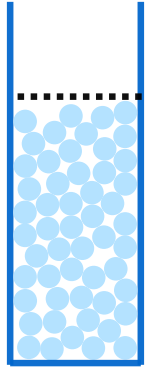
X-ray radiography of granular systems – particle densities and dynamics

X-ray radiography of granular systems – particle densities and dynamics

⋮

Volume fraction

$$\Phi = \frac{V_{\text{Particles}}}{V_{\text{Container}}}$$



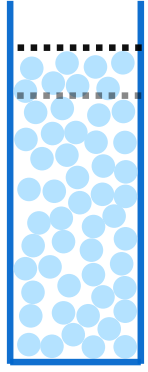
$\Phi = \Phi_{\text{RLP}}$

X-ray radiography of granular systems – particle densities and dynamics

⋮

Volume fraction

$$\Phi = \frac{V_{\text{Particles}}}{V_{\text{Container}}}$$



$\Phi < \Phi_{\text{RLP}}$

$\Phi = \Phi_{\text{RLP}}$

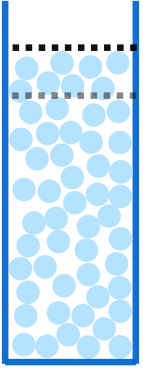
X-ray radiography of granular systems – particle densities and dynamics

Dissipative interactions

Driscoll *et al* (2016)

Volume fraction

$$\Phi = \frac{V_{\text{Particles}}}{V_{\text{Container}}}$$

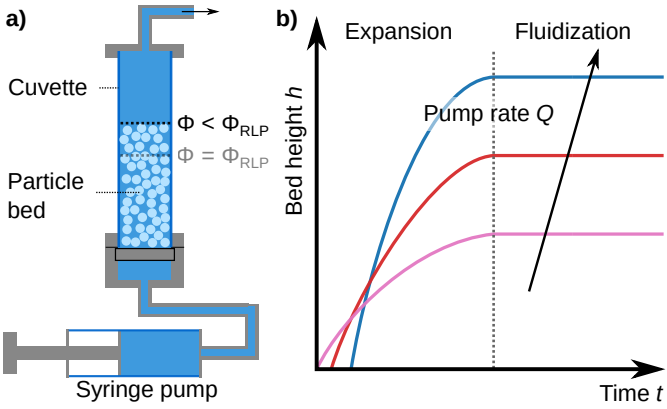


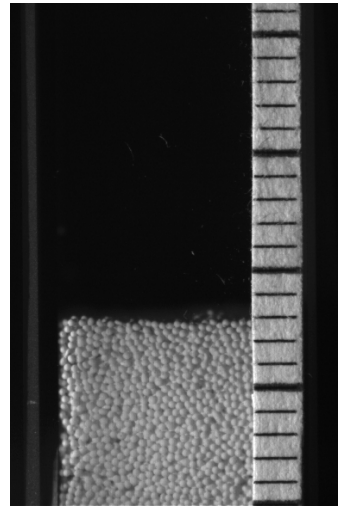
$\Phi < \Phi_{\text{RLP}}$

$\Phi = \Phi_{\text{RLP}}$

X-ray radiography of granular systems – particle densities and dynamics

Fluidized bed

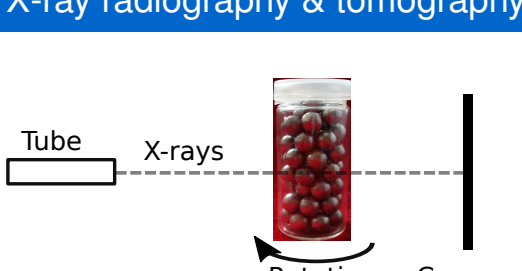


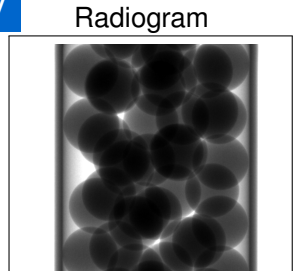


Master thesis Welm Pätzold

Particulate flows are **opaque**

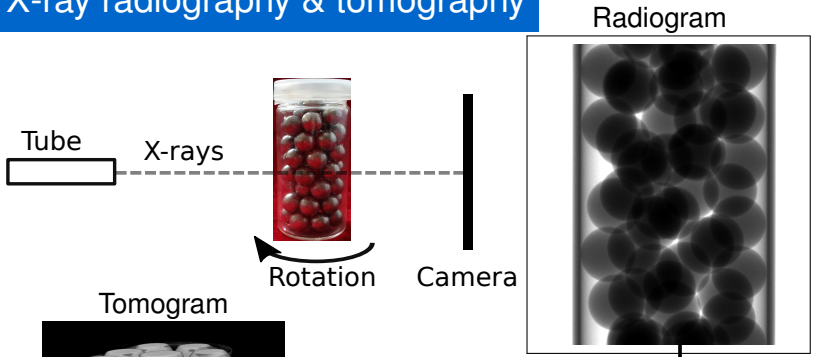
X-ray radiography & tomography





2D projections of 3D object
Short acquisition time
$\rightarrow$ Dynamic system

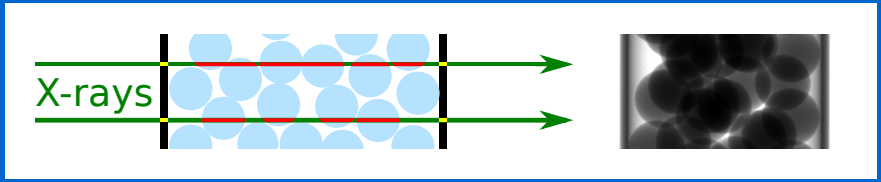
X-ray radiography & tomography



2D projections of 3D object
Short acquisition time
→ Dynamic system

Full 3D information
Long acquisition time
→ Static objects

Measuring the volume fraction of **dynamic** granular systems

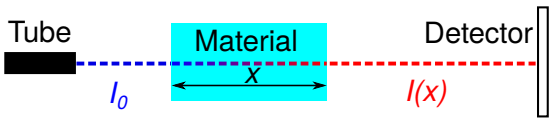


Correction of beam hardening in X-ray radiograms

Baur *et al*, *Rev. Sci. Instrum.* (2019)

In collaboration with Norman Uhlmann, Fraunhofer EZRT

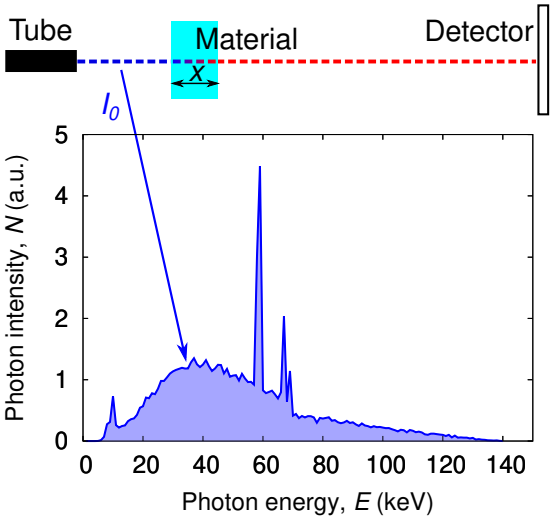
Attenuation of X-rays



Beer-Lambert's law

$I(x) = I_0 \exp(-\mu x)$

Attenuation of X-rays



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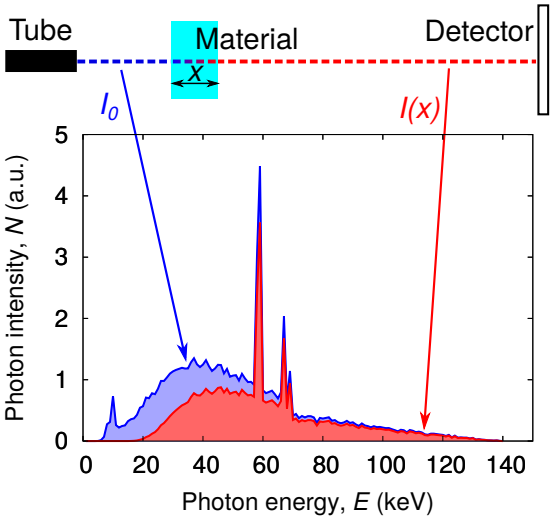
Attenuation of X-rays



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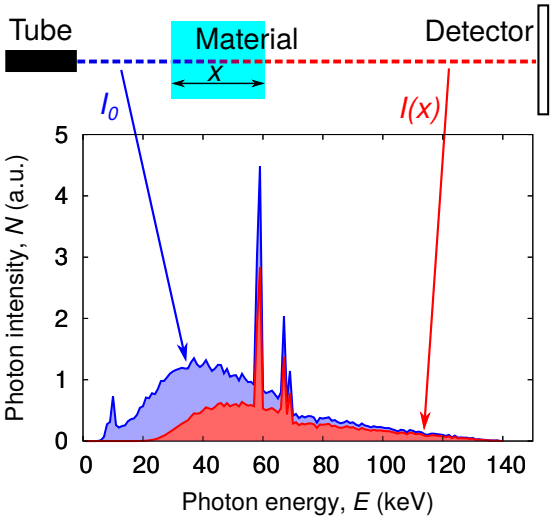
Attenuation of X-rays



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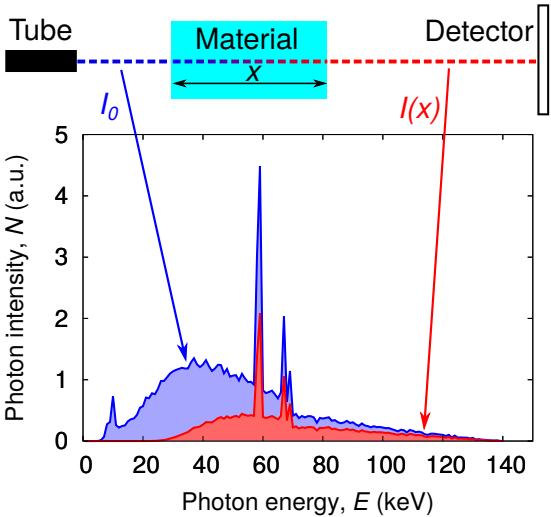
$I(x) = I_0 \exp(-\mu x)$

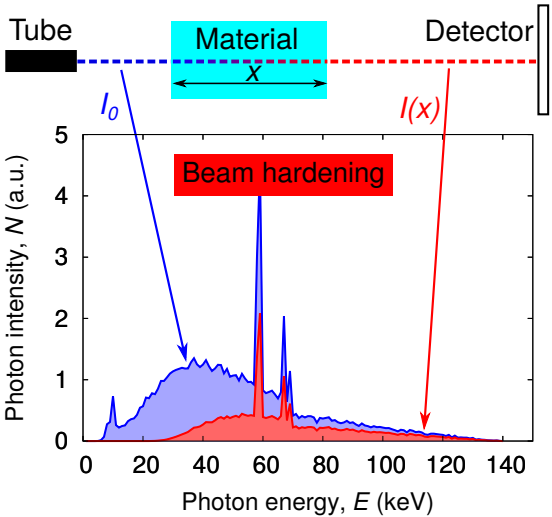
Attenuation of X-rays



Beer-Lambert's law

$I(x) = I_0 \exp(-\mu x)$

Attenuation of X-rays

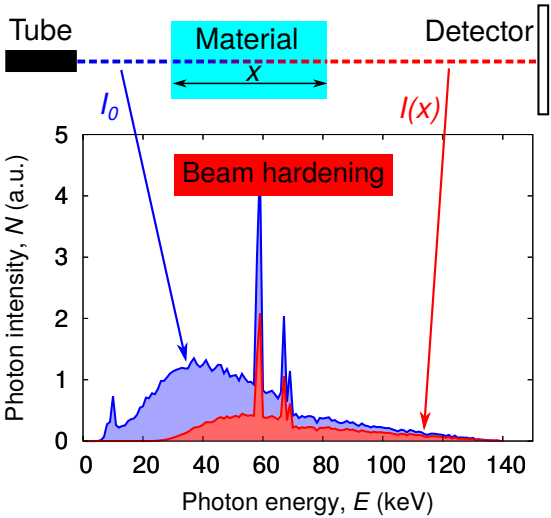


Beer-Lambert's law

~~$I(x) = I_0 \exp(-\mu x)$~~

$$I(x) = I_0 \exp(-\mu_{\text{eff}}(x)\, x)$$

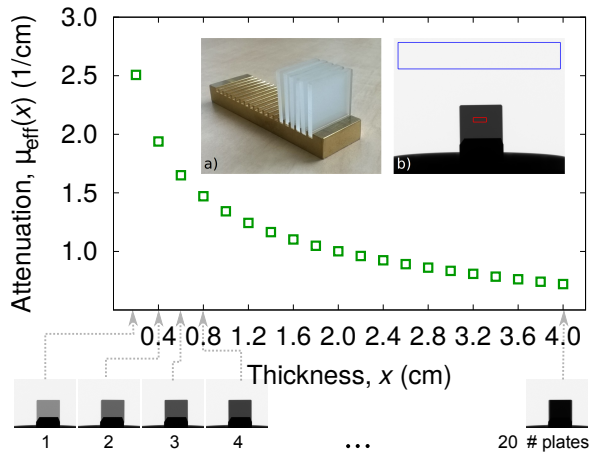
Attenuation of X-rays



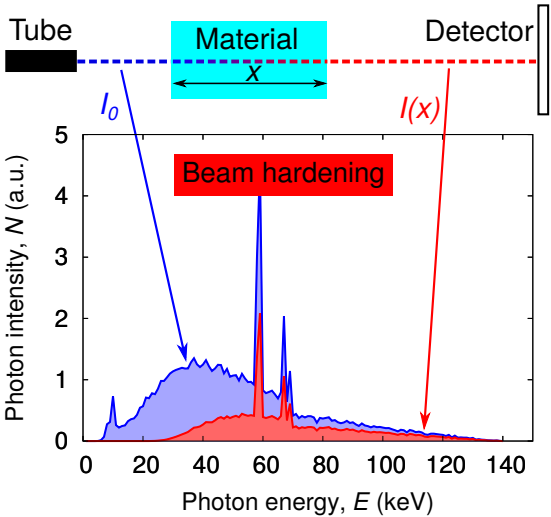
Beer-Lambert's law

$$I(x) = I_0 \exp(-\mu x)$$

$$I(x) = I_0 \exp(-\mu_{\text{eff}}(x)\, x)$$

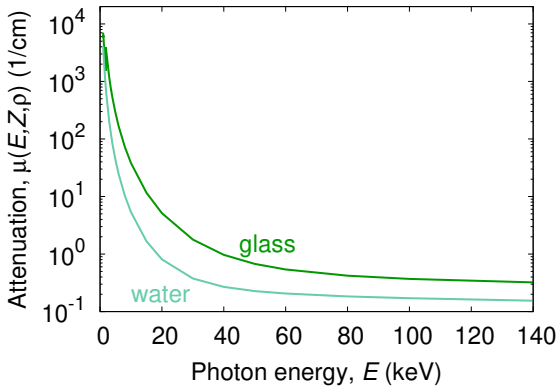


Attenuation of X-rays



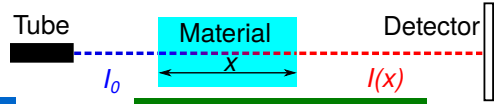
Beer-Lambert's law

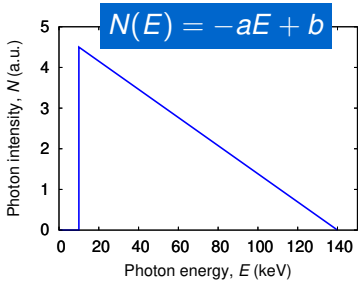
$$I(x) = I_0 \exp(-\mu(E, Z, \rho)\, x)$$

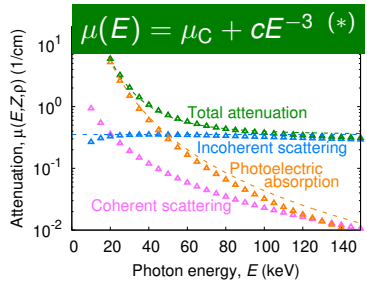


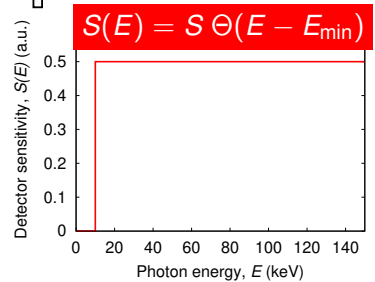
https://www.nist.gov/pml/x-ray-mass-attenuation-coefficients

Modeling of $\mu_{\mathrm{eff}}(x)$





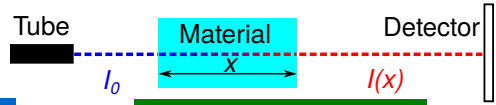


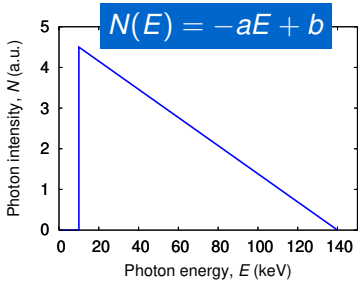


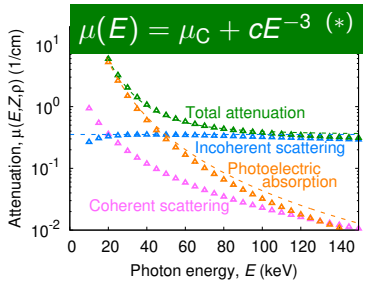
$$I(x) \propto \int N(E) \exp\{-\mu(E, Z, \rho)x\} S(E)\, \mathrm{d}E$$

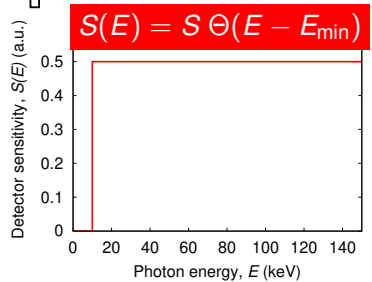
(*) XCOM supplied by NIST

Modeling of $\mu_{\mathrm{eff}}(x)$





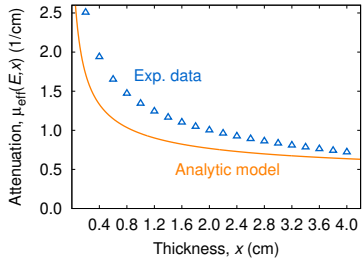




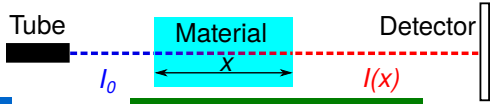
$$I(x) \propto \int N(E) \exp\{-\mu(E,Z,\rho)x\} S(E)\, \mathrm{d}E$$

$$\propto S \int_{E_{\mathrm{min}}}^{E_{\mathrm{max}}} (-aE + b) \exp\{-(\mu_C + cE^{-3})x\} \mathrm{d}E$$

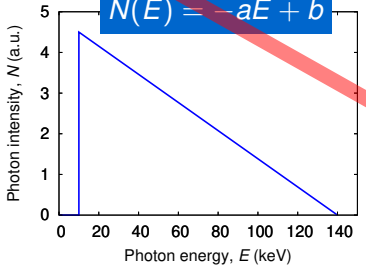
(*) XCOM supplied by NIST



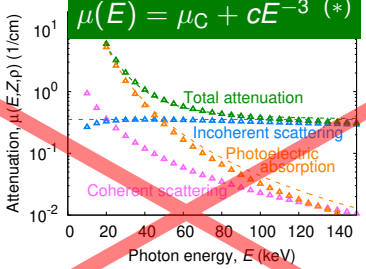
Modeling of $\mu_{\mathrm{eff}}(x)$



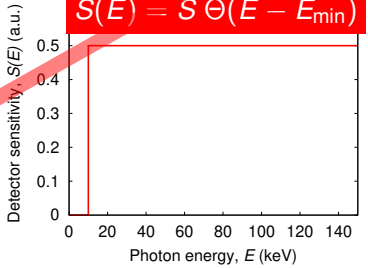
$N(E) = -aE + b$



$\mu(E) = \mu_{\mathrm{C}} + cE^{-3}$ (*)

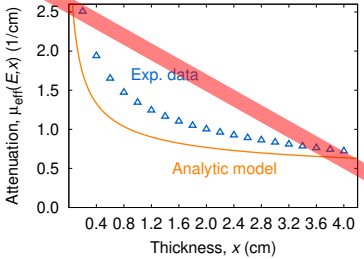


$S(E) = S\,\Theta(E - E_{\mathrm{min}})$



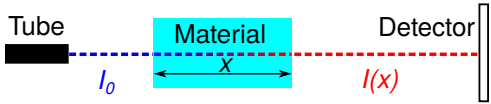
$$I(x) \propto \int N(E)\exp\{-\mu(E,Z,\rho)x\}S(E)\,\mathrm{d}E$$

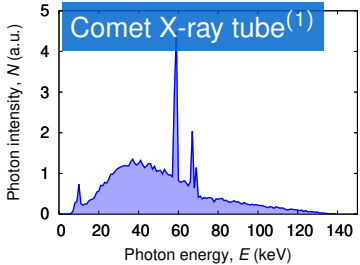
$$\propto S\int_{E_{\mathrm{min}}}^{E_{\mathrm{max}}}(-aE+b)\exp\{-(\mu_{\mathrm{C}}+cE^{-3})x\}\mathrm{d}E$$

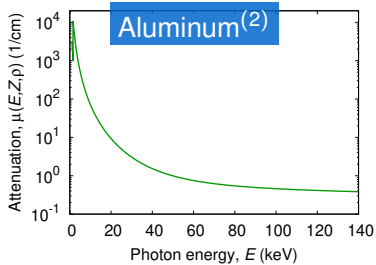


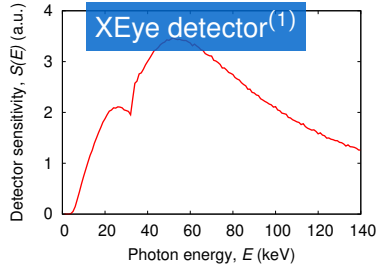
(*) XCOM supplied by NIST

Numerical approx. of $\mu_{\text{eff}}(x)$









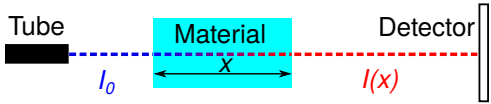
$$I(x) \propto \int N(E) \exp\{-\mu(E, Z, \rho)x\} S(E)\, \mathrm{d}E$$

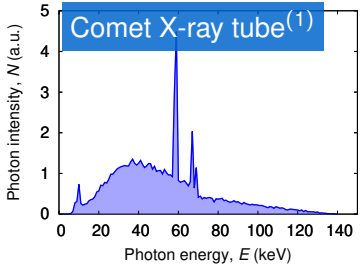
$$\int \rightarrow \sum$$

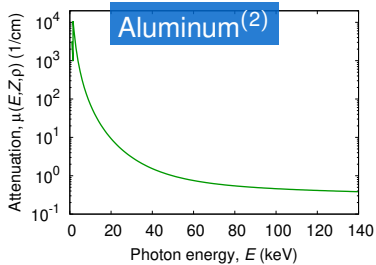
(1) Supplied by Norman Uhlmann, Fraunhofer EZRT

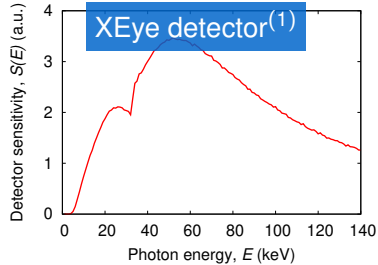
(2) XCOM supplied by NIST

Numerical approx. of $\mu_{\text{eff}}(x)$









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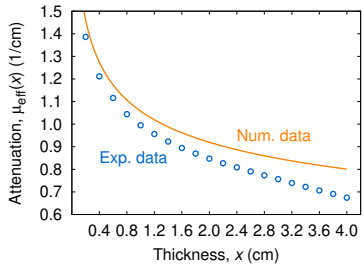
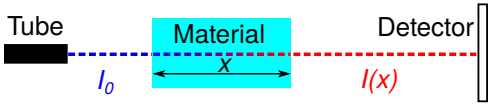
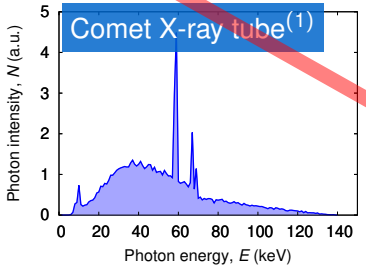


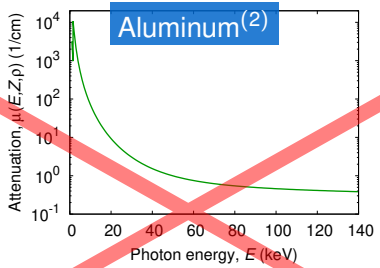
(1) Supplied by Norman Uhlmann, Fraunhofer EZRT

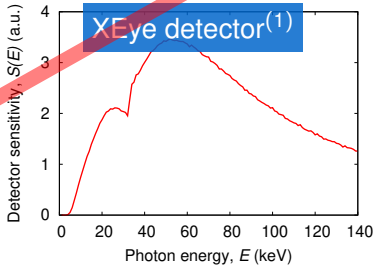
(2) XCOM supplied by NIST

Numerical approx. of $\mu_{\text{eff}}(x)$









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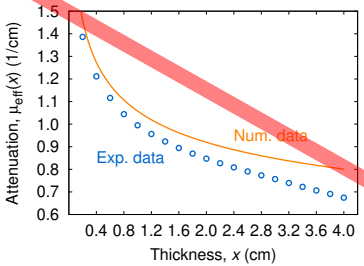
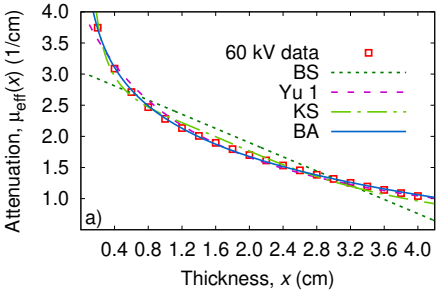
$$\int \rightarrow \sum$$



(1) Supplied by Norman Uhlmann, Fraunhofer EZRT

(2) XCOM supplied by NIST

Heuristic model functions for $\mu_{\text{eff}}(x)$



$$\mu_{\text{eff}}(x) = \mu_0 - \lambda x$$

Bjärngard & Shackford (1994)

$$\mu_{\text{eff}}(x) = \frac{\mu_0}{1+\lambda x}$$

Yu *et al.* (1997)

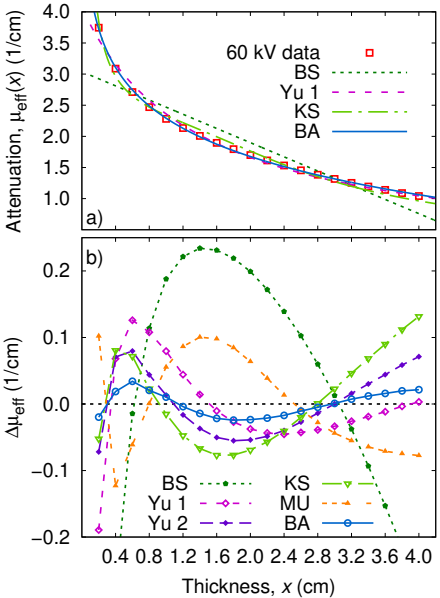
$$\mu_{\text{eff}}(x) = \mu(E_{\text{max}}) + \frac{2\mu_1}{x\sqrt{-\lambda_1^2+4\lambda_2}} \times \left[\arctan\left(\frac{\lambda_1+2\lambda_2 x}{\sqrt{-\lambda_1^2+4\lambda_2}}\right) - \arctan\left(\frac{\lambda_1}{\sqrt{-\lambda_1^2+4\lambda_2}}\right)\right]$$

Kleinschmidt (1999)

$$\boldsymbol{\mu_{\text{eff}}(x) = a + \frac{b}{x^{\alpha}}}$$

Baur *et al.* (2019)
(this work)

Heuristic model functions for $\mu_{\text{eff}}(x)$



$\mu_{\text{eff}}(x) = \mu_0 - \lambda x$ — Bjärngard & Shackford (1994)

$\mu_{\text{eff}}(x) = \frac{\mu_0}{1+\lambda x}$
$\mu_{\text{eff}}(x) = \frac{\mu_0}{(1+\lambda x)^{\beta}}$ — Yu *et al.* (1997)

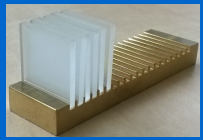
$$\mu_{\text{eff}}(x) = \mu(E_{\text{max}}) + \frac{2\mu_1}{x\sqrt{-\lambda_1^2+4\lambda_2}} \times \left[\arctan\left(\frac{\lambda_1+2\lambda_2 x}{\sqrt{-\lambda_1^2+4\lambda_2}}\right) - \arctan\left(\frac{\lambda_1}{\sqrt{-\lambda_1^2+4\lambda_2}}\right)\right]$$

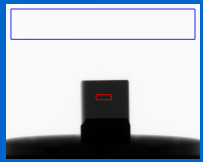
Kleinschmidt (1999)

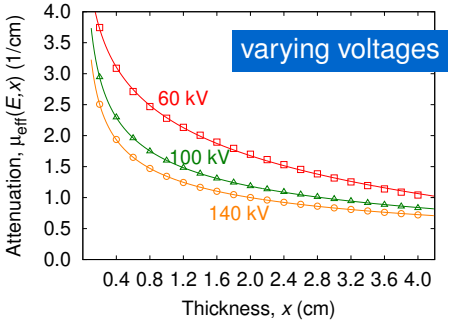
$\mu_{\text{eff}}(x) = -\frac{1}{x}\ln\left[A + B\exp(-x/C)\right]$ — Mudde *et al.* (2008)

$\boldsymbol{\mu_{\text{eff}}(x) = a + \frac{b}{x^{\alpha}}}$ — **Baur *et al.* (2019)** (this work)

Universality of $\mu_{\text{eff}}(x) = a + \frac{b}{x^{\alpha}}$







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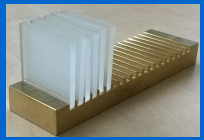

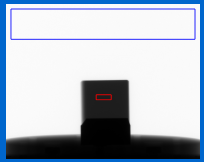

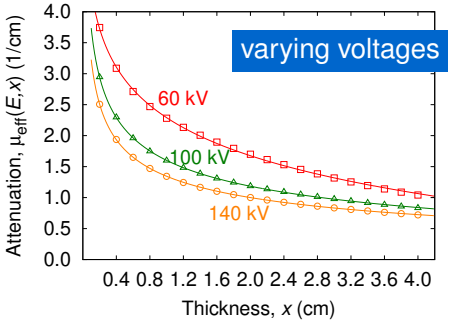

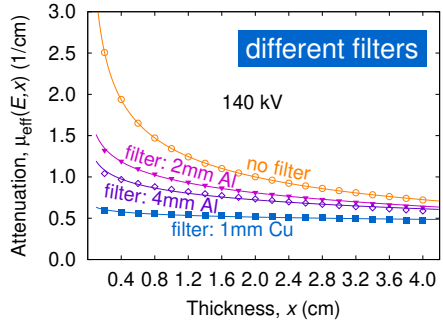

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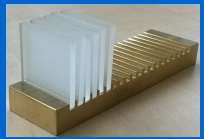

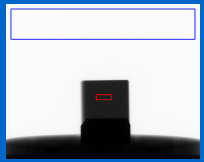

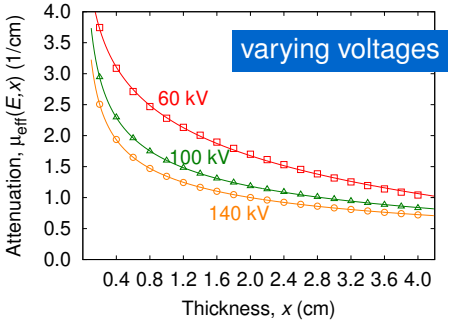

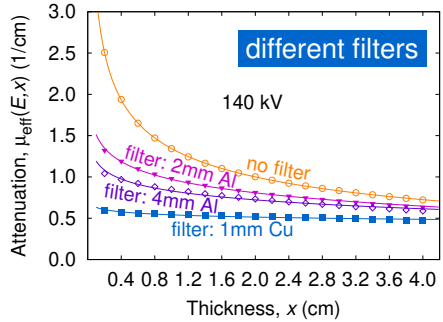

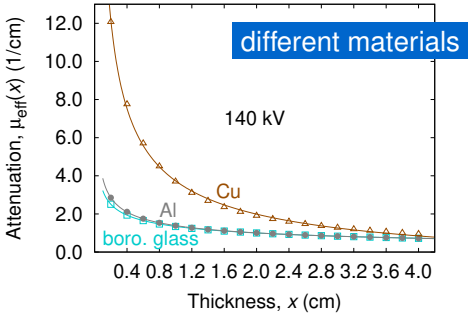

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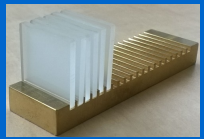
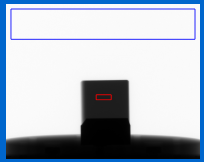
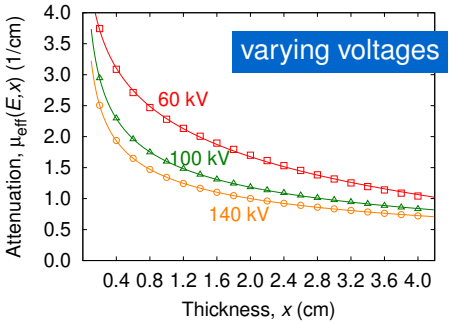
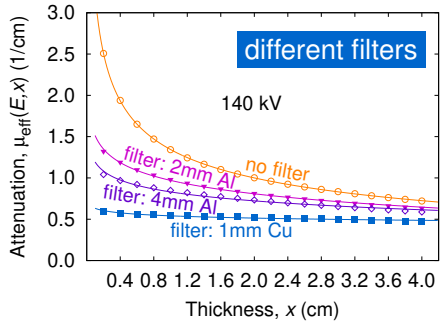
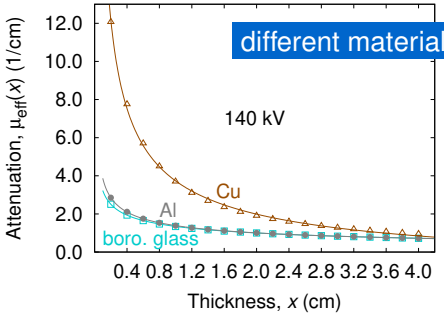
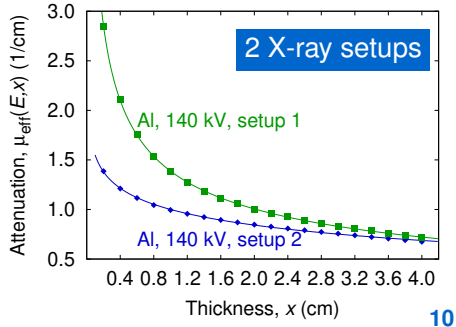












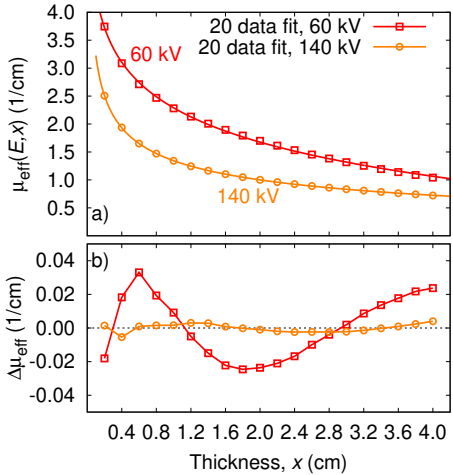
Determining the material thickness x

Generalized Beer-Lambert

$$I(x) = I_0 \exp(-\mu_{\text{eff}}(x)\, x)$$

Model function

$$\mu_{\text{eff}}(x) = a + \frac{b}{x^{\alpha}}$$



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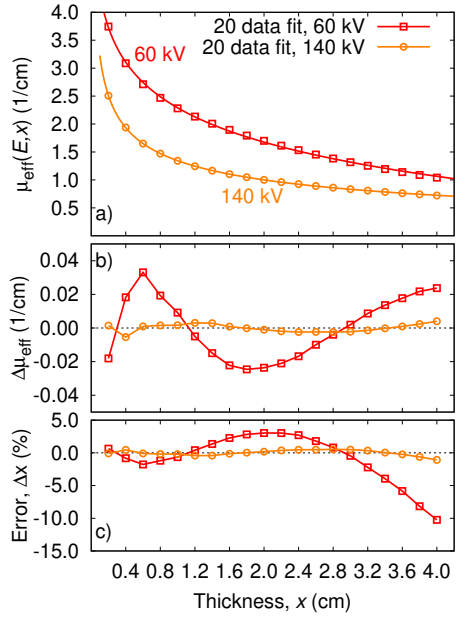
$$ax + bx^{1-\alpha} + \ln\left(\frac{I(x)}{I_0}\right) = 0$$

e.g. Newton's method or look-up table



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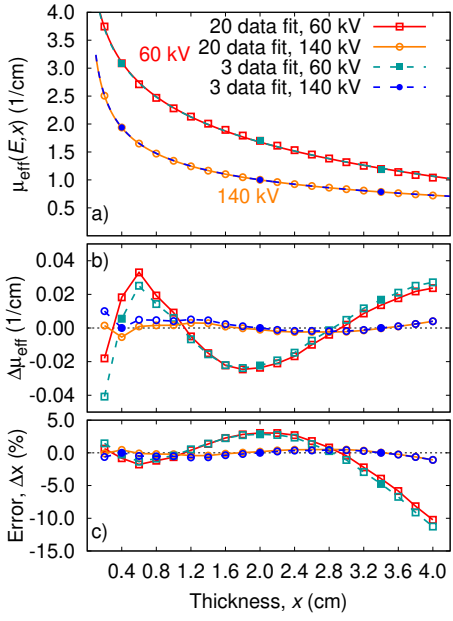

Migrating shear bands in shaken granular matter, Kollmer *et al* (2020)