Control & Instrumentation Lab, Autumn 2021-22

Session 5: Root Locus Technique

Note to the students: Please solve all the problems by hand and then verify using MATLAB. Show to the TAs how you have solved them by hand.

1. Consider the characteristic equation
$$s(s+2)(s+3)+K(s+1)=0$$
Determine $K=0$ and $K=\infty$ points and the number of branches of the root loci.
2. Consider the characteristic equation
$s(s+2)(s+1+j)(s+1-j)+K=0.$
Show the symmetry of the root loci.
3. Consider the characteristic equation
$$s(s+4)(s^2+2s+2)+K(s+1)=0$$
Compute the angles of asymptotes and intersect of asymptotes.
4. Consider the characteristic equation
$$s(s+3)(s^2+2s+2)+K=0$$
Show root loci on the real axis and compute the angle of departure at $s_1=-1+j1$.
5. Consider the characteristic equation
$$s(s+4)(s^2+4s+20)+K=0.$$
Determine the intersections of the root loci with the imaginary axis and breakaway points on the root loci.
6. Construct the root locus, given the characteristic equation
$s(s+5)(s+6)(s^2+2s+2)+K(s+3)=0.$
(a) manually, and (b) using MATLAB
7. A simplified form of the open-loop transfer function of an airplane with an autopilot in the longitudinal mode is
$$G(s)H(s)=\frac{K(s+a)}{s(s-b)(s^2+2\xi\omega_n s+{\omega_n}^2)}, \qquad a>0, \quad b>0$$
Such a system involving an open-loop pole in the right-half s plane may be conditionally stable. Sketch the root loci when $a=b=1. \xi=0.5$ and$\omega_n=4$. Find the range of gain K for stability.

8. Consider the two open loop transfer functions $G_1(s)$ and $G_2(s)$, as shown below:
$$G_1(s)=\frac{1}{s^3+6s^2+45s}$$

$$G_2(s) = \frac{0.075s^2 + s + 1}{s^3 + 3s^2 + 5s}$$

I. Examine the root locus of each of these systems using '***rlocus***' command in *MATLAB* and comment on their closed loop stabilities. Are both the systems stable for all ***K***? If not, find the range of ***K*** for which, each of these systems is stable.

II. Theoretically find the value of ***K*** for which the closed loop characteristic equation of these two systems are identical i.e., if $1 + KG_1(s) = 0$ is the characteristic equation of the first system and $1 + KG_2(s) = 0$ is that of the second system, find the value of ***K*** for which these two equations are identical.

III. Examine the unit step response characteristics of these two systems (e.g., rise time, settling time, peak overshoot) at this value of ***K***, using '***stepinfo***' command in *MATLAB*. Plot the unity feedback step response of the two systems using '***step***' command. Do you observe any anomaly in the nature of the plots? Aren't they supposed to look identical, since the characteristic equations are same? Explain why the step responses of these two systems are not identical, but quite different, even though their characteristic equations are same (take the help of the root loci of two systems to reach your explanation). Comment on the role of the zeros of $G_2(s)$ in your findings.

9. A unity feedback system with open-loop transfer function is given below:

$$G(s) = \frac{K(s^2 - 2s + 2)}{(s+2)(s+3)}$$

Plot root locus and find valid breakaway point, and value of K for stability.

10. Draw the RL of the system, without finding all the breakaway points (BPs). It is already given that one of the BPs = centroid. No need to find the values of BPs.

a) $G(s)H(s) = \frac{K}{s(s^2+2s+1.25)(s+2)}$;

b) $G(s)H(s) = \frac{K}{s(s^2+2s+2)(s+2)}$;

c) $G(s)H(s) = \frac{K}{s(s^2+2s+10)(s+2)}$