

**SPA; Spring 2021-2022 Introduction to Probability  
Assignment-1**

1. A bag contains 4 red, 5 white and 6 black balls. What is the probability that two balls drawn are red and black? Answer  $8/35$
2. Consider rolling a fair die. If we suppose that all six numbers were equally likely to appear, then what is the probability of getting an even number. Answer  $0.5$
3. When A and B are 2 mutually exclusive events such  $P(A)=1/2$ ,  $P(B)=1/3$ , find  $P(A \cup B)$  and  $P(A \cap B)$ . Answer  $5/6$ ,  $0$
4. A lot contains 10 good articles, 4 with minor defects and 2 with major defects. Two articles are chosen from the lot at random (without replacement). Find the probability that (i) both are good (ii) both have major defects (iii) at least 1 is good (iv) at most 1 is good (v) exactly one is good (vi) neither has major defects, (vii) neither is good. Answer  $3/8$ ,  $1/120$ ,  $7/8$ ,  $5/8$ ,  $1/2$ ,  $91/120$ ,  $1/8$
5. A box contains 6 red, 4 white and 5 black balls. A person draws 4 balls from the box at random. Find the probability that among the balls drawn there is at least one ball of each colour. Ans  $48/91$
6. George prepared 4 different letters to be sent to 4 different addresses. For each letter, he prepared an envelope with its correct address. If the letters are placed in the envelopes at random, and the probability that (i) none of the letters is in the correct envelope and (ii) at least 1 letter is in the correct envelope. Ans  $3/8$ ,  $5/8$
7. A bag contains 10 gold and 8 silver coins. Two successive drawings of 4 coins are made such that: (i) coins are replaced before the second trail, (ii) the coins are not replaced before the second trail. Find the probability that the first drawing will give 4 gold and the second 4 silver coins. Ans  $0.0015$ ,  $0.0047$
8. Two persons are competing for the post of the principal of a college. The probabilities that the first and the second persons will win are 0.6 and 0.4 respectively. If the first person wins, the probability of introducing the common model of examination is 0.8 and the corresponding probability if the second person wins is 0.3. What is the probability that the common model examination will be introduced? Ans  $0.60$
9. In 1989 there were three candidates for the position of principal - Mr. Chatterji, Mr. Ajay and Dr. Singh - whose chances of getting the appointment are in the proportion 4:2:3 respectively. The probability that Mr. Chatterji if selected would introduce co-education in the college is 0.3. The probabilities of Mr. Ajay and Dr. Singh doing the same are respectively 0.5 and 0.8. What is the probability that there was co-education in the college in 1990? Ans  $0.51$
10. Two dice are rolled. What is the probability that at least one is a six? If two faces are different, what is the probability that at least one is a six? Ans  $11/36$ ,  $5/18$
11. Two cards are randomly selected from a deck of 52 playing cards. What is the probability that they constitute a pair (that is, they are of the same denomination)? (ii) What is the probability they constitute a pair given that they are of different suits? Ans  $3/51$ ,  $1/13$
12. In a bolt factory machines A;B and C manufacture respectively 25%, 35% and 40% of the total. Of their output 5%, 4%, 2% are defective bolts. A bolt is drawn at random from the product and is found to be defective. What are the probabilities that it was manufactured by machines A, B and C? Ans  $25/69$ ,  $28/69$ ,  $16/69$

13. (i) A gambler has in his pocket a fair coin and a two-headed coin. He selects one of the coin at random, and when he flips it, he shows heads. What is the probability that it is the fair coin?  
(ii) Suppose that he flips the same coin a second time and again it shows heads. Now what is the probability that it is the fair coin? (iii) Suppose that he flips the same coin a third time and it show tails. Now what is the probability that it is the fair coin ? Ans  $1/3, 1/5, 1$

14. Let A and B be two events such that  $P(A) > 0$ , and  $P(A) + P(B) > 1$ . Show that  $P(B | A) \geq 1 - (P(\bar{B})/P(A))$

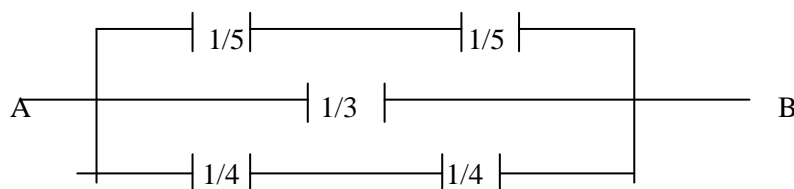
15. In a precision bombing attack there is a **50%** chance that a bomb will strike the target. Two direct hits are required to destroy the target completely. How many bombs must be dropped to give at least **94%** chance of completely destroying the target? Ans 8 bombs

16. A fair die is rolled twice. Let A, B and C be the events  $A = \{5 \text{ or } 6 \text{ on the first roll}\}$ ,  $B = \{5 \text{ on the second roll}\}$ ,  $C = \{\text{both rolls produce the same face value}\}$ . Are A, B and C pairwise independent? Ans Yes

17. A machine to detect improper welds in a fabricating shop detects 80% of all improper welds, but it also incorrectly indicates an improper weld on 5% of all satisfactory welds. Past experience indicates that 10% of all welds are improper. What is the probability that a weld which the machine indicates to be defective is, in fact satisfactory. Ans 0.36

18. If  $P(A^c) = 0.3$ ,  $P(B) = 0.4$  and  $P(A \cap B^c) = 0.5$ , find  $P(B / A \cup B^c)$ . Ans  $2/8$

19. An electric circuit looks as in the figure below, where the numbers indicate the probabilities of failure for the various links, which are all independent. What is the probability that the circuit is closed ?



Ans 0.9475