DIFFERENTIAL EQUATIONS

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ASSIGNMENT 3

Predicting the Age of the Earth using Uranium Dating

Objective

The aim of this project is to estimate the age of the Earth by using **Uranium Dating**.
This scientific method depends on the **radioactive decay** of uranium isotopes (U-238 and U-235) into their stable lead products (Pb-206 and Pb-207).By calculating the ratios of these isotopes, we can infer how much time has passed since the rocks (and Earth) first formed.

Scientific Principle

Radioactive Decay

Radioactive elements decay over time into stable isotopes. The decay follows an exponential law:

$$N(t) = N_o * e^{(-\lambda t)}$$

Where:

- N(t): remaining quantity of the parent isotope at time t,

- N_o: initial quantity,

- λ: decay constant,

- t: time elapsed.

Half-life

The half-life is the time it takes for half of the radioactive substance to decay. It is related to the decay constant by:

$$\lambda = \ln(2) / \text{half-life}$$

Code Explanation

Importing Libraries

numpy: For numerical calculations.

matplotlib.pyplot: For plotting graphs.

Defining Constants

Half-lives of uranium isotopes:

- U-238: 4.468 billion years
- U-235: 0.704 billion years

Decay constants are calculated from the half-lives.

Simulating Radioactive Decay Over Time

Time is simulated from **0 to 5 billion years**.The decay of U-238 and U-235 is modeled using the exponential decay formula.

Plotting the Decay Curves

Plotted graphs showing the decay of U-238 and U-235 over billions of years.Helps visualize how much parent isotope remains over time.

Age Estimation Using Pb/U Ratios

Example ratios:

- Pb-206/U-238 = 0.85
- Pb-207/U-235 = 1.65

Rearranging the decay formula, the estimated ages are:

- **U-238:** ~4.31 billion years
- **U-235:** ~4.55 billion years

Basic Error Analysis

Assuming ±5% uncertainty in ratio measurements:

- **U-238 Age Range:** 4.13 – 4.48 billion years
- **U-235 Age Range:** 4.38 – 4.73 billion years

Results

Isotope Reaction	Pb/U Ratio	Estimated Age (by decay law)	Uncertainty Range (±5%)
$^{238}U \rightarrow {}^{206}Pb$	0.85	~4.31 billion years	4.13 – 4.48 billion years
$^{235}U \rightarrow {}^{207}Pb$	1.65	~4.55 billion years	4.38 – 4.73 billion years

Conclusion

Using uranium dating, the **age of the Earth** is estimated to lie between **4.3 and 4.6 billion years**, consistent with modern scientific findings.The use of two independent uranium decay chains (U-238 and U-235) enhances the reliability of the estimate.Basic error analysis further improves confidence in the range.

Future Scope

- Incorporate other radiometric dating methods (e.g., **Thorium-Lead**, **Potassium-Argon**).
- Analyze **real geological data** from meteorites or Earth's oldest rocks for even greater precision.