ML for Earth Science.

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Q1. $X(s,t) = a^* m(s) + (1-a)^* n(t) + e(s,t)$

$a = 0.25$,

$m(s_1) = 5$ $\quad$ $e(s,t) = \mathcal{N}(0, 0.5)$.
gaussian distribution.

Months	1	2	3	4.	1	2	3	4
S_1	4.5	6.0	7.0	4.7	4.6	7.0	7.2	5.4
S_2	6.2	7.5	8.0	6.5	6.4	7.2	7.7	6.4
	Year 1.				Year 2.			

Month	1	2	3	4	1	2	3	4.
S_1	4.6	5.4	8.5	6.6	5.4	5.8	6.1	5.0
S_2	5.9	6.6	7.6	6.4	6.0	7.5	8.2	6.3
	Year 3				Year 4			

Month	1	2	3	4	1	2	3	4.
S_1	5.1	6.3	7.7	5.1	5.5	5.9	7.9	6.1
S_2	6.0	6.8	7.8	6.8	5.7	7.2	7.9	6.3
	Year 5				Year 6.			

for same month in diff^{rt} years.

$4.5 = 5a + (1-a)\,n(t_1) + e(s_1, t_1)$

$4.6 = 5a + (1-a)\,n(t_1) + e(s_1, t_1)$

$4.6 = \ \vdots$

$5.4 = \ \vdots$

$5.1 = \ \vdots$

$5.5 = \ \vdots$

$29.7 = 30a + 6(1-a)\,n(t_1) + \sum e_i$

divide by 6, 0 when /6.

$4.95 = 5a + (1-a)\,n(t_1)$

$4.95 = 5(0.25) + (1-0.25)\,n(t_1)$

$\boxed{n(t_1) = 4.934}$

$n(t_1) = \dfrac{\bar{X}_{t_1} - a\,m(s_1)}{1-a} = 4.934.$

Similarly for t_2.

$n(t_2) = \dfrac{\text{mean}(6, 7, 5.4, 5.8, 6.3, 6.9) - 0.25 \times 5}{1 - 0.25}$

$= 6.421$

$n(t_3) = \dfrac{\text{mean}(7, 7.2, 8.5, 6.1, 7.7, 7.9) - 1.25}{1 - 0.25}$

$= 8.2$

$$n(t_4) = \frac{\text{mean}(4.7, 5.4, 6.6, 5.0, 5.1, 6.1) - 0.25}{1 - 0.25}$$

$$= 5.644$$

$n(t_1) = 4.934$

$n(t_2) = 6.421$

$n(t_3) = 8.2$

$n(t_4) = 5.644$.

$\therefore$ estimate $m(S_2)$.

$$\begin{cases} X(S_2, t_1) = a\,m(S_2) + (1-a)\,n(t_1) + e_1 \\ X(S_2, t_2) = a\,m(S_2) + (1-a)\,n(t_2) + e_2 \\ \vdots \end{cases}$$

$$\overline{X}_{S_2} = a\,m(S_2) + (1-a)\left(6n(t_1) + 6n(t_2) \cdots n(t_4)\right) + \cancel{\sum e_i}$$

$\swarrow$ 0

$$6.87 = 0.25\,m(S_2) + 0.75 \times 6(n_1 + \cdots n_4)$$

$$\boxed{m(S_2) = -426.102}$$

Q2. weakly stationery wrt covariance:

Day/time	12 midnight	6AM	12 noon	6 pm
1	6	8	25	20
2	7	9	27	22
3	6	7	24	X
4	8	10	25	23
5	5	6	x	19

covariance stationery:-

$\operatorname{cov}(x_1, x_2) \simeq \operatorname{cov}(x_2, x_3) \simeq \operatorname{cov}(x_3, x_4)$.

$$1.75 = \frac{1}{5}\Big[(25-(20.2+x/5))(0) + (27-(20.2+x/5))(1) + (24-(20.2+x/5))(-1) + (25-(20.2+x/5))(2) + (x-(20.2+x/5))(-2)\Big]$$

$\bar{x} = \text{mean} = 80+10+11+x$

$= \frac{101+x}{5} = 20 + (x+1)/5$

$\text{mean} = 8+19+13 = 32+8 = 40/5 = 8$.

$1.75 \times 5 \simeq 27 - \bar{x} - 24 + \bar{x} + 2(25-\bar{x}) - 2(x-\bar{x})$

$8.75 \simeq \underbrace{27-24+50}_{3.} - 2x$.

$x \simeq \frac{53-8.75}{2} \simeq \boxed{22.125}$

$\therefore\ x \simeq 24.$

$\bar{x} = 25$

$cov(x_2, x_3) = 1.25$ } approx same.
$cov(x_1, x_2) = 1.75$ }

$$cov(x_3, x_4) \simeq 1.75 = \frac{1}{5}\left[(20-\bar{x})(0) + (22-\bar{x})(2) + (-1)(X-\bar{X}) + (23-\bar{x})(0) + (19-\bar{x})(-1)\right]$$

$$6.25 = 44 - 2\bar{X} - X + \bar{X} - 19 + \bar{X}$$

$$X = 44 - 19 - 6.25$$

$$X \simeq 18.75$$

$\therefore\ X = 18$

$$\bar{X} = \frac{84 + X}{5} = 20.4$$

$$cov(x_3, x_4) = 1.75$$

$\therefore$ $\boxed{\begin{matrix} x = 24 \\ X = 18 \end{matrix}}$ for covarience stationery.

Q3. $X(s,t) = Y(s) + Z(t) + e(s,t)$.

$m(s)$ = Gaussian process mean func^{n}

$C(s,s') = \exp(-0.1 * \text{dist}^{n}(s,s'))$

$Z(t) = a(t) + Z(t-1)$

$a = N(0,2)$.

$Z(1) = 0$.

$e(s,t)$ = random noise $N(0,1)$.

Lat	Lon	Day 1	2	3	4	5	m(s)	
20	60	6	7	5	5	5	5	
20	61	a	9	8	7	8	6	
20	62	6	8	5	5	6	7	missing
21	60	5	5	4	4	5	c	
21	61	6	7	b	6	6	5	
21	62	4	6	4	4	5	6	

$Y(S) = X(s,t) - Z(t) - e(s,t)$.

$Y(S) = X(s,t) - a(t) - Z(t-1) - e(s,t)$

(2) $\boxed{Y(S) = X_2 - 2a - 0 - e(s,t)}$

at Day 1

$Z(1) = a + Z(0)$.

at Day 2

$Z(2) = 2a + Z(1)$; $Z(2) = 2a$

(1) $$Y(S) = \begin{bmatrix} 5 \\ 6 \\ 7 \\ 5 \\ 6 \end{bmatrix} \begin{bmatrix} 1 & e^{-0.1} & e^{-0.2} & 1 & 1 & 1 \\ 1 & e^{-0.1} & e^{-0.1} & e^{-0.1} & 1 \\ 1 & e^{-0.2} & e^{-0.1} & 1 & 1 \\ 1 & 1 & 1 & e^{-0.1} & 1 \\ 1 & & & & \\ 1 & & & & \end{bmatrix}$$

entering covarience.

6x6 matrix.

comparing (1) & (2), we will get parameters.