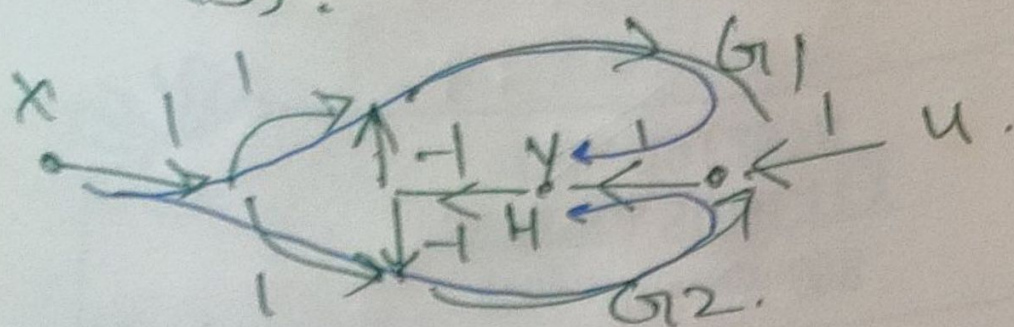


Q1. (b).



$$M_1 = G_1.$$

$$M_2 = G_2.$$

$$\Delta = 1 - (1 - HG_2 - HG_1).$$

$$\Delta_1 = 1 - 0$$

$$\Delta_2 = 1 - 0.$$

$$\frac{Y}{X} = \frac{M_1 \Delta_1 + M_2 \Delta_2}{\Delta}$$

$$= \frac{G_1 + G_2}{H(G_1 + G_2)} = \left(\frac{Y}{H} \right)$$

82.

(2)

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & -2 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.$$

$$\dot{x}_1 = -2x_2 \quad \Rightarrow \quad \cancel{x_1 + 2 =}$$

$$\dot{x}_2 = x_1 - 3x_2$$

$$\ddot{x}_2(0) = -2 - 3 = -5$$

$$\ddot{x}_2 = \dot{x}_1 - 3\dot{x}_2$$

$$\ddot{x}_2 = -2x_2 - 3\dot{x}_2$$

$$\ddot{x}_2 + 3\dot{x}_2 + 2x_2 = 0.$$

$$s^2 X_2(s) - s(0) - (-5) + 3sX_2(s) - 3 + 2X_2(s) = 0.$$

$$X_2(s) [s^2 + 3s + 2] = s - 2.$$

$$X_2(s) = \frac{(s-2)}{(s+1)(s+2)}.$$

$$= \frac{1}{s+2} - \frac{3}{(s+1)(s+2)} \quad \left\{ s+2 - s+1 \right\}$$

$$= \frac{1}{s+2} - 3 \left[\frac{1}{s+1} - \frac{1}{s+2} \right].$$

$$= \frac{4}{s+2} - \frac{3}{s+1}$$

(3)

$$X_2(s) = \frac{4}{s+2} - \frac{3}{s+1}$$

$$x_2(t) = 4e^{-2t} - 3e^{-t}$$

$$\begin{aligned} \dot{x}_1(t) &= -2x_2(t) \\ &= -8e^{-2t} + 6e^{-t} \end{aligned}$$

$$\cancel{x_1(t) = -4e^{-2t} + 6e^{-t}}$$

$$x_1(t) = 4e^{-2t} - 6e^{-t}$$

cross check.

$$x_1(0) = -2 = -2$$

$$x_2(0) = 1 = 1.$$

State Transition

(4)

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & -2 \\ 1 & -3 \end{bmatrix}}_A \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$(sI - A) = \begin{bmatrix} s & +2 \\ -1 & s+3 \end{bmatrix}$$

$$(sI - A)^{-1} = \frac{1}{s^2 + 3s + 2} \begin{bmatrix} s+3 & -2 \\ +1 & s \end{bmatrix}$$

$$(sI - A)^{-1} = \begin{bmatrix} \left(\frac{s+3}{(s+1)(s+2)} \right) & - \left(\frac{2}{(s+1)(s+2)} \right) \\ \left(\frac{+1}{(s+1)(s+2)} \right) & \left(\frac{s}{(s+1)(s+2)} \right) \end{bmatrix}$$

Where

$$\frac{s+3}{(s+1)(s+2)} = \frac{2(s+2) - (s+1)}{(s+1)(s+2)} = \frac{2}{s+1} - \frac{1}{s+2}$$

$$\frac{-2}{(s+1)(s+2)} = \frac{-2}{s+1} + \frac{2}{s+2}$$

$$\frac{+1}{(s+1)(s+2)} = \frac{-1}{s+2} + \frac{1}{s+1}$$

$$\frac{s}{(s+1)(s+2)} = \frac{2}{s+2} - \frac{1}{s+1}$$

(5)

$$(sI - A)^{-1} = \begin{bmatrix} \frac{2}{s+1} - \frac{1}{s+2} & -\frac{2}{s+1} + \frac{2}{s+2} \\ -\frac{1}{s+2} + \frac{1}{s+1} & \frac{2}{s+2} - \frac{1}{s+1} \end{bmatrix}$$

$$\mathcal{L}^{-1}[(sI - A)^{-1}] = e^{At}$$

$$= \begin{bmatrix} 2e^{-s} - e^{-2s} & -2e^{-s} + 2e^{-2s} \\ -e^{-2s} + e^{-s} & 2e^{-2s} - e^{-s} \end{bmatrix}$$

State transition matrix ϕ .

$$x(t) = e^{At} x(0) + \int_0^t e^{A(t-\tau)} B u(\tau) d\tau$$

$$\begin{matrix} x_1(t) \\ x_2(t) \end{matrix} = \begin{bmatrix} 2e^{-t} - e^{-2t} & -2e^{-t} + 2e^{-2t} \\ e^{-t} + e^{-2t} & 2e^{-2t} - e^{-t} \end{bmatrix} \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

$$x_1(t) = \frac{(-4 + 2)e^{-t}}{(1 + 2 + 2)e^{-2t}} = \frac{-6e^{-t}}{4e^{-2t} - 3e^{-t}}$$

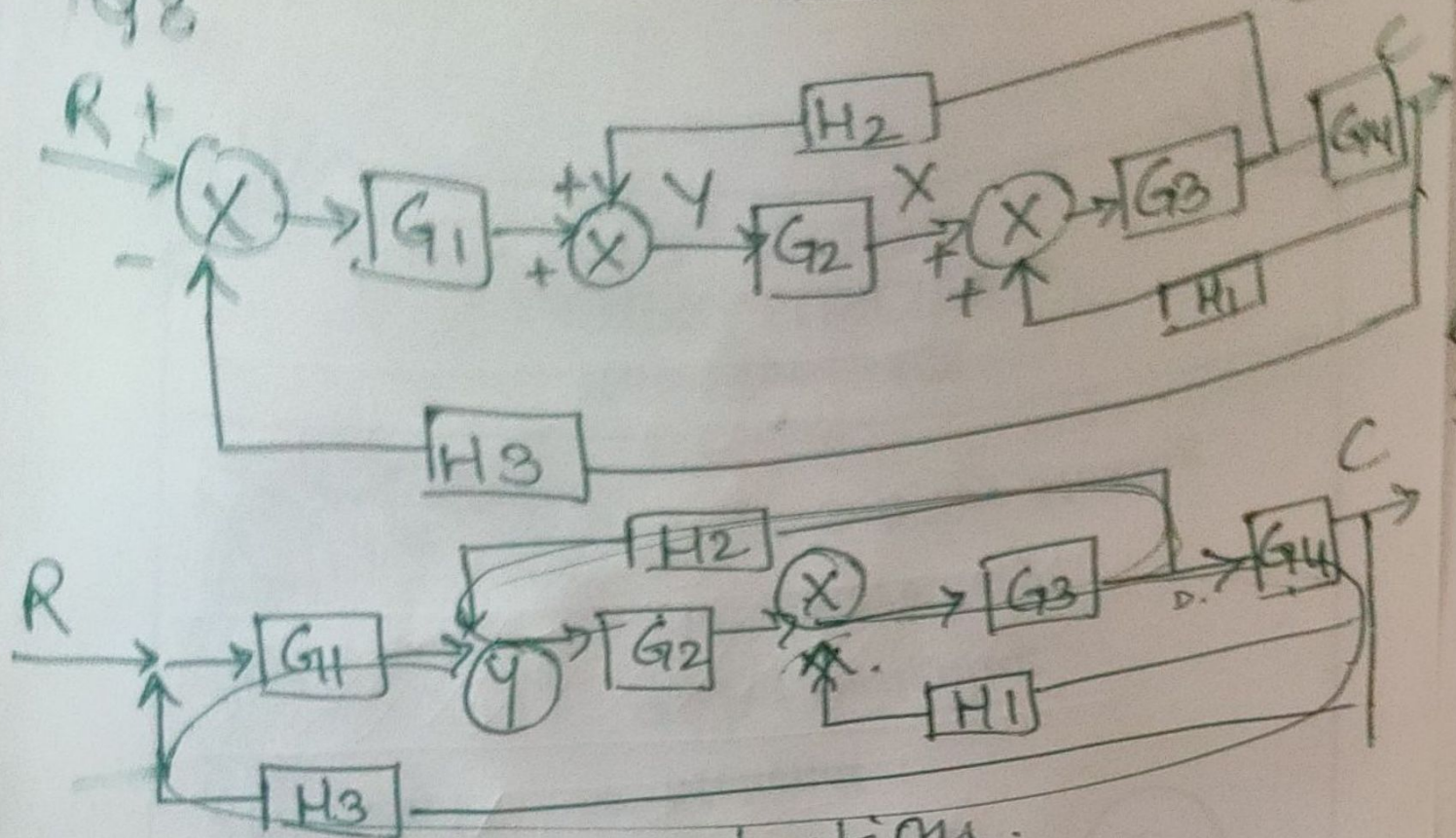
$$x_2(t) = \frac{(1 + 2 + 2)e^{-2t}}{(1 + 2 + 1)e^{-t}} = \frac{4e^{-2t} - 3e^{-t}}{4e^{-2t} - 6e^{-t}}$$

$$x(t) = \begin{bmatrix} 4 \\ 1 \end{bmatrix} e^{-t} + \begin{bmatrix} -2 \\ 1 \end{bmatrix} e^{-2t}$$

$$\begin{cases} x_1(t) = 4e^{-2t} - 6e^{-t} \\ x_2(t) = 4e^{-2t} - 3e^{-t} \end{cases}$$

Q2

6



Block diagram reduction.

$$R \rightarrow \frac{G_1 G_2 G_3 G_4}{1 - G_3 G_4 H_1 + G_2 G_3 H_2 + G_1 G_2 G_3 G_4 H_3} \downarrow C.$$

$$M_1 = G_3 H_2$$

$$M_2 = G_3 G_4 H_3 G_1$$

$$\Delta_1 = 1$$

$$\Delta_2 = 1$$

$$\Delta = 1 - (G_2 G_4 H_1 + G_3 H_2 G_2 + G_3 G_4 H_3 G_1 G_2)$$

$$T = \frac{M_1 \Delta_1 + M_2 \Delta_2}{\Delta}$$

(7)

$$\frac{Y}{X} = \frac{G_3 H_2 + G_3 H_3 G_1 G_4}{1 - (G_3 G_4 H_1 + G_3 H_2 G_2 + G_3 H_3 G_4 G_1 G_2)}$$

Ans.