Introduction to Quantum Information and Quantum Computing
Assignment 1

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Introduction

This report presents the solution to Assignment 1 of the Introduction to Quantum Information and Quantum Computing course.

The goal of this assignment is to illustrate a fundamental feature of quantum mechanics: measurement outcomes do not, in general, possess predetermined values prior to measurement—unless one is willing to accept the possibility of faster-than-light communication.

1 Preparation of a shared state

Alice (A), Bob (B), and Charlie (C) are separated from each other by a considerable distance. A source (S) prepares three qubits in the so-called $|\text{GHZ}\rangle$ state, according to the circuit below, and sends one qubit to each of them. As always, the qubits start in the state

$$|000\rangle \quad (|\text{CBA}\rangle \text{ ordering})$$

before passing through the circuit.

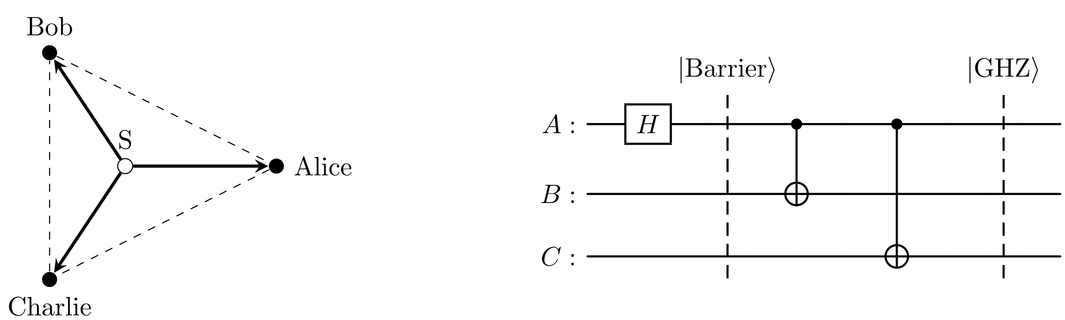

Figure 1: System layout and quantum circuit preparing the GHZ state

(a) i) Let $|\text{Barrier}\rangle$ denote the state at the first barrier. Since the Hadamard gate is applied only to qubit A (the least significant qubit in the $|\text{CBA}\rangle$ ordering), we obtain

$$|000\rangle \xrightarrow{\text{H}_A} |00\rangle\left(\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right) = \frac{1}{\sqrt{2}}|000\rangle + \frac{1}{\sqrt{2}}|001\rangle = |\text{Barrier}\rangle .$$

ii) Starting from $|\text{Barrier}\rangle$, we apply a CNOT gate with A as control and B as target, followed by a CNOT with A as control and C as target:

$$|\text{Barrier}\rangle \xrightarrow{\text{CNOT}_{A,B}} \frac{1}{\sqrt{2}}|000\rangle + \frac{1}{\sqrt{2}}|011\rangle \xrightarrow{\text{CNOT}_{A,C}} \frac{1}{\sqrt{2}}|000\rangle + \frac{1}{\sqrt{2}}|111\rangle = |\text{GHZ}\rangle .$$

At the first barrier, the system is in a superposition of $|000\rangle$ and $|001\rangle$. After the CNOT operations, this superposition is extended across all three qubits, producing the maximally entangled GHZ state.

(b) Measuring Alice's qubit collapses the state:

- If Alice measures 0, Bob and Charlie collapse to $|00\rangle$
- If Alice measures 1, Bob and Charlie collapse to $|11\rangle$

From Bob's or Charlie's local perspective, the outcomes remain random until classical communication occurs.

(c) Since $|\mathrm{GHZ}\rangle$ is an equal superposition of $|000\rangle$ and $|111\rangle$, we expect each outcome with probability 1/2. This behavior is confirmed by simulation, as shown in Figure 2.

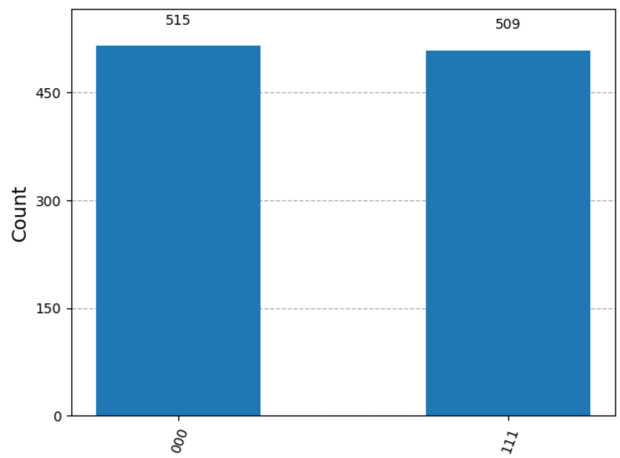

Figure 2: Simulation results for 1024 shots

2 Independent Operations

After receiving their qubits, Alice, Bob, and Charlie independently apply either

$$M_X = H \quad \text{or} \quad M_Y = HS^\dagger,$$

followed by measurement.

(a) When all three apply M_X, the resulting state $|\mathrm{XXX}\rangle$ is produced by the circuit in Figure 3.

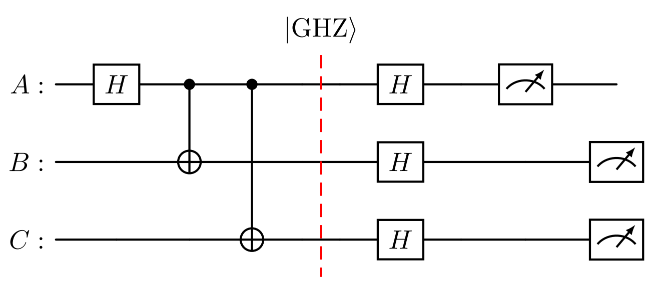

Figure 3: Circuit XXX: all parties apply $M_X = H$

Applying the Hadamard gates yields

$$|\mathrm{XXX}\rangle = \frac{1}{2}\big(\,|000\rangle + |011\rangle + |101\rangle + |110\rangle\,\big).$$

Simulation results are shown in Figure 4.

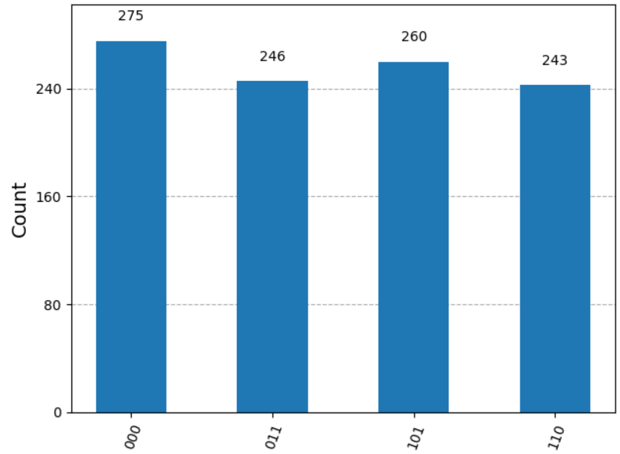

Figure 4: Simulation results for circuit XXX (1024 shots)

(b) The possible outcomes are $|000\rangle$, $|011\rangle$, $|101\rangle$, and $|110\rangle$. In all cases, the sum $a + b + c$ is even.

(c) When exactly one party applies M_X and the others apply M_Y, only four outcomes occur, and the sum $a + b + c$ is always odd.

(d) The outcomes for each configuration are:

- YYX, YXY, XYY:

$$(C, B, A) = (0, 0, 1), (0, 1, 0), (1, 0, 0), (1, 1, 1).$$

These results confirm the GHZ correlations.