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Control & Instrumentation Lab, Autumn 2021-22
Experiment 4

Aim: Stability Analysis

Q1)

1. $M(s) = \frac{20(s-1)}{(s+2)(s^2+4)}$

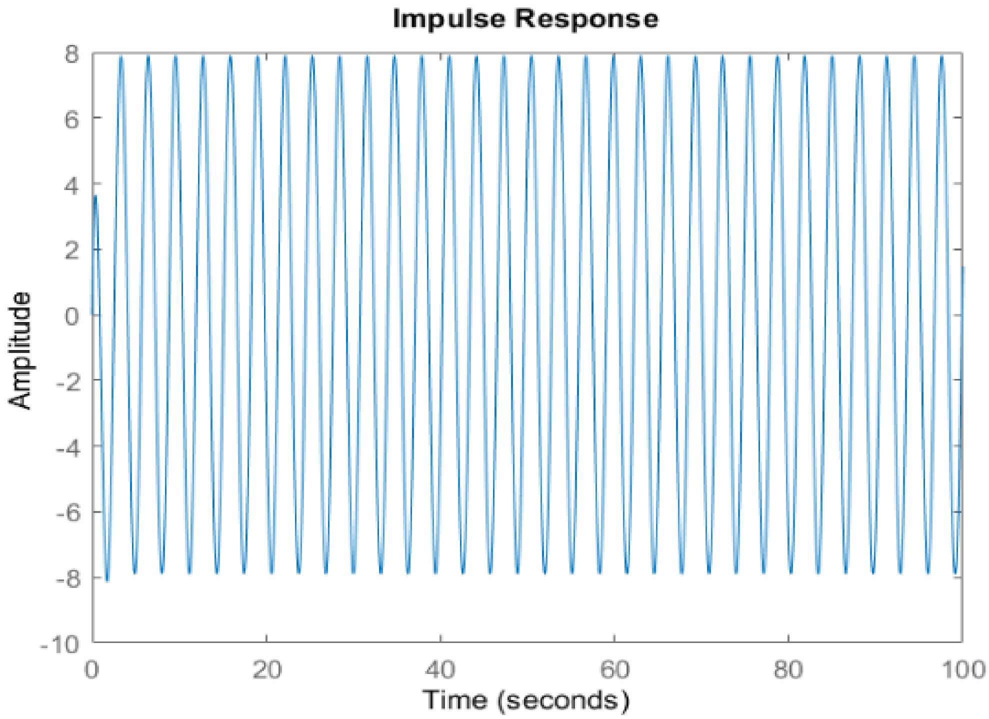
   Is $M(s)$ asymptotically stable, marginal stable or unstable?

As the system has 2 poles on imaginary axis and one pole in LHP, so it is marginally stable.
This can be confirmed in Matlab.

Matlab Code:

```
tf1=zpk(1,[-2,2i,-2i],20);
impulse(tf1,100)
```

The graph is oscillating and never dies out confirming that it is a marginally stable function.



I will be declaring an overall rhc function in matlab which will be used at various places in the given experiment.

Matlab code:

```
function y=rhc(coff,x)
ceoff = length(coff);
rhTableColumn = round(ceoff/2);
rhTable = zeros(ceoff,rhTableColumn);
rhTable(1,:) = coff(1,1:2:ceoff);
if (rem(ceoff,2) ~= 0)
   rhTable(2,1:rhTableColumn - 1) = coff(1,2:2:ceoff);
else
   rhTable(2,:) = coff(1,2:2:ceoff);
end
epssilon=0.0001;
for i = 3:ceoff
   if rhTable(i-1,:) == 0
      order = (ceoff - i);
```

```
        cnt1 = 0;
        cnt2 = 1;
        for j = 1:rhTableColumn - 1
            rhTable(i-1,j) = (order - cnt1) * rhTable(i-2,cnt2);
            cnt2 = cnt2 + 1;
            cnt1 = cnt1 + 2;
        end
    end
    for j = 1:rhTableColumn - 1
        firstElemUpperRow = rhTable(i-1,1);
        rhTable(i,j) = ((rhTable(i-1,1) * rhTable(i-2,j+1))-(rhTable(i-2,1) *
rhTable(i-1,j+1))) / firstElemUpperRow;
    end
    if rhTable(i,1) == 0
        rhTable(i,1) = epssilon;
    end
end
unstablePoles = 0;
for i = 1:ceoff - 1
    if sign(rhTable(i,1)) * sign(rhTable(i+1,1)) == -1
        unstablePoles = unstablePoles + 1;
    end
end
if x==1
    fprintf('\n Routh-Hurwitz Table:\n')
    rhTable %#ok<NOPRT>
end
if unstablePoles == 0
    y=1;
else
    y=0;
end
end
```

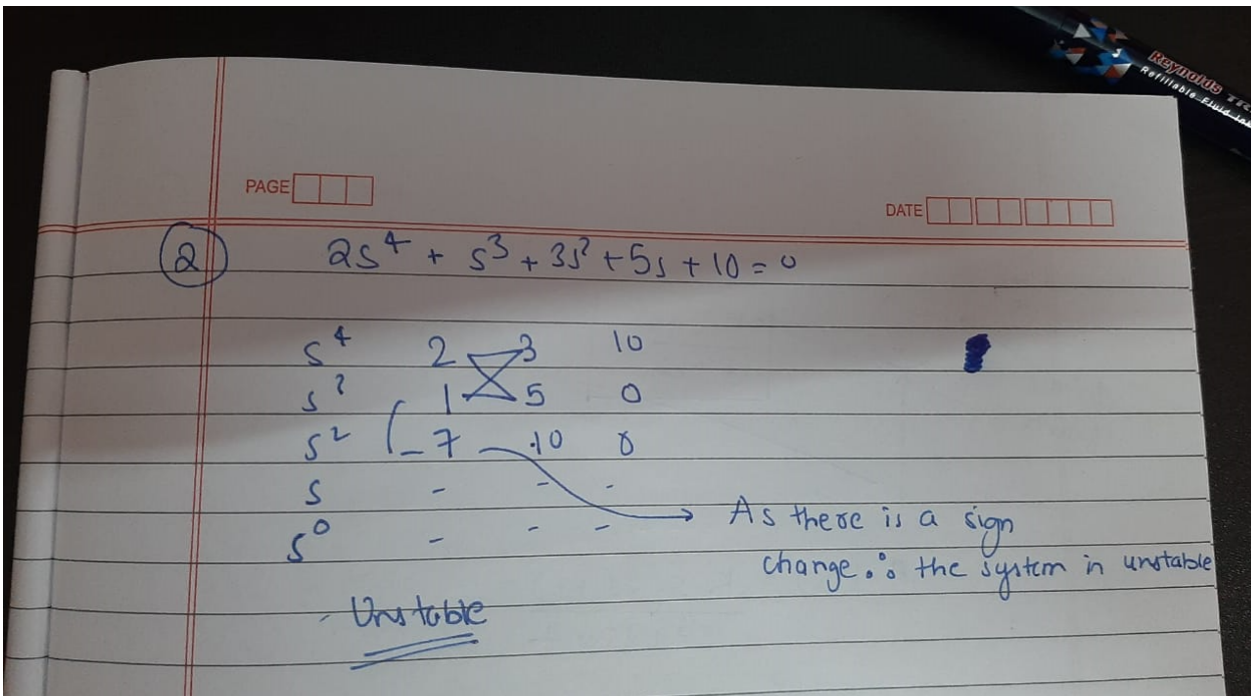
Q2)

2. Consider the characteristic equation

$$2s^4 + s^3 + 3s^2 + 5s + 10 = 0$$

of a system. Using Routh-Hurwitz (RH) criterion determine the stability of system.

By hand-



By Matlab:

Now we implement the Routh-Hurwitz external function to find the stability of the given characteristic equation. In the defined function, the first input is the coefficient of characteristic polynomial. And the 2nd input prints the RH Table when input is 1.

Matlab Code:

```
y1=rhc([2,1,3,5,10],1);
if y1==0
   fprintf("System is unstable \n")
else
   fprintf("System is stable \n")
end
```

Matlab Command line:

```
Routh-Hurwitz Table:

rhTable =

    2.0000    3.0000   10.0000
    1.0000    5.0000         0
   -7.0000   10.0000         0
    6.4286         0         0
   10.0000         0         0

System is unstable
```

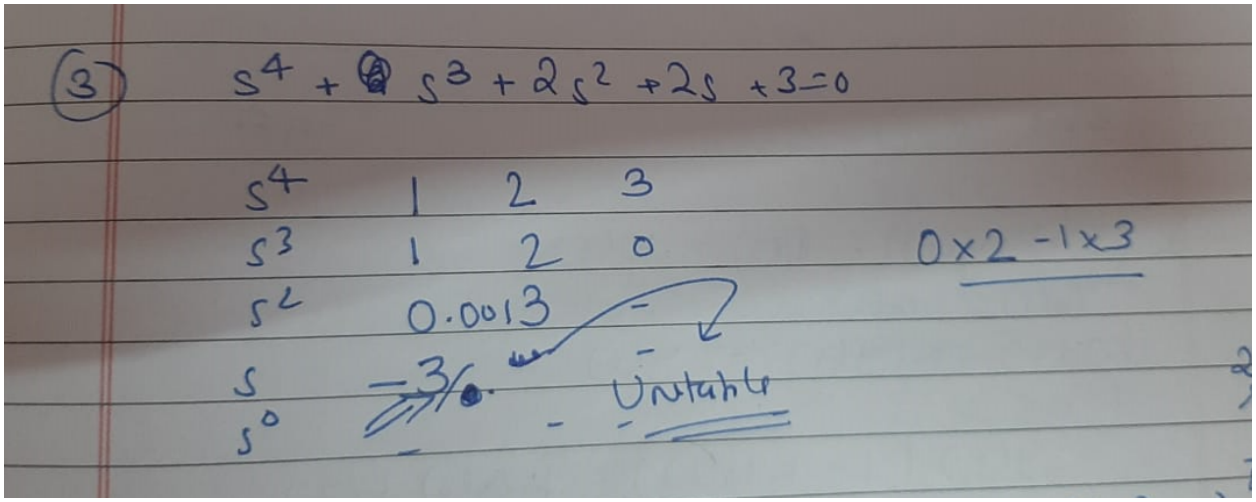
Q3)

3. Consider the characteristic equation

$$s^4 + s^3 + 2s^2 + 2s + 3 = 0$$

of a system. Using RH criterion determine its stability.

By hand-



(3) $s^4 + s^3 + 2s^2 + 2s + 3 = 0$

s^4	1	2	3
s^3	1	2	0
s^2	0.0013	-	
s	$-3/$	-	
s^0		-	

$0\times2 - 1\times3$

Unstable

Matlab Code:

```
y1=rhc([1,1,2,2,3],1);
if y1==0
   fprintf("System is unstable \n")
else
   fprintf("System is stable \n")
end
```

Matlab Command line:

```
Routh-Hurwitz Table:

rhTable =

   1.0e+04 *

    0.0001    0.0002    0.0003
    0.0001    0.0002         0
    0.0000    0.0003         0
   -2.9998         0         0
    0.0003         0         0

System is unstable
```

Q4)

4. Consider the characteristic equation

$$s^5 + 4s^4 + 8s^3 + 8s^2 + 7s + 4 = 0$$

of a system. Using RH criterion determine its stability.

By hand-

4

s^5	1	8	7	0
s^4	4	8	4	0
s^3	6	6	0	0
s^2	4	4	0	0
s	0.001	0	0	
s^0	4			

No sign change

$\therefore$ system is stable.

Matlab Code:

```
y1=rhc([1,4,8,8,7,4],1);
if y1==0
   fprintf("System is unstable \n")
else
   fprintf("System is stable \n")
end
```

Matlab Command Line:

Routh-Hurwitz Table:

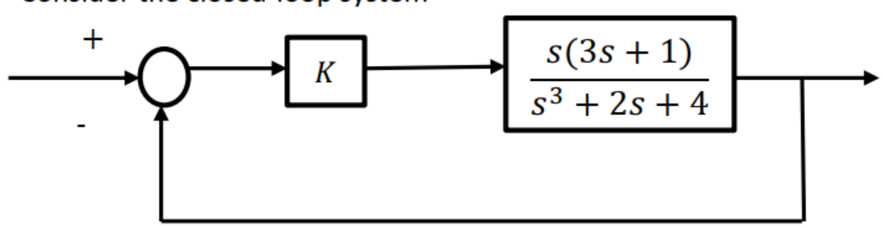
rhTable =

1.0000	8.0000	7.0000
4.0000	8.0000	4.0000
6.0000	6.0000	0
4.0000	4.0000	0
0.0001	0	0
4.0000	0	0

System is stable

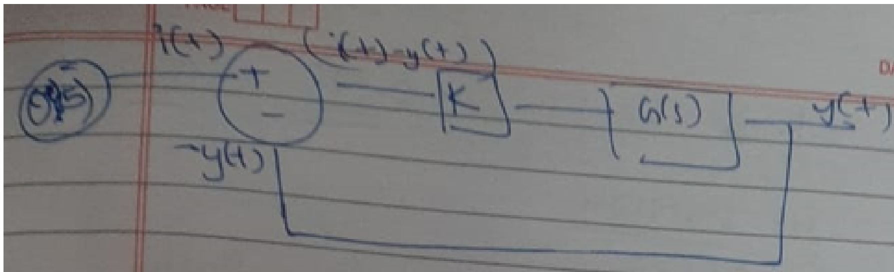
Q5)

5. Consider the closed-loop system



where K is the gain of proportional controller. Determine the admissible values of K for which the closed-loop system is stable. Use RH criterion.

(Q5)



$1 + KG(s) = 0$

$K G(s) \left(i(t) - y(t) \right) = y(t)$

$$1 + \frac{(s^3 + 2s + 4)}{K \, s(3s+1)} = 0$$

$$s^3 + 2s + 4 + Ks(3s+1) = 0$$

$$s^3 + 2s + 4 + 3Ks^2 + Ks = 0$$

$$\boxed{s^3 + 3Ks^2 + (2+K)s + 4 = 0}$$ ☆

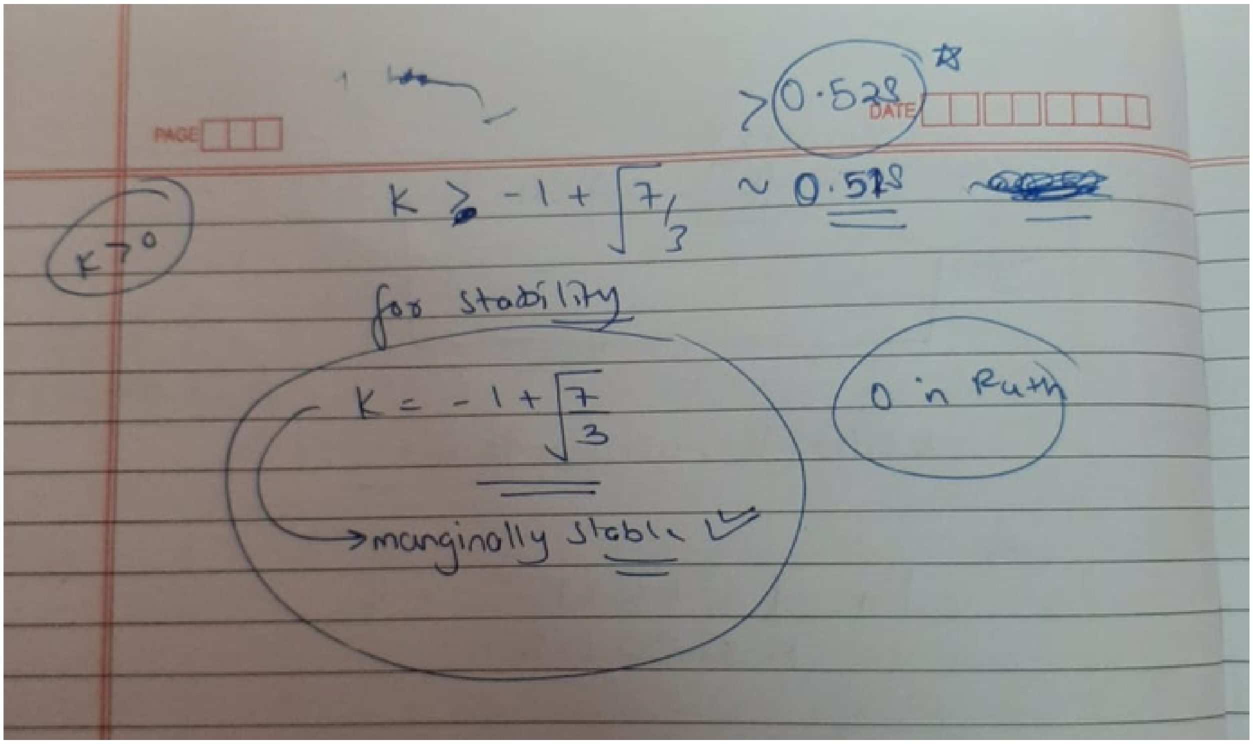
Now RH table;

$K > 0$

s^3	1	$(2+K)$
s^2	$3K$	4
s	$\frac{(3K)(2+K) - 4}{3K}$	0
s^0	$\cancel{\frac{(3K)(2+K) - 4}{3K}}$	4

$$(3K)(2+K) - 4 > 0$$

$$6K + 3K^2 - 4 > 0$$

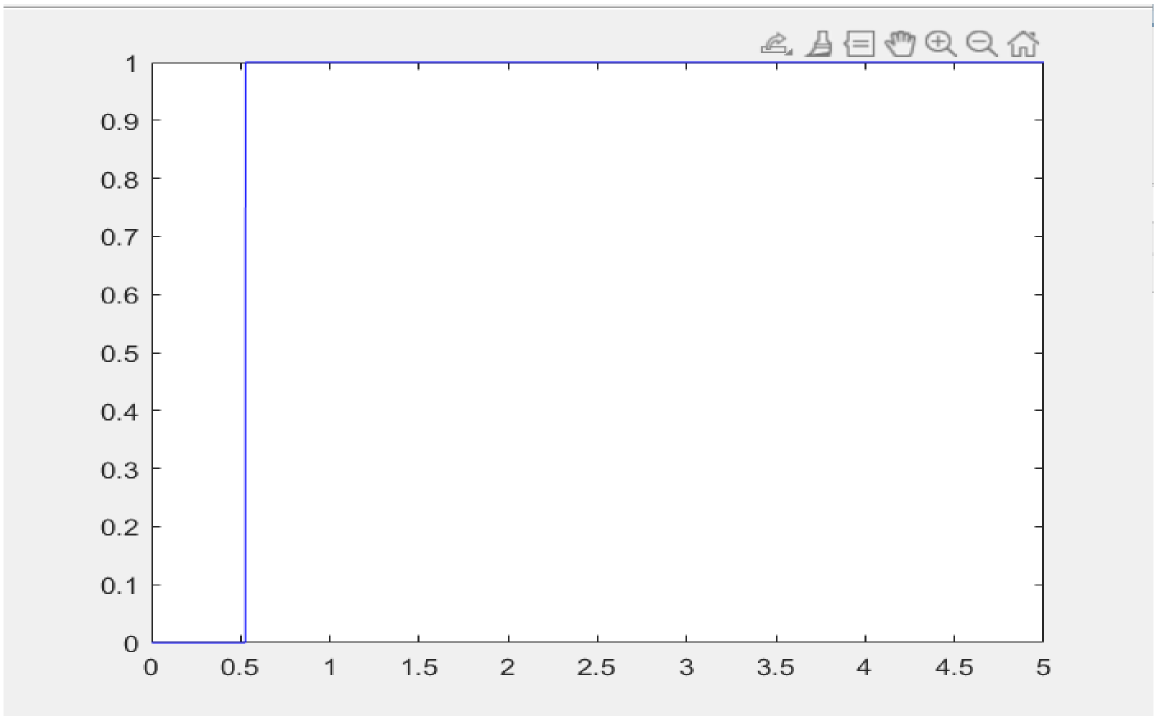


From the above solution, for k>= 0.528 the system is stable. For k=0.528 it is marginally stable.
Now we will verify it using the rhc function we have found by determining stability for variables.

Matlab Code:

```
k=1e-3:1e-3:5;
y=zeros(size(k));
for i =1:length(k)
   y(i)=rhc([1,3*k(i),k(i)+2,4],0);
end
figure;
plot(k,y,"b-")
```

This plot will give 0 when the system is unstable and 1 when the system is stable.



Q6)

6. Consider a disk-storage data-head positioning control system as shown in Fig. 1 where $k > 0$.

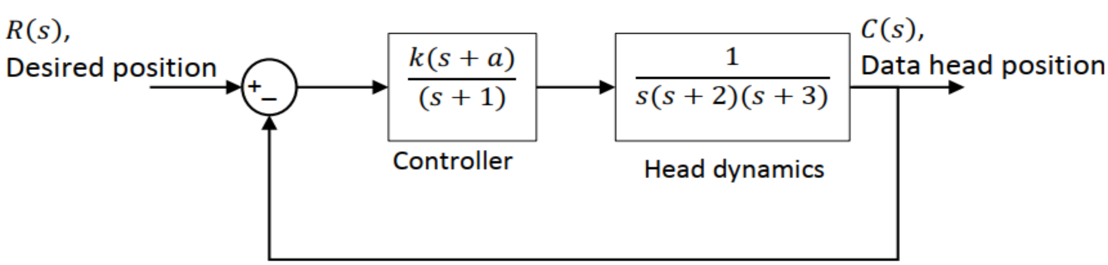


Fig. 1: Disk-storage data-head positioning feedback system

Determine the conditions on k and a for closed-loop stability. Also sketch the stability region. What is the impact reducing the gain a on the head position response for a fixed k?.

(6)

$$1 + \frac{K(s+a)}{(s+1)(s)(s+2)(s+3)} = 0$$

$$\boxed{s^4 + 6s^3 + 11s^2 + (6+K)s + Ka = 0}$$

As $K > 0$ $\therefore a \geq 0$

s^4	1	11	Ka
s^3	6	$(6+K)$	0
s^2	$\frac{6 \times 11 - (6+K)}{6}$	Ka	0
s	$\dfrac{\frac{(60-K)(6+K) - 6K}{6}}{\left(10 - \frac{K}{6}\right)}$	Ka	0
s^0	Ka	0	0

$$10 - K/6 \geq 0$$

$$K/6 \leq 10$$

$$\boxed{0 \leq K \leq 60} \quad \text{— (1)}$$

$$\boxed{0 \leq a \leq 10/K + 3/2 - K/36}$$

$K = 60$, $a = 0$

marginally stable

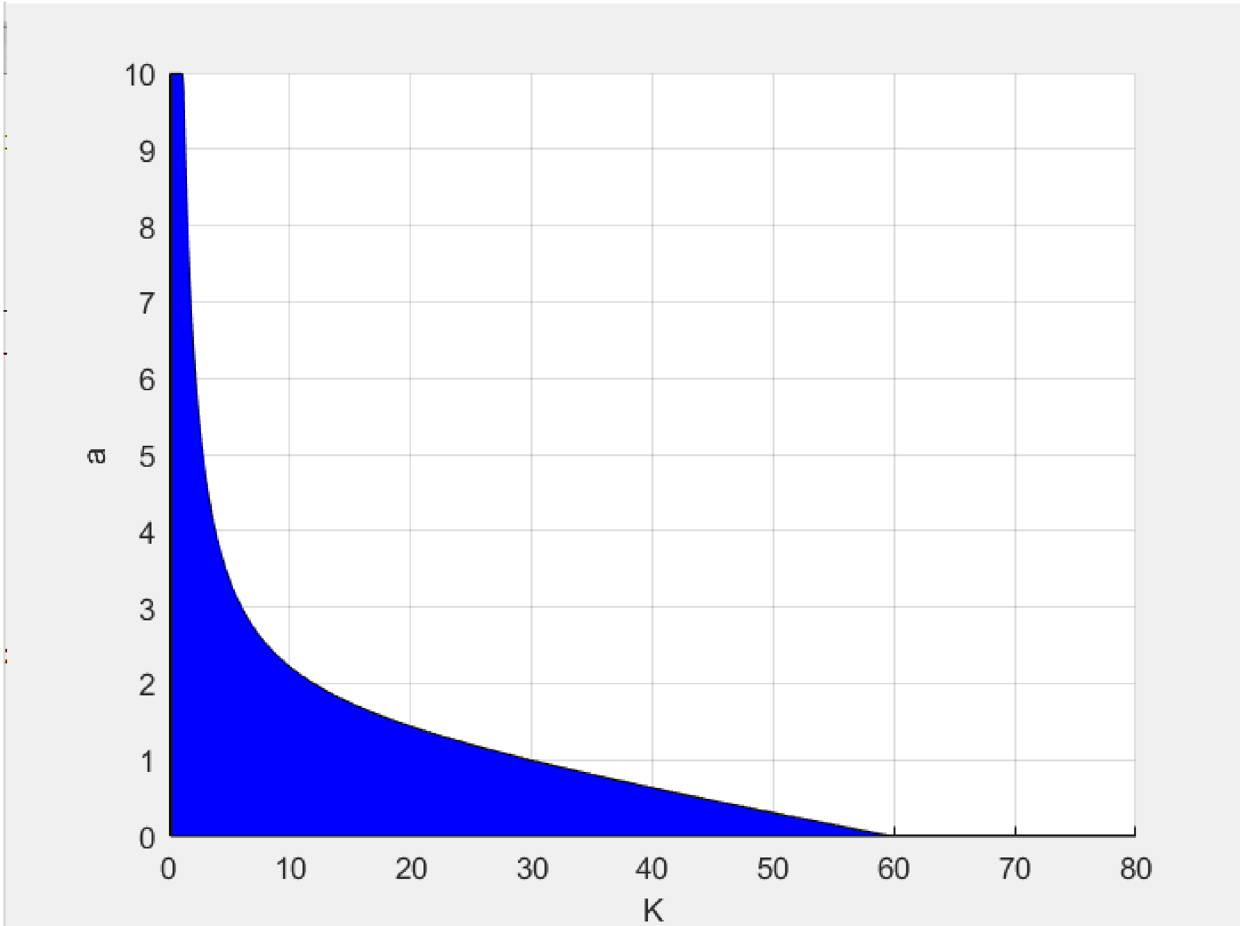
For 0<=k<=60

And 0<=a<=10/k+1.5-k/36

The system is stable.

Matlab code for stability region:

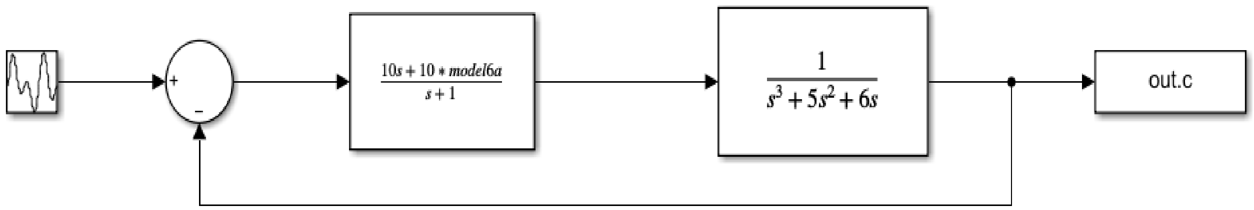
```
k=0.1:0.1:80; %change k from 0 to 10
a=0:0.01:10; %change a from 0 to 10
y=zeros(size(k));
for i=1:length(k)
   for j=1:length(a)-1
      c=rhc([1,6,11,6+k(i),k(i)*a(j)],0);
      b=rhc([1,6,11,6+k(i),k(i)*a(j+1)],0);
      if(c~=b)||(b==1 && j==length(a)-1)
         y(i)=a(j);
      end
   end
end
figure;
patch([k,fliplr(k)],[y,zeros(size(y))],'b')
xlabel("K");
ylabel("a");
grid on;
```



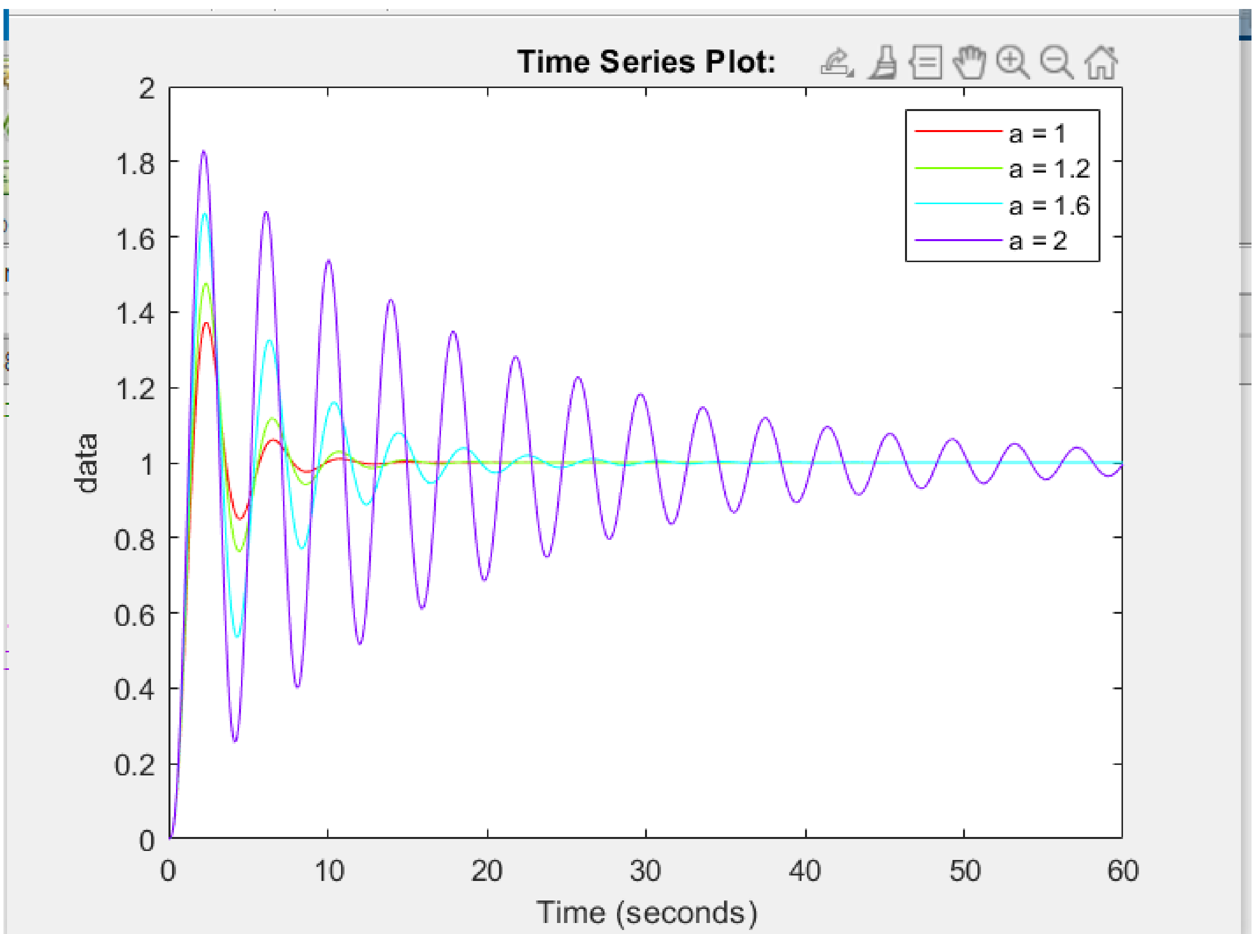
Matlab Code for changing a values with fixed k

```
a=[1,1.2,1.6,2]; %fixed k to 10 and changed a from 1 to 2
CM=hsv(4);
figure;
for i=1:4
   model6a=a(i);
   out=sim("model.slx");
   plot(out.c,"Color",CM(i,:),"DisplayName",strcat("a = ",num2str(model6a)));
   hold on
end
hold off
legend
```

Simulink Model (model.slx)



Plot:



From the above graph it can be seen that as a value increases, amplitude of oscillation also increases and the function saturates slower.

Q7)

7. The attitude control system of a space shuttle rocket is shown in Fig. 2.
   a. Determine the range of gain k and m so that the closed-loop system is stable and plot the region of stability.
   b. Select the values of k and m so that the steady-state error to a ramp input is less than or equal to 10% of the input magnitude.

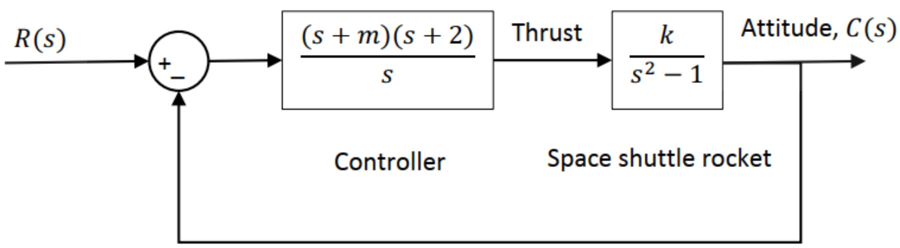
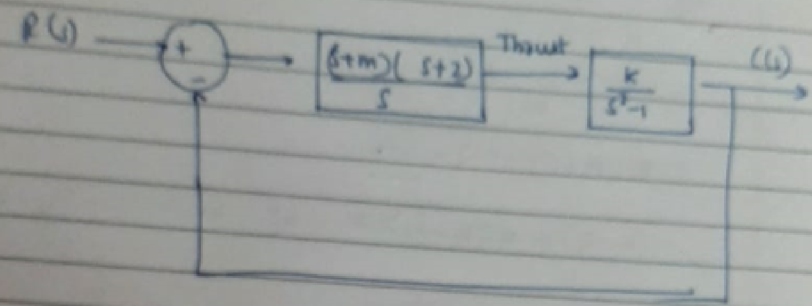


Fig. 2: Attitude control system of a space shuttle

a)

(7)



$\cancel{1 + \frac{K}{(s+1)(s-1)s}(s+m)(s+2)}$

$1 + \frac{K}{(s^2-1)}\frac{(s+m)(s+2)}{s} = 0$

$\boxed{s^3 + Ks^2 + (K(m+2)-1)s + 2Km = 0}$ ☆

$K > 0$ $\qquad K(m+2)-1 > 0$

$2Km > 0$

$m \geq 0$

$m > \frac{1}{K} - 2$

RH Table

s^3	1	$K(m+2)-1$
s^2	K	$2Km$
s^1	$\frac{K(K(m+2)-1)-2Km}{K}$	0
s^0	$2Km$	0

$K(m+2)-1-2m \geq 0$

$k > 0$

$m \geq 0$

$m > \frac{1}{k} - 2$

$k(m+1) - 1 - 2m \geq 0$

$(k-2)m + (2k-1) > 0 \quad \forall \quad m \geq 0 \quad$ if $\underline{\underline{k \geq 2}}$

$\cancel{(k-2)m + (2k-1)}$

$k \geq 2 \quad , m \geq 0$

and

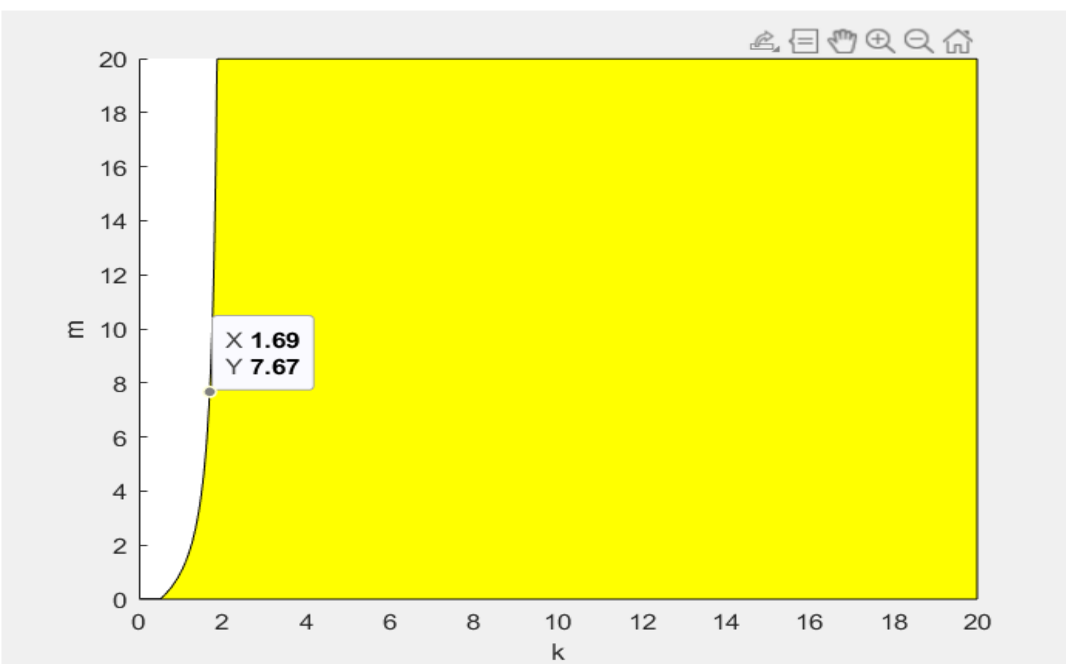
$\frac{1}{2} \leq k < 2 \qquad 0 \leq m \leq \frac{2k-1}{2-k}$

for $m = \frac{2k-1}{2-k} \quad k \in \left(\frac{1}{2}, 2\right)$

System is marginally stable.

Region of Stability

```
k=0.01:0.01:20;
m=0:0.01:20;
y=zeros(size(k));
for i=1:length(k)
   for j=1:length(m)-1
      a=rhc([1,k(i),k(i)*(m(j)+2)-1,2*k(i)*m(j)],0);
      b=rhc([1,k(i),k(i)*(m(j+1)+2)-1,2*k(i)*m(j+1)],0);
      if(a~=b)||(b==1 && j==length(m)-1)
         y(i)=m(j);
      end
   end
end
figure;
patch([k,fliplr(k)],[y,zeros(size(y))],'y')
xlabel("k");
ylabel("m");
```



b)

7 (b)

range of k and m for which steady state i
is less than or equal to 10% of i/p magnitud

$$G_1(s) = \frac{(s+m)(s+2)}{s}$$

$$G_2(s) = \frac{k}{s^2-1}$$

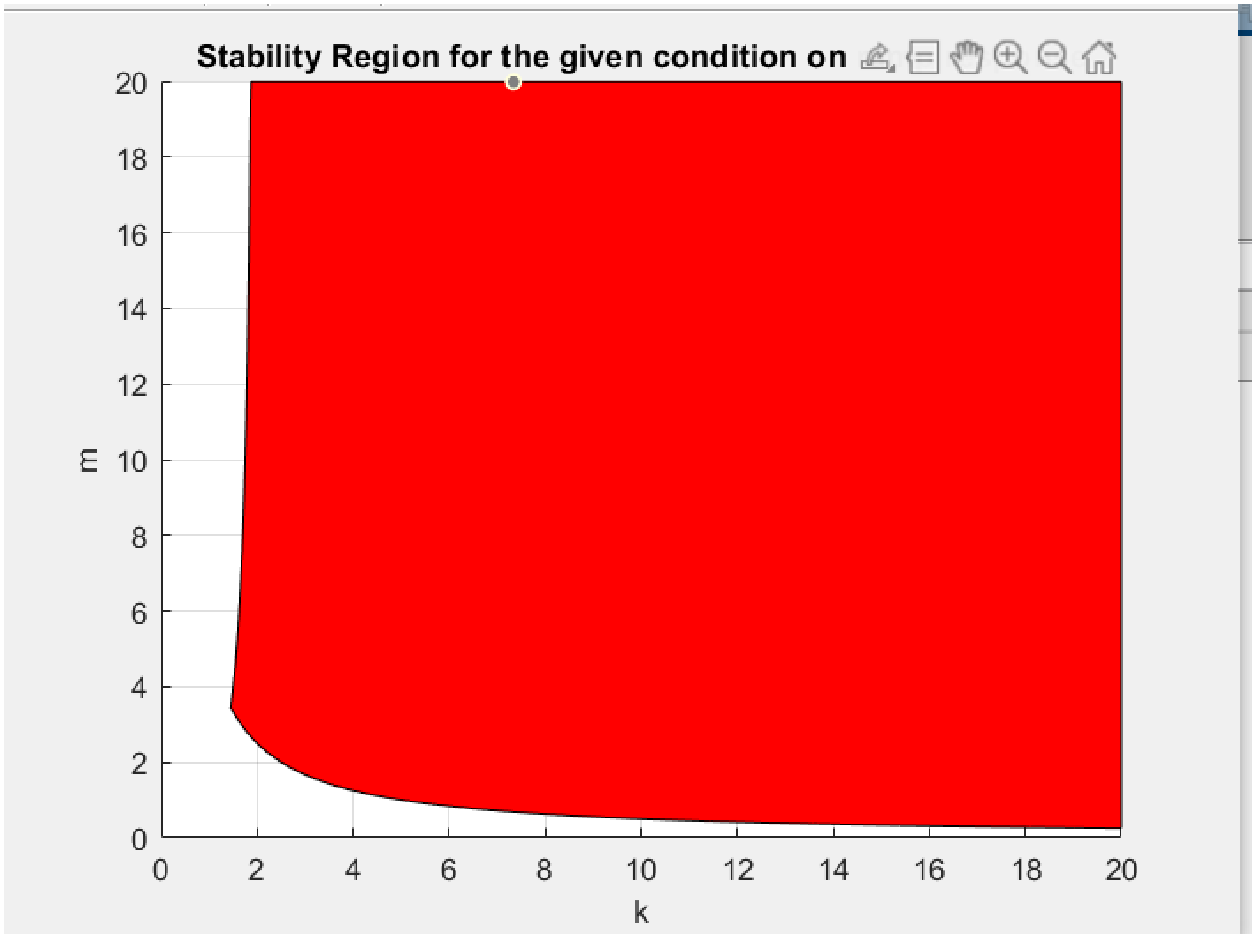
$$e_{ss} = \frac{1}{\lim_{s \to 0} s\, G_1(s)\, G_2(s)}$$

$$= \frac{-1}{2km}$$

$$\frac{-1}{2km} \geq -0.1 \qquad m \geq 5/k$$

Matlab Code:

```
y1=5./k;
Li=(y>=y1);
figure;
patch([k(Li),fliplr(k(Li))],[y(Li),fliplr(y1(Li))],'r');
xlabel("k");
ylabel("m");
grid on
title("Stability Region for the given condition on ramp response");
```


Stability Region for the given condition on
m
k
0
2
4
6
8
10
12
14
16
18
20

Q8)

8. The system shown in Fig. 3 has

$$G_1 = \frac{1}{s(s+2)(s+4)}.$$

Find the following:

a. The value of K_2 for which the inner-loop will have two equal negative real poles and the associated range of K_1 for system stability.

b. The value of K_1 at which the system oscillates and the associated frequency of oscillation.

c. The gain K_1 at which a real closed-loop pole is at $s = 5$. Can the step response, $c(t)$, be approximated by a second order, under-damped response in this case? Why or why not?

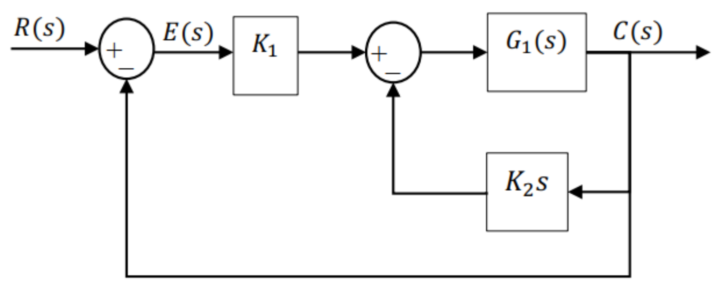


Fig. 3: A two-loop feedback system

a.b.)

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Q-8) $G_1(s) = \dfrac{1}{s(s+2)(s+4)}$

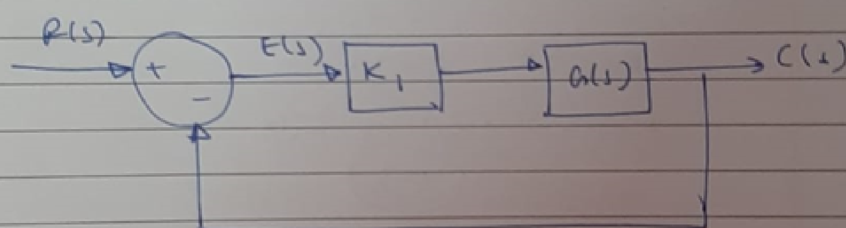
$1 + K_2 s G_1(s) = 0$

→ will have 2 equal -ve roots

$(s+2)(s+4) + K_2 = 0$

$$\boxed{\begin{array}{l} K_2 = 1 \\ s^2 + 6s + 9 = 0 \; \begin{array}{l} -3 \\ -3 \end{array} \Big\} \text{ 2 equal roots} \end{array}}$$

$G(s) = \dfrac{1}{s + s(s+2)(s+4)}$



$1 + K_1 G(s) = 0$

$1 + \dfrac{K_1}{s(s+2)(s+4)+s} = 0$

$s^3 + 6s^2 + 9s + K_1 = 0$ $\quad$ $K_1 > 0$

RH table

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s^3	1	9
s^2	6	K_1
s	$\frac{54-K_1}{6}$	0

$\boxed{0 < K_1 < 54 \text{ stability.}}$

$s^3 + 6s^2 + 9s + 54 = 0$

$(s+6)(s^2+9) = 0$

$K_1 = 54 \quad K_2 = 1$

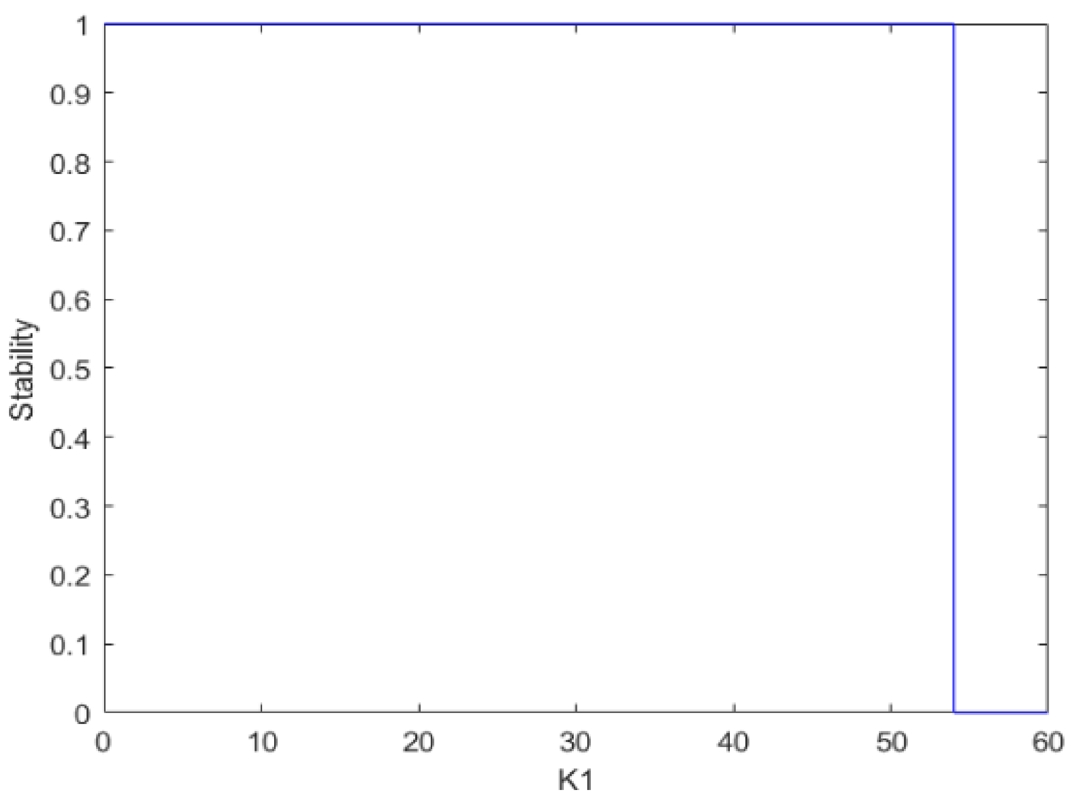
System oscillate with

$w = 3$

Hence, for given value of K2=1 we get the corresponding range of 0<K1<54
At K1=54 the system is marginally stable i.e. oscillates. Frequency of Oscillation will be 3rad/sec.

Verifying using matlab:

```
k1=1e-2:1e-2:60;
y=zeros(size(k));
for i =1:length(k1)
   y(i)=rhc([1,6,9,k1(i)],0);
end
figure;
plot(k1,y,"b-")
xlabel("K1");
ylabel("Stability");
```



System becomes unstable when K1>54

c)As a closed-loop pole at will make the system unstable, we won't be able to approximate the step response as underdamped.

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(c) $s^3 + 6s^2 + (8+K_2)s + K_1 = 0$

$s = 5$ is a pole

$K_1 = -5(63+K_2)$

This result does not have any significance with 2nd part so pole $s = -5$.

Then solving we get,

$K_1 = 5(K_2+3)$

Factorising characteristic eq^n we get,

$(s+5)(s^2+s+K_2+3)=0$

So while one pole is at $s=-5$, both complex poles have real value $= -1/2$, so real pole is 10 times far away from img axis w.r.t img poles.

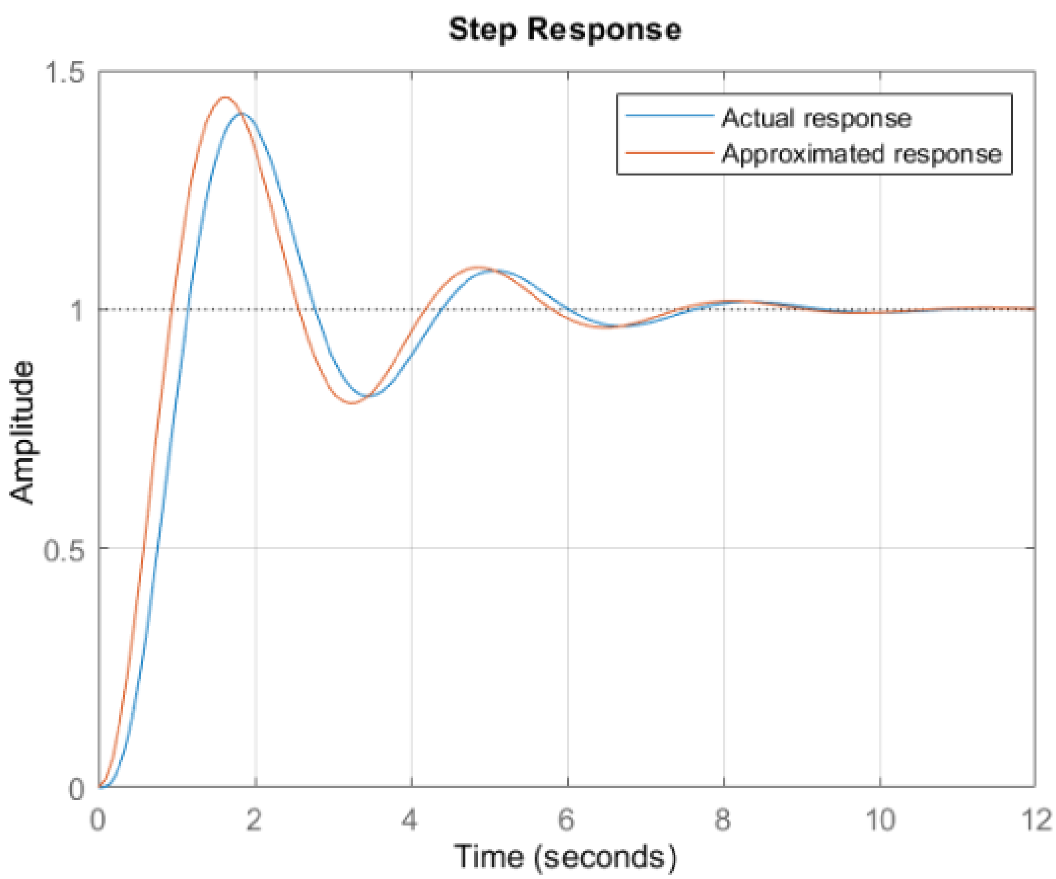
Dominant pole approximation.

$K_2 = 1 \quad K_1 = 20$

$$G(s) = \frac{20}{s^3+6s^2+9s+20}$$

$$G_{app}(s) = \frac{4}{s^2+s+4}$$

```
tf1=tf(20,[1,6,9,20]);
tf2=tf(4,[1,1,4]);
step(tf1,tf2);
legend("Actual response","Approximated response")
grid on
```



Q9)

9. Consider the feedback system shown in Fig. 4. With the help of R-H criterion, determine the range of gain K so that the steady-state error in presence of step reference input remains within 15% of the reference input magnitude.

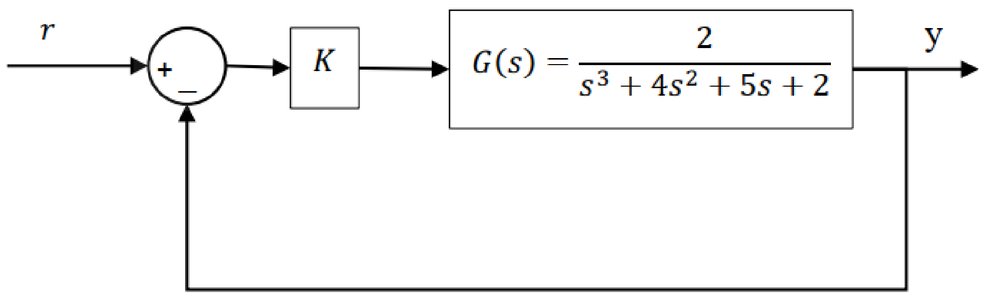
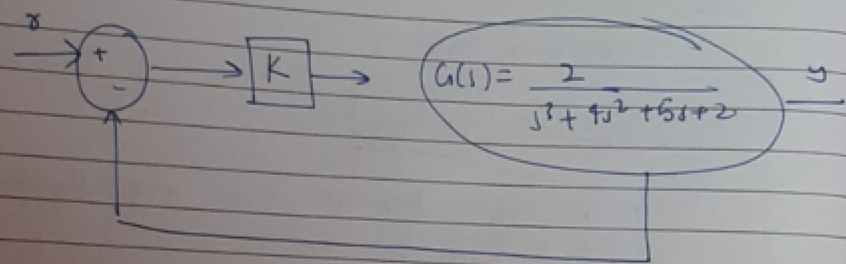


Fig. 4: A feedback system with proportional controller

Q-9)



$$y(s) = \left(\frac{KG(s)}{1+KG(s)}\right) R(s)$$

$$E(s) = \frac{R(s)}{1+KG(s)}$$

$R(s) = 1/s$

$$e_{ss} = \frac{1}{1 + K \lim_{s \to 0} G(s)}$$

$$e_{ss} = \frac{1}{1+K}$$

$G(s) = (-2, -1, -1)$ in LHP.

So, $\frac{1}{1+K} \leq 0.15$

$K \geq 17/3$

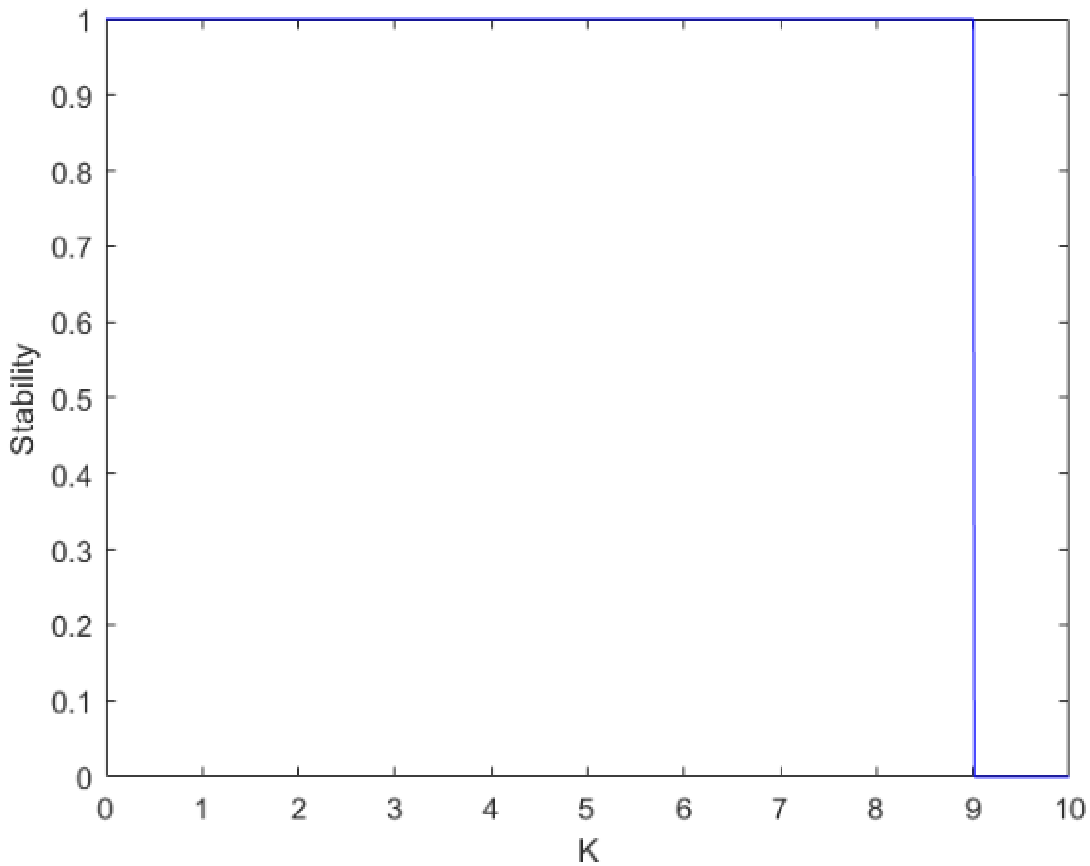
using rhs we can also find the range of K for stability of the system, DATE

Characteristic eq^n:-

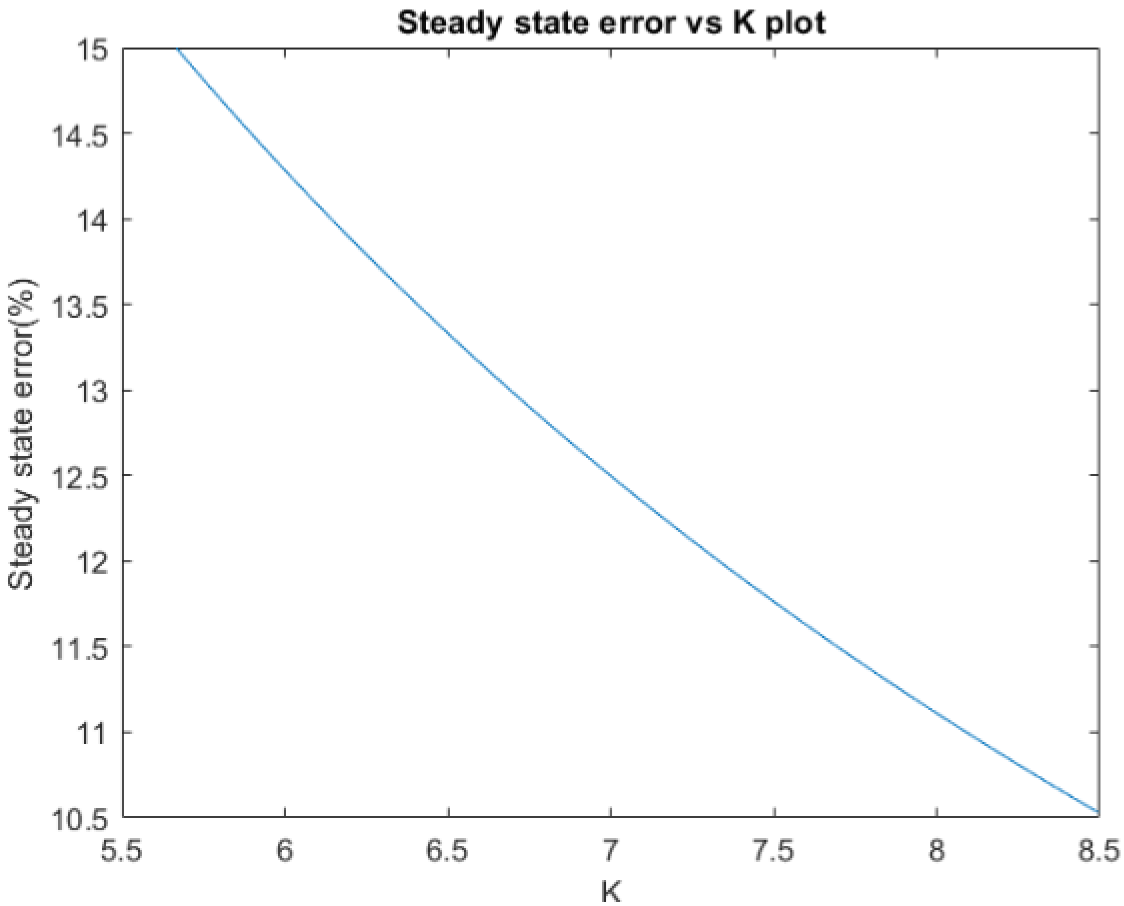
$1 + K G(s) = 0$

$s^3 + 4s^2 + 5s + 2 + 2K = 0$

```
k=0:0.01:10;
y=zeros(size(k));
for i =1:length(k)
   y(i)=rhc([1,4,5,2+2*k(i)],0);
end
figure;
plot(k,y,"b-")
xlabel("K");
ylabel("Stability");
```




```
K=17/3:1e-2:8.5;
error=zeros(size(K));
for i=1:length(K)
   [y,t]=step(tf(2*K(i),[1,4,5,2+2*K(i)]),500);
   error(i)=100*abs(1-y(end));
end
figure;
plot(K,error);
xlabel("K");
ylabel("Steady state error(%)");
title("Steady state error vs K plot");
```


Steady state error vs K plot
Steady state error(%)
15
14.5
14
13.5
13
12.5
12
11.5
11
10.5
5.5
6
6.5
7
7.5
8
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