QS1

part1

$$G(s) = \frac{100(s+2)}{s(s+5)(s+10)}$$

```
tf1_open=tf([100,200],[1,15,50])
```

```
tf1_open =

    100 s + 200
  ---------------
  s^2 + 15 s + 50

Continuous-time transfer function.
```

closed loop transfer function

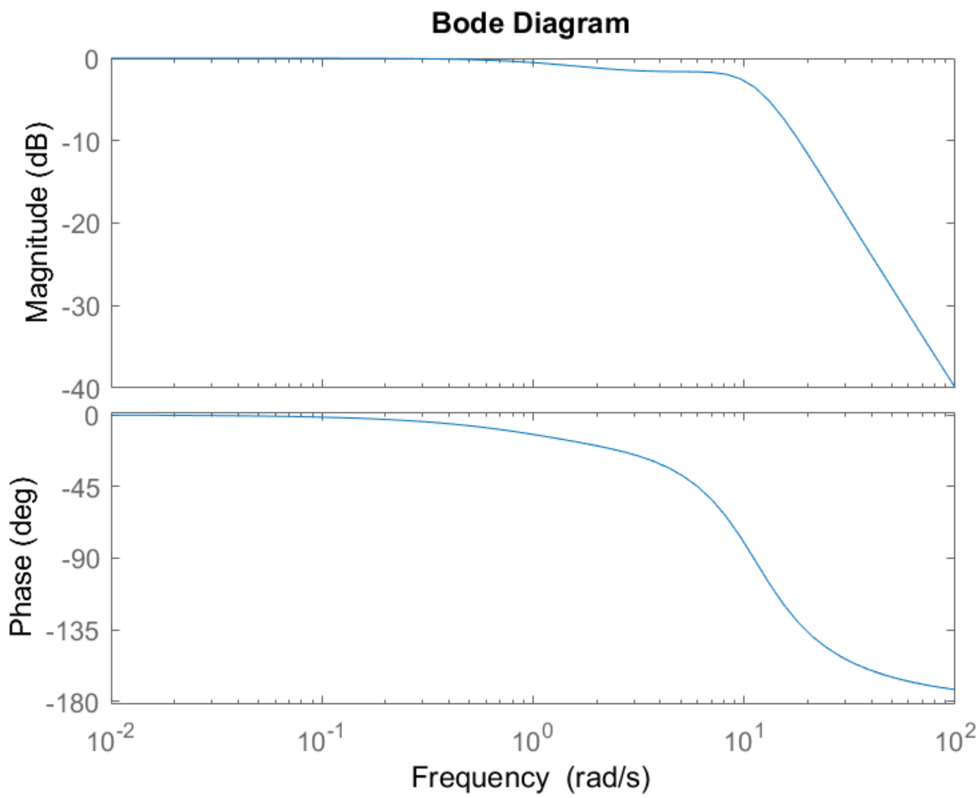
$$\frac{G(s)}{1+G(s)} = \frac{100(s+2)}{s(s+5)(s+10)+100(s+2)}$$

a) Bode plot

```
%Transfer function:
tf1=tf([100,200],[1,15,150,200])
```

```
tf1 =

         100 s + 200
  --------------------------
  s^3 + 15 s^2 + 150 s + 200

Continuous-time transfer function.
```

```
w1=0.01;
w2=100;
bode(tf1,{w1,w2})
```



b) Gain margin, Phase margin, Phase crossover frequency, Gain crossover frequency:-

```
allmargin(tf1)
```

```
ans = struct with fields:
     GainMargin: Inf
    GMFrequency: Inf
    PhaseMargin: -180
    PMFrequency: 0
    DelayMargin: Inf
    DMFrequency: 0
         Stable: 1
```

c) resonant peak, resonance frequency and bandwidth:-

```
[gpeak1,fpeak1]=getPeakGain(tf1) %closed loop property
```

```
gpeak1 = 1.0000
fpeak1 = 0
```

```
bw1=bandwidth(tf1)
```

```
bw1 = 10.4767
```

part2

$$G(s) = \frac{20(s+1)}{s(s+5)(s^2+2s+10)}$$

closed loop transfer function

$$\frac{G(s)}{1+G(s)} = \frac{20(s+1)}{s(s+5)(s^2+2s+10)+20(s+1)}$$

a) Bode plot

```
%Transfer function:
tf2=tf([20,20],[1,7,20,70,20])
```

```
tf2 =

             20 s + 20
  --------------------------------
  s^4 + 7 s^3 + 20 s^2 + 70 s + 20

Continuous-time transfer function.
```

```
bode(tf2,{w1,w2})
```

b) Gain margin, Phase margin, Phase crossover frequency, Gain crossover frequency:-

```
allmargin(tf2)
```

c) resonant peak, resonance frequency and bandwidth:-

```
[gpeak2,fpeak2]=getPeakGain(tf2)
bw2=bandwidth(tf2)
```

part3

$$G(s) = \frac{10(s^2+0.4s+1)}{s(s^2+0.8s+9)}$$

closed loop transfer function

$$\frac{G(s)}{1+G(s)} = \frac{10(s^2+0.4s+1)}{s(s^2+0.8s+9)+10(s^2+0.4s+1)}$$

a) Bode plot:-

```
%Transfer function:
tf3=tf([10,4,10],[1,10.8,13,10])
bode(tf3,{0.01,100})
```

b) Gain margin, Phase margin, Phase crossover frequency, Gain crossover frequency:-

```
allmargin(tf3)
```

c) resonant peak, resonance frequency and bandwidth:-

```
[gpeak3,fpeak3]=getPeakGain(tf3)
bw3=bandwidth(tf3)
```

QS2

open loop $G(s) = \frac{K}{s(s+1)(s+5)}$

closed loop transfer function

$$\frac{G(s)}{1+G(s)} = \frac{K}{s(s+1)(s+5)+K}$$

Transfer functions for K = 10, 20 and 100

```
H1=tf(10,[1,6,5,10])
```

```
H1 =

            10
  ----------------------
  s^3 + 6 s^2 + 5 s + 10

Continuous-time transfer function.
```

```
H2=tf(20,[1,6,5,20])
```

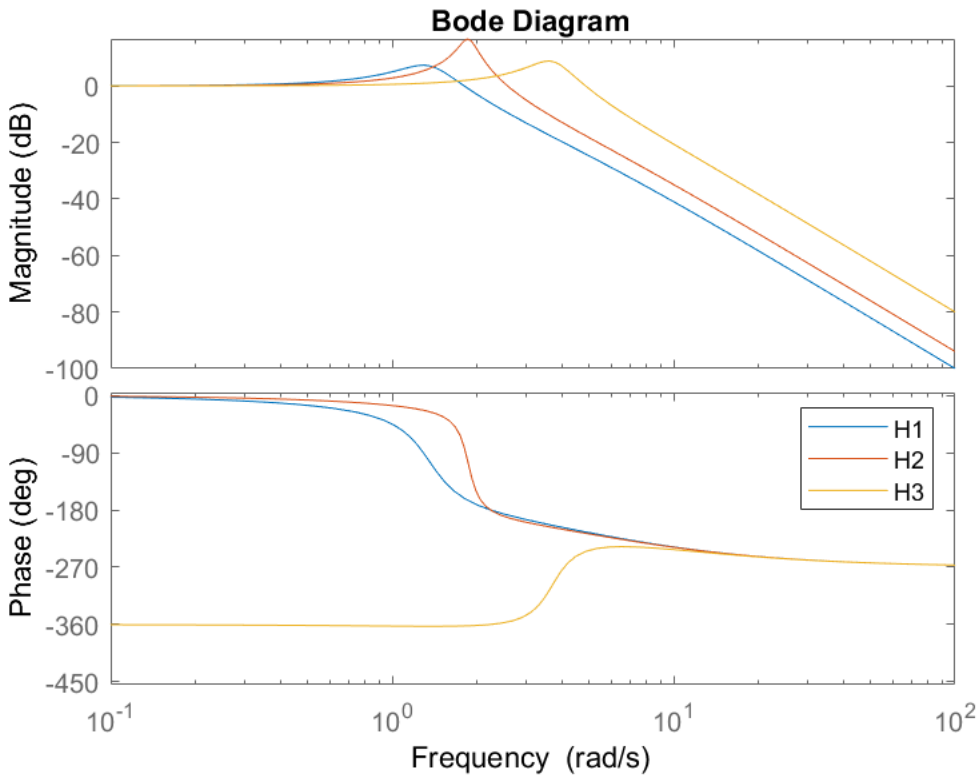
```
H2 =

            20
  ----------------------
  s^3 + 6 s^2 + 5 s + 20

Continuous-time transfer function.
```

```
H3=tf(100,[1,6,5,100])
```

```
H3 =

            100
  -----------------------
  s^3 + 6 s^2 + 5 s + 100

Continuous-time transfer function.
```

a) Bode plot of closed loop systems:-

```
bode(H1,H2,H3)
legend
```



b)

We cannot predict the stability from the frequency response/bode plot of the closed loop system. As we cannot replace s by $j\omega$ in case of K=100 as it is an unstable system which has region of convergence $Re(s) \geq a$ where $a > 0$. Hence we find the gain and phase margin of the open loop system for 3 values of K,

```
H1=tf(10,[1,6,5,0]);
H2=tf(20,[1,6,5,0]);
H3=tf(100,[1,6,5,0]);
%For H1(K=10):
allmargin(H1)
```

```
ans = struct with fields:
      GainMargin: 3
     GMFrequency: 2.2361
     PhaseMargin: 25.3898
     PMFrequency: 1.2271
     DelayMargin: 0.3611
     DMFrequency: 1.2271
          Stable: 1
```

```
%For H2(K=20):
allmargin(H2)
```

```
ans = struct with fields:
      GainMargin: 1.5000
     GMFrequency: 2.2361
     PhaseMargin: 8.9095
     PMFrequency: 1.8147
     DelayMargin: 0.0857
```

```
    DMFrequency: 1.8147
         Stable: 1
```

```
%For H3(K=100):
allmargin(H3)
```

```
ans = struct with fields:
     GainMargin: 0.3000
    GMFrequency: 2.2361
    PhaseMargin: -23.6504
    PMFrequency: 3.9073
    DelayMargin: 1.5024
    DMFrequency: 3.9073
         Stable: 0
```

thus gain and phase margin for K=10, 20 is +ve and also for those values of K the closed loop system is stable.

But for K=100, both gain and phase margins are -ve, so the closed loop system is unstable.

c)

We have to find the minimum value of K for which the system becomes unstable, i.e. the gain margin becomes equal to 0dB and phase margin becomes equal to 0°. At phase crossover, imaginary part of the open loop expression should be 0 when we put $j\omega$ in place of s. S0, $\omega^3 = 5\omega$ and hence, at limiting point, $\omega_{cg} = \omega_{cp} = \sqrt{5}$ and $\frac{K}{-6\omega^2} = -1$ as gain margin is also 0dB. So minimum value of K for which the system is unstable is K=30.

```
k=29.9;
H=tf(k,[1,6,5,0]);
allmargin(H)
```

```
ans = struct with fields:
     GainMargin: 1.0033
    GMFrequency: 2.2361
    PhaseMargin: 0.0713
    PMFrequency: 2.2323
    DelayMargin: 5.5762e-04
    DMFrequency: 2.2323
         Stable: 1
```

```
k=30;
H=tf(k,[1,6,5,0]);
allmargin(H)
```

```
ans = struct with fields:
     GainMargin: 1.0000
    GMFrequency: 2.2361
    PhaseMargin: 9.5374e-06
    PMFrequency: 2.2361
    DelayMargin: 7.4443e-08
    DMFrequency: 2.2361
         Stable: 0
```

After K=30, it has approximately 0 gain and phase margin and same crossover frequencies and closed loop system becomes unstable. At K=30 the closed loop system is marginally stable.

d)

$$G(j\omega) = \frac{K}{-6\omega^2 + j(5\omega - \omega^3)}$$

phase margin = $\frac{\pi}{6}$, Hence, $tan(\frac{7\pi}{6}) = \frac{5-\omega^2}{6\omega}$

So gain crossover frequency is given by the equation,

$$\omega_{cg}^2 + 2\sqrt{3}\,\omega_{cg} - 5 = 0 \;\therefore\; \omega_{cg} = 2\sqrt{2} - \sqrt{3} = 1.0964$$

Also the gain = 1

at $\omega = \omega_{cg}$

$$K = |-6\omega^2 + j(5\omega - \omega^3)| = 8.328$$

$$\omega_{cp} = \sqrt{5}$$

$$\omega = \omega_{cp}$$

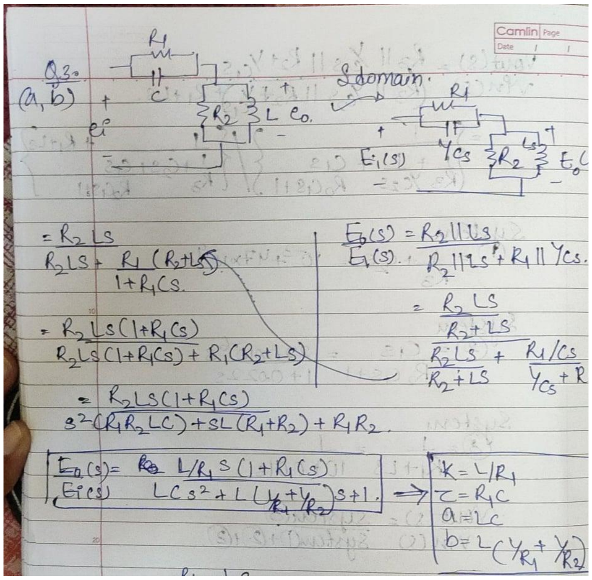
Gain margin = $\frac{6\omega^2}{K} = \frac{30}{8.32798} = 3.60$

Hence Gain margin = 11.1316dB

```
k=8.328;
H=tf(k,[1,6,5,0]);
allmargin(H)
```

```
ans = struct with fields:
      GainMargin: 3.6023
     GMFrequency: 2.2361
     PhaseMargin: 30.0000
     PMFrequency: 1.0964
     DelayMargin: 0.4776
     DMFrequency: 1.0964
          Stable: 1
```

QS3



Q3. (a, b)

S domain

$$\frac{E_o(s)}{E_i(s)} = \frac{R_2 \| Ls}{R_2 \| Ls + R_1 \| 1/Cs}$$

$$= \frac{\frac{R_2 Ls}{R_2 + Ls}}{\frac{R_2 Ls}{R_2 + Ls} + \frac{R_1/Cs}{1/Cs + R_1}}$$

$$= \frac{R_2 Ls}{R_2 Ls + \frac{R_1 (R_2 + Ls)}{1 + R_1 Cs}}$$

$$= \frac{R_2 Ls(1 + R_1 Cs)}{R_2 Ls(1 + R_1 Cs) + R_1(R_2 + Ls)}$$

$$= \frac{R_2 Ls(1 + R_1 Cs)}{s^2(R_1 R_2 LC) + sL(R_1 + R_2) + R_1 R_2}$$

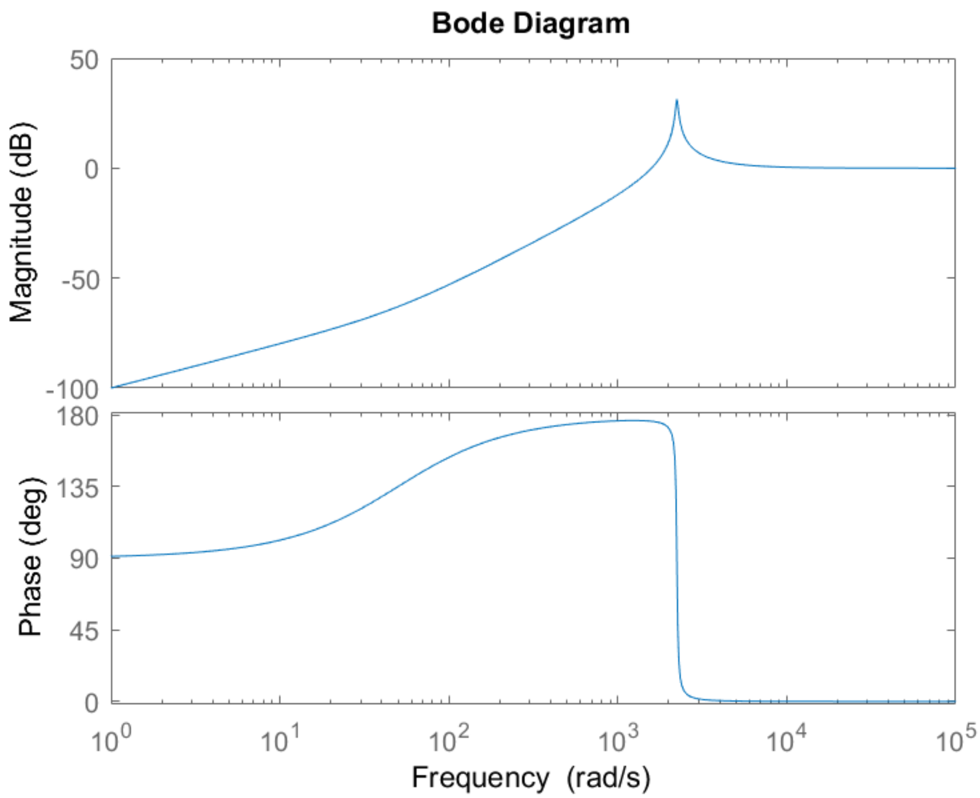
$$\boxed{\frac{E_o(s)}{E_i(s)} = \frac{L/R_1 \, s(1 + R_1 Cs)}{LCs^2 + L(1/R_1 + 1/R_2)s + 1}} \rightarrow \begin{array}{l} k = L/R_1 \\ \tau = R_1 C \\ a = LC \\ b = L(1/R_1 + 1/R_2) \end{array}$$

c)

```
% these system are defined from the calculations shown at end

k=1e-5;
t=0.02;
a=2e-7;
b=1.2e-5;
G=tf([k*t,k,0],[a,b,1])
```

```
G =

     2e-07 s^2 + 1e-05 s
  -------------------------
  2e-07 s^2 + 1.2e-05 s + 1

Continuous-time transfer function.
```

```
bode(G)
```



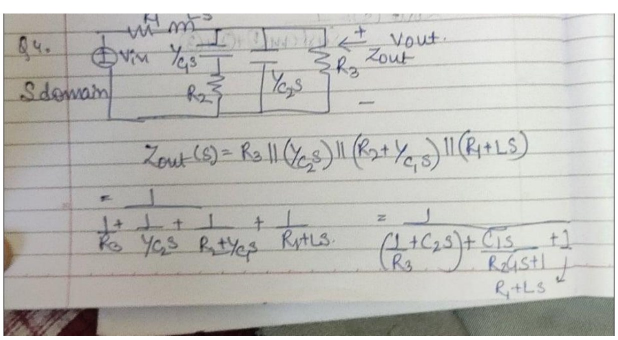
It acts as a high-pass filter. As the phase never crosses -180° so, we cannot calculate gain margin. There is a point before the resonant peak where the gain = 1.

```
allmargin(G)
```

```
ans = struct with fields:
     GainMargin: [1×0 double]
    GMFrequency: [1×0 double]
    PhaseMargin: [-3.9847 -180]
    PMFrequency: [1.5812e+03 Inf]
    DelayMargin: [0.0039 0]
    DMFrequency: [1.5812e+03 Inf]
         Stable: 1
```

So at $\omega = 1.5812 \times 10^3$, the gain = 1(Gain crossover frequency), and the phase margin is equal to -3.9847°.

QS4



$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{R_3 \parallel \frac{1}{C_2 s} \parallel R_2 + \frac{1}{C_1 s}}{\left(R_3 \parallel \frac{1}{C_2 s} \parallel R_2 + \frac{1}{C_1 s}\right) + R_1 + Ls}$$

$$= \left\{ \frac{1}{\frac{1}{R_3} + \frac{1}{1/C_2 s} + \frac{C_1 s}{R_2 C_1 s + 1}} \right\} \Big/ \left\{ \frac{1}{\frac{1}{R_3} + C_2 s + \frac{C_1 s}{R_2 C_1 s + 1}} + R_1 + Ls \right\}$$

System (1) $= \frac{1}{R_3} + C_2 s = 10^{-3} + 47\times10^{-9} s$

System (2) $= \frac{C_1 s}{R_2 C_1 s + 1} = \frac{220\times10^{-6} s}{1 + 0.022 s}$

System (3) $= \frac{1}{R_1 + Ls} = \frac{1}{10 + 0.01 s}$

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{\text{system (3)}}{\text{system (1)} + (2) + (3)}$$

$$Z_{out}(s) = \frac{1}{\text{System (1)} + (2) + (3)}$$

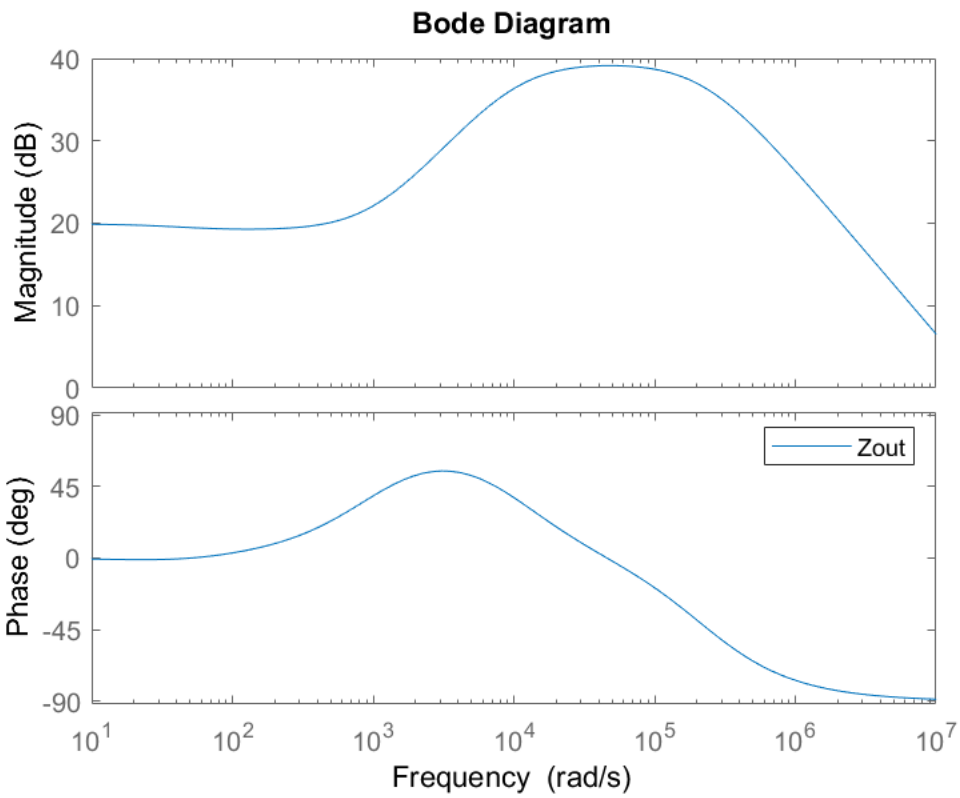
```
% these system are defined from the calculations shown at end

sys1=tf([47e-9,1e-3],1);
sys2=tf([220e-6,0],[0.022,1]);
sys3=tf(1,[0.01,10]);
Zout=1/(sys1+sys2+sys3)
```

```
Zout =

             0.00022 s^2 + 0.23 s + 10
  ---------------------------------------------
  1.034e-11 s^3 + 2.431e-06 s^2 + 0.02443 s + 1.01

Continuous-time transfer function.
```

```
H=sys3/(sys1+sys2+sys3) %Vo/Vin
```

```
H =

                     0.00022 s^2 + 0.23 s + 10
  ----------------------------------------------------------
  1.034e-13 s^4 + 2.441e-08 s^3 + 0.0002686 s^2 + 0.2544 s + 10.1

Continuous-time transfer function.
```

```
bode(Zout)
legend
```




```
bode(H)
legend
```

Thus the circuit works as a low-pass filter.

```
bandwidth(H)
```

```
ans = 8.5241e+03
```

Hence corner frequency of the low-pass filter(-3dB frequency) = **8524.1 rad/sec** = **1.357KHz**

QS5

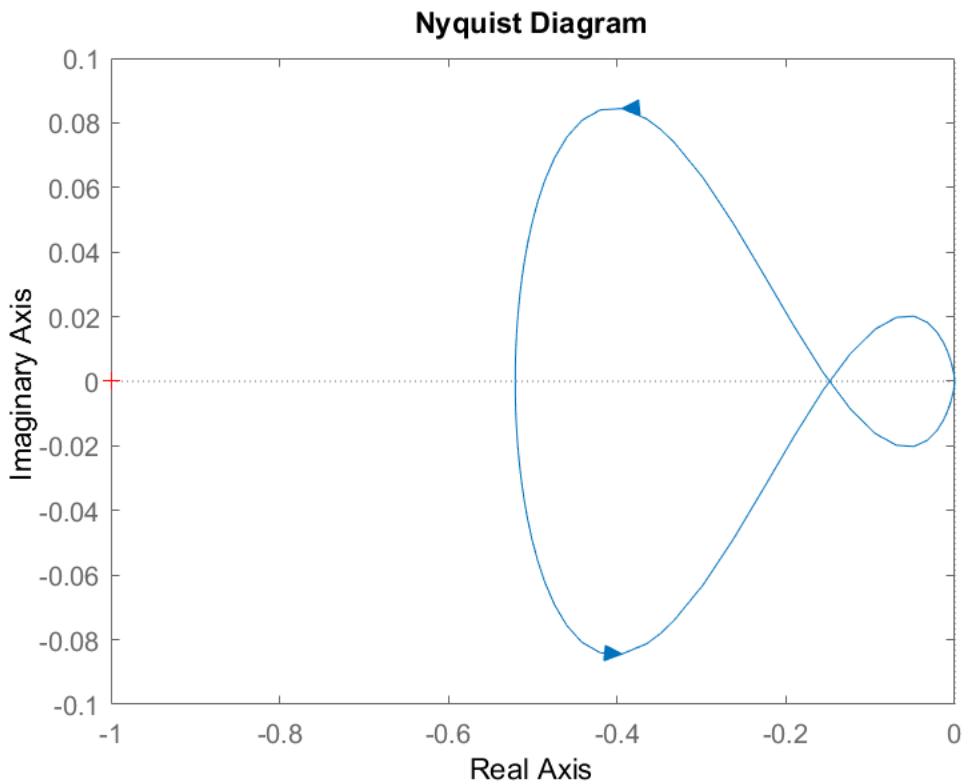
a)

$G(s) = \dfrac{K}{(s+8)(s+6)(s-2)}$ for $K = 50, 100, 336, 350$

Number of open loop poles in RHP, $P = 1$

<u>K=50</u>

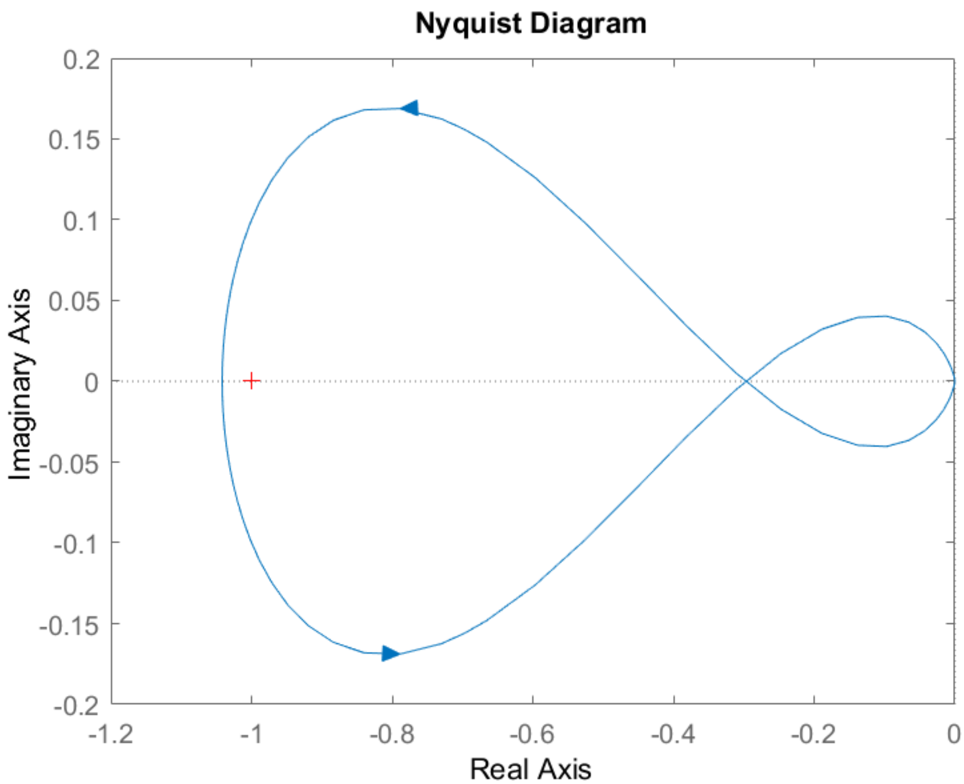
```
nyquist(zpk([],[-8,-6,2],50));
```



The Nyquist plot doesn't encircle the point -1 so $N = 0$. Hence number of zeros of 1+G in RHP is N+P=1. Hence the closed-loop system is **unstable**.

<u>K=100</u>

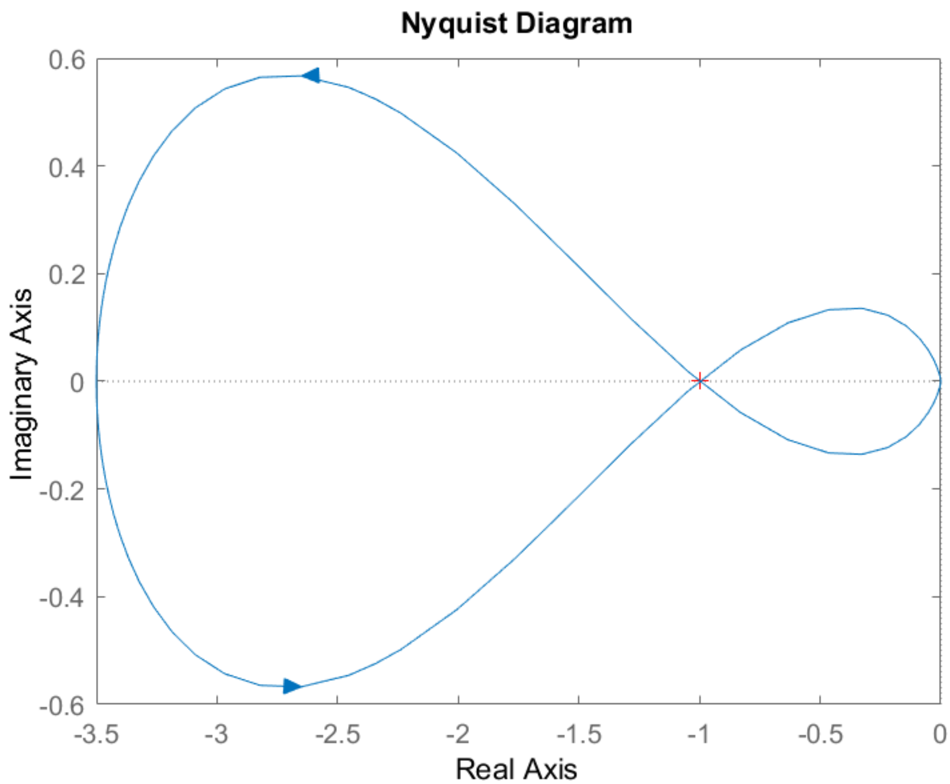
```
nyquist(zpk([],[-8,-6,2],100));
```



One CCW encirclement of -1, hence $N = -1$. Number of zeros of 1+G in RHP is N+P=0. So, the closed-loop system is **stable**.

<u>K=336</u>

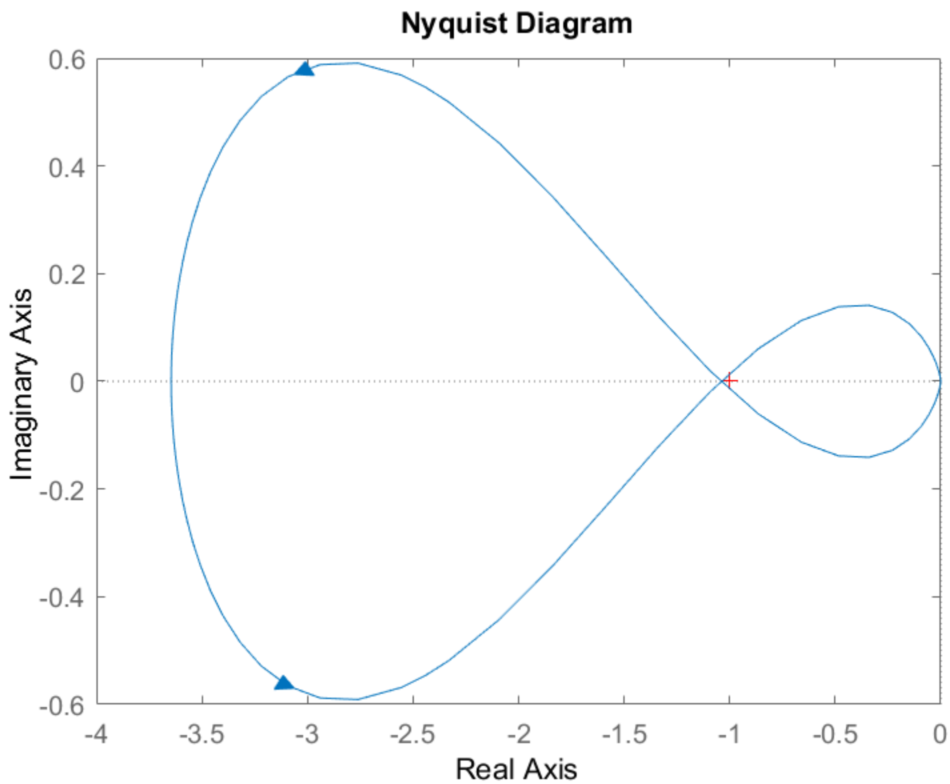
```
nyquist(zpk([],[-8,-6,2],336));
```



The nyquist plot passes through -1, so the system has phase margin and gain margin(in dB) both equal to 0. Also the point is on a CCW loop of the nyquist plot so $N = -1$.Number of open loop zeros of 1+G in RHP is N+P=0. Closed-loop system is **marginally stable**.

K=350

```
nyquist(zpk([],[-8,-6,2],350));
```



One CW encirclement of -1, hence $N = 1$. Number of zeros of 1+G in RHP is N+P=2. So, the closed-loop system is **unstable**.

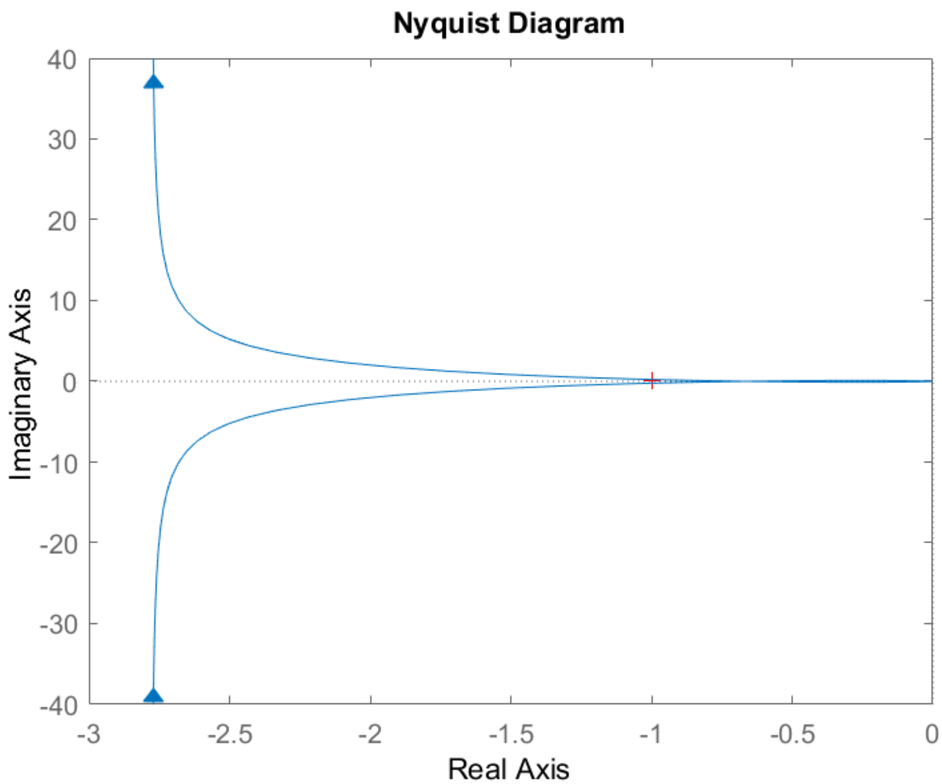
b)

$G(s) = \dfrac{K}{s(s+3)(s+2)}$ for $K = 20, 30, 100$

Number of open loop RHP poles, $P = 0$

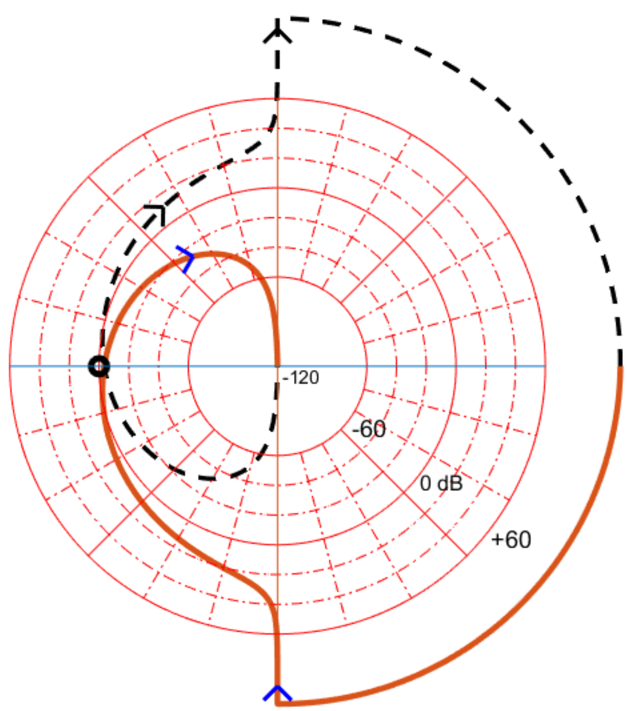
<u>K=20</u>

```
nyquist(zpk([],[0,-3,-2],20));
```




```
nyqlog(zpk([],[0,-3,-2],20));
```

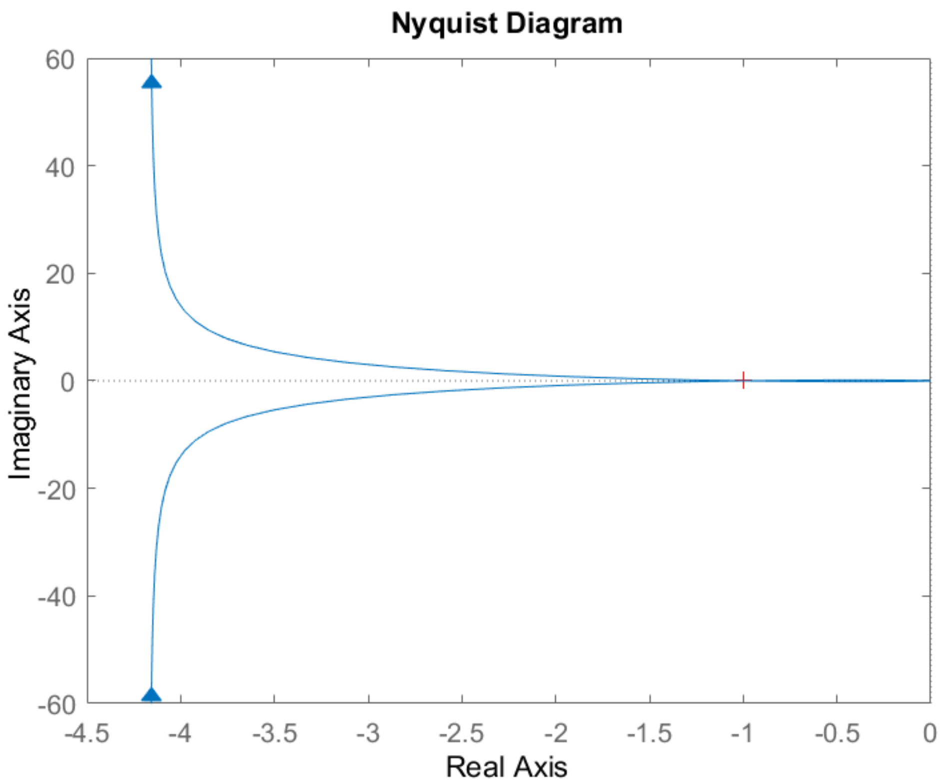
```
Number of poles in RHP of open-loop system: 0
Number of net encirclements around the -1 point:   0
=> Number of poles in RHP of closed-loop system:   0
and no closed-loop poles on Im-axis
=> Closed-loop-system is asymptotically stable
```



As the system has open loop pole on imaginary axis, we used the user-built 'nyqlog' function to determine the complete nyquist diagram including it's behaviour at infinity. As we observe, nyquist plot doesn't encircle -1, so $N = 0$. Hence number of open loop zeros of 1+G in RHP is N+P=0. Closed-loop system is **stable**.

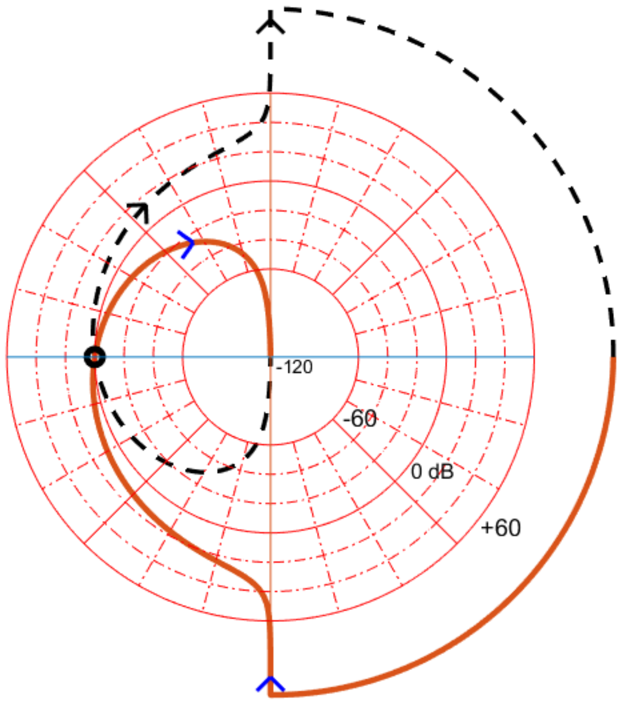
K=30

```
nyquist(zpk([],[0,-3,-2],30));
```




```
nyqlog(zpk([],[0,-3,-2],30));
```

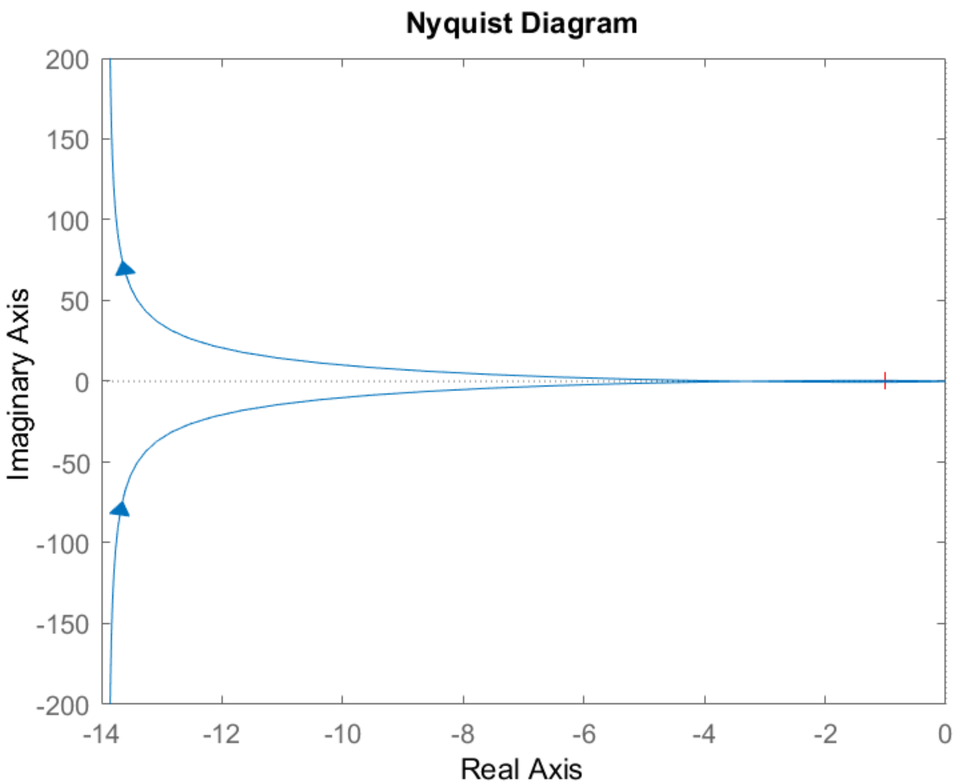
```
Number of poles in RHP of open-loop system: 0
Number of net encirclements around the -1 point:   0
=> Number of poles in RHP of closed-loop system:   0
and no closed-loop poles on Im-axis
=> Closed-loop-system is asymptotically stable
```



As we observe, nyquist plot passes through -1, also it doesn't encircle the -1 point. So $N = 0$. Number of open loop zeros of 1+G in RHP is N+P=0. But as the nyquist plot passes through -1 point, closed-loop system is **marginally stable**.

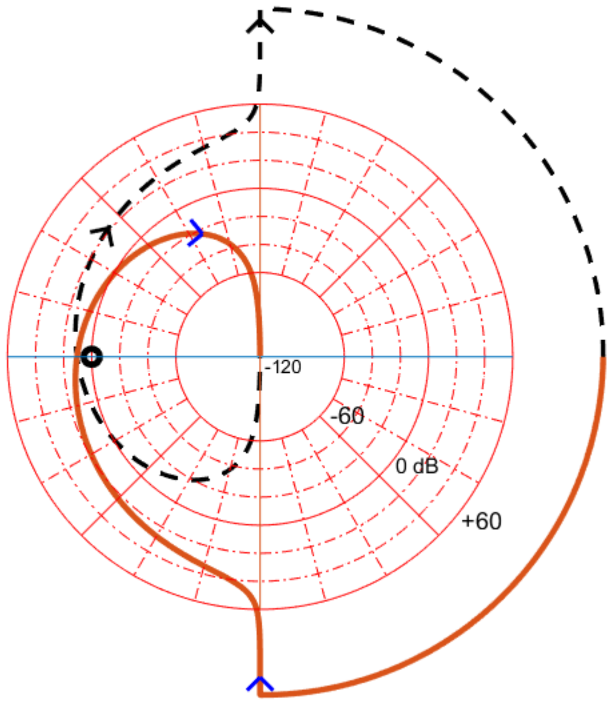
K=100

```
nyquist(zpk([],[0,-3,-2],100));
```




```
nyqlog(zpk([],[0,-3,-2],100));
```

```
Number of poles in RHP of open-loop system: 0
Number of net encirclements around the -1 point:   2
=> Number of poles in RHP of closed-loop system:   2
=> Closed-loop-system is unstable
```



2 CW encirclement of -1, hence $N = 2$. Number of zeros of 1+G in RHP is N+P=2. So, the closed-loop system is **unstable**.

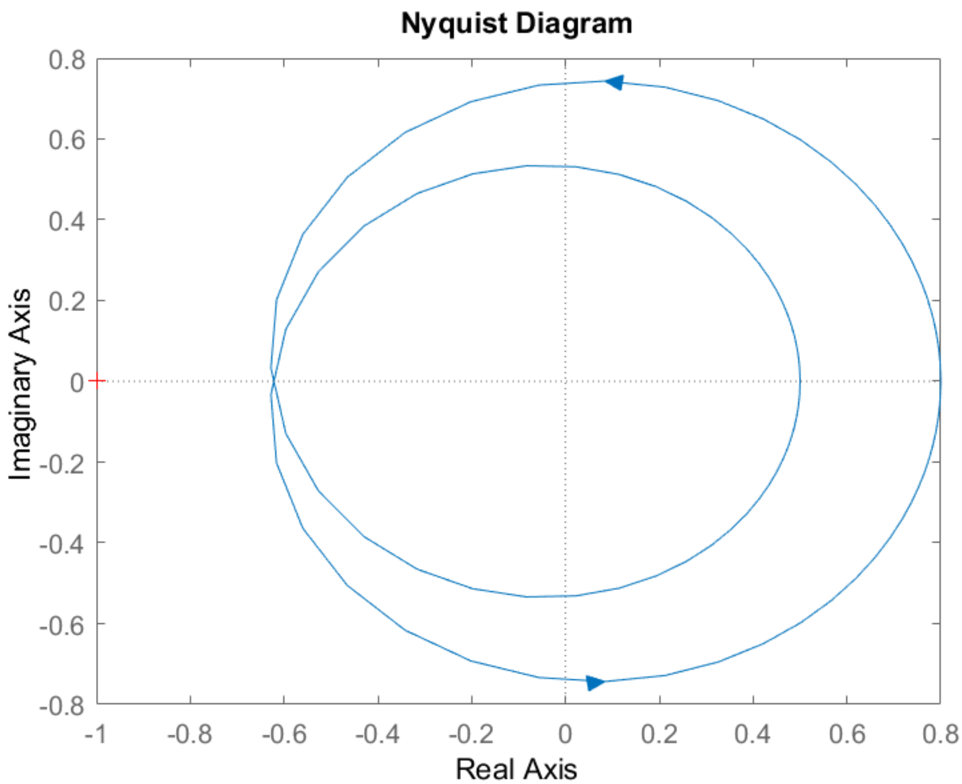
c)

$G(s) = \dfrac{K(s^2 + 10s + 24)}{s^2 - 8s + 15}$, for $K = 0.5, 0.8, 1$

Number of open loop RHP poles, P=2

<u>K=0.5</u>

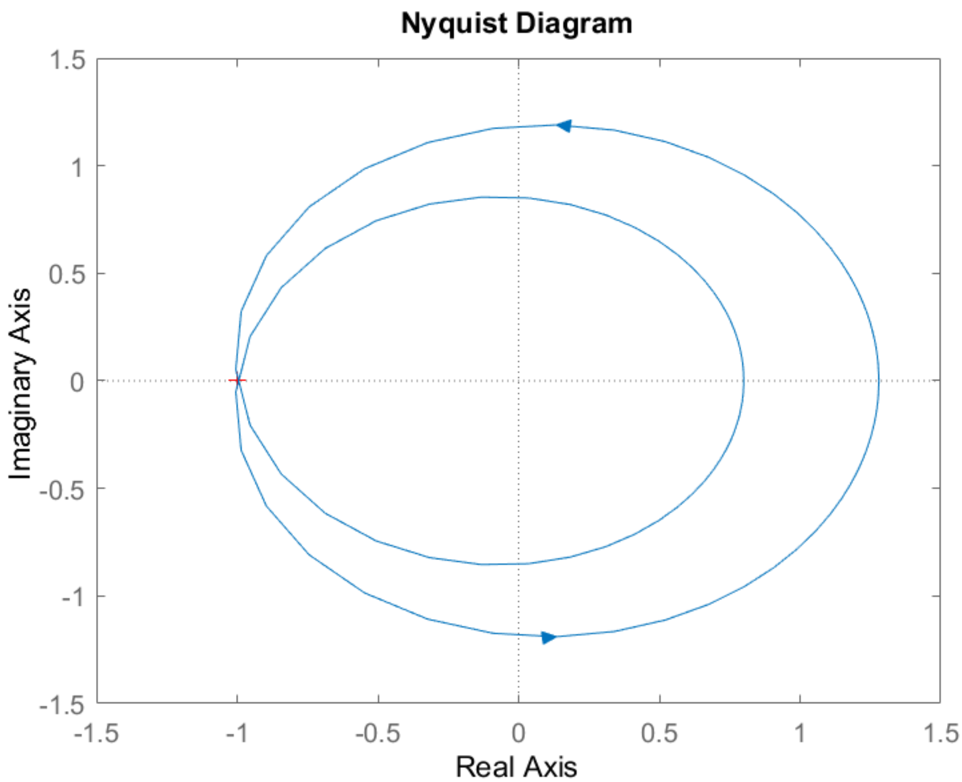
```
nyquist(tf([0.5,5,12],[1,-8,15]));
```



The nyquist plot doesn't encircle the -1 point, hence $N = 0$. Number of zeros of 1+G in RHP is N+P=2. So, the closed-loop system is **unstable**.

K=0.8

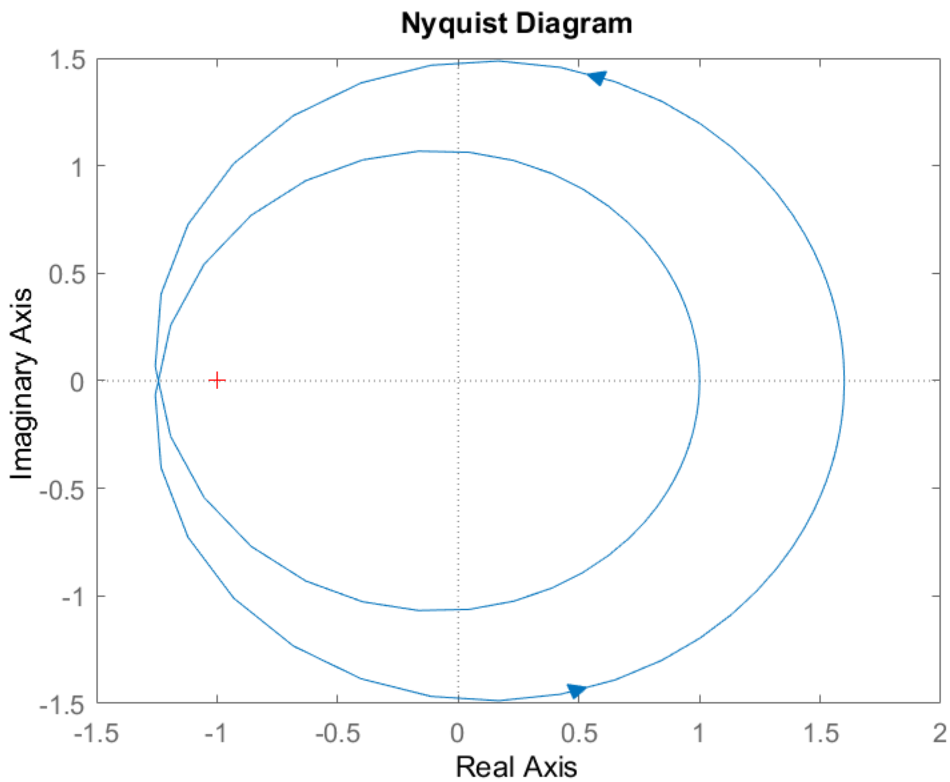
```
nyquist(tf([0.8,8,19.2],[1,-8,15]));
```



The nyquist plot passes through -1, also the point is on 2 CCW loop of the nyquist plot, so $N=-2$. Number of open loop zeros of 1+G in RHP is N+P=0. But as the nyquist plot passes through -1 point, closed-loop system is **marginally stable**.

K=1

```
nyquist(tf([1,10,24],[1,-8,15]));
```



2 CCW encirclement of the -1 point, so $N=-2$. Number of open loop zeros of 1+G in RHP is N+P=0. Hence, the closed-loop system is **stable**.

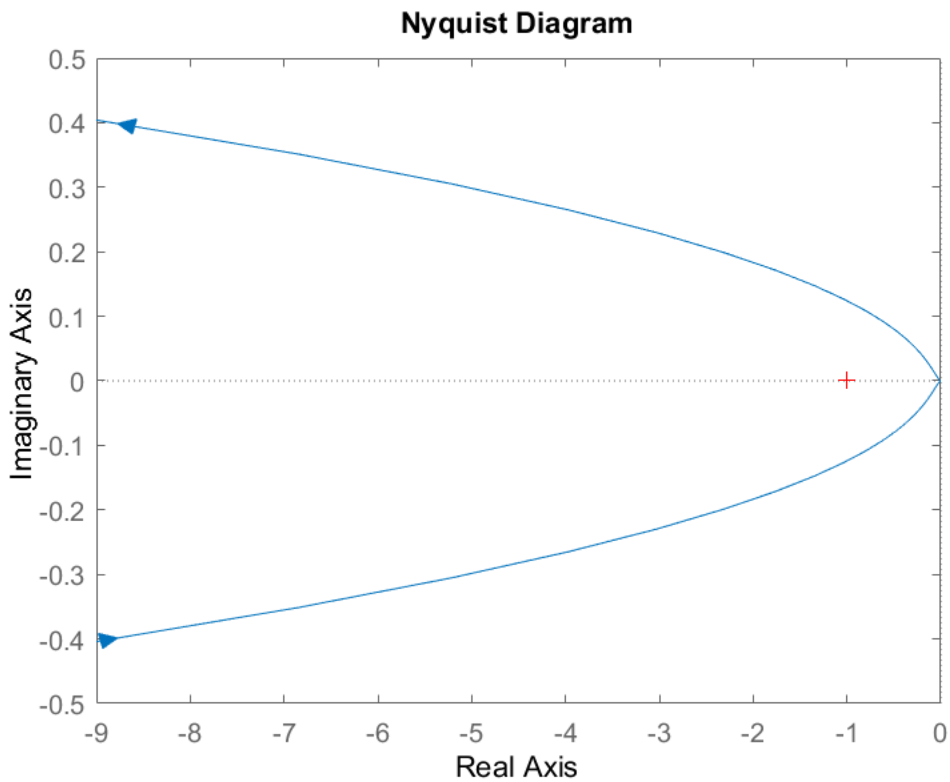
d)

$G(s)=\dfrac{10(s+p)}{s^2(s+3)}$ for $p=2,4$

Number of open loop poles in RHP, $P=0$.

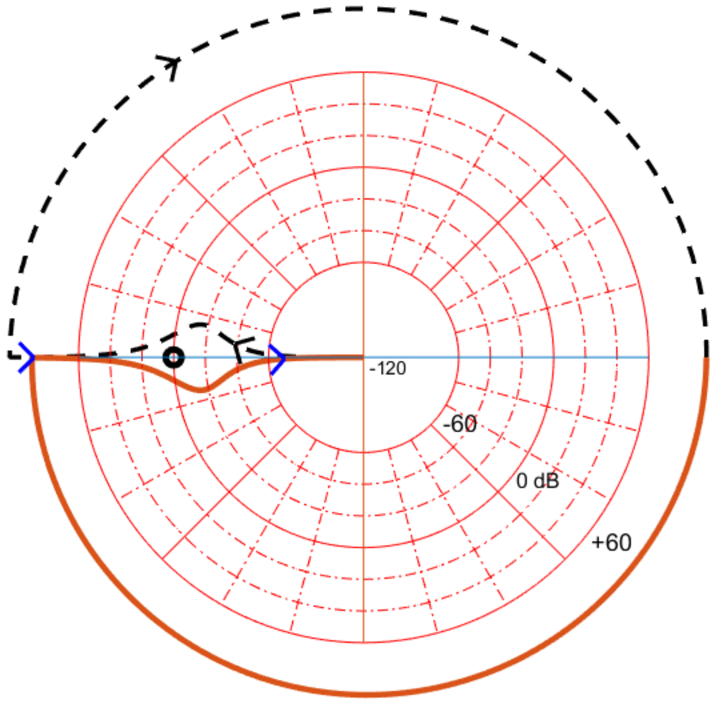
<u>p=2</u>

```
nyquist(zpk(-2,[0,0,-3],1));
```




```
nyqlog(zpk(-2,[0,0,-3],1));
```

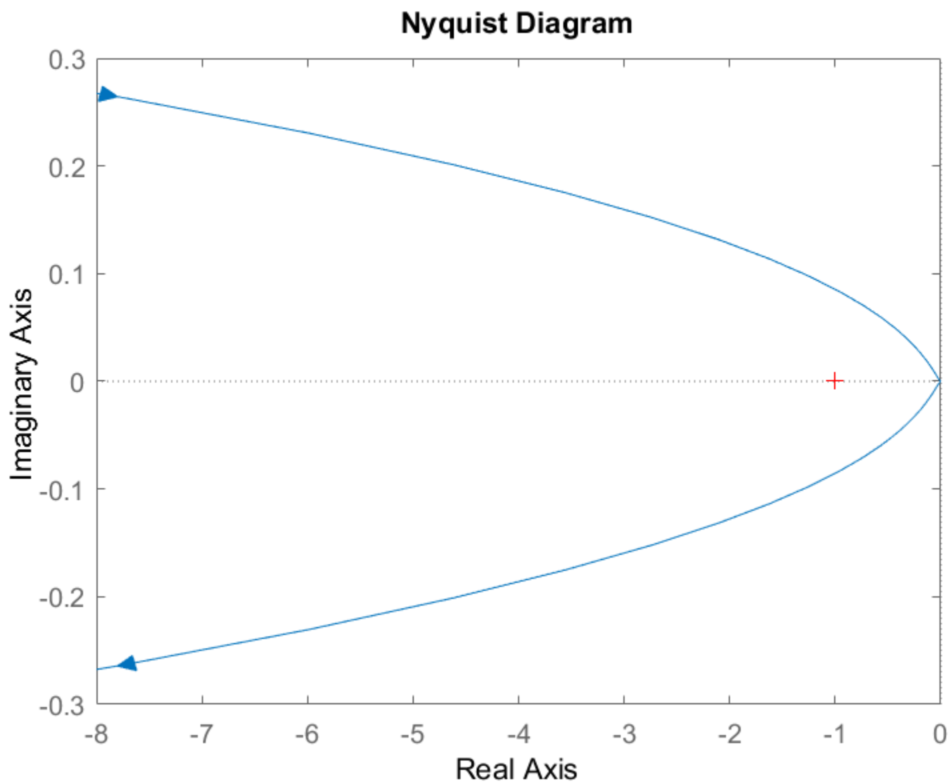
```
Number of poles in RHP of open-loop system: 0
Number of net encirclements around the -1 point:   0
=> Number of poles in RHP of closed-loop system:   0
and no closed-loop poles on Im-axis
=> Closed-loop-system is asymptotically stable
```



The nyquist plot doesn't encircle the -1 point, so $N = 0$. Number of open loop zeros of 1+G in RHP is N+P=0. Hence, the closed-loop system is **stable**.

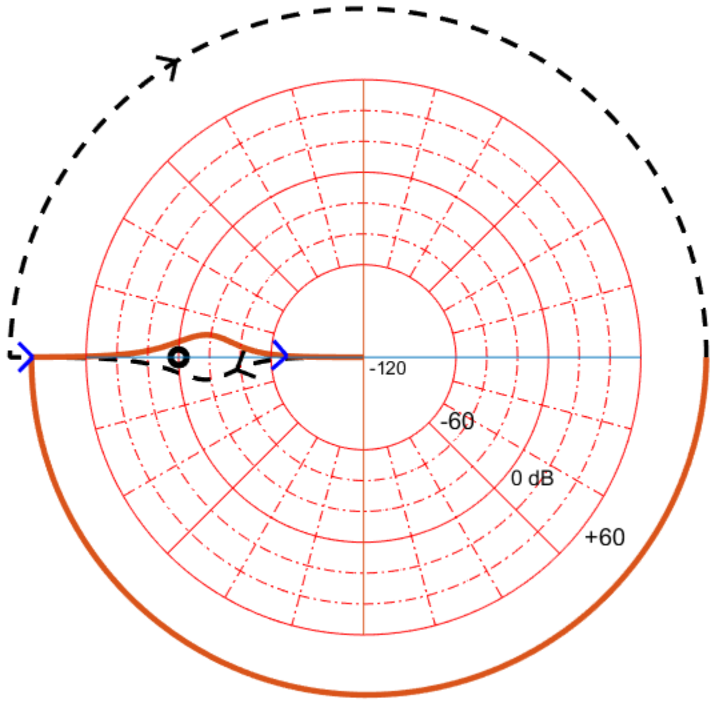
p=4

```
nyquist(zpk(-4,[0,0,-3],1));
```




```
nyqlog(zpk(-4,[0,0,-3],1));
```

```
Number of poles in RHP of open-loop system: 0
Number of net encirclements around the -1 point:   2
=> Number of poles in RHP of closed-loop system:   2
=> Closed-loop-system is unstable
```



2 CW encirclement of the -1 point, so $N = 2$. Number of open loop zeros of 1+G in RHP is N+P=2. Hence, the closed-loop system is **unstable**.

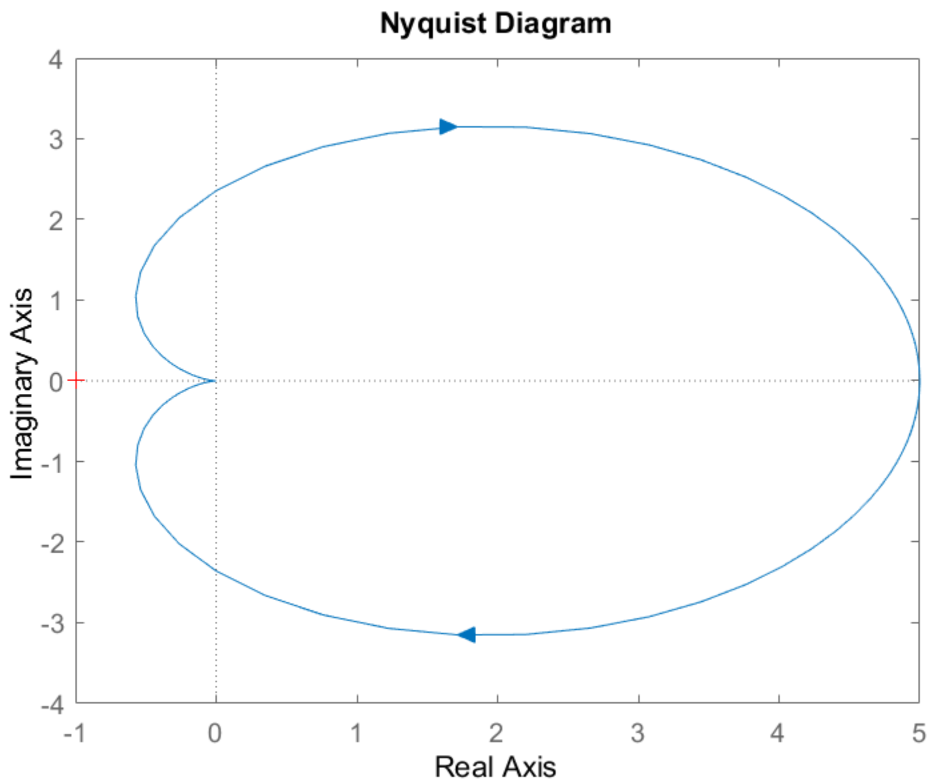
e)

$G(s) = \dfrac{90}{(s+3)(s+6)} e^{-ps}$ for $p = 0, 0.05, 0.5$

Number of open loop poles in RHP, $P = 0$.

p=0

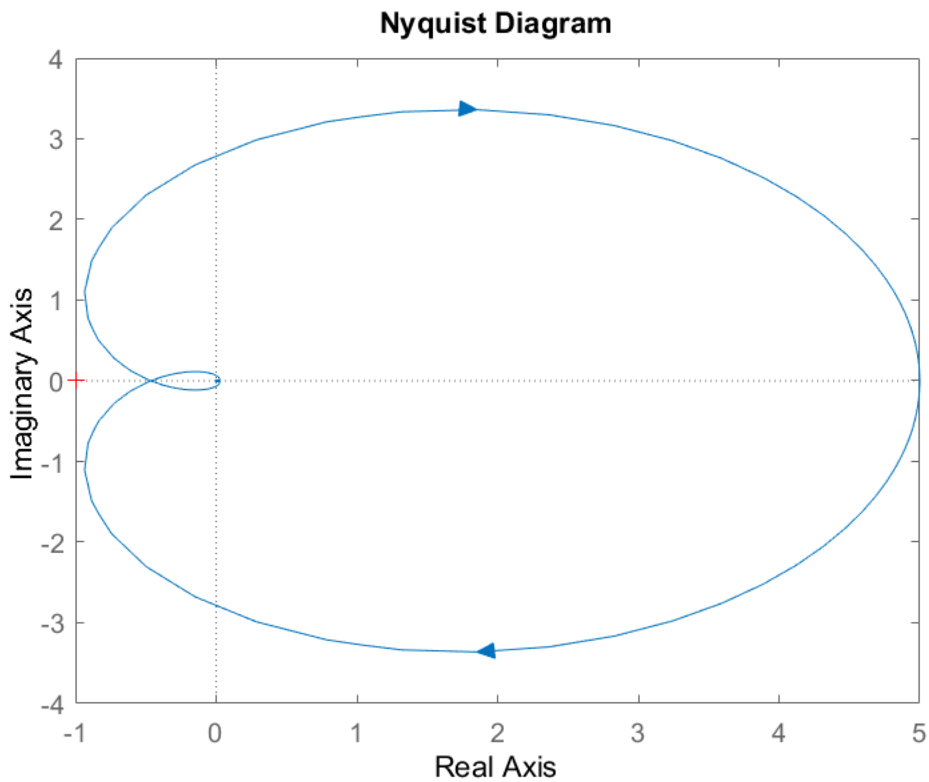
```
nyquist(tf(90,[1,9,18],'InputDelay',0));
```



The nyquist plot doesn't encircle the -1 point, so $N = 0$. Number of open loop zeros of 1+G in RHP is N+P=0. Hence, the closed-loop system is **stable**.

<u>p=0.05</u>

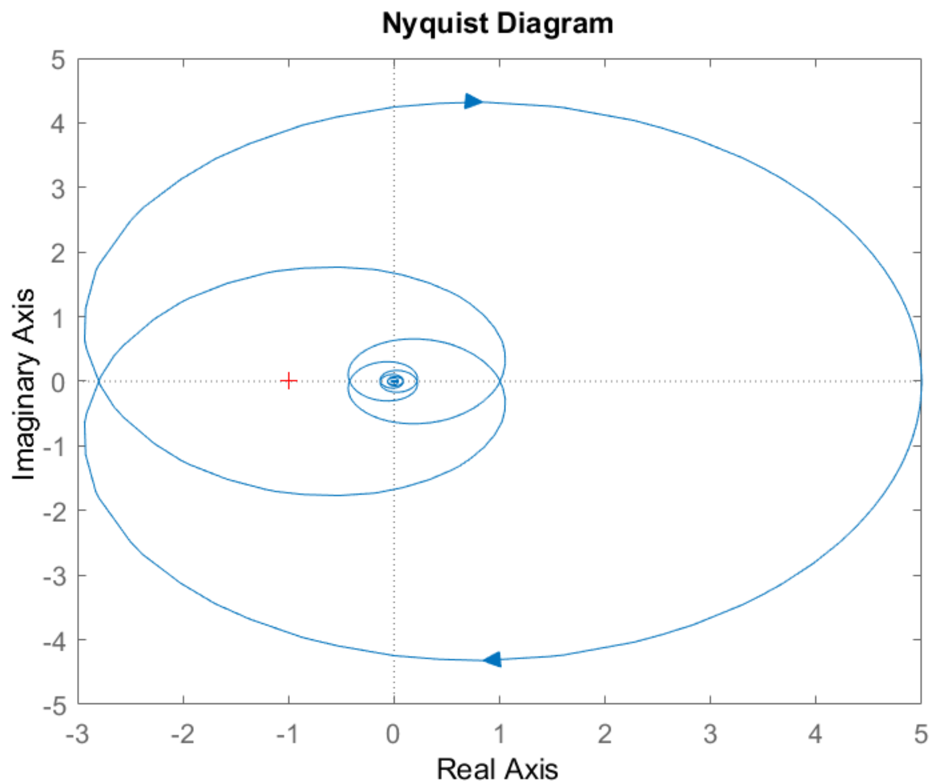
```
nyquist(tf(90,[1,9,18],'InputDelay',0.05));
```



The nyquist plot doesn't encircle the -1 point, so $N = 0$. Number of open loop zeros of 1+G in RHP is N+P=0. Hence, the closed-loop system is **stable**.

<u>p=0.5</u>

```
nyquist(tf(90,[1,9,18],'InputDelay',0.5));
```



2 CW encirclement of the -1 point, so $N = 2$. Number of open loop zeros of 1+G in RHP is N+P=2. Hence, the closed-loop system is **unstable**.