

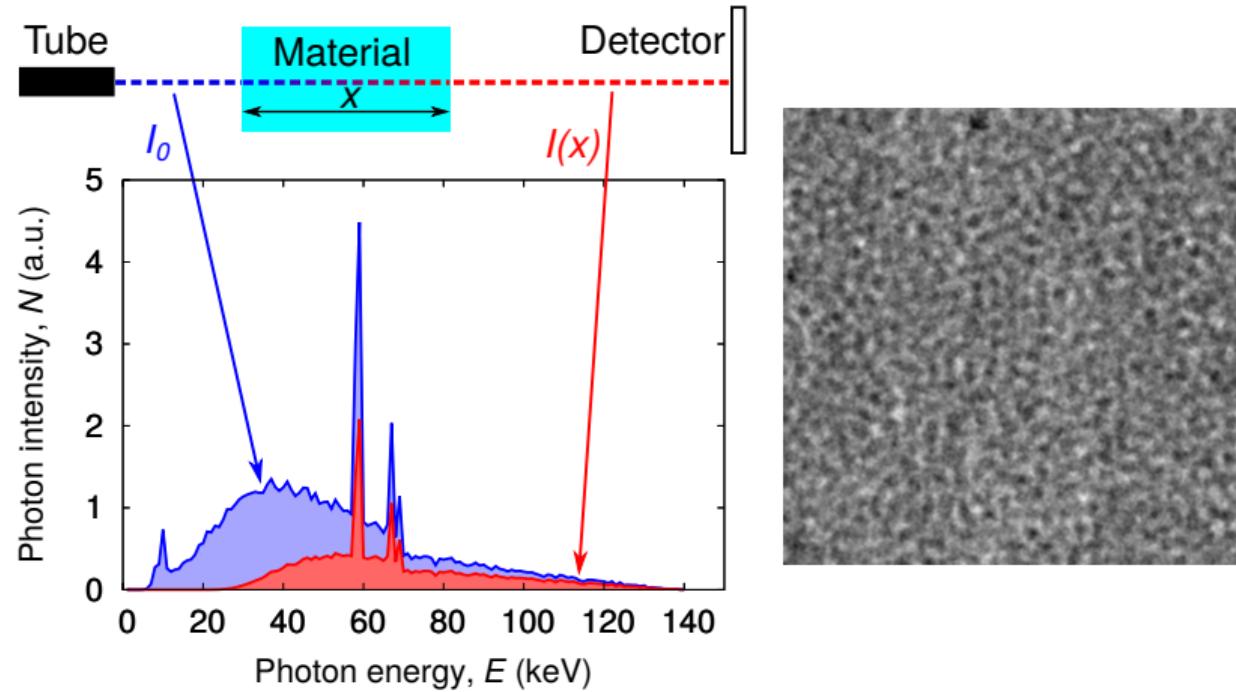


PhD defense  
**Manuel Baur**

Funded by the German  
Federal Ministry for  
Economic Affairs and  
Energy, grant no. 50WM

1653

# X-ray radiography of granular systems – particle densities and dynamics



# X-ray radiography of granular systems – particle densities and dynamics

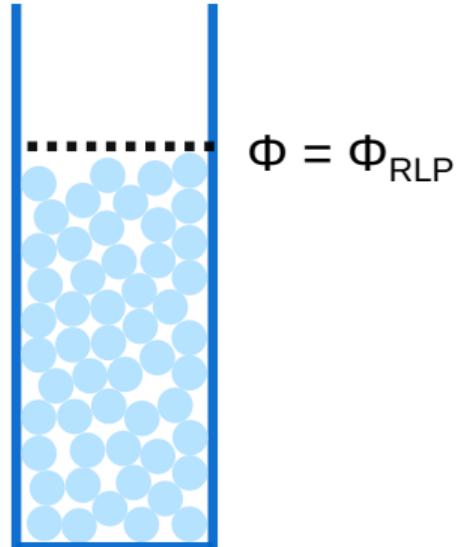


# X-ray radiography of granular systems – particle densities and dynamics



**Volume fraction**

$$\Phi = \frac{V_{\text{Particles}}}{V_{\text{Container}}}$$



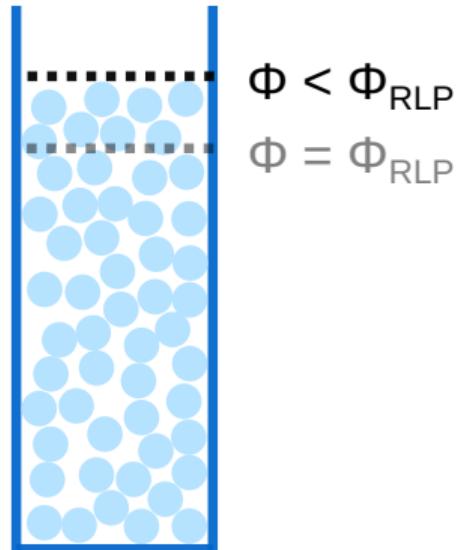
# X-ray radiography of granular systems – particle densities and dynamics



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## Volume fraction

$$\Phi = \frac{V_{\text{Particles}}}{V_{\text{Container}}}$$



# X-ray radiography of granular systems – particle densities and dynamics



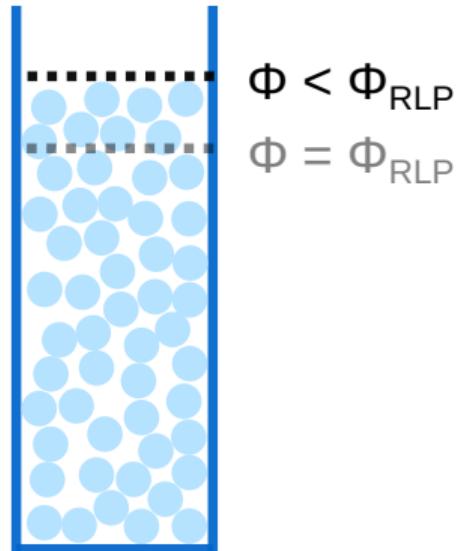
Dissipative interactions



Driscoll *et al* (2016)

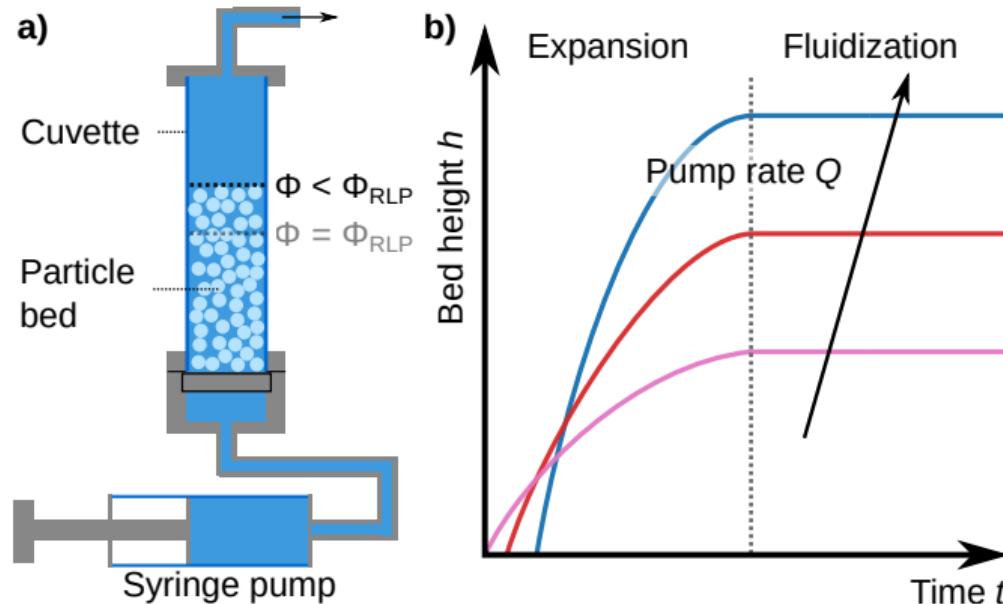
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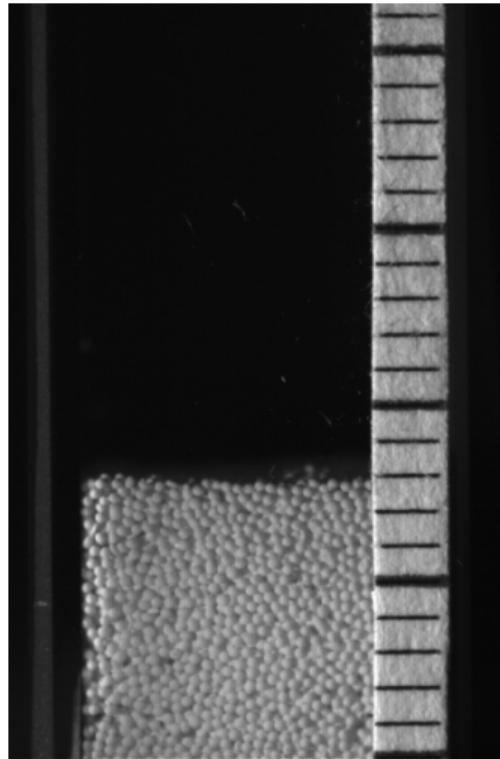
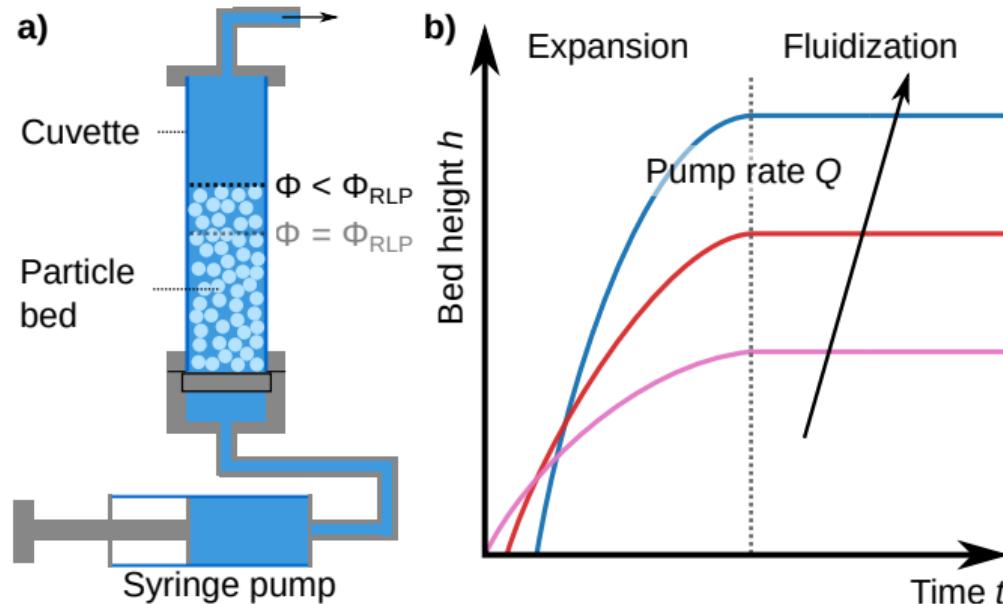
# X-ray radiography of granular systems – particle densities and dynamics

## Liquid fluidized bed



# X-ray radiography of granular systems – particle densities and dynamics

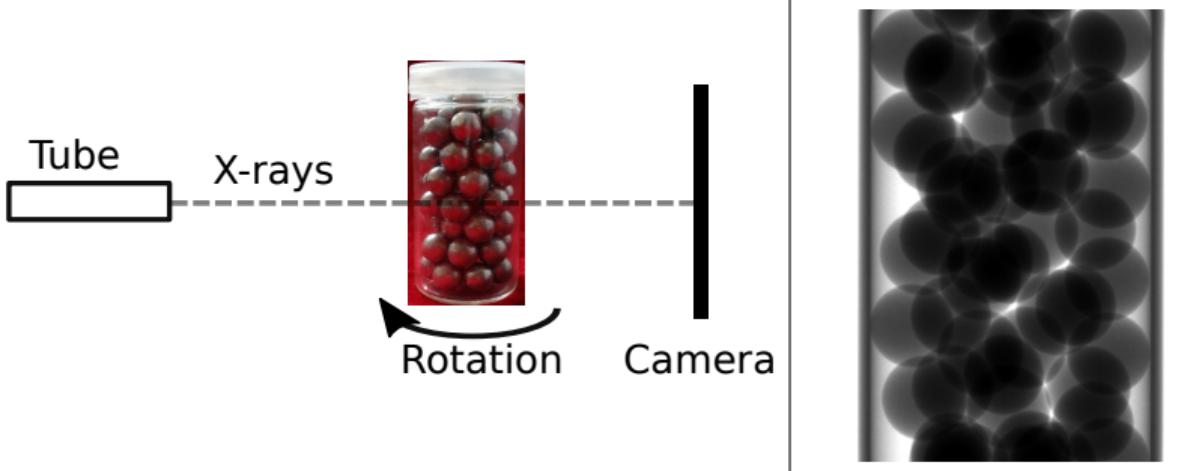
## Liquid fluidized bed



Master thesis Welm Pätzold

Particulate flows are opaque

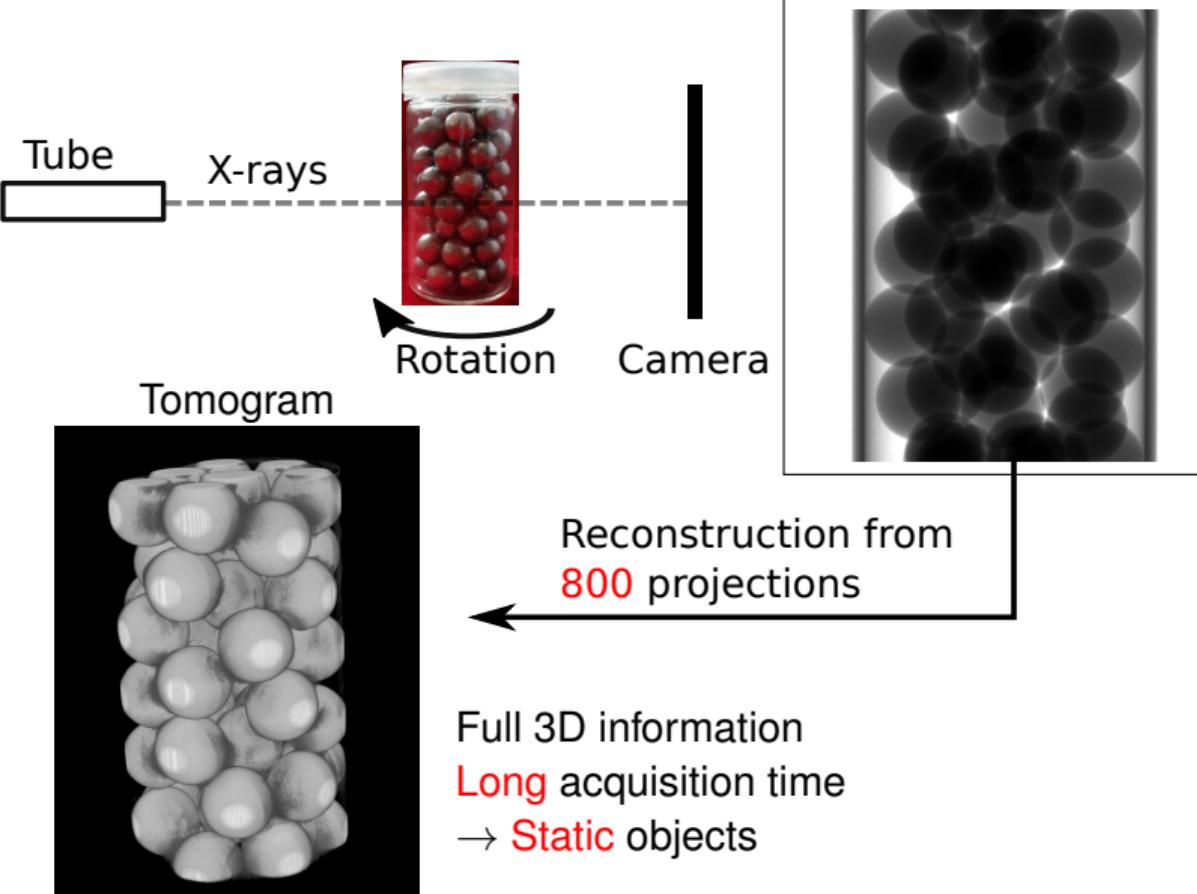
# X-ray radiography & tomography



Radiogram

2D projections of 3D object  
Short acquisition time  
→ Dynamic system

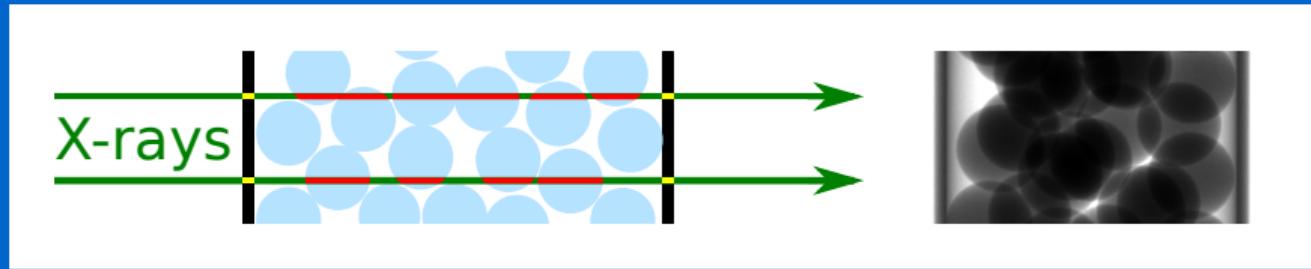
# X-ray radiography & tomography



2D projections of 3D object  
Short acquisition time  
→ Dynamic system

Full 3D information  
Long acquisition time  
→ Static objects

# Measuring the volume fraction of **dynamic** granular systems

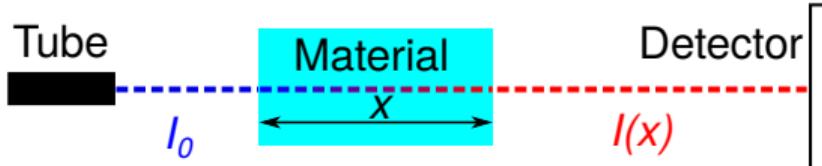


Correction of beam hardening  
in X-ray radiograms

Baur *et al*, *Rev. Sci. Instrum.* (2019)

In collaboration with Norman Uhlmann, Fraunhofer EZRT

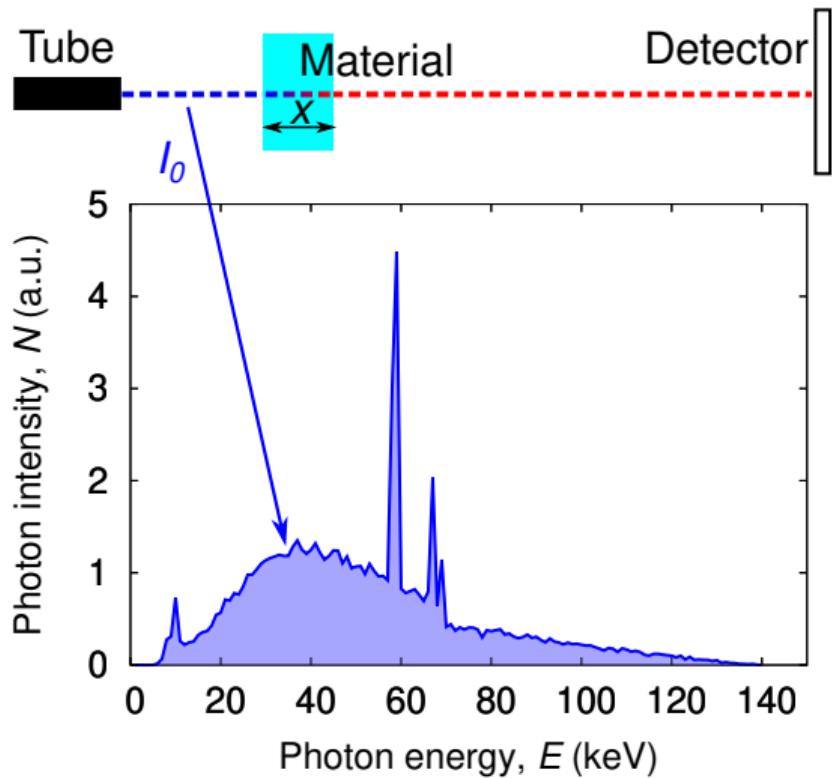
# Attenuation of X-rays



Beer-Lambert's law

$$I(x) = I_0 \exp(-\mu x)$$

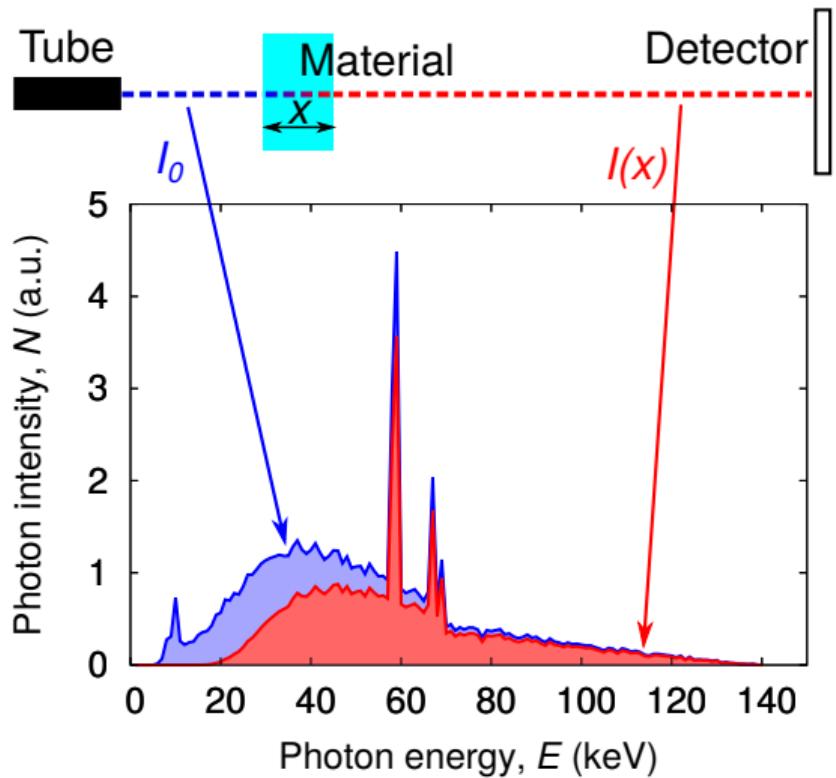
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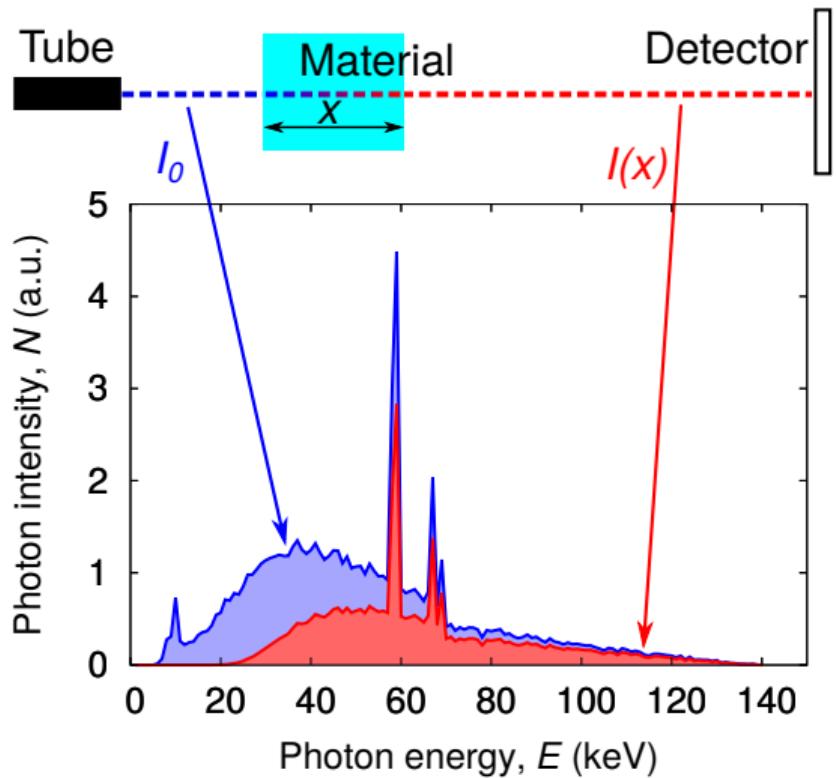
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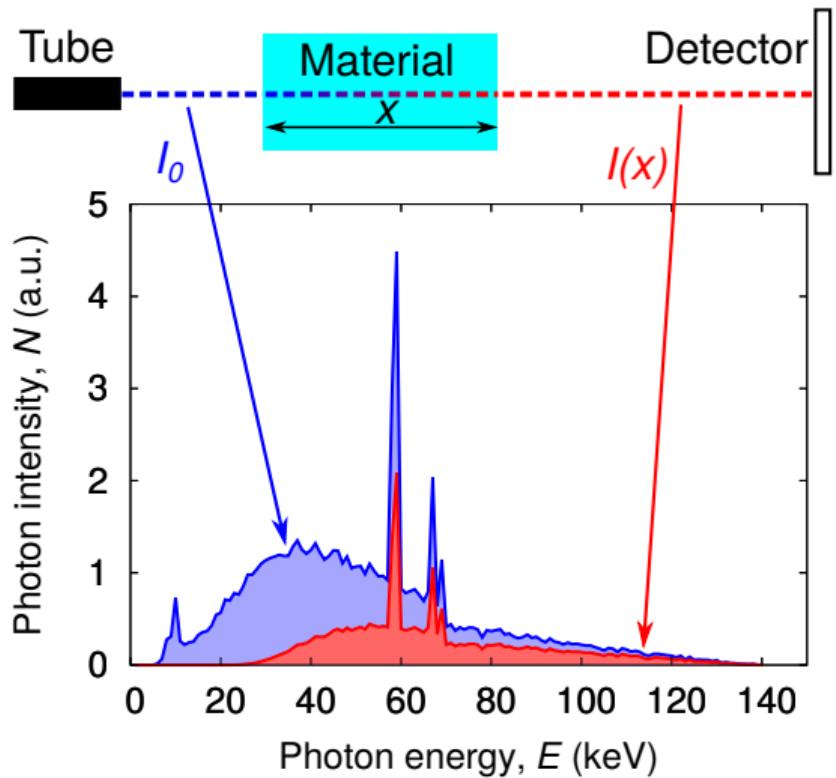
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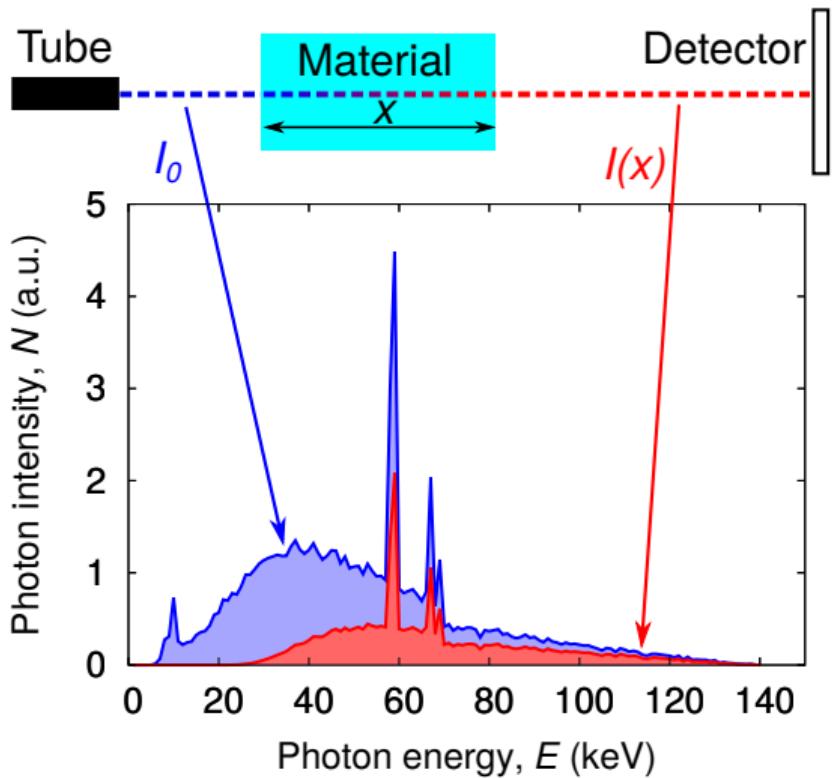
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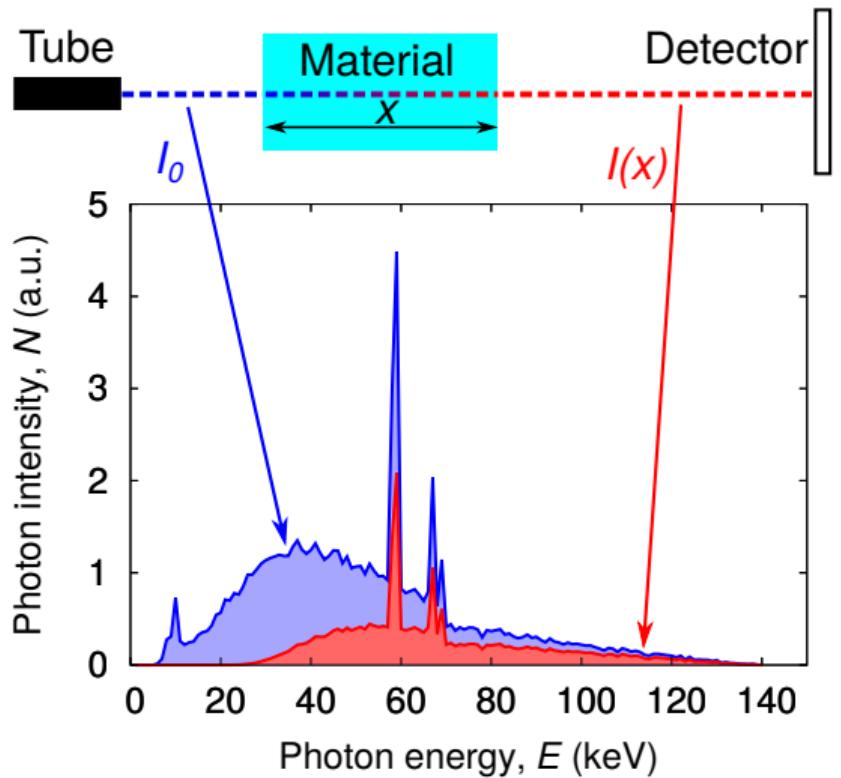
# Attenuation of X-rays



Beer-Lambert's law

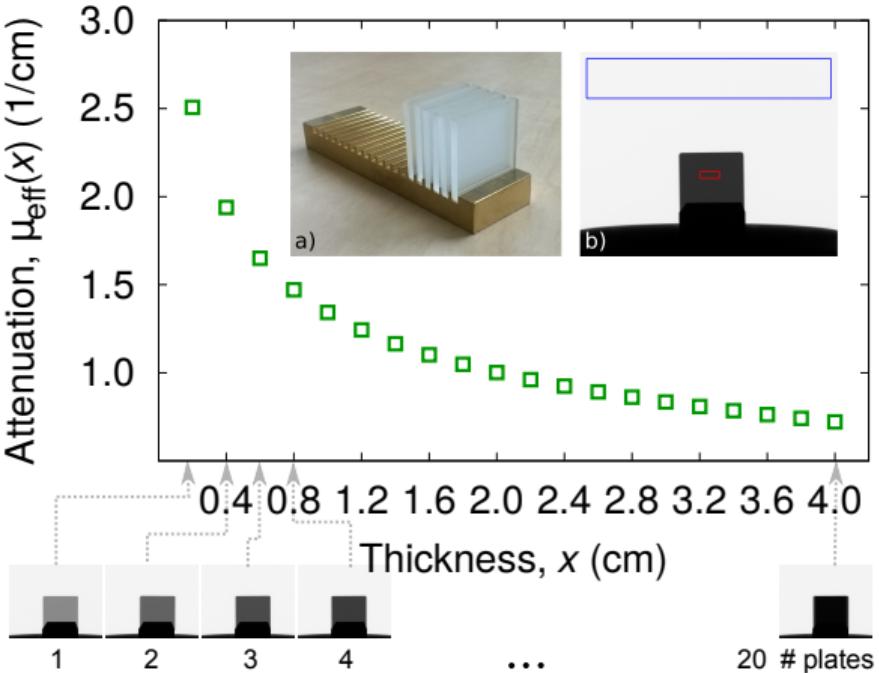
$$\begin{aligned} I(x) &\equiv I_0 \exp(-\mu x) \\ I(x) &= I_0 \exp(-\mu_{\text{eff}}(x) x) \end{aligned}$$

# Attenuation of X-rays

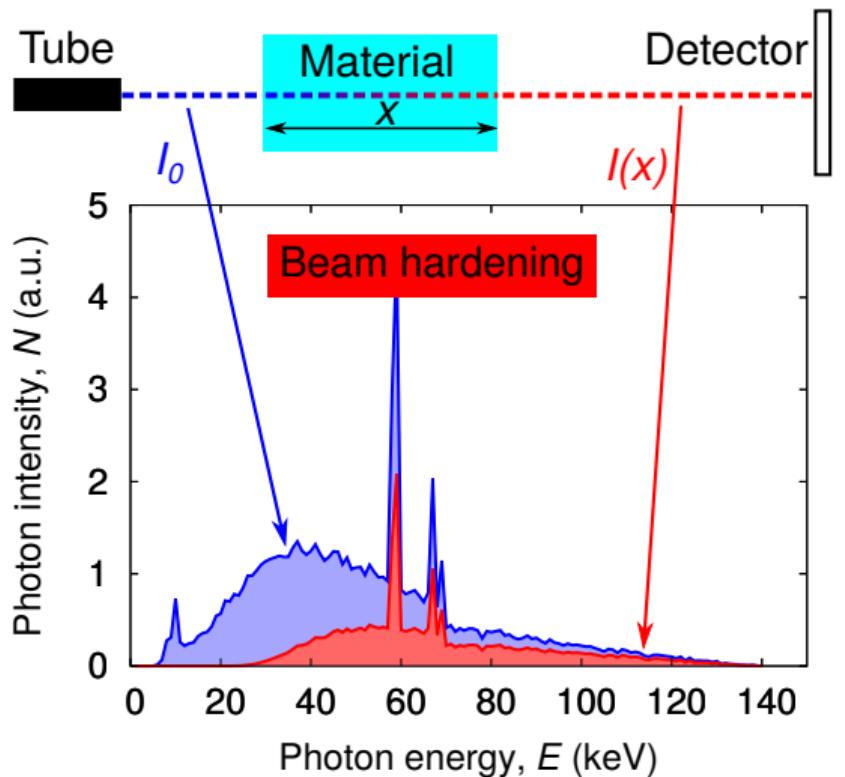


Beer-Lambert's law

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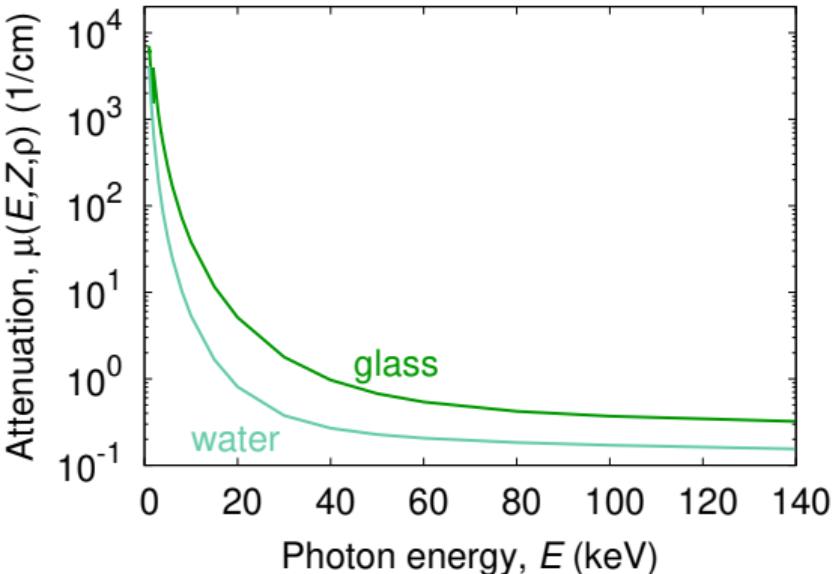


# Attenuation of X-rays



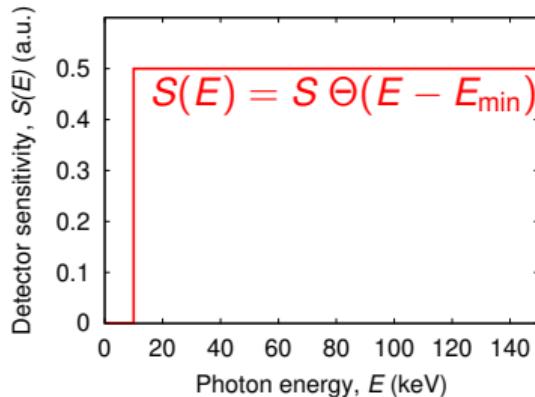
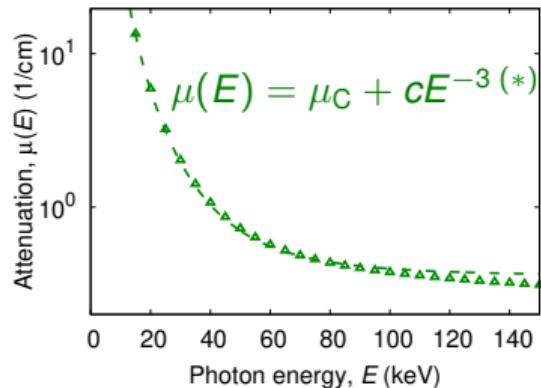
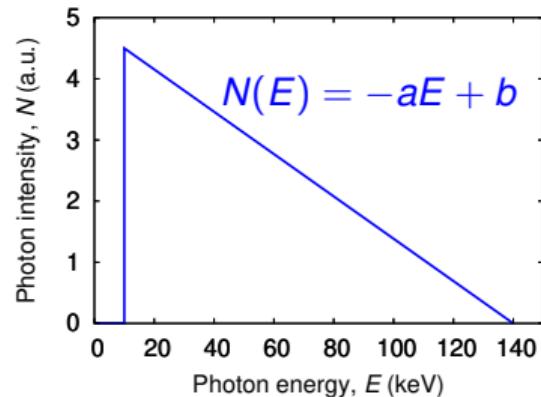
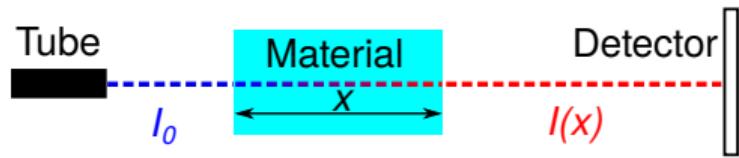
Beer-Lambert's law

$$I(x) = I_0 \exp(-\mu(E, Z, \rho) x)$$



Tabulated attenuation coefficients from NIST.gov

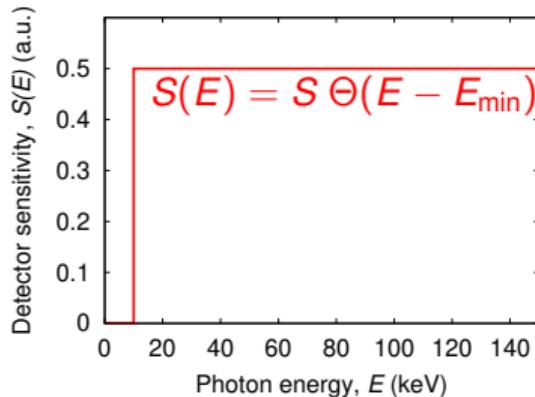
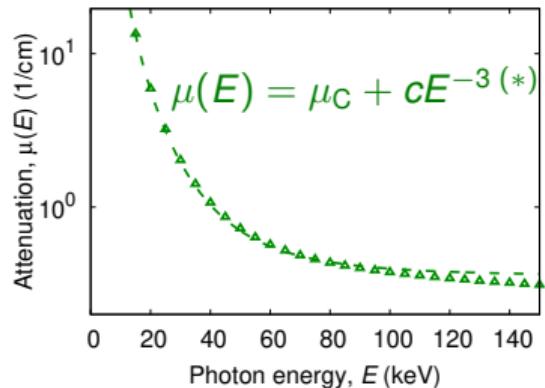
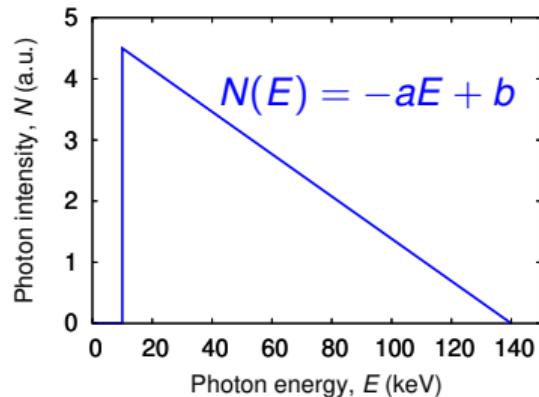
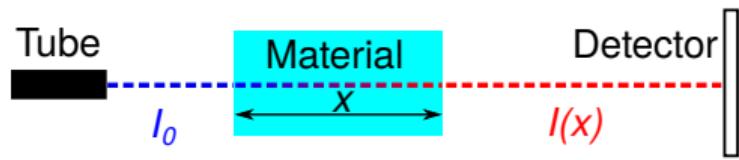
# Modeling of $\mu_{\text{eff}}(x)$



$$I(x) \propto \int N(E) \exp\{-\mu(E)x\} S(E) dE$$

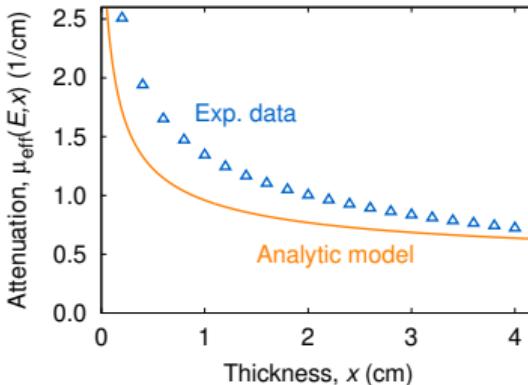
(\*) XCOM supplied by NIST

# Modeling of $\mu_{\text{eff}}(x)$



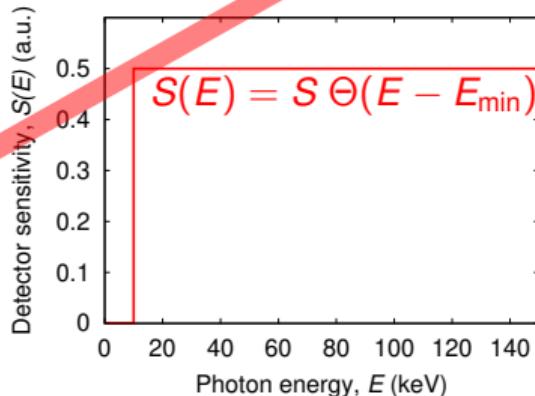
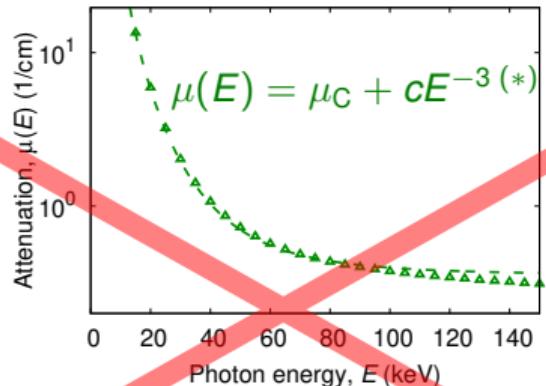
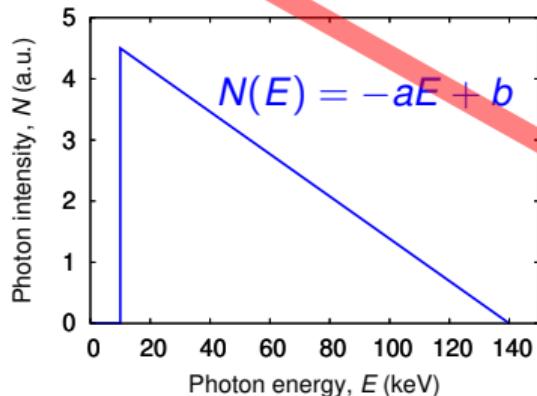
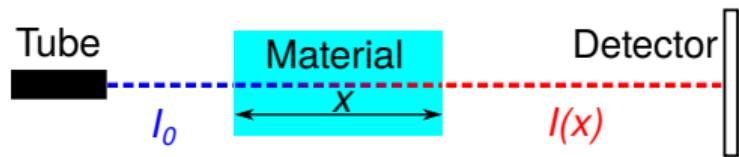
$$I(x) \propto \int N(E) \exp\{-\mu(E)x\} S(E) dE$$

$$\propto S \int_{E_{\min}}^{E_{\max}} (-aE + b) \exp\{-(\mu_C + cE^{-3})x\} dE$$



(\*) XCOM supplied by NIST

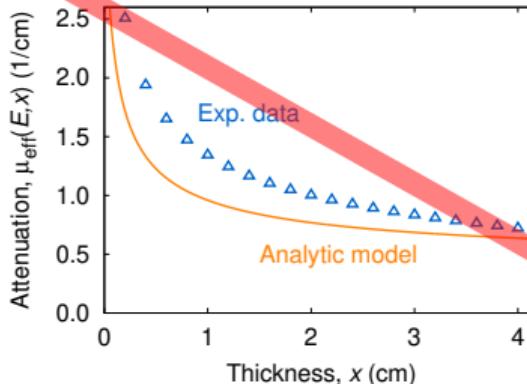
# Modeling of $\mu_{\text{eff}}(x)$



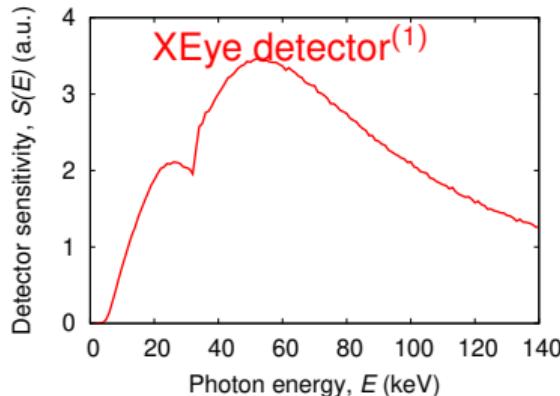
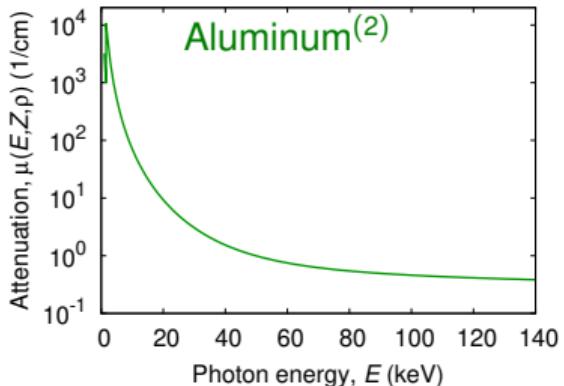
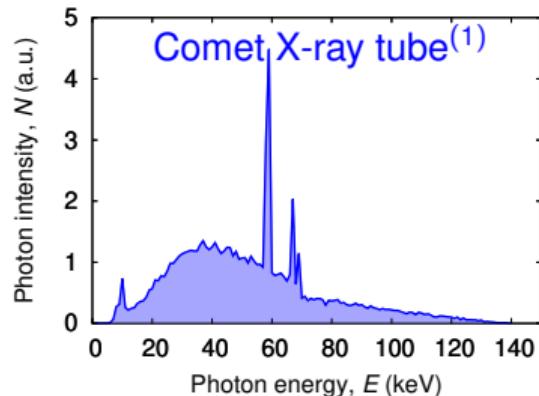
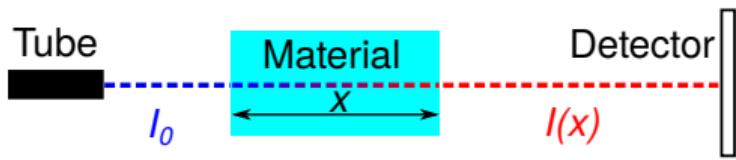
$$I(x) \propto \int N(E) \exp\{-\mu(E)x\} S(E) dE$$

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# Numerical approx. of $\mu_{\text{eff}}(x)$



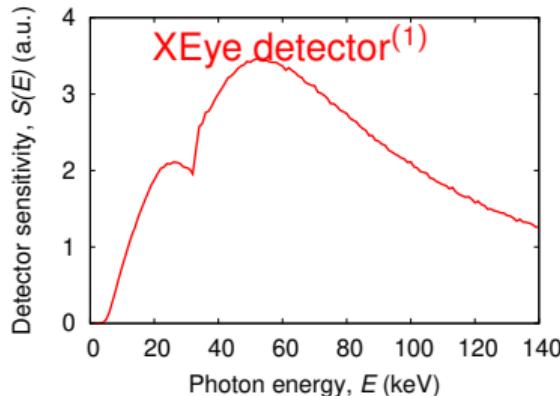
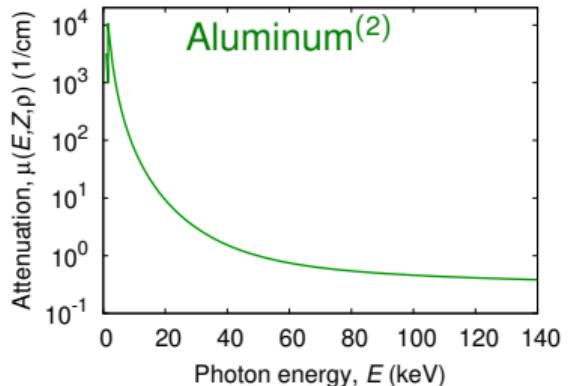
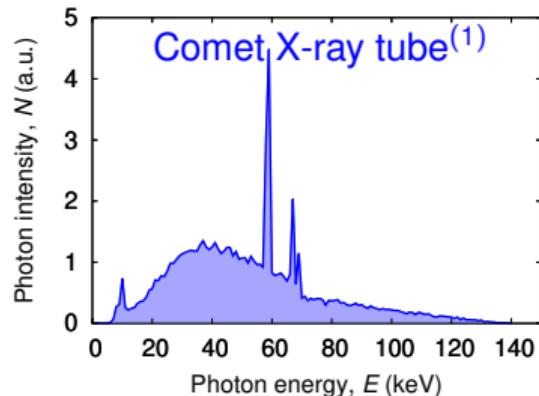
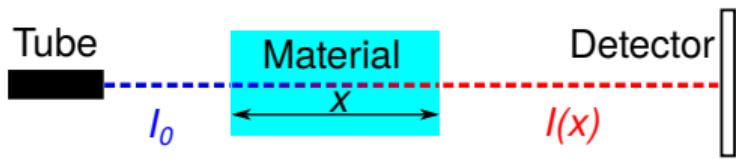
$$I(x) \propto \int \mathbf{N}(E) \exp\{-\mu(E)x\} \mathbf{S}(E) dE$$

$$\int \rightarrow \sum$$

(1) Supplied by Norman Uhlmann, Fraunhofer EZRT

(2) XCOM supplied by NIST

# Numerical approx. of $\mu_{\text{eff}}(x)$

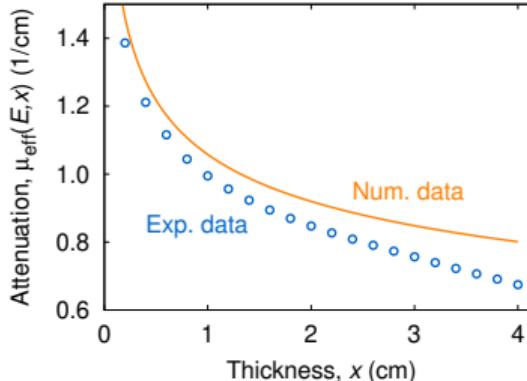


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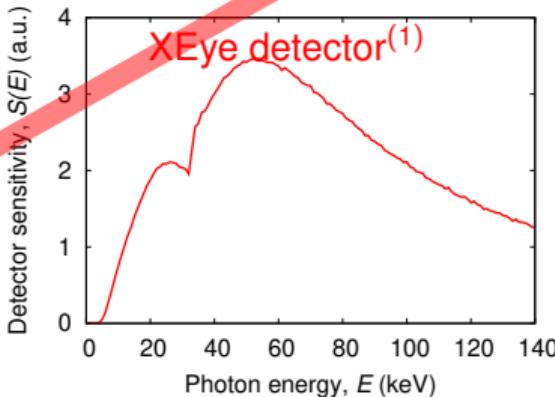
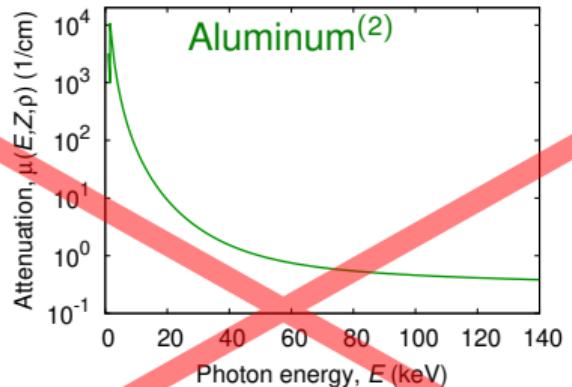
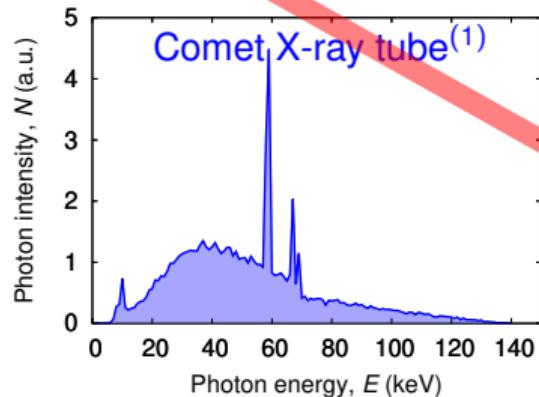
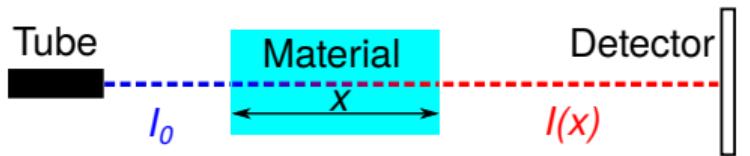
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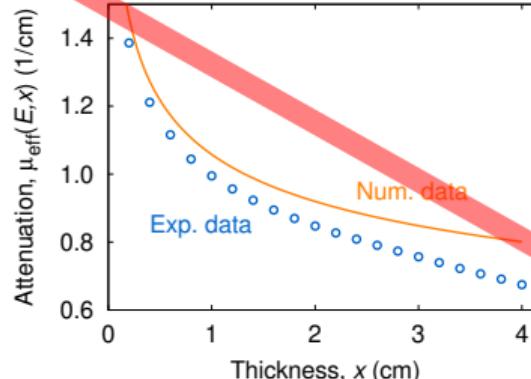


$$I(x) \propto \int N(E) \exp\{-\mu(E)x\} S(E) dE$$

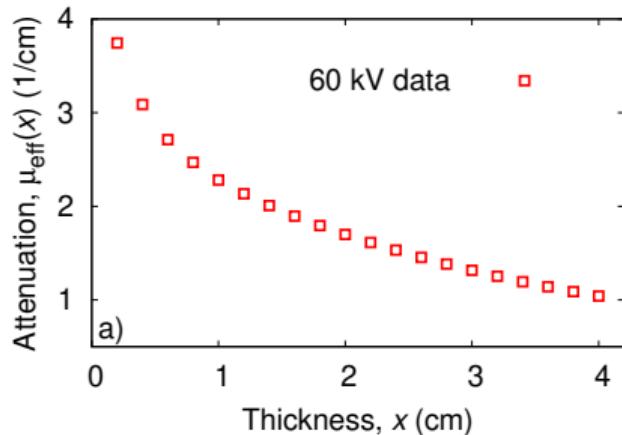
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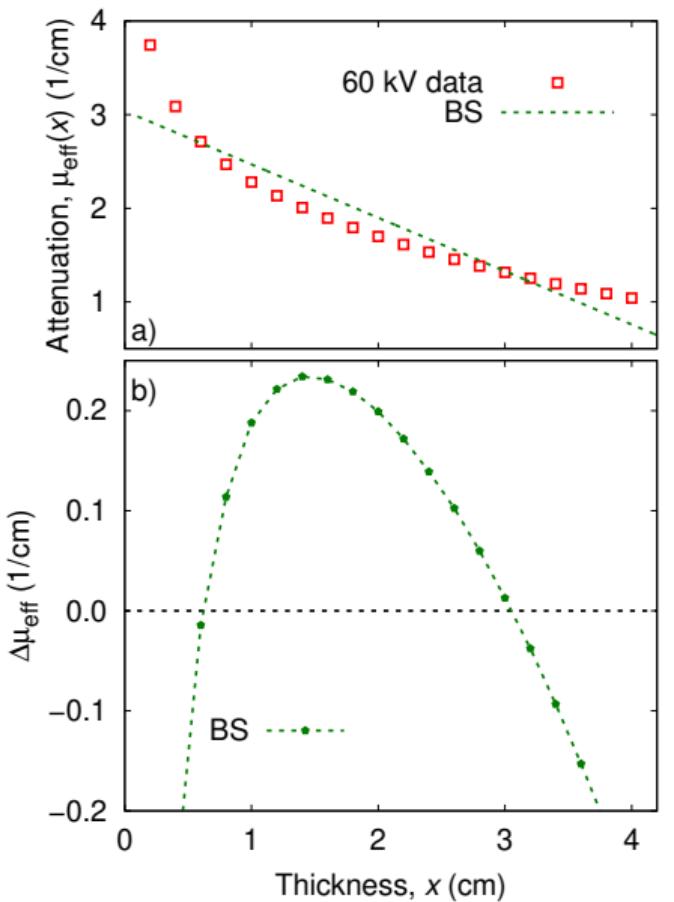
(2) XCOM supplied by NIST



# Heuristic model functions for $\mu_{\text{eff}}(x)$



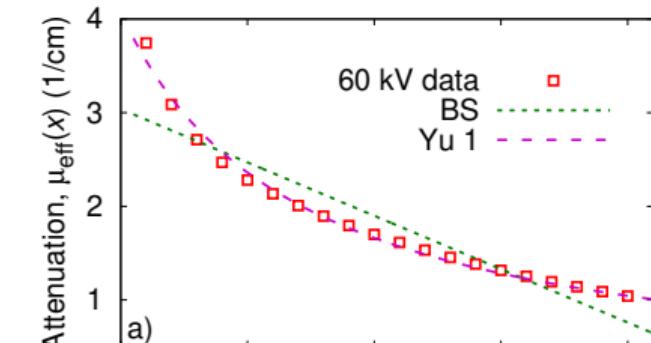
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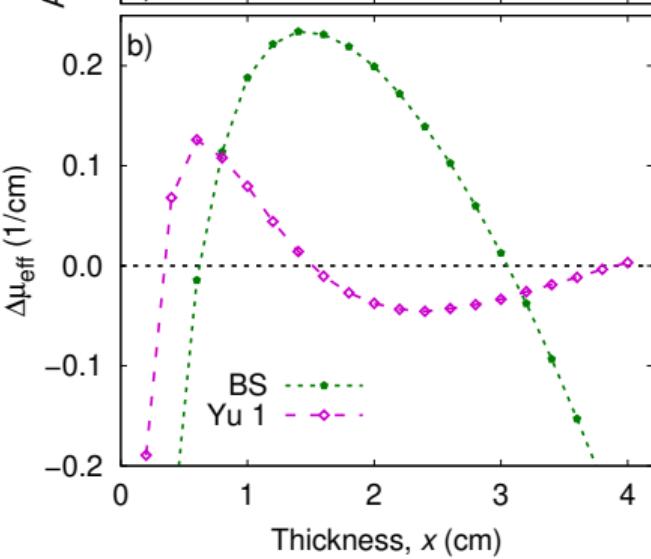
$$\mu_{\text{eff}}(x) = \mu_0 - \lambda x$$

Bjärngard & Shackford  
(1994)

# Heuristic model functions for $\mu_{\text{eff}}(x)$



$$\mu_{\text{eff}}(x) = \mu_0 - \lambda x$$

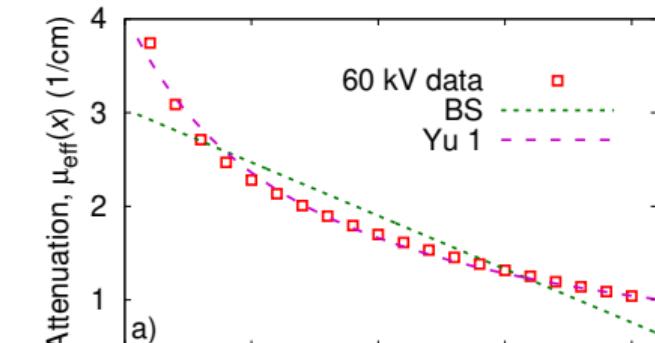


$$\mu_{\text{eff}}(x) = \frac{\mu_0}{1 + \lambda x}$$

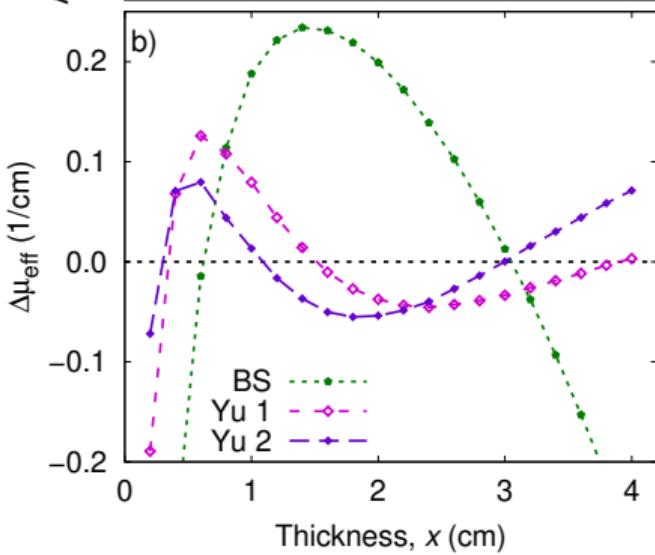
Bjärngard & Shackford  
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Yu *et al.* (1997)

# Heuristic model functions for $\mu_{\text{eff}}(x)$



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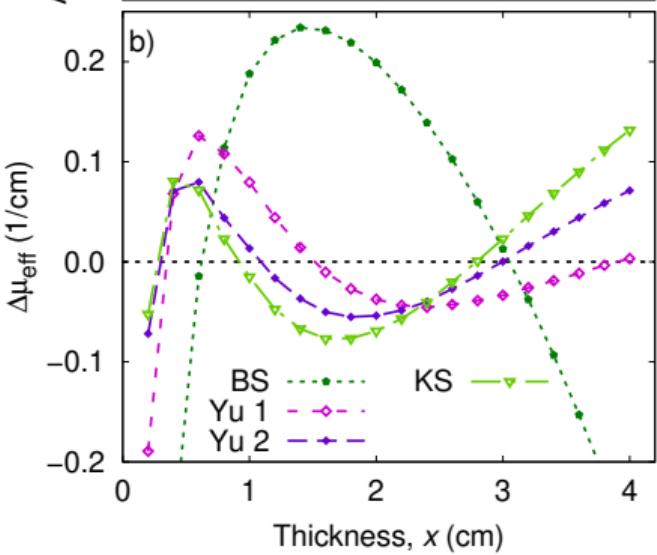
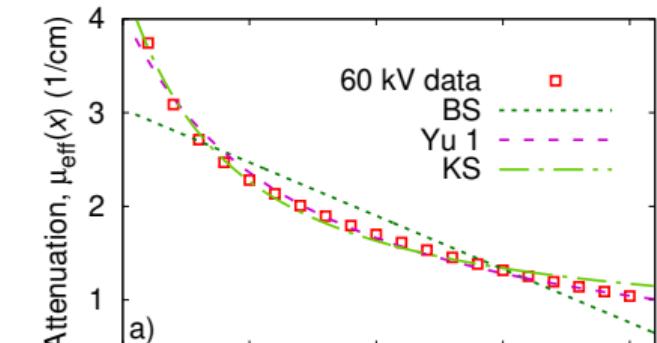
$$\mu_{\text{eff}}(x) = \frac{\mu_0}{1+\lambda x}$$

$$\mu_{\text{eff}}(x) = \frac{\mu_0}{(1+\lambda x)^\beta}$$

Bjärngard & Shackford  
(1994)

Yu *et al.* (1997)

# Heuristic model functions for $\mu_{\text{eff}}(x)$



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Bjärngard & Shackford  
(1994)

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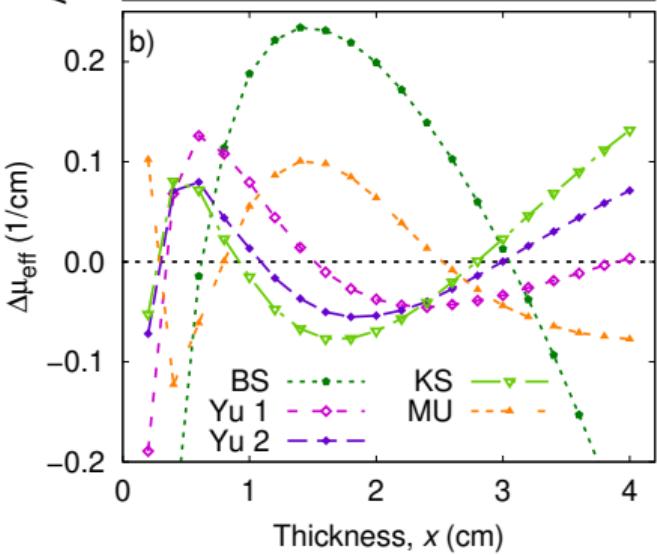
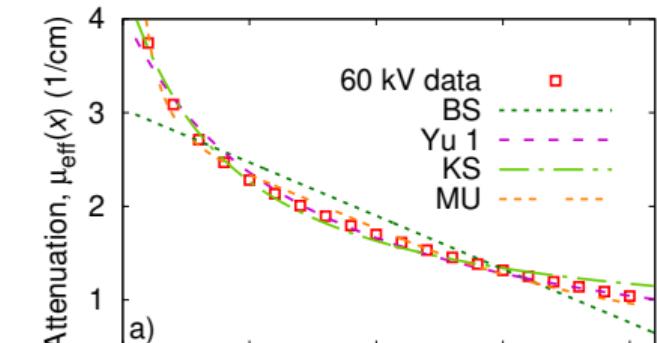
Yu *et al.* (1997)

$$\mu_{\text{eff}}(x) = \mu(E_{\max}) + \frac{2\mu_1}{x\sqrt{-\lambda_1^2 + 4\lambda_2}} \times$$

$$\left[ \arctan\left(\frac{\lambda_1 + 2\lambda_2 x}{\sqrt{-\lambda_1^2 + 4\lambda_2}}\right) - \arctan\left(\frac{\lambda_1}{\sqrt{-\lambda_1^2 + 4\lambda_2}}\right) \right]$$

Kleinschmidt (1999)

# Heuristic model functions for $\mu_{\text{eff}}(x)$



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Bjärngard & Shackford  
(1994)

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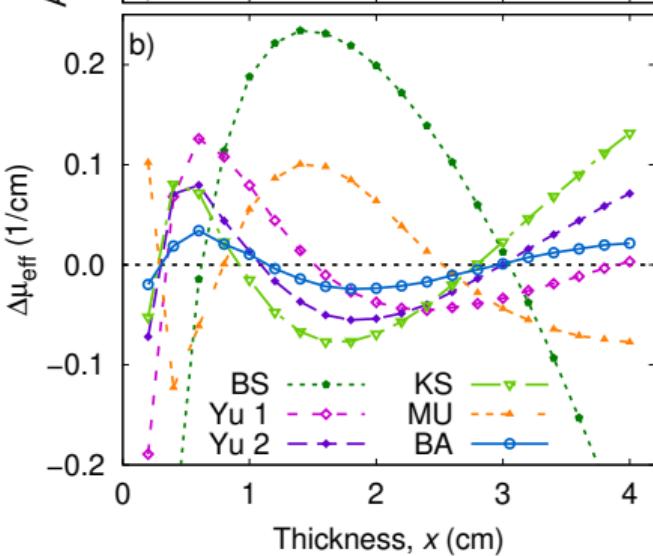
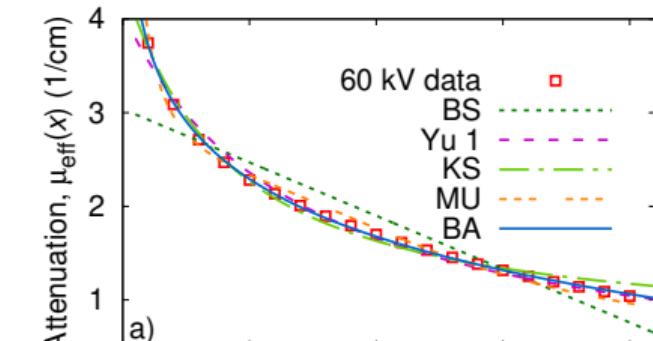
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Kleinschmidt (1999)

$$\mu_{\text{eff}}(x) = -\frac{1}{x} \ln [A + B \exp(-x/C)]$$

Mudde *et al.* (2008)

# Heuristic model functions for $\mu_{\text{eff}}(x)$



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Bjärngard & Shackford  
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Yu *et al.* (1997)

$$\mu_{\text{eff}}(x) = \mu(E_{\text{max}}) + \frac{2\mu_1}{x\sqrt{-\lambda_1^2 + 4\lambda_2}} \times \left[ \arctan\left(\frac{\lambda_1 + 2\lambda_2 x}{\sqrt{-\lambda_1^2 + 4\lambda_2}}\right) - \arctan\left(\frac{\lambda_1}{\sqrt{-\lambda_1^2 + 4\lambda_2}}\right) \right]$$

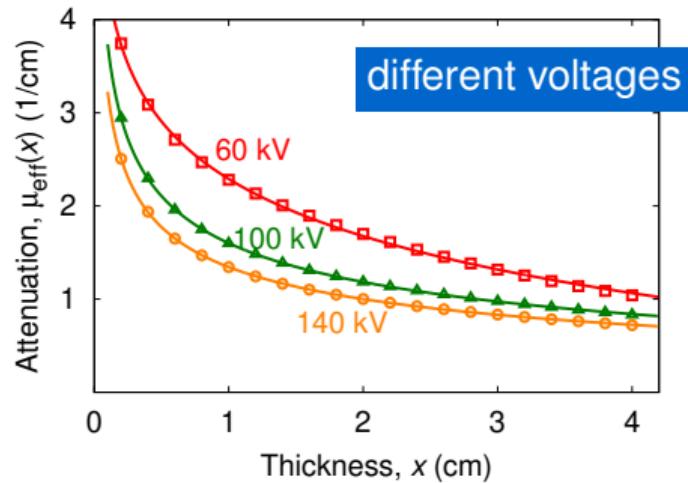
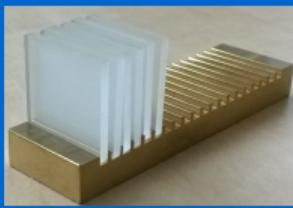
$$\mu_{\text{eff}}(x) = -\frac{1}{x} \ln [A + B \exp(-x/C)]$$

Kleinschmidt (1999)  
Mudde *et al.* (2008)

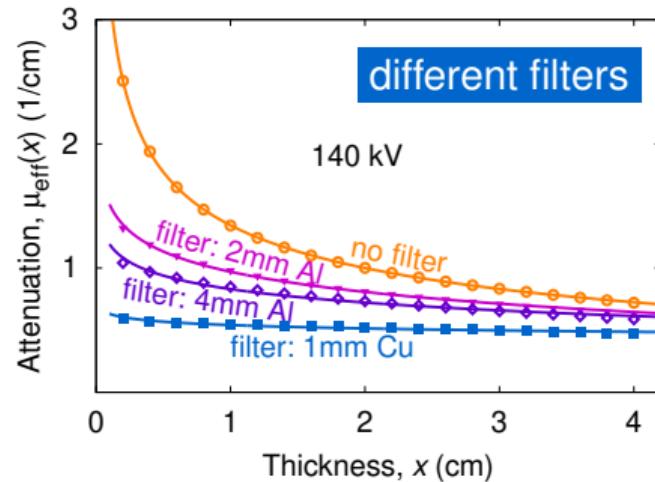
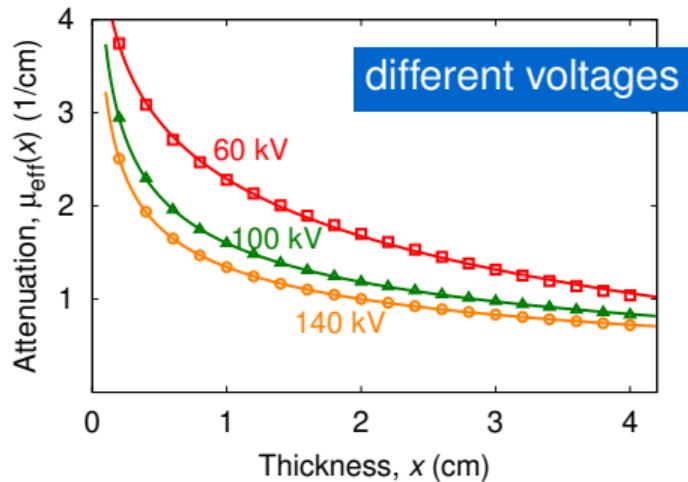
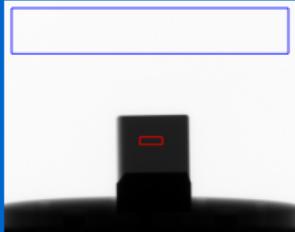
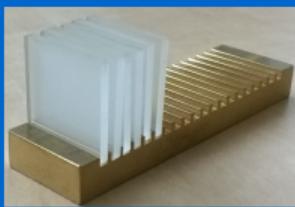
$$\mu_{\text{eff}}(x) = a + \frac{b}{x^\alpha}$$

Baur *et al.* (2019)  
(this work)

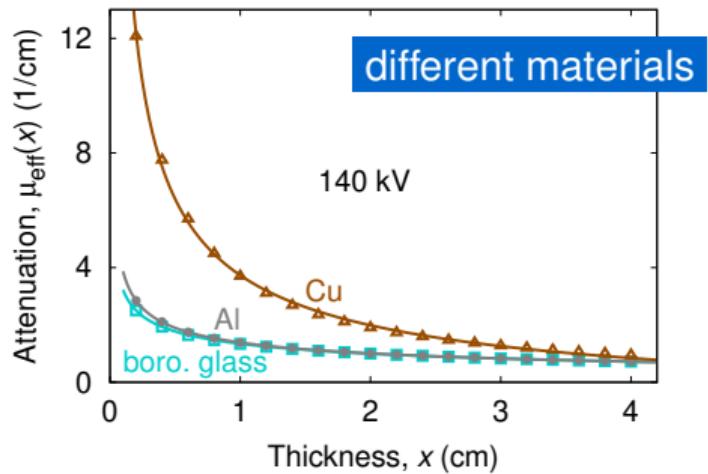
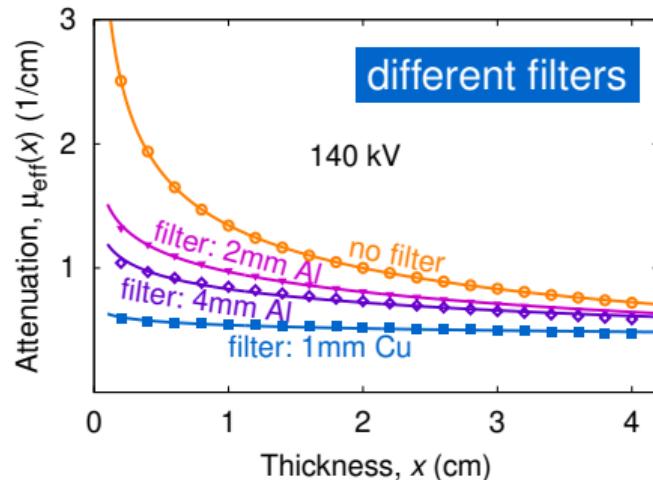
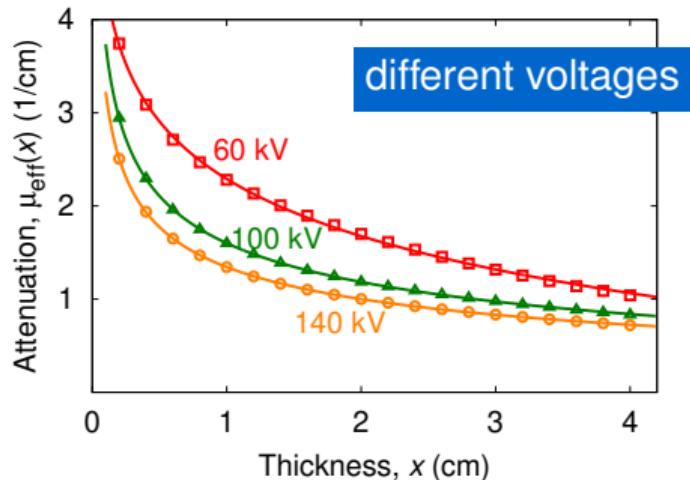
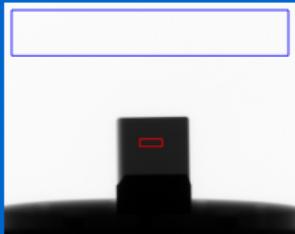
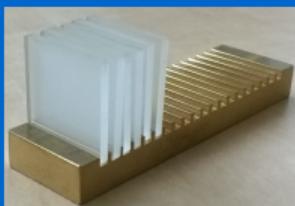
Applicability of  
 $\mu_{\text{eff}}(x) = a + \frac{b}{x^\alpha}$



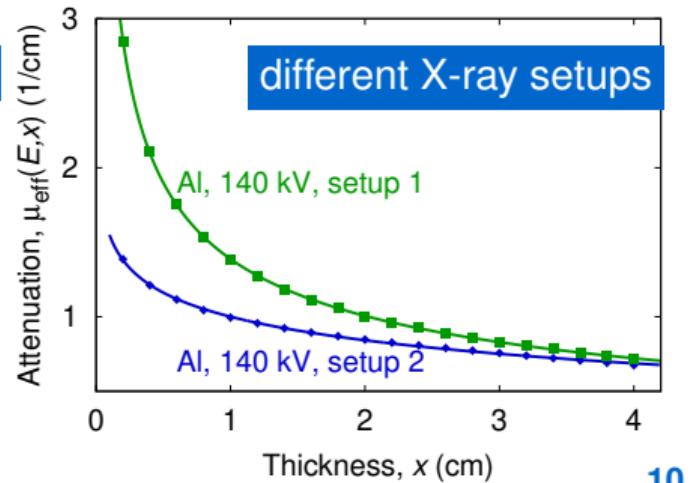
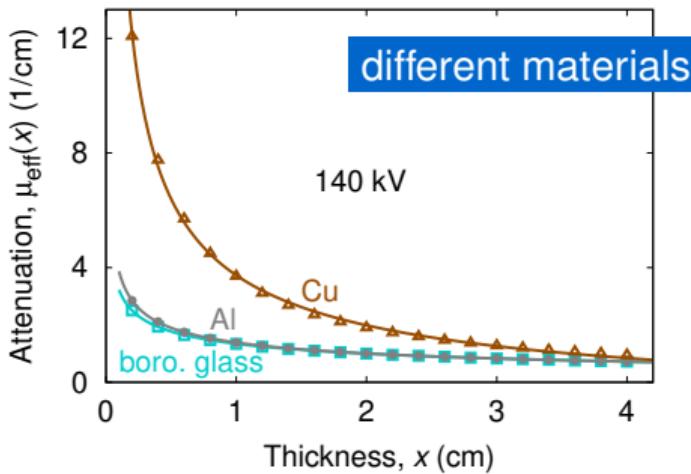
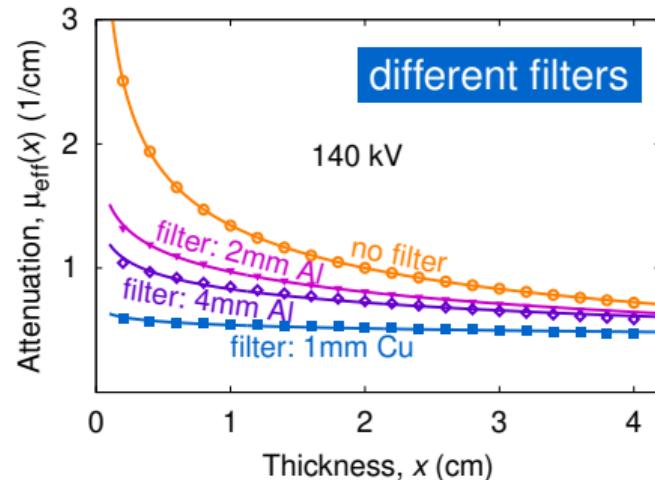
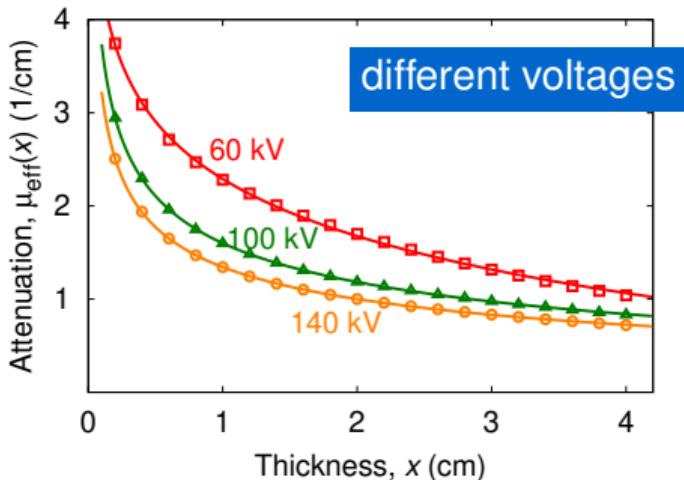
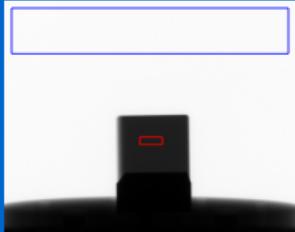
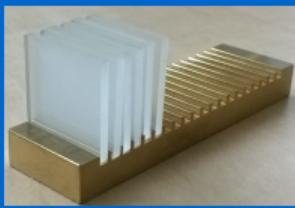
Applicability of  
 $\mu_{\text{eff}}(x) = a + \frac{b}{x^\alpha}$



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 $\mu_{\text{eff}}(x) = a + \frac{b}{x^\alpha}$



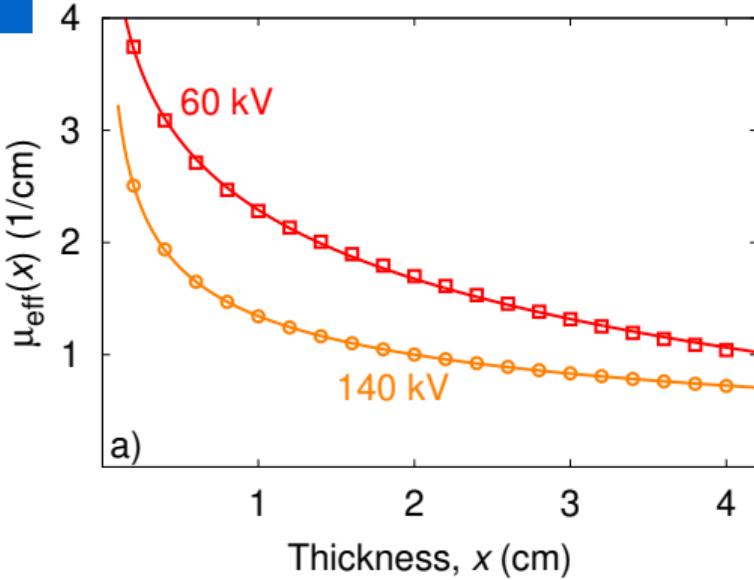
# Determining the material thickness $x$

Generalized Beer-Lambert

$$I(x) = I_0 \exp(-\mu_{\text{eff}}(x) x)$$

Model function

$$\mu_{\text{eff}}(x) = a + \frac{b}{x^\alpha}$$



# Determining the material thickness $x$

Generalized Beer-Lambert

$$I(x) = I_0 \exp(-\mu_{\text{eff}}(x)x)$$

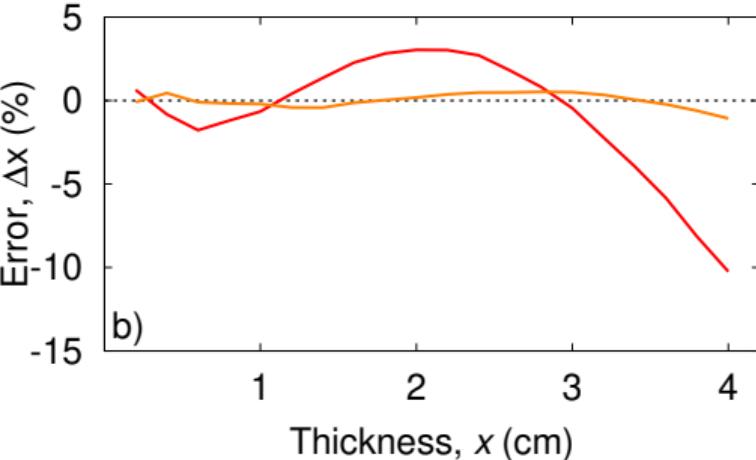
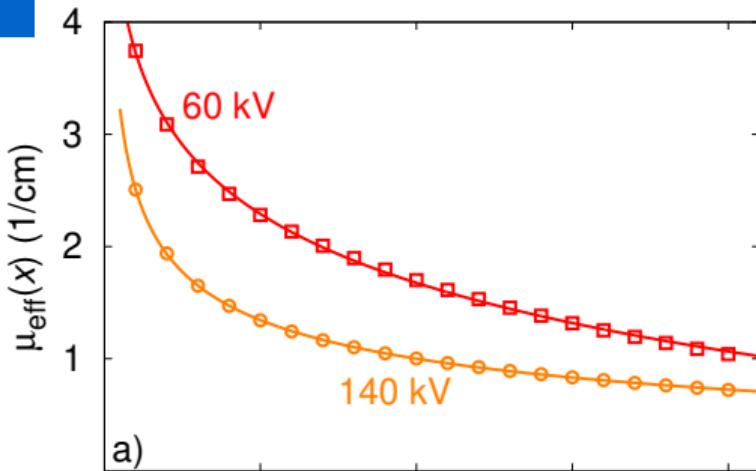
Model function

$$\mu_{\text{eff}}(x) = a + \frac{b}{x^\alpha}$$

Solve

$$ax + bx^{1-\alpha} + \ln\left(\frac{I(x)}{I_0}\right) = 0$$

e.g. Newton's method or look-up table



# Determining the material thickness $x$

Generalized Beer-Lambert

$$I(x) = I_0 \exp(-\mu_{\text{eff}}(x)x)$$

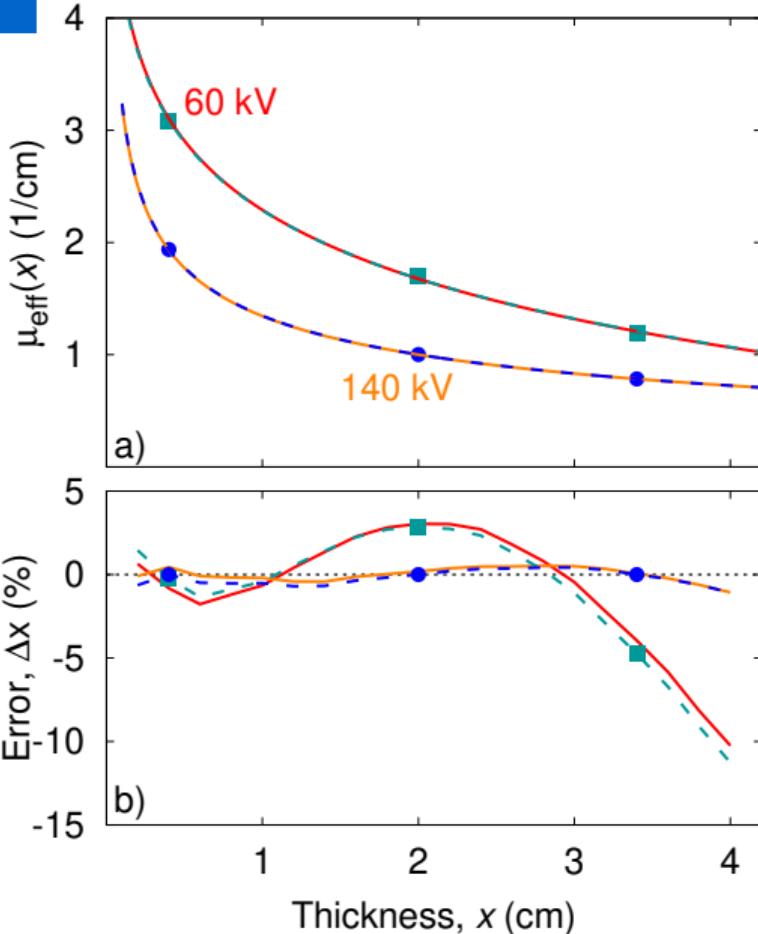
Model function

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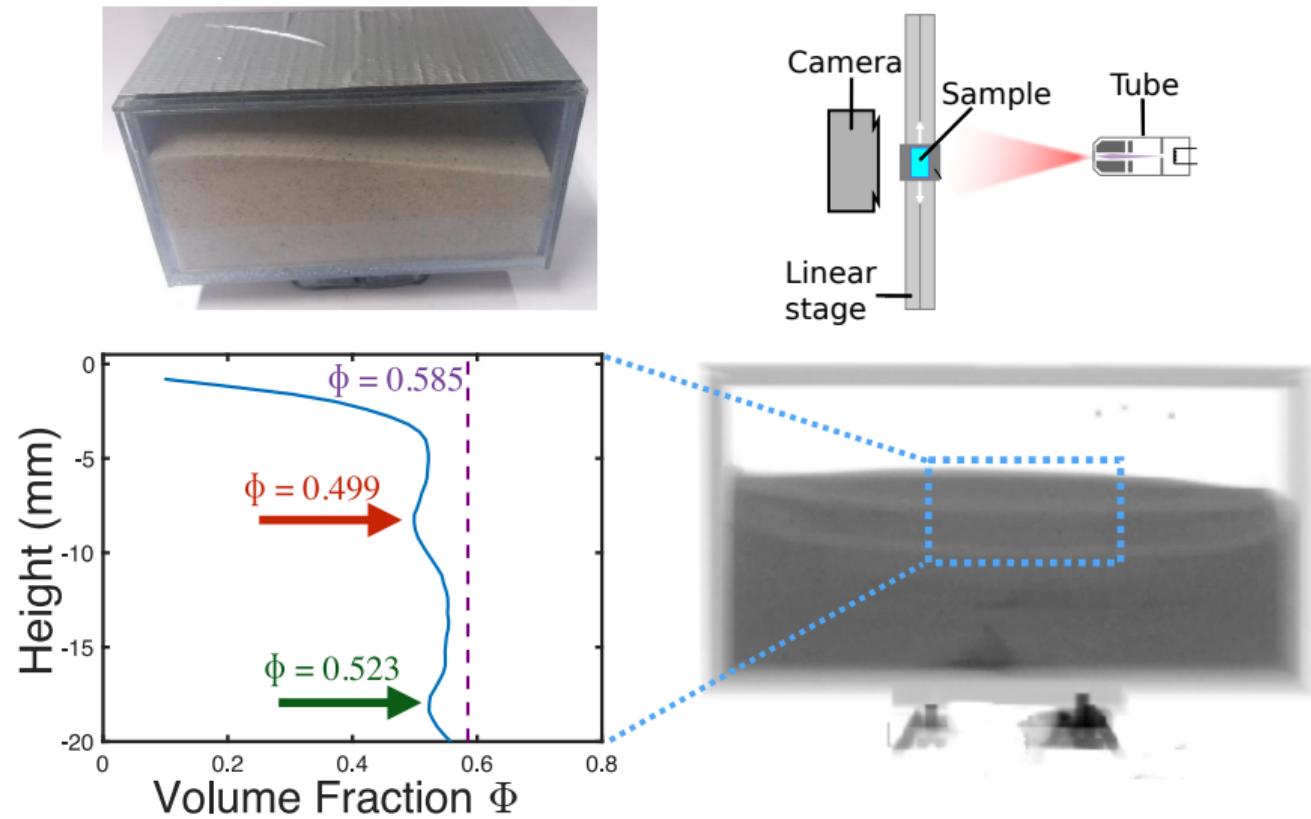
Solve

$$ax + bx^{1-\alpha} + \ln\left(\frac{I(x)}{I_0}\right) = 0$$

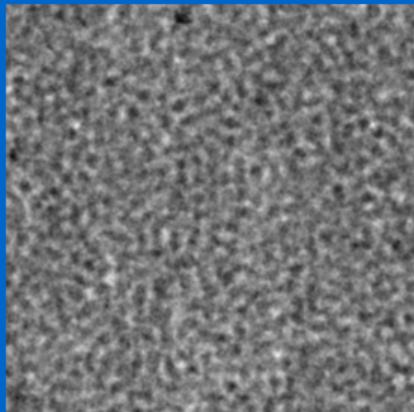
e.g. Newton's method or look-up table



# Migrating shear bands in shaken granular matter, Kollmer *et al* (2020)

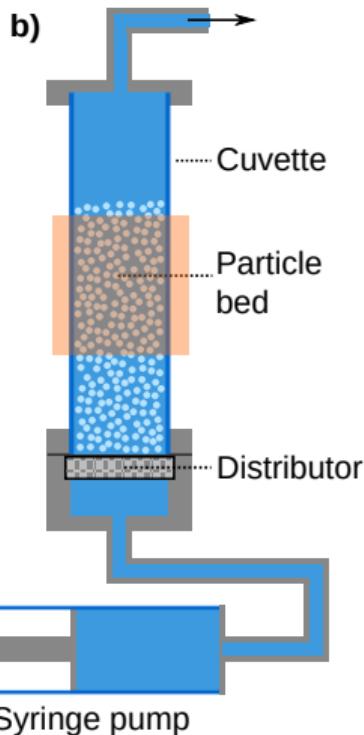
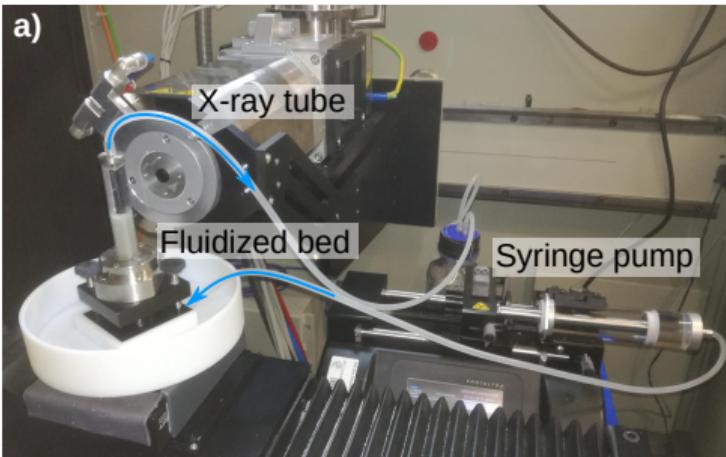


# Measuring granular dynamics with X-ray Digital Fourier Analysis (X-DFA)

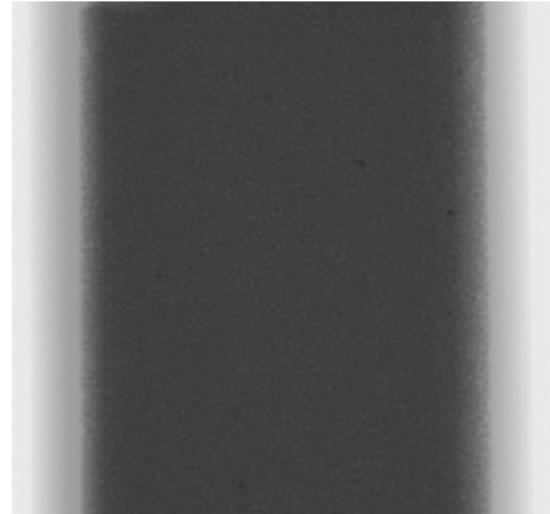


In collaboration with M. Escobedo & S. Egelhaaf, University of Düsseldorf

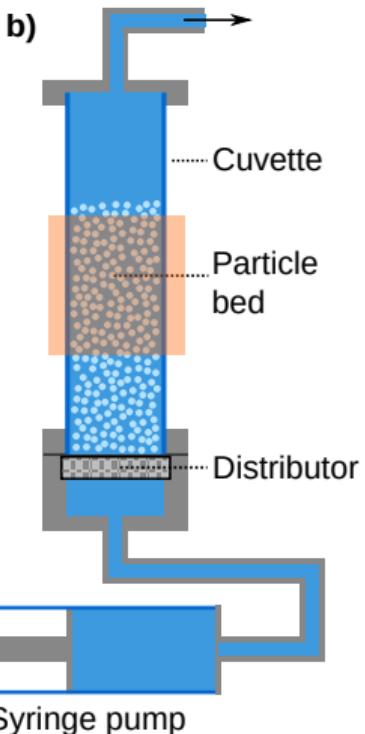
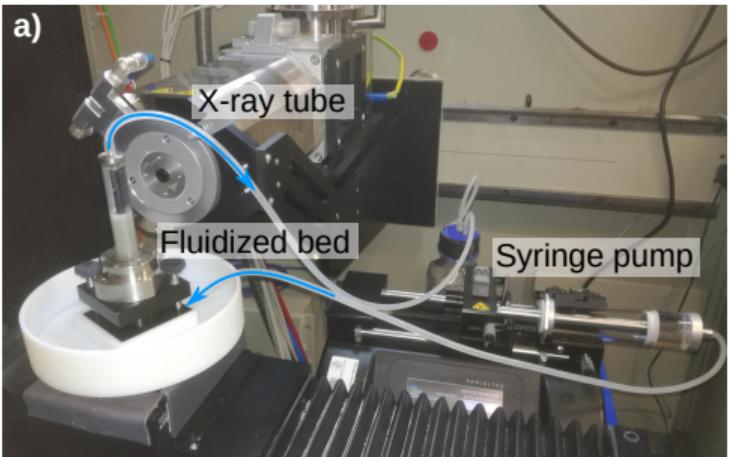
# The system: A liquid fluidized bed



Radiogram

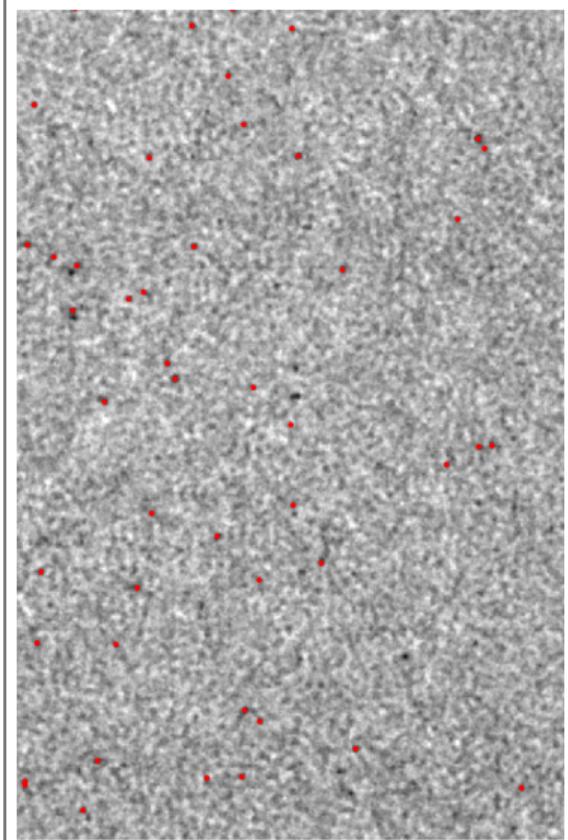


# The system: A liquid fluidized bed

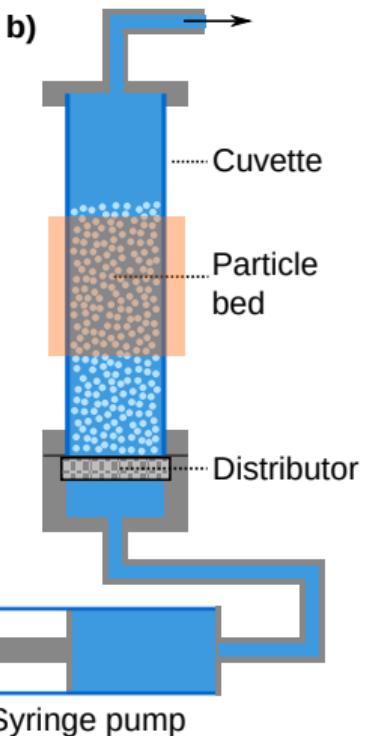
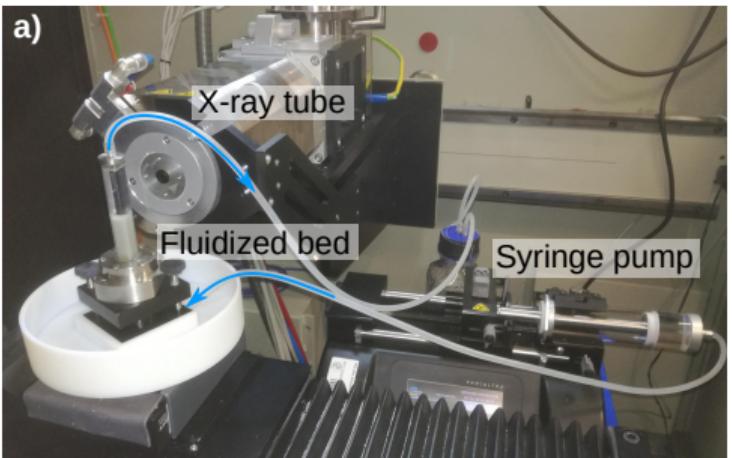


## Particle tracking

Contrast:  $\rho_{\text{tracer}} > \rho_{\text{bed}}$

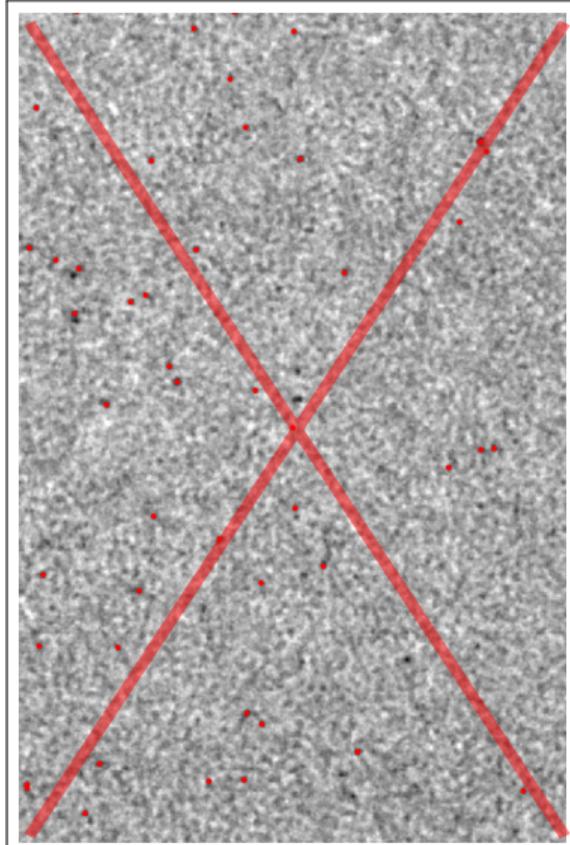


# The system: A liquid fluidized bed

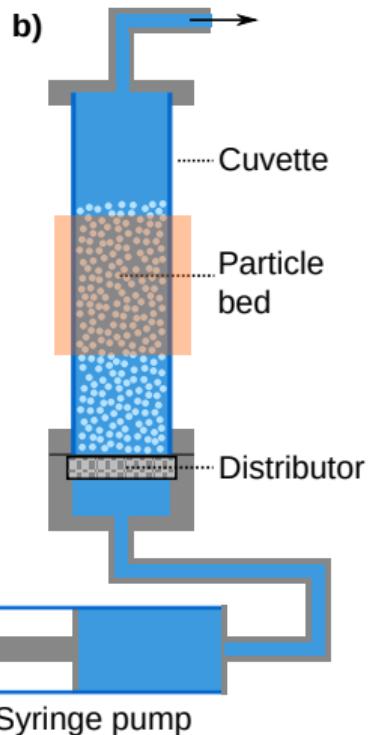
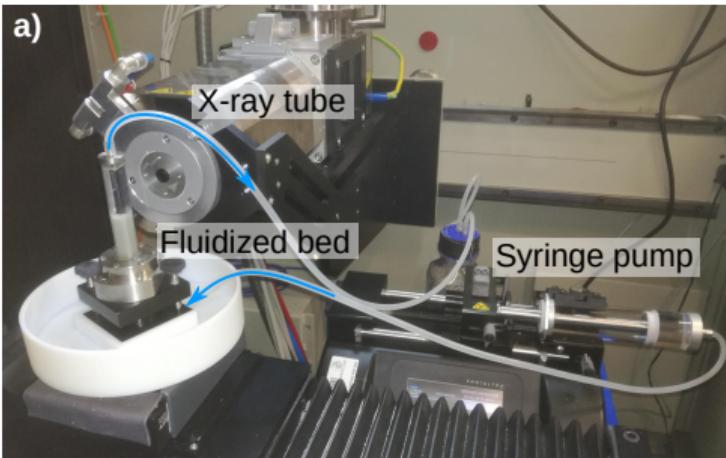


## Particle tracking

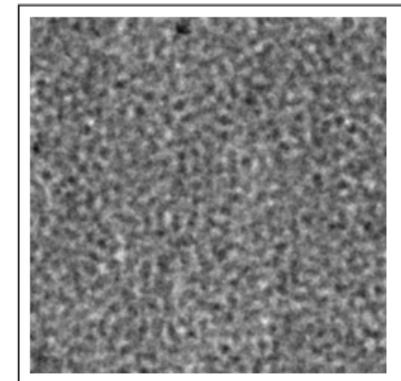
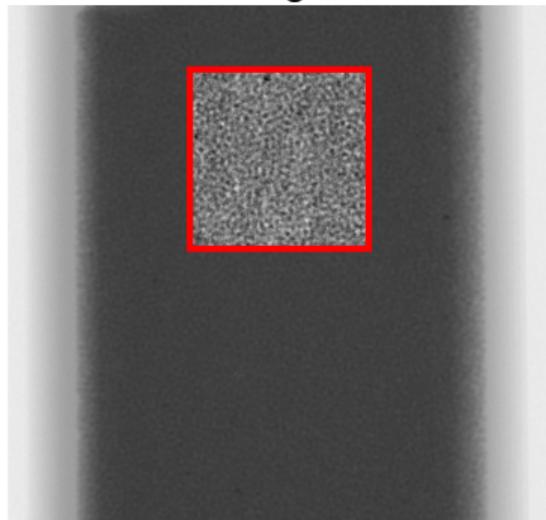
Contrast:  $\rho_{\text{tracer}} > \rho_{\text{bed}}$



# The system: A liquid fluidized bed



Radiogram



## Differential Dynamic Microscopy (DDM)

	<b>Up to now</b>	<b>This work</b>
<b>System</b>	Dispersion, gels	Fluidized bed
<b>Particles</b>	Colloids $< 1 \mu\text{m}$	Granulates ( $150 - 180 \mu\text{m}$ )
<b>Volume fraction</b>	$\Phi \leq 0.33$	$0.45 < \Phi < 0.56$
<b>Imaging</b>	Light microscopy	X-ray radiography
<b>Dynamics</b>	Brownian motion, caging, glassy, collective motion	

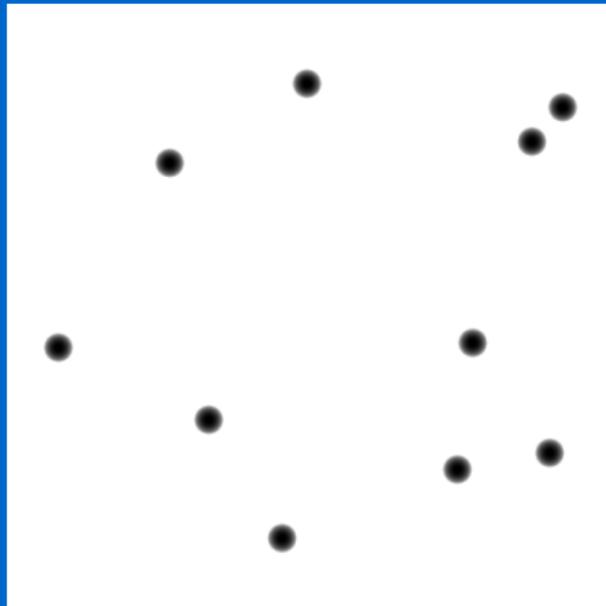
# Extending Differential Dynamic Microscopy (DDM) to X-ray imaging

	<b>Up to now</b>	<b>This work</b>
<b>System</b>	Dispersion, gels	Fluidized bed
<b>Particles</b>	Colloids $< 1 \mu\text{m}$	<b>Granulates (150 – 180) <math>\mu\text{m}</math></b>
<b>Volume fraction</b>	$\Phi \leq 0.33$	$0.45 < \Phi < 0.56$
<b>Imaging</b>	Light microscopy	<b>X-ray radiography</b>
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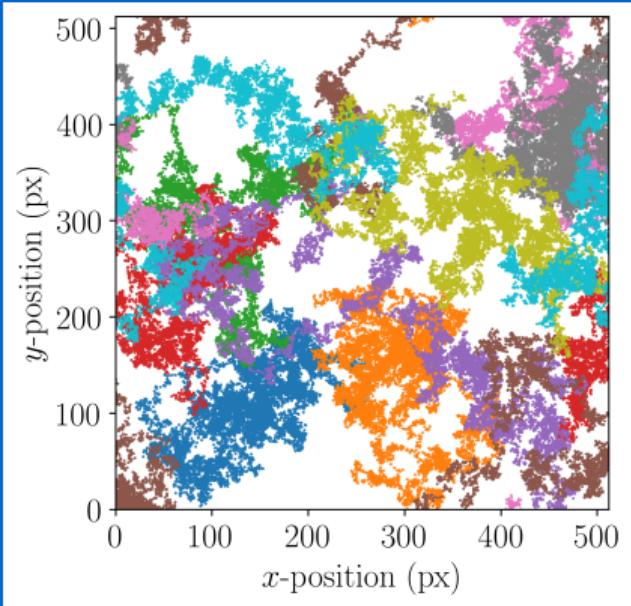
## Digital Fourier Analysis of X-Ray radiograms (X-DFA)

# Introduction to X-ray Digital Fourier Analysis (X-DFA)

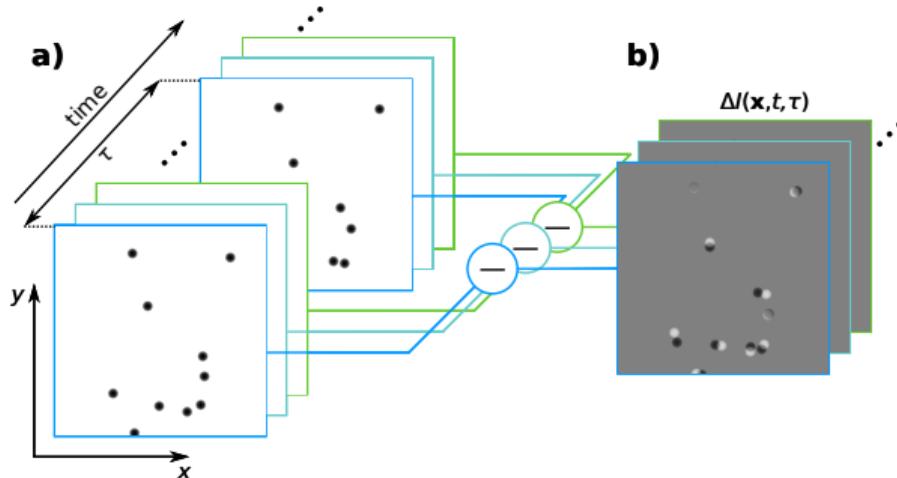
Synthetic radiograms



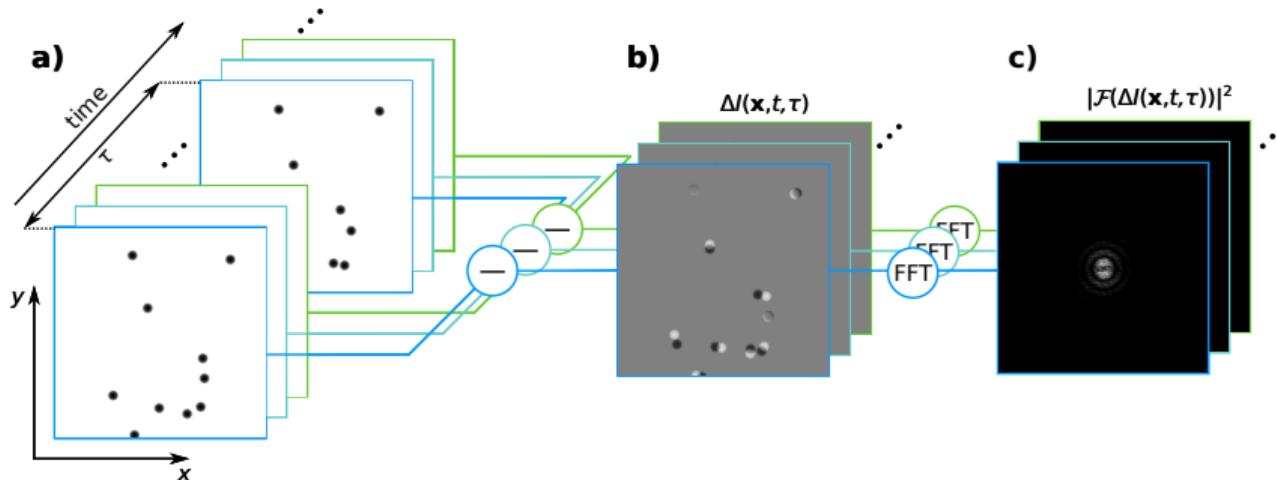
Particle trajectory



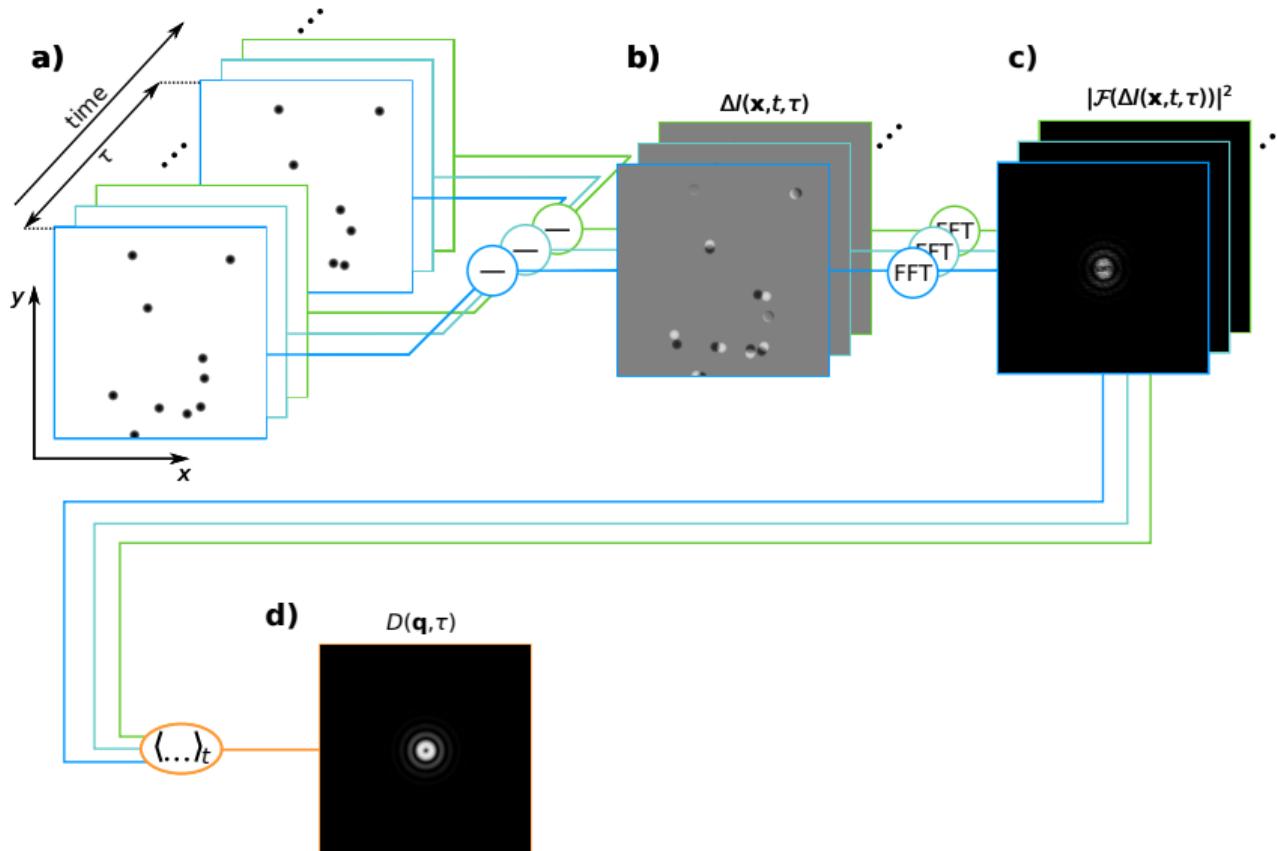
# The image structure function $D(\mathbf{q}, \tau)$



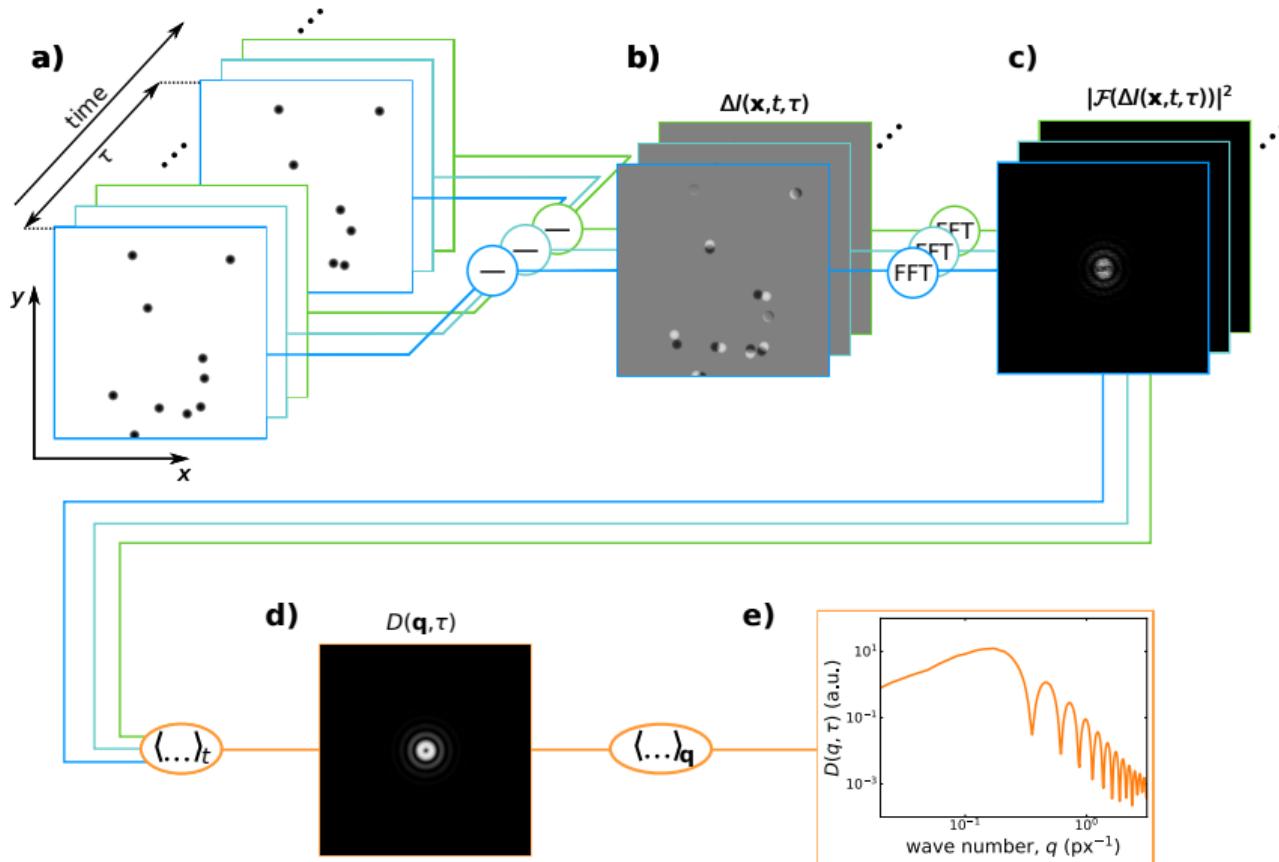
# The image structure function $D(\mathbf{q}, \tau)$



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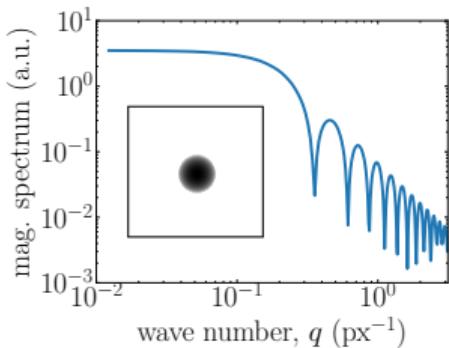
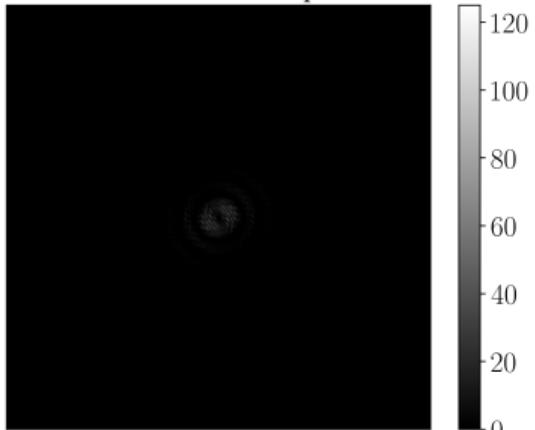


# The image structure function $D(\mathbf{q}, \tau)$



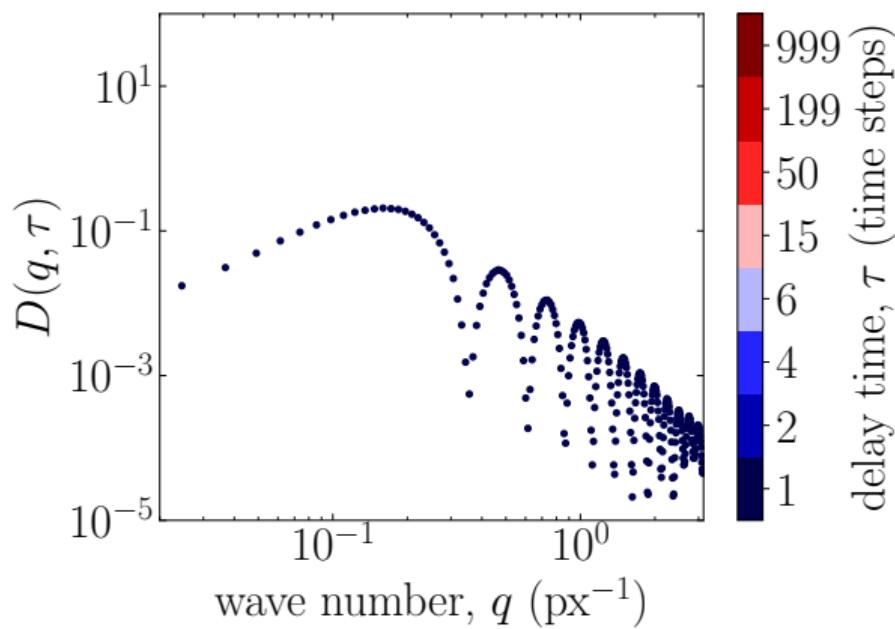
# The image structure function $D(q, \tau)$

$\tau = 1$  time steps



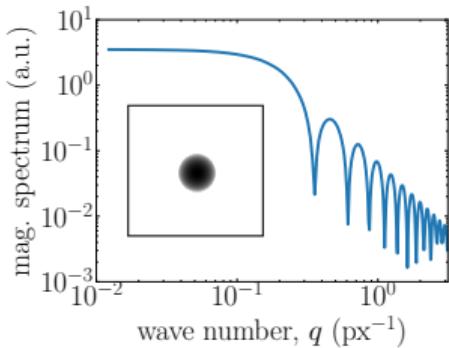
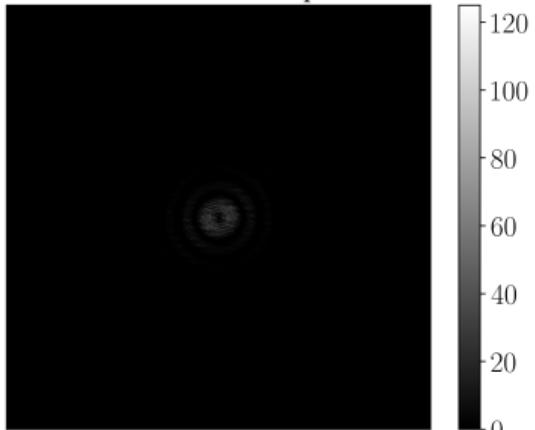
Time averaging:  $D(\mathbf{q}, \tau) = \langle |\mathcal{F}(\Delta I)|^2 \rangle_t$

Azimuthal averaging:  $D(\mathbf{q}, \tau) \rightarrow D(\mathbf{q}, \tau) \dots$



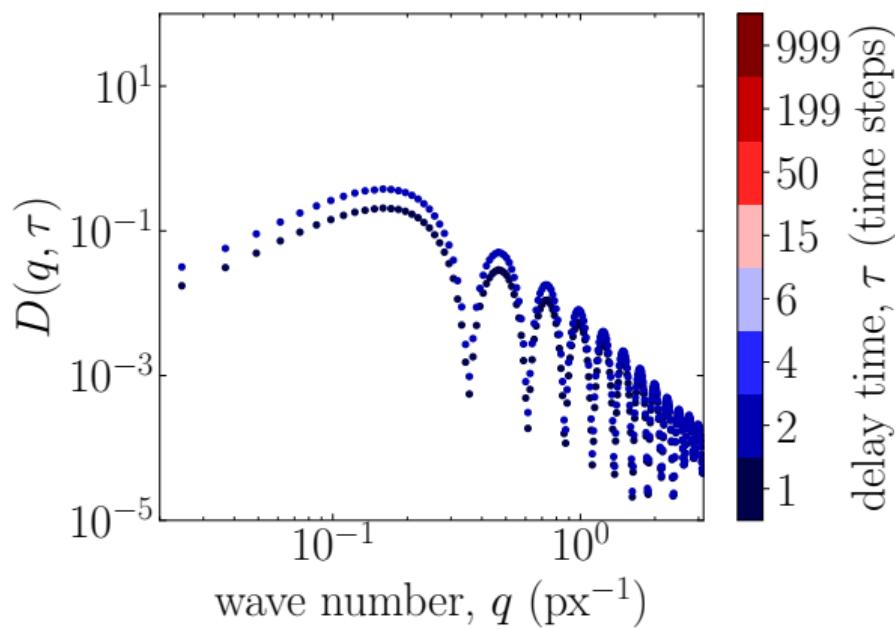
# The image structure function $D(q, \tau)$

$\tau = 2$  time steps



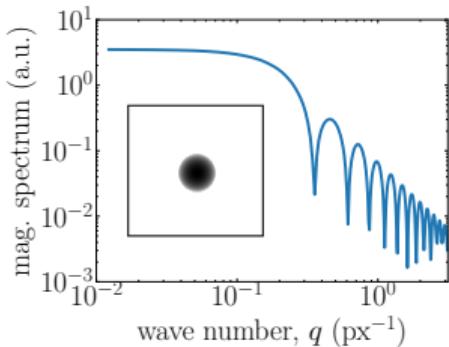
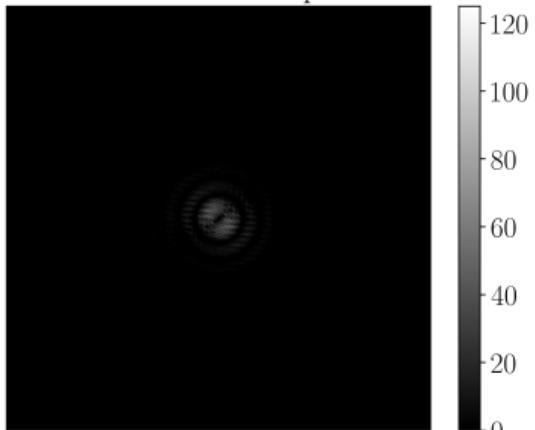
Time averaging:  $D(\mathbf{q}, \tau) = \langle |\mathcal{F}(\Delta I)|^2 \rangle_t$

Azimuthal averaging:  $D(\mathbf{q}, \tau) \rightarrow D(\mathbf{q}, \tau) \dots$



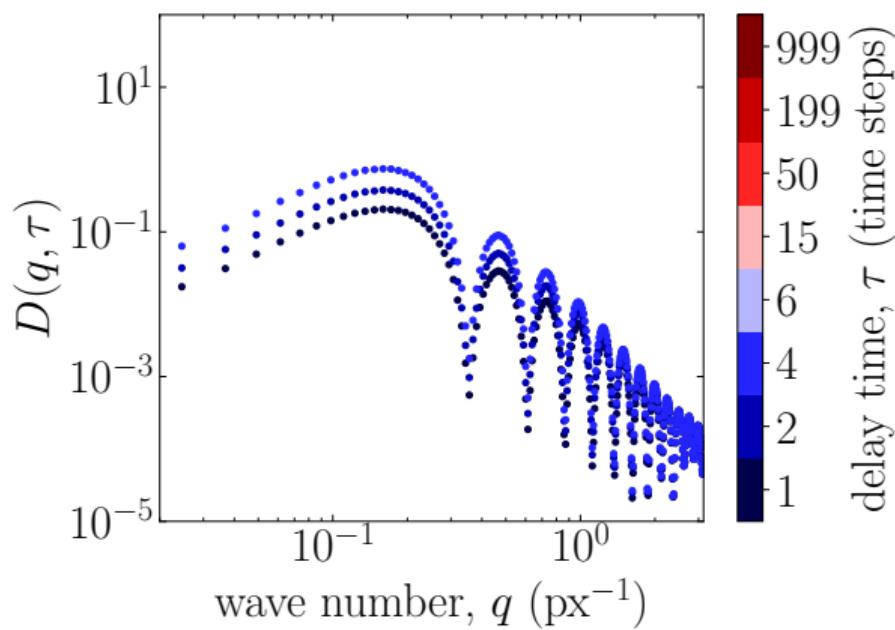
# The image structure function $D(q, \tau)$

$\tau = 4$  time steps



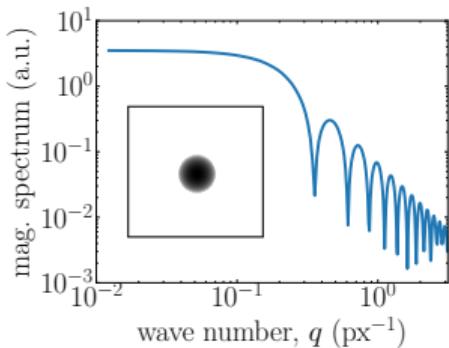
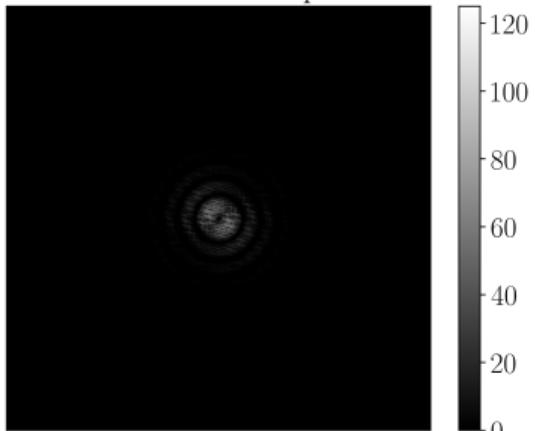
Time averaging:  $D(\mathbf{q}, \tau) = \langle |\mathcal{F}(\Delta I)|^2 \rangle_t$

Azimuthal averaging:  $D(\mathbf{q}, \tau) \rightarrow D(\mathbf{q}, \tau) \dots$



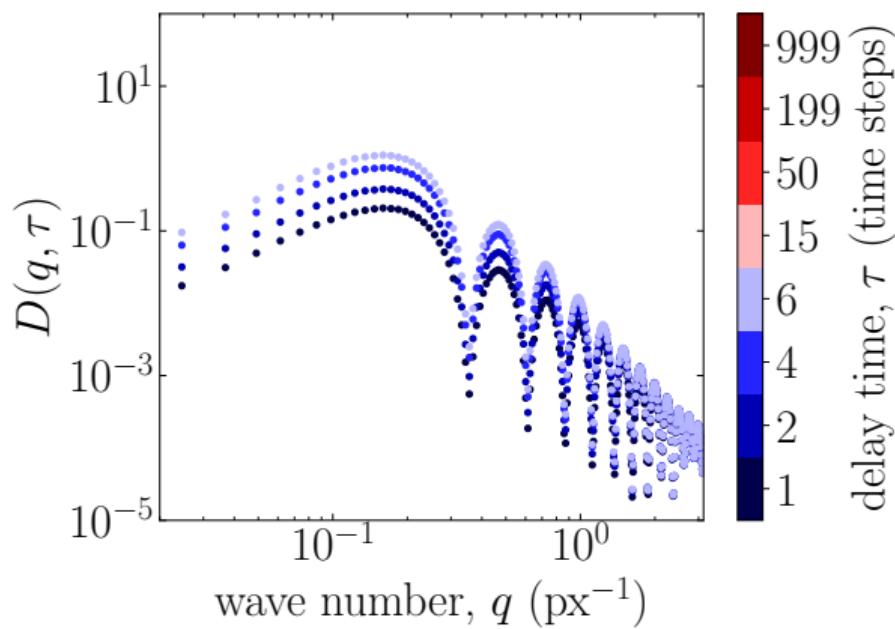
# The image structure function $D(q, \tau)$

$\tau = 6$  time steps

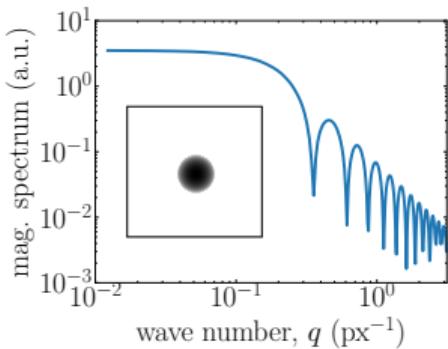
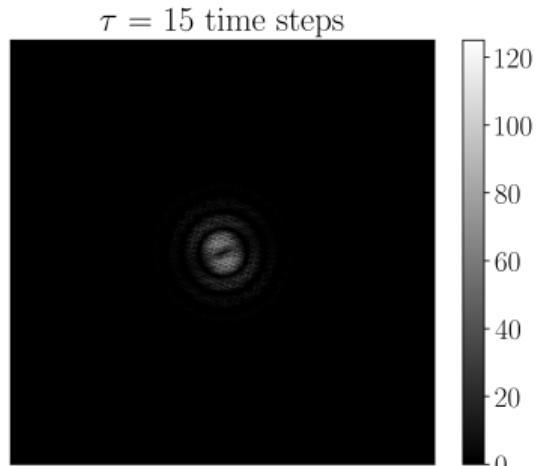


Time averaging:  $D(\mathbf{q}, \tau) = \langle |\mathcal{F}(\Delta I)|^2 \rangle_t$

Azimuthal averaging:  $D(\mathbf{q}, \tau) \rightarrow D(\mathbf{q}, \tau) \dots$

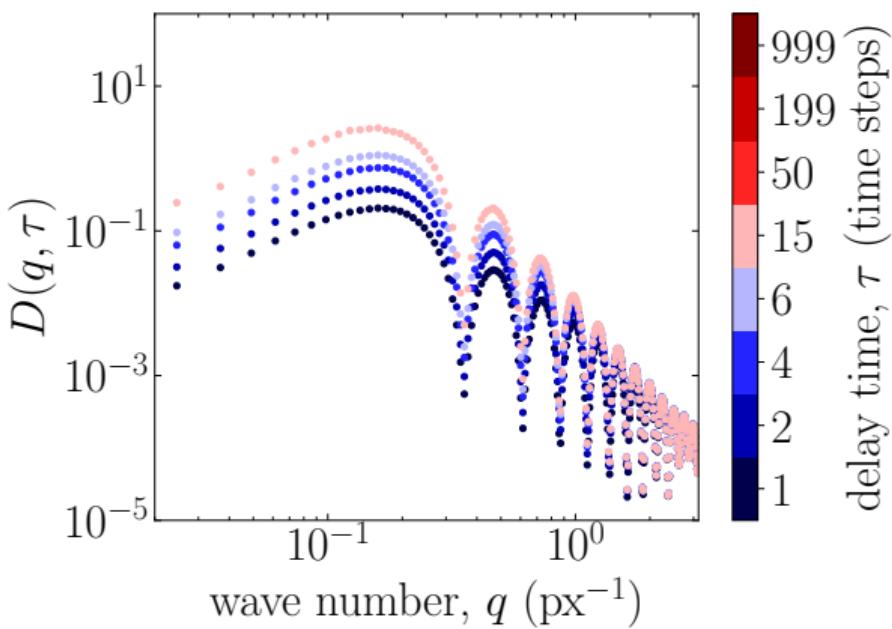


# The image structure function $D(q, \tau)$

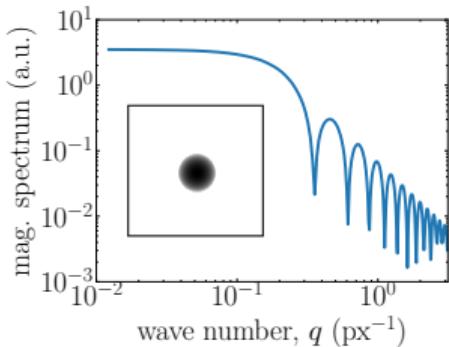
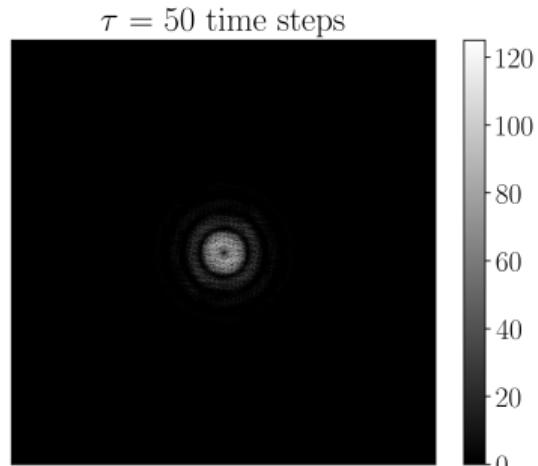


Time averaging:  $D(\mathbf{q}, \tau) = \langle |\mathcal{F}(\Delta I)|^2 \rangle_t$

Azimuthal averaging:  $D(\mathbf{q}, \tau) \rightarrow D(\mathbf{q}, \tau) \dots$

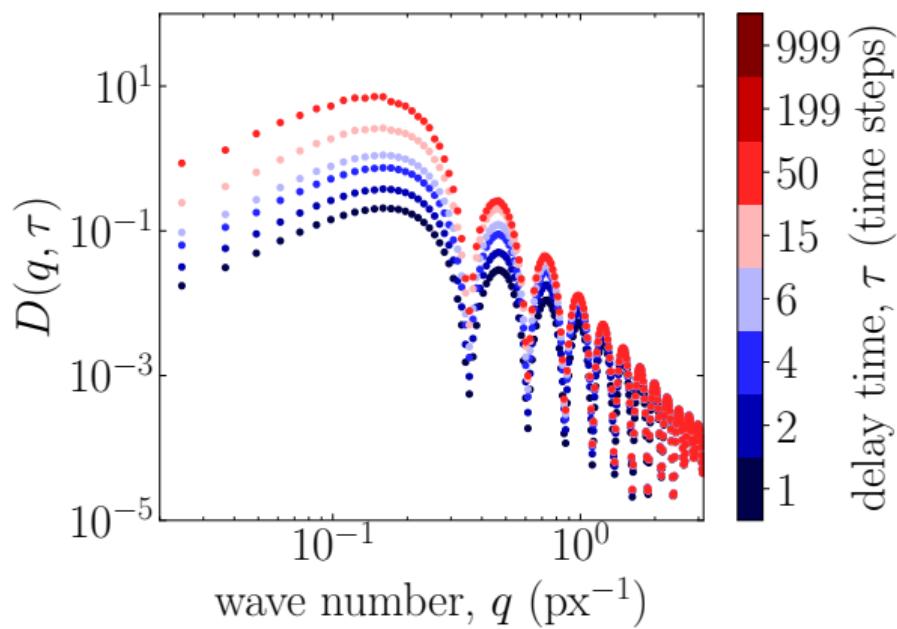


# The image structure function $D(q, \tau)$



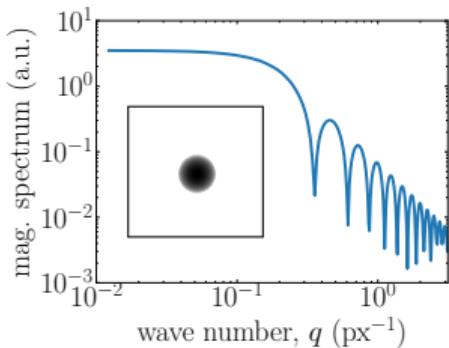
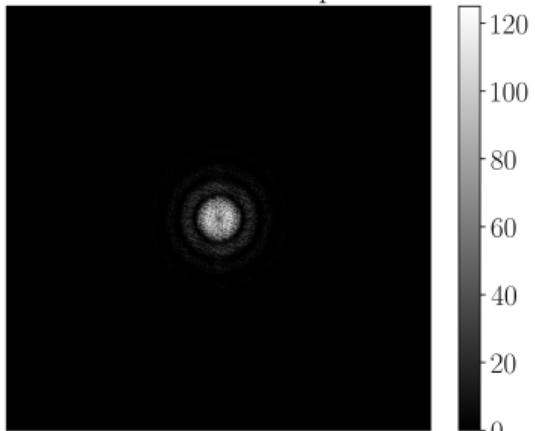
Time averaging:  $D(\mathbf{q}, \tau) = \langle |\mathcal{F}(\Delta I)|^2 \rangle_t$

Azimuthal averaging:  $D(\mathbf{q}, \tau) \rightarrow D(\mathbf{q}, \tau) \dots$



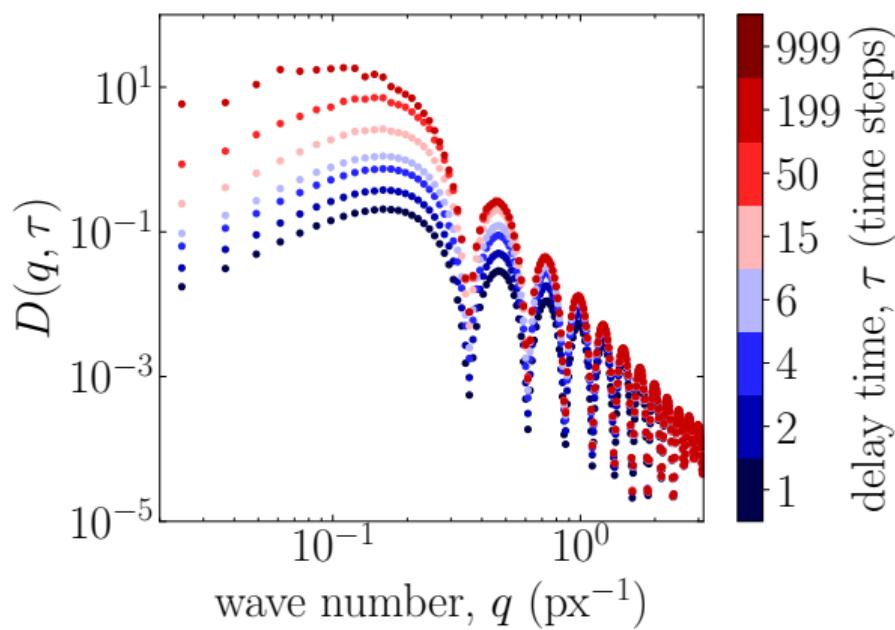
# The image structure function $D(q, \tau)$

$\tau = 199$  time steps

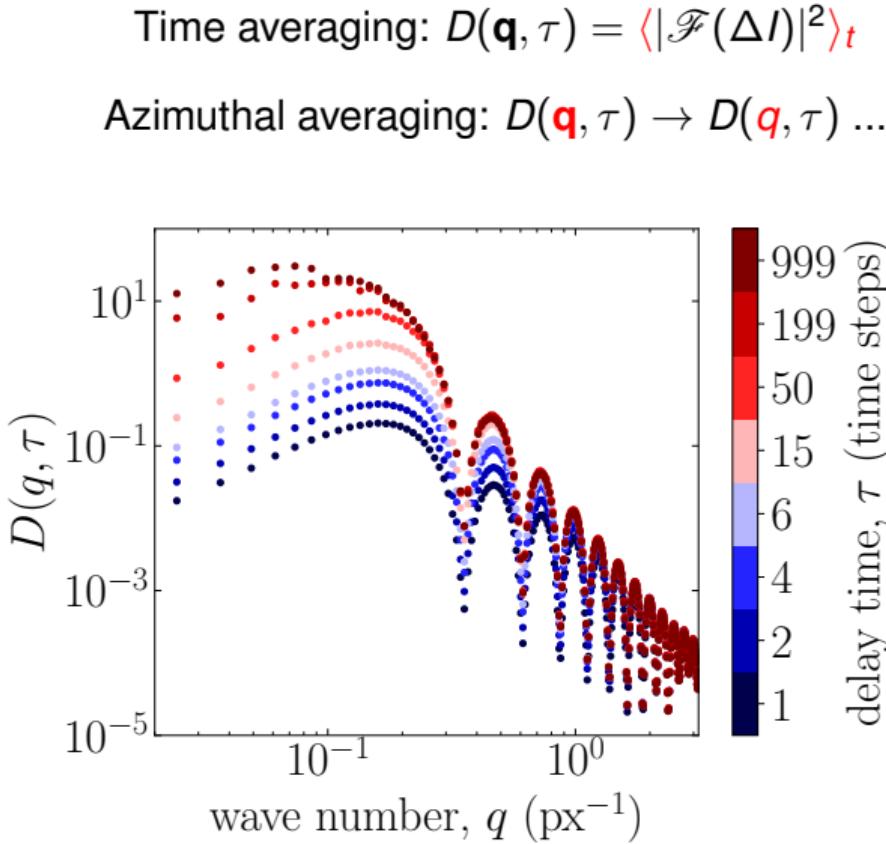
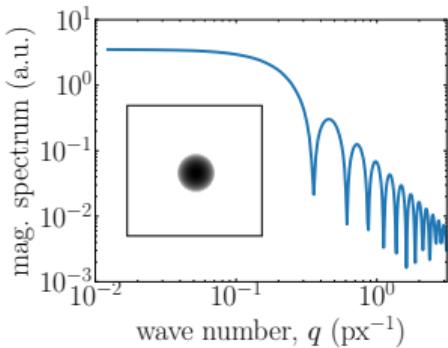
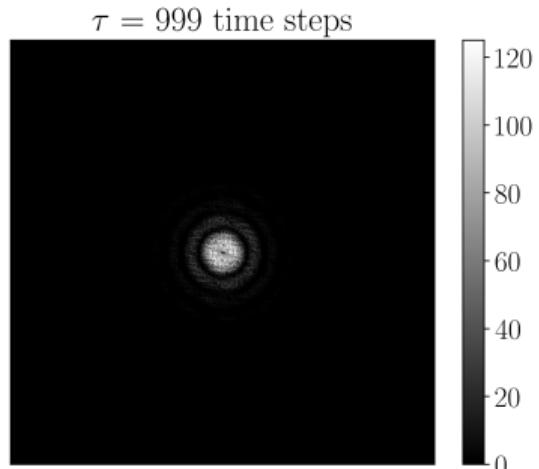


Time averaging:  $D(\mathbf{q}, \tau) = \langle |\mathcal{F}(\Delta I)|^2 \rangle_t$

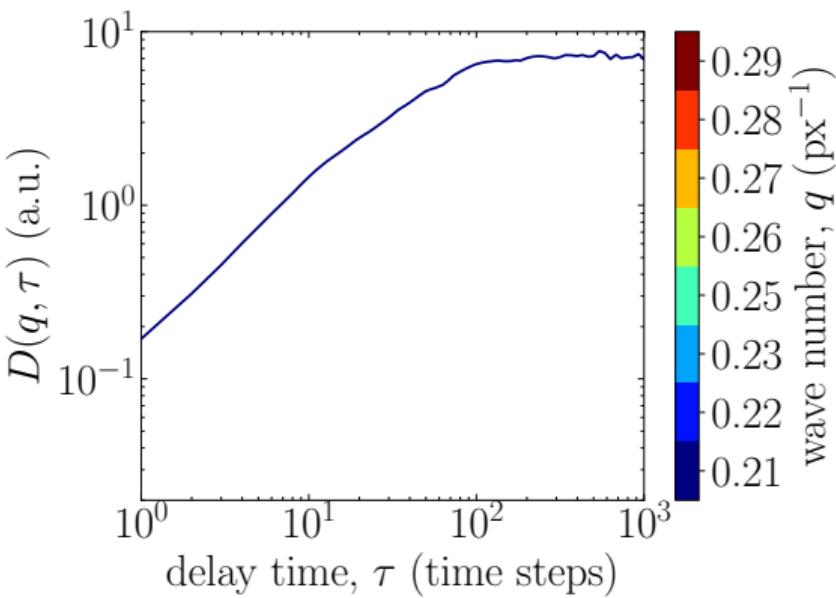
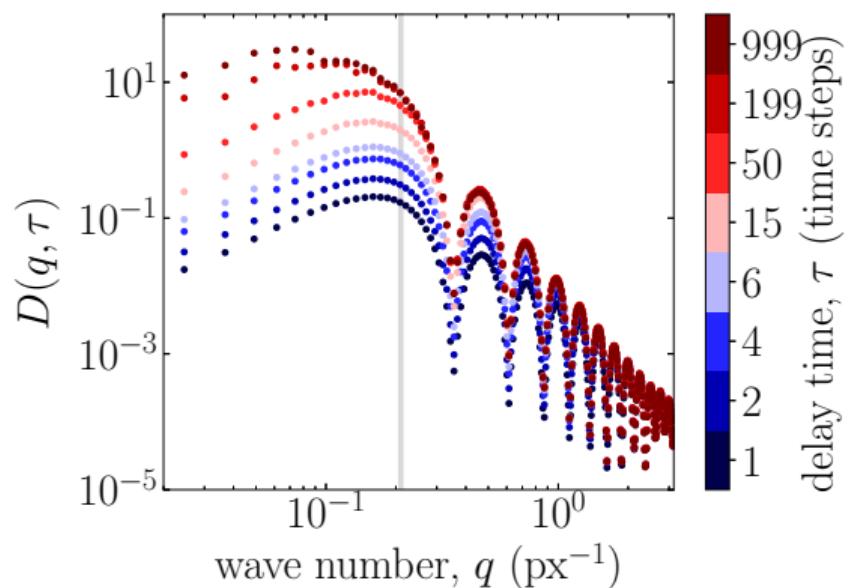
Azimuthal averaging:  $D(\mathbf{q}, \tau) \rightarrow D(\mathbf{q}, \tau) \dots$



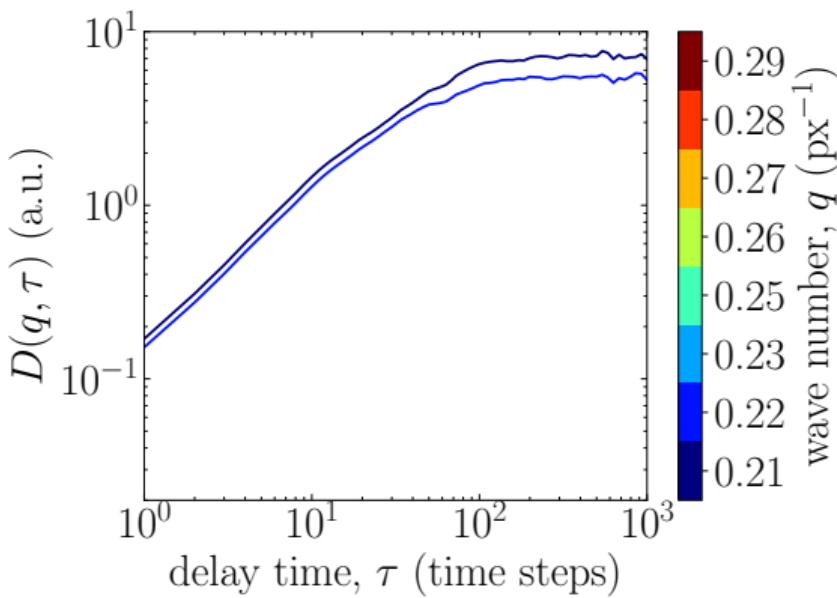
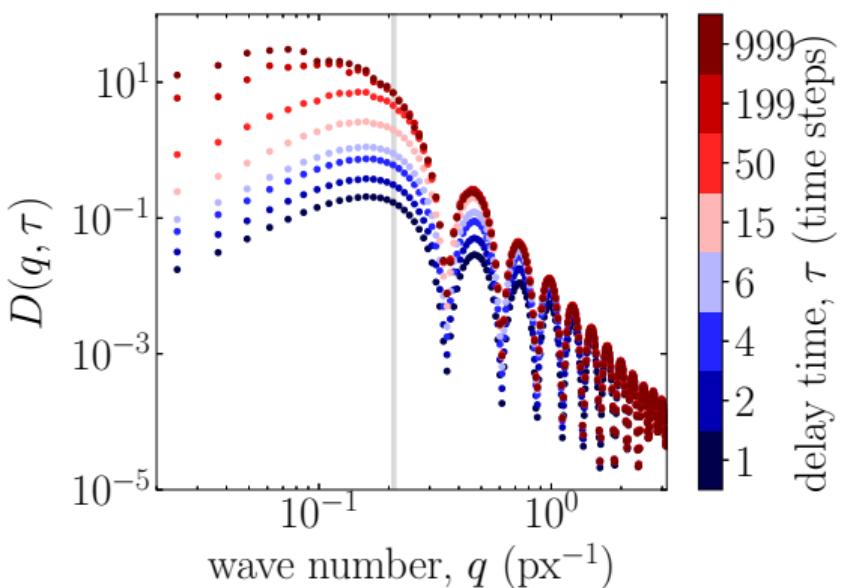
# The image structure function $D(q, \tau)$



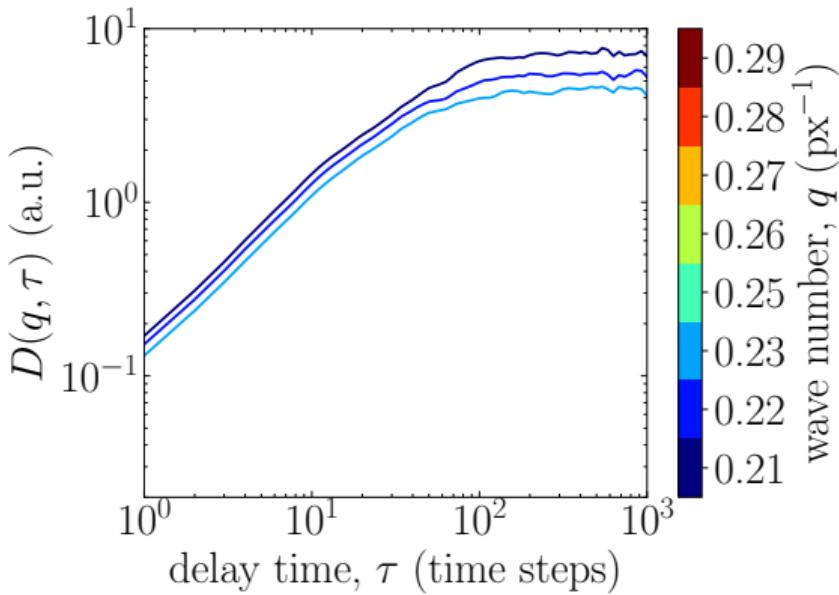
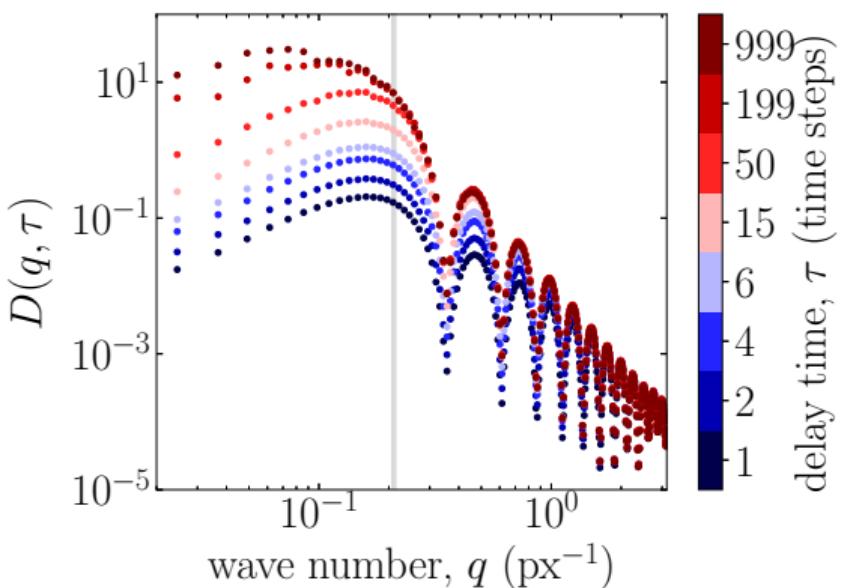
# The image structure function $D(q, \tau)$



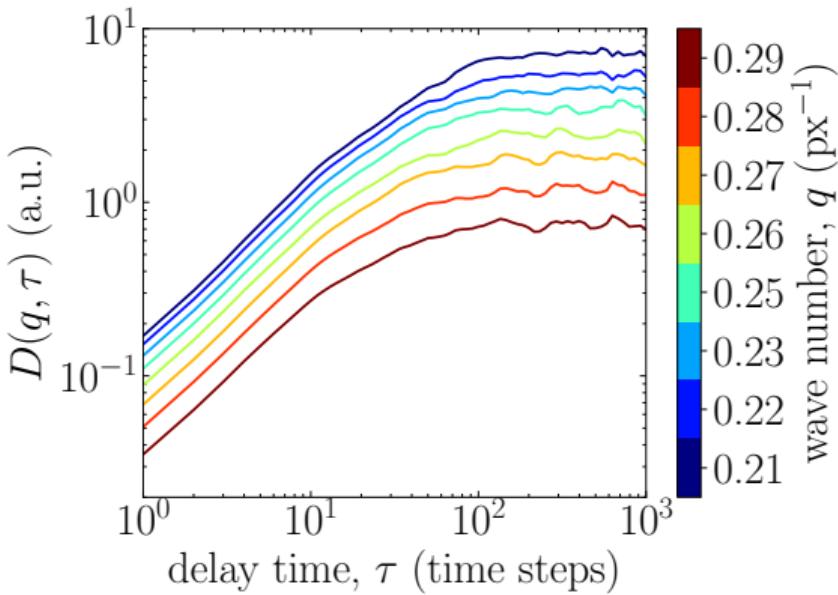
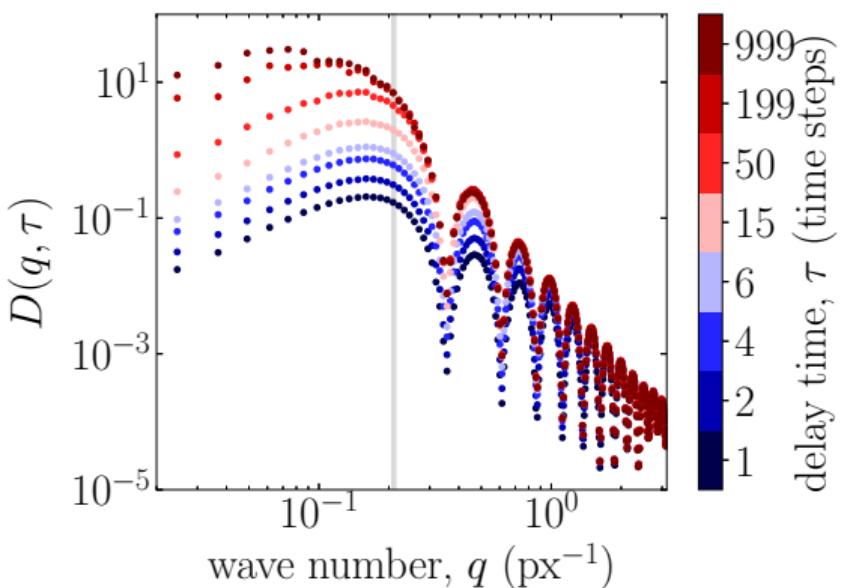
# The image structure function $D(q, \tau)$



# The image structure function $D(q, \tau)$



# The image structure function $D(q, \tau)$



## Linear space invariant imaging

$$D(q, \tau) = \left\langle |I(q, t + \tau) - I(q, t)|^2 \right\rangle_t$$

## Linear space invariant imaging

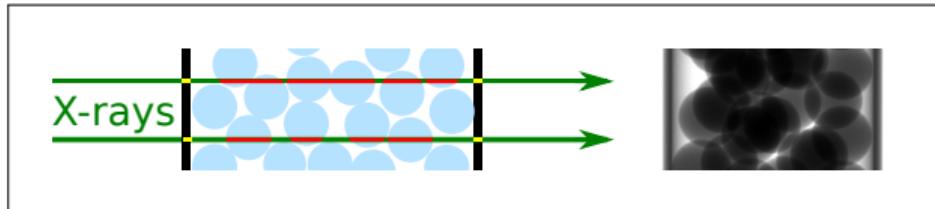
$$\begin{aligned} D(q, \tau) &= \left\langle |I(q, t + \tau) - I(q, t)|^2 \right\rangle_t \\ &= A(q) \left[ 1 - \frac{\left\langle I^*(q, t) I(q, t + \tau) \right\rangle_t}{\left\langle |I(q, t)|^2 \right\rangle_t} \right] + B(q) \end{aligned}$$

## Linear space invariant imaging

$$\begin{aligned} D(q, \tau) &= \left\langle |I(q, t + \tau) - I(q, t)|^2 \right\rangle_t \\ &= A(q) \underbrace{\left[ 1 - \frac{\langle I^*(q, t) I(q, t + \tau) \rangle_t}{\langle |I(q, t)|^2 \rangle_t} \right]}_{\text{Image correlation function}} + B(q) \end{aligned}$$

# Linear space invariant imaging

$$\begin{aligned} D(q, \tau) &= \left\langle |I(q, t + \tau) - I(q, t)|^2 \right\rangle_t \\ &= A(q) \underbrace{\left[ 1 - \frac{\langle I^*(q, t) I(q, t + \tau) \rangle_t}{\langle |I(q, t)|^2 \rangle_t} \right]}_{\text{Image correlation function}} + B(q) \end{aligned}$$



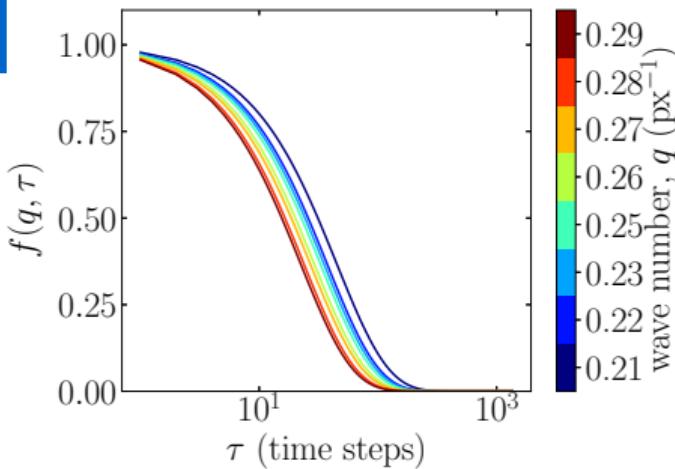
Intermediate scattering function

$$f(q, \tau) = \frac{\langle \rho^*(q, t) \rho(q, t + \tau) \rangle_t}{\langle |\rho(q, t)|^2 \rangle_t}$$

## Intermediate scattering function $f(q, \tau)$

**Brownian motion:**  $f(q, \tau) = \exp(-q^2\tau/\tau_D)$

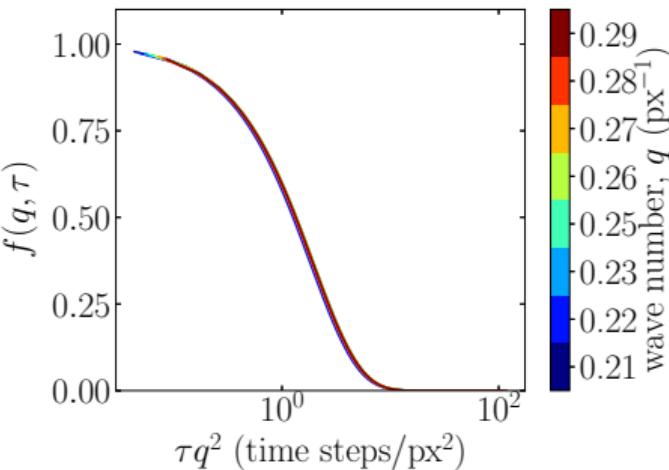
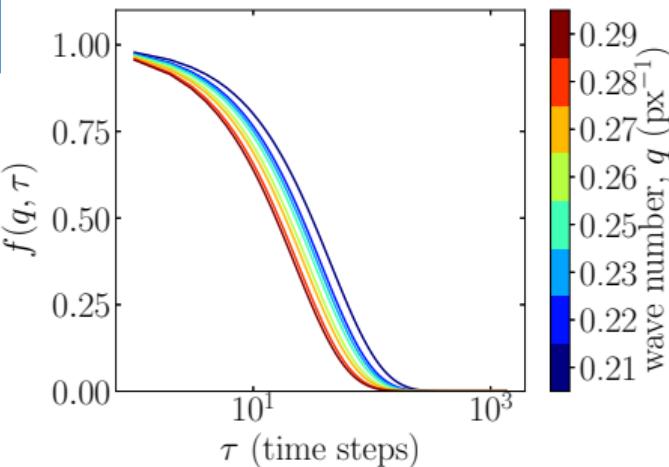
- Non-interacting particles
- Gaussian velocity distribution



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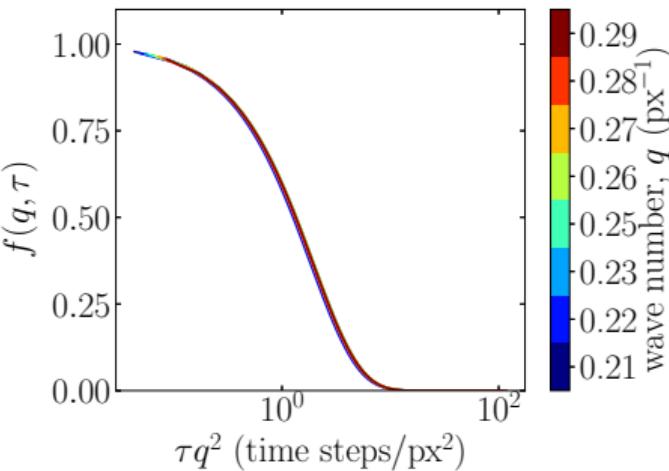
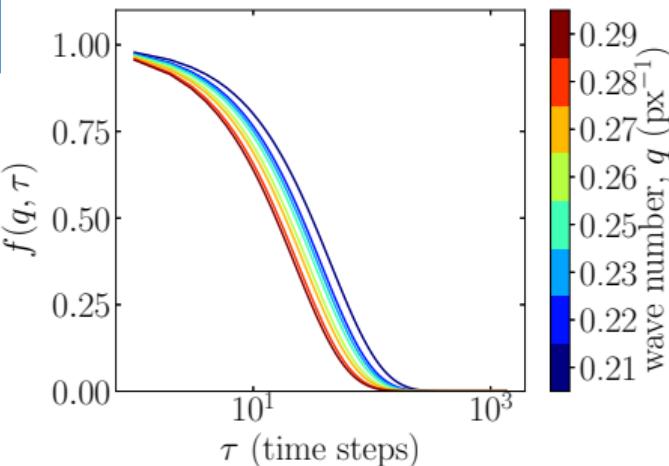
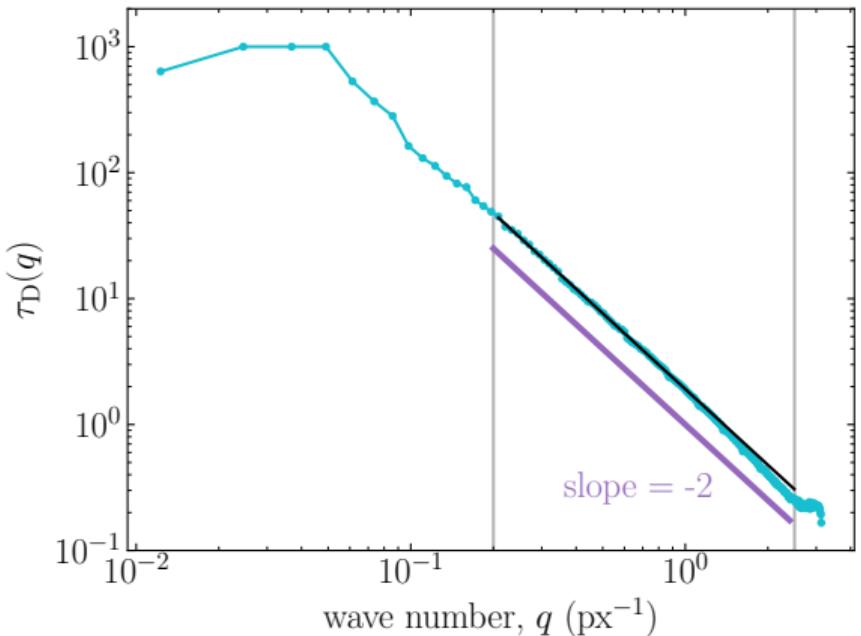
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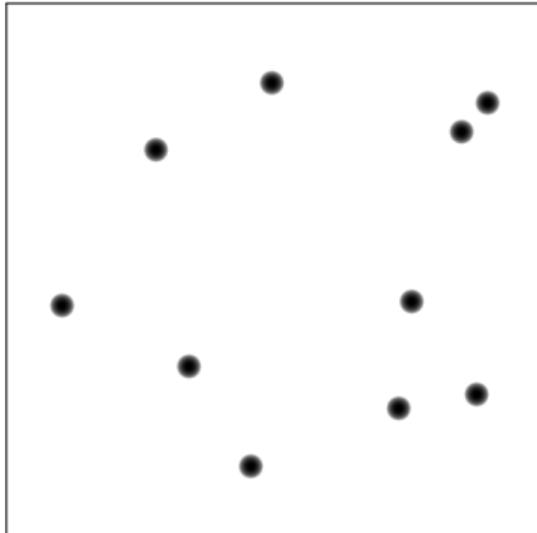
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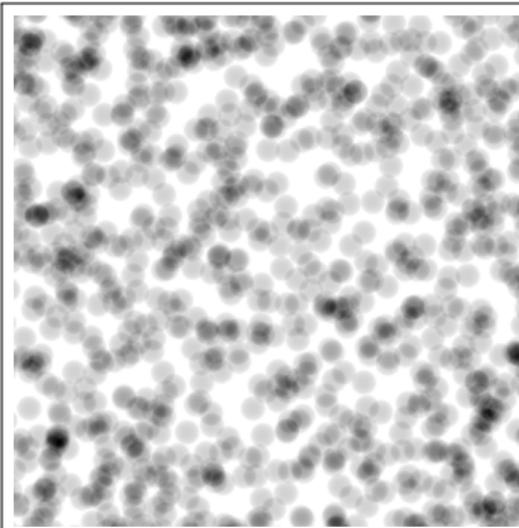


# Accuracy of X-DFA: Varying the number of particles

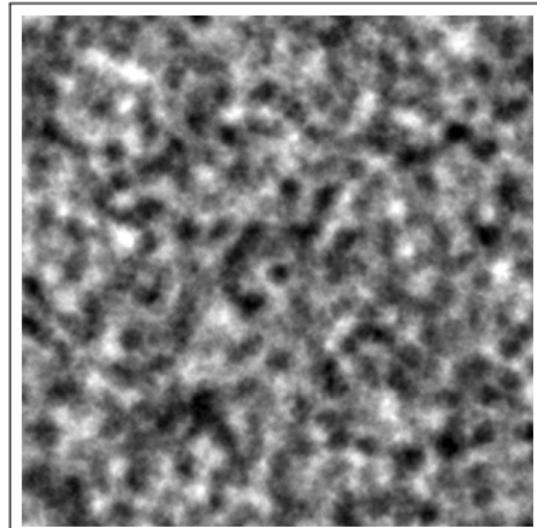
10 particles



1000 particles



100 000 particles



Deviation from the simulation input:

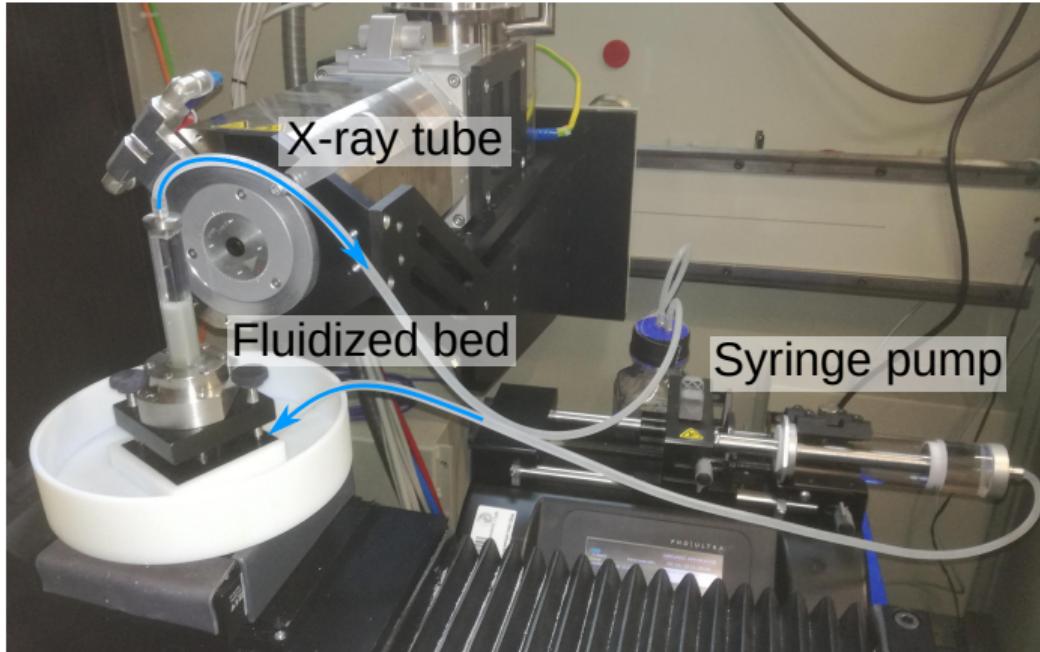
6%

2%

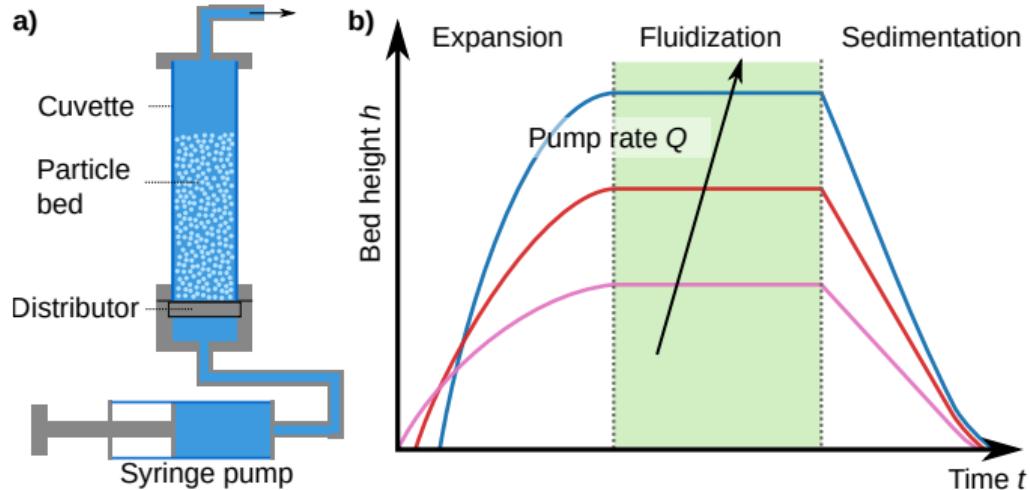
2%

PIV off by  $\approx 650\%$

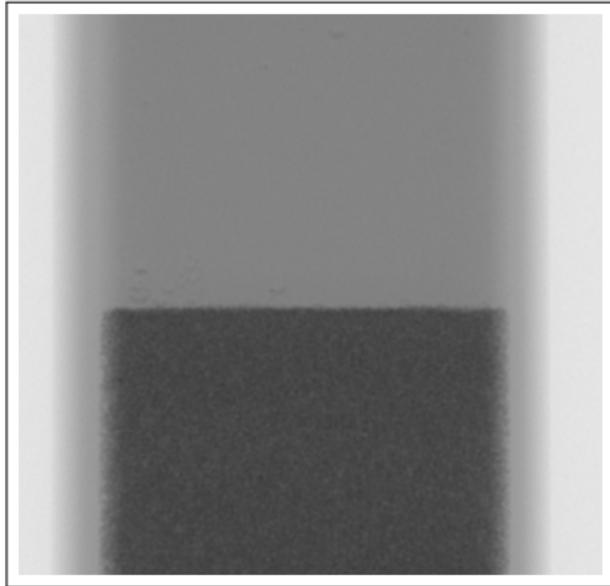
# Experimental validation of X-DFA: A suspension of sedimenting particles



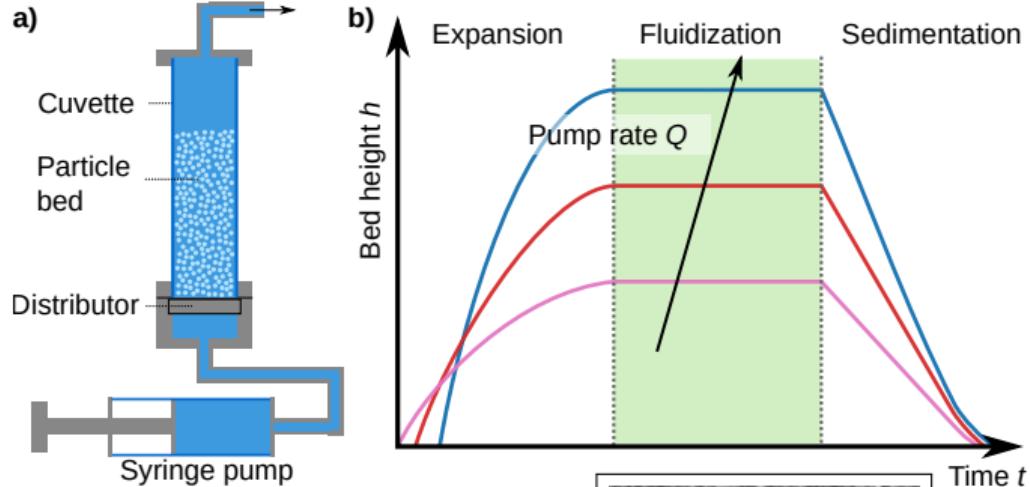
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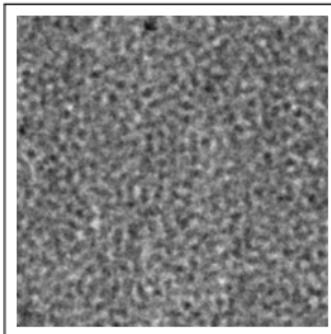
X-ray radiography



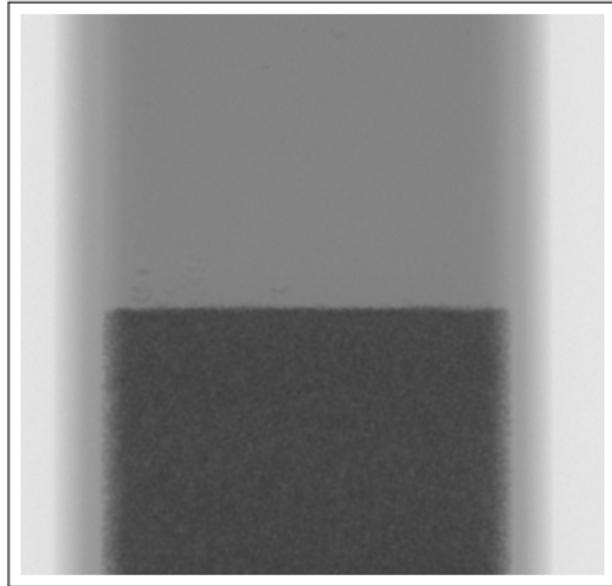
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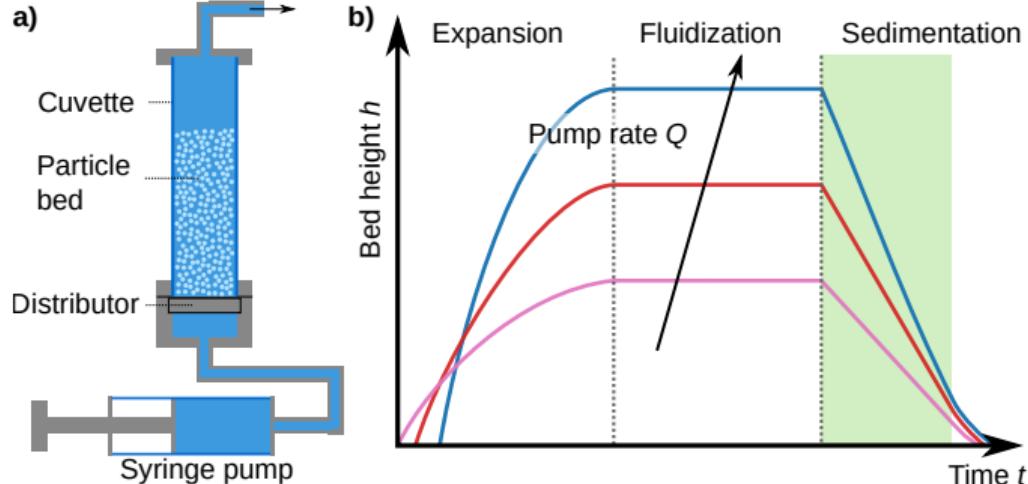
No reliable reference velocity!



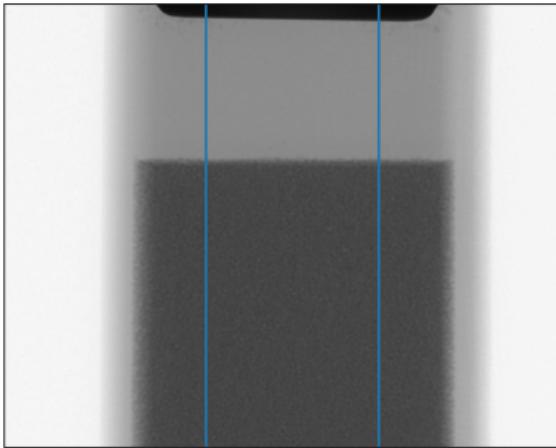
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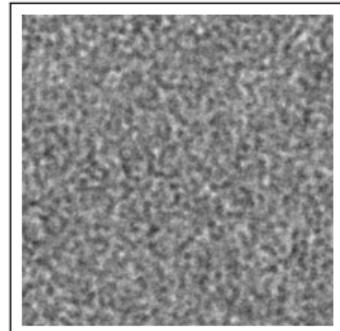
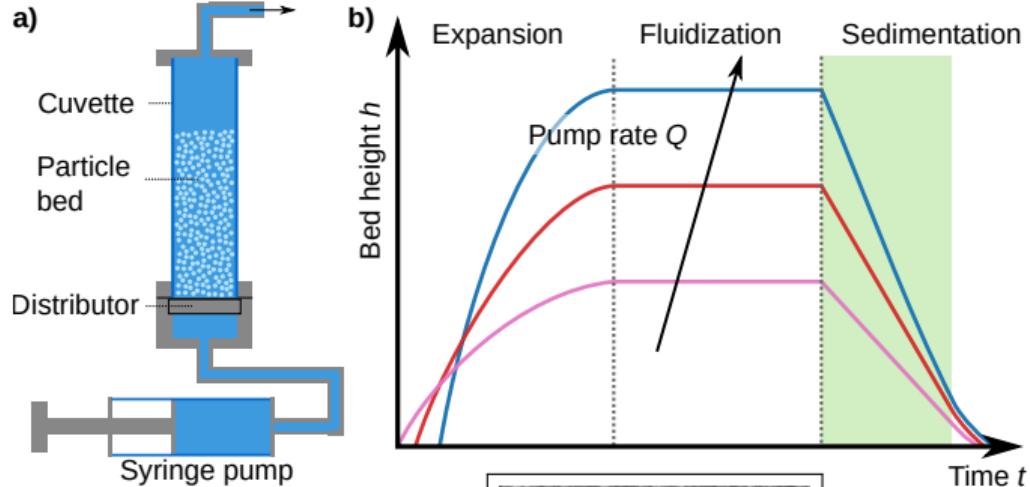
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X-ray radiography

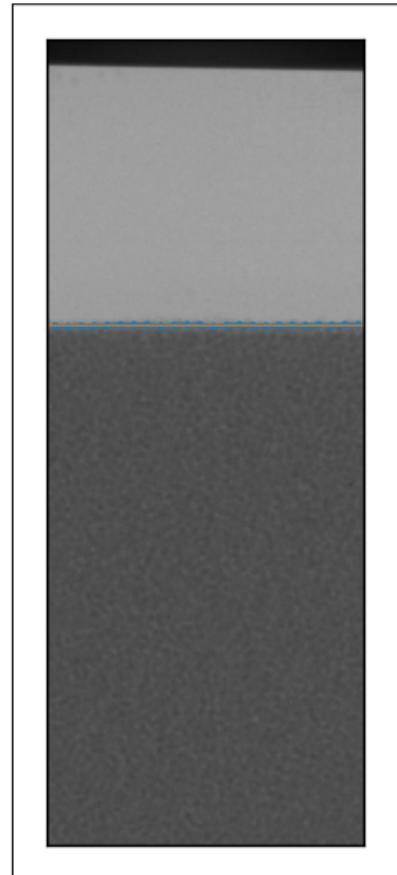


# Experimental validation of X-DFA: A suspension of sedimenting particles



Comparison of  
 $\langle v \rangle_{\text{dfa}}$  and  $\langle v \rangle_{\text{front}}$

X-ray radiography



# X-DFA for a suspension of sedimenting particles<sup>(1)</sup>

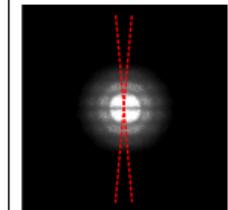


Image structure function:  
 $D(\mathbf{q}, \tau) = \langle |\mathcal{F}[\Delta I(\mathbf{r}, t, \tau)]|^2 \rangle_t$

(1) In collaboration with Manuel Escobedo, University of Düsseldorf

# X-DFA for a suspension of sedimenting particles<sup>(1)</sup>

Intermediate scattering function<sup>(2)</sup>:

$$f(q, \tau) = \cos(q\langle v_s \rangle \tau) \exp\left(-\frac{1}{2}q^2 \delta v^2 \tau^2\right)$$

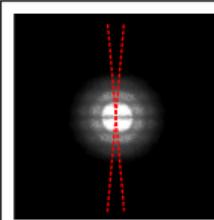
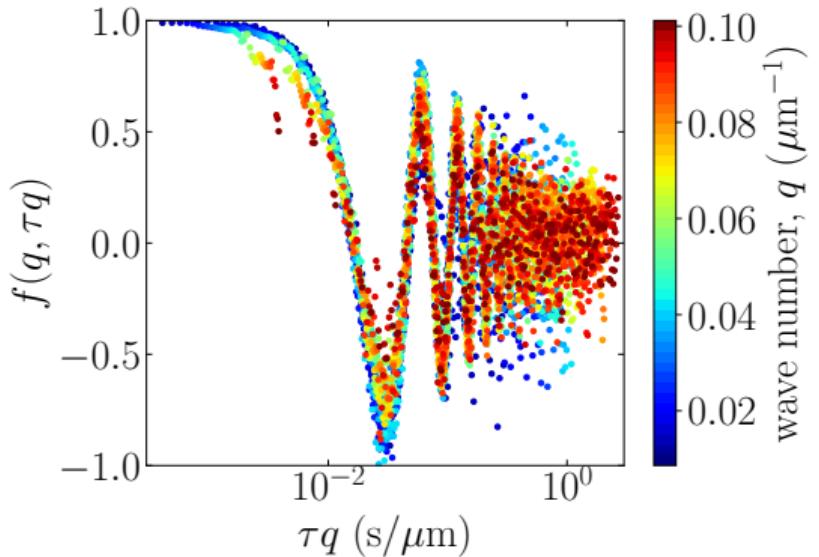


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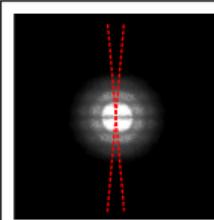
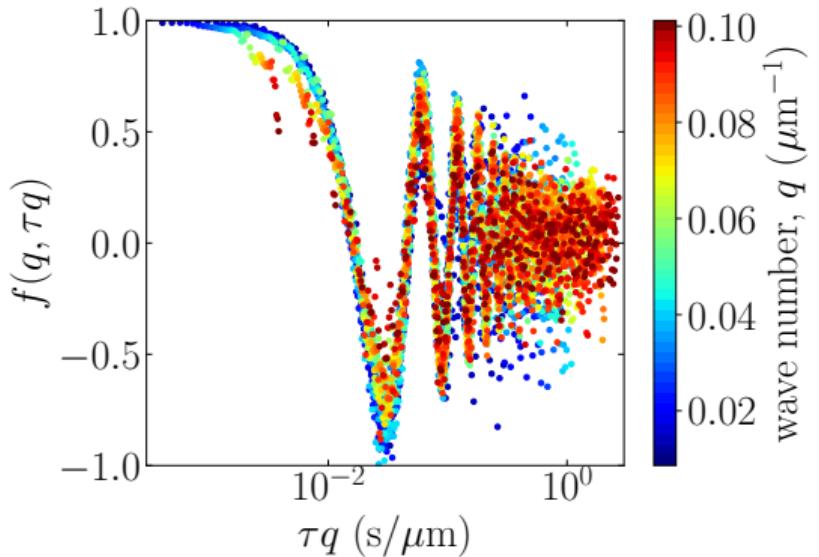


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$$\langle v_s \rangle = q \tau_\nu \quad \delta v = q \tau_{\delta\nu}$$

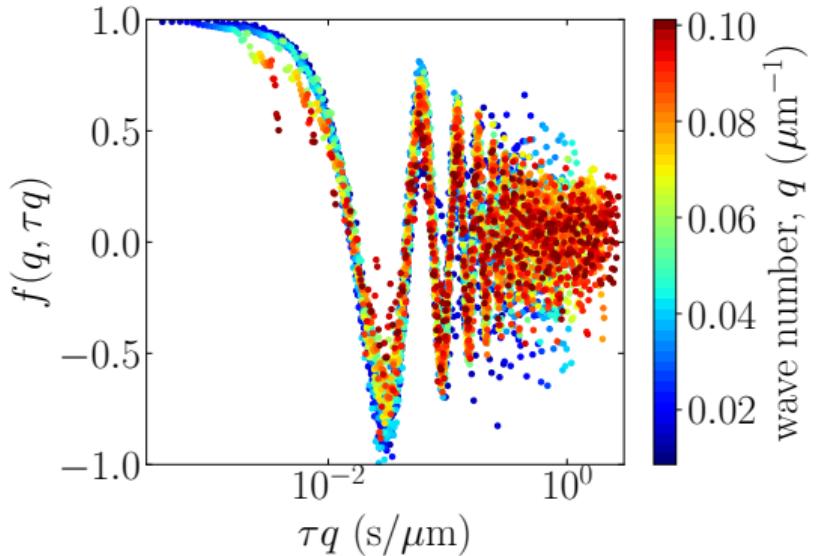
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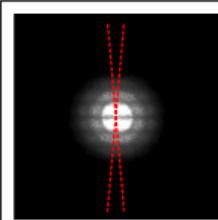
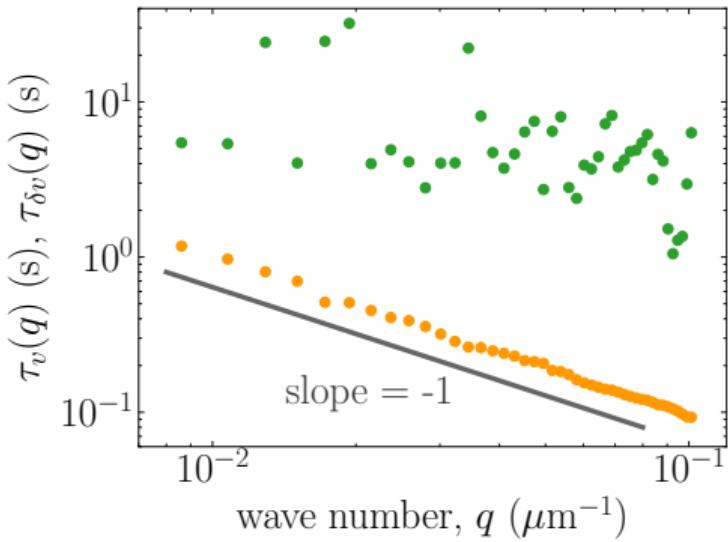


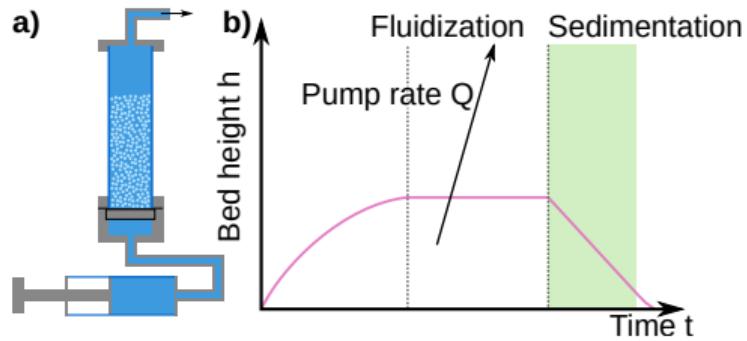
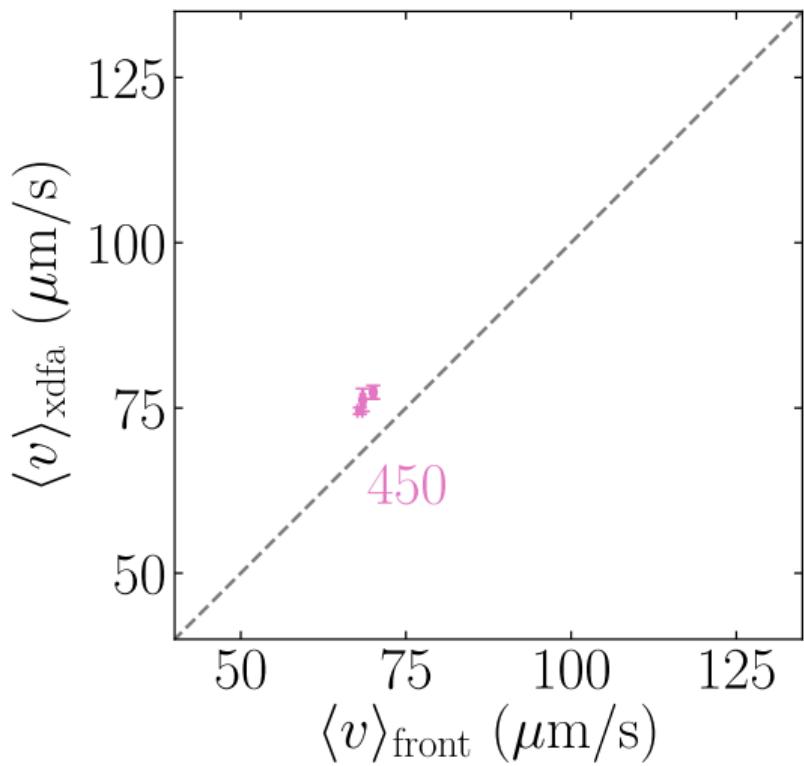
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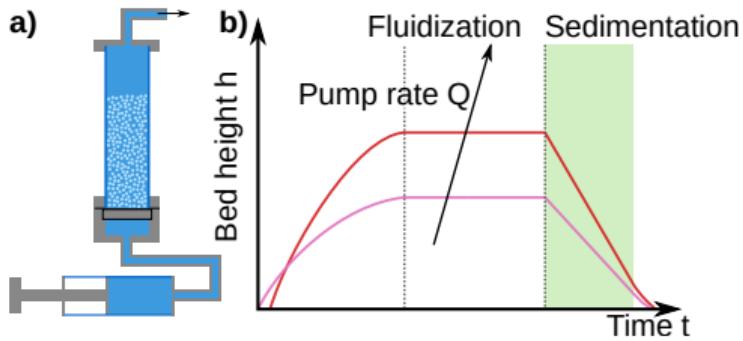
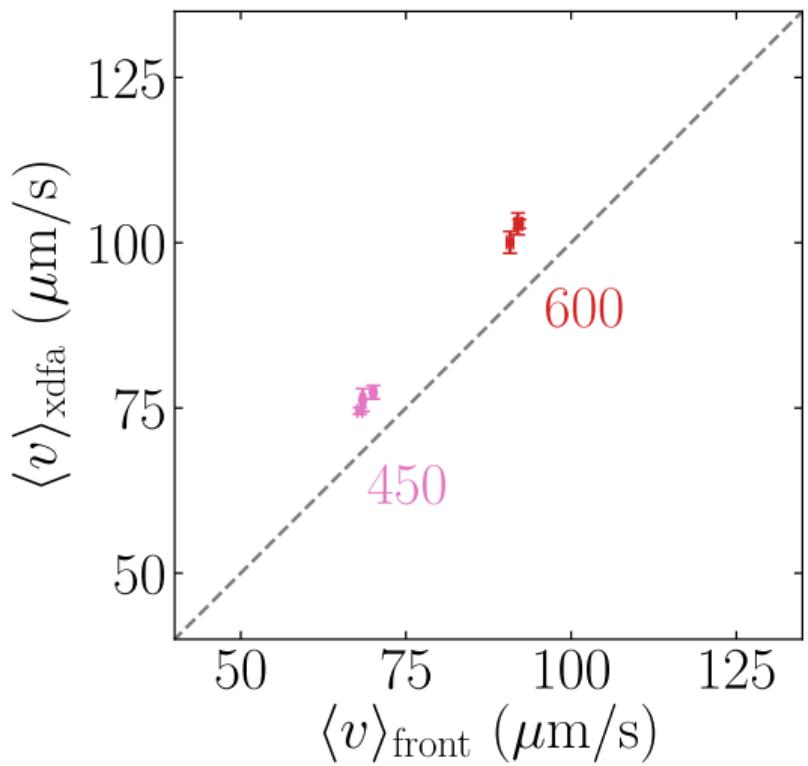


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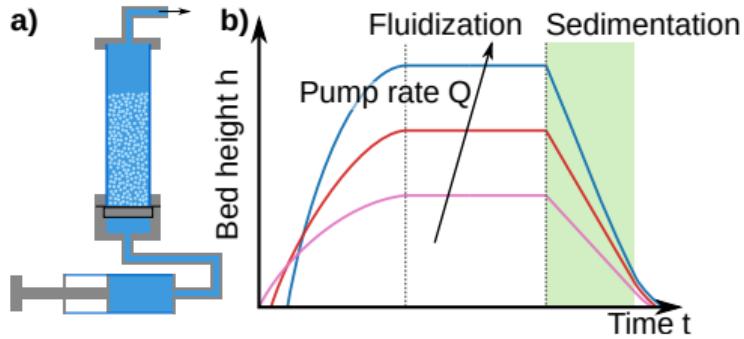
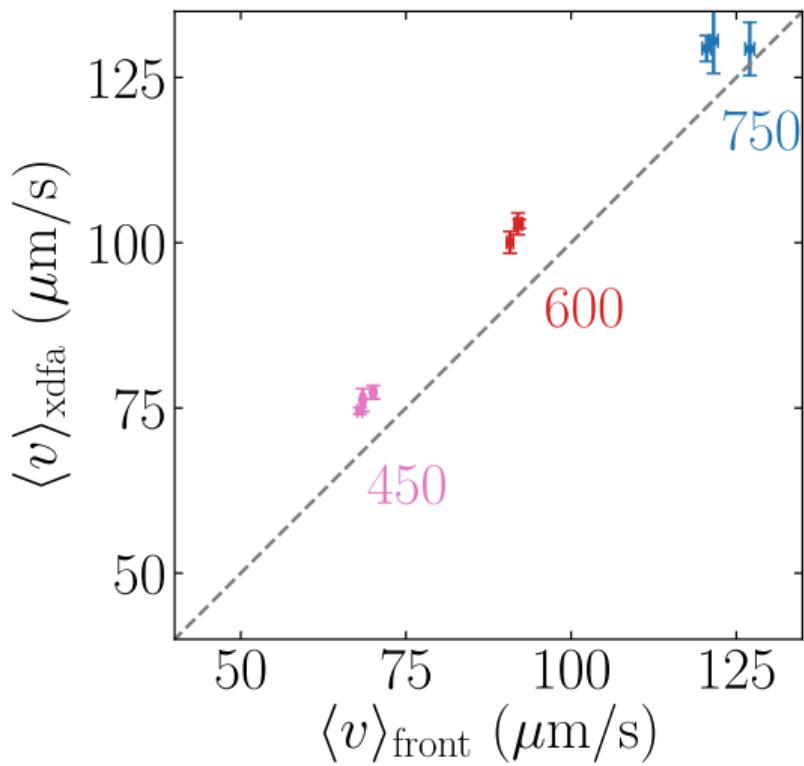
# Front tracking vs. X-DFA



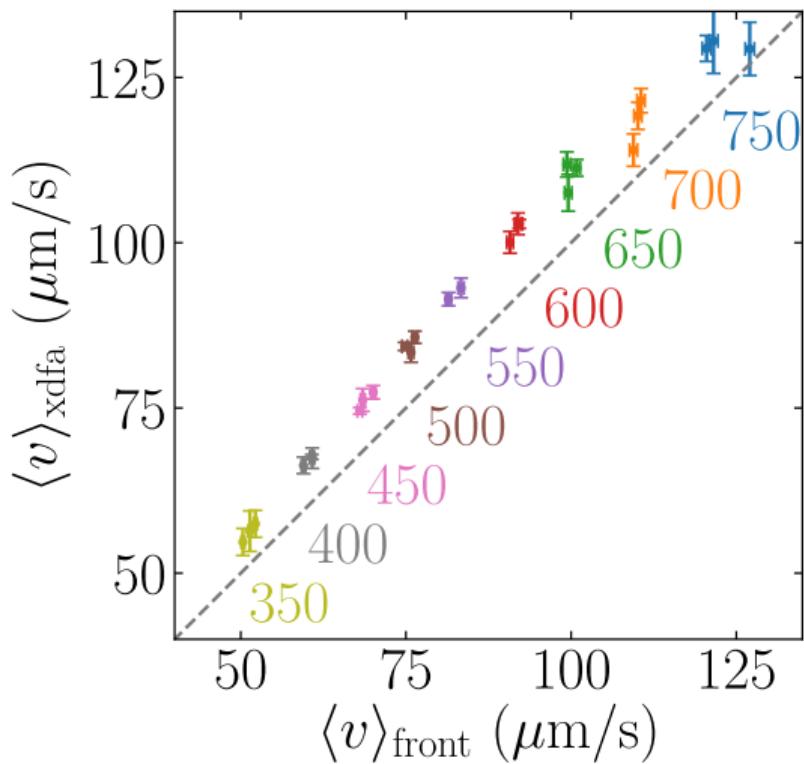
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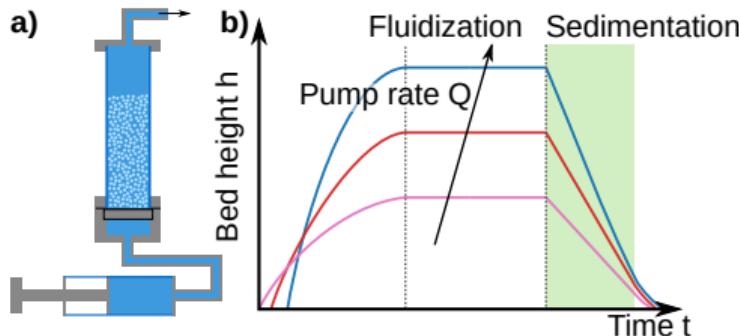
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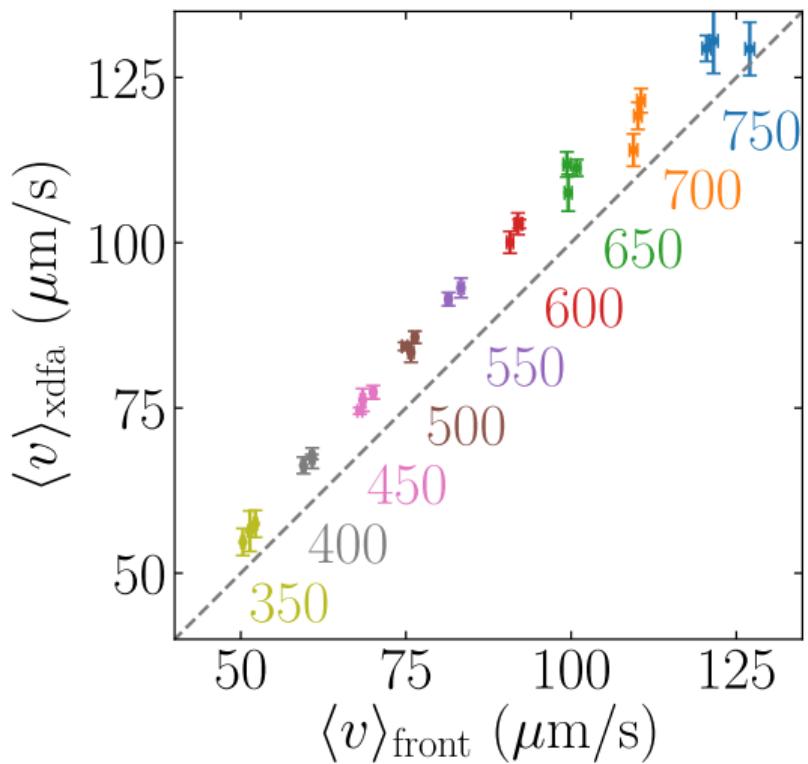
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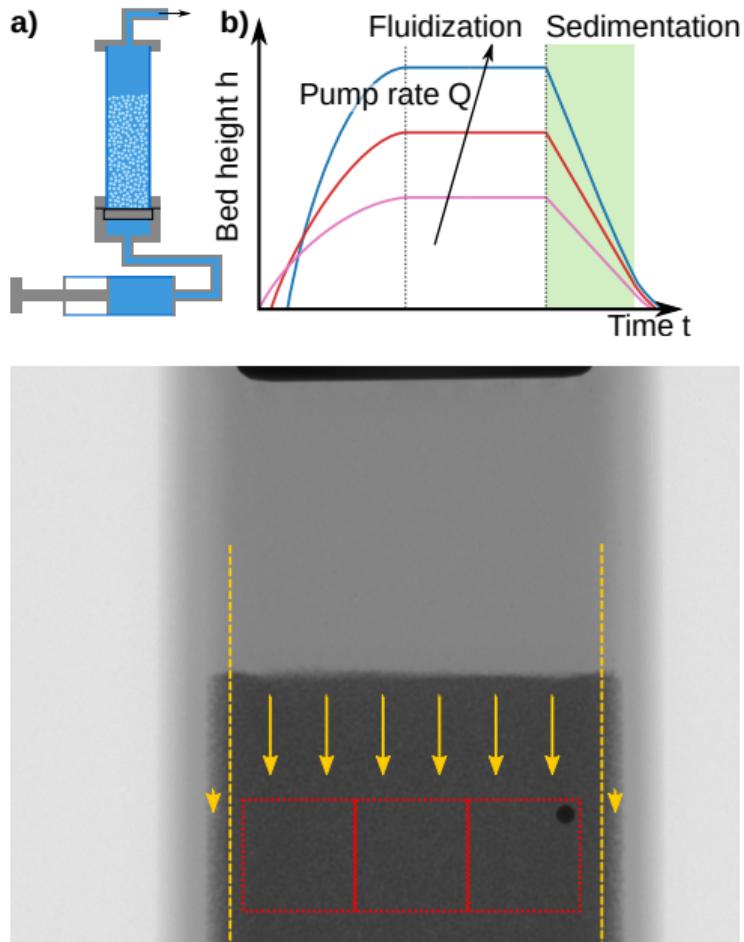
$\langle v \rangle_{\text{xdfa}} > \langle v \rangle_{\text{front}}$  by 9.4%



# Front tracking vs. X-DFA



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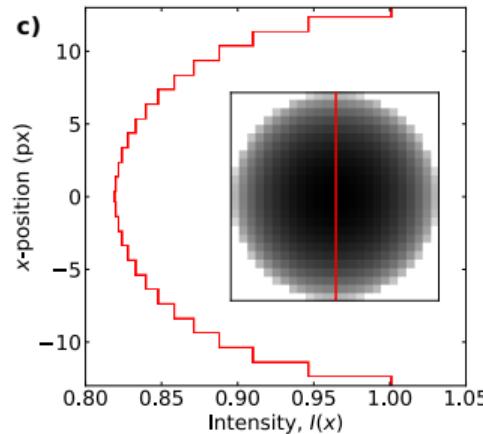
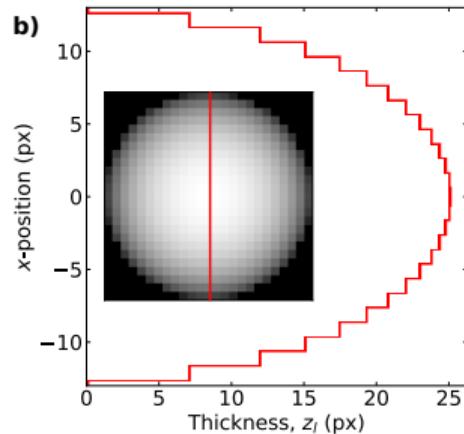
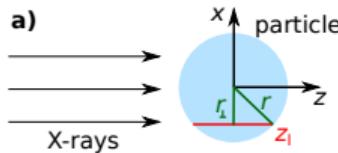
# Conclusion

Thank you for your attention!



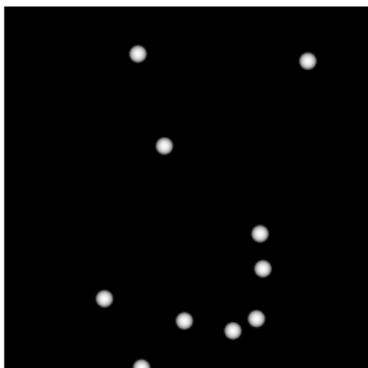
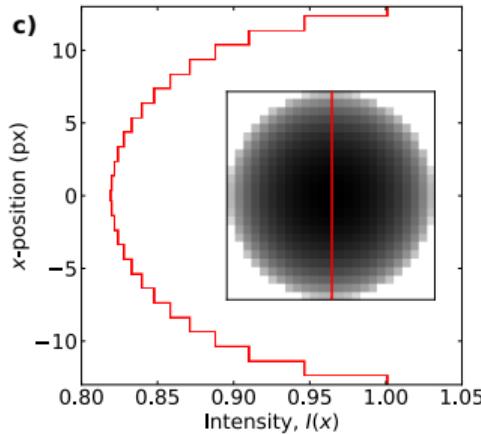
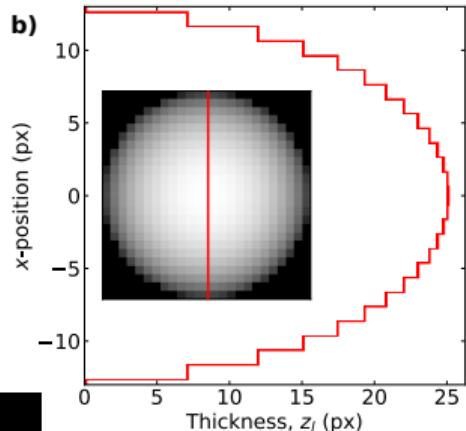
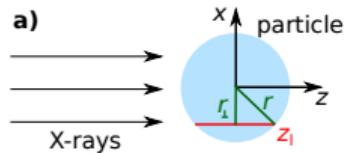
# Backup slides

# Synthetic radiograms

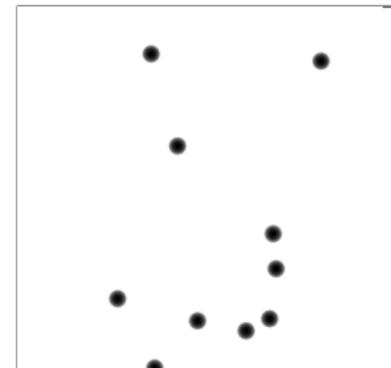


Beer-Lambert  
 $I(z_I) = I_0 \exp(-\mu z)$

# Synthetic radiograms



Beer-Lambert  
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## Linear space invariant imaging

Image correlation function

$$g(\mathbf{q}, \tau) = \frac{\langle I^*(\mathbf{q}, t)I(\mathbf{q}, t + \tau) \rangle_t}{\langle |I(\mathbf{q}, t)|^2 \rangle_t}$$

# Linear space invariant imaging

Image correlation function

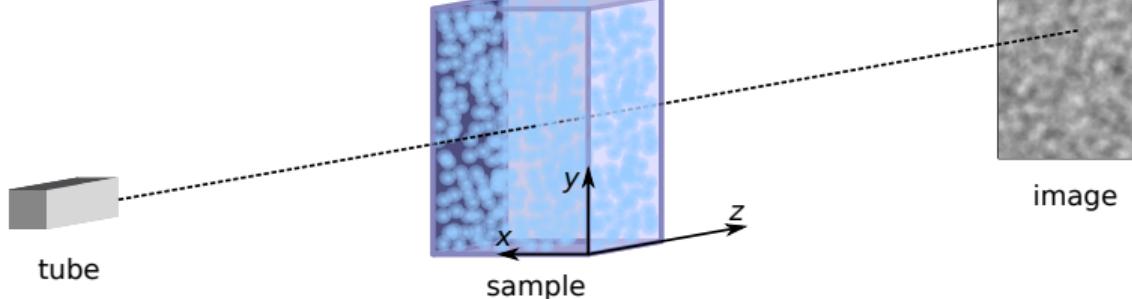
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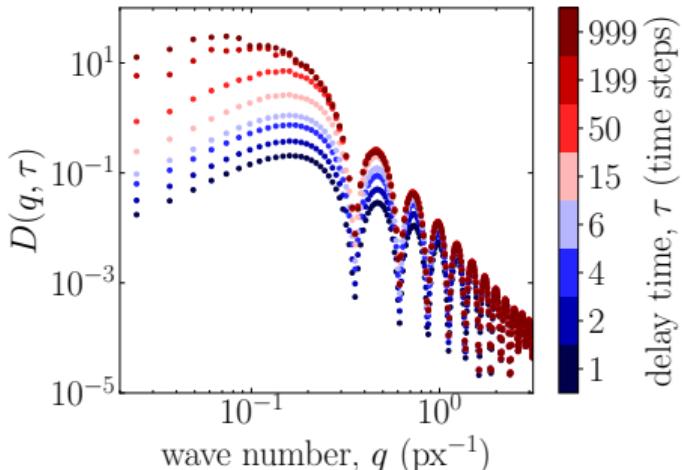
Intermediate scattering function

$$f(\mathbf{q}, \tau) = \frac{\langle \rho^*(\mathbf{q}, t)\rho(\mathbf{q}, t + \tau) \rangle_t}{\langle |\rho(\mathbf{q}, t)|^2 \rangle_t}$$

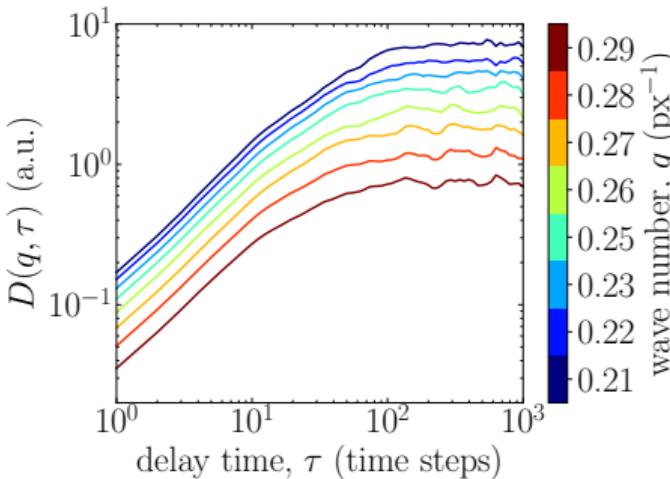
Linear space-invariant imaging:

$$I(\mathbf{r}, t) = I_0 + \int d\mathbf{r}' dz' T(\mathbf{r} - \mathbf{r}', -z') c(\mathbf{r}', z', t)$$





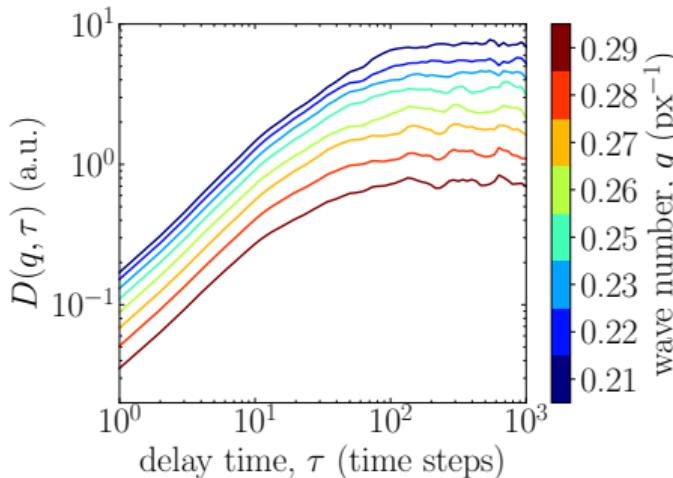
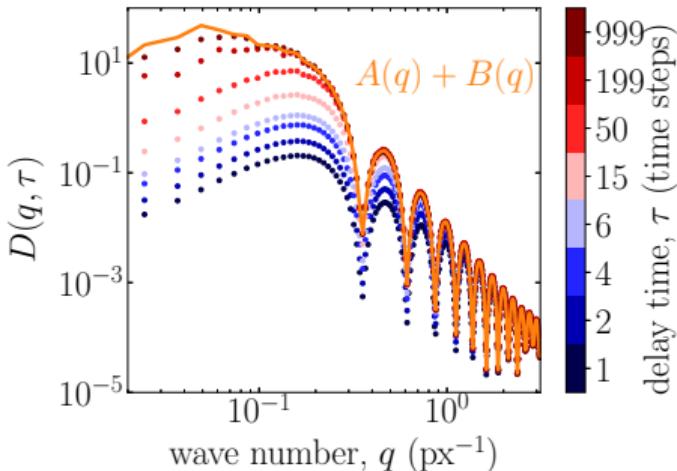
$$\begin{aligned}
 D(q, \tau) &= \left\langle |I(q, t + \tau) - I(q, t)|^2 \right\rangle_t \\
 &= A(q) \underbrace{\left[ 1 - \frac{\langle I^*(q, t) I(q, t + \tau) \rangle_t}{\langle |I(q, t)|^2 \rangle_t} \right]}_{\text{Image correlation function}} + B(q)
 \end{aligned}$$



Linear space invariant imaging

$$f(q, \tau) = \frac{\langle \rho^*(q, t) \rho(q, t + \tau) \rangle_t}{\langle |\rho(q, t)|^2 \rangle_t}$$

Intermediate scattering function



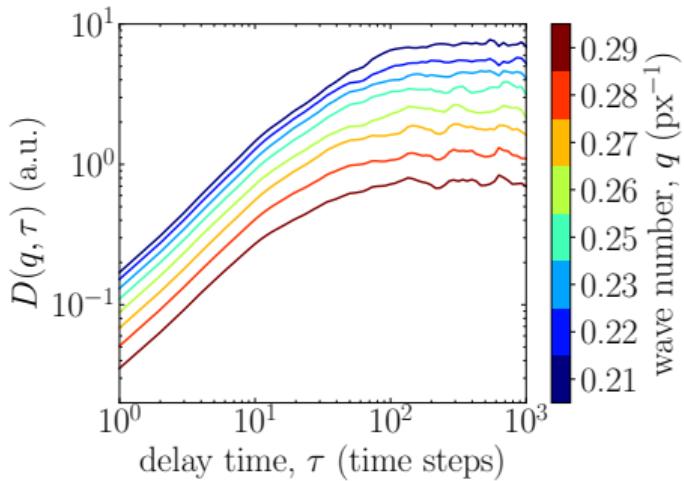
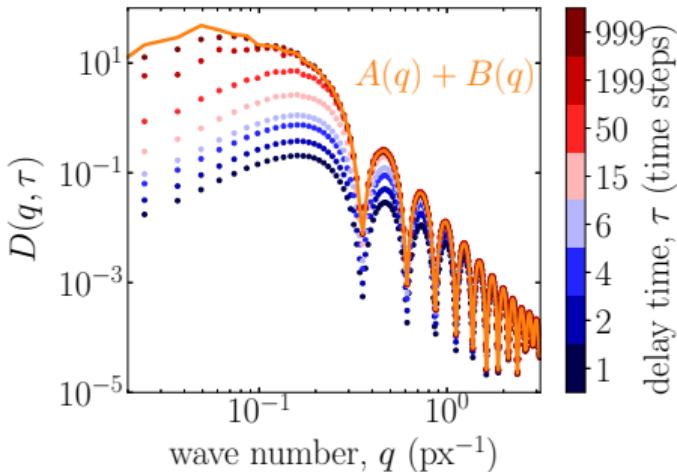
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 \end{aligned}$$

- $D(q, \tau \rightarrow 0) = B(q) = 0$
- $D(q, \tau \rightarrow \infty) = A(q) + B(q)$

### Linear space invariant imaging

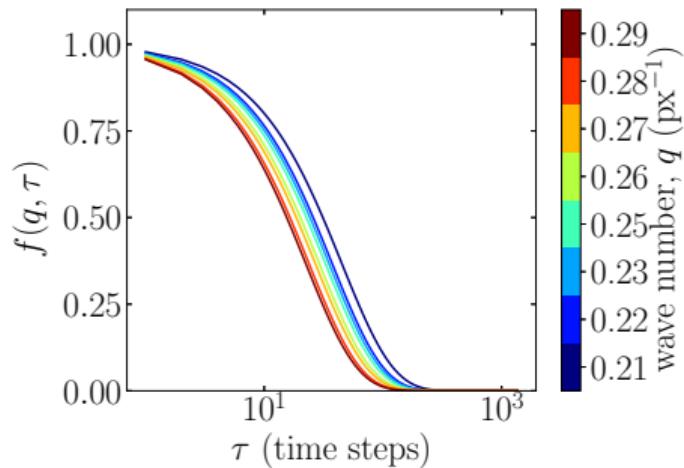
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Intermediate scattering function

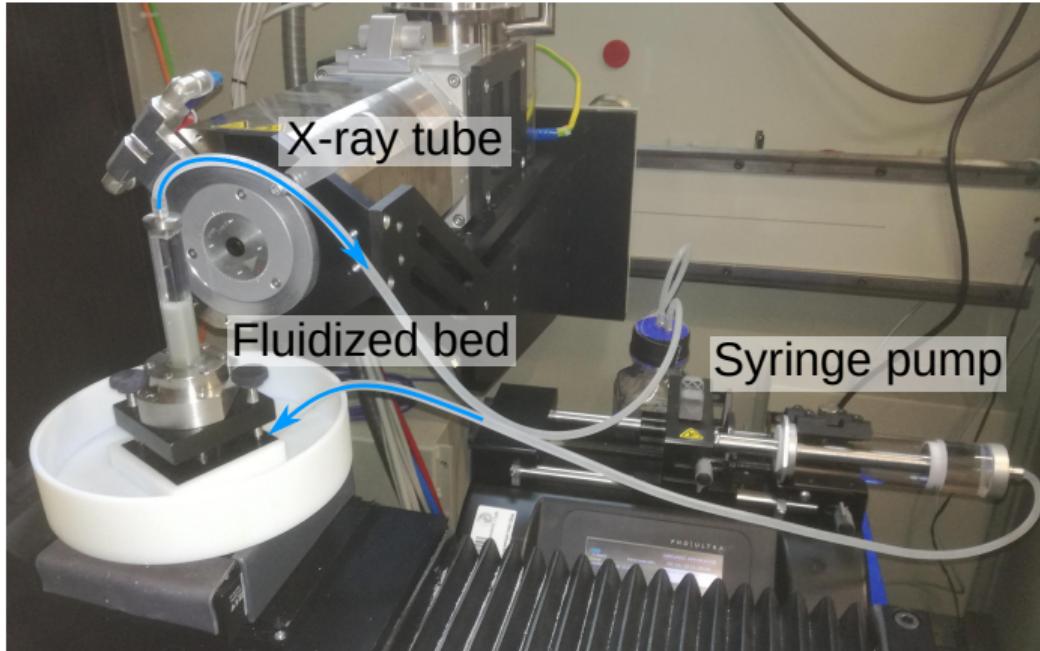


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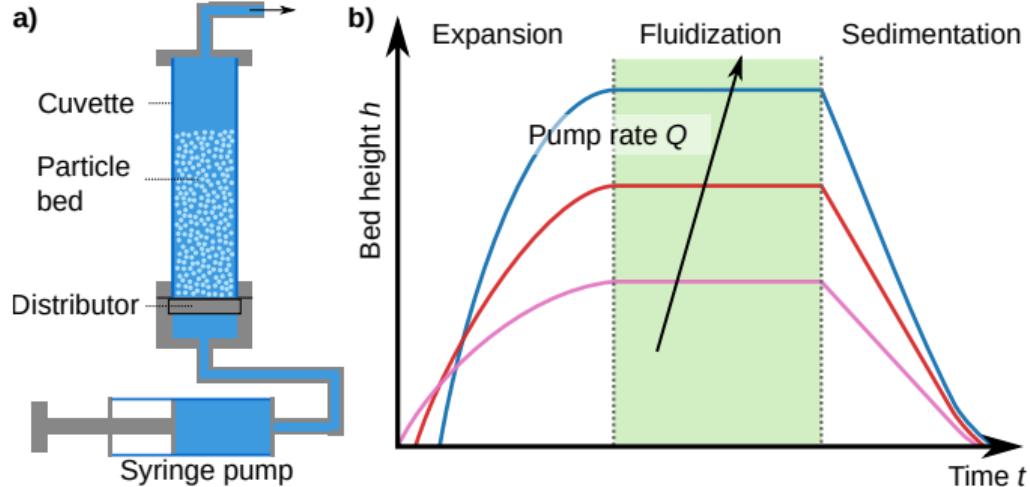
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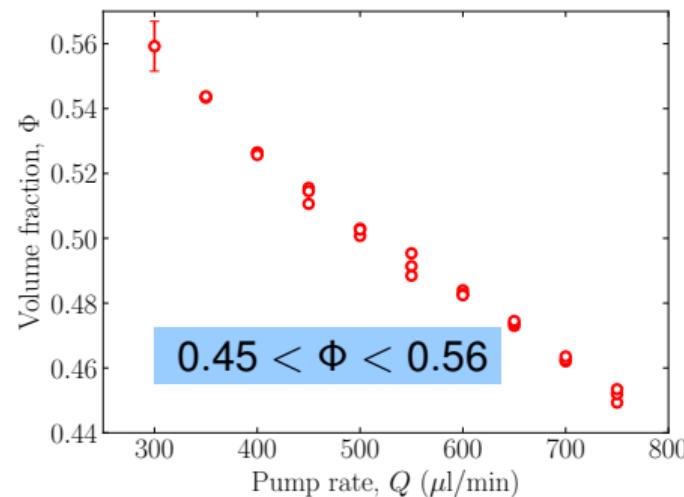
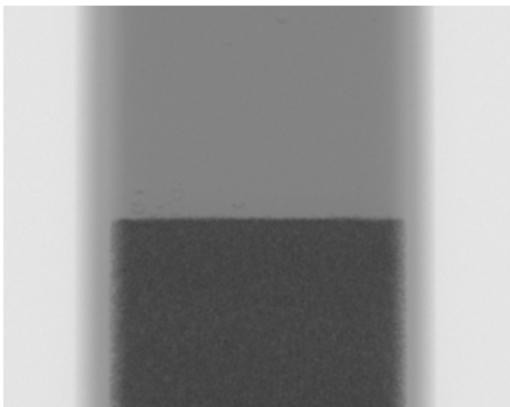
# Experimental validation of X-DFA: A suspension of sedimenting particles



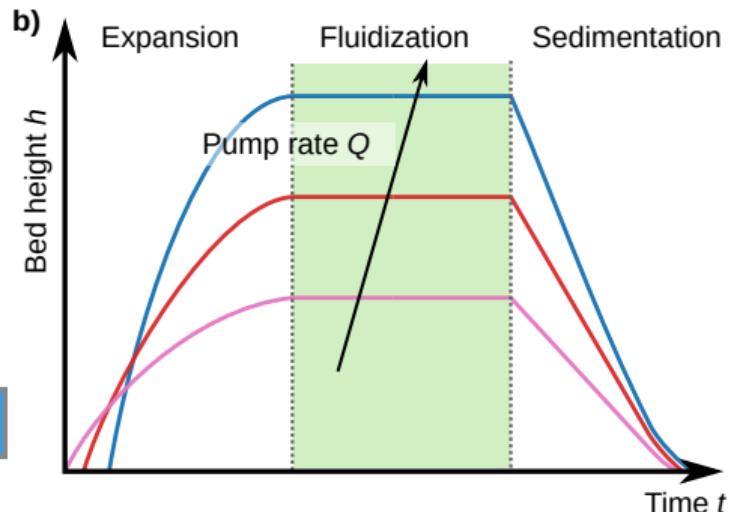
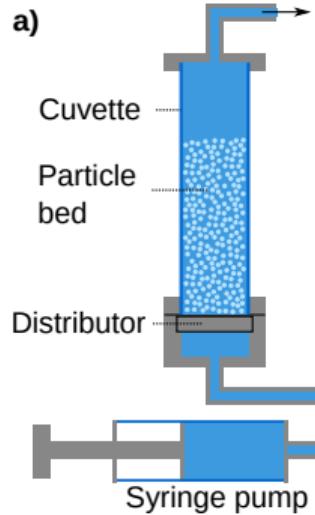
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X-ray radiography

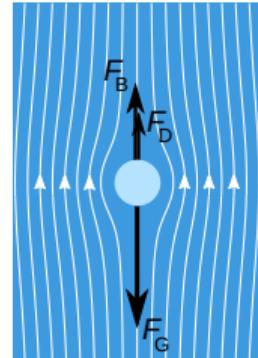


# Liquid fluidized bed: Richardson-Zaki law



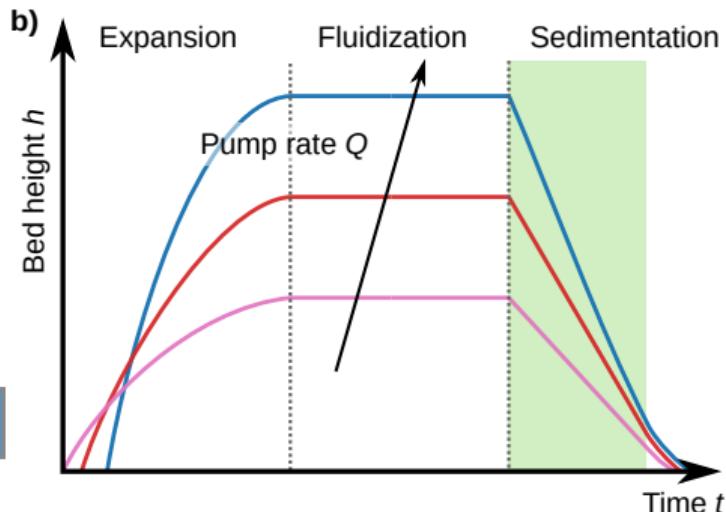
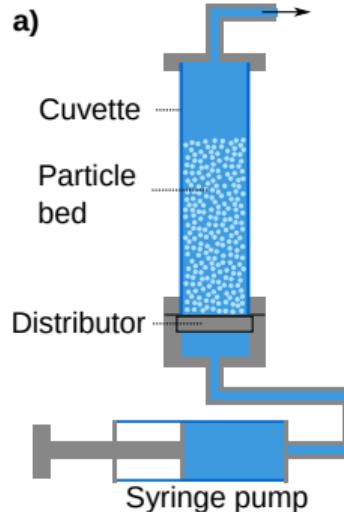
Gravitation      Buoyancy      Drag

$$F_G = F_B + F_D$$

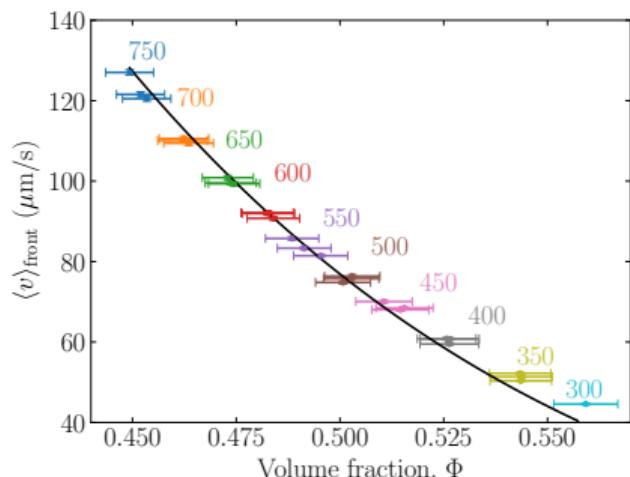
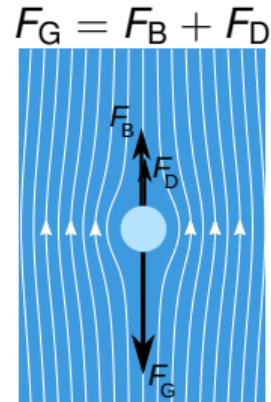


$$\frac{\langle v \rangle_{\text{fluid}}}{v_{\text{Stokes}}} = (1 - \phi)^n$$

# Liquid fluidized bed: Richardson-Zaki law

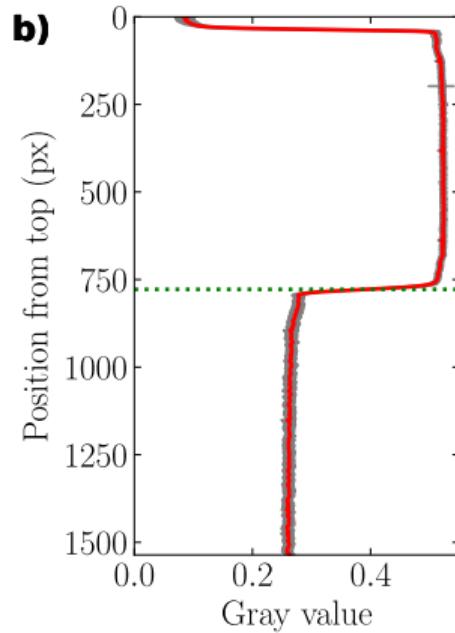
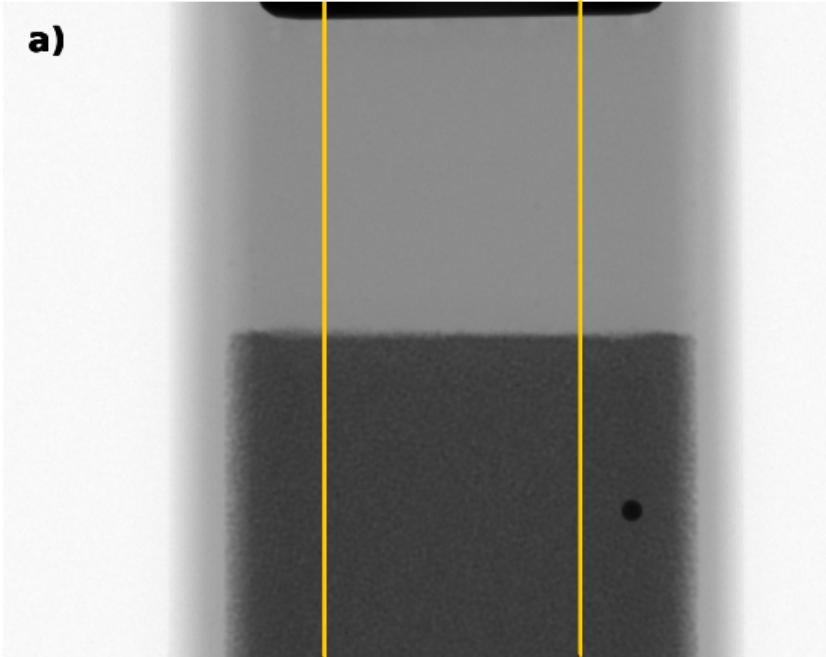


Gravitation      Buoyancy      Drag

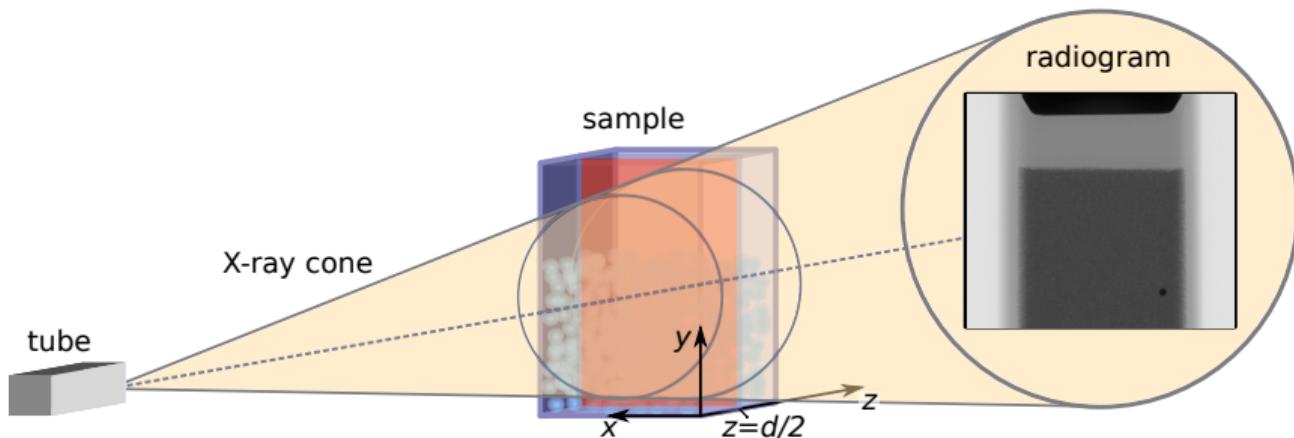
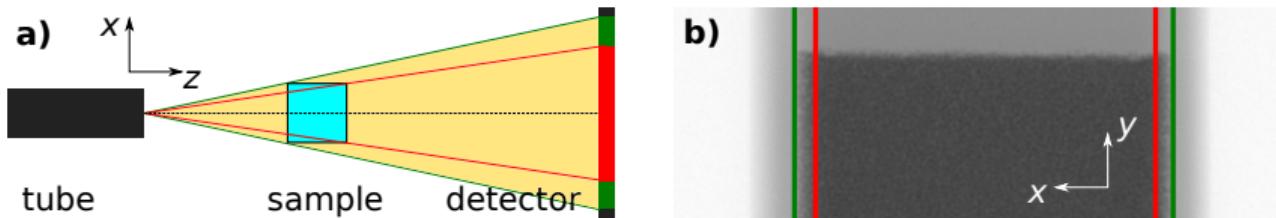


$$\frac{\langle v \rangle_{\text{front}}}{v_{\text{Stokes}}} = (1 - \Phi)^n$$

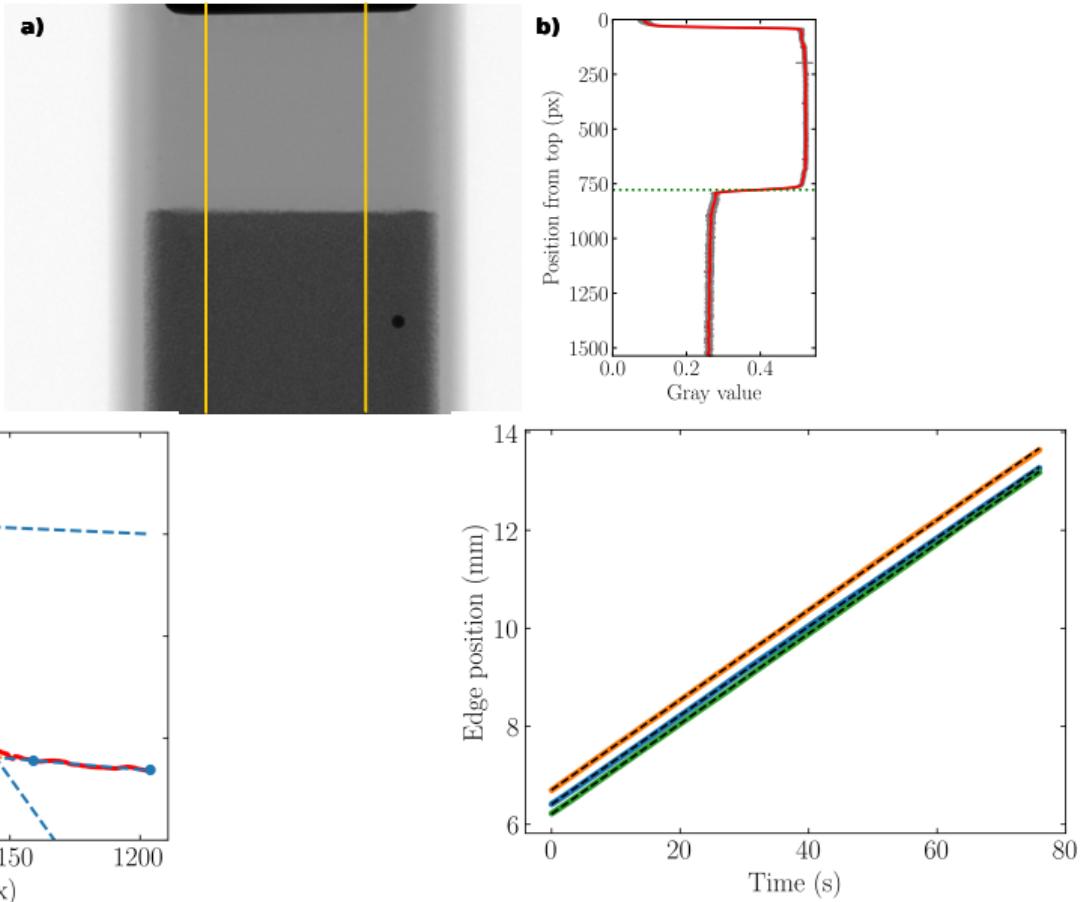
## Tracking of particle front



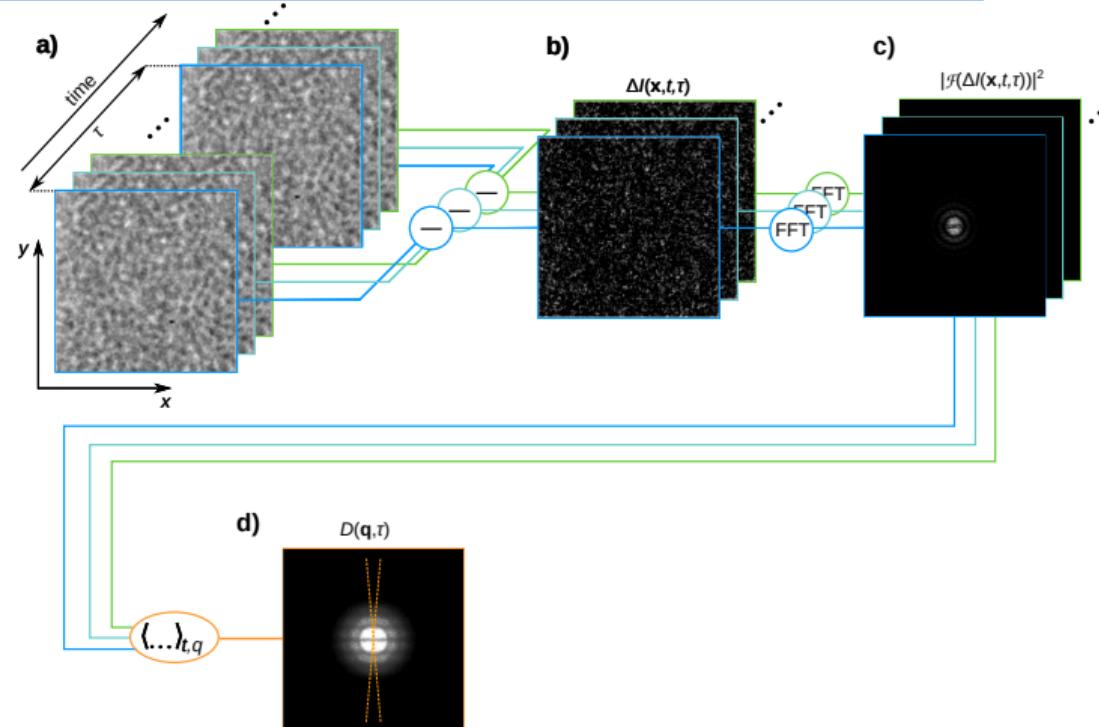
## Tracking of particle front



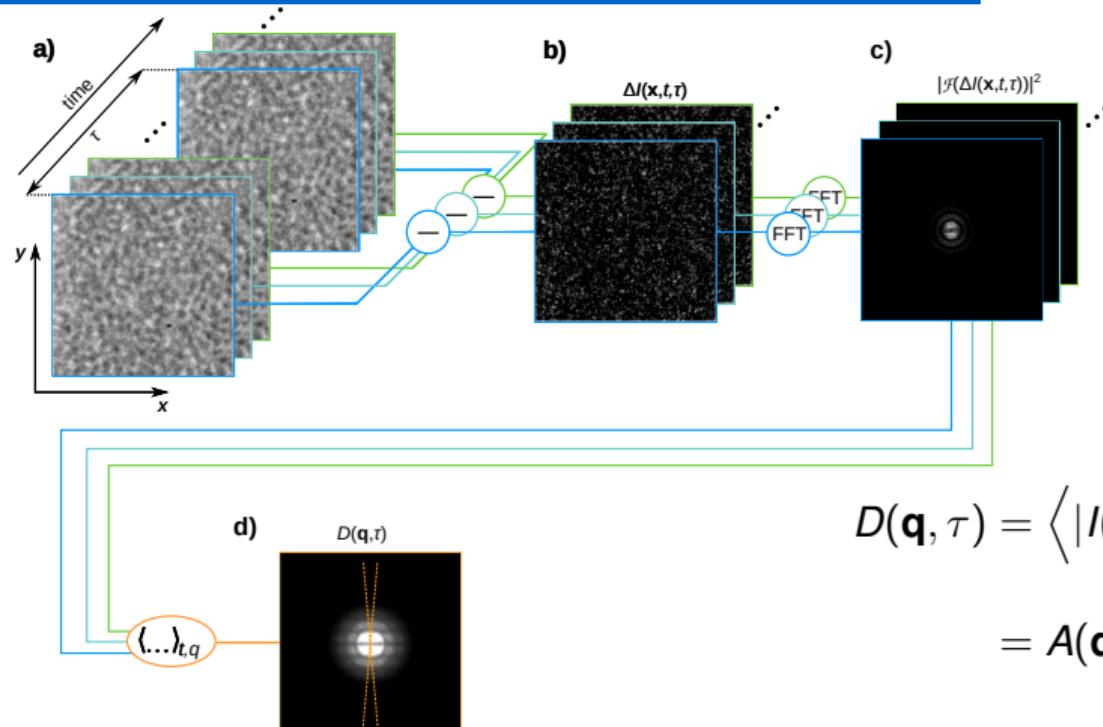
# Tracking of particle front



# The image structure function $D(\mathbf{q}, \tau)$



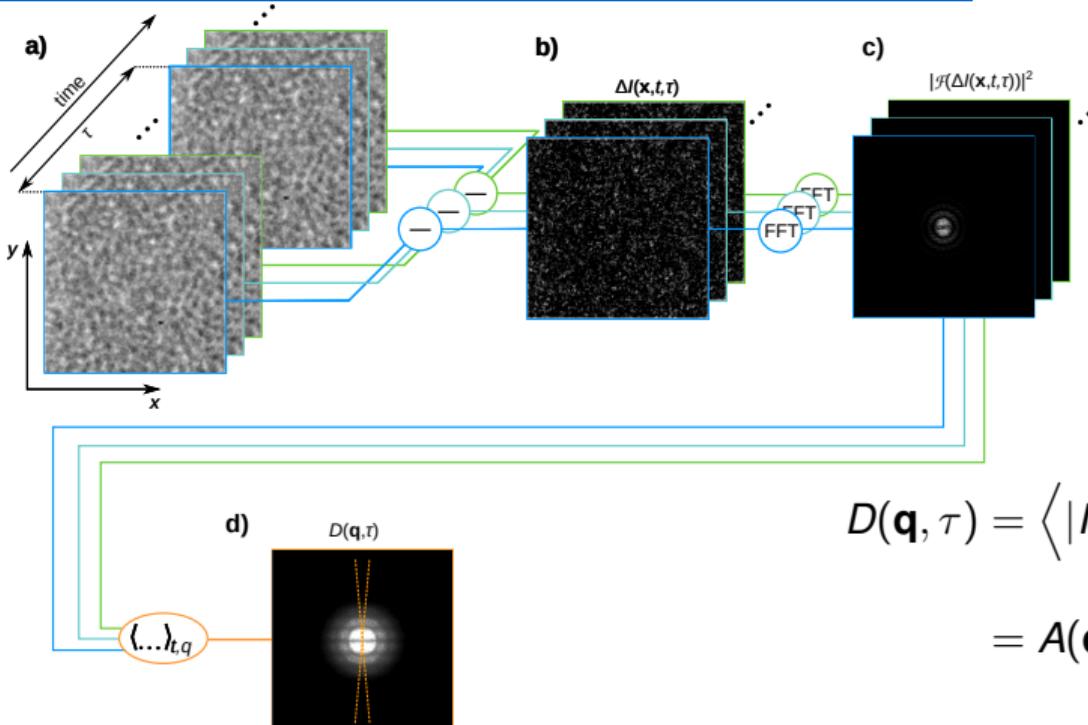
# The image structure function $D(\mathbf{q}, \tau)$



$$\begin{aligned} D(\mathbf{q}, \tau) &= \left\langle |I(\mathbf{q}, t + \tau) - I(\mathbf{q}, t)|^2 \right\rangle_t \\ &= A(\mathbf{q}) \left[ 1 - \frac{\langle I^*(\mathbf{q}, t) I(\mathbf{q}, t + \tau) \rangle_t}{\langle |I(\mathbf{q}, t)|^2 \rangle_t} \right] + B(\mathbf{q}) \end{aligned}$$

- $D(\mathbf{q}, \tau \rightarrow 0) = B(\mathbf{q})$
- $D(\mathbf{q}, \tau \rightarrow \infty) = A(\mathbf{q}) + B(\mathbf{q})$

# The image structure function $D(\mathbf{q}, \tau)$



Linear space invariant imaging

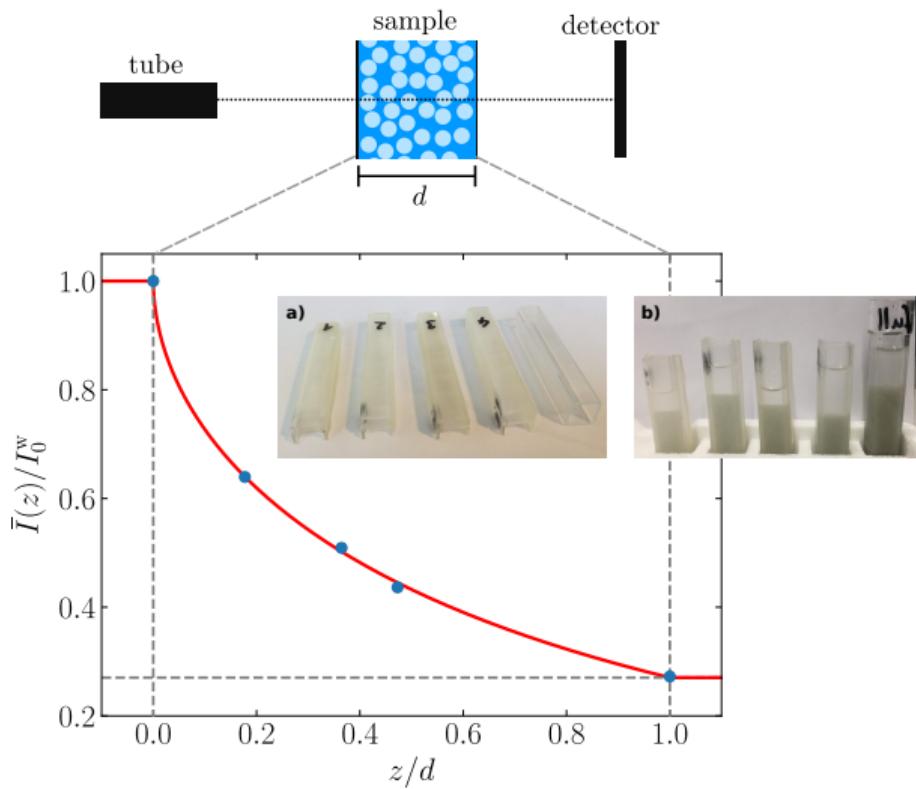
$$f(\mathbf{q}, \tau) = \frac{\langle \rho^*(\mathbf{q}, t)\rho(\mathbf{q}, t + \tau) \rangle_t}{\langle |\rho(\mathbf{q}, t)|^2 \rangle_t}$$

Intermediate scattering function

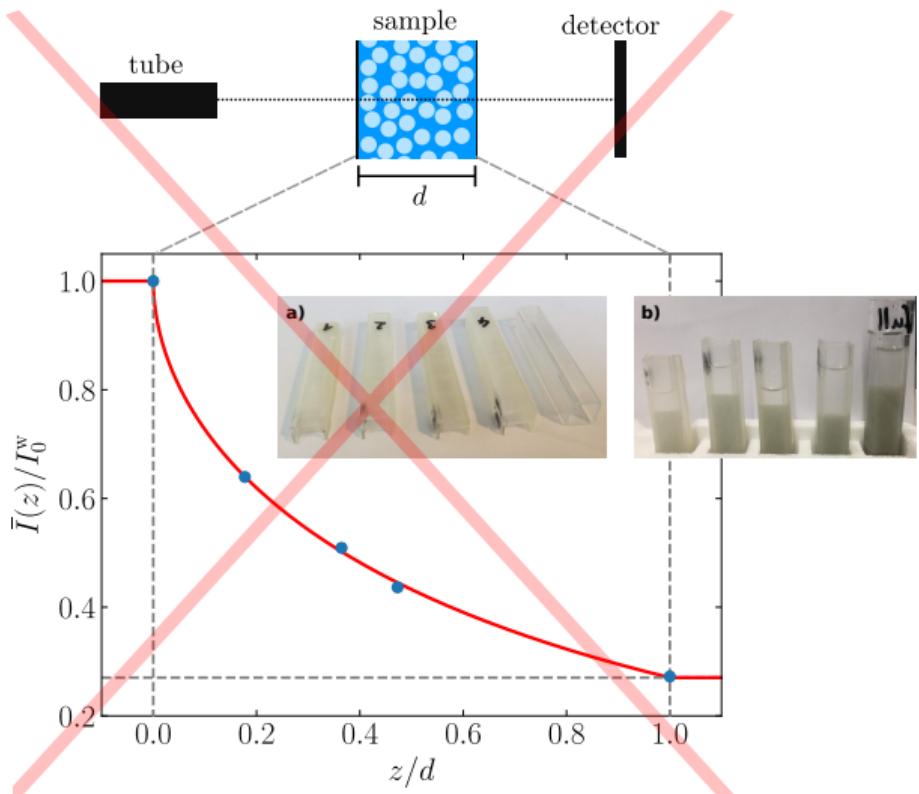
$$\begin{aligned} D(\mathbf{q}, \tau) &= \left\langle |I(\mathbf{q}, t + \tau) - I(\mathbf{q}, t)|^2 \right\rangle_t \\ &= A(\mathbf{q}) \left[ 1 - \frac{\langle I^*(\mathbf{q}, t)I(\mathbf{q}, t + \tau) \rangle_t}{\langle |I(\mathbf{q}, t)|^2 \rangle_t} \right] + B(\mathbf{q}) \end{aligned}$$

- $D(\mathbf{q}, \tau \rightarrow 0) = B(\mathbf{q})$
- $D(\mathbf{q}, \tau \rightarrow \infty) = A(\mathbf{q}) + B(\mathbf{q})$

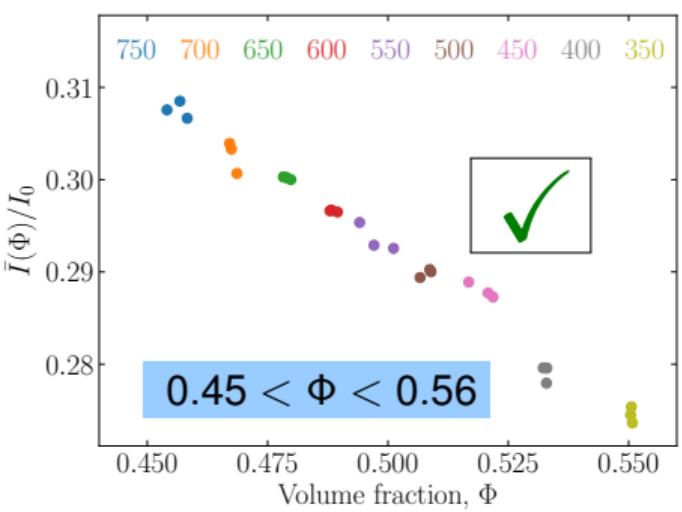
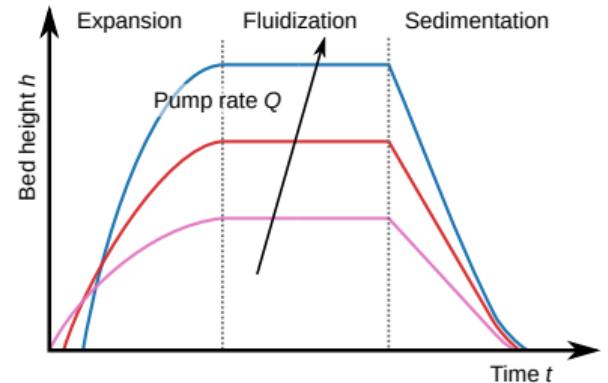
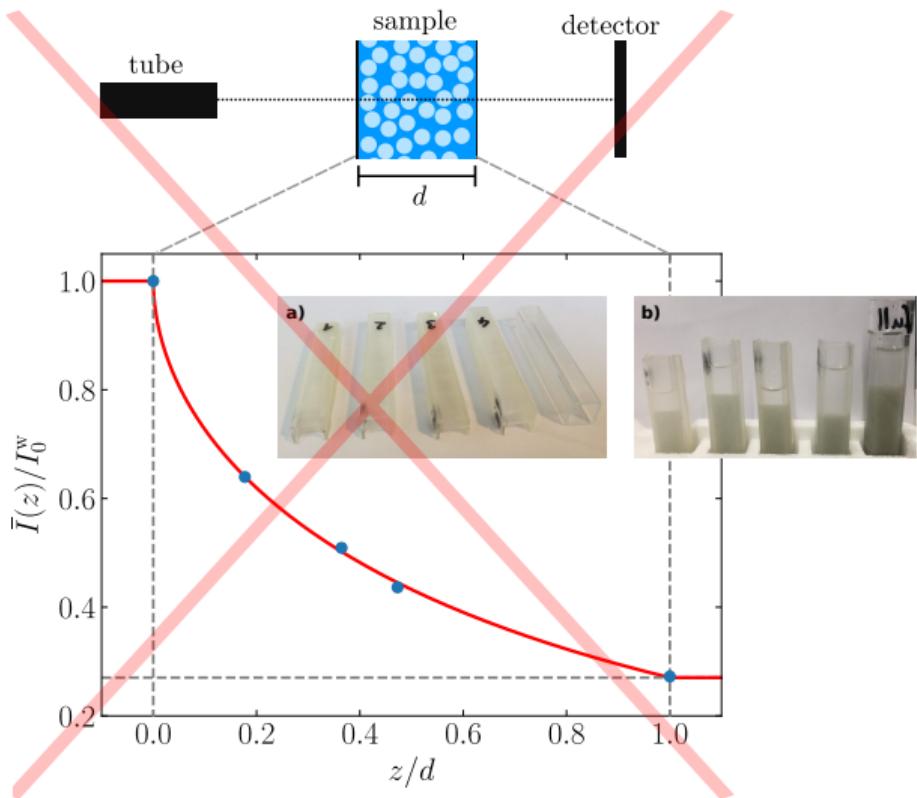
# X-ray imaging – Linear space invariant?



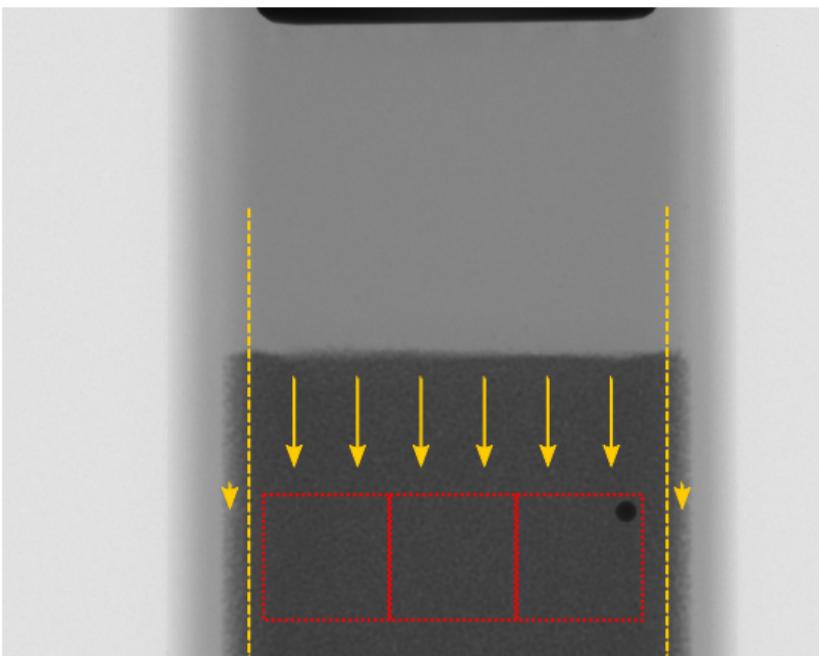
# X-ray imaging – Linear space invariant?



# X-ray imaging – Linear space invariant?

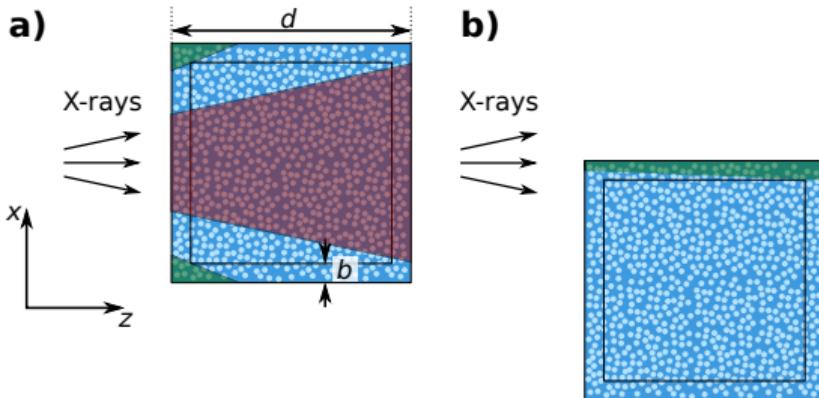


# Estimate width of boundary layer



$\langle v \rangle_{\text{xdfa}} > \langle v \rangle_{\text{front}}$  by 9.4%

$\langle v \rangle_{\text{xdfa}}$  takes two layers into account  
 $\langle v \rangle_{\text{front}}$  takes four layers into account



## Estimation:

Boundary velocity = 0

Else = const.

→  $b \approx 3$  particle diameters