Introduction to Quantum Information and Quantum Computing
Assignment 1

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Introduction

This report presents the solution to Assignment 1 of the Introduction to Quantum Information and Quantum Computing course.

The goal of this assignment is to illustrate a fundamental feature of quantum mechanics: measurement outcomes do not, in general, possess predetermined values prior to measurement—unless one is willing to accept the possibility of faster-than-light communication.

1 Preparation of a shared state

Alice (A), Bob (B), and Charlie (C) are separated from each other by a considerable distance. A source (S) has prepared three qubits in the so-called $|\text{GHZ}\rangle$ state, according to the circuit below, and sent one qubit to each of them. As always, the qubits all start in the $|0\rangle$ state before passing through the circuit.

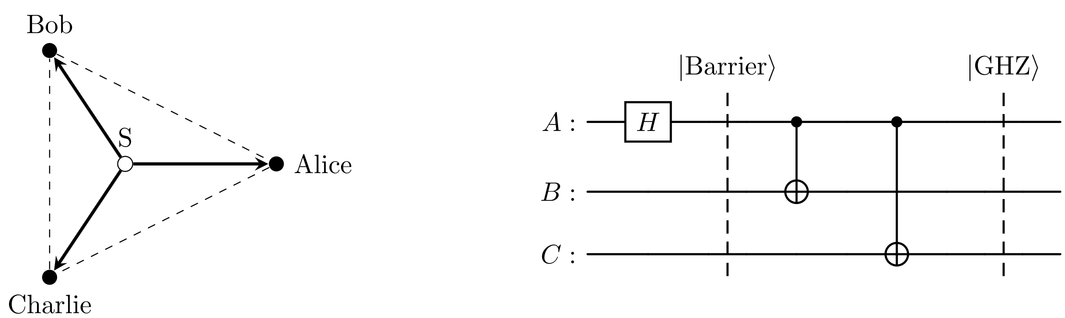

Figure 1: System and circuit representation

(a) i) Let $|\text{Barrier}\rangle$ be the state of the system at the first barrier. Since we start with the qubits $|000\rangle$ ($|\text{CBA}\rangle$ respectively), and we only apply the H gate to the first qubit (A), we get:

$$|000\rangle \xrightarrow{\text{H}_A} |00\rangle \left(\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right) = \frac{1}{\sqrt{2}}|000\rangle + \frac{1}{\sqrt{2}}|001\rangle = |\text{Barrier}\rangle$$

ii) Let $|GHZ\rangle$ be the state of the system at the second barrier (end of the circuit). We start from the $|\text{Barrier}\rangle$ state and apply the CNOT gate first with A as control and B as target, and then with A as control and C as target. As so, we get:

$$|\text{Barrier}\rangle = \frac{1}{\sqrt{2}}|000\rangle + \frac{1}{\sqrt{2}}|001\rangle \xrightarrow{\text{CNOT}_{A,B}} \frac{1}{\sqrt{2}}|000\rangle + \frac{1}{\sqrt{2}}|011\rangle \xrightarrow{\text{CNOT}_{A,C}} \frac{1}{\sqrt{2}}|000\rangle + \frac{1}{\sqrt{2}}|111\rangle = |\text{GHZ}\rangle$$

At the first barrier, the system is in a superposition of $|000\rangle$ and $|001\rangle$, represented by $|\text{Barrier}\rangle$. After applying the CNOT gates, this superposition is extended across all three qubits, resulting in the maximally entangled state $|\text{GHZ}\rangle$.

(b) Measuring Alice's qubit collapses its state:

- If Alice obtains 0, Bob and Charlie's qubits collapse to $|00\rangle$
- If Alice obtains 1, Bob and Charlie's qubits collapse to $|11\rangle$

However, from Bob or Charlie's perspective alone, their measurement outcomes are still random until they compare results with Alice.

(c) As shown previously, the $|\text{GHZ}\rangle$ state is in a superposition of $|000\rangle$ and $|111\rangle$. Thus, we expect to observe $|000\rangle$ half of the time and $|111\rangle$ the other half.

By running the simulation, we confirm that this is the case, as shown by the results in Figure 2.

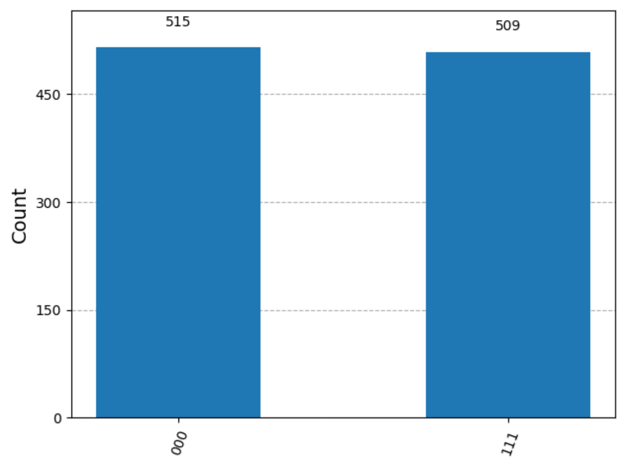

Figure 2: Result of the simulation for 1024 shots

2 Independent Operations

After receiving their qubit, Alice, Bob, and Charlie each apply one of two operations to their qubit: either $M_X = H$ or $M_Y = HS^\dagger$. Immediately afterward, each of them measures their qubit.

(a) Let's start with the case in which all of them apply $M_X = H$, to which we'll call state $|\text{XXX}\rangle$. The following circuit generates state $|\text{XXX}\rangle$:

We expect the $|\text{XXX}\rangle$ state to be:

$$
\begin{aligned}
|\text{GHZ}\rangle &= \frac{1}{\sqrt{2}}|000\rangle + \frac{1}{\sqrt{2}}|111\rangle \\
&\xrightarrow{\text{H}_A} \frac{1}{2}|000\rangle + \frac{1}{2}|001\rangle + \frac{1}{2}|110\rangle - \frac{1}{2}|111\rangle \\
&\xrightarrow{\text{H}_B} \frac{1}{2\sqrt{2}}|000\rangle + \frac{1}{2\sqrt{2}}|010\rangle + \frac{1}{2\sqrt{2}}|001\rangle + \frac{1}{2\sqrt{2}}|011\rangle \\
&\quad + \frac{1}{2\sqrt{2}}|100\rangle - \frac{1}{2\sqrt{2}}|110\rangle - \frac{1}{2\sqrt{2}}|101\rangle + \frac{1}{2\sqrt{2}}|111\rangle \\
&\xrightarrow{\text{H}_C} \frac{1}{2}|000\rangle + \frac{1}{2}|011\rangle + \frac{1}{2}|101\rangle + \frac{1}{2}|110\rangle = |\text{XXX}\rangle
\end{aligned}
$$

By running the simulation, we confirm that this is the case, as shown by the results in Figure 3.

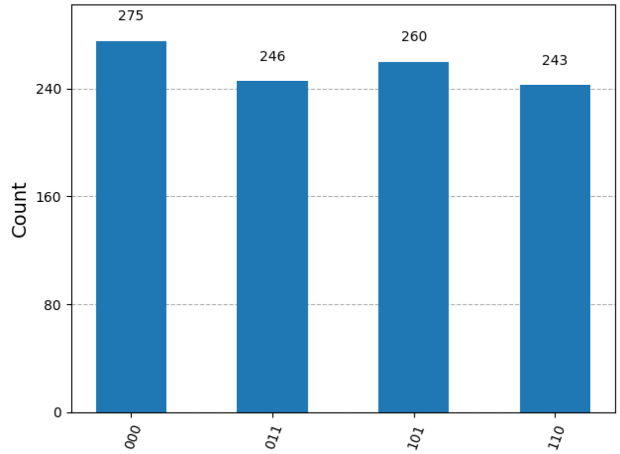

Figure 3: Result of the simulation of 1024 shots for circuit XXX

(b) The possible results are $|000\rangle, |011\rangle, |101\rangle$ and $|110\rangle$ for (C,B,A) = (0,0,0), (0,1,1), (1,0,1), (1,1,0) respectively. It's also possible to confirm that in each case, the sum $a + b + c$ is always even.

(c) Next, we considered the circuits in which only one of the participants applies M_X and the others apply $M_Y = HS^\dagger$.

After simulating these circuits, we observe that only four measurement outcomes are possible for each case, and they are always correlated such that the sum $a + b + c$ is odd.

(d) The possible measurement outcomes for each of these circuits are:

- YYX (Charlie and Bob apply M_Y, Alice applies M_X):

$$(C, B, A) = (0, 0, 1), (0, 1, 0), (1, 0, 0), (1, 1, 1)$$

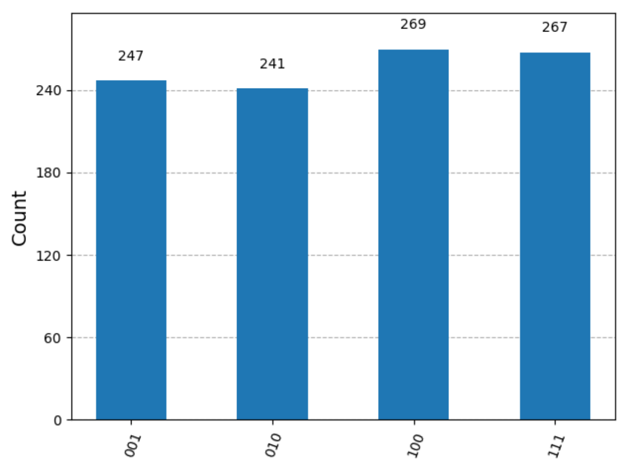

Figure 4: Result of the simulation of 1024 shots for circuit YYX

- YXY (Charlie and Alice apply M_Y, Bob applies M_X):

$$(C, B, A) = (0, 0, 1), (0, 1, 0), (1, 0, 0), (1, 1, 1)$$

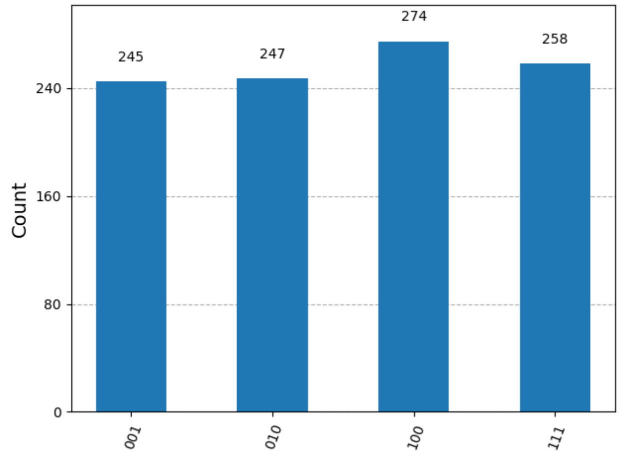

Figure 5: Result of the simulation of 1024 shots for circuit YXY

- XYY (Alice applies M_X, Bob and Charlie apply M_Y):

$$(C, B, A) = (0, 0, 1), (0, 1, 0), (1, 0, 0), (1, 1, 1)$$

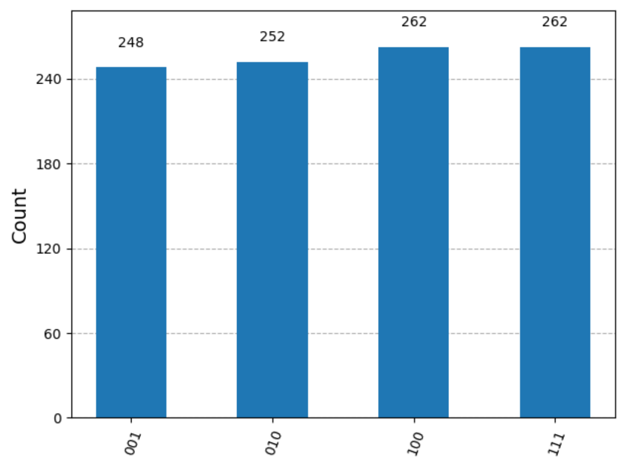

Figure 6: Result of the simulation of 1024 shots for circuit XYY

In each case, the sum $a + b + c$ is always odd, as expected from the GHZ correlations.