Portfolio Efficient Frontier Optimization: A Pioneering Approach Using Monte Carlo Simulation and Non-linear Programming

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1 Introduction

This report outlines a Portfolio Optimization model that employs Monte Carlo Simulation and Quadratic Programming to determine the optimal asset allocation for a given set of financial instruments. The model is implemented in Python, leveraging libraries such as NumPy, pandas, SciPy, Matplotlib, and finance. The model successfully identifies the optimal portfolio by maximizing the Sharpe Ratio, providing valuable insights for investment strategies. The efficient frontier is also plotted to visualize the set of optimal portfolios.

2 Parameters and Variables

- **Tickers**: List of asset symbols.
- **Start_Date, End_Date**: Time range for historical data.
- **Historical_Returns**: Adjusted close prices.
- **Log_Ret**: Logarithmic returns.
- **Weights**: Asset allocation in the portfolio.

3 Simulation Module

The Monte Carlo Simulation is employed to generate a wide range of portfolios with varying asset allocations. The simulation runs for a user-defined number of iterations (*num_sims*) and calculates the following metrics for each portfolio:

$$\text{Portfolio Return} = \sum_i (\text{Log\_Ret}_i \times \text{Weight}_i) \times N$$

$$\text{Portfolio Volatility} = \sqrt{\text{Weight}^T \times (\text{Log\_Ret.cov()} \times N) \times \text{Weight}}$$

$$\text{Sharpe Ratio} = \frac{\text{Portfolio Return}}{\text{Portfolio Volatility}}$$

where N is the number of data points in the historical returns, serving as the annualization factor.

4 Optimization Model

The optimization model employs Sequential Least Squares Quadratic Programming (SLSQP) to maximize the Sharpe Ratio. The model is subject to the constraint that the sum of the asset weights must be equal to one. Mathematically, the optimization problem is formulated as:

$$\begin{aligned} \underset{\text{Weights}}{\text{maximize}} \quad & \frac{\text{Portfolio Return}}{\text{Portfolio Volatility}} \\ \text{subject to} \quad & \sum_i \text{Weight}_i = 1 \end{aligned}$$