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2 authors:

Edward P. J. van den Heuvel
University of Amsterdam

Camiel De Loore

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The Nature of X-ray Binaries III. Evolution of Massive Close Binaries with One Collapsed Component – with a Possible Application to Cygnus X–3*

E. P. J. van den Heuvel

Astronomical Institute, Utrecht, the Netherlands
Astrophysical Institute, Vrije Universiteit, Brussels

C. De Loore

Astrophysical Institute, Vrije Universiteit, Brussels

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Summary. The evolution is computed for close binaries consisting of a massive primary ($15\,M_\odot$ or $21\,M_\odot$) together with a neutron star secondary ($M_\odot$ or $2\,M_\odot$) with orbital periods between 2 and 6 days. Two cases are considered, viz. (i) the neutron star is able to accrete the $10^{-3}\,M_\odot$/yr lost by the primary after this star has started to overflow its Roche lobe, and (ii) accretion onto the neutron star is limited to $\lesssim 10^{-7}\,M_\odot$/yr, due to the critical luminosity; in case (ii) practically all mass lost from the primary is lost from the system or is dispersed around the system. In all cases it is found that before the system stabilizes the primary looses most of its hydrogen-rich outer layers and becomes a hot luminous star, mainly consisting of helium.

In case (i) the secondary becomes a massive black hole and the final binary period is of the order of a few hours. Also in case (ii) the binary period decreases drastically even if the escaping matter carries with it a minor fraction ($<10\%$) of the dynamical energy of the system.

It is suggested that the final systems may be identified with X-ray sources such as Cygnus X–3 ($P_x = 4^h.8$). In case of a black hole (instead of a neutron star) secondary the evolution of the system is expected to follow a similar pattern.

Key words: evolution – close binaries – X-ray sources – collapsed stars

I. Introduction

We consider the evolution of massive close binaries in which one of the components is a neutron star. (In Section VI we also discuss the evolution for the case of a black hole secondary.) Such systems are thought to resemble systems such as Cen X–3 (Pringle and Rees, 1972; van den Heuvel and Heise, 1972 (hereafter referred to as Paper I)) and other massive X-ray binaries such as Cyg X–1 (HD 226868, B 0 Ib, $P = 5^d.6$, Webster and Murdin, 1972), 2U 0900-40 (HD 77581, B 0.5 Ib, $P = 8^d.96$ Hiltner *et al.*, 1972), SMC X–1 (Sanduleak, Nr 160, B 0 I, $P = 3^d.8927$, Schreier *et al.*, 1972a; Liller, 1972; Webster *et al.*, 1972), 2U 1700-37 (HD 153919, O 7 f, $P = 3^d.412$, Jones *et al.*, 1972; Giacconi, 1972) and 2U 0525-06 (θ^2 Orionis, O 9.5 Vp, $P = 21^d.03$, Barbon *et al.*, 1972).

The orbital periods which we consider are chosen such that the systems evolve according to case B (cf. Kippenhahn and Weigert, 1967; Paczyński, 1971a). Two possibilities for the evolution of the secondary are considered: (i) the neutron star can accept all the matter lost by the primary. This may be the case if the neutron star is non-rotating and non-magnetized and/or the accreted matter has no angular momentum. In such a case, accretion rates as high as $10^{-1}\,M_\odot$/yr might be permitted as most of the gain in potential energy of the accreted matter is radiated in the form of neutrino's (Zeldovitch *et al.*, 1972). (Also in the case that accretion is non-spherical, e.g.: channelled by the magnetic field, high accretion rates might be possible as the photon flux in this case will be highly anisotropic), (ii) accretion onto the neutron star is limited to $\lesssim 10^{-7}\,M_\odot$/yr due to the critical luminosity. This might be the case if the neutron star is rotating and magnetized and/or the accreted matter has angular momentum (cf. Pringle and Rees, 1972). In case (ii) practically all the mass lost by the primary will be lost

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from the system or will be forced to form a ring or disk around the system.

We will not further discuss the specific merits of these two assumptions, but adopt as a working hypothesis that they form two physically reasonable alternatives for the accretion processes onto neutron stars or black holes. Before exploring the evolution of the system we give, in Section II, a justification for the assumption that most of the observed X-ray binaries are evolving according to case B or will be doing so in the future.

II. The Critical Period for Case B Evolution in Massive Binaries

We adopt the evolutionary tracks of massive stars computed by Simpson (1971) with the original Schwarzschild criterion in the form $(dT/dP) \leqq (dT/dP)_{ad}$, as these give best agreement with the observed distributions of blue and red supergiants in the Hertzsprung-Russell diagrams of young stellar groups (Chiosi and Summa, 1970; Simpson, 1971). The post-main-sequence tracks for stars of $15\,M_\odot$ and $30\,M_\odot$ show that these stars move on a Kelvin-Helmholtz time scale (τ_{K-H}) towards $\log T_e \sim 4.2$ (onset of helium burning) and from there on slowly expand on a nuclear time scale (Simpson, 1971). From this (together with the radii at the end of core-hydrogen burning) one computes that – for a secondary mass $\geqq M_\odot$ – binaries with primary masses of $15\,M_\odot$ and $30\,M_\odot$ are in case B if $1^d\!.5 \leqq P \leqq 10^d$ and $2^d \leqq P \leqq 29^d$, respectively. None of the optical candidates of the X-ray binaries mentioned in Section I has a spectral type later than B 0.5 I, which means that all are, or have descended from O-type main-sequence stars. Consequently all have $M \geqq 15\,M_\odot$ (cf. Underhill, 1966, p. 140). Also the primary of Cen X–3 has $M \geqq 15\,M_\odot$ (cf. Paper I). The binary periods and masses of these systems therefore indicate that – except for θ^2 Orionis – all these systems are in case B. If the primary of θ^2 Orionis has $M \geqq 24\,M_\odot$, also this system will be in case B. In view of their short binary periods and large X-ray fluxes ($> 10^{36}$ erg/s) the primaries of the first mentioned five X-ray systems are most probably overflowing their Roche lobes, and mass exchange is going on. Only in θ^2 Orionis the primary is certainly not yet filling its Roche lobe, which might explain the small X-ray flux of this source (Barbon *et al.*, 1972).

III. Choice of the Initial Parameters and Method of Computation

We consider systems with primary masses of $15\,M_\odot$ and $21\,M_\odot$ respectively and secondary masses of $M_\odot$ and $2\,M_\odot$ (the unknown upper mass limit for neutron stars does not exclude the latter value, cf. Ruderman, 1972). These secondary masses were chosen, since in Cen X–3 and SMC X–1 the secondaries have masses $\leqq 0.84\,M_\odot$ and $\leqq 1.5\,M_\odot$, respectively (cf. Paper I and Webster *et al.*, 1972). An initial composition $X = 0.60$, $Z = 0.04$ is adopted for the primaries. Just as in paper I the neutron star is expected to have been formed $\lesssim 1.7 \times 10^6$ yr after the first stage of mass exchange. The total main-sequence lifetimes of stars of $15\,M_\odot$ and $21\,M_\odot$ ($X = 0.60$, $Z = 0.04$) are $\sim 8 \times 10^6$ yr and 6×10^6 yr respectively; hence, the neutron stars have ages of $\gtrsim 6.3 \times 10^6$ and $\gtrsim 4.3 \times 10^6$ yr respectively when the OB stars (initially the secondaries) leave the main sequence and the mass exchange begins (cf. Paper I). This moment is taken as the starting point of our computations. At this moment the primary star has a contracting helium core and its envelope is rapidly expanding. The helium cores of stars of $15\,M_\odot$ and $21\,M_\odot$ have masses of $3.7\,M_\odot$ and $6.6\,M_\odot$ respectively. Cores with such masses contract and start helium burning regardless of the amount of mass lost from their envelopes (cf. Paczyński, 1971a). In this case the evolution of the primary star during the mass exchange can simply be computed with the concept of equilibrium models, a method which we will use here (cf. Giannone *et al.*, 1968; Paczyński, 1970); the primary will be able to attain a new stable situation only if it has lost so much mass that its thermal equilibrium radius R_e at the onset of core-helium burning has become equal to or smaller than the radius R_R of its Roche lobe. Both R_e and R_R are functions of the (decreasing) primary mass which will be computed in the following sections. The final primary mass at which the system stabilises is found from the condition $R_e(M_1) = R_R(M_1)$.

IV. The Thermal Equilibrium Radii of Remnants Resulting by Mass Loss from Helium-burning Stars with Masses of 15 $M_\odot$ and 21 $M_\odot$

Such stars represent possible final models of the initial primaries of $15\,M_\odot$ and $21\,M_\odot$, respectively, after these have been stripped of parts of their outer layers. The equilibrium models were calculated in the same way as by Giannone *et al.* (1968), viz: under the assumption that the cores consist purely of helium and the envelopes have the initial composition $X = 0.60$, $Z = 0.04$. The equilibrium radii of the helium burning stars which result by mass loss from the $15\,M_\odot$ star with the $3.7\,M_\odot$ helium core are represented by the fully drawn curve in Fig. 1. For $0.3\,M_\odot \leqq M_1 \leqq 3.7\,M_\odot$ the stars are helium-burning pure helium stars. The radii of these stars were obtained by interpolation between the sequence of models of pure helium stars at zero age calculated by Paczyński (1971b). For $M_1 < 0.3\,M_\odot$ the stars are pure helium white dwarfs, for which the classical mass-radius relation was adopted (cf. Schatzman, 1958). For models with hydrogen-rich envelopes ($M_1 > 3.7\, M_\odot$) the radius R_e increases very rapidly with mass. The figure shows that R_e remains very large unless the $15\, M_\odot$ primary has lost over $9\, M_\odot$ from its envelope. This mass loss will take place on the Kelvin-Helmholtz timescale τ_{K-H} which is given by (Paczyński, 1971a):

$$\tau_{K-H} = 3.1 \times 10^7\, M^2/RL\ \mathrm{yr}\,, \tag{1}$$

where R and L denote the stellar radius and luminosity, respectively. Inserting the values for the $15\, M_\odot$ and $21\, M_\odot$ stars at the moment that they have left the main sequence, one obtains in both cases:

$$\tau_{K-H} \cong 10^4\ \mathrm{yr}\,.$$

Hence, a rate of mass loss of the order of $10^{-3}\, M_\odot/\mathrm{yr}$ is expected. The $R_e(M_1)$ relation for the models resulting by mass loss from the $21\, M_\odot$ primary is very similar to that for the $15\, M_\odot$ primary, the difference being that in this case one has pure helium stars for $M_1 \leqq 6.6\, M_\odot$, and a very rapid increase in R_e for M_1 between $6.6\, M_\odot$ and $10\, M_\odot$.

IV. Evolution of the Systems During the Mass Loss from the Primary

Case (i)

In this case the neutron star will, during the mass exchange, soon pass the (unknown) upper mass limit for neutron stars and become a black hole. As the accretion process onto black holes is expected to proceed very similar to that onto neutron stars (cf. Section VI) we will assume in case (i) that also the black hole will be able to accept all the matter lost by the primary. It seems permissible then to apply the "usual" assumption of close binary evolution, i.e. of conservation of total mass and orbital angular momentum of the system (cf. Paczyński, 1971a), in which case the semi-major axis a of the system changes according to the formula

$$a/a_0 = (M_1 M_2 / M_1^0 M_2^0)^{-2} \tag{2}$$

where M_1 and M_2 indicate the masses of the primary and secondary, respectively, and sub- and super-scripts zero indicate the initial situation. Starting from a given combination of a_0, M_1^0, M_2^0 one can calculate a as a function of M_1 with Eq. (2), using the fact that

$$M_2 = M_1^0 + M_2^0 - M_1\,.$$

R_R then follows from the values of a, M_1, M_2 (Paczyński, 1971a). In Fig. 1 the dashed lines indicate R_R as a function of M_1 for the cases $M_1^0 = 15\, M_\odot$, $M_2^0 = M_\odot$, $P_0 = 2\overset{d}{.}087$ (= in the period of Cen X-3) and $M_1^0 = 15\, M_\odot$, $M_2^0 = 2\, M_\odot$, $P_0 = 2\overset{d}{.}37$, respectively. The figure shows that in these cases R_e becomes $\leqq R_R$ if $M_1 \leqq 3.7\, M_\odot$ and $\leqq 3.9\, M_\odot$, respectively. Consequently, these values indicate the final primary masses at which the system stabilises. Using Eq. (2) and Kepler's third law one then finds that the final periods in these cases are 1.8 and 11.3 hours, respectively. Starting from $P_0 = 5^d$, $M_1^0 = 15\, M_\odot$, $M_2^0 = M_\odot$, the system stabilises at $M_1 = 3.80\, M_\odot$, $P = 4.08$ hours. The changes in shape of the system for this case are depicted in the upper part of Fig. 2. In the same way we calculated the final systems for $M_1^0 = 15\, M_\odot$ and $21\, M_\odot$ and the initial periods and secondary masses indicated in table 1.

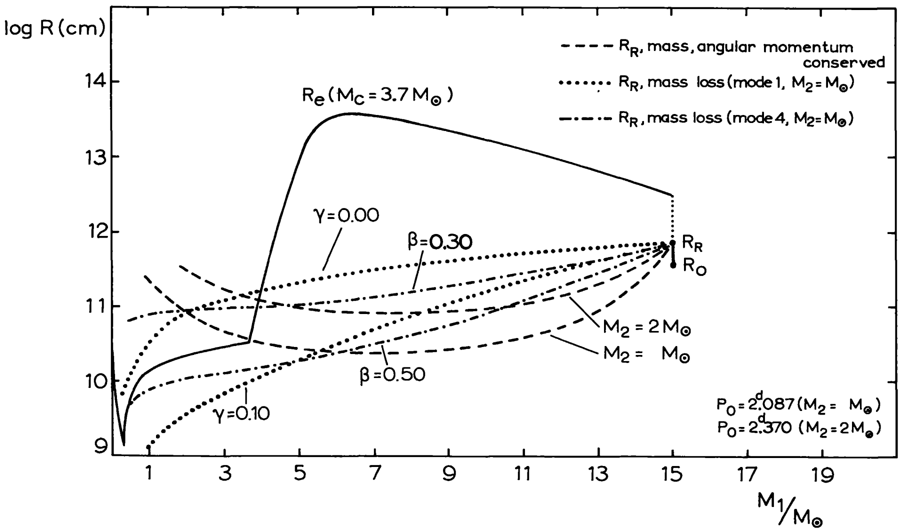

Fig. 1. The thermal equilibrium radius R_e of primaries of mass M_1 which result by mass loss from a core helium burning star of $15\, M_\odot$ (with a helium core of $3.7\, M_\odot$) together with the radius R_R of the Roche lobe for various types of mass exchange. The initial binary periods P_0 and secondary masses M_2 are indicated. The dotted curve labelled $\gamma = 0.00$ indicates the upper limit of R_R in the case of mass loss from the system [case (ii)]

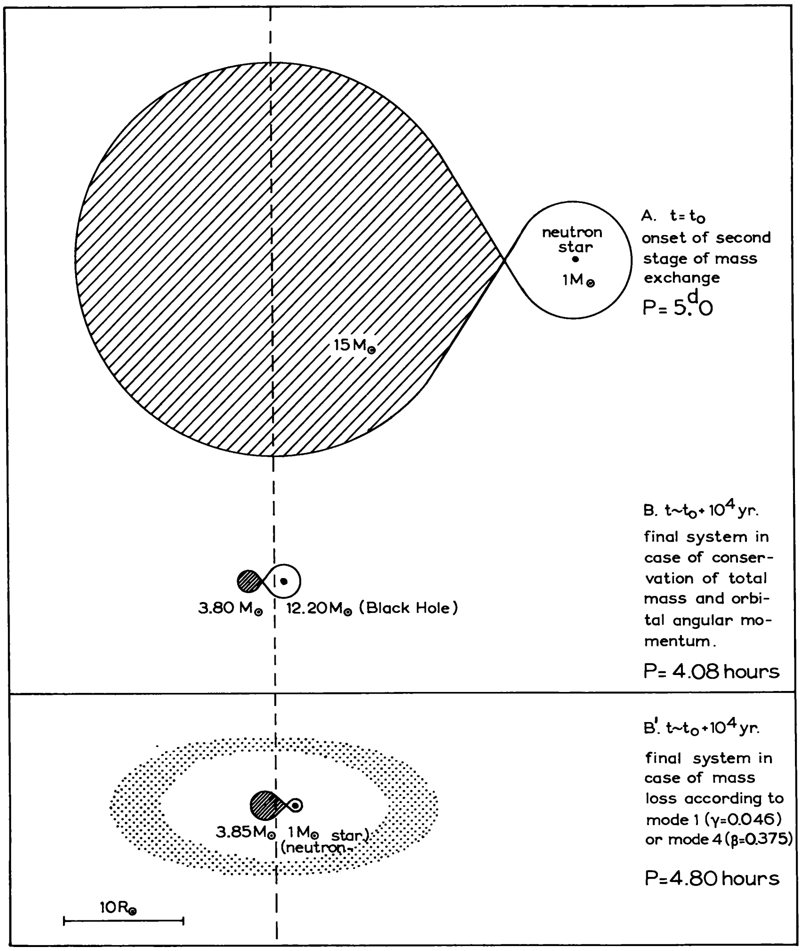

Fig. 2. Two possible ways of evolution of a binary with $M_1^0 = 15\,M_\odot$, $M_2^0 = M_\odot$, $P = 5^{\mathrm{d}}0$. *Upper part:* the result if total mass and orbital angular momentum are conserved [case (i)]. *Lower part:* the final system is case that accretion onto the neutron star is limited due to the critical luminosity [case (ii)] and mass is lost according to the indicated mass loss modes. The dashed line indicates the position of the centre of gravity of the system

The table shows that in all cases the final primaries have $L \geqq 10^4\,L_\odot$ ($M_{\mathrm{bol}} < -5^{\mathrm{m}}$) and are almost pure helium stars.

Case (ii)

As in this case the neutron star can only accrete $\lesssim 10^{-7}\,M_\odot$/yr, while the primary must loose some $10\,M_\odot$ in 10^4 yr, it is permissible to assume $M_2 = \mathrm{const}$ during the mass loss phase. The matter lost by the primary is expected either be lost from the system or will form a ring or disk around the system (cf. Huang, 1963, 1966). (Storage of some $10\,M_\odot$ in a ring around the neutron star seems very unlikely as such a massive ring would be subject to very large perturbations).
We will show that in both cases the period of the system will decrease, independent of the details of the mass loss process. We first consider the case that the lost mass is completely expelled from the system. The energy required to expell it from the system can be drawn from two sources: (i) radiation pressure (due to the electromagnetic flux from the neutron star and its surroundings), (ii) the dynamical energy $E = E_g + E_T$ of the system, (E_g = potential energy, E_T = kinetic energy) where

$$E = -\tfrac{1}{2}\,G M_1 M_2 / a \tag{3}$$

in which G is the gravitational constant.
Let us consider the extreme case that all of the energy required to expell an amount of mass δM_1 from the system is only due to the radiation pressure and other electromagnetic forces. In such a case the system will evolve at $E = \mathrm{const}$. As M_2 practically does not change during the mass loss process, we have during the mass loss

$$E = -\tfrac{1}{2} G M_1 M_2 / a = -\tfrac{1}{2} G M_1^0 M_2 / a_0 \tag{4}$$

so:

$$a/a_0 = M_1 / M_1^0 \tag{5}$$

Table 1. Parameters of final binary systems in case (i); and in case (ii) with mass loss according to mode 1. The periods for $\gamma = 0.00$ are the upper limits of the final binary periods in the case of loss from the system

			Initial system							
			$M_1^0 = 15\,M_\odot$ $M_2^0 = M_\odot$		$M_1^0 = 15\,M_\odot$ $M_2^0 = 2\,M_\odot$		$M_1^0 = 21\,M_\odot$ $M_2^0 = M_\odot$		$M_1^0 = 21\,M_\odot$ $M_2^0 = 2\,M_\odot$	
		Final system	$P_0 = 2^{\rm d}.087$	$5^{\rm d}.00$	$2^{\rm d}.37$	$5^{\rm d}.68$	$2^{\rm d}.50$	$5^{\rm d}.00$	$2^{\rm d}.75$	$5^{\rm d}.50$
Case (i)		P	$1^{\rm h}.8$	$4^{\rm h}.1$	$11^{\rm h}.3$	$24^{\rm h}.6$	$1^{\rm h}.7$	$1^{\rm h}.8$	$3^{\rm h}.7$	$7^{\rm h}.3$
conservation of		$M_1/M_\odot$	3.7	3.8	3.9	4.1	3.8	5.0	6.7	6.8
total mass and		$\log L_1/L_\odot$	4.11	4.14	4.17	4.21	4.13	4.44	4.67	4.68
orbital angular		$M_{\rm bol_1}$	− 5.59	− 5.66	− 5.73	− 5.84	− 5.63	− 6.41	− 6.99	− 7.02
momentum		$\log T_{e_1}$	4.93	4.89	4.80	4.65	4.95	4.99	4.93	4.87
Case (ii)	$\gamma = 0.00$	P	$12^{\rm h}.2$	$27^{\rm h}.1$	$13^{\rm h}.0$	$28^{\rm h}.3$	$19^{\rm h}.7$	$39^{\rm h}.8$	$20^{\rm h}.7$	$42^{\rm h}.0$
mass loss		$M_1/M_\odot$	3.95	4.10	3.95	4.10	7.20	7.35	7.20	7.35
according		$\log L_1/L_\odot$	4.18	4.21	4.18	4.21	4.72	4.82	4.72	4.82
to mode 1		$M_{\rm bol_1}$	− 5.76	− 5.84	− 5.76	− 5.84	− 7.11	− 7.36	− 7.11	− 7.36
		$\log T_{e_1}$	4.76	4.65	4.76	4.65	4.62	4.54	4.62	4.54
	$\gamma = 0.05$	P	$1^{\rm h}.7$	$4^{\rm h}.3$	$4^{\rm h}.4$	$10^{\rm h}.8$	$1^{\rm h}.8$	$3^{\rm h}.7$	$5^{\rm h}.9$	$12^{\rm h}.2$
		$M_1/M_\odot$	3.72	3.83	3.82	3.92	6.65	6.80	6.90	7.05
		$\log L_1/L_\odot$	4.12	4.15	4.15	4.17	4.66	4.68	4.70	4.71
		$M_{\rm bol_1}$	− 5.61	− 5.69	− 5.69	− 5.73	− 6.96	− 7.02	− 7.07	− 7.10
		$\log T_{e_1}$	4.92	4.86	4.86	4.80	4.96	4.87	4.81	4.72

Consequently, in this case, a decreases proportional to M_1. According to Kepler's third law, P then changes according to

$$P/P_0 = (M_1^3(M_1^0 + M_2)/M_1^{0^3}(M_1 + M_2))^{1/2} \tag{6}$$

which implies that also the period *decreases* during the mass loss (as $M_2 \ll M_1$). If part of the energy required to expell a particle from the system is drawn from the dynamical energy of the system, E will decrease during the mass loss process, which implies that a and P will decrease more rapidly with M_1 than given by Eqs. (5) and (6). [The problem in this case is, in principle, analogous to the escape of stars from a cluster due to mutual gravitational interactions: in such a case the remaining cluster becomes more tightly bound (v. Albada, 1968; Spitzer and Thuan, 1972; Ostriker *et al.*, 1972).] Hence, for the case that mass escapes from the system, Eqs. (5) and (6) give the upper limit for the semi-major axis and period of the binary during the escape process, as a function of M_1. From the upper limit of a one then can compute the upper limit $\bar{R}_R$ of the radius of the Roche lobe during the mass loss process. For the combinations $M_1^0 = 15\,M_\odot$, $M_2 = M_\odot$, $P = 2^{\rm d}.087$ and $M_2 = 2\,M_\odot$, $P = 2^{\rm d}.37$, $\bar{R}_R$ as a function of M_1 is represented by the dotted curve labelled $\gamma = 0.00$ in Fig. 1. The figure shows that in case that $R_R = \bar{R}_R$, the system will stabilize at $M_1 = 3.95\,M_\odot$, which – at the given initial periods, yields a final period $P_F = 12^{\rm h}.2$ for $M_2 = M_\odot$ and $P = 13^{\rm h}$ for $M_2 = 2\,M_\odot$. (Similarly for $M_1^0 = 21\,M_\odot$, $M_2 = M_\odot$, $P_0 = 2^{\rm d}.5$, one obtains $P = 19^{\rm h}.7$.) These values represent the upper limits for the final binary period in case of mass loss from the system, for the above given initial system parameters.

In any realistic case of mass loss the gas will always carry with it some amount of angular momentum and hence, also some part of the dynamical energy of the system.

A large variety of "modes" of loss of matter and angular momentum from binaries are possible (cf. Huang, 1963, 1966). We will only consider here some simple modes which appear to us more or less realistic for the systems under consideration. Detailed calculations on the various modes of mass loss will be presented in a separate paper (Paper V of this series). Only a brief description of some modes will be presented here together with the results.

Mode 1

In this case it is assumed that before it is expelled a mass element δM_1 (lost by the primary) is moving in the system at a mean distance a from the center of gravity. As the Roche lobe of the primary has about the same size as a, this seems a reasonable assumption. The energy required to completely expell it (to infinity) is then given by

$$\varepsilon = -G(M_1 + M_2)\,\delta M_1/a \qquad (\delta M_1 < 0) \tag{7}$$

Let us assume that a (small) fraction γ of this energy is drawn from the dynamical energy of the system. Then, if the mass element leaves the system, the decrease in E is given by

$$\delta E = \gamma G(M_1 + M_2)\,\delta M_1/a\,. \tag{8}$$

Together with Eq. (3) this yields

$$\delta E/E = -2\gamma(M_1 + M_2)\,\delta M_1/M_1 M_2 \tag{9}$$

which yields, after integration:

$$a/a_0 = (M_1/M_1^0)^{1+2\gamma} \exp(2\gamma(M_1 - M_1^0)/M_2)) \tag{10}$$

and

$$P/P_0 = ((M_1^0 + M_2)/(M_1 + M_2))^{1/2} (M_1/M_1^0)^{1.5+3\gamma} \cdot \exp(3\gamma(M_1 - M_1^0)/M_2). \quad (11)$$

Hence, in this case a and P decrease roughly exponentially with

$3\gamma(M_1 - M_1^0)/M_2$.

The dotted curves in Fig. 1 show how R_R decreases as a function of M_1 for $\gamma = 0.00$ and $\gamma = 0.10$, starting from $M_1^0 = 15\,M_\odot$, $M_2 = M_\odot$, $P_0 = 2\overset{d}{.}087$. The figure shows that for $\gamma = 0.10$ the R_R-curve does not intersect the R_e-curve for any value of $M_1 < 15\,M_\odot$. Hence, for $\gamma = 0.10$ no stable final solution is possible: the dimensions of the binary system shrink so rapidly in this case, that the system is never able to accomodate a primary in thermal equilibrium. Only for $\gamma \leqq 0.07$ (and the above given initial system parameters) does the R_R-curve intersect the R_e-curve at some M_1-value $< 15\,M_\odot$. Hence, a stable solution occurs only if the escaping matter draws 7% or less of its escape energy from the dynamical energy of the system. For the case $\gamma = 0.07$ the final system has $P = 1.2$ hours. Table 1 summarizes stable final solutions for a variety of combinations of initial system parameters, for $\gamma = 0.00$, 0.05. The lower part of Fig. 2 represents as an example the result for $M_1^0 = 15\,M_\odot$, $M_2 = M_\odot$, $P_0 = 5\overset{d}{.}0$, $\gamma = 0.046$.

Mode 2

In this case the matter lost from the primary is assumed to form a ring around the system [as is, for instance, observed in the case of β Lyrae (Huang, 1966)]. From Huang's (1963) formulae one then finds that, even if the lost matter is deposited in rings with a radius of only twice the momentaneous orbital radius, the orbit shrinks so rapidly that no stable final solutions are possible.

Mode 3

In this case it is assumed that the matter expelled by the primary flows through L_1 towards the secondary and from there is expelled from the system, isotropically with respect to the secondary. In this case the orbit shrinks practically exponentially with $2\,(M_1 - M_1^0)$ and again no stable solutions are found.

Mode 4

(A "weaker" form of mode 3). It is assumed here that a fraction β of a mass element δM_1 lost by the primary reaches L_1 and from there is accelerated outward; it is assumed that it carries with it the angular momentum which it had at L_1. The other part $(1-\beta)\,\delta M_1$ of the mass element is assumed to have been blown away from the primary as soon as it overflowed the Roche lobe; it is assumed that it is blown away symmetrically with respect to the primary, i.e. that it carries with it an amount of angular momentum per unit mass equal to the orbital angular momentum per unit mass of the primary. In Fig. 1 the change of R_R with M_1 is represented for $\beta = 0.50$ and $\beta = 0.30$, for the system with $M_1^0 = 15\,M_\odot$, $M_2 = M_\odot$, $P_0 = 2\overset{d}{.}087$. The figure shows that stable solutions in this case can be obtained only if $\beta \leqq 0.42$. Figure 2 (lower part) shows that for a system with $M_1^0 = 15\,M_\odot$, $M_2 = M_\odot$, $P = 5\overset{d}{.}0$ for $\beta = 0.375$ a final period of 4.80 hours is reached ($M_1 = 3.85\,M_\odot$).

V. Characteristics of Stable Final Systems in the Case of Mass Loss from the System

In view of the large variety of possible modes of mass loss from the system it seems hard to judge what the final solution in the case of mass loss will be. However, as the binary period always will decrease [cf. Eqs. (5) and (6)] one can, even if one does not know the exact mode of mass loss but if one assumes that the system will finally stabilise, calculate the relation between the final period P_F and the final primary mass M_1 at which the system stabilises, as follows.

Stable systems must have $M_1 \leqq \bar{M}_1$, where $\bar{M}_1$ is the M_1-value for which $R_e(M_1) = \bar{R}_R$ (cf. Section IV). For $M_1^0 = 15\,M_\odot$ and $21\,M_\odot$ this corresponds roughly to $M_1 \leqq 4\,M_\odot$ and $M_1 \leqq 7\,M_\odot$, respectively (for $P_0 \leqq 7^d$, $M_2 \leqq 3\,M_\odot$). For each value of M_1 one further knows that for a stable solution holds: $R_e(M_1) = R_R$; since R_R, M_1 and M_2 uniquely determine the value of a and P (cf. Kopal, 1959), one then also knows the orbital period P_F of the corresponding stable final solution. Hence, for a given combination of M_2 and M_1^0 the relation $R_e(M_1)$ uniquely determines the relation between P_F and M_1.

Figure 3 represents P_F as a function of M_1 for $M_1^0 = 15\,M_\odot$ and $21\,M_\odot$, for three values of M_2. The horizontal bars at the top of each curve indicate the upper limit of the final binary period derived from Eq. (6) for the P_0-values indicated in the figure caption. The figure shows that for $M_1^0 = 15\,M_\odot$ a final period of the order of 1^h to $1\overset{h}{.}5$ will result over the entire range in M_1 between $\sim M_\odot$ and $3.7\,M_\odot$. For $M_1^0 = 21\,M_\odot$ the same holds over the entire range between $\sim M_\odot$ and $6.6\,M_\odot$. For $M_1 > 3.7\,M_\odot$ ($> 6.6\,M_\odot$), the presence of the hydrogen-rich envelope causes the steep increase in P_F (cf. Fig. 1).

VI. Evolution in the Case that the Secondary is a Black Hole

In analogy with the case of a neutron star we infer that steady hydro-dynamically inflowing matter without angular momentum will – at distance of several times

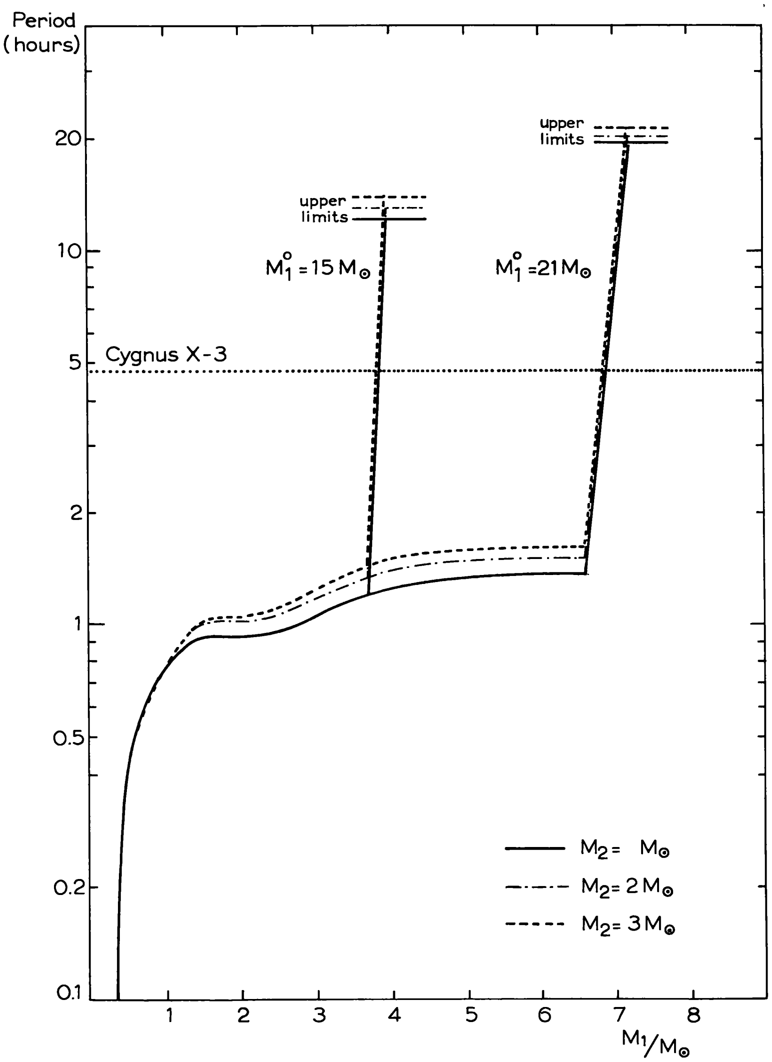

Fig. 3. Binary period as a function of M_1 (final primary mass) for stable final systems resulting by mass loss [case (ii)] from binaries with initial primary masses of $15\,M_\odot$ and $21\,M_\odot$, for three values of the initial secondary mass. The horizontal bars indicate the upper limits of the final binary periods in case (ii) for: $M_1^0 = 15\,M_\odot$ and $M_2 = M_\odot$, $P_0 = 2^d.087$; $M_2 = 2\,M_\odot$, $P_0 = 2^d.37$; $M_2 = 3\,M_\odot$, $P_0 = 2^d.68$ and for: $M_1^0 = 21\,M_\odot$ and $M_2 = M_\odot$, $P_0 = 2^d.50$; $M_2 = 2\,M_\odot$, $P_0 = 2^d.75$; $M_2 = 3\,M_\odot$, $P_0 = 2^d.95$

the Schwarzschild radius – reach densities and temperatures sufficiently high for plasma neutrino emission to become the dominant source of energy emission (cf. Zeldovitch *et al.*, 1972). Hence, in this case one again expects no limitation to the accretion rate. On the other hand, if the accreted matter has angular momentum, it will form a ring around the black hole and only small accretion rates are to be expected – the accretion again being limited by the critical luminosity (cf. Pringle and Rees, 1972). (The critical luminosity is proportional to the mass of the black hole, and the same will hold for the maximum permitted accretion rate). Consequently, for the evolution of binaries with a black hole secondary one can again consider the cases (i) and (ii) mentioned in Section I, and the results of the computations are expected to be similar to those for binaries with a neutron star secondary.

VII. The Case of Cygnus X-3

The X-ray emission of Cyg X–3 shows a stable periodicity of 4.8 hours, which seems suggestive of binary motion (Parsignault *et al.*, 1972). Cyg X–3 is located in a very obscured region of the sky; from 21-cm and 3.7 cm radio observations a distance of $\geqq 10$ kpc but $\leqq 50$ kpc is derived (Braes *et al.*, 1972; Hinteregger *et al.*, 1972). No optical identification is available but the position of the radio source connected with Cyg X–3 closely coincides with an infrared source for which – at $\geqq 10$ kpc distance and assuming a blackbody energy distribution – an M_{bol} value of $\leqq -5^m.5$ is found (Becklin *et al.*, 1972). If the X-ray source, radio source and IR source are indeed the same object, the characteristics of Cyg X–3 show a striking resemblence to those of the final systems after mass exchange or mass loss, as listed in Table 1. Figure 2 illustrates under what conditions a final period of the order of $4^{\mathrm{h}}.8$ may result from a system with $M_1^0 = 15\,M_\odot$, $M_2^0 = M_\odot$, $P_0 = 5^{\mathrm{d}}$. Hence, Cyg X–3 might represent a later stage of evolution of systems such as Cen X–3 and Cyg X–1. The evolution as outlined in Fig. 2 does not explain the radio flares of Cyg X–3.

However, also the close binaries Algol and β Lyrae show radio flares (Wade and Hjellming, 1972a, b). The optical spectrum of these systems shows evidence for gaseous streams (Struve and Sahade, 1957; Huang, 1966; Bolton, 1972) but the radio flares are in general not correlated with the regular photometric variations of these stars, which suggests that the radio flares are a characteristic of the gaseous material itself (Hjellming *et al.*, 1972). A similar explanation for the flares in Cyg X–3 would be completely in line with the fact that the giant radio flares of this source were not accompanied by changes in the over-all X-ray emission (Parsignault *et al.*, 1972). The enormous strength of the flares may, perhaps, be connected with the fact that the amount of circumstellar matter may become very large in X-ray binaries, if the accretion onto the neutron star is limited by the critical luminosity. The sharp low-energy cut off in the spectrum of Cyg X–3 indeed shows that the system is surrounded by a thick cloud of gas.

VIII. Discussion and Conclusions

Due to the adopted jump in helium abundance at the core boundary, the equilibrium radii of models with hydrogen-rich envelopes as calculated in Section IV may be slightly different from radii computed with a more realistic helium abundance profile. Such differences are, however, not expected to greatly affect the character of the final binary systems (cf. Giannone *et al.*, 1968).

From the results in Table 1 and Fig. 3 it seems therefore permissible to conclude that, independent of the assumptions about the accretion process onto neutron stars, massive binary systems with a collapsed component of $M_\odot$ or $2\,M_\odot$ mass will evolve into short to ultra-short period binaries. In the cases of Cen X–3, SMC X–1 and 2U 1700-37 the binary period is such that the primaries are almost certainly filling their Roche lobes. Hence, these primaries should be losing mass at a rate of some $10^{-3}\,M_\odot$/yr and their binary periods should be decreasing at a rate of some 10 to 20 seconds per year [in case (i) as well as (ii)]. Presently, only the periods of Cen X–3 and SMC X–1 seem to be known to a sufficient accuracy to detect such a change within a few years. Also the supergiants in Cyg X–1 and 2U 0900-40 are probably filling their Roche lobes; these systems might therefore provide a similar check and continued observation of their radial velocities seems very useful. The main uncertainty in our computations is reflected by the assumptions (i) and (ii). The assumption (ii) may be the more realistic one, since real neutron stars are rotating and magnetized and since exchanged matter in binaries will always carry some angular momentum. This then would imply that the massive X-ray binaries mentioned in the introduction will evolve with considerable mass loss from the system.

An observational consequence of such mass loss would be that the X-ray binaries will soon become surrounded by a thick cloud of gas. The low-energy cut-off in the spectra of all massive X-ray binaries (cf. Giacconi *et al.*, 1971; Tananbaum, 1972; Schreier *et al.*, 1972a) indeed suggests the presence of circumstellar material.

A final point which merits to be mentioned is the possible energy loss in the form of gravitational waves. From the formulas given by Kraft *et al.* (1962) one computes that this energy loss for the final systems in case (i) and (ii) as depicted in Fig. 2, will be 1.4×10^{34} erg/s and 7×10^{31} erg/s, respectively. These amounts, though large in absolute value, are negligible in comparison with the photon energy losses and will not contribute to a detectable reduction in the binary period during the lifetimes of the systems.

Note added in Proof. Observations by Schreier *et al.* (1973) show that the binary period of Cen X – 3 decreased by about 6 seconds over the past one and a half year. This confirms the above made predictions about the evolution of massive X-ray binaries. If one assumes that the primary star is loosing its envelope on a Kelvin-Helmholtz timescale, the observed decrease in binary period rules out that noticeable accretion onto the neutron star can take place, since in case (i), at $dM/dt = -10^{-3}\,M_\odot$/yr and $M_2 = M_\odot$ the decrease in binary period expected from equation (2) would be 500 seconds per year. On the other hand, for the case that accretion is limited by the critical luminosity, one expects from equation (11) for $\gamma = 0.00$ and a mass loss rate of $10^{-3}\,M_\odot$/yr that the period will decrease by about 5 seconds per yr – in very good agreement with the observations. Hence, the observations seem to confirm that the critical luminosity is the crucial parameter for the accretion process onto neutron stars.

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E. P. J. van den Heuvel
Sterrekundig Instituut
Servaas Bolwerk 13
Utrecht, Holland

C. De Loore
Astrophysisch Instituut der V.U.B.
Adolphe Buyllaan 105
B-1050 Brussel, België