FACULDADE DE CIÊNCIAS E TECNOLOGIA
UNIVERSIDADE DE COIMBRA

Computational Methods for Detection, Estimation and Identification 2022/2023

Assignment 5

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1 Overview

This assignment aims to develop algorithms to estimate planar conic curves from a set of points X. If the conic is parameterized by a six dimensional vector $\omega = (a, b, c, d, e, f)t$ and X lies on the curve, then it follows that (1).

$$y = ax_i^2 + bx_iy_i + cy_i^2 + dx_i + ey_i + f = 0 \qquad (1)$$

The conic curve can also be represented by a 3×3 symmetric matrix Ω. In this case the relation of the previous equation can be rewritten as (2)

$$\begin{bmatrix} x_i & y_i & 1 \end{bmatrix} \underbrace{\begin{bmatrix} a & \frac{b}{2} & \frac{d}{2} \\ \frac{b}{2} & c & \frac{e}{2} \\ \frac{d}{2} & \frac{e}{2} & f \end{bmatrix}}_{=\Omega} \begin{bmatrix} x_i \\ y_i \\ i \end{bmatrix} = 0 \qquad (2)$$

The measured points X are usually contaminated with bidimensional zero mean gaussian noise. We will assume that the noise for each direction X and Y is independent but has the same standard deviation σ.

2 Ground Truth

The first step was to obtain the curves to study.

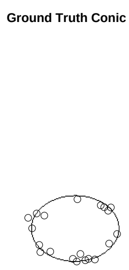

Figure 1: Ground Truth with 30 points and $\sigma = 4$

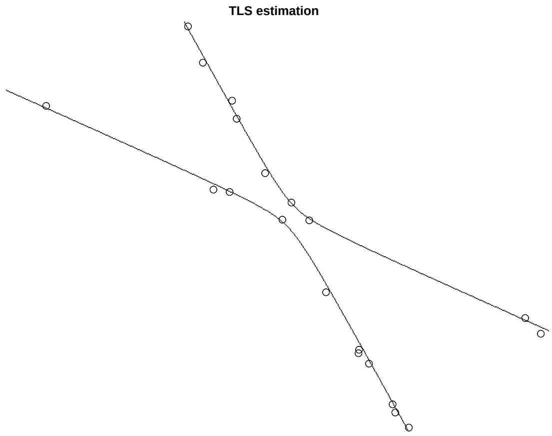

Figure 2: Ground Truth with 30 points and $\sigma = 4$

3 Total Least Squares Estimation

The second task is to obtain a Total Least Square estimation for the model. In order to achieve this task, the first step was to get the G matrix for the specific problem (3).

$$G = \begin{bmatrix} x_1^2 & x_1y_1 & y_1^2 & x_1 & y_1 & 1 \\ x_2^2 & x_2y_2 & y_2^2 & x_2 & y_2 & 1 \\ & \vdots & & & \vdots & \\ x_m^2 & x_my_m & y_m^2 & x_m & y_m & 1 \end{bmatrix}_{m\times 6} \qquad (3)$$

Next we proceeded to do the Singular Value Decomposition of G in order to obtain the last column of V, that gives us the Total Least Squares estimation. The results are shown in 3 and 4.

Figure 3: TLS estimation with 30 points and $\sigma = 4$

In order to see the way TLS reacts to the increase of points, it was attempted a simulation with Monte Carlo the reaction to an increasing number of points. It was not successful.

4 Normalized Total Least Squares Estimation

The second task is to obtain a Normalized Total Least Square estimation for the model. It's similar to the first approach, but requires the previous normalization of the points. It provided the results in 5 and 6.

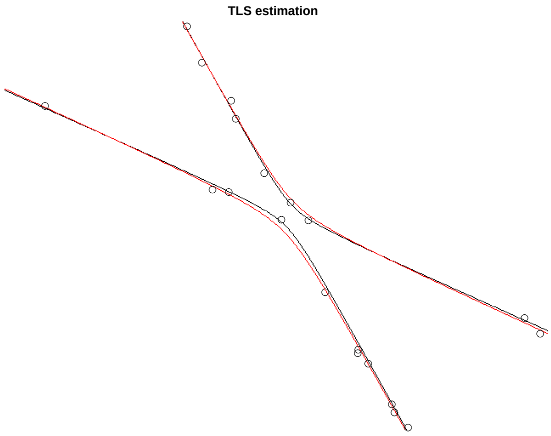

Figure 4: TLS estimation with 30 points and $\sigma = 4$

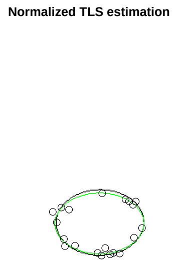

Figure 5: Normalized TLS estimation with 30 points and $\sigma = 4$

5 Levenberg-Marquadt Estimation with Sampson distance

The last task was to obtain an estimation using Levenberg-Marquadt estimation with Sampson distance as the metric for the error. It provided the results in 7 and 8.

6 Conclusions

References

[1] R. C. Aster, B. Borchers, and C. H. Thurber, Parameter Estimation and Inverse Problems. Elsevier, 2018.

[2] Nuno Gonçalves, sebenta_TCDEI_chapter1_linear_inverse_problem University of Coimbra, 2023

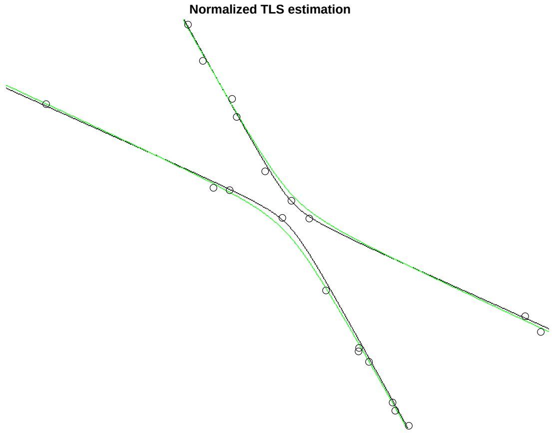

Figure 6: Normalized TLS estimation with 30 points and $\sigma = 4$

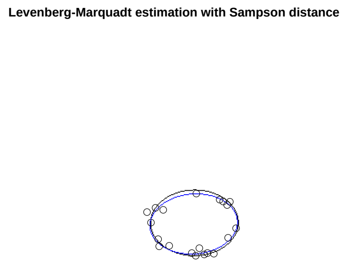

Figure 7: Levenberg-Marquadt Estimation with Sampson with 30 points and $\sigma = 4$

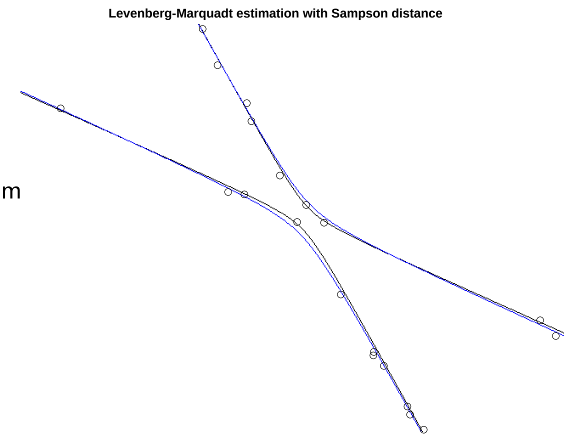

Figure 8: Levenberg-Marquadt Estimation with Sampson with 30 points and $\sigma = 4$

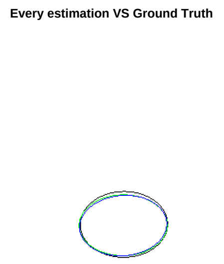

Figure 9: Methods Comparison with 30 points and $\sigma = 4$

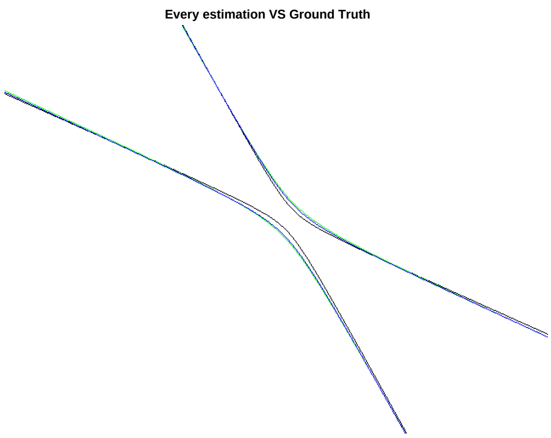

Figure 10: Methods Comparison with 30 points and $\sigma = 4$