Name: Mansi Sodhani

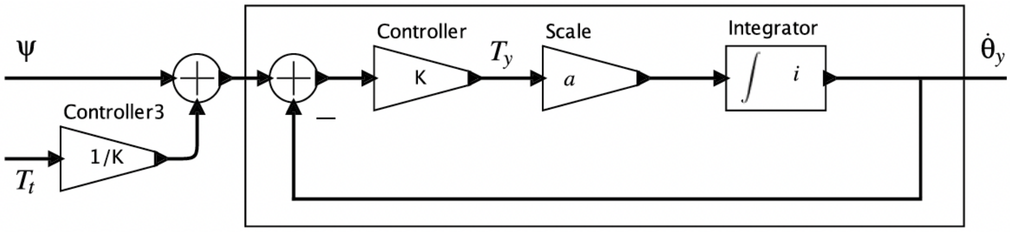
Roll No: 210591 (e.g. 170001) **Dept.:** CSE (e.g. CSE)

Indian Institute of Technology Kanpur
CS637 Embedded and Cyber-Physical Systems
Homework Assignment 1
Deadline: September 13, 2024

Total: **60 marks**

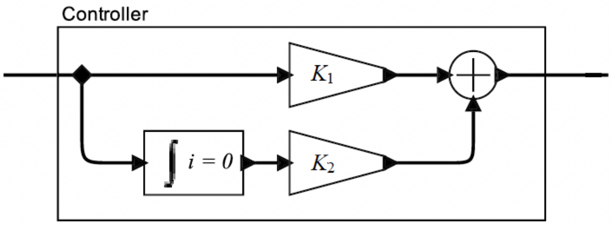
Problem 1. (20 points)

(a) Construct a Simulink model of the helicopter control system shown in the figure below.



Choose some reasonable parameters and plot the actual angular velocity as a function of time, assuming that the desired angular velocity is zero, $\psi(t) = 0$, and that the top-rotor torque is non-zero, $T_t(t) = bu(t)$. Give your plot for three different values of K and discuss how the behavior varies with K.

(b) Modify the model of part (a) to replace the Controller (the simple scale-by-K actor) with the alternative controller shown in the figure below.



This alternative controller is called a proportional-integrator (PI) controller. It has two parameter K_1 and K_2. Experiment with the values of these parameters, give some plots of the behavior with the same inputs as in part (a), and discuss the behavior of this controller in contrast to the one of part (a).

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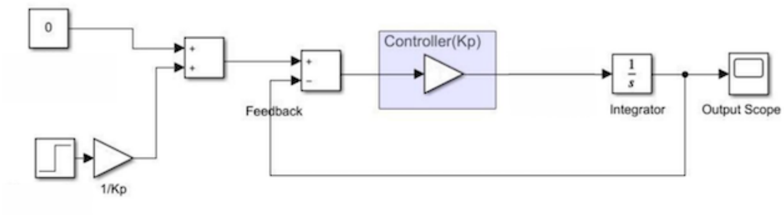
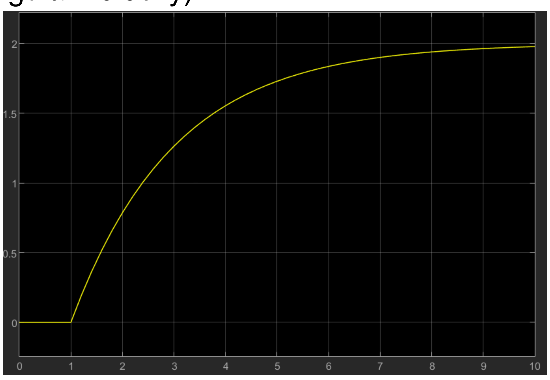
Sol 1 (a)

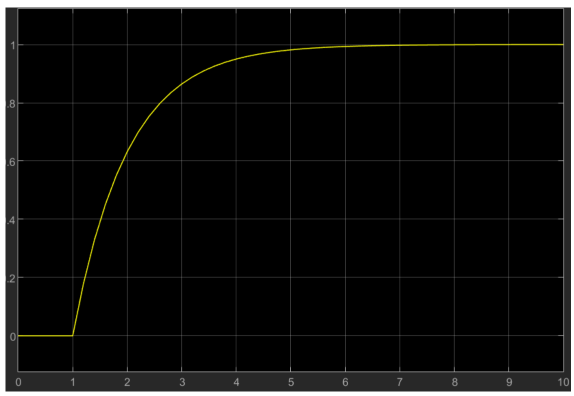


Reference Input (desired angular velocity) = 0, Top Rotor Torque = 1u(t), Kp = Proportional Gain

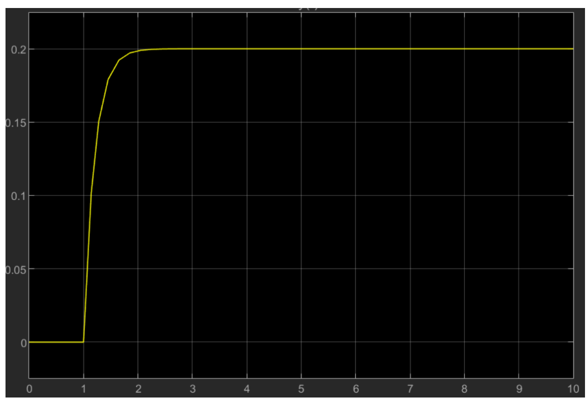
For K = 0.5 (x axis : time, y axis : angular velocity)



For K = 1(x axis : time, y axis : angular velocity)



For K = 5 (x axis : time, y axis : angular velocity)



Observation : As Kp increases, the convergence rate of error increases and the rise time decreases.

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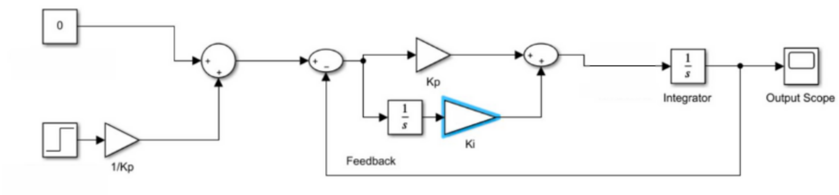
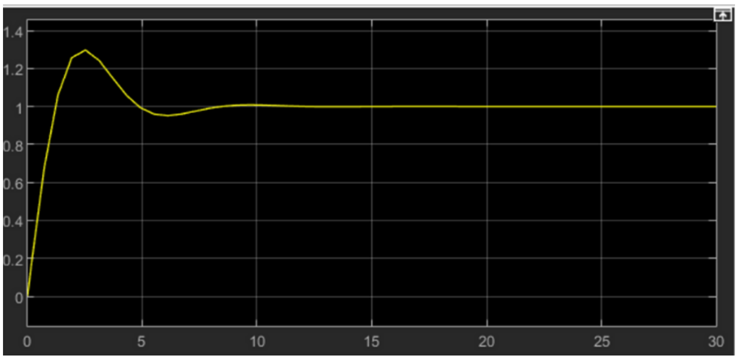
(b) PI controller

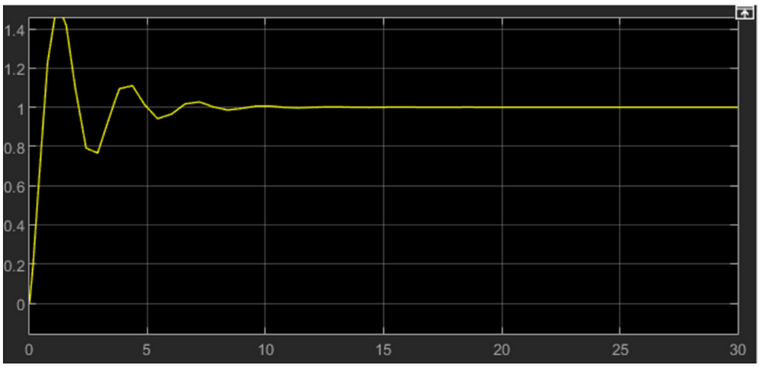


Reference Input = 0, Top Rotor Torque = 1u(t), Kp = Proportional Gain, Ki = Integral Gain

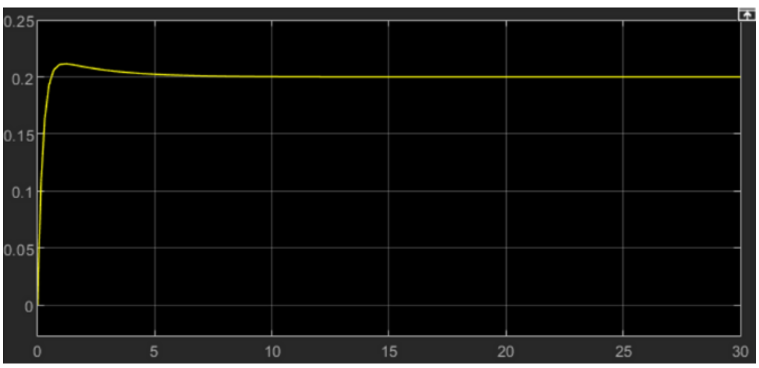
For Kp = 1, Ki = 1(x axis : time, y axis : angular velocity)



For Kp = 1, Ki = 5(x axis : time, y axis : angular velocity)



For Kp = 5, Ki = 2(x axis : time, y axis : angular velocity)



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Roll No: 210591 (e.g. 170001) Dept.: CSE (e.g. CSE)

Observations :

On increasing Kp :
1. Rise time decreases
2. Overshoot decreases

On increasing Ki :
1. Rise time increases
2. Convergence rate of error increases
3. Oscillatory component decreases because of increase in order of transfer function from 1 to 2

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Problem 2. (10 points)

Consider the linearized model of a vehicle steering system given in Example 6.4 and its observer-based controller in Example 7.3 in [AM09]. The states of the linearized model of a vehicle steering system represent the lateral deviation of the vehicle from the x-axis and the angle between the vehicle axis and the x-axis. The output of the linearized model is only the first state. Construct a Simulink model for the vehicle steering system with its controller that includes an observer. Apply a sinusoidal signal as the reference trajectory that specifies the desired deviation of the vehicle from the x-axis with time. Plot the output (lateral deviation of the vehicle from the x-axis) with time.

[AM09] K. J. Astrom and R. M. Murray. Feedback Systems: An Introduction for Scientists and Engineers. Princeton University Press, 2009.
http://www.cds.caltech.edu/ murray/books/AM08/pdf/am08-complete_22Feb09.pdf .

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Sol 2

In the modelling of a vehicle steering system with an observer, we have parameters: A,B,C,L,u,Kr and K. To ensure a stable solution, the parameters K and L must be selected such that the matrices A–BK and A–LC have negative eigenvalues. This condition allows us to determine appropriate values for the matrix K. Since Kr and K are interrelated, one can be derived from the other when evaluating the system at equilibrium in steady-state conditions.

The values of parameters chosen are :

A = [0 , 1 ; 0 , 0]

B = [0.45 ; 1]

c = [1, 0]

C = [1 , 0]

e = [-1.5403 ; -0.2597] (e is the column vector containing eigenvalues of required matrix)

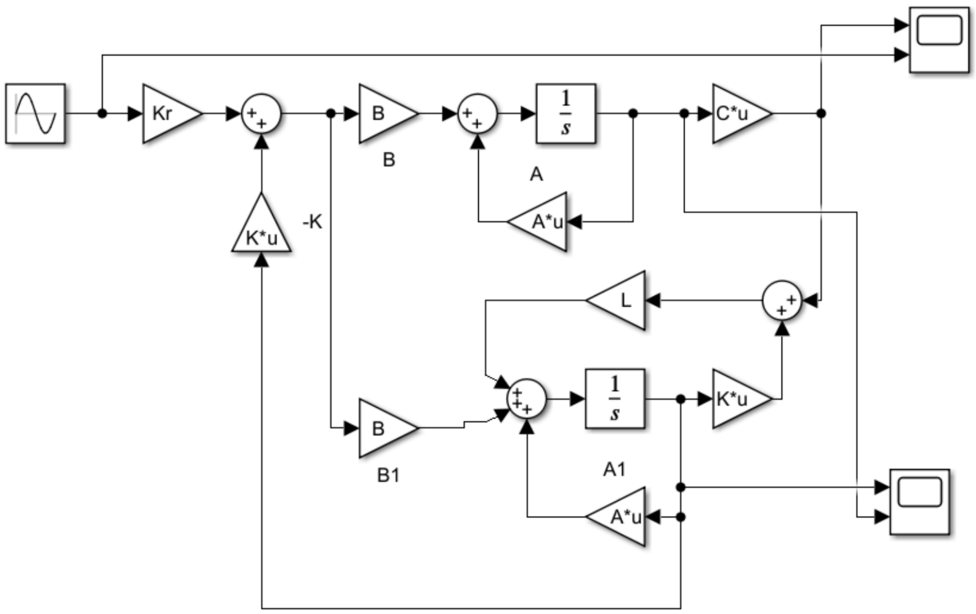
K = [1.5 , 5.5]

Kr = 1.5

L = [1.8 ; 0.4]

r = 1

y = 0.45



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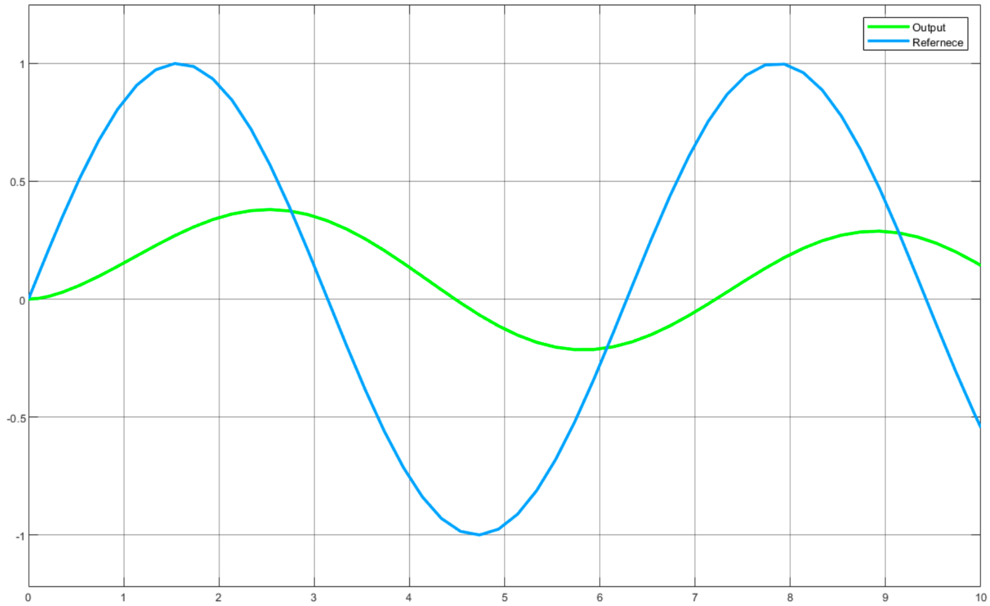
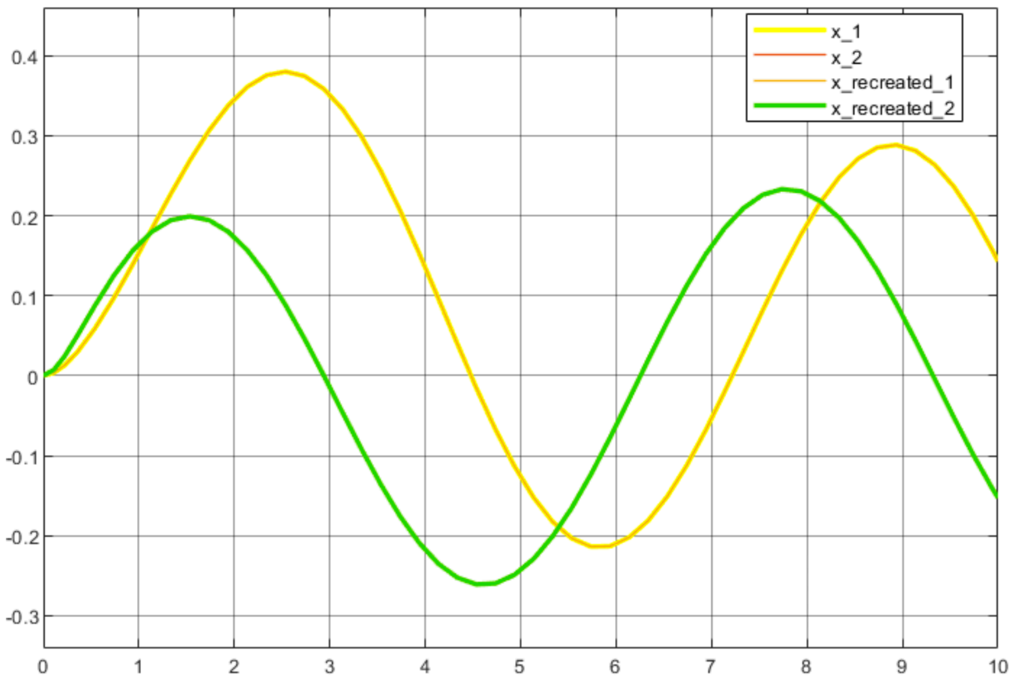
Plots obtained for the model

1. Plot of y vs time (compared with reference signal)
   Desired trajectory - blue and Lateral displacement of vehicle from x-axis - green



2. Plot of recreated state vs time (compared with reference signal)
   In this plot the actual states of the model and states recreated by the observer are super-imposing on one another.



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Model behaviour with respect to values of parameters :

1. Feedback Gain Matrix (K):

• The matrix K controls how much feedback is applied to the system. For the system to be stable, the matrix A−BK must have negative eigenvalues, which means the system will settle down over time.

• If K is larger, the system will respond faster, meaning it will reach its desired state more quickly. But if K is too large, the system might overshoot or become unstable. The key is finding a balance where the system responds quickly but remains stable.

2. Observer Gain Matrix (L):

• The matrix L helps the observer estimate the system's internal states using available measurements. To ensure stability, the matrix A−LC must also have negative eigenvalues, so the observer can estimate the states accurately.

• A larger L will help the observer estimate the states faster, allowing the controller to react more quickly. However, if L is too large, it can make the system more sensitive to noise and errors, which could lead to instability.

3. Reference Gain (Kr):

• Kr helps ensure that the system tracks the desired reference value without any steady-state error.

• Tuning Kr properly helps the system follow the reference input closely. This tuning works alongside the feedback from the matrix K.

4. System Dynamics (A, B, and C matrices):

• These matrices define how the system behaves in response to control inputs. They represent the system's natural tendencies before any feedback is applied.

• Changing the values in these matrices alters how the system behaves. For example, increasing values in B makes the system respond more to control inputs, while changes in C affect what is measured and passed to the observer.

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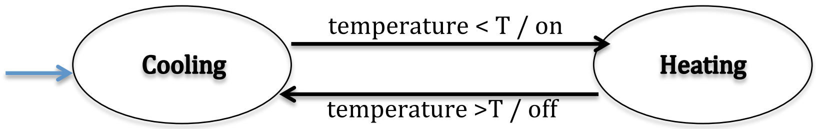
Problem 3. (10 points)

Consider the following model for a thermostat system.



The thermostat has been designed to maintain the temperature of a room at T°C. The model has two states: *cooling* and *heating*. When the system is in the *cooling* state and the temperature of the room goes below T°C, the system generates a signal to switch on a heater and moves to the *heating* state. When the temperature of the room goes over T°C, the system generates a signal to switch off the heater and moves to the *cooling* state.
(a) Represent the system as an actor that takes the current temperature as input and produces a signal to control the heater. The actor uses the set point T as a parameter.
(b) Identify a design problem in the model.
(c) Provide two different remedies to address the problem — one event triggered and one time triggered.
(d) Compare your proposed two solutions in terms of ease of implementation and guaranties on the system behavior.
(e) Provide a timed automaton model of your time-triggered solution.

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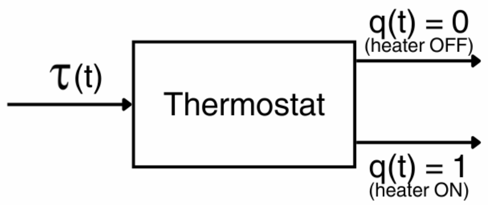
(a) q(t) = 1 if $\tau(t) < T$ (heater ON)

q(t) = 0 if $\tau(t) > T$ (heater OFF)



(b) Design problem in this model is : "Chattering" i.e where the heater rapidly switches on and off when the temperature hovers near the set-point.

(c) Hysteresis Strategy (Event Triggered) : In this approach, instead of turning the heater on or off exactly when the temperature reaches the set point T, we introduce a buffer zone with two thresholds: a higher threshold Th (above the set point) and a lower threshold Tl (below the set point). The heater will only switch on when the temperature falls below Tl and will switch off when the temperature goes above Th.

Dwell Time Strategy (Time Triggered) : In this solution, instead of changing the heater's state (on/off) instantly when the temperature crosses the set point, we introduce a mandatory waiting period, called the dwell time. After the heater turns on or off, it must remain in that state for a certain amount of time before it can switch again, regardless of the temperature fluctuations. This adds a timed delay to avoid rapid changes.

(d) 1. Ease of implementation :

Dwell Time : This solution is more complex to put in place compared to hysteresis. It requires adding a timing mechanism, which might also mean using additional sensors or timers.

Hysteresis : This approach is fairly easy to set up. It only requires basic logic to define the upper and lower temperature limits.

2. Guarantee on system behaviour :

Dwell Time : It is more stable and provides precise control over sequence of event (needs precise timing to avoid delays or performance issues)

Hysteresis : It reduces unnecessary switching but It may lead to steady state error, especially if the hysteresis band is too wide. It is not suitable for systems which require fine tuned control or precise set point tracking.

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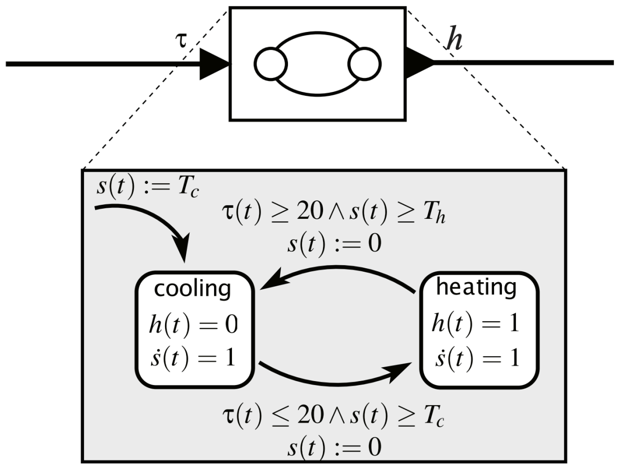
(e) Following is the schematics of timed automata model of thermostat, with a single temperature threshold, 20°C, and minimum times Tc and Th in each mode.



- s(t) is the continuous variable (clock) to measure time, increasing linearly.
- Th is the minimum time the heater stays on after it's turned on, ensuring it doesn't turn off too quickly.
- Tc is the minimum time the heater stays off once it's turned off, ensuring it doesn't turn back on too soon.
- The system starts in cooling mode, and if the temperature drops to 20°C, the heater can turn on if enough time Tc has been passed. The clock s resets.
- Once the heater turns on, it stays on for at least a set time Th, after which it switches back to cooling mode. The clock s resets again.

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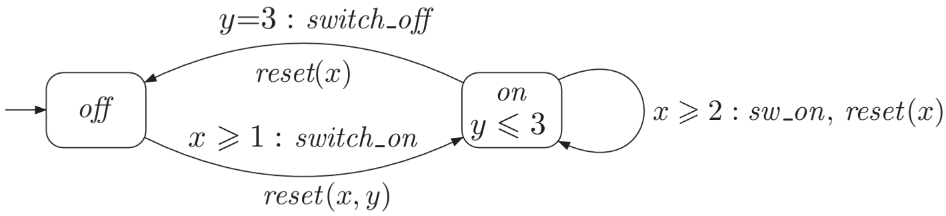
Problem 4. (10 points)

Consider the timed automaton LightSwitch illustrated below:



(a) Determine the transition system TS(LightSwitch).

(b) Check whether LightSwitch is timelock-free and non-zeno.

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Sol 4

(a) The transition system (TS) for the LightSwitch automaton can be described as a set of states and transitions:

States :
• sw_off : This is the initial state, where the light is off.
• sw_on : This state represents when the light is turned on.

Transitions :
• Off to On : When the system is in the "off" state, and the clock variable x reaches or exceeds 1, the system will transition to the "on" state, perform the sw_on action, and reset x to 0.
1. Condition: $x \geq 1$
2. Action: sw_on
3. Reset : x

• On to Off : When the system is in the "on" state, and the clock variable y reaches or exceeds 3, the system will transition to the "off" state, perform the sw_off action, and reset y to 0.
1. Condition: $y \geq 3$
2. Action: sw_off
3. Reset : y

• On to On : When the system is in the "on" state, and the clock variable x exceeds 2, the system will transition to the "on" state, perform the sw_on action, and reset x to 0.
1. Condition: $x > 2$
2. Action: sw_on
3. Reset : x

(b)

Timelock - free :
A timed automaton is timelock - free if it never reaches a state where no further transitions are possible due to time constraints. In the LightSwitch automaton, there are always conditions that allow transitions between states. For example, in the "off" state, the system will transition to "on" when $x \geq 1$. Similarly, when in the "on" state, the system will either transition to "off" when $y \geq 3$, or stay in the "on" state if $x > 2$. Therefore, the automaton is timelock - free.

Non - zeno :
A timed automaton is non - zeno if it doesn't allow an infinite number of transitions in a finite period. The conditions $x \geq 1$ and $y \geq 3$ enforce a minimum time delay between transitions, preventing the system from making an infinite number of transitions within a finite time. This makes the automaton non - zeno.

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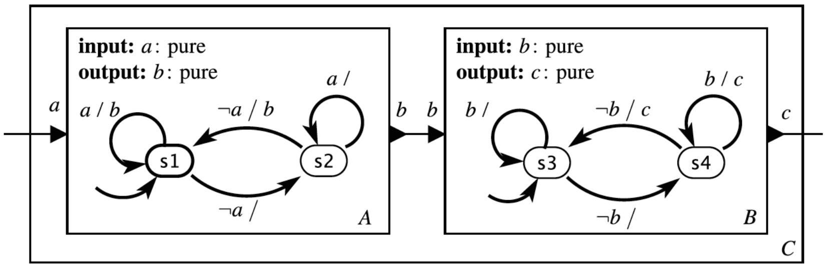
Problem 5. (10 points)

Consider the following synchronous composition of two state machines A and B.



Construct a single state machine C representing the composition. Which states of the composition are unreachable?

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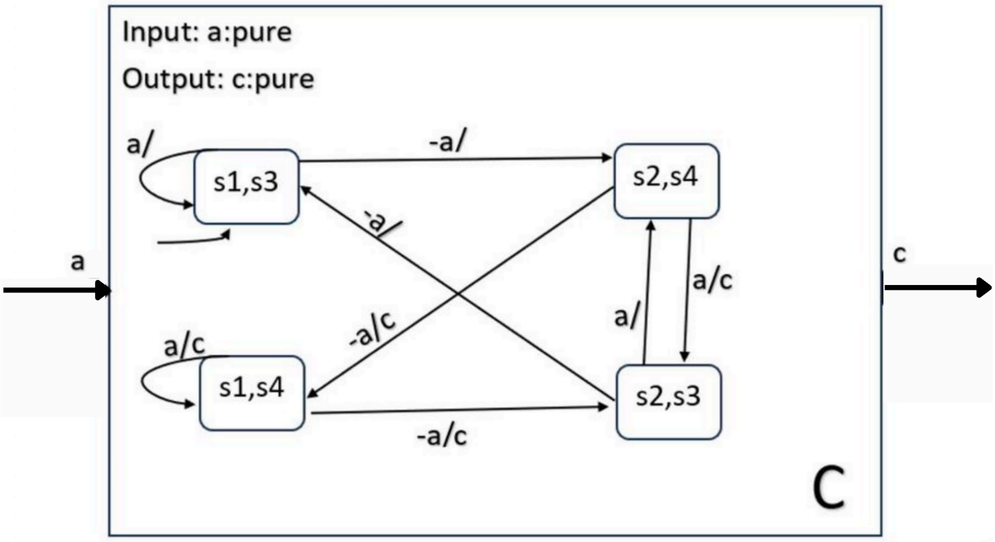
Sol 5

State C = State A x State B = {s1, s2} x {s3, s4}
= {s1, s2}, {s1, s4}, {s2, s3}, {s2, s4}

Initial State of C = (Initial State of A, Initial State of B) where Initial State of B = Output of A

As we can see, all the states are reachable



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