Testing of Hypothesis: Large Sample Tests

Lesson 5: Test of Hypothesis for Difference of Two Population Standard Deviations

If the test statistics are standard deviations s_1 and s_2 of samples of sizes n_1 and n_2 out of two populations with standard deviations σ_1 and σ_2, then the test statistic for the sampling distribution of the differences of two population standard deviations will follow a standard normal distribution and will be given by

$$z = \frac{(s_1 - s_2) - (\sigma_1 - \sigma_2)}{\sqrt{\frac{{\sigma_1}^2}{2n_1} + \frac{{\sigma_2}^2}{2n_2}}} - - - - (1)$$

The steps for testing are:

(i) Formulate null hypothesis, $H_0 : \sigma_1 = \sigma_2$: There is no significant difference between two population standard deviations

(ii) Formulate alternative hypothesis, $H_1 : \sigma_1 \neq \sigma_2$ (or otherwise) : There is a significant difference between two population standard deviations

(iii) Level of significance is : α

(iv) From the two tailed test, the critical region is : $|z| \geq z_{\alpha/2}$

(v) Compute the test statistic z from equation (1)

(vi) Reject H_0 if the computed value of z falls in the critical region, otherwise accept H_0

Problems:

Ex.1. Random samples drawn from two countries gave the following data relating to the heights of adult males:

	Country A	Country B
Standard deviation (inches)	2.58	2.50
No. of individuals	1000	1200

Is the difference between the standard deviations of heights of adult males in two countries significant?

Solution:

Here we have, in country A, $n_1 = 1000$ and $s_1 = 2.58$. Also in country B, $n_2 = 1200$ and $s_2 = 2.5$.

(i) Null hypothesis, $H_0 : \sigma_1 = \sigma_2$: There is no significant difference between two population standard deviations

(ii) Alternative hypothesis, $H_1 : \sigma_1 \neq \sigma_2$: There is a significant difference between two population standard deviations

(iii) $\alpha = 0.05$ (5% level)

(iv) The critical region at 5% level of significance is $|z| \geq 1.96$

(v) $z = \frac{(s_1 - s_2)-(\sigma_1 - \sigma_2)}{\sqrt{\frac{\sigma_1{}^2}{2n_1}+\frac{\sigma_2{}^2}{2n_2}}} = \frac{(2.58 - 2.5)-0}{\sqrt{\frac{2.58^2}{2000}+\frac{2.5^2}{2400}}} = 1.03$

(vi) As computed $|z| < 1.96$, i.e, the test statistic does not lie in the critical region, so H_0 is accepted

Hence, we conclude that the difference between the standard deviations of heights of adult males in two countries is not significant.

Ans.

Ex.2. A large organization produces electric light bulbs in each of its two factories. A test is carried out to check the efficiency of both the factories ascertaining the variability of the life of the bulbs produced by each factory. The results are as follows:

	Factory A	Factory B
Standard deviation of lifetime (hours)	240	220
No. of bulbs in sample	100	200

Determine whether the difference between the variability of life of light bulbs from the two factories is significant (at 1% level).

Solution:

Here in factory A, $n_1 = 100$ and $s_1 = 240$. In factory B, $n_2 = 200$ and $s_2 = 220$.

(i) Null hypothesis, $H_0 : \sigma_1 = \sigma_2$: There is no significant difference between two population standard deviations

(ii) Alternative hypothesis, $H_1 : \sigma_1 \neq \sigma_2$: There is a significant difference between two population standard deviations

(iii) $\alpha = 0.01$

(iv) Critical region at 1% level of significance is $|z| \geq 2.58$

(v) $z = \dfrac{(s_1 - s_2) - (\sigma_1 - \sigma_2)}{\sqrt{\dfrac{{\sigma_1}^2}{2n_1} + \dfrac{{\sigma_2}^2}{2n_2}}} = \dfrac{(240 - 220) - 0}{\sqrt{\dfrac{240^2}{200} + \dfrac{220^2}{400}}} = 0.9889$

(vi) Since computed $|z| < 2.58$, i.e, it does not lie in the critical region, so null hypothesis H_0 is accepted

Hence we conclude that the difference between the variability of life of light bulbs from the two factories is not significant.

Ans.