

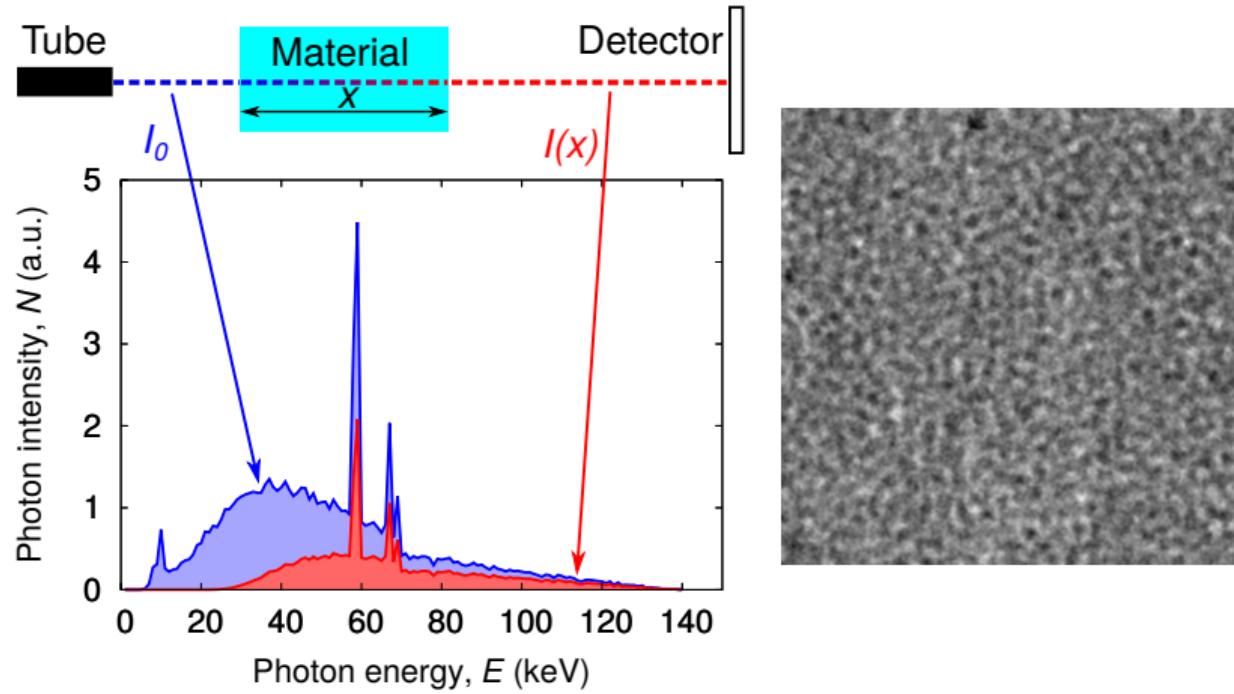


PhD defense
Manuel Baur

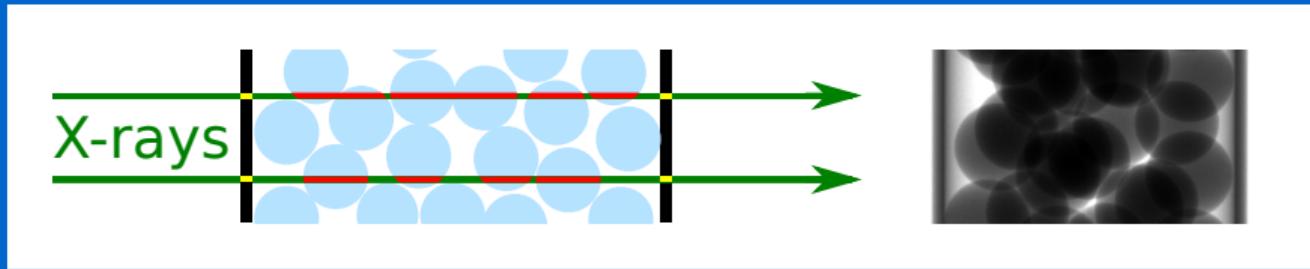
Funded by the German
Federal Ministry for
Economic Affairs and
Energy, grant no. 50WM

1653

X-ray radiography of granular systems – particle densities and dynamics



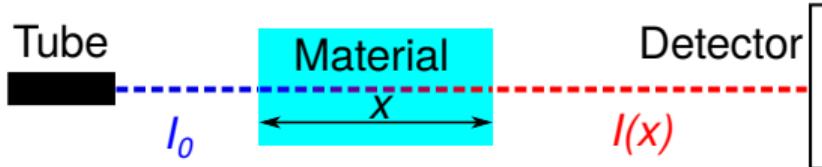
Measuring the volume fraction of **dynamic** granular systems



Correction of beam hardening in X-ray radiograms

In collaboration with Norman Uhlmann, Fraunhofer EZRT

Attenuation of X-rays

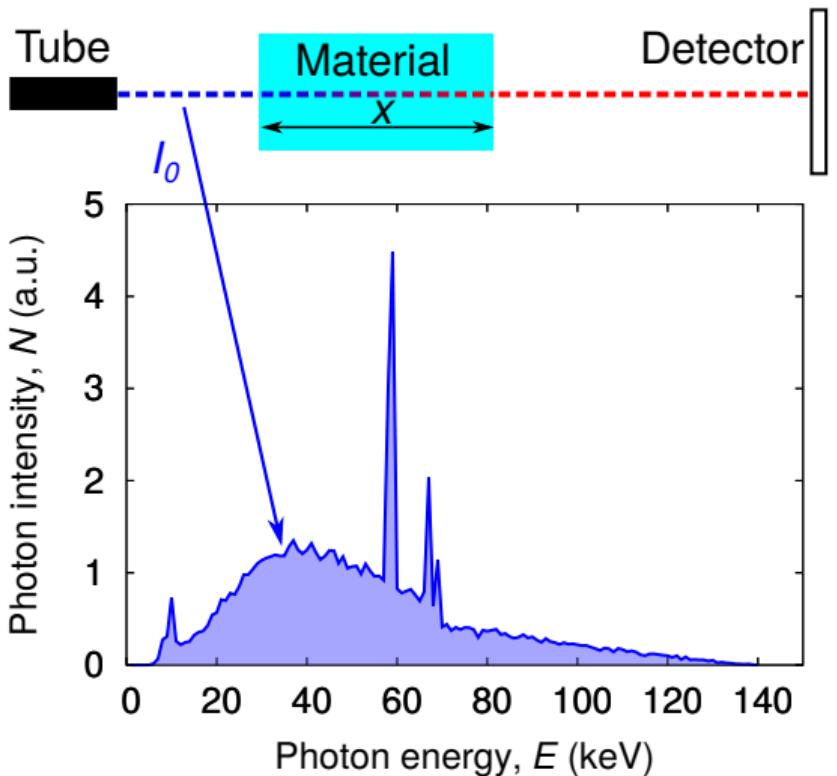


Beer-Lambert's law

$$I(x) = I_0 \exp(-\mu x)$$

$$\text{Thickness: } x = -\frac{1}{\mu} \ln \frac{I(x)}{I_0}$$

Attenuation of X-rays

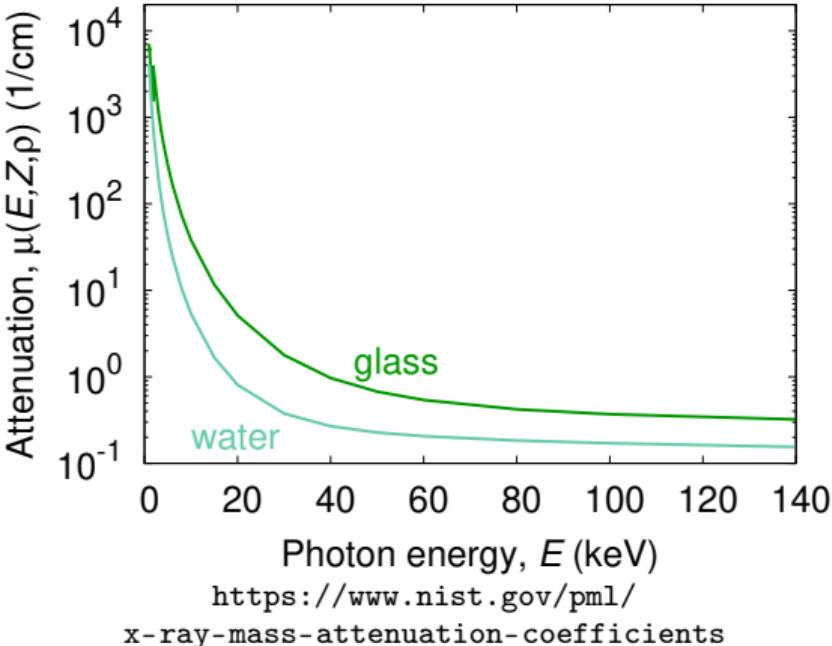


Beer-Lambert's law

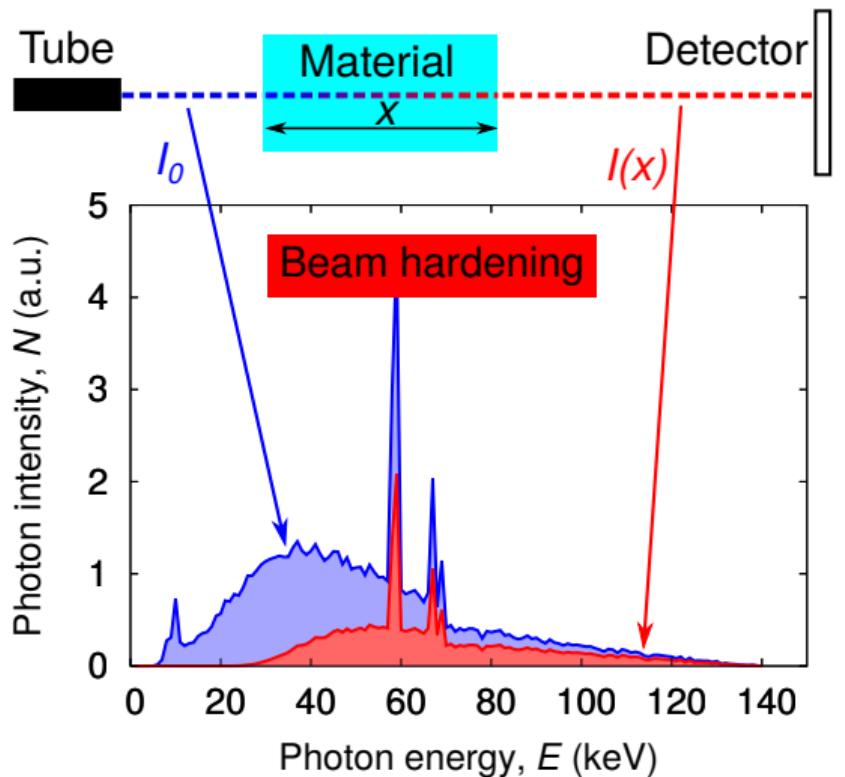
$$I(x) = I_0 \exp(-\mu(E, Z, \rho)x)$$

$\mu \neq \text{const}$

Thickness: $x = ?$



Attenuation of X-rays

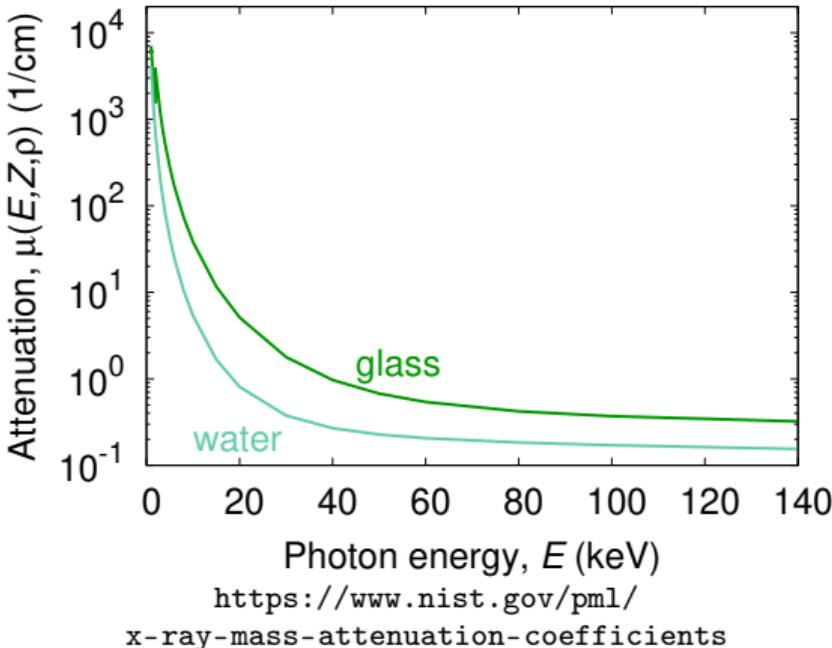


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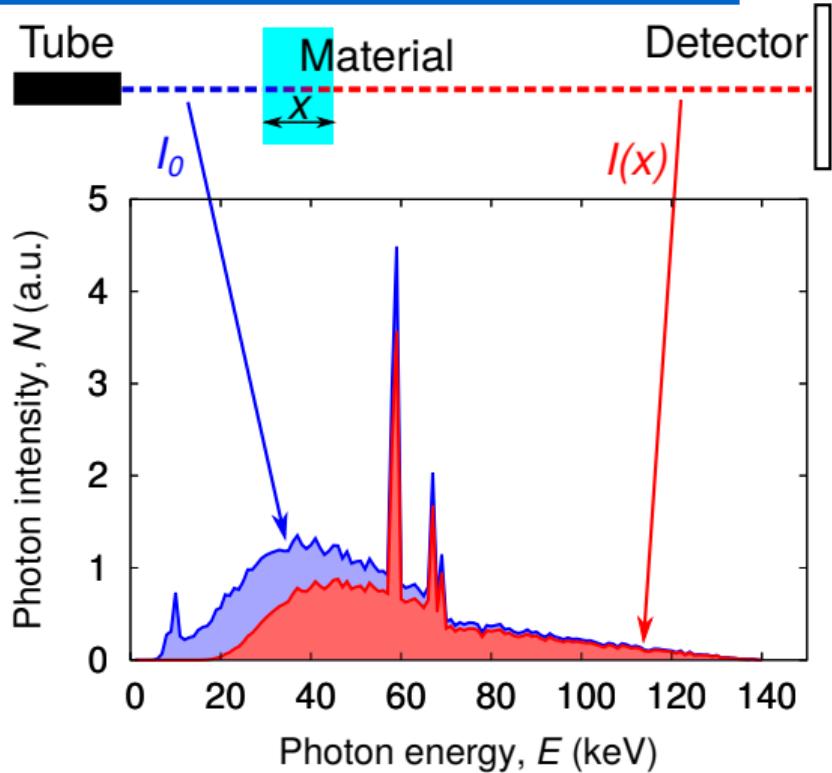
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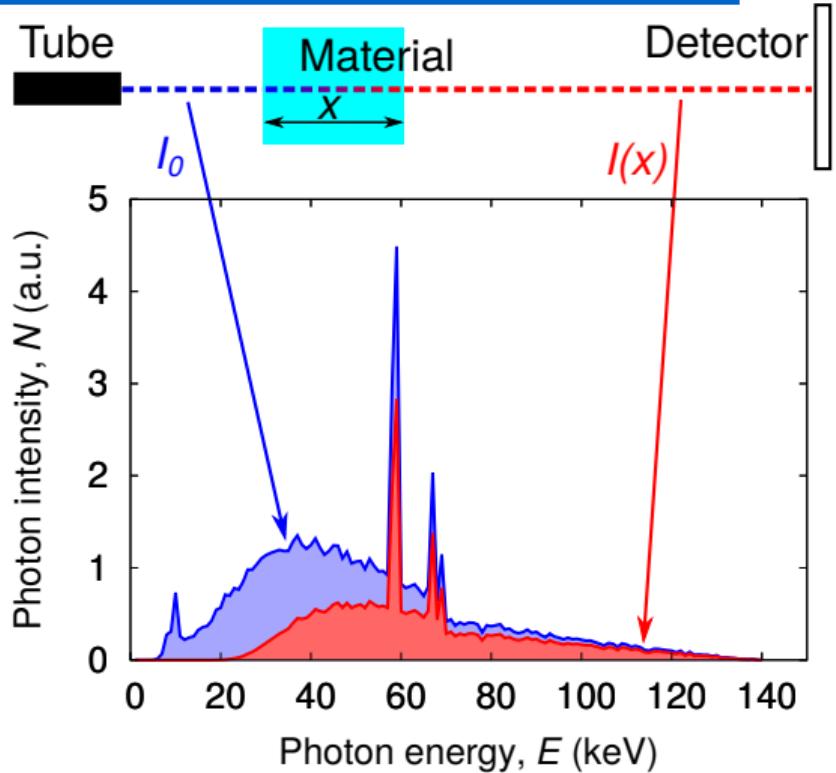
The effective attenuation, μ_{eff}



effective attenuation:

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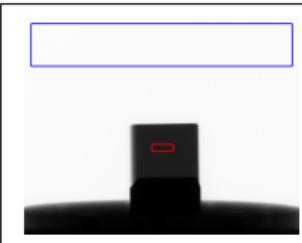
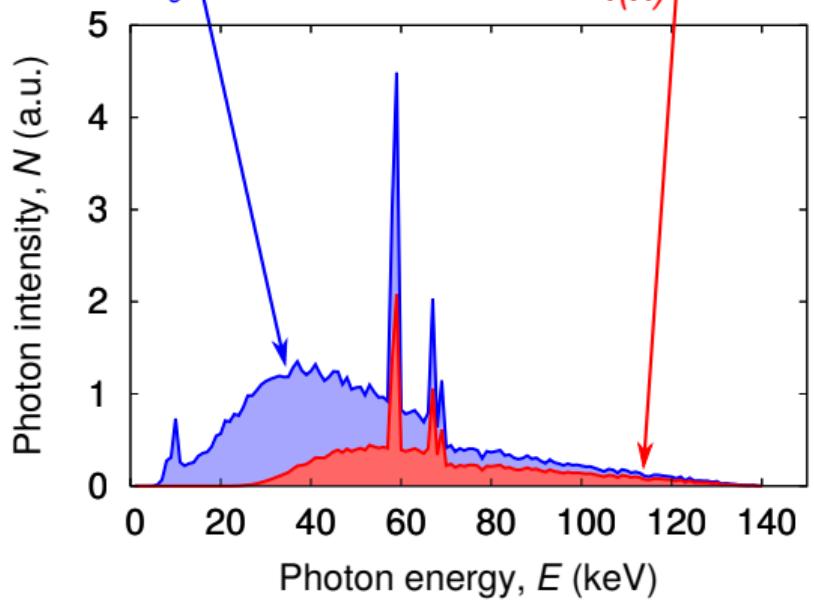
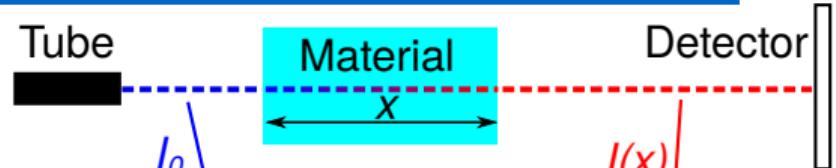
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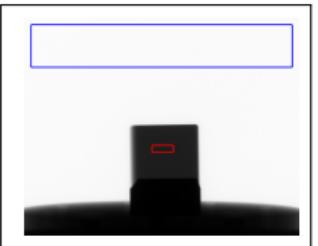
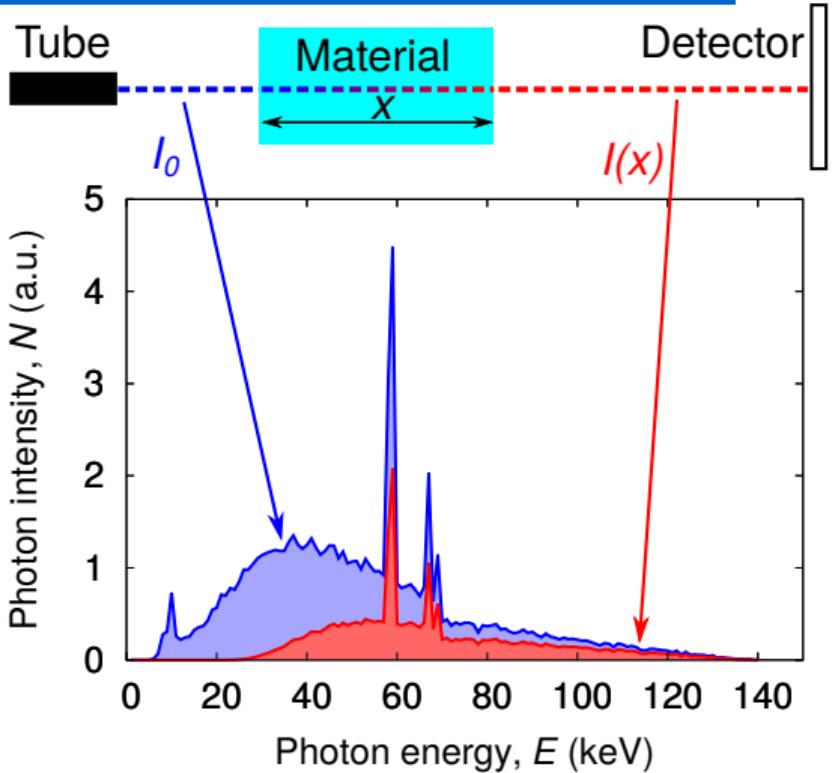
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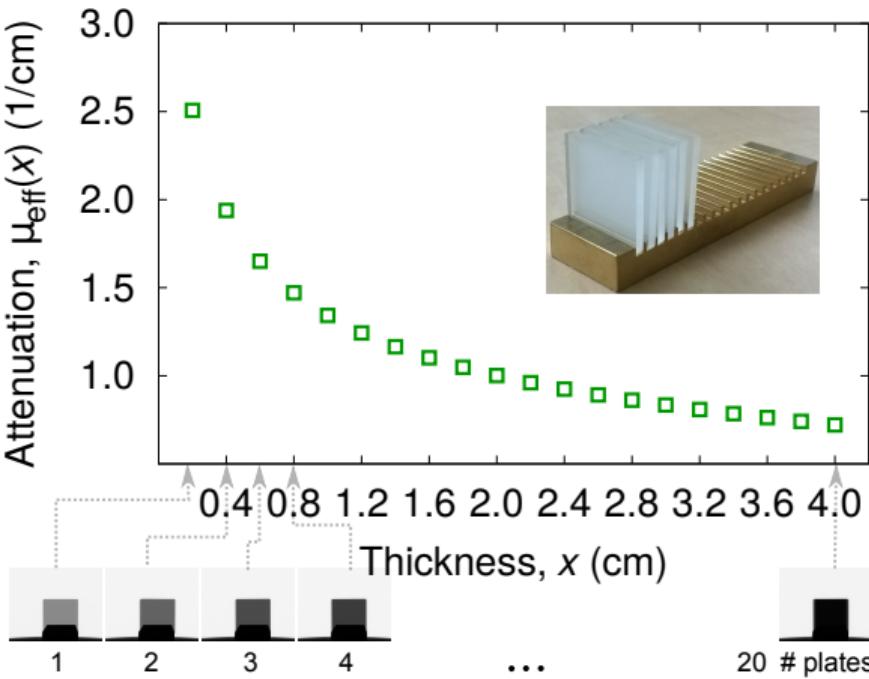
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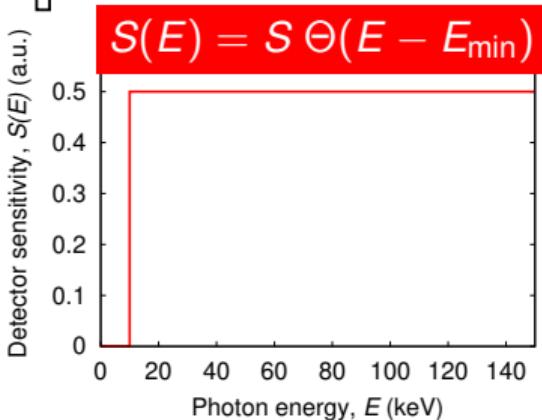
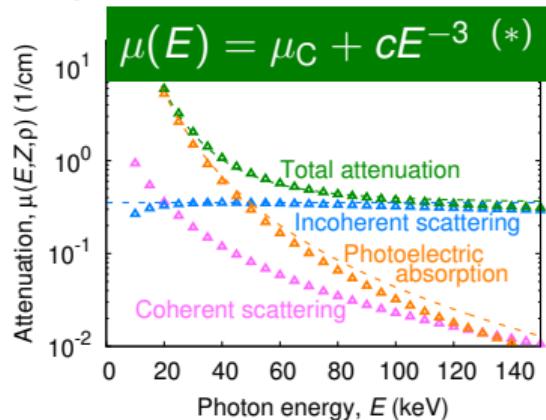
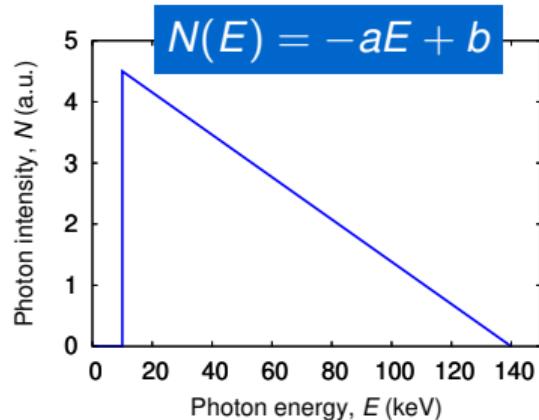
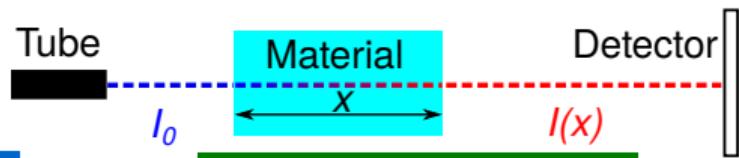


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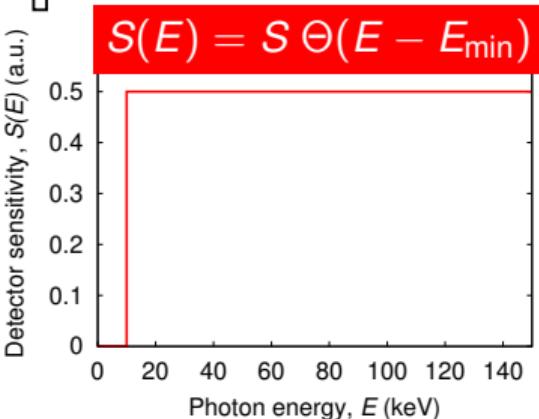
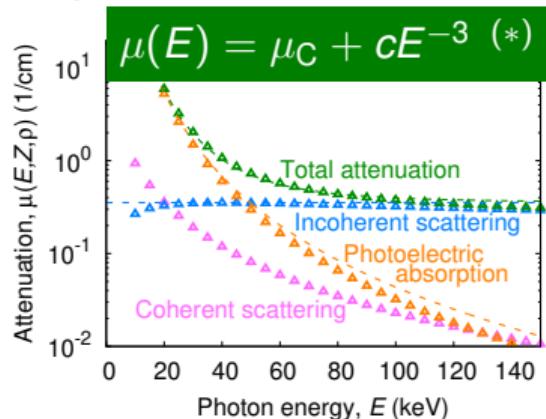
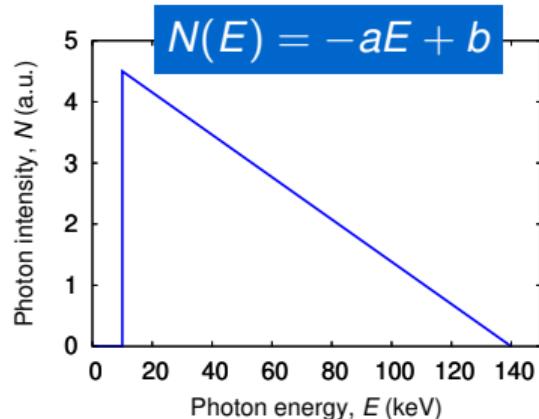
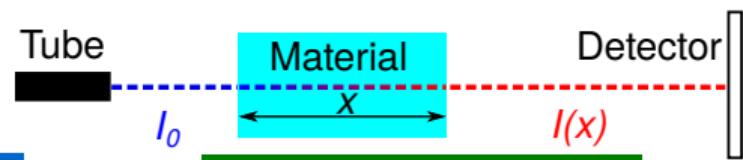
Modeling of μ_{eff}



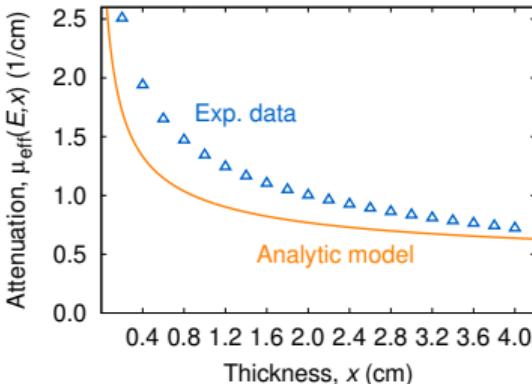
$$I(x) \propto \int N(E) \exp\{-\mu(E, Z, \rho)x\} S(E) dE$$

(*) XCOM supplied by NIST

Modeling of μ_{eff}

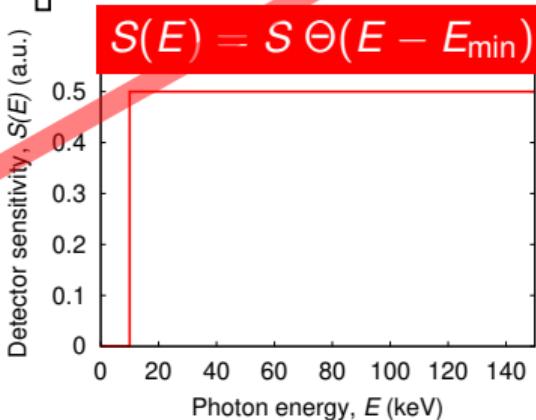
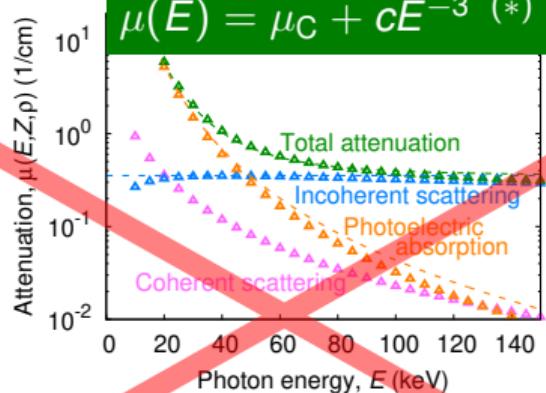
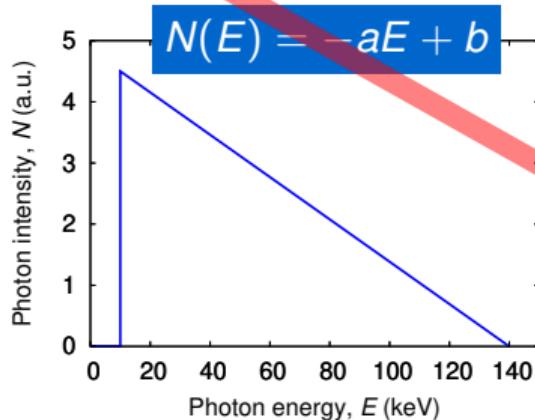
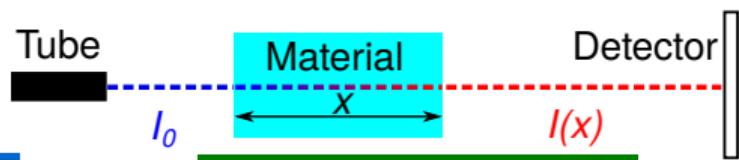


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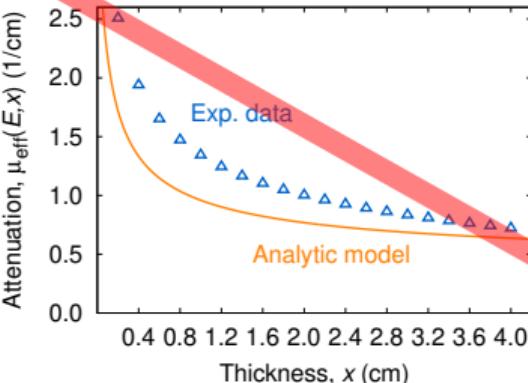


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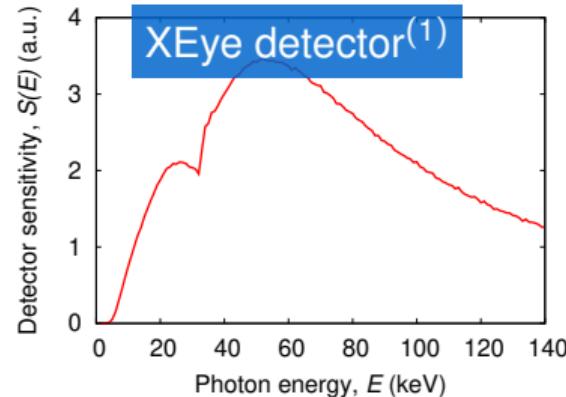
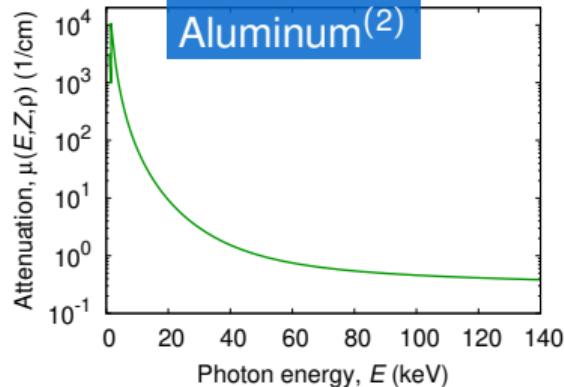
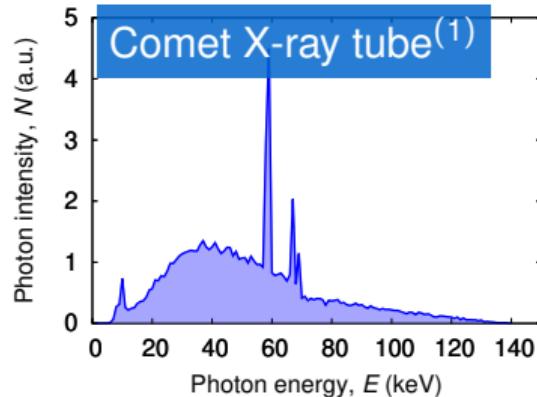
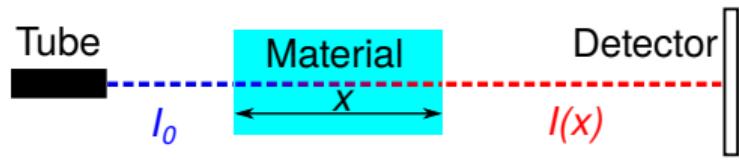


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Numerical approx. of μ_{eff}

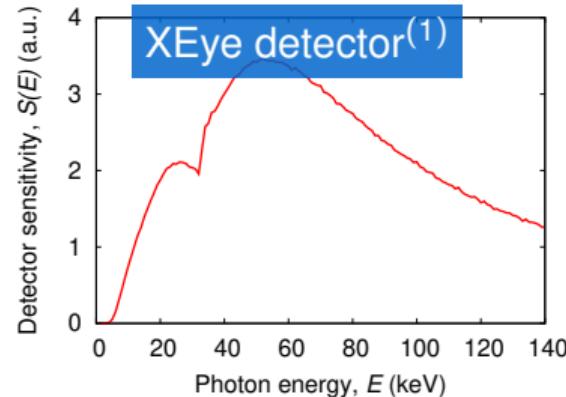
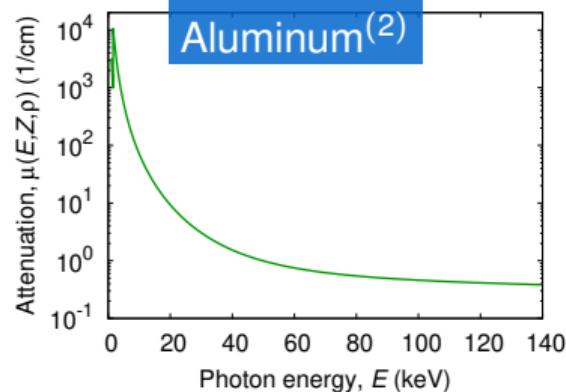
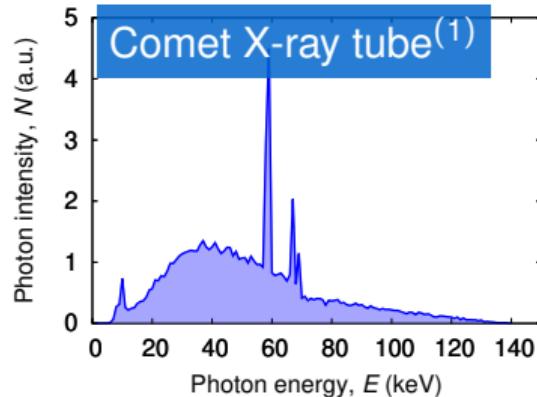
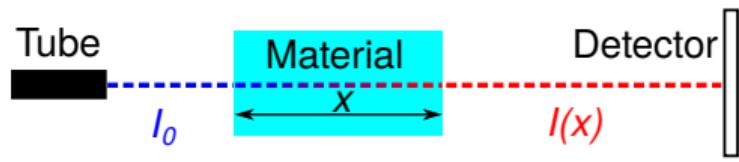


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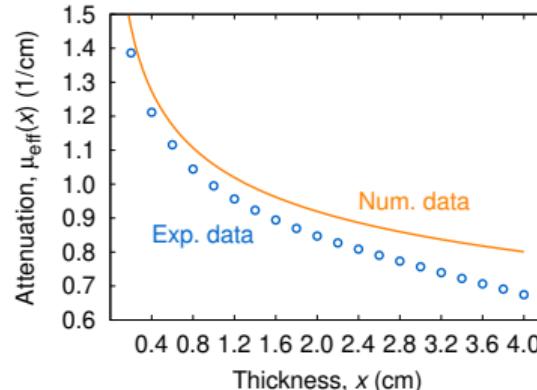
(1) Supplied by Norman Uhlmann, Fraunhofer EZRT

(2) XCOM supplied by NIST

Numerical approx. of μ_{eff}



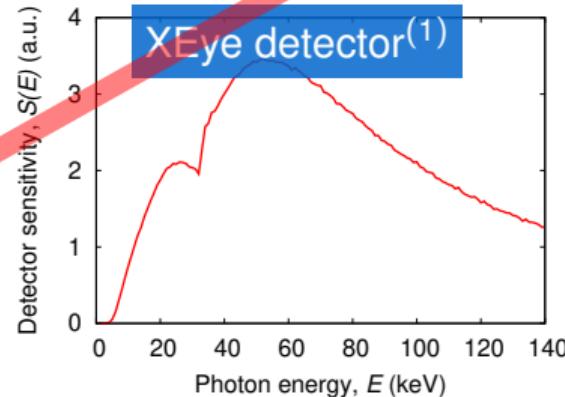
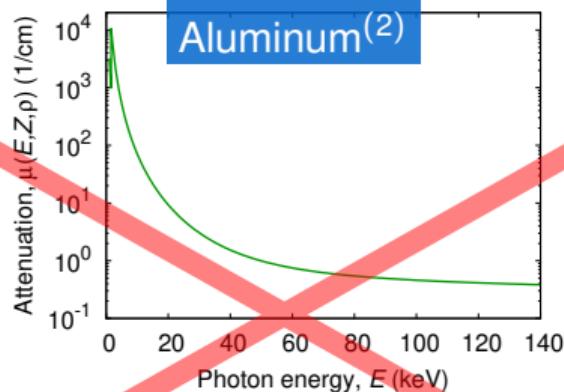
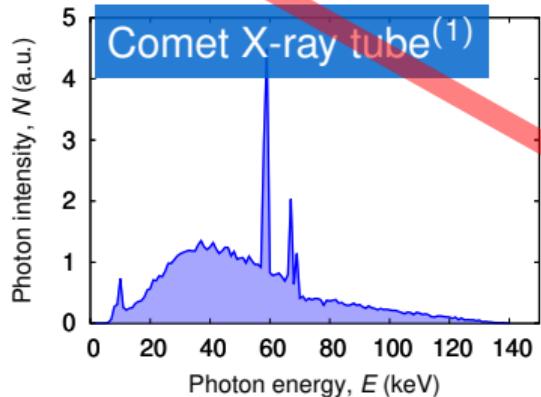
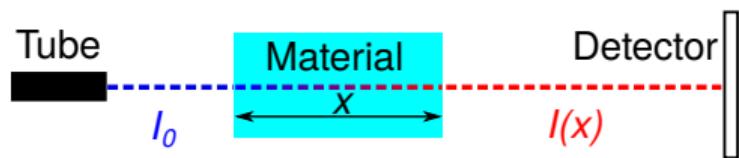
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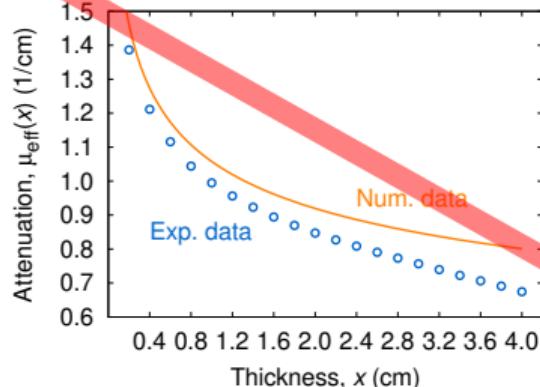
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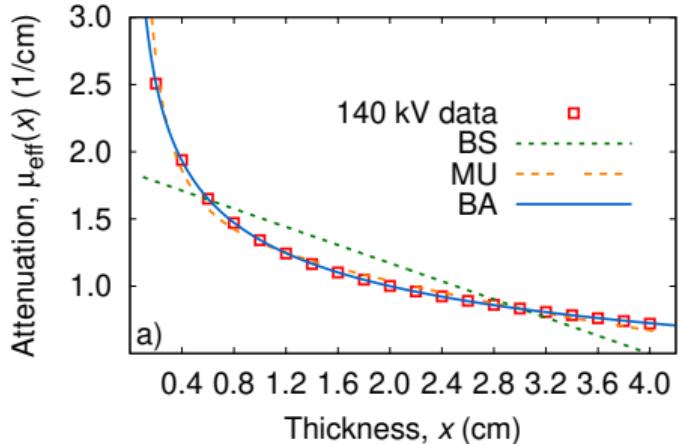
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Heuristic model functions for μ_{eff}



$$\mu_{\text{eff}}(x) = \mu_0 - \lambda x$$

$$\mu_{\text{eff}}(x) = \frac{\mu_0}{1 + \lambda x}$$

$$\mu_{\text{eff}}(x) = \frac{\mu_0}{(1 + \lambda x)^\beta}$$

Bjärngard & Shackford
(1994)

Yu *et al.* (1997)

$$\mu_{\text{eff}}(x) = \mu(E_{\text{max}}) + \frac{2\mu_1}{x\sqrt{-\lambda_1^2 + 4\lambda_2}} \times$$

$$\left[\arctan \left(\frac{\lambda_1 + 2\lambda_2 x}{\sqrt{-\lambda_1^2 + 4\lambda_2}} \right) - \arctan \left(\frac{\lambda_1}{\sqrt{-\lambda_1^2 + 4\lambda_2}} \right) \right]$$

Kleinschmidt (1999)

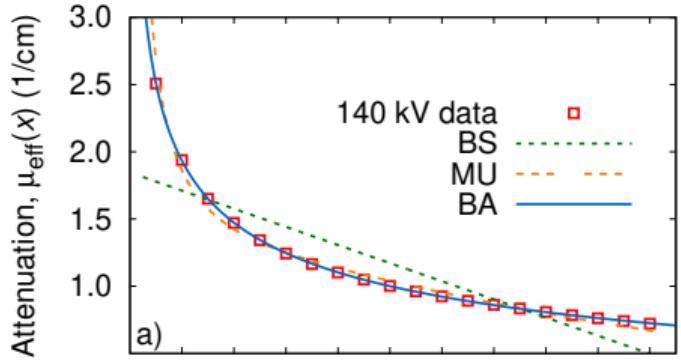
$$\mu_{\text{eff}}(x) = -\frac{1}{x} \ln [A + B \exp(-x/C)]$$

Mudde *et al.* (2008)

$$\mu_{\text{eff}}(x) = a + \frac{b}{x^\alpha}$$

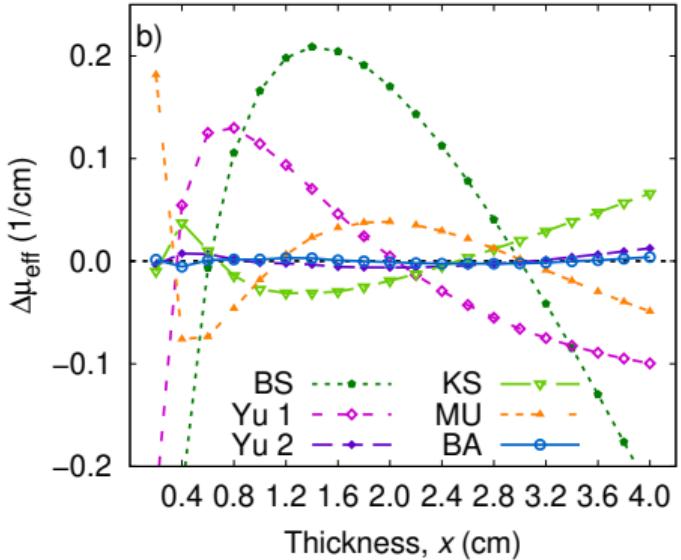
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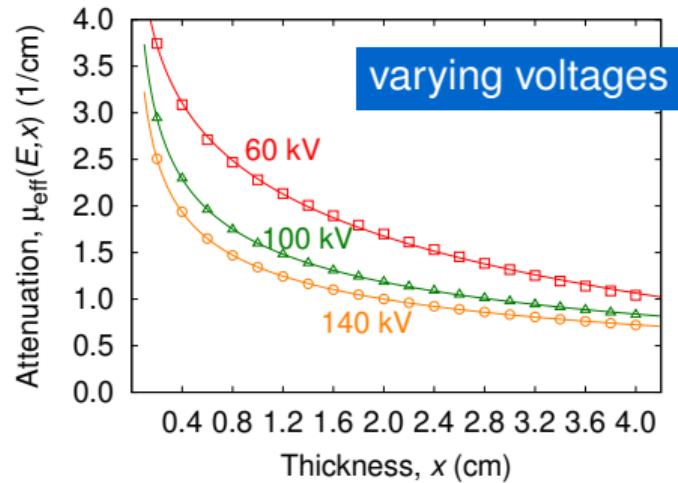
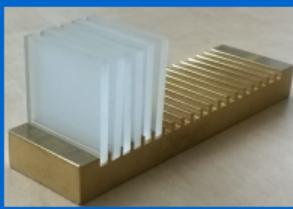
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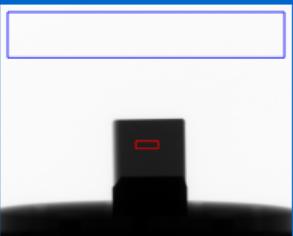
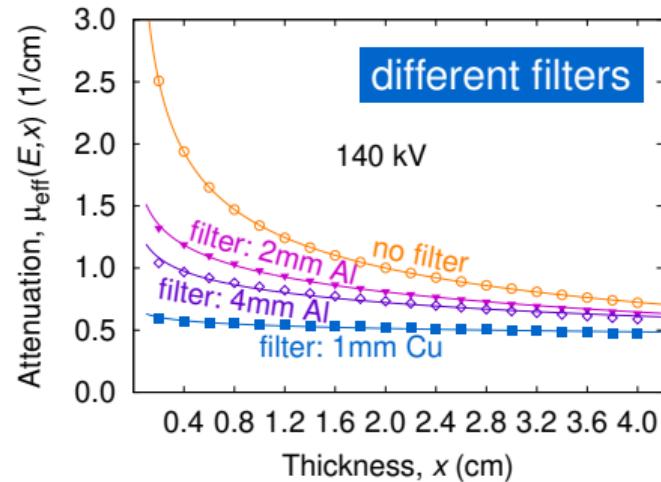
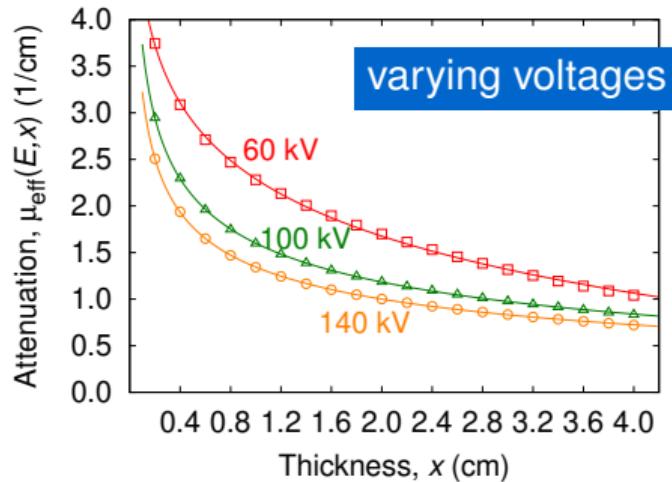
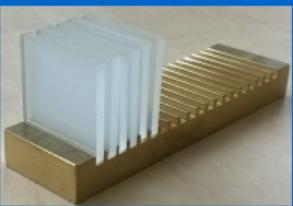
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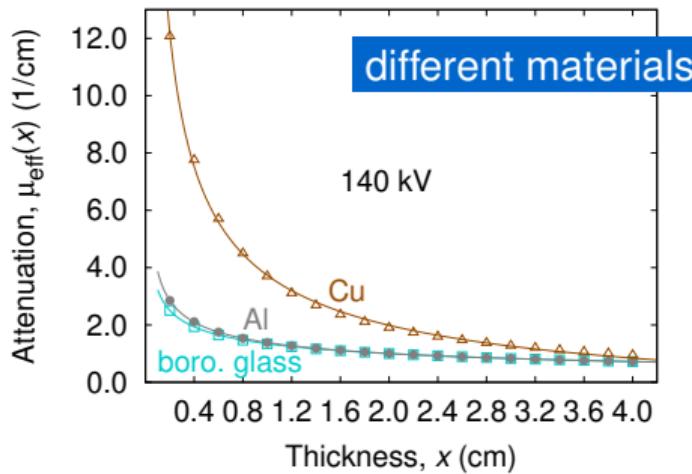
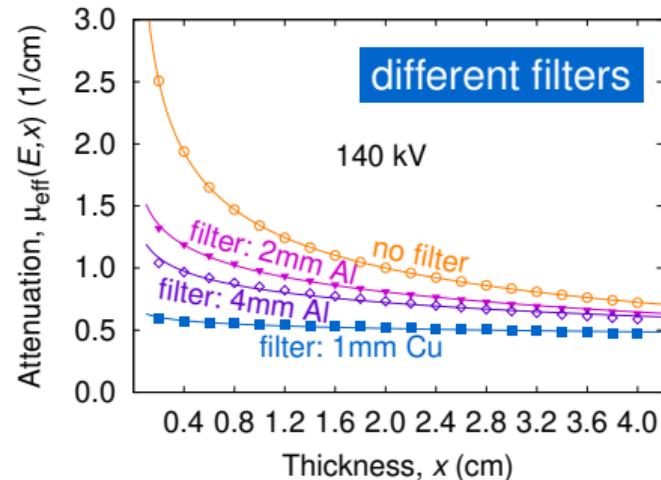
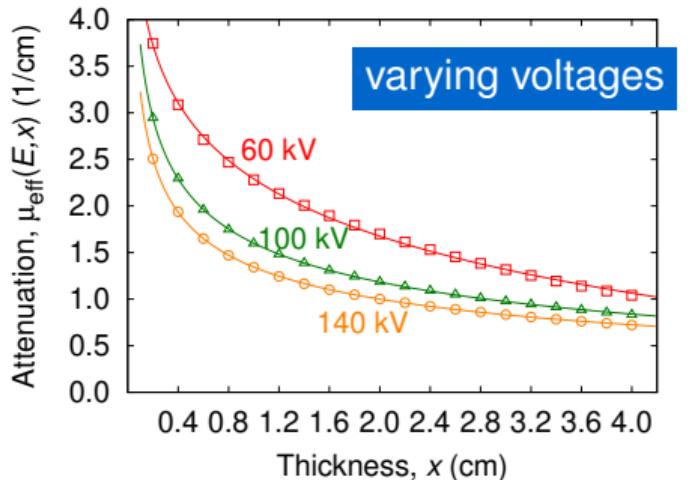
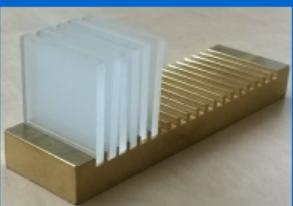
Universality of
 $\mu_{\text{eff}}(x) = a + \frac{b}{x^\alpha}$



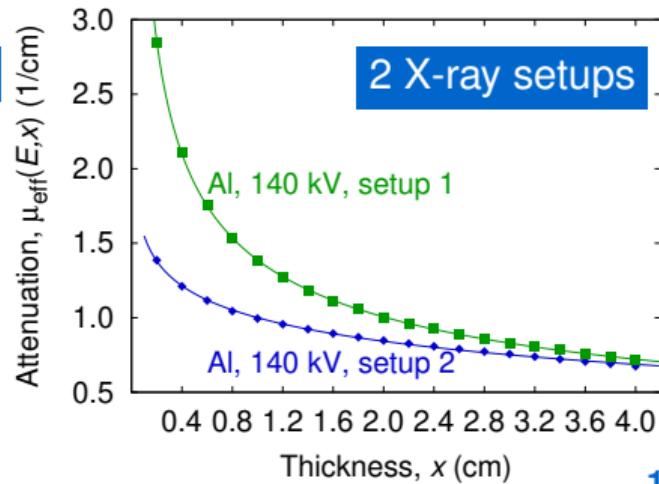
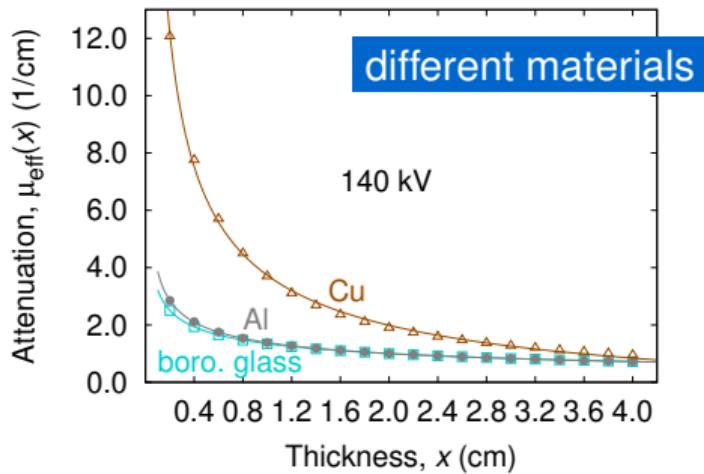
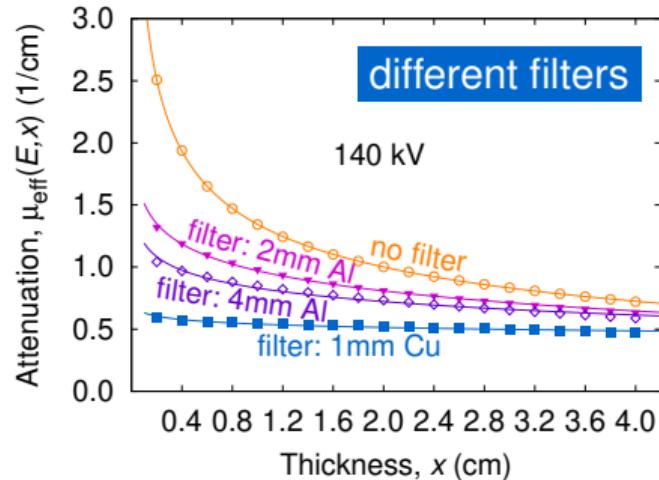
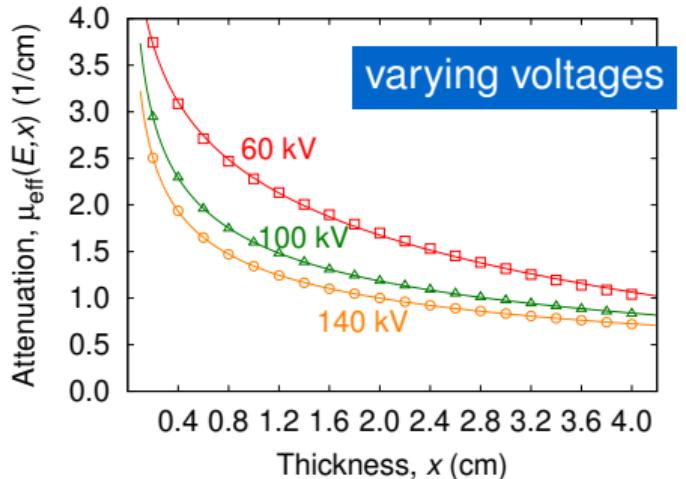
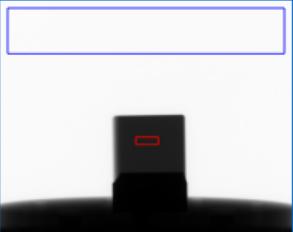
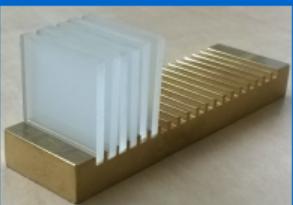
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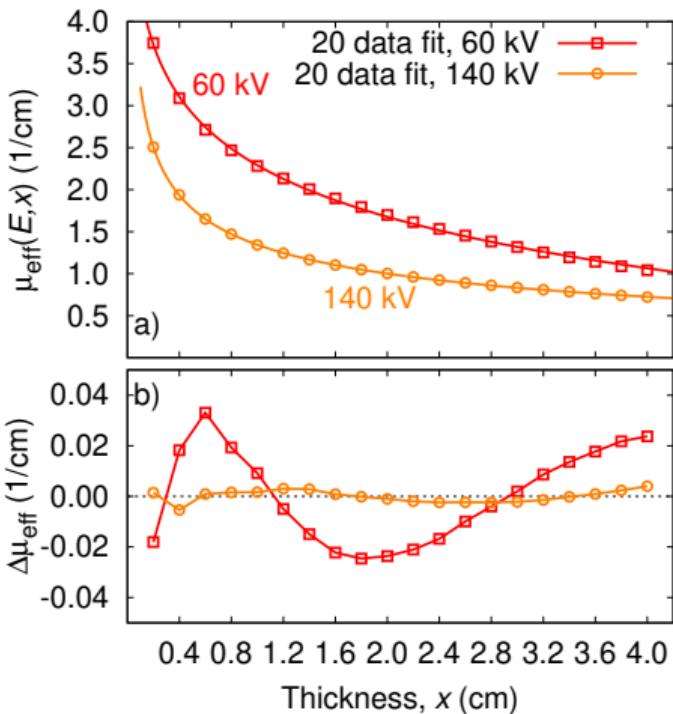
Determining the material thickness x

Generalized Beer-Lambert

$$I(x) = I_0 \exp(-\mu_{\text{eff}}(x) x)$$

Model function

$$\mu_{\text{eff}}(x) = a + \frac{b}{x^\alpha}$$



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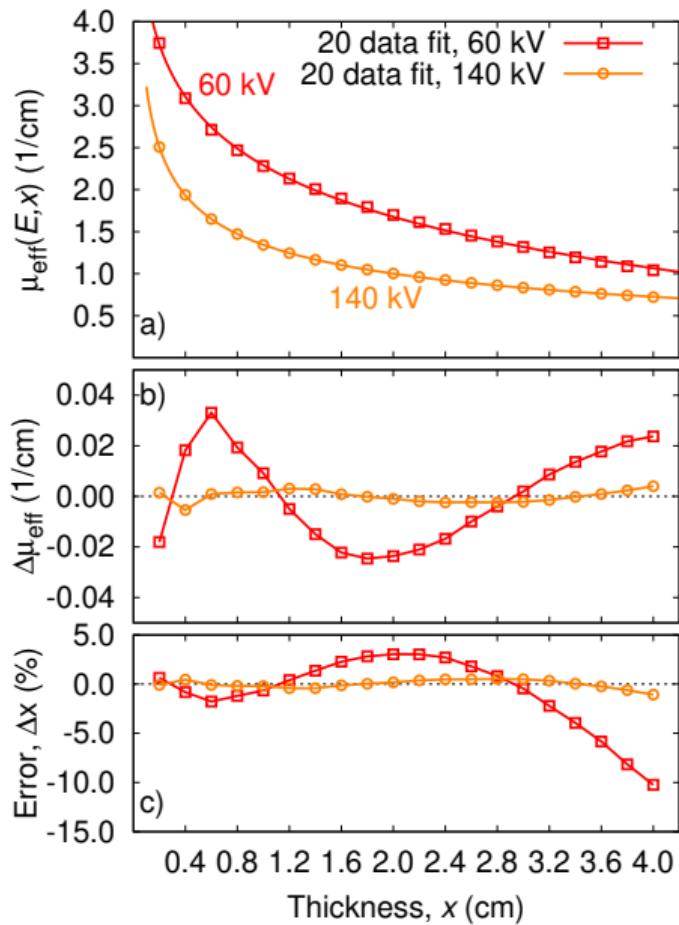
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Solve

$$ax + bx^{1-\alpha} + \ln\left(\frac{I(x)}{I_0}\right) = 0$$

e.g. Newton's method or look-up table



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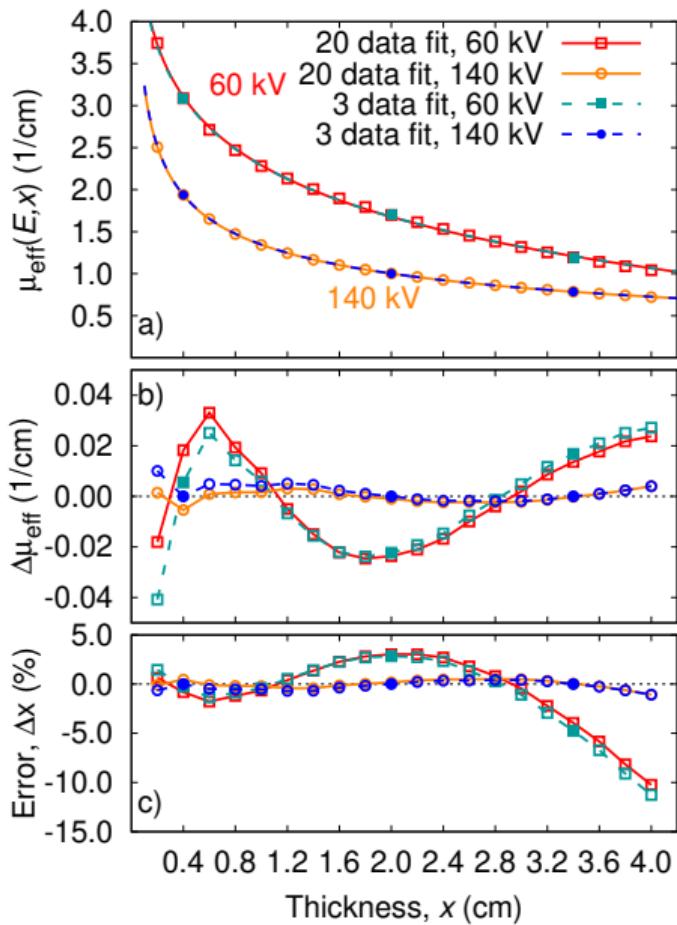
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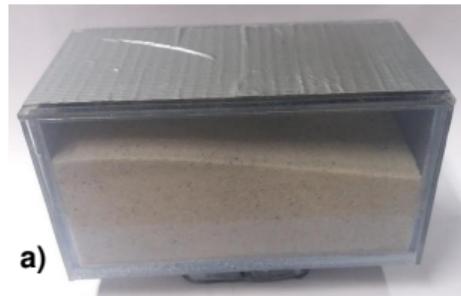
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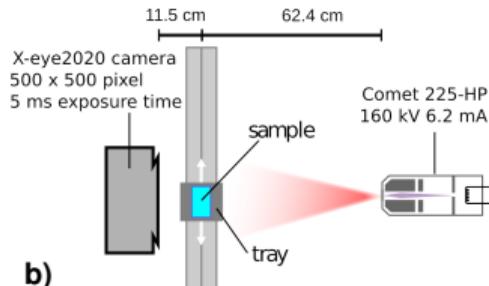
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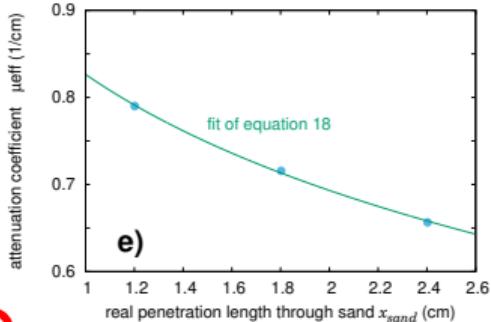
Applications



a)

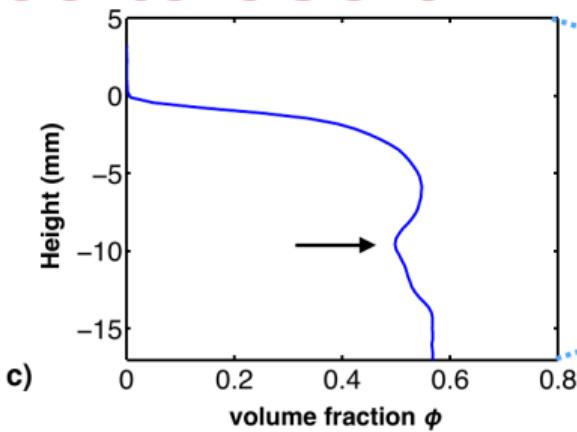


b)

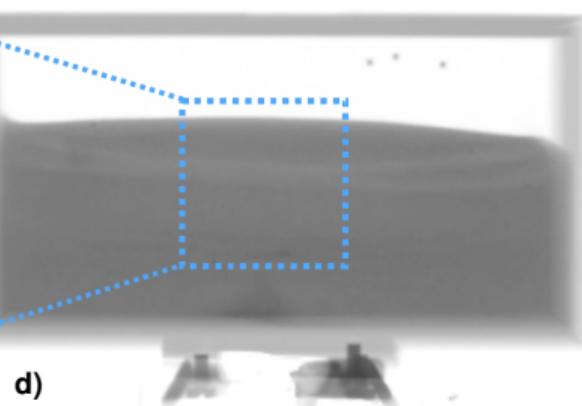


e)

reduce to useful illustrative



c)



d)