

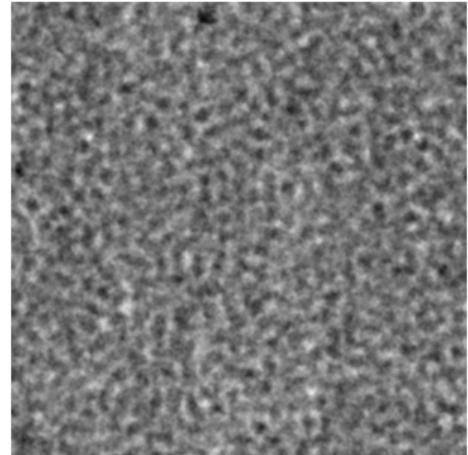
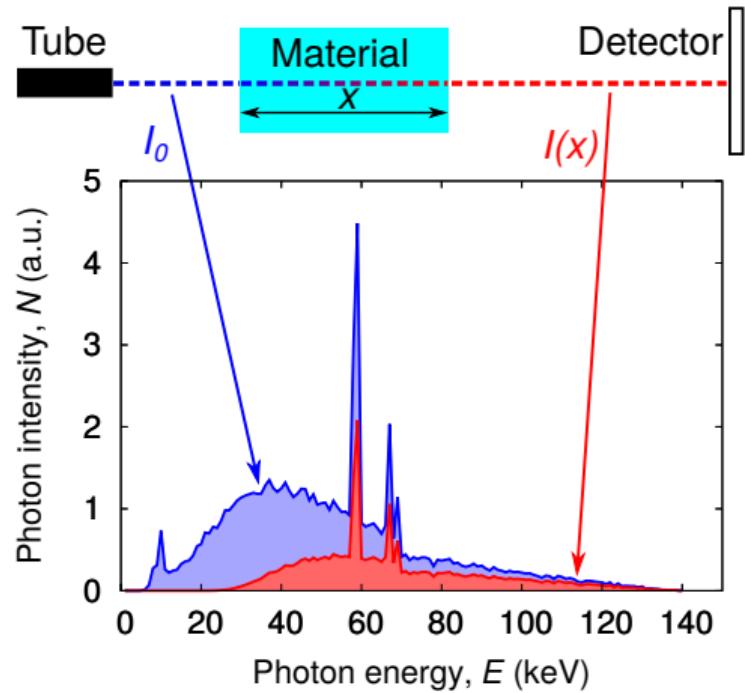


PhD defense  
**Manuel Baur**

Funded by the German  
Federal Ministry for  
Economic Affairs and  
Energy, grant no. 50WM

1653

## X-ray radiography of granular systems – particle densities and dynamics



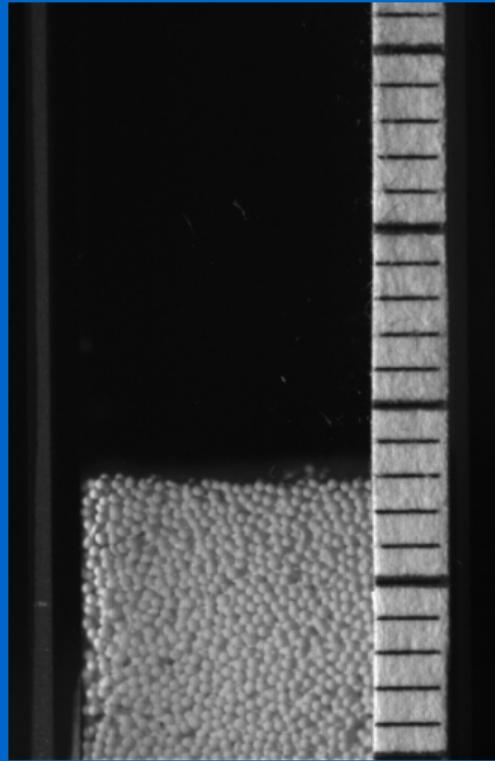
## Example Fluidized bed reactor

Wikipedia: Disadvantage: Lack of understanding: "It is very difficult to predict and calculate the complex mass and heat flows within the bed."

Particle dynamics ↔ densities

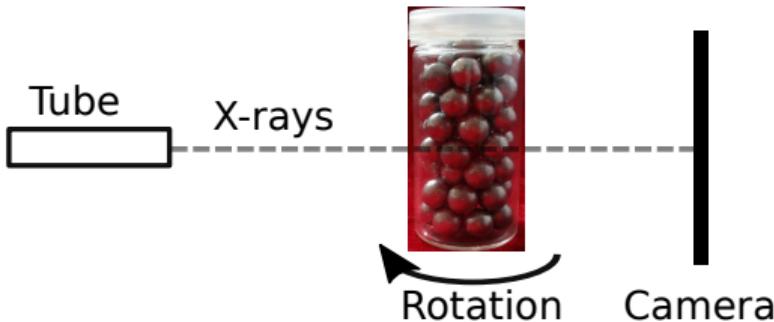
1. Motivate physical system of granular flow (nice challenge to study)
2. Motivate technique of radiography

Particulate flows are **opaque**

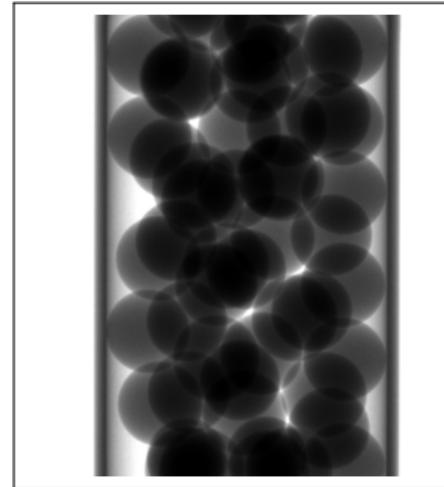


Master thesis Welm Pätzold

# X-ray radiography

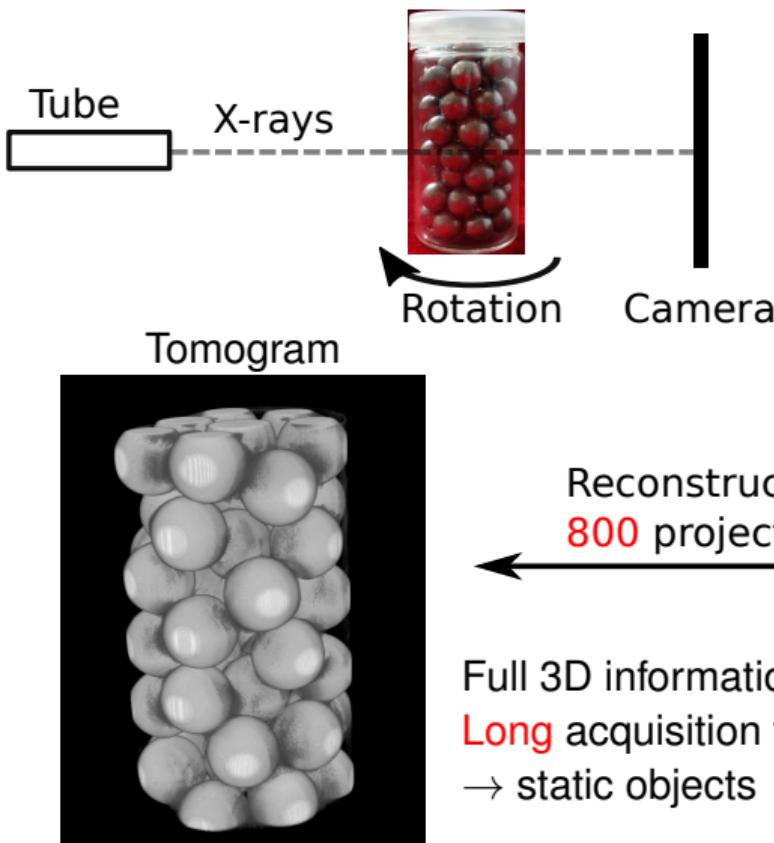


Radiogram



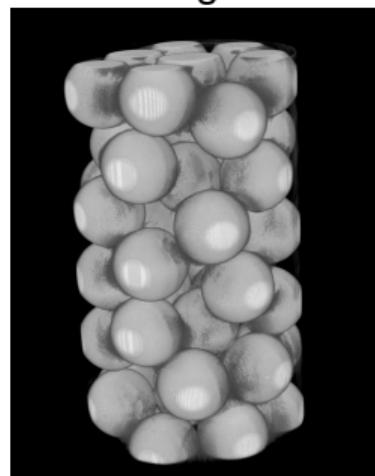
2D projections of 3D object  
Short acquisition time

# X-ray radiography



Radiogram

2D projections of 3D object  
Short acquisition time

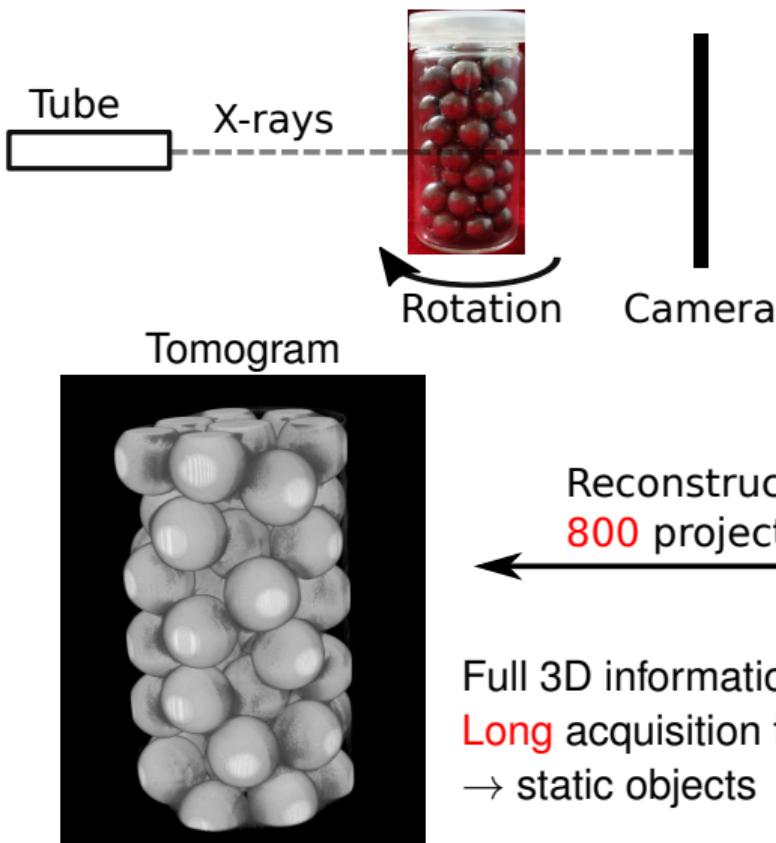


Tomogram

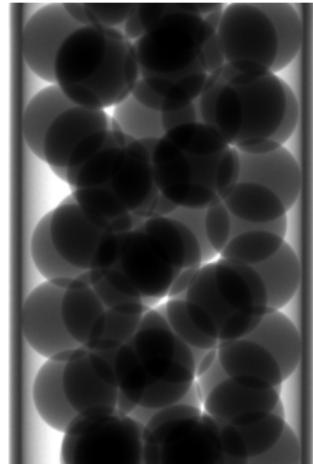
Reconstruction from  
800 projections

Full 3D information  
Long acquisition time  
→ static objects

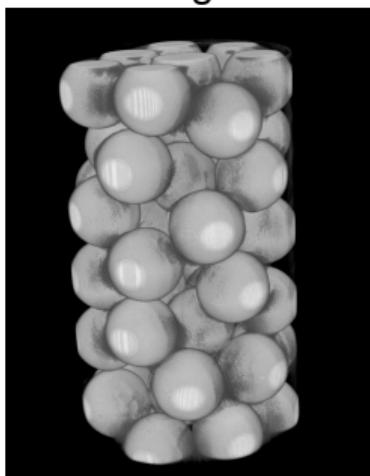
# X-ray radiography



Radiogram

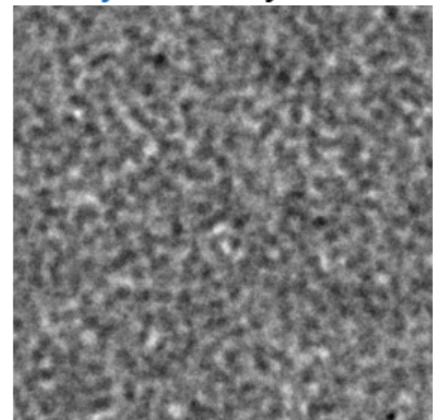


2D projections of 3D object  
Short acquisition time

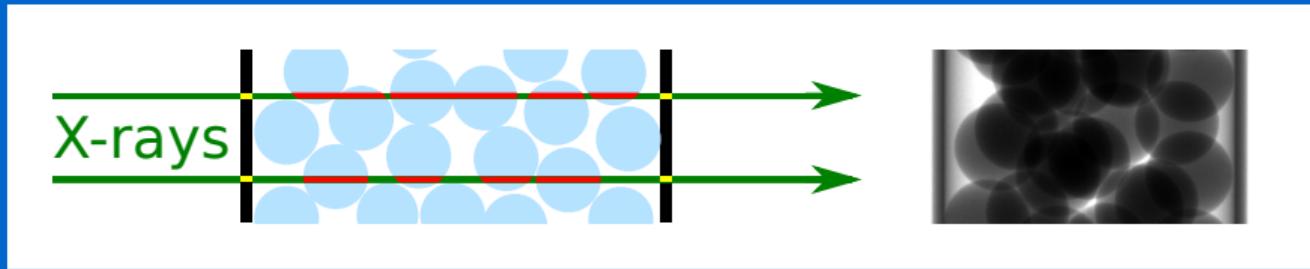


Full 3D information  
Long acquisition time  
→ static objects

Dynamic system



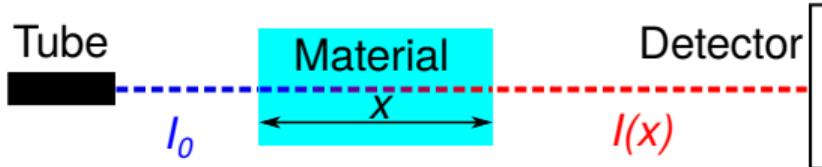
# Measuring the volume fraction of **dynamic** granular systems



## Correction of beam hardening in X-ray radiograms

In collaboration with Norman Uhlmann, Fraunhofer EZRT

# Attenuation of X-rays

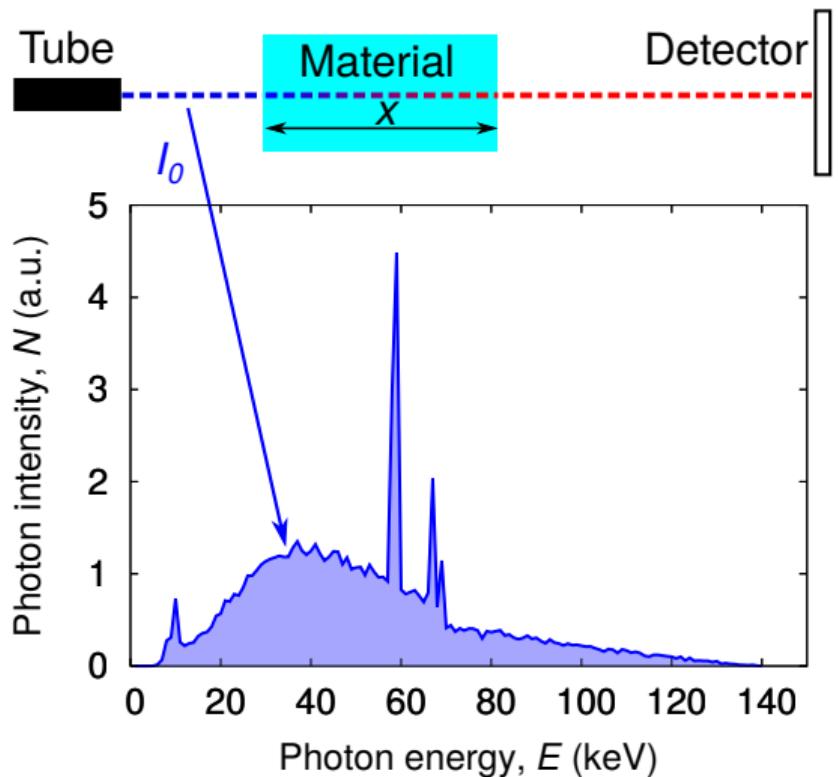


Beer-Lambert's law

$$I(x) = I_0 \exp(-\mu x)$$

$$\text{Thickness: } x = -\frac{1}{\mu} \ln \frac{I(x)}{I_0}$$

# Attenuation of X-rays

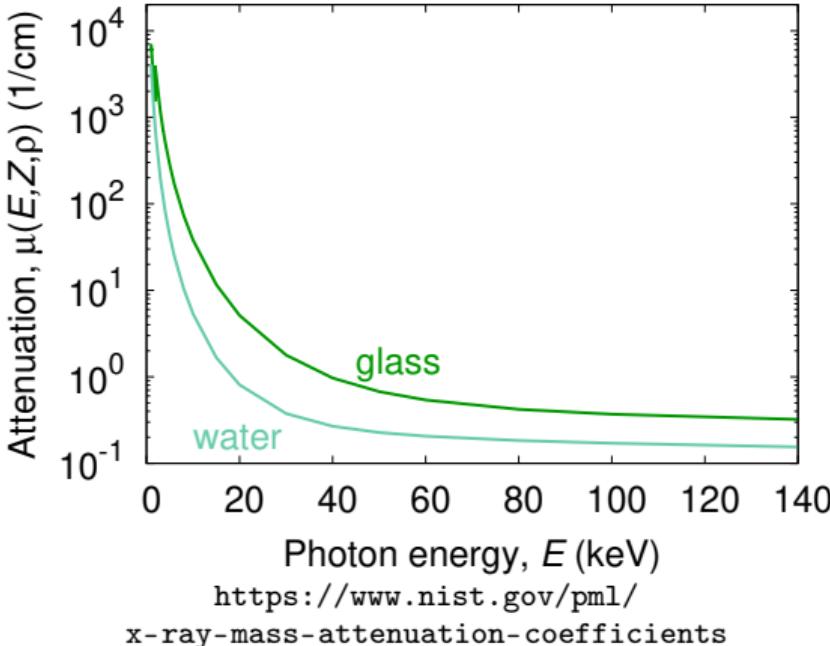


Beer-Lambert's law

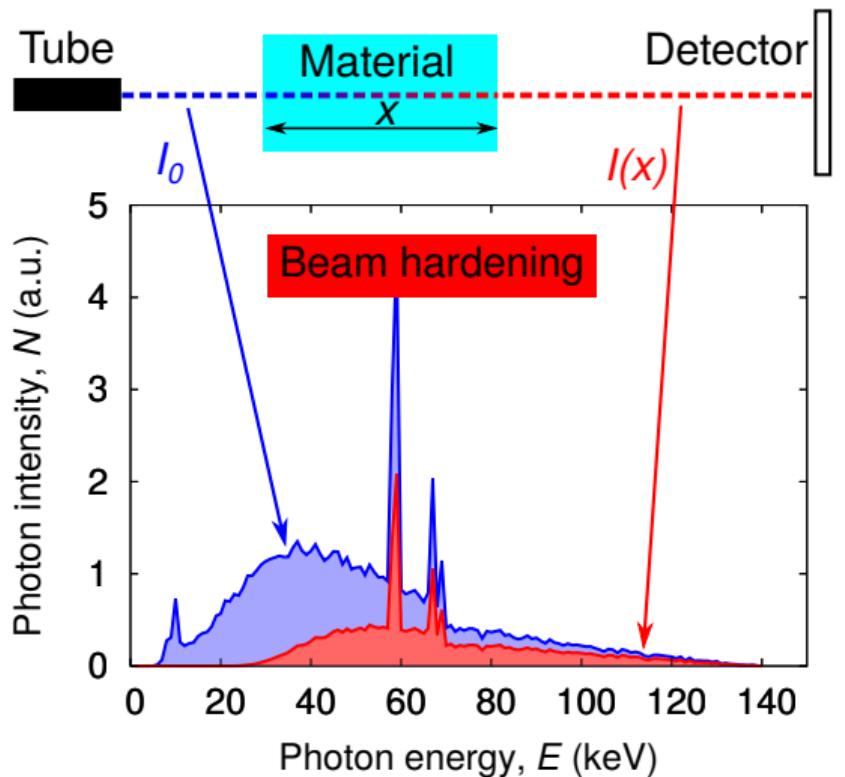
$$I(x) = I_0 \exp(-\mu(E, Z, \rho)x)$$

$\mu \neq \text{const}$

Thickness:  $x = ?$



# Attenuation of X-rays

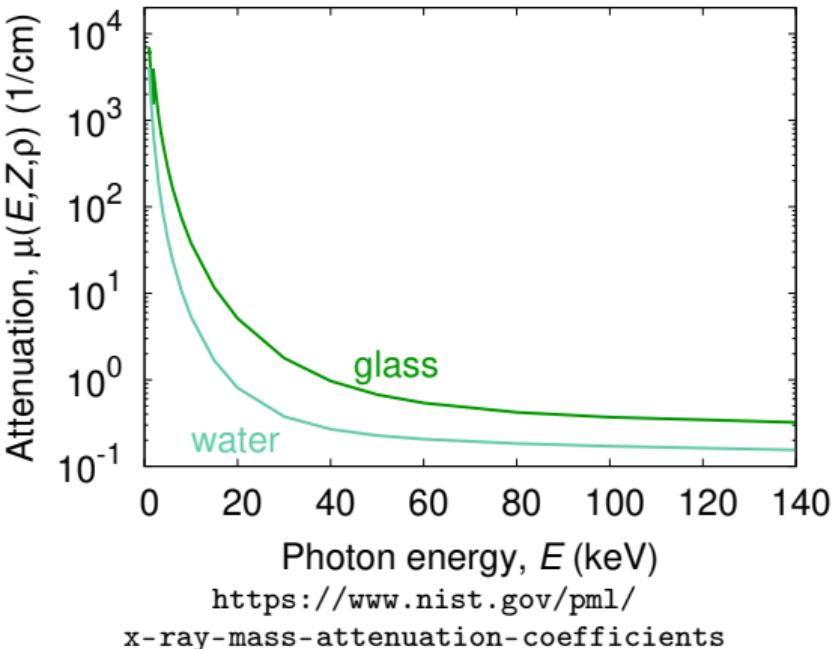


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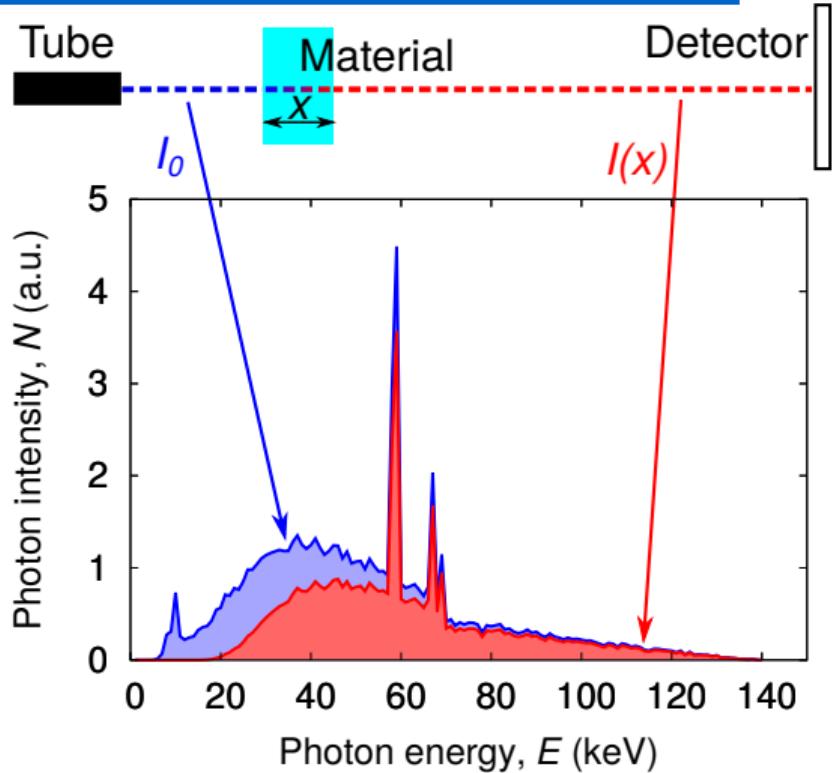
$\mu \neq \text{const}$

Thickness:  $x = ?$



[https://www.nist.gov/pml/  
x-ray-mass-attenuation-coefficients](https://www.nist.gov/pml/x-ray-mass-attenuation-coefficients)

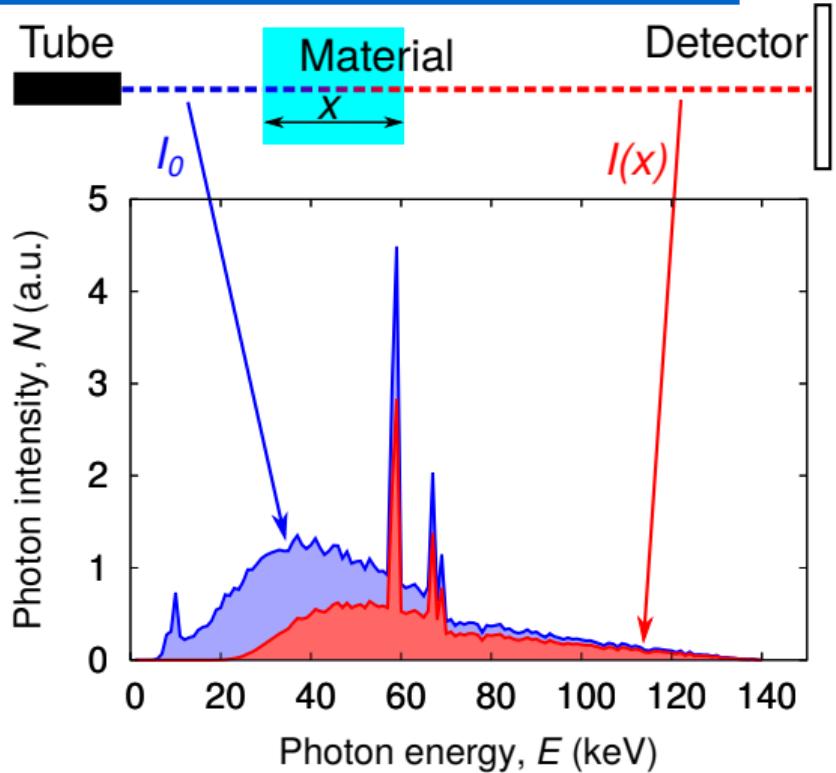
# The effective attenuation, $\mu_{\text{eff}}$



effective attenuation:

$$I(x) = I_0 \exp(-\mu_{\text{eff}}(x) x)$$

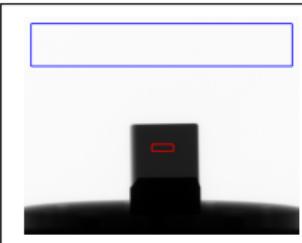
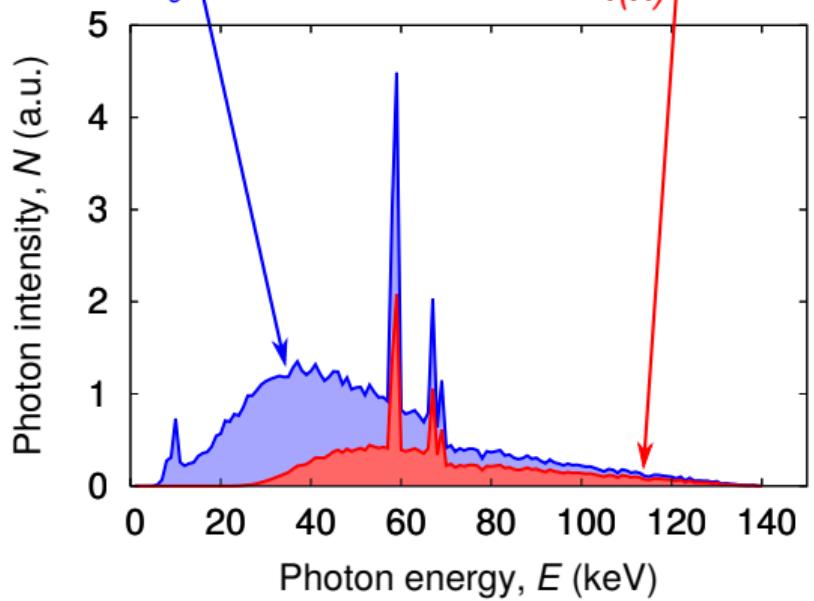
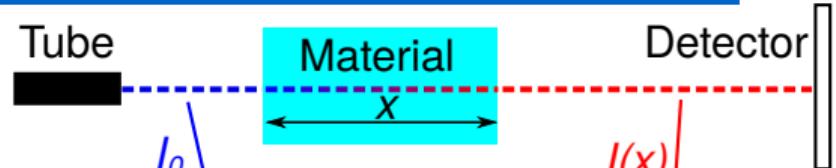
## The effective attenuation, $\mu_{\text{eff}}$



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$$I(x) = I_0 \exp(-\mu_{\text{eff}}(x) x)$$

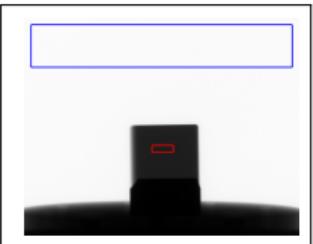
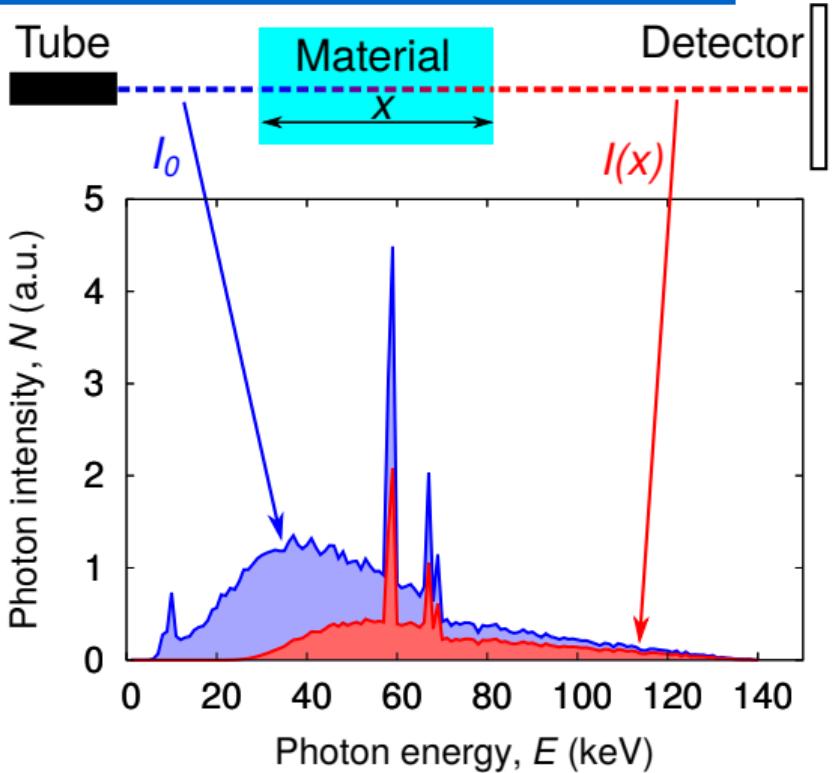
## The effective attenuation, $\mu_{\text{eff}}$



effective attenuation:

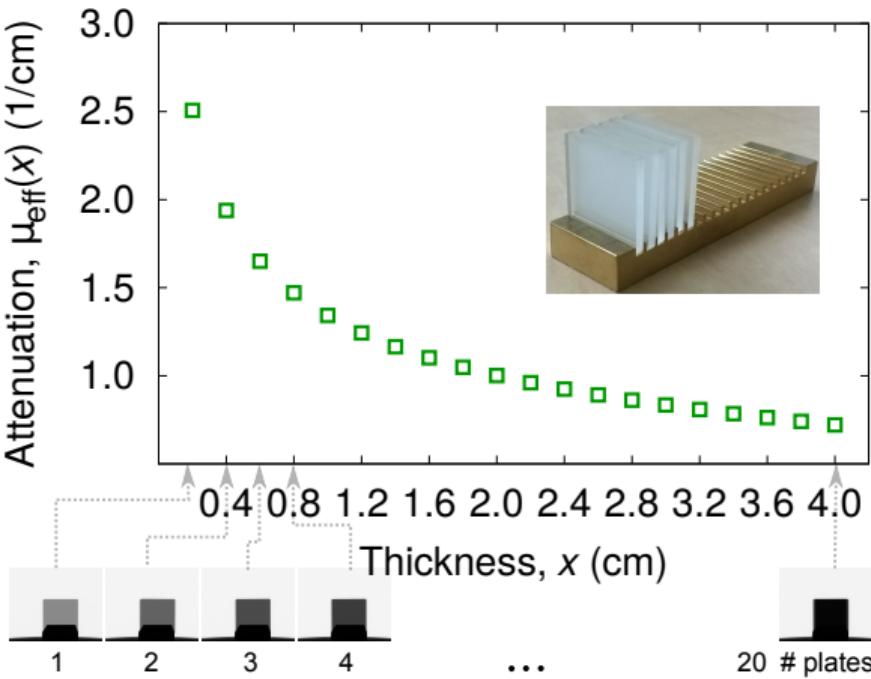
$$I(x) = I_0 \exp(-\mu_{\text{eff}}(x) x)$$

# The effective attenuation, $\mu_{\text{eff}}$

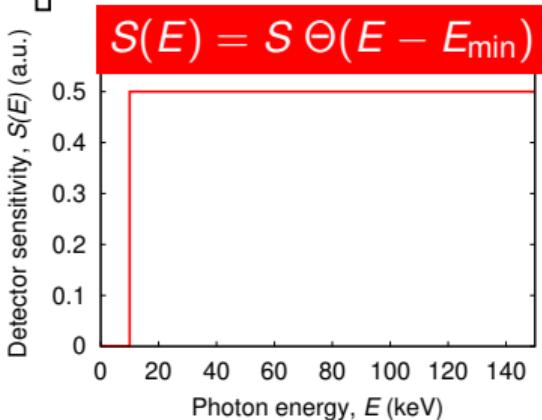
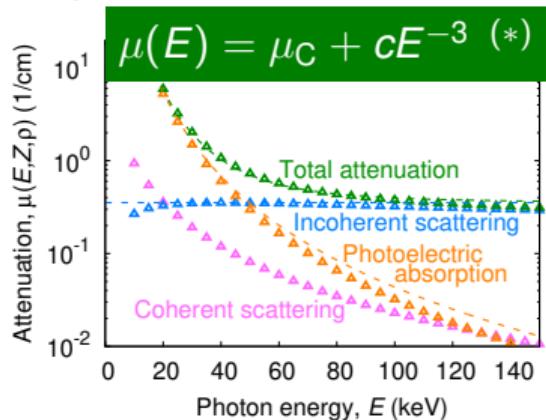
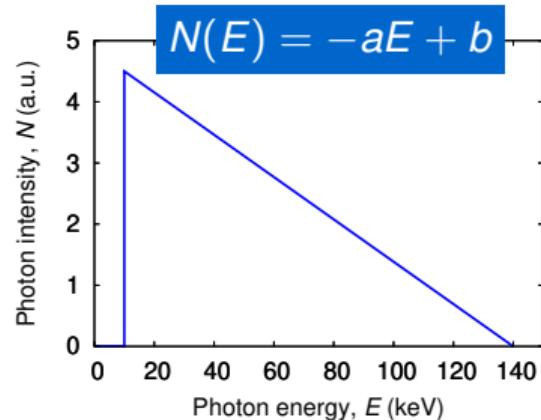
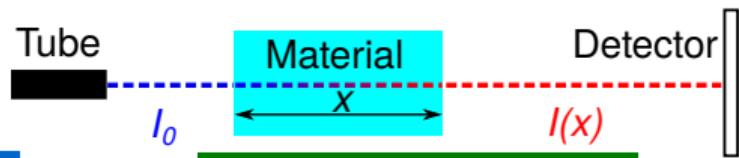


*effective* attenuation:  
 $I(x) = I_0 \exp(-\mu_{\text{eff}}(x)x)$

$$\mu_{\text{eff}}(x) = -\frac{1}{x} \frac{I(x)}{I_0}$$



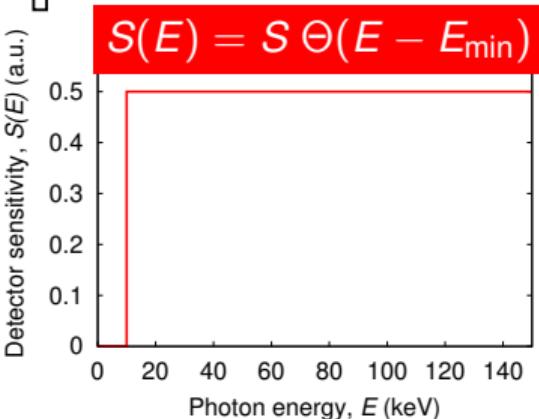
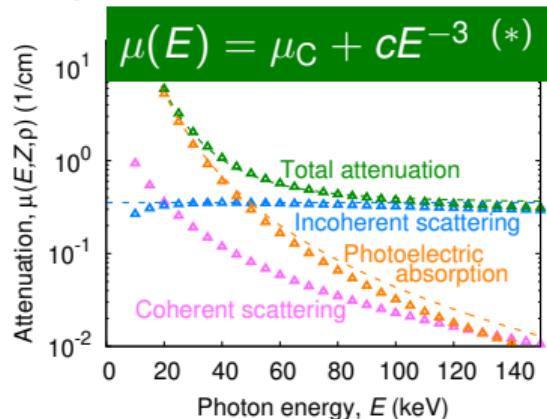
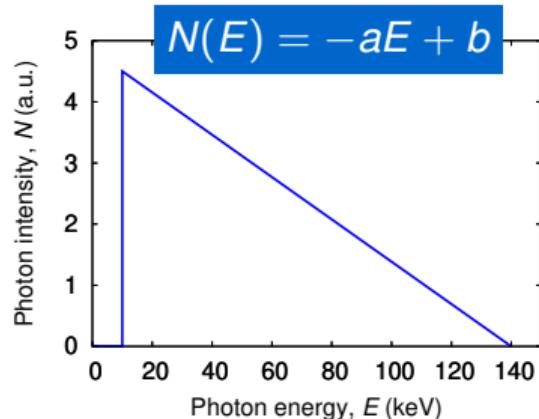
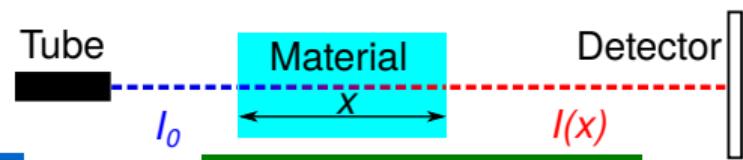
# Modeling of $\mu_{\text{eff}}$



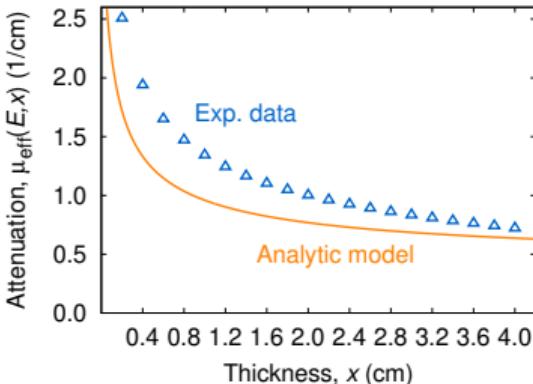
$$I(x) \propto \int N(E) \exp\{-\mu(E, Z, \rho)x\} S(E) dE$$

(\*) XCOM supplied by NIST

# Modeling of $\mu_{\text{eff}}$

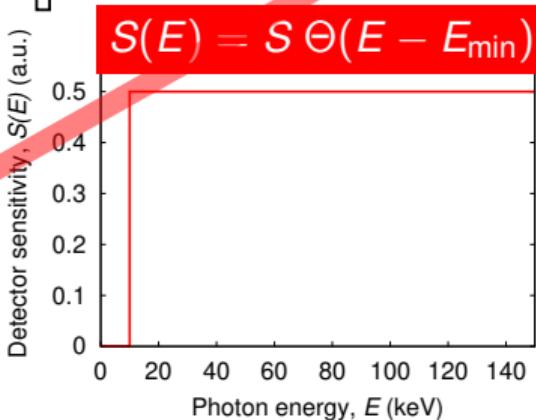
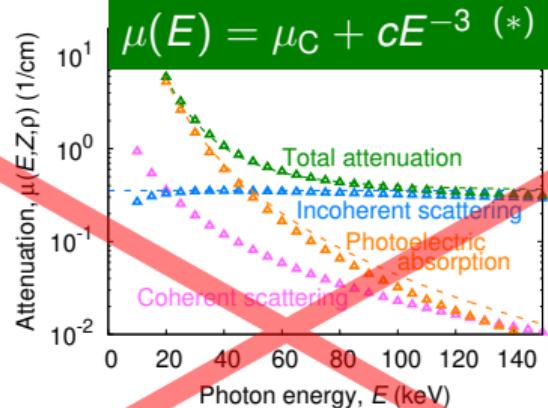
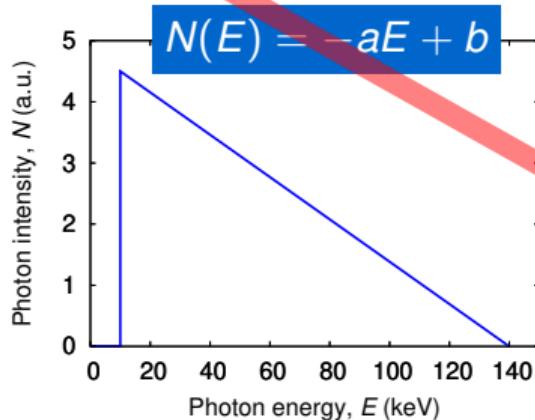
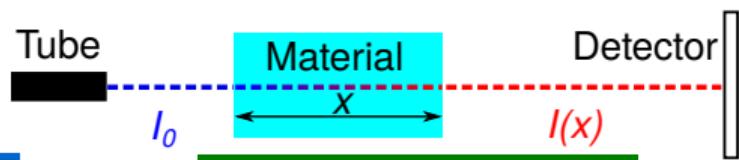


$$I(x) \propto \int N(E) \exp\{-\mu(E, Z, \rho)x\} S(E) dE$$

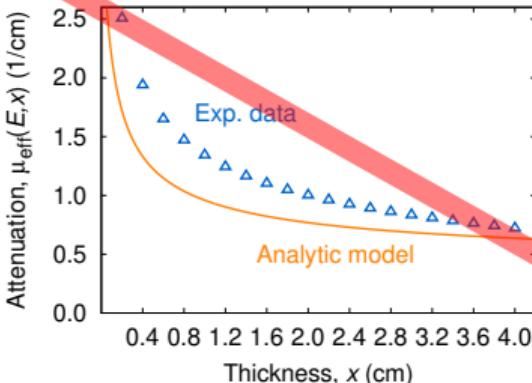


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# Modeling of $\mu_{\text{eff}}$

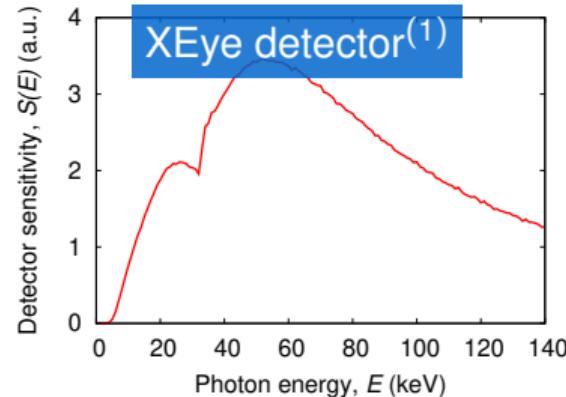
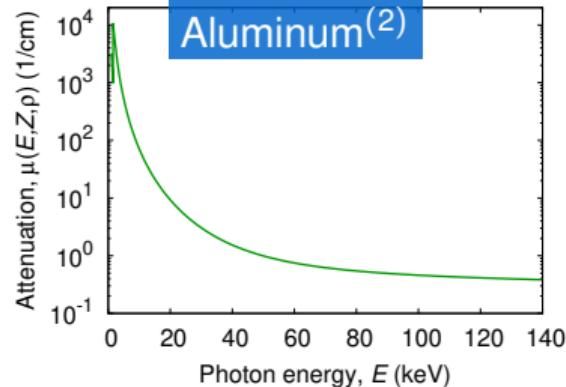
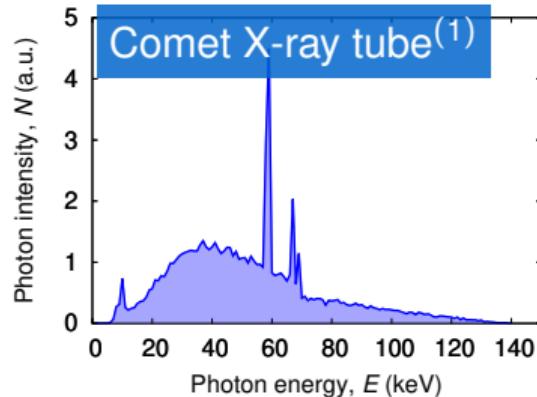
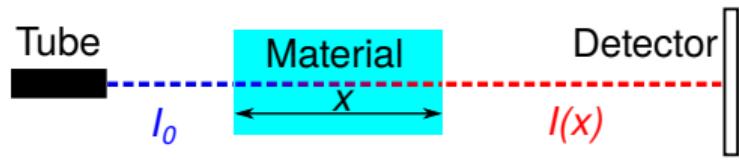


$$I(x) \propto \int N(E) \exp\{-\mu(E, Z, \rho)x\} S(E) dE$$



(\*) XCOM supplied by NIST

# Numerical approx. of $\mu_{\text{eff}}$

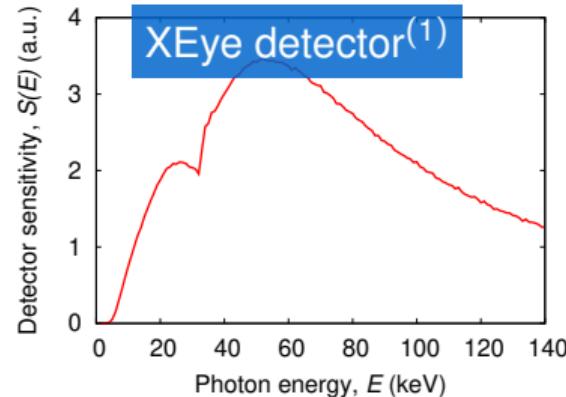
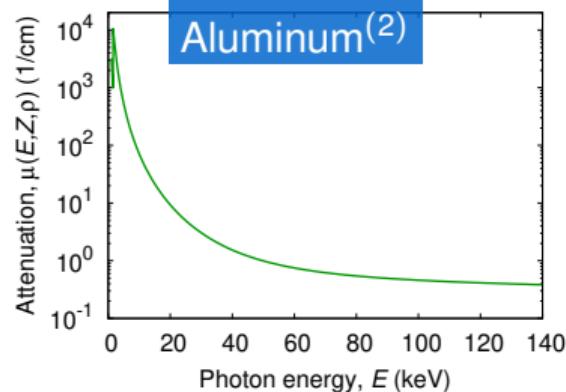
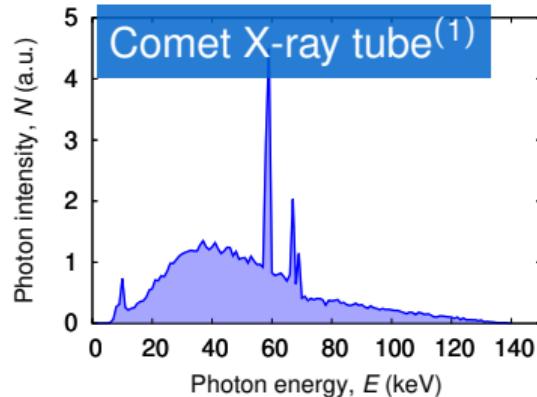
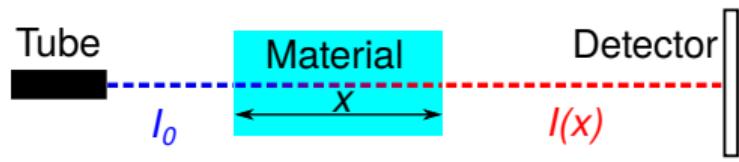


$$I(x) \propto \int N(E) \exp\{-\mu(E, Z, \rho)x\} S(E) dE$$

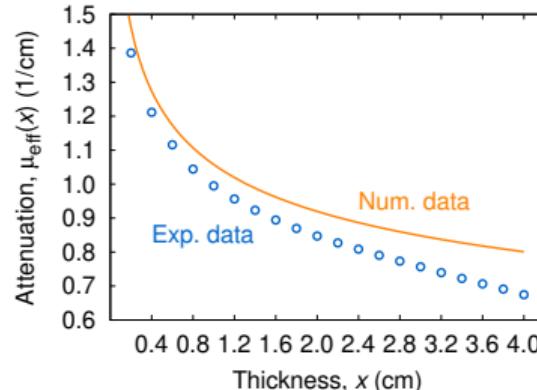
(1) Supplied by Norman Uhlmann, Fraunhofer EZRT

(2) XCOM supplied by NIST

# Numerical approx. of $\mu_{\text{eff}}$



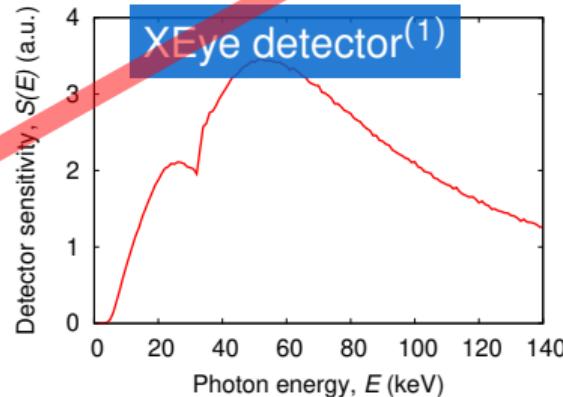
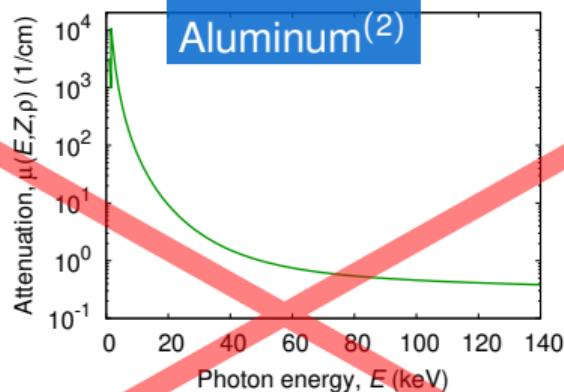
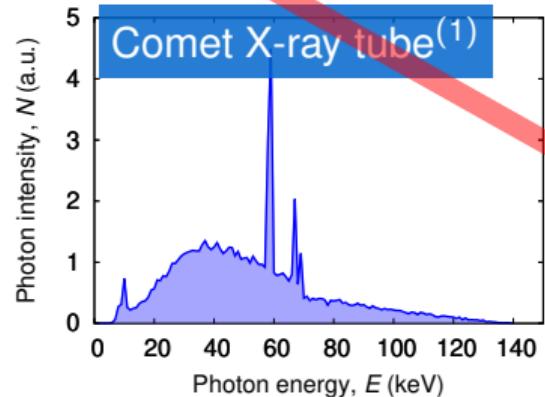
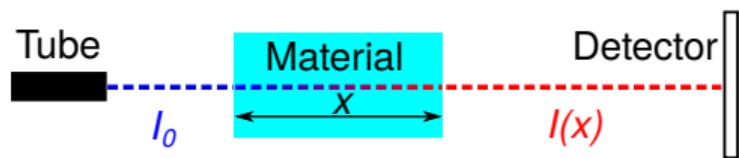
$$I(x) \propto \int N(E) \exp\{-\mu(E, Z, \rho)x\} S(E) dE$$



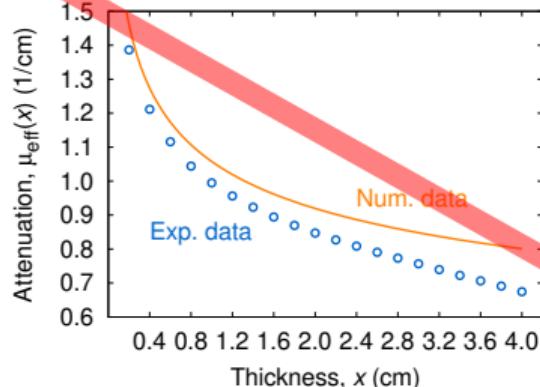
(1) Supplied by Norman Uhlmann, Fraunhofer EZRT

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# Numerical approx. of $\mu_{\text{eff}}$



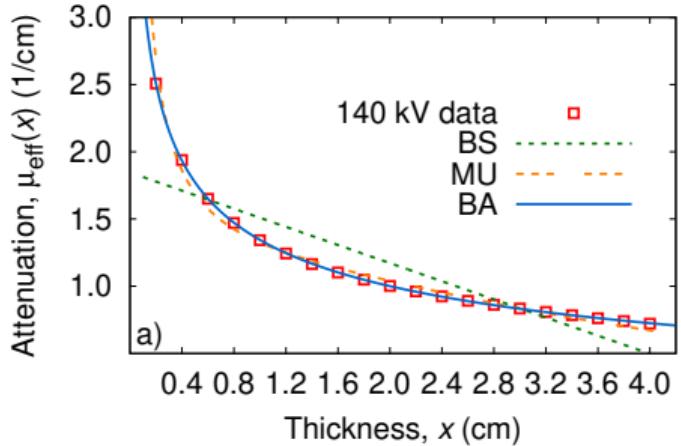
$$I(x) \propto \int N(E) \exp\{-\mu(E, Z, \rho)x\} S(E) dE$$



(1) Supplied by Norman Uhlmann, Fraunhofer EZRT

(2) XCOM supplied by NIST

# Heuristic model functions for $\mu_{\text{eff}}$



$$\mu_{\text{eff}}(x) = \mu_0 - \lambda x$$

$$\mu_{\text{eff}}(x) = \frac{\mu_0}{1 + \lambda x}$$

$$\mu_{\text{eff}}(x) = \frac{\mu_0}{(1 + \lambda x)^\beta}$$

Bjärngard & Shackford  
(1994)

Yu *et al.* (1997)

$$\mu_{\text{eff}}(x) = \mu(E_{\text{max}}) + \frac{2\mu_1}{x\sqrt{-\lambda_1^2 + 4\lambda_2}} \times$$

$$\left[ \arctan \left( \frac{\lambda_1 + 2\lambda_2 x}{\sqrt{-\lambda_1^2 + 4\lambda_2}} \right) - \arctan \left( \frac{\lambda_1}{\sqrt{-\lambda_1^2 + 4\lambda_2}} \right) \right]$$

Kleinschmidt (1999)

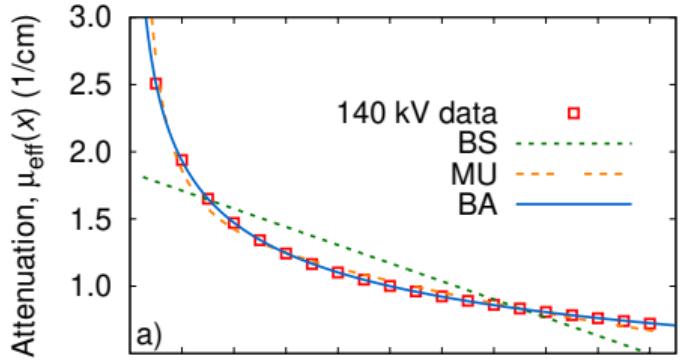
$$\mu_{\text{eff}}(x) = -\frac{1}{x} \ln [A + B \exp(-x/C)]$$

Mudde *et al.* (2008)

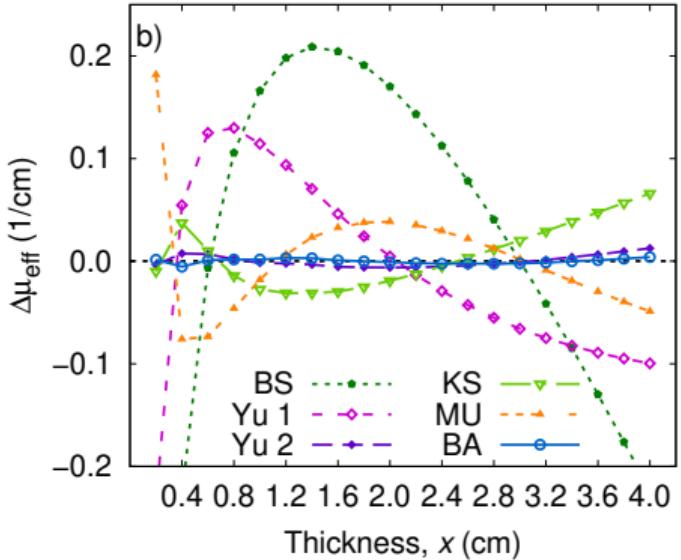
$$\mu_{\text{eff}}(x) = a + \frac{b}{x^\alpha}$$

Baur *et al.* (2019)  
(this work)

# Heuristic model functions for $\mu_{\text{eff}}$



$$\mu_{\text{eff}}(x) = \mu_0 - \lambda x$$



$$\mu_{\text{eff}}(x) = \frac{\mu_0}{1+\lambda x}$$

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Bjärngard & Shackford  
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$$\mu_{\text{eff}}(x) = \mu(E_{\text{max}}) + \frac{2\mu_1}{x\sqrt{-\lambda_1^2+4\lambda_2}} \times \left[ \arctan\left(\frac{\lambda_1+2\lambda_2 x}{\sqrt{-\lambda_1^2+4\lambda_2}}\right) - \arctan\left(\frac{\lambda_1}{\sqrt{-\lambda_1^2+4\lambda_2}}\right) \right]$$

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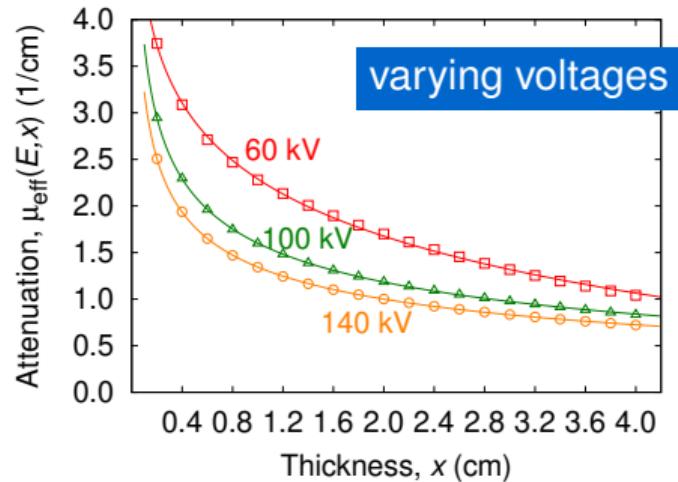
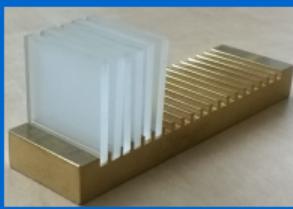
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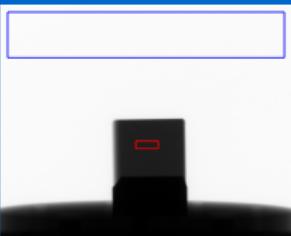
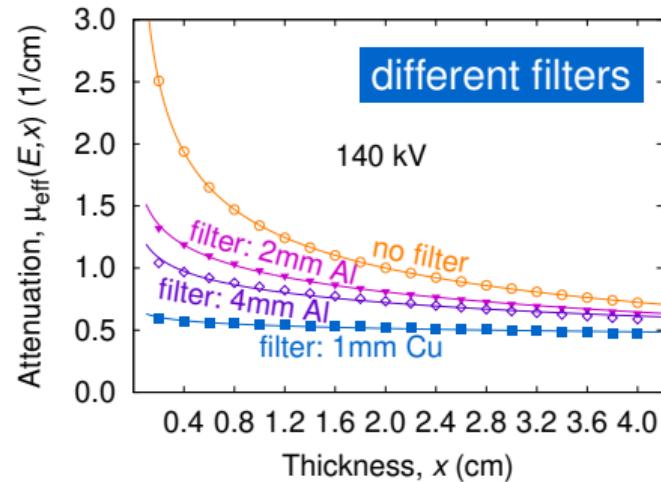
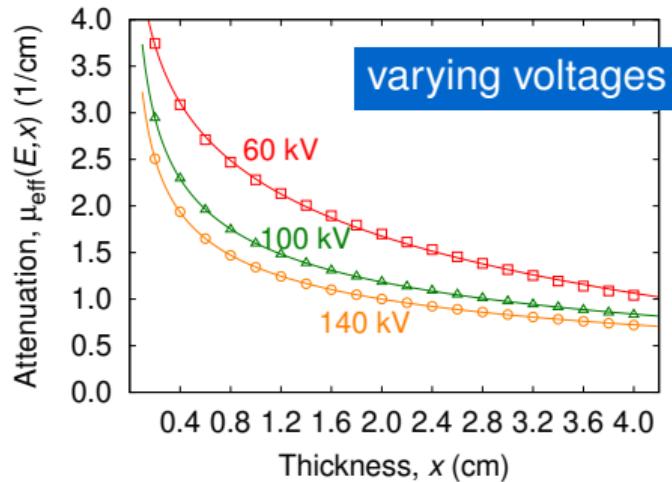
$$\mu_{\text{eff}}(x) = a + \frac{b}{x^\alpha}$$

Baur *et al.* (2019)  
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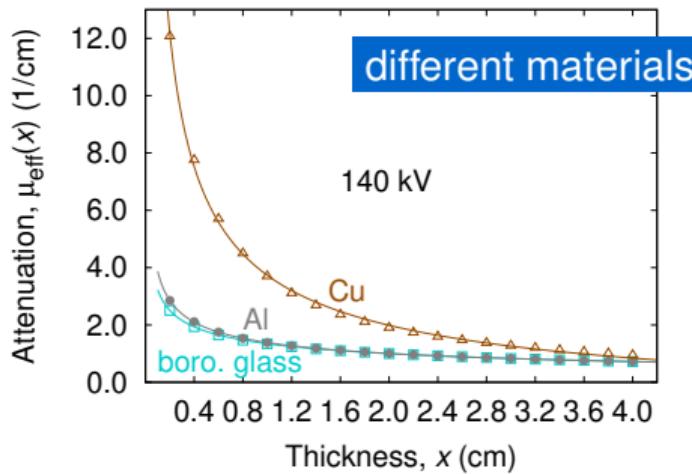
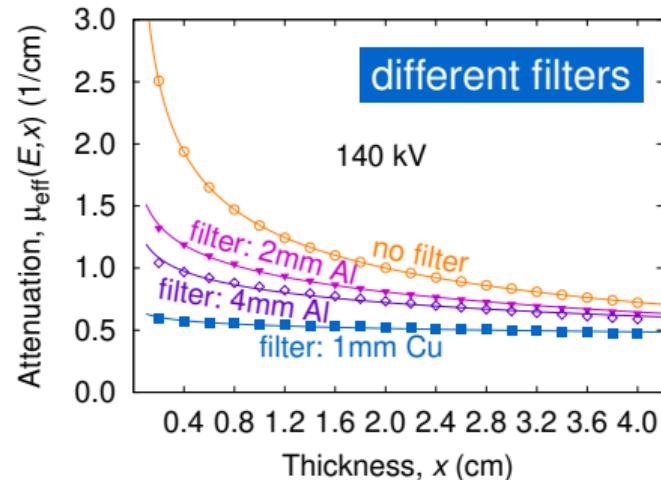
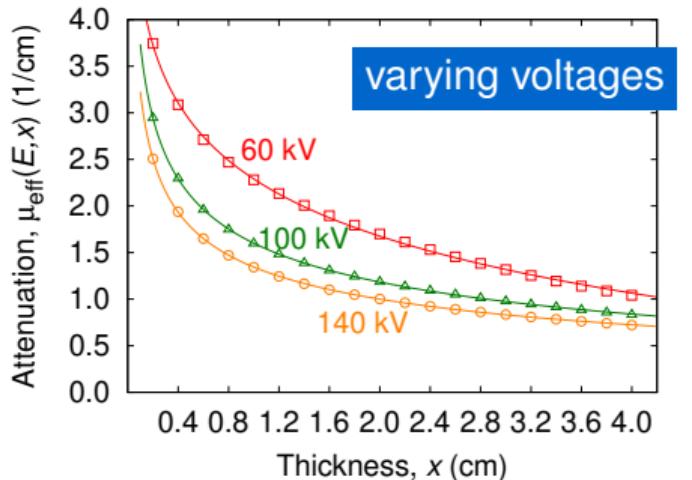
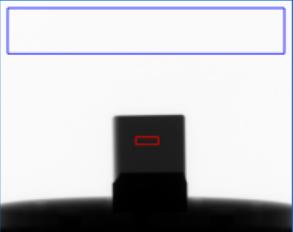
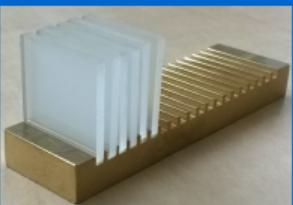
Universality of  
 $\mu_{\text{eff}}(x) = a + \frac{b}{x^\alpha}$



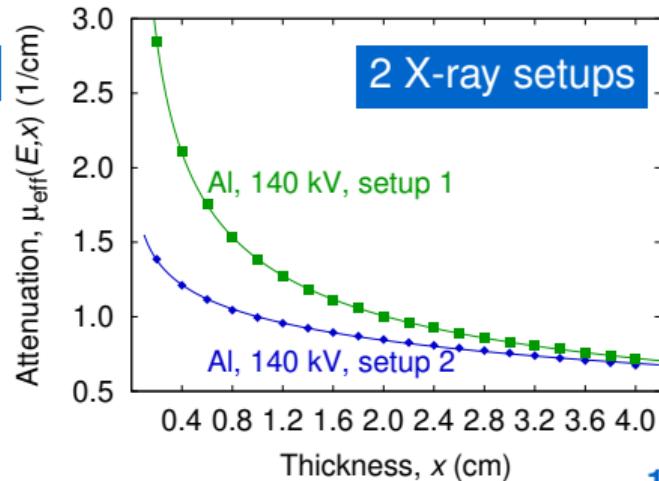
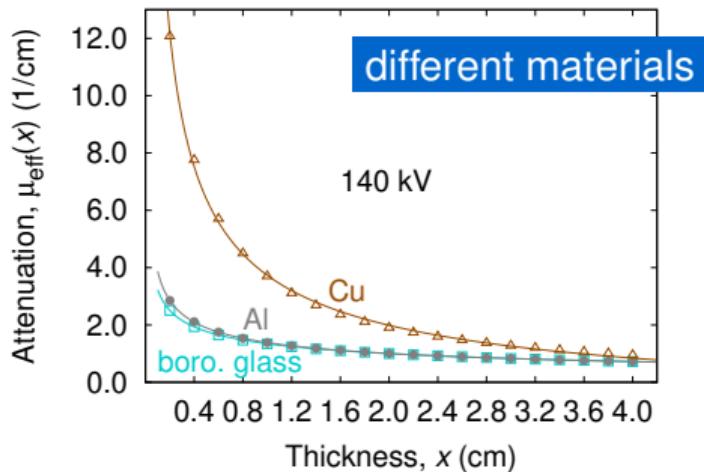
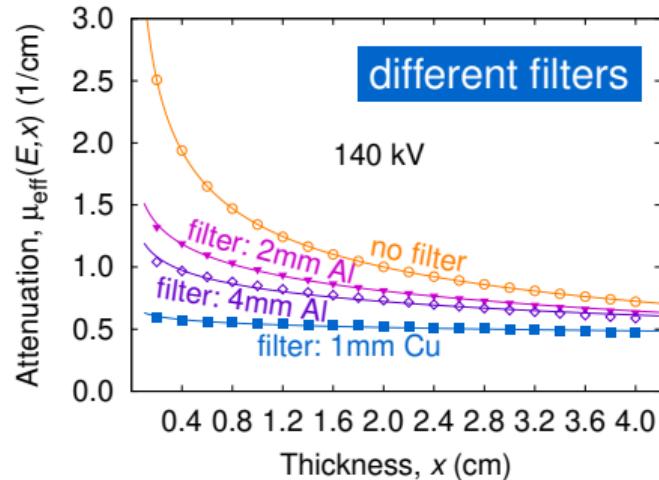
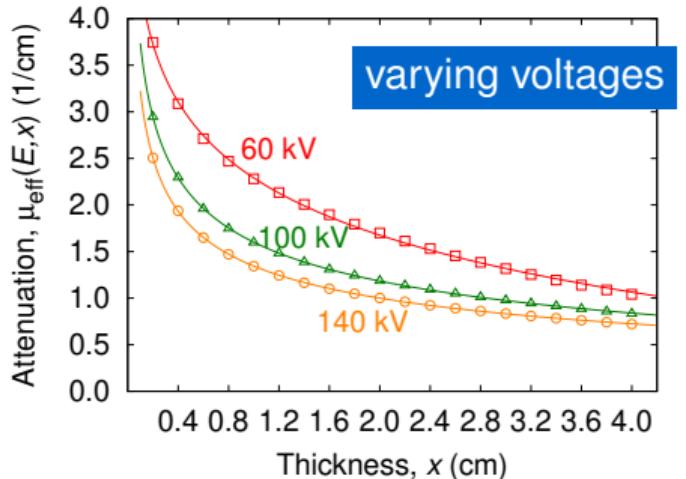
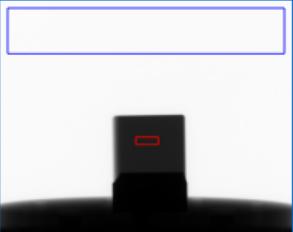
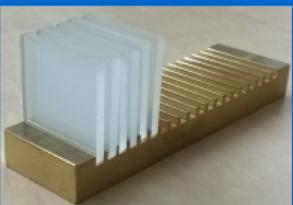
Universality of  
 $\mu_{\text{eff}}(x) = a + \frac{b}{x^\alpha}$



Universality of  
 $\mu_{\text{eff}}(x) = a + \frac{b}{x^\alpha}$



Universality of  
 $\mu_{\text{eff}}(x) = a + \frac{b}{x^\alpha}$



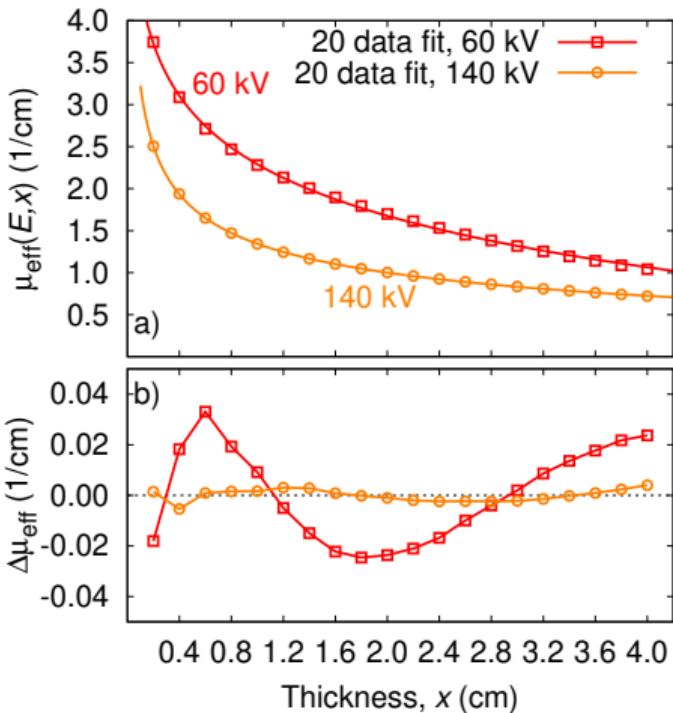
# Determining the material thickness $x$

Generalized Beer-Lambert

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Model function

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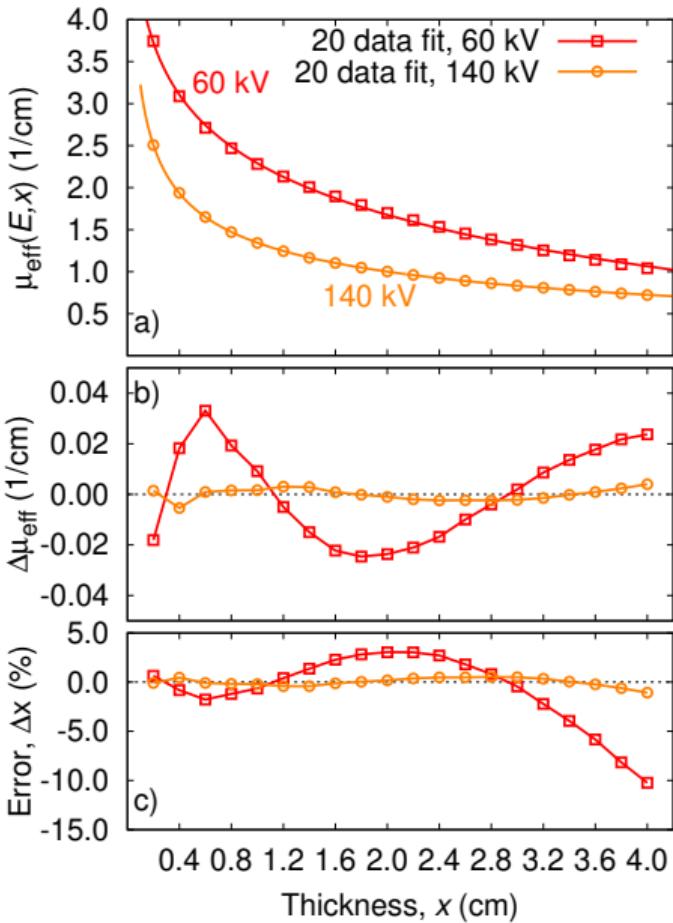
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e.g. Newton's method or look-up table



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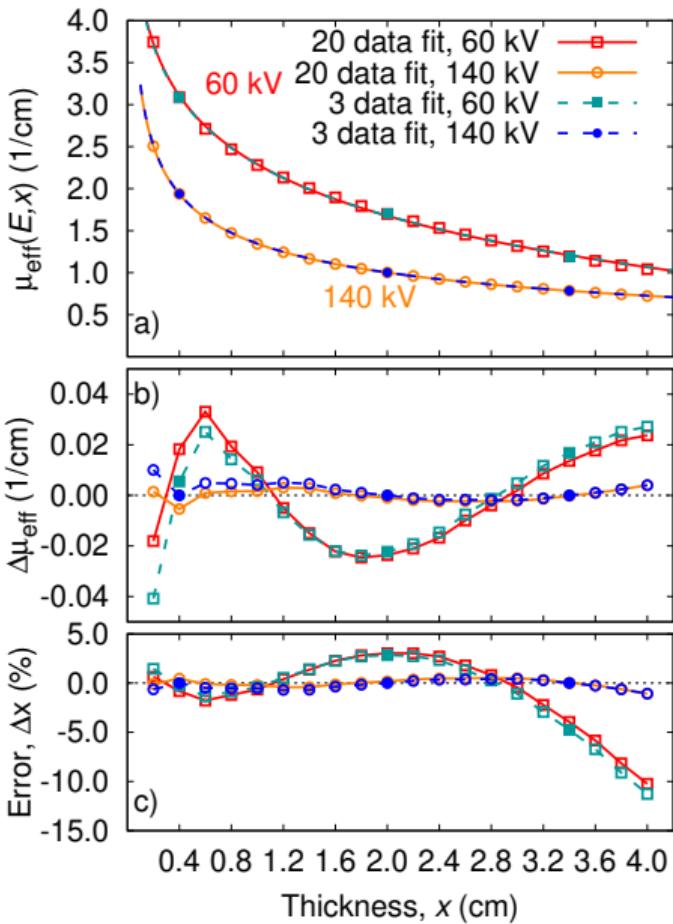
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# Migrating shear bands in shaken granular matter, Kollmer *et al* (2020)

