# Basic Statistics

## Lesson 1: Measures of Central Tendency: Arithmetic Mean

Quantitative data in a mass differ from each other in the following way. They show a tendency to concentrate at certain values, usually in the center of the distribution. Measures of this tendency are called measures of central tendency. There are three main measures of central tendency: mean, median and mode.

### Arithmetic Mean:

This can be defined for the following cases:

(a) Simple Series or Ungrouped Data
(b) Frequency Distribution (Weighted Arithmetic Mean)
     (i)      Discrete/Simple Frequency Distribution
     (ii)      Continuous/Grouped Frequency Distribution

- Simple Series/Ungrouped Data:

Let the $n$ observations or data are given by $x_1, x_2, \dots, x_n$ $(x_i, i = 1, 2, \dots, n)$. The simple arithmetic mean for this data is given by

$$\bar{x} = \frac{x_1 + x_2 + \cdots + x_n}{n} = \frac{1}{n} \sum_{i=1}^{n} x_i$$

- Frequency Distribution:
(i)      Discrete/Simple Frequency Distribution:

Let us consider that $n$ observations $x_i$ are given with $n$ frequencies (or weights) $f_i$ $(i = 1, 2, \dots, n)$. The weighted arithmetic mean for this data is given by

$$\bar{x} = \frac{x_1 f_1 + x_2 f_2 + \cdots + x_n f_n}{N} = \frac{1}{N} \sum_{i=1}^{n} x_i f_i$$

where $N = \sum_{i=1}^{n} f_i$ = total frequency of the distribution

(ii)    <u>Continuous/Grouped Frequency Distribution:</u>

Let us consider that there are $n$ classes given with $n$ frequencies (or weights) $f_i$ $(i = 1, 2, ...., n)$. The weighted arithmetic mean for this data is given by

$$\bar{x} = \frac{1}{N} \sum_{i=1}^{n} x_i f_i$$

Here $x_i$ are the class marks or mid values for each class

<u>Problems:</u>

Ex.1. Calculate the simple and weighted arithmetic mean price/ton of coal purchased by an industry for the half year:

| Month | Jan | Feb | Mar | Apr | May | June |
|---|---|---|---|---|---|---|
| Price/ton (in Rs.) | 42.5 | 51.25 | 50 | 52 | 44.25 | 54 |
| Tons purchased | 25 | 30 | 40 | 50 | 10 | 45 |

Solution: Let us make the table for calculating the A.M.

| Month | Jan | Feb | Mar | Apr | May | June | Total |
|---|---|---|---|---|---|---|---|
| Price/ton (in Rs.) (x) | 42.5 | 51.25 | 50 | 52 | 44.25 | 54 | 294 |
| Tons purchased (f) | 25 | 30 | 40 | 50 | 10 | 45 | 200 |
| fx | 1062.5 | 1537.5 | 2000 | 2600 | 442.5 | 2430 | 10072.5 |

Then the simple A.M $= \frac{1}{n}\sum_{i=1}^{n} x_i = \frac{294}{6} = 49$ Rs.

and the weighted A.M $= \frac{1}{N}\sum_{i=1}^{n} x_i f_i = \frac{10072.5}{200} = 50.3625$ Rs.

Ans.

Ex.2. Calculate the A.M.

| Marks | 0-10 | 10-20 | 20-30 | 30-40 | 40-50 | 50-60 |
|---|---|---|---|---|---|---|
| No. of students | 12 | 18 | 27 | 20 | 17 | 6 |

Solution: Let us make the table for calculating the A.M.

| Marks | 0-10 | 10-20 | 20-30 | 30-40 | 40-50 | 50-60 | Total |
|---|---|---|---|---|---|---|---|
| No. of students (f) | 12 | 18 | 27 | 20 | 17 | 6 | 100 |
| Class mark (x) | 5 | 15 | 25 | 35 | 45 | 55 | - |
| fx | 60 | 270 | 675 | 700 | 765 | 330 | 2800 |

Then the A.M is given by,

$$\bar{x} = \frac{1}{N}\sum_{i=1}^{n} x_i f_i = \frac{2800}{100} = 28$$

Ans.

Ex.3. Find the A.M.

| Age (in years) | 15-19 | 20-24 | 25-29 | 30-34 | 35-39 | 40-44 |
|---|---|---|---|---|---|---|
| No. of persons | 37 | 81 | 43 | 24 | 9 | 6 |

Solution: Let us make the table for calculating the A.M.

| Age (in years) | 15-19 | 20-24 | 25-29 | 30-34 | 35-39 | 40-44 | Total |
|---|---|---|---|---|---|---|---|
| No. of persons (f) | 37 | 81 | 43 | 24 | 9 | 6 | 200 |
| Class boundary | 14.5-19.5 | 19.5-24.5 | 24.5-29.5 | 29.5-34.5 | 34.5-39.5 | 39.5-44.5 | |
| Class mark (x) | 17 | 22 | 27 | 32 | 37 | 42 | - |
| fx | 629 | 1782 | 1161 | 768 | 333 | 252 | 4925 |

Therefore the A.M is given by,

$$\bar{x} = \frac{1}{N}\sum_{i=1}^{n} x_i f_i = \frac{4925}{200} = 24.625$$

Hence the A.M of the distribution is 24.625 years.          Ans.

## Composite Arithmetic Mean:

For two samples of sizes $n_1$ and $n_2$ having A.M $\bar{x}_1$ and $\bar{x}_2$ , the composite arithmetic mean is given by

$$\bar{x} = \frac{n_1 \bar{x}_1 + n_2 \bar{x}_2}{n_1 + n_2}$$

## Problems:

Ex.1. There are two branches of an establishment employing 100 and 80 persons respectively. If the A.M's of the monthly salaries paid by the two branches are Rs. 275 and Rs.225 respectively, find the A.M of the salaries of the employees of the establishment as a whole.

Solution: We need to find the composite mean salary of all the employees, which is given by

$$\bar{x} = \frac{n_1 \bar{x}_1 + n_2 \bar{x}_2}{n_1 + n_2} = \frac{100 \times 275 + 80 \times 225}{100 + 80} = \frac{45500}{180}$$

$$= 252.78 \text{ Rs.}$$

Ans.

Ex.2. The average salary of male employees in a firm was Rs. 5200 and that of female was Rs. 4200. The mean salary of all the employees was Rs. 5000. Find the percentage of male and female employees.

Solution: Let the number of male and female employees are given by $n_1$ and $n_2$ and their mean salaries are given by $\bar{x}_1$ and $\bar{x}_2$. Then we have

$$\bar{x}_1 = 5200, \ \bar{x}_2 = 4200$$

Also the mean salary of all employees is the composite mean $\bar{x} = 5000$. Then we can write

$$5000 = \frac{5200 n_1 + 4200 n_2}{n_1 + n_2}$$

$$\text{or,} \quad \frac{n_1}{n_2} = 4$$

This gives there are 80% male and 20% female employees in the firm. Ans.