Pol. de Newton para valores equi.

$P_n(x)$

$$f(x_0) + \frac{\Delta x_0}{h}(x-x_0) + \frac{\Delta^2 x_0}{2!\,h^2}(x-x_0)(x-x_1) + \dots +$$

$$\frac{\Delta^n x_0}{n!\,h^n}(x-x_0)(x-x_1)\dots(x-x_{n-1}) + E_n.$$

$$E_n = \frac{f^{(n+1)}(\bar{x})}{(n+1)!}(x-x_0)(x-x_1)\dots(x-x_n)$$

Cambio de variable

$$x = x_0 + sh$$
$$x_1 = x_0 + h$$
$$\vdots$$
$$x_{n-1} = x_0 + (n-1)h$$
$$x_n = x_0 + nh$$

$s = 0 \Rightarrow x = x_0$

$s = 1 \Rightarrow x = x_0 + h = x_1$

$s = 2 \Rightarrow x = x_0 + 2h = x_2$

$\vdots$

$s = n \Rightarrow x = x_0 + nh = x_n.$

$\boxed{x - x_0 = sh}$ $\boxed{x - x_1 = sh - h = h(s-1)}$

$$\boxed{x - x_{n-1} = sh - (n-1)h = h(s-(n-1)) = h(s-n+1)}$$

$$\boxed{x - x_n = sh - nh = h(s-n)}$$

Reemplazando.

$$P_n(x_0+sh) \cong f(x_0) + \frac{\Delta x_0}{\cancel{h}}s\cancel{h} + \frac{\Delta^2 x_0}{2\cancel{h^2}}s\cancel{h}\cancel{h}(s-1) + \dots$$

$$+ \dots \frac{\Delta^n x_0}{n!\,\cancel{h^n}}s\cancel{h}.h(s-1)h(s-2)\dots\cancel{h}(s-n+1)$$

$$E_n(x_0+sh) = \frac{f^{(n+1)}(\bar{x})}{(n+1)!}sh.h(s-1).h(s-2)\dots h(s-n)$$

$$P_n(x_0+sh) = f(x_0) + \Delta x_0\, s + \frac{\Delta^2 x_0}{2} s(s-1) + \dots$$

$$+ \dots + \frac{\Delta^n x_0}{n!} s(s-1)(s-2)\dots(s-n+1)$$

$$E_n(x_0+sh) = \frac{f^{n+1}(\bar{x})}{(n+1)!} h^{n+1} s(s-1)(s-2)\dots(s-n)$$

<u>Regla del Trapecio</u>. Pol. grado 1. $\begin{cases} x_0, f(x_0) \\ x_1, f(x_1) \end{cases}$

$$P_1(x_0+sh) \cong f(x_0) + \Delta x_0\, s$$

$$E_1(x_0+sh) = \frac{f''(\bar{x})}{2!} h^2 s(s-1)$$

Integrando:

$[x_0, x_1]$ $\begin{cases} x = x_0 + sh \quad \text{si } x = x_0 = x_0 + sh \Rightarrow \underline{s=0}. \\ x_1 = x_0 + h \Rightarrow \underline{s=1}. \end{cases}$

Si $x = x_0 + sh$.

$dx = h\, ds$.

$$\int_{x_0}^{x_1} P_n(x)\,dx \cong \int_0^1 P_1(x_0+sh)\, h\, ds = h\int_0^1 (f(x_0) + \Delta x_0\, s)\, ds$$

$$= h\left[f(x_0).\, s\Big|_0^1 + \Delta x_0 . \frac{s^2}{2}\Big|_0^1 \right]$$

$$= h\left[f(x_0) + \Delta x_0 \frac{1}{2} \right]$$

$$= h\left[f(x_0) + \frac{1}{2}\left(f(x_1) - f(x_0) \right) \right]$$

$$= \frac{h}{2}\left[f(x_0) + f(x_1) \right] \checkmark$$

Error regla del Trapecio

$$\int_{x_0}^{x_1} E_1(x)\,dx = \int_0^1 E_1(x_0+sh)\,h\,ds = \int_0^1 \frac{f''(\bar{x})}{2!} h^2 s(s-1)\,h\,ds$$

$$E_{trap} = h^3 \cdot \frac{f''(\bar{x})}{2!} \underbrace{\left[\frac{s^3}{3}\Big|_0^1 - \frac{s^2}{2}\Big|_0^1 \right]}_{\frac{1}{3}-\frac{1}{2} = \frac{2-3}{6} = -\frac{1}{6}}$$

$$E_{trap} = -\frac{f''(\bar{x})}{12} h^3 \quad \checkmark$$

Regla de Simpson. (3 puntos) $\Rightarrow P_2(x)$ ④

$$\int_{x_0}^{x_2} P_2(x)\,dx = \int_0^2 \left[f(x_0) + \Delta x_0 . S + \frac{\Delta^2 x_0}{2} \underbrace{S(S-1)}_{S^2 - S} \right] h\,dS .$$

$x = x_0 + S h$

$x_2 = x_0 + 2h$

$$I_{simp} = h . \left[f(x_0) S \Big|_0^2 + \Delta x_0 \frac{S^2}{2} \Big|_0^2 + \frac{\Delta^2 x_0}{2} \left(\frac{S^3}{3} \Big|_0^2 - \frac{S^2}{2} \Big|_0^2 \right) \right]$$

$$I_{simp} = h . \left[2 f(x_0) + \frac{\Delta x_0}{\cancel{2}} \cancel{4}^{2} + \frac{\Delta^2 x_0}{2} \underbrace{\left(\frac{8}{3} - \frac{\cancel{4}^{2}}{\cancel{2}_1} \right)}_{2/3} \right]$$

$\Delta x_0 = f(x_1) - f(x_0)$

$\Delta x_1 = f(x_2) - f(x_1)$

x_0	$f(x_0)$	$f(x_1) - f(x_0)$	$f(x_2) - 2f(x_1) + f(x_0)$
x_1	$f(x_1)$	$f(x_2) - f(x_1)$	
x_2	$f(x_2)$		

$$I_{simp} = h . \left[2 f(x_0) + \Delta x_0\, 2 + \frac{\Delta^2 x_0}{3} \right]$$

$$= h \left[2 f(x_0) + \left(f(x_1) - f(x_0) \right) 2 + \frac{\left(f(x_2) - 2 f(x_1) + f(x_0) \right)}{3} \right]$$

$$= h \left[\cancel{2 f(x_0)} + \underbrace{2 f(x_1)} - \cancel{2 f(x_0)} + \frac{f(x_2)}{3} - \underbrace{\frac{2}{3} f(x_1)} + \frac{f(x_0)}{3} \right]$$

$$2 - \frac{2}{3} = \frac{4}{3}$$

$$I_{simp} = \frac{h}{3} \left[f(x_0) + 4 f(x_1) + f(x_2) \right] \checkmark$$

Escaneado con CamScanner

Error de Simpson.

$$E_2(x_0+sh) = \frac{f^{n+1}(\bar{x})}{(n+1)!}\, sh \cdot h(s-1) \cdot h(s-2)$$

$$\int_0^2 E(x_0+sh)\, h\, ds = \int_0^2 \frac{f'''(\bar{x})}{3!} h^4 \cdot \underbrace{s(s-1)(s-2)}_{\substack{(s^2-s)(s-2) \\ s^3-2s^2-s^2+2s \\ s^3-3s^2+2s}}\, ds.$$

$$E_{Simp} = \frac{f'''(\bar{x})}{3!} h^4 \underbrace{\left[\frac{s^4}{4}\Big|_0^2 - 3\frac{s^3}{3}\Big|_0^2 + \cancel{2}\frac{s^2}{\cancel{2}}\Big|_0^2 \right]}$$

$$\frac{\cancel{16}^{4}}{\cancel{4}_1} - \cancel{3}\left(\frac{8}{\cancel{3}}\right) + 4 = 8 - 8 = 0 \checkmark$$

Magnífico!!

$$\int_0^2 \frac{f^{IV}(\bar{x})}{4!} h^5 \underbrace{s(s-1)(s-2)(s-3)}_{\substack{(s^3-3s^2+2s)(s-3) \\ s^4-3s^3-3s^3+9s^2+2s^2-6s}}\, ds.$$

$$E_{Simp} = \frac{f^{IV}(\bar{x})}{4!} h^5 \cdot \underbrace{\left[\frac{s^5}{5} - 6\frac{s^4}{4} + 11\frac{s^3}{3} - 6\frac{s^2}{2} \Big|_0^2 \right]}$$

$$\frac{32}{5} - \frac{3}{\cancel{2}_1} \cdot \cancel{16}^{8} + \frac{11}{3} 8 - 3 \times 4 = \frac{32}{5} - 36 + \frac{88}{3} = \frac{-4}{15}$$

$$E_{Simp} = \frac{f^{IV}(\bar{x})}{\cancel{1.2.3.4}} h^5 \cdot \left(-\frac{\cancel{4}}{15}\right) = -\frac{1}{90} h^5 f^{IV}(\bar{x}) \checkmark$$