## **Some Important Distributions**

## **Discrete Distributions**

 $ullet X \sim Bernoulli(p) \ ext{PMF:}$ 

$$P_X(k) = \left\{ egin{array}{ll} p & \quad ext{for } k=1 \ 1-p & \quad ext{for } k=0 \end{array} 
ight.$$

CDF:

$$F_X(x) = egin{cases} 0 & ext{for } x < 0 \ 1-p & ext{for } 0 \leq x < 1 \ 1 & ext{for } 1 \leq x \end{cases}$$

Moment Generating Function (MGF):

$$M_X(s) = 1 - p + pe^s$$

Characteristic Function:

$$\phi_X(\omega) = 1 - p + pe^{i\omega}$$

**Expected Value:** 

$$EX = p$$

Variance:

$$\operatorname{Var}(X) = p(1-p)$$

•  $X \sim Binomial(n,p)$ 

$$P_X(k) = inom{n}{k} p^k (1-p)^{n-k} \quad ext{for } k=0,1,2,\cdots,n$$

Moment Generating Function (MGF):

$$M_X(s) = (1-p+pe^s)^n$$

Characteristic Function:

$$\phi_X(\omega)=(1-p+pe^{i\omega})^n$$

**Expected Value:** 

$$EX = np$$

Variance:

$$\operatorname{Var}(X) = np(1-p)$$

MATLAB:

$$R = binornd(n,p)$$

•  $X \sim Geometric(p)$ 

$$P_X(k) = p(1-p)^{k-1} \quad ext{for } k=1,2,3,\dots$$

CDF:

$$F_X(x) = 1 - (1-p)^{\lfloor x 
floor} \quad ext{for } x \geq 0$$

Moment Generating Function (MGF):

$$M_X(s) = rac{pe^s}{1-(1-p)e^s} \quad ext{for } s < -\ln(1-p)$$

Characteristic Function:

$$\phi_X(\omega) = rac{pe^{i\omega}}{1-(1-p)e^{i\omega}}$$

**Expected Value:** 

$$EX = \frac{1}{p}$$

Variance:

$$\mathrm{Var}(X) = rac{1-p}{p^2}$$

MATLAB:

$$R = geornd(p) + 1$$

ullet  $X \sim Pascal(m,p)$  (Negative Binomial)

$$P_X(k)=inom{k-1}{m-1}p^m(1-p)^{k-m}\quad ext{for }k=m,m+1,m+2,m+3,\ldots$$

Moment Generating Function (MGF):

$$M_X(s) = \left(rac{pe^s}{1-(1-p)e^s}
ight)^m \quad ext{for} \quad s < -\log(1-p)$$

Characteristic Function:

$$\phi_X(\omega) = \left(rac{pe^{i\omega}}{1-(1-p)e^{i\omega}}
ight)^m$$

**Expected Value:** 

$$EX = \frac{m}{p}$$

Variance:

$$\mathrm{Var}(X) = \frac{m(1-p)}{p^2}$$

MATLAB:

R = nbinrnd(m, p) + 1

 $ullet X \sim Hypergeometric(b,r,k) \ _{ ext{PMF}}.$ 

$$P_X(x) = rac{inom{b}{x}inom{r}{k-x}}{inom{b+r}{r}} \quad ext{for } x = \max(0,k-r), \max(0,k-r)+1, \ldots, \min(k,b)$$

**Expected Value:** 

$$EX = rac{kb}{b+r}$$

Variance:

$$\mathrm{Var}(X) = rac{kbr}{(b+r)^2} rac{b+r-k}{b+r-1}$$

MATLAB:

R = hygernd(
$$b + r, b, k$$
)

 $ullet X \sim Poisson(\lambda) \ ext{PMF:}$ 

$$P_X(k) = rac{e^{-\lambda}\lambda^k}{k!} \quad ext{for } k=0,1,2,\cdots$$

Moment Generating Function (MGF):

$$M_X(s) = e^{\lambda(e^s-1)}$$

Characteristic Function:

$$\phi_X(\omega) = e^{\lambda \left(e^{i\omega}-1
ight)}$$

**Expected Value:** 

$$EX = \lambda$$

Variance:

$$\operatorname{Var}(X) = \lambda$$

MATLAB:

$$R = poissrnd(\lambda)$$

## **Continuous Distributions**

 $ullet X \sim Exponential(\lambda) \ ext{PDF:}$ 

$$f_X(x)=\lambda e^{-\lambda x},\quad x>0$$

CDF:

$$F_X(x) = 1 - e^{-\lambda x}, \quad x > 0$$

Moment Generating Function (MGF):

$$M_X(s) = \left(1 - rac{s}{\lambda}
ight)^{-1} \quad ext{for} \quad s < \lambda$$

Characteristic Function:

$$\phi_X(\omega) = \left(1 - rac{i\omega}{\lambda}
ight)^{-1}$$

**Expected Value:** 

$$EX = \frac{1}{\lambda}$$

Variance:

$$\mathrm{Var}(X) = rac{1}{\lambda^2}$$

MATLAB:

R = exprnd(
$$\mu$$
), where  $\mu = \frac{1}{\lambda}$ .

 $ullet X \sim Laplace(\mu,b) \ _{ ext{PDF}}.$ 

$$f_X(x) = rac{1}{2b} \mathrm{exp}igg(-rac{|x-\mu|}{b}igg) = \left\{egin{array}{l} rac{1}{2b} \mathrm{exp}\Big(rac{x-\mu}{b}\Big) & ext{if } x < \mu \ rac{1}{2b} \mathrm{exp}\Big(-rac{x-\mu}{b}\Big) & ext{if } x \geq \mu \end{array}
ight.$$

CDF:

$$F_X(x) = egin{cases} rac{1}{2} ext{exp} \Big(rac{x-\mu}{b}\Big) & ext{if } x < \mu \ 1 - rac{1}{2} ext{exp} \Big(-rac{x-\mu}{b}\Big) & ext{if } x \geq \mu \end{cases}$$

Moment Generating Function (MGF):

$$M_X(s) = rac{e^{\mu s}}{1-b^2s^2} \quad ext{for} \quad |s| < rac{1}{b}$$

Characteristic Function:

$$\phi_X(\omega) = rac{e^{\mu i \omega}}{1 + h^2 \omega^2}$$

**Expected Value:** 

$$EX = \mu$$

Variance:

$$Var(X) = 2b^2$$

 $ullet \ X \sim N(\mu, \sigma^2)$  (Gaussian Distribution) PDF:

$$f_X(x) = rac{1}{\sigma\sqrt{2\pi}}e^{-rac{(x-\mu)^2}{2\sigma^2}}$$

CDF:

$$F_X(x) = \Phi\left(rac{x-\mu}{\sigma}
ight)$$

Moment Generating Function (MGF):

$$M_X(s)=e^{\mu s+rac{1}{2}\sigma^2s^2}$$

Characteristic Function:

$$\phi_X(\omega)=e^{i\mu\omega-rac{1}{2}\sigma^2\omega^2}$$

**Expected Value:** 

$$EX = \mu$$

Variance:

$$Var(X) = \sigma^2$$

MATLAB:

$$Z = \text{randn}, R = \text{normrnd}(\mu, \sigma)$$

 $ullet X \sim Beta(a,b) \ _{ ext{PDF}}.$ 

$$f_X(x) = rac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} x^{(a-1)} (1-x)^{(b-1)}, \ \ ext{for} \ 0 \leq x \leq 1$$

Moment Generating Function (MGF):

$$M_X(s) = 1 + \sum_{k=1}^\infty \left(\prod_{r=0}^{k-1} rac{a+r}{a+b+r}
ight) rac{s^k}{k!}$$

**Expected Value:** 

$$EX = \frac{a}{a+b}$$

Variance:

$$\mathrm{Var}(X) = rac{ab}{(a+b)^2(a+b+1)}$$

MATLAB:

$$R = betarnd(a,b)$$

•  $X \sim \chi^2(n)$  (Chi-squared)

Note:

$$\chi^2(n) = Gamma\left(rac{n}{2},rac{1}{2}
ight)$$

PDF:

$$f_X(x)=rac{1}{2^{rac{n}{2}}\Gamma\left(rac{n}{2}
ight)}x^{rac{n}{2}-1}e^{-rac{x}{2}},\quad ext{for }x>0.$$

Moment Generating Function (MGF):

$$M_X(s)=(1-2s)^{-rac{n}{2}}\quad ext{for}\quad s<rac{1}{2}$$

Characteristic Function:

$$\phi_X(\omega)=(1-2i\omega)^{-rac{n}{2}}$$

**Expected Value:** 

$$EX = n$$

Variance:

$$Var(X) = 2n$$

MATLAB:

$$R = chi2rnd(n)$$

•  $X \sim T(n)$  (The t-Distribution) PDF:

$$f_X(x) = rac{\Gamma(rac{n+1}{2})}{\sqrt{n\pi}\Gamma\left(rac{n}{2}
ight)}igg(1+rac{x^2}{n}igg)^{-rac{n+1}{2}}$$

Moment Generating Function (MGF):

undefined

**Expected Value:** 

$$EX = 0$$

Variance:

$$\operatorname{Var}(X) = \frac{n}{n-2} \quad \text{for} \quad n > 2, \quad \infty \quad \text{for } 1 < n \leq 2, \quad \text{undefined} \quad \text{otherwise}$$

MATLAB:

$$R = trnd(n)$$

 $ullet X \sim Gamma(lpha, \lambda) \ ext{ iny PDF:}$ 

$$f_X(x)=rac{\lambda^{lpha}x^{lpha-1}e^{-\lambda x}}{\Gamma(lpha)},\quad x>0$$

Moment Generating Function (MGF):

$$M_X(s) = \left(1 - rac{s}{\lambda}
ight)^{-lpha} \quad ext{for} \quad s < \lambda$$

Expected Value:

$$EX = \frac{\alpha}{\lambda}$$

Variance:

$$\operatorname{Var}(X) = rac{lpha}{\lambda^2}$$

MATLAB:

$$R = gamrnd(\alpha, \lambda)$$

 $oldsymbol{\cdot} X \sim Erlang(k,\lambda) \, [= Gamma(k,\lambda)], k>0 ext{ is an integer}$  PDF:

$$f_X(x)=rac{\lambda^k x^{k-1}e^{-\lambda x}}{(k-1)!},\quad x>0$$

Moment Generating Function (MGF):

$$M_X(s) = \left(1 - rac{s}{\lambda}
ight)^{-k} \quad ext{for} \quad s < \lambda$$

**Expected Value:** 

$$EX = rac{k}{\lambda}$$

Variance:

$$\operatorname{Var}(X) = rac{k}{\lambda^2}$$

 $ullet X \sim Uniform(a,b) \ ext{ PDF:}$ 

$$f_X(x)=rac{1}{b-a},\quad x\in [a,b].$$

CDF:

$$F_X(x) = \left\{ egin{array}{ll} 0 & & x < a \ rac{x-a}{b-a} & & x \in [a,b) \ 1 & & ext{for } x > b \end{array} 
ight.$$

Moment Generating Function (MGF):

$$M_X(s) = \left\{ egin{array}{ll} rac{e^{sb}-e^{sa}}{s(b-a)} & s 
eq 0 \ 1 & s = 0 \end{array} 
ight.$$

Characteristic Function:

$$\phi_X(\omega) = rac{e^{i\omega b} - e^{i\omega a}}{i\omega(b-a)}$$

Expected Value:

$$EX=rac{1}{2}(a+b)$$

Variance:

$$\mathrm{Var}(X) = \frac{1}{12}(b-a)^2$$

MATLAB:

$$U = \text{rand or } R = \text{unifrnd}(a,b)$$

 $\begin{array}{c} \leftarrow \underline{\text{previous}} \\ \underline{\text{next}} \rightarrow \end{array}$