Quantum assignment 2

CS 682

March 2024

1 Task 1

Define a function $f : \{0,\ 1\}^3 \to \{0,\ 1\}$ by $f(x,\ y,\ z) = x \oplus y \oplus z'$. Your task is to create a quantum circuit that implements the Deutsch-Josza algorithm to determine whether the function f is balanced or constant.

i. Remember to provide a detailed explanation of each step in your quantum circuit, i.e., the initial state, all the operations on the quantum state, the evaluation of function f as a black box and the final measurement. (20)

ii. Then, analyze the measurement results to determine whether the function f is constant or balanced. (5)

2 Task 2

i. Design an efficient quantum circuit for half-adder which takes inputs x and y and produces the outputs $x \wedge y$ (logical AND) and $x \oplus y$ (logical XOR). (5)

ii. Using the previous circuit, create a quantum circuit for the operation

$$(x,\ y,\ z) \to (x,\ t,\ x \oplus y \oplus z)$$

where $t = ((x \oplus y) \wedge z) \vee (x \wedge y)$. (15)

iii. Using the above two circuits, create a quantum circuit for adding two numbers a_1a_0 and b_1b_0. (15)

To answer the above questions, you can use the ancilla bits.