Answer (Assignment sheet 5(b) (Joint sheet)

1. (a) $\frac{5}{128}$

(b) $f_x(x) = 3x^2 \quad 0<x<1, \quad f_y(y) = \frac{3}{2}(1-y^2), \quad 0<y<1$

(c) $f(x/y) = \frac{2x}{1-y^2} \quad y \le x \le 1, \quad f(x/y=\frac{1}{2}) = \frac{8}{5}x, \quad \frac{1}{2} \le x \le 1$

(d) Not indep.

(e) $\frac{15}{8}$

(f) $\frac{3}{10}$

2. $f_y(y) = \frac{1}{3} + \frac{1}{6}y, \quad 0<y<2$

$f(x/y) = \frac{x^2 + \frac{xy}{3}}{\frac{1}{6}y + \frac{1}{3}}, \quad 0 \le x \le 1$

$f(x/y=\frac{1}{2}) = \frac{12}{5}\left(x^2 + \frac{x}{6}\right)$

$E\left(X/Y=\frac{1}{2}\right) = \int_0^1 x \cdot \frac{12}{5}\left(x^2+\frac{x}{6}\right)dx = \frac{11}{15} = 0.733.$

3. $E(XY) = \int_0^\infty \int_0^y (xy) \cdot \frac{1}{64} e^{-y/8} dx\,dy = 192$

$E(X) = \int_0^\infty \int_0^y x \cdot \frac{1}{64} e^{-y/8} dx\,dy = 8$

$E(Y) = \int_0^\infty \int_0^y y \cdot \frac{1}{64} e^{-y/8} dx\,dy = 16$

$Cov(X,Y) = 64.$

4. $f_x(x) = \frac{1}{14}(4x+3) \quad 0 \le x \le 2, \quad E(X) = \frac{25}{21}, \quad V(X) = 131/441$

$f_y(y) = \frac{1}{14}(2y+5), \quad 0 \le y \le 2 \quad E(Y) = \frac{23}{21} \quad V(Y) = 143/441$

(a) $E(XY) = \frac{9}{7}$ (b) $Cov(X,Y) = -8/441$

(c) ~~Var~~ $\rho = -0.0585$

5. $c = \frac{6}{19}$

6. do it

7. do it

8. $f_X(x) = x + \frac{1}{2}$ $0 < x < 1$
$f_Y(y) = y + \frac{1}{2}$, $0 < y < 1$,
Find $E(X)$, $E(Y)$, $E(XY)$
$V(X)$, $V(Y)$, $Cov(X,Y)$
then $\rho = -\frac{1}{11}$

9.

10. $f_Y(y) = -\ln y$, $P(Y \le 0.025) = 0.5987$, $P(X \le 0.5 / Y > 0.25) = 0.19038$

11. printing error, $f(x,y) = \frac{1}{2\pi} e^{-\frac{1}{2}(x^2+y^2)}$ $-\infty < x \le y < \infty$
$f_X(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$, $f_Y(y) = e^{-\frac{1}{2}y^2}$.
$X \sim N(0;1)$ $Y \sim N(0,1)$, X & Y are indep
$P[-\sqrt{2} < X+Y < \sqrt{2}]$. → find as $X+Y \sim N(0,2)$.
$P[-1 < Z < 1] =$ see table.

12. $E(Z_1) = 2$, $E(Z_2) = 86$, $E(Z_1 Z_2) = 64$
$\rho_{Z_1,Z_2} = 0.60749$.

13. Hint find $f_X(x)$, $f(y/x)$, then $E[Y/X=x]$.
↳ $x e^{-xy}$ ↳ $\frac{1}{x}$

14. do it.

15. (i) $\text{Cov}(X,Y) = 0$
(ii) Not independent as $P(-1,-1) \neq P_X(-1)\, P_Y(-1)$
$\frac{1}{8} \neq \frac{3}{8} \cdot \frac{3}{8}$.

16. do it

17. do it

18. $f_Y(y) = \frac{1}{\lambda} e^{-\frac{1}{\lambda} y}$, $y \geq 0$.

19. $G\left(\frac{1}{2}, \frac{1}{2}\right)$ Ans = 0.77359

20 $\mu = 3$, $\sigma^2 = 16$

21. $f_{Y_1}(y) = n(1 - F(y))^{n-1} \cdot f(y)$, $f_{Y_2}(y) = n(F(y))^{n-1} f(y)$.
As $F_X(x) = 1 - e^{-2x}$, $f_X(x) = 2e^{-2x}$. find $f_{Y_1}(y)$, $f_{Y_2}(y)$.

22. $f_{UV}(u,v) = \frac{1}{\pi} e^{-\frac{1}{2}\sigma^2 u} \cdot \frac{1}{2(v^2+1)}$ $0 < u < \infty$, $0 < v < \infty$
$f_U(u) = \frac{1}{\pi(v^2+1)}$ $-\infty < v < \infty$.

23. $f_{Y_1,Y_2}(y_1, y_2) = \frac{1}{y_2} \lambda^2 e^{-\lambda y_1}$

24. Done in the class.

25. Similar problem done in the class.

26. do it (standard problem)

27. (a) do it as $X_1 \sim N(6,1)$.
(b) typo Error, $P[X_2 \leq 5 / X_1 = 5]$ → do it
(c) Do it

28. Do it. (a) 0.1891 (b) 0.2024

29. (i) 0.1586
(ii) 0.5249.

30. Use CLT $E(X) = 2$, $V(X) = \frac{1}{12}$ $E(Y) = 5$, $V(Y) = \frac{10}{12}$

$P(Y > 6) \simeq P\left[Z > \frac{6-5}{\sqrt{5/6}}\right] = 0.1379$

31. Use CLT

Hint: Find $E(Y)$, $V(Y)$.

$\bar{Y} = \frac{\sum Y_i}{15}$, Find $E(\bar{Y})$, $V(\bar{Y})$.

Use CLT.

$E(Y) = \frac{1}{4}$, $E(\bar{Y}) = \frac{1}{4}$, $V(Y) = \frac{3}{80}$ $V(\bar{Y}) = \frac{1}{400}$

$P\left(\frac{1}{8} < \bar{Y} < \frac{3}{8}\right) \simeq P(-2.5 < Z < 2.5)$

$= 0.98575$.