

$$\text{Let } E_1 = E_{X,Z \sim P(X,Z)} \quad \text{and } E_2 = E_{X,Z \sim P(X)P(Z)}$$

$$L = -E_1(\ln \sigma f_0) - E_2(\ln \sigma f_0)$$

$$\sigma(f) = \frac{1}{1+e^{-f}} \quad | \quad \sigma(-f) = \frac{1}{1+e^f}$$

$$L = -E_1 \ln\left(\frac{1}{1+e^{-f}}\right) - E_2 \ln(1+e^f) \quad \text{Sub in (-)}$$

$$L = E_1 \ln(1+e^f) + E_2 \ln(1+e^f) \quad \text{Now derivative}$$

$$\frac{dL}{df} = E_1\left(\frac{1}{1+e^{-f}} \cdot -e^{-f}\right) + E_2\left(\frac{1}{1+e^f} e^f\right)$$

$$= E_1\left(\frac{e^{-f}}{1+e^{-f}}\right) + E_2\left(\frac{e^f}{1+e^f}\right)$$

$$\underbrace{\hspace{10em}}_{\text{equivalent to } \sigma(-f)} \quad \underbrace{\hspace{10em}}_{\sigma(f)}$$

$$\frac{dL}{df} = E_1 \sigma(-f) + E_2 \sigma(f)$$

$$= E_{X,Z \sim P(X,Z)} \sigma(f_0(X,Z)) + E_{X,Z \sim P(X)P(Z)} \sigma(f_0(X,Z))$$

$f_0(X,Z)$ . given by probability distributions

$$\frac{dL}{d\theta} = E_1 \left( \frac{P(x,z)}{P(x,z) + P(x)P(z)} \right) + E_2 \left( \frac{P(x)P(z)}{P(x,z) + P(x)P(z)} \right)$$

Use definition of expected value  $E = \int x P(x) dx$

$$\frac{dL}{d\theta} = E_{x,z \sim P(x,z)} \left( \frac{P(x,z)}{P(x,z) + P(x)P(z)} \right) + E_{x,z \sim P(x)P(z)} \left( \frac{P(x)P(z)}{P(x,z) + P(x)P(z)} \right)$$

$$= \iint \frac{P(x,z)P(x,z)}{P(x,z) + P(x)P(z)} dx dz + \iint \frac{P(x)P(z)P(x)P(z)}{P(x,z) + P(x)P(z)} dx dz$$

This minimizes when the two integrands are equal  
lets look at numerators

$$P(x,z)P(x,z) = P(x)P(z)P(x)P(z)$$

which we see we get  $r(x,z) = \frac{P(x,z)}{P(x)P(z)}$

∴  $\frac{dL}{d\theta}$  is minimized when  $f_\theta(x,z) = \ln(r(x,z))$

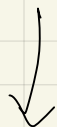
$$r = \frac{P(x,z)}{P(x)P(z)}$$

$$2) \quad L(M_0) = \int dx dy P(x, y) (x - M_0(y))^2$$

$$L(M_\phi) = E_{x, y \sim P(x, y)} (x - M_\phi(y))^2$$

$$\frac{dL}{d\phi} = 0 = E_{x, y} \left( \cancel{2(x - M_\phi(y))} \cdot - \left( \cancel{\frac{dM_\phi}{d\phi}} \right) \right) \quad \text{Cancel at} \\ \text{since } = 0$$

$$M_\phi(y) = E_{x, y \sim P(x, y)}(x) = \underbrace{\int x P(x, y) dy dx}_{E(x|y)}$$



$$M_\phi(y) = E(x|y)$$