Basic Statistics

Lesson 2: Measures of Central Tendency: Median and Mode

Median:

Median of a distribution is the value of the variables which divides it into two equal parts. It is thus a positional average.

Median can be defined for the following cases:

- Simple Series or Ungrouped Data
- Frequency Distribution
  - (a) Discrete/Simple Frequency Distribution
  - (b) Continuous/Grouped Frequency Distribution

- Simple Series/Ungrouped Data:

  (i) Arrange observations in ascending or descending order of magnitude
  (ii) If odd number of observations is given, then the middle value is the median
  (iii) If even number of observations is given, then the A.M of the two middle values is the median

- Frequency Distribution:

(a) Discrete/Simple Frequency Distribution:

(i) Find N/2
(ii) See the cumulative frequency (less than type) just greater than N/2
(iii) The corresponding value of the observation/data is the median

(b) Continuous/Grouped Frequency Distribution:

(i) Find N/2
(ii) See the cumulative frequency (less than type) just greater than N/2
(iii) The corresponding class is the median class

Then the median is given by

$$\text{Median} = l + \frac{h}{f}\left(\frac{N}{2} - c\right)$$

Where l = lower boundary of the median class
h = width of the median class
f = frequency of the median class
c = cumulative frequency of the class preceding the median class

Note:

Cumulative frequency:

Observation	frequency	Cum. Freq. (less than type)	Cum. Freq. (more than type)
x_1	f_1	f_1	$f_1 + f_2 + f_3 + f_4$
x_2	f_2	$f_1 + f_2$	$f_2 + f_3 + f_4$
x_3	f_3	$f_1 + f_2 + f_3$	$f_3 + f_4$
x_4	f_4	$f_1 + f_2 + f_3 + f_4$	f_4

Problems:

Ex.1. Find median of 3.1, 2.6, 5, 4.7, 2.4, 3.9, 5.1 and 3.6.

Solution:

(i) Let us arrange the observations in ascending order of magnitude
2.4, 2.6, 3.1, 3.6, 3.9, 4.7, 5 and 5.1
(ii) There are 8 observations and the two middle values are 3.6 and 3.9
(iii) Median will be = $\frac{3.6+3.9}{2} = 3.75$

Ans.

Ex.2. Obtain the median:

Obs.	1	2	3	4	5	6	7	8	9
Frequency	8	10	11	16	20	25	15	9	6

Solution: Let us make the table for calculating the median:

Obs.	1	2	3	4	5	6	7	8	9
Frequency	8	10	11	16	20	25	15	9	6
C.F	8	18	29	45	65	90	105	114	120

(i) Here N/2 = total frequency/2 = 120/2 = 60
(ii) Cumulative Frequency (C.F) just greater than 60 is 65
(iii) Therefore the corresponding observation 5 will be the median

Ans.

Ex.3. Find the median wage of the distribution:

Wages (in Rs.)	2000-3000	3000-4000	4000-5000	5000-6000	6000-7000
No. of workers	3	5	20	10	5

Solution: Let us make the table for calculating the median:

Wages (in Rs.)	2000-3000	3000-4000	4000-5000	5000-6000	6000-7000
No. of workers	3	5	20	10	5
C.F.	3	8	28	38	43

(i) Here N/2 = 43/2 = 21.5
(ii) Cumulative Frequency (C.F) just greater than 21.5 is 28
(iii) The corresponding class 4000-5000 is median class

Then the median is given by $= l + \frac{h}{f}\left(\frac{N}{2} - c\right)$

$$= 4000 + \frac{1000}{20}(21.5 - 8)$$

$= 4675$ Rs.

Ans.

Ex.4. An incomplete frequency distribution is given with N = 229 and median = 46. Determine the missing frequencies.

Obs.	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Freq.	12	30	?	65	?	25	18

Solution: Let us assume that the two missing frequencies are x and y. Let us make the table for calculation:

Obs.	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Freq.	12	30	x	65	y	25	18
C.F.	12	42	$42 + x$	$107 + x$	$107 + x + y$	$132 + x + y$	$150 + x + y$

We have, $150 + x + y = 229$ or, $x + y = 79$ -------- (1)

Also we know that the median is 46, which means the medial class will be 40 – 50. Then we can write

$$46 = 40 + \frac{10}{65}\left[\frac{229}{2} - (42 + x)\right]$$

Solving this, we get $x = 33.5 \cong 34$. Then from equation (1), we get $y = 45$

Thus, the two missing frequencies are 34 and 45.

Ans.

Mode:

Mode is the value which occurs most frequently in a set of observations and around which, other items of the set cluster densely. In other words, mode is the value of the variable which is predominant in the series.

Mode can be defined for the following cases:

- Frequency Distribution

(a) Discrete/Simple Frequency Distribution:

In this case, mode is the observation with highest frequency.

(b) Continuous/Grouped Frequency Distribution:

In this case, we need to first identify the modal class. This is the class with the highest frequency. Then the mode is given by,

$$\text{Mode} = l + \frac{h(f_1 - f_0)}{2f_1 - (f_0 + f_2)}$$

Where l = lower boundary of the modal class
h = width of the modal class
f_1 = frequency of the modal class
f_0 = frequency of the previous class
f_2 = frequency of the next class

Note: (i) The relation between mean, median and mode is given by

Mean – Mode = 3 (Mean - Median)

(ii) For a symmetrical frequency distribution, all three of them coincide

Problems:

Ex.1. Find mode of the following distribution:

Class	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50
Frequency	10	14	19	17	13

Solution: The class with the highest frequency is 20 – 30, hence this is the modal class. Then the mode is given by

$$\text{Mode} = l + \frac{h(f_1 - f_0)}{2f_1 - (f_0 + f_2)}$$
$$= 20 + \frac{10(19 - 14)}{38 - (14 + 17)} = 27.143$$

Ans.