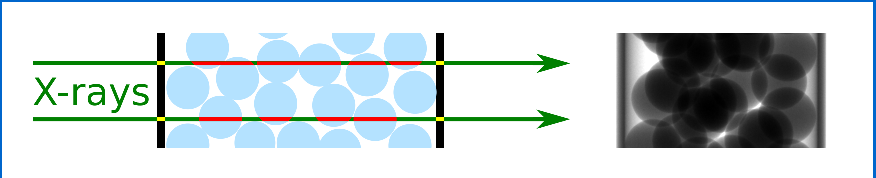
Measuring the volume fraction of **dynamic** granular systems

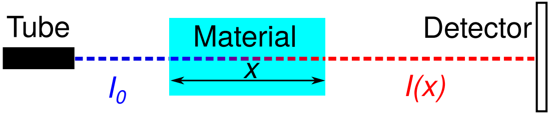


Correction of beam hardening in X-ray radiograms

Baur *et al*, *Rev. Sci. Instrum.* (2019)

In collaboration with Norman Uhlmann, Fraunhofer EZRT

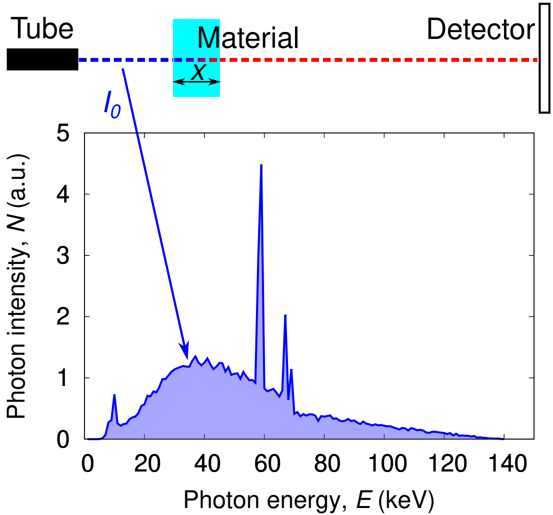
Attenuation of X-rays



Beer-Lambert's law

$$I(x) = I_0 \exp(-\mu\, x)$$

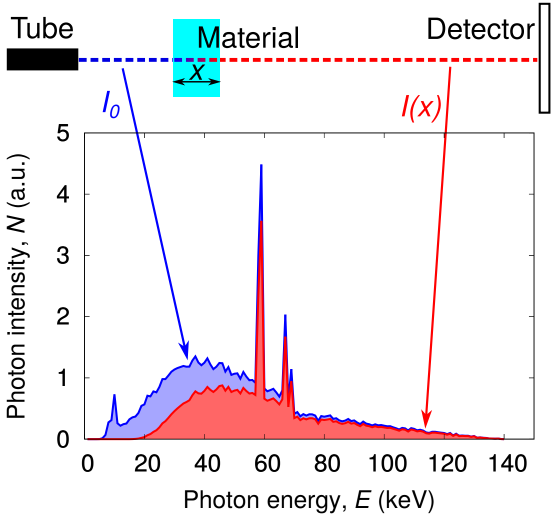
Attenuation of X-rays



Beer-Lambert's law

$$I(x) = I_0 \exp(-\mu x)$$

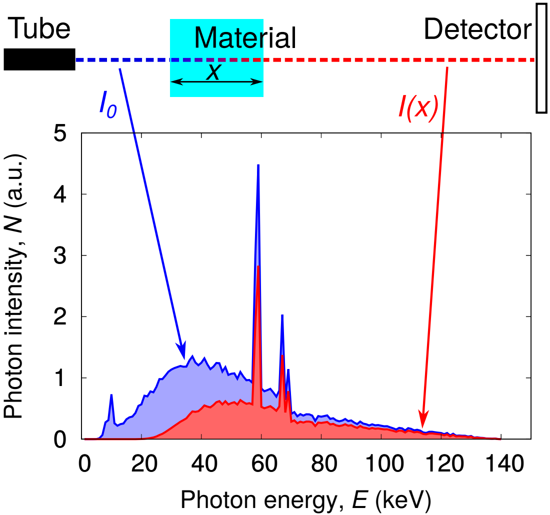
Attenuation of X-rays



Beer-Lambert's law

$$I(x) = I_0 \exp(-\mu x)$$

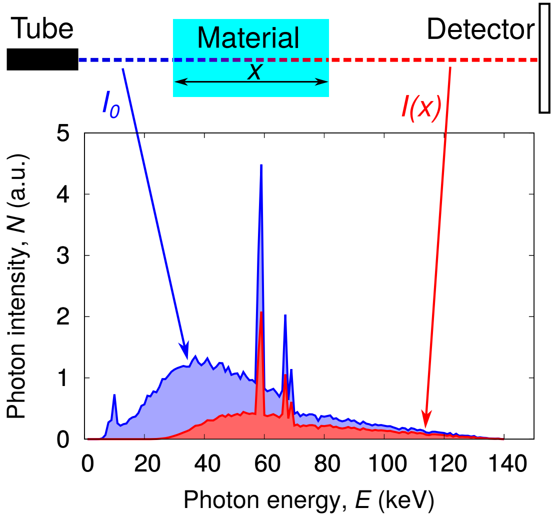
Attenuation of X-rays



Beer-Lambert's law

$$I(x) = I_0 \exp(-\mu\, x)$$

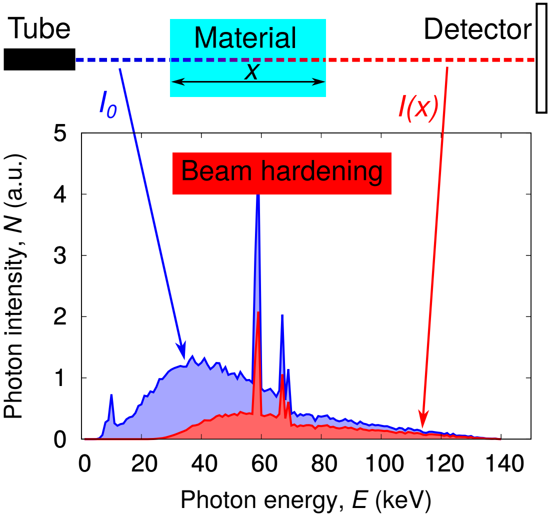
Attenuation of X-rays



Beer-Lambert's law

$$I(x) = I_0 \exp(-\mu\, x)$$

Attenuation of X-rays

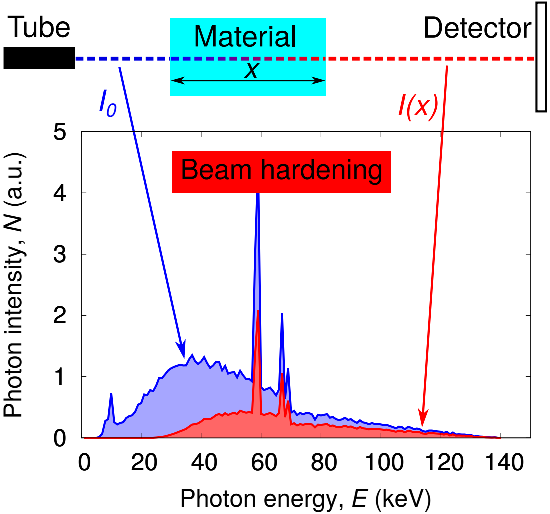


Beer-Lambert's law

$$\cancel{I(x) = I_0 \exp(-\mu\, x)}$$

$$I(x) = I_0 \exp(-\mu_{\text{eff}}(x)\, x)$$

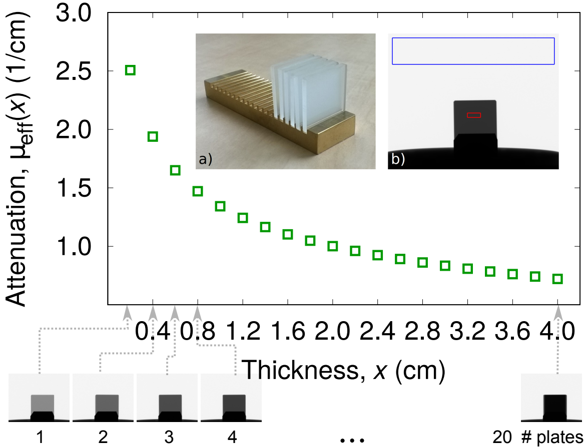
Attenuation of X-rays



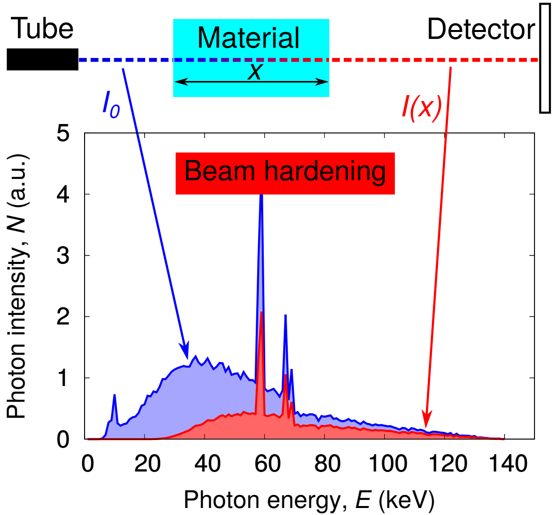
Beer-Lambert's law

$$\cancel{I(x) = I_0 \exp(-\mu\, x)}$$

$$I(x) = I_0 \exp(-\mu_{\mathrm{eff}}(x)\, x)$$

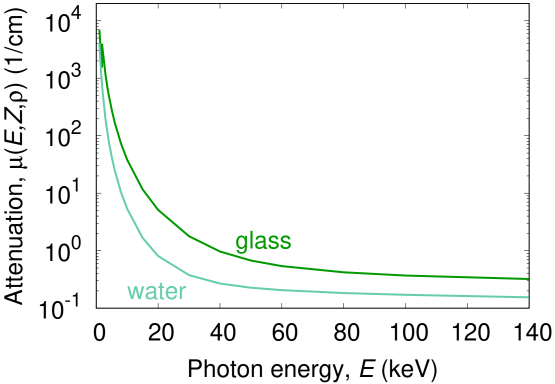


Attenuation of X-rays



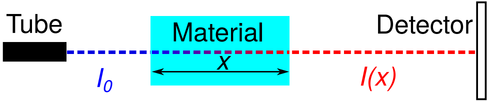
Beer-Lambert's law

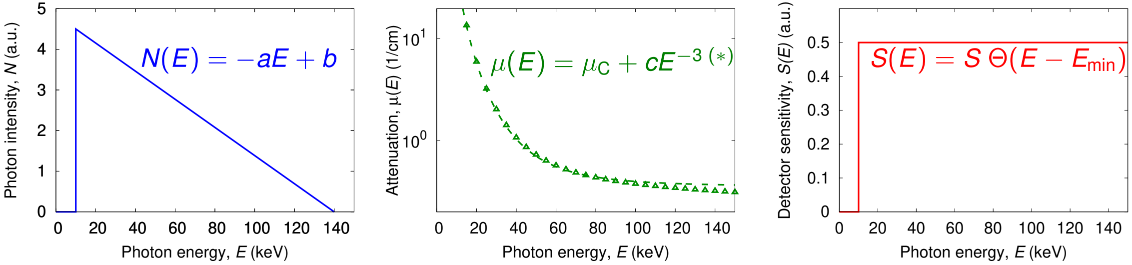
$$I(x) = I_0 \exp(-\mu(E, Z, \rho)\, x)$$



Tabulated attenuation coefficients from NIST.gov

Modeling of $\mu_{\mathrm{eff}}(x)$

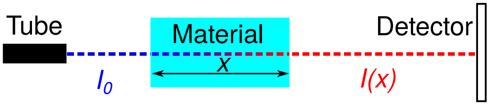


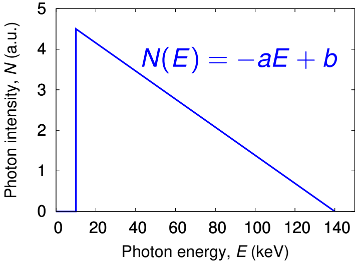


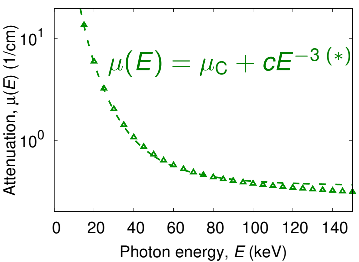
$$I(x) \propto \int N(E)\,\exp\{-\mu(E)x\}\,S(E)\,\mathrm{d}E$$

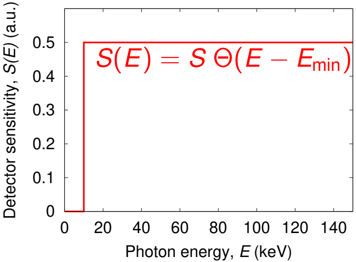
(∗) XCOM supplied by NIST

Modeling of $\mu_{\mathrm{eff}}(x)$



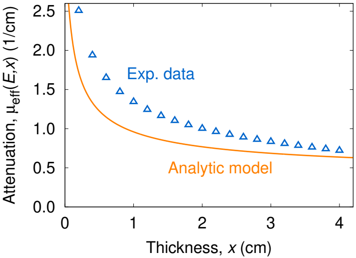






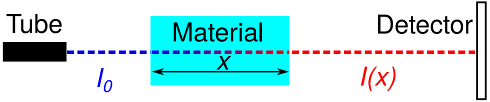
$$I(x) \propto \int N(E)\exp\{-\mu(E)x\}S(E)\,\mathrm{d}E$$

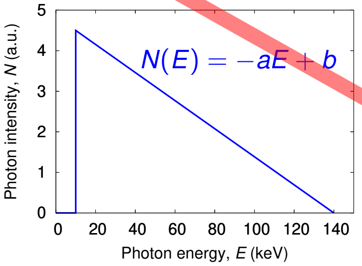
$$\propto S\int_{E_{\mathrm{min}}}^{E_{\mathrm{max}}}(-aE+b)\exp\{-(\mu_C+cE^{-3})x\}\,\mathrm{d}E$$

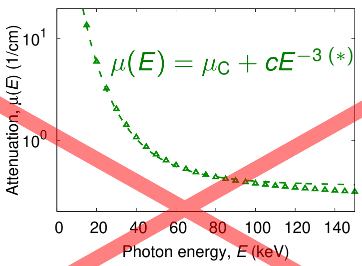


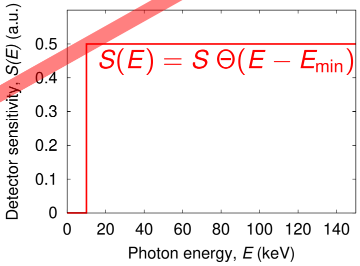
(∗) XCOM supplied by NIST

Modeling of $\mu_{\mathrm{eff}}(x)$



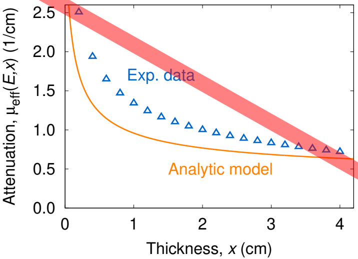






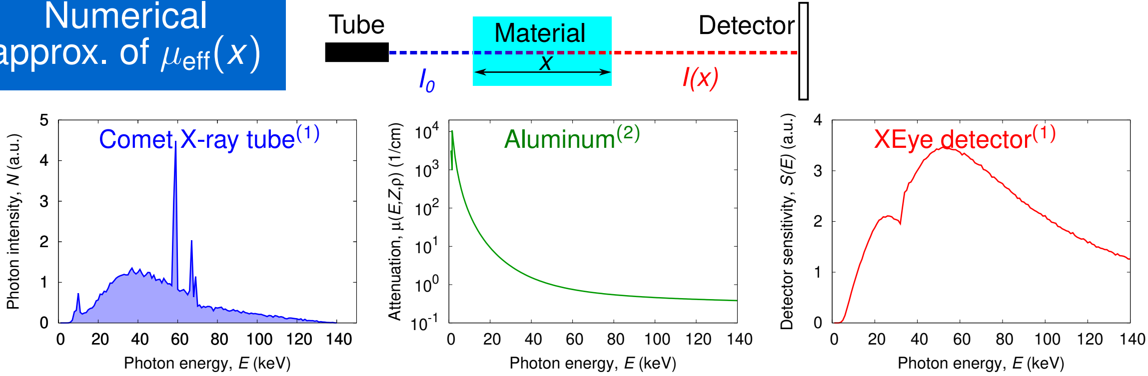
$$I(x) \propto \int N(E) \exp\{-\mu(E)x\} S(E)\, \mathrm{d}E$$

$$\propto S \int_{E_{\mathrm{min}}}^{E_{\mathrm{max}}} (-aE + b) \exp\{-(\mu_C + cE^{-3})x\}\, \mathrm{d}E$$



(∗) XCOM supplied by NIST

Numerical approx. of $\mu_{\text{eff}}(x)$



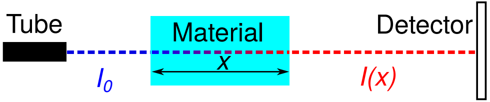
$$I(x) \propto \int N(E) \exp\{-\mu(E)x\} S(E) \, \mathrm{d}E$$

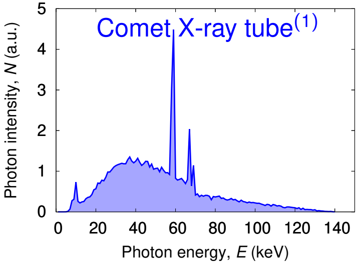
$$\int \rightarrow \sum$$

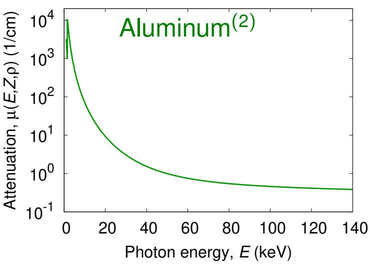
(1) Supplied by Norman Uhlmann, Fraunhofer EZRT

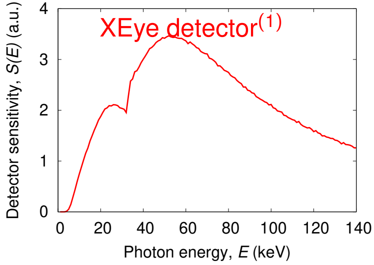
(2) XCOM supplied by NIST

Numerical approx. of $\mu_{\mathrm{eff}}(x)$



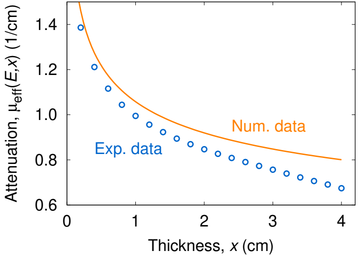






$$I(x) \propto \int N(E)\, \exp\{-\mu(E)x\}\, S(E)\, \mathrm{d}E$$

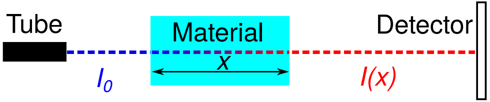
$$\int \rightarrow \sum$$

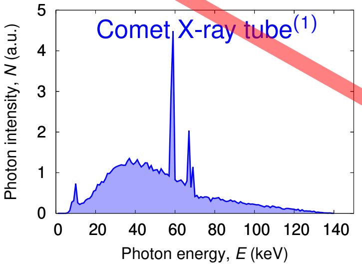


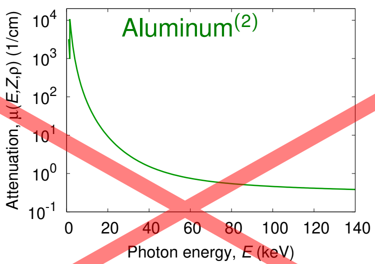
(1) Supplied by Norman Uhlmann, Fraunhofer EZRT

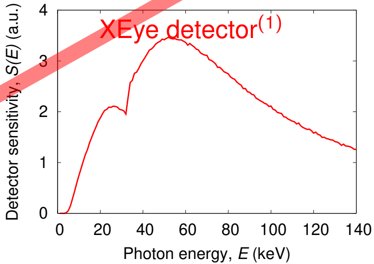
(2) XCOM supplied by NIST

Numerical approx. of $\mu_{\mathrm{eff}}(x)$



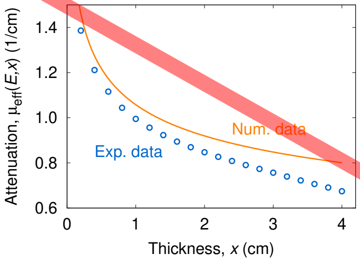






$$I(x) \propto \int N(E)\exp\{-\mu(E)x\}S(E)\,\mathrm{d}E$$

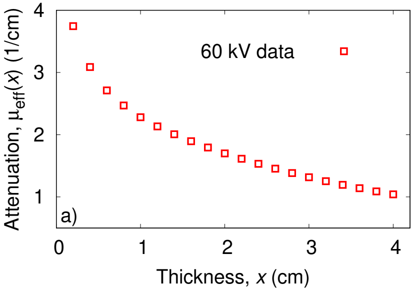
$$\int \rightarrow \sum$$



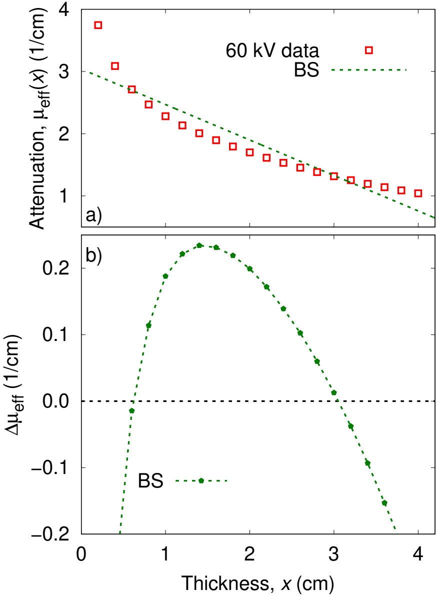
(1) Supplied by Norman Uhlmann, Fraunhofer EZRT

(2) XCOM supplied by NIST

Heuristic model functions for $\mu_{\text{eff}}(x)$



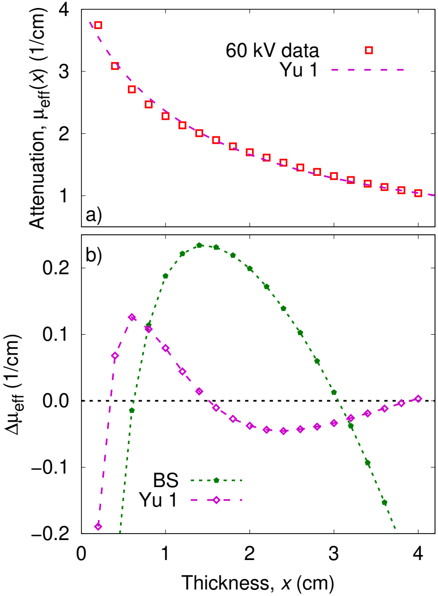
Heuristic model functions for $\mu_{\rm eff}(x)$



$$\mu_{\rm eff}(x) = \mu_0 - \lambda x$$

Bjärngard & Shackford (1994)

Heuristic model functions for $\mu_{\mathrm{eff}}(x)$



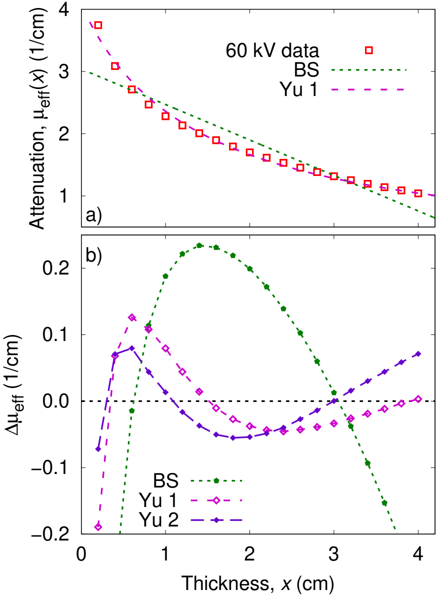
$$\mu_{\mathrm{eff}}(x) = \mu_0 - \lambda x$$

Bjärngard & Shackford (1994)

$$\mu_{\mathrm{eff}}(x) = \frac{\mu_0}{1+\lambda x}$$

Yu *et al.* (1997)

Heuristic model functions for $\mu_{\text{eff}}(x)$



$$\mu_{\text{eff}}(x) = \mu_0 - \lambda x$$

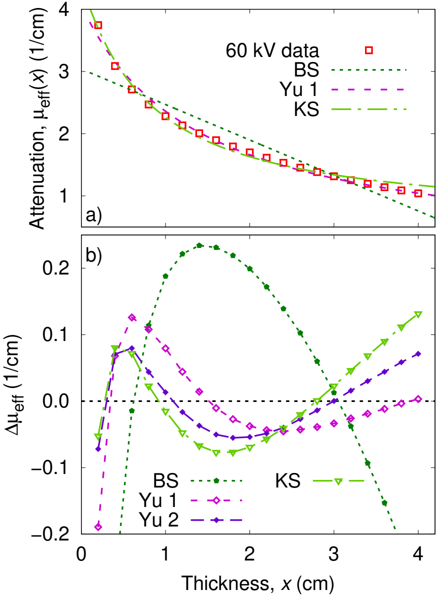
Bjärngard & Shackford (1994)

$$\mu_{\text{eff}}(x) = \frac{\mu_0}{1+\lambda x}$$

$$\mu_{\text{eff}}(x) = \frac{\mu_0}{(1+\lambda x)^\beta}$$

Yu *et al.* (1997)

Heuristic model functions for $\mu_{\text{eff}}(x)$



$$\mu_{\text{eff}}(x) = \mu_0 - \lambda x$$

Bjärngard & Shackford (1994)

$$\mu_{\text{eff}}(x) = \frac{\mu_0}{1+\lambda x}$$

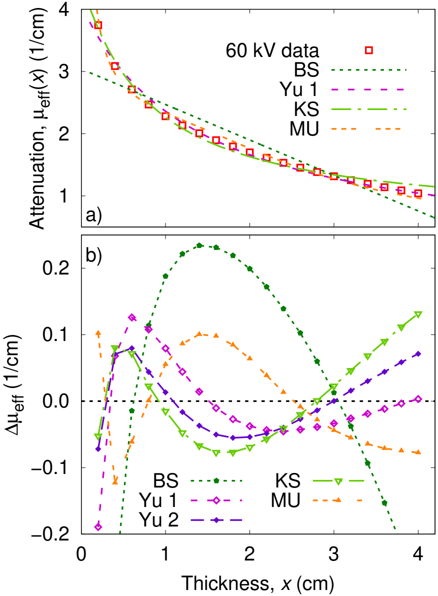
$$\mu_{\text{eff}}(x) = \frac{\mu_0}{(1+\lambda x)^{\beta}}$$

Yu *et al.* (1997)

$$\mu_{\text{eff}}(x) = \mu(E_{\text{max}}) + \frac{2\mu_1}{x\sqrt{-\lambda_1^2+4\lambda_2}} \times \left[\arctan\left(\frac{\lambda_1+2\lambda_2 x}{\sqrt{-\lambda_1^2+4\lambda_2}}\right) - \arctan\left(\frac{\lambda_1}{\sqrt{-\lambda_1^2+4\lambda_2}}\right)\right]$$

Kleinschmidt (1999)

Heuristic model functions for $\mu_{\text{eff}}(x)$



$$\mu_{\text{eff}}(x) = \mu_0 - \lambda x$$

Bjärngard & Shackford (1994)

$$\mu_{\text{eff}}(x) = \frac{\mu_0}{1+\lambda x}$$

$$\mu_{\text{eff}}(x) = \frac{\mu_0}{(1+\lambda x)^{\beta}}$$

Yu *et al.* (1997)

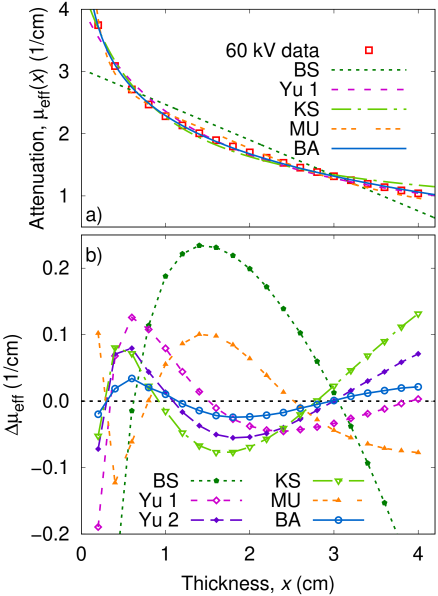
$$\mu_{\text{eff}}(x) = \mu(E_{\text{max}}) + \frac{2\mu_1}{x\sqrt{-\lambda_1^2+4\lambda_2}} \times \left[\arctan\left(\frac{\lambda_1+2\lambda_2 x}{\sqrt{-\lambda_1^2+4\lambda_2}}\right) - \arctan\left(\frac{\lambda_1}{\sqrt{-\lambda_1^2+4\lambda_2}}\right)\right]$$

Kleinschmidt (1999)

$$\mu_{\text{eff}}(x) = -\frac{1}{x} \ln\left[A + B\exp(-x/C)\right]$$

Mudde *et al.* (2008)

Heuristic model functions for $\mu_{\text{eff}}(x)$



$$\mu_{\text{eff}}(x) = \mu_0 - \lambda x$$

Bjärngard & Shackford (1994)

$$\mu_{\text{eff}}(x) = \frac{\mu_0}{1+\lambda x}$$

$$\mu_{\text{eff}}(x) = \frac{\mu_0}{(1+\lambda x)^\beta}$$

Yu *et al.* (1997)

$$\mu_{\text{eff}}(x) = \mu(E_{\max}) + \frac{2\mu_1}{x\sqrt{-\lambda_1^2+4\lambda_2}} \times \left[\arctan\left(\frac{\lambda_1+2\lambda_2 x}{\sqrt{-\lambda_1^2+4\lambda_2}}\right) - \arctan\left(\frac{\lambda_1}{\sqrt{-\lambda_1^2+4\lambda_2}}\right)\right]$$

Kleinschmidt (1999)

$$\mu_{\text{eff}}(x) = -\frac{1}{x}\ln\left[A + B\exp(-x/C)\right]$$

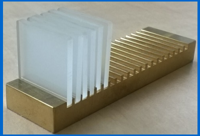
Mudde *et al.* (2008)

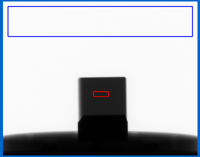
$$\boldsymbol{\mu_{\text{eff}}(x) = a + \frac{b}{x^\alpha}}$$

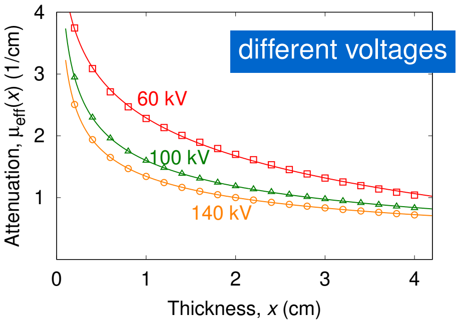
Baur *et al.* (2019)

(this work)

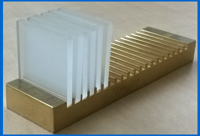
Applicability of $\mu_{\text{eff}}(x) = a + \frac{b}{x^\alpha}$

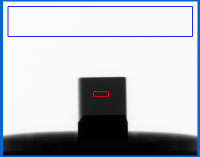


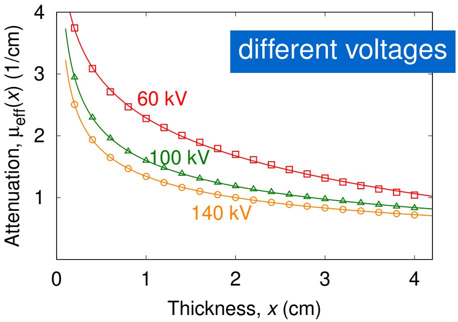


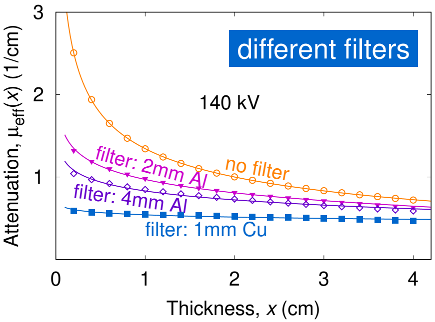


Applicability of $\mu_{\text{eff}}(x) = a + \frac{b}{x^{\alpha}}$

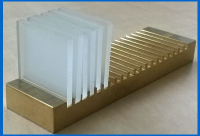


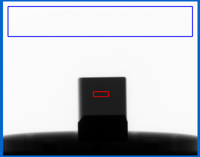


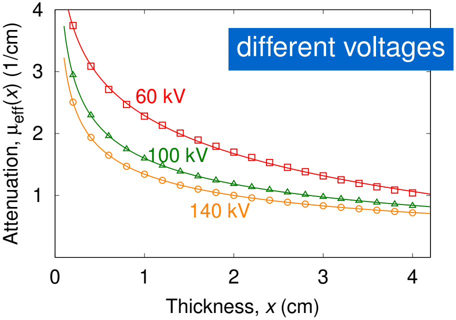


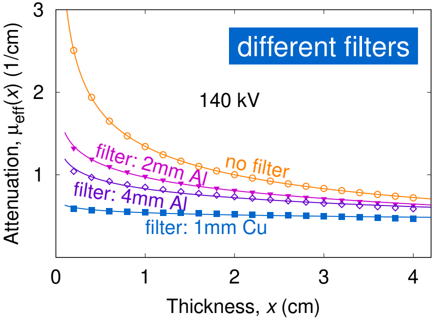


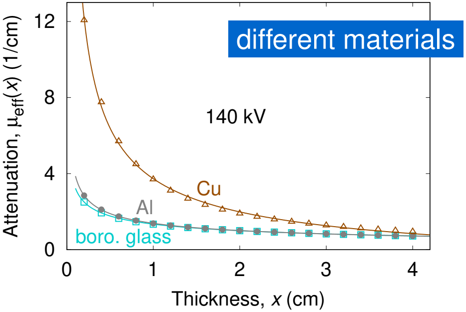
Applicability of $\mu_{\text{eff}}(x) = a + \frac{b}{x^{\alpha}}$



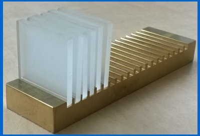


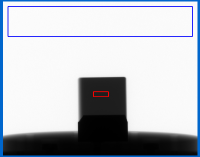


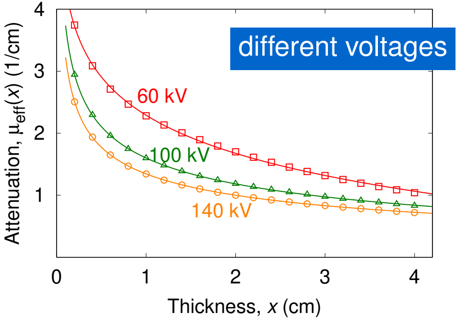


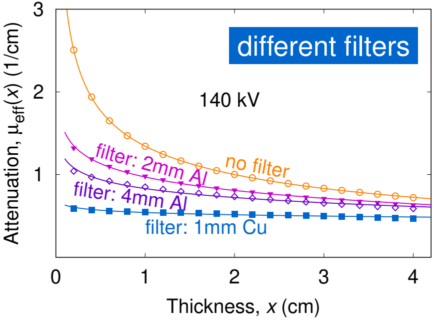


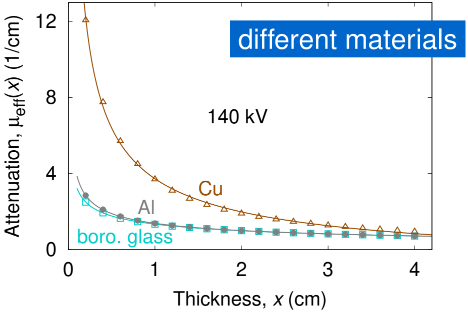
Applicability of $\mu_{\text{eff}}(x) = a + \frac{b}{x^{\alpha}}$

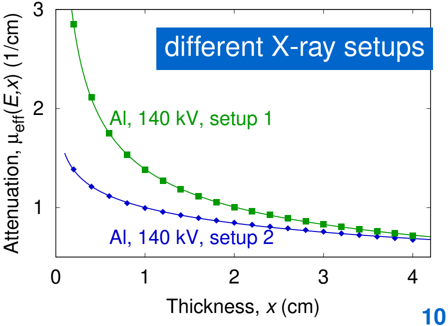












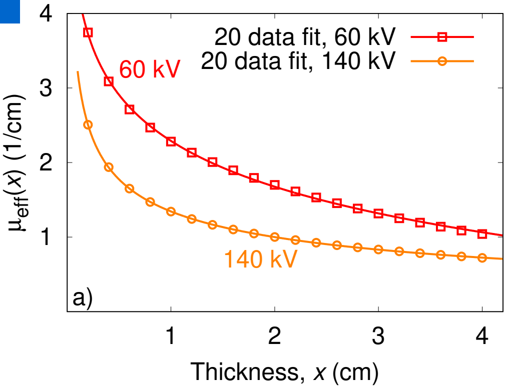
Determining the material thickness x

Generalized Beer-Lambert

$$I(x) = I_0 \exp(-\mu_{\mathrm{eff}}(x)\, x)$$

Model function

$$\mu_{\mathrm{eff}}(x) = a + \frac{b}{x^{\alpha}}$$



Determining the material thickness x

Generalized Beer-Lambert

$$I(x) = I_0 \exp(-\mu_{\text{eff}}(x)\, x)$$

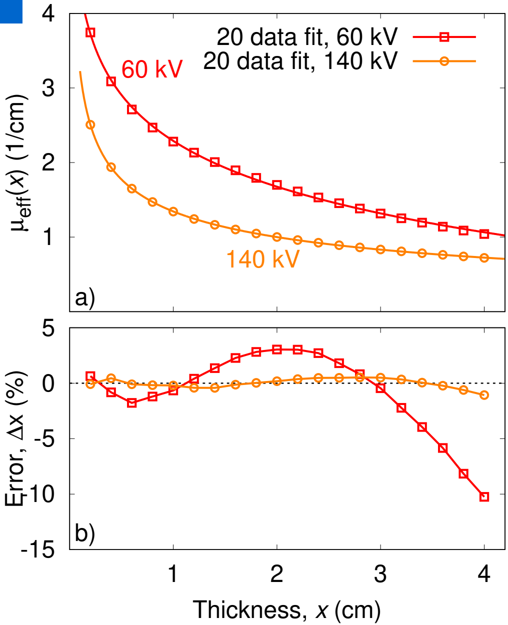
Model function

$$\mu_{\text{eff}}(x) = a + \frac{b}{x^\alpha}$$

Solve

$$ax + bx^{1-\alpha} + \ln\left(\frac{I(x)}{I_0}\right) = 0$$

e.g. Newton's method or look-up table



Determining the material thickness x

Generalized Beer-Lambert

$$I(x) = I_0 \exp(-\mu_{\text{eff}}(x)\, x)$$

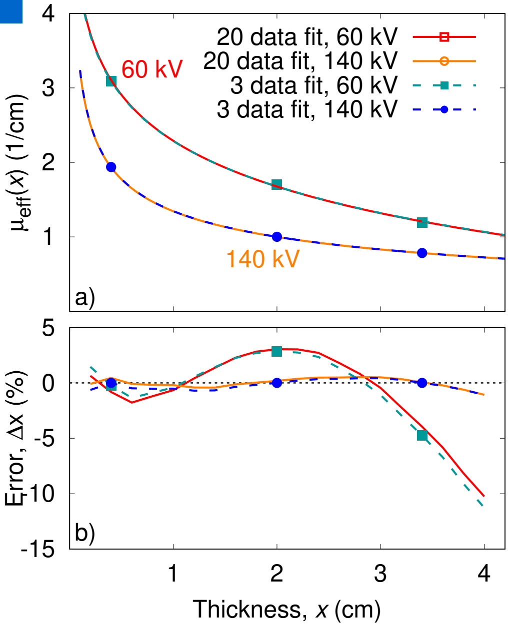
Model function

$$\mu_{\text{eff}}(x) = a + \frac{b}{x^{\alpha}}$$

Solve

$$ax + bx^{1-\alpha} + \ln\left(\frac{I(x)}{I_0}\right) = 0$$

e.g. Newton's method or look-up table



Migrating shear bands in shaken granular matter, Kollmer *et al* (2020)