Q1

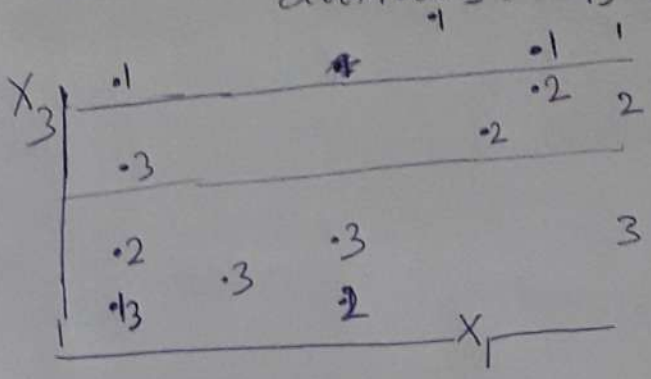
X_1	X_2	X_3	X_4	Y
1	2	1	4	1
5	2	1	4	2
1	2	1	2	3
1	2	8	4	1
1	5	3	2	2
5	2	3	4	3
9	2	8	4	1
8	2	5	4	2
3	2	2	3	3

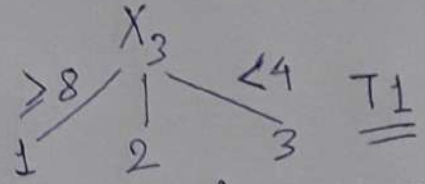
Common Validation Set

6	9	9	8	1
9	8	7	6	2
1	9	5	5	3

Tree 1

Chosen attributes: X_1, X_3



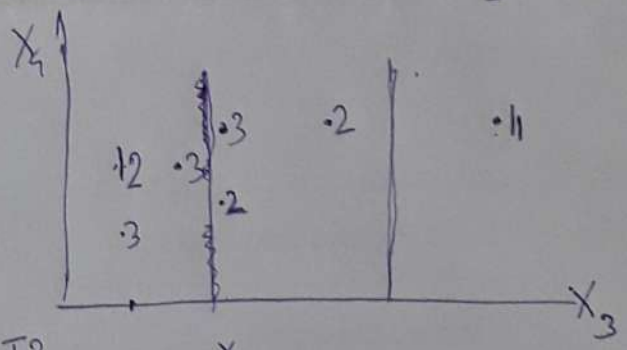


T1

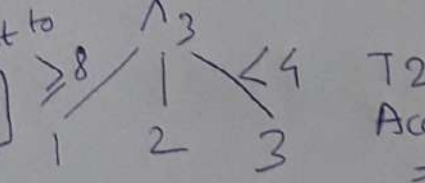
Accuracy on CVS = 2/3 (6,9,9,8 and 9,8,7,6 correct)

Tree 2

Chosen attributes: X_3 vs X_4



[T1, T2 turn out to be same!]

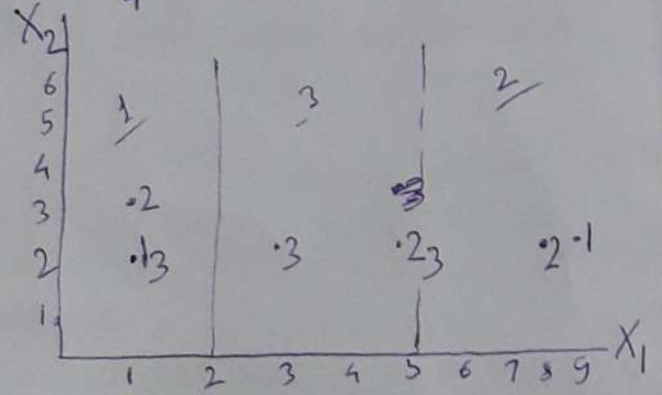


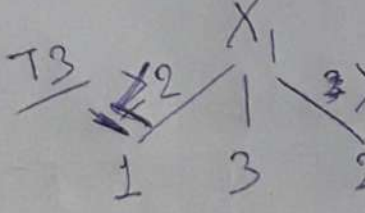
T2

Accuracy on CVS = 2/3

Tree 3

Chosen attributes X_1 vs X_2





T3

Accuracy on CVS = 1/3 (9,8,7,6 correct)

Test set

	T1 (2/3)	T2 (2/3)	T3 (1/3)	F
7, 1, 9, 3	1	1	2	1
-4, 9, 1, 3	3	3	1	3
12, -3, -9, 18	3	3	2	3

Q2

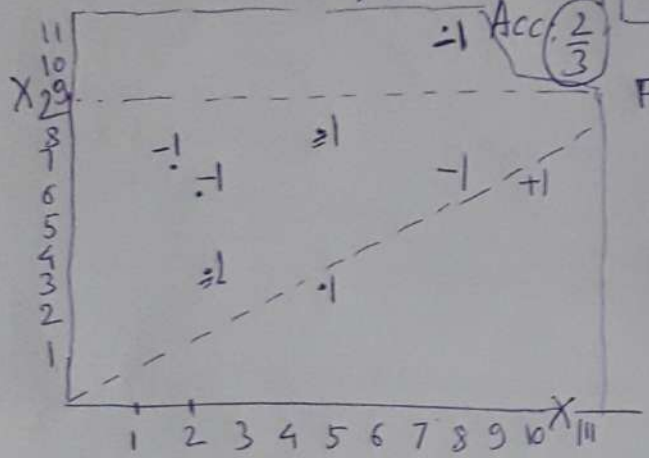
	1	2	3	4	5	6
X1	2·3	4·8	2·1	4·5	7·2	1·7
X2	3·8	3·4	5·8	7·3	11·6	6·2
Y	1	1	-1	1	-1	-1

	IT0	IT1	IT2	IT3
w_1	1/6	1/4	1/3	1/7
w_2	1/6	1/8	1/4	3/28
w_3	1/6	1/8	1/4	3/14
w_4	1/6	1/4	1/3	1/7
w_5	1/6	1/8	5/12	5/28
w_6	1/6	1/8	1/4	3/14

C1 $y = \text{sign}(x_1 - x_2)$

CORRECT: 2, 3, 5, 6
WRONG: 1, 4

$\alpha_1 = \frac{1}{2}\log\frac{2/3}{1/3} = \frac{1}{2}\log 2$



After normalization
CORRECT: 1/8
WRONG: 1/4

For correctly classified examples
$w = \frac{1}{6}\exp\left(-\frac{1}{2}\log 2\right) = \frac{1}{6}\exp\left(\log\frac{1}{\sqrt{2}}\right) = \frac{1}{6\sqrt{2}}$

For wrongly classified examples
$w = \frac{1}{6}\exp\left(\frac{1}{2}\log 2\right) = \frac{\sqrt{2}}{6}$

Next, choose a classifier to correctly classify 1 and 4.

C2 $y = \text{sign}(x_1 - 2{\cdot}2)$

CORRECT: 1, 2, 3, 4, 6
WRONG: 5

$\alpha_2 = \frac{1}{2}\log 5$

For 1, 4, $w_2 = \frac{1}{4\sqrt{5}}$

For 5, $w_2 = \frac{1}{8}\times\sqrt{5} = \frac{\sqrt{5}}{8}$

For 2, 3, 6.
$w_2 = \frac{1}{8}\cdot\frac{1}{8}\cdot\exp\left(-\frac{1}{2}\log 5\right)$
$= \frac{1}{8}\frac{1}{\sqrt{5}}$

Normalize to 1

C3 $y = \text{sign}(9 - x_2)$

CORRECT: 1, 2, 4, 5
WRONG: 3, 6

$\alpha_3 = \frac{1}{2}\log 2$

Wts:
1 $1/3 \exp\left(-\frac{1}{2}\log 2\right) = \frac{1}{3\sqrt{2}}$
2 $1/4 \exp\left(-\frac{1}{2}\log 2\right) = \frac{1}{4\sqrt{2}}$
3 $1/4 \exp\left(\frac{1}{2}\log 2\right) = \frac{\sqrt{2}}{4}$
4 $1/3 \exp\left(-\frac{1}{2}\log 2\right) = \frac{1}{3\sqrt{2}}$
5 $5/12 \exp\left(-\frac{1}{2}\log 2\right) = \frac{5}{12\sqrt{2}}$
6 $1/4 \exp\left(\frac{1}{2}\log 2\right) = \frac{\sqrt{2}}{4}$.

Normalize

FINAL CLASSIFICATION

	1	-1	
(2·3, 3·8)	$\frac{1}{2}\log 5 + \frac{1}{2}\log 2$	$\frac{1}{2}\log 2$	✓ (1)
(4·8, 3·4)	FULL		✓ (1)
(2·1, 5·8)	$\frac{1}{2}\log 2$	$\frac{1}{2}\log 2 + \frac{1}{2}\log 5$	✓ (-1)
(4·5, 7·3)	$\frac{1}{2}\log 5 + \frac{1}{2}\log 2$	$\frac{1}{2}\log 2$	✓ (1)
(7·2, 11·6)	$\frac{1}{2}\log 5$	$\frac{1}{2}\log 2 + \frac{1}{2}\log 2$	X (-1)
(1·7, 6·2)	$\frac{1}{2}\log 2$	$\frac{1}{2}\log 2 + \frac{1}{2}\log 5$	✓ (-1)

OVERALL ACCURACY ON TRAINING SET $= 5/6$

Q3

X_1	X_2	y
4	2	1
2	3	1
3	6	1
4	4	1
6	2	-1
8	5	-1
7	1	-1
6	6	-1

$X_2 = 0$: $\boxed{W_0 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}}$

Iter 1

Pt 1: $\hat{y} = \text{sign}\left(\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 2 \\ 1 \end{pmatrix}\right) = \text{sign}(2) = 1$ ✓

Pt 2: $\hat{y} = \text{sign}\left(\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 3 \\ 1 \end{pmatrix}\right) = \text{sign}(3) = 1$ ✓

Pt 3: $\hat{y} = \text{sign}\left(\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 6 \\ 1 \end{pmatrix}\right) = \text{sign}(6) = 1$ ✓

Pt 4: $\hat{y} = \text{sign}\left(\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 4 \\ 1 \end{pmatrix}\right) = \text{sign}(4) = 1$ ✓

Pt 5: $\hat{y} = \text{sign}\left(\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 6 \\ 2 \\ 1 \end{pmatrix}\right) = \text{sign}(2) = 1$ ✗

$\boxed{W_1 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} - \begin{pmatrix} 6 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} -6 \\ -1 \\ -1 \end{pmatrix}}$

Pt 6: $\hat{y} = \text{sign}\left(\begin{pmatrix} -6 \\ -1 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 8 \\ 5 \\ 1 \end{pmatrix}\right) = \text{sign}(-54) = -1$ ✓

Pt 7: $\hat{y} = \text{sign}\left(\begin{pmatrix} -6 \\ -1 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 7 \\ 1 \\ 1 \end{pmatrix}\right) = \text{sign}(-44) = -1$ ✓

Pt 8: $\hat{y} = \text{sign}\left(\begin{pmatrix} -6 \\ -1 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 6 \\ 6 \\ 1 \end{pmatrix}\right) = \text{sig}(-43) = -1$ ✓

Ite 2

Pt 1: $\hat{y} = \text{sign}\left(\begin{pmatrix} -6 \\ -1 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 2 \\ 1 \end{pmatrix}\right) = \text{sign}(-27) = -1$ ✗

$\boxed{W_2 = \begin{pmatrix} -6 \\ -1 \\ -1 \end{pmatrix} + \begin{pmatrix} 4 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}}$

Pt 2: $\hat{y} = \text{sign}\left(\begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}\right) = \text{sign}(-1) = -1$ ✗

$\boxed{W_3 = \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 4 \\ 1 \end{pmatrix}}$

Pt 3: $\hat{y} = \text{sign}\left(\begin{pmatrix} 0 \\ 4 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 6 \\ 1 \end{pmatrix}\right) = \text{sign}(25) = 1$ ✓

Pt 4: $\hat{y} = \text{sign}\left(\begin{pmatrix} 0 \\ 4 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 4 \\ 1 \end{pmatrix}\right) = \text{sign}(17) = 1$ ✓

Pt 5: $\hat{y} = \text{sign}\left(\begin{pmatrix} 0 \\ 4 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 6 \\ 2 \\ 1 \end{pmatrix}\right) = \text{sign}(9) = 1$ ✗

$\boxed{W_4 = \begin{pmatrix} 0 \\ 4 \\ 1 \end{pmatrix} + \begin{pmatrix} 6 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} -6 \\ 2 \\ 0 \end{pmatrix}}$

Pt 6: $\hat{y} = \text{sign}\left(\begin{pmatrix} -6 \\ 2 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 8 \\ 5 \\ 1 \end{pmatrix}\right) = \text{sign}(-38) = -1$ ✓

Pt 7: $\hat{y} = \text{sign}\left(\begin{pmatrix} -6 \\ 2 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 7 \\ 1 \\ 1 \end{pmatrix}\right) = \text{sign}(-40) = -1$ ✓

Pt 8: $\hat{y} = \text{sign}\left(\begin{pmatrix} -6 \\ 2 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 6 \\ 6 \\ 1 \end{pmatrix}\right) = \text{sign}(-24) = -1$ ✓

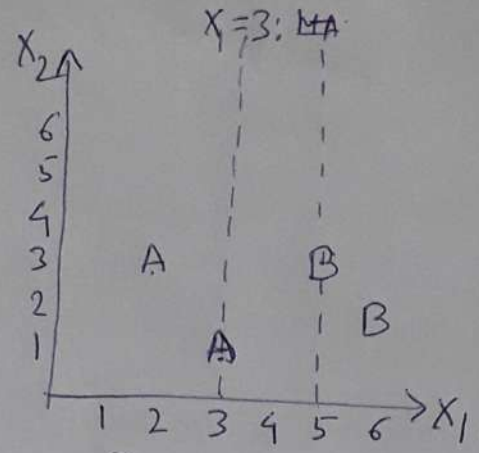
Ite 3

Pt 1: $\hat{y} = \text{sign}\left(\begin{pmatrix} -6 \\ 2 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 2 \\ 1 \end{pmatrix}\right) = \text{sign}(-20) = -1$ ✗

$\boxed{W_5 = \begin{pmatrix} -6 \\ 2 \\ 0 \end{pmatrix} + \begin{pmatrix} 4 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} -2 \\ 4 \\ 1 \end{pmatrix}}$

Q4

	X_1	X_2	y
1	2	3	A
2	3	1	A(SV)
3	5	3	B(SV)
4	6	2	B



MARGINAL CLASSIFIER FOR CLASS A:
$X_1 = 3$

MARGINAL CLASSIFIER FOR CLASS B: $X_1 = 5$

OPTIMAL CLASSIFIER: $X_1 = 4$

MARGIN = $|+1| = 2$

$$W = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \alpha_2 (+1) \cdot \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix} + \alpha_3 (+1) \begin{pmatrix} 5 \\ 3 \\ 1 \end{pmatrix}$$

[CONSIDERING 'A' AS -1, 'B' AS +1]

$$\boxed{W = \sum_{i=1}^{N} \alpha_i y_i X_i}$$

$\alpha_i = 0$ for non-support vectors.

So $\alpha_1 = \alpha_4 = 0$.

$$\Rightarrow \begin{aligned} 5\alpha_3 - 3\alpha_2 &= 1 \\ 3\alpha_3 - \alpha_2 &= 0 \end{aligned} \Rightarrow \boxed{\begin{aligned} \alpha_2 &= -3/4 \\ \alpha_3 &= -1/4 \end{aligned}}$$

~~$\alpha_3 + \alpha_2 = ?$~~

~~$\alpha_3 - \alpha_2 = 0$~~

So, $W = \begin{pmatrix} 1 \\ 0 \\ 1/2 \end{pmatrix}$ $\boxed{X = 1/2}$

G5

ID	X1	X2	Y	α
1	-4	3	B	0
2	-6	1	B	0
3	1	2	B	5/8
4	6	0	B	0
5	-3	-2	A	-3/8
6	2	1	A	5/8
7	-2	-3	A	3/8
8	2	-5	A	0

CONSIDER $B=1$, $A=-1$.

MAXMARGIN CLASSIFIER:

$$W = \sum_{i=1}^{8} \alpha_i y_i X_i$$

$$= \underbrace{\left(\frac{5}{8}\right)(1)\begin{pmatrix}1\\2\\1\end{pmatrix}}_{3} + \underbrace{\left(-\frac{3}{8}\right)(-1)\begin{pmatrix}-3\\-2\\1\end{pmatrix}}_{5} + \underbrace{\left(\frac{5}{8}\right)(-1)\begin{pmatrix}2\\1\\1\end{pmatrix}}_{6} + \underbrace{\left(\frac{3}{8}\right)(-1)\begin{pmatrix}-2\\-3\\1\end{pmatrix}}_{7} = \begin{pmatrix}-1\\1\\0\end{pmatrix}$$

SO, MARGINAL CLASSIFIERS: $\begin{pmatrix}-1\\1\\1\end{pmatrix}$ AND $\begin{pmatrix}-1\\1\\-1\end{pmatrix}$.

MISCLASSIFIED POINTS:

Pt 4: $\hat{y} = \text{sign}\left(\begin{pmatrix}-1\\1\\0\end{pmatrix}\cdot\begin{pmatrix}6\\0\\1\end{pmatrix}\right) = \text{sign}(-6) = -1$ (A) (WRONG).

Pt 5: $\hat{y} = \text{sign}\left(\begin{pmatrix}-1\\1\\0\end{pmatrix}\cdot\begin{pmatrix}-3\\-2\\1\end{pmatrix}\right) = \text{sign}(1) = 1$ (B) (WRONG).

CALCULATION OF SLACK VARIABLES:

Pt 4: $y_4 W^T X_4 \geq 1-\xi_4 \Rightarrow (1)\cdot\left(\begin{pmatrix}-1\\1\\0\end{pmatrix}\cdot\begin{pmatrix}6\\0\\1\end{pmatrix}\right) \geq 1-\xi_4 \Rightarrow -6 \geq 1-\xi_4$

$\Rightarrow \boxed{\xi_4 = 7}$

Pt 5: $y_5 W^T X_5 \geq 1-\xi_5 \Rightarrow (-1)\left(\begin{pmatrix}-1\\1\\0\end{pmatrix}\cdot\begin{pmatrix}-3\\-2\\1\end{pmatrix}\right) \geq 1-\xi_5$

$\Rightarrow -1 \geq 1-\xi_5$

$\Rightarrow \boxed{\xi_5 = 2}$

$\xi_i = 0$ for all other points