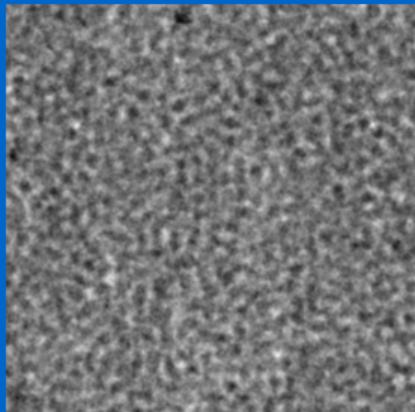
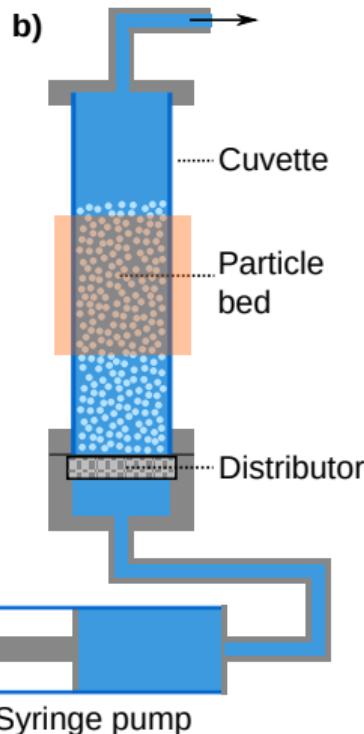
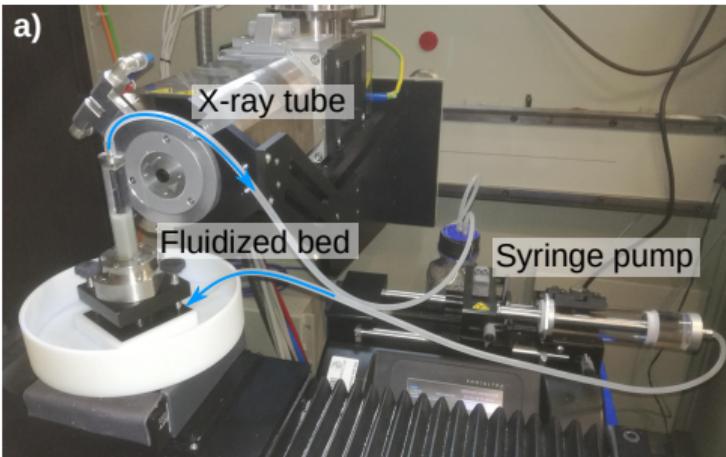


Measuring granular dynamics with X-ray Digital Fourier Analysis (X-DFA)

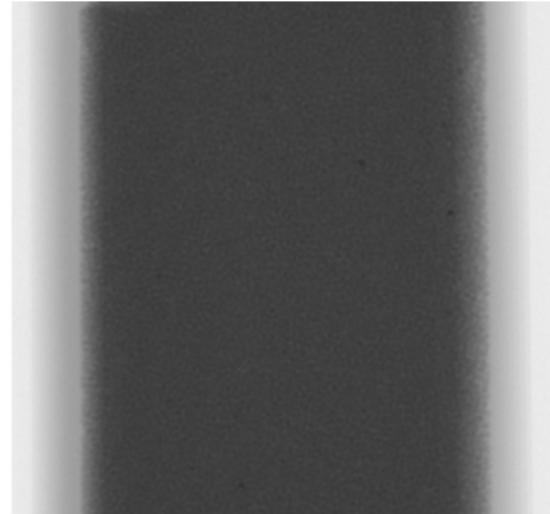


In collaboration with M. Escobedo & S. Egelhaaf, University of Düsseldorf

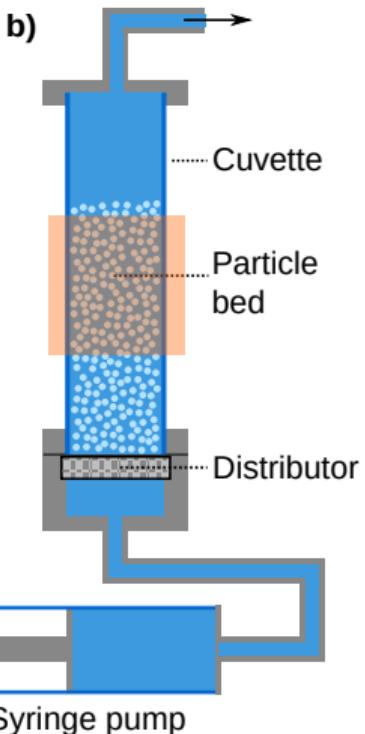
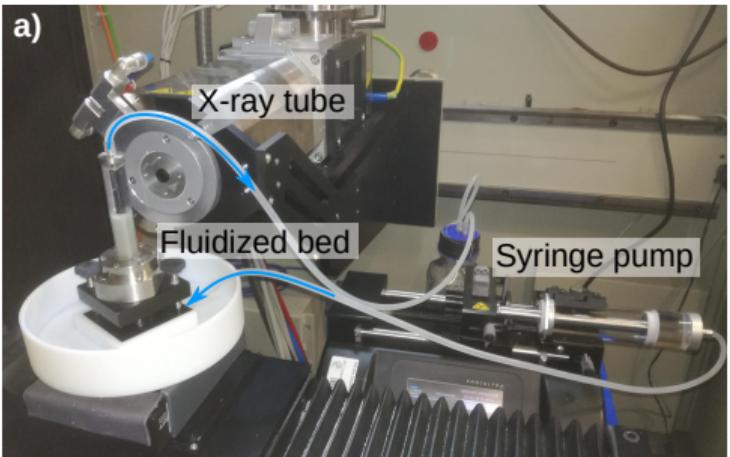
The system: A liquid fluidized bed



Radiogram

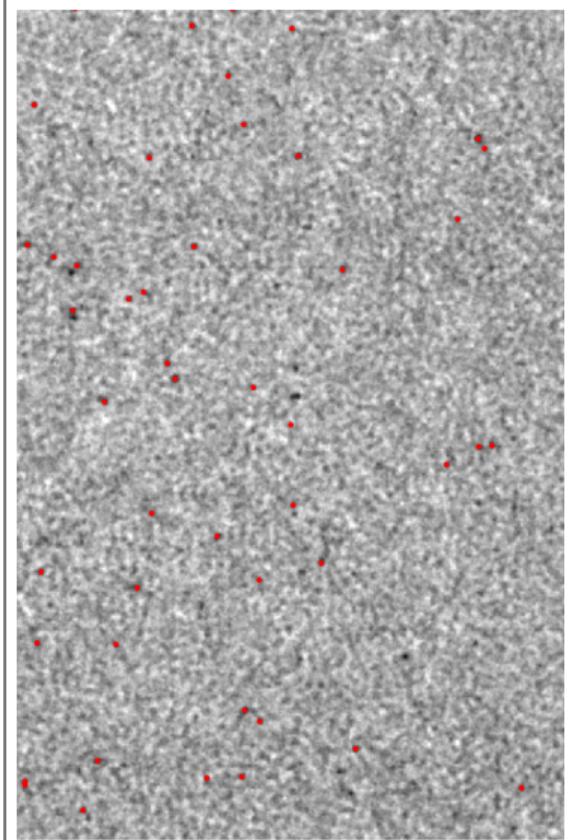


The system: A liquid fluidized bed

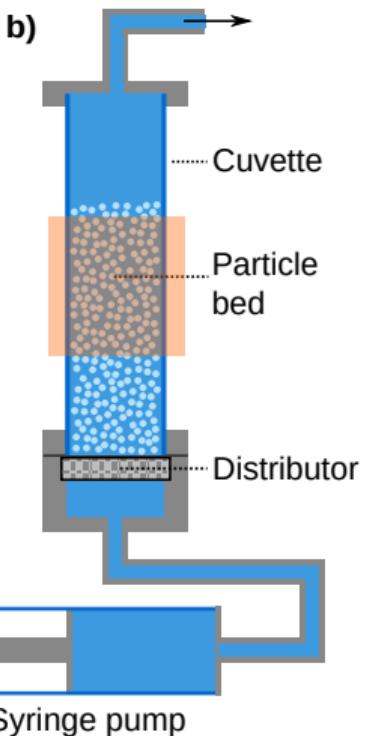
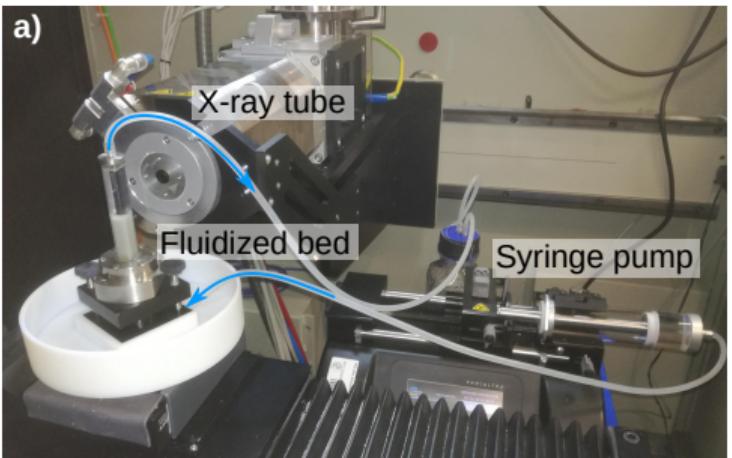


Particle tracking

Contrast: $\rho_{\text{tracer}} > \rho_{\text{bed}}$

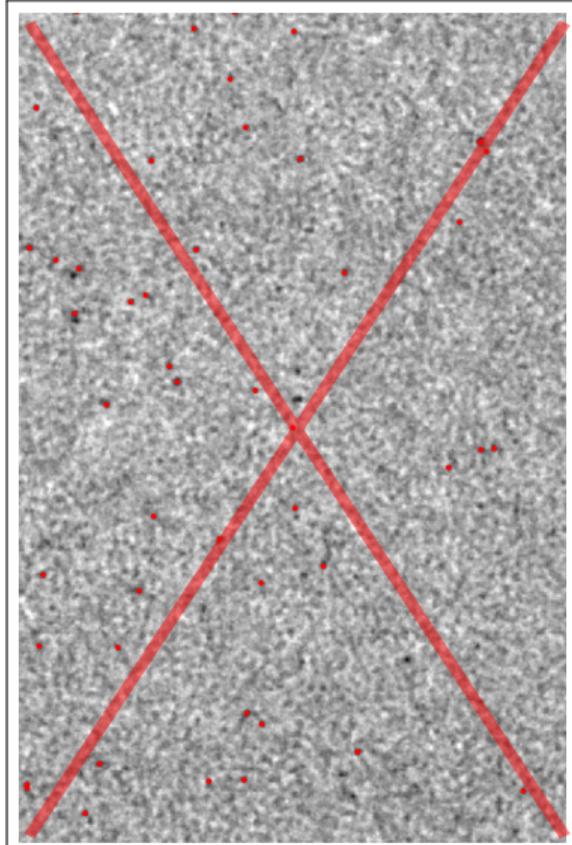


The system: A liquid fluidized bed

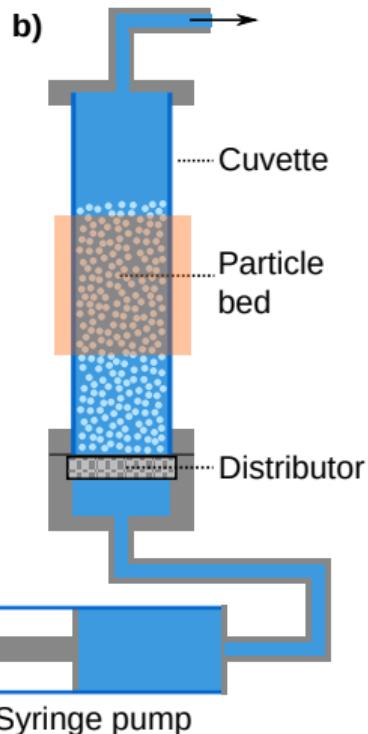
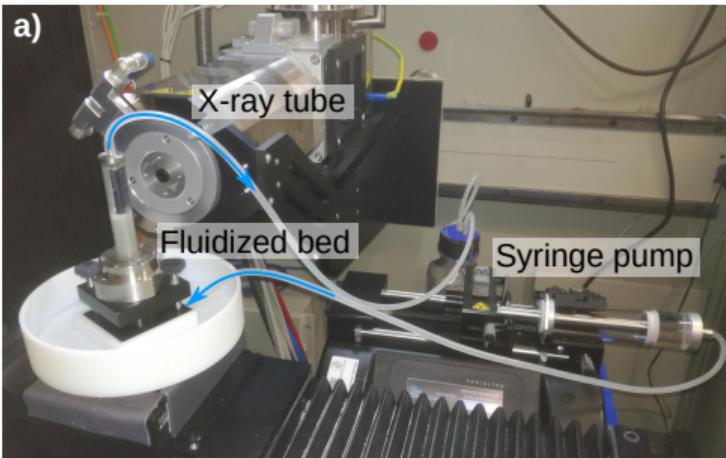


Particle tracking

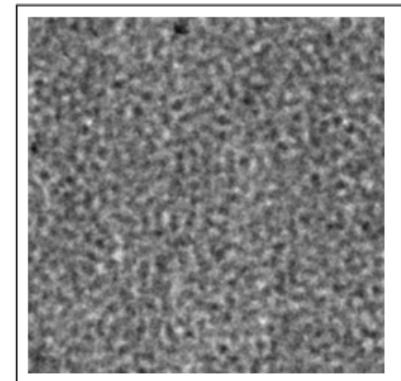
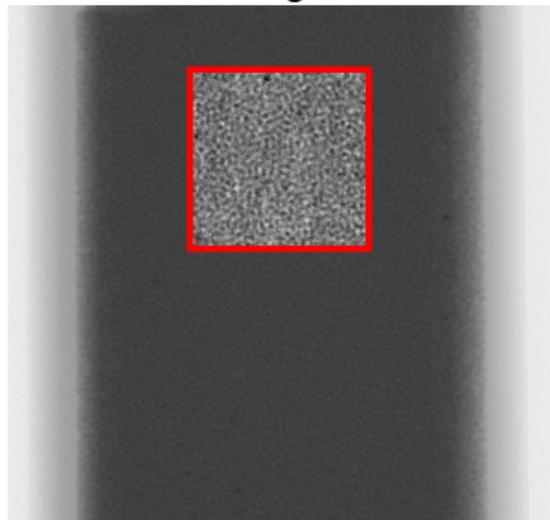
Contrast: $\rho_{\text{tracer}} > \rho_{\text{bed}}$



The system: A liquid fluidized bed



Radiogram



Differential Dynamic Microscopy (DDM)

	Up to now	This work
System	Dispersion, gels	Fluidized bed
Particles	Colloids $< 1 \mu\text{m}$	Granulates ($150 - 180 \mu\text{m}$)
Volume fraction	$\Phi \leq 0.33$	$0.45 < \Phi < 0.56$
Imaging	Light microscopy	X-ray radiography
Dynamics	Brownian motion, caging, glassy, collective motion	

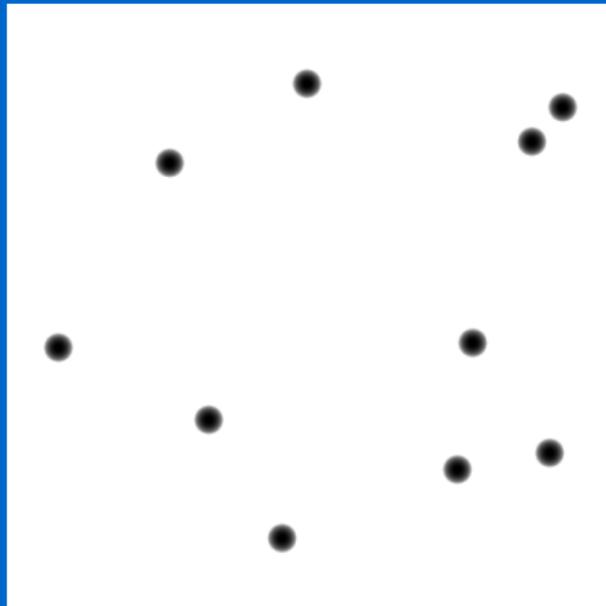
Extending Differential Dynamic Microscopy (DDM) to X-ray imaging

	Up to now	This work
System	Dispersion, gels	Fluidized bed
Particles	Colloids $< 1 \mu\text{m}$	Granulates (150 – 180) μm
Volume fraction	$\Phi \leq 0.33$	$0.45 < \Phi < 0.56$
Imaging	Light microscopy	X-ray radiography
Dynamics	Brownian motion, caging, glassy, collective motion	

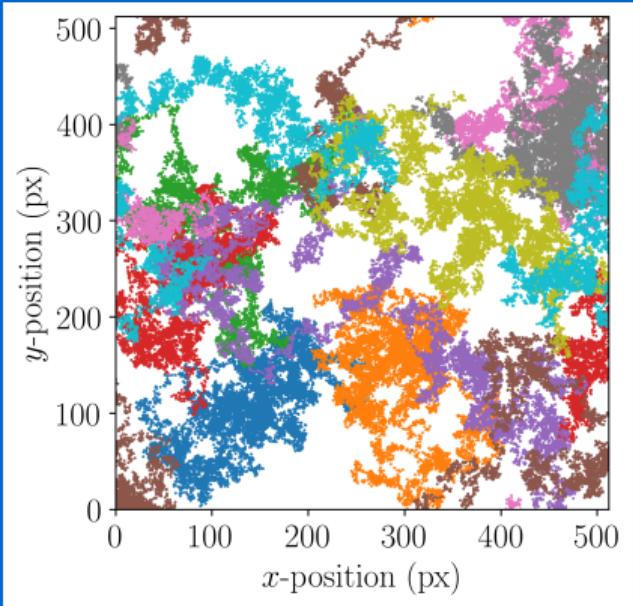
Digital Fourier Analysis of X-Ray radiograms (X-DFA)

Introduction to X-ray Digital Fourier Analysis (X-DFA)

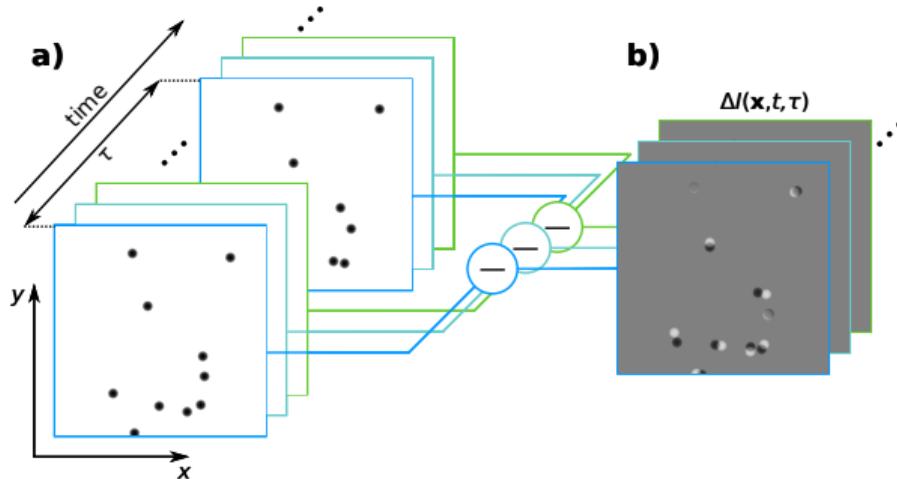
Synthetic radiograms



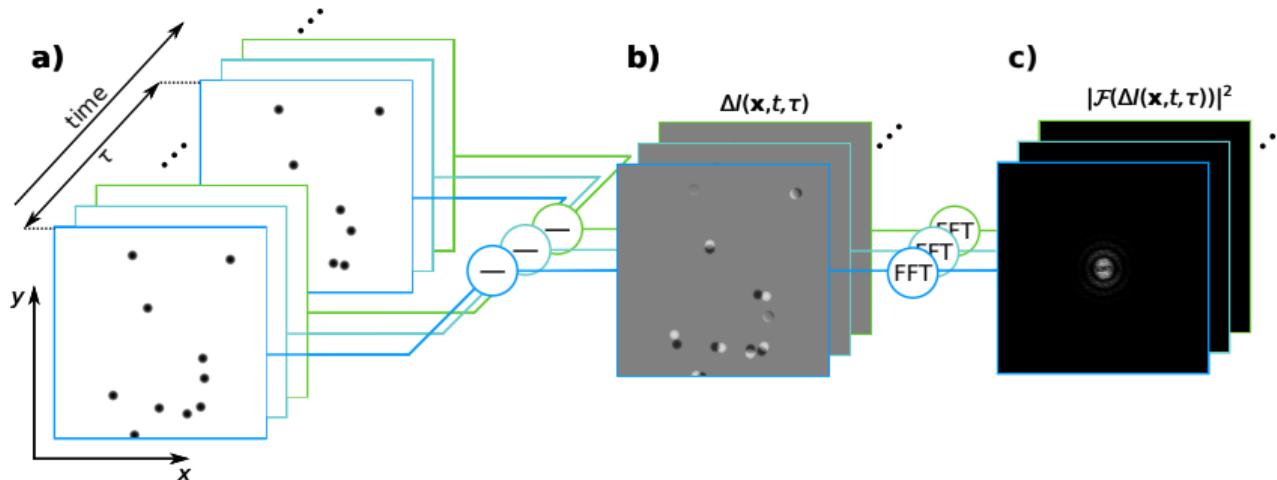
Particle trajectory



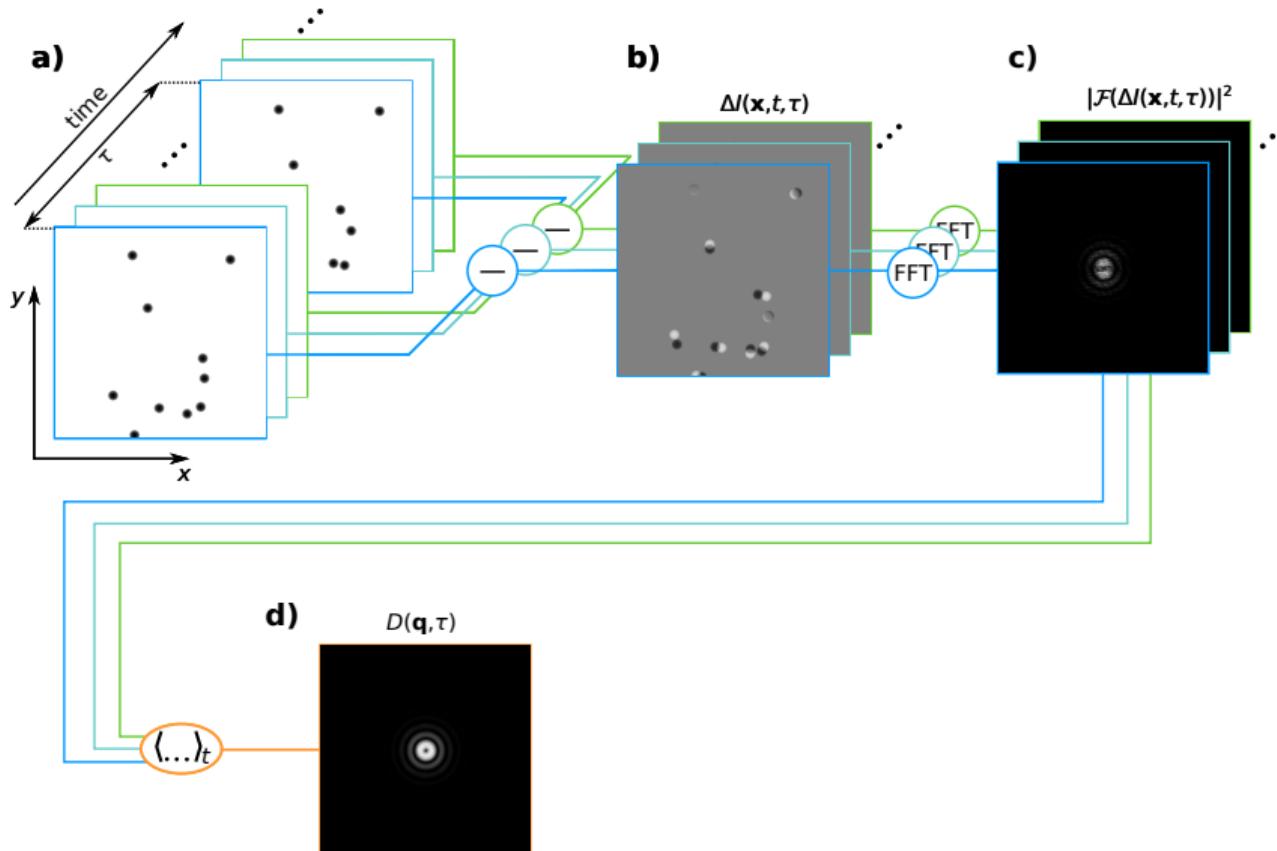
The image structure function $D(\mathbf{q}, \tau)$



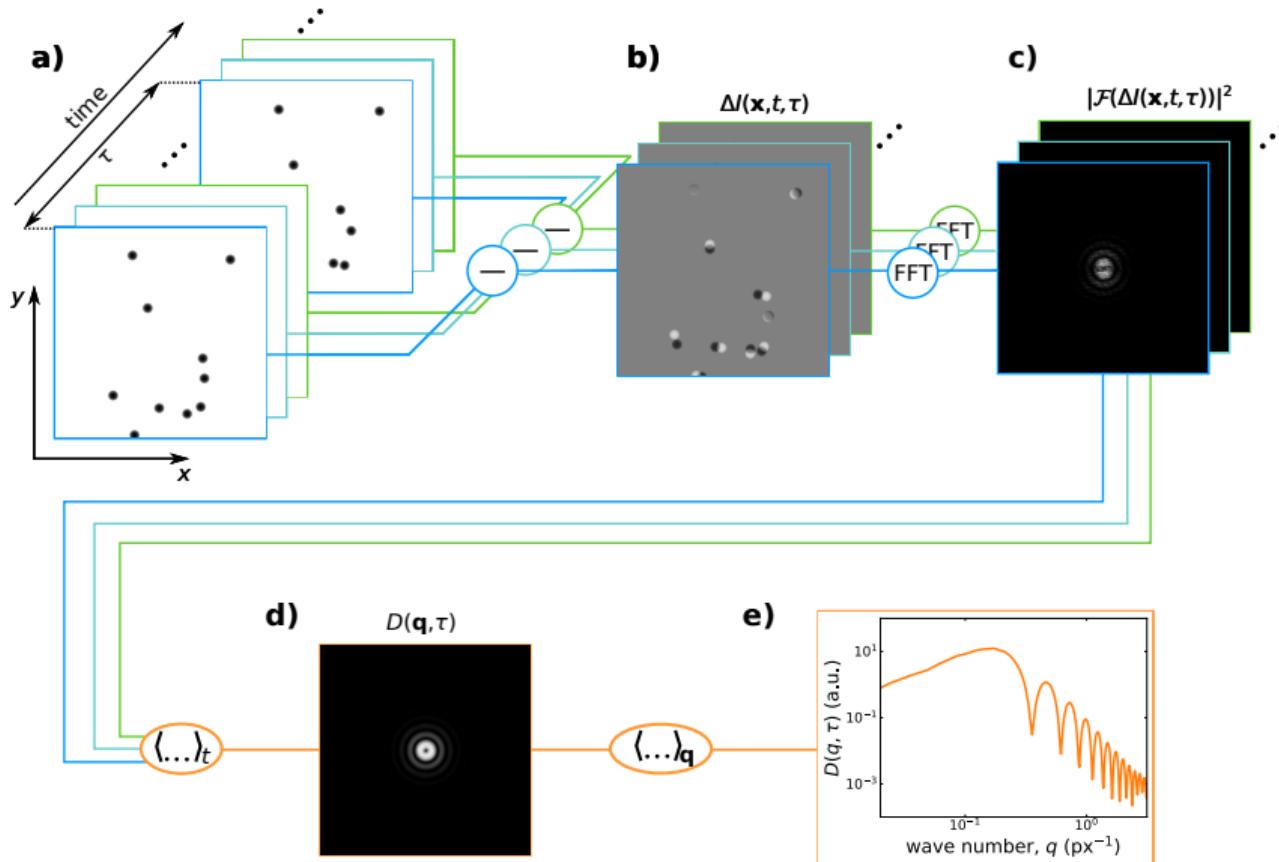
The image structure function $D(\mathbf{q}, \tau)$



The image structure function $D(\mathbf{q}, \tau)$

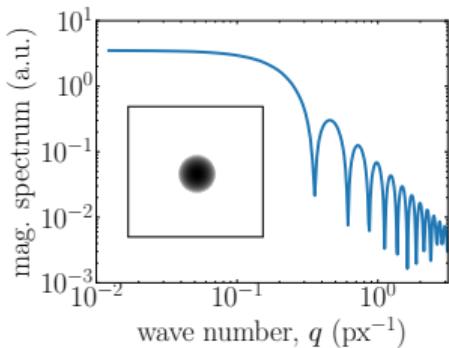
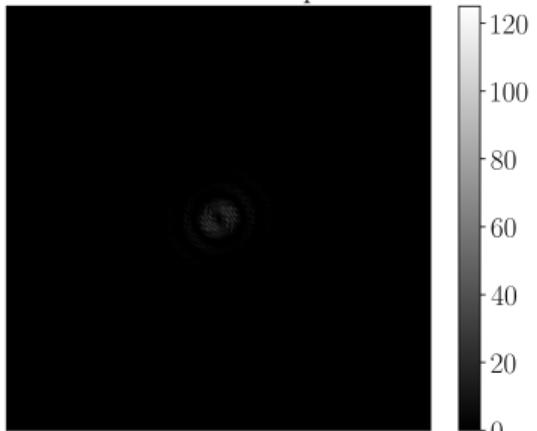


The image structure function $D(\mathbf{q}, \tau)$



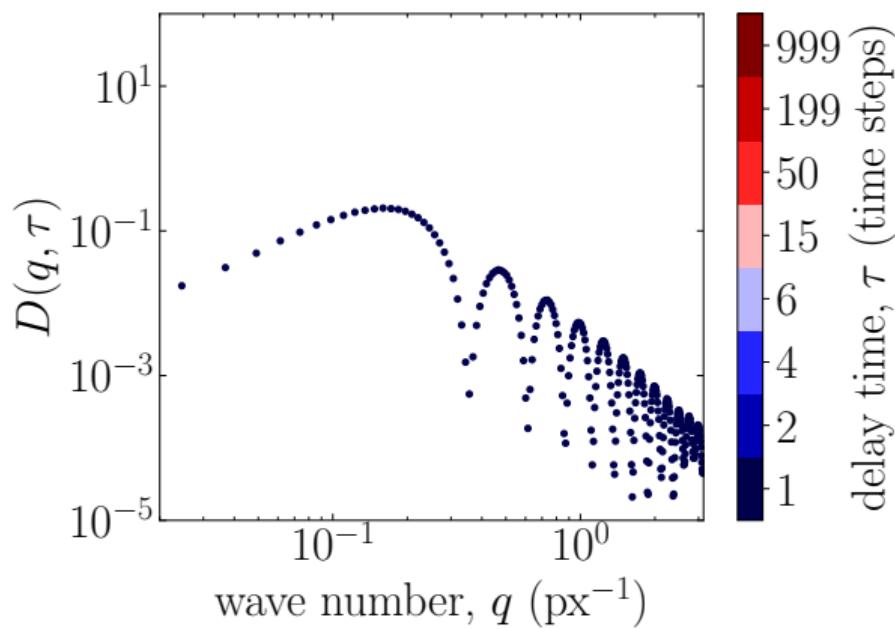
The image structure function $D(q, \tau)$

$\tau = 1$ time steps



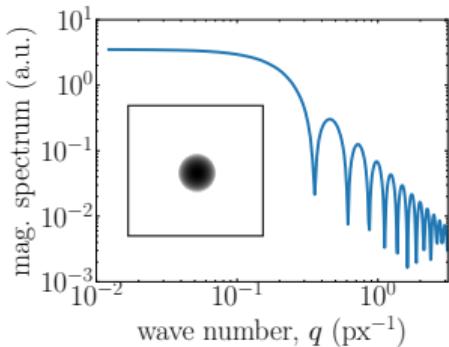
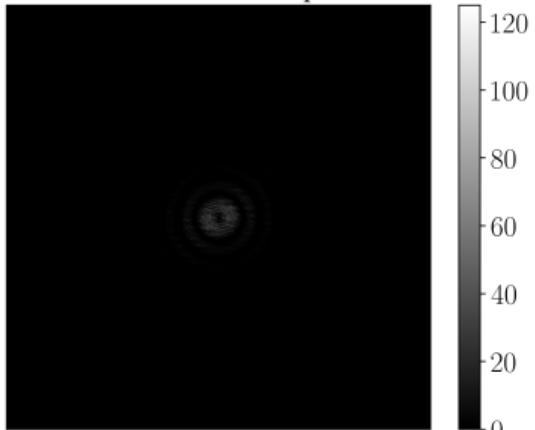
Time averaging: $D(\mathbf{q}, \tau) = \langle |\mathcal{F}(\Delta I)|^2 \rangle_t$

Azimuthal averaging: $D(\mathbf{q}, \tau) \rightarrow D(\mathbf{q}, \tau) \dots$



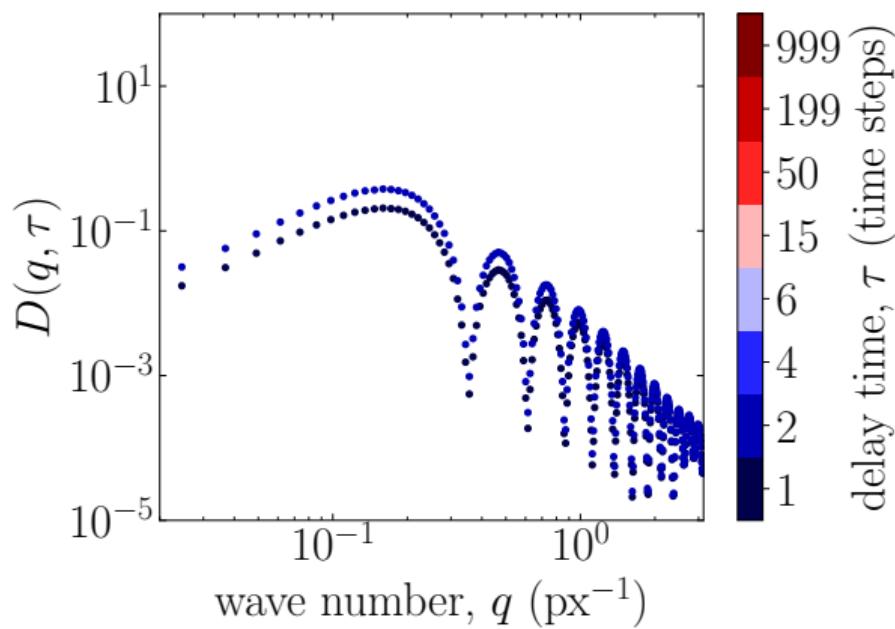
The image structure function $D(q, \tau)$

$\tau = 2$ time steps



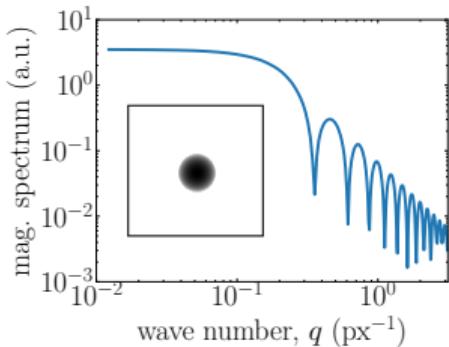
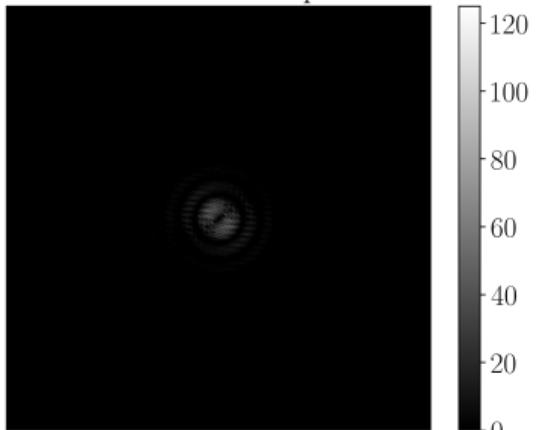
Time averaging: $D(\mathbf{q}, \tau) = \langle |\mathcal{F}(\Delta I)|^2 \rangle_t$

Azimuthal averaging: $D(\mathbf{q}, \tau) \rightarrow D(\mathbf{q}, \tau) \dots$



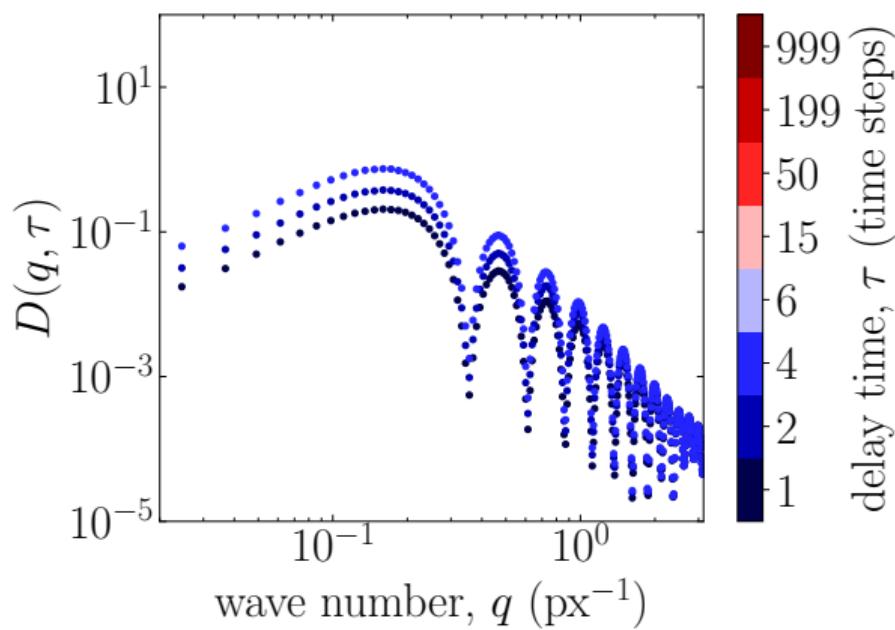
The image structure function $D(q, \tau)$

$\tau = 4$ time steps



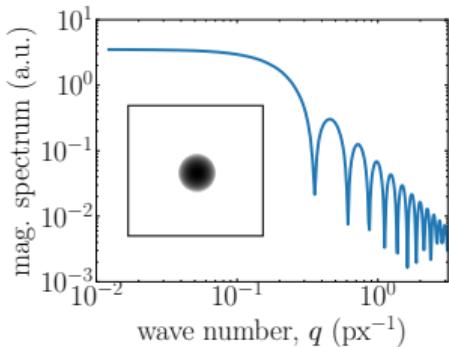
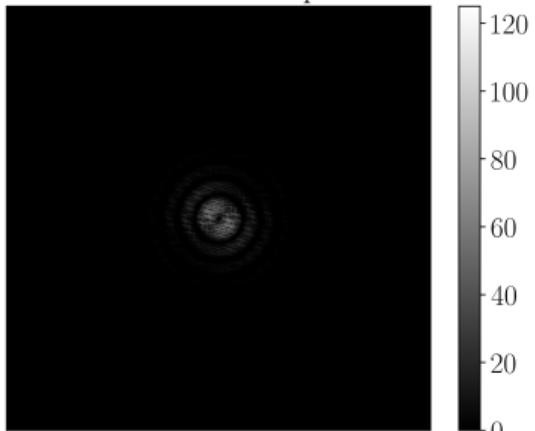
Time averaging: $D(\mathbf{q}, \tau) = \langle |\mathcal{F}(\Delta I)|^2 \rangle_t$

Azimuthal averaging: $D(\mathbf{q}, \tau) \rightarrow D(\mathbf{q}, \tau) \dots$



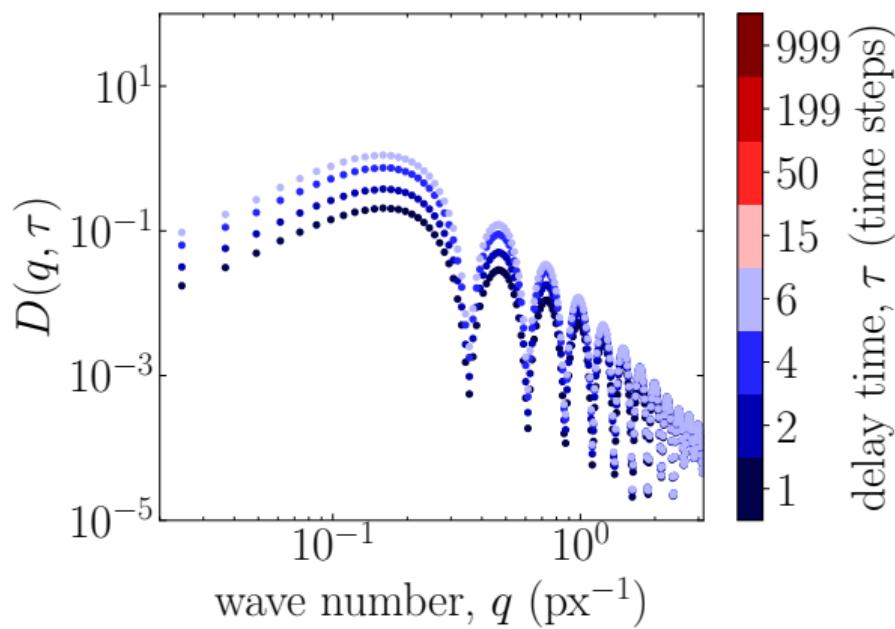
The image structure function $D(q, \tau)$

$\tau = 6$ time steps

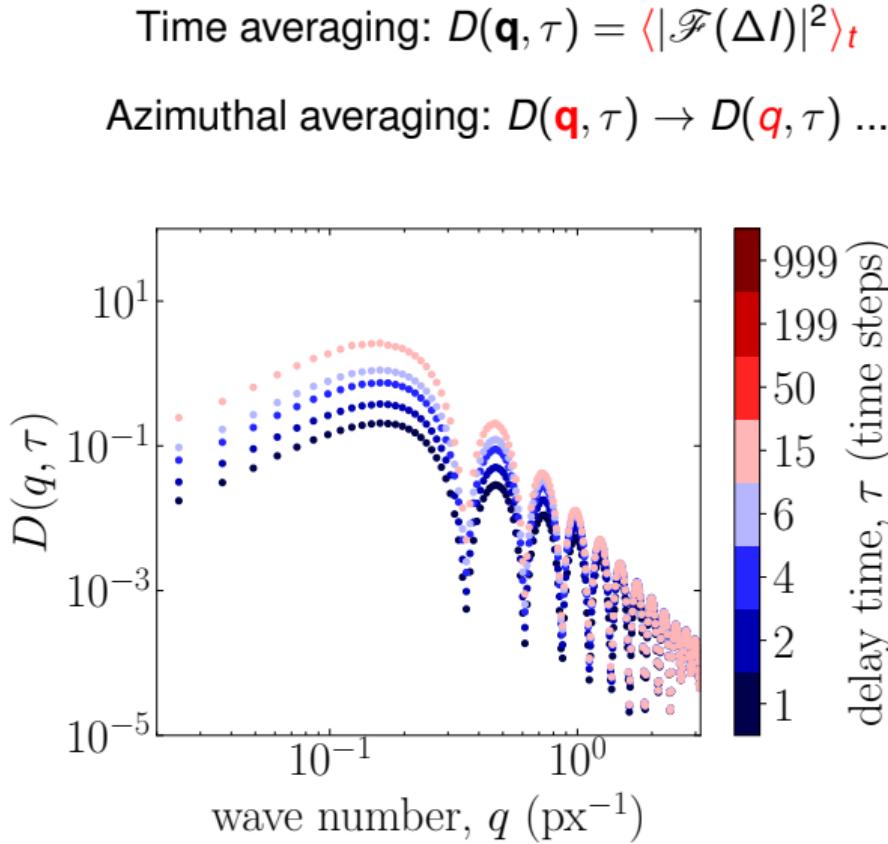
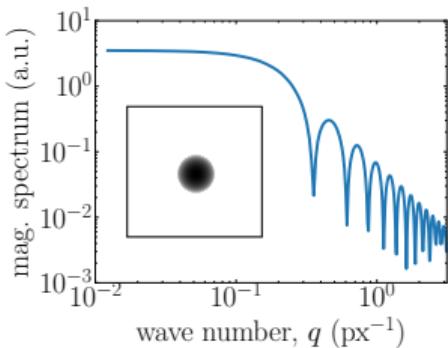
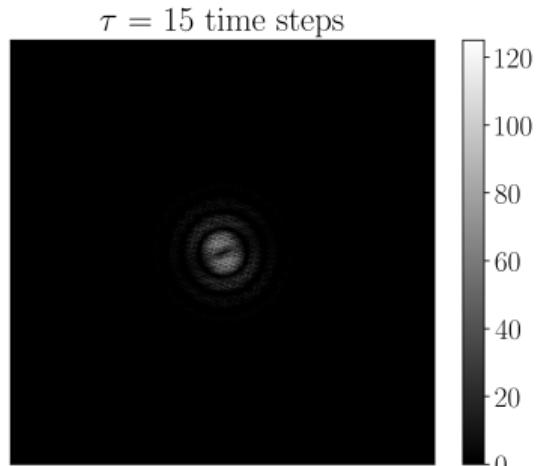


Time averaging: $D(\mathbf{q}, \tau) = \langle |\mathcal{F}(\Delta I)|^2 \rangle_t$

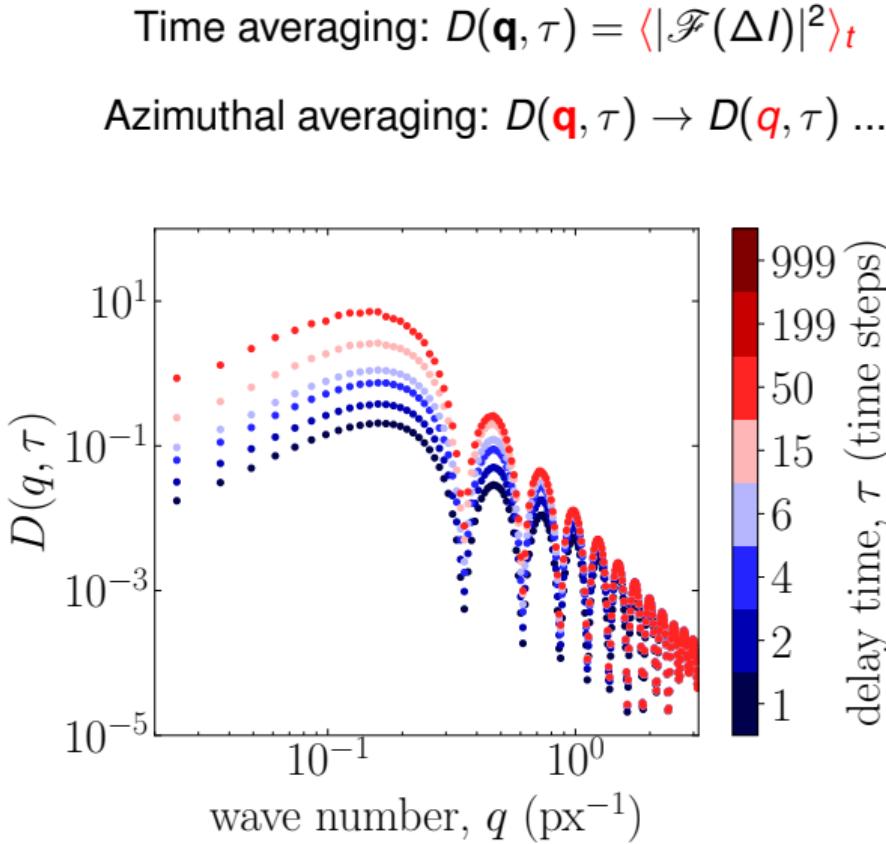
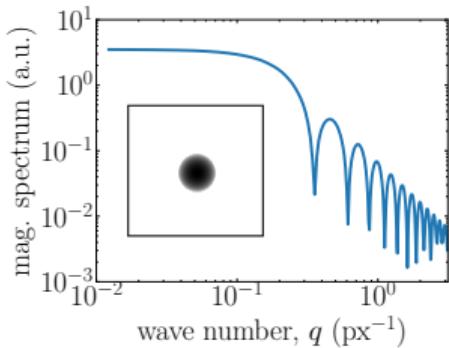
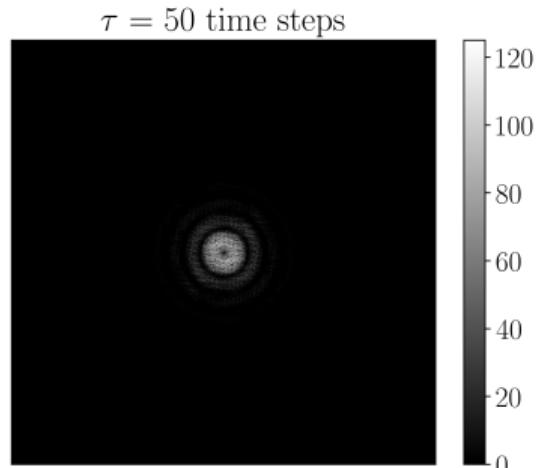
Azimuthal averaging: $D(\mathbf{q}, \tau) \rightarrow D(\mathbf{q}, \tau) \dots$



The image structure function $D(q, \tau)$

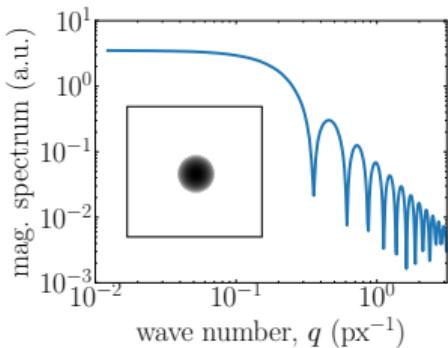
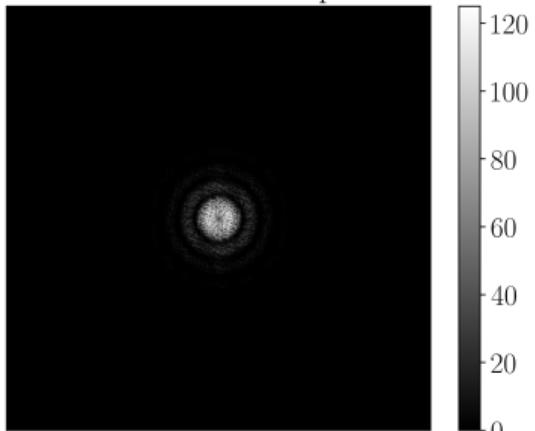


The image structure function $D(q, \tau)$



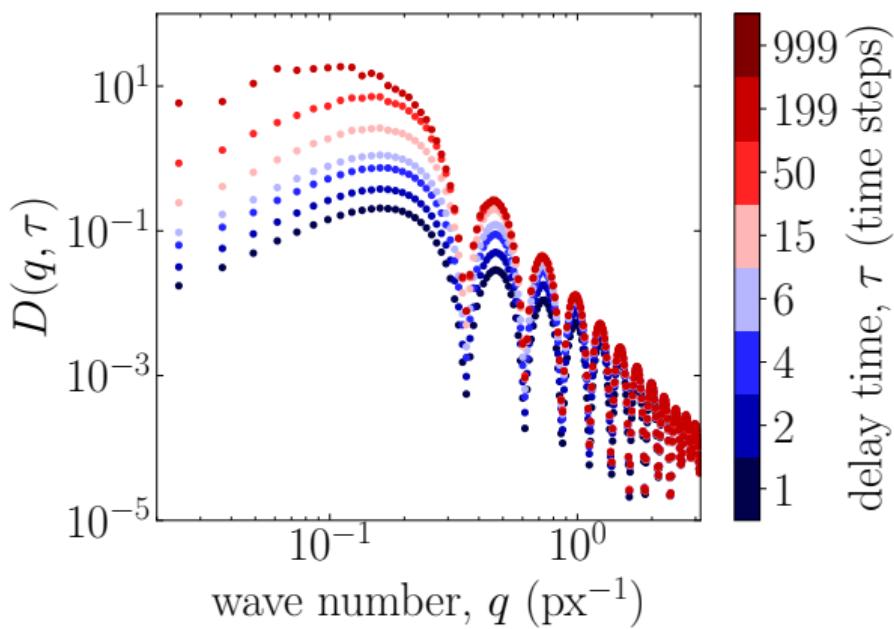
The image structure function $D(q, \tau)$

$\tau = 199$ time steps

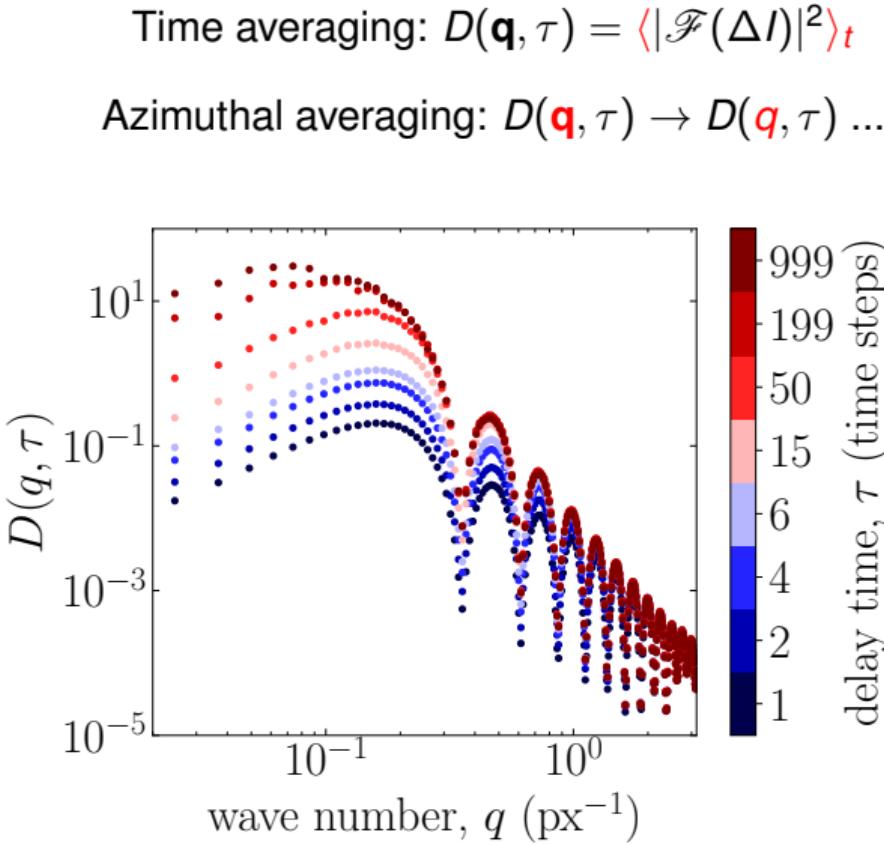
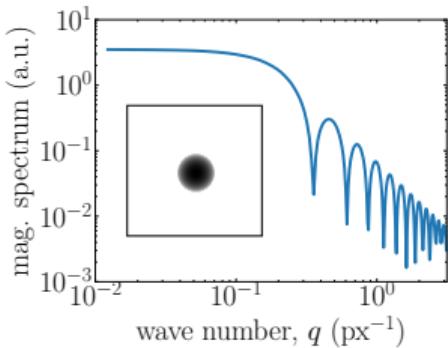
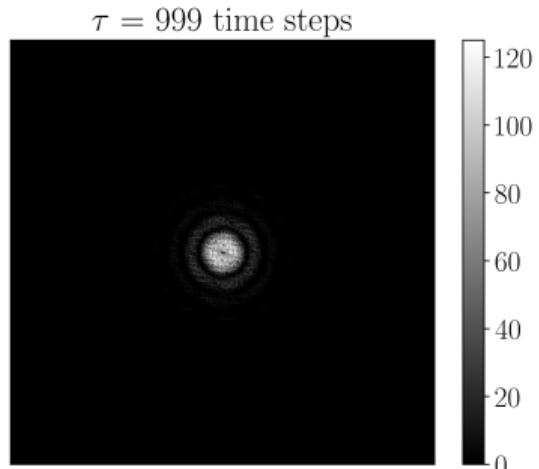


Time averaging: $D(\mathbf{q}, \tau) = \langle |\mathcal{F}(\Delta I)|^2 \rangle_t$

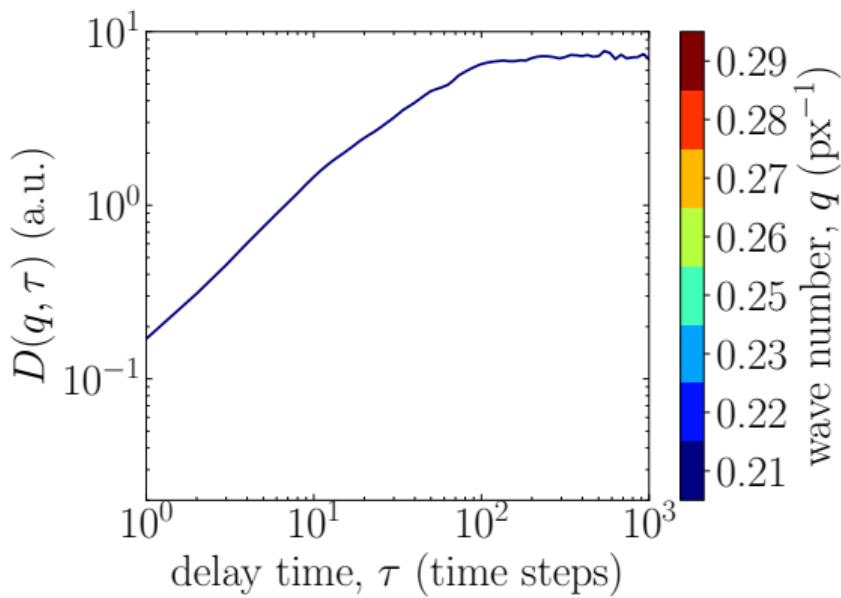
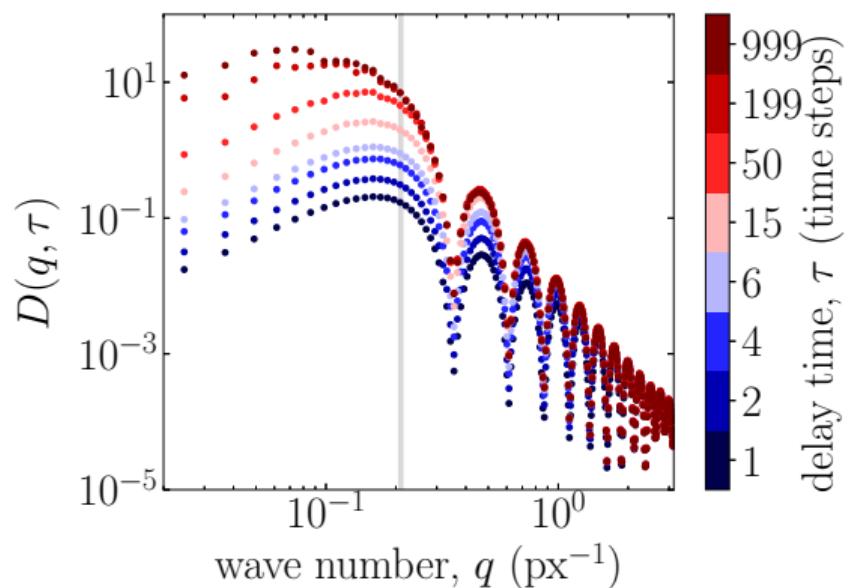
Azimuthal averaging: $D(\mathbf{q}, \tau) \rightarrow D(\mathbf{q}, \tau) \dots$



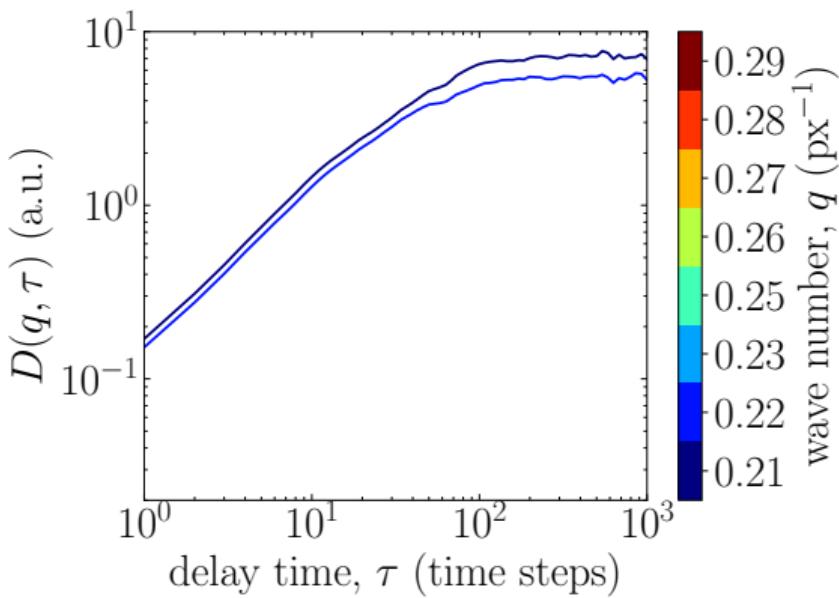
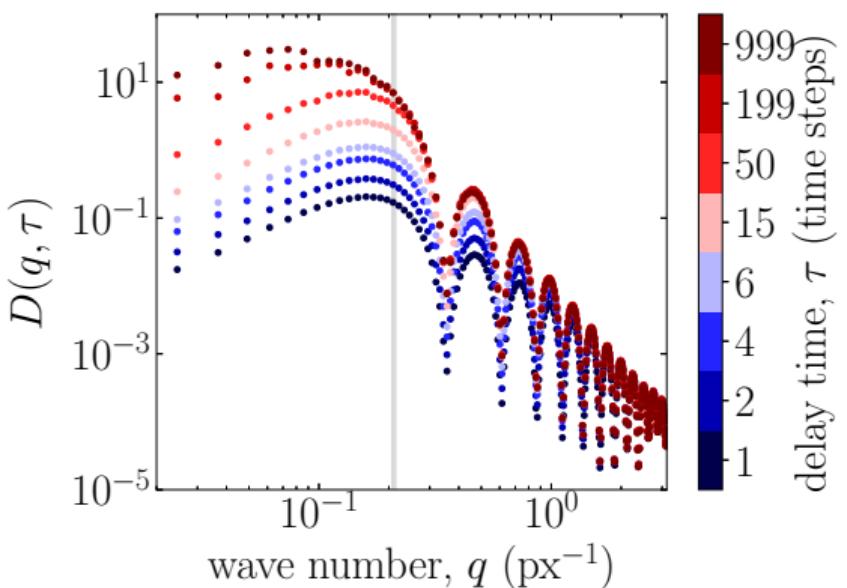
The image structure function $D(q, \tau)$



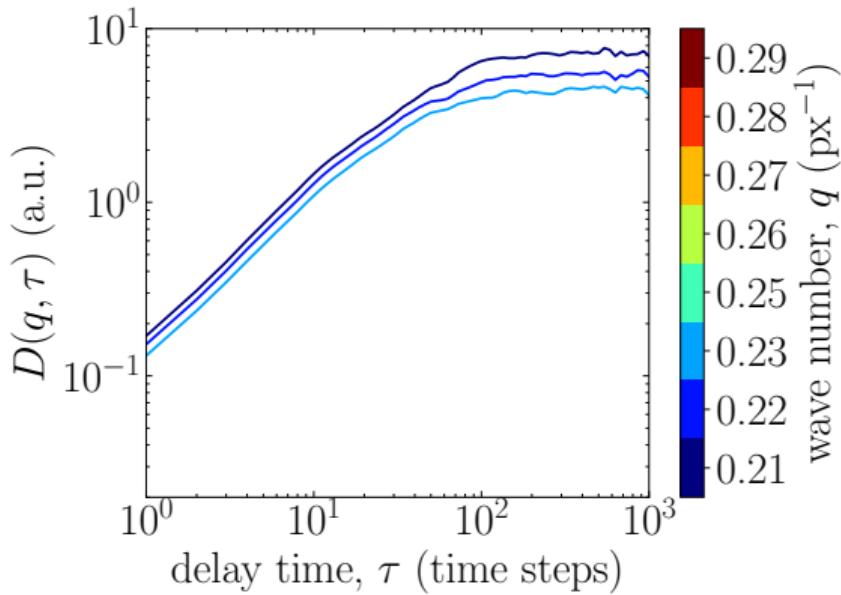
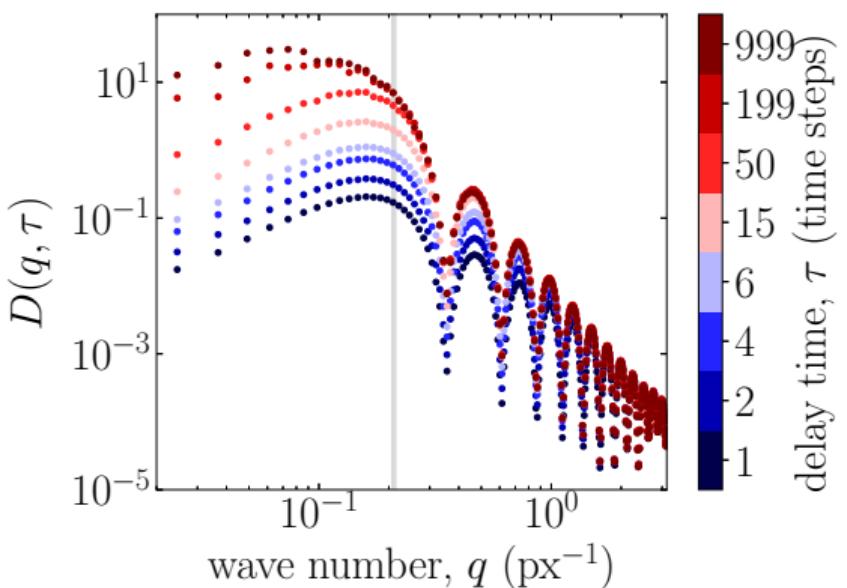
The image structure function $D(q, \tau)$



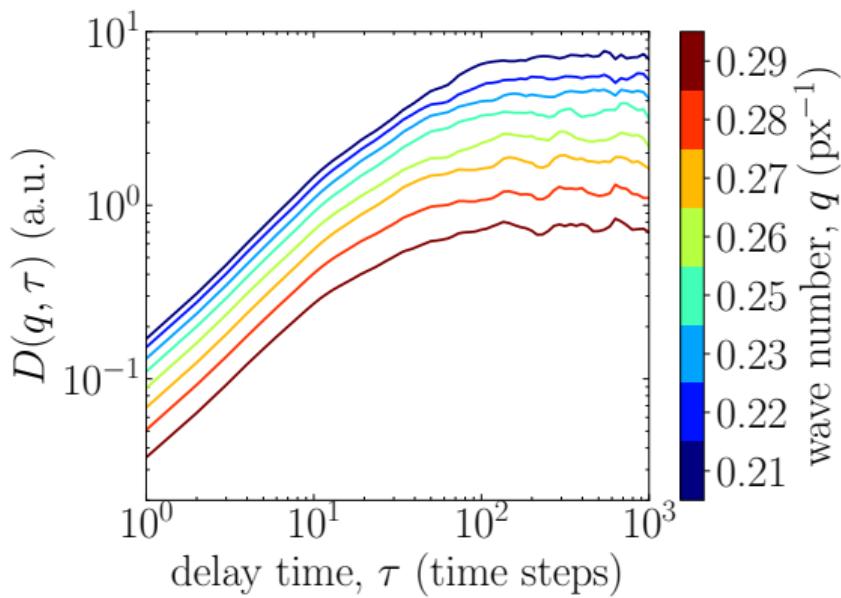
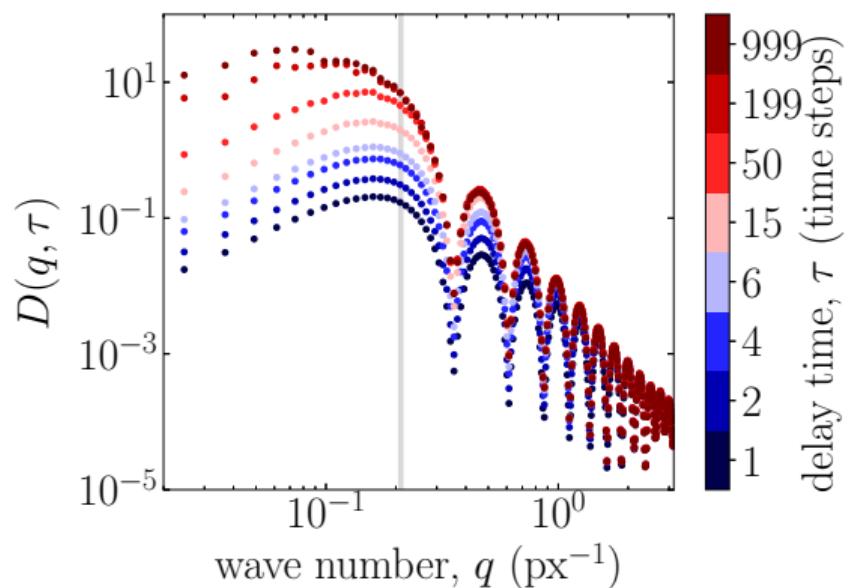
The image structure function $D(q, \tau)$



The image structure function $D(q, \tau)$



The image structure function $D(q, \tau)$



Linear space invariant imaging

$$D(q, \tau) = \left\langle |I(q, t + \tau) - I(q, t)|^2 \right\rangle_t$$

Linear space invariant imaging

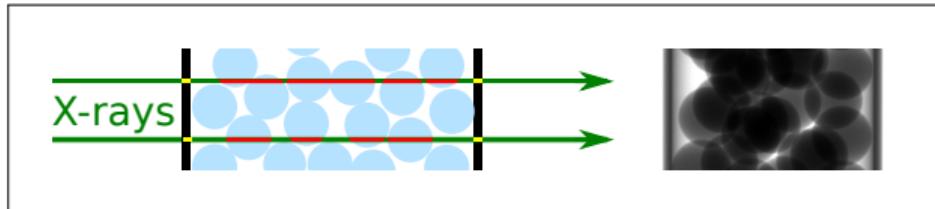
$$\begin{aligned} D(q, \tau) &= \left\langle |I(q, t + \tau) - I(q, t)|^2 \right\rangle_t \\ &= A(q) \left[1 - \frac{\left\langle I^*(q, t) I(q, t + \tau) \right\rangle_t}{\left\langle |I(q, t)|^2 \right\rangle_t} \right] + B(q) \end{aligned}$$

Linear space invariant imaging

$$\begin{aligned} D(q, \tau) &= \left\langle |I(q, t + \tau) - I(q, t)|^2 \right\rangle_t \\ &= A(q) \underbrace{\left[1 - \frac{\langle I^*(q, t) I(q, t + \tau) \rangle_t}{\langle |I(q, t)|^2 \rangle_t} \right]}_{\text{Image correlation function}} + B(q) \end{aligned}$$

Linear space invariant imaging

$$\begin{aligned} D(q, \tau) &= \left\langle |I(q, t + \tau) - I(q, t)|^2 \right\rangle_t \\ &= A(q) \underbrace{\left[1 - \frac{\langle I^*(q, t) I(q, t + \tau) \rangle_t}{\langle |I(q, t)|^2 \rangle_t} \right]}_{\text{Image correlation function}} + B(q) \end{aligned}$$

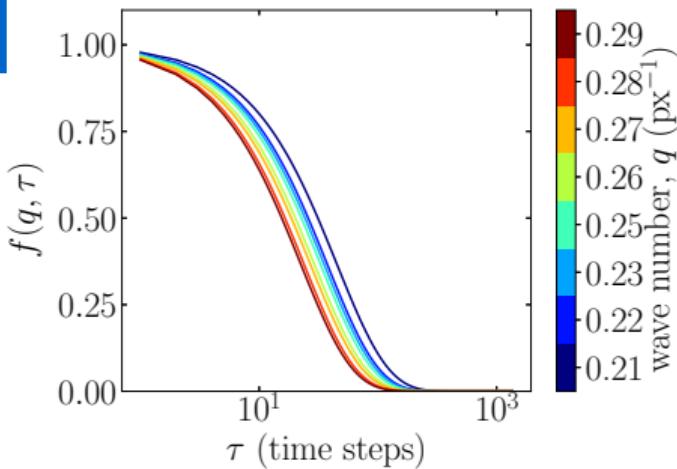


Intermediate scattering function

$$f(q, \tau) = \frac{\langle \rho^*(q, t) \rho(q, t + \tau) \rangle_t}{\langle |\rho(q, t)|^2 \rangle_t}$$

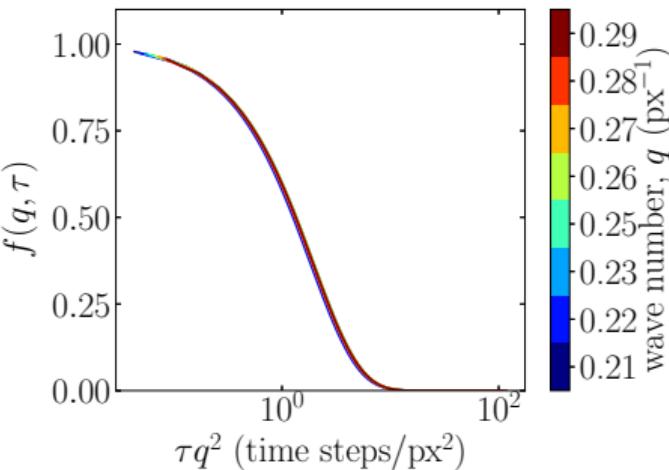
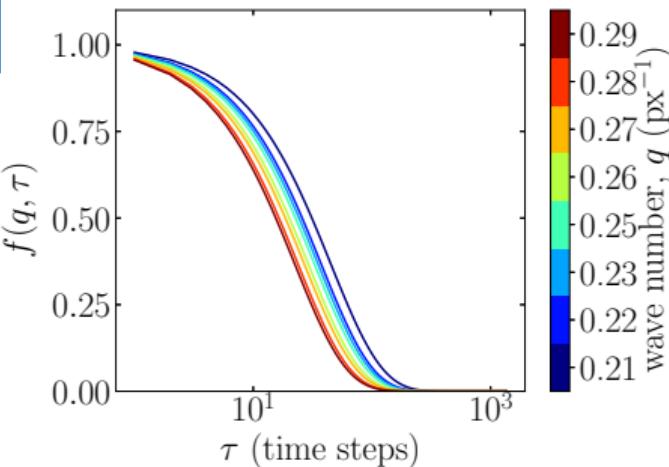
Intermediate scattering function $f(q, \tau)$

Brownian motion:
 $f(q, \tau) = \exp(-q^2 \tau / \tau_D)$



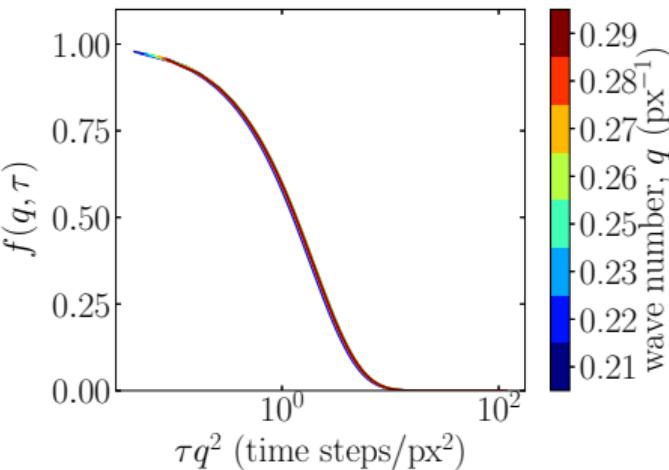
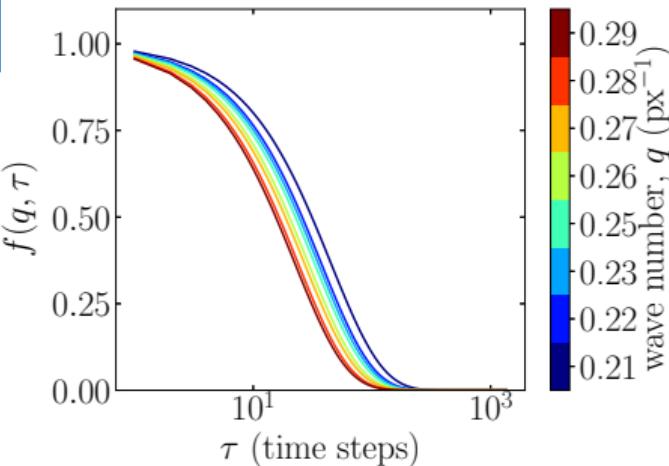
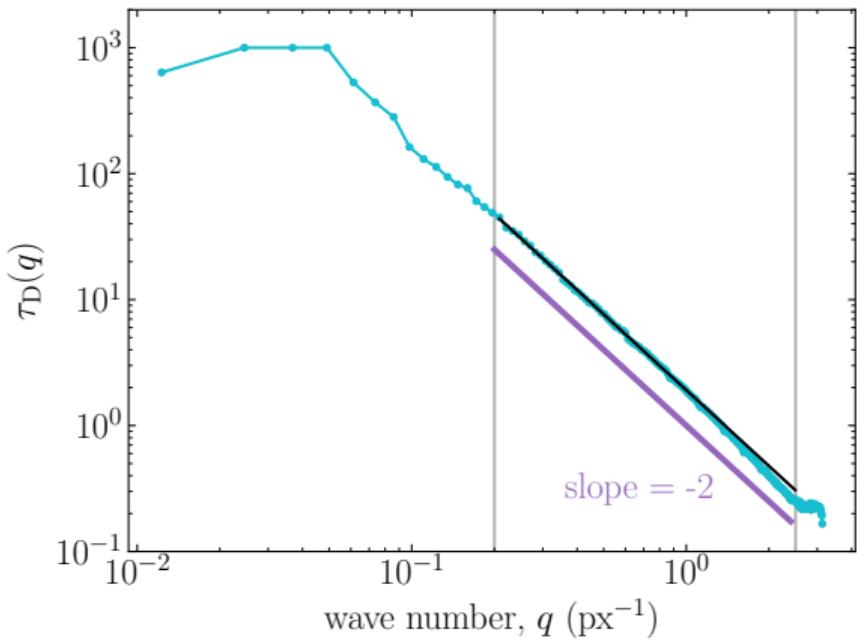
Intermediate scattering function $f(q, \tau)$

Brownian motion:
 $f(q, \tau) = \exp(-q^2 \tau / \tau_D)$



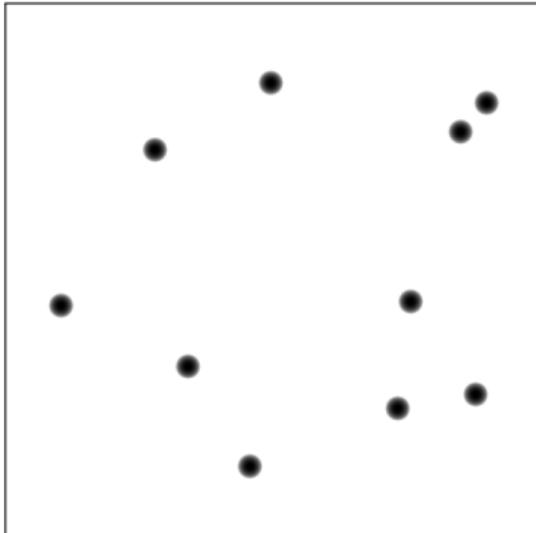
Intermediate scattering function $f(q, \tau)$

Brownian motion:
 $f(q, \tau) = \exp(-q^2 \tau / \tau_D)$

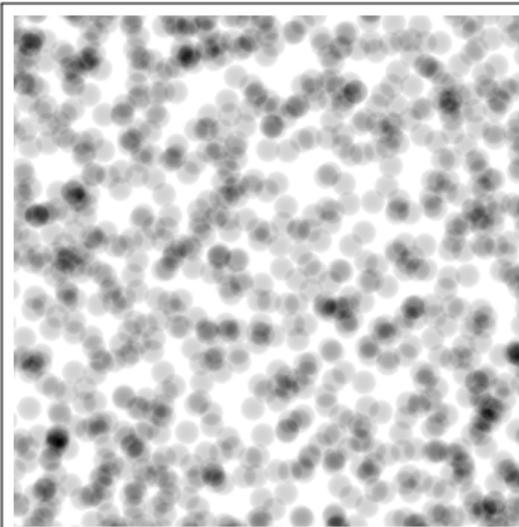


Accuracy of X-DFA: Varying the number of particles

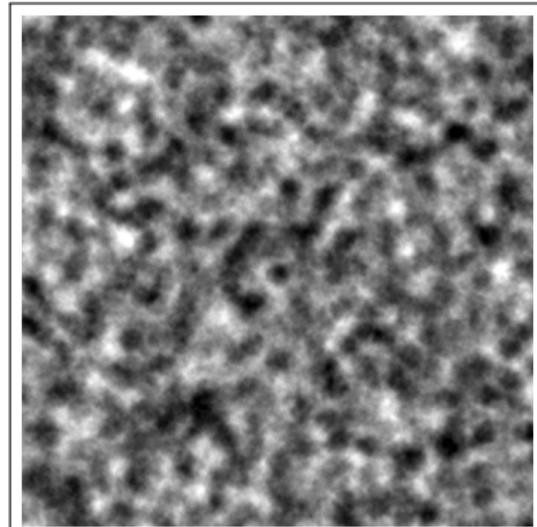
10 particles



1000 particles



100 000 particles



Deviation from the simulation input:

6%

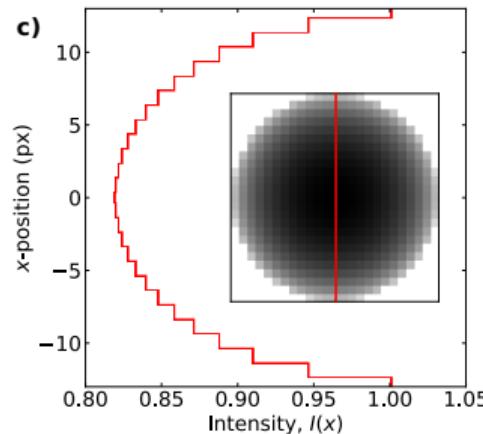
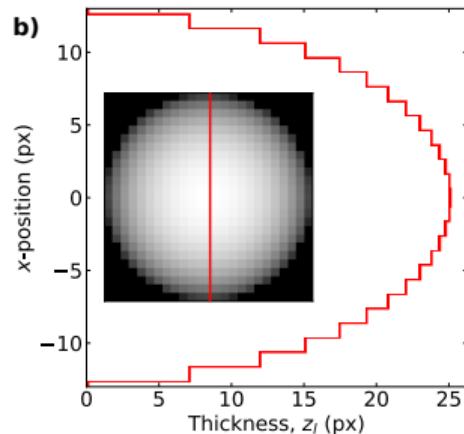
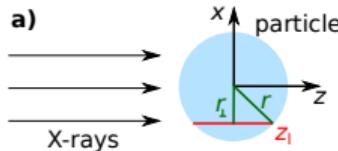
2%

2%

PIV off by $\approx 650\%$

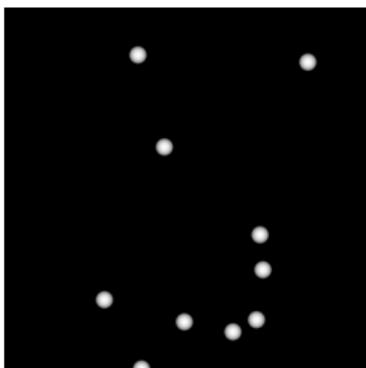
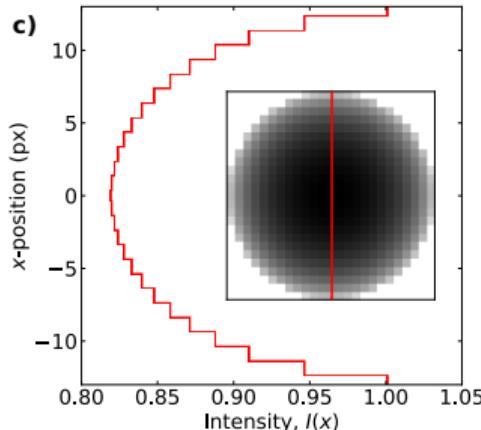
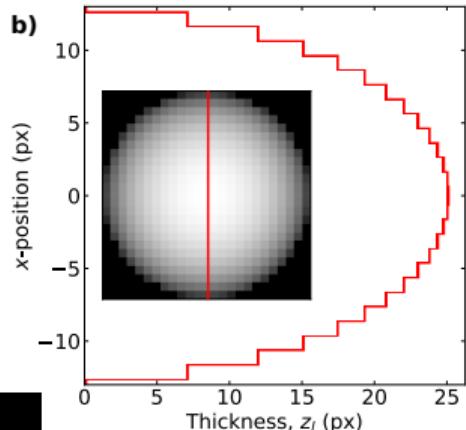
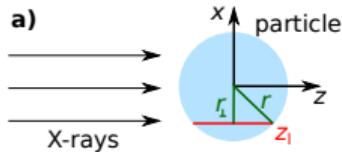
Backup slides

Synthetic radiograms

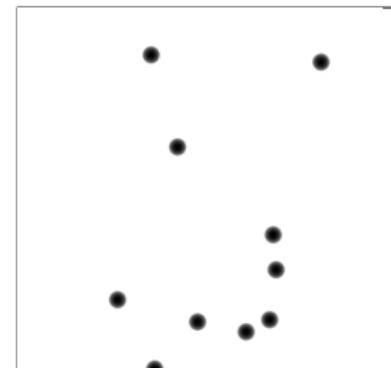


Beer-Lambert
 $I(z_I) = I_0 \exp(-\mu z)$

Synthetic radiograms



Beer-Lambert
 $I(z_I) = I_0 \exp(-\mu z)$



Linear space invariant imaging

Image correlation function

$$g(\mathbf{q}, \tau) = \frac{\langle I^*(\mathbf{q}, t)I(\mathbf{q}, t + \tau) \rangle_t}{\langle |I(\mathbf{q}, t)|^2 \rangle_t}$$

Linear space invariant imaging

Image correlation function

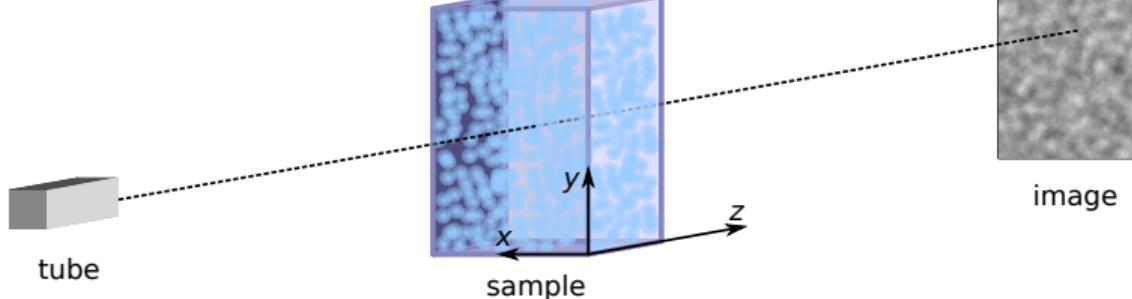
$$g(\mathbf{q}, \tau) = \frac{\langle I^*(\mathbf{q}, t)I(\mathbf{q}, t + \tau) \rangle_t}{\langle |I(\mathbf{q}, t)|^2 \rangle_t}$$

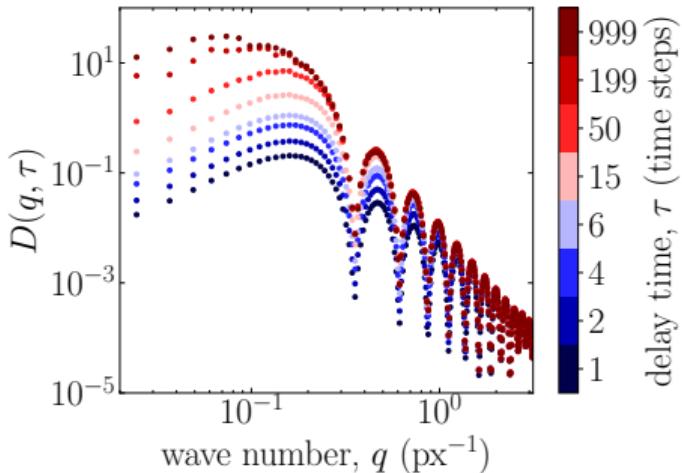
Intermediate scattering function

$$f(\mathbf{q}, \tau) = \frac{\langle \rho^*(\mathbf{q}, t)\rho(\mathbf{q}, t + \tau) \rangle_t}{\langle |\rho(\mathbf{q}, t)|^2 \rangle_t}$$

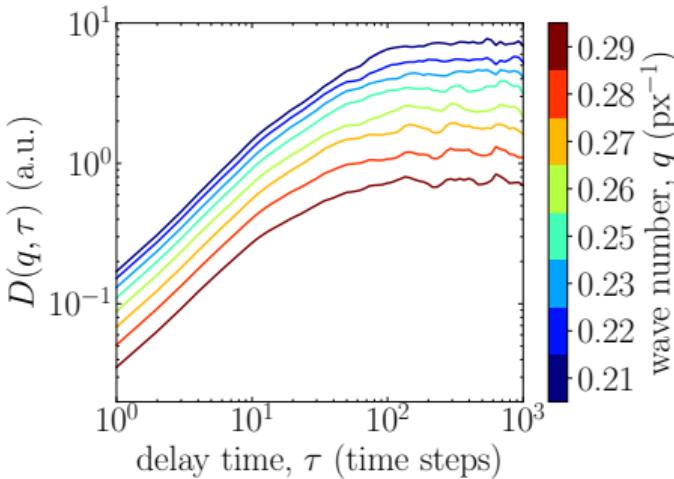
Linear space-invariant imaging:

$$I(\mathbf{r}, t) = I_0 + \int d\mathbf{r}' dz' T(\mathbf{r} - \mathbf{r}', -z') c(\mathbf{r}', z', t)$$





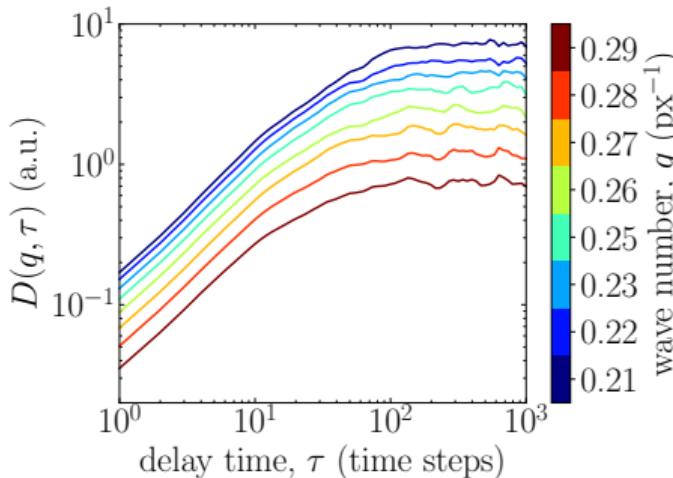
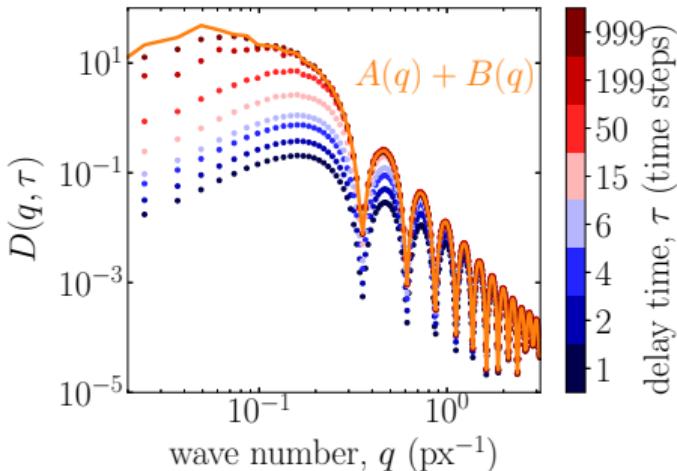
$$\begin{aligned}
 D(q, \tau) &= \left\langle |I(q, t + \tau) - I(q, t)|^2 \right\rangle_t \\
 &= A(q) \underbrace{\left[1 - \frac{\langle I^*(q, t) I(q, t + \tau) \rangle_t}{\langle |I(q, t)|^2 \rangle_t} \right]}_{\text{Image correlation function}} + B(q)
 \end{aligned}$$



Linear space invariant imaging

$$f(q, \tau) = \frac{\langle \rho^*(q, t) \rho(q, t + \tau) \rangle_t}{\langle |\rho(q, t)|^2 \rangle_t}$$

Intermediate scattering function



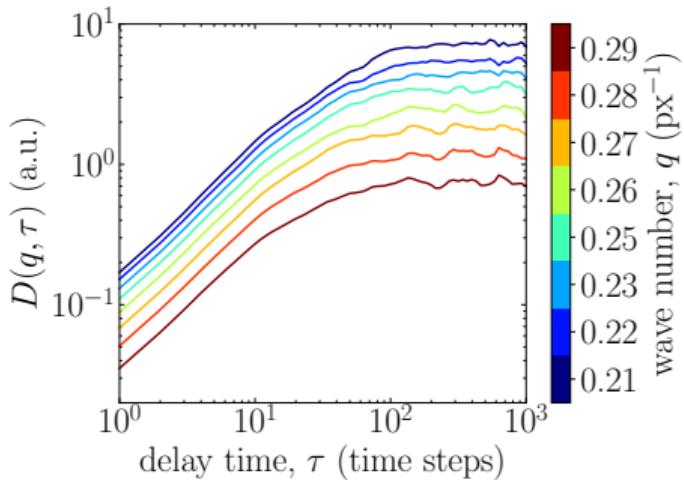
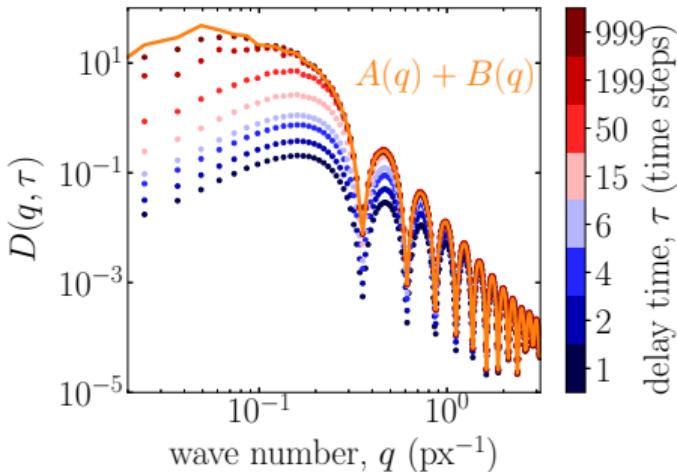
$$\begin{aligned}
 D(q, \tau) &= \left\langle |I(q, t + \tau) - I(q, t)|^2 \right\rangle_t \\
 &= A(q) \left[1 - \frac{\langle I^*(q, t) I(q, t + \tau) \rangle_t}{\langle |I(q, t)|^2 \rangle_t} \right] + B(q)
 \end{aligned}$$

- $D(q, \tau \rightarrow 0) = B(q) = 0$
- $D(q, \tau \rightarrow \infty) = A(q) + B(q)$

Linear space invariant imaging

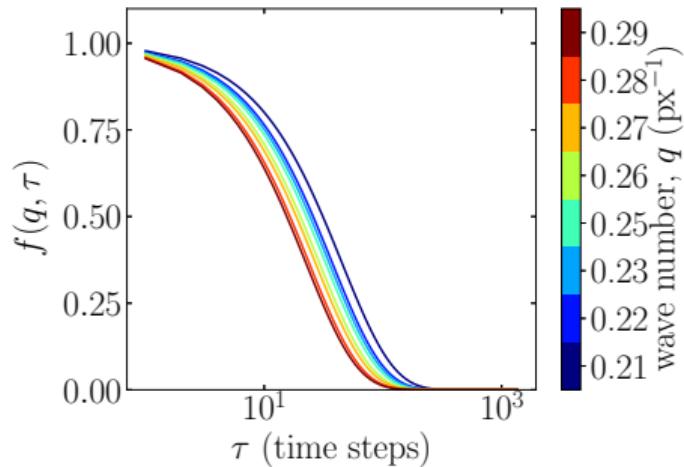
$$f(q, \tau) = \frac{\langle \rho^*(q, t) \rho(q, t + \tau) \rangle_t}{\langle |\rho(q, t)|^2 \rangle_t}$$

Intermediate scattering function

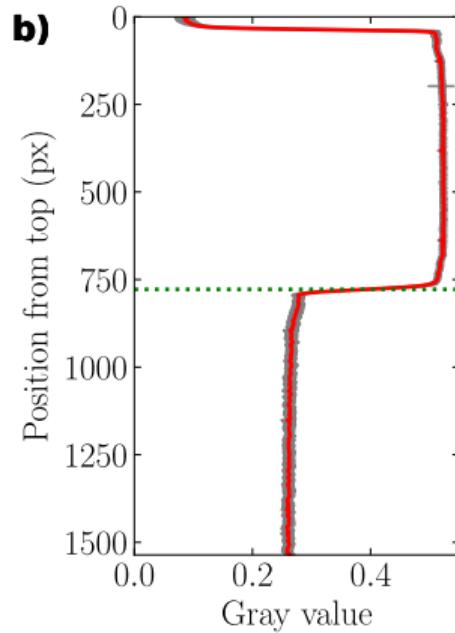
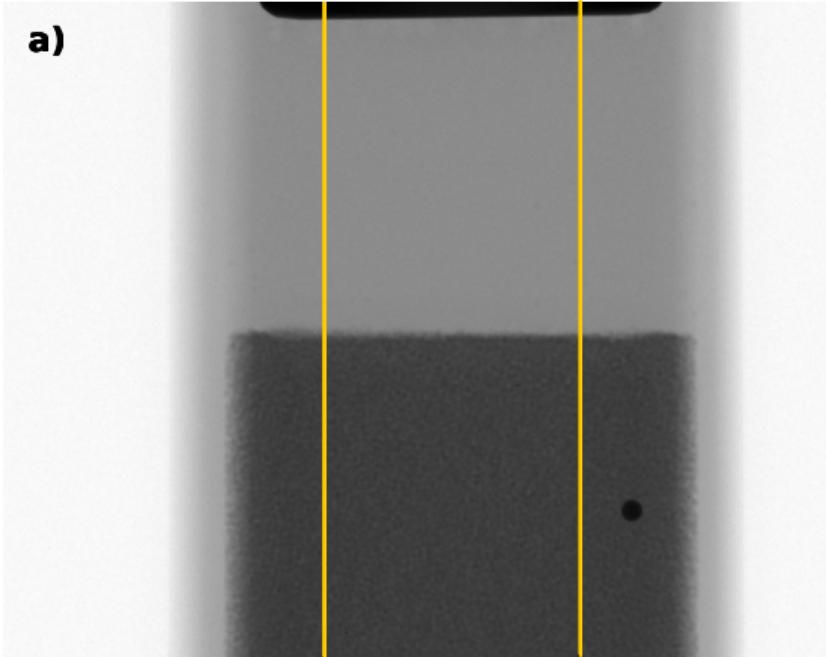


$$\begin{aligned} D(q, \tau) &= \left\langle |I(q, t + \tau) - I(q, t)|^2 \right\rangle_t \\ &= A(q) \left[1 - \frac{\langle I^*(q, t) I(q, t + \tau) \rangle_t}{\langle |I(q, t)|^2 \rangle_t} \right] + B(q) \end{aligned}$$

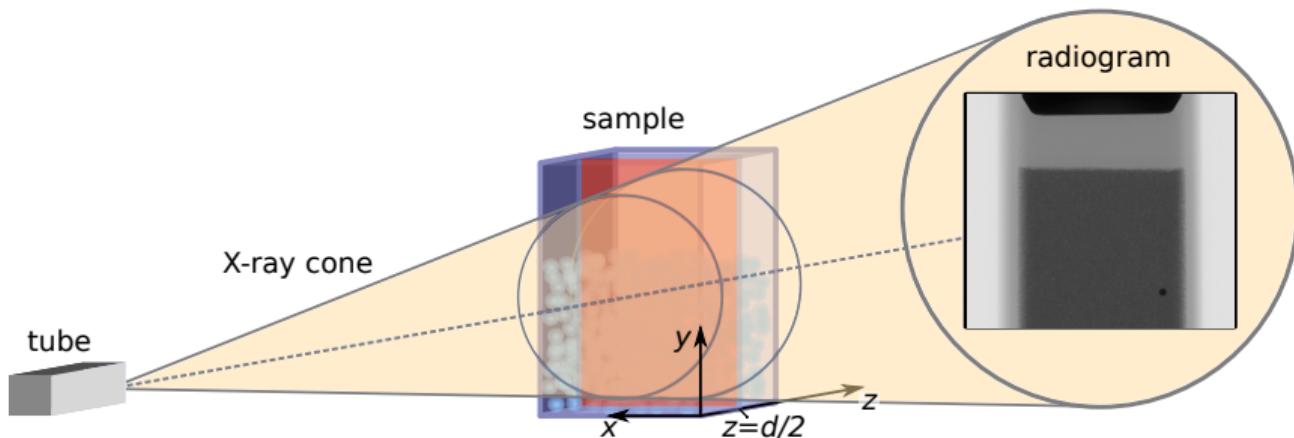
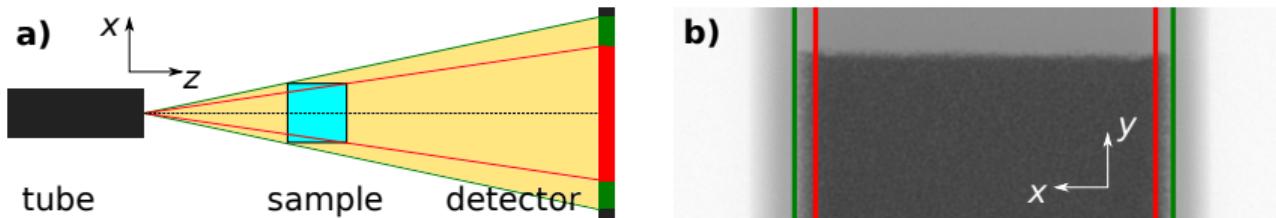
- $D(q, \tau \rightarrow 0) = B(q) = 0$
- $D(q, \tau \rightarrow \infty) = A(q) + B(q)$



Tracking of particle front



Tracking of particle front



Tracking of particle front

