Stochastic Rainfall Generators

16th March, 2020

AI60002

Simulation: Recap

- Process-based Models

  Try to create a mathematical model for the entire process

  Requires detailed knowledge about physics of the process

- Statistical Models

  Try to reproduce only the observable part of the process irrespective of the physics behind it

  Requires detailed knowledge about the statistical properties of the observables

Stochastic Rainfall Generator: Recap

- Create a model to generate/simulate synthetic values of rainfall
- Simulation is not same as prediction: point-wise matching is not required
- Statistical properties of the simulated values should match the corresponding properties of observed/historical values
- Stochastic: to utilize the uncertainty/noise inherent to the process
- General approach:

  Build a probabilistic model

  Create "synthetic" observations by sampling repeatedly from it

Key Statistics to be evaluated

- Proportion of wet days
- Intensity of rainfall during wet days
- Mean/max length of wet and dry spells
- Mean intensity of rainfall during wet and dry spells
- *Number of extreme rainfall events*
- The above statistics separately for each month

Key Statistics to be evaluated

- Proportion of wet days
- Intensity of rainfall during wet days
- Mean/max length of wet and dry spells
- Mean intensity of rainfall during wet and dry spells
- *Number of extreme rainfall events*
- The above statistics separately for each month
- *The above statistics separately for each location*
- *Spatial correlation across locations*

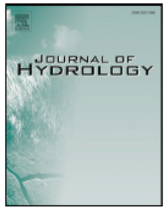
Journal of Hydrology 574 (2019) 181–192

Contents lists available at ScienceDirect

Journal of Hydrology

journal homepage: www.elsevier.com/locate/jhydrol

ELSEVIER



Research papers

A daily spatially explicit stochastic rainfall generator for a semi-arid climate

Ying Zhao[a,*], Mark A. Nearing[b], D. Phillip Guertin[a]

[a] *School of Natural Resources and the Environment, University of Arizona, Tucson, AZ 85719, USA*
[b] *USDA-Agricultural Research Service, Southwest Watershed Research Center, Tucson, AZ 85719, USA*

ARTICLE INFO

This manuscript was handled by A. Bardossy, Editor-in-Chief, with the assistance of Uwe Haberlandt, Associate Editor

Keywords:
Rainfall generator
Spatial
Semi-arid
Convective storm
Markov chain

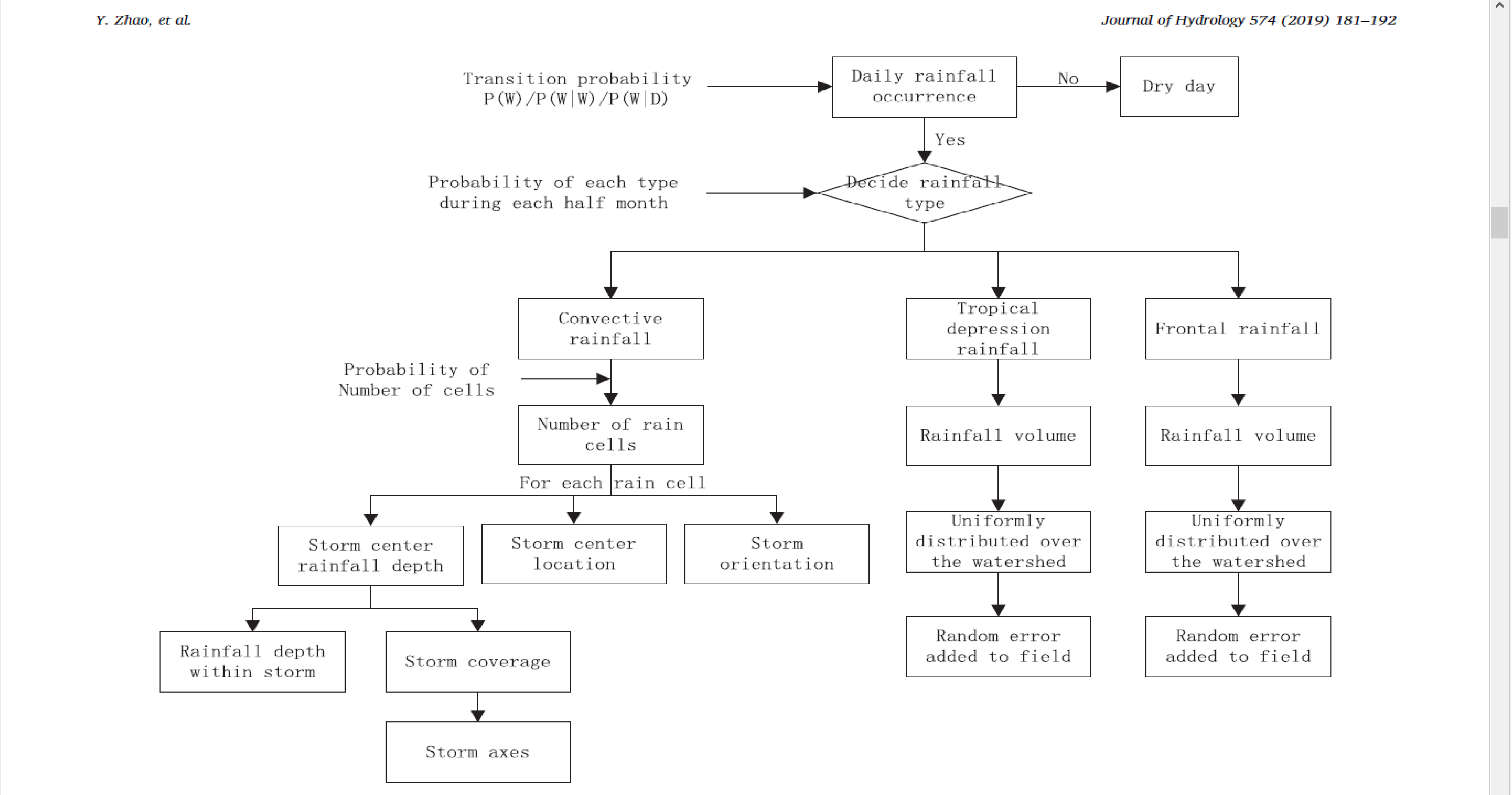
ABSTRACT

Many semi-arid regions of the world experience rainfall patterns characterized by high spatial variability. Accurate spatial representation of different types of rainfall will facilitate the application of distributed hydrological models in these areas. This study presents a daily, spatially distributed, stochastic rainfall generator based on a first-order Markov chain model, calibrated using 50 years of rainfall observations at 88 gages from 1967 through 2016 in the 148-km^2 Walnut Gulch Experimental Watershed. Three types of rainfall, including convective, frontal, and tropical depression storms, were simulated separately in the generator using biweekly parameterization. Convective storms were simulated based on an elliptical shape rain cell conceptual model, whereas frontal and tropical depression storms were simulated as uniform rainfall fields over the whole watershed with introduced random variability. The rainfall generator was evaluated by comparing the mean statistics of 30 sets of 50-year simulated data versus the 50-year rain gage observed data. Most individual storm statistics and aggregated seasonal rainfall statistics were similar to the measured rainfall observations. The long-term mean values of both summer and winter rainfall amount were statistically satisfactory. This model can serve as a guide for application in areas with convective, frontal, and tropical depression storms.

Simulation Field

- A relatively small semi-arid region in USA at altitude
- Months of July, August, and September, accounts for approximately 60% of the total annual amount of rainfall
- Frontal storms during the non-summer months account for approximately 35% of the annual precipitation.
- The remaining 5% of the annual rainfall falls in the form of tropical depression storms
- The summer rain often forms as convective storms, with relatively short duration but high intensity, and cover a limited spatial extent
- The winter frontal storms are, however, usually of long duration but low intensity, and usually cover the whole watershed more uniformly

Simulation Model



Model Parameters

Table 1
Transition probabilities, probabilities for three types of rainfall, and the probabilities for multiple events in all 24 half month periods.

Half month		1	2	3	4	5	6	7	8	9	10	11	12
Transition probabilities	P(W)	0.2053	0.2225	0.2547	0.1914	0.1880	0.1338	0.1147	0.0960	0.0960	0.1175	0.1240	0.2640
	P(W|W)	0.4740	0.5000	0.5602	0.4776	0.4752	0.3738	0.4186	0.4167	0.3750	0.5106	0.4624	0.6111
	P(W|D)	0.1359	0.1431	0.1503	0.1237	0.1215	0.0952	0.0753	0.0619	0.0664	0.0652	0.0761	0.1377
Probabilities for types of rainfall	Convective	0	0	0	0	0	0	0	0	0	0	0	0
	Frontal	1	1	1	1	1	1	1	1	1	1	1	1
	Tropical	0	0	0	0	0	0	0	0	0	0	0	0
Probabilities for multiple events	1	0.7309	0.7155	0.6589	0.7629	0.6751	0.7312	0.6690	0.8808	0.8217	0.7066	0.7868	0.8031
	2	0.1651	0.1626	0.2321	0.1626	0.2188	0.1680	0.2242	0.1022	0.1426	0.1726	0.1745	0.1663
	3	0.1040	0.1219	0.1089	0.0745	0.1061	0.1008	0.1068	0.0170	0.0357	0.1208	0.0388	0.0306
	4												
	5												
Parameter of distributions*	μ (mm or 10^5 m^3)	2.6923	2.8794	2.1721	1.8628	2.6981	2.0318	1.5667	1.6052	1.4837	1.6541	2.3326	2.1434
	σ (mm)												

Half month		13	14	15	16	17	18	19	20	21	22	23	24
Transition probabilities	P(W)	0.6213	0.7738	0.7547	0.6438	0.5320	0.2733	0.2160	0.1950	0.1440	0.1613	0.2173	0.2288
	P(W|W)	0.7854	0.8336	0.8269	0.7592	0.7118	0.5561	0.5185	0.4615	0.4444	0.3884	0.5215	0.5191
	P(W|D)	0.3521	0.5635	0.5326	0.4316	0.3276	0.1651	0.1327	0.1304	0.0935	0.1176	0.1329	0.1410
Probabilities for types of rainfall	Convective	1	1	1	1	0.9876	0.9876	0	0	0	0	0	0
	Frontal	0	0	0	0	0	0	0.9876	0.9876	0.9876	0.9876	1	1
	Tropical	0	0	0	0	0.0124	0.0124	0.0124	0.0124	0.0124	0.0124	0	0
Probabilities for multiple events	1	0.6449	0.6499	0.6450	0.6842	0.7133	0.6966	0.7409	0.7515	0.7496	0.7596	0.7801	0.6929
	2	0.2187	0.2227	0.2171	0.2178	0.1633	0.1678	0.1766	0.1792	0.2019	0.1492	0.1367	0.1906
	3	0.0961	0.0911	0.0953	0.0713	0.0867	0.0807	0.0824	0.0694	0.0485	0.0912	0.0832	0.1165
	4	0.0362	0.0300	0.0310	0.0255	0.0250	0.0323						
	5	0.0042	0.0062	0.0115	0.0013	0.0117	0.0226						
Parameter of distributions*	μ (mm or 10^5 m^3)	1.5314	1.7461	1.6551	1.6105	1.6272	1.3189	2.4079	2.4877	2.5405	2.0375	3.2548	2.3647
	σ (mm)	1.4235	1.4680	1.4851	1.4827	1.4576	1.5344						

* (1) July–September (13–18): lognormal distribution for convective rainfall maximum depth, unit: mm. (2) Other months (1–12, 19–24): exponential distribution for frontal rainfall volume, unit: 10^5 m^3. (3) September–November (17–22): μ of exponential distribution for tropical depression rainfall is 4.2643×10^6 m^3.

Comparisons

Table 4
Observed and simulated rainfall totals for summer months of six gages (mm).

Gage ID	Observed						Simulated					
	13	34	44	46	62	80	13	34	44	46	62	80
Mean	186.7	192.2	194.6	199.5	194.5	189.7	196.3	187.1	185.8	191.8	186.5	195.0
Std. dev.	60.4	63.7	58.6	61.4	52.2	66.8	100.4	99.7	101.6	102.6	96.3	100.7
Max	336.6	345.7	345.9	410.5	327.3	380.0	508.3	561.7	623.5	617.3	534.2	511.9
Min	89.8	70.2	81.0	77.7	88.8	75.4	3.6	1.5	7.2	8.2	10.4	9.1
Range	246.8	275.5	264.9	332.7	238.5	304.5	504.8	560.2	616.3	609.1	523.9	502.8
Skewness	0.5	0.4	0.4	0.5	−0.2	0.6	0.5	0.6	1.0	0.9	0.6	0.6

Table 5
Observed and simulated rainfall totals for non-summer months of six gages (mm).

Gage ID	Observed						Simulated					
	13	34	44	46	62	80	13	34	44	46	62	80
Mean	122.7	120.7	121.6	132.8	116.9	120.9	122.6	122.0	122.0	122.7	121.6	122.1
Std. dev.	59.7	65.8	61.1	65.1	64.3	59.8	34.2	34.1	33.7	34.7	34.0	34.3
Max	266.4	308.9	295.0	318.8	300.4	282.8	265.0	305.8	282.0	280.8	264.7	256.8
Min	19.8	18.0	13.2	16.3	10.7	12.2	42.3	41.9	42.3	33.2	44.7	30.3
Range	246.6	290.8	281.8	302.5	289.7	270.6	222.7	263.9	239.8	247.6	219.9	226.5
Skewness	0.6	0.9	0.9	0.8	1.2	0.7	0.5	0.6	0.4	0.5	0.6	0.4

Observed and simulated median length of dry and wet spells (day).

	Annual	Summer	Non-summer		Annual	Summer	Non-summer
Dry_observed	4.2	2.2	7.2	Wet_observed	1.0	1.0	1.0
Dry_simulated	4.0	2.0	6.0	Wet_simulated	2.0	3.0	1.0

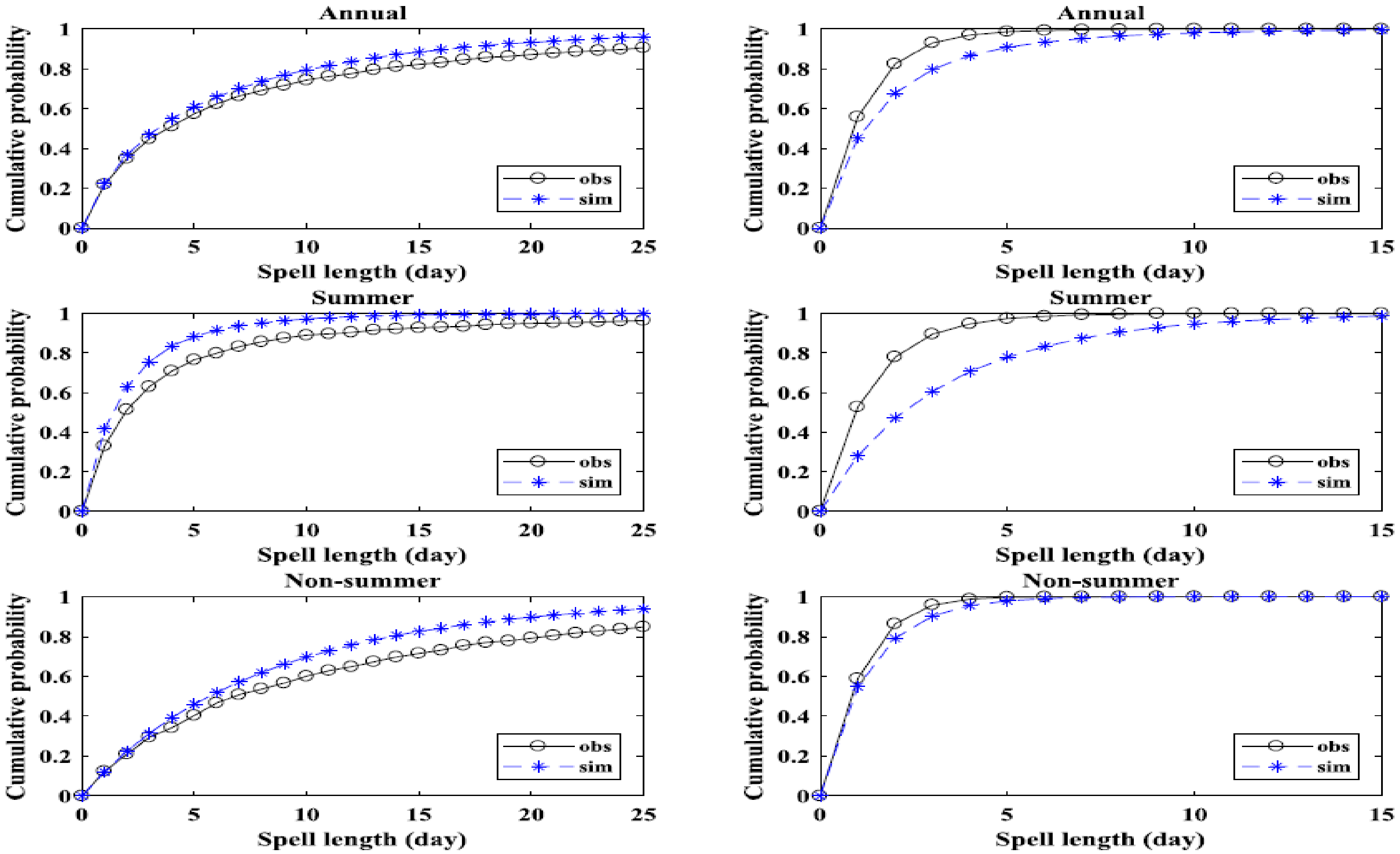


Fig. 7. CDFs of observed and simulated dry and wet spell length for annual, summer and non-summer periods, (1) first column: dry spell, (2) second column: wet spell.

Stoch Environ Res Risk Assess (2015) 29:347–356
DOI 10.1007/s00477-014-0911-6

ORIGINAL PAPER

Coupled stochastic weather generation using spatial and generalized linear models

Andrew Verdin · Balaji Rajagopalan ·
William Kleiber · Richard W. Katz

Published online: 5 July 2014

Abstract We introduce a stochastic weather generator for the variables of minimum temperature, maximum temperature and precipitation occurrence. Temperature variables are modeled in vector autoregressive framework, conditional on precipitation occurrence. Precipitation occurrence arises via a probit model, and both temperature and occurrence are spatially correlated using spatial Gaussian processes. Additionally, local climate is included by spatially varying model coefficients, allowing spatially evolving relationships between variables. The method is illustrated on a network of stations in the Pampas region of Argentina where nonstationary relationships and historical spatial correlation challenge existing approaches.

Keywords Spatial correlation · Pampas · Precipitation · Temperature · Weather simulation

ensembles of input sequences, which are typically daily weather, resulting in ensembles of system variables and their probability density functions that provide estimates of risk that are useful for decision making. Historic data is often limited in space and time hence the risk estimates based solely on them do not accurately reflect the underlying variability. Therefore, robust generation of weather sequences that capture the underlying variability is essential. Generating random weather sequences that are statistically consistent with historical observations is known as stochastic weather generation.

Crop models for agriculture planning, hydrologic models for generating streamflow needed for water resources management, and erosion models for land erosion management (Wallis and Griffiths 1997; Richardson 1981; Richardson and Wright 1984; Wilks 1998; Wilks and Wilby 1999; Friend et al. 1997) have motivated the development of stochastic weather generators over the

Stochastic Model - temperature

- Condition the bivariate temperature process on precipitation occurrence.
- Precipitation largely occurs due to large scale atmospheric movement, while surface temperatures are highly controlled by local climate factors and by whether or not precipitation occurs

$$Z_N(s,t) = \beta_N(s)'\mathbf{X}_N(s,t) + W_N(s,t)$$
$$Z_X(s,t) = \beta_X(s)'\mathbf{X}_X(s,t) + W_X(s,t).$$

- The first component is a local regression on some covariate vector X
- Regression parameters β are specific to location
- The weather component (denoted by W for weather) generates variability and spatial correlation via a multivariate normal Gaussian process.

Stochastic Model - rainfall

- The precipitation process is broken into two components: the occurrence O(s,t), and the intensity or amount, A(s,t) at location s on day t.
- Occurrence process is modelled as a probit: $O(s,t) = \mathbb{1}_{[W_O(s,t) \geq 0]}$

  where the latent process Wo(s,t) is a Gaussian Process.

  If the latent process is positive, it rains at location s, else it doesn't
- The latent process has mean function that is a regression on some covariates
- Rainfall intensity is spatially correlated by imposing a zero-mean Gaussian process WA(s,t) with covariance function CA(h,t) $A(s,t) = G_{s,t}^{-1}(\Phi(W_A(s,t)))$

Modeling Choices

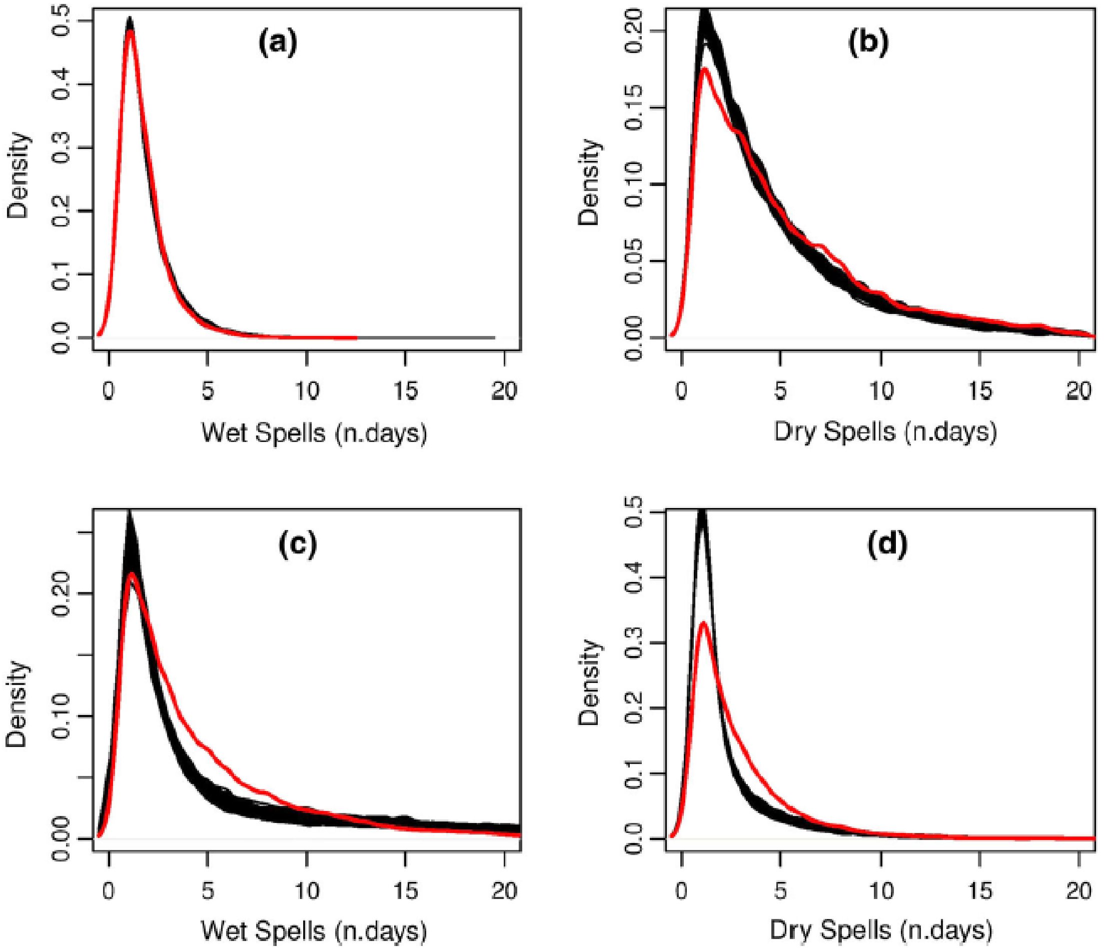
- For each of max, min temperature and precipitation intensity we have a covariate vector

$$\mathbf{X}_N(s,t) = (1, \cos(2\pi t/365), \sin(2\pi t/365), r(t), Z_N(s, t-1), Z_X(s, t-1), O(s,t))'.$$

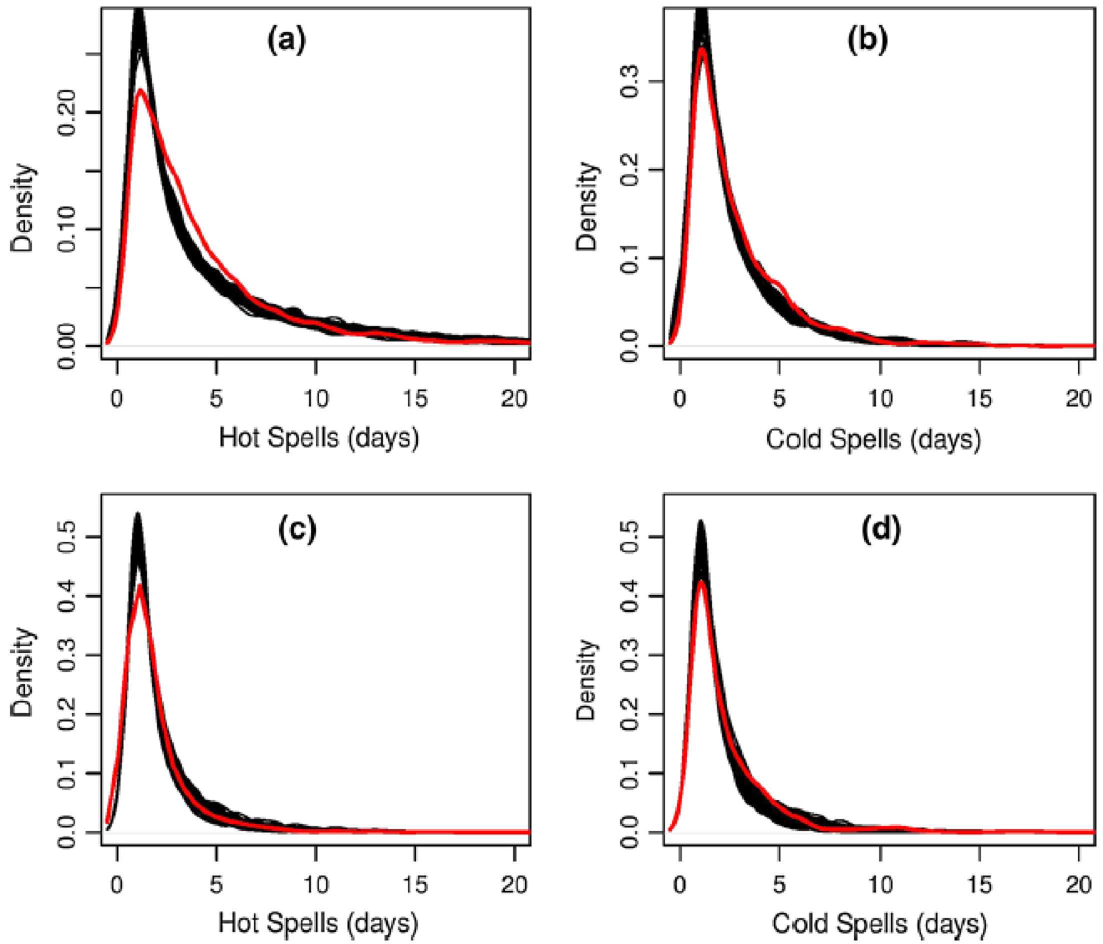
$$\mathbf{X}_O(s,t) = (1, \cos(2\pi t/365), \sin(2\pi t/365), O(s, t-1))',$$

- The linear coefficients β are estimated through least-square regression
- Different covariance functions can be tried out for the Gaussian Processes

Results

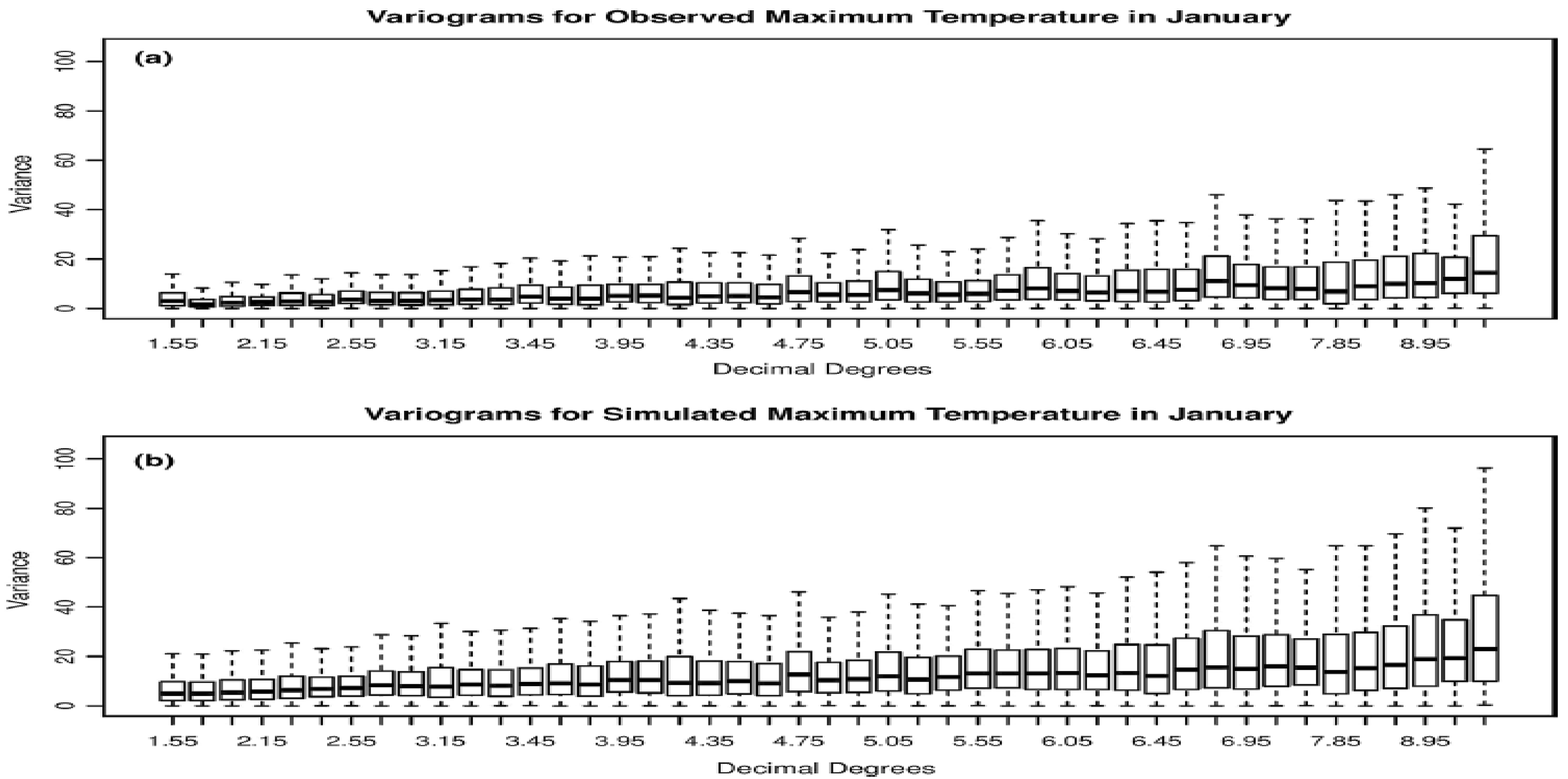


(a)
(b)
(c)
(d)
Density
Wet Spells (n.days)
Dry Spells (n.days)
0.0 0.1 0.2 0.3 0.4 0.5
0.00 0.05 0.10 0.15 0.20
0.00 0.10 0.20
0 5 10 15 20



(a)
(b)
(c)
(d)
Density
Hot Spells (days)
Cold Spells (days)
0.00 0.10 0.20
0.0 0.1 0.2 0.3
0.0 0.1 0.2 0.3 0.4 0.5
0 5 10 15 20

Results


Variograms for Observed Maximum Temperature in January
(a)
Variance
0
20
40
60
80
100
1.55
2.15
2.55
3.15
3.45
3.95
4.35
4.75
5.05
5.55
6.05
6.45
6.95
7.85
8.95
Decimal Degrees
Variograms for Simulated Maximum Temperature in January
(b)
Variance
0
20
40
60
80
100
1.55
2.15
2.55
3.15
3.45
3.95
4.35
4.75
5.05
5.55
6.05
6.45
6.95
7.85
8.95
Decimal Degrees