Some Important Distributions

Discrete Distributions

- $X \sim Bernoulli(p)$
  PMF:

$$P_X(k) = \begin{cases} p & \text{for } k = 1 \\ 1-p & \text{for } k = 0 \end{cases}$$

  CDF:

$$F_X(x) = \begin{cases} 0 & \text{for } x < 0 \\ 1-p & \text{for } 0 \leq x < 1 \\ 1 & \text{for} 1 \leq x \end{cases}$$

  Moment Generating Function (MGF):

$$M_X(s) = 1 - p + pe^s$$

  Characteristic Function:

$$\phi_X(\omega) = 1 - p + pe^{i\omega}$$

  Expected Value:

$$EX = p$$

  Variance:

$$\mathrm{Var}(X) = p(1-p)$$

- $X \sim Binomial(n, p)$
  PMF:

$$P_X(k) = \binom{n}{k} p^k (1-p)^{n-k} \quad \text{for } k = 0, 1, 2, \cdots, n$$

  Moment Generating Function (MGF):

$$M_X(s) = (1 - p + pe^s)^n$$

  Characteristic Function:

$$\phi_X(\omega) = (1 - p + pe^{i\omega})^n$$

  Expected Value:

$$EX = np$$

  Variance:

$$\mathrm{Var}(X) = np(1-p)$$

MATLAB:

R = binornd(n,p)

- $X \sim Geometric(p)$

  PMF:

$$P_X(k) = p(1-p)^{k-1} \quad \text{for } k = 1, 2, 3, \ldots$$

  CDF:

$$F_X(x) = 1 - (1-p)^{\lfloor x \rfloor} \quad \text{for } x \geq 0$$

  Moment Generating Function (MGF):

$$M_X(s) = \frac{pe^s}{1-(1-p)e^s} \quad \text{for } s < -\ln(1-p)$$

  Characteristic Function:

$$\phi_X(\omega) = \frac{pe^{i\omega}}{1-(1-p)e^{i\omega}}$$

  Expected Value:

$$EX = \frac{1}{p}$$

  Variance:

$$\text{Var}(X) = \frac{1-p}{p^2}$$

  MATLAB:

R = geornd(p)+1

- $X \sim Pascal(m,p)$ (Negative Binomial)

  PMF:

$$P_X(k) = \binom{k-1}{m-1} p^m (1-p)^{k-m} \quad \text{for } k = m, m+1, m+2, m+3, \ldots$$

  Moment Generating Function (MGF):

$$M_X(s) = \left(\frac{pe^s}{1-(1-p)e^s}\right)^m \quad \text{for} \quad s < -\log(1-p)$$

  Characteristic Function:

$$\phi_X(\omega) = \left(\frac{pe^{i\omega}}{1-(1-p)e^{i\omega}}\right)^m$$

  Expected Value:

$$EX = \frac{m}{p}$$

  Variance:

$$\mathrm{Var}(X) = \frac{m(1-p)}{p^2}$$

MATLAB:

R = nbinrnd(m,p)+1

- $X \sim Hypergeometric(b, r, k)$
  PMF:

$$P_X(x) = \frac{\binom{b}{x}\binom{r}{k-x}}{\binom{b+r}{k}} \quad \text{for } x = \max(0, k-r), \max(0, k-r)+1, \ldots, \min(k, b)$$

Expected Value:

$$EX = \frac{kb}{b+r}$$

Variance:

$$\mathrm{Var}(X) = \frac{kbr}{(b+r)^2}\frac{b+r-k}{b+r-1}$$

MATLAB:

R = hygernd($b+r$,b,k)

- $X \sim Poisson(\lambda)$
  PMF:

$$P_X(k) = \frac{e^{-\lambda}\lambda^k}{k!} \quad \text{for } k = 0, 1, 2, \cdots$$

Moment Generating Function (MGF):

$$M_X(s) = e^{\lambda(e^s-1)}$$

Characteristic Function:

$$\phi_X(\omega) = e^{\lambda(e^{i\omega}-1)}$$

Expected Value:

$$EX = \lambda$$

Variance:

$$\mathrm{Var}(X) = \lambda$$

MATLAB:

R = poissrnd(λ)

Continuous Distributions

- $X \sim Exponential(\lambda)$
  PDF:

$$f_X(x) = \lambda e^{-\lambda x}, \quad x > 0$$

CDF:

$$F_X(x) = 1 - e^{-\lambda x}, \quad x > 0$$

Moment Generating Function (MGF):

$$M_X(s) = \left(1 - \frac{s}{\lambda}\right)^{-1} \quad \text{for} \quad s < \lambda$$

Characteristic Function:

$$\phi_X(\omega) = \left(1 - \frac{i\omega}{\lambda}\right)^{-1}$$

Expected Value:

$$EX = \frac{1}{\lambda}$$

Variance:

$$\text{Var}(X) = \frac{1}{\lambda^2}$$

MATLAB:

R = exprnd(μ), where $\mu = \frac{1}{\lambda}$.

- $X \sim Laplace(\mu, b)$
  PDF:

$$f_X(x) = \frac{1}{2b}\text{exp}\left(-\frac{|x-\mu|}{b}\right) = \begin{cases} \frac{1}{2b}\text{exp}\left(\frac{x-\mu}{b}\right) & \text{if } x < \mu \\ \frac{1}{2b}\text{exp}\left(-\frac{x-\mu}{b}\right) & \text{if } x \geq \mu \end{cases}$$

CDF:

$$F_X(x) = \begin{cases} \frac{1}{2}\text{exp}\left(\frac{x-\mu}{b}\right) & \text{if } x < \mu \\ 1 - \frac{1}{2}\text{exp}\left(-\frac{x-\mu}{b}\right) & \text{if } x \geq \mu \end{cases}$$

Moment Generating Function (MGF):

$$M_X(s) = \frac{e^{\mu s}}{1 - b^2 s^2} \quad \text{for} \quad |s| < \frac{1}{b}$$

Characteristic Function:

$$\phi_X(\omega) = \frac{e^{\mu i \omega}}{1 + b^2\omega^2}$$

Expected Value:

$$EX = \mu$$

Variance:

$$\mathrm{Var}(X) = 2b^2$$

- $X \sim N(\mu, \sigma^2)$ (Gaussian Distribution)
  PDF:

$$f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

CDF:

$$F_X(x) = \Phi\left(\frac{x-\mu}{\sigma}\right)$$

Moment Generating Function (MGF):

$$M_X(s) = e^{\mu s + \frac{1}{2}\sigma^2 s^2}$$

Characteristic Function:

$$\phi_X(\omega) = e^{i\mu\omega - \frac{1}{2}\sigma^2\omega^2}$$

Expected Value:

$$EX = \mu$$

Variance:

$$\mathrm{Var}(X) = \sigma^2$$

MATLAB:

Z = randn, R = normrnd(μ,σ)

- $X \sim Beta(a, b)$
  PDF:

$$f_X(x) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} x^{(a-1)}(1-x)^{(b-1)}, \quad \text{for } 0 \le x \le 1$$

Moment Generating Function (MGF):

$$M_X(s) = 1 + \sum_{k=1}^{\infty} \left(\prod_{r=0}^{k-1} \frac{a+r}{a+b+r} \right) \frac{s^k}{k!}$$

Expected Value:

$$EX = \frac{a}{a+b}$$

Variance:

$$\mathrm{Var}(X) = \frac{ab}{(a+b)^2(a+b+1)}$$

MATLAB:

R = betarnd(a,b)

- $X \sim \chi^2(n)$ (Chi-squared)

Note:

$$\chi^2(n) = Gamma\left(\frac{n}{2}, \frac{1}{2}\right)$$

PDF:

$$f_X(x) = \frac{1}{2^{\frac{n}{2}}\Gamma\left(\frac{n}{2}\right)} x^{\frac{n}{2}-1} e^{-\frac{x}{2}}, \quad \text{for } x > 0.$$

Moment Generating Function (MGF):

$$M_X(s) = (1-2s)^{-\frac{n}{2}} \quad \text{for} \quad s < \frac{1}{2}$$

Characteristic Function:

$$\phi_X(\omega) = (1-2i\omega)^{-\frac{n}{2}}$$

Expected Value:

$$EX = n$$

Variance:

$$\text{Var}(X) = 2n$$

MATLAB:

R = chi2rnd(n)

- $X \sim T(n)$ (The t-Distribution)

  PDF:

$$f_X(x) = \frac{\Gamma\left(\frac{n+1}{2}\right)}{\sqrt{n\pi}\Gamma\left(\frac{n}{2}\right)}\left(1+\frac{x^2}{n}\right)^{-\frac{n+1}{2}}$$

  Moment Generating Function (MGF):

$$\text{undefined}$$

  Expected Value:

$$EX = 0$$

  Variance:

$$\text{Var}(X) = \frac{n}{n-2} \quad \text{for} \quad n > 2, \quad \infty \quad \text{for } 1 < n \leq 2, \quad \text{undefined} \quad \text{otherwise}$$

  MATLAB:

  R = trnd(n)

- $X \sim Gamma(\alpha, \lambda)$

  PDF:

$$f_X(x) = \frac{\lambda^\alpha x^{\alpha-1} e^{-\lambda x}}{\Gamma(\alpha)}, \quad x > 0$$

  Moment Generating Function (MGF):

$$M_X(s) = \left(1 - \frac{s}{\lambda}\right)^{-\alpha} \quad \text{for} \quad s < \lambda$$

Expected Value:

$$EX = \frac{\alpha}{\lambda}$$

Variance:

$$\text{Var}(X) = \frac{\alpha}{\lambda^2}$$

MATLAB:

R = gamrnd(α,λ)

- $X \sim Erlang(k, \lambda)\,[= Gamma(k, \lambda)], k > 0$ is an integer

PDF:

$$f_X(x) = \frac{\lambda^k x^{k-1} e^{-\lambda x}}{(k-1)!}, \quad x > 0$$

Moment Generating Function (MGF):

$$M_X(s) = \left(1 - \frac{s}{\lambda}\right)^{-k} \quad \text{for} \quad s < \lambda$$

Expected Value:

$$EX = \frac{k}{\lambda}$$

Variance:

$$\text{Var}(X) = \frac{k}{\lambda^2}$$

- $X \sim Uniform(a, b)$

PDF:

$$f_X(x) = \frac{1}{b-a}, \quad x \in [a, b]$$

CDF:

$$F_X(x) = \begin{cases} 0 & x < a \\ \frac{x-a}{b-a} & x \in [a, b) \\ 1 & \text{for } x \geq b \end{cases}$$

Moment Generating Function (MGF):

$$M_X(s) = \begin{cases} \frac{e^{sb} - e^{sa}}{s(b-a)} & s \neq 0 \\ 1 & s = 0 \end{cases}$$

Characteristic Function:

$$\phi_X(\omega) = \frac{e^{i\omega b} - e^{i\omega a}}{i\omega(b-a)}$$

Expected Value:

$$EX = \frac{1}{2}(a+b)$$

Variance:

$$\mathrm{Var}(X) = \frac{1}{12}(b-a)^2$$

MATLAB:

U = rand or R = unifrnd(a,b)

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