Economics of Conflict, Network Theory and the case for Ancient Greece

Dr. Cong Chen, Public Health England
Dr. Manuela Dal Borgo, BA Posdoc Fellow

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Motivation - Classical approaches

1. Tactics and strategy: Sun Tzu (5^{th} c. BC), Carl von Clausewitz (1835)

2. Rulers and Ruled: Thomas Hobbes (1651), Niccolo Machiavelli (1513)

3. Cost of conflict: Thomas Schelling (1960)

4. Analysis of historical experience: Dziubinski, Goyal and Minarsch (2017), Levine and Modica (2013), Hoffman (2015)

5. Hegemony versus multiple kingdoms: Betts (2013), Mearsheimer (2001), Waltz (1979)

6. Buffer states: Chay and Ross (1986)

Historical examples - Dziubinski, Goyal and Minarsch (2017), Mearsheimer (2014), Walt (2014)

7. Overstretched empire: Kennedy (1987), Mearsheimer and Walt (2016)

8. Technology: Niccolo Machiavelli, Thucydides (5^{th} c. BC), Hirshleifer (1991), Hoffman (2015)

Motivation: Theory

Recent theoretical research on Warfare and strategy

- Resource Wars:
  - Dynamic conflict models w/ *two-actors*
    - Recent - Acemoglu, Golosov, Tsyvinski, Yared (2012), Caselli, Morelli, Rohner (2015), Novta (2016)
    - Survey - Baliga and Sjostrom (2012)
    - CSF - Tullock (1980), Fu and Pan (2015)

- Resources and a Network:
  - One shot model - Franke and Öztürk (2015), Kovenock and Roberson (2012), Jackson and Nei (2015), Konig, Rohner, Thoenig and Zilibotti (2014)
  - Dynamic Conflict Networks – Hirshleifer (1995), Huremovic (2014), Dziubinski, Goyal and Vigier (2016)
    - De Jong, Ghiglino and Goyal (2014) versus Dziubinski, Goyal and Minarsch (2017)

Mathematical setup of the basic game

building on Dziubiński, Goyal, Minarsch (2017)

- Let V be the set of vertices, N the set of players.
- $E \subset V^2$ is the set of edges, which is symmetric ($(a, b) \in E \Rightarrow (b, a) \in E$). Thus $G = (V, E)$ is an undirected graph, which we may assume is connected.
- The network also contains a resource function $r\colon V \to \mathbb{R}_+$, specifying the resources available at each vertex $v \in V$. We write r_v for $r(v)$.
- The state of the game at any point is described by an ownership function $\mathbb{O}\colon V \to N$ specifying which player controls each node.

Mathematical setup (cont.)

- An attack by player i on player j involves first computing $R_k(\mathbb{O}) = \sum_{\mathbb{O}(v)=k} r_v$ namely the resource owned by each player, for $k \in \{i, j\}$.
- Then the Tullock contest function $\mathrm{c}(R_i, R_j) = \frac{R_i^\gamma}{R_i^\gamma + R_j^\gamma}$, where γ is a globally set parameter, gives the probability that i wins the attack. Note that this is symmetric, and is said to be rich-rewarding when $\gamma > 1$ and poor-rewarding when $\gamma < 1$.

Rules of the basic game as described in [1]

- Turns are labelled $1, 2, \ldots \in \mathbb{N}$.
- At the start of any turn let the set of remaining players $P = \{n \in \mathbb{N} \colon \mathbb{O}(n) \neq \emptyset\}$.
- At the start of turn t a player is chosen at random with uniform probability from P, and may choose to attack or not attack. If he chooses *not* to attack then a player not yet chosen in turn t is chosen at random. If all players in $P(t)$choose not to attack in turn t then the game ends in peace.

Basic game rules (cont.)

- A ruler i who chooses to attack must nominate another ruler j such that i and j own vertices in contact, i.e. $\left(\mathbb{O}^{-1}(i)\times\mathbb{O}^{-1}(j)\right) \cap E \neq \emptyset$.
- In the above expression recall that $\mathbb{O}^{-1}(i)$ is the set of vertices controlled by player i, $\mathbb{O}^{-1}(j)$ the set of vertices controlled by player j, and E is the set of edges in the network G.
- We apply the contest function to determine the winner. The winner gains control of $\mathbb{O}^{-1}(i) \cup \mathbb{O}^{-1}(j)$, that is all nodes held by both players.

Basic game rules (cont.)

- If the attacker i wins, he may immediately choose another player to attack, and so on until he chooses not to attack or is defeated.
- If the defender j wins, the turn ends and we move to turn $t + 1$.

Assumptions of the basic game

- Each player is assumed to be unboundedly rational.
- Each player has total knowledge of the state of the game.
- Players have linear utility in resources held at the end of the game.

Value calculations

- In the previous work, any game would terminate in at most $|N|$ turns, as in any turn either the game ends due to peace or at least one player is eliminated in the conflict.
- As [1] points out, when $\gamma > 1$ and the graph is connected the game always ends in hegemony, i.e. with just one player remaining, the following is also from the paper.
- The game is a zero-sum game, so the sum of the values of the game for each player is the sum of the resources available, $\mathrm{R} = \sum_{v \in V} r_v$. When $\gamma = 1$ the expected value to a player from attacking another is the same as the value of the player before attacking, so the value of the game to each player is his starting resources.

Value calculations (cont.)

- When $\gamma > 1$ an upper bound for the expected value for each player i is attained if he successfully takes the first turn and attempts to end the game in that turn by conquering all opposition. This value is
$$R\left(\max_{g\in S_N:g(1)=i\wedge\bigwedge_{m\in N\setminus\{1\}}\left(\mathbb{O}^{-1}(\{n\in N:n<m\})\times\mathbb{O}^{-1}\left(g(m)\right)\cap E\neq\emptyset\right)}\left[\prod_{j>1}c\left(\sum_{k\in\{l\in N:l<j\}}R_{g(k)},R_{g(j)}\right)\right]\right)$$
- In other words, the total resources available multiplied by the greatest probability, over all legal sequences the player could choose to attack, of success. As it is a zero-sum game we can get a lower bound for the value for a player by subtracting from R the upper bounds for each other player. A simpler lower bound is given by pitting the player against the universe, which gives
$$Rc(R_i, R - R_i)$$
- For finite games these can be calculated, though the complexity grows as $|N|!$, i.e. faster than exponentially.

Value calculations (cont.)

- As this game terminates, there is a finite game tree which can be searched.
- We can use this to analyse this turn-by-turn.
- Let $V_i(P)$be the value to player i of position P, $V(P)$ the vector of all such values.
- We know $\sum_i V_i(P) = R$ for every possible position P.
- We also know $V(P)$ for every position where the game ends (and there are finitely many of these).

Value calculations (cont.)

- If we assume that players' willingness to attack and preferred target does not depend on whether other players have opted not to attack that turn, then the value calculation is as follows.
- If no players wish to attack then the value is determined by the resources available to each player.
- If

The scenario we're developing

- Each turn only has at most one contest.
- The game does not end, turns continue for $t \in \{1, 2, 3, \ldots\} = \omega$.
- This means we should study the system at a range of timescales, short-term, medium-term and asymptotic.
- There are a number of interactions which mean equilibria are mostly dynamic and changing rather than static.

Descriptive differences w/ Dziubinski et al. (2017)

- Conflict happens between adjacent nodes:
  - while the contest function where players commit all their resources is unchanged, <u>the stakes are now only the nodes directly involved, so each turn does not lead to elimination of a player.</u>
- A victorious attacker does not immediately get another turn.
  - This means that a ruler’s strategy cannot be considered in isolation - he must account for other players’ possible moves.

Extensions

- We add:
  - the ability of actors to defect
  - implement more complex derivations of the costs of war.
- Our initial hypotheses are:
  - hegemonic states are still possible under threat of defection and
  - investments into the peaceful productive economy support the conflict readiness of the actors.
- We test these hypotheses through simulation modelling to better understand the influence of:
  - transport links
  - technology
  - economic growth

Extensions to defection

Defection:

- If a player controls more than k of the total resources, each of his nodes has a chance p_exit to rebel and become an independent player.
- This is evaluated at the end of each turn where each player has had an opportunity to attack, whether there has been an attack or not (e.g. if one player owns all resources).
- Inter-state and civil war are modelled as equivalent.

Definitions of variables

In a complete graph, where all the nodes are of roughly equal value, we normalise the map so the total of all resources is 1. We also assume that gamma > 1 is high and rich-rewarding, so rich players are much more likely to win.

T is the threshold, empires above T start losing nodes to rebellion. These nodes become new, independent players.

p is the probability that a node which may defect defects in every turn.

N is the number of nodes in the graph.

Let x be the current biggest player, and Rx his resources. Rx(t) is his resources at turn t, the current turn.

Let n be the number of currently active players.

Basic trends in Rx

If Rx(t) < T, then x is not subject to the defection tax, so E(Rx(t+1)) >= Rx(t) and E(Rx(t+1)) <= Rx(t) + 1/(n-1) max_(x doesn’t own v) r_v

By our assumption that the nodes are of roughly equal value, the right-hand side is approximately Rx(t) + 1/[N(n-1)], and for arbitrarily large gamma x grows this fast.

Applying defection

Now look at when $T < Rx(t) < 1$. Then $E(Rx(t+1))$ is approximately $(1-p)Rx(t) + 1/[N(n-1)]$.

Suppose a steady state average R exists, then it should satisfy

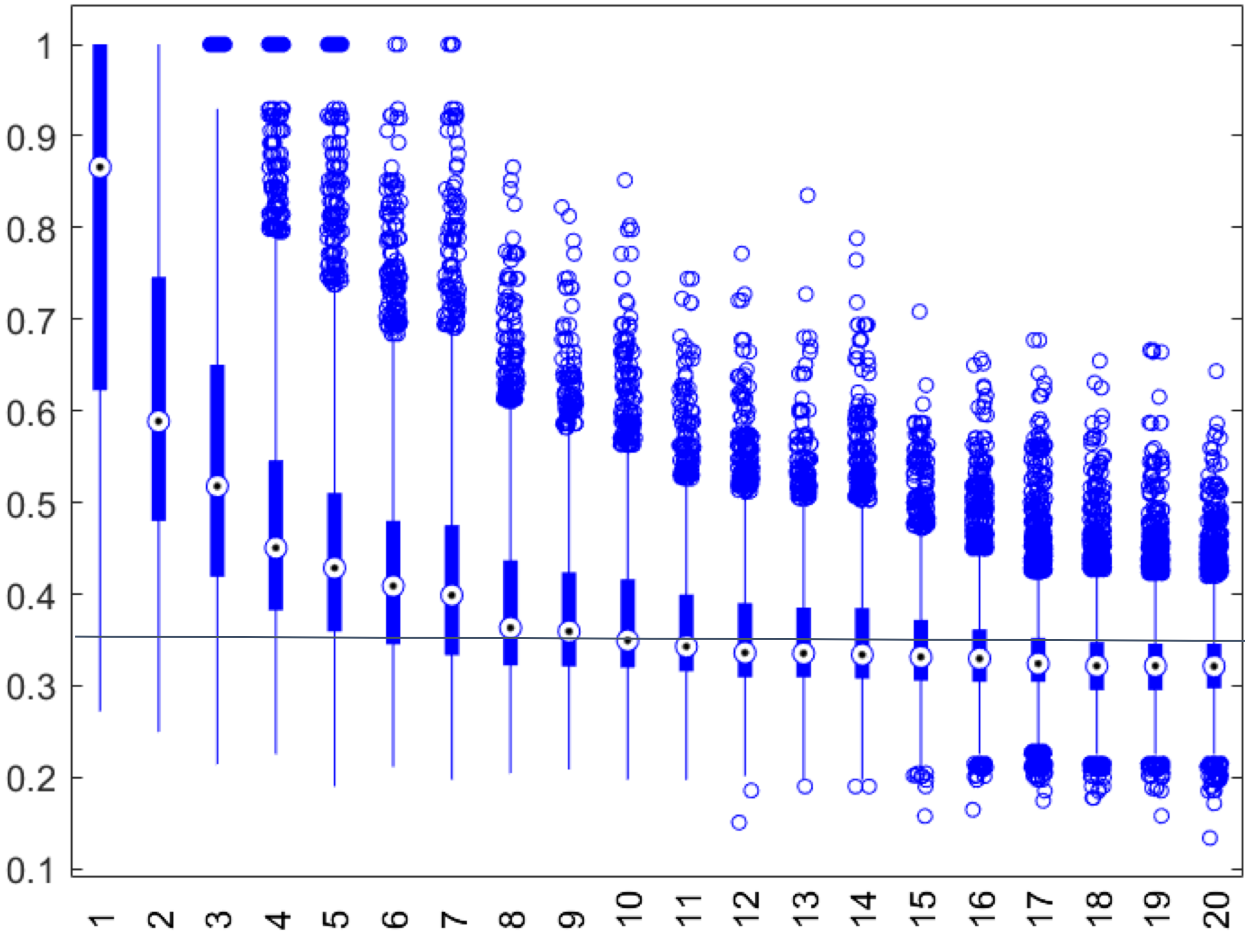
$R = (1-p)R + 1/[N(n-1)]$.

I.e. $R = 1/[N(n-1)p]$

For instance, if $p = 0.02$, $N = 10$, $n - 1 = 2$, then $R = 2.5$. This is nonsense.

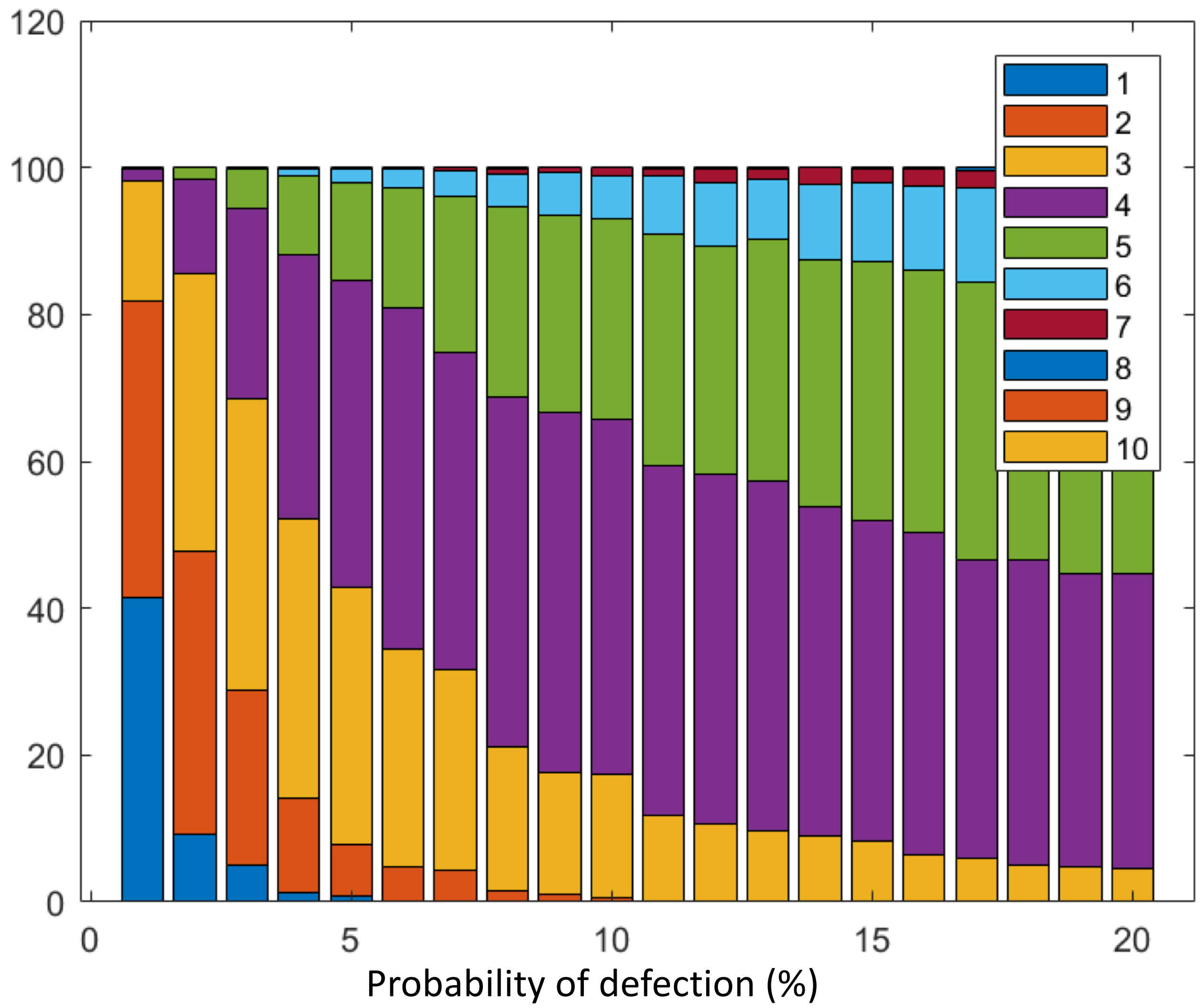
In this case since neither $Rx < 1$ is not a stable position and Rx wants to increase, we say hegemony ($n = 1$) is a stable state.

Simulation results - Rx



Probability of defection (%)

Simulation results - n



Observations

For sufficiently low probability of defection relative to the size of the map, hegemony is stable.

As the probability of defection increases, there is a period when an intermediate state of strength is stable.

At higher probabilities of defection players stabilise with resources just under the threshold, and the number of players stabilises, increasing with the size of the map.

Further observations

Each node represents what can be contested in a single campaign - as logistics capabilities (military or otherwise) increase, it is possible to fight over more at once, so the effective world is smaller with fewer nodes. In this way the development of technology can support the rise of hegemonies.

In a sufficiently large map, empires which reach the threshold are overstretched and unstable, and cannot stably grow further. These are the implications of imperial overreach. Similar results found in Dziubinski, Goyal, Minarsch 2017, who use a cost of conflict parameter and external threats, instead of internal instability.

We now want to add the idea that resources are divisible into two units, and the probability of defection is affected by the division of resources into productive and military efforts.

Stability of hegemony with rebellion

We provide sufficient conditions for a one-player hegemony to be stable despite frequent revolts.

In short we need the expected resources of the largest player to increase with every turn up to all resources available.

Negative changes: rebellion in the player’s nodes, losses in war

Positive changes: gains in war

Instability of large empires leads to multiple-player stability

Observing properties of the previous result we see that if the expected decrease due to rebellions in large empires is enough to make hegemony unstable and discourage players from exceeding what they control, but the trend is still for players to increase in power, then the position is stable with multiple players.

Consequences for imperial overreach:

This leads to a static situation unlike the stable situation with one player, and is thus more stable over time.

Extensions to technology of conflict

Conflict technology:

- Each player may allocate his resources to *fighting* or *productive* effort.
- Each node increases by a fraction with peace and also trade.

Motivation:

- classic ‘guns and butter’ problem interpreted as a public goods problem
    - Levine and Modica (2013) - Army is assumed to be loyal to institutions not persons.
    - Hausken (2003) - Production and Conflict models compared to Rent-seeking models
        - Garfinkel, Skaperdas, Syropoulos (2012) - Benefits accrue depending on producers cooperating or defecting where international trade takes place in insecure environments
        - Cournot Duopoly model where states can trade to collude or compete (Hirshleifer, 1991) which tends to a PD (Abada and Ehrenamann, 2016). The evolution can lead to both cooperation and inefficiency.
- Closed form characterisation of NE.

Incorporating division of effort for trade

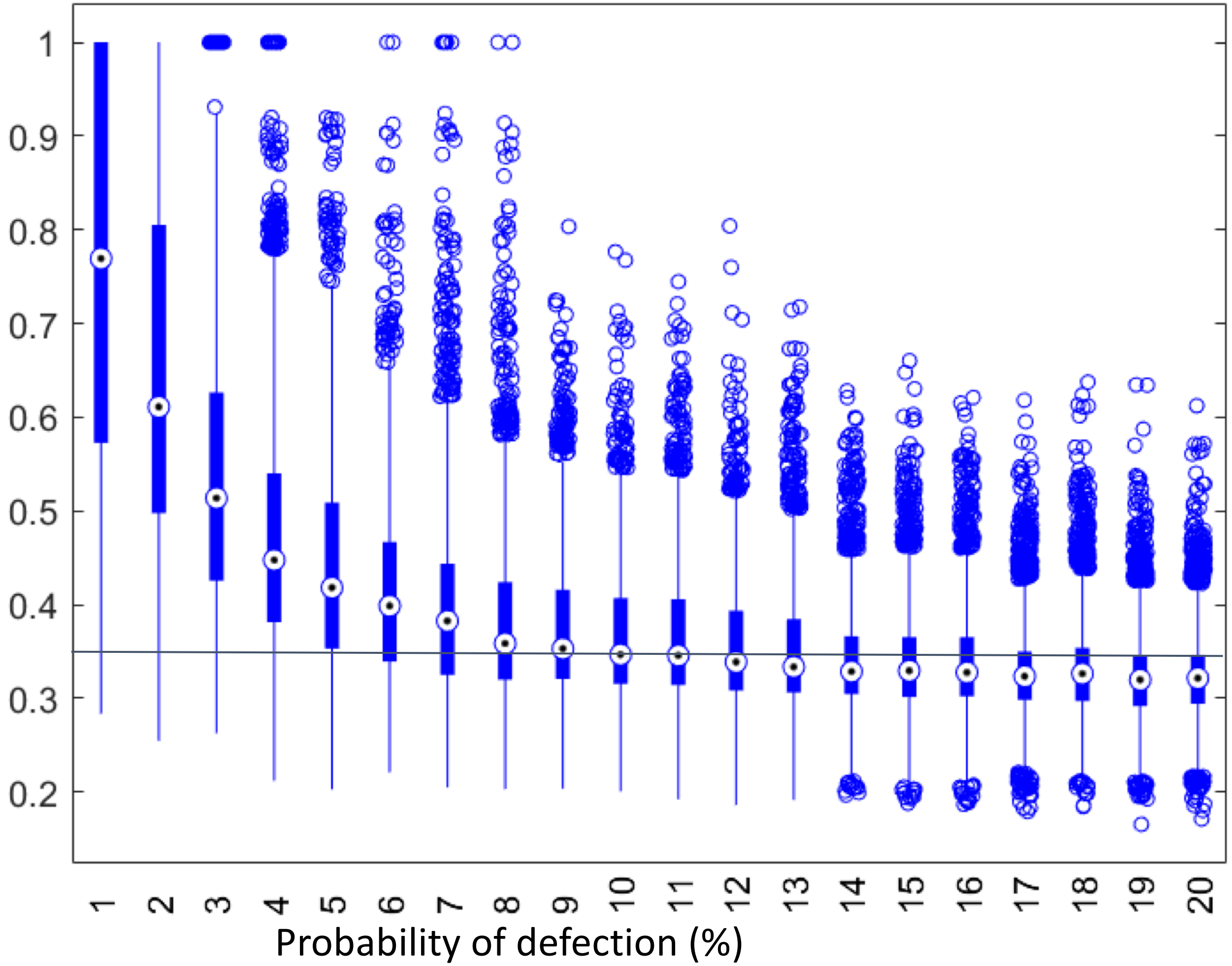
- At the start of each turn, each player allocates a proportion of his resources to fighting and productive efforts.
- The probability that a node will leave a large empire is now not a constant, but proportional to the resources allocated for fighting.
- Each player also adopts a stance towards every other player at the start of the turn, indicating whether they will cooperate or defect in trading.
- Each turn, the resources available for economic growth at each node are determined by the proportion of resources not allocated for fighting.

The trade model continued

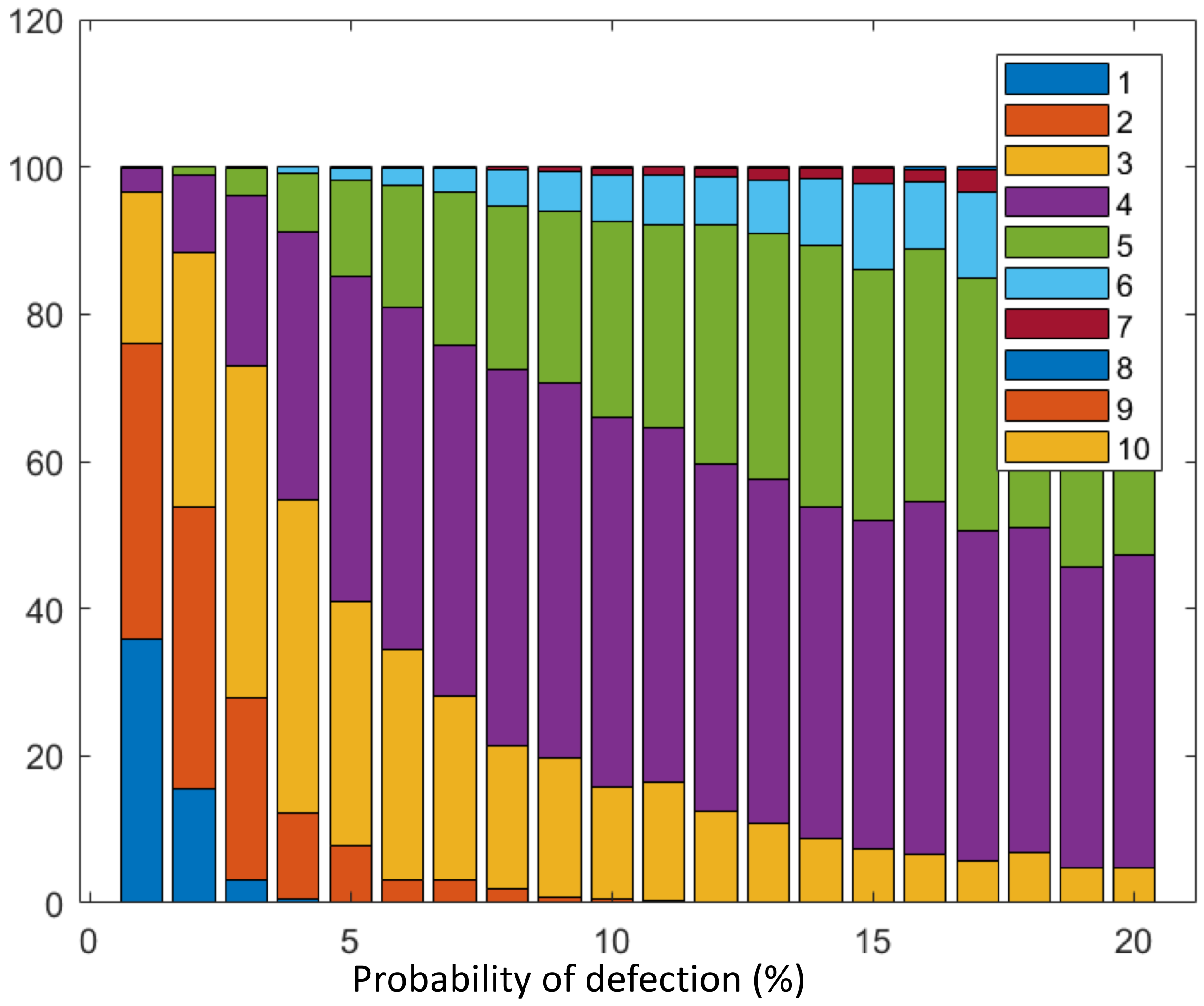
Every pair of adjacent nodes plays a game according to their owners' attitudes towards each other. In this case we approximate the Cournot duopoly model by the prisoner's dilemma.

The nodes grow in proportion to the payoff and the resources both nodes have allocated to trade. Internal trade pays off similarly to external trade, but will usually involve all cooperation.

Simulations with trade - Rx



Simulations with trade - n

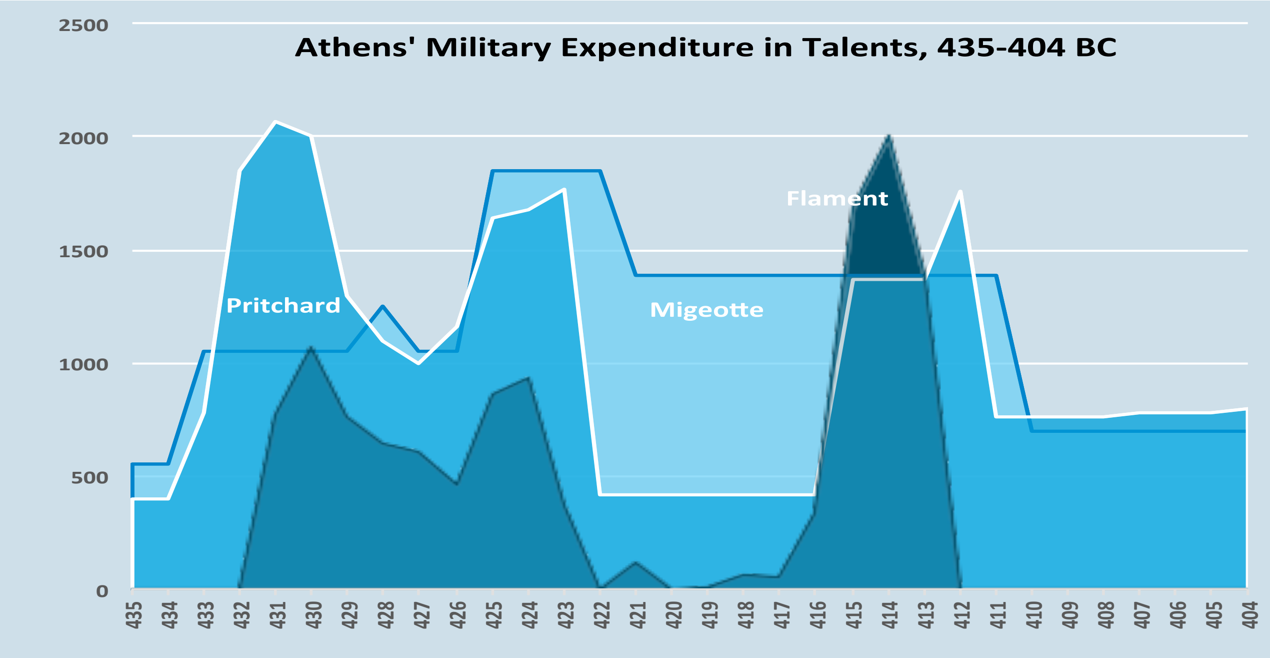


Fitting models to historical data

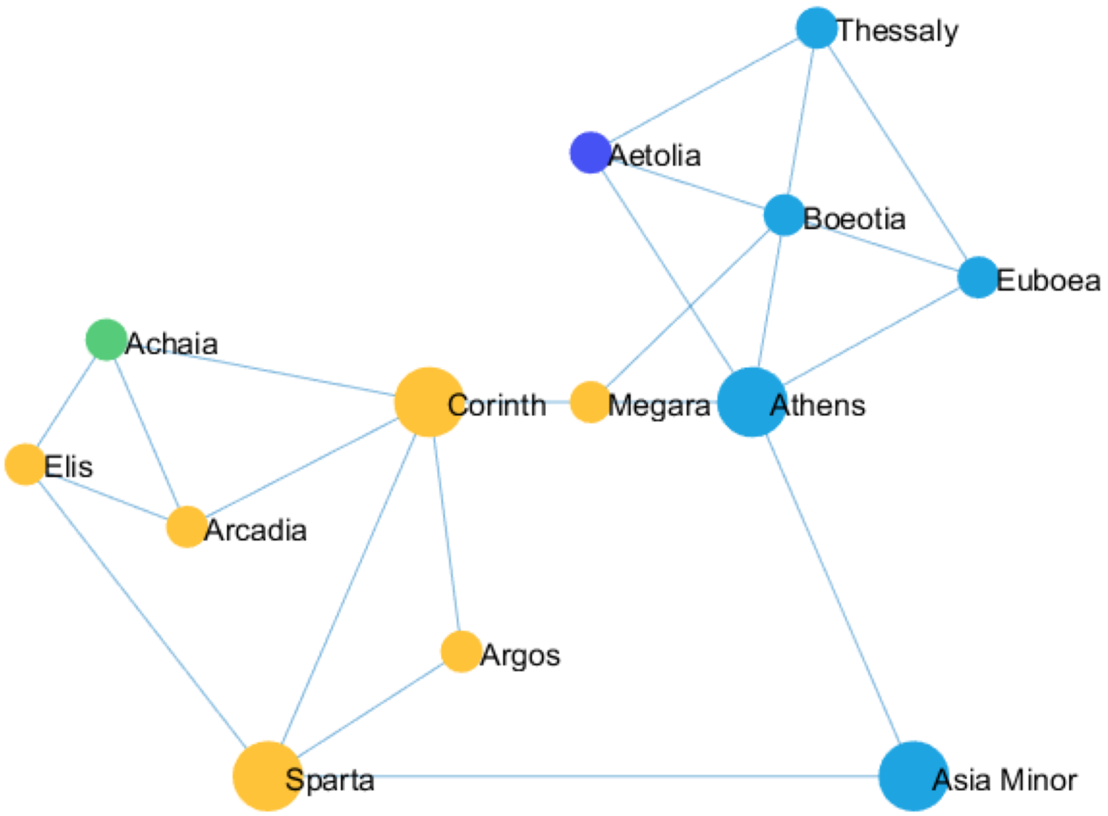
History only happens once, but two states may fight many times.

Given a network, can we find what resource distribution/other parameters make the historical events most likely? How does this match up with reality?

This graph was created from data collected archaeologically or collated from ancient historians. They demonstrate completely different versions of Athens’ financial decision-making toward military expenditure during the Peloponnesian war 431-404 BC.



The Peloponnesian War using our model



Outcomes with trade and defection

The Spartan alliance adopts a policy of 20% allocation to trade and 80% to fighting, other states split 50%-50%.

Number of times as first hegemon in a 300-turn simulation, out of 500 runs		
'Athens'	20%	~100 runs
'Achaia'	5%	~22 runs
'Sparta'	27%	~135 runs
Leavers	43%	>200 runs
No-one	5%	~24 runs

Observations

More highly connected nodes experience much higher growth, as they have more opportunities for trade profit, and can eventually eclipse smaller nodes.

Trade does not greatly change the stable number of players in a complete graph.