Testing of Hypothesis: Small Sample Tests

Lesson 8: Test of Hypothesis for an Observed Sample Correlation Coefficient

Let us consider a random sample of *n* observations (x_i, y_i) from a bivariate normal population and let *r* is the observed correlation coefficient in the sample. It is required to test if this sample correlation coefficient *r* is significant of any correlation ρ in the population. This means whether the population correlation coefficient ρ is zero and the observed value of *r* has arisen due to fluctuations of sampling. The test statistic

$$t = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}} ------(1)$$

is considered to test the hypothesis which follows **Students' t-distribution** with ν = $(n-2)$ degrees of freedom.

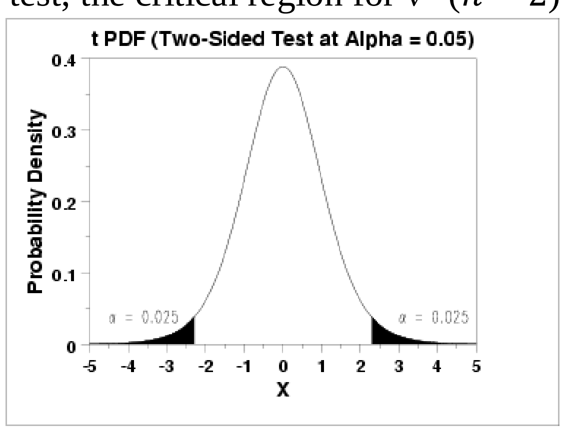
The steps for the testing are:

(i) Formulate null hypothesis, H_0 : ρ = 0 : The variables are uncorrelated in the population

(ii) Formulate alternative hypothesis, H_1 : ρ $\neq 0$ (or otherwise) : The variables are correlated in the population

(iii) Level of significance is α (say 5%)

(iv) From the two tailed test, the critical region for ν=$(n-2)$ degrees of freedom



(v) Compute the test statistic t from equation (1)

(vi) Reject H_0 if the computed value of t falls in the critical region, otherwise accept H_0

Problems:

Ex.1. A random sample of 18 pairs of observations from a bivariate normal population gives a correlation coefficient 0.3. Is it likely that the variables are uncorrelated in the population? (Test at 5% significance level)

Solution:

Here the sample size is $n = 18\ (< 30)$, hence small sample test is to be applied

The sample correlation coefficient $r = 0.3$

Degrees of freedom $\nu = (18 - 2) = 16$

(i) Null hypothesis, $H_0 : \rho = 0$

(ii) Alternative hypothesis, $H_1 : \rho \neq 0$

(iii) $\alpha = 0.05$

(iv) Critical region at 5% level of significance for 16 degrees of freedom is $|t| \geq 2.12$

(v) $t = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}} = \frac{0.3\sqrt{18-2}}{\sqrt{1-(0.3)^2}} = 1.26$

(vi) As computed $|t| < 2.12$, therefore H_0 is accepted

Hence, we conclude that the variables are uncorrelated in the population.

Ans.

Ex.2. The correlation coefficient between income and food expenditure for sample of 7 household from a low income group is 0.9. Using 1% level of significance, test whether the correlation coefficient between incomes and food expenditure is positive. Assume that the population of both variables are normally distributed.

Solution:

Here the sample size is $n = 7\ (< 30)$, hence small sample test is to be applied

The sample correlation coefficient r = 0.9

Degrees of freedom ν = (7 – 2) = 5

(i) Null hypothesis, H_0 : ρ = 0

(ii) Alternative hypothesis, H_1 : ρ $>$ 0

(iii) α = 0.01

(iv) Critical region at 1% level of significance for 5 degrees of freedom is $t \geq 3.365$

(v) $t = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}} = \frac{0.9\sqrt{7-2}}{\sqrt{1-(0.9)^2}} = 4.61$

(vi) As computed t > 3.365, therefore H_0 is rejected

Hence, we conclude that the correlation coefficient between the incomes and food expenditure is indeed positive.

Ans.

Ex.3. Find the least value of *r* in a sample of 27 pairs from a bivariate normal population which is significant at 5% level.

Solution:

Here the sample size is $n = 27\ (< 30)$, hence small sample test is to be applied

Degrees of freedom ν = (27 – 2) = 25

The value of *r* for *n* = 27 will be significant at 5% level, if

$$|t| = \left|\frac{r\sqrt{n-2}}{\sqrt{1-r^2}}\right| \geq 2.06$$

Squaring both sides and putting $n = 27$, we have,

$$\frac{r^2(27-2)}{1-r^2} \geq 4.2436$$

or, $25r^2 \geq 4.2436 - 4.2436r^2$

or, $29.2436r^2 \geq 4.2436$

or, $r^2 \geq 0.145$

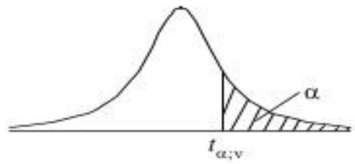
or, $|r| \geq 0.381$

Hence the least value of r is 0.381.

Ans.

Table of the Student's *t*-distribution

The table gives the values of $t_{\alpha;\nu}$ where
$\Pr(T_\nu > t_{\alpha;\nu}) = \alpha$, with ν degrees of freedom



ν \ α	0.1	0.05	0.025	0.01	0.005	0.001	0.0005
1	3.078	6.314	12.076	31.821	63.657	318.310	636.620
2	1.886	2.920	4.303	6.965	9.925	22.326	31.598
3	1.638	2.353	3.182	4.541	5.841	10.213	12.924
4	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	1.319	1.714	2.069	2.500	2.807	3.485	3.767
24	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	1.296	1.671	2.000	2.390	2.660	3.232	3.460
120	1.289	1.658	1.980	2.358	2.617	3.160	3.373
∞	1.282	1.645	1.960	2.326	2.576	3.090	3.291