Special Probability Distribution

Lesson 2: Poisson Distribution

Poisson distribution is a limiting form of binomial distribution, where the number of trials (n) is very large and consequently the probability (p) of getting success in a trial becomes very small. Then by defining $\lambda = np$, a finite positive number, we can get the probability that a random variable X will have x successes by the following expression of the probability mass function

$$p(x) = P(X = x) = \frac{e^{-\lambda}\lambda^{x}}{x!}, \quad x = 0, 1, 2, \dots \dots$$

$$= 0, \qquad \text{otherwise}$$

It is to be noted that

(i) $p(x) \geq 0 \ \forall \ x = 0, 1, 2, \dots \dots$

(ii) $\sum_{x=0}^{\infty} p(x) = 1$

Note:

(i) Poisson distribution is a discrete distribution

(ii) λ is called the parameter of the distribution

(iii) Mean of the distribution = μ = $E(X) = \lambda$

(iv) Variance of the distribution = $\sigma^2 = V(X) = \lambda$

(v) Skewness of the distribution = $\frac{1}{\sqrt{\lambda}}$

(vi) Kurtosis of the distribution = $\frac{1}{\lambda}$

Problems:

Ex.1. A random variable X follows Poisson distribution with parameter 3. Find the probability that the variable assumes the values (i) 0, 1, 2, 3 (ii) less than 3 and (iii) at least 2.

Solution: As X follows Poisson distribution with mean = $\lambda = 3$, therefore the probability mass function for X will be given by

$$P(X = x) = \frac{e^{-3}3^x}{x!}, x = 0, 1, 2, \dots \dots$$

(i) The probabilities that the variable assumes the values 0, 1, 2 and 3 are given by

$$P(X = 0) = e^{-3} = 0.0498$$

$$P(X = 1) = \frac{e^{-3}3^1}{1!} = 0.1494$$

$$P(X = 2) = \frac{e^{-3}3^2}{2!} = 0.2241$$

$$P(X = 3) = \frac{e^{-3}3^3}{3!} = 0.2241$$

(ii) The probability that the variable assumes the values less than 3 will be

$P(X < 3) = P(X = 0) + P(X = 1) + P(X = 2)$

= 0.4233.

(iii) The probability that the variable assumes at least 2 is given by

$P(X \geq 2) = P(X = 2) + P(X = 3) + P(X = 4) + \cdots \dots \dots$

$= 1 - [P(X = 0) + P(X = 1)]$

= 0.8008.

Ans.

Ex.2. In a company, there are 250 workers. The probability of a worker remain absent on any day is 0.02. Find the probability that on a day, 7 workers are absent.

Solution: It is given that n = 250, p = 0.02, then $\lambda = np = 5$.

Let us consider that X = number of workers remaining absent on any one day

Then the probability that on any day, x workers will be absent is given by

$$P(X = x) = \frac{e^{-5}5^x}{x!}, x = 0, 1, 2, \ldots\ldots, 250$$

Therefore, the required probability that on a day, 7 workers will be absent will be

$$P(X = 7) = \frac{e^{-5}5^7}{7!} \cong 0.014.$$

Ans.

Ex.3. Potholes on a highway are real problems. The past experience suggests that there are on the average, 2 potholes per mile after a certain amount of usage. It is assumed that the Poisson process is applied to the random variable 'number of potholes'. What is the probability that no more than 4 potholes will occur in a given section of 5 miles?

Solution:

Let us consider that X = number of potholes in 5 miles. It is given that the average number of potholes per mile = 2. Then for a section of 5 miles, the average number of potholes = $\lambda = 10$.

Thus the probability that there will be x potholes in the chosen section of the highway will be given by

$$P(X = x) = \frac{e^{-10}10^x}{x!}, x = 0, 1, 2, \ldots$$

Following the above distribution, the probability that no more than 4 potholes will occur in the given section

$$P(X \leq 4) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4)$$

$$\cong 0.02925.$$

Ans.

Ex.4. A business firm receives on an average 2.5 telephone calls per day during the time period 10 – 10.05 AM. Find the probability that on a certain day, the firm receives (i) no call (ii) exactly 4 calls, during the same period assuming Poisson distribution.

Solution: Let us consider that X = number of telephone calls received by the firm on a day during the mentioned time period.

It is given that $\lambda = 2.5$. Subsequently, the probability that the firm receives x calls on a day during the same time period is given by

$$P(X = x) = \frac{e^{-2.5}(2.5)^x}{x!}, x = 0, 1, 2, \ldots$$

(i) The probability that the firm receives no call on a day during the given period

$= P(X = 0) = e^{-2.5} = 0.0821$

(ii) The probability that the firm receives exactly 4 calls on a day during the given period $= P(X = 4) = \frac{e^{-2.5}(2.5)^4}{4!} = 0.1336$.

Ans.

Ex.5. The random variable X is distributed in a Poisson form. If $P(X = 1) = P(X = 2)$, what is $P(X = 0 \text{ or } 1)$.

Solution: We know that the probability mass function for a random variable following Poisson distribution is given by

$$P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}, \quad x = 0, 1, 2, \ldots \ldots$$

According to the problem

$$P(X = 1) = P(X = 2)$$

$$\text{or, } \quad \frac{e^{-\lambda}\lambda^1}{1!} = \frac{e^{-\lambda}\lambda^2}{2!}$$

$$\text{or, } \quad \lambda = 2$$

Hence the required probability $P(X = 0 \text{ or } 1) = P(X = 0) + P(X = 1) = 3e^{-2}$.

Ans.