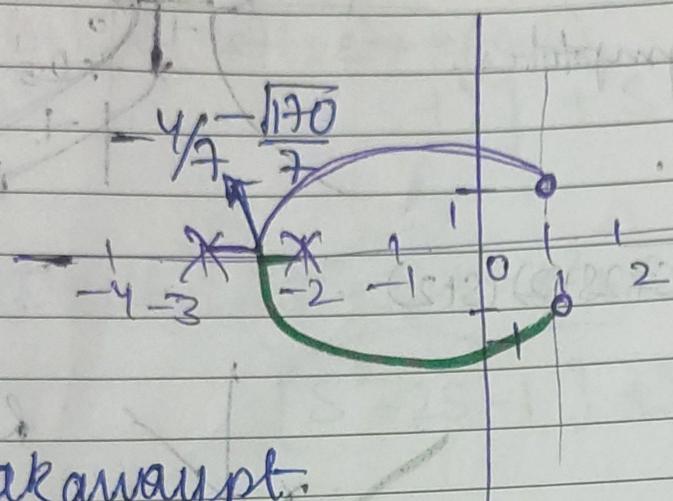
$$G(s) = \frac{K(s^2 - 2s + 2)}{(s+2)(s+3)}$$

 poles
 $-2, -3$

BP between

 2 poles $(-2, -3)$

zeros

 $-1-j, -1+j$

 value of
 K for
 stability

breakaway pt.

$$1 + K(s^2 + 2s + 2) = 0 \\ (s+2)(s+3)$$

$$\frac{dK}{ds} = 0$$

$$-(7s^2 + 8s - 22) = 0 \quad \rightarrow \text{roots } \left\{ \frac{-4 \pm \sqrt{170}}{7} \right\}$$

characteristic eqn

$$(s+2)(s+3) + K(s^2 - 2s + 2) = 0$$

$$(1+K)s^2 + (5-2K)s + (6+2K) = 0$$

$$1+K>0 \\ K>-1$$

$$5-2K>0 \\ K<2.5$$

$$6+2K>0 \\ K>-3$$

 all
 coef of
 same sign.

$$-1 < K < 2.5$$

$$1+K<0 \\ K<-1$$

$$5-2K<0 \\ K>2.5$$

$$6+2K<0 \\ K<-3$$

No mutual solution.

(Q) 10. characteristic eqn.

$$G(s)H(s) = K$$

(a)

$$s(s^2 + 2s + 1.25)(s+2)$$

Centroid
= -1

one of
breakaway
points.

pole

$$0, -2, -1 + 0.5j$$

$$-1 - 0.5j$$

2 BP betw

$$0, -2$$

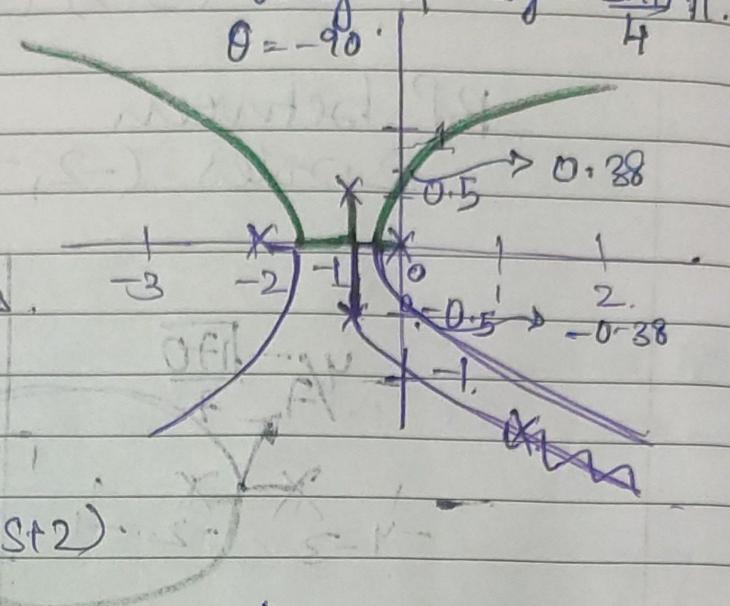
real axis.

(imaginary points
closer to centroid).

4 asymptotes.

$$(b) G(s)H(s) = K$$

$$s(s^2 + 2s + 2)(s+2)$$



poles

$$0, -2, -1-j$$

$$-1+j$$

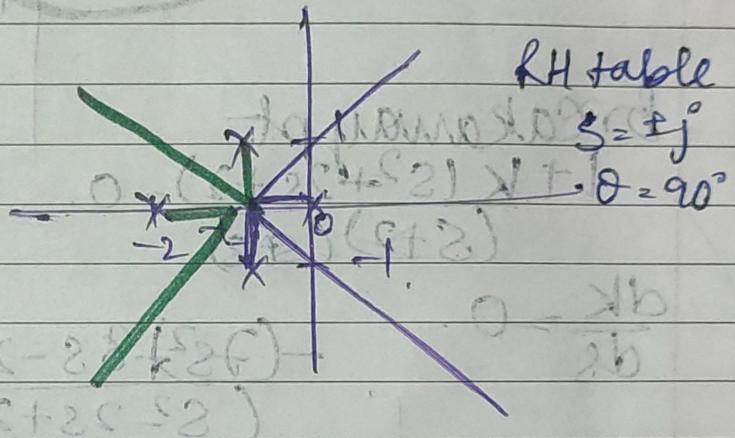
BP at
centroid

Symmetric

4 asymptotes

RH table

$$s = \pm j$$



(c)

$$G(s)H(s) = K(s+2)(s^2 + 2s + 2)$$

$$K = s(s+2)(s^2 + 2s + 10)$$

poles

$$0, -2, -1-3j$$

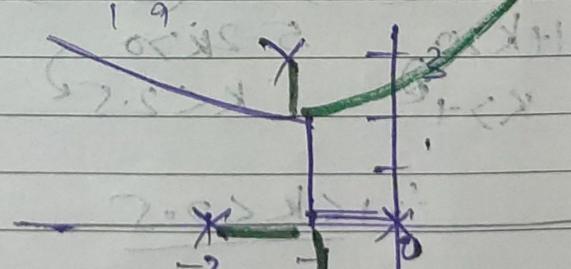
$$-1+3j$$

4 asymptotes.

RH table

$$K = \pm j$$

$$\theta = -90^\circ$$



2 BP betw
-1-3j
-1+3j

(real points

closer to centroid)