Control & Instrumentation Lab Assignment 7

SWARNENDU PAUL19EE3FP18

Identification of LTI systems using Frequency Response data

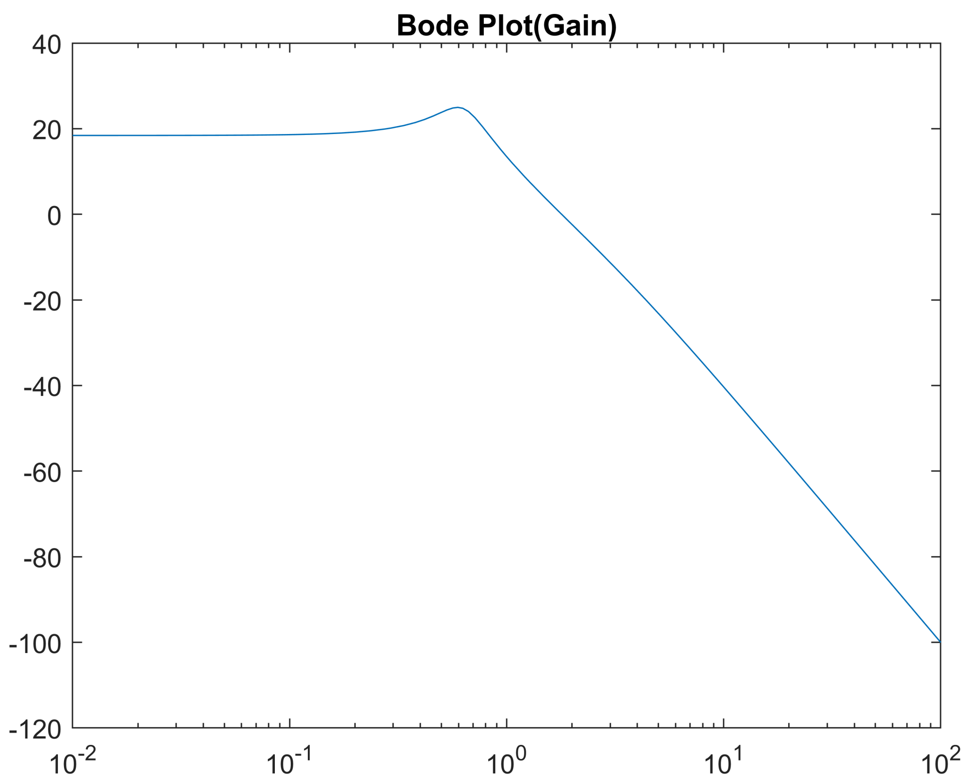
From the provided data we need to estimate the unknown tranfer function. So, let's begin with changing the dimension of the given data that can be used easily for further calculation.

```
Mag_G=squeeze(Mag_G);
Phase_G_deg=squeeze(Phase_G_deg);
```

Now, the bode diagram of the given system is shown.

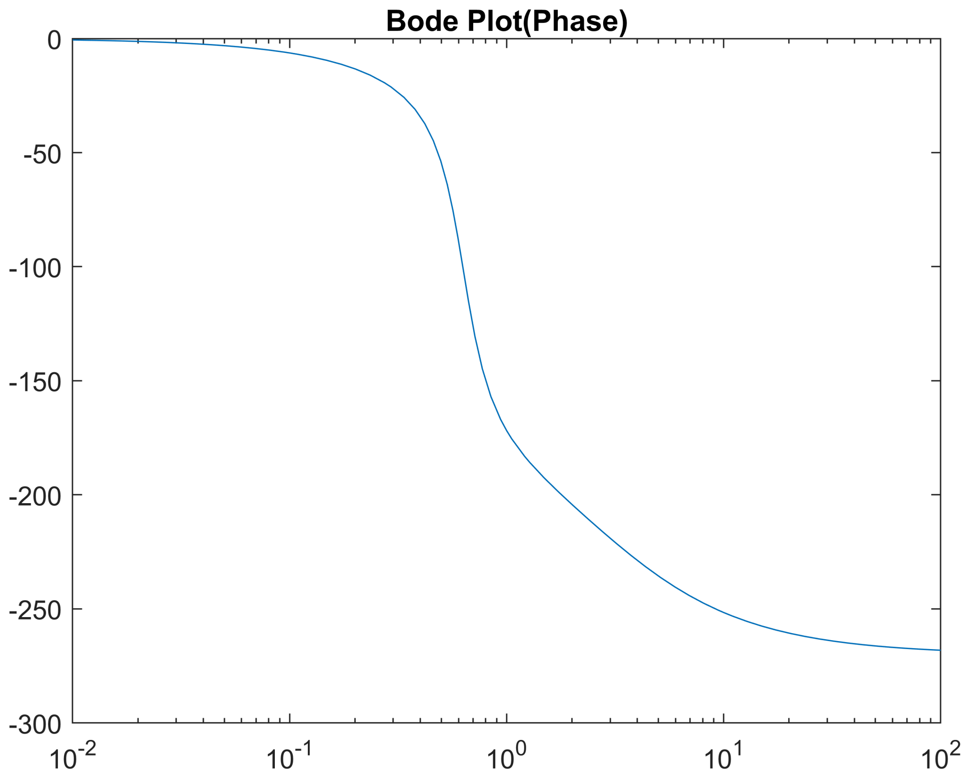
Gain plot:-

```
figure;
semilogx(W,Mag_G);
title("Bode Plot(Gain)")
```



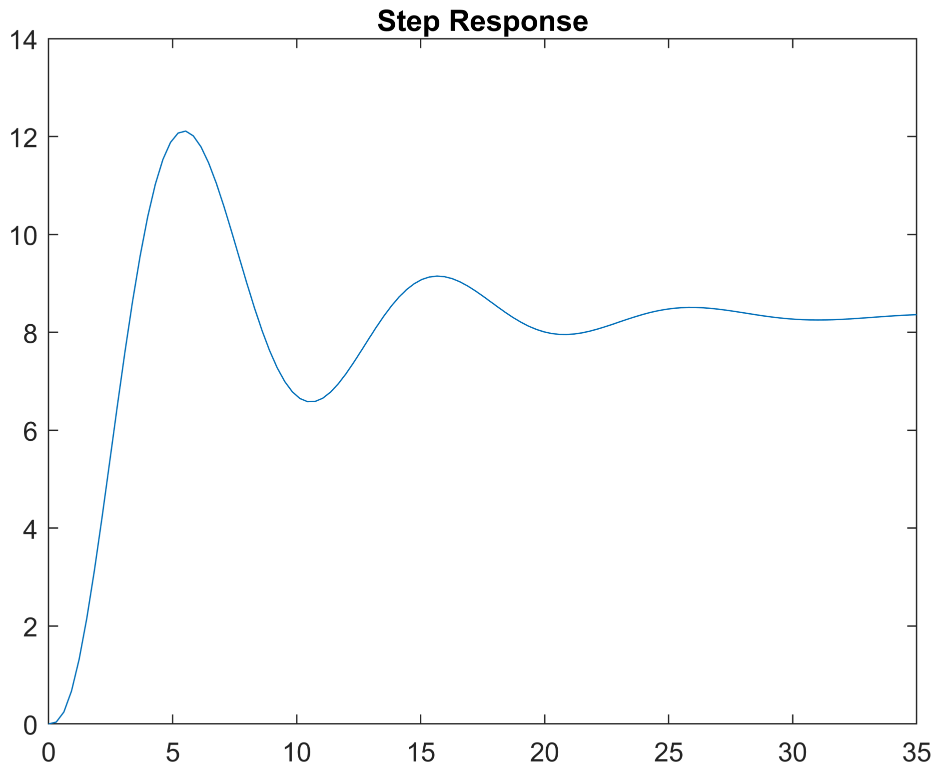
Phase plot:-

```
figure;
semilogx(W,Phase_G_deg);
title("Bode Plot(Phase)")
```



Also the step response of the system,

```
figure;
plot(t,y)
title("Step Response")
```



From the given data we can observe that, when $\omega \approx 0$ magnitude of gain is 18.4183dB or 8.3352 which is almost same as the steady state value of the step response(8.3607).

This is a confirmation that both the data provided belongs to the same LTI system.

From the phase plot, we can observe there are 3 excess poles in the system that contributes -270° to the phase as ω increases.

By observation the step response and bode gain plot of the system looks similar to a 2nd order underdamped system. Now to confirm our assumption lets check the position of peaks and troughs in the step response.

```
pk=[];
tr=[];
for i=2:114
    if (y(i-1)<y(i)) && (y(i+1)<y(i))
        pk=[pk;t(i)]; %#ok<*AGROW>
    end
    if (y(i-1)>y(i)) && (y(i+1)>y(i))
        tr=[tr;t(i)];
    end
end
```

Hence position of peaks are at $t =$

```
pk
```

```
pk = 3×1
     5.5262
    15.6576
    25.7890
```

position of troughs are at $t =$

```
tr
```

```
tr = 3×1
    10.4384
    20.8768
    31.0081
```

Which confirms our assumption as the damped frequency is equal to 0.6202 . Also this confirms the absence of zeros in the LTI system, as no change in step response data in the initial stage due to presence of zeros can be observed. The system behaves like an underdamped 2nd order system.

Hence **the system has 3 poles**(2 imaginary, 1 real) and 0 zeros which can be reduced to a 2nd order underdamped system using dominant pole approximation.

From the bode plot data we can find the damping ratio ζ and natural frequency ω_n.

To find that, we need to find the frequency of maximum gain of the given LTI system, which will be same as the 2nd order underdamped system we get from dominant pole approximation.

```
[M,in]=max(Mag_G);
```

Hence the peak gain of the system is, $G =$

```
db2mag(M)
```

```
ans = 17.7377
```

And frequency at which peak occurs, $\omega =$

```
W(in)
```

```
ans = 0.5958
```

We know,

Peak Gain, $G = \frac{1}{2\zeta\sqrt{1-\zeta^2}}$ which is equal to $\frac{17.7377}{8.3352} = 2.128$ here

And $\omega_{peak} = \omega_n\sqrt{1-2\zeta^2}$ which is equal to 0.5958

Solving we get,

damping ratio $\zeta = 0.242171$

natural frequency $\omega_n = 0.634151$

Hence damped frequency $\omega_d = \omega_n \sqrt{1-\zeta^2} = 0.61527$ which is almost same as the frequency we calculated from the step response data(0.6202).

Hence two poles of the system are,

$-0.15 \pm 0.615i$

Now, we can find the other real pole from the phase data of the LTI system. At $\omega = \omega_n$ the phase is equal to -90°.

But from the given data we find that,

when $\omega = 0.627828$ phase = -100.047°

when $\omega = 0.66663$ phase = -115.044°

so approximately at $\omega = \omega_n = 0.634151$ phase will be -102.553°.

let's assume $-p$ is a pole of the LTI system. Then,

phase contribution of $\frac{1}{s+p}$ will be $-tan^{-1}\frac{\omega}{p}$

Hence, $tan^{-1}\frac{0.634151}{p} = 12.553^\circ$

Solving we get,

$p = 2.948$

So, from all the above calculations, our estimated system has,

no zeros, **poles at** $-2.948, -0.15 \pm 0.615i$, **DC Gain** = 8.3352

So the predicted LTI system is given by,

```
p1=-2.948;
p2=-0.15+0.615i;
p3=-0.15-0.615i;
sys_pred=zpk([],[p1,p2,p3],8.3352*(-p1)*p2*p3)
```

```
sys_pred =

             9.8467
  ------------------------------
  (s+2.948) (s^2 + 0.3s + 0.4007)

Continuous-time zero/pole/gain model.
```

```
[mag_pred,phase_pred,W]=bode(sys_pred,W);
mag_pred=squeeze(mag_pred);
phase_pred=squeeze(phase_pred);
```

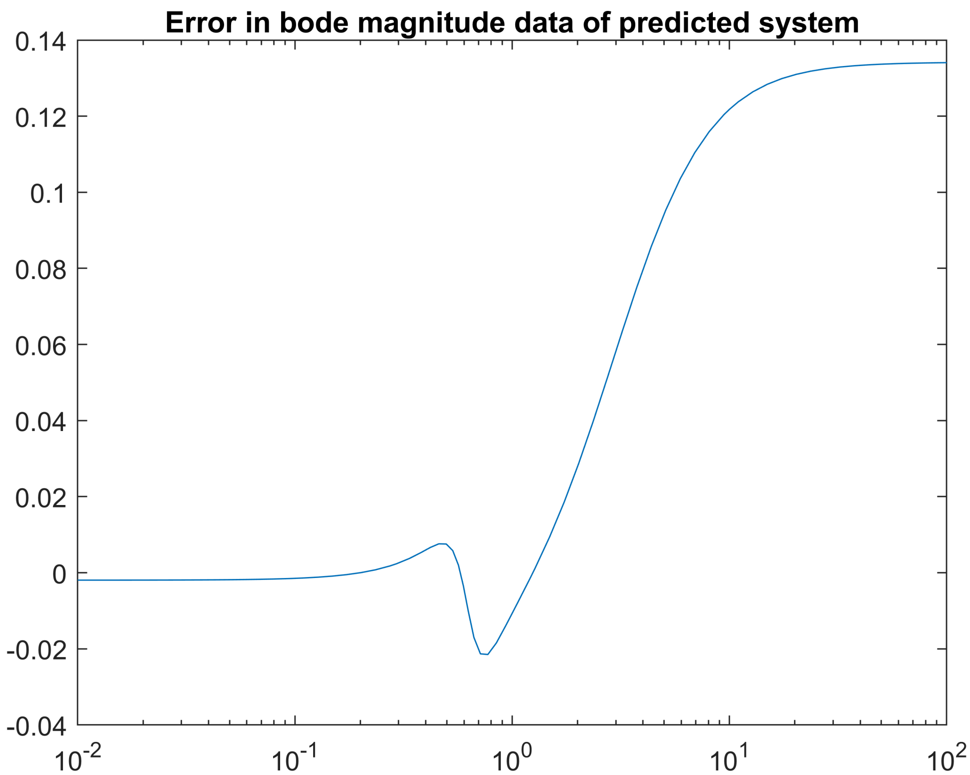
```
mag_pred=mag2db(mag_pred);
```

Now, to determine the accuracy of our calculation we can plot the errors in our prediction.

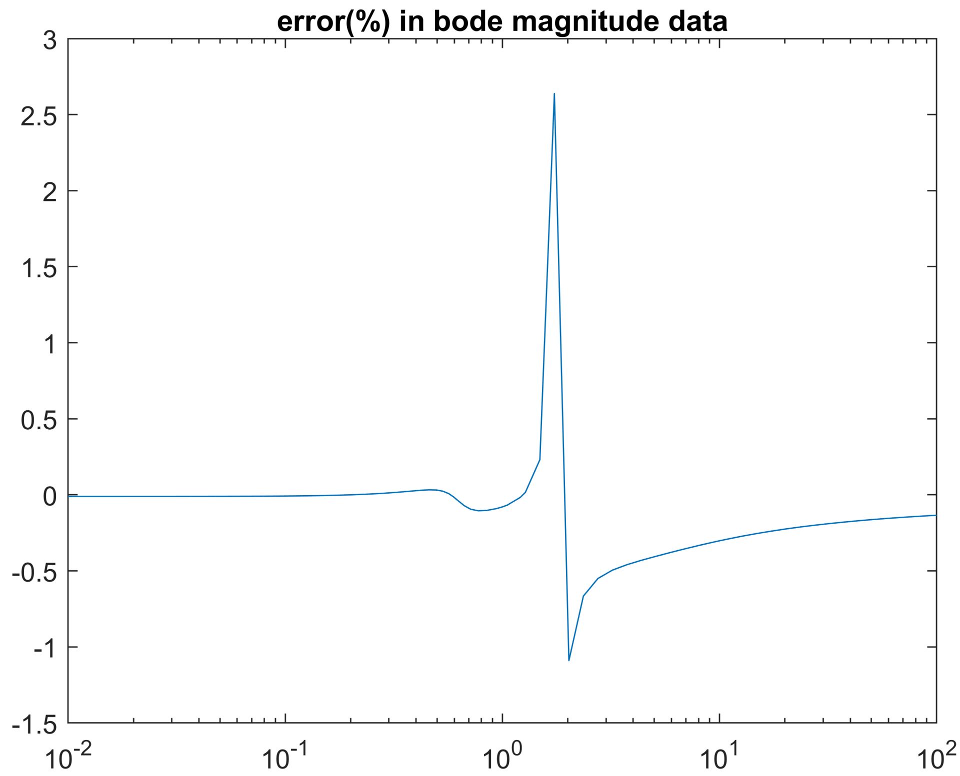
```
err_mag=Mag_G-mag_pred;
err_phase=Phase_G_deg-phase_pred;
```

error in magnitude:-

```
figure;
semilogx(W,err_mag)
title("Error in bode magnitude data of predicted system");
```



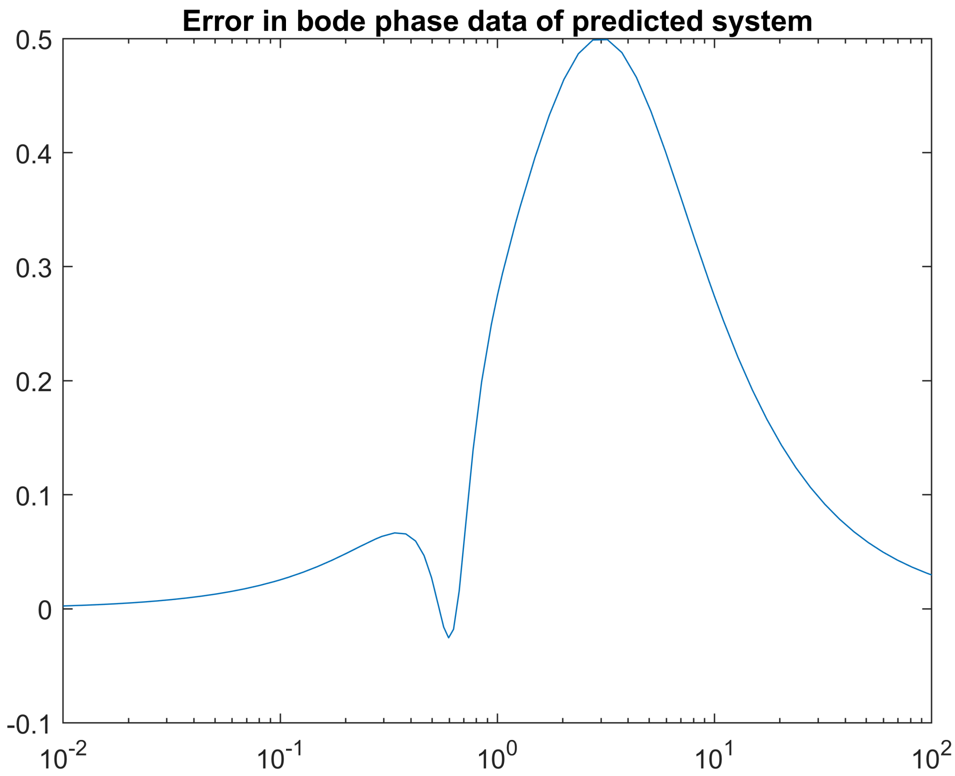

```
figure;
semilogx(W,(err_mag*100)./Mag_G)
title("error(%) in bode magnitude data")
```



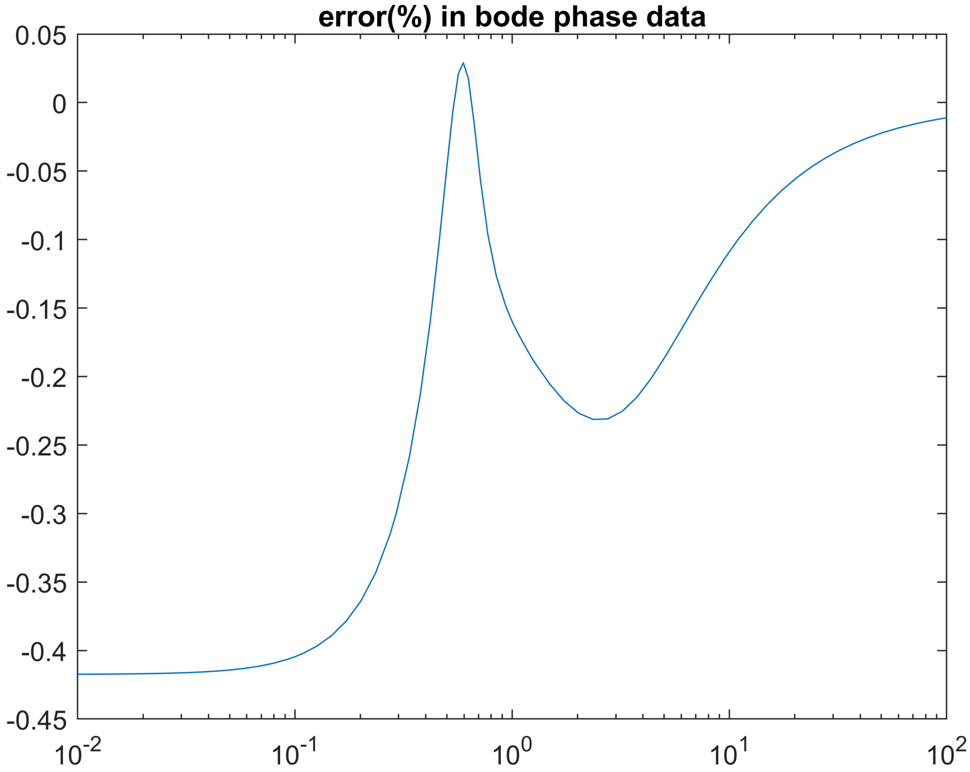
So, we can observe that, the increase in percentage error near 1 is due to the magnitude of the system being almost equal to 0dB. Otherwise our predicted system is within $\pm 2\%$ error.

error in phase:-

```
figure;
semilogx(W,err_phase)
title("Error in bode phase data of predicted system");
```




```
figure;
semilogx(W,(err_phase*100)./Phase_G_deg)
title("error(%) in bode phase data");
```



Phase data is also within $\pm 2\%$ error.

We can also compare the step response data.

```
fprintf("Predicted System:-")
```

```
Predicted System:-
```

```
stepinfo(sys_pred)
```

```
ans = struct with fields:
        RiseTime: 2.0507
    SettlingTime: 26.4287
     SettlingMin: 6.5838
     SettlingMax: 12.1134
       Overshoot: 45.3277
      Undershoot: 0
            Peak: 12.1134
        PeakTime: 5.5262
```

```
fprintf("Given system:-")
```

```
Given system:-
```

```
stepinfo(y,t,8.3352)
```

```
ans = struct with fields:
        RiseTime: 2.0508
    SettlingTime: 26.3713
     SettlingMin: 6.5837
     SettlingMax: 12.1111
```

```
        Overshoot: 45.3002
       Undershoot: 0
             Peak: 12.1111
         PeakTime: 5.5262
```

Hence, step response data is also comparable.

We can conclude that the unknown LTI system is,

```
sys_pred
```

```
sys_pred =

              9.8467
  ------------------------------
  (s+2.948) (s^2 + 0.3s + 0.4007)

Continuous-time zero/pole/gain model.
```

Confirmation:-

Using the system identification toolbox, we can verify our calculations.

```
data=iddata(y,ones([115,1]),[],"SamplingInstants",t)
```

```
data =

Time domain data set with 115 samples.
Sample time: 0.307011 seconds

Outputs      Unit (if specified)
   y1

Inputs       Unit (if specified)
   u1
```

```
sys=tfest(data,3,0)
```

```
sys =

  From input "u1" to output "y1":
              10
  ---------------------------
  s^3 + 3.3 s^2 + 1.3 s + 1.2

Continuous-time identified transfer function.

Parameterization:
   Number of poles: 3   Number of zeros: 0
   Number of free coefficients: 4
   Use "tfdata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Estimated using TFEST on time domain data "data".
Fit to estimation data: 100%
FPE: 2.199e-30, MSE: 1.947e-30
```

In zero-pole-gain form:-

```
poles=pole(sys)
```

```
poles = 3×1 complex
  -3.0000 + 0.0000i
  -0.1500 + 0.6144i
  -0.1500 - 0.6144i
```

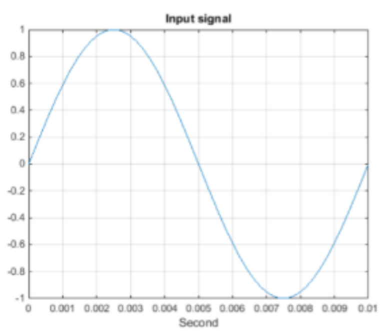
```
sys_zpk=zpk([],poles,10);
```

```
sys_zpk =

             10
  ------------------------
  (s+3) (s^2 + 0.3s + 0.4)

Continuous-time zero/pole/gain model.
```

Hence by observation we can confirm that our predicted system is almost same as the unknown LTI system.

Question 1:-

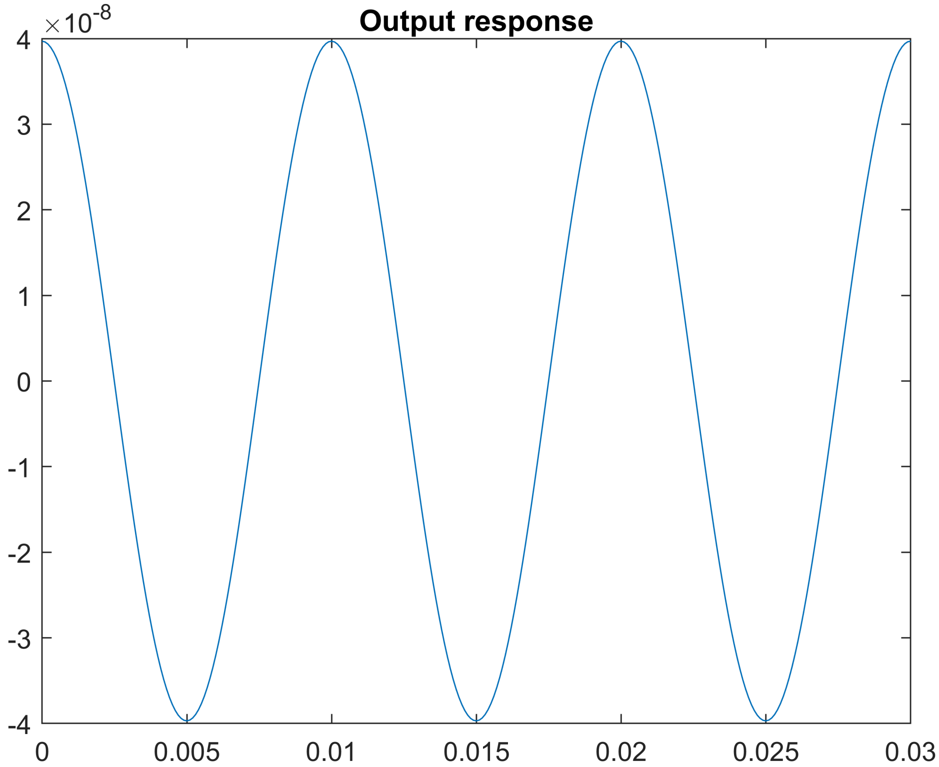
The given sinusoidal input:-



Which has $\omega = 2\pi \times 100$

from bode data we can find magnitude and phase for given ω

```
w=2*pi*100;
[m,p,w]=bode(sys_pred,w);
T=0:0.0001:0.03;
u=m*sin(2*pi*100*T+p*pi/180);
figure;
plot(T,u)
title("Output response")
```



Magnitude and phase error plots are shown above.

Question 4:-

Identified transfer function in pole-zero-gain form:

```
sys_pred
```

```
sys_pred =

              9.8467
  ------------------------------
  (s+2.948) (s^2 + 0.3s + 0.4007)

Continuous-time zero/pole/gain model.
```