Basic Probability

Lesson 1: Introduction to Classical Probability

If an experiment is repeated under essentially homogeneous and similar conditions, we come across two types of situations:

(i) The result or "outcome" is unique or certain

(ii) The result is not unique but may be one of the several possible outcomes

The phenomena covered in (i) are called deterministic or predictable. For example, for a perfect gas, PV = constant, provided the temperature remains the same.

The phenomena covered in (ii) are called probabilistic or unpredictable. For example, in tossing of a coin, one is not sure if a head or a tail would be obtained. In such cases, we talk about the chance or probability of happening of an event which is taken to be quantitative measure of certainty.

Some Basic Terminologies:

- Random Experiment: If in each trial of an experiment conducted under identical conditions, the result is not unique, but may be any one of the possible results, then such an experiment is called a random experiment. For example, tossing of a coin, throwing a die etc.

- Trial and Event: Any particular performance of a random experiment is known as trial and any one of the several possible results is called an event or a case or an outcome. For example, tossing of a coin is a trial and getting either a head or a tail is an event.

- Exhaustive Events: The total number of possible outcomes in any trial is called exhaustive events. For example, in tossing of a coin, there are two exhaustive cases, head and tail.

- Favorable Events: The number of outcomes ensuring the happening of a particular event in a trial is known as favorable events. For example, in throwing of two dice, the number of favorable cases to get the sum 5 is: (1, 4), (4, 1), (2, 3), (3, 2), i.e, 4.

- Mutually exclusive Events: Events are called mutually exclusive if they cannot happen simultaneously in the same trial. For example, in tossing a coin, head and tail are mutually exclusive.

- Equally Likely Events: Events are called equally likely if taking into consideration of all the relevant evidences, there is no reason to expect one in preference to the others. For example, in tossing an unbiased coin, the two faces head and tail are equally likely.

- Independent Events: Two events are called independent to each other, if the occurrence (or non-occurrence) of one event is not affected by the knowledge of the occurrence of the other event. For example, in tossing an unbiased coin, the event of getting a head in the first toss is independent of getting a head in the second, third and subsequent throws.

Mathematical or Classical or 'a priori' Definition of Probability:

If a trial results in n exhaustive, mutually exclusive and equally likely events, where m of them are favorable to the happening of an event E, then the probability or chance p of happening of the event E is given by

$$p = P(E) = \frac{\text{no. of favorable cases}}{\text{no. of exhaustive events}} = \frac{m}{n}$$

------------------ (1)

Note:

(i) Sometimes, equation (1) can also be rephrased as
(a) The odds in favor of happening of the event E are $m : (n - m)$
(b) The odds against happening of the event E are $(n - m) : m$

(ii) If the happening of an event E is called a 'success', then the non-happening of E will be called 'failure' and it is denoted by $\bar{E}$. Subsequently, the probability of $\bar{E}$ will then be given by

$$P(\bar{E}) = \frac{\text{no. of cases favorable to the non} - \text{happening of E}}{\text{no. of exhaustive events}}$$
$$= \frac{n-m}{n} = 1 - \frac{m}{n} = 1 - p = q \quad \text{(say)}$$

Hence we can say, $p + q = 1$, $0 \leq p \leq 1, 0 \leq q \leq 1$.

(iii) If P(E) = 1, then E is called a certain event and if P(E) = 0, then E is called an impossible event.

Limitations of Classical Definition:

The classical definition breaks down in the following cases:

(i) If the various outcomes are not equally likely
(ii) If the exhaustive number of cases in a trial is infinite

Problems:

Ex.1. What is the chance that a leap year selected at random will contain 53 Sundays?

Solution: In a leap year, there are 52 complete weeks and 2 extra days. These two extra days can be:

(Sunday, Monday), (Monday, Tuesday), (Tuesday, Wednesday), (Wednesday, Thursday), (Thursday, Friday), (Friday, Saturday) and (Saturday, Sunday)

Hence, there are total 7 possible outcomes out of which 2 cases are such that they contain Sunday. Therefore, we have, $m = 2$ and $n = 7$.

Thus, the chance that the year will contain 53 Sundays is, $p = \frac{2}{7}$

Ans.

Ex.2. (i) From a pack of 52 cards, three cards are drawn at random. Find the chance that they are a king, a queen and a jack.

(ii) Four cards are drawn from a pack of cards. Find the probability that there are two spades and two hearts.

Solution:

(i) From a pack of 52 cards, 3 cards can be chosen in ${}^{52}C_3$ ways. Thus, the exhaustive number of cases = ${}^{52}C_3$.

We know that there are 4 kings, 4 queens and 4 jacks in a pack of cards. 1 king can be drawn out of 4 kings in ${}^{4}C_1$ way. Similarly, 1 queen and 1 jack can be drawn from 4 queens and 4 jacks in ${}^{4}C_1$ ways each. Further, each way of drawing a king is associated to each of the ways of drawing a queen and a jack. Hence the total number of favorable cases are = ${}^{4}C_1 \times {}^{4}C_1 \times {}^{4}C_1$.

Thus the probability = $\frac{{}^{4}C_1 \times {}^{4}C_1 \times {}^{4}C_1}{{}^{52}C_3}$

(ii) Four cards can be drawn from a pack of cards in ${}^{52}C_4$ ways. Thus, the exhaustive number of cases = ${}^{52}C_4$.

Out of 13 spades, 2 can be drawn in ${}^{13}C_2$ ways and similarly out of 13 hearts, 2 can be drawn in ${}^{13}C_2$ ways. Thus the total number of favorable cases are = ${}^{13}C_2 \times {}^{13}C_2$.

Hence the required probability = $\frac{{}^{13}C_2 \times {}^{13}C_2}{{}^{52}C_4}$

Ans.

Ex.3. From 25 tickets marked with the first 25 numerals, one is drawn at random. Find the chance that (i) it is a multiple of 5 or 7 (ii) it is a multiple of 3 or 7.

Solution:

(i) Among the first 25 numerals, numbers which are multiple of 5 are: 5, 10, 15, 20, 25, i.e 5 numbers and numbers which are multiple of 7 are: 7, 14, 21, i.e, 3 numbers. Hence total number of favorable cases = 5 + 3 = 8. Thus the required probability = 8/25.

(ii) In a similar way, numbers which are multiple of 3 are: 3, 6, 9, 12, 15, 18, 21, 24, i.e, there are 8 numbers. We have already seen that there are 3 multiples of 7. But the number 21 is common to both sets of multiples. Hence the total favorable cases = 8 + 3 – 1 = 10. Thus the required probability = 10/25 = 2/5.

Ans.

Ex.4. If the letters of the word "REGULATIONS" are arranged at random, what is the chance that there will be exactly 4 letters between R and E?

Solution: Out of 11 letters from the word, 2 letters can be arranged in ${}^{11}P_2$ ways. This gives the exhaustive number of cases. Let us see all possible arrangements where there can be exactly 4 letters between R and E:

(i) R in 1st place; E is 6th place
(ii) R in 2nd place; E in 7th place
(iii) R in 3rd place; E in 8th place
(iv) R in 4th place; E in 9th place
(v) R in 5th place; E in 10th place
(vi) R in 6th place; E in 11th place

Also, R and E can interchange their positions. So, there are total 2 $\times$ 6 = 12 favorable cases. Hence the required probability = $\frac{12}{{}^{11}P_2} = \frac{12}{110} = \frac{6}{55}$

Ans.