Let $E_1 = \mathbb{E}_{x,z \sim P(x,z)}$ and $E_2 = \mathbb{E}_{x,z \sim p(x)p(z)}$

$$L = -E_1(\ln \sigma f_\theta) - E_2(\ln \sigma f_\theta)$$

$$\sigma(f) = \frac{1}{1+e^{-f}} \quad \Big| \quad \sigma(-f) = \frac{1}{1+e^{f}}$$

$$L = -E_1 \ln\left(\frac{1}{1+e^{-f}}\right) - E_2 \ln(1+e^{f})$$ Sub in (-)

$$L = E_1 \ln(1+e^{-f}) + E_2 \ln(1+e^{f})$$ Now derivative

$$\frac{dL}{d\theta} = E_1\left(\frac{1}{1+e^{-f}} \cdot -e^{-f}\right) + E_2\left(\frac{1}{1+e^{f}} e^{f}\right)$$

$$= E_1\left(\underbrace{\frac{e^{-f}}{1+e^{-f}}}_{\text{equivalent to } \sigma(-f)}\right) + E_2\left(\underbrace{\frac{e^{f}}{1+e^{f}}}_{\sigma(f)}\right)$$

$$\frac{dL}{d\theta} = E_1 \sigma(-f) + E_2 \sigma(f)$$

$$= \mathop{E}_{x,z \sim P(x,z)} \sigma(f_\theta(x,z)) + \mathop{E}_{x,z \sim p(x)p(z)} \sigma(f_\theta(x,z))$$

$f_\theta(x,z)$. given by probability distributions

$$\frac{dL}{d\theta} = E_1\left(\frac{P(x,z)}{P(x,z) + P(x)P(z)} \right) + E_2\left(\frac{P(x)P(z)}{P(x,z) + P(x)P(z)} \right)$$

Use defintion of expectation value $\quad E = \int x\, P(x)\, dx$

$$\frac{dL}{d\theta} = E_{x,z \sim P(x,z)}\left(\quad \right) + E_{x,z \sim P(x)P(z)}\left(\quad \right)$$

$$= \iint \frac{P(x,z)\,P(x,z)}{P(x,z) + P(x)P(z)}\, dx\, dz + \iint \frac{P(x)P(z)\; P(x)P(z)}{P(x,z) + P(x,z)}\, dx\, dz$$

This minimizes when the two integrads are equivalent

lets look at numerators

$$P(x,z)\,P(x,z) = P(x)P(z)\; P(x)P(z)$$

which we see we get $\quad r(x,z) = \dfrac{P(x,z)}{P(x)P(z)}$

$\therefore \quad \dfrac{dL}{d\theta}$ is minimized when $\quad f_\theta(x,z) = \ln(r(x,z))$

$$r = \frac{P(x,z)}{P(x)P(z)}$$

2) $L(M_\phi) = \int dx\,dy\, P(x,y)\,(x - M_\phi(y))^2$

$$L(M_\phi) = E_{x,y \sim P(x,y)} (x - M_\phi(y))^2$$

$$\frac{dL}{d\phi} = 0 = E_{xy}\left(\cancel{2}(x - M_\phi(y)) \circ -\left(\cancel{\frac{dM_\phi}{d\phi}}\right)\right)$$

Cancel out since $= 0$

$$M_\phi(y) = E_{x,y \sim P(x|y)}(x) = \underbrace{\int x\, P(x|y)\, dy\, dx}_{E(x|y)}$$

$$\boxed{M_\phi(y) = E(x|y)}$$