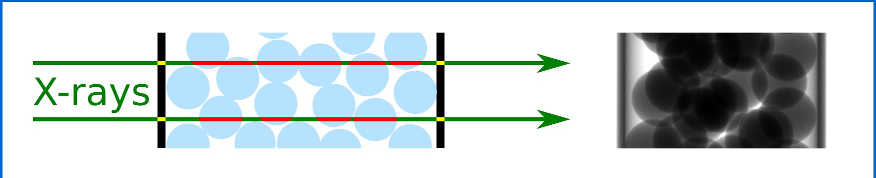
Measuring the volume fraction of **dynamic** granular systems

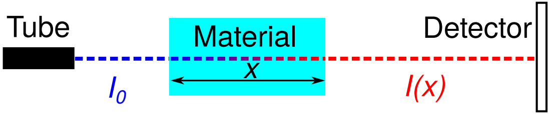


Correction of beam hardening in X-ray radiograms

Baur *et al*, *Rev. Sci. Instrum.* (2019)

In collaboration with Norman Uhlmann, Fraunhofer EZRT

Attenuation of X-rays

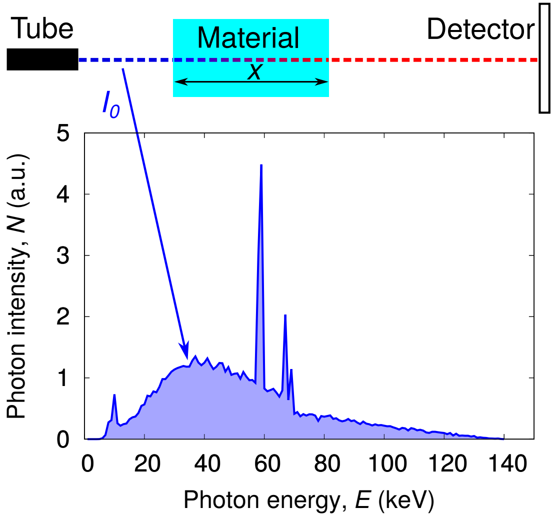


Beer-Lambert's law

$$I(x) = I_0 \exp(-\mu\, x)$$

Thickness: $x = -\frac{1}{\mu} \ln \frac{I(x)}{I_0}$

Attenuation of X-rays

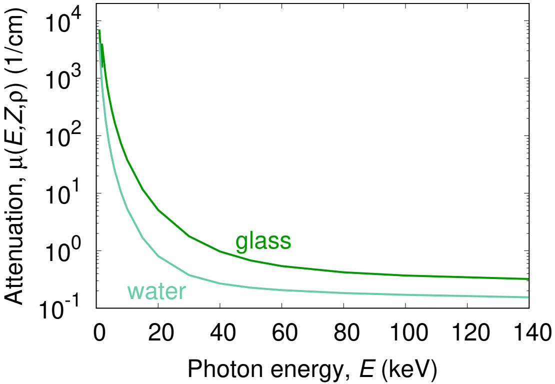


Beer-Lambert's law

$$I(x) = I_0 \exp(-\mu(E, Z, \rho)\, x)$$

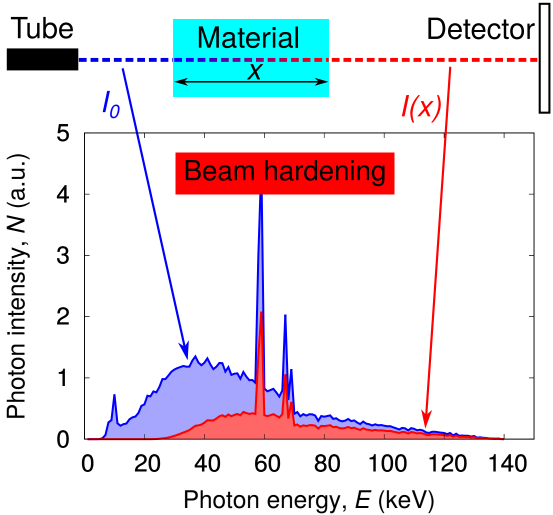
$$\mu \neq \text{const}$$

Thickness: $x = ?$



`https://www.nist.gov/pml/x-ray-mass-attenuation-coefficients`

Attenuation of X-rays



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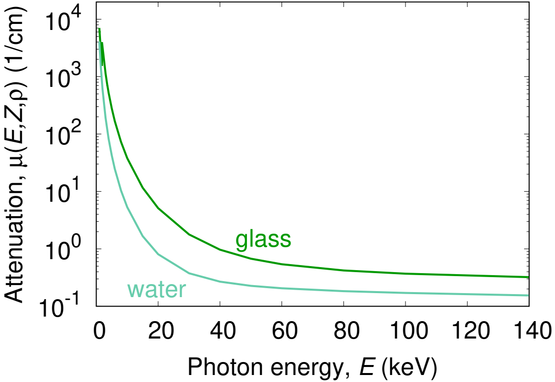
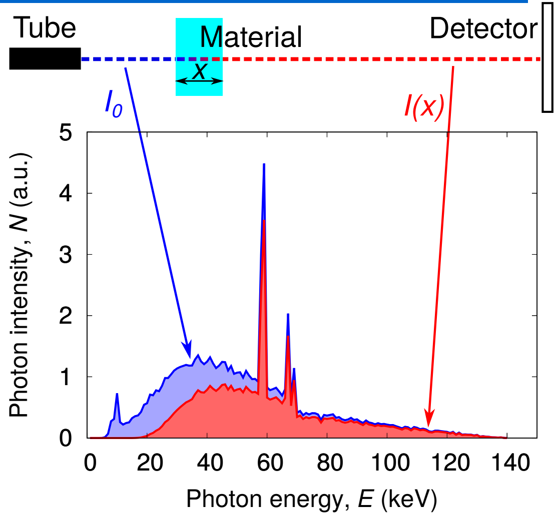
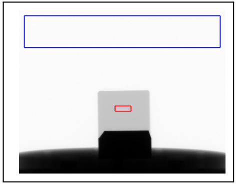


`https://www.nist.gov/pml/x-ray-mass-attenuation-coefficients`

The effective attenuation, $\mu_{\text{eff}}(x)$

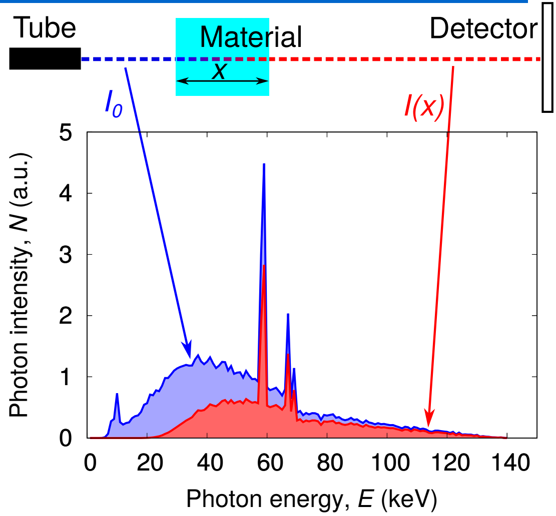


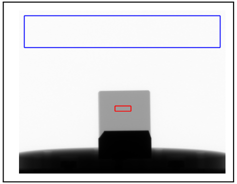


effective attenuation:

$$I(x) = I_0 \exp(-\mu_{\text{eff}}(x)\, x)$$

The effective attenuation, $\mu_{\mathrm{eff}}(x)$

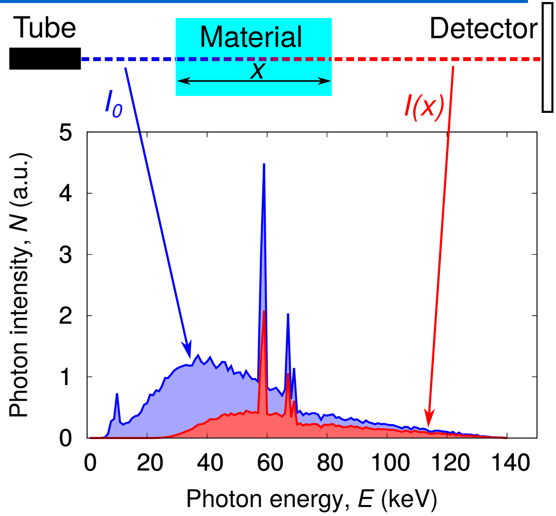


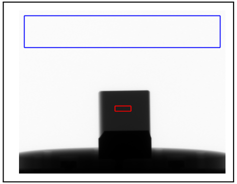


effective attenuation:

$$I(x) = I_0 \exp(-\mu_{\mathrm{eff}}(x)\, x)$$

The effective attenuation, $\mu_{\text{eff}}(x)$

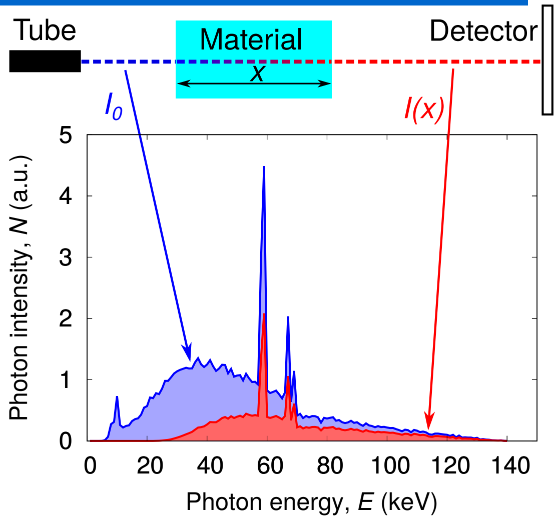


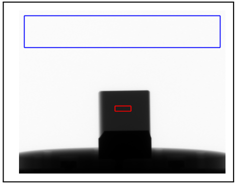


effective attenuation:

$$I(x) = I_0 \exp(-\mu_{\text{eff}}(x)\, x)$$

The effective attenuation, $\mu_{eff}(x)$

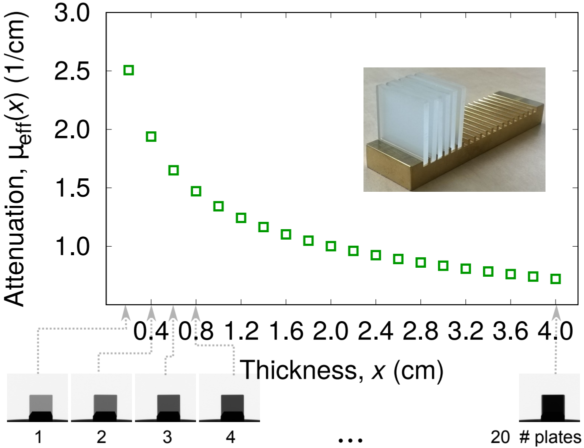




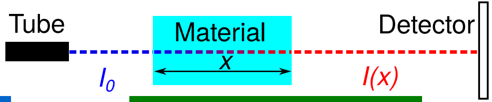
effective attenuation:

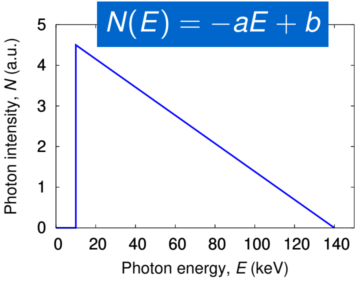
$$I(x) = I_0 \exp(-\mu_{eff}(x)\, x)$$

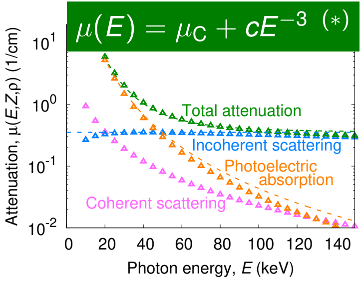
$$\mu_{eff}(x) = -\frac{1}{x}\,\frac{I(x)}{I_0}$$

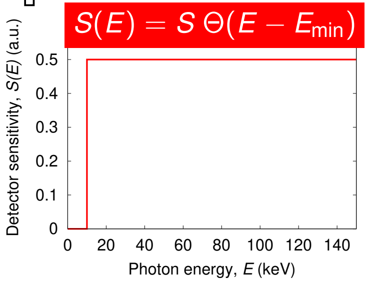


Modeling of $\mu_{\text{eff}}(x)$





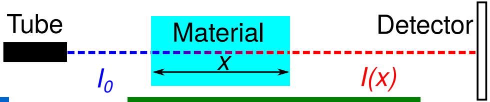




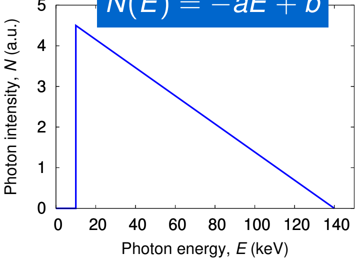
$$I(x) \propto \int N(E) \exp\{-\mu(E, Z, \rho)x\} S(E)\, \mathrm{d}E$$

(∗) XCOM supplied by NIST

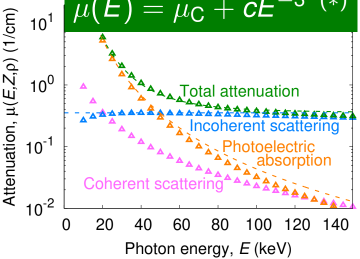
Modeling of $\mu_{\text{eff}}(x)$



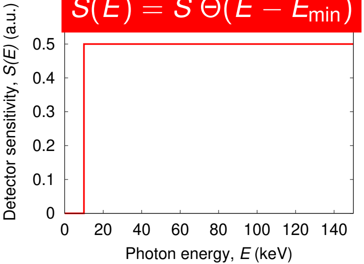
$$N(E) = -aE + b$$



$$\mu(E) = \mu_C + cE^{-3} \quad (*)$$

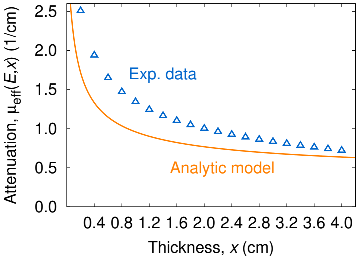


$$S(E) = S\,\Theta(E - E_{\text{min}})$$



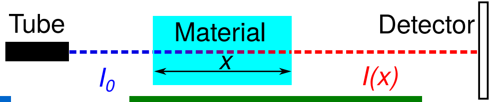
$$I(x) \propto \int N(E)\exp\{-\mu(E,Z,\rho)x\}S(E)\,\mathrm{d}E$$

$$\propto S\int_{E_{\text{min}}}^{E_{\text{max}}}(-aE+b)\exp\{-(\mu_C + cE^{-3})x\}\mathrm{d}E$$

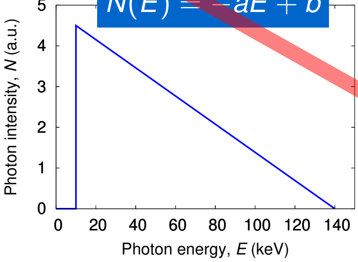


(∗) XCOM supplied by NIST

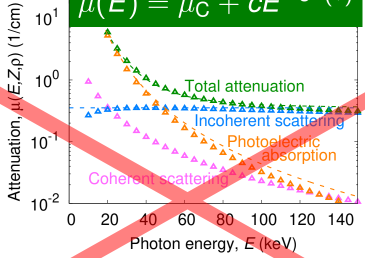
Modeling of $\mu_{\mathrm{eff}}(x)$



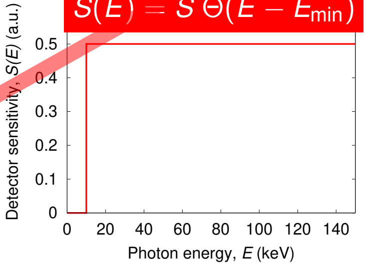
$N(E) = -aE + b$



$\mu(E) = \mu_C + cE^{-3}$ $(*)$

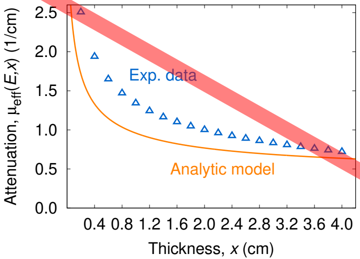


$S(E) = S\,\Theta(E - E_{\mathrm{min}})$



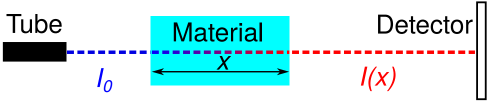
$$I(x) \propto \int N(E) \exp\{-\mu(E, Z, \rho)x\} S(E)\, \mathrm{d}E$$

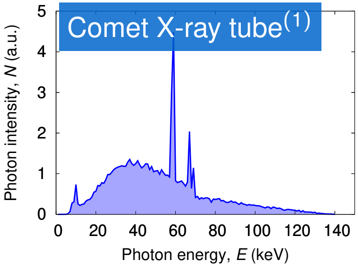
$$\propto S \int_{E_{\mathrm{min}}}^{E_{\mathrm{max}}} (-aE + b) \exp\{-(\mu_C + cE^{-3})x\} \mathrm{d}E$$

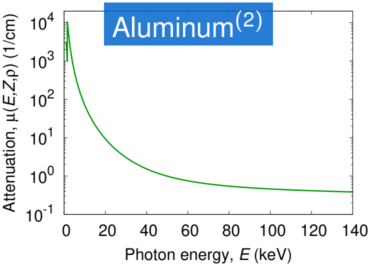


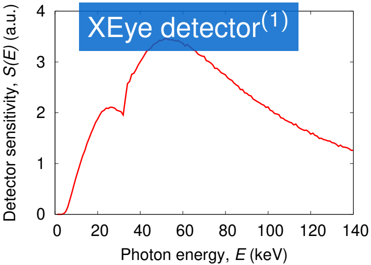
$(*)$ XCOM supplied by NIST

Numerical approx. of $\mu_{\text{eff}}(x)$









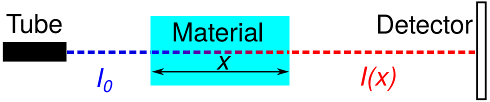
$$I(x) \propto \int N(E) \exp\{-\mu(E,Z,\rho)x\} S(E) \, \mathrm{d}E$$

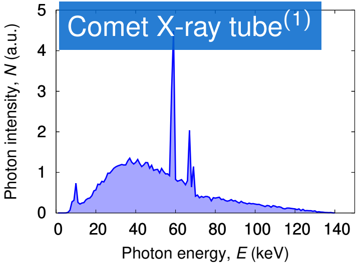
$$\int \to \sum$$

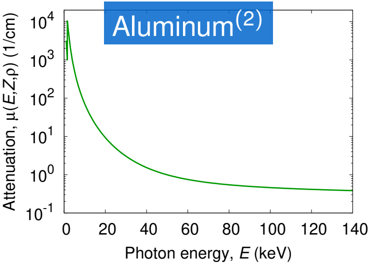
(1) Supplied by Norman Uhlmann, Fraunhofer EZRT

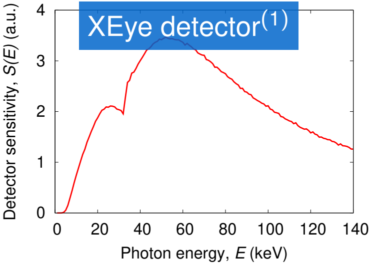
(2) XCOM supplied by NIST

Numerical approx. of $\mu_{\mathrm{eff}}(x)$









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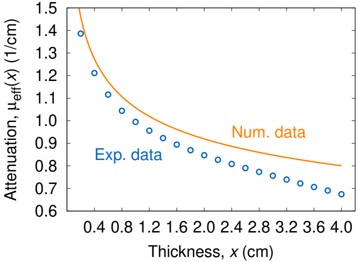

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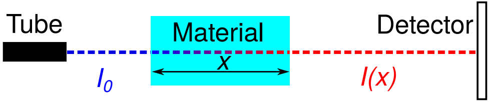
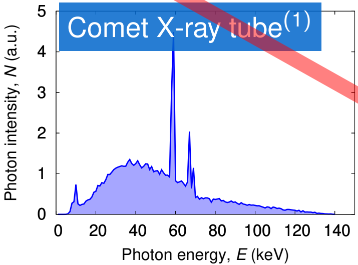
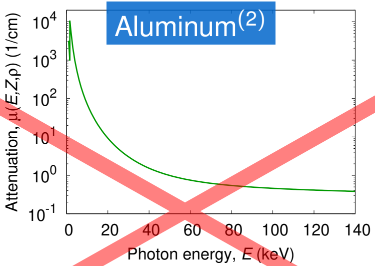
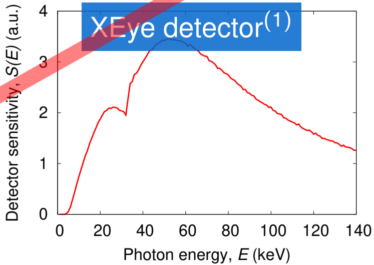


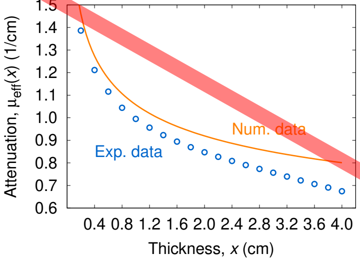






$$I(x) \propto \int N(E)\,\exp\{-\mu(E,Z,\rho)x\}\,S(E)\,\mathrm{d}E$$

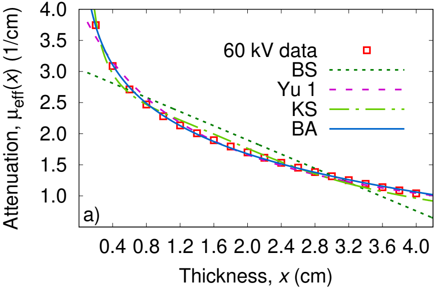
$$\int \to \sum$$



(1) Supplied by Norman Uhlmann, Fraunhofer EZRT

(2) XCOM supplied by NIST

Heuristic model functions for $\mu_{\text{eff}}(x)$



$$\mu_{\text{eff}}(x) = \mu_0 - \lambda x$$

Bjärngard & Shackford (1994)

$$\mu_{\text{eff}}(x) = \frac{\mu_0}{1+\lambda x}$$

Yu *et al.* (1997)

$$\mu_{\text{eff}}(x) = \mu(E_{\text{max}}) + \frac{2\mu_1}{x\sqrt{-\lambda_1^2+4\lambda_2}} \times \left[\arctan\left(\frac{\lambda_1+2\lambda_2 x}{\sqrt{-\lambda_1^2+4\lambda_2}}\right) - \arctan\left(\frac{\lambda_1}{\sqrt{-\lambda_1^2+4\lambda_2}}\right)\right]$$

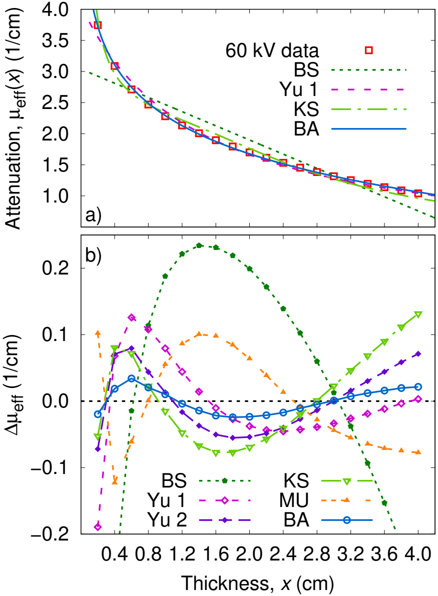
Kleinschmidt (1999)

$$\boldsymbol{\mu_{\text{eff}}(x) = a + \frac{b}{x^{\alpha}}}$$

Baur *et al.* (2019)

(this work)

Heuristic model functions for $\mu_{\text{eff}}(x)$



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$$\mu_{\text{eff}}(x) = \frac{\mu_0}{(1+\lambda x)^\beta}$$

Yu *et al.* (1997)

$$\mu_{\text{eff}}(x) = \mu(E_{\text{max}}) + \frac{2\mu_1}{x\sqrt{-\lambda_1^2+4\lambda_2}} \times \left[\arctan\left(\frac{\lambda_1+2\lambda_2 x}{\sqrt{-\lambda_1^2+4\lambda_2}}\right) - \arctan\left(\frac{\lambda_1}{\sqrt{-\lambda_1^2+4\lambda_2}}\right)\right]$$

Kleinschmidt (1999)

$$\mu_{\text{eff}}(x) = -\frac{1}{x}\ln\left[A + B\exp(-x/C)\right]$$

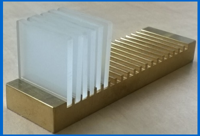
Mudde *et al.* (2008)

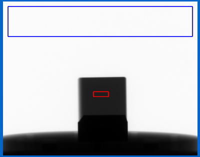
$$\boxed{\mu_{\text{eff}}(x) = a + \frac{b}{x^\alpha}}$$

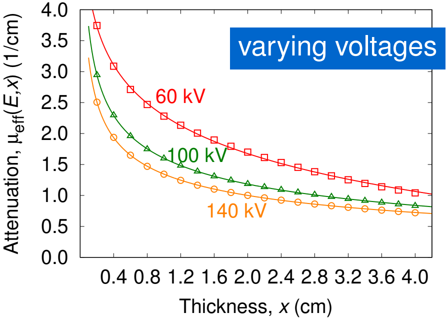
Baur *et al.* (2019)

(this work)

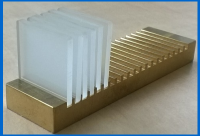
Universality of $\mu_{\text{eff}}(x) = a + \frac{b}{x^{\alpha}}$

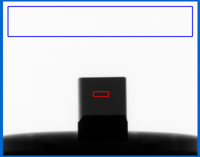


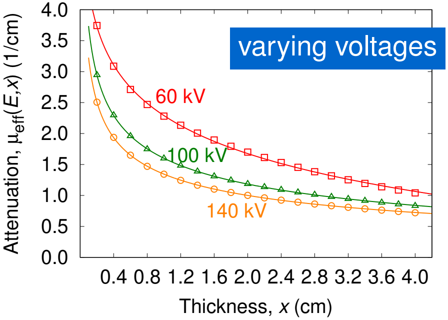


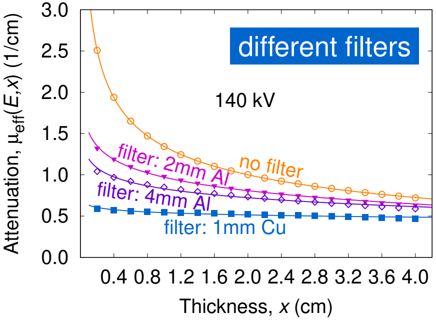


Universality of $\mu_{\mathrm{eff}}(x) = a + \frac{b}{x^{\alpha}}$

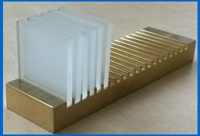


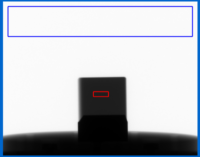


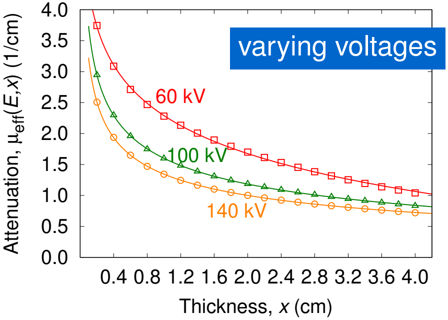


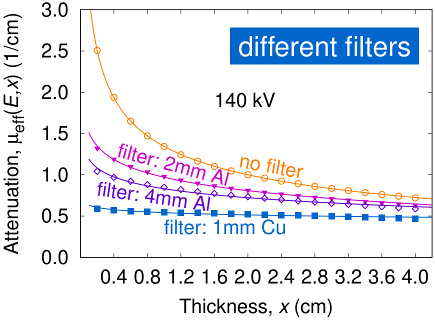


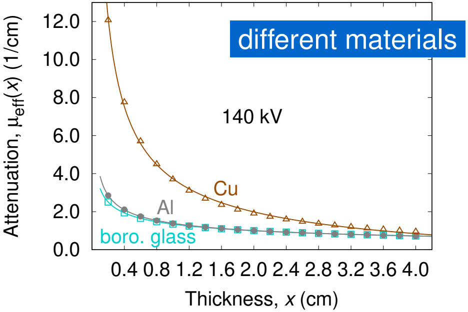
Universality of $\mu_{\text{eff}}(x) = a + \frac{b}{x^\alpha}$



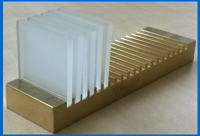


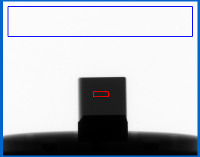


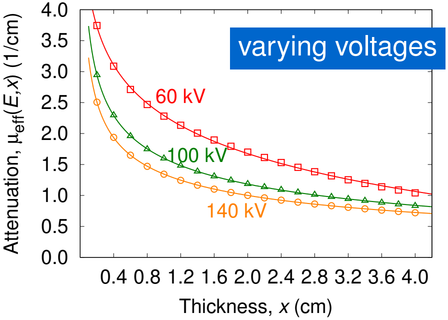


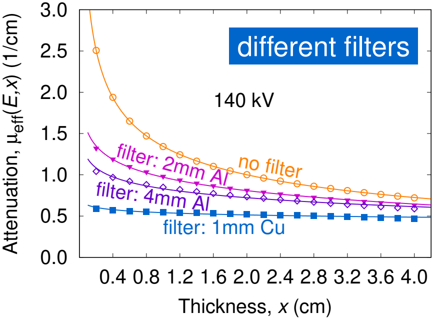


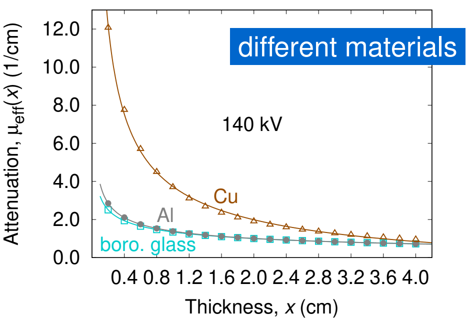
Universality of $\mu_{\text{eff}}(x) = a + \frac{b}{x^{\alpha}}$

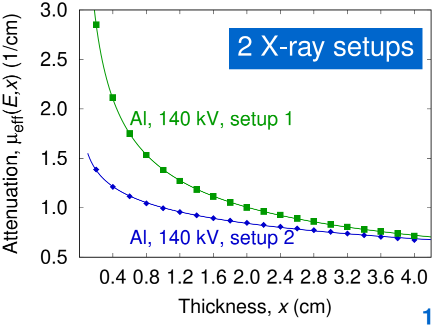












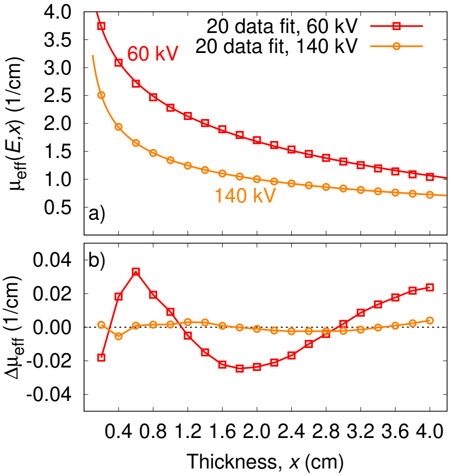
Determining the material thickness x

Generalized Beer-Lambert

$$I(x) = I_0 \exp(-\mu_{\mathrm{eff}}(x)\, x)$$

Model function

$$\mu_{\mathrm{eff}}(x) = a + \frac{b}{x^{\alpha}}$$



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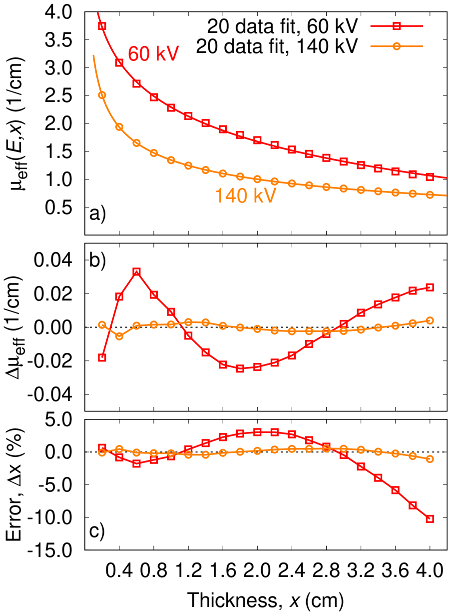
e.g. Newton's method or look-up table



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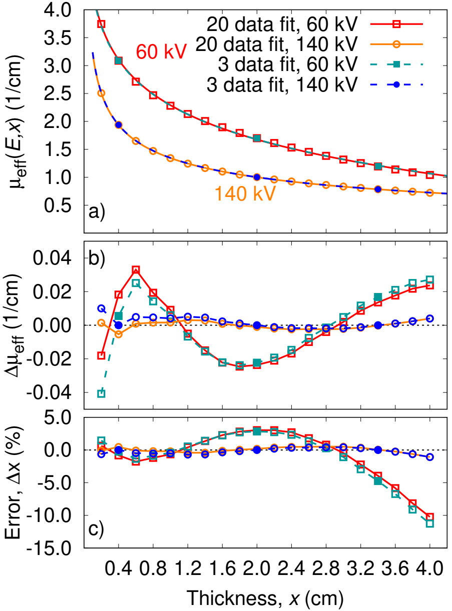

Migrating shear bands in shaken granular matter, Kollmer *et al* (2020)