CSE Test Mansi Uniyal
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KGCSE05

1
A.

Q1. $\hat{G}(s) = \frac{1}{s-2}\left(1 + \frac{ks}{s+a}\right)$.

PI controller.
-ve feed back

nominal plant ⇝ $G(s) = \frac{1}{s-2}$.

$C(s) = k_p + \frac{k_i}{s}$

gain to place all closed loop poles at $s=-1$ = ?

closed loop TF $= \frac{G(s)\,C(s)}{1 + C(s)\,G(s)}$.

$= \frac{(1/s-2)\,(k_p + k_i/s)}{1 + \frac{1}{s-2}\left(k_p + \frac{k_i}{s}\right)}$.

$= \frac{sk_p + k_i}{s(s-2) + sk_p + k_i}$

$= \frac{sk_p + k_i}{s^2 + s(k_p - 2) + k_i}$ ⇝ pole at $s=-1$.

$\left\{ \begin{matrix} k_p = 4 \\ k_i = 1 \end{matrix} \right\}$

∴ closed loop TF with $s=-1$ pole $= \frac{4s+1}{s^2+2s+1}$

$$\boxed{= \frac{4s+1}{(s+1)^2}}$$

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1. B.

$$\left|W_c(j\omega)S(j\omega)\right| = \left|\frac{\hat{G}(s) - 1}{G(s)}\right| = \left|\frac{ks}{s+a}\right|$$

$$= \left|\frac{kj\omega}{j\omega + a}\right|.$$

$$\Rightarrow \frac{k\omega}{\sqrt{a^2+\omega^2}}.$$ but, $S(j\omega) < 1 \; \forall \omega$

$$\frac{k\omega}{\sqrt{a^2+\omega^2}} < \left|W_c(j\omega)\right|.$$

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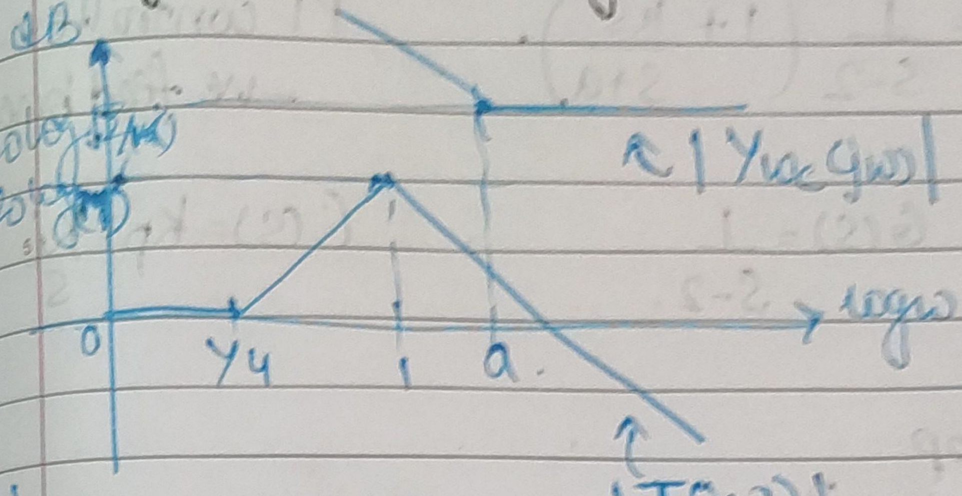
(1)

B.

Assymptotic bode magnitude plot:



$$F(s) = \frac{G(s)C(s)}{1+G(s)C(s)} = \frac{4s+1}{(s+1)^2}$$

$$\left\{ W_c(s) = \frac{ks}{s+a}. \right.$$

$T(jw)$ will be below $1/W_c(jw)$

(i)

$$\left|\frac{1}{W_c(jw)}\right|_{w=a} = \frac{\sqrt{a^2+a^2}}{ka} = \sqrt{2}/k.$$

$$\sqrt{2}/k > \sqrt{17}/2 \;;\; \boxed{k < 2\sqrt{2}/\sqrt{17}}$$

$$|T(jw)|_{w=1} = \frac{4jw+1}{(jw+1)^2} = \frac{1+4j}{(1+j)^2} = \frac{\sqrt{17}}{2} = M.$$

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(2)
A.

Q2. unstable non-minimum phase plant
$G(s) = \frac{s-2}{s^3}$.

Normalised closed loop (DC gain=1). ref. i/p to o/p model
poles at $s=-1$. = ?.
2DOF controller.

as relative order of plant = 2, min. relative order of T_{yr} = 2

1 finite zero in $G(s)$.

$T_{yr} = \frac{\alpha(s-2)}{(s+1)^3}$, $\alpha = -1/2$.

controller
pole order ⇒ $m = n-1 = 3-1 = 2$

$\delta(s) = (s+1)^3(s+4)$
$= (s^3+3s^2+3s+1)(s+4)$
$= s^5 + 11s^4 + 43s^3 + 73s^2 + 56s + 16$.

order 5

a.k + c.h.

$\therefore\ s^3(s^2+k_1s+k_0) + (s-2)(h_2s^2+h_1s+h_0)$
$= s^5 + k_1s^4 + (k_0+h_2)s^3 + (h_1-2h_2)s^2 + (h_0-2h_1)s - 2h_0$.

$\therefore\ -2h_0 = 16$ $\boxed{h_0 = -8}$
$h_1 - 2h_2 = 73$,
~~$h_0-2h_1 = 56$~~
$k_0 + h_2 = 43$.
$h_0 - 2h_1 = 56$.

$\boxed{h_1 = -32}$
$\boxed{k_1 = 1}$
$\boxed{h_2 = -52.5}$
$\boxed{k_0 = 95.5}$

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19EE10039 (2) B

$n(s) = -52.5s^2 - 32s - 8$
$k(s) = s^2 + 11s + 95.5$
$q(s) = \alpha(s+4)^2 = -\frac{1}{2}(s+4)^2$
$= -0.5s^2 - 4s - 8$

controller 2DOF:
$$u = \left(\frac{52.5s^2 + 32s + 8}{s^2 + 11s + 95.5}\right) y - \left(\frac{0.5s^2 + 4s + 8}{s^2 + 11s + 95.5}\right) r.$$

1 DOF:
$$u = -\left(\frac{52.5s^2 + 32s + 8}{s^2 + 11s + 95.5}\right)(r - y).$$

loop transfer fun^{cn}

~~2DOF~~
$$L(s) = \frac{ch}{ak} = \frac{(s-2)}{s^3}\left[\frac{-52.5s^2 - 32s - 8}{s^3 + 11s + 95.5}\right]$$
$$= \frac{-52.5s^3 + 73s^2 + 56s + 16}{s^5 + 11s^4 + 95.5s^3}$$

r to y:
1DOF:
$$\frac{y}{r} = \frac{ch}{\delta} = \frac{(s-2)(-52.5s^2 - 32s - 8)}{(s+1)^3(s+4)^2}$$

2DOF
$$\frac{y}{r} = \frac{ca}{\delta} = \frac{-\frac{1}{2}(s-2)(s+4)^2}{(s+1)^3(s+4)^2} = \frac{-\frac{1}{2}(s-2)}{(s+1)^3}.$$

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(2) C.

In 1DOF ref i/p o/p tf
2 extra zeros $(-0.3047 \pm 0.2491i)$
are present which
are closer to imaginary
axis.
$\therefore$ 1 DOF controller overshoot
will be present.

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(3) A.

Q3. $G(s) = \frac{s+0.5}{(s-1)^2}$ $K_p = 6$.
-ve feedback loop.

lower side gain margin = ?

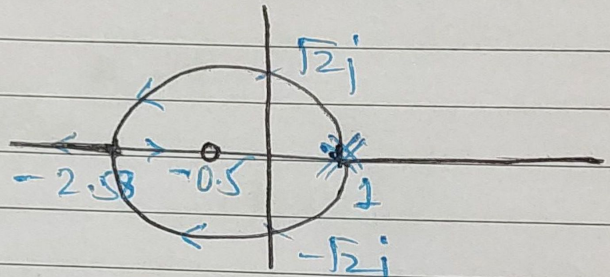
upper side gain margin = ?.

$L(s) = k\,C(s)G(s) = k\frac{6(s+0.5)}{(s-1)^2}$

poles = 1, 1
zeros = -0.5.

∴ real axis root locus should be from -0.5 to -∞
∴ 1 break in point betn -0.5 to -∞.

Root locus:



img. crossing at

$(s-1)^2 + 6k(s+0.5) = 0.$

$s^2 - 2s + 1 + 6ks + 3k = 0$

$s^2 + s(6k-2) + (3k+1) = 0$

when $6k - 2 = 0$
$k = 1/3.$

∴ lower margin gain $= 1/3 = 20\log 1/3.$
$= -9.54$ dB.

upper margin gain $= \infty$

Sensitivity:

$L(s) = \frac{6(s+0.5)}{(s-1)^2}$,

$S = \frac{1}{1+L}$

$= \frac{1}{1+\frac{6(s+0.5)}{(s-1)^2}}$ Ans.

$S = \frac{(s-1)^2}{(s-1)^2 + 6(s+0.5)}.$

$= \frac{(s-1)^2}{s^2 - 2s + 1 + 6s + 3} = \frac{(s-1)^2}{s^2 + 4s + 4} = \frac{(s-1)^2}{(s+2)^2}.$

continued

3 A

Q3. $G(S) = \frac{S+0.5}{(S-1)^2}$. $K_p = 6$.

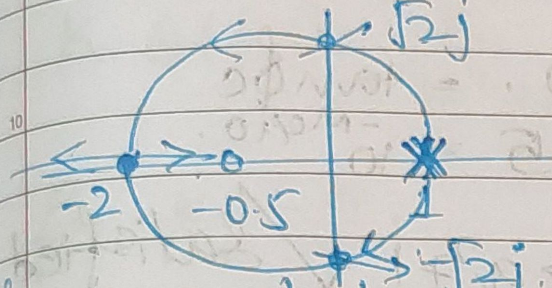
$L(S) = C(S)\,G(S) = \frac{6(S+0.5)}{(S-1)^2}$.

Root locus:

$1 + K\frac{6(S+0.5)}{(S-1)^2} = 0$.

open loop poles $= 1, 1$

zeros $= -0.5$.



for asymtotes: $\sigma = \frac{1+1+0.5}{2-1} = 2.5$

$\theta = \frac{(2n+1)180}{2-1} = 180$

Real axis:

$\frac{dK}{ds} = 0$; $\frac{d}{ds}\left(\frac{-(S-1)^2}{6(S+0.5)}\right) = 0$.

$\cancel{6}(S+0.5)(-2(S-1)) + (S-1)^2\cancel{6} = 0$.

~~(S+0.5)(-2(S+1)) + (S+1)^2 = 0~~

~~S = 0, ...~~

$-2(S+0.5) + S-1 = 0$

$2S+1 = S-1$.

$S = -2$. ✓

$S-1 = 0$

$S = 1$ ✗.

$\therefore S = -2$

break in point.

imaginary axis crossing

$(S-1)^2 + 6k(S+0.5) = 0$.

$S^2 + S(6k-2) + 3k - 1 = 0$

③ c continued

$$\begin{array}{c|cc} s^2 & 1 & 3k+1 \\ s^1 & 6k-2 & 0 \\ s^0 & 3k+1 & \end{array}$$

$6k=2$ $\quad k = 1/3$

$A(s) \Rightarrow s^2 + 3k+1 = 0$ $\qquad s^2+2=0$

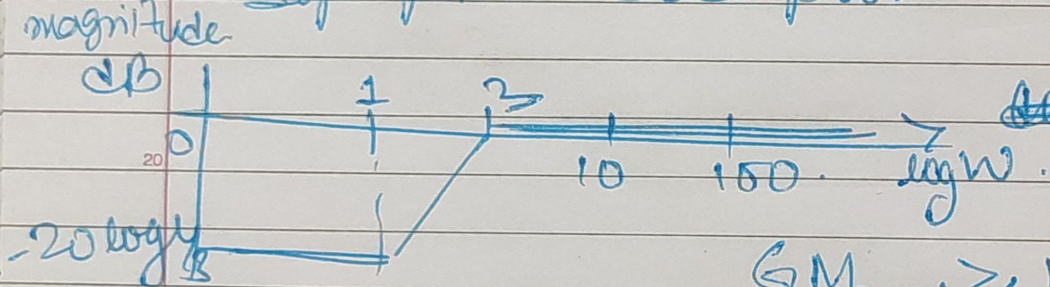
$s = \pm\sqrt{2}\,j$.

Sensitivity func^n $\Rightarrow \dfrac{1}{1+L(s)}$

$$= \frac{(s-1)^2}{(s-1)^2+6s+3} = \frac{(s-1)^2}{s^2+1-2s+6s+3}$$

$$= \frac{(s-1)^2}{(s+2)^2} = \left(\frac{s-1}{s+2}\right)^2.$$

Asymptotic bode plot:



at $w=1$ mag: 40dB ~~dogw~~ rise with ~~decade~~ decade.

$GM_{max} \geq \dfrac{MS}{MS-1}$. $\qquad$ from ∞ notion $MS=1$.

$GM_{min} \leq \dfrac{MS}{MS+1}$

$GM_{min} \leq 1/2$

$PM \geq 2\sin^{-1}\dfrac{1}{2MS}$

$\geq 2\sin^{-1} 1/2$

$\geq 60^\circ$

$\therefore GM = 0.33 = 1/3$. also satisfies the condition.

min. phase margin $= 60^\circ$

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Q4. $G(s) = \frac{1}{s^2-1}$. | $e_{ss} = 10\%$

unstable plant. | $\frac{1}{1+k_p} = \pm 0.1$

$K_p = \lim_{s \to 0} G(s)\,C(s)\,k$ $\qquad k_p = 9$ or -11

$= k/-1$

$k = -9$. or 11

$K\,G(j\omega) = \frac{-K}{\omega^2+1}$ $\qquad \omega_c = 5$ rad

for $k = -9$.

$-9\,G(j\omega) = \frac{9}{\omega^2+1}$ $\qquad \angle K G(j\omega) = 0'$

$|kG(j\omega)| = \frac{9}{26} = -9.2$ dB

(-ve in dB)

for $k = 11$.

$K G(j\omega) = \frac{-11}{\omega^2+1}$.

$\angle K G(j\omega) = -180'$

$|K G(j\omega)| = 11/26$.

$\phi_{nom} = -180° + 30 = -150°$

for $k = -9$, $\phi_{com} < \phi_{mc}$. $\quad M_c < 0$. Not satisfied

for $k = 11$, $\phi_{com} > \phi_{mc}$. $\quad M_c < 0$ Lead controller used.

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4 B.

$MC = 7.471 dB$

$\phi_c = \phi_{com} - \phi_{mc}$.

$= 30°$

$C(s) = \frac{1 + s/a}{1 + s/\alpha a}$ $\alpha > 1$.

Thus quadratic equation becomes:

$(q^2 - c + 1)\alpha^2 + 2q^2 c\alpha + c(q^2 c + c - 1) = 0$

$q = \tan 30° = 0.577. = \tan\phi_c$

$c = 10^{0.7471} = 5.585 = 10^{-Mc/10}$.

$c > q^2 + 1$ ✓ satisfied.

as, $5.585 > 1.3329$ (+ve)

$\therefore$ $\underline{\alpha = 3.379}$

$a = \frac{w_c}{\alpha}\sqrt{\frac{\alpha^2 c}{c-1}} = \underline{1.669}$

$\left\{ C(s) = \frac{1 + s/1.669}{1 + s/5.634} \right\}$

Ans $C(s) = \frac{(1.669 + s) \times 1}{(5.634 + s)(3.3756)}$