Lab 3.

Q1. $M = \frac{20(s-1)}{(s+2)(s^2+4)}$

$(s+2) \to$ 1 LHP. $=(-2)$ pole

$(s^2+4) \to$ 2 imaginary poles

$\therefore$ M(s) is marginally stable.

Q2. $2s^4+s^3+3s^2+5s+10=0$.

using rh criterion :

$-\begin{vmatrix} 2 & 3 \\ 1 & 5 \end{vmatrix} = -7$ $\quad -\begin{vmatrix} 2 & 10 \\ 1 & 0 \end{vmatrix} = 10$

$-\frac{1}{-7}\begin{vmatrix} 1 & 5 \\ -7 & 10 \end{vmatrix} = \frac{45}{7}$.

s^4	2	3	10
s^3	1	5	0
s^2	-7	10	0
s	45/7	0	0
s^0	10	0	0

as there is 2 sign change

$\therefore$ system UNSTABLE

Q3. $s^4+s^3+2s^2+2s+3=0$.

s^4	1	2	3
s^3	1	2	0
s^2	0	3	0
s^1	-3	0	0
s^0	0	0	0

UNSTABLE.

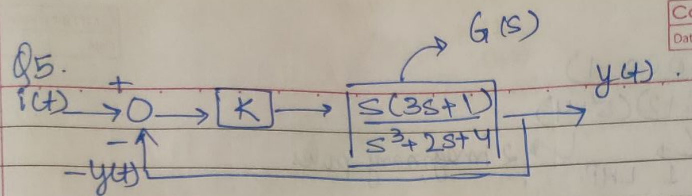
Q4. $s^5+4s^4+8s^3+7s+4=0$.

s^5	1	8	7	0
s^4	4	8	4	0
s^3	6	6	0	0
s^2	4	4	0	0
s^1	0	0	0	0
s^0	4	0	0	0

No sign change

$\therefore$ System is STABLE.

Q5.



K → gain of proportional controller

$1 + KG(s) = 0$

$K(G(s))\,(i(t) - y(t)) = y(t).$

$$1 + \frac{s^3 + 2s + 4}{Ks(3s+1)} = 0$$

$s^3 + 2s + 4 + Ks(3s+1) = 0$

$s^3 + (2 + K)s + 4 + 3Ks^2 = 0$

$\underline{K > 0}$. rh table.

s^3	1	$2+K$
s^2	$3K$	4
s^1	$-\frac{1}{3K}\begin{vmatrix} 1 & K+2 \\ 3K & 4 \end{vmatrix}$	0
s^0		

$\rightarrow -\frac{1}{3K}(4 - 3K^2 - 6K).$

as $K > 0$

$3K^2 + 6K - 4 > 0$

$K > -1 + \sqrt{7/3} \approx 0.53$

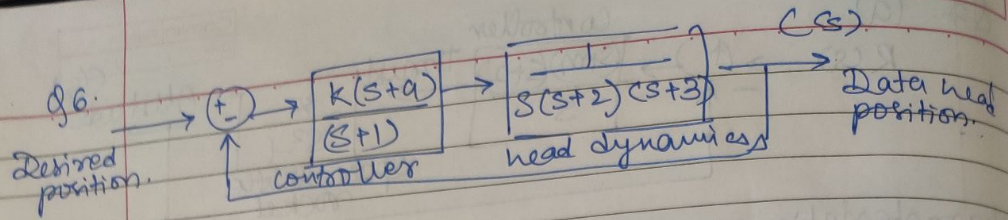
when $K = -1 + \sqrt{7/3}$ both elements of 3rd row is 0. stable (marginally).

s^3	1	$K+2$
s^2	$3K$	4
s	$6K$	0
1	4	0

$\therefore \boxed{K \geq -1 + \sqrt{7/3}}$

System stable for.

Q6.



characteristic equation:

$$1 + \frac{K(s+a)}{s(s+1)(s+2)(s+3)} = 0.$$

$$s^4 + 6s^3 + 11s^2 + (6+K)s + Ka = 0.$$

as $K > 0$ $\therefore a \geq 0$.

s^4	1	11	Ka
s^3	6	$6+K$	0
s^2	$-\frac{1}{6}\begin{vmatrix} 1 & 11 \\ 6 & 6+K \end{vmatrix}$	$-\frac{1}{6}\begin{vmatrix} 1 & Ka \\ 6 & 0 \end{vmatrix}$	0
s^1	$\frac{6Ka - (6+K)\left(\frac{10-K}{6}\right)}{10 - K/6}$	0	
		Ka.	0
s^0	Ka	0	0

$\to Ka.$

$\to 10 - K/6.$

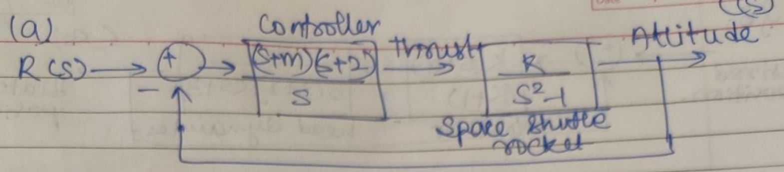
$\therefore 10 - \frac{K}{6} \geq 0.$ $\therefore$ $0 < K \leq 60.$

at $K = 60$ $a = 0$
system $\to$ marginally stable

$$0 \leq a \leq \frac{10}{K} + \frac{3}{2} - \frac{K}{36}.$$

$\therefore$ Stable region :- $K \in (0, 60]$
$a \in [0, 10]$.

Q7. (a)



characteristic equation:

$$1 + \frac{K(s+m)(s+2)}{s(s^2-1)} = 0.$$

$$s^3 - s + Ks^2 + K(m+2)s + 2Km = 0.$$

$$s^3 + Ks^2 + (K(m+2)-1)s + 2Km = 0.$$

for stability:

$K>0$, $K(m+2)-1>0 \longrightarrow m > \frac{1}{K} - 2.$

$2Km \geq 0. \quad \therefore m \geq 0.$

s^3	1	$K(m+2)-1$
s^2	K	$2Km$
s^1	$\frac{-1}{K}\begin{vmatrix} 1 & K(m+2)-1 \\ K & 2Km \end{vmatrix}$	$0.$

$$\underbrace{(K-2)m + 2K-1}\ \begin{cases} >0. \ \forall\ m \geq 0 & \text{if } K \geq 2. \\ <0\ \forall\ m \leq 0 & \text{if } K < \frac{1}{2}. \\ \geq 0\ \forall\ m \leq \frac{2K-1}{2-K}. & K \in [\frac{1}{2}, 2) \end{cases}$$

System is marginally stable $\left\{ m = \frac{2K-1}{2-K} \quad K \in [\frac{1}{2}, 2) \right\}$

$\therefore$ System stable for $K \geq 2, m \geq 0.$

or

$K \in [\frac{1}{2}, 2)$, $0 \leq m \leq \frac{2K-1}{2-K}$

Q 7.
(b) $G_1(s) = \frac{(s+m)(s+2)}{s}$

$G_2(s) = \frac{K}{s^2-1}$.

when system stable

steady state error $\xrightarrow{\text{ramp response}}$ $e = \frac{1}{\lim_{s\to 0} sG_1(s)G_2(s)}$.

$= -1/2Km$.

given bet^n ± 0.1

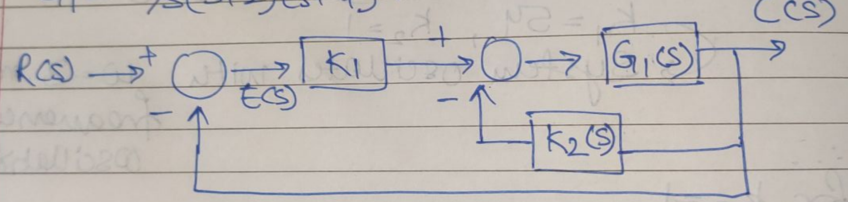
K, m are both +ve

so; $\frac{-1}{2Km} \geq -0.1$

$\boxed{m \geq 5/K}$

Q8. $G_1 = 1/s(s+2)(s+4)$.

(a,b)



$1 + K_2 s G_1(s) = 0$.

→ will have 2 equal −ve roots.

$(s+2)(s+4) + K_2 = 0$.

$G(s) = \frac{1}{s(1+(s+2)(s+4))}$

$K_2 = 1$

$s^2 + 6s + 9 = 0$ → -3, -3 } 2 equal roots.

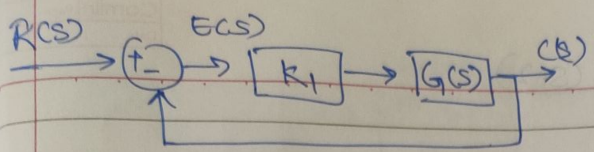
∴

Q 7.

char

for K

2



$1 + K_1\, G(s) = 0$

$$1 + \frac{K_1}{s(s+2)(s+4) + s} = 0.$$

$s^3 + 6s^2 + 9s + K_1 = 0$ $\qquad K_1 > 0.$

$$\begin{array}{lll} s^3 & 1 & 9 \\ s^2 & 6 & K_1 \\ s & \frac{54 - K_1}{6} & 0 \end{array}$$

$0 < K_1 < 54$
for stable system.

$s^3 + 6s^2 + 9s + 54 = 0$
$(s+6)(s^2+9) = 0$

$K_1 = 54$, $K_2 = 1$
→ System oscillates with $\omega = 3$ rad/sec
frequency of oscillation.

∴ for $K_2 = 1$
→ $K_1 \in (0, 54)$

at $K_1 = 54$ → system is marginally stable.
(oscillates).

if K_2 not specified
$\omega = \sqrt{8 + K_2}$ rad/sec
when $K_1 = 6(8 + K_2)$.

Q8 (c) as closed loop pole at will make system unstable, we won't able to approximate step response as underdamped.

$$s^3+6s^2+(8+K_2)s+K_1=0$$

$S=5 \rightarrow$ pole

$K_1=-S(6S+K_2)$.

So

$K_1=5(K_2+3)$

$(S+5)(S^2+S+K_2+3)=0$.

1 pole at $s=-5$
both complex poles have real $=-1/2$.
real pole is 10 times away from img axis.

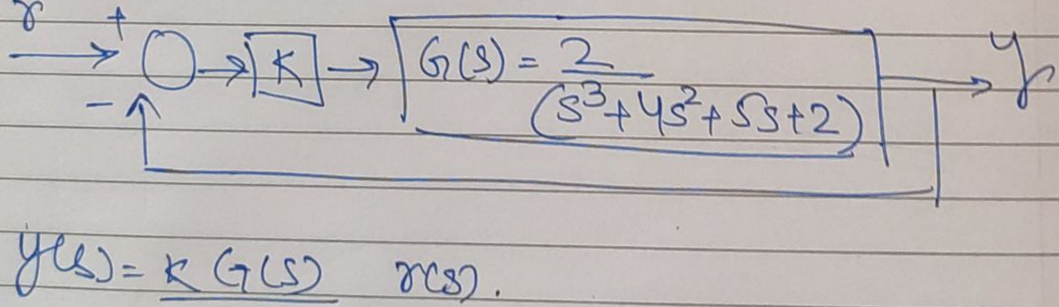
Dominant pole approximation

$K_2=1 \quad K_1=20$.

$$G(s)=\frac{20}{s^3+6s^2+9s+20}.$$

$$G_{app}(s)=\frac{4}{s^2+s+4}.$$

Q9.



$$y(s)=\frac{K\,G(s)}{1+K\,G(s)}\,r(s).$$

$$E(s)=\frac{R(s)}{1+K\,G(s)}. \qquad R(s)=1/s.$$

$$e=\frac{1}{1+K\lim_{s\to 0}G(s)}.$$

$e = \frac{1}{1+K}$.

$G(s) = (-2, -1, -1)$ in LHP.

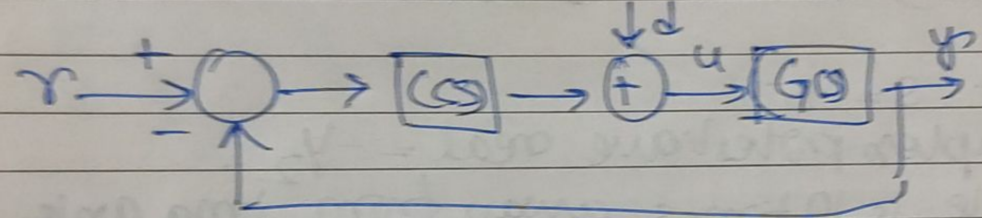
$\frac{1}{1+K} \leqslant 0.15$

$K \geqslant 17/3$

Characteristic equ^n $\quad 1+K\,G(s)=0$

$s^3+4s^2+5s+2+2k=0$

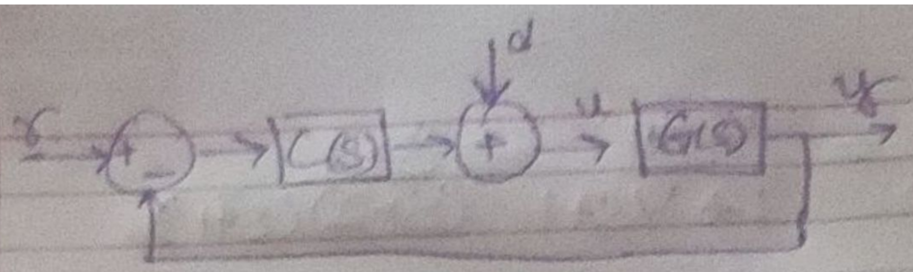
Q10



$G(s) = 1/s-1$

$C(s) = \frac{s-1}{s+1}$.

case 1.

Q10. 

$R(s) = D(s) = \frac{1}{s}$

$$G(s)\big(C(s)(R(s)-Y(s)) + D(s)\big) = Y(s).$$

$$Y(s) = U(s)G(s).$$

$r \to y$

$$G(s)C(s)R(s) = Y(s)\big(1 + G(s)C(s)\big)$$

$$\boxed{\frac{Y(s)}{R(s)} = \frac{GC}{1+GC}}$$

$d \to y$ $\quad GD = Y(1+GC)$

$$\boxed{\frac{Y}{D} = \frac{G}{1+GC}}$$

$r \to u$ $\quad \frac{UG}{R} = \frac{GC}{1+GC}. \qquad \boxed{\frac{U}{R} = \frac{C}{1+GC}}$

$d \to u.$ $\quad \frac{GU}{D} = \frac{G}{1+GC} \qquad \boxed{\frac{U}{D} = \frac{1}{1+GC}}$

Case ① $\quad C = \frac{S-1}{S+1} \qquad G = \frac{1}{S-1}$

$r \to y$ - $\frac{Y}{R} = \frac{1/S+1}{1+1/S+1} = \frac{1}{S+2}.$ $\qquad Y = \frac{1}{S+2}\cdot\frac{1}{S}$

$$Y = \frac{1}{2}\left(\frac{1}{S} - \frac{1}{S+2}\right)$$

$$\boxed{y(t) = \frac{1}{2}\left(u(t) - e^{-2t}u(t)\right)}$$

$d \to y$

$$\frac{Y}{D} = \frac{1/(s+1)}{1+1/(s+1)} \Rightarrow \frac{s+1}{(s+2)(s+1)}$$

$$Y = \frac{\cancel{s+1}}{(s+2)\cancel{(s+1)}(s)} \rightarrow \underbrace{Y = \frac{-1}{2s} + \frac{2}{3(s-1)} - \frac{1}{6(s+2)}}$$

$$y(t) = \left(\frac{-1}{2} + \frac{2}{3}e^{t} - \frac{1}{6}e^{-2t}\right)u(t)$$

$d \to u$.

$$\frac{U}{D} = \frac{1}{1+1/(s+1)} = \frac{s+1}{s+2} \qquad U = \frac{s+1}{s(s+2)}$$

$$U = \frac{1}{2}\left(\frac{1}{s+2} + \frac{1}{s}\right) \qquad \underbrace{u(t) = \left(\frac{1}{2} + \frac{1}{2}e^{-2t}\right)u(t)}$$

$r \to u$.

$$\frac{U}{R} = \frac{(s-1)/(s+1)}{1+1/(s+1)} = \frac{s-1}{s+2}$$

$$U = \frac{s-1}{(s+2)s} = \frac{3}{2}\frac{1}{s+2} - \frac{1}{2s} \qquad \underbrace{u(t) = \left(\frac{3}{2}e^{-2t} - \frac{1}{2}\right)u(t)}$$

case ② $G(s) = \frac{s-1}{s+1}$ $C(s) = 1/s-1$

$d \to u$

$$\frac{U}{D} = \frac{1}{1+GC} \xrightarrow{\mathcal{L}^{-1}} \underbrace{u(t) = \left(\frac{1}{2} + \frac{1}{2}e^{-2t}\right)u(t)}_{\text{remains the same.}}$$

$r \to y$

$$\frac{Y}{R} = \frac{GC}{1+GC} \to \underbrace{y(t) = \left(\frac{1}{2} - \frac{1}{2}e^{-2t}\right)u(t)}_{\text{remains the same}}$$

$r \to u$

$$\frac{U}{R} = \frac{1/s-1}{1+1/(s-1)} \to$$

Same as case ① $d \to y$.

$$\underbrace{u(t) = \left(\frac{-1}{2} + \frac{2}{3}e^{t} - \frac{e^{-2t}}{6}\right)u(t)}$$

$d \to y$

$$\frac{Y}{D} = \frac{s-1/s+1}{1+s-1/s+1} \to$$

Same as case ① $r \to u$.

$$\underbrace{y(t) = \left(\frac{3}{2}e^{-2t} - \frac{1}{2}\right)u(t)}$$