Control & Instrumentation Lab, Autumn 2021-22

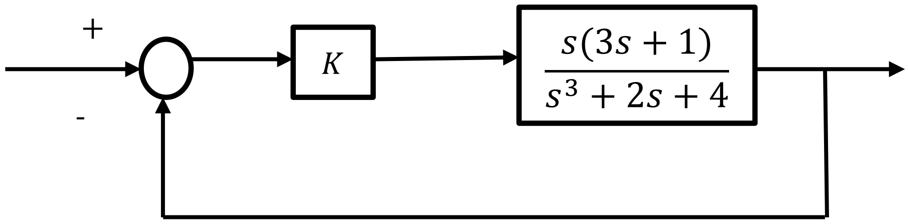
Session 4: Stability Analysis

Note: Please solve all the problems by hand and then verify using MATLAB.

1. $M(s) = \frac{20(s-1)}{(s+2)(s^2+4)}$

   Is $M(s)$ asymptotically stable, marginal stable or unstable?

2. Consider the characteristic equation
   $$2s^4 + s^3 + 3s^2 + 5s + 10 = 0$$
   of a system. Using Routh-Hurwitz (RH) criterion determine the stability of system.
3. Consider the characteristic equation
   $$s^4 + s^3 + 2s^2 + 2s + 3 = 0$$
   of a system. Using RH criterion determine its stability.
4. Consider the characteristic equation
   $$s^5 + 4s^4 + 8s^3 + 8s^2 + 7s + 4 = 0$$
   of a system. Using RH criterion determine its stability.
5. Consider the closed-loop system



   where K is the gain of proportional controller. Determine the admissible values of K for which the closed-loop system is stable. Use RH criterion.

6. Consider a disk-storage data-head positioning control system as shown in Fig. 1 where $k > 0$.

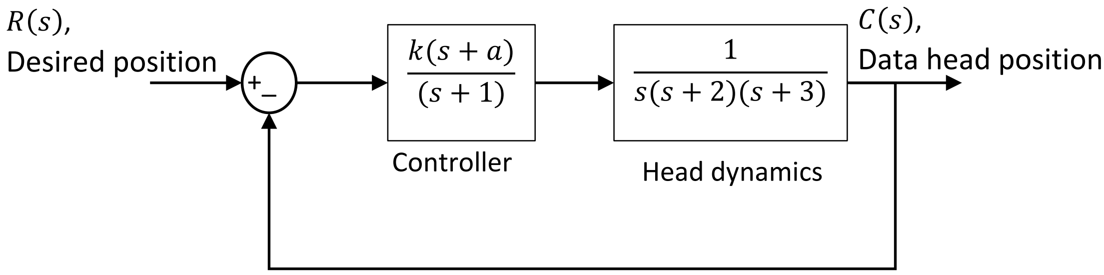


Fig. 1: Disk-storage data-head positioning feedback system

   Determine the conditions on k and a for closed-loop stability. Also sketch the stability region. What is the impact reducing the gain a on the head position response for a fixed k?.

7. The attitude control system of a space shuttle rocket is shown in Fig. 2.
   a. Determine the range of gain k and m so that the closed-loop system is stable and plot the region of stability.
   b. Select the values of k and m so that the steady-state error to a ramp input is less than or equal to 10% of the input magnitude.

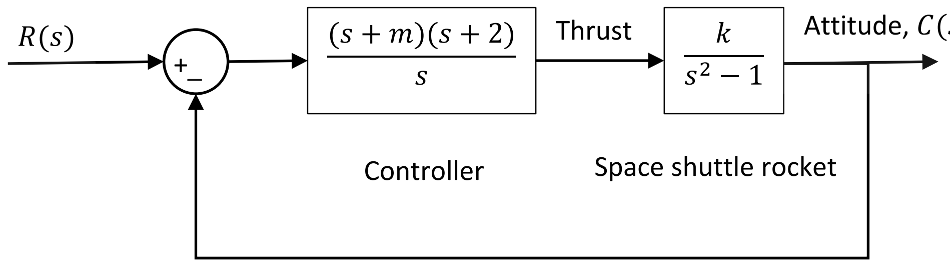


Fig. 2: Attitude control system of a space shuttle

8. The system shown in Fig. 3 has

$$G_1 = \frac{1}{s(s+2)(s+4)}.$$

   Find the following:
   a. The value of K_2 for which the inner-loop will have two equal negative real poles and the associated range of K_1 for system stability.
   b. The value of K_1 at which the system oscillates and the associated frequency of oscillation.
   c. The gain K_1 at which a real closed-loop pole is at $s = 5$. Can the step response, $c(t)$, be approximated by a second order, under-damped response in this case? Why or why not?

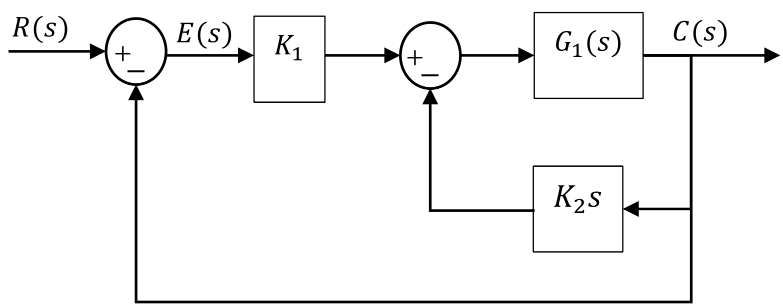


Fig. 3: A two-loop feedback system

9. Consider the feedback system shown in Fig. 4. With the help of R-H criterion, determine the range of gain K so that the steady-state error in presence of step reference input remains within 15% of the reference input magnitude.

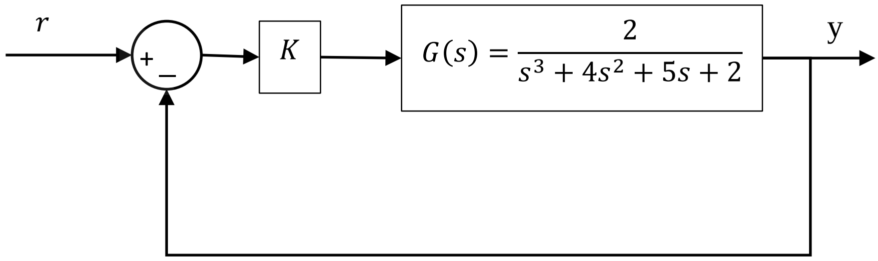


Fig. 4: A feedback system with proportional controller

10. Consider the feedback system shown in Fig. 5. Suppose $G(s) = 1/(s-1)$ and $C(s) = \frac{s-1}{s+1}$. Check the step responses from r-to-y, d-to-y, r-to-u, and d-to-u. Now consider $G(s) = \frac{s-1}{s+1}$ and $C(s) = 1/(s-1)$ and check the above responses. Can you draw some conclusions from these two cases?

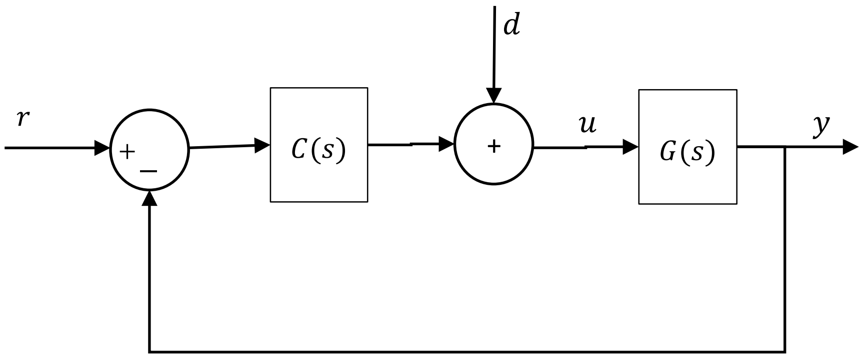


Fig. 5: A general feedback system