Problems : 1. Suppose 4 married couples are arranged to be seated in a row in randomly selected order. What is the probability that no couple is together (i.e. wife & husband are seated together) ?

Sol^n. $E \rightarrow$ no married couple is together.

$E^c \rightarrow$ at least one couple is together

$A_i \rightarrow$ i^{th} couple sits together $i = 1, 2, 3, 4$

$$E^c = \bigcup_{i=1}^{4} A_i$$

$$P(A_i) = \frac{2 \times 7!}{8!}, \quad i = 1, 2, 3, 4$$

$$P(A_i \cap A_j) = \frac{2^2 \times 6!}{8!}, \quad i < j$$

$$P(A_i \cap A_j \cap A_k) = \frac{2^3 \times 5!}{8!}, \quad i < j < k$$

$$P\left(\bigcap_{i=1}^{4} A_i\right) = \frac{2^4 \times 4!}{8!}$$

$$P(E^c) = P\left(\bigcup_{i=1}^{4} A_i\right) = \sum P(A_i) - \sum_{i<j}\sum P(A_i \cap A_j)$$

$$+ \sum_{i<j<k}\sum\sum P(A_i \cap A_j \cap A_k) - P\left(\bigcap_{i=1}^{4} A_i\right)$$

$$= \frac{23}{35}$$

* Ex.

$$P(E) = \frac{12}{35} \simeq 0.34$$

2. Three players A, B, C, throw a fair dice in order ABC, ABC, ... What is the prob that

(i) A is the second player to get a six for the first time?

(ii) A is the last player ....

(iii) A is the first ....

<u>Sol^n</u>: A gets a thrown on $(3r+1)^{th}$ trial

$r = 0, 1, 2, \ldots$. If he is second,

then he will throw on $(3r+1)^{th}$ trial, $r = 1, 2 \ldots$

On $(r+1)$ trials that A gets to throw, he

does not get 6 on r trials & 6 on $(r+1)^{th}$

trial. $\rightarrow \left(\frac{5}{6}\right)^r \cdot \frac{1}{6}$.

B may get at least one six in his r trials

with prob. $\left\{1 - \left(\frac{5}{6}\right)^r\right\}$

C will not any six in his r trials w/p $\left(\frac{5}{6}\right)^r$

$P(\text{A gets 6 after } \underbrace{B}_{C} \text{ but before } \underbrace{C}_{B})$

$$= \left(\frac{5}{6}\right)^{2r} \left\{ 1 - \left(\frac{5}{6}\right)^{r} \right\} \cdot \frac{1}{6}$$

$P(\text{A is second to get a 6 for the } 1^{\text{st}} \text{ time})$

$$= 2 \sum_{r=1}^{\infty} \left(\frac{5}{6}\right)^{2r} \left\{ 1 - \left(\frac{5}{6}\right)^{r} \right\} \frac{1}{6} = \frac{300}{1001}$$

$$\simeq 0.2997$$

(ii) P(A is last to get a six for the first time)

$$= \sum_{r=1}^{\infty} \left(\frac{5}{6}\right)^r \left\{1 - \left(\frac{5}{6}\right)^r\right\}^2 \cdot \frac{1}{6} = \frac{305}{1001}$$

$$\approx 0.3047$$

(iii) P(A is the first to get a six for the 1st time)

$$= \frac{1}{6} + \left(\frac{5}{6}\right)^3 \cdot \frac{1}{6} + \left(\frac{5}{6}\right)^6 \cdot \frac{1}{6} + \cdots$$

$$= \frac{36}{91} \simeq \frac{396}{1001} \approx 0.40$$

3. Consider all families with two children and assume that boys and girls are equally likely. (i) If a family is chosen at random and is found to have a boy, what is the prob that the other one is also a boy?

(ii) If a child is chosen at random from these families and is found to be a boy, what is the prob that the other child in the family is also a boy?

Sol^n $b \to$ boy, $g -$ girl

(i) $\Omega = \{(b, b), (b, g), (g, b), (g, g)\}$

$A \to$ family has a boy

$P(A) = \frac{3}{4}$, $B \to$ Second child is also a boy

$P(A \cap B) = \frac{1}{4}$, $P(B|A) = 1/3$.

(ii) $\Omega^* = \{b_b, b_g, g_b, g_g\}$

$A \to$ child is a boy, $P(A) = \frac{1}{2}$

$B \rightarrow$ child has a brother

$P(A \cap B) = \frac{1}{4}$, $P(B|A) = \frac{1}{2}$

4\.. 2 kinds of tubes $\longrightarrow$ electronic gadget

If one of each kind is defective, it will not function

$P(\text{first kind is def}) = 0.1$

$P(\text{second kind is def}) = 0.2$

It is known that two tubes are defective.

What is the prob that tube is still working?

Sol^n $A \rightarrow$ two tubes are defective

$$P(A) = (0.1)^2 + (0.2)^2 + 2(0.1)(0.2)$$
$$= 0.09$$

$B \rightarrow$ gadget is still working

$$P(A \cap B) = (0.1)^2 + (0.2)^2 = 0.05$$

$$P(B|A) = \frac{5}{9}.$$

5. Suppose all circuits in a machine are either from brand A or B.

% of defective circuits from A is 5%.

% . . . . . . B is 1%.

We inspect randomly two circuits. If the first is found to be defective, what is the probability the second is also defective?

$D_1 \rightarrow$ first one is defective

$$P(D_1) = P(D_1 | A)\, P(A) + P(D_2 | B)\, P(B)$$

$$= \frac{5}{100} \times \frac{1}{2} + \frac{1}{100} \times \frac{1}{2} = 0.03$$

$D_2 \rightarrow$ second one is defective

$$P(D_1 \cap D_2) = P(D_1 \cap D_2 | A)\, P(A) + P(D_1 \cap D_2 | B)\, P(B)$$

$$= \left(\frac{5}{100}\right)^2 \times \frac{1}{2} + \left(\frac{1}{100}\right)^2 \times \frac{1}{2} = \frac{13}{(100)^2}$$

$$P(D_2 \mid D_1) = \frac{13}{300} > P(D_1) = \frac{3}{100}$$

$G_1 \rightarrow$ first is good

$G_2 \rightarrow$ second is good

$$P(G_1) = \frac{95}{100} \times \frac{1}{2} + \frac{99}{100} \times \frac{1}{2} = \frac{97}{100} = 0.97$$

$$P(G_2 \cap G_1) = \left(\frac{95}{100}\right)^2 \times \frac{1}{2} + \left(\frac{99}{100}\right)^2 \times \frac{1}{2}$$

$$P(G_2 \mid G_1) = 0.9704 > 0.97$$