CS425
Homework 2

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1 Solution 1

The code is attached with the solutions, the file name for the code is 210591_Q1.py. To run this file :
1. Ensure that Python is installed on your system.
2. In the Terminal/ Commnd Prompt, navigate to the directory where this file is located.
3. Run the script : python 210591_Q1.py.
4. Output will be displayed.

2 Solution 2

Let us assume that the window size is n = 2^k with the sender transmitting frames in the order without loss of generality (0,1,2...n-1). After receiving all of this successfully, the receiver sends an acknowledgement signal and expects the next window in the same order. If the acknowledgment signal is lost due to some reasons, the sender runs out of time which causes the sender to transmit frames from the previous window. But as the receiver was expecting frames of the next window, he accepts this in the next window, resulting in the duplication of the previous window which was unintended. This ambiguity introduces error.
Now if the window size is n = 2^k - 1 with a similar situation as described above, the receiver would expect frame number n, but the sender begins with frame 0 during the re-transmission, preventing the error occurred in the previous case.
So, the window size limited to 2^k - 1 and not 2^k.

3 Solution 3

The maximum window size is 2^{k-1}.
Let the maximum window size be w. There are two possible cases: one with an overlap between consecutive windows, and the other without an overlap.
Case 1 : With Overlap
Let's assume the first window has frame numbers from 0 to w - 1, and the next window has frame numbers from w onwards with at least 1 overlap. If the receiver successfully receives the first window and sends an acknowledges signal it but the sender doesn't receive this due to some reasons and re-transmits the first window. But as the receiver was expecting frames of the next window, he accepts this in the next window, which causes the confusion about which window the overlapping frame belongs to. Due to this reason we cannot have a window size such which causes consecutive windows to overlap.
Case 2 : Without Overlap
In this case there will be no ambiguities as there's no confusion regarding the window to which a frame belongs.
So to determine the window size we will consider case 2 i.e. no overlap condition. As there's no overlap, so the total number of frames in 2 consecutive windows = 2w (distinct). With a k bit sequence, there can be at most 2^k frames (distinct).
So 2w $\leq 2^k$ i.e. w $\leq 2^{k-1}$

4 Solution 4

Given : Propagation Delay (t_{prop})= 20 ms; Data Rate (r) = 4kbps; Efficiency(U) $\geq$ 0.5, the Transmission Time(t_{trans}) = $\frac{s}{r}$ where s = Frame Size

We know that U = $\frac{1}{1+2a}$ using these 2 equations : U = $\frac{1}{1+2a} \geq 0.5 \implies 1 + 2a \leq 2 \implies a \leq 0.5$

Also a = $\frac{t_{prop}}{t_{trans}} \implies \frac{4kbps \cdot 20ms}{s} \leq 0.5 \implies s \geq 160$ bits

So for an efficiency of at least 50%, the frame size must be at least 160 bits.

5 Solution 5

(a) The probability of bit error (p) = 10^{-3} $\implies$ The probability of bit error = 1 - p = 0.999. As the bits are independent of each other
So the probability that the received frame contains no errors = $(1-p)^4 = (0.999)^4 \approx$ **0.996**

(b) The probability that the received frame contains at least one error = 1 - (probability that the received frame contains no error) $\approx$ 1 - 0.996 $\approx$ **0.004** [using the result from part (a)]

(c) With the addition of 1 parity bit, we now have 5 bits received in a frame. We know that this parity bit can detect odd number of errors and cannot detect even number of errors. So here we must determine the probability of scenarios involving either 2 or 4 errors (excluding the case with 0 errors). 1, 3, or 5 errors can be identified by checking the parity of the 5 bits.
The probability that the frame is received with errors that are not detected = P(2 errors) + P(4 errors)

P(2 errors) = Choose 2 bits from the 5 $\cdot$ probability that these 2 bits have error $\cdot$ probability that the rest 3 bits have no error = $\binom{5}{2} \cdot p^2 \cdot (1-p)^3 = 9.97 \cdot 10^{-6}$

P(4 errors) = Choose 4 bits from the 5 $\cdot$ probability that these 4 bits have error $\cdot$ probability that the rest 1 bit has no error = $\binom{5}{4} \cdot p^4 \cdot (1-p) = 4.995 \cdot 10^{-12}$

So the probability that the frame is received with errors that are not detected $\approx$ $\mathbf{9.97 \cdot 10^{-6}}$

6 Solution 6

Firstly, M will be padded with $|P|$ - 1 number of 0's, now using this as dividend and P as divisor, we will perform division (modulo 2). The obtained remainder is the CRC. **CRC = 11010**. Long division :

```
            10110110
      110011)1110001100000
             110011
              010111
              000000
               101111
               110011
                111000
                110011
                 010110
                 000000
                  101100
                  110011
                   111110
                   110011
                    011010
                    000000
                     11010
```

7 Solution 7

(a) As M = 10010011011 $\implies$ M(x) = $x^{10} + x^7 + x^4 + x^3 + x + 1$
P(x) = $x^4 + x + 1$ $\implies$ P = 10011 i.e. $|P| = 5$
So shifting M by $|P|$ - 1 = 4 bits, i.e. $x^4 \cdot$ M(x) = $x^{14} + x^{11} + x^8 + x^7 + x^5 + x^4$
Dividing $x^4 \cdot$ M(x) by P(x) :

$$
\begin{array}{r}
x^{10} + x^6 + x^4 + x^2 \\
x^4 + x + 1 \overline{)\, x^{14} + x^{11} + x^8 + x^7 + x^5 + x^4} \\
\underline{x^{14} + x^{11} + x^{10}} \qquad\qquad \\
x^{10} + x^8 + x^7 \qquad\qquad \\
\underline{x^{10} + x^7 + x^6} \qquad\qquad \\
x^8 + x^6 + x^5 + x^4 \\
\underline{x^8 + x^5 + x^4} \qquad \\
x^6 \qquad\qquad\qquad \\
\underline{x^6 + x^3 + x^2} \qquad \\
x^3 + x^2 \qquad
\end{array}
$$

R(x) = x^3 + x^2 $\implies$ R = 1100, the encoding for M will be MR = **100100110111100**.

(b) The string W = 000110110111100 is received where we have flipped the 1^{st} and 5^{th} bits of MR.
W(x) = x^{11} + x^{10} + x^8 + x^7 + x^5 + x^4 + x^3 + x^2
To check if there's an error, we divide W(x) by P(x). As there's an error, the remainder obtained must be non-zero and if the remainder we get after dividing as non-zero, then this implies that the error is detected. Dividing W(x) by P(x) :

$$
\begin{array}{r}
x^7 + x^6 + x^3 + x^2 + x \\
x^4 + x + 1 \overline{)\, x^{11} + x^{10} + x^8 + x^7 + x^5 + x^4 + x^3 + x^2} \\
\underline{x^{11} + x^8 + x^7} \qquad\qquad\qquad \\
x^{10} + x^5 + x^4 \qquad\qquad\qquad \\
\underline{x^{10} + x^7 + x^6} \qquad\qquad\qquad \\
x^7 + x^6 + x^5 + x^4 + x^3 \\
\underline{x^7 + x^4 + x^3} \qquad\qquad \\
x^6 + x^5 + x^2 \qquad\qquad \\
\underline{x^6 + x^3 + x^2} \qquad\qquad \\
x^5 + x^3 \qquad\qquad \\
\underline{x^5 + x^2 + x} \qquad\qquad \\
x^3 + x^2 + x \qquad
\end{array}
$$

R(x) = x^3 + x^2 + x i.e. R = 1110 which is non zero, **so the error will be detected**.

(c) The receive string W = (100100110111100) $\oplus$ (100110000000000) = 000010110111100
W(x) = x^{10} + x^8 + x^7 + x^5 + x^4 + x^3 + x^2
Again we follow the same steps as before.

$$
\begin{array}{r}
x^6 + x^4 + x^2 \\
x^4 + x + 1 \overline{)\, x^{10} + x^8 + x^7 + x^5 + x^4 + x^3 + x^2} \\
\underline{x^{10} + x^7 + x^6} \qquad\qquad \\
x^8 + x^6 + x^5 + x^4 \\
\underline{x^8 + x^5 + x^4} \qquad \\
x^6 + x^3 + x^2 \qquad \\
\underline{x^6 + x^3 + x^2} \qquad \\
0 \qquad\qquad\qquad
\end{array}
$$

R(x) = 0 i.e. R = 0000, **so the error cannot be detected**.