Problem Set 4: due 23:59 on 29 Nov

Related reading

Ghisellini Ch2, Ch5, Ch6, R&L Sections 7.1-7.4

Problem 1 – Flat-spectrum jet "signature": final project prep!

Note: in this problem you will learn how to quantitatively reproduce the flat spectrum of jets as covered in class. It is also exactly what you will need to do for part of the final project, the only difference is that you will add in special relativity and tune the values to match some real data. Please therefore aim to finish parts (a) through (e) in the coming WCs, and work on the code development while you have plenty of time to get help from the TAs!

Imagine a perfectly conical jet (fixed opening angle of 10°) originating from a black hole a distance D away from us, that emits isotropically, and with constant bulk velocity along its axis, v (please ignore special relativistic effects for now). You will need to assume that the total energy and magnetic and particle fluxes are conserved.

Assume accretion power $L_j = \eta \dot{M} c^2$ enters the jets as moving plasma (just think about one jet) at the base with the bulk velocity given above. The base of the jets have a radius of r_0. For this problem let's not worry about protons, so assume the energy density of the plasma is split equally ("equipartition") between the magnetic field and electrons, which have a distribution $n_e(E_e) = CE_e^{-p}$ erg^{-1} cm^{-3} where p=2 and $E_e = \gamma_e m_e c^2$.

(a) Assume the magnetic field is mainly torroidal, meaning mostly 'wrapped' around the jets (hint: you need this to calculate the conservation of magnetic flux along the jets). How do B and n_e evolve along the jets as a function of r?

According to R&L (Rybicki & Lightman), for a powerlaw of electrons the synchrotron self-absorption coefficient is given by:

$$\alpha_\nu \propto CB^{(p+2)/2}\nu^{-(p+4)/2}, \tag{1}$$

where C is the same electron distribution normalization mentioned above.

b) What is the expression for the optical depth τ in terms of lengthscale r and ν? (*hint: think units*).

c) At the photosphere (defined where $\tau = 1$), what is the relationship between r and ν? (*hint: reduce all quantities to their dependencies on either r or ν*).
According to R&L, for a power law of electrons the total radiated power per volume per frequency is:

$$P_\nu \approx \frac{10^{-22}CB}{(p+1)}\left(\frac{10^{-7}\nu}{B}\right)^{-(p-1)/2} \quad \text{erg cm}^{-3}\text{s}^{-1}\text{ Hz}^{-1} \tag{2}$$

d) Express the isotropic flux density at Earth in terms of dependence on r, D and ν.

e) Use your answer to (c) to find the flux density of only the photosphere as a function of frequency, what is its dependence on ν?

f) Code this problem up by dividing your jet into slices (making sure you use even bins in logarithmic space!) , loop through the slices and calculate the spectum from each slice, then add to a total spectrum for the entire jet (or technically both jets). Ideally you will plot each component as well as the total to check that your ν_{SSA} is doing what you expect. To test your code, use an accretion power appropriate for Sgr A*, $r_0 = 10r_g$ and $v = 0.3c$ (but at this point don't Lorentz transform anything, it's not relativistic enough to be a huge effect). Normalise your initial L_j such that either F_ν in Janskys (10^{-23} erg/cm^{-2}/s/Hz) or total power per frequency bin (erg/s/Hz) matches the values for Sgr A* at 10^{12} Hz.

Problem 2 – Bremsstrahlung revisited

(a) Use exactly the same geometry discussed in the lecture, for an electron travelling with velocity v passing by an ion of charge Ze with closest approach b. Make the same assumptions about the small angle and calculate the Coulomb force to derive an expression for $\ddot{\vec{d}}$. So far this should be familiar, but keep the full geometrical terms rather than making assumptions.

(b) Plug your expression for $\ddot{\vec{d}}$ into the expression derived in class for $\tilde{d}(\omega)$, and this time use an integral table to find an exact analytic solution for the integral. You should end up with an exponential term.

(c) Use your derived expression to find the bremsstrahlung spectrum $dW/d\omega$ from the single electron–ion system, which should only be in terms of some constants and b, v and ω. Plot this function with *labelled* axes showing the spectrum as a function of ω.

Problem 3 – Compton scattering off the CMB

Very high energy electrons, with $E_e = 100$ GeV have been observed at the top of the Earth's atmosphere. While traveling through space, these electrons must have undergone inverse Compton scattering with the microwave photons which are at a temperature of $T = 2.73$ K. Estimate the characteristic timescale in which these electrons lose their energy. In what waveband do the upscattered photons typically show up?

Problem 4 – Sebastian Heinz's amazing IC tutorial!: final project prep

My friend and colleague at University of Wisconsin in the US has developed a fantastic inverse Compton tutorial Jupyter notebook! Eventually you will use the routines there to build into your final projects so it's time to familiarise yourselves with it first.

Please open the `Inverse_Compton_Monte-Carlo_Tutorial.ipynb` Jupyter notebook and read your way through to the Tutorial section 4. Complete part 4.1. Discuss in WC if you have any problems with this and make sure you understand it before moving onto the next part.