MantaPay Protocol Specification
v0.4.0

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Abstract

MantaPay is an implementation of a *decentralized anonymous payment* scheme based on the MANTA$_{\text{DAP}}$ protocol outlined in the original MANTA whitepaper.

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1 Introduction

TODO: add introductory remarks

2 Notation

The following notation is used throughout this specification:

- Type is the type of types[1].
- If $x : T$ then x is a value and T is a type, denoted $T : \mathsf{Type}$, and we say that x *has type* T.
- Bool is the type of booleans with values True and False.
- For any types $A : \mathsf{Type}$ and $B : \mathsf{Type}$ we denote the *type of functions* from A to B as $A \to B : \mathsf{Type}$.
- For any types $A : \mathsf{Type}$ and $B : \mathsf{Type}$ we denote the *product type* over A and B as $A \times B : \mathsf{Type}$ with constructor $(-,-) : T \to (S \to T \times S)$.
- For any type $T : \mathsf{Type}$, we define $\mathsf{Option}(T) : \mathsf{Type}$ as the inductive type with constructors:

$$\begin{aligned} \mathsf{None} &: \mathsf{Option}(T) \\ \mathsf{Some} &: T \to \mathsf{Option}(T) \end{aligned}$$

- We denote the *type of distributions* over a type $T : \mathsf{Type}$ as $\mathfrak{D}(T) : \mathsf{Type}$. A value x sampled from $\mathfrak{D}(T)$ is denoted $x \sim \mathfrak{D}(T)$ and the fact that the value x belongs to the range of $\mathfrak{D}(T)$ is denoted $x \in \mathfrak{D}(T)$. So namely, $y \in \{x \mid x \sim \mathfrak{D}(T)\} \leftrightarrow y \in \mathfrak{D}(T)$.
- Depending on the context, the notation $|\cdot|$ denotes either the absolute value of a quantity, the length of a vector, the number of characters in a string, or the cardinality of a set.

3 Concepts

3.1 Assets

The Asset is the fundamental currency object in the $\mathsf{MantaPay}$ protocol. An asset $a : \mathsf{Asset}$ is a tuple

$$a = (a.\mathsf{id}, a.\mathsf{value}) : \mathsf{AssetId} \times \mathsf{AssetValue}$$

The $\mathsf{MantaPay}$ protocol is a *decentralized anonymous payment* scheme which facilitiates the private ownership and private transfer of Asset objects. The $\mathsf{AssetId}$ field encodes the type of currency being used, and the $\mathsf{AssetValue}$ encodes how many units of that currency are being used, in the standard base unit of that currency.

Whenever an Asset is being used in a public setting, we simply refer to it as an Asset, but when the $\mathsf{AssetId}$ and/or $\mathsf{AssetValue}$ of a particular Asset is meant to be hidden from public view, we refer to the Asset as either, *secret*, *private*, *hidden*, or *shielded*.

Assets form the basic units of *transactions* which consume Assets on input, transform them, and return Assets on output. To preserve the economic value stored in Assets, the sum of the input $\mathsf{AssetValues}$ must balance the sum of the output $\mathsf{AssetValues}$, and all assets in a single transaction must have the same $\mathsf{AssetId}$[2].

3.2 Addresses

In order for participants in the $\mathsf{MantaPay}$ protocol to send and receive Assets, they must create secret and public *addresses* according to an *address scheme*. For $\mathsf{MantaPay}$, the address scheme consists of a *spending key* sk, a *viewing key* vk, and a *receiving key* rk. The keys have the following uses/properties:

- Access to a receiving key rk represents the ability to send Assets to the owner of the associated sk.
- Access to a viewing key vk represents the ability to reveal shielded Asset information for Assets belonging to the owner of the associated sk.
- Access to a spending key sk represents the ability to spend Assets that were received under the associated receiving key rk.

See § 4.2 for more information on how these keys are constructed and used for spending, viewing, and receiving Assets.

[1] By *type of types*, we mean the type of *first-level* types in some family of type universes. Discussion of the type theory necessary to make these notions rigorous is beyond the scope of this paper.

[2] It is beyond the scope of this paper to discuss transactions with inputs and outputs that feature different $\mathsf{AssetIds}$, like those that would be featured in a *decentralized anonymous exchange*.

3.3 Ledger

Ensuring that Assets maintain their economic value is not only dependent on transactions preserving inputs and outputs, but also that Assets are not *double-spent*. The *double-spending problem* can be solved by using a public ledger[3] that keeps track of the flow of Assets from one participant to the other. Unfortunately, using a public ledger alone does not allow participants to remain anonymous, so MantaPay extends the public ledger by adding a special account called the ShieldedAssetPool. The ShieldedAssetPool is responsible for keeping track of the Assets which have been anonymized by the protocol.

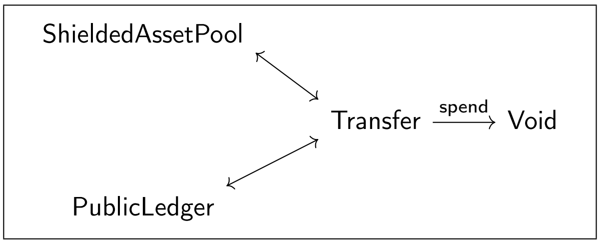

Figure 1: Lifecycle of an Asset.

Assets can be in one of three states, public (tracked by the PublicLedger), allocated (spendable subset of the ShieldedAssetPool), or spent (voided Assets). By way of the § 4.3 Transfer Protocol, Assets can be sent to and from the PublicLedger and the ShieldedAssetPool.

The ShieldedAssetPool is made up of four parts:

1. ShieldedAssetPool Balance: The MantaPay ledger contains a collection of Assets which represent the combined economic value of the ShieldedAssetPool and the PublicLedger. The ShieldedAssetPool Balance is the subset of this total value that has been anonymized by the MantaPay protocol.
2. § 3.3.1 UTXO Set: A collection of claims to subsets of the ShieldedAssetPool, each owned by participants of the MantaPay protocol.
3. § 3.3.2 EncryptedNotes: For each UTXO there is a matching EncryptedNote which contains information necessary to spend the Asset, which is commited in the UTXO, but can only be decrypted by the recipient of the Asset, specifically, the correct viewing key vk. See § 3.2 for more.
4. § 3.3.3 VoidNumber Set: A collection of commitments keeping track of those UTXOs which have participated in exactly one instance of the Transfer Protocol.

An Asset is in the public state if it belongs to the PublicLedger. An Asset is in the allocated state if a UTXO for the Asset is a member of the UTXO Set, but its matching VoidNumber is **not** in the VoidNumber Set. An Asset is in the spent state if it was allocated in the past, but its matching VoidNumber is now in the VoidNumber Set.

The operation of the different parts of the ShieldedAssetPool is elaborated in the following subsections.

3.3.1 UTXO Set

3.3.2 EncryptedNotes

3.3.3 VoidNumber Set

4 Abstract Protocol

4.1 Abstract Cryptographic Schemes

Definition 4.1.1. A *commitment scheme* COM is defined by the schema:

$$\begin{aligned}
\mathsf{Trapdoor} &: \mathsf{Type} \\
\mathsf{Input} &: \mathsf{Type} \\
\mathsf{Output} &: \mathsf{Type} \\
\mathsf{TrapdoorDistribution} &: \mathfrak{D}(\mathsf{Trapdoor}) \\
\mathsf{commit} &: \mathsf{Trapdoor} \times \mathsf{Input} \to \mathsf{Output}
\end{aligned}$$

with the following properties:

- **Binding**: It is infeasible to find an $x, y : \mathsf{Input}$ and $r, s : \mathsf{Trapdoor}$ such that $x \neq y$ and $\mathsf{commit}(r, x) = \mathsf{commit}(s, y)$.

[3] A public (or private) ledger is not enough to solve the *double-spending problem*. A *consensus mechanism* is also required to ensure that all participants agree on the current state of the ledger. The *consensus mechanism* that secures the MantaPay ledger is beyond the scope of this paper.

- **Hiding**: For all $x, y : \mathsf{Input}$, the distributions $\{\mathsf{commit}(r, x) \mid r \sim \mathsf{TrapdoorDistribution}\}$ and $\{\mathsf{commit}(r, y) \mid r \sim \mathsf{TrapdoorDistribution}\}$ are *computationally indistinguishable*.

Notation: For convenience we refer to $\mathsf{COM.commit}(r, x)$ by $\mathsf{COM}_r(x)$.

Definition 4.1.2. A *hash function* CRH is defined by the schema:

$$\begin{aligned} \mathsf{Input} &: \mathsf{Type} \\ \mathsf{Output} &: \mathsf{Type} \\ \mathsf{hash} &: \mathsf{Input} \to \mathsf{Output} \end{aligned}$$

with the following properties:

- **Pre-Image Resistance**: For a given $y : \mathsf{Output}$, it is infeasible to find $x : \mathsf{Input}$ such that $\mathsf{hash}(x) = y$.
- **Collision Resistance**: It is infeasible to find an $x_1, x_2 : \mathsf{Input}$ such that $x_1 \neq x_2$ and $\mathsf{hash}(x_1) = \mathsf{hash}(x_2)$.

Notation: For convenience we refer to $\mathsf{CRH.hash}(x)$ by $\mathsf{CRH}(x)$.

Definition 4.1.3. A *symmetric-key encryption scheme* SYM is defined by the schema:

$$\begin{aligned} \mathsf{Key} &: \mathsf{Type} \\ \mathsf{Plaintext} &: \mathsf{Type} \\ \mathsf{Ciphertext} &: \mathsf{Type} \\ \mathsf{encrypt} &: \mathsf{Key} \times \mathsf{Plaintext} \to \mathsf{Ciphertext} \\ \mathsf{decrypt} &: \mathsf{Key} \times \mathsf{Ciphertext} \to \mathsf{Option}(\mathsf{Plaintext}) \end{aligned}$$

with the following properties:

- **Validity**: For all keys $k : \mathsf{Key}$ and plaintexts $p : \mathsf{Plaintext}$, we have that

$$\mathsf{decrypt}(k, \mathsf{encrypt}(k, p)) = \mathsf{Some}(p)$$

- **TODO**: hiding, one-time encryption security?

Definition 4.1.4. A *key-agreement scheme* KA is defined by the schema:

$$\begin{aligned} \mathsf{PublicKey} &: \mathsf{Type} \\ \mathsf{SecretKey} &: \mathsf{Type} \\ \mathsf{SharedSecret} &: \mathsf{Type} \\ \mathsf{derive} &: \mathsf{SecretKey} \to \mathsf{PublicKey} \\ \mathsf{agree} &: \mathsf{SecretKey} \times \mathsf{PublicKey} \to \mathsf{SharedSecret} \end{aligned}$$

with the following properties:

- **Agreement**: For all $\mathsf{sk}_1, \mathsf{sk}_2 : \mathsf{SecretKey}$, $\mathsf{agree}(\mathsf{sk}_1, \mathsf{derive}(\mathsf{sk}_2)) = \mathsf{agree}(\mathsf{sk}_2, \mathsf{derive}(\mathsf{sk}_1))$
- **TODO**: security properties

Definition 4.1.5. A *key-derivation function* KDF defined over a symmetric-key encryption scheme SYM and a key-agreement scheme KA is a function of type:

$$\mathsf{KDF} : \mathsf{KA.SharedSecret} \to \mathsf{SYM.Key}$$

Definition 4.1.6. An *integrated encryption scheme* IES is a hybrid encryption scheme made of up a symmetric-key encryption scheme SYM, a key-agreement scheme KA, and a KDF to convert from KA.SharedSecret to SYM.Key. We can define the following encryption/decryption algorithms:

- Encryption: Given a secret key $\mathsf{sk} : \mathsf{KA.SecretKey}$, a public key $\mathsf{pk} : \mathsf{KA.PublicKey}$, and plaintext $p : \mathsf{SYM.Plaintext}$, we produce the pair

$$m := (\mathsf{KA.derive}(\mathsf{sk}), \mathsf{SYM.encrypt}(\mathsf{KDF}(\mathsf{KA.agree}(\mathsf{sk}, \mathsf{pk})), p)) : \mathsf{KA.PublicKey} \times \mathsf{SYM.Ciphertext}$$

- Decryption: Given a secret key $\mathsf{sk} : \mathsf{KA.SecretKey}$, and an encrypted message, as above, $m := (\mathsf{pk}, c) : \mathsf{KA.PublicKey} \times \mathsf{SYM.Ciphertext}$, we can decrypt m, producing the plaintext,

$$p := \mathsf{SYM.decrypt}(\mathsf{KDF}(\mathsf{KA.agree}(\mathsf{sk}, \mathsf{pk})), c) : \mathsf{Option}(\mathsf{SYM.Plaintext})$$

which should decrypt successfully if the $\mathsf{KA.PublicKey}$ that m was encrypted with is the derived key of $\mathsf{sk} : \mathsf{KA.SecretKey}$.

Notation: We denote the above *encrypted message* type as $\mathsf{Message} := \mathsf{KA.PublicKey} \times \mathsf{SYM.Ciphertext}$, and the above two algorithms by

$$\begin{aligned} \mathsf{encrypt} &: \mathsf{KA.SecretKey} \times \mathsf{KA.PublicKey} \times \mathsf{SYM.Plaintext} \to \mathsf{Message} \\ \mathsf{decrypt} &: \mathsf{KA.SecretKey} \times \mathsf{SYM.Ciphertext} \to \mathsf{Option}(\mathsf{SYM.Plaintext}) \end{aligned}$$

TODO: security properties, combine with SYM and KA properties, like the fact that some of these keys should be ephemeral, etc.
TODO: add explicit message authentication

Definition 4.1.7. A *key-diversification scheme* KDIV over a key-agreement scheme KA is defined by the schema:

$$\begin{aligned} \mathsf{public} &: \mathsf{KA.SecretKey} \times \mathsf{KA.PublicKey} \times \mathsf{KA.PublicKey} \to \mathsf{KA.PublicKey} \\ \mathsf{secret} &: \mathsf{KA.PublicKey} \times \mathsf{KA.SecretKey} \times \mathsf{KA.SecretKey} \to \mathsf{KA.SecretKey} \end{aligned}$$

Notation: We refer to the first argument to a KDIV function as the *diversifier* and we write it as a subscript

$$\mathsf{public}_d(x, y) := \mathsf{public}(d, x, y) \text{ and } \mathsf{secret}_d(x, y) := \mathsf{secret}(d, x, y)$$

For convenience we also write KDIV_d to mean $\mathsf{KDIV.public}_d$ or $\mathsf{KDIV.secret}_d$, when the context is clear.

Every KDIV also has the following properties:

- **Derivation Invariance**: For any diversifier $d : \mathsf{KA.SecretKey}$ and pair of secret keys $(\mathsf{sk}_1, \mathsf{sk}_2)$ we have

$$\mathsf{KDIV}_d(\mathsf{KA.derive}(\mathsf{sk}_1), \mathsf{KA.derive}(\mathsf{sk}_2)) = \mathsf{KA.derive}(\mathsf{KDIV}_{\mathsf{KA.derive}(d)}(\mathsf{sk}_1, \mathsf{sk}_2))$$

- **TODO**: security properties?

Definition 4.1.8. A *non-interactive zero-knowledge proving system* ZKPS is defined by the schema:

$$\begin{aligned} \mathsf{Statement} &: \mathsf{Type} \\ \mathsf{ProvingKey} &: \mathsf{Type} \\ \mathsf{VerifyingKey} &: \mathsf{Type} \\ \mathsf{PublicInput} &: \mathsf{Type} \\ \mathsf{SecretInput} &: \mathsf{Type} \\ \mathsf{Proof} &: \mathsf{Type} \\ \mathsf{keys} &: \mathsf{Statement} \to \mathfrak{D}(\mathsf{ProvingKey} \times \mathsf{VerifyingKey}) \\ \mathsf{prove} &: \mathsf{Statement} \times \mathsf{ProvingKey} \times \mathsf{PublicInput} \times \mathsf{SecretInput} \to \mathfrak{D}(\mathsf{Option}(\mathsf{Proof})) \\ \mathsf{verify} &: \mathsf{VerifyingKey} \times \mathsf{PublicInput} \times \mathsf{Proof} \to \mathsf{Bool} \end{aligned}$$

Notation: We use the following notation for a ZKPS:

- We write the Statement and ProvingKey arguments of prove in the superscript and subscript respectively,

$$\mathsf{prove}^P_{\mathsf{pk}}(x, w) := \mathsf{prove}(P, \mathsf{pk}, x, w)$$

- We write the VerifyingKey argument of verify in the subscript,

$$\mathsf{verify}_{\mathsf{vk}}(x, \pi) := \mathsf{verify}(\mathsf{vk}, x, \pi)$$

- We say that $(x, w) : \mathsf{PublicInput} \times \mathsf{SecretInput}$ has the property of being a satisfying input whenever

$$\mathsf{satisfying}(x, w) := \exists \pi : \mathsf{Proof}, \mathsf{Some}(\pi) \in \mathsf{prove}^P_{\mathsf{pk}}(x, w)$$

Every ZKPS has the following properties for a fixed statement $P : \mathsf{Statement}$ and keys $(\mathsf{pk}, \mathsf{vk}) \sim \mathsf{keys}(P)$:

- **Completeness**: For all $(x, w) : \mathsf{PublicInput} \times \mathsf{SecretInput}$, if there exists a proof $\pi : \mathsf{Proof}$, such that $\mathsf{Some}(\pi) \in \mathsf{prove}^P_{\mathsf{pk}}(x, w)$, then $\mathsf{verify}_{\mathsf{vk}}(x, \pi) = \mathsf{True}$.
- **Knowledge Soundness**: For any polynomial-size adversary $\mathcal{A}$,

$$\mathcal{A} : \mathsf{ProvingKey} \times \mathsf{VerifyingKey} \to \mathfrak{D}(\mathsf{PublicInput} \times \mathsf{Proof})$$

there exists a polynomial-size extractor $\mathcal{E}_{\mathcal{A}}$

$$\mathcal{E}_{\mathcal{A}} : \mathsf{ProvingKey} \times \mathsf{VerifyingKey} \to \mathfrak{D}(\mathsf{SecretInput})$$

such that the following probability is negligible:

$$\Pr\left[\begin{array}{c} \mathsf{satisfying}(x, w) = \mathsf{False} \\ \mathsf{verify}_{\mathsf{vk}}(x, w) = \mathsf{True} \end{array} \middle| \begin{array}{l} (\mathsf{pk}, \mathsf{vk}) \sim \mathsf{keys}(P) \\ (x, \pi) \sim \mathcal{A}(\mathsf{pk}, \mathsf{vk}) \\ w \sim \mathcal{E}_{\mathcal{A}}(\mathsf{pk}, \mathsf{vk}) \end{array}\right]$$

- **Statistical Zero-Knowledge**: There exists a stateful simulator $\mathcal{S}$, such that for all stateful distinguishers $\mathcal{D}$, the difference between the following two probabilities is negligible:

$$\Pr\left[\begin{array}{c} \mathsf{satisfying}(x, w) = \mathsf{True} \\ \mathcal{D}(\pi) = \mathsf{True} \end{array} \middle| \begin{array}{l} (\mathsf{pk}, \mathsf{vk}) \sim \mathsf{keys}(P) \\ (x, w) \sim \mathcal{D}(\mathsf{pk}, \mathsf{vk}) \\ \mathsf{Some}(\pi) \sim \mathsf{prove}^P_{\mathsf{pk}}(x, w) \end{array}\right] \text{ and } \Pr\left[\begin{array}{c} \mathsf{satisfying}(x, w) = \mathsf{True} \\ \mathcal{D}(\pi) = \mathsf{True} \end{array} \middle| \begin{array}{l} (\mathsf{pk}, \mathsf{vk}) \sim \mathcal{S}(P) \\ (x, w) \sim \mathcal{D}(\mathsf{pk}, \mathsf{vk}) \\ \pi \sim \mathcal{S}(x) \end{array}\right]$$

- **Succinctness**: For all $(x, w) : \mathsf{PublicInput} \times \mathsf{SecretInput}$, if $\mathsf{prove}(P, \mathsf{pk}, x, w) = \mathsf{Some}(\pi)$, then $|\pi| = \mathcal{O}(1)$, and $\mathsf{verify}(\mathsf{vk}, x, \pi)$ runs in time $\mathcal{O}(|x|)$.

4.2 Addresses and Key Components

Given a choice of IES for the Transfer protocol we have the following definitions:

Definition 4.2.1. A SpendingKey is the following pair of keys:

$$\begin{aligned} \mathsf{view} &: \mathsf{IES.KA.SecretKey} \\ \mathsf{spend} &: \mathsf{IES.KA.SecretKey} \end{aligned}$$

The first secret key is called the ViewingKey.

Definition 4.2.2. A ReceivingKey is the following pair of keys:

$$\begin{aligned} \mathsf{view} &: \mathsf{IES.KA.PublicKey} \\ \mathsf{spend} &: \mathsf{IES.KA.PublicKey} \end{aligned}$$

which is derived from a spending key $\mathsf{sk} : \mathsf{SpendingKey}$ by deriving each component:

$$\begin{aligned} \mathsf{rk.view} &:= \mathsf{KA.derive}(\mathsf{sk.view}) \\ \mathsf{rk.spend} &:= \mathsf{KA.derive}(\mathsf{sk.spend}) \end{aligned}$$

4.3 Transfer Protocol

Definition 4.3.1. A Sender is the following tuple:

$$\begin{aligned} \mathsf{sk} &: \mathsf{SpendingKey} \\ \tilde{d} &: \mathsf{IES.KA.PublicKey} \\ \mathsf{trapdoor} &: \mathsf{IES.KA.PublicKey} \\ \mathsf{asset} &: \mathsf{Asset} \\ \mathsf{cm} &: \mathsf{UTXO} \\ \mathsf{cm}_{\mathsf{root}} &: \mathsf{MerkleTree.Root} \\ \mathsf{cm}_{\mathsf{path}} &: \mathsf{MerkleTree.Path} \\ \mathsf{vn} &: \mathsf{VoidNumber} \end{aligned}$$

A Sender is constructed from a spending key $\mathsf{sk} : \mathsf{SpendingKey}$ and an encrypted message $\mathsf{note} : \mathsf{EncryptedNote}$ by the following algorithm:

$$
\begin{aligned}
S.\mathsf{sk} &:= \mathsf{sk} \\
\tilde{d}, c &:= \mathsf{note} \\
\mathsf{Some}(\mathsf{asset}) &:= \mathsf{IES.decrypt}(S.\mathsf{sk.view}, c) \\
S.\mathsf{asset} &:= \mathsf{asset} \\
S.\tilde{d} &:= \tilde{d} \\
S.\mathsf{trapdoor} &:= \mathsf{KA.derive}(\mathsf{KDIV}_{\tilde{d}}(S.\mathsf{sk.view}, S.\mathsf{sk.spend})) \\
S.\mathsf{cm} &:= \mathsf{COM}_{S.\mathsf{trapdoor}}(S.\mathsf{asset}) \\
\mathsf{Some}(\mathsf{cm}_{\mathsf{root}}, \mathsf{cm}_{\mathsf{path}}) &:= \mathsf{MerkleTree.contains}(S.\mathsf{cm}) \\
S.\mathsf{cm}_{\mathsf{root}} &:= \mathsf{cm}_{\mathsf{root}} \\
S.\mathsf{cm}_{\mathsf{path}} &:= \mathsf{cm}_{\mathsf{path}} \\
S.\mathsf{vn} &:= \mathsf{COM}_{S.\tilde{d}}(S.\mathsf{sk.view} \,||\, S.\mathsf{sk.spend})
\end{aligned}
$$

Definition 4.3.2. A $\mathsf{SenderPost}$ is the following tuple:

$$
\begin{aligned}
\mathsf{cm}_{\mathsf{root}} &: \mathsf{MerkleTree.Root} \\
\mathsf{vn} &: \mathsf{VoidNumber}
\end{aligned}
$$

Definition 4.3.3. A $\mathsf{Receiver}$ is the following tuple:

$$
\begin{aligned}
\mathsf{rk} &: \mathsf{ReceivingKey} \\
d &: \mathsf{IES.KA.SecretKey} \\
\mathsf{trapdoor} &: \mathsf{IES.KA.PublicKey} \\
\mathsf{asset} &: \mathsf{Asset} \\
\mathsf{cm} &: \mathsf{UTXO} \\
\mathsf{note} &: \mathsf{EncryptedNote}
\end{aligned}
$$

A $\mathsf{Receiver}$ is constructed from a receving key $\mathsf{rk} : \mathsf{ReceivingKey}$, an asset $\mathsf{asset} : \mathsf{Asset}$, and a chosen diversifier $d : \mathsf{IES.KA.SecretKey}$ by the following algorithm:

$$
\begin{aligned}
R.\mathsf{rk} &:= \mathsf{rk} \\
R.d &:= d \\
R.\mathsf{trapdoor} &:= \mathsf{KDIV}_{R.d}(R.\mathsf{rk.view}, R.\mathsf{rk.spend}) \\
R.\mathsf{asset} &:= \mathsf{asset} \\
R.\mathsf{cm} &:= \mathsf{COM}_{R.\mathsf{trapdoor}}(R.\mathsf{asset}) \\
R.\mathsf{note} &:= \mathsf{IES.encrypt}(R.d, R.\mathsf{rk.view}, \mathsf{asset})
\end{aligned}
$$

Definition 4.3.4. A $\mathsf{ReceiverPost}$ is the following tuple:

$$
\begin{aligned}
\mathsf{cm} &: \mathsf{UTXO} \\
\mathsf{note} &: \mathsf{EncryptedNote}
\end{aligned}
$$

Definition 4.3.5. A $\mathsf{Transfer}$ is a 4-tuple of finite sets $(T_{\mathsf{source}}, T_{\mathsf{sender}}, T_{\mathsf{receiver}}, T_{\mathsf{sink}})$ made up of Source, Sender, $\mathsf{Receiver}$, and Sink tuples respectively. The *shape* of a $\mathsf{Transfer}$ is the 4-tuple of cardinalities of those sets

$$
\left(|T_{\mathsf{source}}|, |T_{\mathsf{sender}}|, |T_{\mathsf{receiver}}|, |T_{\mathsf{sink}}|\right)
$$

In order for a $\mathsf{Transfer}$ to be considered *valid*, it must adhere to the following constraints:

- **Same Id**: All the $\mathsf{AssetId}$s in the $\mathsf{Transfer}$ must be equal.
- **Balanced**: The sum of input $\mathsf{AssetValue}$s must be equal to the sum of output $\mathsf{AssetValue}$s.

- **Well-formed Senders**: All of the $\mathsf{Senders}$ in the $\mathsf{Transfer}$ must be constructed according to the above Sender definition.
- **Well-formed Receivers**: All of the $\mathsf{Receivers}$ in the $\mathsf{Transfer}$ must be constructed according to the above $\mathsf{Receiver}$ definition.

In order to prove that these constraints are satisfied for a given $\mathsf{Transfer}$, we build a zero-knowledge proof which will witness that the $\mathsf{Transfer}$ is valid and should be accepted by the ledger. It is not necessary to prove that the encryption of $\mathsf{Receiver.note}$ and the decryption of a note from the ledger are valid. Deviation from the protocol in encryption or decryption does not reduce the security of the protocol for honest participants.

Definition 4.3.6. ($\mathsf{Transfer\ Validity\ Statement}$) A transfer $T : \mathsf{Transfer}$ is considered *valid* if and only if

1. All the $\mathsf{AssetIds}$ in T are equal:
$$\left| \left(\bigcup_{x \in T_{\mathsf{source}}} x.\mathsf{asset.id} \right) \cup \left(\bigcup_{S \in T_{\mathsf{sender}}} S.\mathsf{asset.id} \right) \cup \left(\bigcup_{R \in T_{\mathsf{receiver}}} R.\mathsf{asset.id} \right) \cup \left(\bigcup_{x \in T_{\mathsf{sink}}} x.\mathsf{asset.id} \right) \right| = 1$$
2. The sum of input $\mathsf{AssetValues}$ is equal to the sum of output $\mathsf{AssetValues}$:
$$\left(\sum_{x \in T_{\mathsf{source}}} x.\mathsf{asset.value} \right) + \left(\sum_{S \in T_{\mathsf{sender}}} S.\mathsf{asset.value} \right) = \left(\sum_{R \in T_{\mathsf{receiver}}} R.\mathsf{asset.value} \right) + \left(\sum_{x \in T_{\mathsf{sink}}} x.\mathsf{asset.value} \right)$$
3. For all $S \in T_{\mathsf{sender}}$, S is well-formed:
$$\begin{aligned} S.\mathsf{trapdoor} &= \mathsf{KA.derive}(\mathsf{KDIV}_{S.\tilde{d}}(S.\mathsf{sk.view}, S.\mathsf{sk.spend})) \\ S.\mathsf{cm} &= \mathsf{COM}_{S.\mathsf{trapdoor}}(S.\mathsf{asset}) \\ S.\mathsf{vn} &= \mathsf{COM}_{S.\tilde{d}}(S.\mathsf{sk.view} \,||\, S.\mathsf{sk.spend}) \\ \mathsf{MerkleTree.verify}(S.\mathsf{cm}, S.\mathsf{cm}_{\mathsf{root}}, S.\mathsf{cm}_{\mathsf{path}}) &= \mathsf{True} \end{aligned}$$
4. For all $(i, R) \in \mathsf{enumerate}(T_{\mathsf{receiver}})$ R is well-formed at index i with respect to FAIR:
$$\begin{aligned} R.d &= \mathsf{CRH}(i \,||\, R.\mathsf{rk.view} \,||\, R.\mathsf{rk.spend} \,||\, \mathsf{FAIR}) \\ R.\mathsf{trapdoor} &= \mathsf{KDIV}_{R.d}(R.\mathsf{rk.view}, R.\mathsf{rk.spend}) \\ R.\mathsf{cm} &= \mathsf{COM}_{R.\mathsf{trapdoor}}(R.\mathsf{asset}) \end{aligned}$$

where FAIR is the following fair randomness constant:
$$\mathsf{FAIR} := \mathsf{CRH}(\mathsf{Concat}_{S \in T_{\mathsf{sender}}}(S.\mathsf{cm}))$$

TODO: The $\mathsf{VoidNumber}$ trapdoor is not a good choice, since the range of possible trapdoor values would be publicly known (the diversifiers in this case are posted to the ledger), so it would remove the hiding property of the commitment.

Definition 4.3.7. A $\mathsf{TransferPost}$ is the following tuple:

5 Concrete Protocol

5.1 Constants

TODO: add constants for the protocol

5.2 Concrete Cryptographic Schemes

TODO: add names of cryptographic scheme implementations

5.2.1 Commitments

5.2.2 Hash Functions

5.2.3 Encryption

5.2.4 Zero-Knowledge Proving Systems

5.2.4.1 Groth16

5.2.4.2 PLONK

6 Differences from $\text{M}\textsc{anta}_{\texttt{DAP}}$

6.1 Reusable Addresses

TODO: compare old one-time address protocol to reusable addresses and why reusable is better

6.2 Transfer Circuit Unification

TODO: compare new single transfer circuit to the many old circuits

7 Acknowledgements

TODO: add acknowledgements

8 References

TODO: add references