Assignment 5

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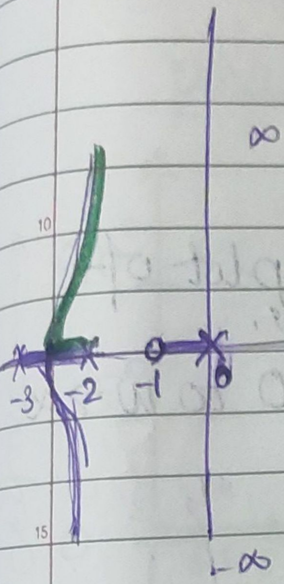
Q1. characteristic equation

$S(S+2)(S+3) + K(S+1) = 0$.

transfer func^n $K = -\dfrac{(S+1)}{S(S+2)(S+3)}$ $\rightarrow 0$ ~~pole~~ zero : $S = -1$

∞ pole : $S = 0, -2, -3$



∴ pole at 0 goes to -1 as $K \to \infty$ and other poles $\to \infty$ with asymptotes at -2. with angles of $90°$ & $270°$

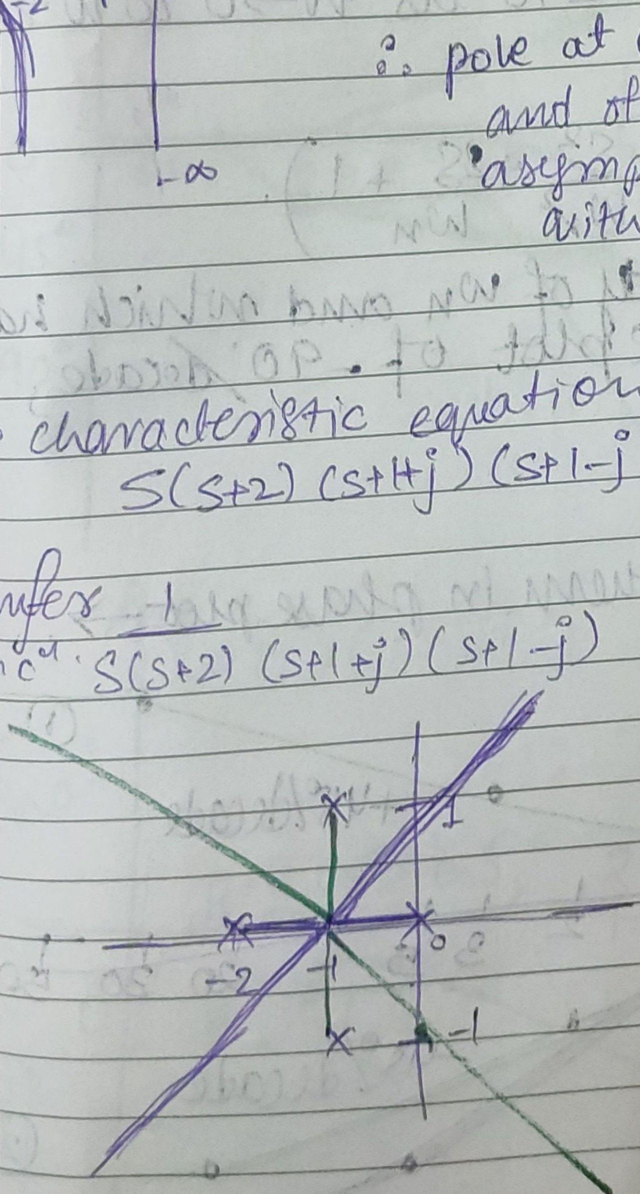
Q2. characteristic equation

$S(S+2)(S+1+j)(S+1-j) + k = 0$

pole $\rightarrow 0, -2, -1-j, -1+j$

transfer func^n $\dfrac{1}{S(S+2)(S+1+j)(S+1-j)}$

zero X



∴ can be seen from the plot there is symmetry about -1 point.

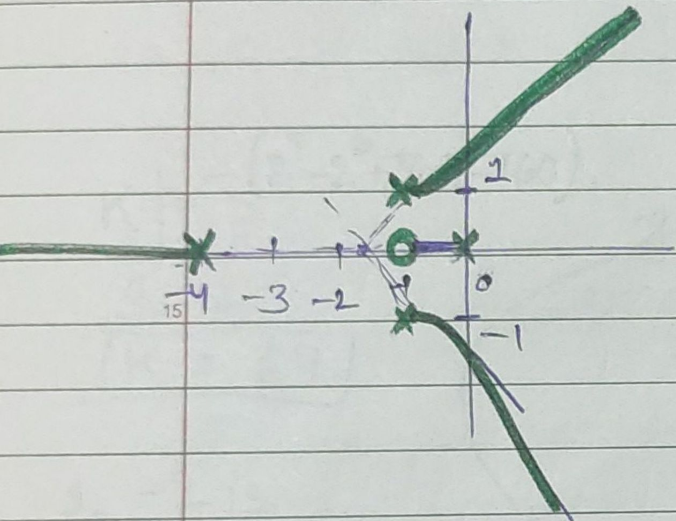
Q3. characteristic eqn

$$S(S+4)(S^2+2S+2)+K(S+1)=0,$$

transfer func^n → $\dfrac{S+1}{S(S+4)(S^2+2S+2)}$.

centroid $= \dfrac{0-4-1-1+1}{4-1} = \dfrac{-5}{3}$ → intersect of asymptotes.

asymptote angles, $\theta = \dfrac{(2q+1)180^\circ}{4-1} = 60^\circ, 180^\circ, 300^\circ$



3 asymptotes intersect at centroid.

Q4. characteristic eqn

$$S(S+3)(S^2+2S+2)+K=0$$

transfer func^n $= \dfrac{1}{S(S+3)(S^2+2S+2)}$.

zero → X
pole $0, -3, -1+j, -1-j$.

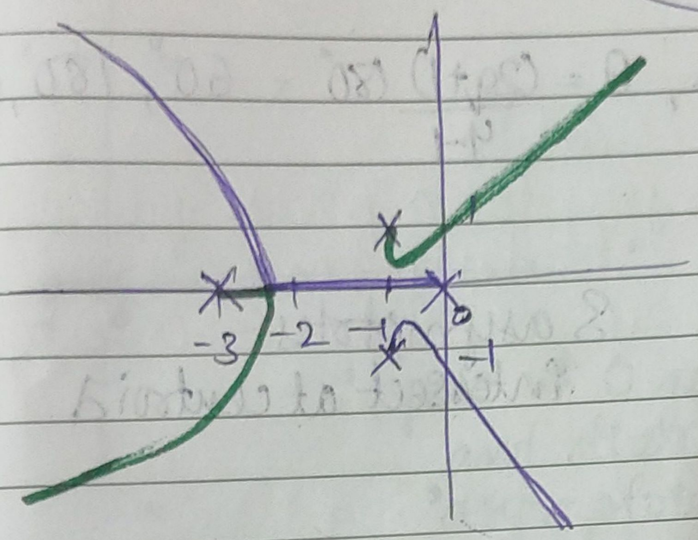
If odd number of open loop poles and zeros exists to the left side of point on real axis then point is root locus branch.

So on real axis root loci will be present bet^n 2 poles $0, -3$.

for departure angle of pole $-1+j$ (s_1)

pole at 0 makes 135° with s_1.
pole at -3 makes $\tan^{-1}(1/2)$ with s_1
pole at $-1-j$ makes 90° with s_1.

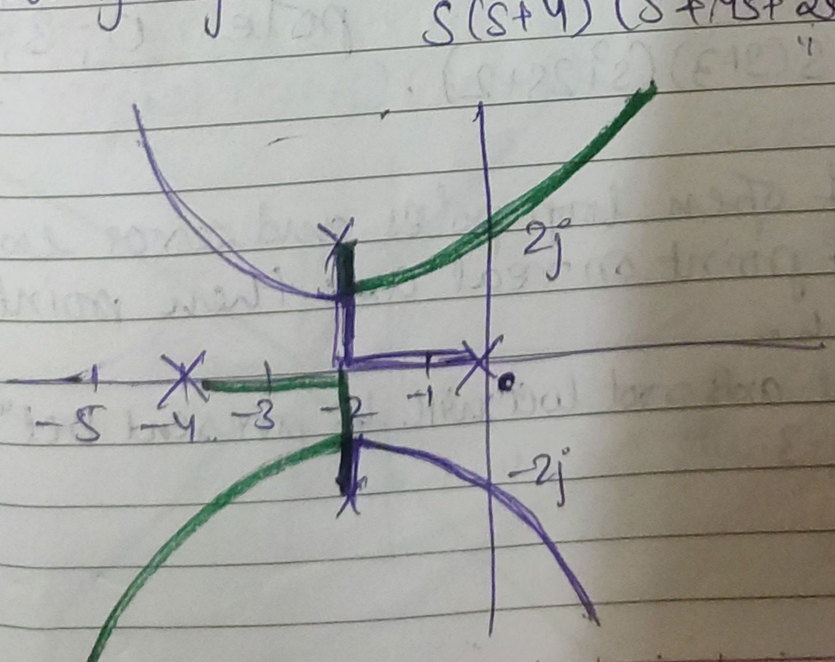
$\therefore$ angle of departure $= 180^\circ - 135^\circ - 90^\circ - \tan^{-1}(1/2) =$
$= -71.56^\circ$



Q5. characteristic eqn:
$s(s+4)(s^2+4s+20)+k=0$

transfer fun^n $\dfrac{1}{s(s+4)(s^2+4s+20)}$

pole
$0, -4,$
$-2-2j,$
$-2+2j.$



intersection with imaginary axis using RH table.

K = ? for row is 0.

∴ at, K=260, we have 2 poles on imaginary axis ([illegible]).

for K=260. intersection with imaginary axis at $\longrightarrow \pm\sqrt{10}j$

Breakaway point:

$$K = -(s^4+8s^3+36s^2+80s)$$

$$\frac{dK}{ds} = 0.$$

$$4s^3+24s^2+\cancel{64}78s+80=0$$

$$\cancel{2s^3+12s^2+36}$$

$$s^3+6s^2+18s+20=0.$$

$$s=-2$$

$$K\Big|_{s=-2} = -(2^4-2^6+36\times4-160).$$

$$\boxed{K=64.}$$

∴ −2 is breakaway point on real axis.

Q6. characteristic eqn.

$$s(s+5)(s+6)(s^2+2s+2)+k(s+3)=0$$

transfer func^n = $\dfrac{s+3}{s(s+5)(s+6)(s^2+2s+2)}$.

pole: 0, −5, −6, −1−j, −1+j

Zeros: −3.

∴ 5 root locus & 4 of them will go to ∞ as K increases

If odd number of open loop poles and zeros exits to the left side of a point on real axis, then the point is on root locus branch.
So on real axis root loci will be present bet^n 0, −3 and −5, −6.

Centroid $\alpha = 2.5$

angle of asymptotes = 45°, 135°, 225°, 315°.

Breakaway point bet^n −5 and −6.

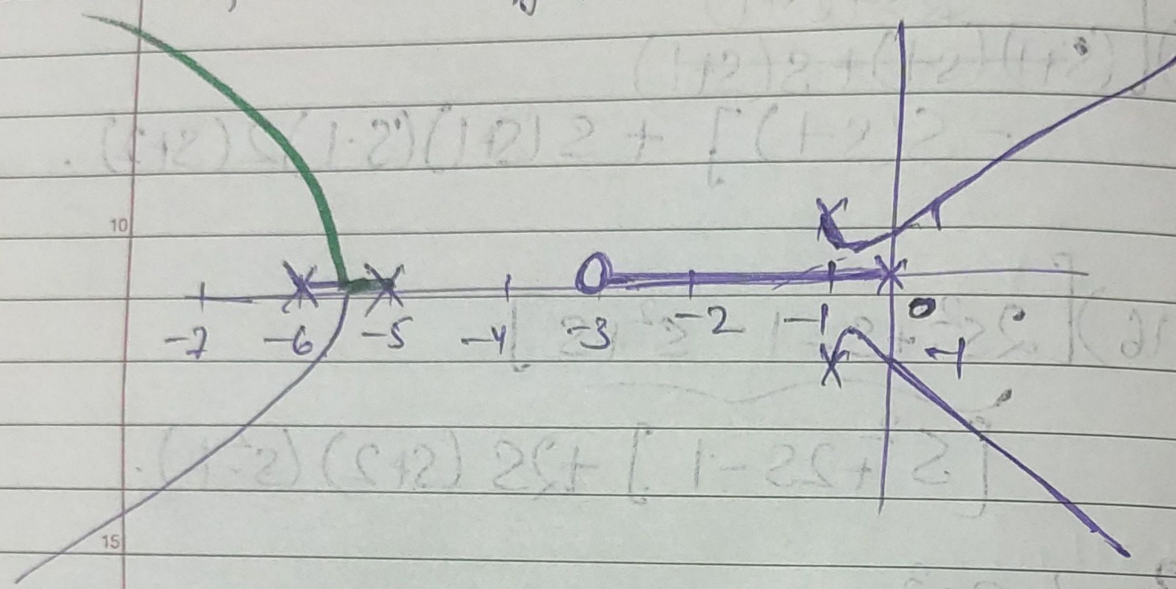
Angle of departure for −1+j. is

$$180^\circ - (180 - \tan^{-1} 1) - 90^\circ - \tan^{-1} 1/4 - \tan^{-1} 1/5 + \tan^{-1} 1/2 = \boxed{-43.78^\circ}$$

So root locus of pole at 0 will end at −3 pole at $-1+j$ will start with $-43.78°$ and follow $45°$ asymptote from centroid.

pole at $-1-j$ will reflect $-1+j$ locus wrt real axis.

poles at $-5, -6$ will breakaway and follow asymptotes.



Q7. $G(s)H(s) = \dfrac{K(s+a)}{s(s-b)(s^2+2\zeta\omega_n s+\omega_n^2)}$.

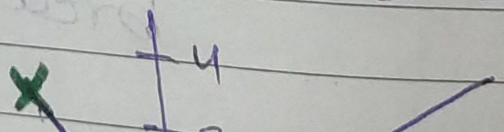
$$= \frac{K(s+1)}{s(s-1)(s^2+4s+16)}$$

Zeros: -1.

poles: $0, 1, -2+2\sqrt{3}j, -2-2\sqrt{3}j$

gain for stability = ?

0.47, -2.26 } breakaway points.



$$K = \frac{s(s-1)(s^2+4s+16)}{(s+1)}$$

$$\frac{dK}{ds} = \frac{(s+1)\left[(s-1)(s^2+4s+16) + s(s^2+4s+16) + s(s-1)(2s+4)\right] - s(s-1)(s^2+4s+16)}{(s+1)^2}$$

$$0 = \frac{(s^2+4s+16)}{(s+1)^2}\left[\cancel{(s+1)(s-1) + s(s+1)}^{\,(2s-1)(s+1)} - s(s-1)\right] + s(s+1)(s-1)2(s+2).$$

$$= (s^2+4s+16)\underbrace{\left[2s^2+s-1-s^2+s\right]}_{\left[s^2+2s-1\right]} + 2s(s+2)(s^2-1).$$

$$= s^4 + 8s^2 - s^2 + 2s^3 + 4s^3 - 4s + 16s^2 + 32s - 16 + 2s(s^3+2s^2-s-2).$$

$$= s^4 + 6s^3 + 23s^2 + 28s - 16 + 2s^4 + 4s^3 - 2s^2 - 4s$$

$$= \underline{3s^4 + 10s^3 + 21s^2 + 24s - 16}.$$

$\hookrightarrow$ $\underbrace{s = -2.263,\ 0.448}_{\text{breakaway points}}$

continued.

Q7

characteristic eqn.

$1 + G(s)(H(s)) = 0.$

$s(s-1)(s^2+4s+16) + k(s+1) = 0.$

open loop poles $\rightarrow 0, +1, -2 \pm 2\sqrt{3}j$.

Zeros $\rightarrow$ -1

centroid $= \dfrac{0+1-2-2\sqrt{3}j-2+2\sqrt{3}j+1}{3}$

$= -2/3.$

angle of assymptotes $\dfrac{(2n+1)\pi}{3} = 60, 180, 300 \ldots$

Breakaway points

$s = 0.448, -2.26.$

Routh table. Img. axis cross over

$s^4 + 3s^3 + 12s^2 + s(k-16) + k = 0$

$$\begin{array}{c|cc}
s^4 & 1 & 12 \quad k \\
s^3 & 3 & k-16 \\
s^2 & \frac{52-k}{3} & k \longrightarrow A(s) \\
s & \frac{-k^2+59k-832}{52-k} & \longrightarrow \text{should be } 0. \\
1 & k &
\end{array}$$

$k = \dfrac{59}{2} \pm \dfrac{3\sqrt{17}}{2}$

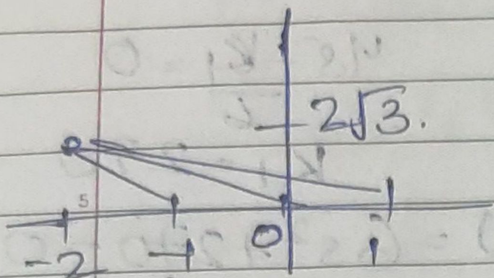
$k = 35.68$, or 23.32.

$A(s) = \dfrac{52-k}{3}s^2 + k = 0$

$s = \sqrt{\dfrac{3k}{k-52}}$

$s = \pm 2.56j$, or $\pm 1.2j$

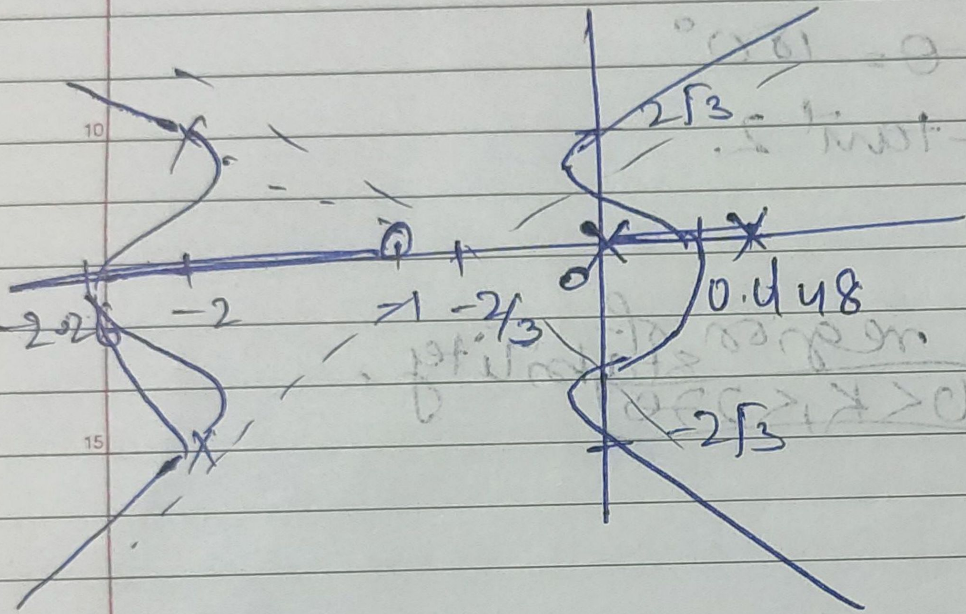
angle of departene.



$180 - \tan^{-1}(2\sqrt{3}) - 180 + \tan^{-1}\sqrt{3}$

$-180 + \tan^{-1}(2/\sqrt{3}) - 90 - \theta$

$\underline{\theta \simeq -54.8^\circ}$



$K - 16 > 0$

$K > 0$

$\frac{52-K}{3} > 0$

$\boxed{16 < K < 52}$

$\frac{K^2 - 59K + 832}{K-52} > 0$

$\boxed{K^2 - 59K + 832 < 0}$

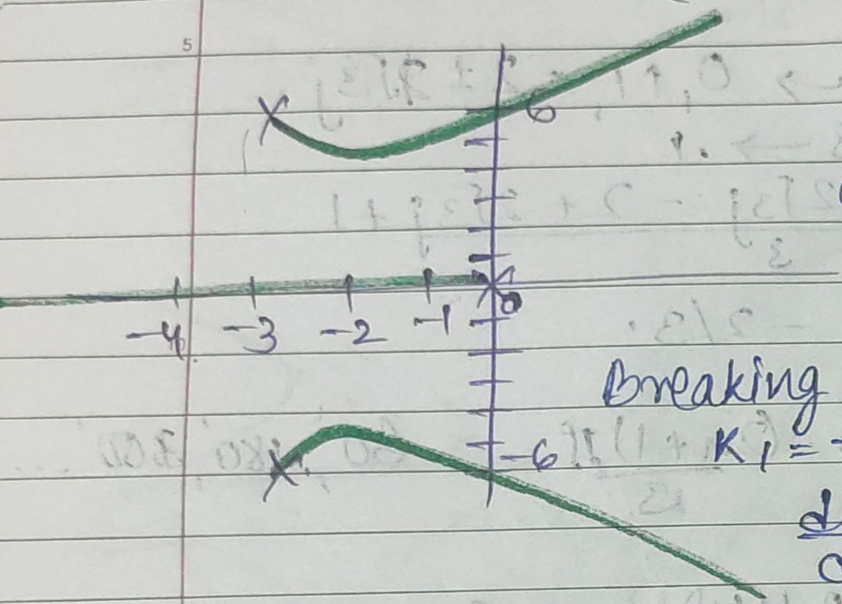
$K > 23.32$ or $K < 35.68$

range for K for stability

$\boxed{23.32 < K < 35.68}$

Q8. $G_1(S) = \frac{1}{S^3+6S^2+45S} = \frac{1}{S(S^2+6S+45)}$ 9 36

$= \frac{1}{S((S+3)^2+6^2)}$



pole
$0, -3-6j, -3+6j$.

centroid = -2.
angle of asymptotes = 60, 180, 300 ...

Breaking pt.
$K_1 = -(S^3+6S^2+45S)$
$\frac{dK_1}{ds} = 0$. $\boxed{S = -2 \pm 3.32j}$
∴ No breaking pt.

$G_2(S) = \frac{0.075S^2 + 8.S + 1}{S^3+3S^2+5S} = \frac{\frac{3}{\frac{1000}{4}}(S^2+S+1.}{S(S^2+3S+5)}$.

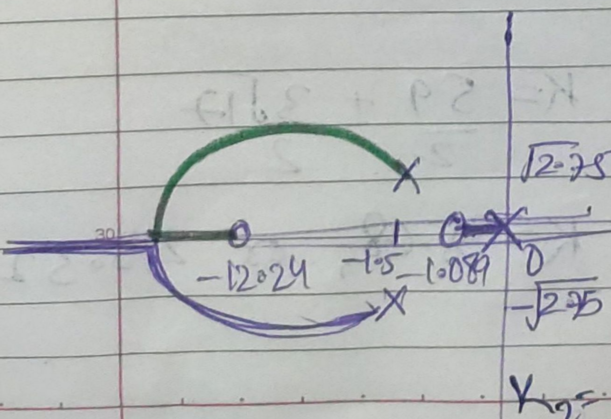
$= \frac{3S^2+40s+40}{40s(s^2+3s+5)}$.

Zero $= \frac{-40 \pm \sqrt{40^2 - 40\times3\times4}}{6}$ $40(40-12)$ 28

$\begin{cases} -1.089, \\ -12.244 \end{cases}$

2.25
2.75

poles
$\left. \begin{array}{l} 0, -1.5+\sqrt{2.75}j \\ -1.5-\sqrt{2.75}j \end{array} \right\}$.



centroid = 10.3
angle of asym = $(2n+1)\pi$
= 180, 360 ...

Breaking away pt
$K_2 = \frac{-(s^3+3s^2+5s)}{(0.075s^2+5+1)}$ $\frac{dK_2}{dS} = 0$ $s = -0.46 \pm 1.06j$, -23.63, -2.123

continued.

Q8 Routh table 1.

s^3	1	45	
s^2	6	K_1	$\rightarrow A(s)$
s	$45 - K_1/6$		$\rightarrow 0$
1	K_1		

$45 - \frac{K_1}{6} = 0$

$K_1 = 270$

$A(s) = 6s^2 + 270 = 0$

$\underline{s = \pm 6.71j}$

angle of departure

$= -\theta_1 - 90 - \theta = 180°$

$\hookrightarrow 180 - \tan^{-1} 2.$

$\underline{\theta = -26.56°}$

$K_1 > 0$

$45 - \frac{K_1}{6} > 0.$ $\boxed{0 < K_1 < 270}$ region of stability.

Routh table 2.

s^3	1	$5 + K_2$	
s^2	$3 + 0.075K_2$	K_2	$\rightarrow A(s).$
s	$\frac{0.075K_2^2 + 2.375K_2 + 15}{3 + 0.075K_2}$		$\rightarrow 0$
1	K_2		

$\hookrightarrow K_2 = -8.71, -22.95$

$A(s) = 3s^2 + 0.075K_2 s^2 + K_2 = 0$

No img. axis cross over

$$\left.\begin{array}{l} 3 + 0.075K_2 > 0 \\ 5 + K_2 > 0 \\ K_2 > 0 \\ \frac{0.075K_2^2 + 2.375K_2 + 15}{3 + 0.075K_2} > 0 \end{array}\right\} \rightarrow \boxed{K_2 > 0}$$

stability region.

$K_2 > 0$

II) value of k for identical characterist

$$1 + K G_1(s) = 1 + K G_2(s).$$

$$s^3 + 6s^2 + 45s + K = s^3 + (3 + 0.075K)s^2 + (5 + K)s + K.$$

$0.075K + 3 = 6$ $\quad$ $K + 5 = 45$

$K = 40$ $\quad$ $K = 40.$

con

Q8 R

s^3

s^2

s

1

angle

dep

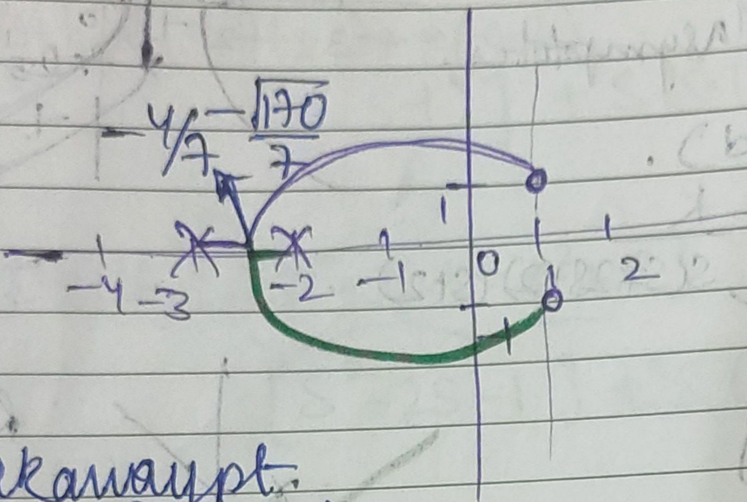
Q9. unity feedback system.

$$G(s) = \frac{K(s^2-2s+2)}{(s+2)(s+3)}$$

poles
$-2, -3$

zeros
$-1-j, -1+j$

BP between 2 poles $(-2, -3)$.



value of K for stability.

breakaway pt.

$$1 + \frac{K(s^2+2s+2)}{(s+2)(s+3)} = 0$$

$$\frac{dk}{ds} = 0 \qquad \frac{-(7s^2+8s-22)}{(s^2-2s+2)^2} = 0 \rightarrow \text{roots} \left\{ \frac{-4}{7} \pm \frac{\sqrt{170}}{7} \right\}$$

characteristic eqn

$$(s+2)(s+3) + k(s^2-2s+2) = 0$$

$$(1+k)s^2 + (5-2k)s + (6+2k) = 0$$

all coef of same sign.

$1+k>0$, $k>-1$ $\qquad 5-2k>0$, $k<2.5$ $\qquad 6+2k>0$, $k>-3$

$$\therefore -1 < k < 2.5$$

$1+k<0$, $k<-1$ $\qquad 5-2k<0$, $k>2.5$ $\qquad 6+2k<0$, $k<-3$

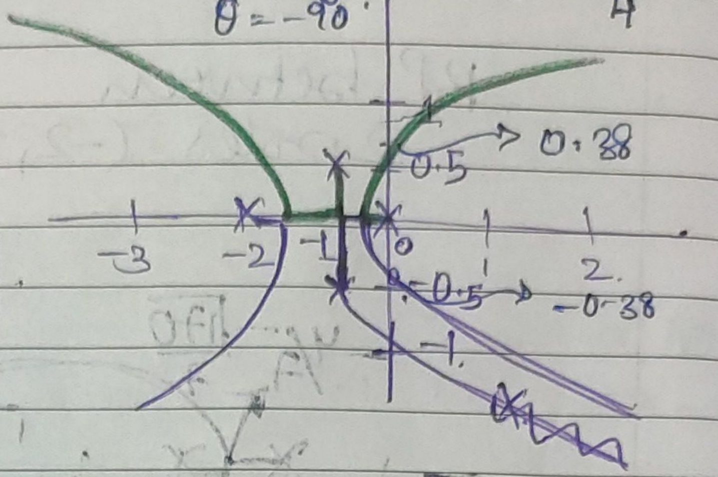
No mutual solution.

centroid $= \boxed{-1}$ → one of breakaway points.

Q 10. characteristic eqⁿ.

(a) $G(s)H(s) = \dfrac{K}{s(s^2+2s+1.25)(s+2)}$

angle of asy $= \dfrac{(2n+1)}{4}\pi$.



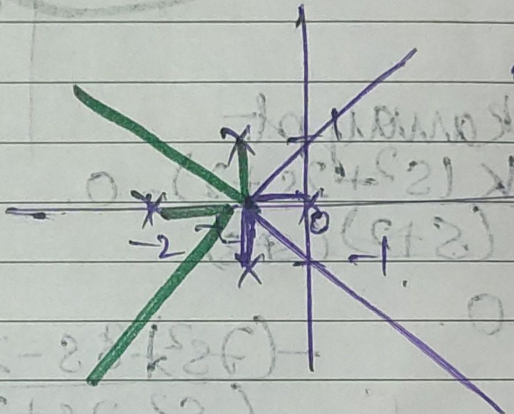
pole

$0, -2, -1+0.5j$
$-1-0.5j$

2 BP betⁿ
0, -2.
real axis.
(imaginary points closer to centroid).

4 asymptotes.

(b) $G(s)H(s) = \dfrac{K}{s(s^2+2s+2)(s+2)}$



RH table
$s = \pm j$
$\theta = 90°$

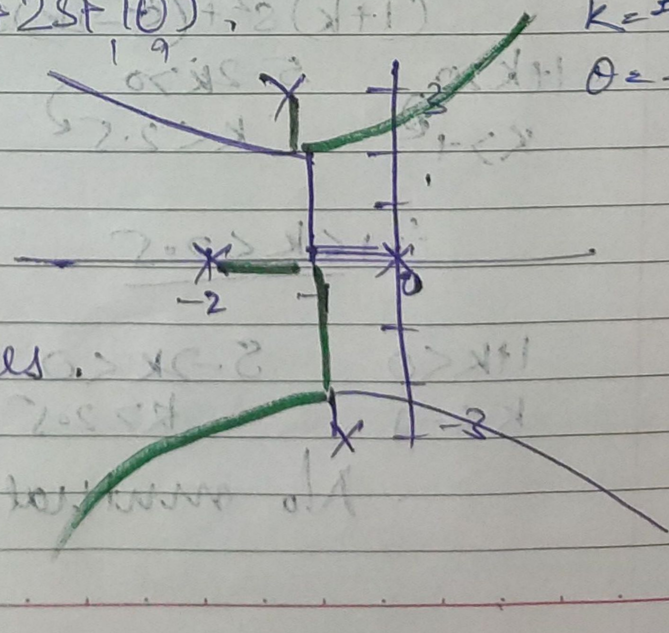
poles
$0, -2, -1-j$
$-1+j$.

BP at centroid -1. } Symmetric 4 assymptotes

(c) $G(s)H(s) = \dfrac{K}{s(s+2)(s^2+2s+10)}$

RH table
$K = \pm\sqrt{3}j$
$\theta = -90°$

poles
$0, -2, -1-3j$
$-1+3j$.



4 asymptodes.

2 BP betⁿ $-1-3j$
$-1+3j$.

~~centroid~~
(real points closer to centroid)