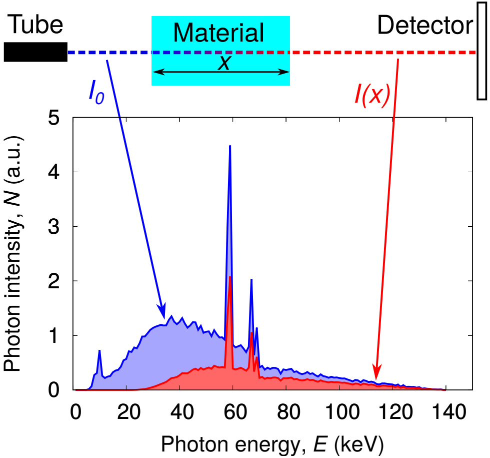
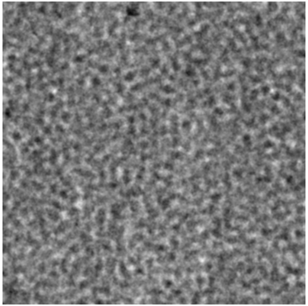
FRIEDRICH-ALEXANDER
UNIVERSITÄT
ERLANGEN-NÜRNBERG

Institute for
Multiscale Simulation
MSS

X-ray radiography of granular systems – particle densities and dynamics





PhD defense

Manuel Baur

Funded by the German Federal Ministry for Economic Affairs and Energy, grant no. 50WM 1653

Granular flows

Example Fluidized bed reactor

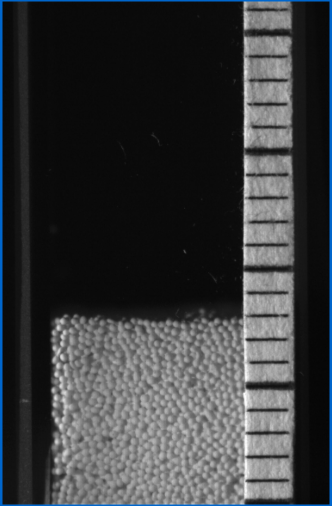
Wikipedia: Disadvantage: Lack of understanding: "It is very difficult to predict and calculate the complex mass and heat flows within the bed."

Particle dynamics $\leftrightarrow$ densities

1. Motivate physical system of granular flow (nice challenge to study)
2. Motivate technique of radiography

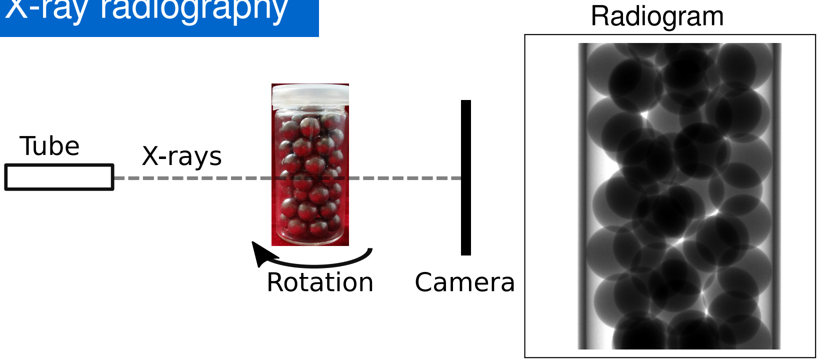
X-ray radiography of granular systems

Particulate flows are **opaque**



Master thesis Welm Pätzold

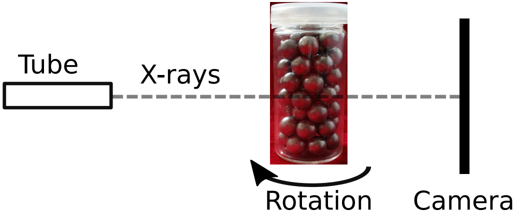
X-ray radiography



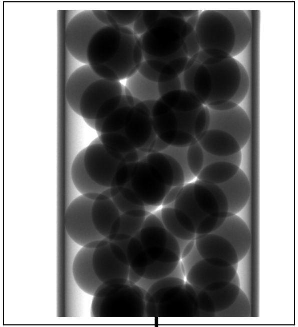
2D projections of 3D object

Short acquisition time

X-ray radiography



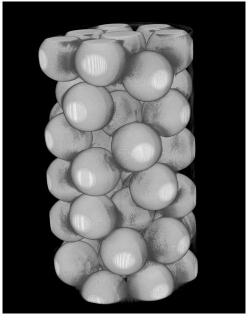
Radiogram



2D projections of 3D object
Short acquisition time

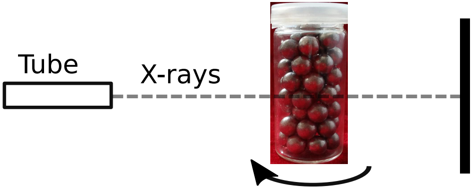
Reconstruction from
800 projections

Tomogram

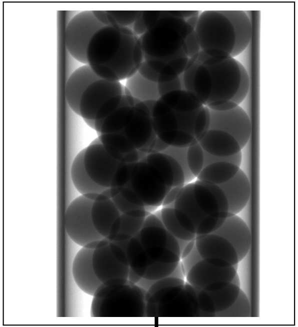


Full 3D information
Long acquisition time
→ static objects

X-ray radiography



Radiogram



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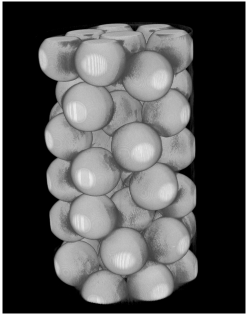


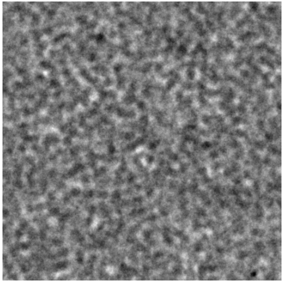
Reconstruction from
800 projections

Full 3D information

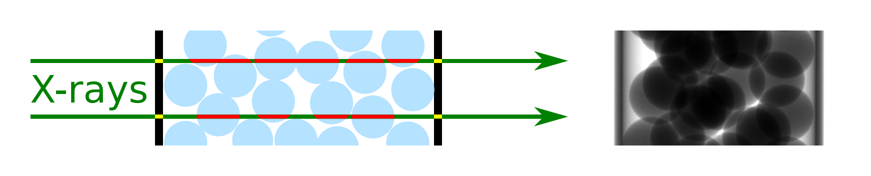
Long acquisition time

→ static objects

Dynamic system



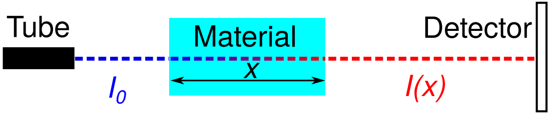
Measuring the volume fraction of **dynamic** granular systems



Correction of beam hardening in X-ray radiograms

In collaboration with Norman Uhlmann, Fraunhofer EZRT

Attenuation of X-rays

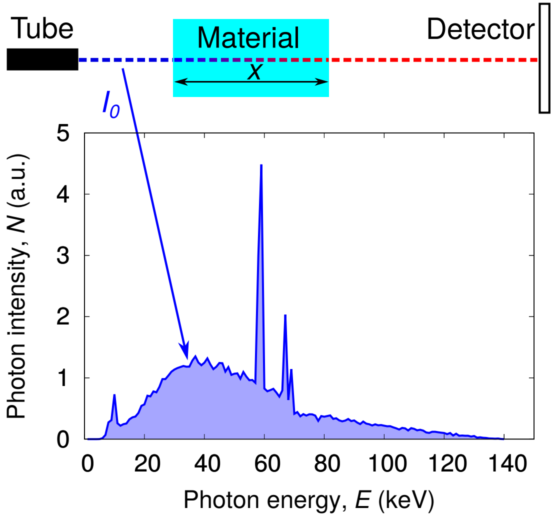


Beer-Lambert's law

$$I(x) = I_0 \exp(-\mu\, x)$$

Thickness: $x = -\frac{1}{\mu} \ln \frac{I(x)}{I_0}$

Attenuation of X-rays

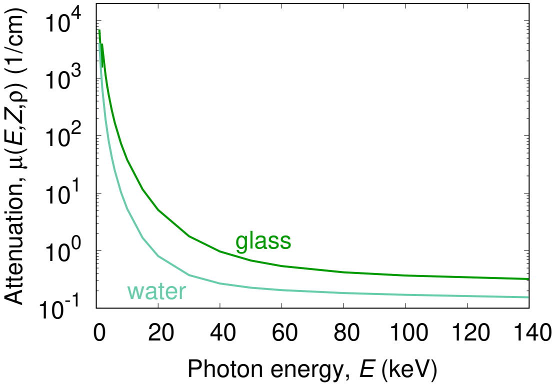


Beer-Lambert's law

$$I(x) = I_0 \exp(-\mu(E, Z, \rho)\, x)$$

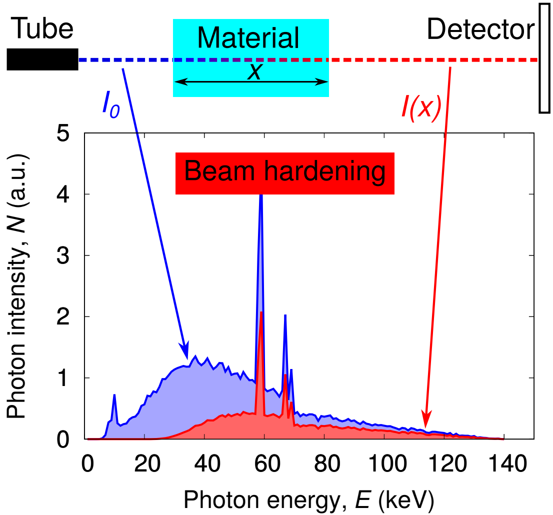
$$\mu \neq \text{const}$$

Thickness: $\boxed{x = ?}$



https://www.nist.gov/pml/
x-ray-mass-attenuation-coefficients

Attenuation of X-rays



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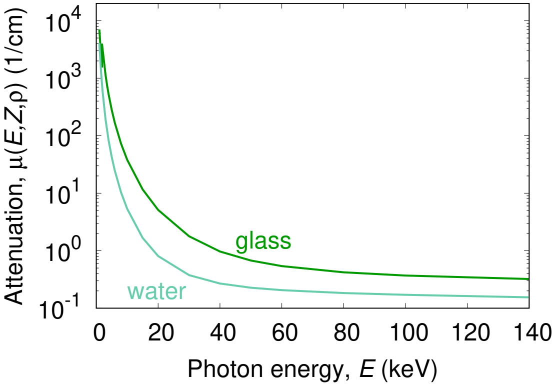

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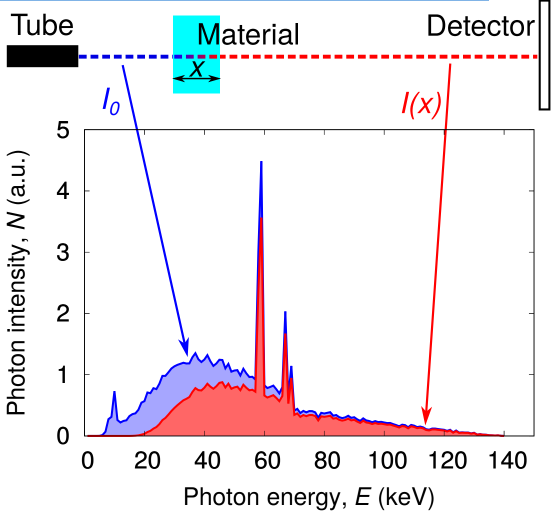
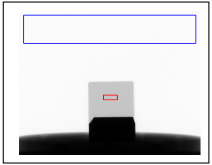
The effective attenuation, μ_{eff}

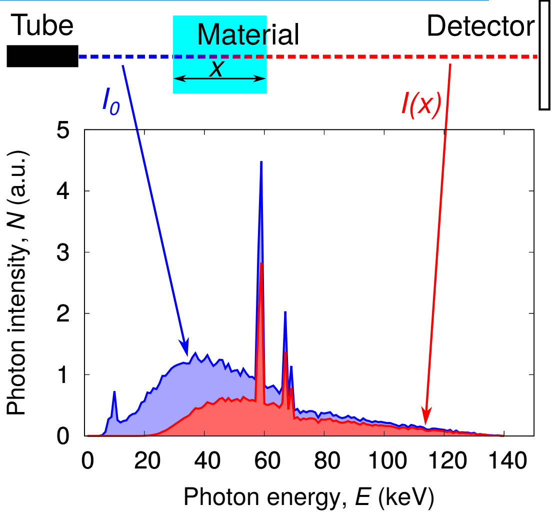


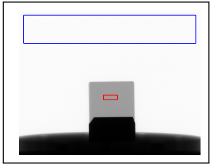


effective attenuation:

$$I(x) = I_0 \exp(-\mu_{\text{eff}}(x)\, x)$$

The effective attenuation, μ_{eff}





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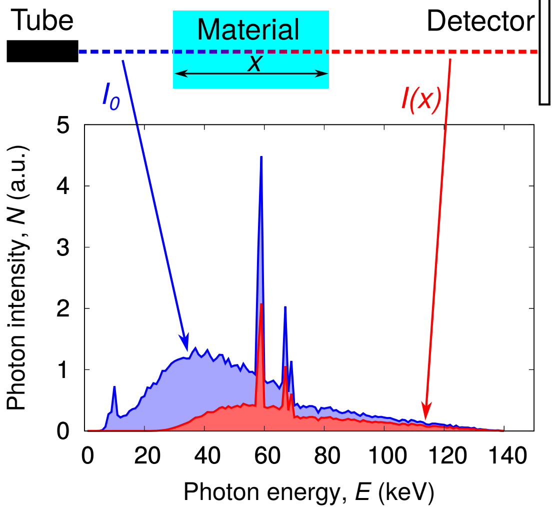
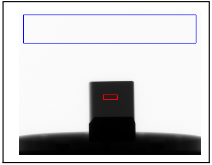
The effective attenuation, μ_{eff}





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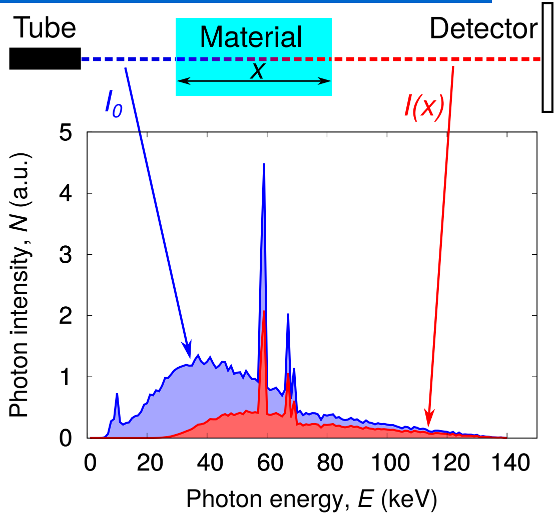
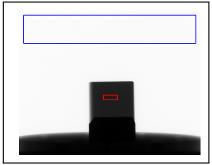
The effective attenuation, μ_{eff}

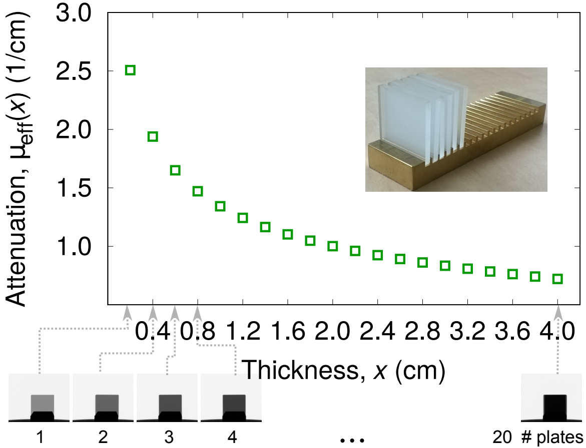




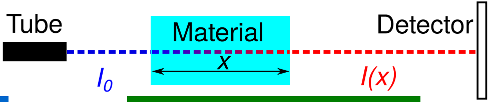
effective attenuation:

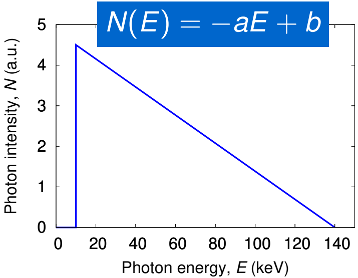
$$I(x) = I_0 \exp(-\mu_{\mathrm{eff}}(x)\, x)$$

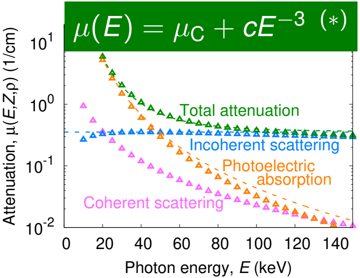
$$\mu_{\mathrm{eff}}(x) = -\frac{1}{x}\frac{I(x)}{I_0}$$

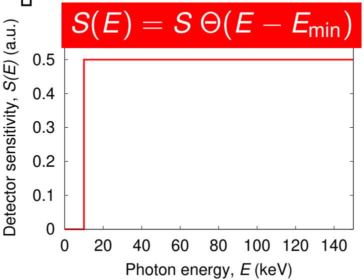


Modeling of μ_{eff}





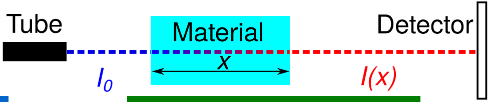


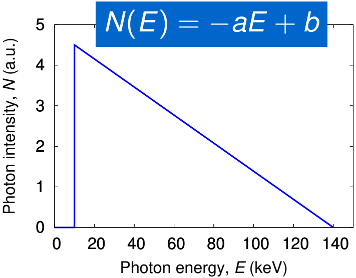


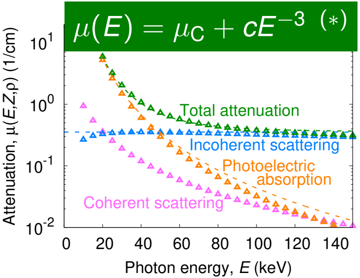
$$I(x) \propto \int N(E) \exp\{-\mu(E, Z, \rho)x\} S(E)\, \mathrm{d}E$$

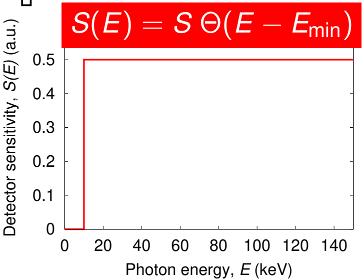
(∗) XCOM supplied by NIST

Modeling of μ_{eff}









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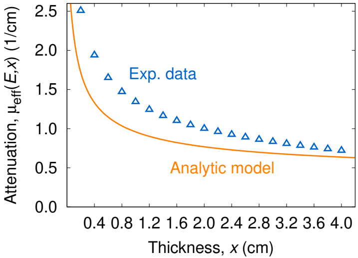
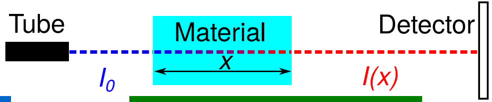
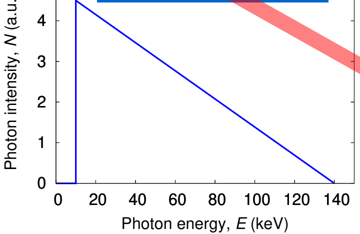


$(*)$ XCOM supplied by NIST

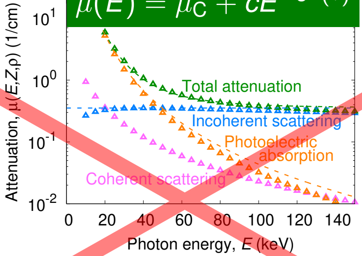
Modeling of μ_{eff}



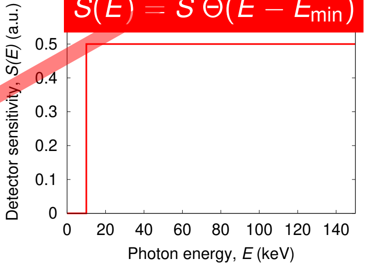
$N(E) = -aE + b$



$\mu(E) = \mu_C + cE^{-3}$ (∗)



$S(E) = S\,\Theta(E - E_{\mathrm{min}})$



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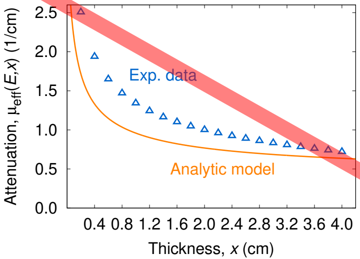
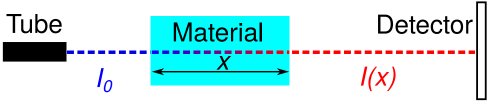
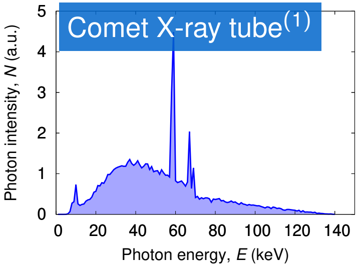
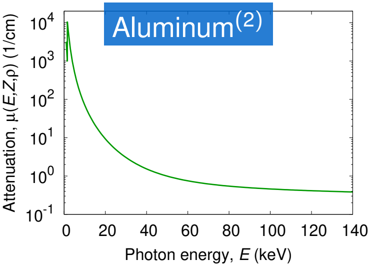


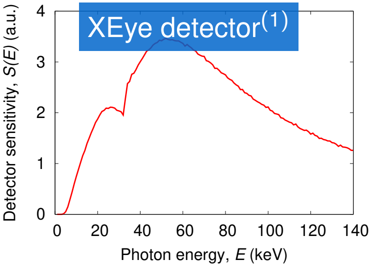
(∗) XCOM supplied by NIST

Numerical approx. of μ_{eff}







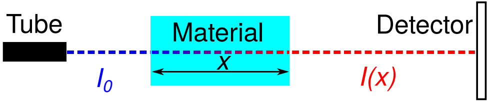


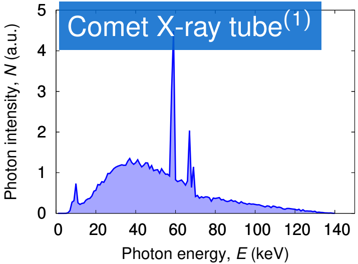
$$I(x) \propto \int N(E) \exp\{-\mu(E, Z, \rho) x\} S(E) \, \mathrm{d}E$$

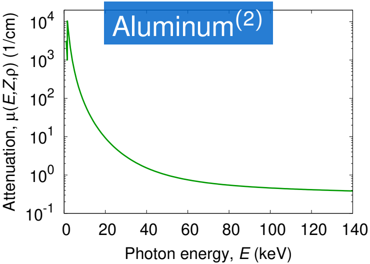
(1) Supplied by Norman Uhlmann, Fraunhofer EZRT

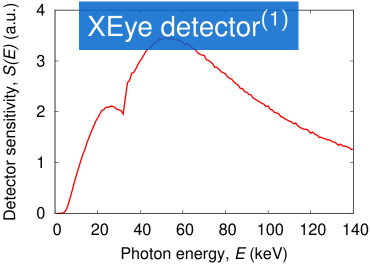
(2) XCOM supplied by NIST

Numerical approx. of μ_{eff}









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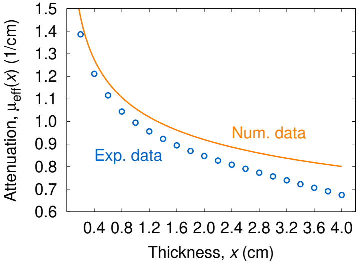

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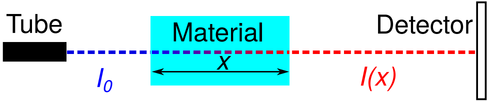

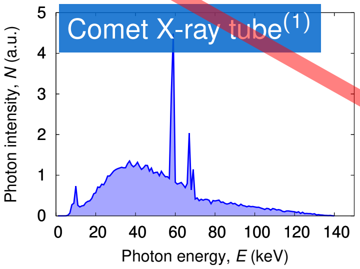

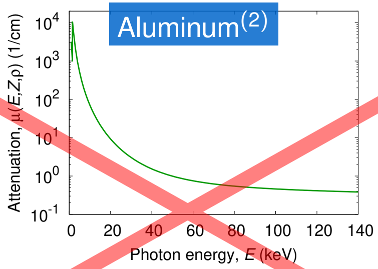

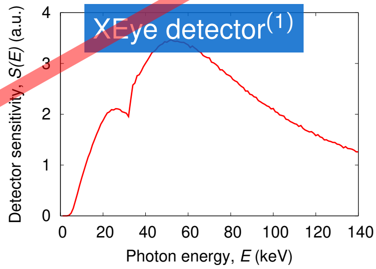

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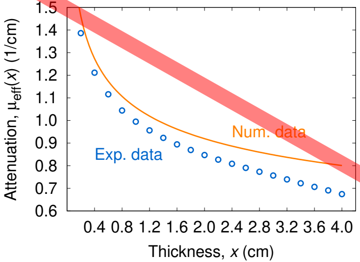

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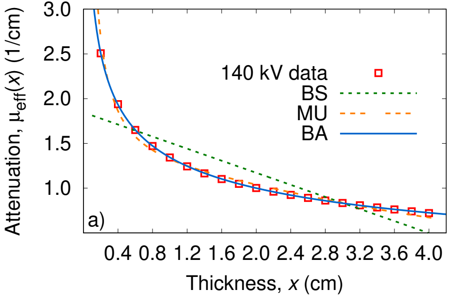
Heuristic model functions for μ_{eff}



$$\mu_{\text{eff}}(x) = \mu_0 - \lambda x$$

Bjärngard & Shackford (1994)

$$\mu_{\text{eff}}(x) = \frac{\mu_0}{1+\lambda x}$$

$$\mu_{\text{eff}}(x) = \frac{\mu_0}{(1+\lambda x)^\beta}$$

Yu *et al.* (1997)

$$\mu_{\text{eff}}(x) = \mu(E_{\text{max}}) + \frac{2\mu_1}{x\sqrt{-\lambda_1^2+4\lambda_2}} \times \left[\arctan\left(\frac{\lambda_1+2\lambda_2 x}{\sqrt{-\lambda_1^2+4\lambda_2}}\right) - \arctan\left(\frac{\lambda_1}{\sqrt{-\lambda_1^2+4\lambda_2}}\right)\right]$$

Kleinschmidt (1999)

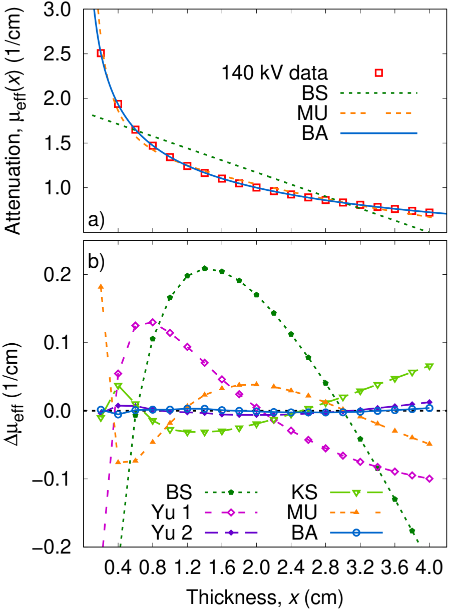
$$\mu_{\text{eff}}(x) = -\frac{1}{x}\ln\left[A + B\exp(-x/C)\right]$$

Mudde *et al.* (2008)

$$\mu_{\text{eff}}(x) = a + \frac{b}{x^\alpha}$$

Baur *et al.* (2019)
(this work)

Heuristic model functions for μ_{eff}



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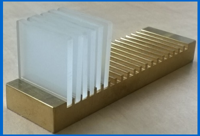
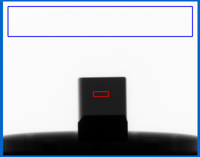
$$\boldsymbol{\mu_{\text{eff}}(x) = a + \frac{b}{x^\alpha}}$$

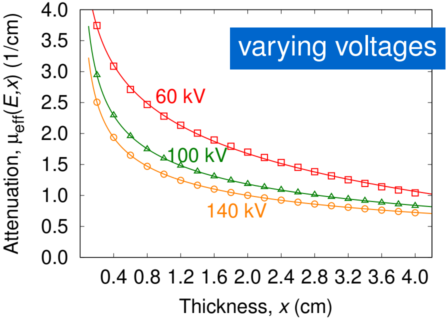
Baur *et al.* (2019)

(this work)

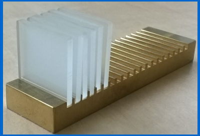
Universality of $\mu_{\text{eff}}(x) = a + \frac{b}{x^{\alpha}}$

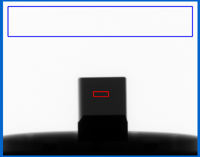


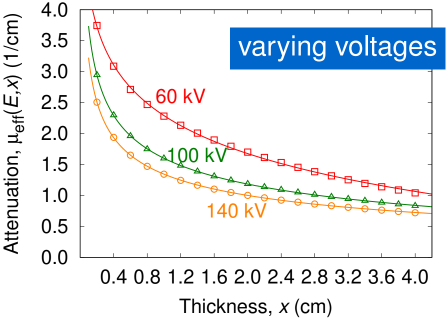


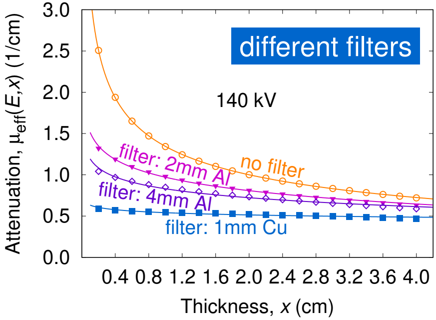


Universality of $\mu_{\text{eff}}(x) = a + \frac{b}{x^{\alpha}}$









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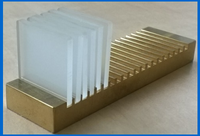

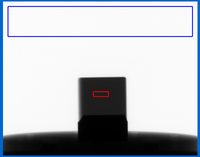

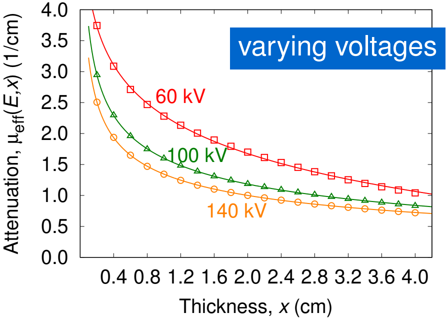

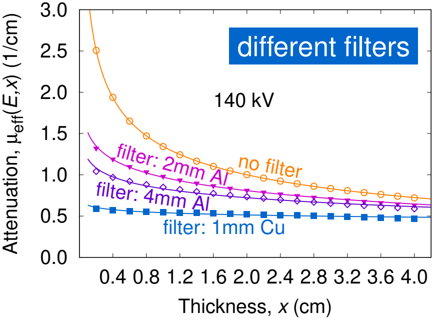

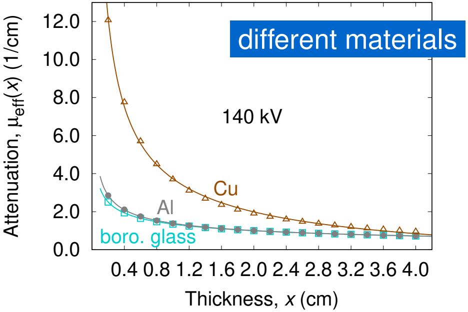

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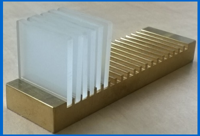
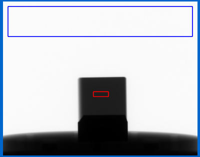
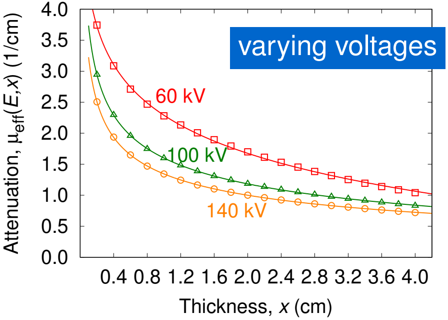
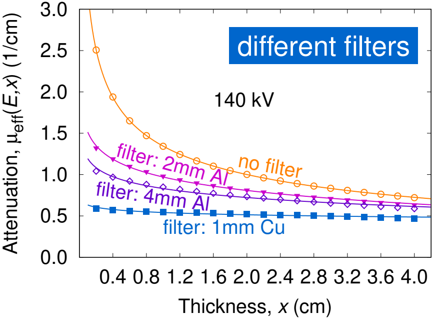
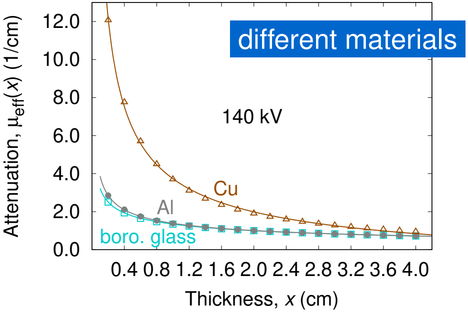
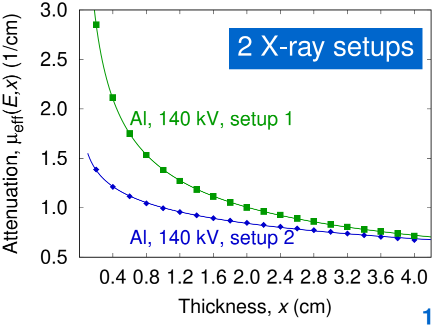












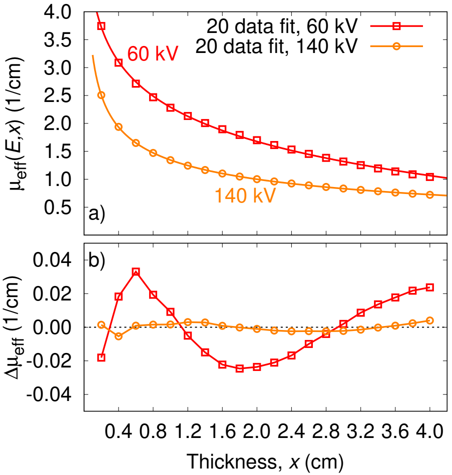
Determining the material thickness x

Generalized Beer-Lambert

$$I(x) = I_0 \exp(-\mu_{\text{eff}}(x)\, x)$$

Model function

$$\mu_{\text{eff}}(x) = a + \frac{b}{x^{\alpha}}$$



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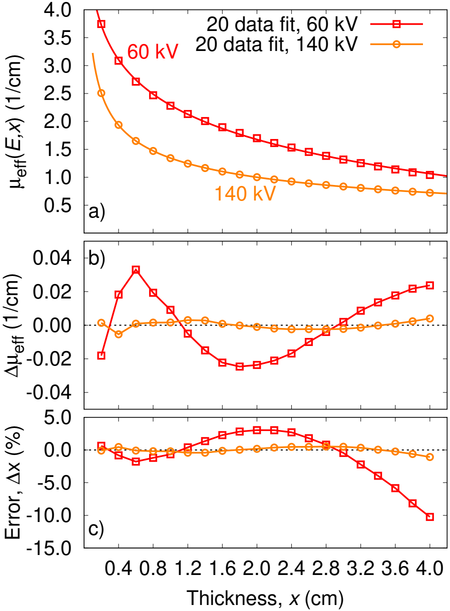
$$ax + bx^{1-\alpha} + \ln\left(\frac{I(x)}{I_0}\right) = 0$$

e.g. Newton's method or look-up table



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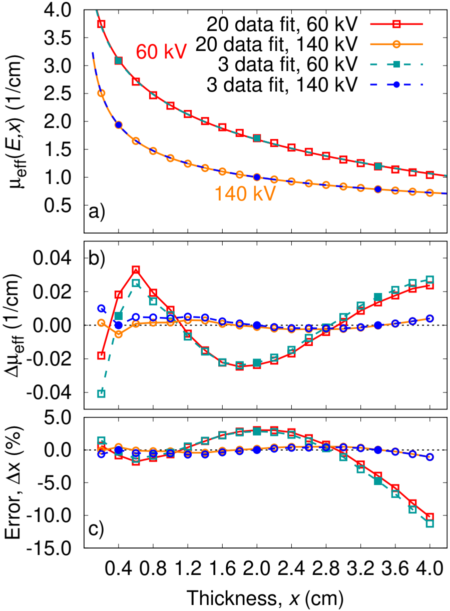

Migrating shear bands in shaken granular matter, Kollmer *et al* (2020)