Last Assignment

$X|0\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$ $|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$X|1\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}}$ $|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

1) a) $|\psi\rangle = \frac{8}{17}|0\rangle - \frac{15}{17}|1\rangle$

Amplitude de $|1\rangle = -\frac{15}{17}$

$P(|1\rangle) = \left|-\frac{15}{17}\right|^2 = \frac{15^2}{17^2} = \frac{225}{289} \simeq 0.77855$

b)

$P(|0\rangle) = \left|\frac{8}{17}\right|^2 = \frac{64}{289} \simeq 0.22145$ $64 + 225 = 289$

$P(|1\rangle) + P(|0\rangle) = \frac{225}{289} + \frac{64}{289} = \frac{289}{289} = 1$

c)

$X(|\psi\rangle) = \frac{8}{17}X(|0\rangle) - \frac{15}{17}X(|1\rangle) =$

$= \frac{8}{17}\left(\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right) - \frac{15}{17}\left(\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right) =$

$= \frac{8}{17\sqrt{2}}|0\rangle + \frac{8}{17\sqrt{2}}|1\rangle - \frac{15}{17\sqrt{2}}|0\rangle + \frac{15}{17\sqrt{2}}|1\rangle =$

$= \frac{-7}{17\sqrt{2}}|0\rangle + \frac{23}{17\sqrt{2}}|1\rangle$

$P(|0\rangle) = \left|\frac{-7}{17\sqrt{2}}\right|^2 = \frac{7^2}{17^2 \cdot 2} = \frac{49}{289 \cdot 2} = \frac{49}{578} \simeq 0.08478$

$P(|1\rangle) = \left|\frac{23}{17\sqrt{2}}\right|^2 = \frac{23^2}{17^2 \cdot 2} = \frac{529}{289 \cdot 2} = \frac{529}{578} \simeq 0.91522$

$P(|0\rangle) + P(|1\rangle) = \frac{49}{578} + \frac{529}{578} = \frac{578}{578} = 1$

d) Se cada pessoa de 5 tem um qubit, precisamos de

2^n, $n = 5$ estados $\Rightarrow 2^5 = 32$ estados

$|A\rangle, |B\rangle, |C\rangle, |D\rangle, |E\rangle \Rightarrow |ABCDE\rangle$

2) a) $|a\rangle = H(|0\rangle)$ $\quad$ $|c\rangle = \sqrt{Y}(|0\rangle)$

$|b\rangle = H(|1\rangle)$ $\quad$ $|d\rangle = \sqrt{Y}(|1\rangle)$

$$H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \rightarrow \text{Standard Hadamard Matrix / Gate}$$

$$\sqrt{Y} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

$$\sqrt{Y}\cdot\sqrt{Y} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}\cdot\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} \cancel{1-1} & \cancel{-1-1} \\ 1+1 & \cancel{-1+1} \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix} =$$

$$= \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \Rightarrow \frac{1}{i}Y$$

$$|a\rangle = H(|0\rangle) = \frac{|0\rangle + |1\rangle}{\sqrt{2}} = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

$$|b\rangle = H(|1\rangle) = \frac{|0\rangle - |1\rangle}{\sqrt{2}} = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$$

2x2 2x1

2x1

$$|c\rangle = \sqrt{Y}|0\rangle = \frac{1}{\sqrt{2}}\cdot\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}\cdot\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1+0 \\ 1+0 \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix} =$$

$$= \frac{1}{\sqrt{2}}\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \frac{1}{\sqrt{2}}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

$$|d\rangle = \sqrt{Y}|1\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}\cdot\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} 0-1 \\ 0+1 \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} -1 \\ 1 \end{bmatrix} =$$

$$= \frac{1}{\sqrt{2}}\left(\begin{bmatrix} -1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = -\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \frac{1}{\sqrt{2}}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = -\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

$\|a\rangle = \frac{1}{\sqrt{2}}\|0\rangle + \frac{1}{\sqrt{2}}\|1\rangle$	$\|b\rangle = \frac{1}{\sqrt{2}}\|0\rangle - \frac{1}{\sqrt{2}}\|1\rangle$	$\|c\rangle = \frac{1}{\sqrt{2}}\|0\rangle + \frac{1}{\sqrt{2}}\|1\rangle$	$\|d\rangle = -\frac{1}{\sqrt{2}}\|0\rangle + \frac{1}{\sqrt{2}}\|1\rangle$
$[1, 1]$	$[1, -1]$	$[1, 1]$	$[-1, 1]$

b) Not possible to distinguish experimentally. $P(|a\rangle) = P(|b\rangle) = P(|c\rangle) = P(|d\rangle)$

c) $$H|0\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle \xrightarrow{H} \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)\right) + \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)\right) =$$

2 vezes

$$= \frac{1}{2}|0\rangle + \cancel{\frac{1}{2}|1\rangle} + \frac{1}{2}|0\rangle - \cancel{\frac{1}{2}|1\rangle} = \frac{2}{2}|0\rangle = |0\rangle$$

1 → $\sqrt{Y}|0\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$ $\qquad$ $\sqrt{Y}|1\rangle = -\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$

2 → $\sqrt{Y}$ ↓

$$\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\right) + \frac{1}{\sqrt{2}}\left(-\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\right) =$$

$$= \cancel{\frac{1}{2}|0\rangle} + \frac{1}{2}|1\rangle \cancel{- \frac{1}{2}|0\rangle} + \frac{1}{2}|1\rangle = |1\rangle$$

3 → $\sqrt{Y}$ ↓

$$\sqrt{Y}|1\rangle = -\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

4 → $\sqrt{Y}$ ↓

$$-\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\right) + \frac{1}{\sqrt{2}}\left(-\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\right) =$$

$$= -\frac{1}{2}|0\rangle \cancel{- \frac{1}{2}|1\rangle} - \frac{1}{2}|0\rangle \cancel{+ \frac{1}{2}|1\rangle} = -|0\rangle$$

phase shift
↓
indistinguishable

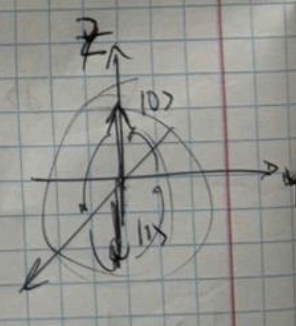
5 → $\sqrt{Y}$ ↓

$$-\sqrt{Y}|0\rangle = -\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$$

6 → $\sqrt{Y}$ ↓

$$-\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\right) - \frac{1}{\sqrt{2}}\left(-\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\right) =$$

$$= \cancel{-\frac{1}{2}|0\rangle} - \frac{1}{2}|1\rangle \cancel{+ \frac{1}{2}|0\rangle} - \frac{1}{2}|1\rangle = -|1\rangle$$



7 → $\sqrt{Y}$ ↓

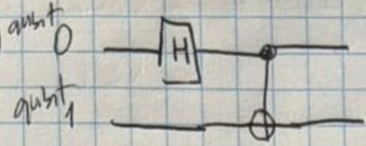
$$\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle =$$

8 → $\sqrt{Y}$ ↓

$$\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\right) - \frac{1}{\sqrt{2}}\left(-\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\right) =$$

$$= \frac{1}{2}|0\rangle \cancel{+ \frac{1}{2}|1\rangle} + \frac{1}{2}|0\rangle \cancel{- \frac{1}{2}|1\rangle} = |0\rangle$$

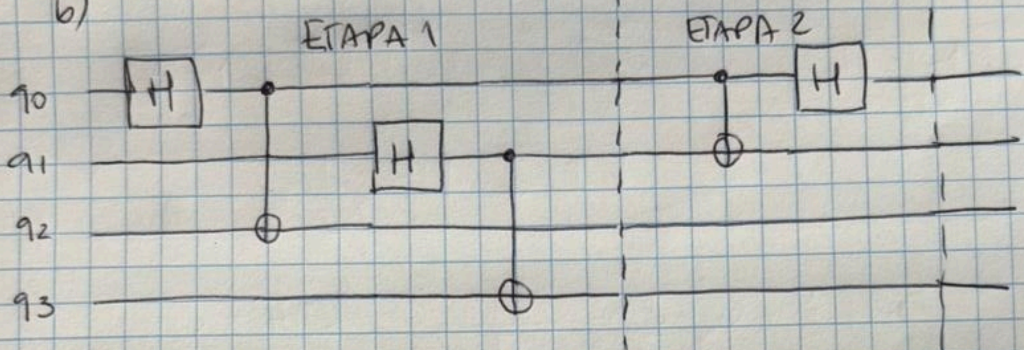
3) a)



$$\overset{10}{|00\rangle} \xrightarrow{H_0} |0\rangle \cdot \frac{|0\rangle + |1\rangle}{\sqrt{2}} = \frac{|00\rangle}{\sqrt{2}} + \frac{|01\rangle}{\sqrt{2}}$$

$$\xrightarrow{CNOT_{(0,1)}} \frac{|00\rangle}{\sqrt{2}} + \frac{|11\rangle}{\sqrt{2}} = \frac{|00\rangle + |11\rangle}{\sqrt{2}} = \frac{1}{\sqrt{2}} \begin{array}{cc} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} & \begin{array}{c} 00 \\ 01 \\ 10 \\ 11 \end{array} \end{array}$$

b)



Comecemos no estado $|0000\rangle \Rightarrow |q_3 q_2 q_1 q_0\rangle$

$$|0000\rangle \xrightarrow{H_{q_0}} |000\rangle \frac{|0\rangle + |1\rangle}{\sqrt{2}} \xrightarrow{CNOT(q_0, q_2)} \frac{|0000\rangle + |0001\rangle}{\sqrt{2}} =$$

$$= \frac{|0000\rangle + |0101\rangle}{\sqrt{2}}$$

$$\xrightarrow{H(q_1)} = \frac{|00\rangle}{\sqrt{2}} \left(\frac{|0\rangle + |1\rangle}{\sqrt{2}} \right) \cdot |0\rangle + \frac{|01\rangle}{\sqrt{2}} \cdot \left(\frac{|0\rangle + |1\rangle}{\sqrt{2}} \right) \cdot |1\rangle =$$

$$= \frac{|0000\rangle}{2} + \frac{|0010\rangle}{2} + \frac{|0101\rangle}{2} + \frac{|0111\rangle}{2}$$

$$CNOT(q_1, q_3) \longrightarrow \frac{|0000\rangle}{2} + \frac{|1010\rangle}{2} + \frac{|0101\rangle}{2} + \frac{|1111\rangle}{2}$$

$$CNOT(q_0, q_1) \longrightarrow \frac{|0000\rangle}{2} + \frac{|1010\rangle}{2} + \frac{|0111\rangle}{2} + \frac{|1101\rangle}{2}$$

$$H(q_0) \longrightarrow \frac{|000\rangle}{2} \frac{|0\rangle + |1\rangle}{\sqrt{2}} + \frac{|101\rangle}{2} \frac{|0\rangle + |1\rangle}{\sqrt{2}} + \frac{|011\rangle}{2} \frac{|0\rangle - |1\rangle}{\sqrt{2}} +$$

$$+ \frac{|110\rangle}{2} \frac{|0\rangle - |1\rangle}{\sqrt{2}} =$$

$$= \frac{|0000\rangle}{2\sqrt{2}} + \frac{|0001\rangle}{2\sqrt{2}} + \frac{|1010\rangle}{2\sqrt{2}} + \frac{|1011\rangle}{2\sqrt{2}} + \frac{|0110\rangle}{2\sqrt{2}} - \frac{|0111\rangle}{2\sqrt{2}} + \frac{|1100\rangle}{2\sqrt{2}} - \frac{|1101\rangle}{2\sqrt{2}}$$

0000 +
0001 +
0010
0011
0100
0101
0110 +
0111 -
1000
1001
1010 +
1011 +
1100 +
1101 -
1110
1111