

## Statistical Techniques

### Assignment No. 3

#### Binomial Distribution:

1.  $A$  and  $B$  plays a game in which their chances of winning are in the ratio 3 : 2. Find  $A$ 's chance of winning at least three games out of the five games played. [Ans: 0.68]

#### Poisson Distribution:

1. A car hire firm has two cars which it hires out day by day. The number of demands for a car on each day is distributed as Poisson variate with mean 1.5. Calculate the proportion of days on which (i) neither car is used, and (ii) some demand is refused. [Ans: (i) 0.2231; (ii) 0.19126]
2. A manufacturer of cotton pins knows that 5% of his product is defective. If he sells cotton pins in boxes of 100 and guarantees that not more than 10 pins will be defective, what is the approximate probability that a box will fail to meet the guaranteed quality? [Ans :  $1 - \sum_{x=0}^{10} \frac{5^x e^{-5}}{x!}$ ]
3. Binary data are transmitted over a noisy communication channel in block of 16 binary digits. The probability that a received digit is in error as a result of channel noise is 0.01. Assume that the errors occurring in various digit positions within a block are independent. (i) Find the mean and the variance of the number of errors per block. (ii) Find the probability that the number of errors per block is greater than or equal to 4. [Ans: (i) 0.16, 0.158 (ii)  $0.165 \times 10^{-4}$ ]
4. Suppose the moment generating function of  $X$  is  $M_X(t) = e^{3(e^t - 1)}$ . Find  $\mathbb{P}(X = 0)$ . [Ans:  $e^{-3}$ ]

#### Geometric Distribution:

1. The probability that a student pilot passes the written test for a private pilot's license is 0.7. Find the probability that the student will pass the test (i) on the third try (ii) before the fourth try.
2. In an urn there are  $r$  red balls and  $b$  blue balls. Balls are selected at random with replacement so that the first blue ball is obtained. What is the probability that exactly  $k$  draws needed for the first blue ball. What is the probability that at least  $m$  draws are needed for the first blue ball.

#### Negative Binomial Distribution:

1. Suppose the probability that a bit transmitted through a digital communication channel and received in error is 0.1. Assuming that the transmissions are independent events, find the probability that the third error occurs at the  $10^{th}$  bit. [Ans : 0.017]
2. An electronic scale in an automated filling operation stops the manufacturing line after three underweight packages are detected. Suppose that the probability of an underweight package is 0.001 and each fill is independent. (i) What is the mean number of fills before the line is stopped? (ii) What is the  $SD$  of the number of fills before the line is stopped? [Ans: (i) 3000; (ii) 1431.18]

### Exponential Distribution:

1. If on the average three trucks arrive per hour to be unloaded at a warehouse. Find the probability that the time between the arrivals of successive trucks will be less than 5 minutes. [Ans:  $1 - e^{-\frac{1}{4}}$ ]
2. Suppose that the length of the time that a new long-life bulb burns is exponentially distributed with a mean of 5 years. A homeowner has two such bulbs. She plans to place one bulb in the outside light of her home and to replace it with the second bulb when the first burns out. What is probability that bulbs will burn for atleast 15 years before the second one burns out? [Ans:  $4e^{-3}$ ]
3. The time between arrivals of taxis at a busy intersection is exponentially distributed with a mean of 10 minutes. (i) What is the probability that you wait longer than one hour for a taxi ? (ii) Suppose you have already been waiting for one hour for a taxi, what is the probability that one arrives within the next 10 minutes? [Ans: (i) 0.0025; (ii) 0.6321]
4. What are the pdf, mean and the variance of  $X$  if the MGF of  $X$  is  $\phi_X(t) = \frac{3}{3-t}$ . [ $\mu = \frac{1}{3}$ ;  $\sigma^2 = \frac{1}{9}$ ]

### Gamma Distribution:

### $\chi^2$ -square Distribution

1. In the manufacturing of computer memory chips, company A produces one defective chip for every nine good chips. Let  $X$  be time to failure (in months) of chips. It is known that  $X$  is an exponential r.v. with parameter  $\lambda = \frac{1}{2}$  for a defective chip and  $\lambda = \frac{1}{10}$  with a good chip. Find the probability that a chip purchased randomly will fail before (i) six months of use; and (ii) one year of use. [Ans: (i) 0.501 (ii) 0.729]
2. In a biological study with rats, a dose-response investigation is used to determine the effect of the dose of a toxicant on their survival time. The toxicant is one that is frequently discharged into the atmosphere from jet fuel. For a ceratin dose of the toxicant the study determines that the survival time, in weeks, has a Gamma distribution with  $\alpha = 5$  and  $\beta = 10$ . What is the probability that a rat survives no longer than 60 weeks? [Ans: 0.715]
3. In a biomedical research activity it was determined that the survival time in weeks of an animal when subjected to a certain exposure of gamma radiation has a Gamma distribution with  $\alpha = 5$  and  $\beta = 10$ .
  - (a) What is the mean survival time of a randomly selected animal of the type used in the experiment? Also find the standard deviation of survival time? [Ans:  $\mu = 50$ ;  $\sigma = \sqrt{500}$ ]
  - (b) What is the probability that an animal survives more than 30 weeks? [Ans: 0.8155]
4. If 15 observations are taken independently from a chi-square distribution with four degrees of freedom, find the probability that atmost three of the 15 observations exceed 7.779. [Ans: 0.9444]

### Normal Distribution:

1. A production line manufactures 1000-ohm ( $\Sigma$ ) resistors that have 10 percent tolerance. Let  $X$  denote the resistance of a resistor. Assuming that  $X$  is a normal RV with mean 1000 and variance 2500, find the probability that a resistor picked at random will be rejected. [Ans : 0.045]
2. A soft-drink machine is regulated so that it discharges an average of 200 milliliters per cup. If the amount of drink is normally distributed with a standard deviation of 15 milliliters.
  - (a) What fraction of the cups will contain more than 224 milliliters? [Ans: 0.0548]
  - (b) What is the probability that a cup contains between 191 and 209 milliliters? [Ans: 0.4514]
  - (c) How many cups will probably overflow if 230 ml cups are used for the next 1000 drinks?
  - (d) Below what value do we get the smallest 25% of the drinks? [Ans: 189.95 ml]
3. The life in years of a certain type of electrical switch has an exponential distribution with an average life of 2. If 100 of these switches are installed in different systems, what is the probability that at most 30 fail during the first year? [Ans:  $\mu = 39.35$ ;  $\sigma = 4.885$ ; 0.0352]
4. Two independent random variables  $X$  and  $Y$  are normally distributed with means 1 and 2 and SDs 3 and 4 respectively. If  $Z = X - Y$ , write the density function of  $Z$ . State median, SD, and mean of the distribution of  $Z$ . Show that  $\mathbb{P}(Z + 1 \leq 0) = 0.5$ . [Ans:  $\mu = \text{median} = -1$ ;  $\sigma = 5$ ]
5. The MGF of a normal random variable  $X$  is  $e^{3t+8t^2}$ . Find  $\mathbb{P}(-1 < X < 9)$ . [Ans: 0.7745]

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