Curve Fitting

Lesson 2: Exponential Curve and Power Curve

Exponential Curve:

Let the equation of the best fit exponential curve be $y = ae^{bx}$ --------- (1)

Taking log (base 10) on both sides, we get

$$\log_{10} y = \log_{10} a + bx\log_{10} e$$

The above equation can then be written as

$$Y = A + Bx - - - - (2)$$

where $\log_{10} y = Y,\ \log_{10} a = A$ and $b\log_{10} e = B$. Equation (2) is a straight line. To fit the given data (x_i, y_i), i = 1, 2, …., n into this linear equation, we need the two normal equations, which will be modified this time as

$$\sum_{i=1}^{n} Y_i = nA + B\sum_{i=1}^{n} x_i$$

and $$\sum_{i=1}^{n} x_i Y_i = A\sum_{i=1}^{n} x_i + B\sum_{i=1}^{n} {x_i}^2$$

Solving the above two normal equations, we shall get the values of A and B and consequently a and b. Substituting the values of a and b in equation (1), we get the exponential curve of best fit to the given data set (x_i, y_i), i = 1, 2, …., n.

Problems:

Ex.1. Find the best fitting curve $y = ae^{bx}$ to the given data:

x	0	2	4
y	5.012	10	31.62

Solution: The best fitting curve to the given data is provided as $y = ae^{bx}$. Let us take log (base 10) on both sides. Then we get

$$\log_{10} y = \log_{10} a + bx\log_{10} e$$

The above equation is rewritten as

$$Y = A + Bx$$

where $\log_{10} y = Y$, $\log_{10} a = A$ and $b\log_{10} e = B$. Hence the normal equations for this case will be

$$\sum_{i=1}^{n} Y_i = nA + B\sum_{i=1}^{n} x_i$$

and $$\sum_{i=1}^{n} x_i Y_i = A\sum_{i=1}^{n} x_i + B\sum_{i=1}^{n} {x_i}^2$$

Let us now make the table for calculation:

x_i	y_i	$Y_i = \log_{10} y_i$	$x_i Y_i$	${x_i}^2$
0	5.012	0.7	0	0
2	10	1	2	4
4	31.62	1.5	6	16
6	**-**	**3.2**	**8**	**20**

From the normal equations, we have

3.2 = 3A + 6B

and 8 = 6A + 20B

Solving we get, A = 0.67 and B = 0.2.

That means a = $10^A = 10^{0.67} = 4.677$ and b = $\frac{B}{\log_{10} e} = \frac{0.2}{0.4343} = 0.4605$

Thus the best fit curve is given by: $y = 4.677e^{0.4605x}$

Ans.

Ex.2. Fit an exponential curve $y = ab^x$ to the following data:

x	1	2	3	4	5	6	7	8
y	1	1.2	1.8	2.5	3.6	4.7	6.6	9.1

Solution: The curve of best fit is given as $y = ab^x$. Let us take log (base 10) on both sides. Then we get

$$\log_{10}y = \log_{10}a + x\log_{10}b$$

The above equation can be written as

$$Y = A + Bx$$

where $\log_{10}y = Y,$ $\log_{10}a = A$ and $\log_{10}b = B$. Therefore the normal equations for this case will be given by

$$\sum_{i=1}^{n} Y_i = nA + B\sum_{i=1}^{n} x_i$$

and $$\sum_{i=1}^{n} x_iY_i = A\sum_{i=1}^{n} x_i + B\sum_{i=1}^{n} {x_i}^2$$

Now, let us make the table for calculation:

x_i	y_i	$Y_i = \log_{10}y_i$	x_iY_i	${x_i}^2$
1	1	0	0	1
2	1.2	0.0792	0.1584	4
3	1.8	0.2553	0.7659	9
4	2.5	0.3979	1.5916	16
5	3.6	0.5563	2.7815	25
6	4.7	0.6721	4.0326	36
7	6.6	0.8195	5.7365	49
8	9.1	0.959	7.672	64
36	**-**	**3.7393**	**22.7385**	**204**

Thus the normal equations become

$$3.7393 = 8A + 36B$$

$$\text{and}\quad 22.7385 = 36A + 204B$$

Solving we get, A = - 0.1662 and B = 0.1408 and consequently a = 0.682 and b = 1.383. Thus the best fit exponential curve becomes $y = 0.682 \times 1.383^x$.

Ans.

Power Curve/Geometric Curve:

Let the curve of best fit is $y = ax^b$ ------ (3)

Taking log (base 10) on both sides, we get

$$\log_{10} y = \log_{10} a + b\log_{10} x$$

which is similar to

$$Y = A + bX - - - - (4)$$

Here $\log_{10} y = Y,\ \log_{10} a = A$ and $\log_{10} x = X$. Equation (4) is linear. Hence the two normal equations will be

$$\sum_{i=1}^{n} Y_i = nA + b\sum_{i=1}^{n} X_i$$

$$\text{and}\quad \sum_{i=1}^{n} X_i Y_i = A\sum_{i=1}^{n} X_i + b\sum_{i=1}^{n} {X_i}^2$$

Solving the above two normal equations, we shall get the values of A and b and from the expression of A, we can get the value of a. Substituting the values of a and b in equation (3), we get the power curve of best fit to the given data set (x_i, y_i), i = 1, 2, …., n.

Problems:

Ex.1. Fit the curve $y = ax^b$ to the given data:

x	61	26	7	2.6
y	350	400	500	600

Solution: The curve of best fit is given by $y = ax^b$. Taking log (base 10) on both sides, we get

$$\log_{10}y = \log_{10}a + b\log_{10}x$$

which is similar to $Y = A + bX$. Here $\log_{10}y = Y$, $\log_{10}a = A$ and $\log_{10}x = X$. The two normal equations are

$$\sum_{i=1}^{n} Y_i = nA + b\sum_{i=1}^{n} X_i$$

and $$\sum_{i=1}^{n} X_iY_i = A\sum_{i=1}^{n} X_i + b\sum_{i=1}^{n} {X_i}^2$$

Let us make the table for calculation:

x_i	y_i	$X_i = \log_{10}x_i$	$Y_i = \log_{10}y_i$	${X_i}^2$	X_iY_i
61	350	1.7853	2.5441	3.187	4.542
26	400	1.415	2.6021	2.002	3.682
7	500	0.8451	2.699	0.714	2.281
2.6	600	0.415	2.7782	0.172	1.153
-	**-**	**4.4604**	**10.6234**	**6.075**	**11.658**

The normal equations thus become

$$10.6234 = 4A + 4.4604b$$

and $$11.658 = 4.4604A + 6.075b$$

Solving these equations, we get A = 2.845 and b = - 0.1697. From the expression of A, we get a = 699.8. Hence the best fit power curve is $y = 699.8x^{-0.1697}$.

Ans.