Testing of Hypothesis: Small Sample Tests

Lesson 6: Test of Hypothesis for a Single Population Mean μ (population variance σ^2 is unknown)

Let $\bar{x}$ is the mean of a sample of size n (< 30) drawn from a normal population with mean μ and unknown variance. Then the unbiased estimator of the population variance σ^2 is given by

$$U^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2 = \frac{1}{n-1} \times n \times \text{sample variance}$$

$$= \frac{n}{n-1} s^2 \ (s = \text{sample standard deviation})$$

Thus the estimate of σ is given by $U = \sqrt{\frac{n}{n-1}}s$.

Then by Central Limit Theorem, the sampling distribution of $\bar{x}$ is normally distributed with mean μ and standard deviation $\frac{U}{\sqrt{n}} = \frac{s}{\sqrt{n-1}}$ and the test statistic for this case is given by

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n-1}}} ------(1)$$

with ν = (n – 1) degrees of freedom

Properties of t-Distribution:

- Symmetrical about mean
- Mean = 0 = skewness, standard deviation = $\sqrt{\frac{\nu}{\nu-2}}$, kurtosis > 0 (leptokurtic)
- When the degrees of freedom ν → ∞, then t-distribution approaches standard normal distribution

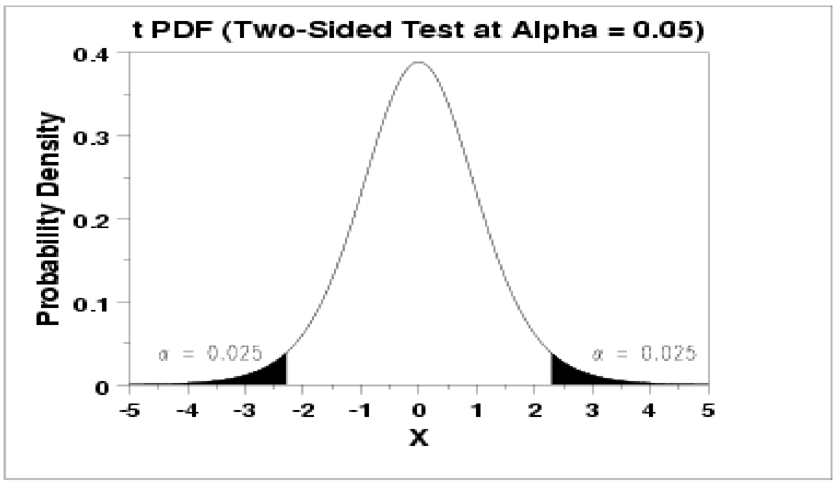
The steps for the testing are:

(i) Formulate null hypothesis, H_0 : $\mu = \mu_1$: There is no significant difference between sample mean and population mean

(ii) Formulate alternative hypothesis, H_1 : $\mu \neq \mu_1$ (or otherwise) : There is a significant difference between sample mean and population mean

(iii) Level of significance is : α (say 5%)

(iv) From the two tailed test, the critical region for $\nu = (n - 1)$ degrees of freedom is :



(v) Compute the test statistic t

(vi) Reject H_0 if the computed value of t falls in the critical region, otherwise accept H_0

<u>Note:</u>

For small sample tests, if σ is unknown, then it can be replaced by sample standard deviation s provided that the population is a normal distribution.

Problems:

Ex.1. A random sample of size 20 from a normal population gives a sample mean of 42 and sample standard deviation of 6. Test the hypothesis that the population mean is 44 (at 5% significant level).

Solution:

Here the population mean $\mu = 44$

The sample size $n = 20$ (< 30), hence small sample test is to be applied

The sample mean $\bar{x} = 42$

The sample standard deviation $s = 6$

Degrees of freedom $\nu = (20 - 1) = 19$

Since the population is normally distributed with an unknown standard deviation σ, hence σ can be replaced by sample standard deviation s in computation of the test statistic t.

(i) Null hypothesis, $H_0 : \mu = 44$

(ii) Alternative hypothesis, $H_1 : \mu \neq 44$

(iii) $\alpha = 0.05$

(iv) Critical region at 5% level of significance for 19 degrees of freedom is $|t| \geq 2.093$ [refer the t-table in last slide]

(v) $t = \dfrac{\bar{x}-\mu}{\frac{s}{\sqrt{n-1}}} = \dfrac{42-44}{\frac{6}{\sqrt{19}}} = -1.45$

(vi) As computed $|t| < 2.093$, therefore H_0 is accepted

Hence, we conclude that the population mean is 44.

Ans.

Ex.2. The mean weekly sales of soap bars in departmental stores was 146.3 bars per store. After an advertising campaign, the mean weekly sales in 22 stores for a typical week increased to 153.7 with standard deviation of 17.2. Was the advertising campaign successful? (at 5% level)

Solution:

Here the population mean $\mu = 146.3$

The sample size $n = 22$ (< 30), hence small sample test is to be applied

The sample mean $\bar{x} = 153.7$

The sample standard deviation $s = 17.2$

Degrees of freedom $\nu = (22 - 1) = 21$

Let us assume that the sales of soap bars is normally distributed with an unknown variance. Then the population standard deviation σ can be replaced by the sample standard deviation s in computation of the test statistic t.

(i) Null hypothesis, $H_0 : \mu = 146.3$

(ii) Alternative hypothesis, $H_1 : \mu > 146.3$, i.e, the mean weekly sales is improved, which means that the campaign was successful

(iii) $\alpha = 0.05$

(iv) Critical region at 5% level of significance for 21 degrees of freedom is $t \geq 1.721$ [refer the t-table]

(v) $t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n-1}}} = \frac{153.7 - 146.3}{\frac{17.2}{\sqrt{21}}} = 1.97$

(vi) As the computed value of $t > 1.721$, hence H_0 is rejected

Thus we conclude that the advertising campaign was indeed successful.

Ans.

Ex.3. Ten individuals were chosen at random from a normal population and their heights were found to be in inches as 63, 63, 66, 67, 68, 69, 70, 70, 71 and 71. Test the hypothesis that the mean height of the population is 66 inches.

Solution:

Here the population mean μ = 66 inches

The sample size n = 10 (< 30), hence small sample test is to be applied

The sample mean $\bar{x} = \frac{1}{10} \sum_{i=1}^{10} x_i = 67.8$ inches

The sample standard deviation s = $\sqrt{\frac{1}{10} \sum_{i=1}^{10}(x_i - \bar{x})^2}$ = 2.857 inches

Degrees of freedom ν = (10 – 1) = 9

Since the population is normally distributed with an unknown standard deviation σ, hence σ can be replaced by sample standard deviation s in computation of the test statistic t.

(i) Null hypothesis, H_0 : μ = 66

(ii) Alternative hypothesis, H_1 : μ ≠ 66

(iii) α = 0.05 (we take 5% level of significance)

(iv) Critical region at 5% level of significance for 9 degrees of freedom is |t| ≥ 2.262 [refer the t-table]

(v) $t = \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n-1}}} = \frac{67.8-66}{\frac{2.857}{\sqrt{9}}} = 1.89$

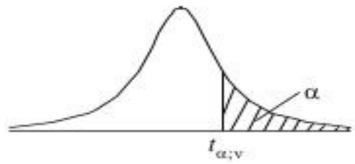
(vi) Since computed $|t|$ < 2.262, hence H_0 is accepted

Hence we conclude that the mean height of the population is 66 inches.

Ans.

Table of the Student's *t*-distribution

The table gives the values of $t_{\alpha;\nu}$ where
$\Pr(T_\nu > t_{\alpha;\nu}) = \alpha$, with ν degrees of freedom



ν \ α	0.1	0.05	0.025	0.01	0.005	0.001	0.0005
1	3.078	6.314	12.076	31.821	63.657	318.310	636.620
2	1.886	2.920	4.303	6.965	9.925	22.326	31.598
3	1.638	2.353	3.182	4.541	5.841	10.213	12.924
4	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	1.319	1.714	2.069	2.500	2.807	3.485	3.767
24	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	1.296	1.671	2.000	2.390	2.660	3.232	3.460
120	1.289	1.658	1.980	2.358	2.617	3.160	3.373
∞	1.282	1.645	1.960	2.326	2.576	3.090	3.291