Assignment 5

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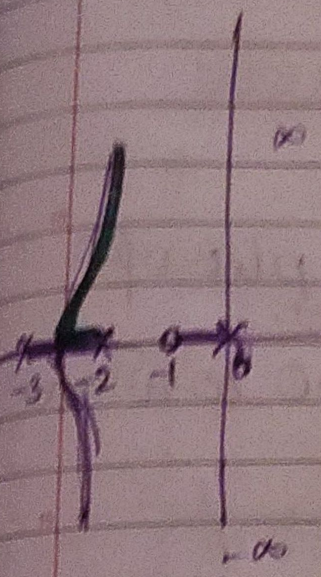
Q1. characteristic equation
$s(s+2)(s+3) + k(s+1) = 0$

transfer function $k \cdot \frac{(s+1)}{s(s+2)(s+3)} \to 0$ zero : $s = -1$

∞ ~~pole~~ pole : $s = 0, -2, -3$



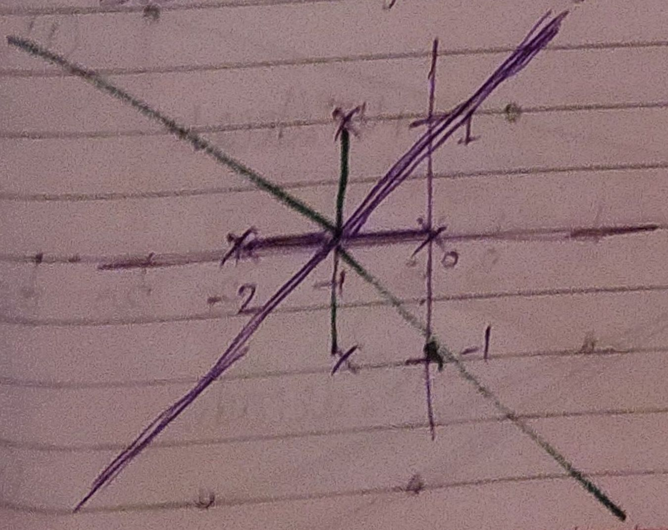
∴ pole at 0 goes to 1 as $k \to \infty$ and others poles $\to \infty$ with asymptotes at -2 with angles of 90° & 270°

Q2. characteristic equation
$S(s+2)(s+1+j)(s+1-j) + k = 0$

transfer function $\frac{1}{S(s+2)(s+1+j)(s+1-j)}$

pole $\to 0, -2, -1-j, -1+j$

zero X



∴ can be seen from the plot there is symmetry about -1 point.

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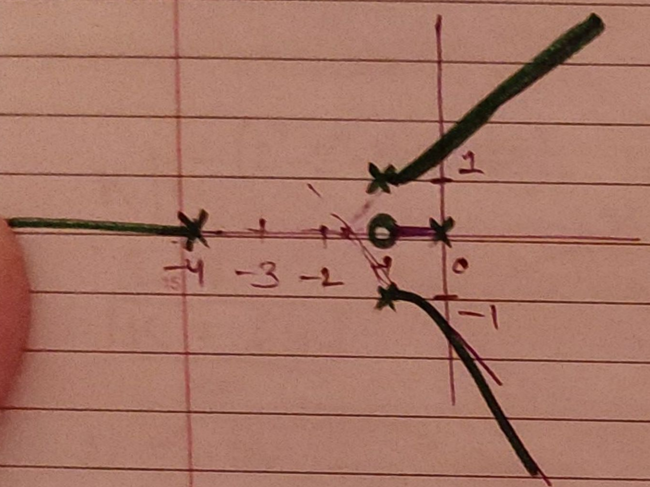
Q3. characteristic eqn

$s(s+4)(s^2+2s+2)+K(s+1)=0$.

transfer funcn $\rightarrow$ $\dfrac{s+1}{s(s+4)(s^2+2s+2)}$.

centroid $= \dfrac{0-4-1-1+1}{4-1} = \dfrac{-5}{3}$ $\rightarrow$ intersect of asymptotes.

asymptote angles, $\theta = \dfrac{(2q+1)180°}{4-1} = 60°, 180°, 300°$



3 asymptotes intersect at centroid.

Q4. characteristic equn.

$s(s+3)(s^2+2s+2)+K=0$

transferfuncn

$= \dfrac{1}{s(s+3)(s^2+2s+2)}$.

zero $-$ X

pole $0, -3, -1+j, -1-j$.

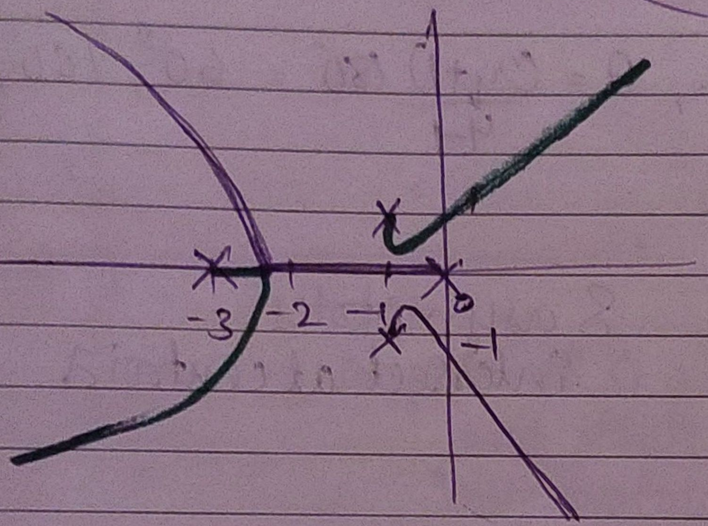
If odd number of open loop poles and zeros exists to the left side of point on real axis then point is root locus branch.

So on real axis root loci will be present betn 2poles 0, -3.

for departure angle of pole $\underset{(S_1)}{-1+j}$.

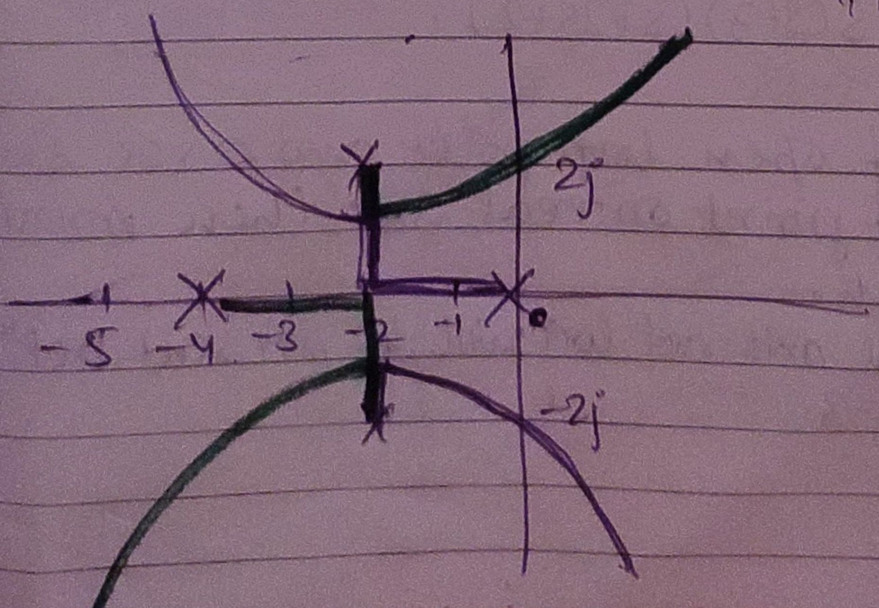
pole at 0 makes 135° with s_1.
pole at -3 makes $\tan^{-1}(1/2)$ with s_1
pole at $-1-j$ makes 90° with s_1.

$\therefore$ angle of departure $= 180^\circ - 135^\circ - 90^\circ - \tan^{-1}(1/2) =$
$= -71.56^\circ$



Q5. characteristic eqn.
$s(s+4)(s^2+4s+20)+k=0$

transfer func^n $\quad \dfrac{1}{s(s+4)(s^2+4s+20)}$ → pole
0, −4,
−2−2j,
−2+2j.



Intersection with imaginary axis using RH table.

$K = ?$ for row is 0

$\therefore$ at, $K = 260$, we have 2 poles on imaginary axis $(\pm\sqrt{10}j)$.

for $K = 260$, intersection with imaginary axis at $\to \pm\sqrt{10}j$

Breakaway point:

$$K = -(s^4 + 8s^3 + 36s^2 + 80s)$$

$$\frac{dK}{ds} = 0.$$

$$4s^3 + 24s^2 + \cancel{64}\,72s + 80 = 0.$$

$$\cancel{2s^3 + 12s^2 + 36}$$

$$s^3 + 6s^2 + 18s + 20 = 0.$$

$$s = -2$$

$$K\Big|_{s=-2} = -(2^4 - 2^6 + 36\times4 - 160).$$

$$\boxed{K = 64.}$$

$\therefore -2$ is breakaway point on real axis.

Q6. characteristic eqn
$$s(s+5)(s+6)(s^2+2s+2) + k(s+3) = 0.$$

transfer func^n = $\dfrac{s+3}{s(s+5)(s+6)(s^2+2s+2)}$.

pole:
$0, -5, -6, -1-j,$
$-1+j$.

Zeros
-3.

∴ 5 root locus & 4 of them will go to ∞ as K increases

If odd number of open loop poles and zeros exits to the left side of a point on real axis, then the point is on root locus branch.
So on real axis, root loci will be present bet^n $0, -3$ and $-5, -6$.

Centroid $\alpha = 2.5$
angle of asymptotes = $45°, 135°, 225°, 315°$.

Breakaway point bet^n -5 and -6.

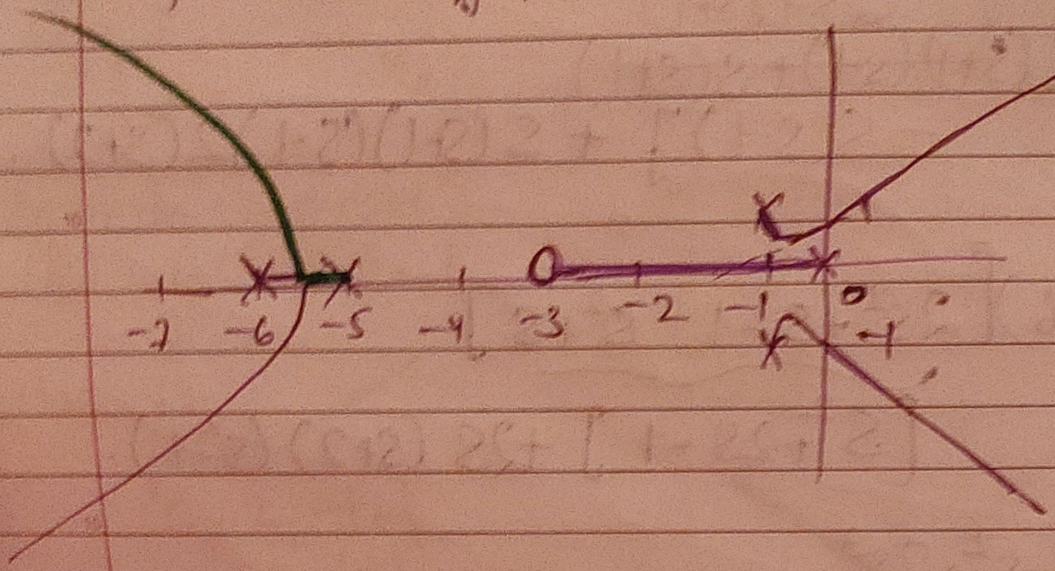
Angle of departure for $-1+j$. is $180° - (180 - \tan^{-1} 1)$
$- 90° - \tan^{-1} 1/4$
$- \tan^{-1} 1/5$
$+ \tan^{-1} 1/2$
$= \boxed{-43.78°}$

So root locus of pole at 0 will end at -3

pole at $-1+j$ will start with $-43.78°$ and follow $45°$ asymptote from centroid.

pole at $-1-j$ will reflect $-1+j$ locus wrt real axis.

poles at $-5, -6$ will breakaway and follow asymptotes.



Q7. $G(s)H(s) = \dfrac{K(s+a)}{s(s-b)(s^2+2\xi\omega_n s+\omega_n^2)}$.

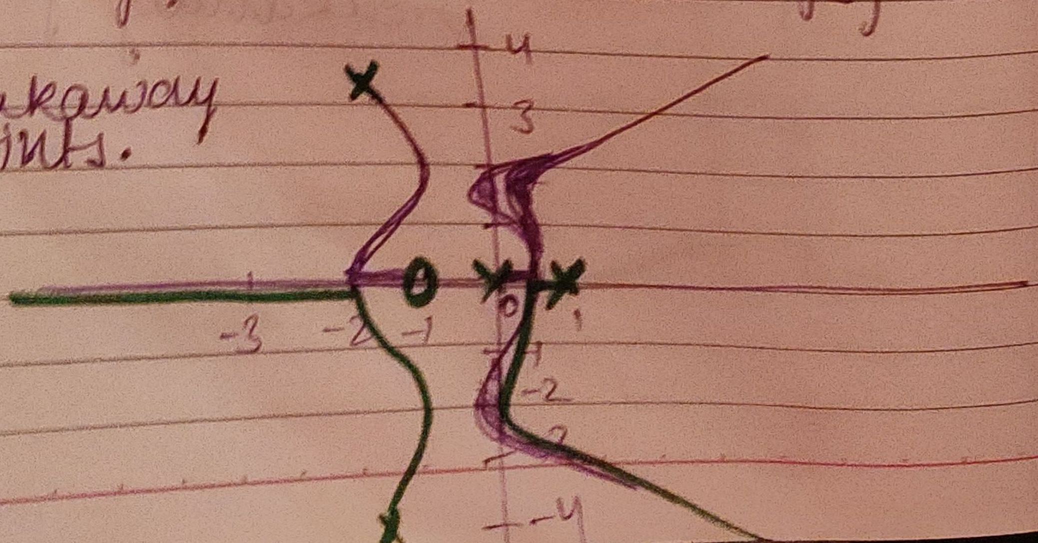
$= \dfrac{K(s+1)}{s(s-1)(s^2+4s+16)}$

Zeros -1.

poles $0, 1, -2+2\sqrt{3}j$, $-2-2\sqrt{3}j$

gain for stability?

$\left.\begin{matrix}0.47\\-2.26\end{matrix}\right\}$ breakaway points.



$$K = \frac{s(s-1)(s^2+4s+16)}{(s+1)}$$

$$\frac{dK}{ds} = \frac{(s+1)\left[(s-1)(s^2+4s+16) + s(s^2+4s+16) + s(s-1)(2s+4)\right] - s(s-1)(s^2+4s+16)}{(s+1)^2}$$

$$0 = \frac{(s^2+4s+16)}{(s+1)^2}\Big[\overset{(2s-1)(s+1)}{\cancel{(s+1)(s-1)+s(s+1)}} - s(s-1)\Big] + s(s+1)(s-1)2(s+2).$$

$$= (s^2+4s+16)\underbrace{\left[2s^2+s-1-s^2+s\right]}$$

$$\left[s^2+2s-1\right] + 2s(s+2)(s^2-1).$$

$$= s^4+8s^2-s^2+2s^3$$
$$+4s^3-4s+16s^2$$
$$+32s-16 + 2s(s^3+2s^2-s-2).$$

$$= s^4+6s^3+23s^2+28s-16+2s^4+4s^3-2s^2-4s$$

$$= 3s^4+10s^3+21s^2+24s-16.$$

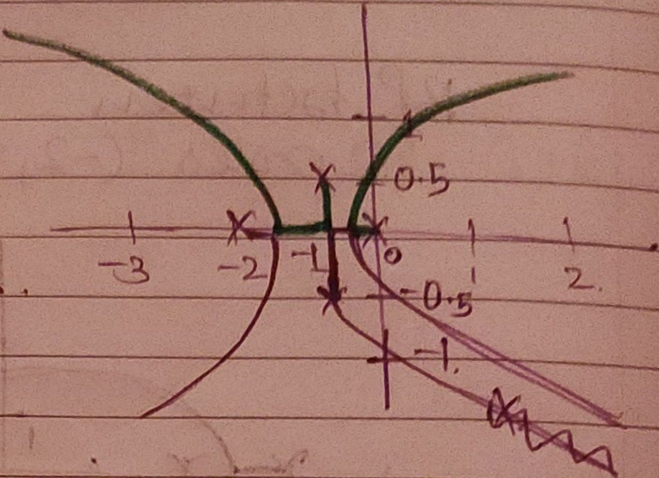
$$\hookrightarrow s = -2.263,\ 0.448$$

breakaway points

Q 10. characteristic eqn.

(a) $G(S)\,H(S) = \dfrac{K}{S(S^2+2S+1.25)(S+2)}$

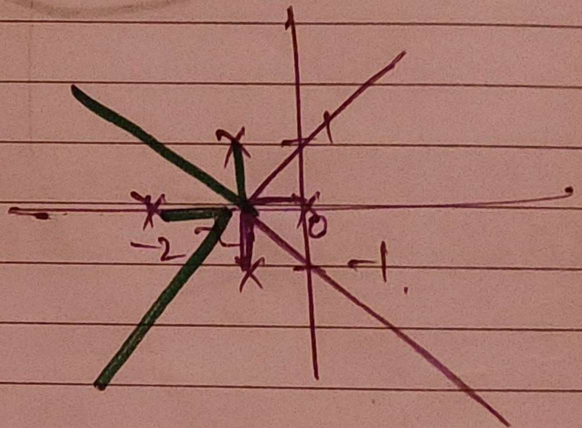
pole
$0, -2, -1+0.5j$
$-1-0.5j$



2 BP betn
$0, -2$
real axis.
4 asymptotes.
(imaginary points closer to centroid).

(b) $G(S)\,H(S) = \dfrac{K}{S(S^2+2S+2)(S+2)}$

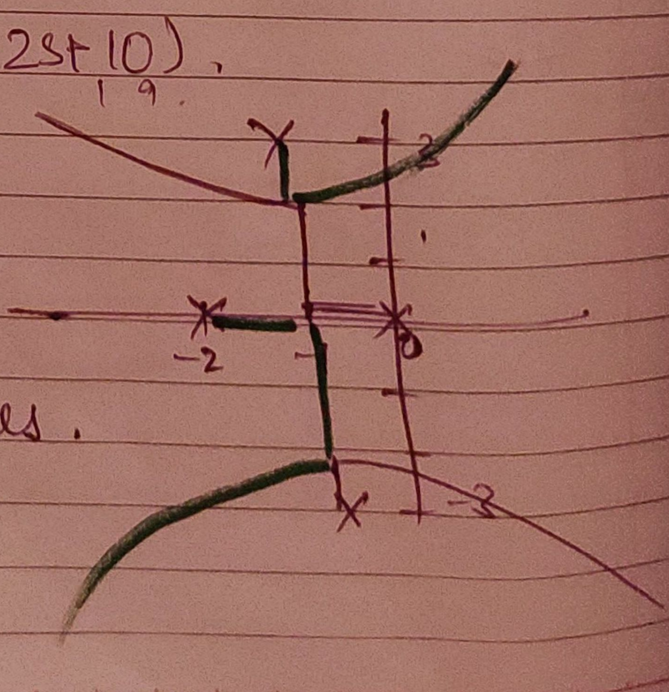
poles
$0, -2, -1-j$
$-1+j$



BP at centroid -1.
Symmetric
4 asymptotes

(c) $G(S)\,H(S) = \dfrac{K}{S(S+2)(S^2+2S+10)}$
1 9

poles
$0, -2, -1-3j$
$-1+3j$



4 asymptotes.

2 BP betn $-1-3j$,
$-1+3j$

(real points closer to centroid)

Q8.

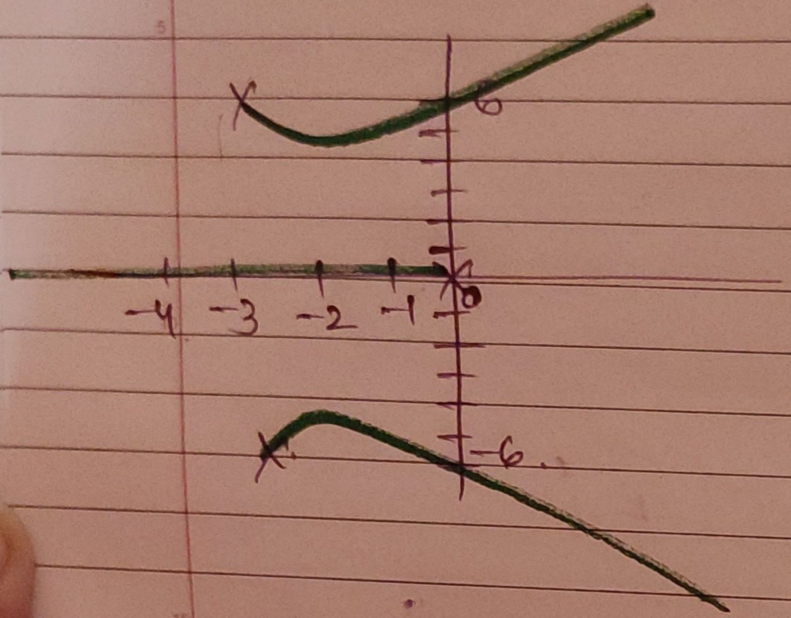
$$G_1(s) = \frac{1}{s^3+6s^2+45s} = \frac{1}{s(s^2+6s+45)}$$

9 36

$$= \frac{1}{s((s+3)^2+6^2)}.$$

pole

$0, -3-6j, -3+6j.$



$$G_2(s) = \frac{0.075s^2+\cancel{0.}s+1}{s^3+3s^2+5s.} = \frac{\overset{3}{\cancel{0.075}}(s^2+s+1.}{\underset{4}{\cancel{1000}}s(s^2+3s+5)}.$$

$$= \frac{3s^2+40s+40}{40s(s^2+3s+5)}.$$

poles

2.25
2.75

$0, -1.5+\sqrt{2.75}j$
$-1.5-\sqrt{2.75}j.$

Zero

$$= \frac{-40 \pm \sqrt{40^2 - 40\times 3\times 4}}{6}$$

40 (40-12)
28

$-1.089,$
-12.244