Dynamic Programming, Greedy Algorithms

Q4. Find the maximum alignment for X = dcdcbacbbb and Y = acdccabdbb by using the Smith-Waterman algorithm (see slides). Execute the pseudocode algorithm and fill the necessary tables H and P in a bottom-up fashion. Reconstruct the strings X‘ and Y‘ using the tables H and P.

Solution:

Table for H:

0	0	0	0	0	0	0	0	0	0	0
0	-1	-1	2	1	0	-1	-1	2	1	0
0	-1	1	1	4	3	2	1	1	1	0
0	-1	0	3	3	3	2	1	3	2	1
0	-1	1	2	5	5	4	3	2	2	1
0	-1	0	1	4	4	4	6	5	4	4
0	2	1	0	3	3	6	5	5	4	3
0	1	4	3	2	5	5	5	4	4	3
0	0	3	3	2	4	4	7	6	6	6
0	-1	2	2	2	3	3	6	6	8	8
0	-1	1	1	1	2	2	5	5	8	10

Table for P:

-	d	d	d	l	l	d	d	d	l	l
-	d	d	u	d	d	l	l	u	d	d
-	d	u	d	u	d	d	d	d	l	l
-	d	d	u	d	d	l	l	u	d	d
-	d	u	u	u	d	d	d	l	d	d
-	d	l	u	u	d	d	u	d	d	d
-	u	d	l	d	d	u	d	d	d	d
-	u	u	d	d	u	d	d	l	d	d
-	d	u	d	d	u	d	d	d	d	d
-	d	u	d	d	u	d	d	d	d	d

Exercise 15.1-2: Show, by means of a counter example, that the following "greedy" strategy does not always determine an optimal way to cut rods. Define the density of a rod of length i to be pi/i, that is, its value per inch. The greedy strategy for a rod of length n cuts off a first piece of length i, where 1 <= i <= n, having maximum density. It then continues by applying the greedy strategy to the remaining piece of length n – i.

Solution:

Let’s us consider the length of the rod (n) to be 6 i.e., n = 6

Now consider the prices to be as follows: p1 = 0, p2 = 1, p3 = 5, p4 = 8, p5 = 2

Now, according to the greedy algorithm, it will consider the height first i.e., p4. The ratio is pi/i = 8/6.

The left height is 6 - 4 = 2

Therefore, for the height of left which is equal to 2, Greedy algorithm will consider the value of p2.

Therefore, the total profit will be 8 + 1 = 9

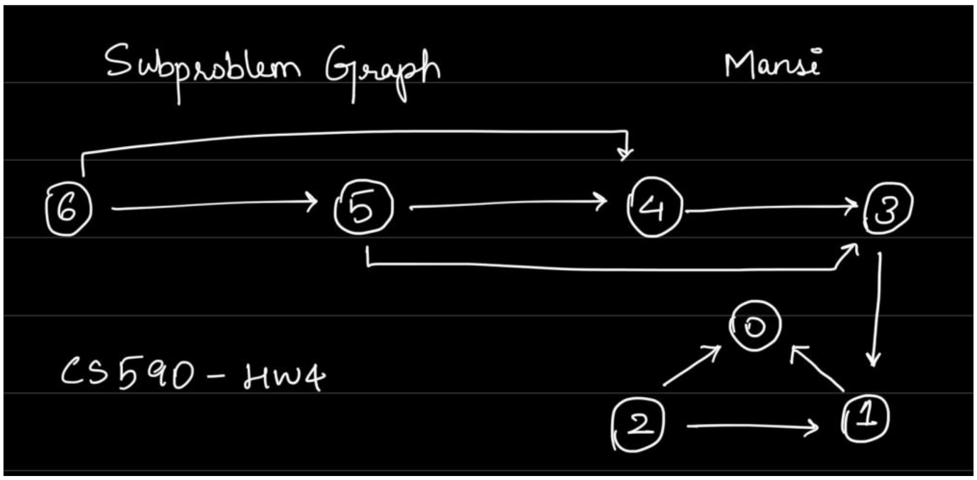
But a better solution is also possible when we consider as follows:- p3 + p3 = 6

By doing that, we can get a profit of 10 i.e., 5 + 5 = 10

Exercise 15.1-5: The Fibonacci numbers are defined by recurrence (3.22). Give an O(n) time dynamic-programming algorithm to compute the n-th Fibonacci number. Draw the subproblem graph. How many vertices and edges are in the graph?

Solution:

```
FIBONACCI(n)
        let fib[0. . n] be a new array
        fib[0] = 1
        fib[1] = 1
        for i = 2 to n
                fib[i] = fib[i - 1] + fib[i - 2]
        return fib[n]
```



The above graph is for the value of n = 6.

Vertex: There are n + 1 vertices in the subproblem graph, i.e., 0, 1, 2, 3, 4, 5, 6

Edges: There are 2n – 2 edges in the subproblem graph.

Exercise 15.4-1: Determine an LCS of ⟨1,0,0,1,0,1,0,1⟩ and ⟨0,1,0,1,1,0,1,1,0⟩

0	0	0	0	0	0	0	0	0	0
0	0	1	1	1	1	1	1	1	1
0	1	1	2	2	2	2	2	2	2
0	1	1	2	2	2	3	3	3	3
0	1	2	2	3	3	3	4	4	4
0	1	2	3	3	3	4	4	4	5
0	1	2	3	4	4	4	5	5	5
0	1	2	3	4	4	5	5	5	6
0	1	2	3	4	5	5	6	6	6

The LCS is ⟨1,0,0,1,1,0⟩ or ⟨1,0,1,0,1,0⟩.