

Assignments5

QS1

```
sys = zpk(-1, [0, -2, -3], 1)
```

```
sys =  
  
      (s+1)  
-----  
s (s+2) (s+3)
```

Continuous-time zero/pole/gain model.

```
pole(sys)
```

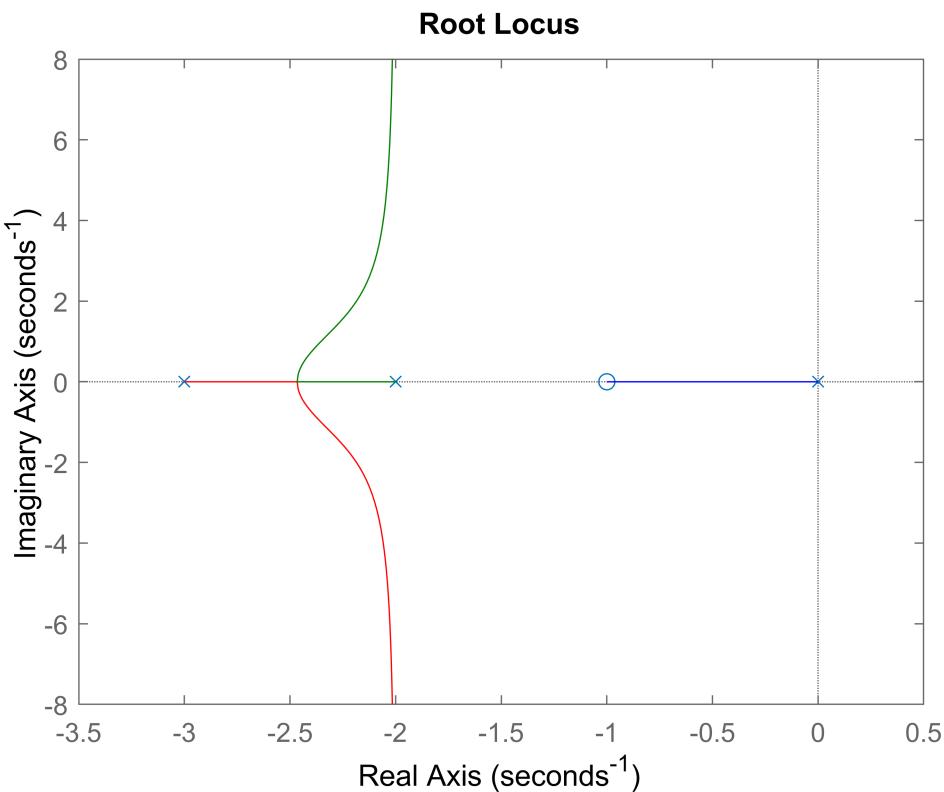
```
ans = 3x1  
    0  
   -2  
   -3
```

```
zero(sys)
```

```
ans = -1
```

```
figure;
```

```
rlocus(sys)
```



QS2

```
sys = zpk([], [0, -2, -1+1i, -1-1i], 1)
```

```
sys =
```

$$\frac{1}{s(s+2)(s^2 + 2s + 2)}$$

Continuous-time zero/pole/gain model.

```
pole(sys)
```

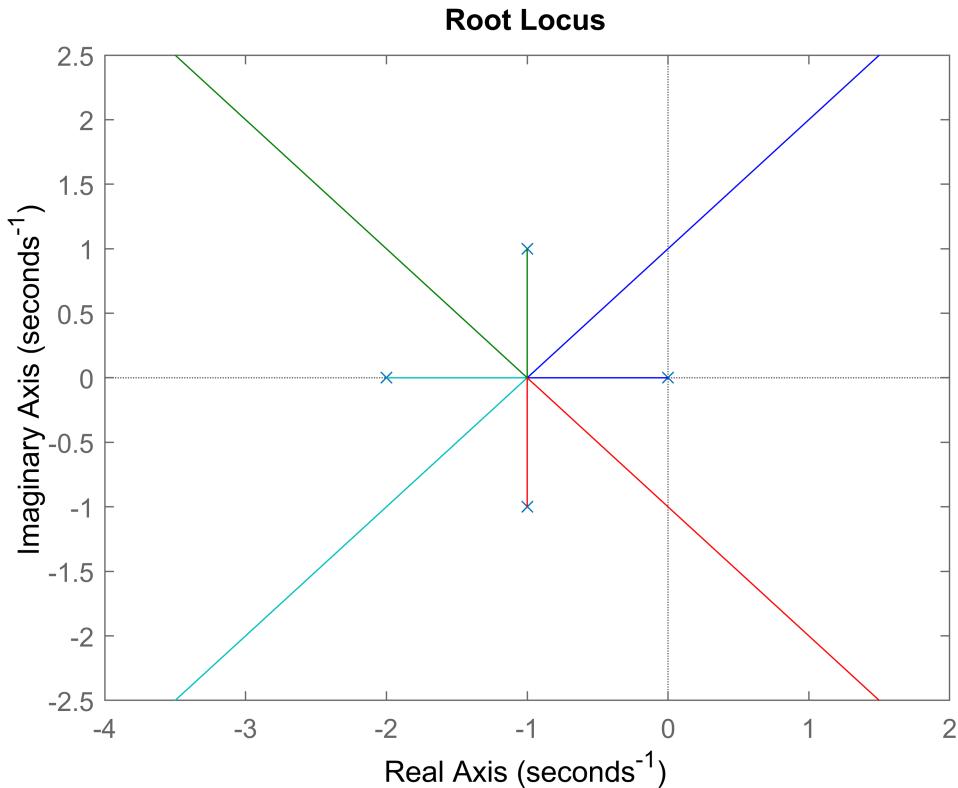
```
ans = 4x1 complex
0.0000 + 0.0000i
-2.0000 + 0.0000i
-1.0000 + 1.0000i
-1.0000 - 1.0000i
```

```
zero(sys)
```

```
ans =
```

0x1 empty double column vector

```
figure;
rlocus(sys)
```



QS3

```
sys = zpk(-1, [0, -4, -1+1i, -1-1i], 1)
```

```
sys =  
      (s+1)  
-----  
s (s+4) (s^2 + 2s + 2)
```

Continuous-time zero/pole/gain model.

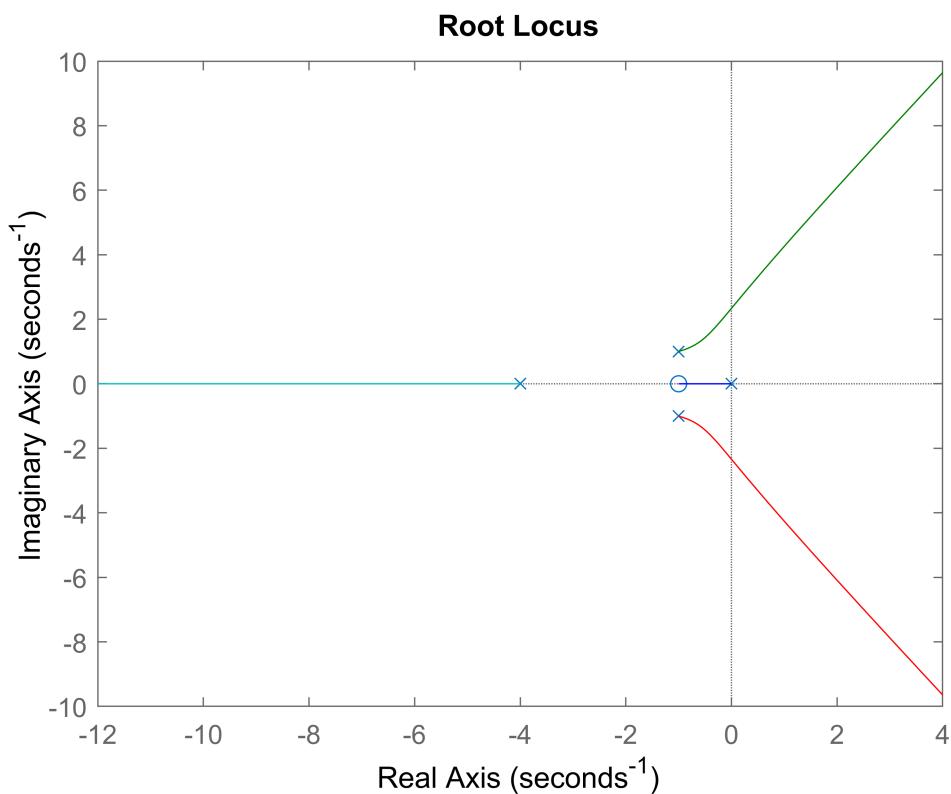
```
pole(sys)
```

```
ans = 4x1 complex  
0.0000 + 0.0000i  
-4.0000 + 0.0000i  
-1.0000 + 1.0000i  
-1.0000 - 1.0000i
```

```
zero(sys)
```

```
ans = -1
```

```
rlocus(sys)
```



QS4

```
sys = zpk([], [0, -3, -1+1i, -1-1i], 1)
```

```
sys =
```

```
1
```

```
s (s+3) (s^2 + 2s + 2)
```

Continuous-time zero/pole/gain model.

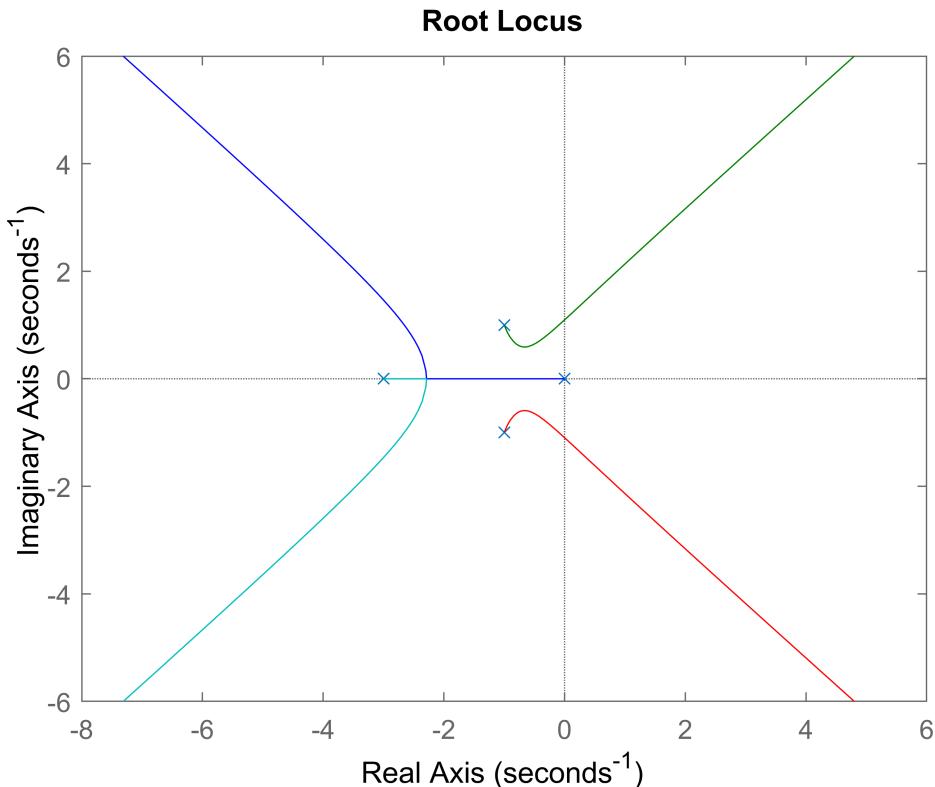
```
pole(sys)
```

```
ans = 4x1 complex  
0.0000 + 0.0000i  
-3.0000 + 0.0000i  
-1.0000 + 1.0000i  
-1.0000 - 1.0000i
```

```
zero(sys)
```

```
ans =  
0x1 empty double column vector
```

```
rlocus(sys)
```



QS5

```
sys = zpk([], [0, -4, -2+4i, -2-4i], 1)
```

```
sys =
```

```
1  
-----  
s (s+4) (s^2 + 4s + 20)
```

Continuous-time zero/pole/gain model.

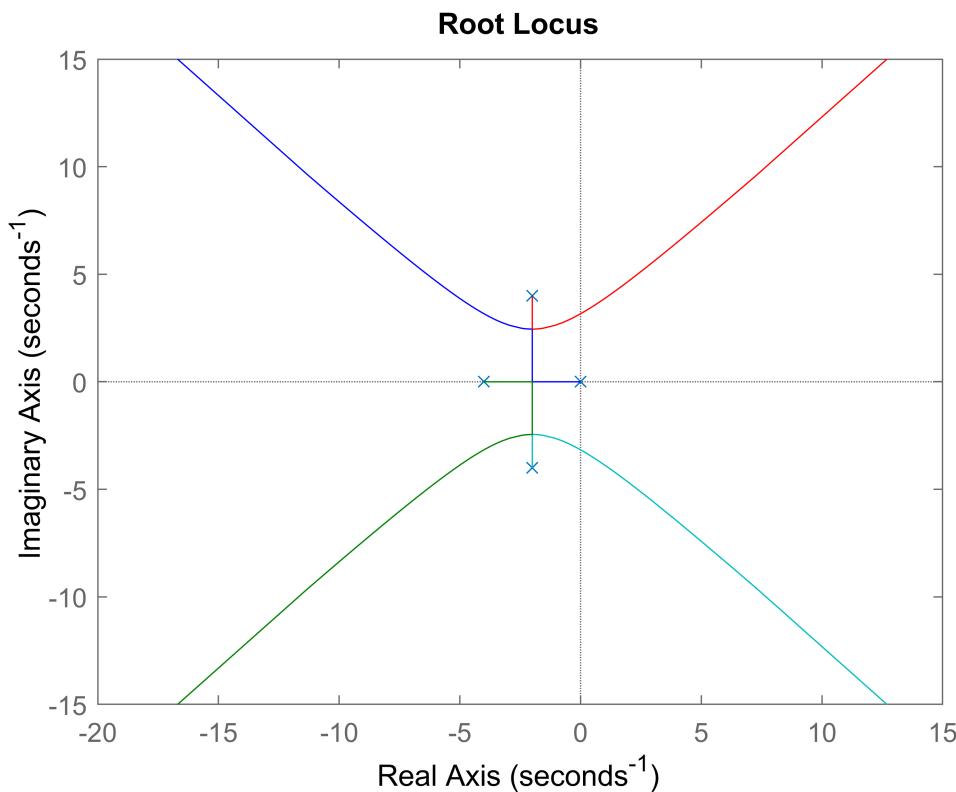
```
pole(sys)
```

```
ans = 4x1 complex
0.0000 + 0.0000i
-4.0000 + 0.0000i
-2.0000 + 4.0000i
-2.0000 - 4.0000i
```

```
zero(sys)
```

```
ans =
0x1 empty double column vector
```

```
rlocus(sys)
```



QS6

```
sys = zpk(-3, [0, -5, -6, -1+1i, -1-1i], 1)
```

```
sys =
```

$$\frac{(s+3)}{s(s+5)(s+6)(s^2 + 2s + 2)}$$

Continuous-time zero/pole/gain model.

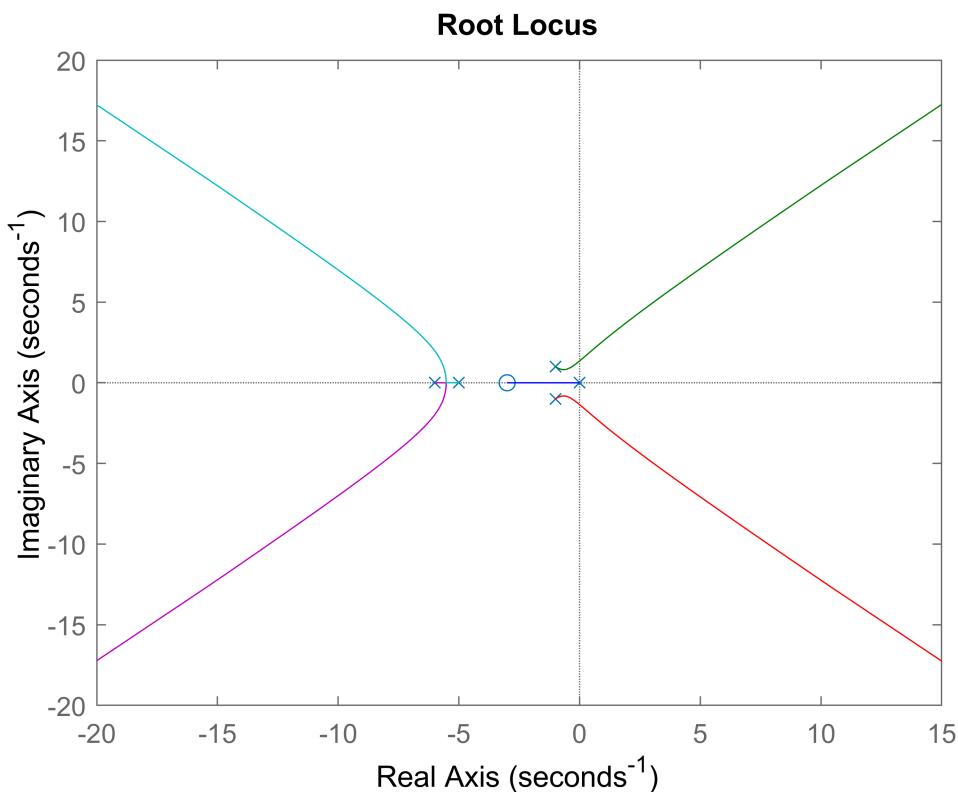
```
pole(sys)
```

```
ans = 5x1 complex
 0.0000 + 0.0000i
 -5.0000 + 0.0000i
 -6.0000 + 0.0000i
 -1.0000 + 1.0000i
 -1.0000 - 1.0000i
```

```
zero(sys)
```

```
ans = -3
```

```
rlocus(sys)
```



QS7

```
sys = zpk(-1, [0, 1, -2+sqrt(3)*2i, -2-sqrt(3)*2i], 1)
```

```
sys =
```

$$\frac{(s+1)}{s(s-1)(s^2 + 4s + 16)}$$

```
Continuous-time zero/pole/gain model.
```

```
pole(sys)
```

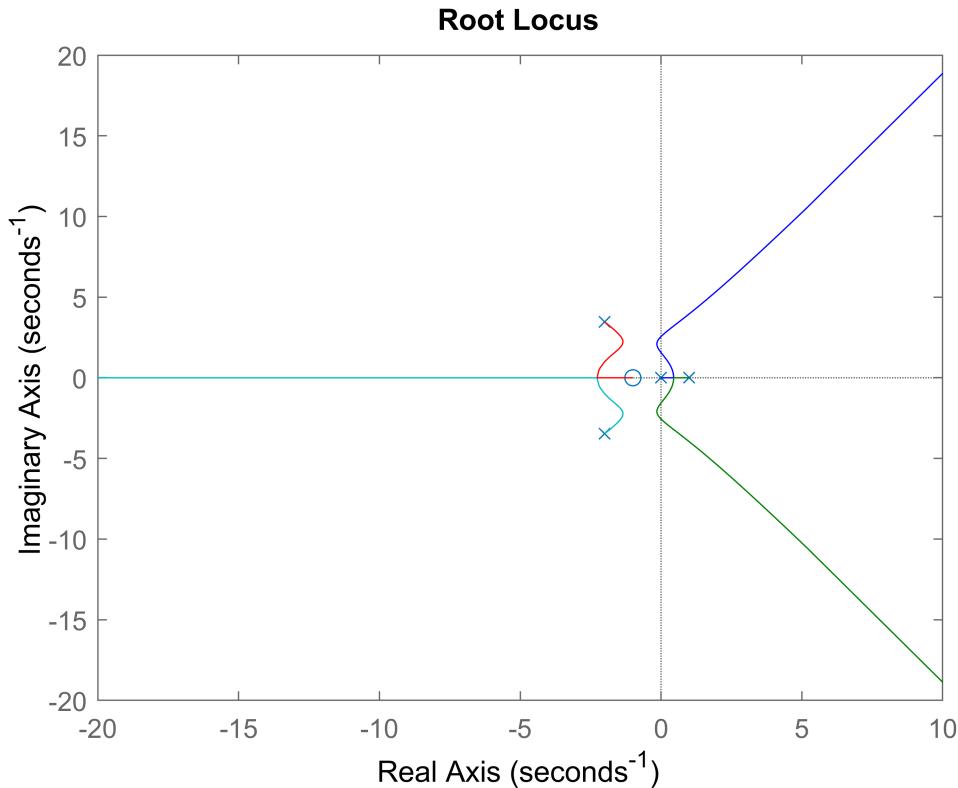
```
ans = 4x1 complex
 0.0000 + 0.0000i
```

```
1.0000 + 0.0000i
-2.0000 + 3.4641i
-2.0000 - 3.4641i
```

```
zero(sys)
```

```
ans = -1
```

```
rlocus(sys)
```



QS8

```
% 1
sys1 = tf(1, [1, 6, 45, 0])
```

```
sys1 =
```

$$\frac{1}{s^3 + 6s^2 + 45s}$$

```
Continuous-time transfer function.
```

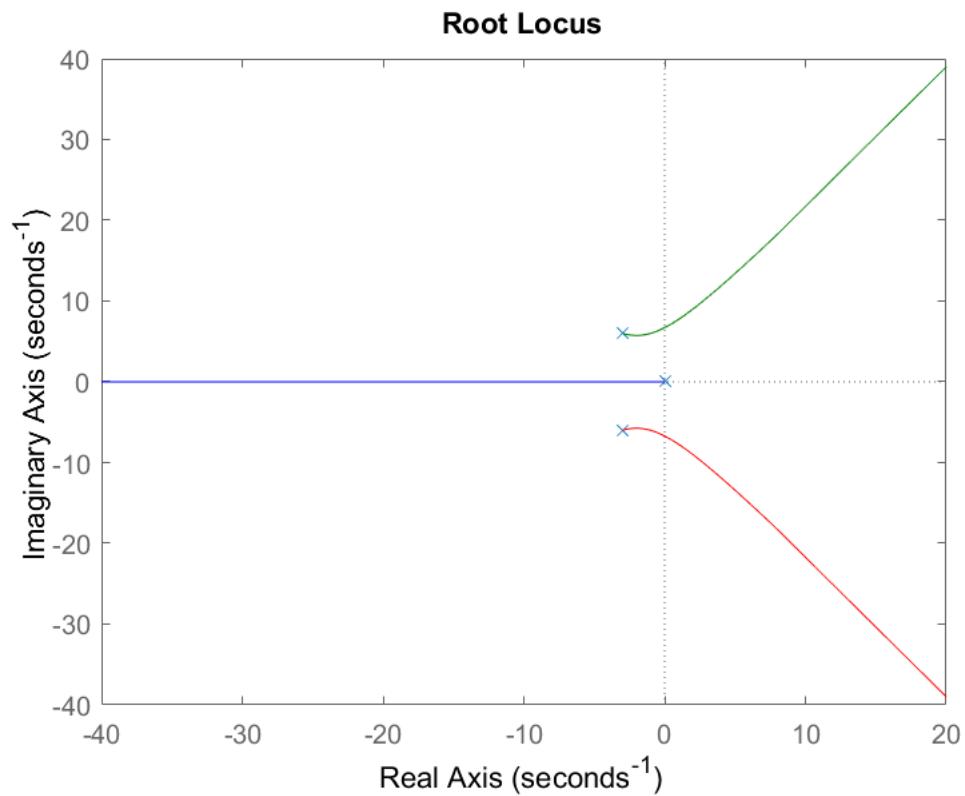
```
sys2 = tf([0.075, 1, 1], [1, 3, 5, 0])
```

```
sys2 =
```

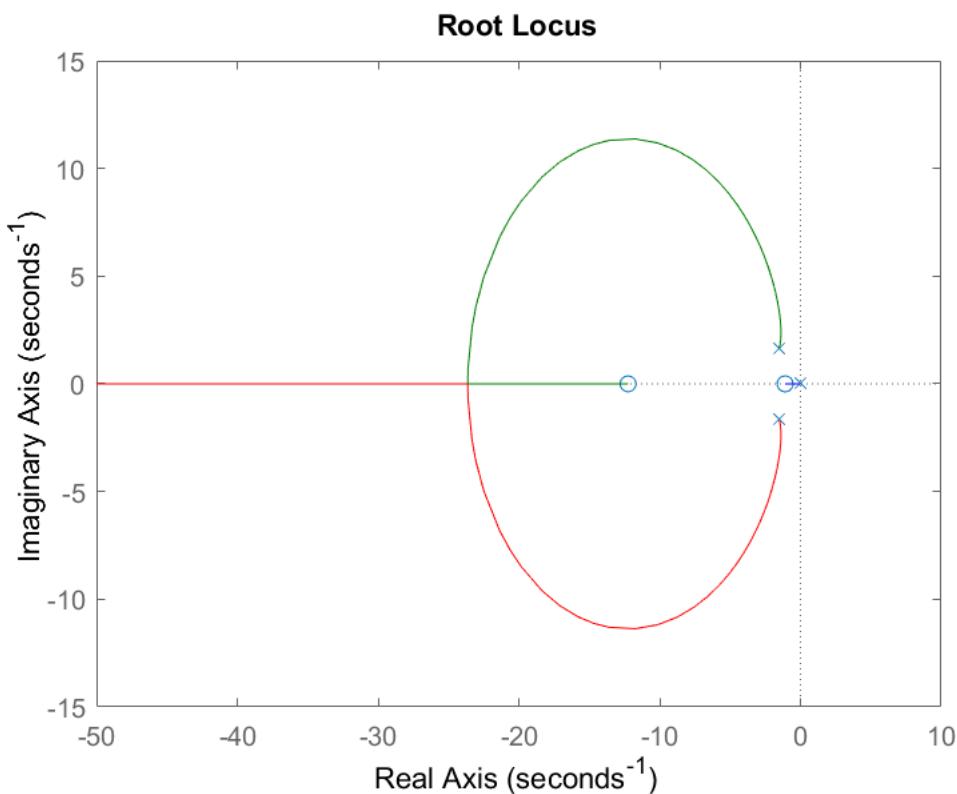
$$\frac{0.075s^2 + s + 1}{s^3 + 3s^2 + 5s}$$

Continuous-time transfer function.

```
rlocus(sys1)
```



```
rlocus(sys2)
```



```
% 2
```

```
% 3
```

```
tf1 = tf(1, [1, 6, 45, 0])
```

```
tf1 =
```

$$\frac{1}{s^3 + 6s^2 + 45s}$$

Continuous-time transfer function.

```
tf2 = tf([0.075, 1, 1], [1, 3, 5, 0])
```

```
tf2 =
```

$$\frac{0.075s^2 + s + 1}{s^3 + 3s^2 + 5s}$$

Continuous-time transfer function.

```
K = 40;
```

```
sys1 = (K* tf1)/(1 + K * tf1) %closed loop
```

```
sys1 =
```

```
40 s^3 + 240 s^2 + 1800 s
```

```
-----  
s^6 + 12 s^5 + 126 s^4 + 580 s^3 + 2265 s^2 + 1800 s
```

Continuous-time transfer function.

```
sys2 = (K* tf2)/(1 + K * tf2) %closed loop
```

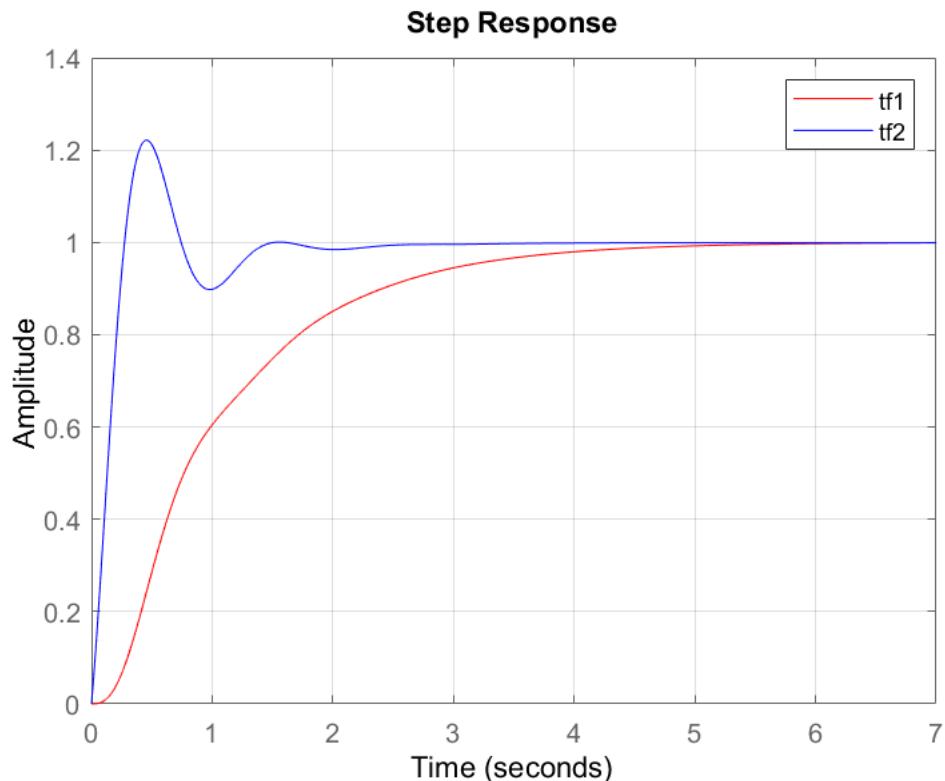
```
sys2 =
```

```
3 s^5 + 49 s^4 + 175 s^3 + 320 s^2 + 200 s
```

```
-----  
s^6 + 9 s^5 + 68 s^4 + 205 s^3 + 345 s^2 + 200 s
```

Continuous-time transfer function.

```
step(sys1, 'r', sys2, 'b')  
hold on;  
legend('tf1','tf2');  
grid on;
```



```
info1 = stepinfo(sys1)
```

```
info1 = struct with fields:  
    RiseTime: 2.1128  
SettlingTime: 4.0172  
SettlingMin: 0.9020  
SettlingMax: 0.9997  
    Overshoot: 0  
Undershoot: 0  
        Peak: 0.9997  
PeakTime: 8.0683
```

```
info2 = stepinfo(sys2)
```

```
info2 = struct with fields:  
    RiseTime: 0.2085  
    SettlingTime: 1.3415  
    SettlingMin: 0.8979  
    SettlingMax: 1.2217  
    Overshoot: 22.1650  
    Undershoot: 0  
    Peak: 1.2217  
    PeakTime: 0.4605
```

QS9

```
sys = tf([1, -2, 2],[1, 5, 6])
```

```
sys =  
  
s^2 - 2 s + 2  
-----  
s^2 + 5 s + 6
```

Continuous-time transfer function.

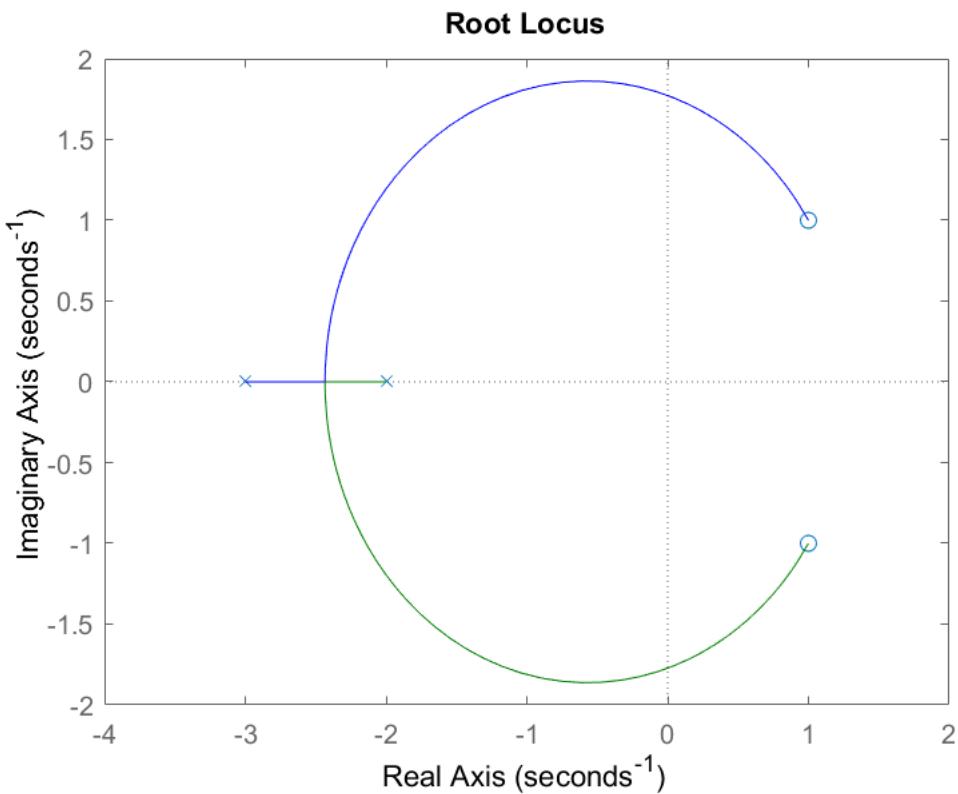
```
pole(sys)
```

```
ans = 2×1  
-3.0000  
-2.0000
```

```
zero(sys)
```

```
ans = 2×1 complex  
1.0000 + 1.0000i  
1.0000 - 1.0000i
```

```
figure  
rlocus(sys)
```



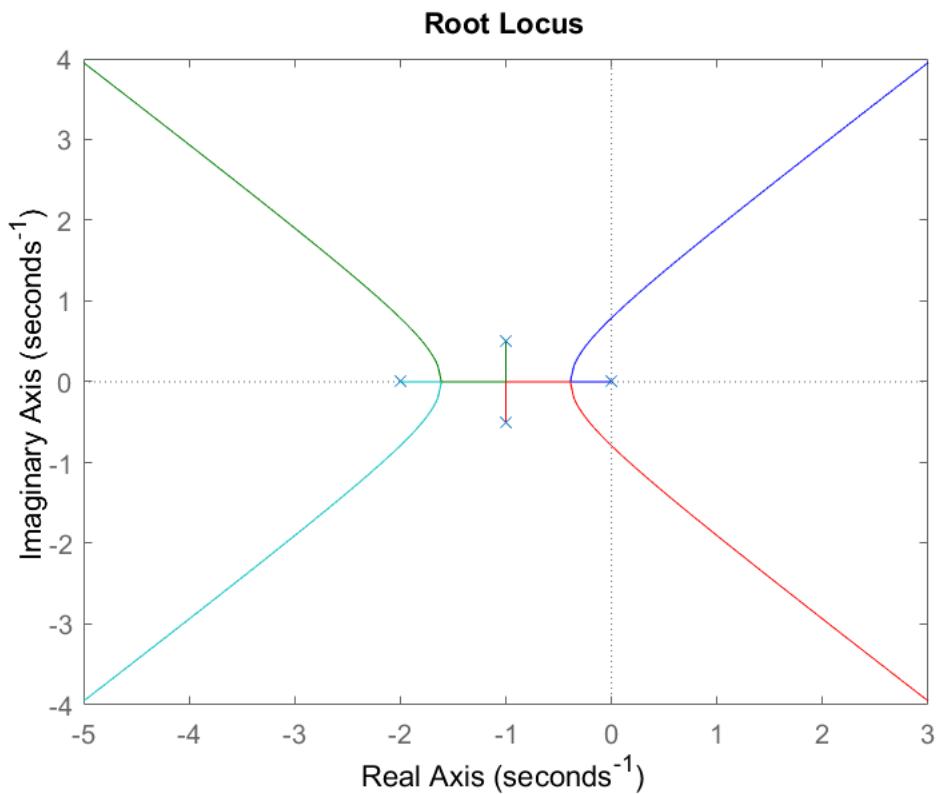
QS10

```
% a
sysa = zpk([], [0, -2, -1 + 0.5*1i, -1 - 0.5*1i], 1)
```

```
sysa =
1
-----
s (s+2) (s^2 + 2s + 1.25)
```

Continuous-time zero/pole/gain model.

```
figure
rlocus(sysa)
```



```
% b
sysb = zpk([], [0, -2, -1 + 1i, -1 - 1i], 1)
```

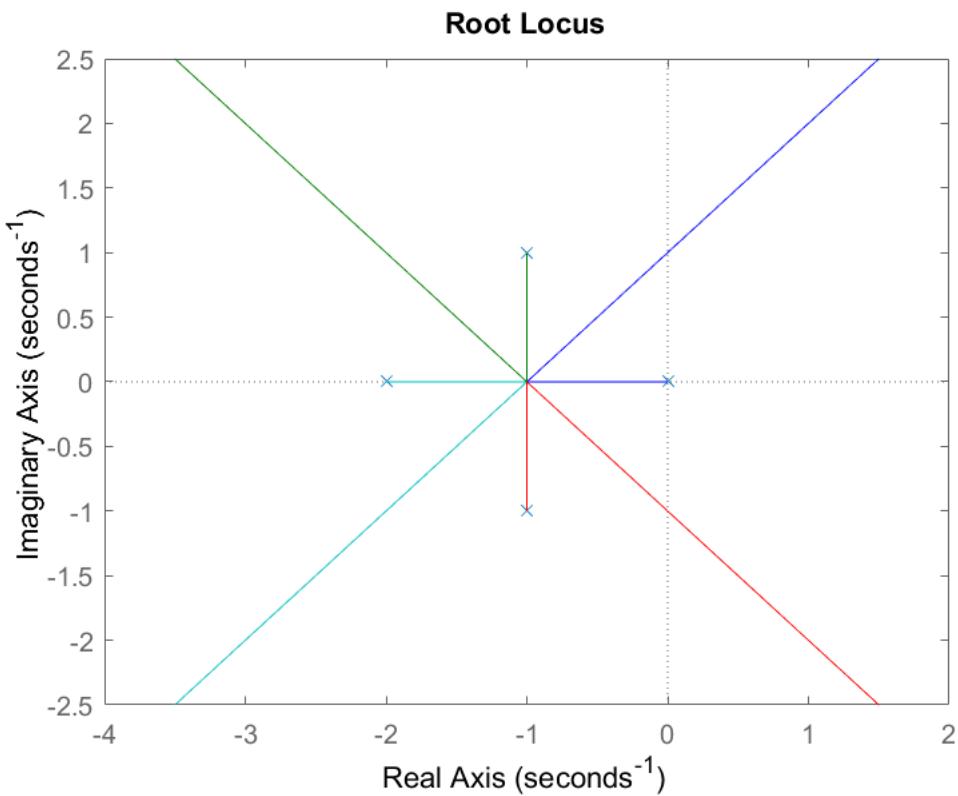
```
sysb =

```

$$\frac{1}{s(s+2)(s^2 + 2s + 2)}$$

Continuous-time zero/pole/gain model.

```
rlocus(sysb)
```

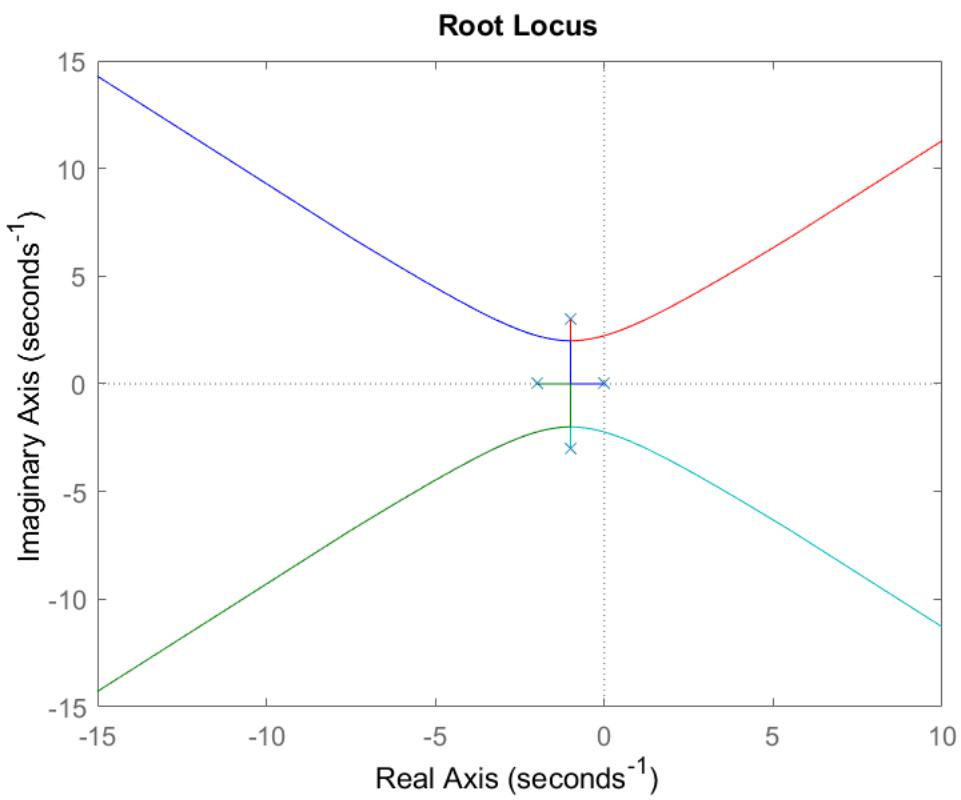


```
% c
sysc = zpk([], [0, -2, -1 + 3i, -1 - 3i], 1)
```

```
sysc =
1
-----
s (s+2) (s^2 + 2s + 10)
```

Continuous-time zero/pole/gain model.

```
rlocus(sysc)
```



Assignment 5

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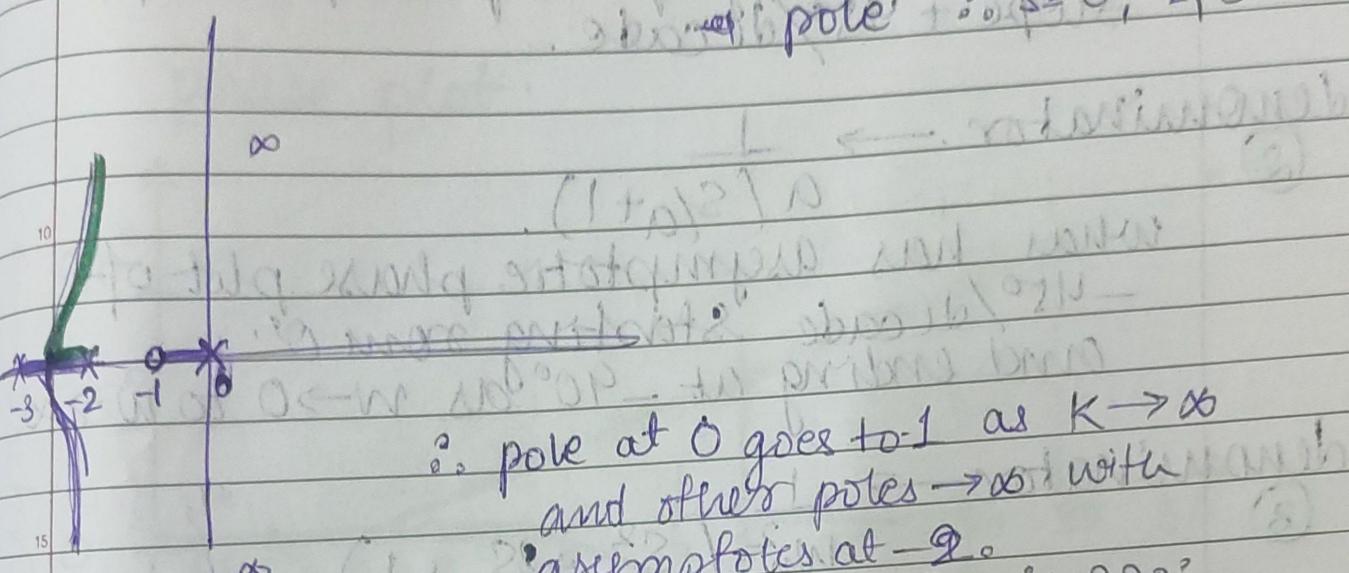
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Date / /

Q1. characteristic equation

$$S(S+2)(S+3) + K(S+1) = 0$$

$$\text{Transfer funcn. } K = -(S+1) \rightarrow 0 \quad \text{pole zero} \quad \therefore S = -1$$

to following state
as ~~zero~~
pole $\therefore S = 0, -2, -3$



\therefore pole at 0 goes to -1 as $K \rightarrow \infty$
and other poles $\rightarrow \infty$ with
asymptotes at -90° and 270°
with angles of 90° & 270°

Q2. characteristic equation

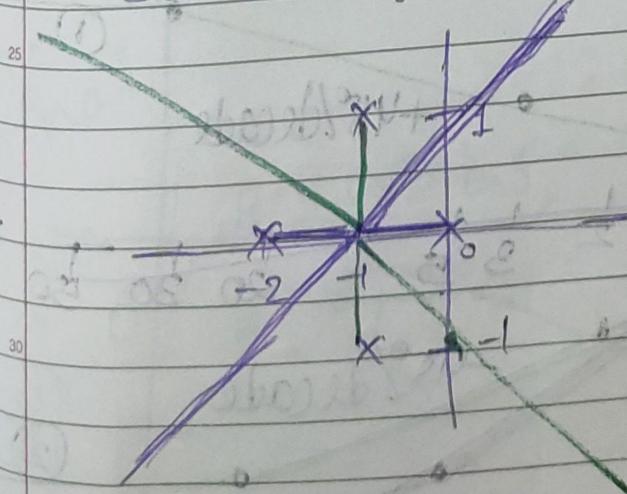
$$S(S+2)(S+1+j)(S+1-j) + K = 0$$

Transfer funcn.

$$S(S+2)(S+1+j)(S+1-j)$$

pole $\rightarrow 0, -2, -1-j, -1+j$

zero X



\therefore can be seen from
the plot there is
symmetry about -1
point.

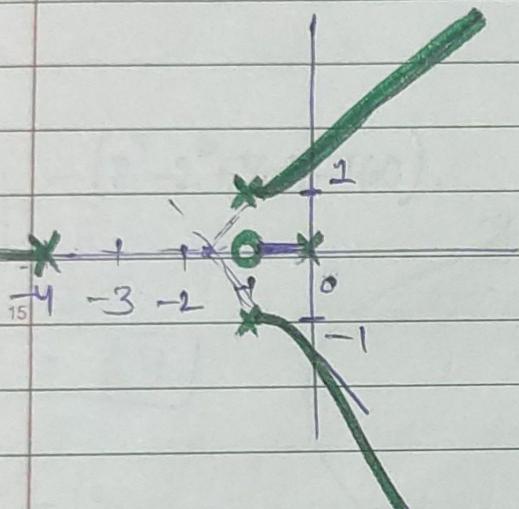
Q3. characteristic eqn⁴

$$s(s+4)(s^2 + 2s + 2) + K(s+1) = 0,$$

transfer funcⁿ $\rightarrow \frac{s+1}{s(s+4)(s^2 + 2s + 2)}$

5 centroid = $\frac{0 - 4 - 1 - 1 + 1}{4 - 1} = \frac{-5}{3}$ \rightarrow intersect of asymptotes

asymptote angles, $\theta = \frac{(2g+1)}{4-1} 180^\circ = 60^\circ, 180^\circ, 300^\circ$



8 asymptotes
intersect at centroid.

Q4. characteristic eqn⁴

$$s(s+3)(s^2 + 2s + 2) + K = 0$$

transfer funcⁿ

$$= \frac{8}{s(s+3)(s^2 + 2s + 2)}$$

zero \rightarrow
pole $0, -3, -1+j, -1-j$.

25 If odd number of open loop poles and zeros exists to the left side of point on real axis then point is root locus branch.

So on real axis root loci will be present betⁿ

30 2 poles $0, -3$.

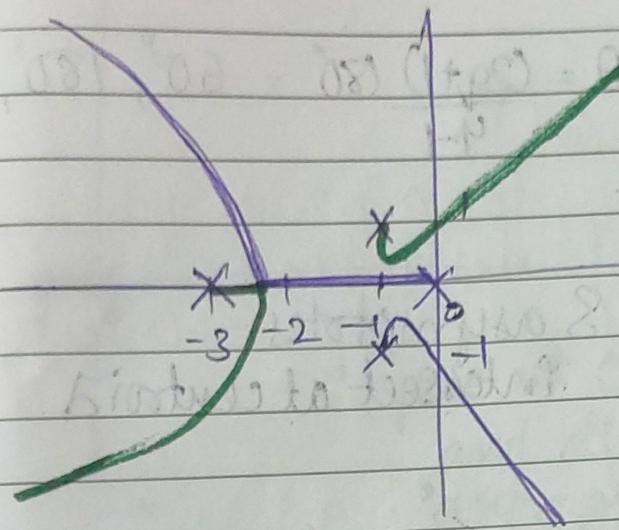
for departure angle of pole $-1+j$. (S1)

pole at 0 makes 135° with s_1 .

pole at -3 makes $\tan^{-1}(1/2)$ with s_1

pole at $-1-j$ makes 90° with s_1 .

$$\text{Q. angle of departure} = 180^\circ - 135^\circ - 90^\circ - \tan^{-1}(1/2) = -71.56^\circ$$



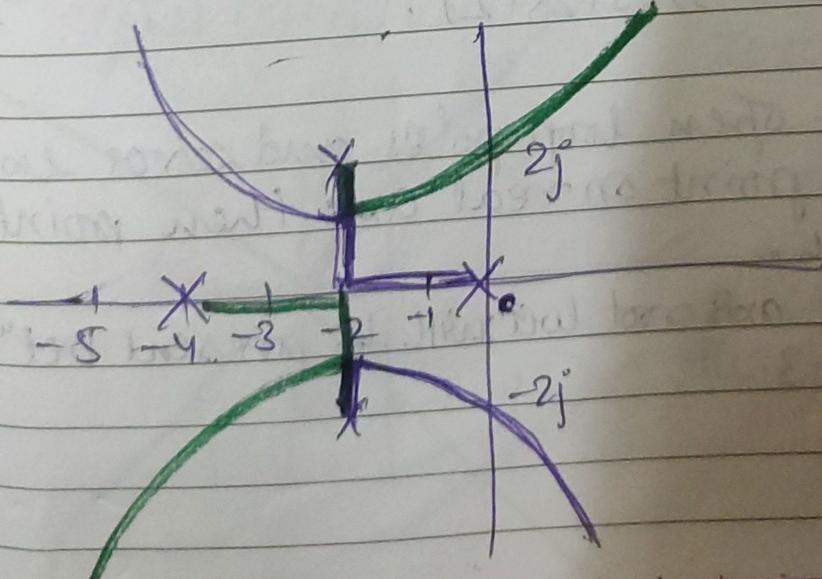
Q5. characteristic eqn⁴

$$S(S+4)(S^2+4S+20)+K=0$$

transfer func^a

$$\frac{1}{S(S+4)(S^2+4S+20)}$$

\rightarrow pole
 $0, -4,$
 $-2-2j,$
 $-2+2j.$



intersection with
imaginary axis
using RH table.

$$K=2 \text{ for now is } 0.$$

∴ at, $K=260$, we have 2 poles
on imaginary axis $(0, \pm j)$.

for $K=260$. intersection with
real & imaginary axis at $\rightarrow \pm j10j$

$$K = -(2^7 - 2^6 + 36 \times 4 - 160).$$

$$K = -(S^4 + 8S^3 + 36S^2 + 80S)$$

$$S_{25} = 2$$

$$\boxed{K = 64}$$

$$\text{at } K = 0, \text{ i.e. } 260$$

$$4S^3 + 24S^2 + 60S + 80 = 0$$

$$2S^3 + 12S^2 + 36S + 20 = 0$$

$$S^3 + 6S^2 + 18S + 20 = 0.$$

∴ -2 is

breakaway
point on real
axis.

Breakaway
point

$$2S = -2$$

Q6. Characteristic eqn.

$$s(s+5)(s+6)(s^2+2s+2) + k(s+3) = 0$$

Transfer func. = $s+3$
 $s(s+5)(s+6)(s^2+2s+2)$.

pole

$$0, -5, -6, -1-j, -1+j$$

Zeros

$$-3$$

5 root locus
& 4 of them will go
to ∞ as k increases

If odd number of
open loop poles and
zeros exists to the left
side of a point on
real axis, then the point is
on root locus branch.

So on real axis, root loci will be
present betw $0, -3$ and $-5, -6$.

Centroid $\alpha = 2.5$

angle of asymptotes = $45^\circ, 135^\circ, 225^\circ, 315^\circ$.

Breakaway point betw -5 and -6 .

Angle of departure for $-1+j$ is

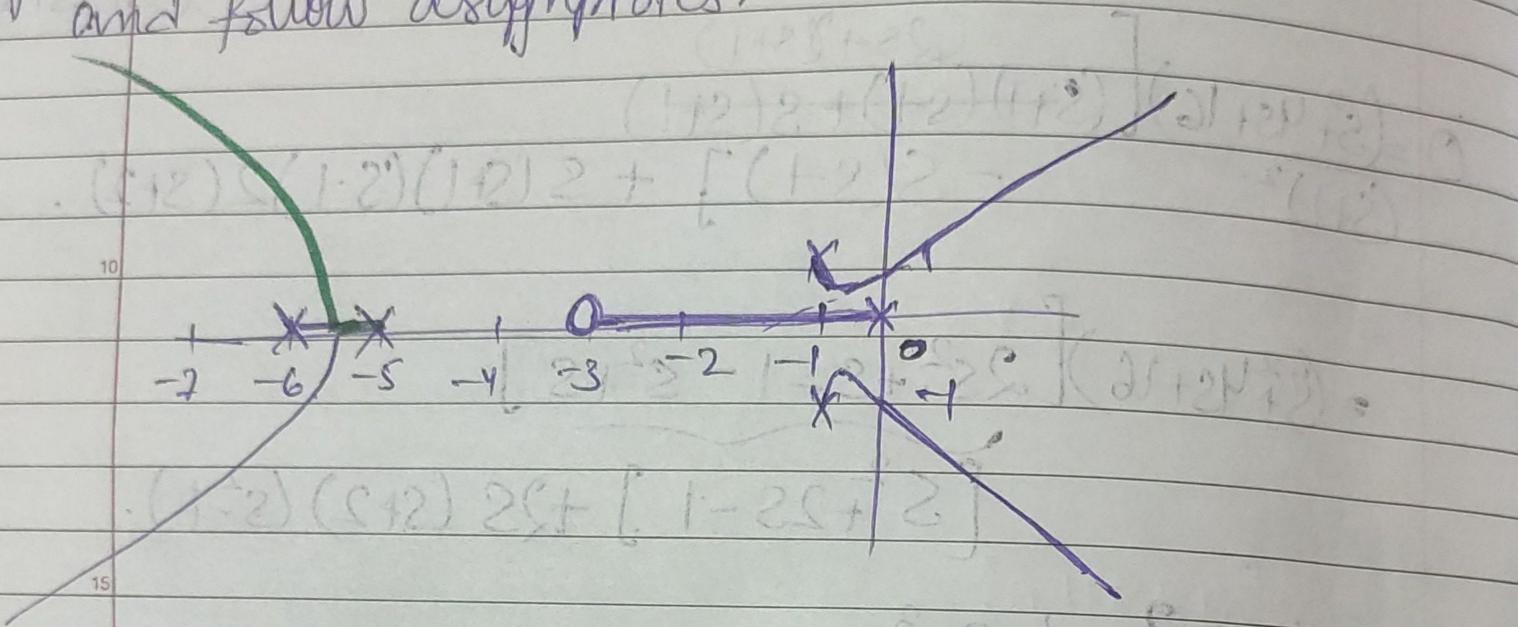
$$\begin{aligned} & 180^\circ - (180 - \tan^{-1} 1) \\ & - 90^\circ - \tan^{-1} y_4 \\ & - \tan^{-1} y_3 \\ & + \tan^{-1} y_2 \\ & = -43.78^\circ \end{aligned}$$

So root locus of pole
at 0 will end at -3

pole at $-1+j$ will start with -43.78°
and follow 45° asymptote from centroid.

pole at $-1-j$ will reflect $-1+j$ locus wst real axis,

poles at $-5, -6$ will breakaway
and follow asymptotes.



$$(Q7) G(s)H(s) = \frac{K(s+a)}{s(s-b)(s^2 + 2\zeta\omega_n s + \omega_n^2)}$$

$$= \frac{K(s+1)}{(s+1)(s^2 + 4s + 16)}$$

Zeros

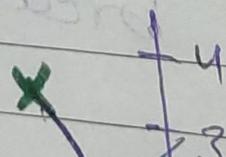
-1.

gain for stability?

poles

$$0, 1, -2 + 2\sqrt{3}j, -2 - 2\sqrt{3}j$$

0.47 ?
-2.263 breakaway points.



$$K = \frac{S(S+1)(S^2 + 4S + 16)}{(S+1)}$$

$$\frac{dK}{ds} = \frac{(S+1)}{(S+1)^2} \left[(S+1)(S^2 + 4S + 16) + S(S^2 + 4S + 16) \right] - \frac{S}{(S+1)} (S+1)(2S+4)$$

$$0 = \frac{(S^2 + 4S + 16)}{(S+1)^2} \left[(S+1)(S+1) + S(S+1) - S(S+1) \right] + S(S+1)(S-1)2(S+2).$$

$$= (S^2 + 4S + 16) \left[\underbrace{2S^2 + S - 1 - S^2 + S}_{[S^2 + 2S - 1]} + 2S(S+2)(S^2 - 1) \right].$$

$$= S^4 + 8S^3 - S^2 + 2S^3 + 4S^3 - 4S + 16S^2 + 32S - 16 + 2S(S^3 + 2S^2 - S - 2).$$

$$= S^4 + 6S^3 + 2S^2 + 28S - 16 + 2S^4 + 4S^3 - 2S^2 - S$$

$$= 3S^4 + 10S^3 + 21S^2 + 24S - 16.$$

$$\hookrightarrow S = -2.263, 0.448$$

breakaway points

continued.

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characteristic eqn

$$1 + G(s)(H(s)) = 0.$$

$$s(s+1)(s^2 + 4s + 16) + K(s+1) = 0.$$

open loop poles $\rightarrow 0, -1, -2 \pm 2\sqrt{3}j$

Zeros $\rightarrow 1$

$$\text{centroid} = \frac{0+1-2-2\sqrt{3}j}{3} = -2 + 2\sqrt{3}j + 1 \\ = -2/3.$$

angle of asymptotes $\frac{(m+1)\pi}{3} = 60^\circ, 180^\circ, 300^\circ$

Breakaway points

$$s = 0.448, -2.26.$$

Routh table.

long axis crosses over

s^4	1	12	K	$s^4 + 3s^3 + 12s^2 + s(K-16)$
				$+K=0$

s^3	3	$K-16$	$(2+28F^2)20N$
-------	---	--------	----------------

s^2	$\frac{s^2-K}{3}$	K	$\rightarrow A(s)$
-------	-------------------	-----	--------------------

s	$\frac{-K^2+59K-832}{s^2-K}$	\rightarrow should be 0.
-----	------------------------------	----------------------------

K

$$K = \frac{59}{2} \pm \frac{3\sqrt{17}}{2}$$

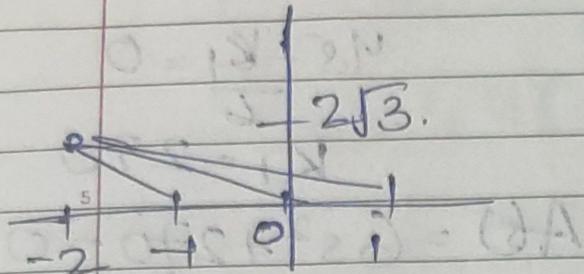
$$A(s) = \underline{s^2 - K} s^2 + K = 0$$

$$K = 35.68, \text{ or } 23.32.$$

$$s = \sqrt{\frac{3K}{K-52}}$$

$$s = \pm 2.86j, \text{ or } \pm 1.02j$$

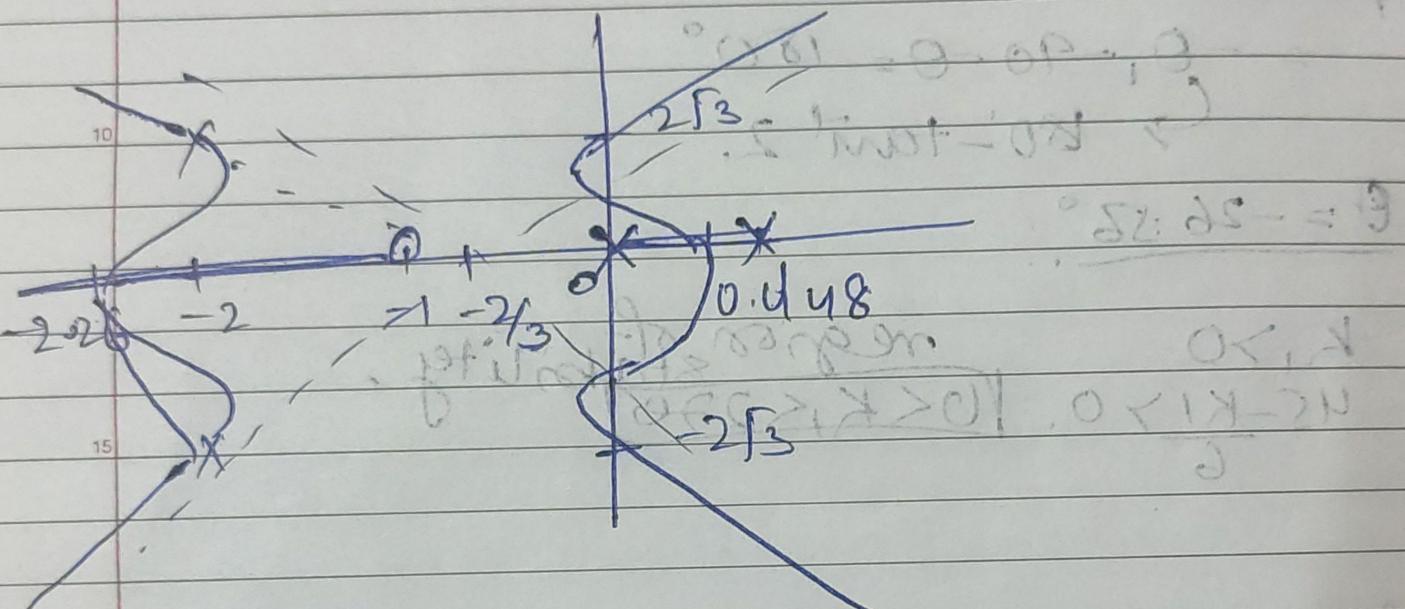
angle of departure.



$$180 - \tan^{-1}(2\sqrt{3}) = 180 + \tan^{-1}\sqrt{3}$$

$$-180 + \tan^{-1}(2\sqrt{3}) = 90 - 0^\circ$$

$$\theta_M = -54.8^\circ$$



$$k - 16 > 0$$

$$k > 0$$

$$\frac{52 - k}{3} > 0$$

$$16 < k < 52$$

$$k^2 - 59k + 832 > 0$$

$$(2)(1) k - 52$$

$$k^2 - 59k + 832 < 0$$

range for k for stability

$$23.32 < k < 35.68$$

$$k > 23.32 \quad \text{or} \quad k < 35.68$$

Q8.

$$G_1(s) = \frac{1}{s^3 + 6s^2 + 45s} = \frac{1}{s(s^2 + 6s + 45)}$$

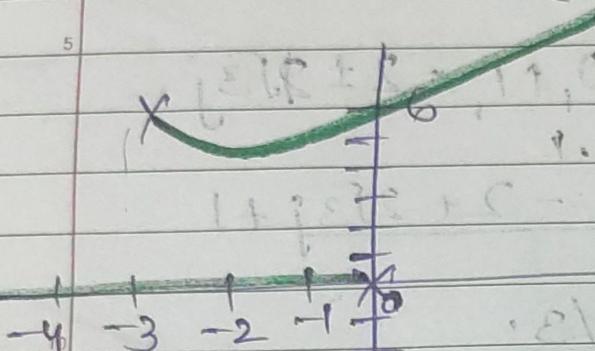
$$= \frac{1}{s((s+3)^2 + 6^2)}$$

pole

$$0, -3 - 6j, -3 + 6j.$$

$$\text{centroid} = -2$$

$$\text{angle of asymptotes} = 60^\circ, 180^\circ, 300^\circ$$



Breaking pt.

$$K_1 = -(s^3 + 6s^2 + 45s)$$

$$\frac{dK_1}{ds} = 0$$

no breaking pt.

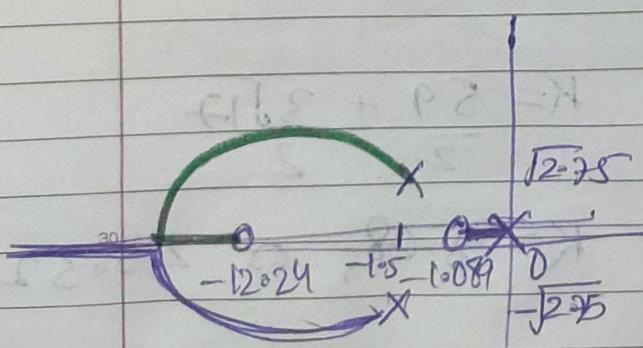
$$G_2(s) = \frac{0.025s^2 + 0.5s + 1}{s^3 + 3s^2 + 5s} = \frac{0.025(s^2 + 5s + 1)}{s(s^2 + 3s + 5)}$$

$$= \frac{3s^2 + 40s + 40}{40s(s^2 + 3s + 5)}$$

Poles

$$0, -1.5 + \sqrt{2.75}j, -1.5 - \sqrt{2.75}j$$

$$\left\{ \begin{array}{l} s = -1.089 \\ -12.244 \end{array} \right.$$



$$\text{centroid} = 10.3$$

$$\text{angle of asympt} = (2n+1)\pi$$

$$= 180^\circ, 360^\circ$$

Breaking away pt

$$K_2 = \frac{(s^3 + 3s^2 + 5s)}{(0.025s^2 + 5s + 1)}$$

$$\frac{dK_2}{ds} = 0 \Rightarrow 0.46 \pm 0.06j, \frac{23.63}{-2.12}$$

continued.

Q8 Routh table 1.

s^3	1	45
s^2	6	$K_1 \rightarrow A(s)$
s	$45 - K_1$	$\rightarrow 0$
1	K_1	

$$45 - K_1 = 0$$

$$K_1 = 270$$

$$A(s) = 6s^2 + 270 = 0$$

$$s = \pm 6.91 j$$

angle of departure

$$= -0, -90 - 0 = 180^\circ$$

$\hookrightarrow 180^\circ - \tan^{-1} 2$.

$$\theta = -26.56^\circ$$

$$K_1 > 0$$

$$\frac{45 - K_1}{6} > 0. \quad [0 < K_1 < 270]$$

region of stability.

Routh Table 2.

s^3	1	$5 + K_2$
s^2	$3 + 0.075K_2$	$K_2 \rightarrow A(s)$
s	$0.075K_2^2 + 2.375K_2 + 15$	$\rightarrow 0$
1	$3 + 0.075K_2$	

$$K_2 < 85 < 2$$

$$A(s) = 3s^2 + 0.075K_2 s^2 + K_2 = 0$$

NO img. axis cross over

$$3 + 0.075K_2 > 0$$

$$3 + K_2 > 0$$

$$K_2 > 0$$

$$0.075K_2^2 + 2.375K_2 + 15 > 0$$

$$3 + 0.075K_2 > 0$$

}

$$K_2 > 0$$

stability region.

II) value of k for identical character

Camlin	Page
Date	/ /

$$1 + K G_1(s) = 1 + K G_2(s).$$

$$s^3 + 6s^2 + 45s + K = s^3 + (3 + 0.075K)s^2 + (5 + K)s + K.$$

$$0.075K + 3 = 6$$

$$K = 40$$

$$K + 5 = 45$$

$$K = 40.$$

Q9.

Unity feedback system

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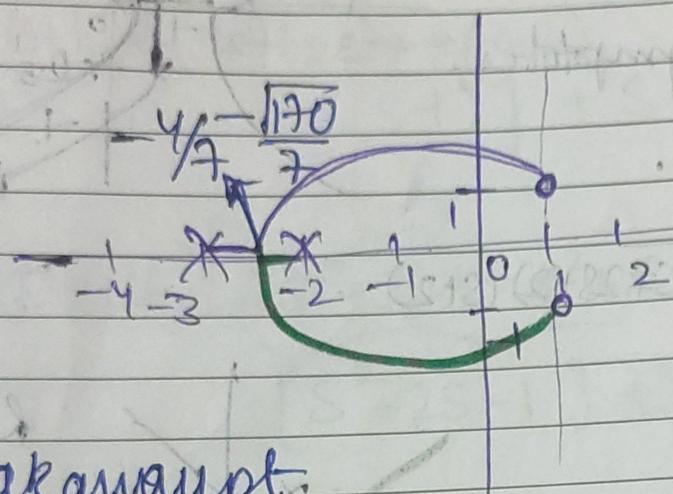
$$G(s) = \frac{K(s^2 - 2s + 2)}{(s+2)(s+3)}$$

 poles
 $-2, -3$

BP between

 2 poles $(-2, -3)$

zeros

 $-1-j, -1+j$

 value of
 K for
 stability

breakaway pt.

$$1 + K(s^2 + 2s + 2) = 0$$

$$(s+2)(s+3)$$

$$\frac{dK}{ds} = 0$$

$$-(7s^2 + 8s - 22) = 0$$

$$(s^2 + 2s + 2)^2$$

$$\rightarrow \text{roots } \left\{ \frac{-4 \pm \sqrt{170}}{7} \right\}$$

characteristic eqn

$$(s+2)(s+3) + K(s^2 - 2s + 2) = 0$$

$$(1+K)s^2 + (5-2K)s + (6+2K) = 0$$

$$1+K > 0$$

$$5-2K > 0$$

$$6+2K > 0$$

$$K > -1$$

$$K < 2.5$$

$$K > -3$$

 all
 coef of
 same sign.

$$-1 < K < 2.5$$

$$1+K < 0$$

$$5-2K < 0$$

$$6+2K < 0$$

$$K < -1$$

$$K > 2.5$$

$$K < -3$$

No mutual solution.

(Q) 10. characteristic eqn.

$$G(s)H(s) = K$$

(a)

$$s(s^2 + 2s + 1.25)(s+2)$$

Centroid
= -1

one of
breakaway
points.

pole

$$0, -2, -1 + 0.5j$$

$$-1 - 0.5j$$

2 BP betw

$$0, -2$$

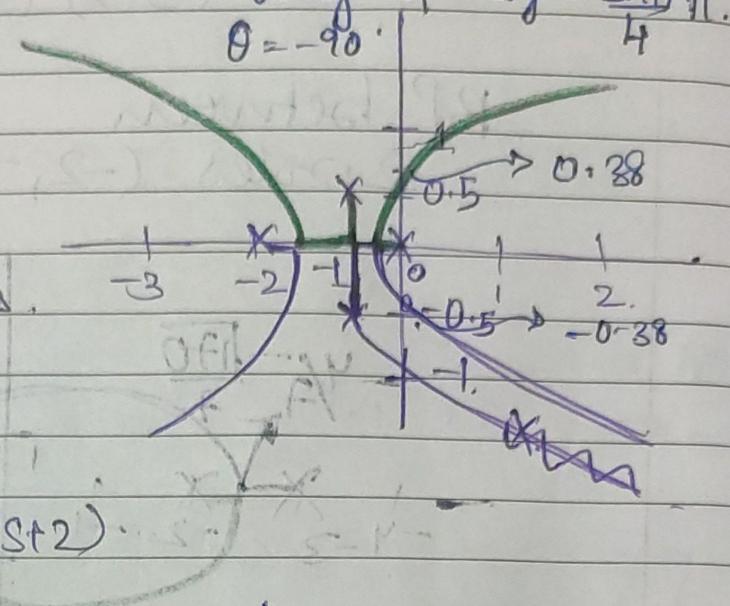
real axis.

(imaginary points
closer to centroid).

4 asymptotes.

$$(b) G(s)H(s) = K$$

$$s(s^2 + 2s + 2)(s+2)$$



poles

$$0, -2, -1-j$$

$$-1+j$$

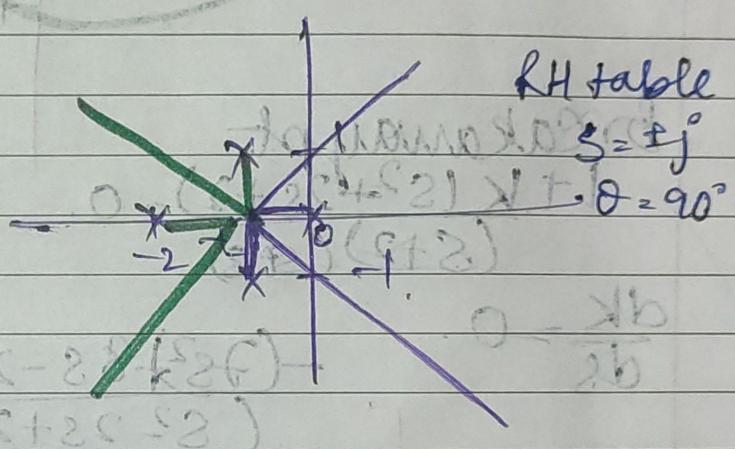
BP at
centroid

Symmetric

4 asymptotes

RH table

$$s = \pm j$$



(c)

$$G(s)H(s) = K(s+2)(s^2 + 2s + 2)$$

$$K = s(s+2)(s^2 + 2s + 10)$$

poles

$$0, -2, -1-3j$$

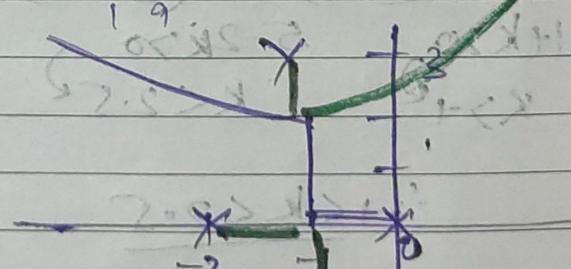
$$-1+3j$$

4 asymptotes.

RH table

$$K = \pm j$$

$$\theta = -90^\circ$$



2 BP betw
-1-3j
-1+3j

(real points

closer to centroid)