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A fuzzy approach to real option valuation[☆]

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Abstract

To have a *real option* means to have the possibility for a certain period to either choose for or against making an investment decision, without binding oneself up front. The real option rule is that one should invest today only if the net present value is high enough to compensate for giving up the value of the option to wait. Because the option to invest loses its value when the investment is irreversibly made, this loss is an opportunity cost of investing. The main question that a management group must answer for a deferrable investment opportunity is: *How long do we postpone the investment*, *if we can postpone it*, *up to T time periods?* In this paper we shall introduce a (heuristic) real option rule in a fuzzy setting, where the present values of expected cash flows and expected costs are estimated by trapezoidal fuzzy numbers. We shall determine the optimal exercise time by the help of possibilistic mean value and variance of fuzzy numbers.

Keywords: Option pricing; Real options; Trapezoidal fuzzy numbers; Possibilistic mean value; Possibilistic variance

1. Probabilistic real option valuation

Options are known from the financial world where they represent the right to buy or sell a financial value, mostly a stock, for a predetermined price (the exercise price), without having the obligation to do so. The actual selling or buying of the underlying value for the predetermined price is called exercising your option. One would only exercise the option if the underlying value is higher than the

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exercise price in case of a call option (the right to buy) or lower than the exercise price in the case of a put option (the right to sell). In 1973 Black and Scholes [4] made a major breakthrough by deriving a differential equation that must be satisfied by the price of any derivative security dependent on a non-dividend paying stock. For risk-neutral investors the *Black–Scholes pricing formula* for a call option is

$$C_0 = S_0 N(d_1) - X e^{-rT} N(d_2),$$

where

$$d_1 = \frac{\ln(S_0/X) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}, \quad d_2 = d_1 - \sigma\sqrt{T},$$

and where, C_0 is the option price, S_0 is the current stock price, $N(d)$ is the probability that a random draw from a standard normal distribution will be less than d, X is the exercise price, r is the annualized continuously compounded rate on a safe asset with the same maturity as the expiration of the option, T is the time to maturity of the option (in years) and σ denotes the standard deviation of the annualized continuously compounded rate of return of the stock.

In 1973 Merton [10] extended the Black–Scholes option pricing formula to dividends-paying stocks as

$$C_0 = S_0 e^{-\delta T} N(d_1) - X e^{-rT} N(d_2), \tag{1}$$

where

$$d_1 = \frac{\ln(S_0/X) + (r - \delta + \sigma^2/2)T}{\sigma\sqrt{T}}, \quad d_2 = d_1 - \sigma\sqrt{T},$$

where δ denotes the dividends payed out during the life-time of the option.

Real options in option thinking are based on the same principles as financial options. In real options, the options involve "real" assets as opposed to financial ones [1]. To have a "real option" means to have the possibility for a certain period to either choose for or against making an investment decision, without binding oneself up front. For example, owning a power plant gives a utility the opportunity, but not the obligation, to produce electricity at some later date.

Real options can be valued using the analogue option theories that have been developed for financial options, which is quite different from traditional discounted cashflow investment approaches. In traditional investment approaches investments activities or projects are often seen as *now or never* and the main question is whether to go ahead with an investment *yes or no* [2].

Formulated in this way it is very hard to make a decision when there is uncertainty about the exact outcome of the investment. To help with these tough decisions valuation methods as *net present value* (NPV) or *discounted cash flow* (DCF) have been developed. And since these methods ignore the value of flexibility and discount heavily for external uncertainty involved, many interesting and innovative activities and projects are cancelled because of the uncertainties.

However, only a few investment projects are now or never. Often it is possible to delay, modify or split up the project in strategic components which generate important learning effects (and therefore reduce uncertainty). And in those cases option thinking can help [8]. The new rule, derived from option pricing theory (1), is that you should invest today only if the net present value is high enough to compensate for giving up the value of the option to wait. Because the option to invest loses its value when the investment is irreversibly made, this loss is an opportunity cost of investing. Following Leslie and Michaels [7] we will compute the value of a real option by

$$\mathrm{ROV} = S_0 \mathrm{e}^{-\delta T} N(d_1) - X \mathrm{e}^{-rT} N(d_2), \tag{2}$$

where

$$d_1 = \frac{\ln(S_0/X) + (r - \delta + \sigma^2/2)T}{\sigma\sqrt{T}}, \quad d_2 = d_1 - \sigma\sqrt{T}$$

and where ROV denotes the current real option value, S_0 is the present value of expected cash flows, X is the (nominal) value of fixed costs, σ quantifies the uncertainty of expected cash flows, and δ denotes the value lost over the duration of the option.

We illustrate the main difference between the traditional (passive) NPV decision rule and the (active) real option approach by an example quoted from [7, p. 10]:

> ... another oil company has the opportunity to acquire a five-year licence on block. When developed, the block is expected to yield 50 million barrels of oil. The current price of a barrel of oil from this field is \$10 and the present value of the development costs is \$600 million. Thus the NPV of the project opportunity is
>
> $$50 \text{ million} \times \$10 - \$600 \text{ million} = -\$100 \text{ million}.$$

Faced with this valuation, the company would obviously pass up the opportunity. But what would option valuation make of the same case? To begin with, such a valuation would recognize the importance of uncertainty, which the NPV analysis effectively assumes away. There are two major sources of uncertainty affecting the value of the block: the quantity and the price of the oil. One can make a reasonable estimate of the quantity of the oil by analyzing historical exploration data in geologically similar areas. Similarly, historical data on the variability of oil prices are readily available.

Assume for the sake of argument that these two sources of uncertainty jointly result in a 30 percent standard deviation (σ) around the growth rate of the value of operating cash inflows. Holding the option also obliges one to incur the annual fixed costs of keeping the reserve active—let us say, \$15 million. This represents a dividend-like payout of three percent (i.e. 15/500) of the value of the assets.

We already know that the duration of the option, T, is five years and the risk-free rate, r, is 5 percent, leading us to estimate option value at

$$\begin{aligned} \mathrm{ROV} &= 500 \times \mathrm{e}^{-0.03\times 5} \times 0.58 - 600 \times \mathrm{e}^{-0.05\times 5} \times 0.32 \\ &= \$251 \text{ million} - \$151 \text{ million} = \$100 \text{ million}. \end{aligned}$$

The main question that a company must answer for a deferrable investment opportunity is: *How long do we postpone the investment up to T time periods*? To answer this question, Benaroch and Kauffman [3, p. 204] suggested the following decision rule for optimal investment strategy: Where the maximum deferral time is T, make the investment (exercise the option) at time t^*, $0 \leqslant t^* \leqslant T$, for which the option, C_{t^*}, is positive and attends its maximum value,

$$C_{t^*} = \max_{t=0,1,\ldots,T} C_t = V_t e^{-\delta t} N(d_1) - X e^{-rt} N(d_2), \tag{3}$$

where

$$V_t = \mathrm{PV}(\mathrm{cf}_0, \ldots, \mathrm{cf}_T, \beta_P) - \mathrm{PV}(\mathrm{cf}_0, \ldots, \mathrm{cf}_t, \beta_P) = \mathrm{PV}(\mathrm{cf}_{t+1}, \ldots, \mathrm{cf}_T, \beta_P),$$

that is

$$V_t = \mathrm{cf}_0 + \sum_{j=1}^{T} \frac{\mathrm{cf}_j}{(1+\beta_P)^j} - \mathrm{cf}_0 - \sum_{j=1}^{t} \frac{\mathrm{cf}_j}{(1+\beta_P)^j} = \sum_{j=t+1}^{T} \frac{\mathrm{cf}_j}{(1+\beta_P)^j},$$

and cf_t denotes the expected cash flow at time t, and β_P is the risk-adjusted discount rate (or required rate of return on the project).

Of course, this decision rule has to be reapplied every time new information arrives during the deferral period to see how the optimal investment strategy might change in light of the new information.

It should be noted that the fact that real options are like financial options does not mean that they are the same. Real options are concerned about strategic decisions of a company, where degrees of freedom are limited to the capabilities of the company. In these strategic decisions different stakeholders play a role, especially if the resources needed for an investment are significant and thereby the continuity of the company is at stake. Real options therefore, always need to be seen in the larger context of the company, whereas financial options can be used freely and independently.

2. Possibilistic mean value and variance of fuzzy numbers

A fuzzy number A is a fuzzy set of the real line with a normal, (fuzzy) convex and continuous membership function of bounded support. The family of fuzzy numbers will be denoted by $\mathscr{F}$. For any $A \in \mathscr{F}$ we shall use the notation $[A]^\gamma = [a_1(\gamma), a_2(\gamma)]$ for γ-level sets of A. Fuzzy numbers can also be considered as possibility distributions [6]. If $A \in \mathscr{F}$ is a fuzzy number and $x \in \mathbb{R}$ a real number then $A(x)$ can be interpreted as the degree of possibility of the statement 'x is A'.

Definition 1. A fuzzy set $A \in \mathscr{F}$ is called a trapezoidal fuzzy number with core $[a, b]$, left width α and right width β if its membership function has the following form:

$$A(t) = \begin{cases} 1 - \dfrac{a-t}{\alpha} & \text{if } a - \alpha \leqslant t < a, \\ 1 & \text{if } a \leqslant t \leqslant b, \\ 1 - \dfrac{t-b}{\beta} & \text{if } b < t \leqslant b + \beta, \\ 0 & \text{otherwise,} \end{cases}$$

and we use the notation $A=(a,b,\alpha,\beta)$. It can easily be shown that

$$[A]^\gamma =,[a-(1-\gamma)\alpha, b+(1-\gamma)\beta], \quad \forall\gamma\in[0,1].$$

The support of A is $(a-\alpha, b+\beta)$.

A trapezoidal fuzzy number with core $[a,b]$ may be seen as a context-dependent description (α and β define the context) of the property "the value of a real variable is approximately in $[a,b]$". If $A(t)=1$ then t belongs to A with degree of membership one (i.e. $a\leqslant t\leqslant b$), and if $A(t)=0$ then t belongs to A with degree of membership zero (i.e. $t\notin(a-\alpha,b+\beta)$, t is *too far* from $[a,b]$), and finally if $0<A(t)<1$ then t belongs to A with an intermediate degree of membership (i.e. t is *close enough* to $[a,b]$). In a possibilistic setting $A(t)$, $t\in\mathbb{R}$, can be interpreted as the degree of possibility of the statement "t is approximately in $[a,b]$".

Let $[A]^\gamma=[a_1(\gamma),a_2(\gamma)]$ and $[B]^\gamma=[b_1(\gamma),b_2(\gamma)]$ be fuzzy numbers and let $\lambda\in\mathbb{R}$ be a real number. Using the extension principle we can verify the following rules for addition and scalar multiplication of fuzzy numbers

$$[A+B]^\gamma=[a_1(\gamma)+b_1(\gamma),a_2(\gamma)+b_2(\gamma)],\ [\lambda A]^\gamma=\lambda[A]^\gamma. \tag{4}$$

Let $A\in\mathscr{F}$ be a fuzzy number with $[A]^\gamma=[a_1(\gamma),a_2(\gamma)], \gamma\in[0,1]$. In [5] we introduced the (crisp) possibilistic mean (or expected) value of A as

$$E(A)=\int_0^1 \gamma(a_1(\gamma)+a_2(\gamma))\,\mathrm{d}\gamma=\frac{\int_0^1\gamma\cdot(a_1(\gamma)+a_2(\gamma)/2)\,\mathrm{d}\gamma}{\int_0^1\gamma\,\mathrm{d}\gamma},$$

i.e., $E(A)$ is nothing else but the level-weighted average of the arithmetic means of all γ-level sets, that is, the weight of the arithmetic mean of $a_1(\gamma)$ and $a_2(\gamma)$ is just γ. It can easily be proved that $E:\mathscr{F}\to\mathbb{R}$ is a linear function (with respect to operations (4)). In [5] we also introduced the (possibilistic) variance of $A\in\mathscr{F}$ as

$$\sigma^2(A)=\int_0^1\gamma\left(\left[\frac{a_1(\gamma)+a_2(\gamma)}{2}-a_1(\gamma)\right]^2+\left[\frac{a_1(\gamma)+a_2(\gamma)}{2}-a_2(\gamma)\right]^2\right)\mathrm{d}\gamma$$
$$=\frac{1}{2}\int_0^1\gamma(a_2(\gamma)-a_1(\gamma))^2\,\mathrm{d}\gamma.$$

i.e. the possibilistic variance of A is defined as the expected value of the squared deviations between the arithmetic mean and the endpoints of its level sets.

It is easy to see that if $A=(a,b,\alpha,\beta)$ is a trapezoidal fuzzy number then

$$E(A)=\int_0^1\gamma[a-(1-\gamma)\alpha+b+(1-\gamma)\beta]\,\mathrm{d}\gamma=\frac{a+b}{2}+\frac{\beta-\alpha}{6}.$$

and

$$\sigma^2(A) = \frac{(b-a)^2}{4} + \frac{(b-a)(\alpha+\beta)}{6} + \frac{(\alpha+\beta)^2}{24}.$$

3. A hybrid approach to real option valuation

Usually, the present value of expected cash flows cannot be characterized by a single number. However, our experiences with the Waeno research project on giga-investments [2] show that managers are able to estimate the present value of expected cash flows by using a trapezoidal possibility distribution of the form

$$\tilde{S}_0 = (s_1, s_2, \alpha, \beta),$$

i.e. the most possible values of the present value of expected cash flows lie in the interval $[s_1, s_2]$ (which is the core of the trapezoidal fuzzy number $\tilde{S}_0$), and $(s_2+\beta)$ is the upward potential and $(s_1-\alpha)$ is the downward potential for the present value of expected cash flows.

In a similar manner one can estimate the expected costs by using a trapezoidal possibility distribution of the form

$$\tilde{X} = (x_1, x_2, \alpha', \beta'),$$

i.e. the most possible values of expected cost lie in the interval $[x_1, x_2]$ (which is the core of the trapezoidal fuzzy number $\tilde{X}$), and $(x_2+\beta')$ is the upward potential and $(x_1-\alpha')$ is the downward potential for expected costs.

Note 1. *The possibility distribution of expected costs and the present value of expected cash flows could also be represented by nonlinear (e.g. Gaussian) membership functions. However, from a computational point of view it is easier to use linear membership functions and, more importantly, our experience shows that senior managers prefer trapezoidal fuzzy numbers to Gaussian ones when they estimate the uncertainties associated with future cash inflows and outflows.*

In these circumstances we suggest the use of the following (heuristic) formula for computing fuzzy real option values

$$\mathrm{FROV} = \tilde{S}_0 \mathrm{e}^{-\delta T} N(d_1) - \tilde{X} \mathrm{e}^{-rT} N(d_2), \tag{5}$$

where

$$d_1 = \frac{\ln(E(\tilde{S}_0)/E(\tilde{X})) + (r-\delta+\sigma^2/2)T}{\sigma\sqrt{T}}, \quad d_2 = d_1 - \sigma\sqrt{T}, \tag{6}$$

and where, $E(\tilde{S}_0)$ denotes the possibilistic mean value of the present value of expected cash flows, $E(\tilde{X})$ stands for the possibilistic mean value of expected costs and $\sigma := \sigma(\tilde{S}_0)$ is the possibilistic

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variance of the present value expected cash flows. Using formulas (4) for arithmetic operations on trapezoidal fuzzy numbers we find

$$\begin{aligned}\mathrm{FROV} &= (s_1, s_2, \alpha, \beta)\mathrm{e}^{-\delta T}N(d_1) - (x_1, x_2, \alpha', \beta')\mathrm{e}^{-rT}N(d_2)\\ &= (s_1\mathrm{e}^{-\delta T}N(d_1) - x_2\mathrm{e}^{-rT}N(d_2), s_2\mathrm{e}^{-\delta T}N(d_1) - x_1\mathrm{e}^{-rT}N(d_2),\\ &\quad \alpha\mathrm{e}^{-\delta T}N(d_1) + \beta\mathrm{e}^{-rT}N(d_2), \beta\mathrm{e}^{-\delta T}N(d_1) + \alpha'\mathrm{e}^{-rT}N(d_2)). \qquad (7)\end{aligned}$$

Note 2. *It is clear that in* (6) *we cannot use* $E(\tilde{S}_0/\tilde{X})$, *because* $\tilde{S}_0/\tilde{X}$ *may not be a fuzzy number.*

Note 3. *The value of the fuzzy real option will also be a fuzzy number* (*being a weighted difference of fuzzy numbers*). *What is then the relation between the probabilistic real option value, computed by* (2), *and the fuzzy real option value, computed by* (5)? *It can easily be shown* [9] *that if the probabilistic density function in* (2) *and the possibility distributions of* $\tilde{S}_0 = (S_0, 1)$ *and* $\tilde{X} = (X, 1)$ *are of symmetric triangular form then*

$$\mathrm{ROV} = E(\mathrm{FROV}),$$

that is, the probabilistic real option value coincides with the possibilistic expected value of the fuzzy real option. In the general case, possibility distributions do not satisfy the properties of probabilistic density functions (*and vice versa, probability density functions are mostly sub-normal*), *therefore* ROV *and* E(FROV) *cannot be compared* (*since one of them may not be defined*).

We have a specific context for the use of the real option valuation method with fuzzy numbers, which is the main motivation for our approach. Giga-investments require a basic investment exceeding 300 million euros and they normally have a life length of 15–25 years. The standard approach with the NPV of DCF methods is to assume that uncertain revenues and costs associated with the investment can be estimated as probabilistic values, which in turn are based on historic time series and observations of past revenues and costs.

It should be clear that the relevance of historic data diminishes very quickly after 2–3 years and that it is not worthwhile to claim that the time series have any predictive value after 5 years (and even much more so for 15–25 years ahead). In the classical real options valuation methods this problem has been met by assuming that either (i) the stock market is able to find a price for the impact of the investment on the stock market value of the shares of the investing company (somehow doubtful for 15–25 years ahead), or that (ii) the uncertain revenues and costs are purely stochastic phenomena (described by, for instance, geometric Brownian motion).

We have discovered that giga-investments actually influence the end-user markets in *non-stochastic ways* and that they are normally significant enough to have an impact on market strategies, on technology strategies, on competitive positions and on business models. Thus, the use of assumptions on purely stochastic phenomena is not well-founded.

Example 1. Suppose we want to find a fuzzy real option value under the following assumptions:

$$\tilde{S}_0 = (\$400 \text{ million}, \$600 \text{ million}, \$150 \text{ million}, \$150 \text{ million}),$$

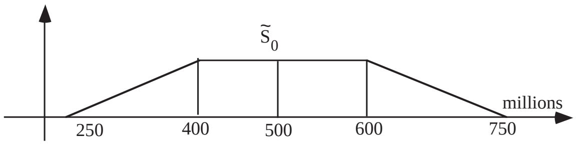

Fig. 1. The possibility distribution of present values of expected cash flow.

$r=5\%$ per year, $T=5$ years, $\delta=0.03$ per year and

$$\tilde{X} = (\$550 \text{ million}, \$650 \text{ million}, \$50 \text{ million}, \$50 \text{ million}),$$

First calculate

$$\sigma(\tilde{S}_0)=\sqrt{\frac{(s_2-s_1)^2}{4}+\frac{(s_2-s_1)(\alpha+\beta)}{6}+\frac{(\alpha+\beta)^2}{24}} = \$154.11 \text{ million},$$

i.e. $\sigma(\tilde{S}_0)=30.8\%$,

$$E(\tilde{S}_0) = \frac{s_1+s_2}{2} + \frac{\beta-\alpha}{6} = \$500 \text{ million},$$

and

$$E(\tilde{X}) = \frac{x_1+x_2}{2} + \frac{\beta'-\alpha'}{6} = \$600 \text{ million},$$

furthermore,

$$N(d_1)=N\left(\frac{\ln(600/500)+(0.05-0.03+0.308^2/2)\times 5}{0.308\times\sqrt{5}}\right) = 0.589,$$

$$N(d_2)=0.321.$$

Thus, from (5) we obtain the fuzzy value of the real option as

$$\text{FROV} = (\$40.15 \text{ million}, \$166.58 \text{ million}, \$88.56 \text{ million}, \$88.56 \text{ million}).$$

The expected value of FROV is \$103.37 million and its most possible values are bracketed by the interval

$$[\$40.15 \text{ million}, \$166.58 \text{ million}]$$

the downward potential (i.e. the maximal possible loss) is \$48.41 *million*, and the upward potential (i.e. the maximal possible gain) is \$255.15 *million* (Figs. 1, 2).

From Fig. 3 we can see that the set of most possible values of fuzzy real option [40.15, 166.58] is quite big. It follows from the huge uncertainties associated with cash inflows and outflows.

In the following we shall generalize the probabilistic decision rule for optimal investment strategy to a fuzzy setting: Where the maximum deferral time is T, make the investment (exercise the option)

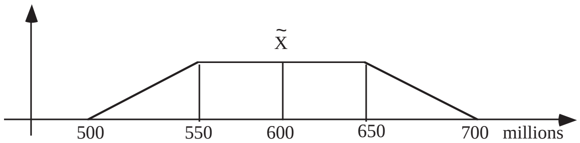

Fig. 2. The possibility distribution of expected costs.

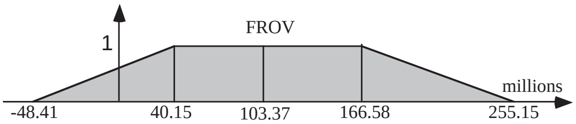

Fig. 3. The possibility distribution of real option values.

at time t^*, $0 \leqslant t^* \leqslant T$, for which the option, $\widetilde{C}_{t^*}$, attends its maximum value,

$$\tilde{C}_{t^*} = \max_{t=0,1,\ldots,T} \tilde{C}_t = \tilde{V}_t \mathrm{e}^{-\delta t} N(d_1) - \tilde{X} \mathrm{e}^{-rt} N(d_2), \tag{8}$$

where

$$\tilde{V}_t = \mathrm{PV}(\widetilde{\mathrm{cf}}_0, \ldots, \widetilde{\mathrm{cf}}_T, \beta_P) - \mathrm{PV}(\widetilde{\mathrm{cf}}_0, \ldots, \widetilde{\mathrm{cf}}_t, \beta_P) = \mathrm{PV}(\widetilde{\mathrm{cf}}_{t+1}, \ldots, \widetilde{\mathrm{cf}}_T, \beta_P),$$

that is,

$$\tilde{V}_t = \widetilde{\mathrm{cf}}_0 + \sum_{j=1}^{T} \frac{\widetilde{\mathrm{cf}_j}}{(1+\beta_P)^j} - \widetilde{\mathrm{cf}}_0 - \sum_{j=1}^{t} \frac{\widetilde{\mathrm{cf}}_j}{(1+\beta_P)^j} = \sum_{j=t+1}^{T} \frac{\widetilde{\mathrm{cf}}_j}{(1+\beta_P)^j},$$

where $\widetilde{\mathrm{cf}}_t$ denotes the expected (fuzzy) cash flow at time t, β_P is the risk-adjusted discount rate (or required rate of return on the project). However, to find a maximizing element from the set

$$\{\tilde{C}_0, \tilde{C}_1, \ldots, \tilde{C}_T\},$$

is not an easy task because it involves ranking of trapezoidal fuzzy numbers.

In our computerized implementation we have employed the following value function to order fuzzy real option values, $\widetilde{C}_t = (c_t^{\mathrm{L}}, c_t^{\mathrm{R}}, \alpha_t, \beta_t)$, of trapezoidal form:

$$v(\tilde{C}_t) = \frac{c_t^{\mathrm{L}} + c_t^{\mathrm{R}}}{2} + r_A \cdot \frac{\beta_t - \alpha_t}{6},$$

where $r_A \geqslant 0$ denotes the degree of the investor's risk aversion. If $r_A = 0$ then the (risk neutral) investor compares trapezoidal fuzzy numbers by comparing their possibilistic expected values, i.e. he does not care about their downward and upward potentials.

4. Real options for strategic planning

There has been a suspicion for some time that capital invested in very large projects, with an expected life cycle of more than a decade is not very productive and that the overall activity around them is not very profitable. There is even some fear (which actually was articulated in the Waeno research program in Finland) that giga-investments in production facilities may consume capital, which could be used more effectively and more profitably in investment opportunities offered by the global financial markets. If the opportunities offered by a global market are used as reference points, arguments may be construed to show that long-term investments in production facilities will destroy capital. If this conclusion was to be made by risk-taking investors, the consequences may be disastrous for major parts of the traditional industry, which—by the way—forms the backbone of the economy and the welfare of most industrial nations.

This discussion was real and urgent during the boom of the so-called Internet economy in 1998–2000, during which the market values of dotcom-start ups exceeded the market values of multinational corporations running tens of pulp and paper mills in a dozen countries.

Giga-investments made in the paper- and pulp industry, in the heavy metal industry and in other base industries, today face scenarios of slow (or even negative) growth (2–3% p.a.) in their key markets and a growing over-capacity in Europe. The energy sector faces growing competition with lower prices and cyclic variations of demand. There is also some statistics, which shows that productivity improvements in these industries have slowed down to 1–2% p.a., which opens the way for effective competitors to gain footholds in their main markets.

Giga-investments compete for major portions of the risk-taking capital, and as their life is long, compromises are made on their short-term productivity. The short-term productivity may not be high, as the life-long return of the investment may be calculated as very good. Another way of motivating a giga-investment is to point to strategic advantages, which would not be possible without the investment and thus will offer some indirect returns.

There are other issues. Global financial markets make sure that capital cannot be used non-productively, as its owners are offered other opportunities and the capital will move (often quite fast) to capture these opportunities. The capital market has learned 'the American way', i.e. there is a shareholder dominance among the actors, which has brought (often quite short-term) shareholder return to the forefront as a key indicator of success, profitability and productivity.

There are lessons learned from the Japanese industry, which point to the importance of immaterial investments. These lessons show that investments in buildings, production technology and supporting technology will be enhanced with immaterial investments, and that these are even more important for reinvestments and for gradually growing maintenance investments.

The core products and services produced by giga-investments are enhanced with life-time service, with gradually more advanced maintenance and financial add-on services. These make it difficult to actually assess the productivity and profitability of the original giga-investment, especially if the products and services are repositioned to serve other or emerging markets.

New technology and enhanced technological innovations will change the life cycle of a giga-investment. The challenge is to find the right time and the right innovation to modify the life cycle in an optimal way. Technology providers are involved through-out the life cycle of a giga-investment, which should change the way in which we assess the profitability and the productivity of an investment.

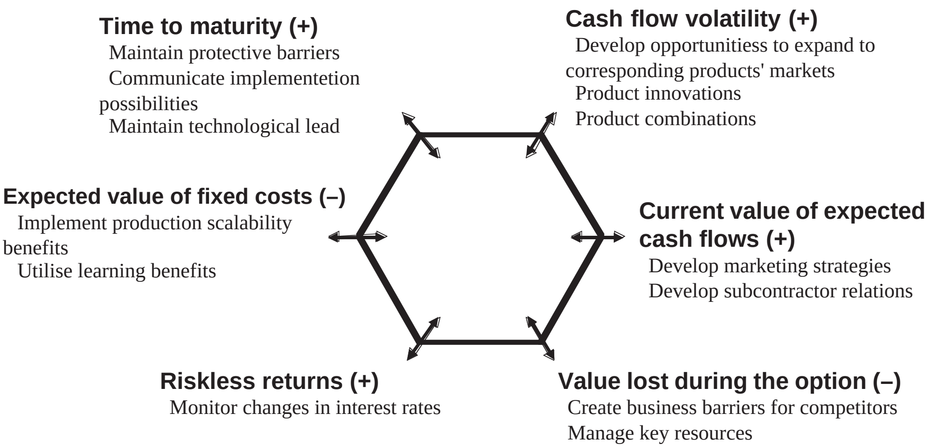

Fig. 4. The impact of 6 factors on the real option values. The (+/−) shows an increase or a decrease of the ROV.

Decision trees are excellent tools for making financial decisions where a lot of vague information needs to be taken into account. They provide an effective structure in which alternative decisions and the implications of taking those decisions can be laid down and evaluated. They also help us to form an accurate, balanced picture of the risks and rewards that can result from a particular choice.

In our empirical cases we have represented strategic planning problems by dynamic decision trees, in which the nodes are projects that can be deferred or postponed for a certain period of time. Using the theory of real options we have been able to identify the optimal path of the tree, i.e. the path with the biggest real option value in the end of the planning period.

5. Nordic Telekom Inc.

The World's telecommunications markets are undergoing a revolution. In the next few years mobile phones may become the World's most common means of communication, opening up new opportunities for systems and services. Characterized by large capital investment requirements under conditions of high regulatory, market, and technical uncertainty, the telecommunications industry faces many situations where strategic initiatives would benefit from real options analysis.

As the FROV method is applied to the telecom markets context and to the strategic decisions of a telecom corporation we will have to understand in more detail how the real option values are formed. In Fig. 4 the impact of a number of factors is summarized on the formation of the real option values.

The FROV will increase with an increasing volatility of cash flow estimates. The corporate management can be proactive and find (i) ways to expand to new markets, (ii) product innovations and (iii) (innovative) product combinations as end results of their strategic decisions. If the current value of expected cash flows will increase, then the FROV will increase. A proactive management can influence this by (for instance) developing market strategies or developing subcontractor relations.

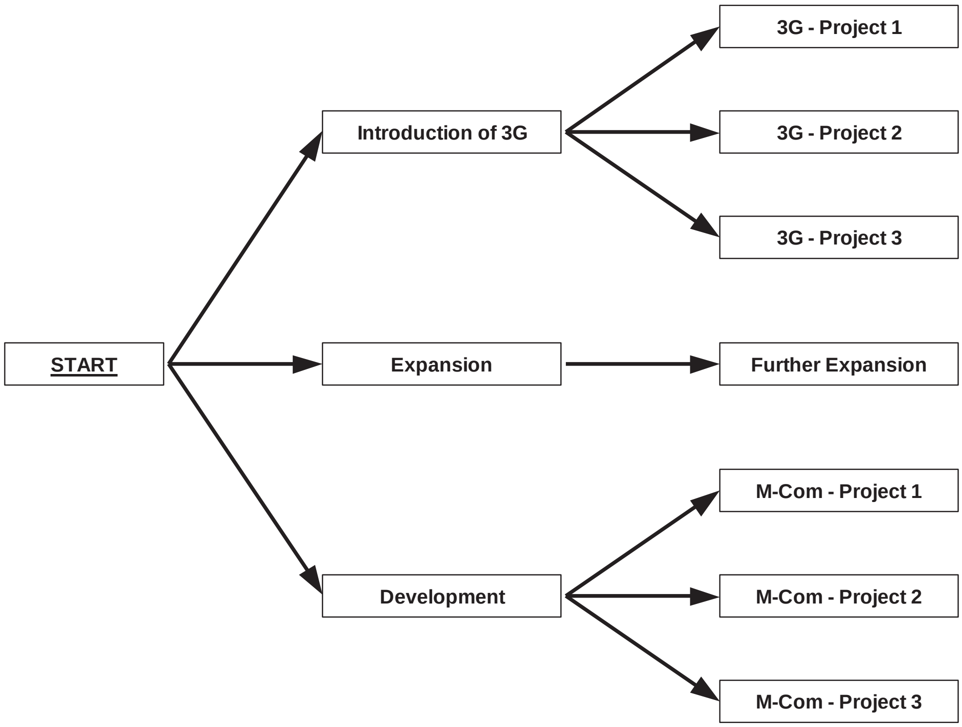

Fig. 5. A simplified decision tree for Nordic telecom Inc.

The FROV will decrease if value is lost during the postponement of the investment, but this can be countered by either creating business barriers for competitors or by better managing key resources. An increase in risk-less returns will increase the FROV, and this can be further enhanced by closely monitoring changes in the interest rates.

If the expected value of fixed costs goes up, the FROV will decrease as opportunities of operating with less cost are lost. This can be countered by using the postponement period to explore and implement production scalability benefits and/or to utilise learning benefits.

The longer the time to maturity, the greater will be the FROV. A proactive management can make sure of this development by (i) maintaining protective barriers, (ii) communicating implementation possibilities and (iii) maintaining a technological lead.

The following example outlines the methodology used (to keep confidentiality we have modified the real setup) in the Nordic Telekom Inc. (NTI) case:

Example 2. Nordic Telekom Inc. is one of the most successful mobile communications operators in Europe[3] and has gained a reputation among its competitors as a leader in quality, innovations in wireless technology and in building long-term customer relationships.

[3] NTI is a fictional corporation, but the dynamic tree model of strategic decisions has been successfully implemented for the 4 Finnish companies which participate in the Waeno project on giga-investments.

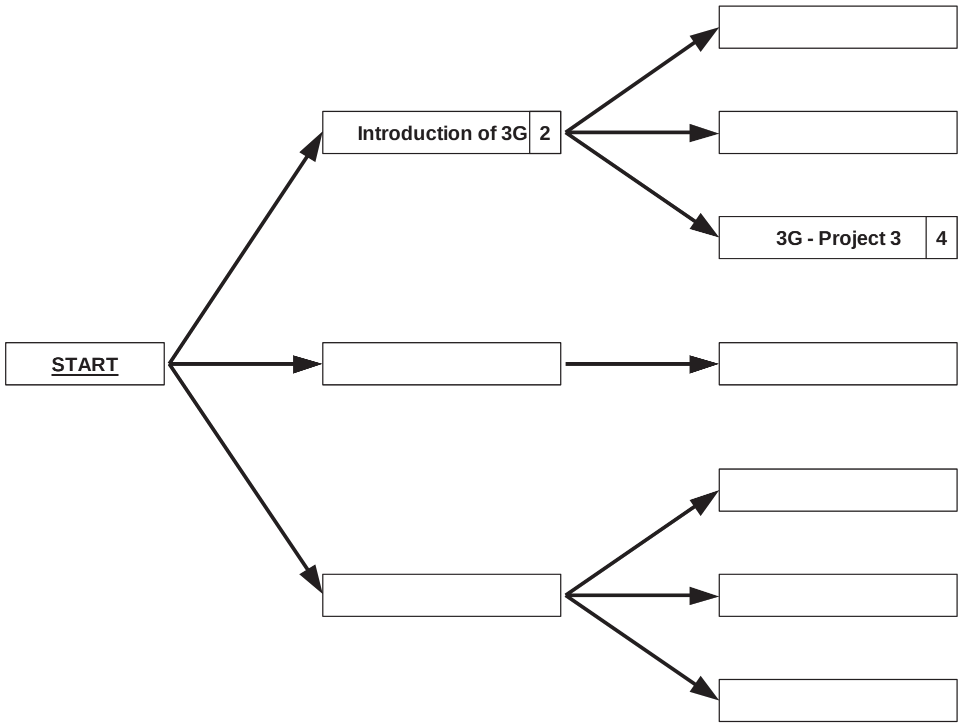

Fig. 6. The optimal path: Introduction of 3G solutions in 2005 (after two years deferral time) and then proceed with 3G-Project 3 in 2009 (after four years deferral time).

Still it does not have a dominating position in any of its customer segments, which is not even advisable in the European Common market, as there are always 4–8 competitors with sizeable market shares. NTI would, nevertheless, like to have a position which would be dominant against any chosen competitor when defined for all the markets in which NTI operates. NTI has associated companies that provide GSM services in five countries and one region: Finland, Norway, Sweden, Denmark, Estonia and the St. Petersburg region.

We consider strategic decisions for the planning period 2003–2011. There are three possible alternatives for NTI: (i) introduction of third generation mobile solutions (3G); (ii) expanding its operations to other countries; and (iii) developing new m-commerce solutions. The introduction of a 3G system can be postponed by a maximum of two years, the expansion may be delayed by maximum of one year and the project on introduction of new m-commerce solutions should start immediately.

Our goal is to maximize the company's cash flow at the end of the planning period (year 2011). In our computerized implementation we have represented NTI's strategic planning problem by a dynamic decision tree, in which the future expected cash flows and costs are estimated by trapezoidal fuzzy numbers. Then using the theory of fuzzy real options we have computed the real option values for all nodes of the dynamic decision tree. Then we have selected the path with the biggest real option value in the end of the planning period (see Figs. 5–8).

3G - Project 3

Length of the Project :	**3**
Maximum Deferral Time of the Project :	**5**

r Risk-adjusted Discount Rate	**δ** Rate of Return Shortfall	**σ** Volatility of Expected Revenues
8.50%	3.25%	15.29%

t Time (in years)	**V** Expected Revenues (in millions)				**X** Investment Cost (in millions)			
	a	*b*	*α*	*β*	*a*	*b*	*α*	*β*
0	€ 95.00	€ 105.00	€ 5.00	€ 5.00	€ 740.00	€ 760.00	€ 15.00	€ 15.00
1	€ 108.00	€1 20.00	€ 8.00	€ 5.00				
2	€ 125.00	€1 32.00	€ 12.00	€ 7.00				
3	€ 135.00	€1 42.00	€ 15.00	€ 5.00				
4	€ 145.00	€1 51.00	€ 4.00	€ 4.00				
5	€ 150.00	€1 60.00	€ 10.00	€ 11.00				
6	€ 145.00	€1 51.00	€ 4.00	€ 4.00				
7	€ 135.00	€1 42.00	€ 15.00	€ 5.00				
8	€ 130.00	€1 38.00	€ 10.00	€ 8.00				

Fig. 7. Input data for the optimal (3G, 3G-Project 3) path.

6. Conclusions

Despite its appearance, the fuzzy real options model is quite practical and useful. Standard work in the field uses probability theory to account for the uncertainties involved in future cash flow estimates. This may be defended for financial options, for which we can assume the existence of an efficient market with numerous players and numerous stocks for trading, which may justify the assumption of the validity of the laws of large numbers and thus the use of probability theory. The situation for real options is quite different. The option to postpone an investment (which in our case is a very large—so-called—giga-investment) will have consequences, differing from efficient markets, as the number of players producing the consequences is quite small.

The imprecision we encounter when judging or estimating future cash flows is not stochastic in nature, and the use of probability theory gives us a misleading level of precision and a notion that

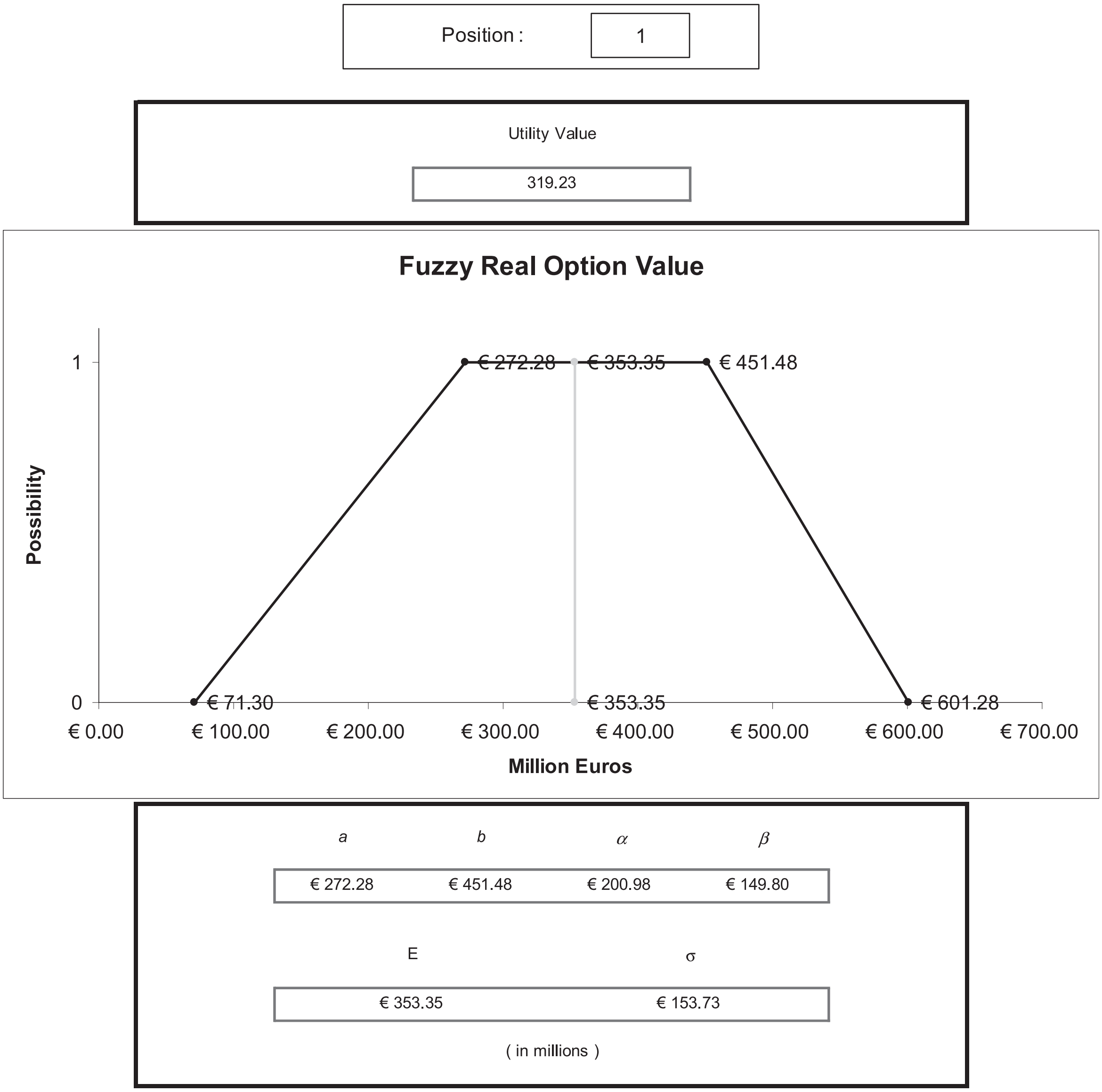

Fig. 8. The fuzzy real option value of the 3G-Project 3.

consequences somehow are repetitive. This is not the case, the uncertainty is genuine, i.e. we simply do not know the exact levels of future cash flows. Without introducing fuzzy real option models it would not be possible to formulate this genuine uncertainty. The proposed model that incorporates subjective judgments and statistical uncertainties may give investors a better understanding of the problem when making investment decisions.

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