Statistical Techniques

Assignment -2

1. Suppose a coin having probability 0.7 of coming up heads is tossed three times. Let $X$ denote the number of heads that appear in the three tosses. Determine the probability mass function of $X$.

2. Find the value of $c$ and the distribution function $F(x)$ given the p.d.f of $X$:

$$f(x) = \begin{cases} cx & , \text{ if } 0 \le x < 2 \\ 2c & , \text{ if } 2 \le x < 4 \\ 6c - cx & , \text{ if } 4 \le x < 6 \end{cases} \quad ; \quad F(x) = \begin{cases} 0 & , \quad x < 0 \\ \frac{1}{2} & , \quad 0 \le x < 1 \\ 1 & , \quad 1 \le x < \infty \end{cases}$$

3. What is the probability mass function of $X$ if the distribution function $F(x)$ is given as above.

4. A random variable $X$ has the following probability distribution:

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| $p(x)$ | 0 | $k$ | $2k$ | $2k$ | $3k$ | $k^2$ | $2k^2$ | $7k^2+k$ |

(i) Find $k$, (ii) Evaluate $P(X < 6)$, $P(0 < X < 5)$, (iii) If $P(X \le c) \ge \frac{1}{2}$, find minimum value of $c$ (iv) Determine the distribution function of $X$. [**Ans** : $(i)$ $\frac{1}{10}$; $(ii)$ $\frac{81}{100}, \frac{4}{5}$; $(iii)$ $c = 4$]

5. Consider the probability distribution of $X$:

$$P(X = x) = \begin{cases} \frac{x}{15} & , \quad x = 1, 2, 3, 4, 5 \\ 0 & , \quad \text{otherwise} \end{cases}$$

Find $(i)$ $P(X = 1 \text{ or } 2)$ $(ii)$ $P\{\frac{1}{2} < X < \frac{5}{2} \mid X > 1\}$ [**Ans:** $(i)$ $\frac{1}{5}$; $(ii)$ $\frac{1}{7}$]

6. A random variable $X$ has the probability density function:

$$f(x) = \begin{cases} 2x & , \quad 0 \le x < 1 \\ 0 & , \quad \text{otherwise} \end{cases}$$

Find $(i)$ $P\left(\frac{1}{4} < X < \frac{1}{2}\right)$ $(ii)$ $P\left(X > \frac{3}{4} \mid X > \frac{1}{2}\right)$. [**Ans:** $(i)$ 3/16, $(ii)$ 7/12]

7. Let $f_1$ and $f_2$ be pdf's and $\theta_1 + \theta_2 = 1$. Is $g(x) = \theta_1 f_1(x) + \theta_2 f_2(x)$ a pdf?

8. For the probability density function of $X$

$$f(x) = \begin{cases} \frac{2(b+x)}{b(a+b)} & , \quad 0 - b \le x < 0 \\ \frac{2(a-x)}{a(a+b)} & , \quad 0 \le x < a \end{cases}$$

Find mean, median and variance. $\left[\textbf{Ans} : (i)\ \frac{a-b}{3};\ (ii)\ \frac{a^2+b^2+ab}{18};\ (iii)\ a - \sqrt{\frac{a(a+b)}{2}}\right]$

9. A continuous random variable $X$ has pdf $f(x) = A + Bx$, $0 \le x \le 1$, If $\mu = 1/2$, find $A$ and $B$.

10. Show that if a random variable $X$ has the p.d.f $f(x) = \frac{1}{2}e^{-|x|}$ for $-\infty < x < \infty$. Find mean, variance of $X$. [**Ans:** $M_X(t) = \frac{1}{1-t^2}$, $\mu = 0$, $\mu_2 = 2$, $\mu_3 = 0$, $\mu_4 = 24$ ]