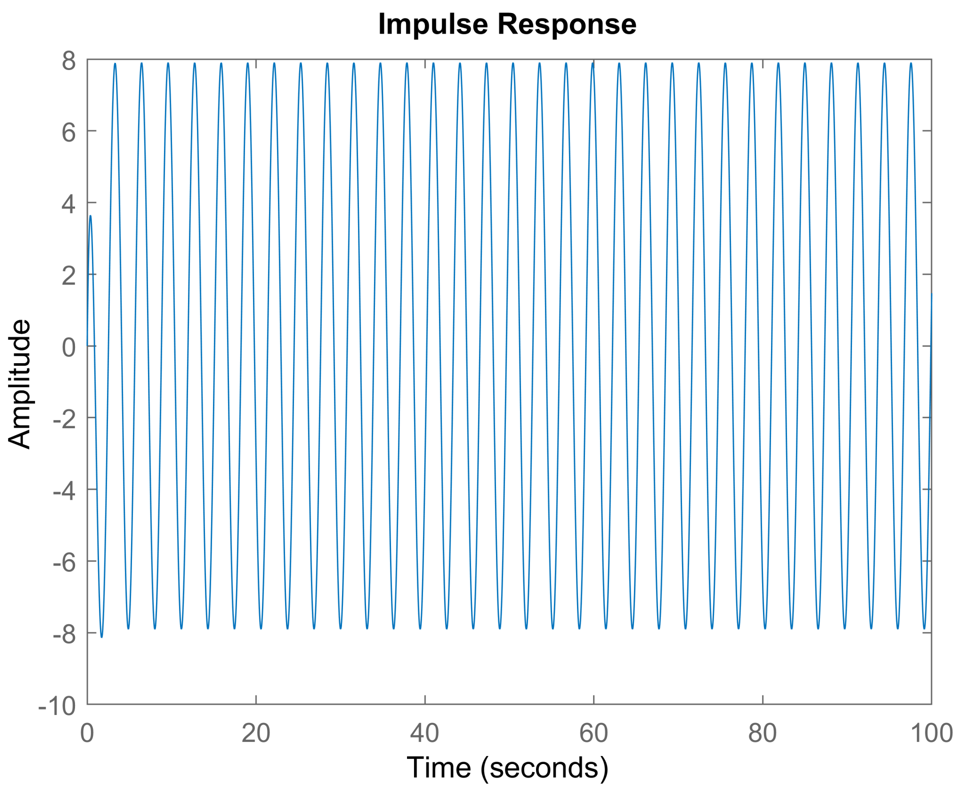
Qs1

$$M(s) = \frac{20(s-1)}{(s+2)(s^2+4)}$$

Poles at $-2, \pm 2j$:- 2 poles on imaginary axis and one pole in LHP, so it is marginally stable, as seen from the impulse response of the transfer function.

```
tf1=zpk(1,[-2,2i,-2i],20);

impulse(tf1,100)
```



QS. 2

Characteristic equation,

$$2s^4 + s^3 + 3s^2 + 5s + 10 = 0$$

```
y1=rhc([2,1,3,5,10],1);
```

```
 Routh-Hurwitz Table:
rhTable = 5×3
    2.0000    3.0000   10.0000
    1.0000    5.0000         0
   -7.0000   10.0000         0
    6.4286         0         0
```

```
   10.0000         0         0
```

```
if y1==0
    fprintf("System is unstable \n")
else
    fprintf("System is stable \n")
end
```

```
System is unstable
```

QS. 3

Characteristic equation,

$$s^4 + s^3 + 2s^2 + 2s + 3 = 0$$

```
y2=rhc([1,1,2,2,3],1);
```

```
 Routh-Hurwitz Table:
rhTable = 5×3
10^4 ×
    0.0001    0.0002    0.0003
    0.0001    0.0002         0
    0.0000    0.0003         0
   -2.9998         0         0
    0.0003         0         0
```

```
if y2==0
    fprintf("System is unstable \n")
else
    fprintf("System is stable \n")
end
```

```
System is unstable
```

QS. 4

Characteristic equation,

$$s^5 + 4s^4 + 8s^3 + 8s^2 + 7s + 4 = 0$$

```
y3=rhc([1,4,8,8,7,4],1);
```

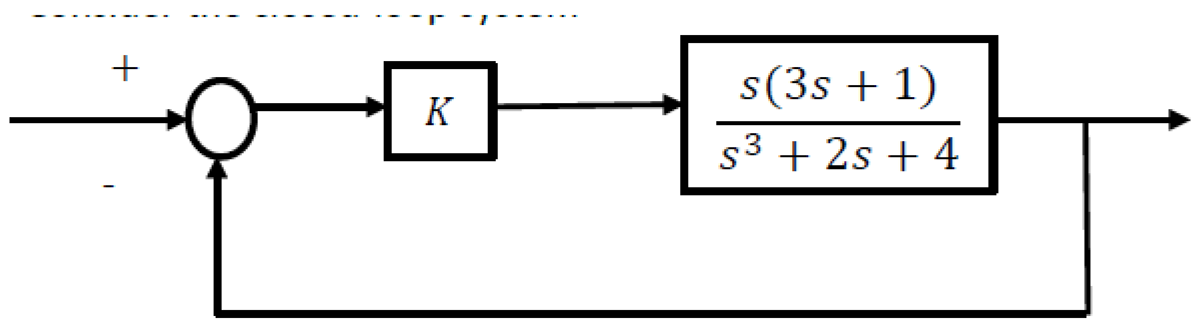
```
 Routh-Hurwitz Table:
rhTable = 6×3
    1.0000    8.0000    7.0000
    4.0000    8.0000    4.0000
    6.0000    6.0000         0
    4.0000    4.0000         0
    0.0001         0         0
    4.0000         0         0
```

```
if y3==0
    fprintf("System is unstable \n")
else
    fprintf("System is stable \n")
```

```
end
```

```
System is stable
```

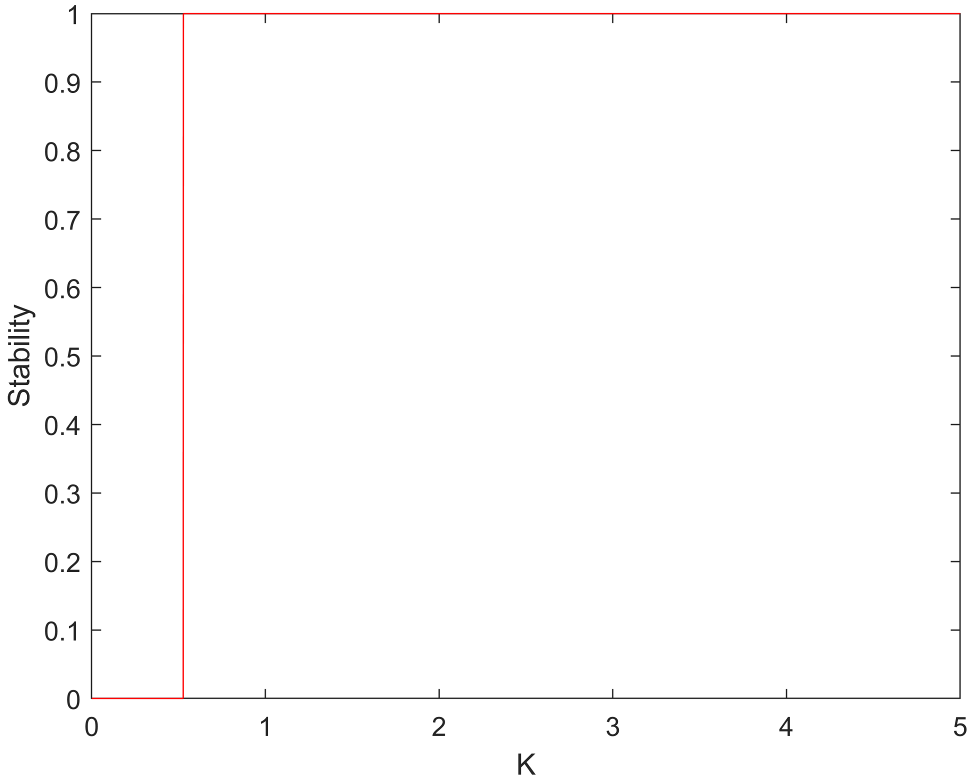
QS. 5

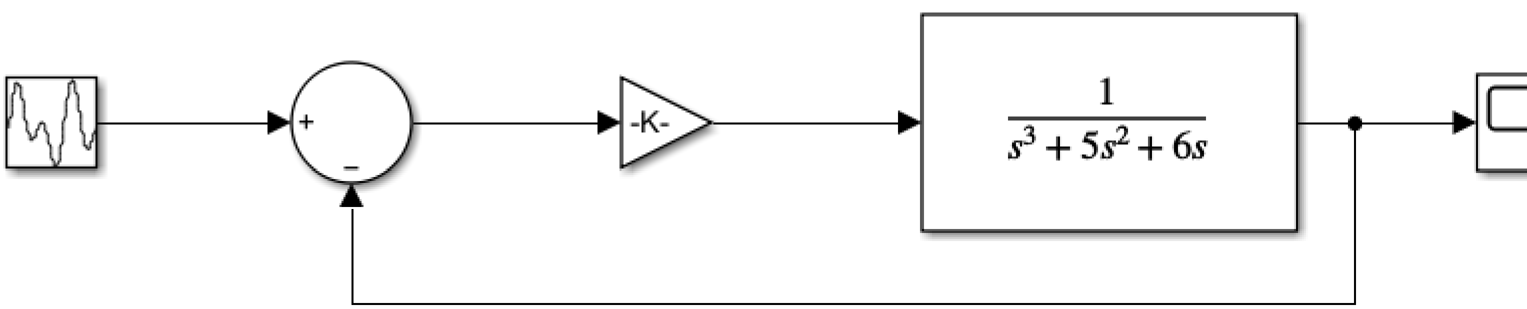


For $K \geq -1 + \sqrt{\frac{7}{3}}$ the system is stable

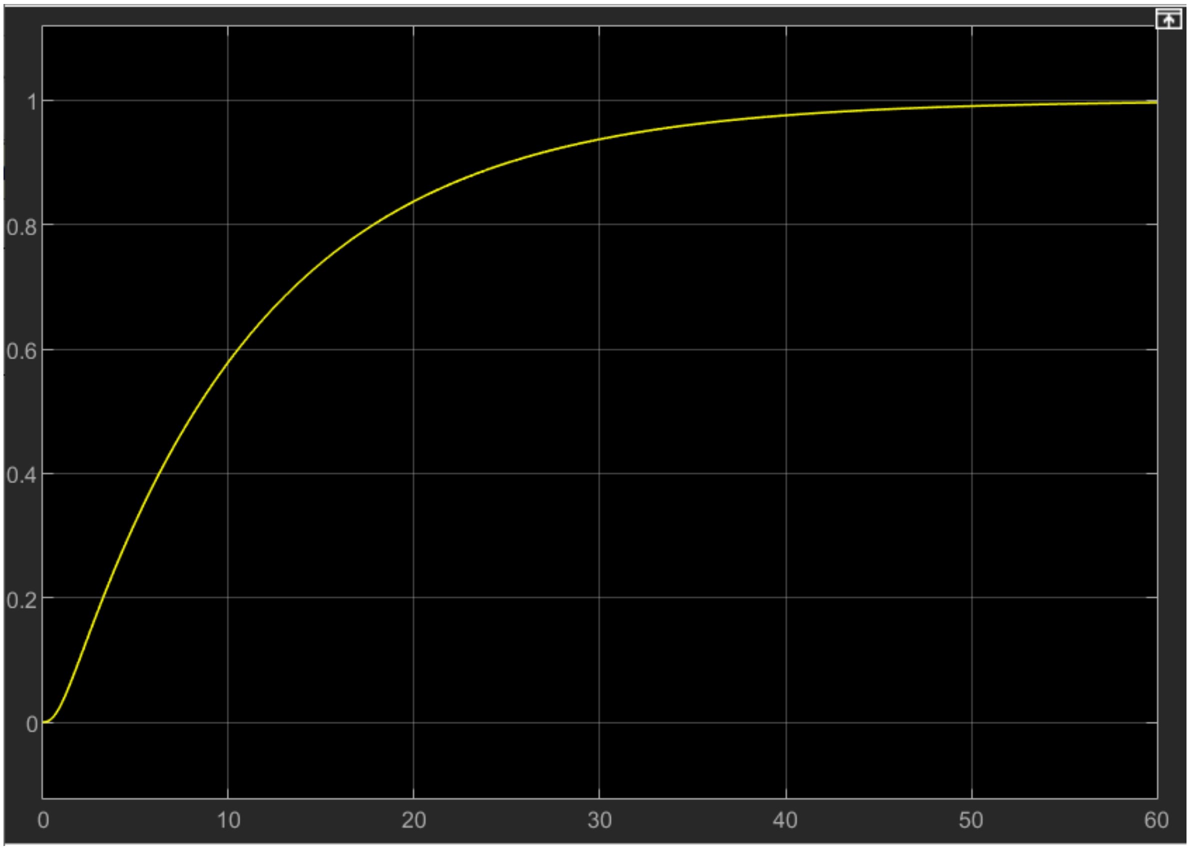
For $K = -1 + \sqrt{\frac{7}{3}}$ it is marginally stable.

```
k=1e-3:1e-3:5;
y=zeros(size(k));

for i =1:length(k)
    y(i)=rhc([1,3*k(i),k(i)+2,4],0);
end
figure;
plot(k,y,"r-")
xlabel("K");
ylabel("Stability");
```





stable response on k >0.527



QS. 6

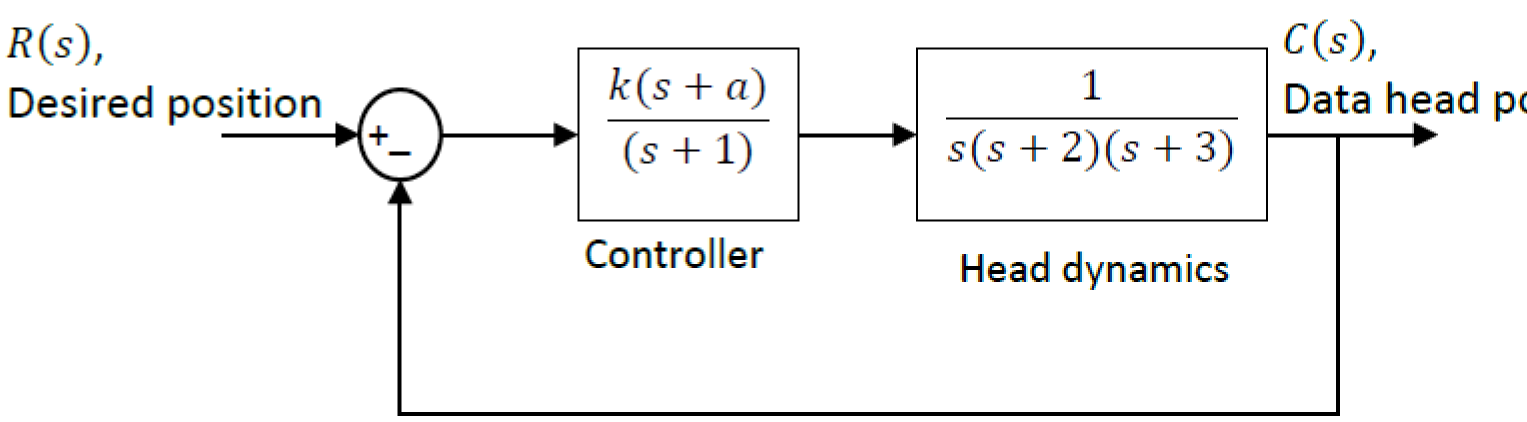


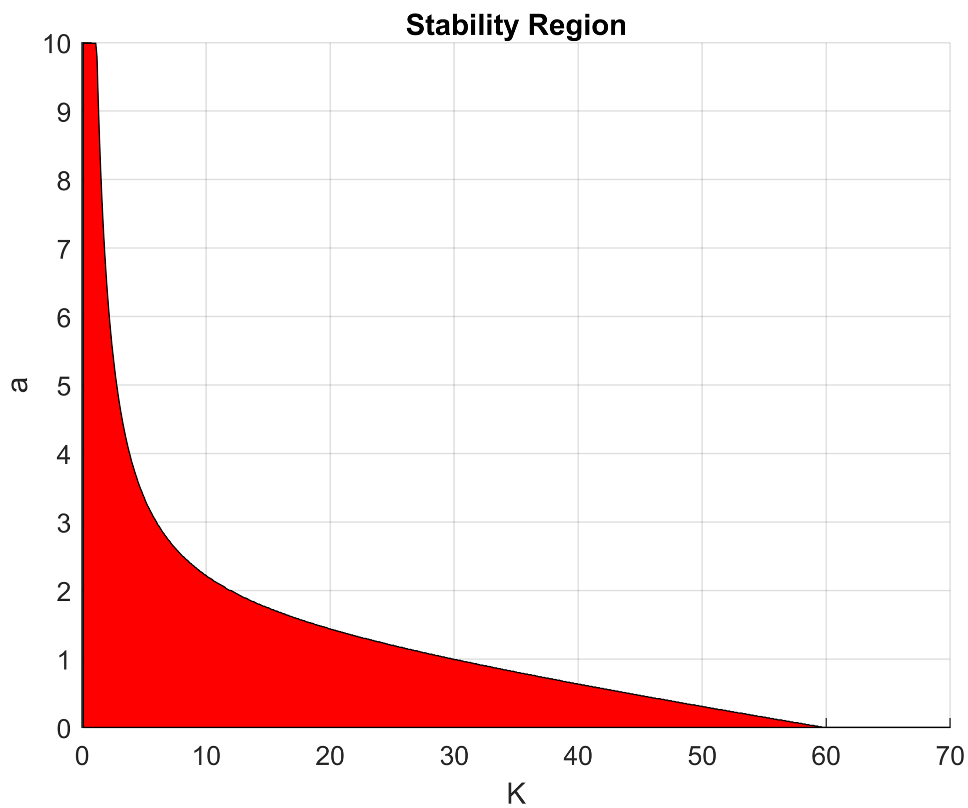
Fig. 1: Disk-storage data-head positioning feedback system

$0 < K \le 60$; $0 \le a \le \frac{10}{k} + \frac{3}{2} - \frac{k}{36}$; The system is stable

$K = 60$; $a = 0$; The system is marginally stable with pole at 0

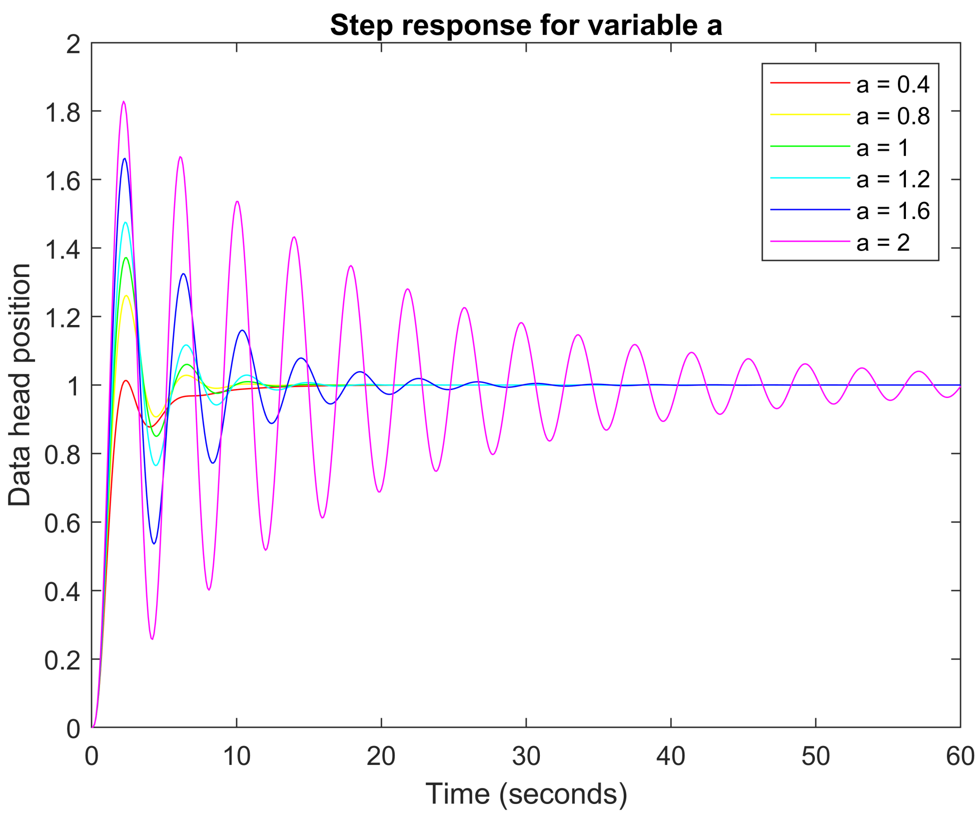
```
k=0.1:0.1:70;
a=0:0.01:10;
y=zeros(size(k));
```

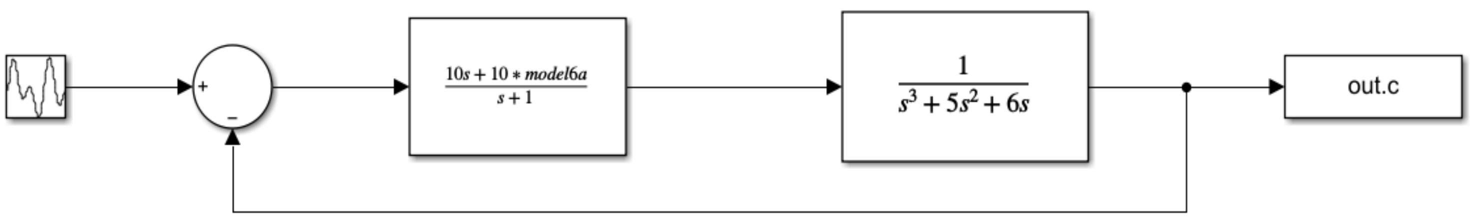
```
for i=1:length(k)
    for j=1:length(a)-1
        c=rhc([1,6,11,6+k(i),k(i)*a(j)],0);
        b=rhc([1,6,11,6+k(i),k(i)*a(j+1)],0);
        if(c~=b)||(b==1 && j==length(a)-1)
            y(i)=a(j);
        end
    end
end

figure;
patch([k,fliplr(k)],[y,zeros(size(y))],'r')
xlabel("K");
ylabel("a");
title("Stability Region");
grid on
```




```
A=[0.4,0.8,1.0,1.2,1.6,2.0];
CM=hsv(6);
figure;

for i=1:6
    model6a=A(i);
    out=sim("model_6.slx");
    plot(out.c,"Color",CM(i,:),"DisplayName",strcat("a = ",num2str(model6a)));
    hold on
end
```

```
hold off
legend
ylabel("Data head position");
title("Step response for variable a");
```





Reducing the value of a, reduces the overshoot and oscillation

For higher value of a, the system reaches the steady state value faster

but it oscillates for long time

QS.7

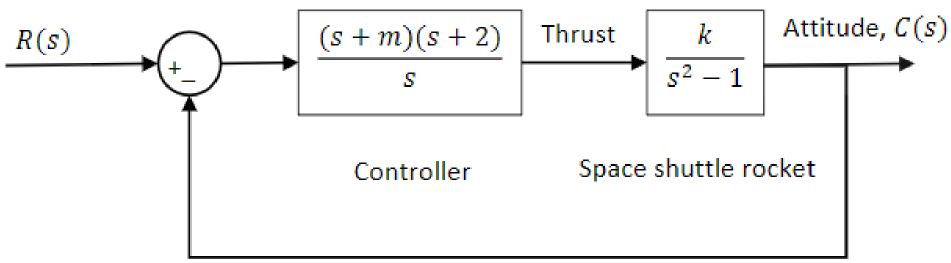
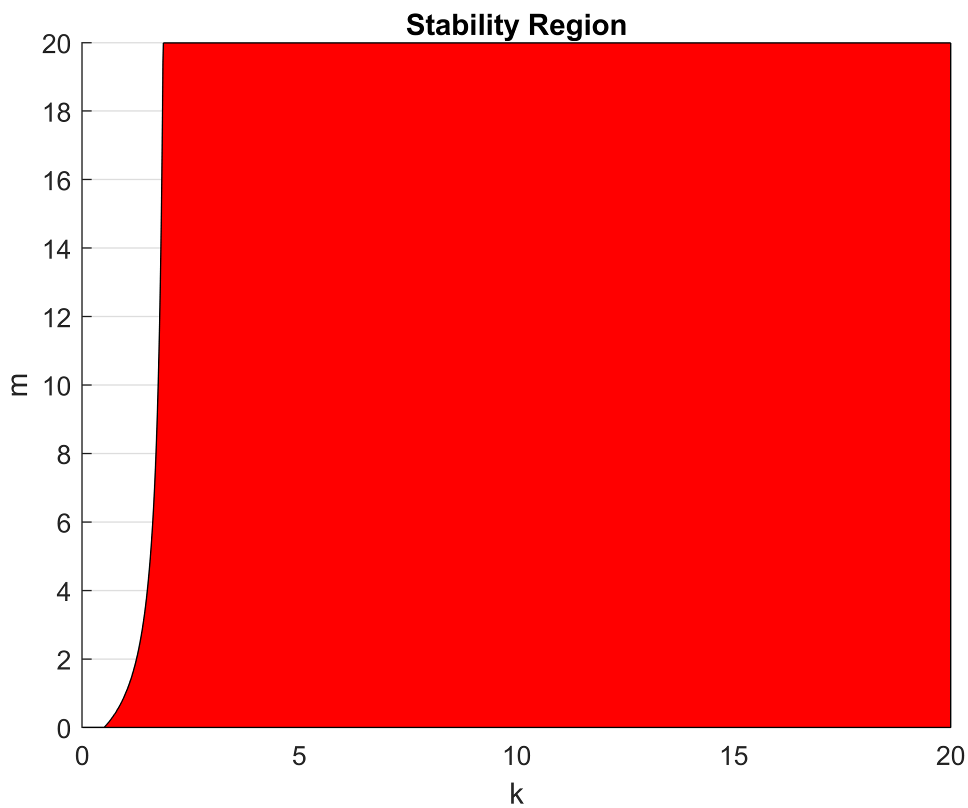


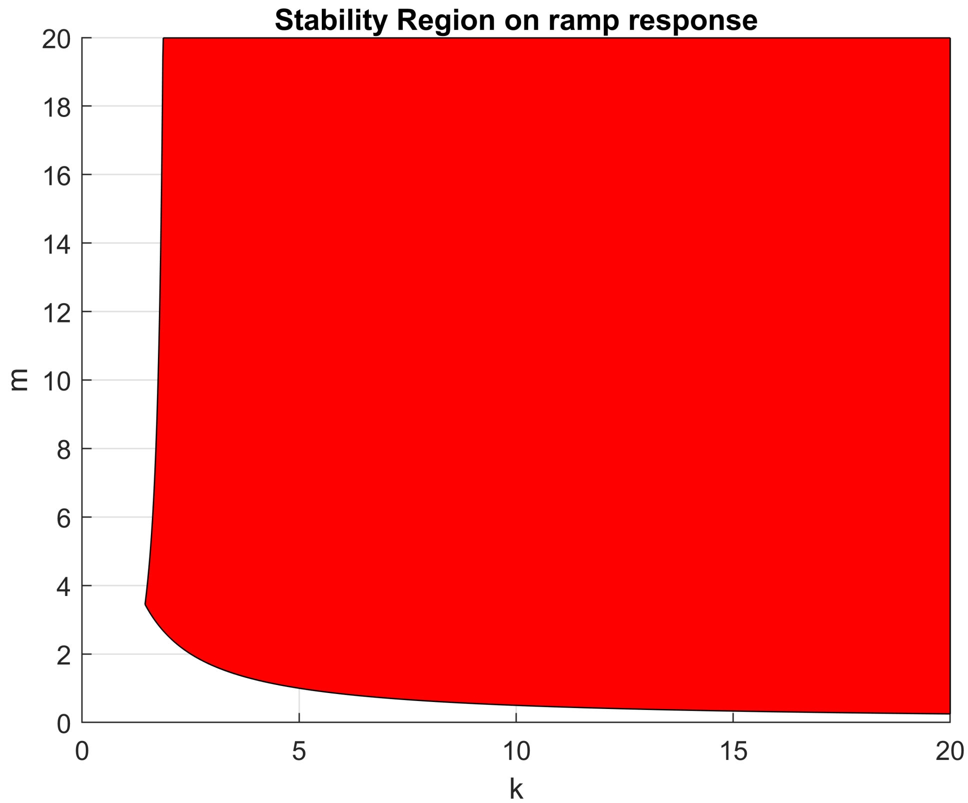
Fig. 2: Attitude control system of a space shuttle

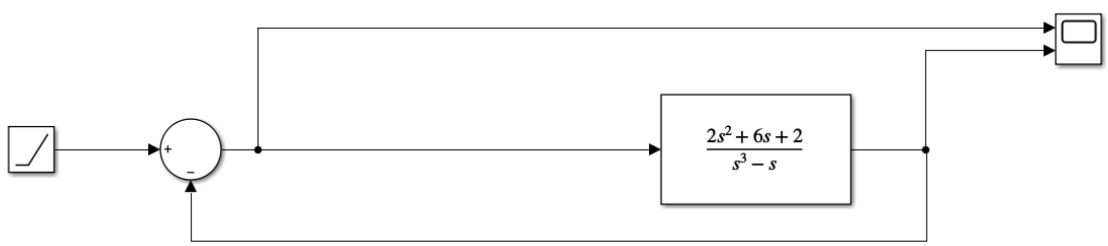
```
k=0.01:0.01:20;
m=0:0.01:20;
y=zeros(size(k));

for i=1:length(k)
    for j=1:length(m)-1
        a=rhc([1,k(i),k(i)*(m(j)+2)-1,2*k(i)*m(j)],0);
        b=rhc([1,k(i),k(i)*(m(j+1)+2)-1,2*k(i)*m(j+1)],0);
        if(a~=b)||(b==1 && j==length(m)-1)
            y(i)=m(j);
        end
    end
end

figure;
patch([k,fliplr(k)],[y,zeros(size(y))],'r')
xlabel("k");
ylabel("m");
title("Stability Region");
grid on
```

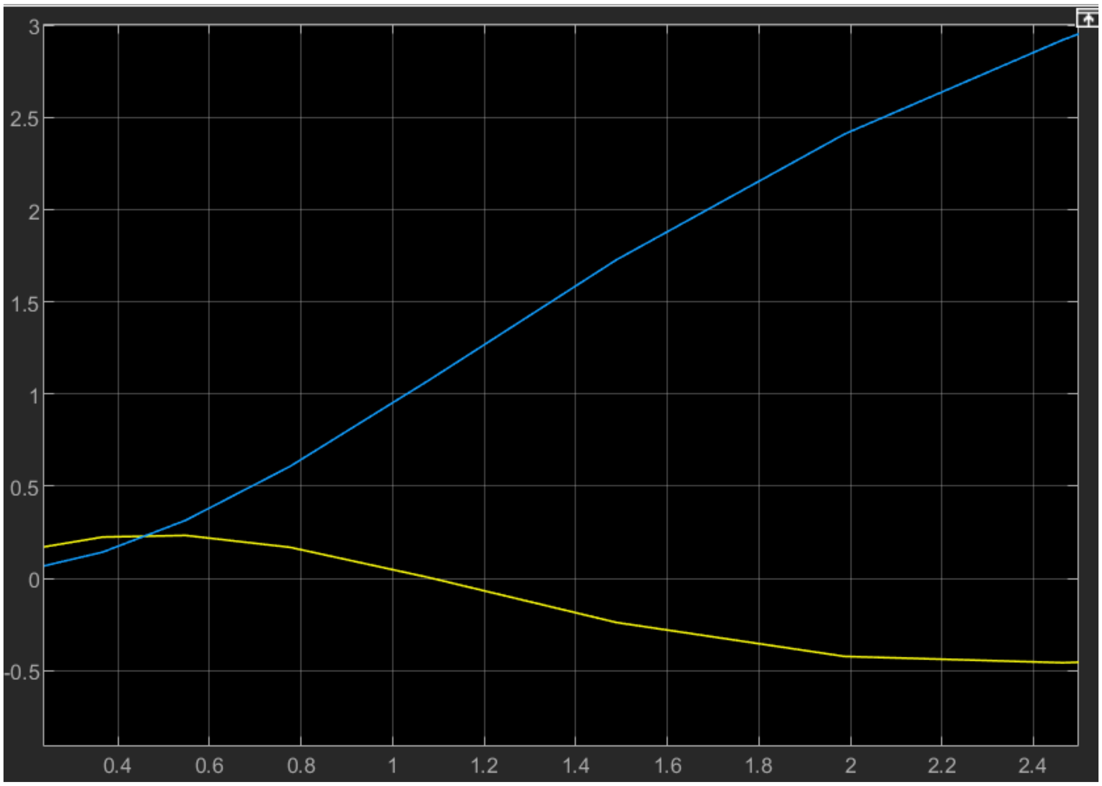


Stability region:

```
y1=5./k;
Li=(y>=y1);
figure;

patch([k(Li),fliplr(k(Li))],[y(Li),fliplr(y1(Li))],'r');
xlabel("k");
ylabel("m");
grid on
title("Stability Region on ramp response");
```


Stability Region on ramp response
m
k
0
2
4
6
8
10
12
14
16
18
20
0
5
10
15
20


$\frac{2s^2 + 6s + 2}{s^3 - s}$



QS. 8

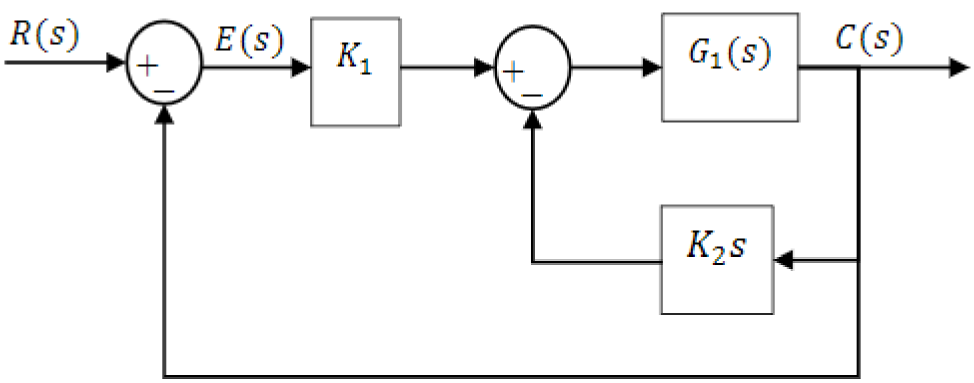


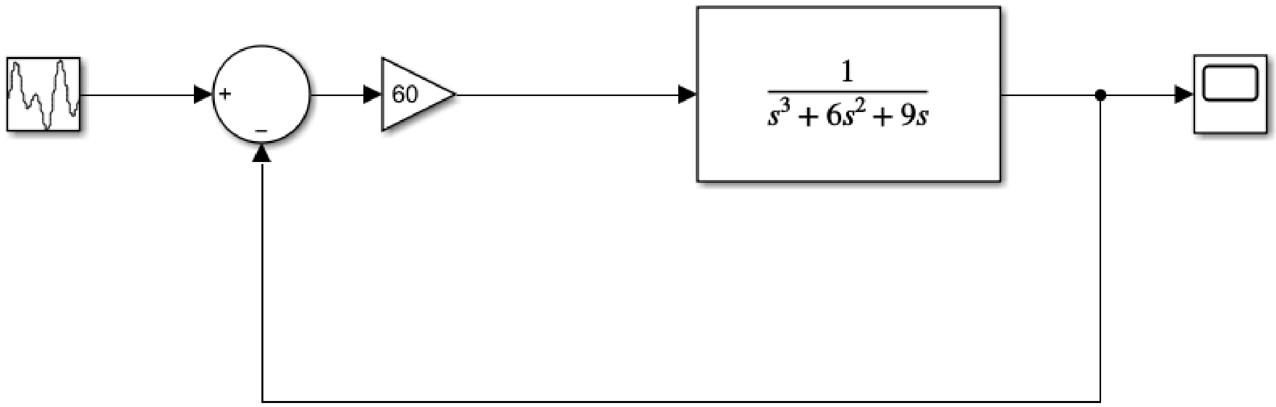
Fig. 3: A two-loop feedback system

At K2=1, value of K1 vaaries in the range of 0<K1<54

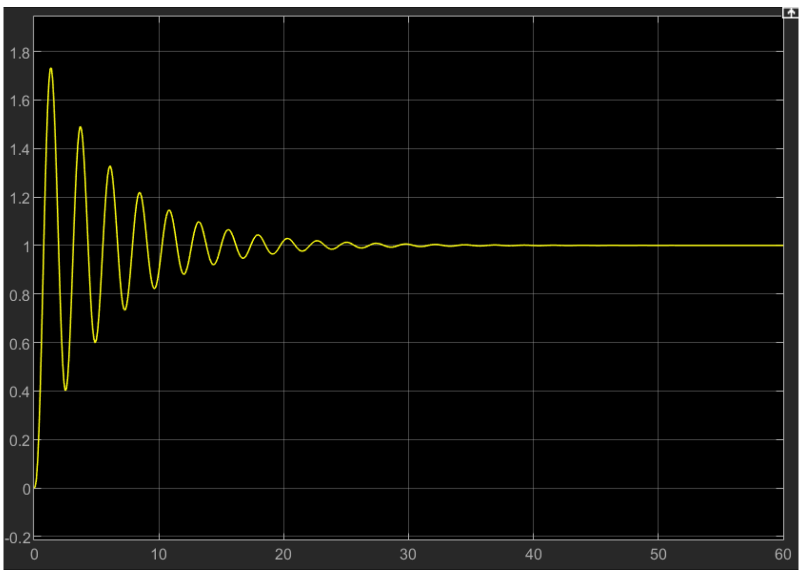
At K1=54, the system is marginally stable i.e. oscillates

Frequency of Oscillation will be 3rad/sec.

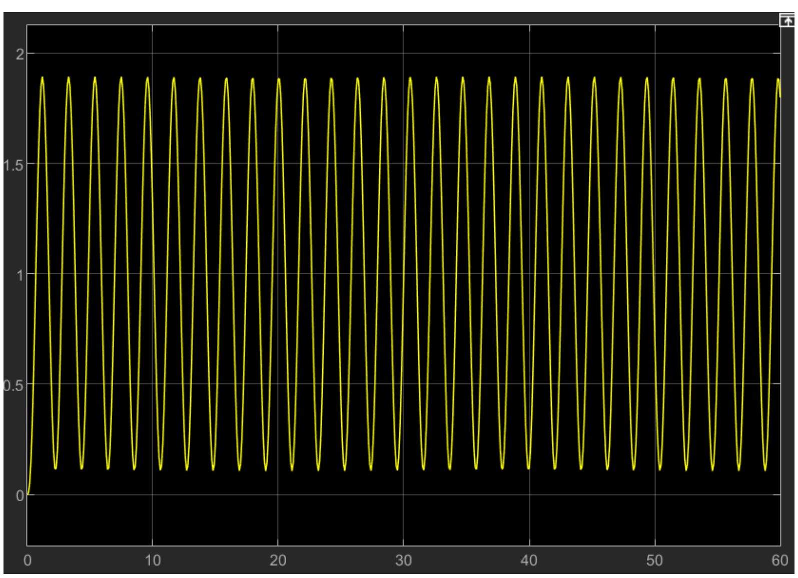
System becomes unstable when K1>54



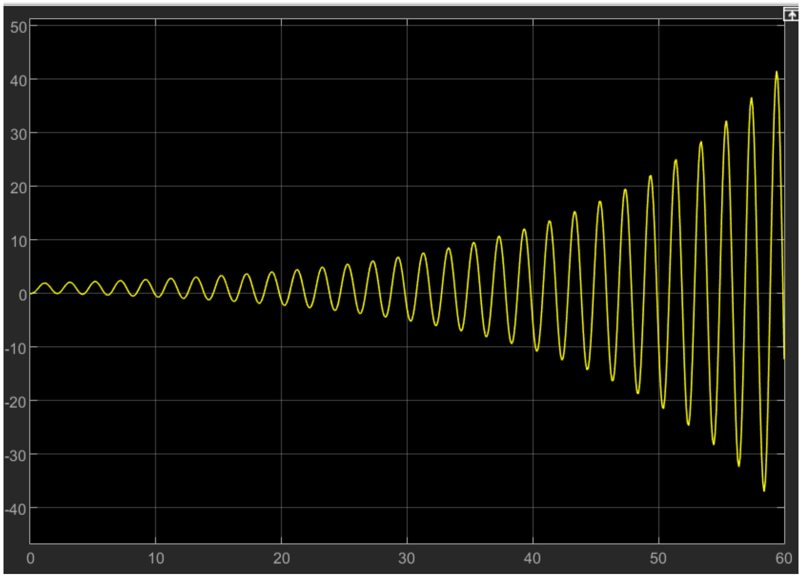
K1=40, stable



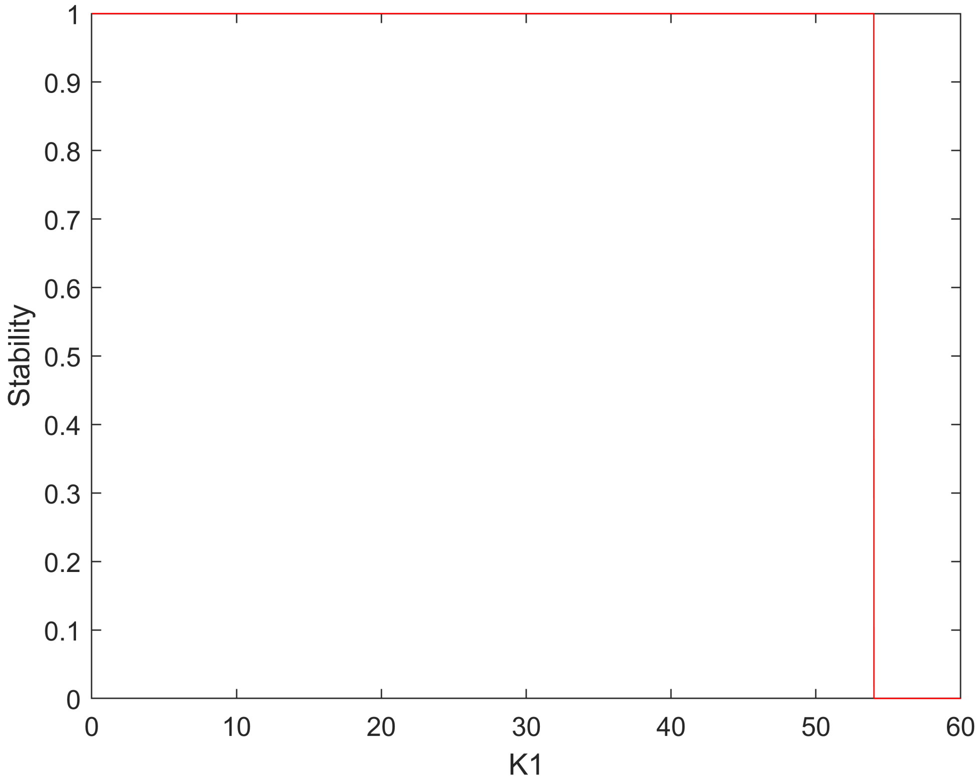
K1=54, marginally stable



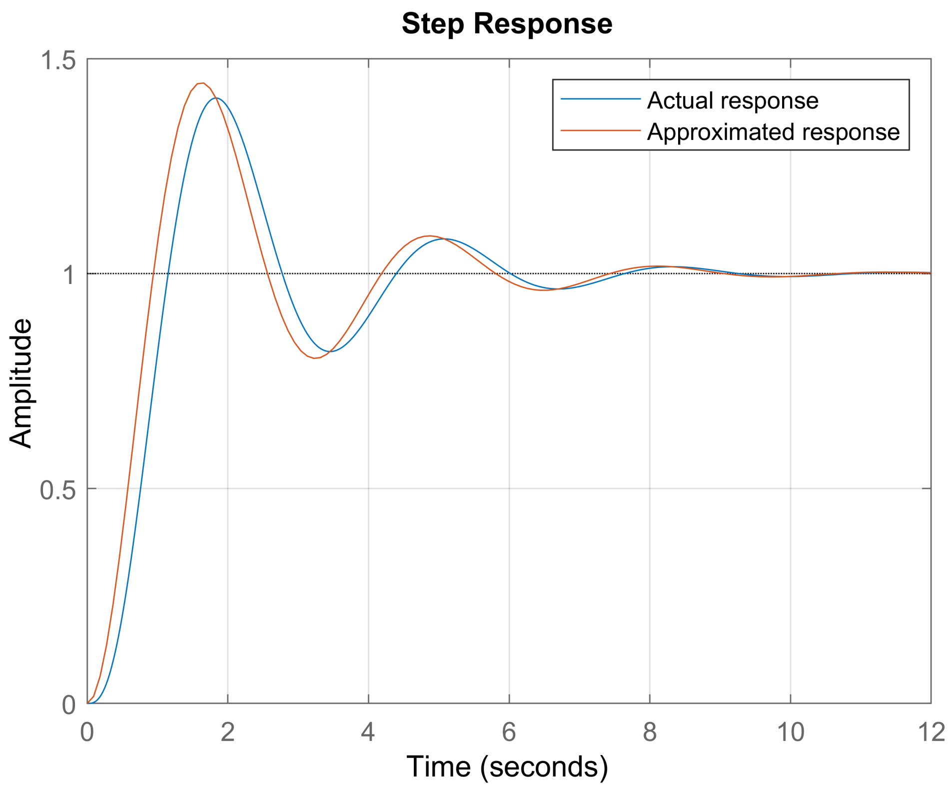
K1=60, unstable




```
%a,b

k1=1e-2:1e-2:60;
y=zeros(size(k));

for i =1:length(k1)
    y(i)=rhc([1,6,9,k1(i)],0);
end

figure;
plot(k1,y,"r-")
xlabel("K1");
ylabel("Stability");
```




```
%c

tf1=tf(20,[1,6,9,20]);
tf2=tf(4,[1,1,4]);
step(tf1,tf2);
legend("Actual response","Approximated response")
grid on
```



QS. 9

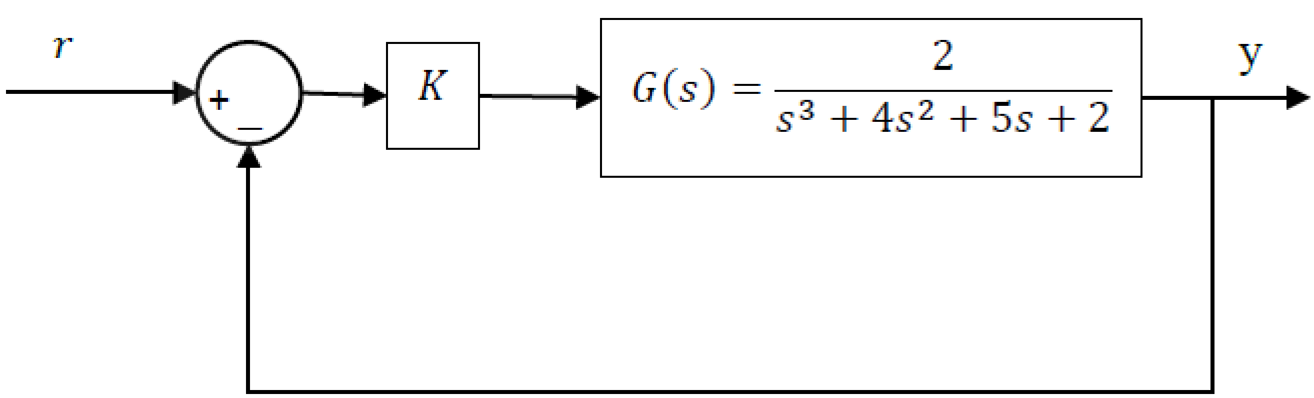
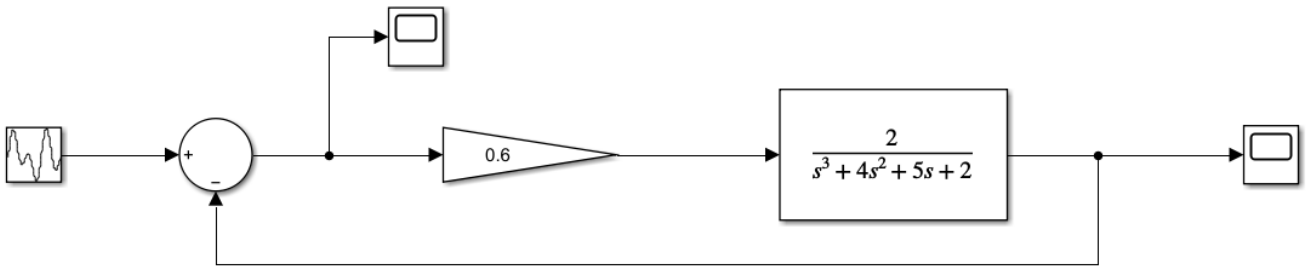
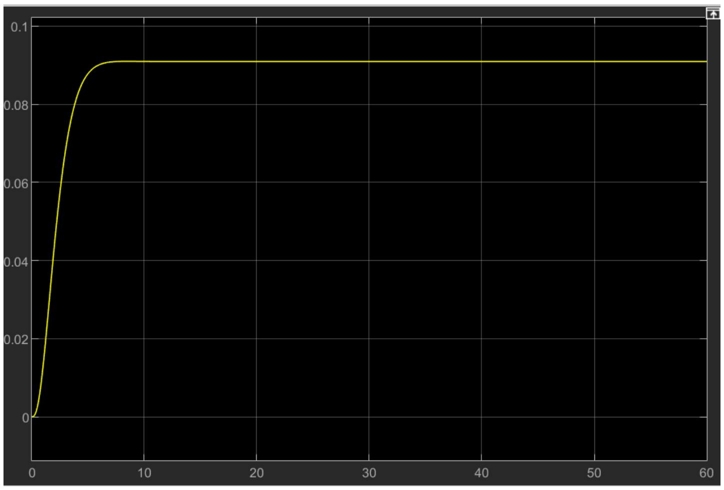


Fig. 4: A feedback system with proportional controller

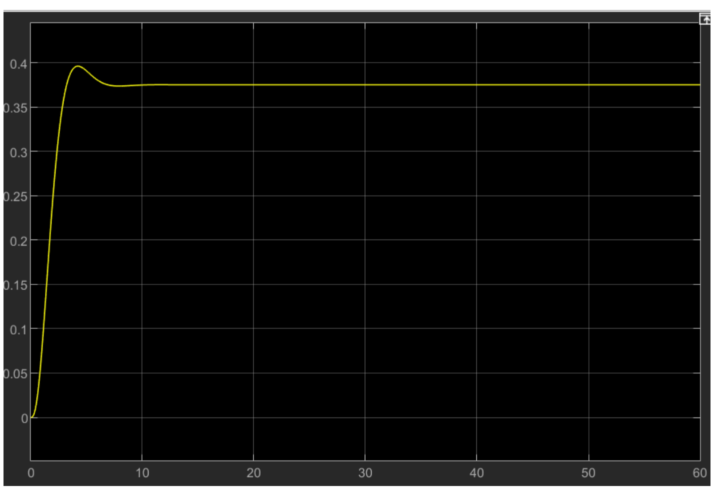


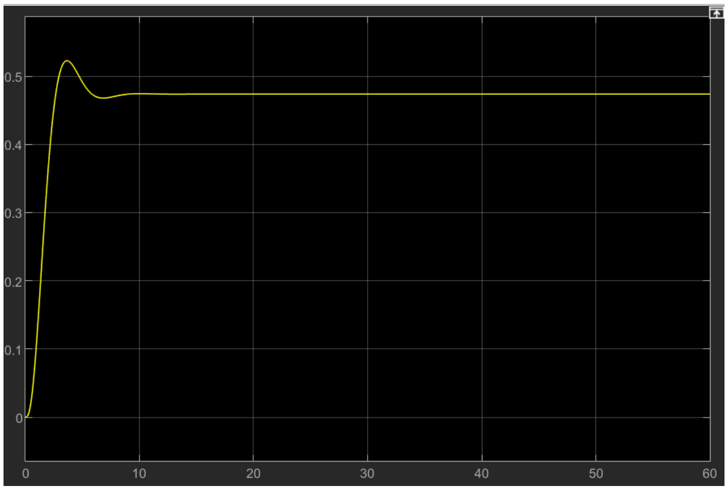
Y curve(output)

Gain=0.1,

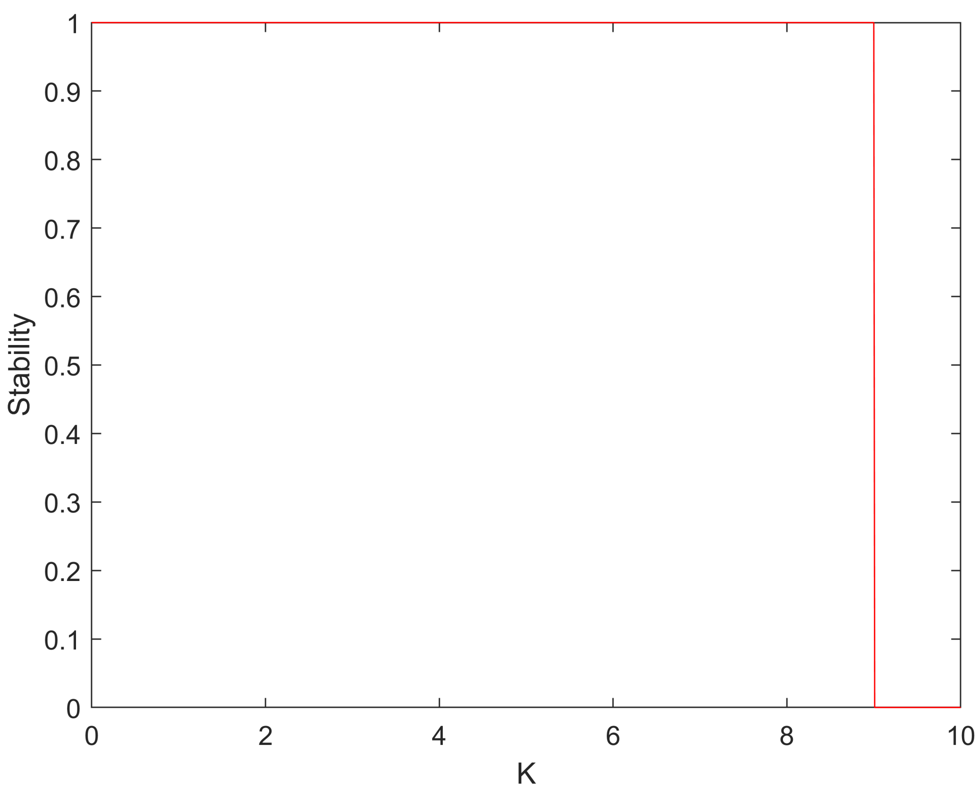


Gain=0.6,



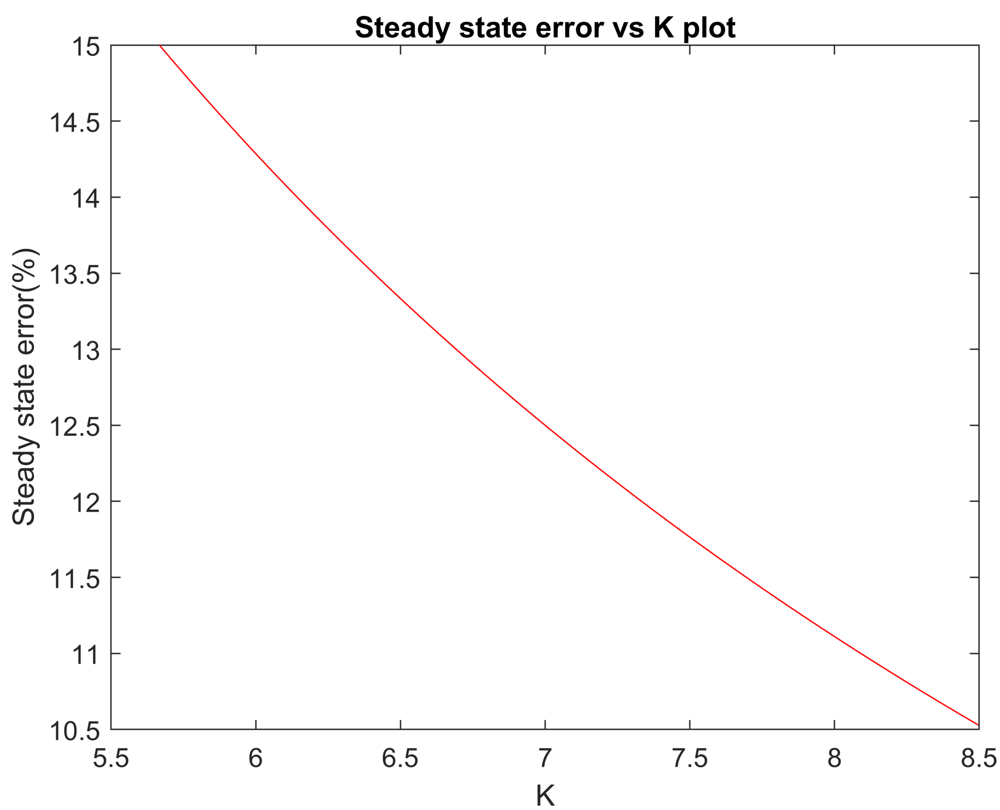
Gain=0.9, 


```
k=0:0.01:10;
y=zeros(size(k));

for i =1:length(k)
y(i)=rhc([1,4,5,2+2*k(i)],0);
end

figure;
plot(k,y,"r-")
xlabel("K");
ylabel("Stability");
```

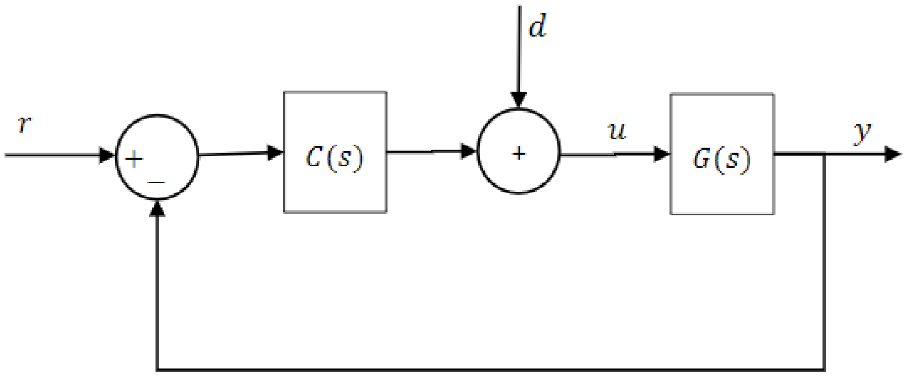


plot e_{ss} vs K graph.

```
% The error gradually decreases till.
% Then the system becomes unstable

K=17/3:1e-2:8.5;
error=zeros(size(K));

for i=1:length(K)
    [y,t]=step(tf(2*K(i),[1,4,5,2+2*K(i)]),500);
    error(i)=100*abs(1-y(end));
end

figure;
plot(K,error,'r');
xlabel("K");
ylabel("Steady state error(%)");
title("Steady state error vs K plot");
```



QS. 10



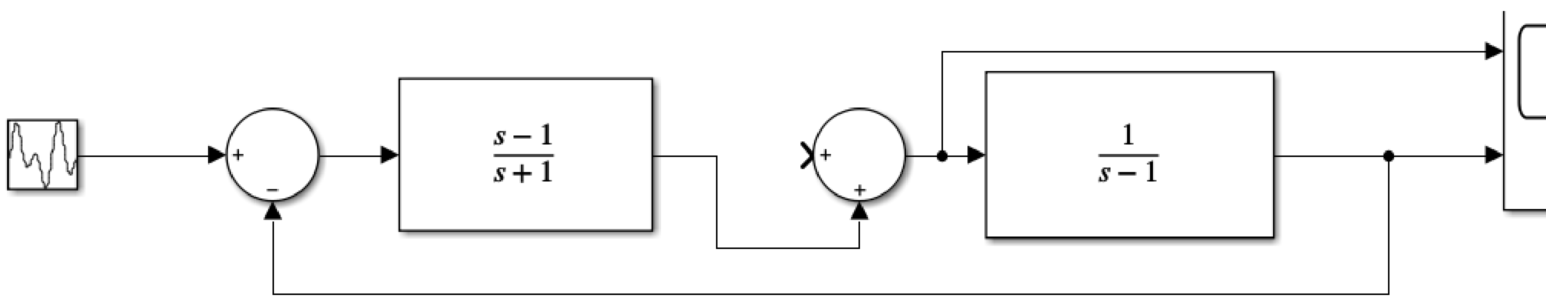
Case-I:

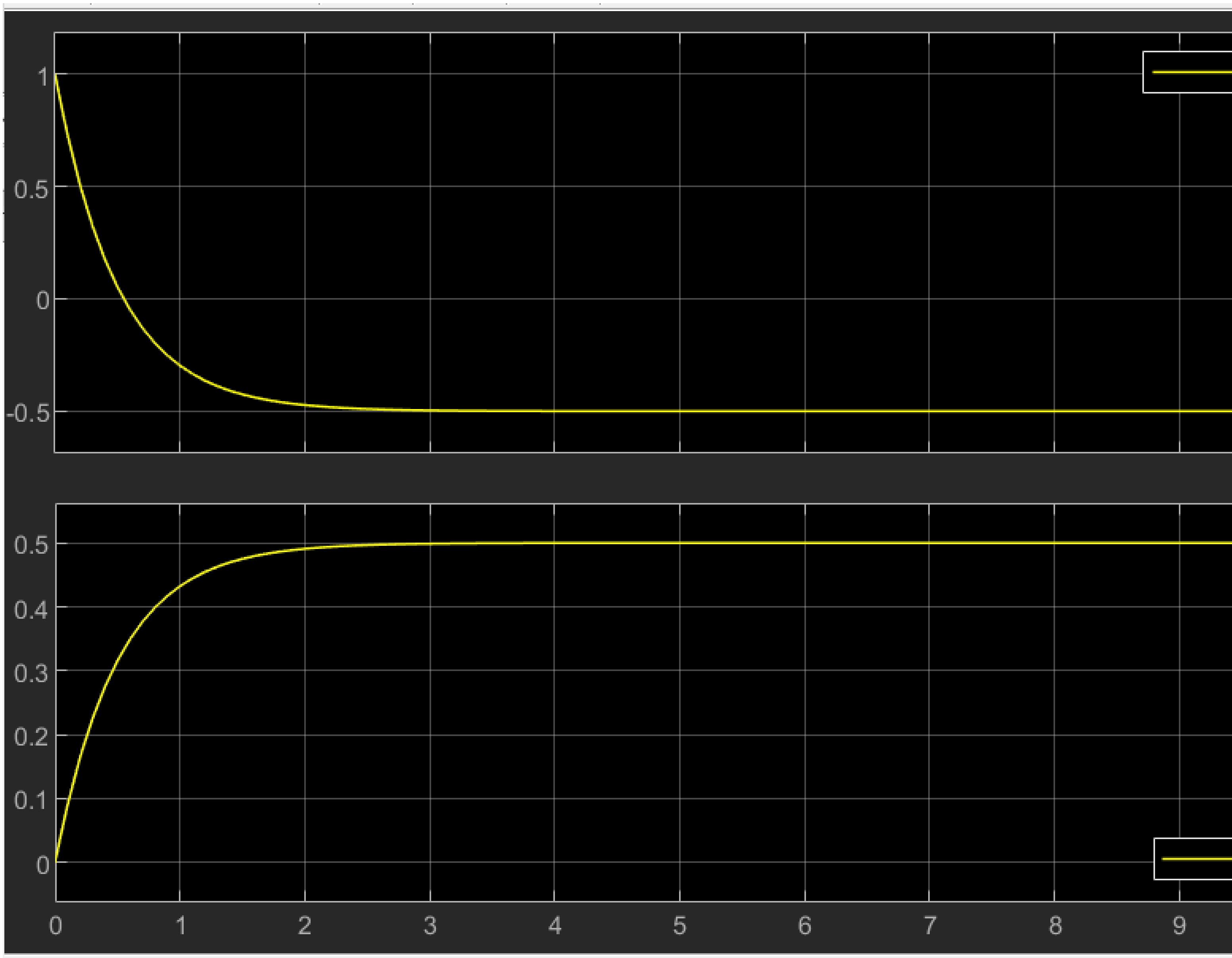
$$G(s) = \frac{1}{s-1}$$

$$C(s) = \frac{s-1}{s+1}$$

r-to-y and r-to-u:

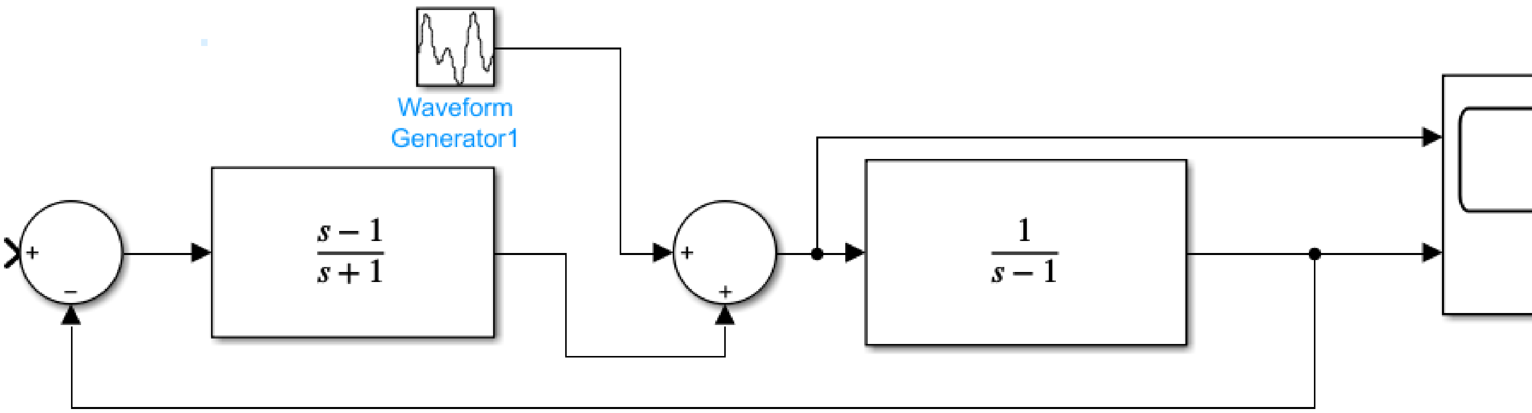
Model:

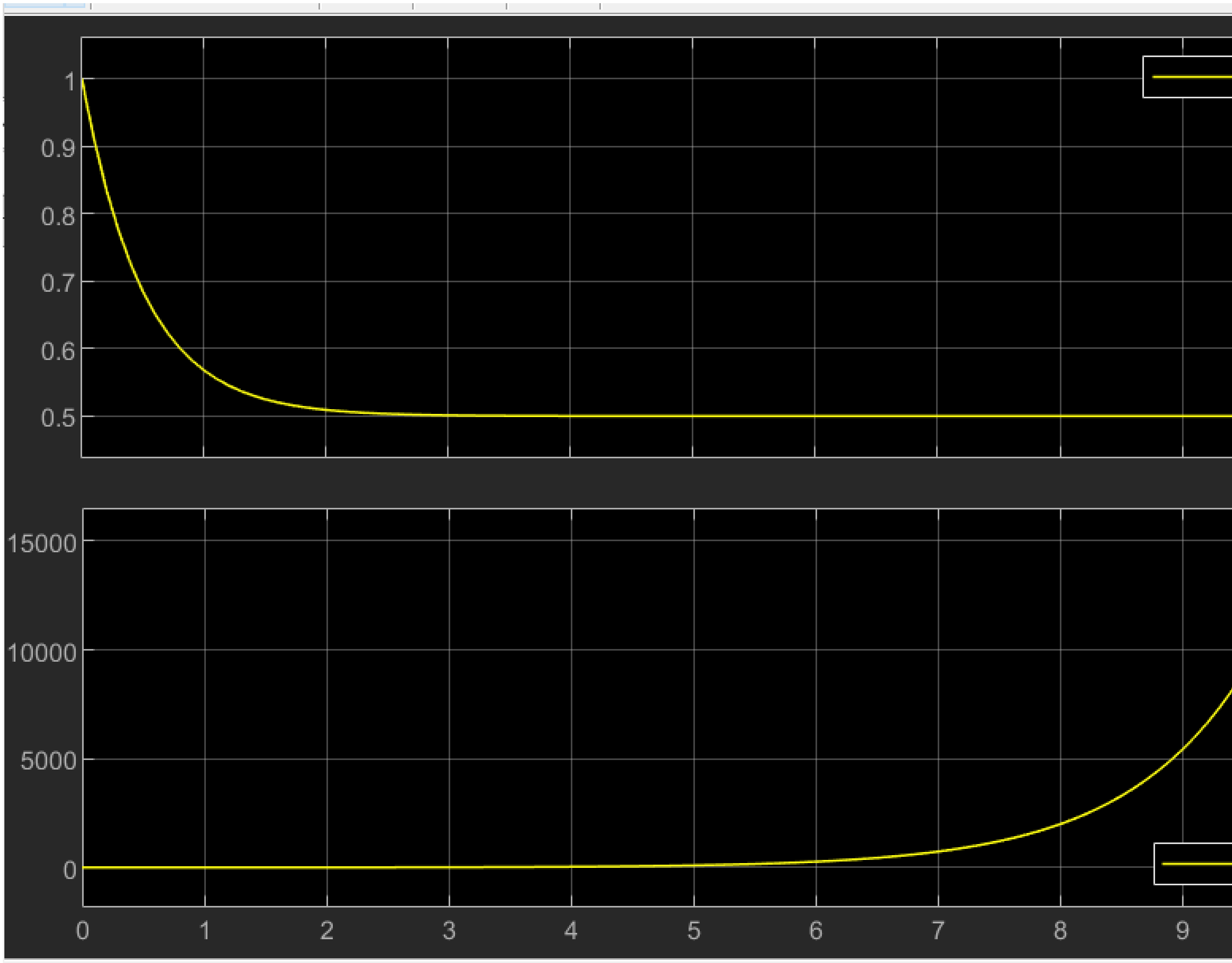




d-to-y and d-to-u:

Model:





With d as disturbance in system, the system behaves uncontrollably,

as u(t) is unstable for d-to-u.

So for even a little disturbance in system, output at u will reach saturation,

changing the system output y.

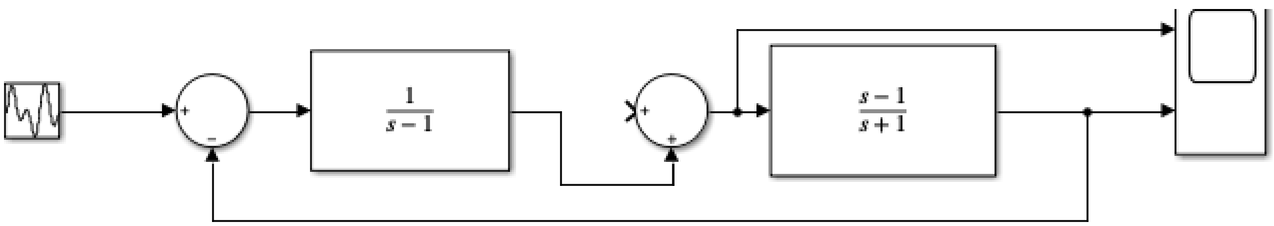
Case II:

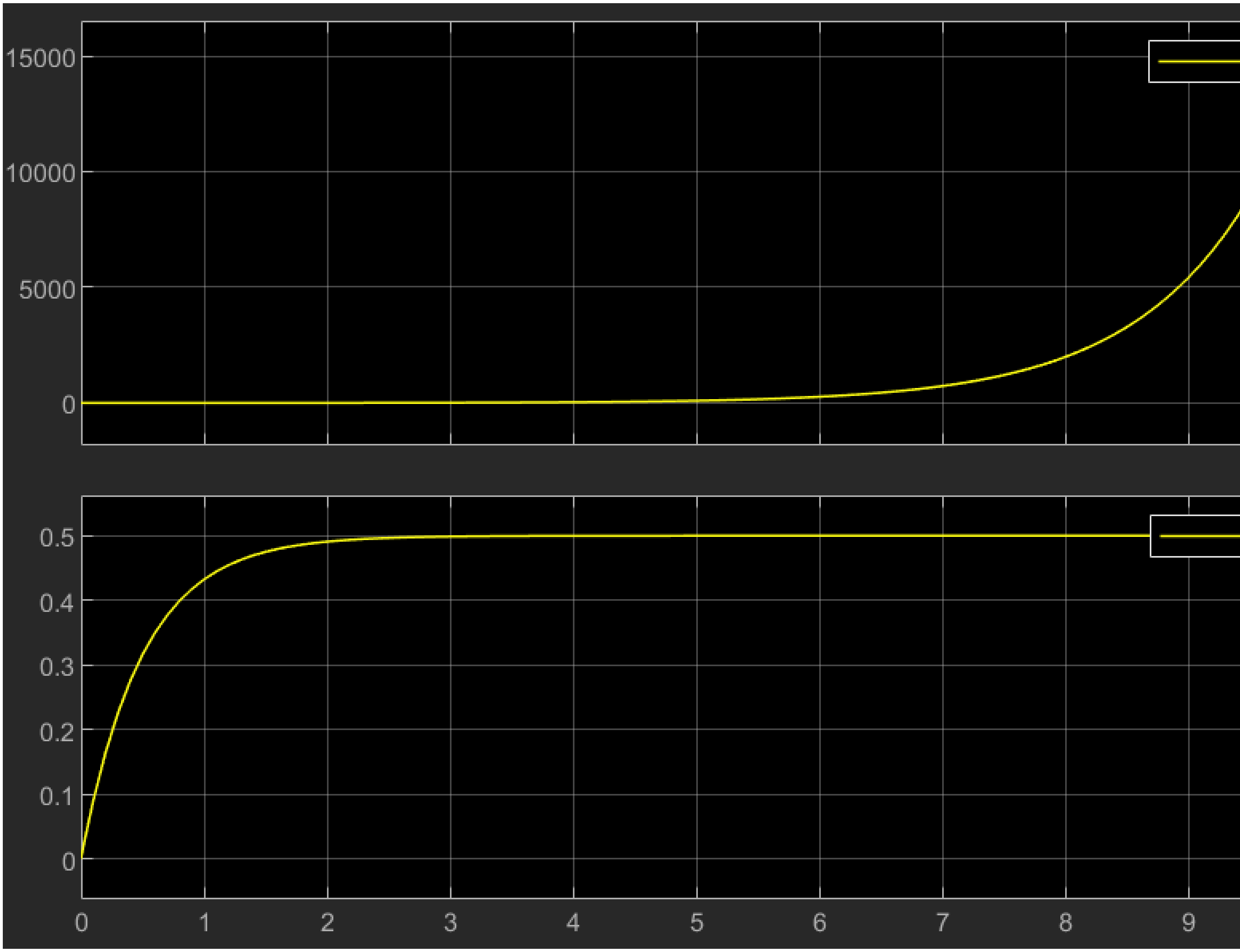
$$G(s) = \frac{s-1}{s+1}$$

$$C(s) = \frac{1}{s-1}$$

r-to-y and r-to-u:

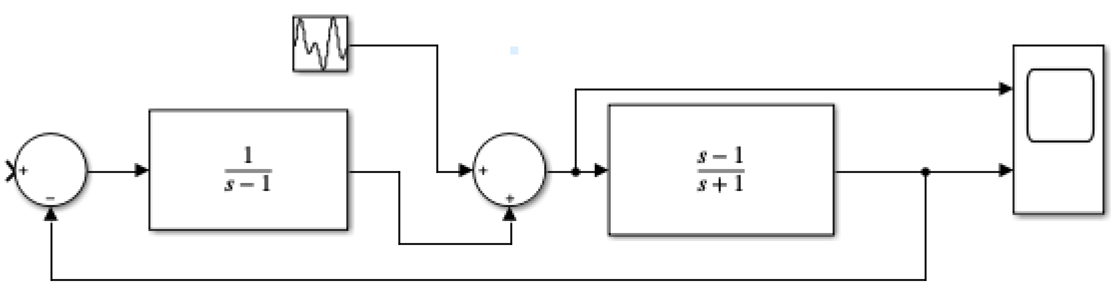
Model:

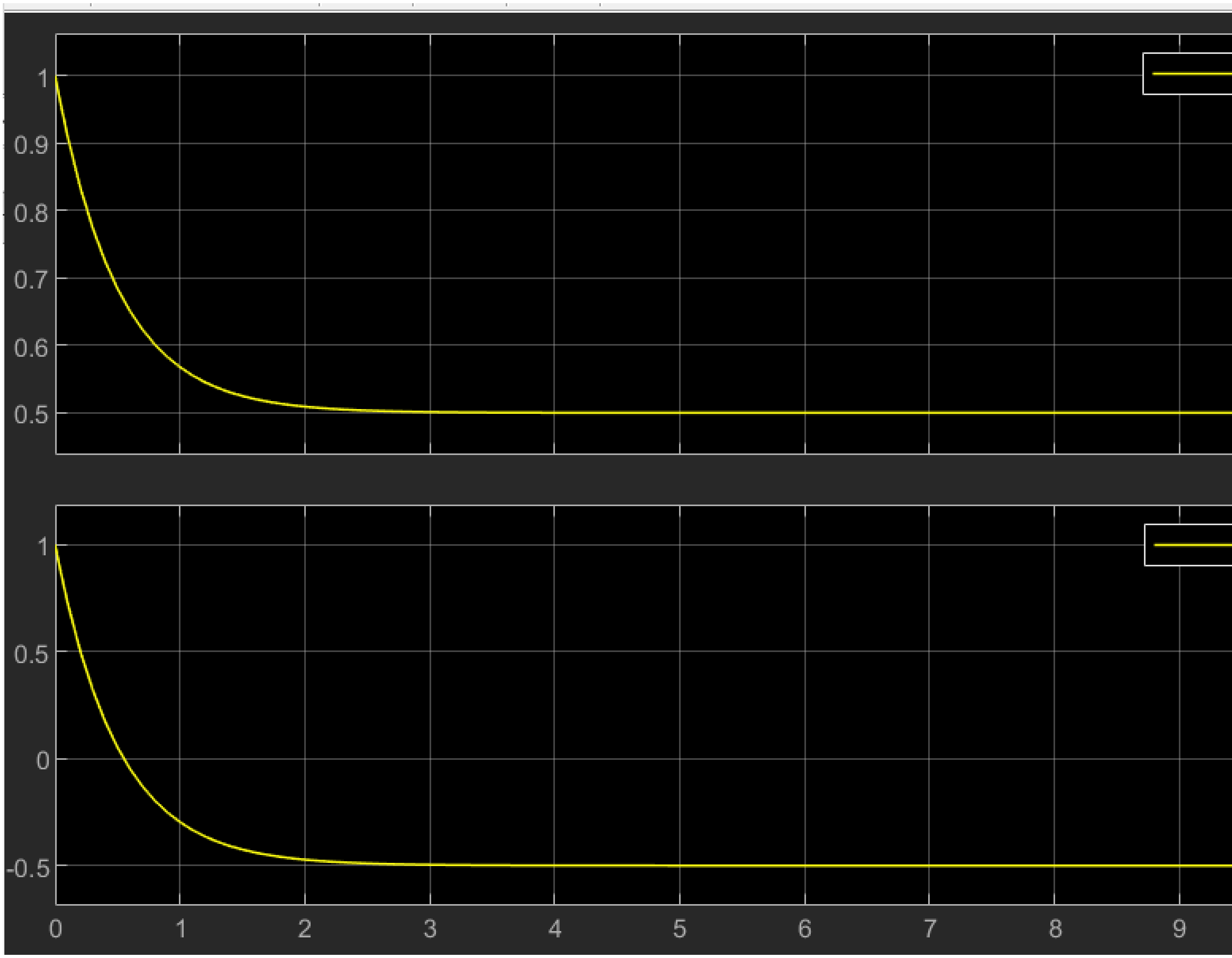




d-to-y and d-to-u:

Model:





Disturbance is controlled, as both output d-to-u and d-to-y are stable.

But the output response at u for reference input is uncontrollable,

which will reach saturation faster and change the output y.

```
function y=rhc(coeffVector,x)
ceoffLength = length(coeffVector);
rhTableColumn = round(ceoffLength/2);
rhTable = zeros(ceoffLength,rhTableColumn);
rhTable(1,:) = coeffVector(1,1:2:ceoffLength);
if (rem(ceoffLength,2) ~= 0)
    rhTable(2,1:rhTableColumn - 1) = coeffVector(1,2:2:ceoffLength);
else
    rhTable(2,:) = coeffVector(1,2:2:ceoffLength);
end
epss=0.0001;
for i = 3:ceoffLength
```

```
    if rhTable(i-1,:) == 0
        order = (ceoffLength - i);
        cnt1 = 0;
        cnt2 = 1;
        for j = 1:rhTableColumn - 1
            rhTable(i-1,j) = (order - cnt1) * rhTable(i-2,cnt2);
            cnt2 = cnt2 + 1;
            cnt1 = cnt1 + 2;
        end
    end
    for j = 1:rhTableColumn - 1
        firstElemUpperRow = rhTable(i-1,1);
        rhTable(i,j) = ((rhTable(i-1,1) * rhTable(i-2,j+1))-(rhTable(i-2,1) * rhTable(i-1,j+1))
    end
    if rhTable(i,1) == 0
        rhTable(i,1) = epss;
    end
end
unstablePoles = 0;
for i = 1:ceoffLength - 1
    if sign(rhTable(i,1)) * sign(rhTable(i+1,1)) == -1
        unstablePoles = unstablePoles + 1;
    end
end
if x==1
    fprintf('\n Routh-Hurwitz Table:\n')
    rhTable %#ok<NOPRT>
end
if unstablePoles == 0
    y=1;
else
    y=0;
end
end
```

Lab 3.

Q1. $M = \dfrac{20(S-1)}{(S+2)(S^2+4)}$

$\hookrightarrow$ 1 LHP. $= (-2)$ pole

$\hookrightarrow$ 2 imaginary poles

$\therefore$ M(S) is marginally stable.

Q2. $2s^4+s^3+3s^2+5s+10=0$.

using rh criterion :

$-\begin{vmatrix} 2 & 3 \\ 1 & 5 \end{vmatrix} = -7$ $\quad -\begin{vmatrix} 2 & 10 \\ 1 & 0 \end{vmatrix} = 10$

$-\dfrac{1}{-7}\begin{vmatrix} 1 & 5 \\ -7 & 10 \end{vmatrix} = \dfrac{45}{7}$.

s^4	2	3	10
s^3	1	5	0
s^2	-7	10	0
s	45/7	0	0
s^0	10	0	0

as there is a sign change

$\therefore$ system UNSTABLE

Q3. $s^4+s^3+2s^2+2s+3=0$.

s^4	1	2	3
s^3	1	2	0
s^2	0	3	0
s^1	-3	0	0
s^0	0	0	0

UNSTABLE.

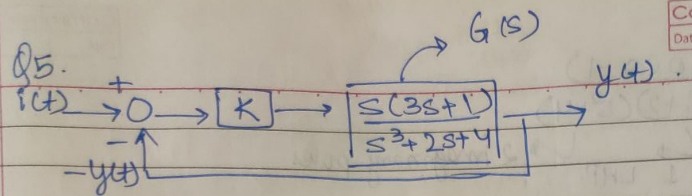
Q4. $s^5+4s^4+8s^3+7s+4=0$.

s^5	1	8	7	0
s^4	4	8	4	0
s^3	6	6	0	0
s^2	4	4	0	0
s^1	0	0	0	0
s^0	4	0	0	0

No sign change

$\therefore$ System is STABLE.

Q5.



K → gain of proportional controller

$1 + KG(s) = 0$

$K(G(s))(i(t) - y(t)) = y(t)$.

$$1 + \frac{s^3+2s+4}{Ks(3s+1)} = 0$$

$$s^3+2s+4+Ks(3s+1)=0$$

$$s^3+(2+K)s+4+3Ks^2=0$$

$\underline{K>0}$. rh table.

s^3	1	$2+K$
s^2	$3K$	4
s^1	$-\frac{1}{3K}\begin{vmatrix}1 & K+2\\ 3K & 4\end{vmatrix}$	0
s^0		

$\rightarrow -\frac{1}{3K}(4-3K^2-6K)$.

as $K>0$

$3K^2+6K-4>0$

$K > -1+\sqrt{7/3}$ ~ 0.53

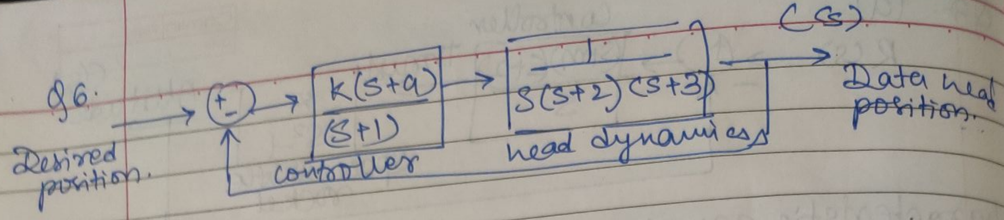
when $K = -1+\sqrt{7/3}$ both elements of 3rd row is 0. stable (marginally).

s^3	1	$K+2$
s^2	$3K$	4
s	$6K$	0
1	4	0

$\therefore \boxed{K \geq -1+\sqrt{7/3}}$

System stable for.

Q6.



characteristic equation:

$$1+\frac{K(s+a)}{s(s+1)(s+2)(s+3)}=0.$$

$$s^4+6s^3+11s^2+(6+K)s+Ka=0.$$

as $K>0$ $\therefore a\geq 0.$

s^4	1	11	Ka
s^3	6	$6+K$	0
s^2	$-\frac{1}{6}\begin{vmatrix}1 & 11\\ 6 & 6+K\end{vmatrix}$	$-\frac{1}{6}\begin{vmatrix}1 & Ka\\ 6 & 0\end{vmatrix}$	0
s^1	$\frac{6Ka-(6+K)\left(\frac{10-K}{6}\right)}{10-K/6}$	0	
	Ka.	0	
s^0	Ka	0	0

$\to Ka.$

$\to 10-K/6.$

$\therefore 10-\frac{K}{6}\geq 0.$ $\therefore$

$0<K\leq 60.$

at $K=60$ $a=0$

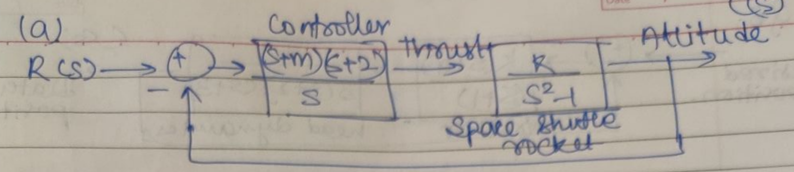
system $\to$ marginally stable

$$0\leq a\leq \frac{10}{K}+\frac{3}{2}-\frac{K}{36}.$$

$\therefore$ Stable region :- $K\in(0,60]$

$a\in[0,10].$

Q7. (a)



characteristic equation:

$$1 + \frac{K(s+m)(s+2)}{s(s^2-1)} = 0.$$

$$s^3 - s + Ks^2 + K(m+2)s + 2Km = 0.$$

$$s^3 + Ks^2 + (K(m+2)-1)s + 2Km = 0.$$

for stability:

$K > 0$, $\quad K(m+2) - 1 > 0. \longrightarrow m > 1/K - 2.$

$2Km \geq 0. \quad \therefore m \geq 0.$

s^3	1	$K(m+2)-1$
s^2	K	$2Km$
s^1	$\frac{-1}{K}\begin{vmatrix} 1 & K(m+2)-1 \\ K & 2Km \end{vmatrix}$	$0.$

$$\underbrace{(K-2)m + 2K - 1}_{} \longrightarrow \begin{cases} > 0. \ \forall \ m \geq 0 & \text{if } K \geq 2. \\ < 0 \ \forall \ m \leq 0 & \text{if } K < 1/2. \\ \geq 0 \ \forall \ m \leq \frac{2K-1}{2-K}. & K \in [1/2, 2) \end{cases}$$

System is marginally stable $\left\{ m = \frac{2K-1}{2-K} \quad K \in [1/2, 2) \right\}$

$\therefore$ System stable for

$K \geq 2, m \geq 0.$

or

$K \in [1/2, 2)$, $0 \leq m \leq \frac{2K-1}{2-K}$

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Q7.
(b) $G_1(s) = \frac{(s+m)(s+2)}{s}$

$G_2(s) = \frac{K}{s^2-1}$.

steady state error $\longrightarrow$ ramp response $e = \frac{1}{\lim_{s\to 0} sG_1(s)G_2(s)}$. when system stable

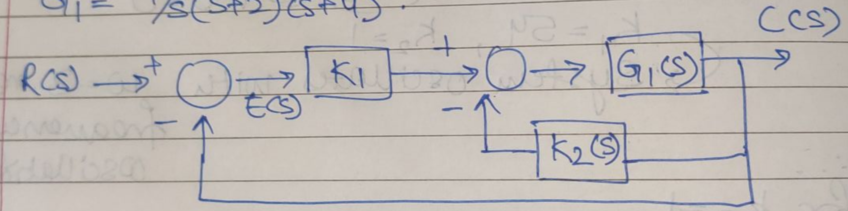
$= -1/2Km$.

given bet^n ± 0.1

K, m are both +ve
so; $\frac{-1}{2Km} \geq -0.1$

$\boxed{m \geq 5/K}$

Q8. $G_1 = 1/s(s+2)(s+4)$.
(a,b)



$1 + K_2 s G_1(s) = 0$.
↳ will have 2 equal -ve roots.

$(s+2)(s+4) + K_2 = 0$.

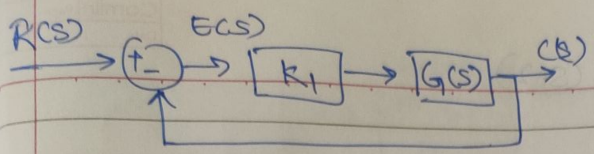
$G(s) = \frac{1}{s(1+(s+2)(s+4))}$

$K_2 = 1$

$s^2 + 6s + 9 = 0$ → -3, -3 } 2 equal roots.

∴

Q7.
char
for
K
2



$1 + K_1 G(s) = 0$

$1 + \dfrac{K_1}{s(s+2)(s+4)+s} = 0.$

$s^3 + 6s^2 + 9s + K_1 = 0$ $\qquad K_1 > 0.$

s^3	1	9
s^2	6	K_1
s	$\frac{54-K_1}{6}$	0

$0 < K_1 < 54$
for stable system.

$s^3 + 6s^2 + 9s + 54 = 0$
$(s+6)(s^2+9) = 0$

$K_1 = 54, \quad K_2 = 1$
↳ System oscillates with $\omega = 3$ rad/sec frequency of oscillation.

$\therefore$ for $K_2 = 1$
↳ $K_1 \in (0, 54)$

at $K_1 = 54$. ↝ system is marginally stable. (oscillates).

if K_2 not specified.
$\omega = \sqrt{8 + K_2}$ rad/sec
when $K_1 = 6(8 + K_2)$.

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Q8 (c) as closed loop pole at will make system unstable, we won't able to approximate step response as underdamped.

$$s^3+6s^2+(8+k_2)s+k_1=0$$

$S=5 \rightarrow$ pole

$k_1 = -S(6S+k_2)$.

So

$k_1 = 5(k_2+3)$

$(S+5)(S^2+S+k_2+3)=0$.

1 pole at $s=-5$

both complex poles have real $= -1/2$.

real pole is 10 times away from img axis.

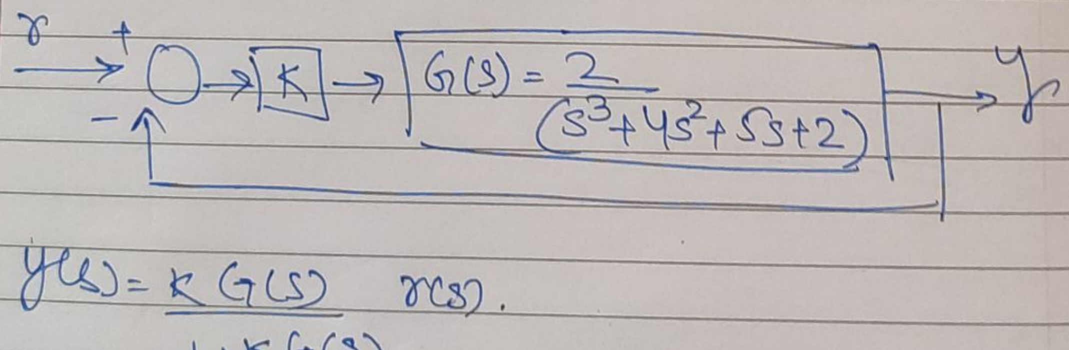
Dominant pole approximation

$k_2=1 \quad k_1=20$.

$$G(s)=\frac{20}{s^3+6s^2+9s+20}.$$

$$G_{app}(s)=\frac{4}{s^2+s+4}.$$

Q9.



$$y(s)=\frac{K\,G(s)}{1+K\,G(s)}\,r(s).$$

$$E(s)=\frac{R(s)}{1+K(G(s)}.$$

$R(s)=1/s$.

$$e=\frac{1}{1+K\lim_{s\to 0}G(s)}.$$

$$e = \frac{1}{1+K}.$$

$G(s) = (-2, -1, -1)$ in LHP.

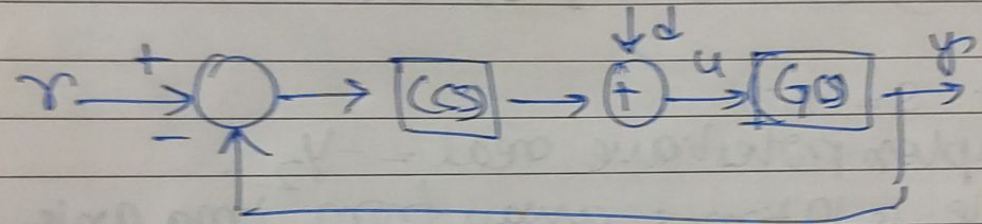
$$\frac{1}{1+K} \leq 0.15$$

$$K \geq 17/3$$

Characteristic equ^n $\quad 1 + K\,G(s) = 0$

$$s^3 + 4s^2 + 5s + 2 + 2k = 0.$$

Q10

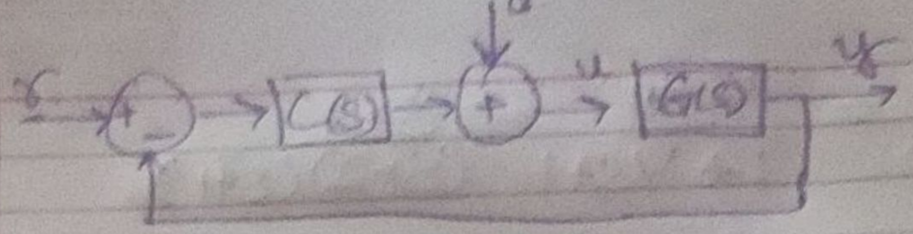


$G(s) = 1/s-1$

$C(s) = \dfrac{s-1}{s+1}.$

case 1.

Q10.



$R(s) = D(s) = 1/s$

$G(s)\big(C(s)(R(s)-Y(s)) + D(s)\big) = Y(s).$

$Y(s) = U(s)G(s).$

$r \to y$

$G(s)C(s)R(s) = Y(s)\big(1 + G(s)C(s)\big)$

$$\boxed{\frac{Y(s)}{R(s)} = \frac{GC}{1+GC}}$$

$d \to y$ $\quad GD = Y(1+GC)$

$$\boxed{\frac{Y}{D} = \frac{G}{1+GC}}$$

$r \to u$ $\quad \frac{UG}{R} = \frac{GC}{1+GC}$ $\qquad \boxed{\frac{U}{R} = \frac{C}{1+GC}}$

$d \to u$ $\quad \frac{GU}{D} = \frac{G}{1+GC}$ $\qquad \boxed{\frac{U}{D} = \frac{1}{1+GC}}$

Case ① $C = \frac{S-1}{S+1}$ $\quad G = \frac{1}{S-1}$

$r \to y$ - $\frac{Y}{R} = \frac{1/(S+1)}{1 + 1/(S+1)} = \frac{1}{S+2}.$ $\qquad Y = \frac{1}{S+2} \cdot \frac{1}{S}$

$Y = \frac{1}{2}\left(\frac{1}{S} - \frac{1}{S+2}\right)$

$$\boxed{y(t) = \frac{1}{2}\left(u(t) - e^{-2t}u(t)\right)}$$

$d \rightarrow y$

$$\frac{Y}{D} = \frac{1/(s+1)}{1+1/(s+1)} \Rightarrow \frac{s+1}{(s+2)(s+1)}$$

$$Y = \frac{\cancel{s+1}}{(s+2)\cancel{(s+1)}(s)} \rightarrow Y = \frac{-1}{2s} + \frac{2}{3(s-1)} - \frac{1}{6(s+2)}$$

$$y(t) = \left(\frac{-1}{2} + \frac{2}{3}e^{t} - \frac{1}{6}e^{-2t}\right)u(t)$$

$d \rightarrow u$.

$$\frac{U}{D} = \frac{1}{1+1/(s+1)} = \frac{s+1}{s+2} \qquad U = \frac{s+1}{s(s+2)}$$

$$U = \frac{1}{2}\left(\frac{1}{s+2} + \frac{1}{s}\right) \qquad u(t) = \left(\frac{1}{2} + \frac{1}{2}e^{-2t}\right)u(t)$$

$r \rightarrow u$.

$$\frac{U}{R} = \frac{(s-1)/(s+1)}{1+1/(s+1)} = \frac{s-1}{s+2}$$

$$U = \frac{s-1}{(s+2)s} = \frac{3}{2}\cdot\frac{1}{s+2} - \frac{1}{2s} \qquad u(t) = \left(\frac{3}{2}e^{-2t} - \frac{1}{2}\right)u(t)$$

case ② $G(s) = \frac{s-1}{s+1}$ $C(s) = 1/s-1$

$d \to u$

$\frac{U}{D} = \frac{1}{1+GC} \Rightarrow$ $\underbrace{u(t) = \left(\frac{1}{2} + \frac{1}{2}e^{-2t}\right)u(t)}_{\text{remains the same.}}$

$r \to y$ $\frac{Y}{R} = \frac{GC}{1+GC}$ $\rightarrow$ $\underbrace{y(t) = \left(\frac{1}{2} - \frac{1}{2}e^{-2t}\right)u(t)}_{\text{remains the same}}$

$r \to u$ $\frac{U}{R} = \frac{1/s-1}{1+1/s+1}$ $\rightarrow$ Same as case ① $d \Rightarrow y$.

$\underbrace{u(t) = \left(-\frac{1}{2} + \frac{2}{3}e^{t} - \frac{e^{-2t}}{6}\right)u(t)}$

$d \to y$ $\frac{Y}{D} = \frac{s-1/s+1}{1+s-1/s+1}$ $\rightarrow$ Same as case ① $r \to u$.

$\underbrace{y(t) = \left(\frac{3}{2}e^{-2t} - \frac{1}{2}\right)u(t)}$