Chi Square (χ^2) Distribution

The square of a standard normal variate is called a chi-square variate with 1 degree of freedom. Thus, if we know that X is a normal variate, i.e, $X \sim N(\mu, \sigma^2)$, then we know that $Z = \frac{X-\mu}{\sigma}$ is a standard normal variate, i.e, $Z \sim N(0,1)$. In that case, $Z^2 = \left(\frac{X-\mu}{\sigma}\right)^2$ will be a chi-square variate with 1 degree of freedom.

Definition:

A random variable X is said to follow χ^2 distribution if its probability density function is of the form

$$f(x) = \frac{e^{-x/2}}{2^{n/2}\ \Gamma(n/2)} x^{\left(\frac{n}{2}-1\right)}, \quad (0 \leq x < \infty)$$

The parameter of the distribution is 'n' (> 0) and is called the number of degrees of freedom.

Properties:

1. Mean = n, mode = $n-2$, standard deviation = $\sqrt{2n}$
2. Positively skewed distribution starting from 0 to ∞, $S_k = \sqrt{\frac{2}{n}}$

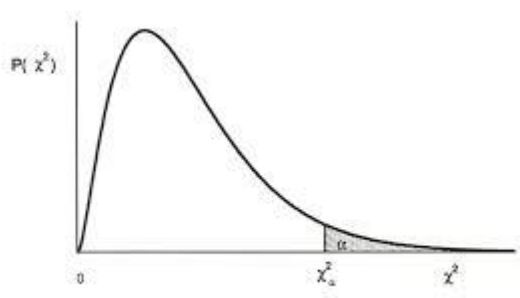

Figure J.1: The χ^2 distribution

3. If the degrees of freedom is large, then $\sqrt{2\chi^2} - \sqrt{2n-1}$ approximates the standard normal distribution

4. If $Z_1, Z_2, \dots \dots, Z_n$ are n independent standard normal variates, then $\sum_{i=1}^{n} {Z_i}^2$ will be a χ^2 variate with 'n' degrees of freedom

<u>Note</u>:

This distribution is used in both large sample and small sample tests. It is used mainly in:

(a) Test for goodness of fit

(b) Test for independence of attributes

(c) Test for a specified standard deviation (small sample test)