

```

clc
%QS1: Defined the function (funct1.m)

%QS2: a) dy/dt=2
[t_1,y_1, time_1]=funct1(@myODE, [0 5], 0, 0.1);
time_1

time_1 = 0.0081

y_1

y_1 = 1x51
    0      0.2000      0.4000      0.6000      0.8000      1.0000      1.2000      1.4000 ...

```

```

t_1

t_1 = 1x51
    0      0.1000      0.2000      0.3000      0.4000      0.5000      0.6000      0.7000 ...

```

```

[t_2,y_2, time_2]=funct1(@myODE, [0 5], 0, 0.01);
time_2

time_2 = 7.9050e-04

y_2

y_2 = 1x501
    0      0.0200      0.0400      0.0600      0.0800      0.1000      0.1200      0.1400 ...

```

```

[t_3,y_3, time_3]=funct1(@myODE, [0 5], 0, 0.001);
time_3

time_3 = 0.0092

y_3

y_3 = 1x5001
    0      0.0020      0.0040      0.0060      0.0080      0.0100      0.0120      0.0140 ...

```

```

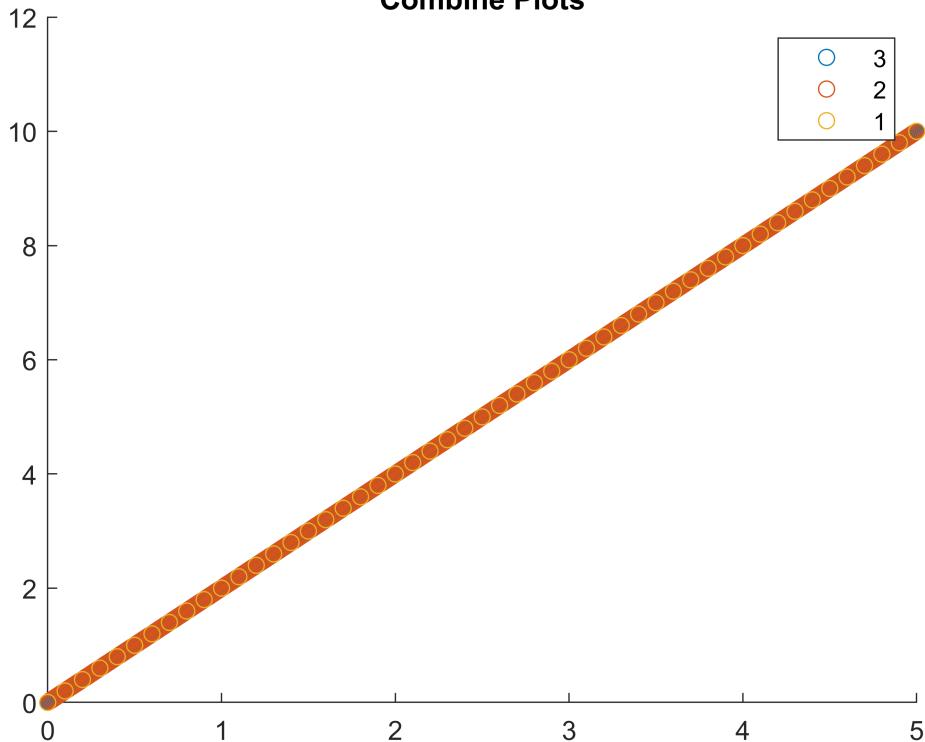
scatter(t_3,y_3)
hold on
scatter(t_2,y_2)
hold on
scatter(t_1,y_1)

title('Combine Plots')
legend('0.001','0.01','0.1')

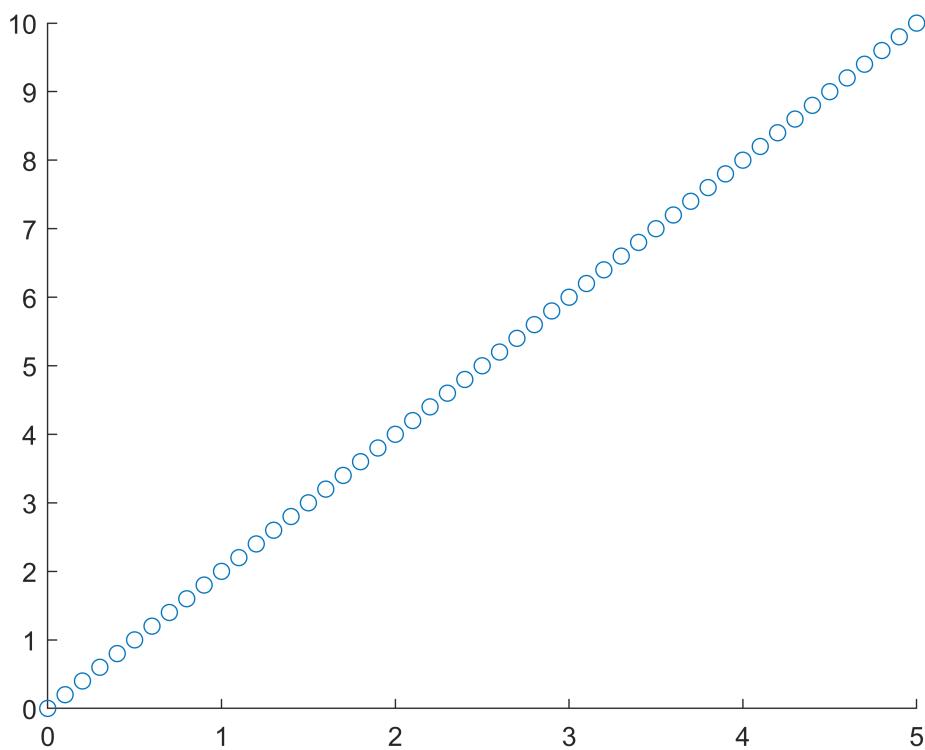
hold off

```

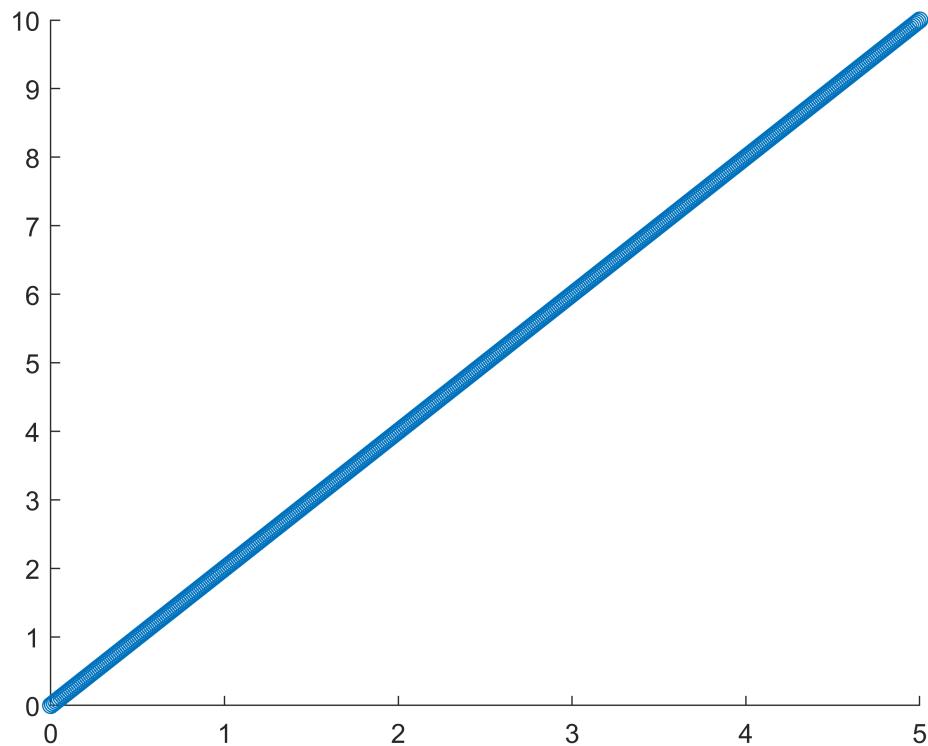
Combine Plots



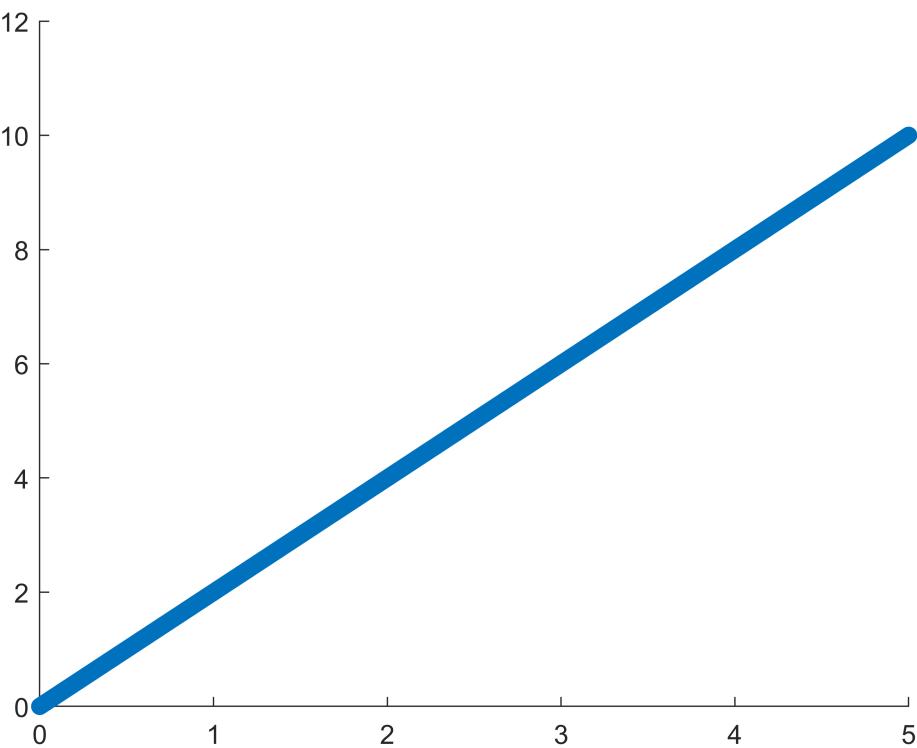
```
scatter(t_1,y_1)
```



```
scatter(t_2,y_2)
```



```
scatter(t_3,y_3)
```



%QS2: b) $dy/dt = 3t^2$

```
[t_1,y_1, time_1]=funct1(@myODE, [0 5], 0, 0.1);  
time_1
```

time_1 = 0.0094

y_1

$y_1 = \begin{bmatrix} 1 \\ 52 \end{bmatrix}$

```
[t_2,y_2, time_2]=funct1(@myODE, [0 5], 0, 0.01);  
time_2
```

time 2 = 0.0015

y_2

```
y_2 = 1x502
     0      0    0.0000    0.0000    0.0000    0.0001    0.0002    0.0003 ...
```

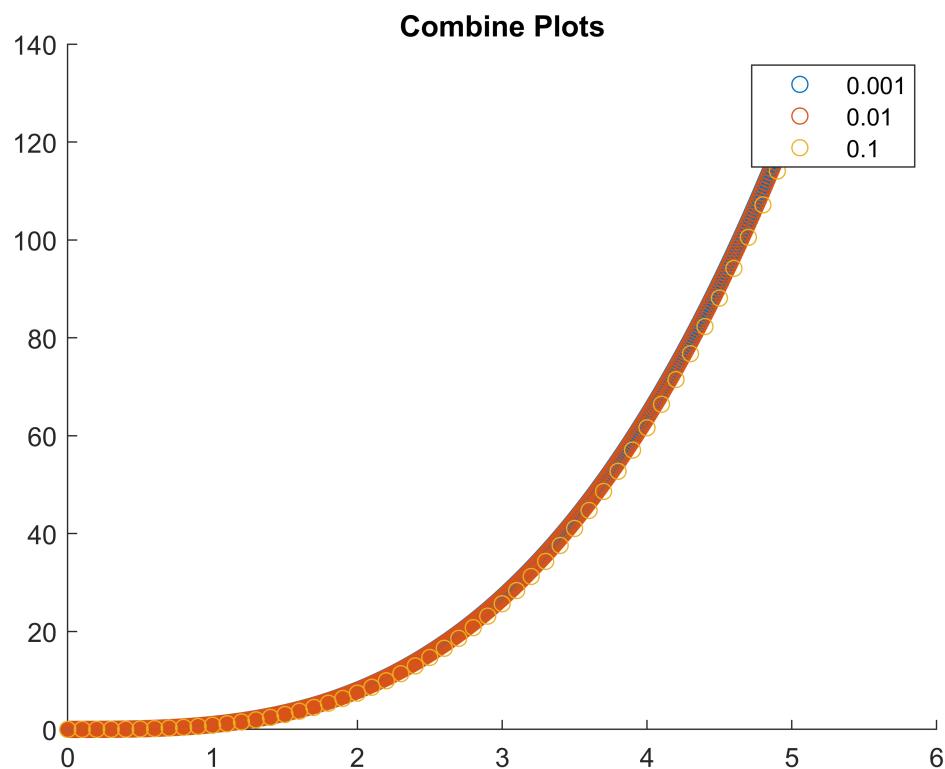
```
[t_3,y_3, time_3]=funct1(@myODE, [0 5], 0, 0.001);  
time_3
```

time 3 = 0.0184

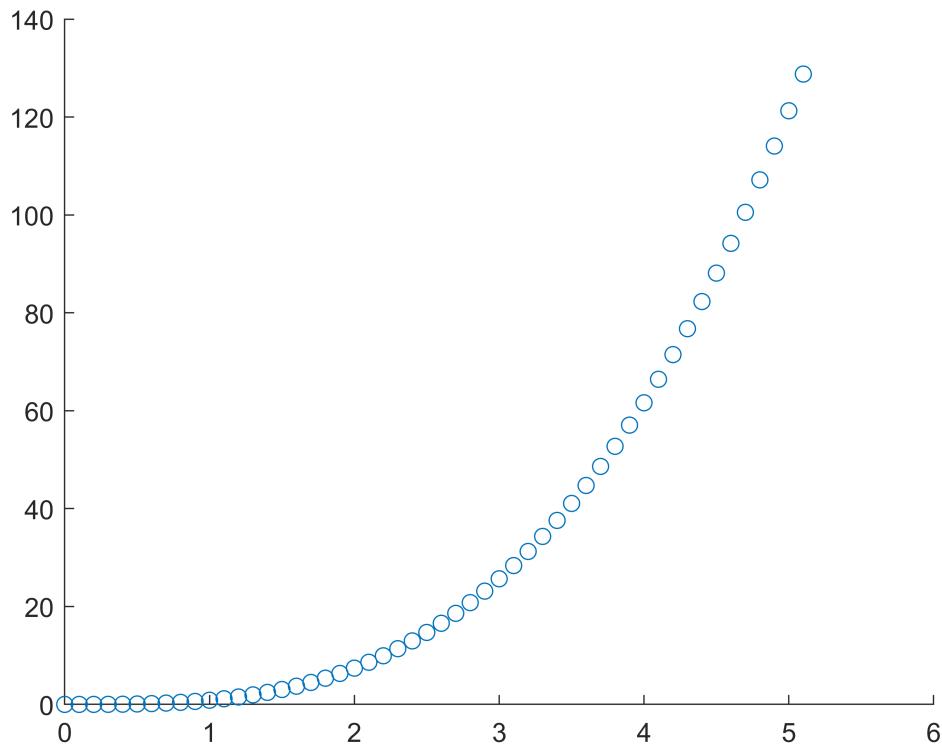
y 3

```
y_3 = 1x5001  
0 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 ...
```

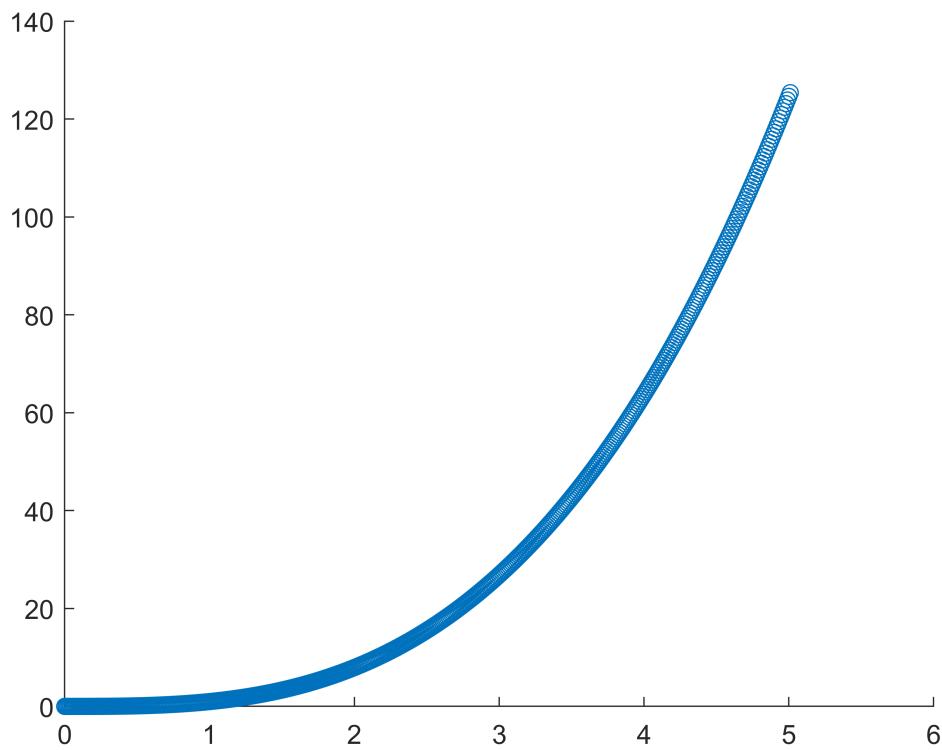
```
scatter(t_3,y_3)  
hold on  
scatter(t_2,y_2)  
hold on  
scatter(t_1,y_1)  
  
title('Combine Plots')  
legend('0.001','0.01','0.1')  
  
hold off
```



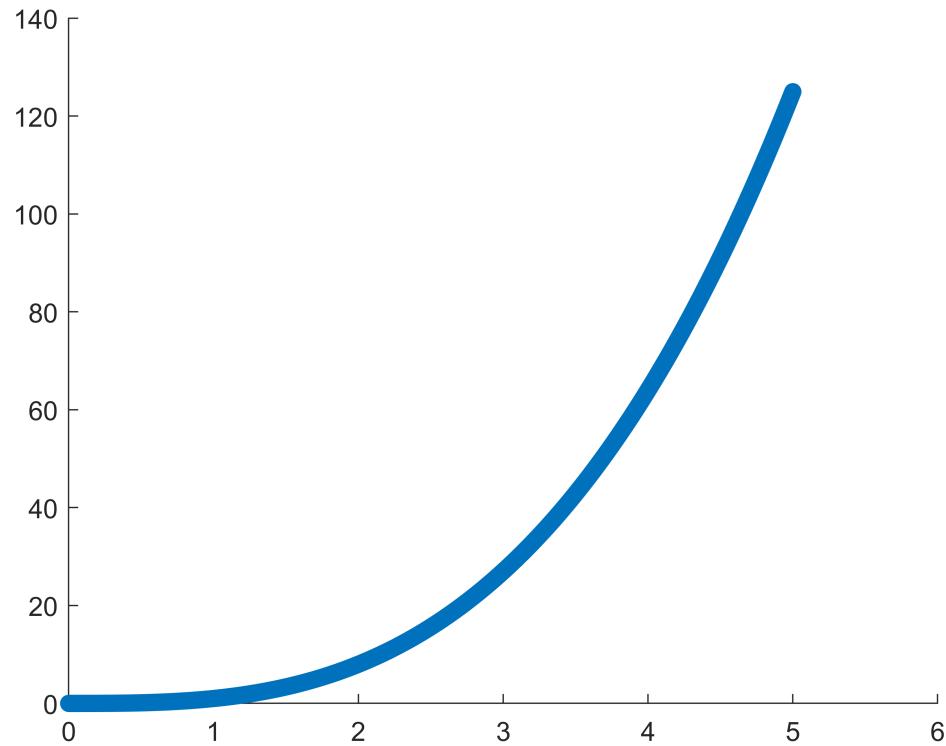
```
scatter(t_1,y_1)
```



```
scatter(t_2,y_2)
```



```
scatter(t_3,y_3)
```



```
tic  
dist_1=norm(y_1-t_1.^3)
```

```
dist_1 = 12.6628
```

```
toc
```

```
Elapsed time is 0.017718 seconds.
```

```
tic  
dist_2=norm(y_2-t_2.^3)
```

```
dist_2 = 3.7750
```

```
toc
```

```
Elapsed time is 0.017854 seconds.
```

```
tic  
dist_3=norm(y_3-t_3.^3)
```

```
dist_3 = 1.1861
```

```
toc
```

```
Elapsed time is 0.009693 seconds.
```

```
% with the increase in h value the accuracy of function reduces, error function increases, the
```

```
% number of datapoints reduces, and the computational time increases
```

```
%QS3: defined the function (funct2.m)
```

```
%QS4:
```

```
[t_1,y_1, time_1]=funct2(@myODE, [0 5], 0, 0.1);  
time_1
```

```
time_1 = 0.0043
```

```
y_1
```

```
y_1 = 1x52  
0 0.0010 0.0080 0.0270 0.0640 0.1250 0.2160 0.3430 ...
```

```
[t_2,y_2, time_2]=funct2(@myODE, [0 5], 0, 0.01);  
time_2
```

```
time_2 = 0.0053
```

```
y_2
```

```
y_2 = 1x502  
0 0.0000 0.0000 0.0000 0.0001 0.0001 0.0002 0.0003 ...
```

```
[t_3,y_3, time_3]=funct2(@myODE, [0 5], 0, 0.001);  
time_3
```

```
time_3 = 0.0497
```

```
y_3
```

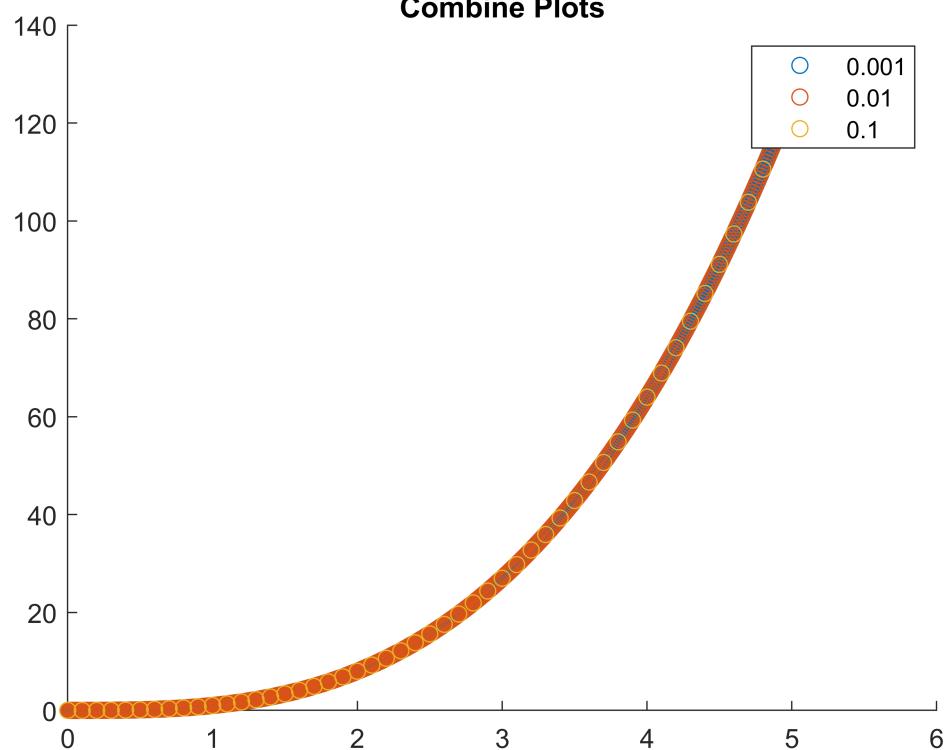
```
y_3 = 1x5001  
0 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 ...
```

```
scatter(t_3,y_3)  
hold on  
scatter(t_2,y_2)  
hold on  
scatter(t_1,y_1)
```

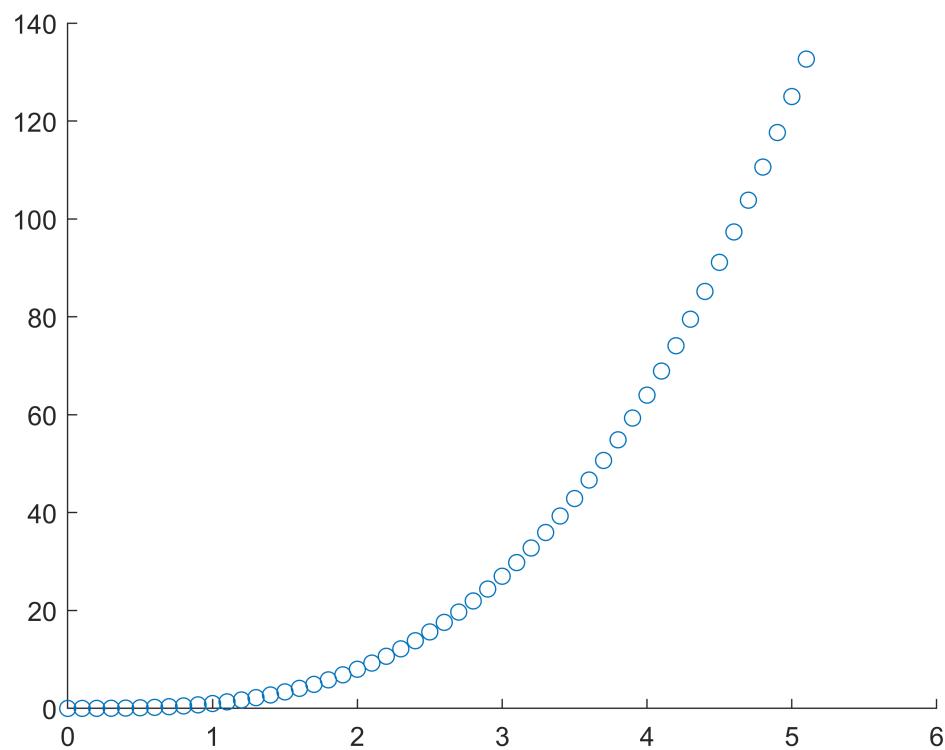
```
title('Combine Plots')  
legend('0.001','0.01','0.1')
```

```
hold off
```

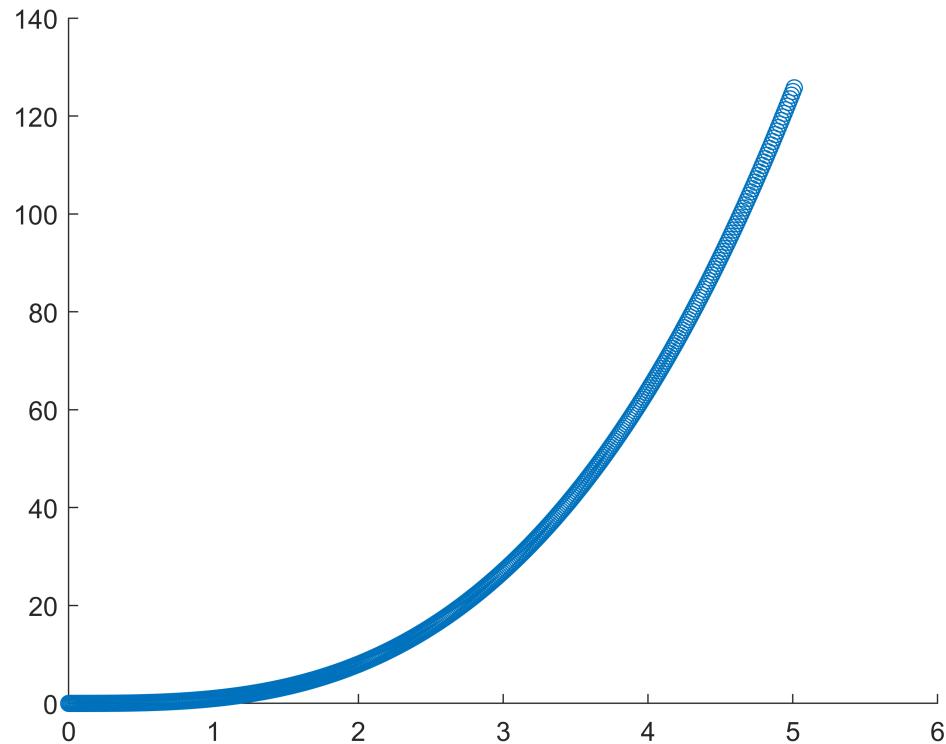
Combine Plots



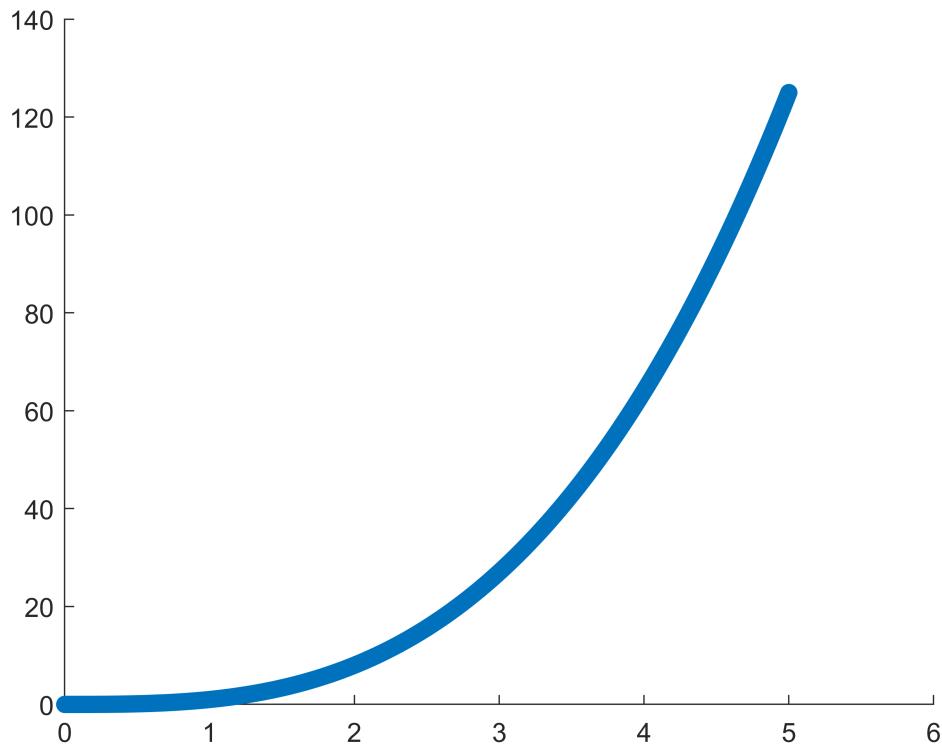
```
scatter(t_1,y_1)
```



```
scatter(t_2,y_2)
```



```
scatter(t_3,y_3)
```



```
tic  
dist_1=norm(y_1-t_1.^3)
```

```
dist_1 = 3.9074e-13
```

```
toc
```

```
Elapsed time is 0.011848 seconds.
```

```
tic  
dist_2=norm(y_2-t_2.^3)
```

```
dist_2 = 2.0548e-11
```

```
toc
```

```
Elapsed time is 0.009676 seconds.
```

```
tic  
dist_3=norm(y_3-t_3.^3)
```

```
dist_3 = 2.6125e-10
```

```
toc
```

```
Elapsed time is 0.007808 seconds.
```

```
% with the RK4 method it provides higher accuracy, very low value of error  
% however higher time of
```

```
% computation wrt Euler method
```

%QS5: Pendulum

```
g = 9.8;
L = 2;
time = [0,20];
f = @(t,y) funct3(t,y,g,L);
[t,y] = ode45(f,time,[2 0]);
```



```
[t1,y1, time1] = funct1(f,time_span,[2 0],0.1);
time1
```

```
time1 = 0.0024
```

```
[t2,y2, time_1] = funct1(f,time_span,[2 0],0.01);
time_1
```

```
time_1 = 0.0099
```

```
[t3,y3, time2] = funct2(f,time_span,[2 0],0.1);
time2
```

```
time2 = 0.0019
```

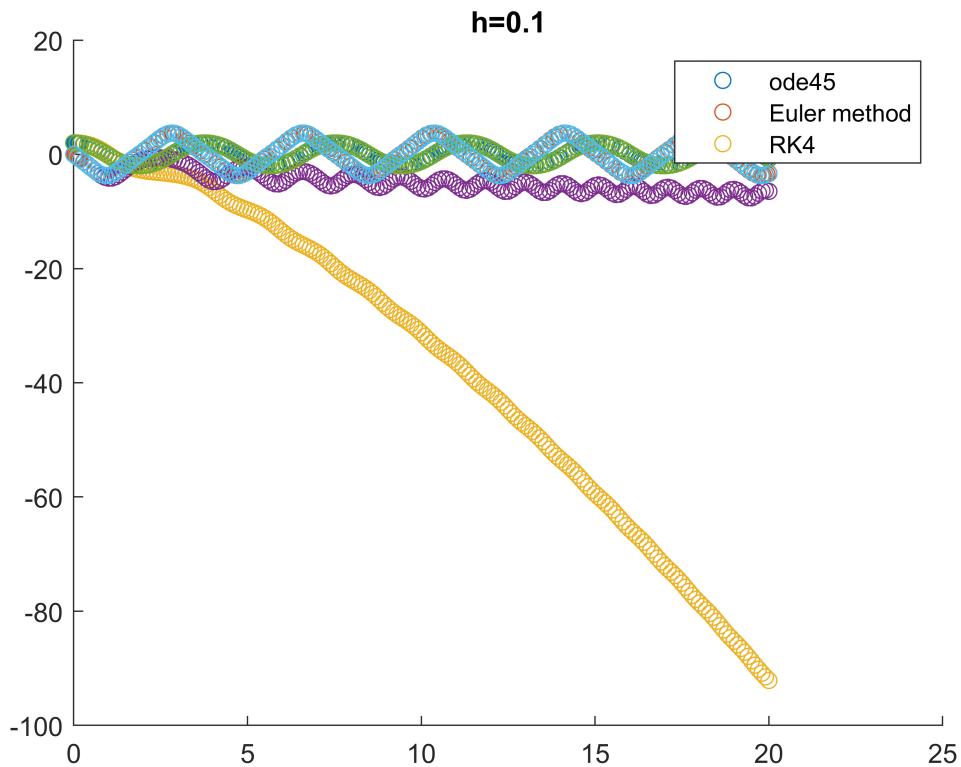
```
[t4,y4, time_2] = funct2(f,time_span,[2 0],0.01);
time_2
```

```
time_2 = 0.0148
```

```
scatter(t,y)
hold on
scatter(t1,y1)
hold on
scatter(t3,y3)

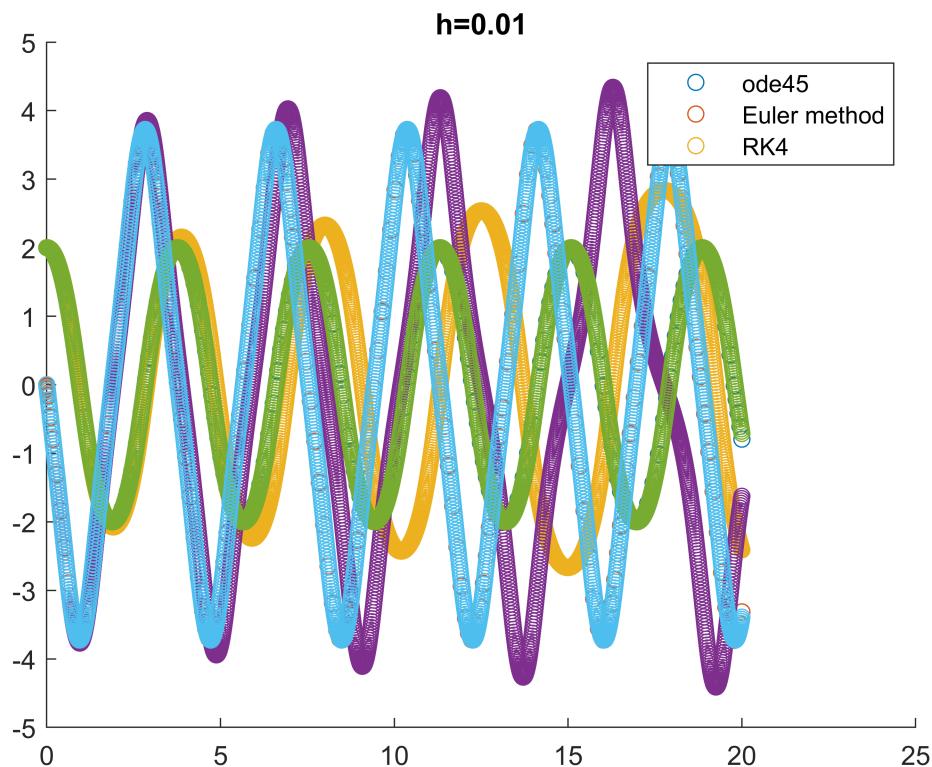
title('h=0.1')
legend('ode45','Euler method','RK4')

hold off
```



```
scatter(t,y)
hold on
scatter(t2,y2)
hold on
scatter(t4,y4)
title('h=0.01')
legend('ode45','Euler method','RK4')

hold off
```



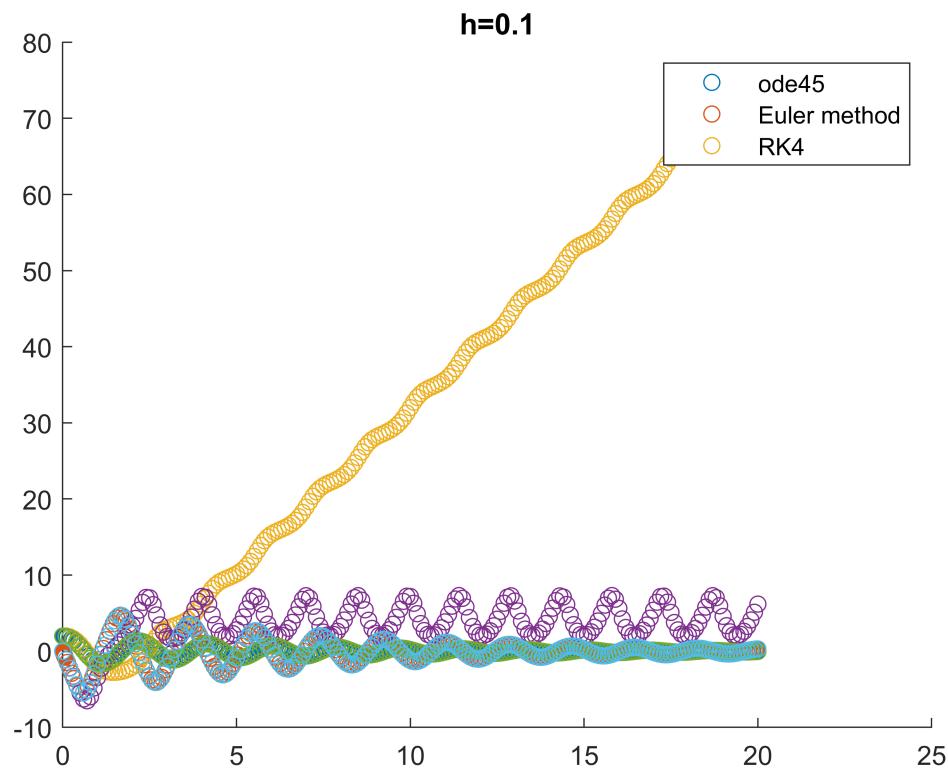
```
m=1;
b=0.3;
time = [0,20];
f = @(t,y) funct4(t,y,g,L,m,b);
[t,y] = ode45(f,time_span,[2 0]);

[t1,y1,time1] = funct1(f,time_span,[2 0],0.1);
[t2,y2,time_1] = funct1(f,time_span,[2 0],0.01);
[t3,y3,time2] = funct2(f,time_span,[2 0],0.1);
[t4,y4,time_2] = funct2(f,time_span,[2 0],0.01);

scatter(t,y)
hold on
scatter(t1,y1)
hold on
scatter(t3,y3)

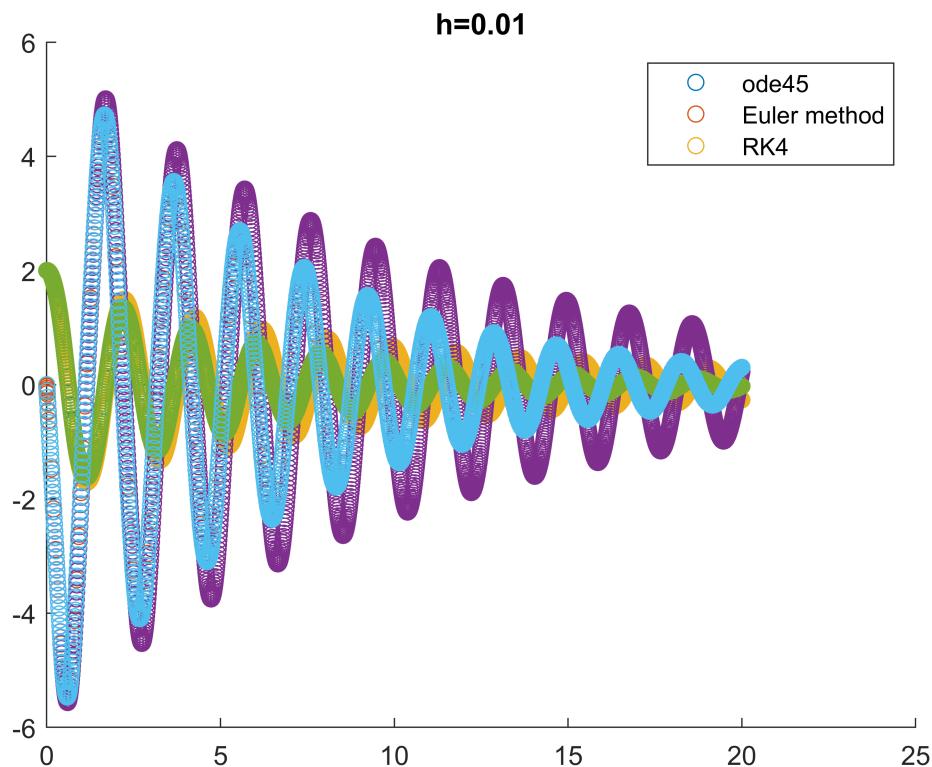
title('h=0.1')
legend('ode45','Euler method','RK4')

hold off
```



```
scatter(t,y)
hold on
scatter(t2,y2)
hold on
scatter(t4,y4)
title('h=0.01')
legend('ode45','Euler method','RK4')

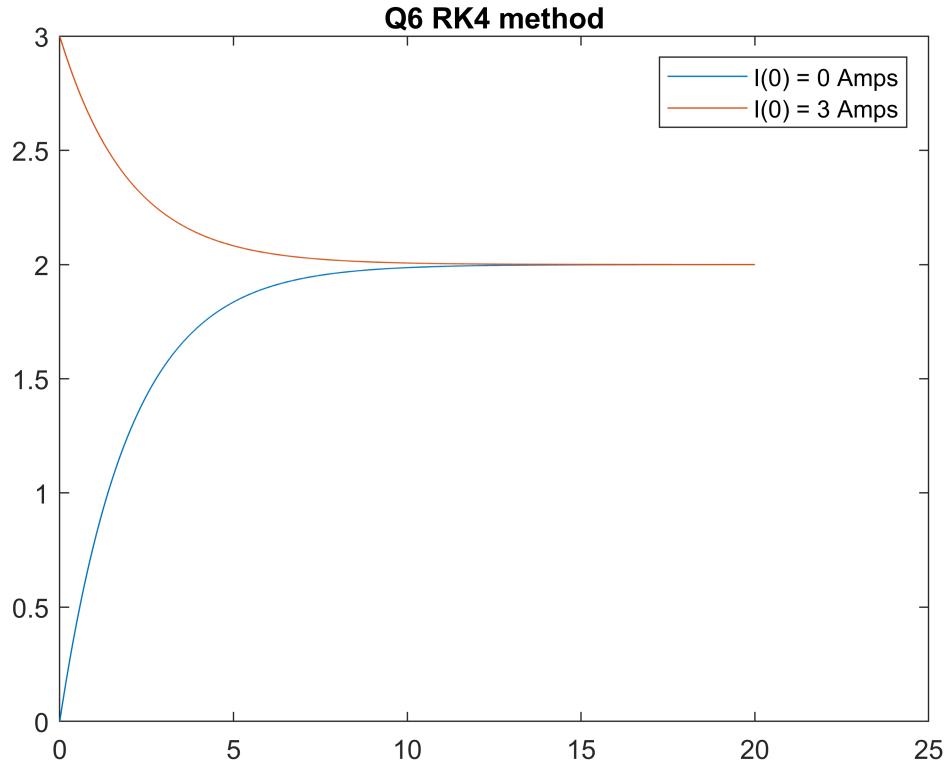
hold off
```



```
%% Q6
```

```
time_span = [0 20];
R = 0.5;
L = 1;
f = @(t,y) (1-y*R)/L;
[t_1,y_1] = funct2(f,time_span,0,0.001);
[t_2,y_2] = funct2(f,time_span,3,0.001);

figure
plot(t_1,y_1,t_2,y_2);
legend("I(0) = 0 Amps","I(0) = 3 Amps")
title('Q6 RK4 method')
```



```
%results same are theoretical
```

```
%% Q7
```

```
y1 = -1:0.05:1;
y2 = -1:0.05:1;

[Y1 Y2] = meshgrid(y1,y2);
y1dot = Y2;
y2dot = -(g/L)*sin(Y1);

subplot(1,3,1);
quiver(Y1,Y2,y1dot,y2dot);
title("undamped");
xlabel("y1");
ylabel("y2");
hold on

f = @(t,y) funct3(t,y,g,L);
[t, y] = ode45(f,[0,20],[0,0.5]);
end_idx = size(y)
```

```
end_idx = 1x2
    337      2
```

```
plot(y(:,1),y(:,2));
plot(y(1,1),y(1,2), 'o', y(end,1),y(end,2), 's');
```

```

hold off

m=1;
b=0;
y1dot_2 = Y2;
y2dot_2 = -((b/m)*Y2 + ((m*g)/(L*(m-2*b)))*sin(Y1));

subplot(1,3,2);
quiver(Y1,Y2,y1dot_2,y2dot_2);
title("damped b=0");
xlabel("y1");
ylabel("y2");
hold on

f = @(t,y) funct4(t,y,g,L,m,b);
[t, y] = ode45(f,[0,20],[0,0.5]);

plot(y(:,1),y(:,2));
plot(y(1,1),y(1,2), 'o',y(end,1),y(end,2), 's');
hold off

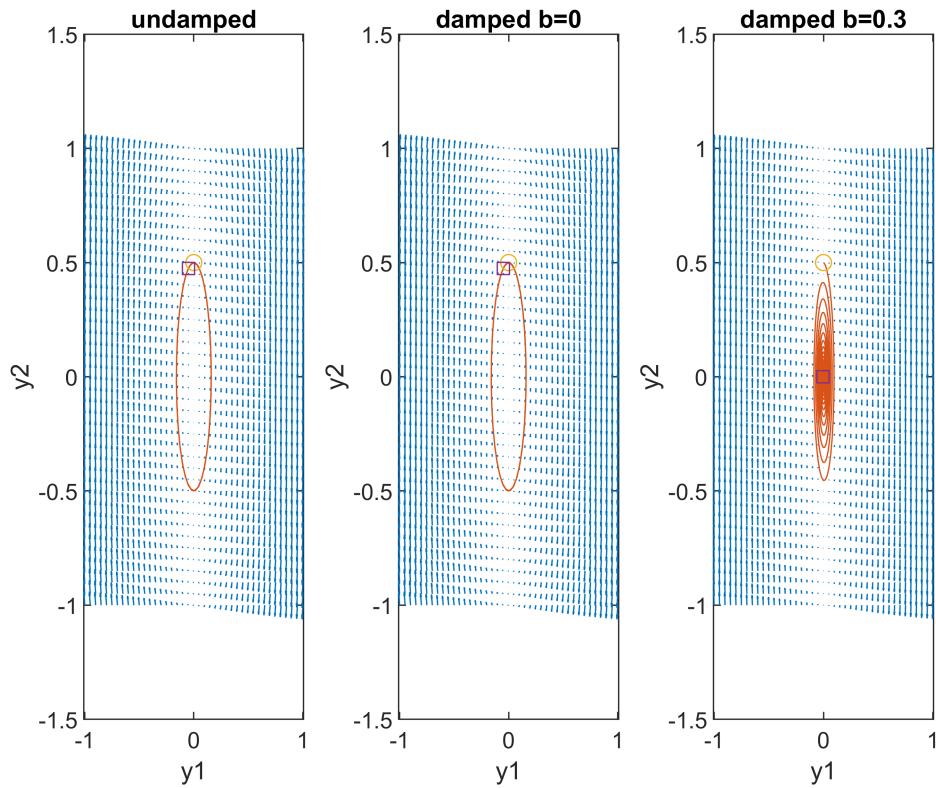
b= 0.3;
y1dot_2 = Y2;
y2dot_2 = -((b/m)*Y2 + ((m*g)/(L*(m-2*b)))*sin(Y1));

subplot(1,3,3);
quiver(Y1,Y2,y1dot_2,y2dot_2);
title("damped b=0.3");
xlabel("y1");
ylabel("y2");
hold on

f = @(t,y) funct4(t,y,g,L,m,b);
[t, y] = ode45(f,[0,20],[0,0.5]);

plot(y(:,1),y(:,2));
plot(y(1,1),y(1,2), 'o',y(end,1),y(end,2), 's');
hold off

```



```
%% Q8
```

```
% (a)
```

```
range = -1:0.08:1;
A = [-1,1;-1,-1];
```

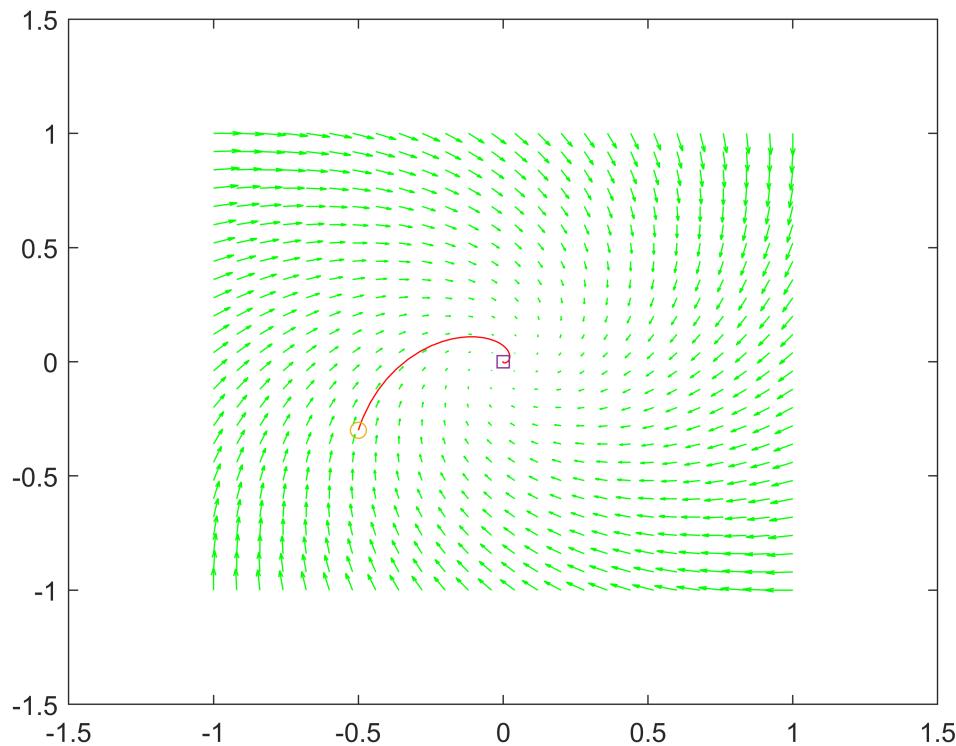
```
eig1 = eig(A);
eig1
```

```
eig1 = 2x1 complex
-1.0000 + 1.0000i
-1.0000 - 1.0000i
```

```
[X1,X2] = meshgrid(range,range);

X1dot = A(1,1)*X1 + A(1,2)*X2;
X2dot = A(2,1)*X1 + A(2,2)*X2;
[t,x] = ode45(@(t,x) A*x,[0,10],[-0.5,-0.3]);

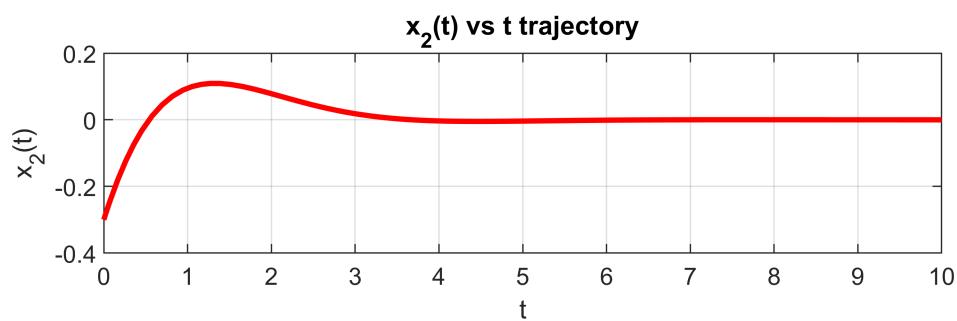
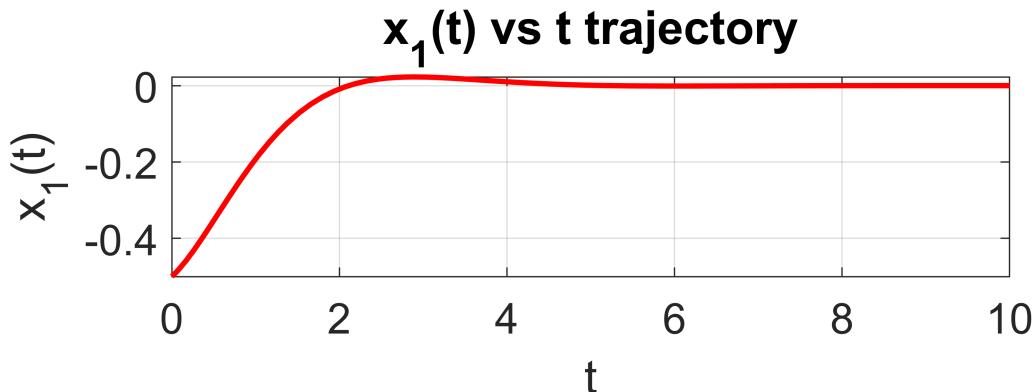
figure
quiver(X1,X2,X1dot,X2dot,'g')
hold on
plot(x(:,1),x(:,2),'r');
plot(x(1,1),x(1,2),'o',x(end,1),x(end,2),'s');
hold off
```



```

figure
%x1(t) vs t plot
subplot(2,1,1)
plot(t,x(:,1),"LineWidth",2,"Color","r");
grid on
title("x_1(t) vs t trajectory");
ylabel("x_1(t)");
xlabel("t");
set(gca,'FontSize',16);
%x2(t) vs t plot
subplot(2,1,2)
plot(t,x(:,2),"LineWidth",2,"Color","r");
grid on
title("x_2(t) vs t trajectory");
ylabel("x_2(t)");
xlabel("t");

```



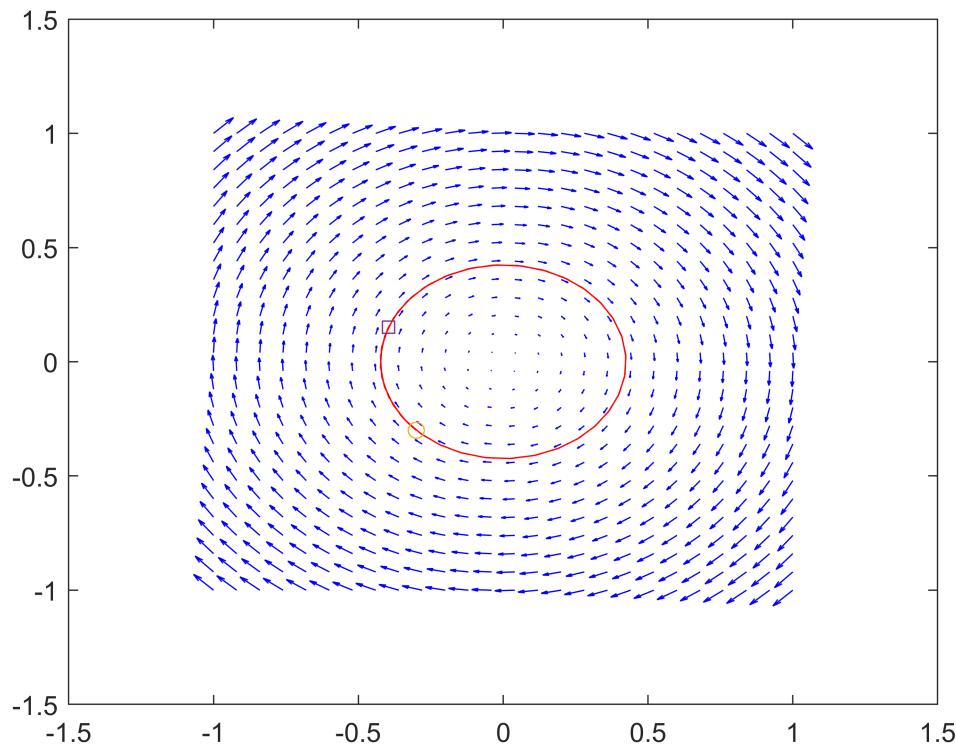
```
% (b)
range = -1:0.08:1;
A = [0,1;-1,0];
eig2 = eig(A);
eig2
```

```
eig2 = 2×1 complex
 0.0000 + 1.0000i
 0.0000 - 1.0000i
```

```
[X1,X2] = meshgrid(range,range);

X1dot = A(1,1)*X1 + A(1,2)*X2;
X2dot = A(2,1)*X1 + A(2,2)*X2;
[t,x] = ode45(@(t,x) A*x,[0,20],[-0.3,-0.3]);

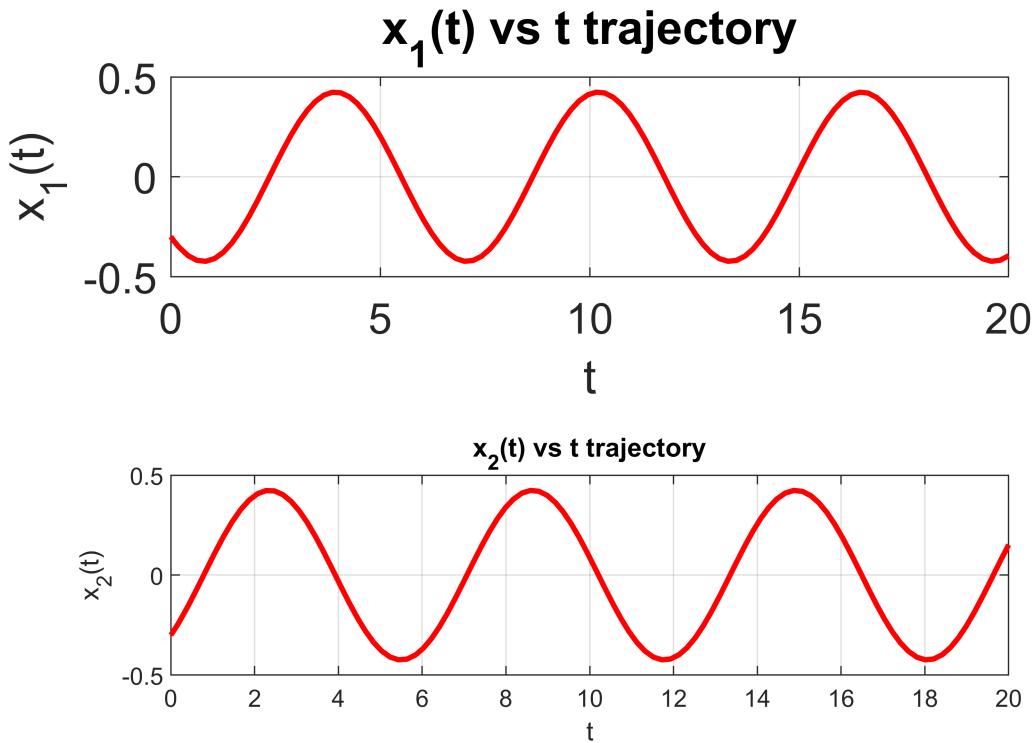
figure
quiver(X1,X2,X1dot,X2dot,'b')
hold on
plot(x(:,1),x(:,2),'r');
plot(x(1,1),x(1,2),'o',x(end,1),x(end,2),'s');
hold off
```



```

figure
%x1(t) vs t plot
subplot(2,1,1)
plot(t,x(:,1),"LineWidth",2,"Color","r");
grid on
title("x_1(t) vs t trajectory");
ylabel("x_1(t)");
xlabel("t");
set(gca,'FontSize',16);
% x2(t) vs t plot
subplot(2,1,2)
plot(t,x(:,2),"LineWidth",2,"Color","r");
grid on
title("x_2(t) vs t trajectory");
ylabel("x_2(t)");
xlabel("t");

```



```
% (c)
range = -2000:80:2000;
A = [1,1;-1,1];
eig3 = eig(A);
eig3

eig3 = 2x1 complex
 1.0000 + 1.0000i
 1.0000 - 1.0000i

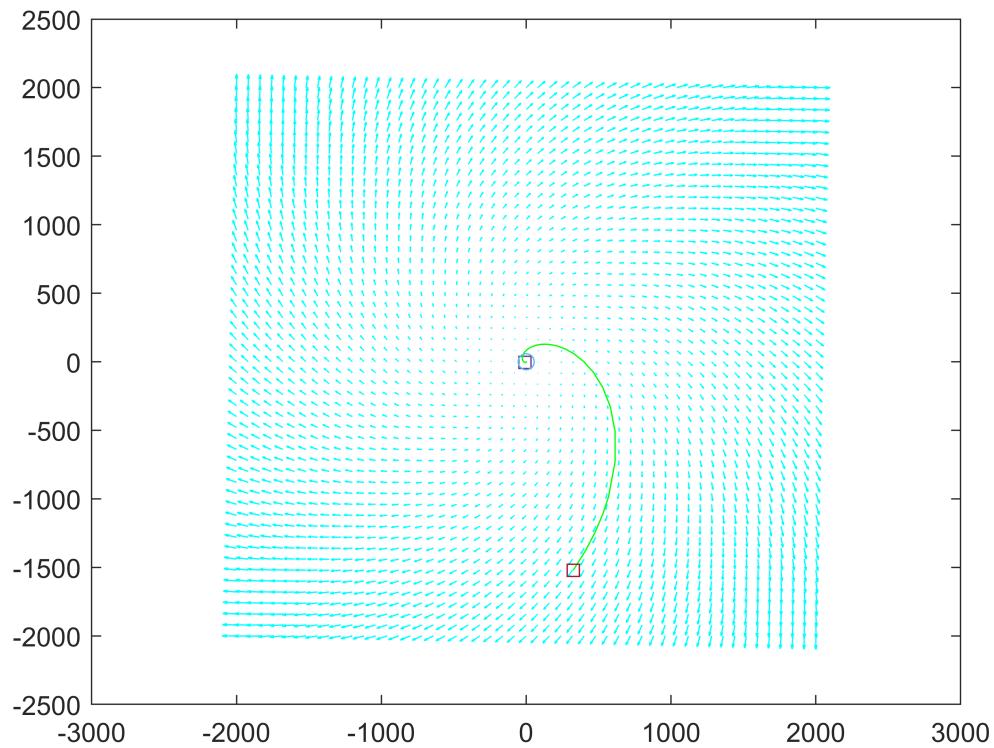
[X1,X2] = meshgrid(range,range);

X1dot = A(1,1)*X1 + A(1,2)*X2;
X2dot = A(2,1)*X1 + A(2,2)*X2;

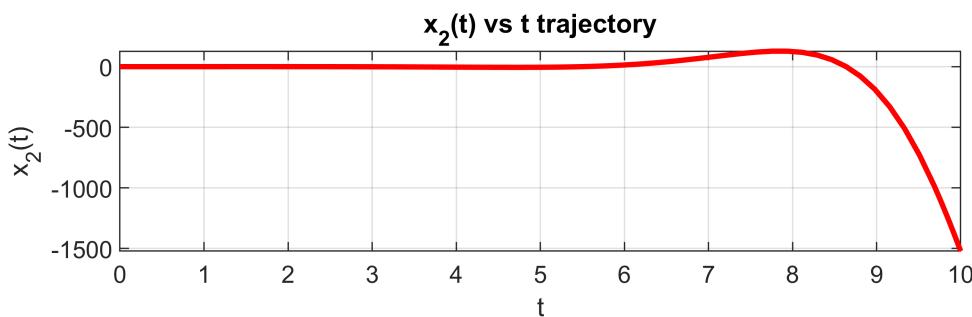
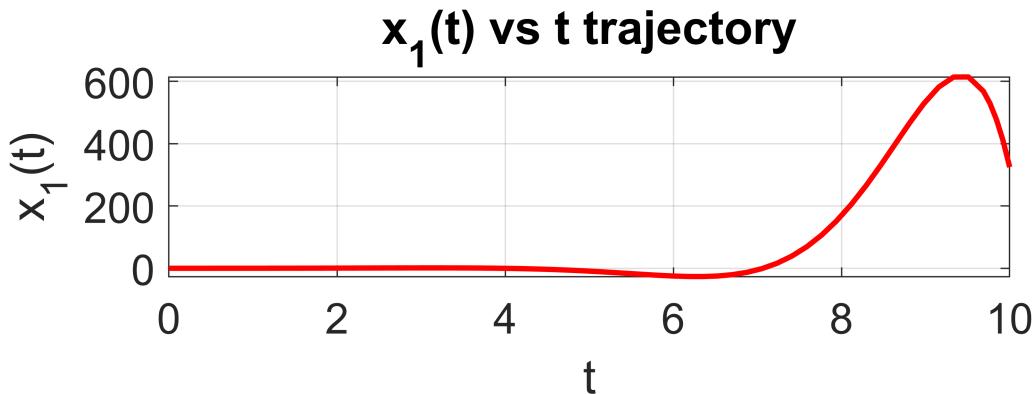
figure
quiver(X1,X2,X1dot,X2dot, 'c')

hold on
[t,x] = ode45(@(t,x) A*x,[0,5],[-0.05,0.05]);
plot(x(:,1),x(:,2), 'r');
plot(x(1,1),x(1,2), 'o',x(end,1),x(end,2), 's');
[t,x] = ode45(@(t,x) A*x,[0,10],[-0.05,0.05]);
plot(x(:,1),x(:,2), 'g');
plot(x(1,1),x(1,2), 'o',x(end,1),x(end,2), 's');
```

```
hold off
```



```
figure
% $x_1(t)$  vs t plot
subplot(2,1,1)
plot(t,x(:,1),"LineWidth",2,"Color","r");
grid on
title("x_1(t) vs t trajectory");
ylabel("x_1(t)");
xlabel("t");
set(gca,'FontSize',16);
% $x_2(t)$  vs t plot
subplot(2,1,2)
plot(t,x(:,2),"LineWidth",2,"Color","r");
grid on
title("x_2(t) vs t trajectory");
ylabel("x_2(t)");
xlabel("t");
```



```
% (d)
range = -20:0.8:20;
A = [1,3;1,-1];
eig4 = eig(A);
eig4
```

```
eig4 =
2
-2
```

```
[X1,X2] = meshgrid(range,range);

X1dot = A(1,1)*X1 + A(1,2)*X2;
X2dot = A(2,1)*X1 + A(2,2)*X2;

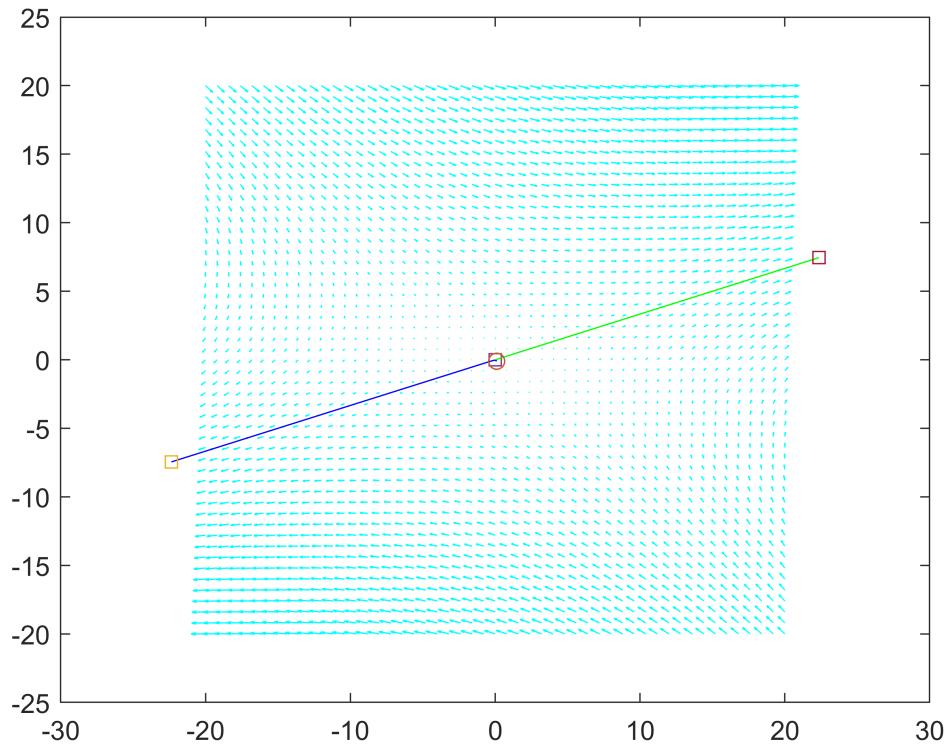
figure
quiver(X1,X2,X1dot,X2dot, 'c')

hold on
[t,x] = ode45(@(t,x) A*x,[0,10],[0.1,-0.1]);
plot(x(:,1),x(:,2), 'r');
plot(x(1,1),x(1,2), 'o',x(end,1),x(end,2), 's');
[t,x] = ode45(@(t,x) A*x,[0,4],[0.11,-0.1]);
plot(x(:,1),x(:,2), 'g');
plot(x(1,1),x(1,2), 'o',x(end,1),x(end,2), 's');
[t,x] = ode45(@(t,x) A*x,[0,4],[0.1,-0.11]);
```

```

plot(x(:,1),x(:,2), 'b');
plot(x(1,1),x(1,2), 'o',x(end,1),x(end,2), 's');
hold off

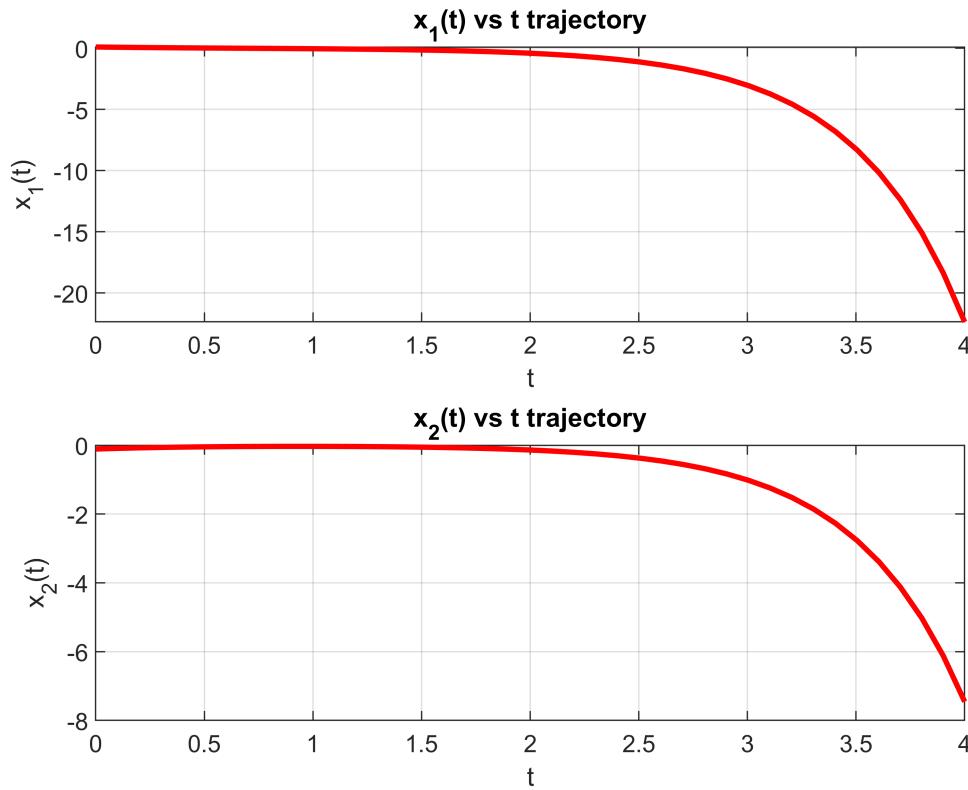
```



```

figure
% $x_1(t)$  vs t plot
subplot(2,1,1)
plot(t,x(:,1),"LineWidth",2,"Color","r");
grid on
title("x_1(t) vs t trajectory");
ylabel("x_1(t)");
xlabel("t");
% $x_2(t)$  vs t plot
subplot(2,1,2)
plot(t,x(:,2),"LineWidth",2,"Color","r");
grid on
title("x_2(t) vs t trajectory");
ylabel("x_2(t)");
xlabel("t");

```



```
%Real eigenvalues provide direction in space and complex generated the
%circular motion
%in first equation, complex eigen with negative part generates damping spiral
%in second equation, complex eigen with 0 real part generates periodic
%motion
%in third equation, complex eigen with positive part generates amplifying spiral
%in fourth equation, negative real eigen generates towards origin vector
```