Computational Methods for Detection, Estimation and Identification

ASSIGNMENT 4

MsC in Electrical and Computer Engineering

DEEC - University of Coimbra

2022/2023

Experimental Work

Quadric surfaces are analytically modelled by a quadratic equation in the space coordinates - x, y and z. This type of surfaces comprises spherical, elliptical, hyperbolic (of two sheets) and parabolic surfaces. Quadric surfaces are often used in robotics, vision and graphics problems. The intersection or two arbitrary quadrics is a polynomial of 8th degree in one single parameter θ and it is commonly called *quartic*.

Figure 1 shows one example of a quartic curves derived by the intersection of two torus.

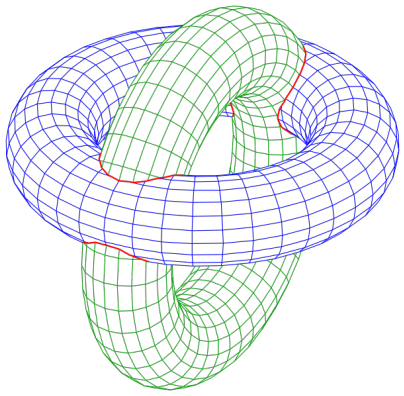

Figure 1: Intersection of two quadrics parameterised as an 8th degree polynomial

For this assignment consider the case of an arbitrary polynomial of degree 8 defined by:

$$y = a\theta^8 + b\theta^7 + c\theta^6 + d\theta^5 + e\theta^4 + f\theta^3 + g\theta^2 + h\theta \tag{1}$$

for which an experiment is able to measure the values of θ and y.

1. You can find enclosed a Matlab function named *SimulationPoly8* (for details make *help SimulationPoly8*). This function simulates readings of data pairs (y_i, θ_i), corresponding to

points belonging to a polynomial of 8th degree. You can define the standard deviation of the additive gaussian noise, as well as the percentage of outliers in the data set.

In this work proposal we want you to use the SimulationPoly8 function to generate data vector with 10, 16, and 32 points and analyse the fitting to a polynomial of 8th degree given by equation 1.

For this first analysis, consider that the measurements are affected by Gaussian noise of zero mean and standard deviation σ, thus considering that the measurement experiences are iid. Consider standard deviations of 0.015, 0.15 and 0.3.

For the fitting mode, consider the solution given by the normal LS.

For each value of the noise standard deviation, perform the following studies:

2. Use the singular value decomposition (SVD) to obtain the generalised inverse solution. Compare it with the LS.

3. Compute the confidence intervals of the solution.

4. Analyse the singular values of the **G** matrix and the condition number. Discuss the results obtained.

5. Test the discrete Picard condition and obtain the Truncated SVD solution. Discuss the results obtained.

6. Analyse the model resolution and discuss the results.

7. Consider at least one example using 32 points and a small percentage of outliers ($\sim 10\%$). Analyse the robustness of the solution. Is there a simple way to make this estimation robust? Implement and test it.

Write a **small report** of the assignment and submit it, with the **code scripts**, in the system until the deadline defined in the system. The report should show the theoretical derivations and provide explanations to the computed values and the comparisons made.