Basic Statistics

Lesson 6: Additional Measures of Dispersion

Range:

The difference between two extreme values of the observations is known as range of the distribution.

Quartile Deviation/Semi-interquartile Range:

This measure of dispersion is defined as

$$Q = \frac{1}{2}(Q_3 - Q_1)$$

Here Q_1 = 1st quartile $\rightarrow$ Find $\frac{N}{4}$ and then the method for locating median

Similarly, Q_3 = 3rd quartile $\rightarrow$ Find $\frac{3N}{4}$ and then the method for locating median

Note:

(i) The 3 points dividing a distribution into four equal parts are called quartiles. In this way, Q_2 = 2nd quartile $\rightarrow$ Find $\frac{2N}{4} = \frac{N}{2}$ and then the method to find median.

(ii) The 9 points dividing a distribution into ten equal parts are called deciles $\left(\frac{N}{10}, \frac{2N}{10}, \frac{3N}{10}, \dots \dots, \frac{9N}{10}\right)$

(iii) The 99 points dividing a distribution into hundred equal parts are called percentiles $\left(\frac{N}{100}, \frac{2N}{100}, \frac{3N}{100}, \dots \dots, \frac{99N}{100}\right)$

(iv) The quartiles, deciles and percentiles are known as the partition values.

Mean Deviation:

(a) Simple Series/Ungrouped Data:

Let us consider that n observations x_i $(i = 1, 2, \ldots, n)$ are given. Then the mean deviation about an average is given by

$$\text{Mean Deviation} = \frac{1}{n}\sum_{i=1}^{n}|x_i - \bar{x}|$$

Here $\bar{x}$ can be either A.M or median or mode of the distribution (whichever average is applicable).

(b) Discrete/Simple Frequency Distribution

Let us consider that n observations x_i are given with n frequencies f_i $(i = 1, 2, \ldots, n)$. Then the mean deviation about an average is given by

$$\text{Mean Deviation} = \frac{1}{N}\sum_{i=1}^{n} f_i|x_i - \bar{x}|$$

(c) Continuous/Grouped Frequency Distribution:

Let us consider that n classes or intervals are given with n frequencies f_i $(i = 1, 2, \ldots, n)$. Then the mean deviation about an average is given by

$$\text{Mean Deviation} = \frac{1}{N}\sum_{i=1}^{n} f_i|x_i - \bar{x}|$$

In this case, x_i will be the class mark or mid value for each class.

Root Mean Square Deviation:

(a) Simple Series/Ungrouped Data:

Let us consider that n observations x_i $(i = 1, 2, \ldots, n)$ are given. Then the root mean square deviation is given by

$$s = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i - A)^2}$$

where A is any arbitrary point.

(b) Discrete/Simple Frequency Distribution

Let us consider that n observations x_i are given with n frequencies f_i $(i = 1, 2, \dots, n)$. Then the root mean square deviation is given by

$$s = \sqrt{\frac{1}{N}\sum_{i=1}^{n} f_i(x_i - A)^2}$$

(c) Continuous/Grouped Frequency Distribution:

Let us consider that n classes or intervals are given with n frequencies f_i $(i = 1, 2, \dots, n)$. Then the root mean square deviation is given by

$$s = \sqrt{\frac{1}{N}\sum_{i=1}^{n} f_i(x_i - A)^2}$$

In this case, x_i will be the class mark or mid value for each class.

Note:

The mean square deviation = s^2

Problems:

Ex.1. Calculate the mean deviation about the median for the following observations:

46, 79, 26, 85, 39, 65, 99, 29, 56, 72

Solution: Let us arrange the observations in ascending order of magnitude:

26, 29, 39, 46, 56, 65, 72, 79, 85, 99

Then median = $\frac{56+65}{2} = 60.5 \rightarrow \sum_{i=1}^{n}|x_i - \text{median}| = 204$

Subsequently the mean deviation about the median is given by

$$\mathrm{M.D} = \frac{1}{n}\sum_{i=1}^{n}|x_i - \text{median}| = \frac{204}{10} = 20.4$$

Ans.

Ex.2. Calculate the (i) quartile deviation and (ii) mean deviation about mean for the following distribution:

Marks	0-10	10-20	20-30	30-40	40-50	50-60	60-70
No of students	6	5	8	15	7	6	3

Solution: Let us construct the table for calculation:

Marks	0-10	10-20	20-30	30-40	40-50	50-60	60-70	Total
No of students (f_i)	6	5	8	15	7	6	3	50
x_i	5	15	25	35	45	55	65	-
C.F	6	11	19	34	41	47	50	-
$x_i f_i$	30	75	200	525	315	330	195	1670
$\lvert x_i - \bar{x}\rvert$	28.4	18.4	8.4	1.6	11.6	21.6	31.6	-
$f_i\lvert x_i - \bar{x}\rvert$	170.4	92	67.2	24	81.2	129.6	94.8	659.2

(i) The quartile deviation is given by

$$Q = \frac{1}{2}(Q_3 - Q_1) - - - - (1)$$

To evaluate Q_1 and Q_3, we shall use the formula for median, where $\frac{N}{2}$ will be replaced by $\frac{N}{4}$ and $\frac{3N}{4}$ respectively. The C.F just greater than $\frac{N}{4} = \frac{50}{4} = 12.5$ is 19, therefore 20 – 30 is the class containing Q_1 and we can write

$$Q_1 = 20 + \frac{10}{8}(12.5 - 11) = 21.8$$

Similarly, the C.F just greater than $\frac{3N}{4} = 37.5$ is 41, therefore 40 – 50 is the class containing Q_3 and we can write

$$Q_3 = 40 + \frac{10}{7}(37.5 - 34) = 45$$

Hence from equation (1), $Q = \frac{1}{2}(45 - 21.8) = 11.6$

(ii) The mean of the distribution is given by

$$\bar{x} = \frac{\sum_{i=1}^{n} f_i x_i}{N} = \frac{1670}{50} = 33.4$$

Then mean deviation about the mean will be given by

$$\text{Mean Deviation} = \frac{1}{N}\sum_{i=1}^{n} f_i |x_i - \bar{x}| = \frac{659.2}{50} = 13.184$$

Ans.

Relative Measures of Dispersion:

(i) Coefficient of Quartile Deviation = 100 × $\frac{\text{Quartile Deviation}}{\text{Median}}$ %

(ii) Coefficient of Mean Deviation = 100 × $\frac{\text{Mean Deviation about } \bar{x}}{\bar{x}}$ %

where $\bar{x}$ can be mean, median or mode around which the deviations are taken.