Dynamic Degree of Freedom Reduction and Analysis of Finite Element Assembly using Characteristic Constraint Modes with Component Mode Synthesis Modelling

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Abstract

In this project report, an effort is made to reduce the Component Mode Synthesis model obtained using Craig-Bampton method over a Multi body assembly. Here the method Characteristic Constraint Mode method proposed by Castanier, Tan and Pierre was followed to reduce the size of (Craig-Bampton) CB model further. The CC modes were obtained by performing an eigen value analysis on the partition of assembled reduced mass and stiffness matrix, corresponding to the constraint (interface) modes. Hence the overall reduction in the node size of the assembled structure is obtained first by truncating the interior (normal) nodes of the substructures sing the Craig-Bampton method followed by truncation of the interface (boundary) nodes using Characteristic Constraint mode method. The following analysis was performed over a multi body structure assembled using 4 substructures. The results in the form of comparison between CB and CC mode method is shown, along with plotting of the Characteristic Constraint modes which represent the dynamic response of the interface. The CC mode method accuracy is increased further by increasing the number of CC modes accounted in the secondary eigen value analysis after CB method. In this way an overall effort is made to save the computation memory and cost by reducing the size of CMS model.

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1. Introduction

1.1 Background

Generally, when dynamic modelling on a huge and complex structure is performed, there often comes a restriction or difficulty in performing the Finite Element Analysis on the entire structure in one go. Whenever a finite element structure obtained has too many Dynamic Degrees of Freedom (DDoF), analysis becomes impossible due to computational costs or node limitations of any FEA software used at frontend. So it often becomes necessary to divide the system into smaller subsystems, perform a dynamic modal analysis of each subsystem and then assemble it back to obtain dynamic response of the entire complex system.

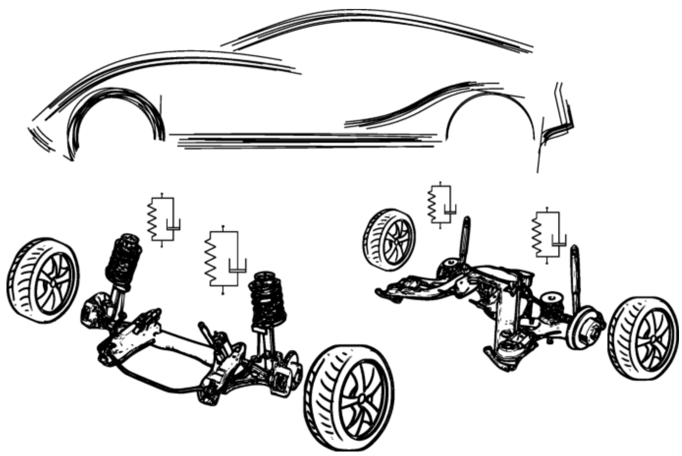

Figure 1: Dynamic Sub-structuring of a Car [1]

Consider an example as shown in figure [1]. Here a highly complex Finite element structure like a car can be broken down into individual subsystems like Car hood, Chassis, Suspension, Wheel axle and the wheels. Dynamic analysis of these components can be done separately and then assembled back considering the interfacing nodes. Evaluation of dynamic behavior of large and complex structure is possible due to this. Such dynamic sub structuring

Allows reducing or to that sense removing the local subsystems with little to no significant effect of the final assembled subsystem. When the complex structure has identical sub-parts, such dynamic sub structuring allows the analysis of these subparts very efficiently by just replicating the modal analysis of single component. Also during Product development or when there are modification or changes in the sub-parts of the product, only that component needs to be reanalyzed, which costs low computational resources.

CMS or Component Mode Synthesis is one such Dynamic sub structuring method which was developed as an efficient way to perform dynamic modelling and analysis of complex global structures. The selected substructure modes are classified here into two categories viz. the normal modes (interior nodes) and the interface nodes, as the boundary nodes, which form a set of static vectors at the interface. The interface nodes also account for coupling. Here each individual component is dynamically reduced by modal analysis done on each individually, before assembling. This reduces computational costs and provides significant order reduction compared to FEA on entire structure. Works on CMS carried by Craig and Seiso are more recognized. A reduction basis is generally used to reduce the no. of modes involved in the analysis. Depending upon the choice of bases vectors, we have Craig-Bampton (Fixed interface) method and Craig-Chang (Free interface) method. Craig-Bampton method is more accurate compared to the rest CMS methods. Here component normal modes are calculated by considering the interface modes fixed. A constraint mode shape is the static deflection induced by giving a unit displacement at the interface. However here there is order reduction of the normal modes but no order reduction for the interfacing modes. There is a chance of FEA model getting dominated by the interface modes than the normal modes. This can make large models difficult to use. In one of the methods proposed by Castanier, Tan and Pierre, the size of CMS model is reduced by applyling eigen mode analysis on the constraint modes. These resultant eigen vectors are termed as Characteristic Constraint Modes (CC) Modes. These modes are then truncated further to get a highly reduced CMS model. Furthermore the in the method proposed by Castanier, Tan and Pierre, authors have showcased a way to perform Dynamic sub structuring via CC Modes on 2 substructures.

1.2 Objective

Here the main objective of this paper is to demonstrate the use of Characteristic Constraint Modes method for Component mode synthesis, on (multiple) more than 2 substructures.

Since Craig Bampton yields more accurate results in all CMS methods, the CB method is followed for initial reduction of normal modes (interior modes). This is followed by reduction over Constraint (Boundary) modes. Assembly is done taking two substructures at a time and merging them to form one. In this way an attempt is made to assemble multiple substructures to form a single assembly. Thus overall reduced modes will help decreasing the size of final CMS model, computational costs, and increasing the computation speed for the software using this particular backend MATLAB/OCTAVE coding. However there will be slight increase in the error as compared to actual value, and the ordinary CB method performed over relatively large CMS model.

1.3 Outline

The report initially discusses the methodology, where the mathematical equations and Matrix transformations required for Craig Bampton method for multi body assembly are discussed initially. Later, CC modes are evaluated based on the reduced mass and stiffness matrix obtained using CB method, and necessary matrix element transformations are done to reduce the CMS (Component Mode Synthesis) model further. In the results section, eigen values obtained using Octave codes by both the approaches are compared and plotted. Also the interface state as reflected by CC modes are plotted. Further the total reduction in figures is showcased for he example carried out in previous sections and finally concluded with the effectiveness of the CC mode method for the Multi body system as well.

2. Methodology

2.1 Component Mode Synthesis using Craig-Bampton method

The method adopted in this paper is demonstrated using a multiple component assembly, i.e 4 substructures as shown in the figure-

Initially we take each substructure individually 1, 2, 3 and 4 for Modal analysis using CMS by Craig-Bampton method. Here we assume that dynamic behavior of the system can be broken down into 2 ways. Static constraint modes resulting by applying unit displacement on the interface (boundary) Degrees of freedom. Second is the interior vibration modes by fixing the boundary degrees of freedom [2].

$$u = u_{static} + u_{dynamic} \qquad (1)$$

The structure breakdown is as shown below-

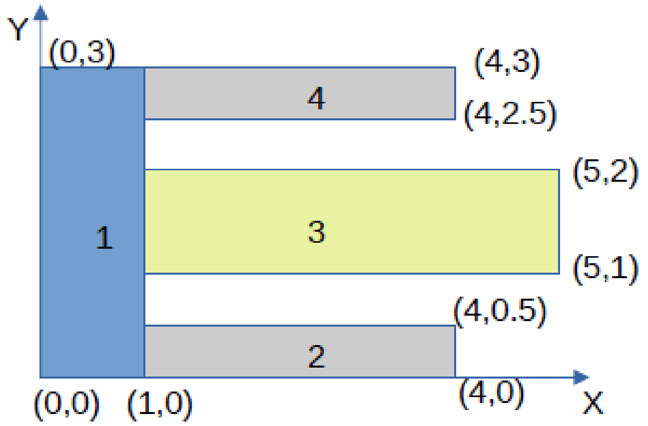

Figure 2: Multiple Component Assembly

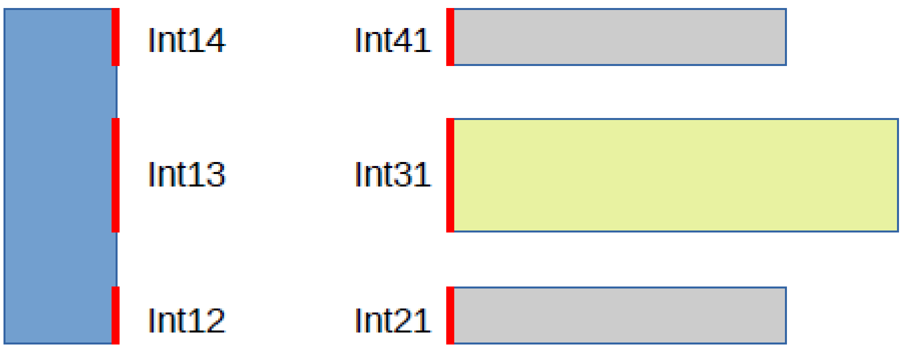

Figure 3: Sub-structuring of the assembly

Considering the Substructure 1, node dimensions are 1342 x 1342, within the dimension of 3x1.

The dimensions of interface nodes are: Interface 14 as 11 x 1, within 2.5 to 3 units over the boundary highlighted. Similarly for Interface 13, the interface nodes are as 21 x 1 and for Interface 12, the interface nodes are 11 x 1 size for Interface 13.

The Mass and Stiffness matrix of 1 are rearranged with internal and boundary nodes separately as follows:

$$M1 = \begin{bmatrix} Mii & Mib \\ Mbi & Mbb \end{bmatrix} \ \& \ K1 = \begin{bmatrix} Kii & Kib \\ Kbi & Kbb \end{bmatrix}$$

The equation of motion is then given by:

$$\begin{bmatrix} Mii & Mib \\ Mbi & Mbb \end{bmatrix} \begin{pmatrix} \ddot{ui} \\ \ddot{ub} \end{pmatrix} + \begin{bmatrix} Kii & Kib \\ Kbi & Kbb \end{bmatrix} \begin{pmatrix} ui \\ ub \end{pmatrix} = \begin{pmatrix} 0 \\ gb \end{pmatrix} \text{------------------(1)}$$

The equation of motion after CB (Craig-Bampton) Transformation is given by [2]:

$$u = R_{CB} \times q$$

Pre-multiplying by R_{CB}^T ,

$$\widehat{M}\ddot{q} + \widehat{K}q = \hat{g}$$

In Matrix form:

$$\begin{bmatrix} \widehat{M_{\eta\eta}} & \widehat{M_{\eta b}} \\ \widehat{M_{b\eta}} & \widehat{M_{bb}} \end{bmatrix} \begin{pmatrix} \ddot{\eta} \\ \ddot{ub} \end{pmatrix} + \begin{bmatrix} \widehat{K_{\eta\eta}} & \widehat{K_{\eta b}} \\ \widehat{K_{b\eta}} & \widehat{K_{bb}} \end{bmatrix} \begin{pmatrix} \eta \\ ub \end{pmatrix} = \begin{pmatrix} 0 \\ gb \end{pmatrix} \text{----------------(2)}$$

This equation can be obtained be solving equation (1) as,

$$M_{ii}\ddot{u}_i + M_{ib}\ddot{u}_b + K_{ii}u_i + K_{ib}u_b = 0 \text{----------------------(3)}$$

Neglecting the inertia terms,

$$K_{ii}u_i + K_{ib}u_b = 0$$

$$u_i = -K_{ii}^{-1}K_{ib}$$

$$u_b = \psi_c u_b, \qquad \psi_c = -K_{ii}^{-1}K_{ib}$$

$$where\ \psi_c\ is\ static\ condensation\ marix$$

$$u_{static} = \begin{pmatrix} u_i \\ u_b \end{pmatrix} = \begin{pmatrix} \psi_c \\ I \end{pmatrix} u_b \text{ (static part of the response)-------------(4)}$$

To consider the dynamic response of the system, static modes are augmented by fixed vibration interface modes. They are obtained by putting ub=0 in equation (1).

$$M_{ii}\ddot{u}_i + K_{ii}u_i = 0$$

Mass normalized eigen vectors can be obtained by solving the eigen value problem. Out of which few modes can be considered for further transformations [2].

$$u_i = \phi\eta \text{---------------------(5)}$$

Dynamic response is then given as-

$$u_{dynamic} = \begin{pmatrix} u_i \\ 0 \end{pmatrix} = \begin{pmatrix} \phi \\ 0 \end{pmatrix} u_b \text{ (dynamic part of response)}$$

Thus total response of system is given by,

$$u = \begin{pmatrix} u_i \\ u_b \end{pmatrix} = u_{static} + u_{dynamic} \text{---------------------------(6)}$$

$$= \begin{pmatrix} \psi_c \\ I \end{pmatrix} u_b + \begin{pmatrix} \phi \\ 0 \end{pmatrix} \eta$$

$$= \begin{bmatrix} \phi & \psi_c \\ 0 & I \end{bmatrix} \begin{pmatrix} \eta \\ u_b \end{pmatrix}$$

$$u = R_{CB} q$$

Where, $R_{CB} = \begin{bmatrix} \phi & \psi_c \\ 0 & I \end{bmatrix}$ $and\ q = [\eta\ u_b]^T$---------------------(7)

Pre multiplying $R_{CB}{}^T to\ equation\ (1) we\ get\ equation\ (2)$

$$\begin{bmatrix} \widehat{M_{\eta\eta}} & \widehat{M_{\eta b}} \\ \widehat{M_{b\eta}} & \widehat{M_{bb}} \end{bmatrix} \begin{pmatrix} \ddot{\eta} \\ \ddot{ub} \end{pmatrix} + \begin{bmatrix} \widehat{K_{\eta\eta}} & \widehat{K_{\eta b}} \\ \widehat{K_{b\eta}} & \widehat{K_{bb}} \end{bmatrix} \begin{pmatrix} \eta \\ ub \end{pmatrix} = \begin{pmatrix} 0 \\ gb \end{pmatrix} \text{-----------------(8)}$$

Using this we reduced the M1 and K1 matrices to 53 x 53 nodes, from 1342 x 1342 nodes. The same process is followed for reducing substructure 2 from 462 x 462 to 21 x 21. Substructure 3 from 882 x 882 nodes to 31 x 31 nodes. Substructure 4 from 462 x 462 to 21 x 21. In every case first 10 modes where taken to form transformation matrix in equation (5).

2.2 Intermediate assembly of the substructures

After the Craig Brampton reduction done for every substructure. The substructures 2, 3, 4 where assembled to form 1 single substructure which can be imagined to combine with Substructure 1.

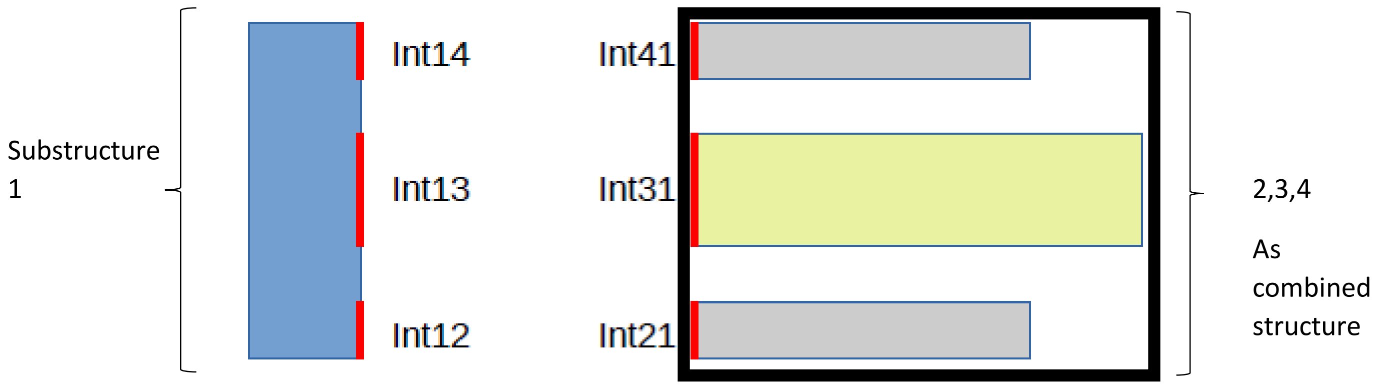

Figure 4: Assembling 2, 3 and 4

For substurctures of 2, 3 and 4 are combined by keeping each reduced sub matrices diagonally to each other as follows:

$$\widehat{M}234 = \begin{bmatrix} & I & & \widehat{M}_{\eta b}^2 & 0 & 0 \\ & & & 0 & \widehat{M}_{\eta b}^3 & 0 \\ & & & 0 & 0 & \widehat{M}_{\eta b}^4 \\ \widehat{M}_{b\eta}^2 & 0 & 0 & \widehat{M}_{bb}^2 & 0 & 0 \\ 0 & \widehat{M}_{b\eta}^3 & 0 & 0 & \widehat{M}_{bb}^3 & 0 \\ 0 & 0 & \widehat{M}_{b\eta}^4 & 0 & 0 & \widehat{M}_{bb}^4 \end{bmatrix} = \begin{bmatrix} I & \widehat{M}_{\eta b}^{234} \\ \widehat{M}_{b\eta}^{234} & \widehat{M}_{bb}^{234} \end{bmatrix} \text{-----------------(9)}$$

$$\widehat{K}234 = \begin{bmatrix} \widehat{K}_{\eta\eta}^{2} & 0 & 0 & \widehat{K}_{\eta b}^{2} & 0 & 0 \\ 0 & \widehat{K}_{\eta\eta}^{3} & 0 & 0 & \widehat{K}_{\eta b}^{3} & 0 \\ 0 & 0 & \widehat{K}_{\eta\eta}^{4} & 0 & 0 & \widehat{K}_{\eta b}^{4} \\ \widehat{K}_{b\eta}^{2} & 0 & 0 & \widehat{K}_{bb}^{2} & 0 & 0 \\ 0 & \widehat{K}_{b\eta}^{3} & 0 & 0 & \widehat{K}_{bb}^{3} & 0 \\ 0 & 0 & K_{b\eta}^{4} & 0 & 0 & \widehat{K}_{bb}^{4} \end{bmatrix} = \begin{bmatrix} \widehat{K}_{\eta\eta}^{234} & \widehat{K}_{\eta b}^{234} \\ \widehat{K}_{b\eta}^{234} & \widehat{K}_{bb}^{234} \end{bmatrix} \text{--------------------(10)}$$

The combined matrices formed are of the size 53 x 53 nodes. After the assembly of 2,3 and 4 substructures, now as M234 and K234, we assemble it with M1 and K1.

$$\widehat{M}1 = \begin{bmatrix} I & \widehat{M}_{\eta b}^{1} \\ \widehat{M}_{b\eta}^{1} & \widehat{M}_{bb}^{1} \end{bmatrix} \; and \; \widehat{K}1 = \begin{bmatrix} \widehat{K}_{\eta\eta}^{1} & \widehat{K}_{\eta b}^{1} \\ \widehat{K}_{b\eta}^{1} & \widehat{K}_{bb}^{1} \end{bmatrix} \text{----------------------(11)}$$

$$\widehat{M}234 = \begin{bmatrix} I & \widehat{M}_{\eta b}^{234} \\ \widehat{M}_{b\eta}^{234} & \widehat{M}_{bb}^{234} \end{bmatrix} \; and \; \widehat{K}1 = \begin{bmatrix} \widehat{K}_{\eta\eta}^{234} & \widehat{K}_{\eta b}^{234} \\ \widehat{K}_{b\eta}^{234} & \widehat{K}_{bb}^{234} \end{bmatrix} \text{----------------------(12)}$$

After assembling, we get,

$$\widehat{M}1234 = \begin{bmatrix} I & 0 & \widehat{M}_{\eta b}^{1} \\ 0 & I & \widehat{M}_{\eta b}^{234} \\ \widehat{M}_{b\eta}^{1} & \widehat{M}_{b\eta}^{234} & \widehat{M}_{bb}^{1} + \widehat{M}_{bb}^{234} \end{bmatrix} \text{-----------------(13)}$$

$$\widehat{K}1234 = \begin{bmatrix} \widehat{K}_{\eta\eta}^{1} & 0 & \widehat{K}_{\eta b}^{1} \\ 0 & \widehat{K}_{\eta\eta}^{234} & \widehat{K}_{\eta b}^{234} \\ \widehat{K}_{b\eta}^{1} & \widehat{K}_{b\eta}^{234} & \widehat{K}_{bb}^{1} + \widehat{K}_{bb}^{234} \end{bmatrix} \text{------------------(14)}$$

This is final assembled Mass and stiffness matrices, from which Eigen modes and eigen values, leading to natural frequencies at the nodes can be obtained, using just CB (Craig Bampton) method.

2.3 CMS reduction using Characteristic Constraints (CC) method

In order to further reduce the matrices, we use the Characteristic Constraints (CC) method [3]. Here we first find the eigen modes using boundary modes as follows

$$(\widehat{K}_{bb}^{1} + \widehat{K}_{bb}^{234})_c \phi^{cc} = \lambda(\widehat{M}_{bb}^{1} + \widehat{M}_{bb}^{234})_c \phi^{cc} \text{------------------(15)}$$

From the set of eigen modes obtained, we select first 10, and further perform transformation to both Mass and stiffness matrices as follows

$$\widehat{M}CC = \begin{bmatrix} I & 0 & \widehat{M}_{\eta b}^{1} \phi^{cc} \\ 0 & I & \widehat{M}_{\eta b}^{234} \phi^{cc} \\ {\phi^{cc}}^{T} \widehat{M}_{b\eta}^{1} & {\phi^{cc}}^{T} \widehat{M}_{b\eta}^{234} & {\phi^{cc}}^{T} (\widehat{M}_{bb}^{1} + \widehat{M}_{bb}^{234}) \phi^{cc} \end{bmatrix} \text{----------------(16)}$$

$$\widehat{K}CC = \begin{bmatrix} \widehat{K}_{\eta\eta}^{1} & 0 & 0 \\ 0 & \widehat{K}_{\eta\eta}^{2} & 0 \\ 0 & 0 & {\phi^{cc}}^{T} (\widehat{K}_{bb}^{1} + \widehat{K}_{bb}^{234}) \phi^{cc} \end{bmatrix} \text{-----------------(17)}$$

Using this technique, we further reduced the modes of our entire assembled structure to just 50 x 50 (again this can be varied, based on the eigen modes chosen for equation (5) and during obtaining CC modes). We can then obtain eigen modes, eigen values and natural frequencies of the assembled structure with reduced computation cost [3].

3. Results

Comparison in the eigen values obtained using Craig Bampton method and Craig Bampton with CC method is shown-

Table 1: Comparison between Craig Bampton and the CC Mode method

Sr. No.	Craig Bampton	Characteristic Constraint	Error (%)
1	-0.000000000060875	0	-100
2	-0.0000000001341	0	-100
3	1	1	0
4	1	1	0
5	1	1	0
6	1	1	0
7	1	1	0
8	1	1	0
9	1	1	0
10	1	1	0
11	1	1	0
12	1	1	0
13	1	1	0
14	1	1	0
15	1	1	0
16	1	1	0
17	1	1	0
18	1	1	0
19	1	1	0
20	1	1	0
21	1	1	0
22	1	1	0
23	1	1	0
24	1	1	0
25	1	1	0
26	1	1	0
27	1	1	0
28	1	1	0
29	1	1	0
30	1	1	0
31	1522000	1522000	0
32	1598600	1598600	0
33	1639400	1639400	0
34	1896900	1896900	0
35	2066100	2066100	0
36	2180300	2180300	0
37	2269400	2269400	0
38	2389600	2389600	0
39	2720200	2720200	0
40	2790900	2790900	0
41	48641000	44816000	7.86

42	49734000	48640000	2.199
43	44816000	49734000	-10.7
45	57023000	57023000	0
46	71108000	71106000	0.00281
47	82410000	82408000	0.00242
48	109070000	109070000	0
49	114740000	114740000	0
50	116250000	116250000	0

So we see that using the ordinary CB method we get 83 eigen values. Whereas using Characteristic Constraint method, we get 50 eigen values. The error in top 50 values using both are showcased above. At many points we see both the methods yield exactly the same result. Be it the lower modes or the higher modes. The only advantage using CC method is we save the computation space and resources, by further reducing, without compromising much in the accuracy.

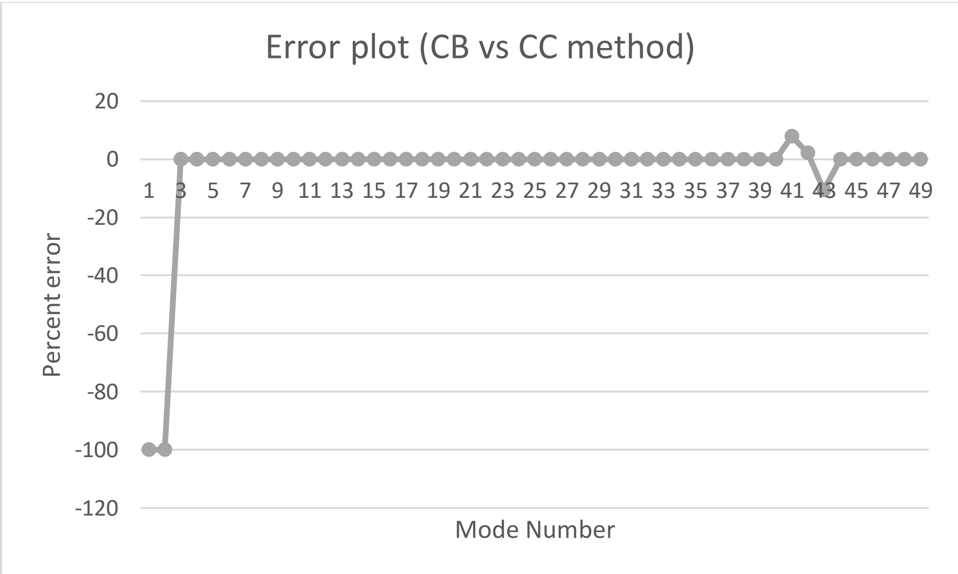

Figure 5: Error plot for comparison between CB and Characteristic Constraint method for CC=10 modes

Now with increase in the CC modes considered for secondary mode analysis, the accuracy of eigen value will be expected to increase as we are now increasing the CMS model size. So highest 10 eigen values are plotted with CC modes as 10 (Series 1), 15 (Series 2), 20 (Series 3), 25 (Series 4) and 30 (Series 5) in 3 dimensional view as shown in Figure 6.

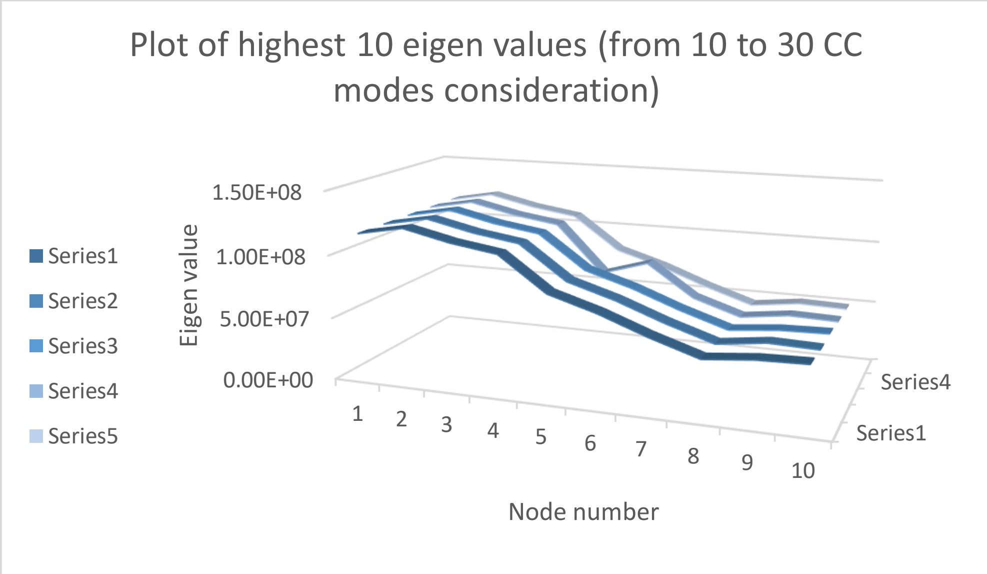

Figure 6: Plot of highest 10 eigen values (from 10 to 30 CC modes consideration)

To compare the variation of eigen values in various CC modes we take 30 CC mode values as standard, and plot the error in other series respectively as shown in the Figure 7.

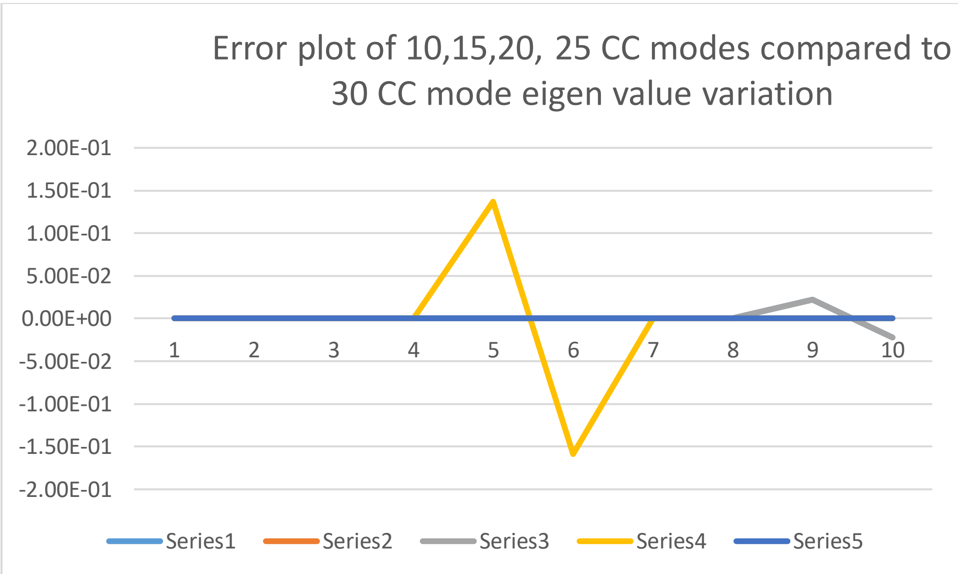

Figure 7: Error plot of 10, 15, 20, 25 CC modes compared to 30 CC mode eigen value variation

The CC Mode values (Eigen modes) can be interpreted as displacement condition at the interface. So 4 such states of interface are shown in the following figures, which can explain the flow of vibration energy between at the interface of the structure.

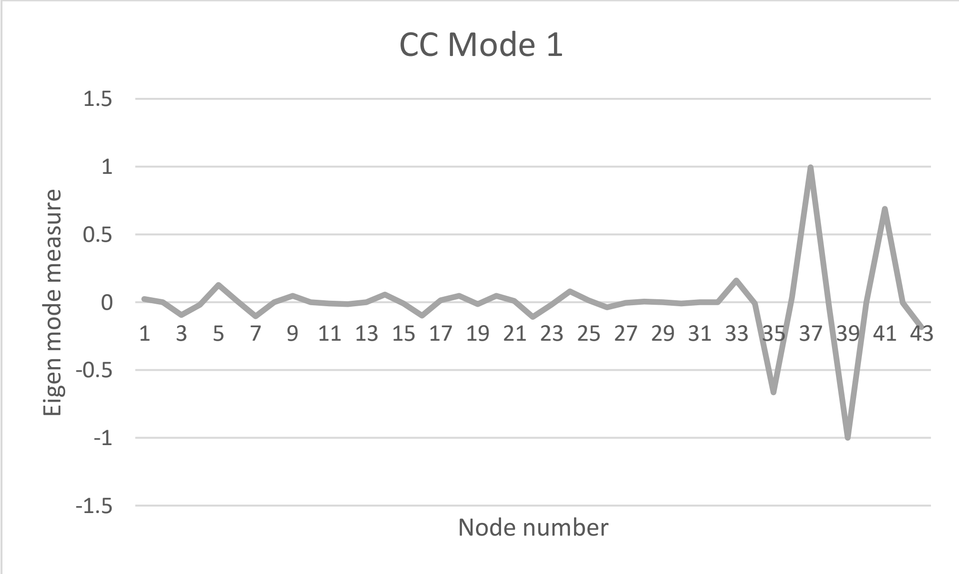

Figure 8: CC Mode 1

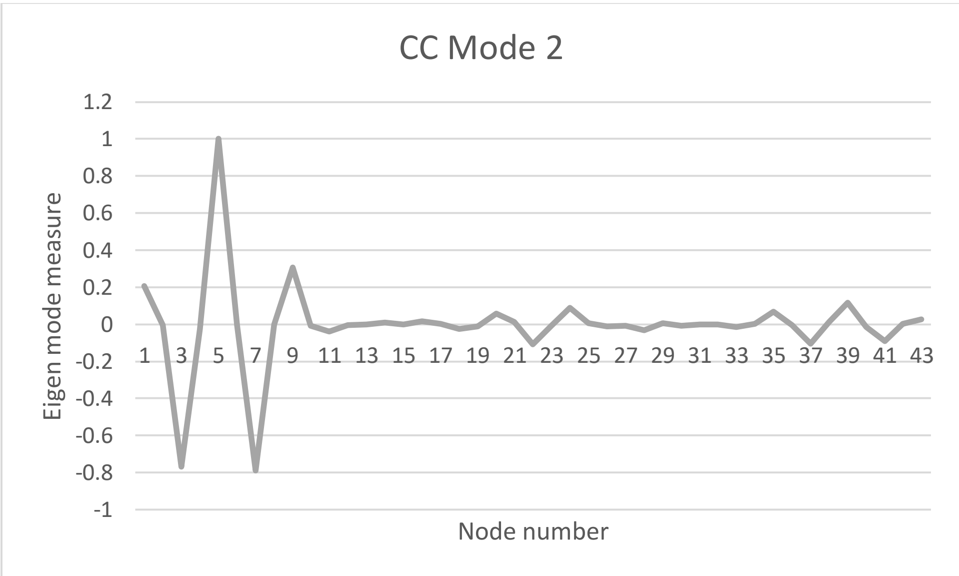

Figure 9: CC Mode 2

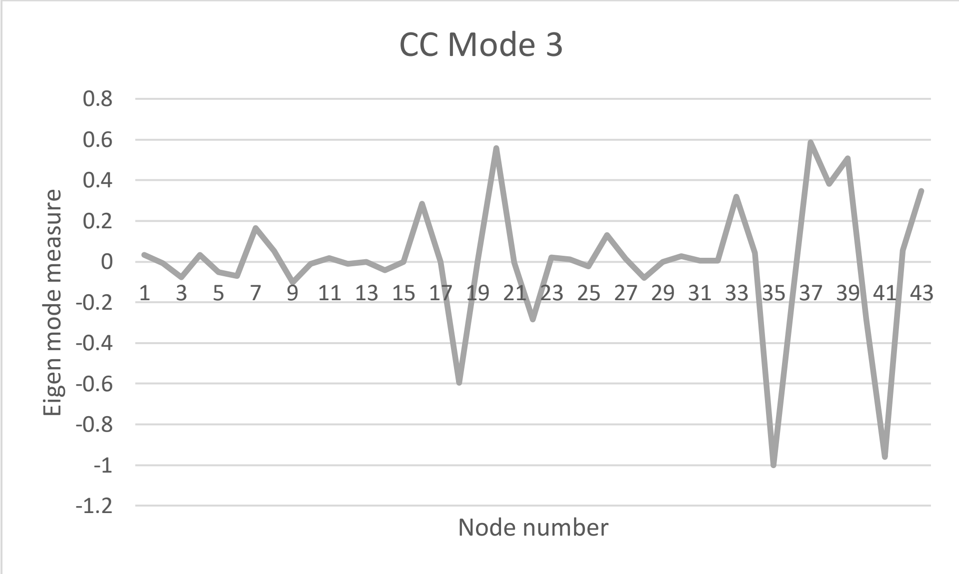

Figure 10: CC Mode 3

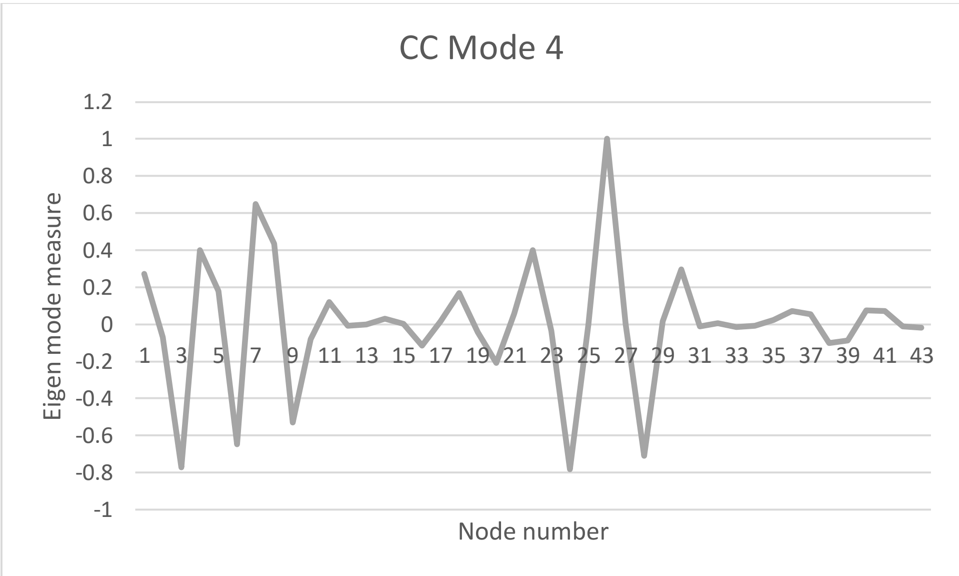

Figure 11: CC Mode 4

4. Discussion

From the results obtained using Craig Bampton and Characteristic Constraint methd, we see that both of them closely match at many points. The average error obtained from first 50 eigen values is 0.0132 % (Neglecting the first two error values).

However few modes are missed due to the reduction obtained in CC mode method. To just compare the total reduction which was able to be obtained from original structure to Craig Bampton to CC mode method, following figures will be helpful.

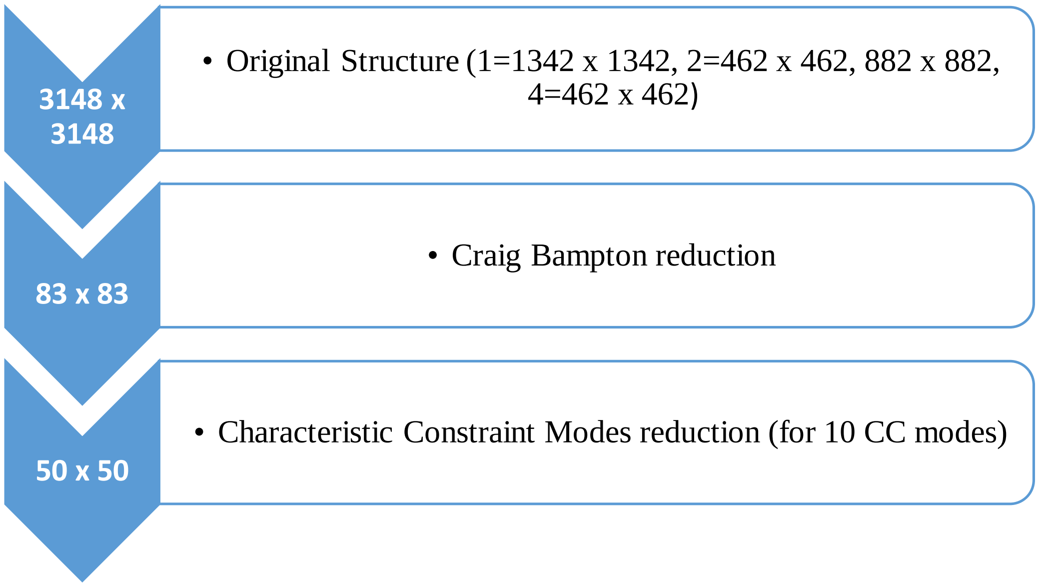

Figure 12: Total reduction obtained

So this makes us clear about the huge computation space, followed by computation costs that can be saved, when dealing with very large scale structures. In order to increase the accuracy in the CMS model using CC mode method, one can increase the number of CC modes used for secondary modal value analysis, as shown in figures 6 and 7.

5. Original Work

In the work demonstrated above, the methods like Craig Bampton and Characteristic Constraint methods were followed from reference papers [2] and [3], as they are well defined and accurate methods compared to other CMS (Component Mode Synthesis) methods available. The method proposed by Castanier, Tan and Pierre [3], was demonstrated on assembly of 2 substructures. So here in the above work, an effort was made to demonstrate the CC mode method to Multi body assembly (More than 2). The CC Mode method is demonstrated using 4 substructure assembly. The results obtained were compared with the Craig Bampton method, to track the correctness of the approach followed.

6. Conclusion

In the work stated above, effort was made to reduce the size of original node structure to Craig-Bampton CMS model and then further reducing the size using CC mode method for a multi body assembly. The proposed model represents the interface modes and capture the characteristic motion at the interface, as shown in figures 6, 7, 8 and 9. The Craig Bampton method deals with reduction of the normal (interior modes) of the substructures and the CC mode, which if used in the combination with Craig Bampton causes reduction by truncation of the interface modes. The overall reduction thus achieved leads to a highly reduced CMS model as explained with the figure 10. The CC modes allow an insight into the physical vibration energy transmission at the interface. The CC modal analysis can be referred to as secondary modal analysis which is incorporated into CMS model after the primary modal analysis. The error values as observed into the eigen values of CC reduced CMS model (Table 1) can be improved by selecting more modes into the secondary analysis, however again it depends upon how we optimize between the computation memory constraint and the accuracy expected.

7. References

[1]https://www.google.com/url?sa=i&url=https%3A%2F%2Flink.springer.com%2Farticle%2F10.1007%2Fs40799-020-00360-1&psig=AOvVaw0uXFyHjxYl15TwUJKJfj5X&ust=1607082365559000&source=images&cd=vfe&ved=0CA0QjhxqFwoTCIDyicnese0CFQAAAAAdAAAAABAD

[2] Roy Craig, Mervyn Bampton. Coupling of Substructures for Dynamic Analyses. AIAA Journal,American Institute of Aeronautics and Astronautics, 1968, 6 (7), pp.1313-1319.

[3] Matthew Castanier, Yung-Chang Tan, Christophe Pierre. Characteristic Constraint Modes for Com-ponent Mode Synthesis. AIAA Journal, American Institute of Aeronautics and Astronautics, 2001,39, pp.1182-1187.

[4] https://github.com/ManthanND/Dynamic_Analysis_CCMode