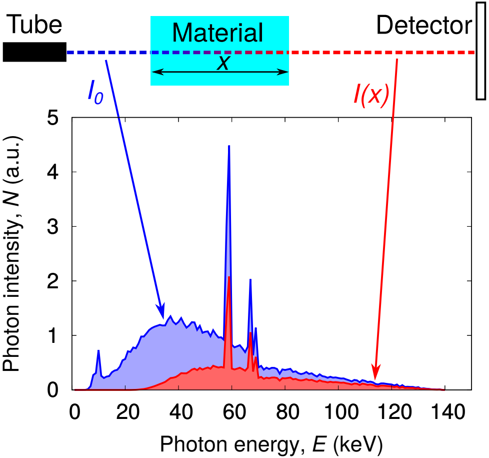
FRIEDRICH-ALEXANDER
UNIVERSITÄT
ERLANGEN-NÜRNBERG

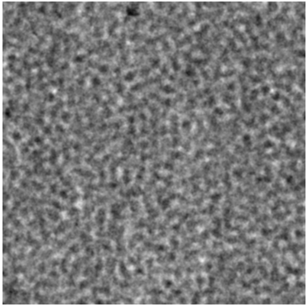
Institute for
Multiscale Simulation
MSS

X-ray radiography of granular systems – particle densities and dynamics

PhD defense
Manuel Baur

Funded by the German Federal Ministry for Economic Affairs and Energy, grant no. 50WM 1653





X-ray radiography of granular systems – particle densities and dynamics

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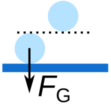
⋮

Granular materials are athermal

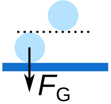


$$E_{pot} \approx 10^{10} E_{thermal}$$

X-ray radiography of granular systems – particle densities and dynamics

⋮

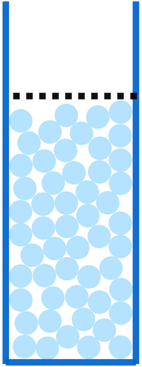
Granular materials are athermal



$$E_{pot} \approx 10^{10} E_{thermal}$$

Volume fraction

$$\Phi = \frac{V_{Particles}}{V_{Container}}$$

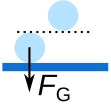


$$\Phi = \Phi_{RLP}$$

X-ray radiography of granular systems – particle densities and dynamics

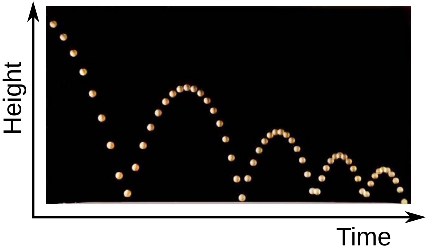
⋮

Granular materials are athermal



$$E_{pot} \approx 10^{10} E_{thermal}$$

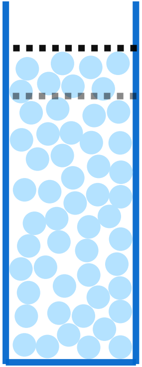
Dissipative interactions



Driscoll *et al* (2016)

Volume fraction

$$\Phi = \frac{V_{\text{Particles}}}{V_{\text{Container}}}$$

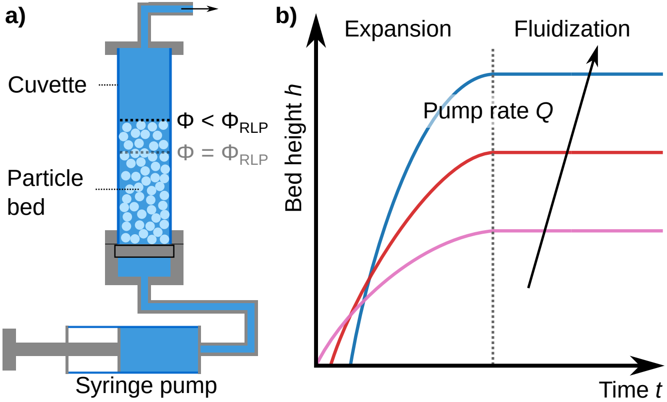


$\Phi < \Phi_{RLP}$

$\Phi = \Phi_{RLP}$

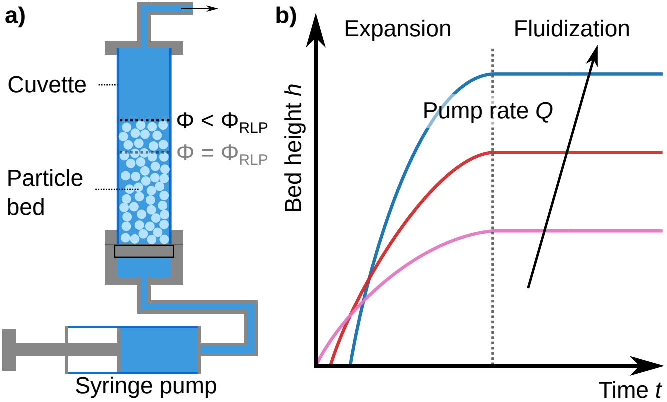
X-ray radiography of granular systems – particle densities and dynamics

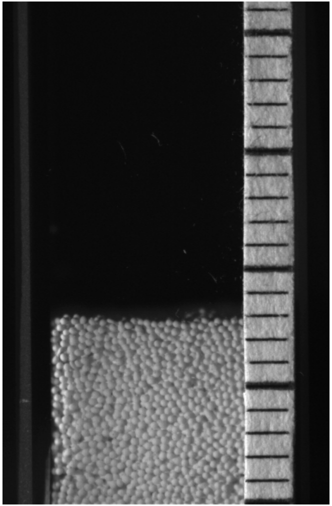
Liquid fluidized bed



X-ray radiography of granular systems – particle densities and dynamics

Liquid fluidized bed

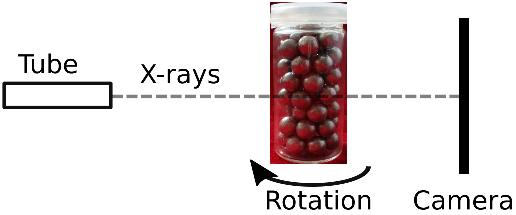




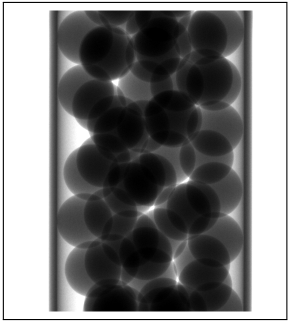
Master thesis Welm Pätzold

Particulate flows are **opaque**

X-ray radiography & tomography



Radiogram

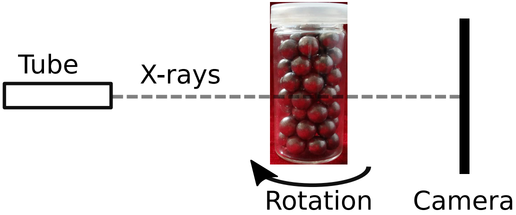


2D projections of 3D object

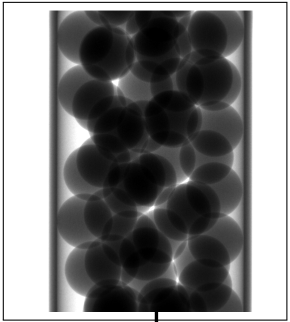
Short acquisition time

$\rightarrow$ Dynamic system

X-ray radiography & tomography



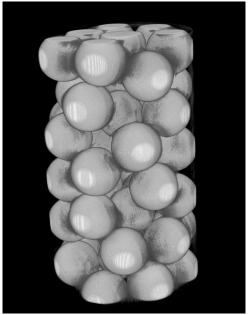
Radiogram



2D projections of 3D object
Short acquisition time
→ Dynamic system

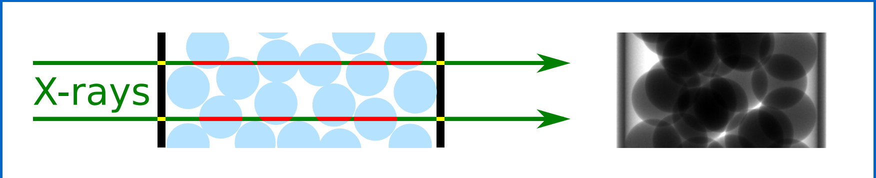
Reconstruction from
800 projections

Tomogram



Full 3D information
Long acquisition time
→ Static objects

Measuring the volume fraction of **dynamic** granular systems

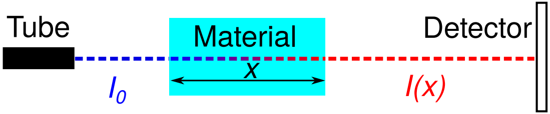


Correction of beam hardening in X-ray radiograms

Baur *et al*, *Rev. Sci. Instrum.* (2019)

In collaboration with Norman Uhlmann, Fraunhofer EZRT

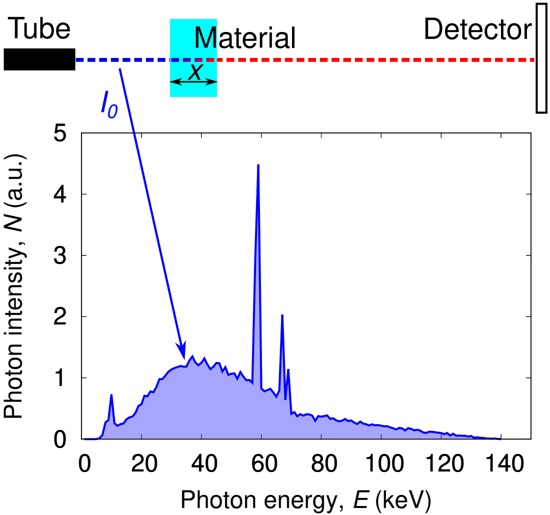
Attenuation of X-rays



Beer-Lambert's law

$$I(x) = I_0 \exp(-\mu\, x)$$

Attenuation of X-rays



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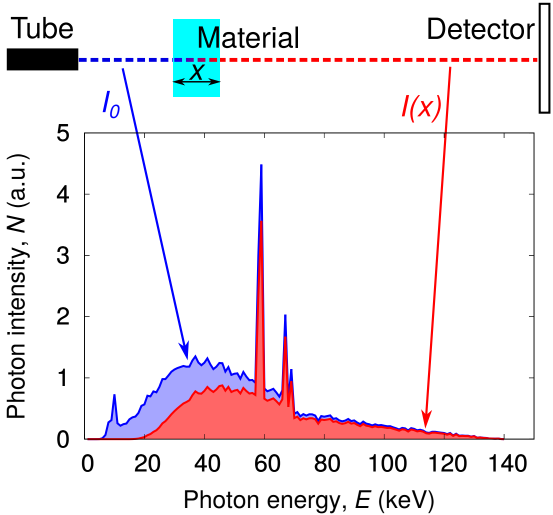

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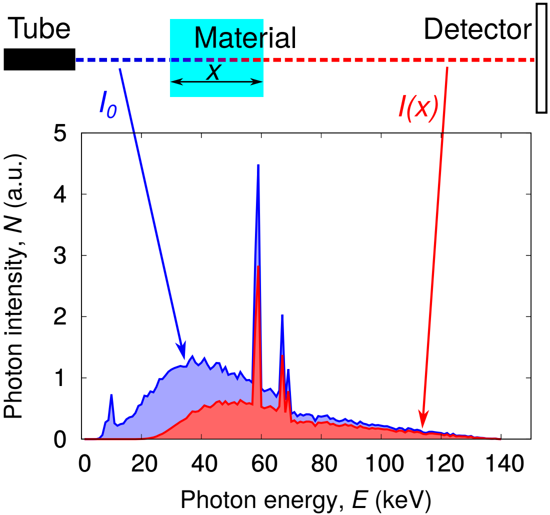

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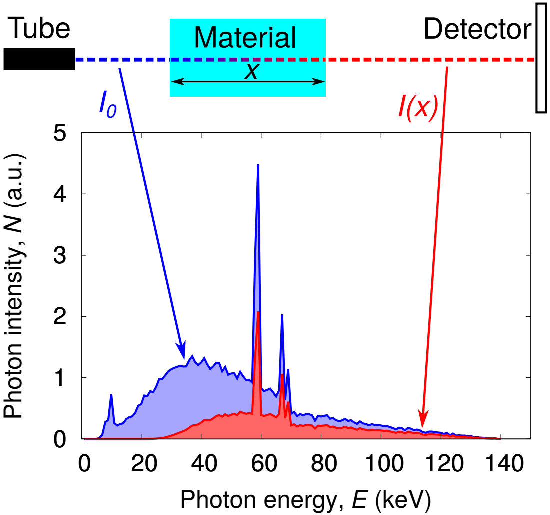

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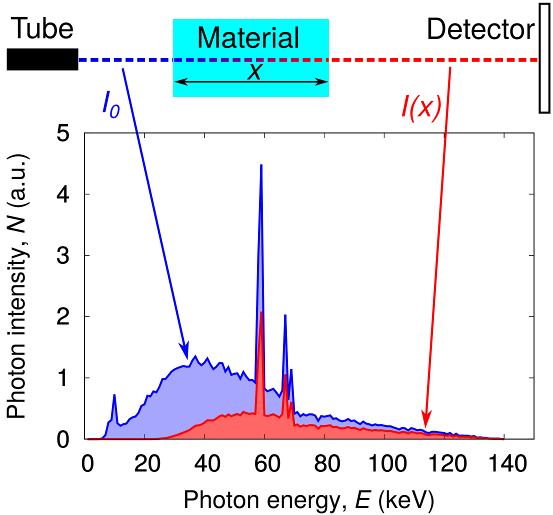
Attenuation of X-rays

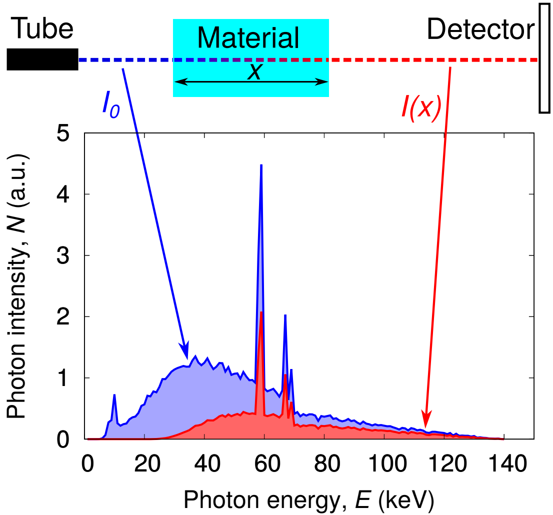


Beer-Lambert's law

$$\cancel{I(x) = I_0 \exp(-\mu\, x)}$$

$$I(x) = I_0 \exp(-\mu_{\text{eff}}(x)\, x)$$

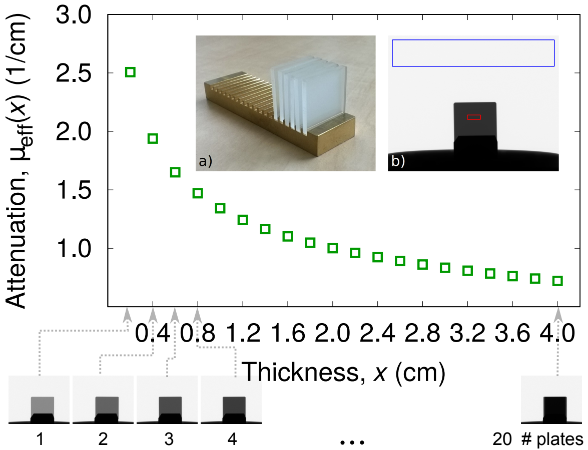
Attenuation of X-rays



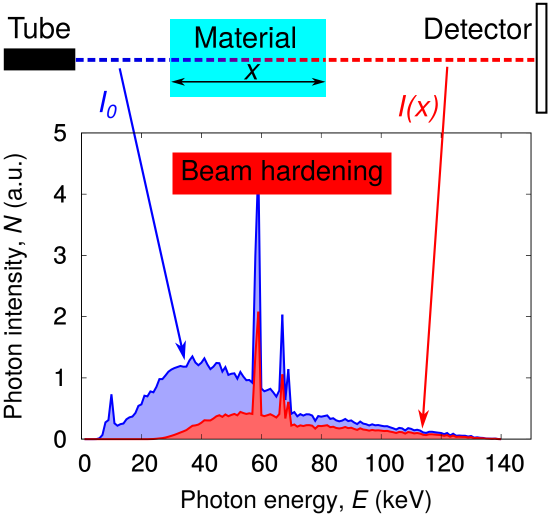
Beer-Lambert's law

$$\cancel{I(x) = I_0 \exp(-\mu\, x)}$$

$$I(x) = I_0 \exp(-\mu_{\text{eff}}(x)\, x)$$

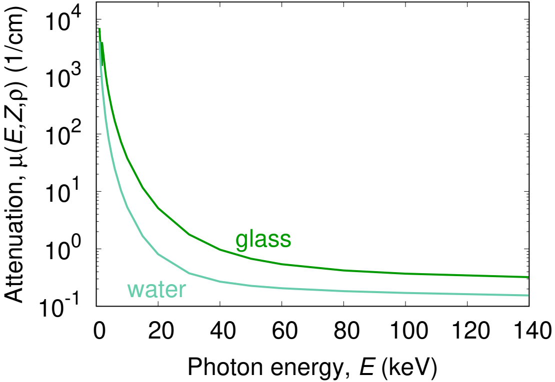


Attenuation of X-rays



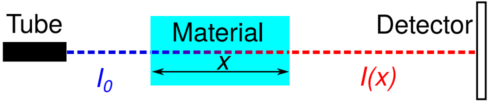
Beer-Lambert's law

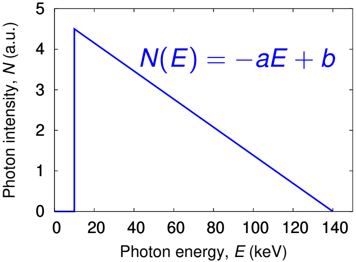
$$I(x) = I_0 \exp(-\mu(E, Z, \rho)\, x)$$

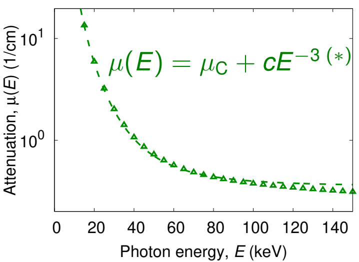


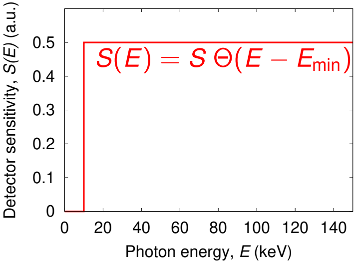
Tabulated attenuation coefficients from NIST.gov

Modeling of $\mu_{\text{eff}}(x)$





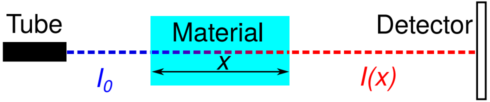


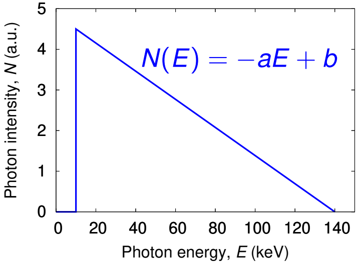


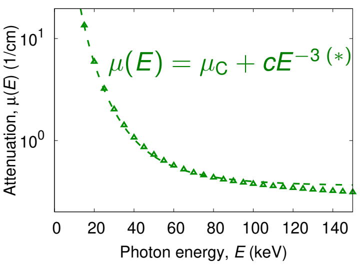
$$I(x) \propto \int N(E) \exp\{-\mu(E)x\} S(E) \, \mathrm{d}E$$

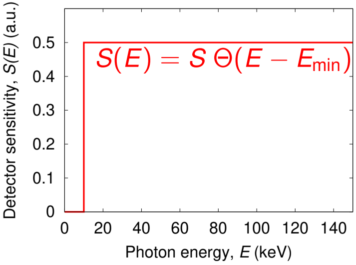
(∗) XCOM supplied by NIST

Modeling of $\mu_{\text{eff}}(x)$



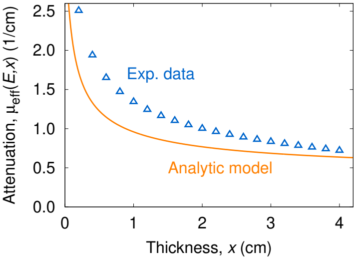






$$I(x) \propto \int N(E)\exp\{-\mu(E)x\}S(E)\,\mathrm{d}E$$

$$\propto S\int_{E_{\min}}^{E_{\max}}(-aE+b)\exp\{-(\mu_C + cE^{-3})x\}\,\mathrm{d}E$$



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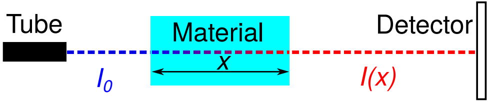
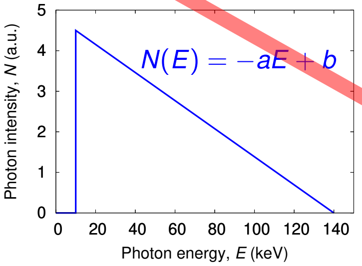
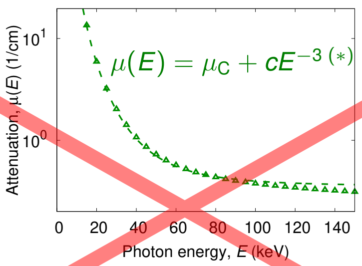
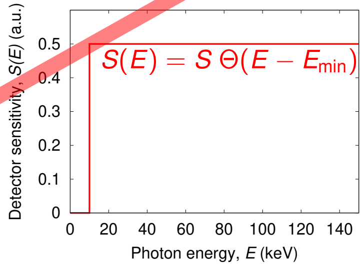
Modeling of $\mu_{\mathrm{eff}}(x)$









$$I(x) \propto \int N(E)\exp\{-\mu(E)x\}S(E)\,\mathrm{d}E$$

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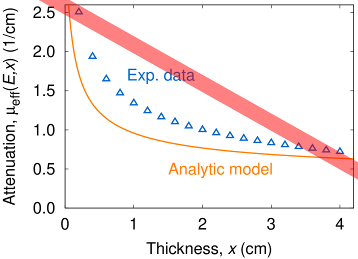

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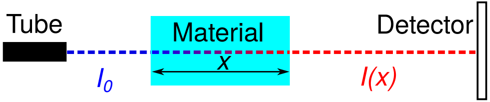
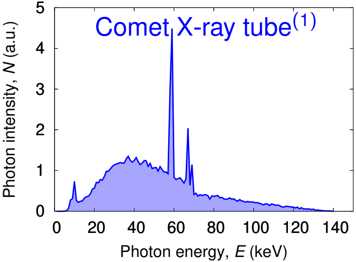
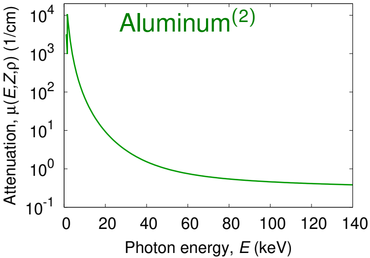
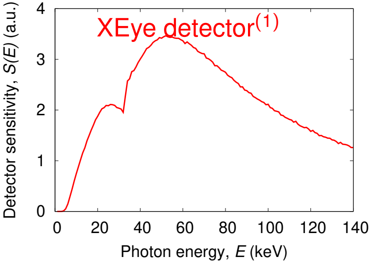
Numerical approx. of $\mu_{\text{eff}}(x)$









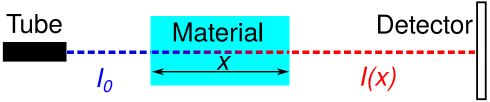
$$I(x) \propto \int N(E)\, \exp\{-\mu(E)x\}\, S(E)\, \mathrm{d}E$$

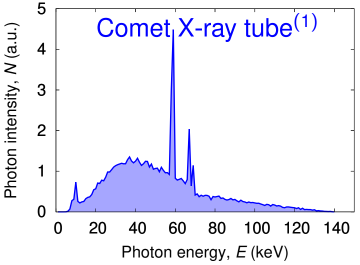
$$\int \to \sum$$

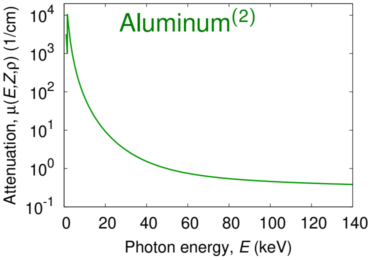
(1) Supplied by Norman Uhlmann, Fraunhofer EZRT

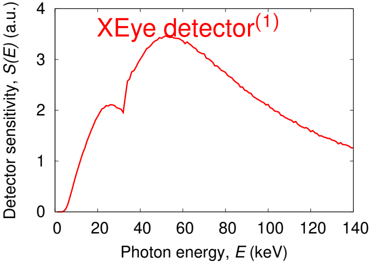
(2) XCOM supplied by NIST

Numerical approx. of $\mu_{\text{eff}}(x)$









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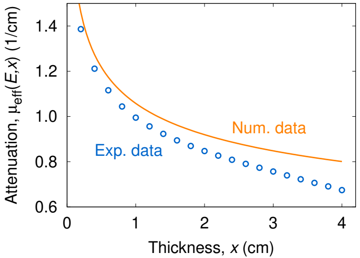

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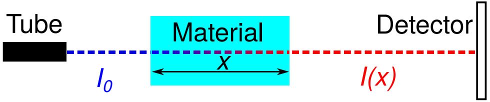
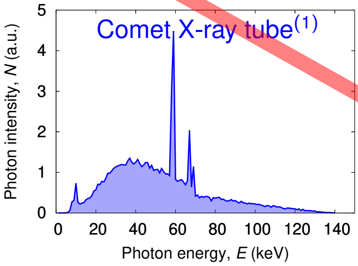
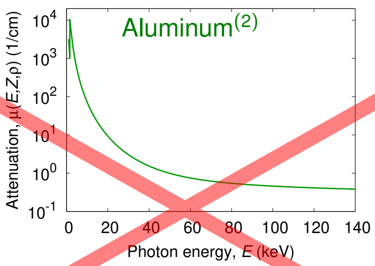
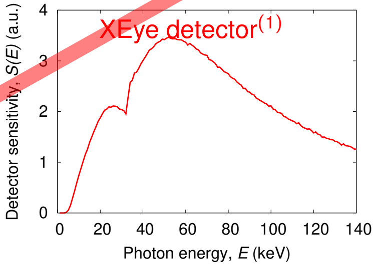
Numerical approx. of $\mu_{\mathrm{eff}}(x)$









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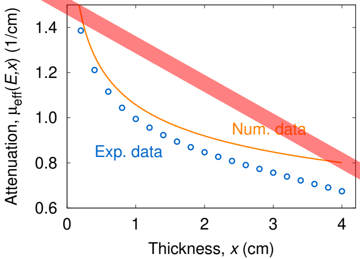

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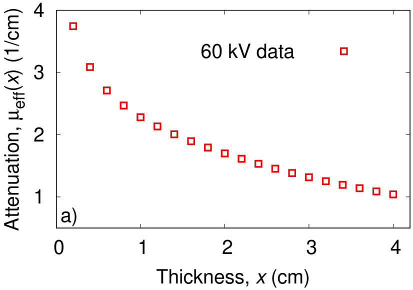
Heuristic model functions for $\mu_{\mathrm{eff}}(x)$



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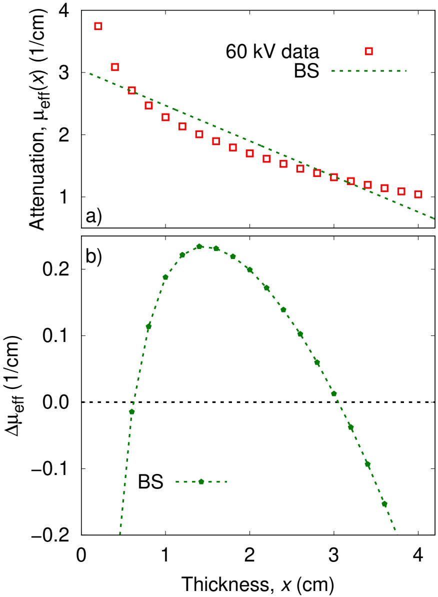
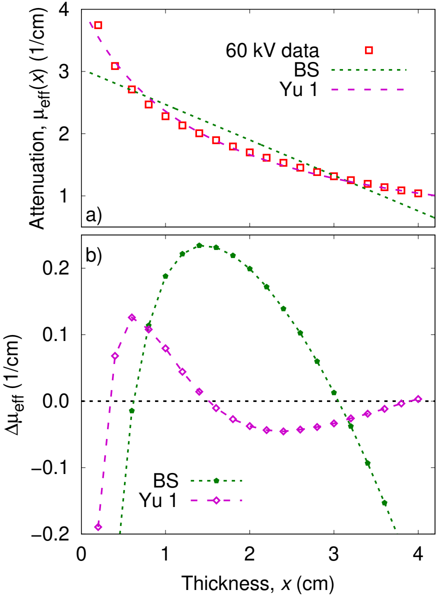


$$\mu_{\text{eff}}(x) = \mu_0 - \lambda x$$

Bjärngard & Shackford (1994)

Heuristic model functions for $\mu_{\text{eff}}(x)$



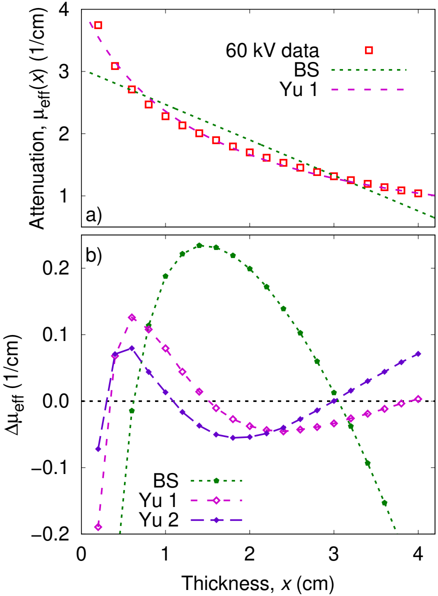
$$\mu_{\text{eff}}(x) = \mu_0 - \lambda x$$

Bjärngard & Shackford (1994)

$$\mu_{\text{eff}}(x) = \frac{\mu_0}{1+\lambda x}$$

Yu *et al.* (1997)

Heuristic model functions for $\mu_{\rm eff}(x)$



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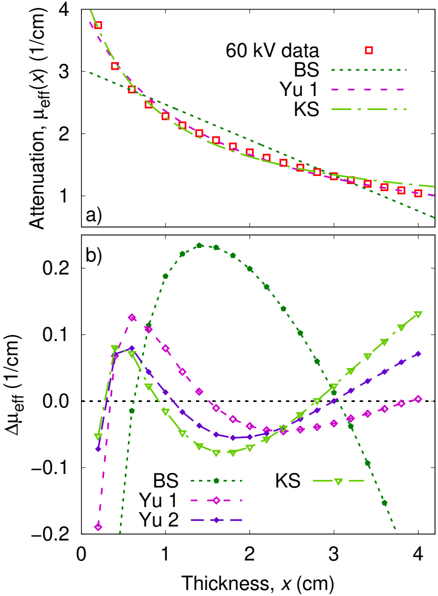
$$\mu_{\rm eff}(x) = \frac{\mu_0}{(1+\lambda x)^\beta}$$

Yu *et al.* (1997)

Heuristic model functions for $\mu_{\text{eff}}(x)$



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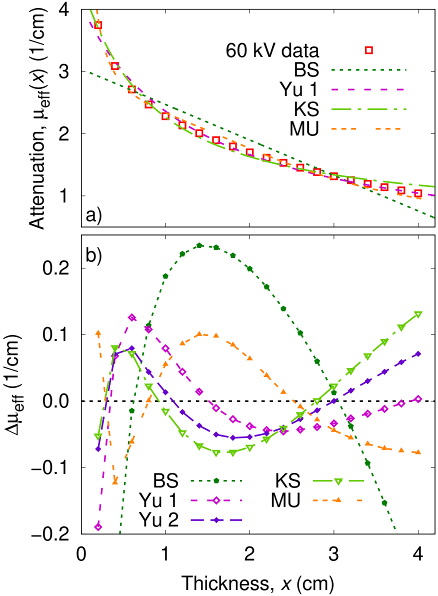
$$\mu_{\text{eff}}(x) = \frac{\mu_0}{(1+\lambda x)^\beta}$$

Yu *et al.* (1997)

$$\mu_{\text{eff}}(x) = \mu(E_{\max}) + \frac{2\mu_1}{x\sqrt{-\lambda_1^2+4\lambda_2}} \times \left[\arctan\left(\frac{\lambda_1+2\lambda_2 x}{\sqrt{-\lambda_1^2+4\lambda_2}}\right) - \arctan\left(\frac{\lambda_1}{\sqrt{-\lambda_1^2+4\lambda_2}}\right)\right]$$

Kleinschmidt (1999)

Heuristic model functions for $\mu_{\mathrm{eff}}(x)$



$$\mu_{\mathrm{eff}}(x) = \mu_0 - \lambda x$$

Bjärngard & Shackford (1994)

$$\mu_{\mathrm{eff}}(x) = \frac{\mu_0}{1+\lambda x}$$

$$\mu_{\mathrm{eff}}(x) = \frac{\mu_0}{(1+\lambda x)^{\beta}}$$

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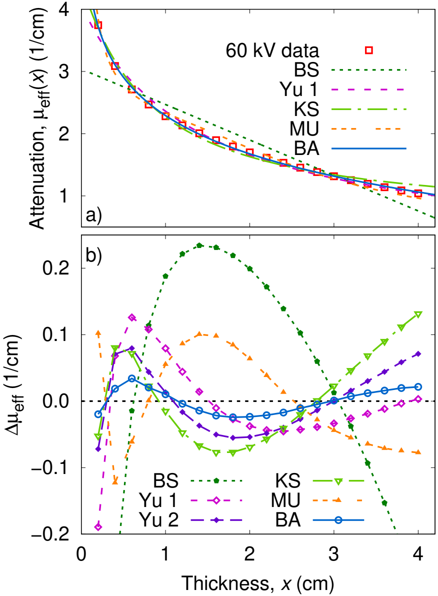

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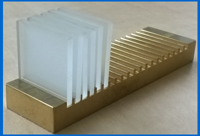
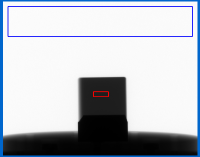
$$\boxed{\mu_{\text{eff}}(x) = a + \frac{b}{x^\alpha}}$$

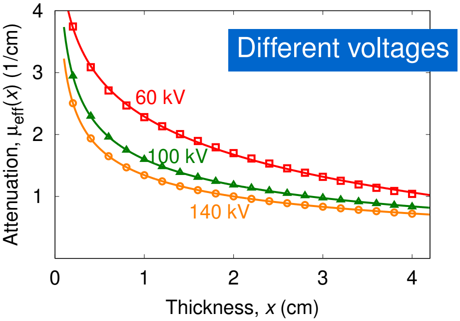
Baur *et al.* (2019)

(this work)

Applicability of $\mu_{\text{eff}}(x) = a + \frac{b}{x^\alpha}$







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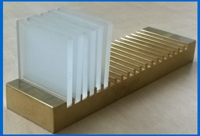

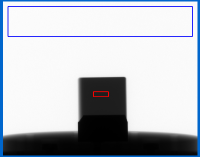

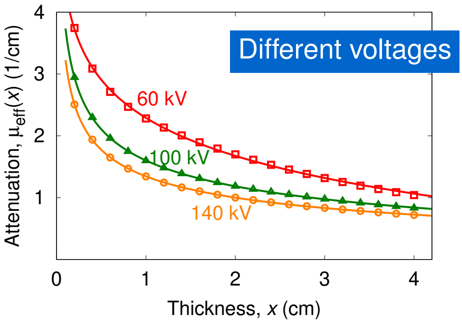

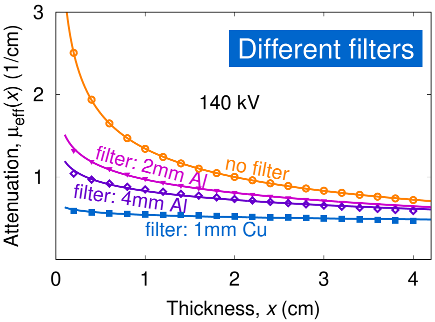

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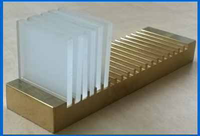
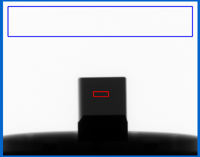
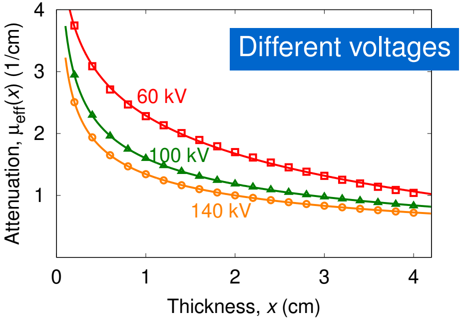
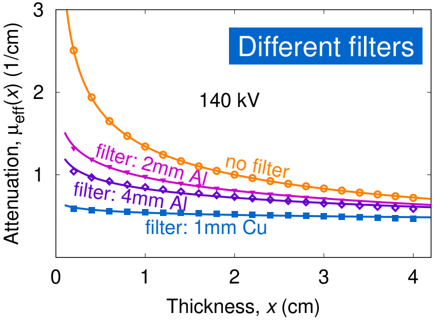
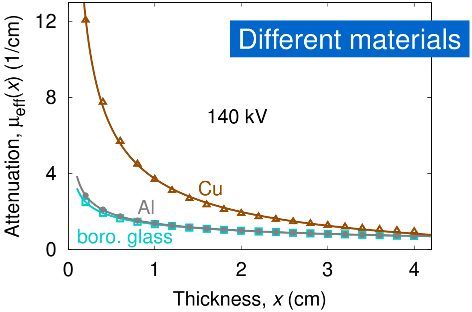


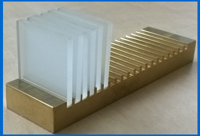


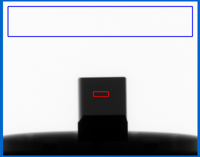


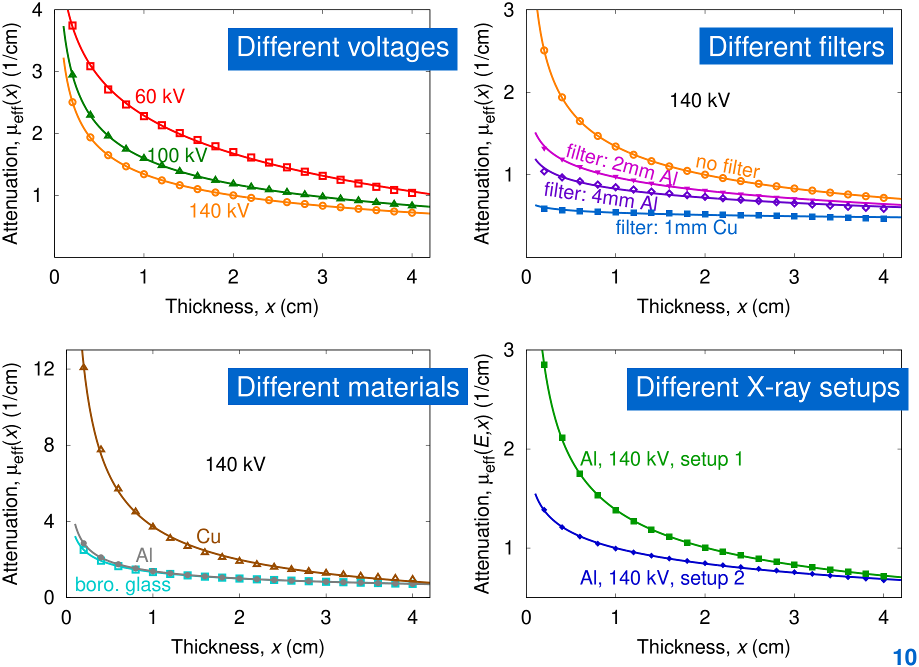




Applicability of $\mu_{\text{eff}}(x) = a + \frac{b}{x^{\alpha}}$







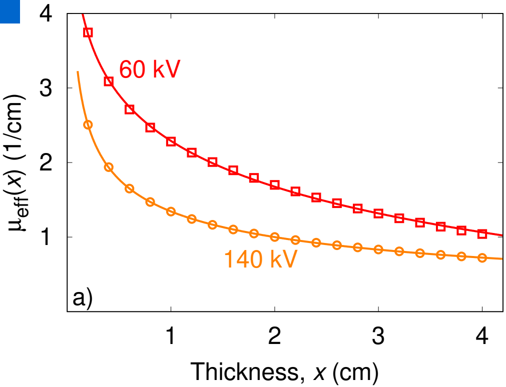
Determining the material thickness x

Generalized Beer-Lambert

$$I(x) = I_0 \exp(-\mu_{\text{eff}}(x)\, x)$$

Model function

$$\mu_{\text{eff}}(x) = a + \frac{b}{x^{\alpha}}$$



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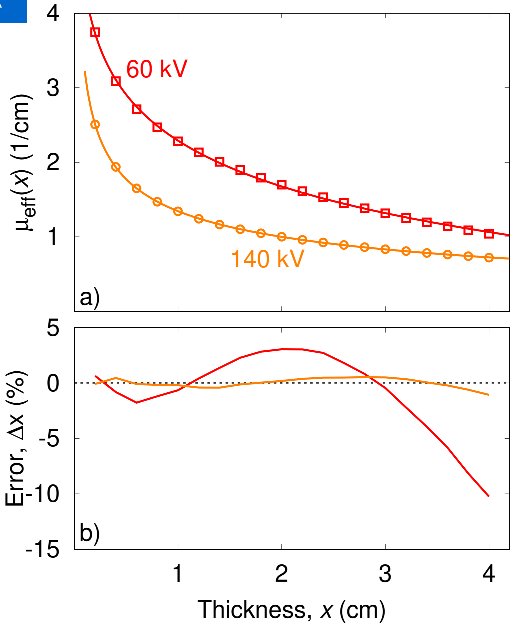
$$ax + bx^{1-\alpha} + \ln\left(\frac{I(x)}{I_0}\right) = 0$$

e.g. Newton's method or look-up table



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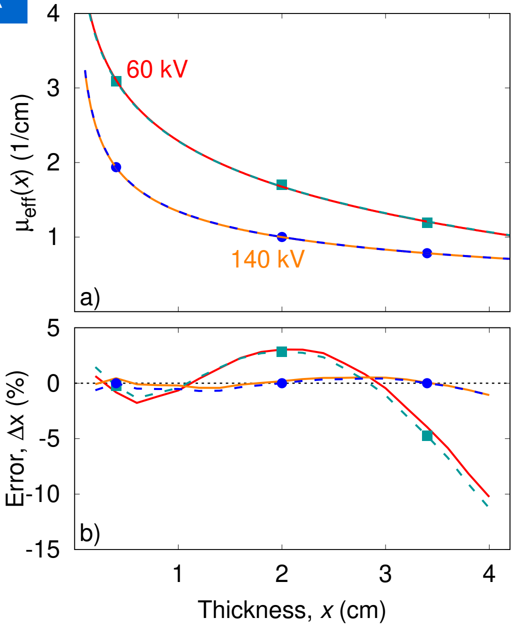
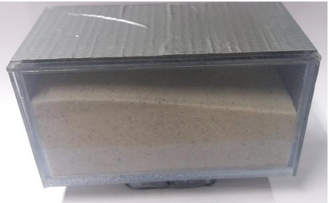
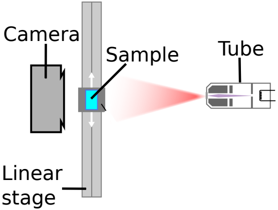
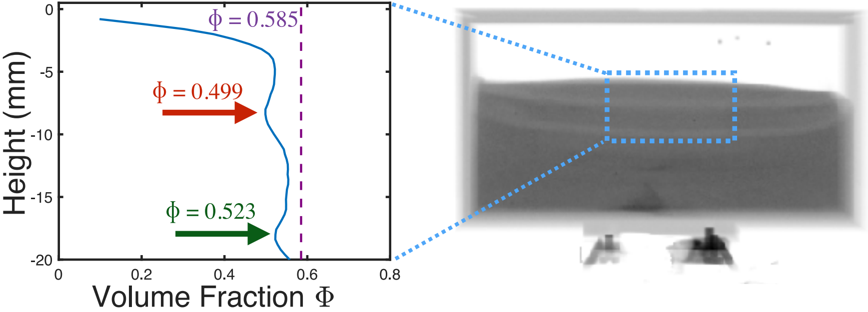


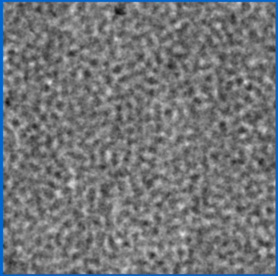
Migrating shear bands in shaken granular matter, Kollmer *et al* (2020)





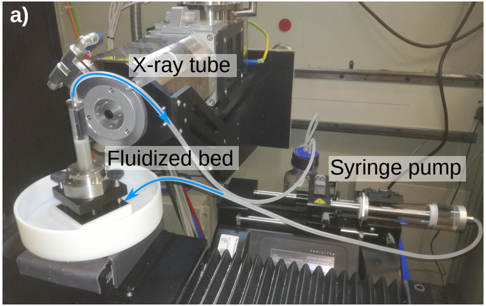


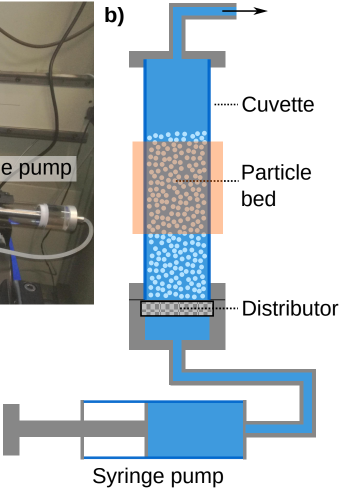
Measuring granular dynamics with
X-ray Digital Fourier Analysis (X-DFA)



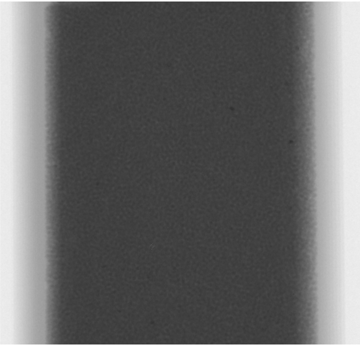
In collaboration with M. Escobedo & S. Egelhaaf, University of Düsseldorf

The system: A liquid fluidized bed





Radiogram



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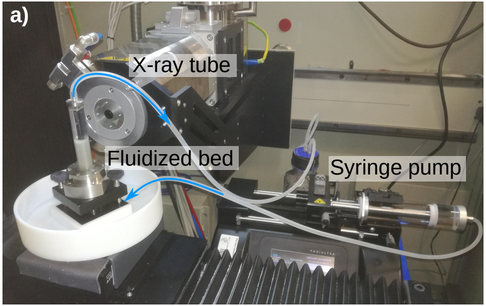
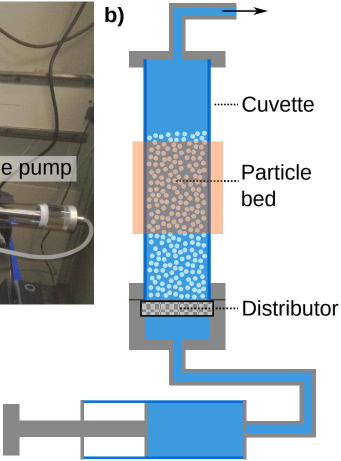
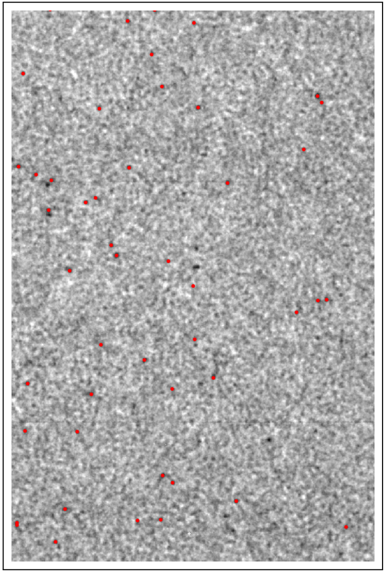




Particle tracking

Contrast: $\rho_{\text{tracer}} > \rho_{\text{bed}}$



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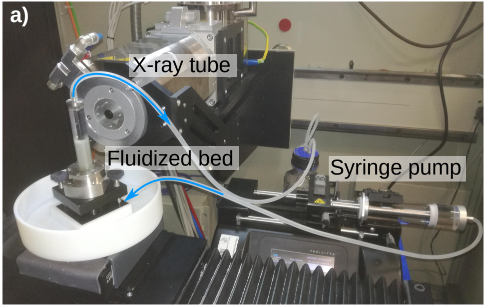

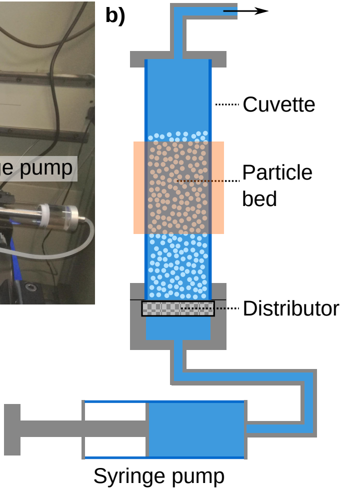

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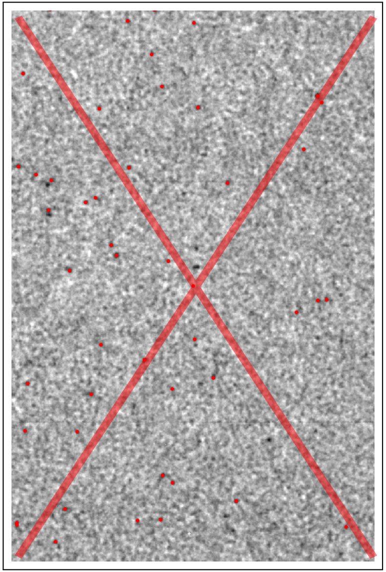

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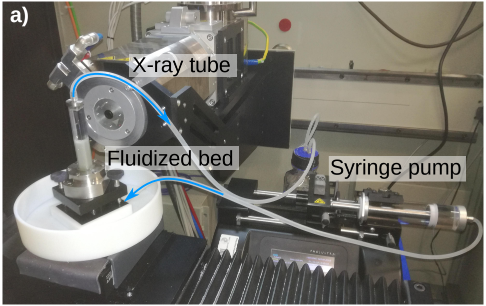
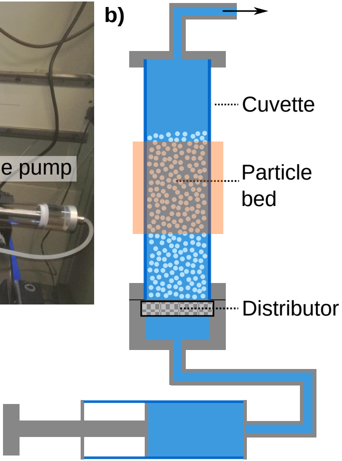
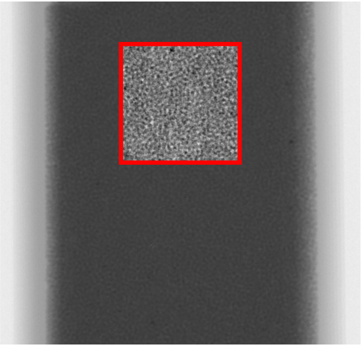
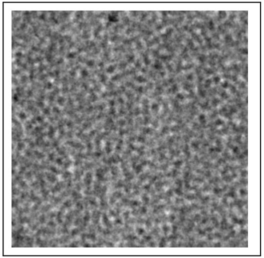




Radiogram





Differential Dynamic Microscopy (DDM)

	Up to now	This work
System	Dispersion, gels	Fluidized bed
Particles	Colloids $< 1\ \mu$m	Granulates $(150 - 180)\ \mu$m
Volume fraction	$\Phi \leq 0.33$	$0.45 < \Phi < 0.56$
Imaging	Light microscopy	X-ray radiography
Dynamics	Brownian motion, caging, glassy, collective motion	

Giavazzi *et al*, PRE **80**, 031403 (2009)

Extending
Differential Dynamic Microscopy (DDM)
to X-ray imaging

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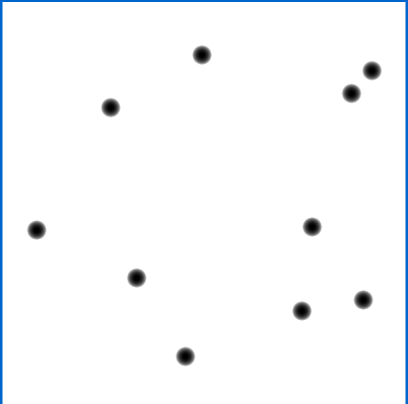
Digital Fourier Analysis of X-Ray radiograms (X-DFA)

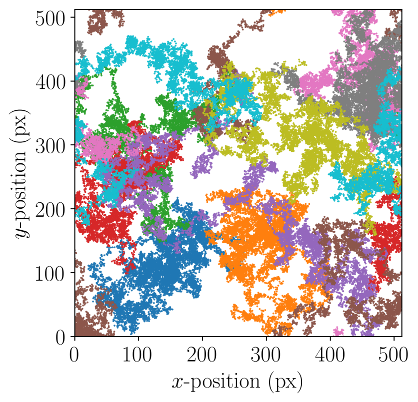
Giavazzi *et al*, PRE **80**, 031403 (2009)

Introduction to
X-ray Digital Fourier Analysis (X-DFA)

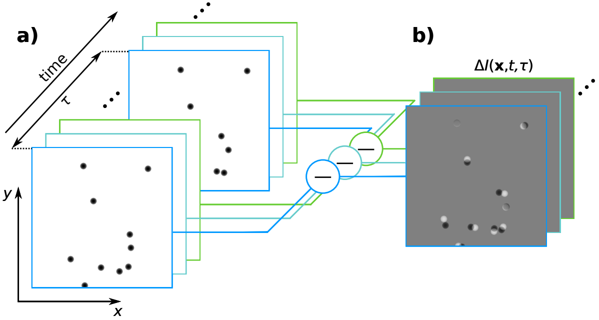
Synthetic radiograms



Particle trajectory



The image structure function $D(\mathbf{q}, \tau)$



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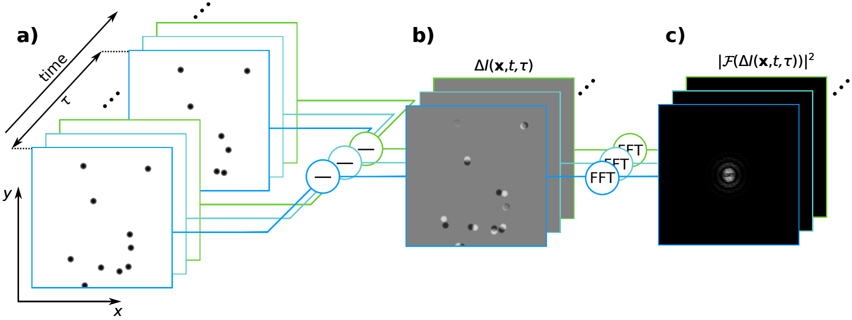

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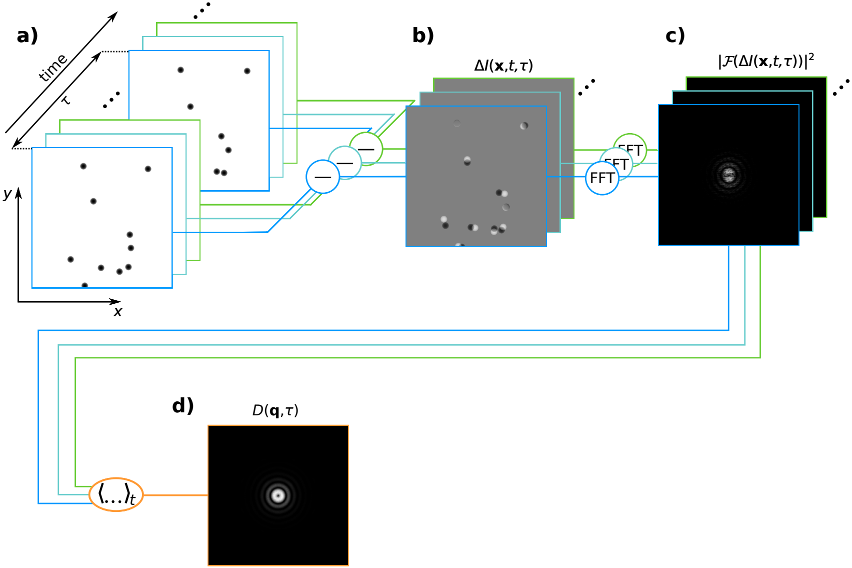

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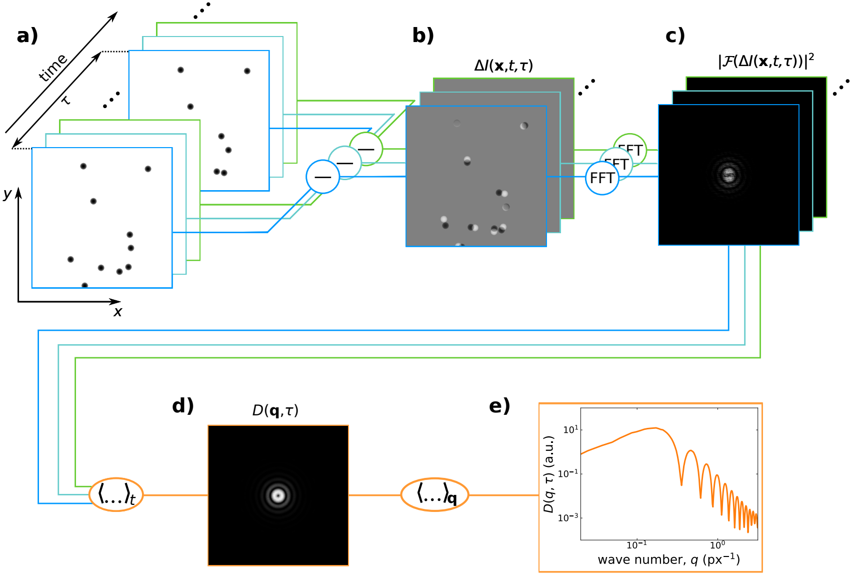

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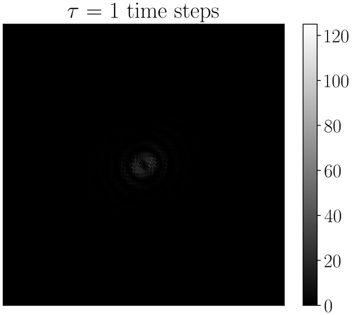
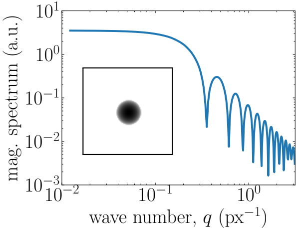
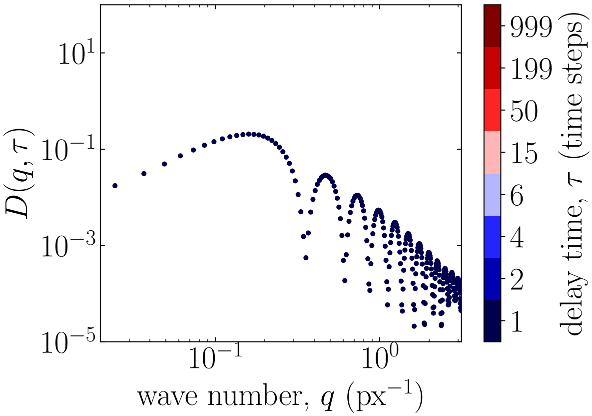


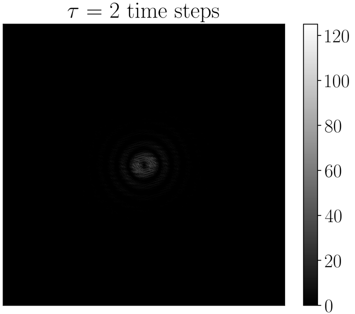


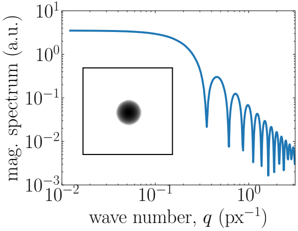
Time averaging: $D(\mathbf{q}, \tau) = \langle |\mathscr{F}(\Delta I)|^2 \rangle_t$

Azimuthal averaging: $D(\mathbf{q}, \tau) \to D(q, \tau)$ ...



The image structure function $D(q,\tau)$





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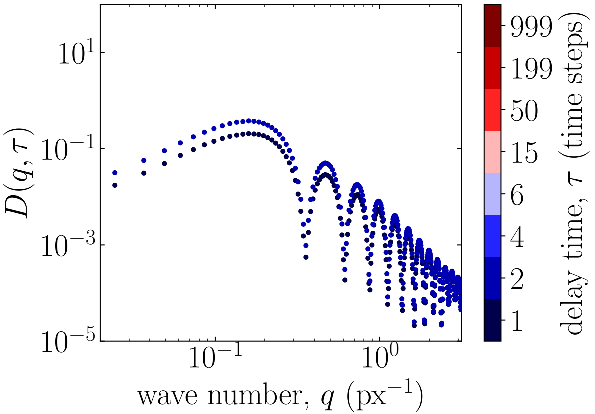

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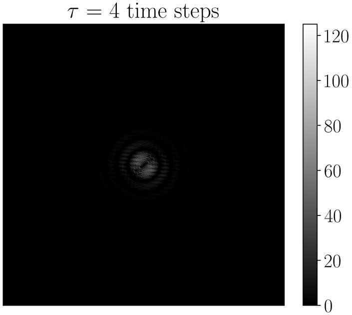

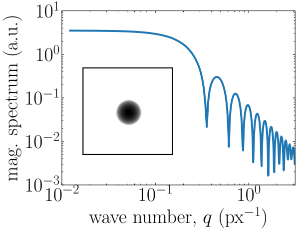

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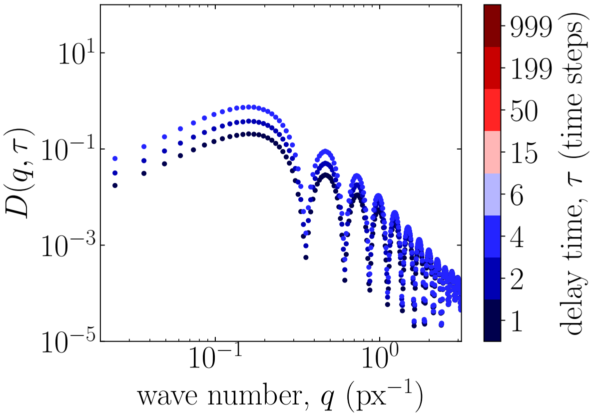

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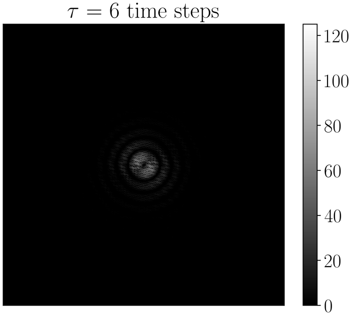

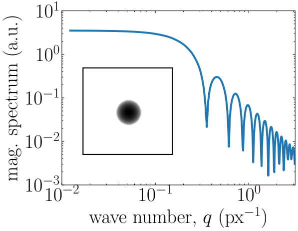

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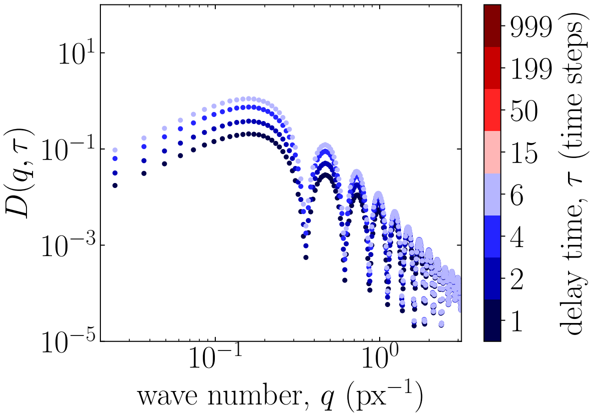

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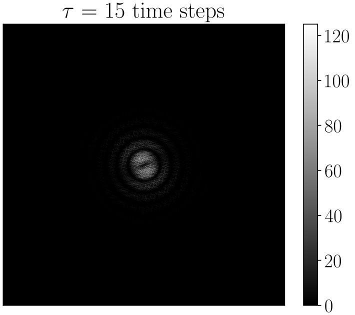

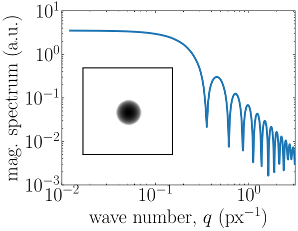

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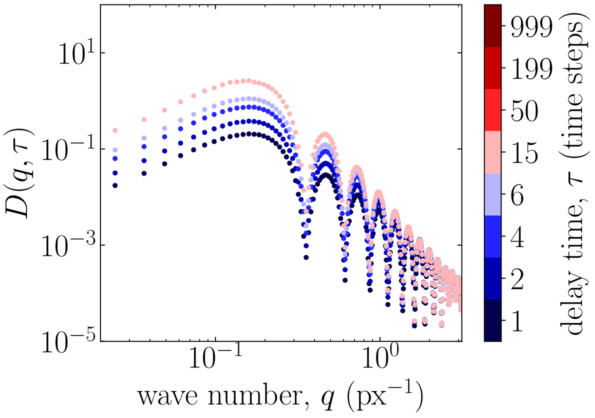
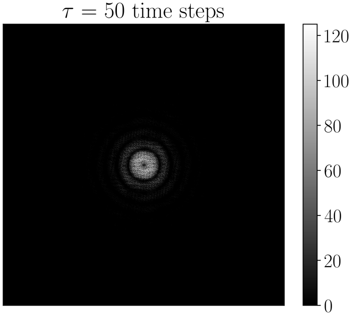
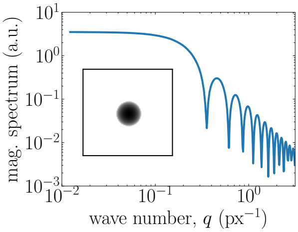


The image structure function $D(q, \tau)$





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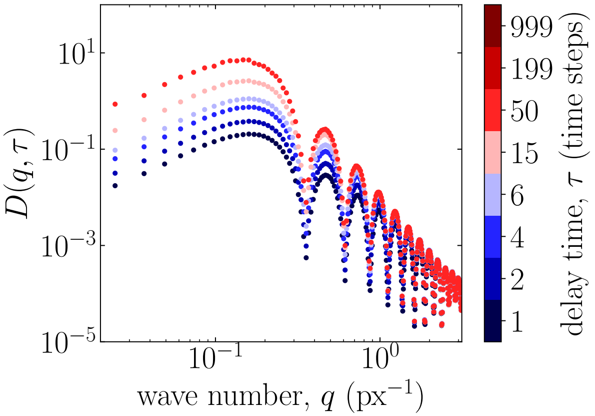

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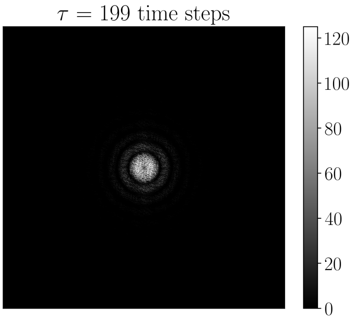

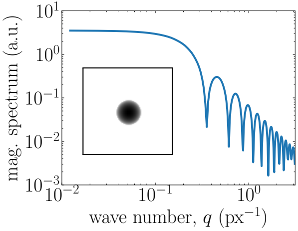

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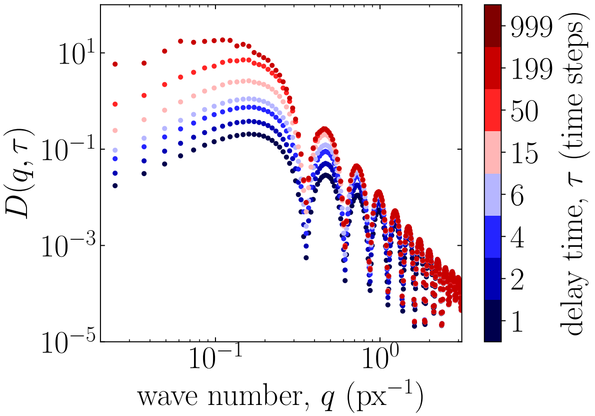

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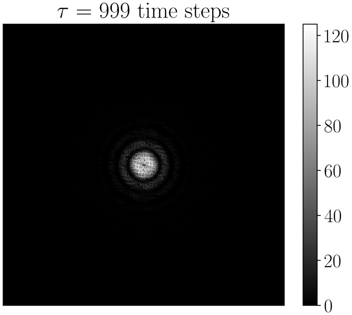

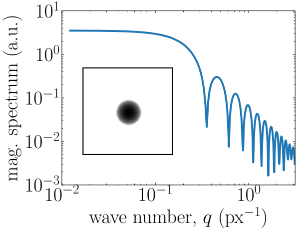

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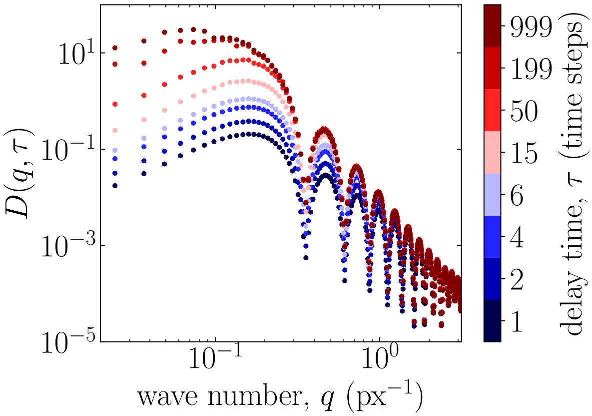

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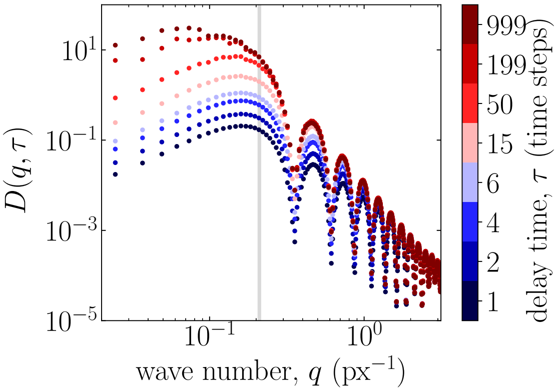

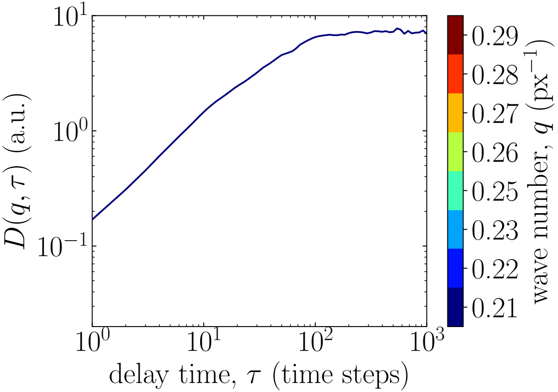

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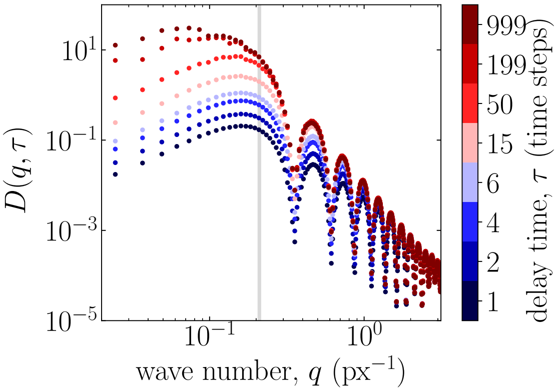

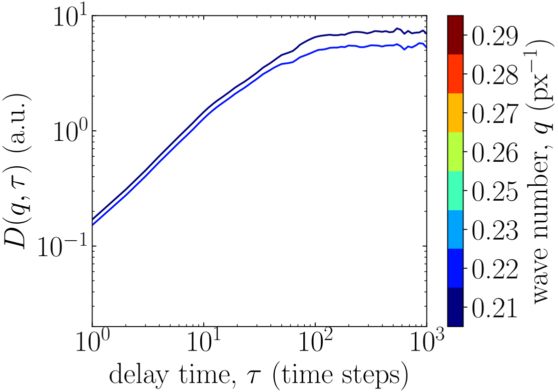

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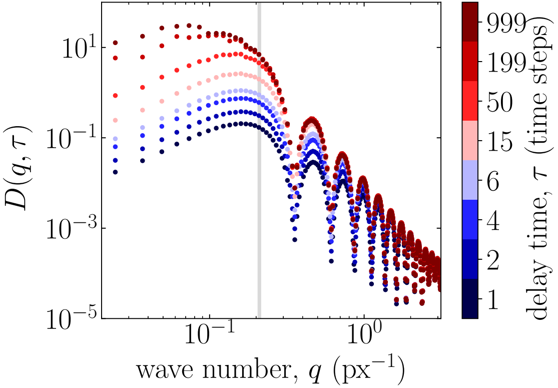

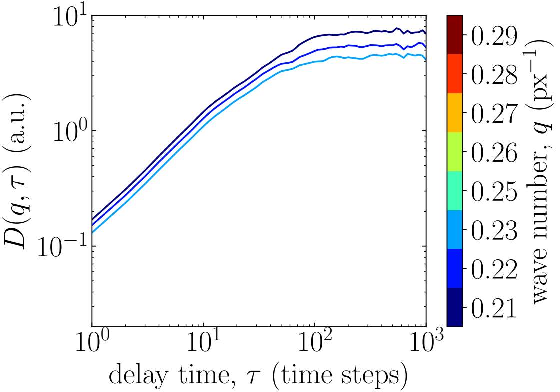

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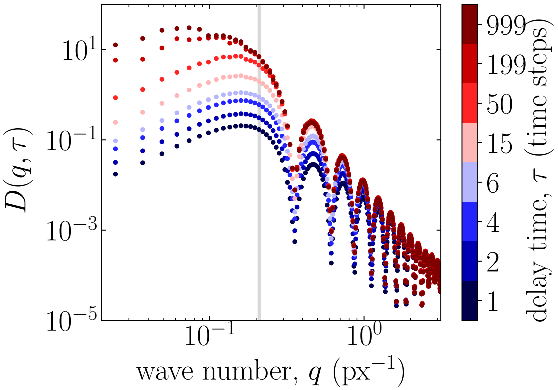

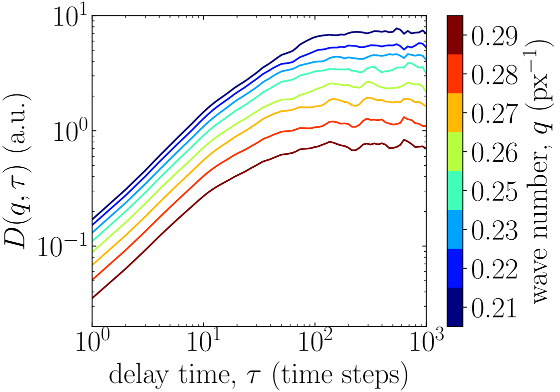

Linear space invariant imaging

$$D(q,\tau) = \left\langle \left| I(q, t+\tau) - I(q,t) \right|^2 \right\rangle_t$$

Linear space invariant imaging

$$
\begin{aligned}
D(q,\tau) &= \left\langle \left| I(q,t+\tau) - I(q,t) \right|^2 \right\rangle_t \\
&= A(q)\left[1 - \frac{\left\langle I^*(q,t) I(q,t+\tau) \right\rangle_t}{\left\langle |I(q,t)|^2 \right\rangle_t}\right] + B(q)
\end{aligned}
$$

Linear space invariant imaging

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$$= A(q)\left[1 - \underbrace{\frac{\left\langle I^*(q,t)I(q,t+\tau)\right\rangle_t}{\left\langle |I(q,t)|^2\right\rangle_t}}_{\text{Image correlation function}}\right] + B(q)$$

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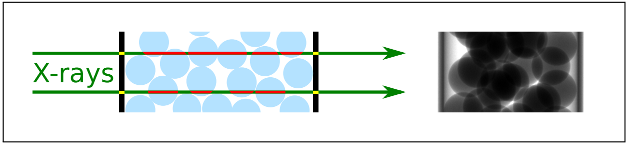
Image correlation function



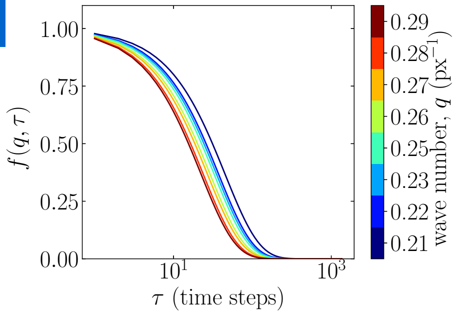
Intermediate scattering function

$$f(q,\tau) = \frac{\langle \rho^*(q,t)\rho(q,t+\tau)\rangle_t}{\langle |\rho(q,t)|^2\rangle_t}$$

Intermediate scattering function $f(q, \tau)$

Brownian motion: $f(q, \tau) = \exp(-q^2\tau/\tau_\mathrm{D})$

- Non-interacting particles
- Gaussian velocity distribution



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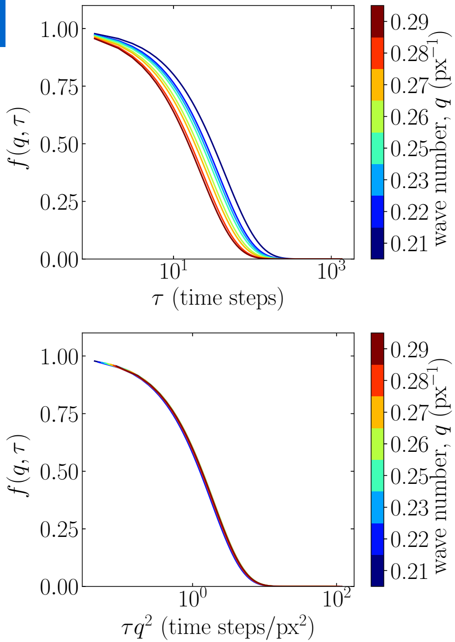

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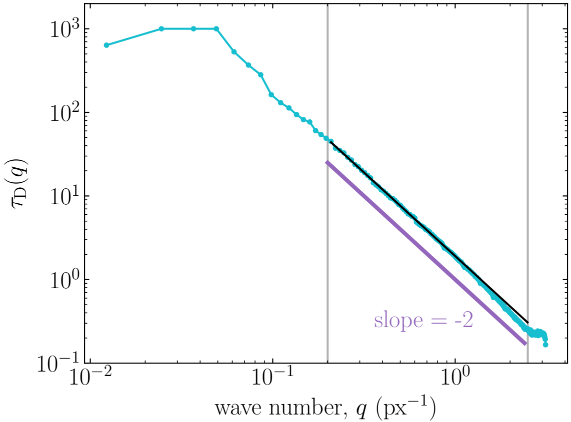
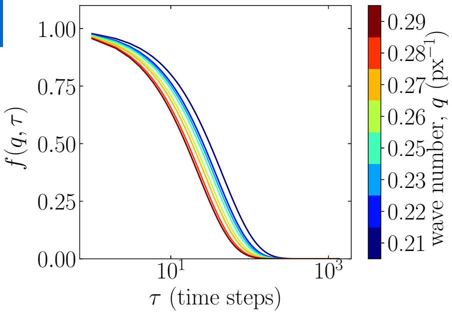
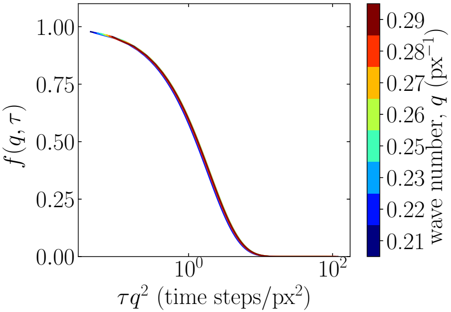
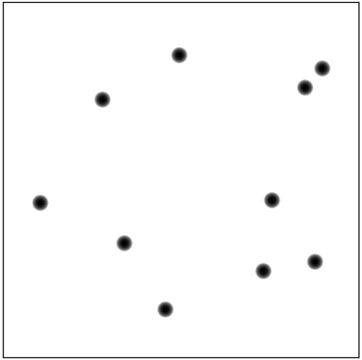




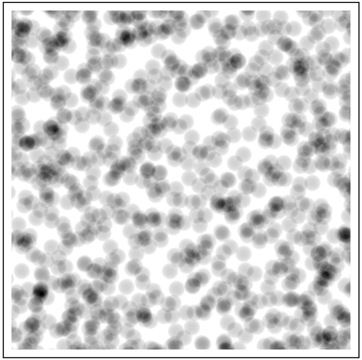


Accuracy of X-DFA: Varying the number of particles

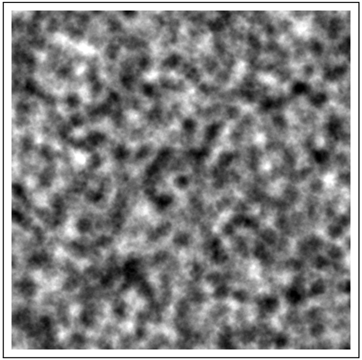
10 particles



1000 particles



100 000 particles



Deviation from the simulation input:

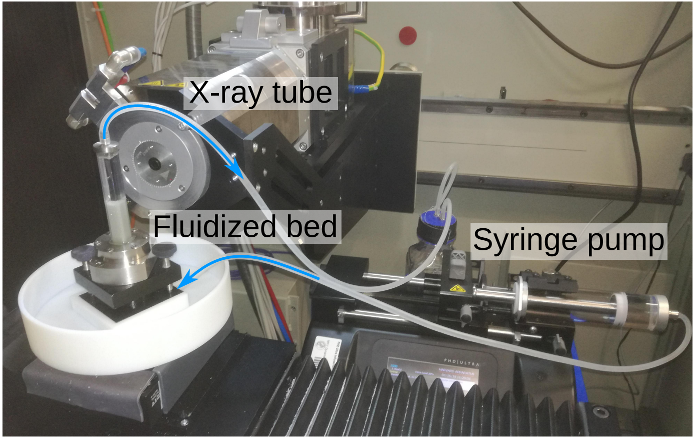
6%

2%

2%

PIV off by $\approx$ 650%

Experimental validation of X-DFA:
A suspension of sedimenting particles



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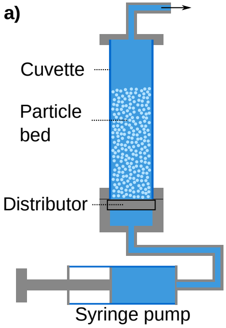
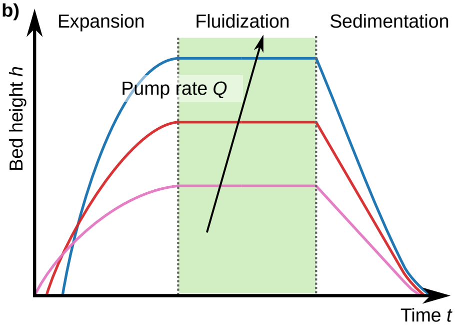
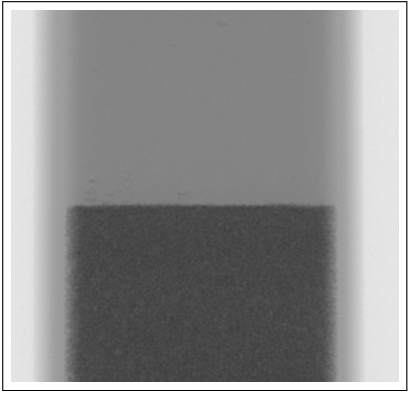




X-ray radiography



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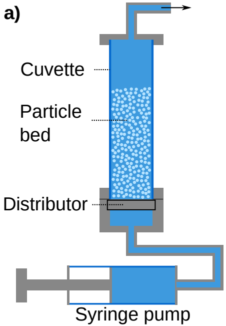
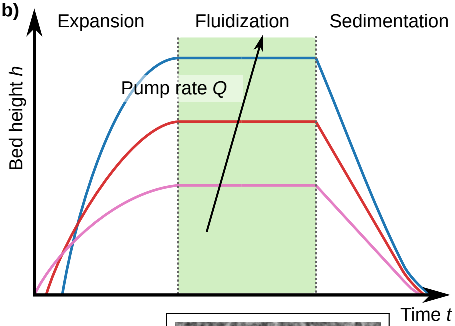
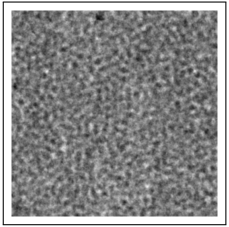




No reliable reference velocity!



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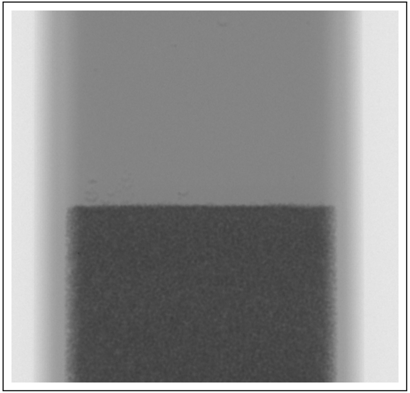

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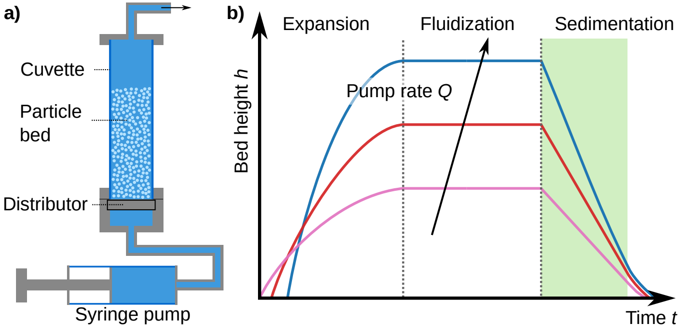

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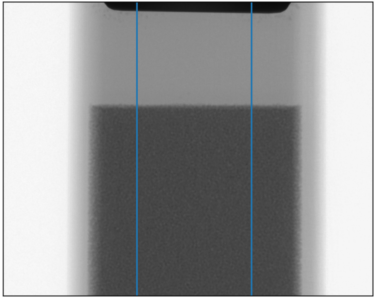

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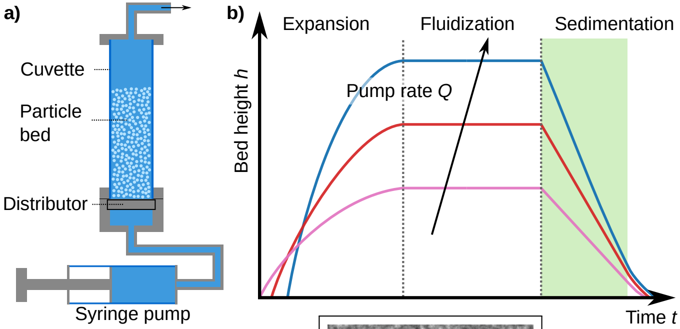
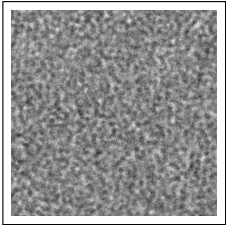
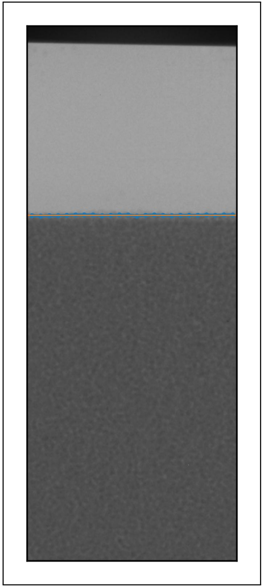




Comparison of $\langle v \rangle_{\mathrm{xdfa}}$ and $\langle v \rangle_{\mathrm{front}}$

X-ray radiography



X-DFA for a suspension of sedimenting particles[1]

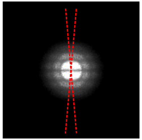


Image structure function:
$D(\mathbf{q}, \tau) = \langle |\mathcal{F}[\Delta I(\mathbf{r}, t, \tau)]|^2 \rangle_t$

(1) In collaboration with Manuel Escobedo, University of Düsseldorf

X-DFA for a suspension of sedimenting particles[1]

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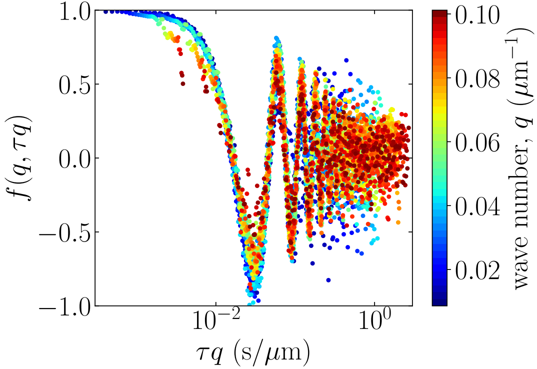
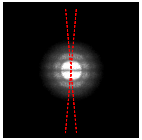
$$f(q,\tau) = \cos(q\langle v_s \rangle \tau)\exp\left(-\frac{1}{2}q^2\delta v^2\tau^2\right)$$





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(2) Kohyama *et al.* (2008)

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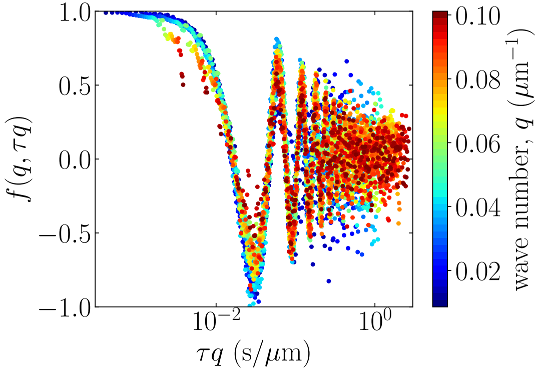

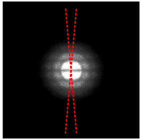

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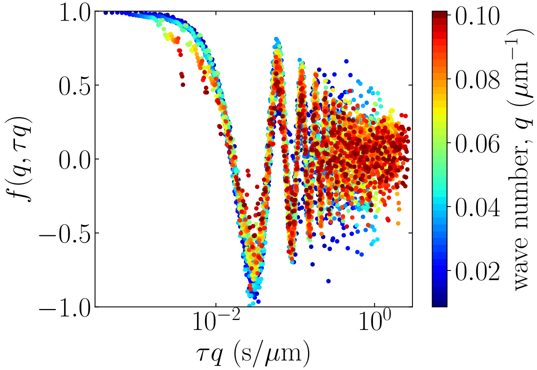

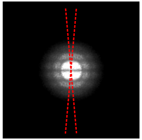

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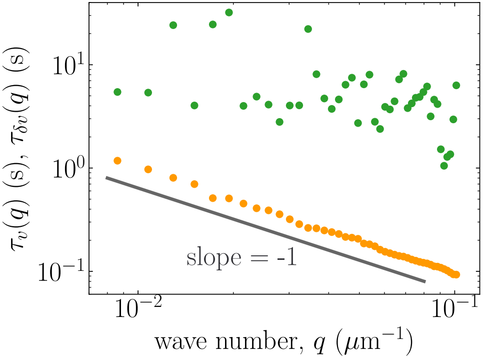

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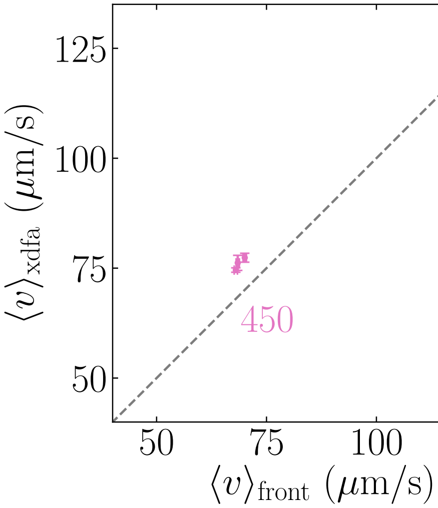
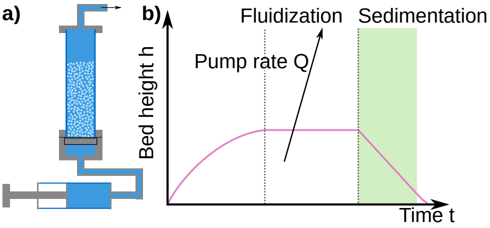
Front tracking vs. X-DFA





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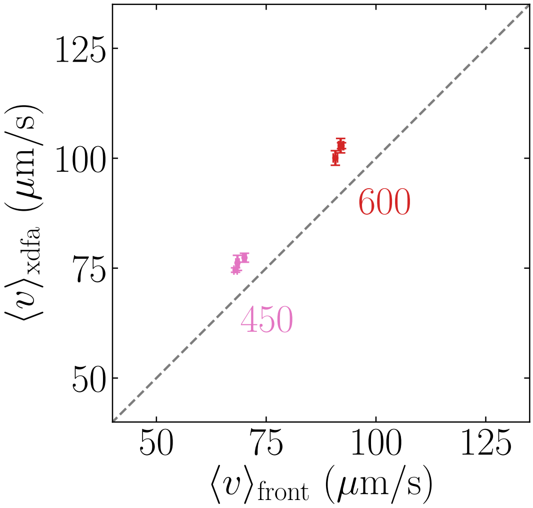

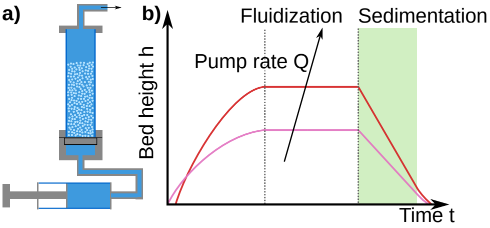

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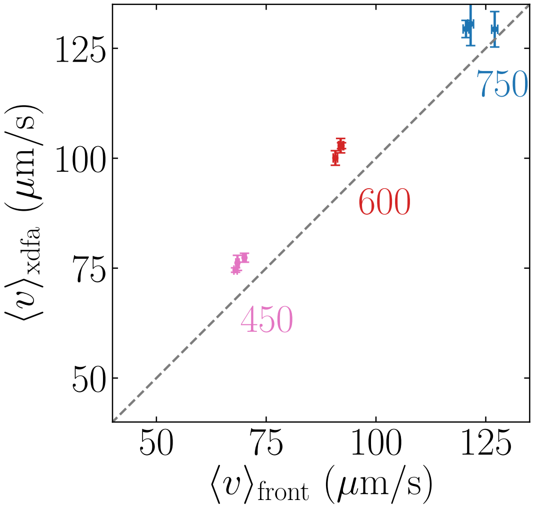

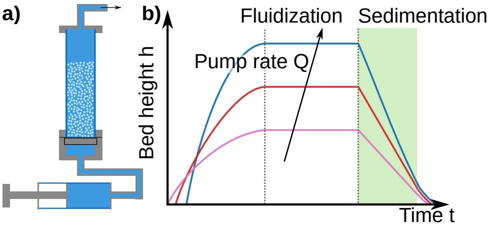

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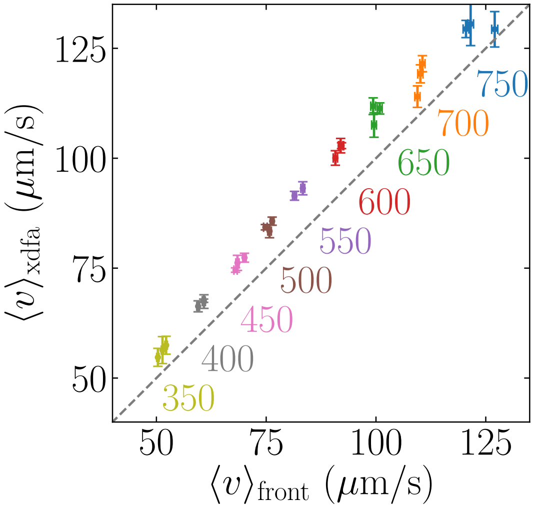
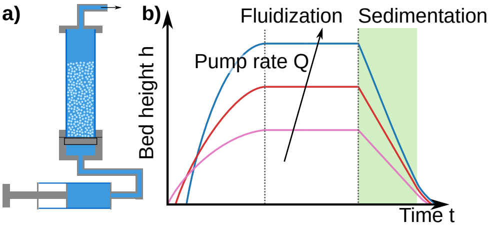
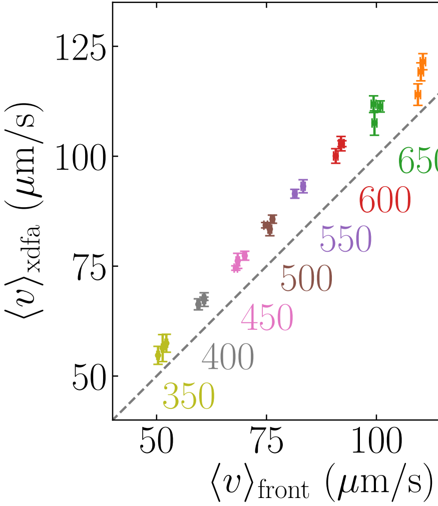




$\langle v \rangle_{\mathrm{xdfa}} > \langle v \rangle_{\mathrm{front}}$ by 9.4%

Front tracking vs. X-DFA



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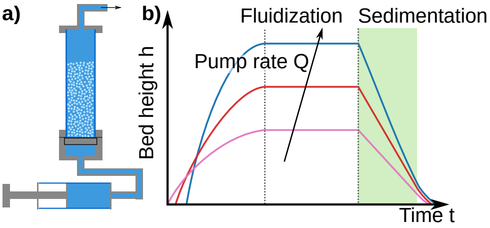
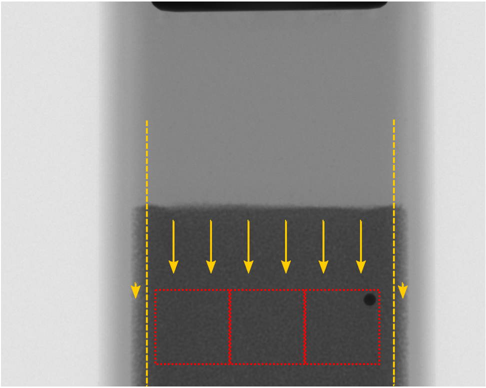
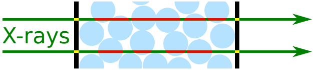
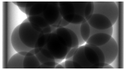


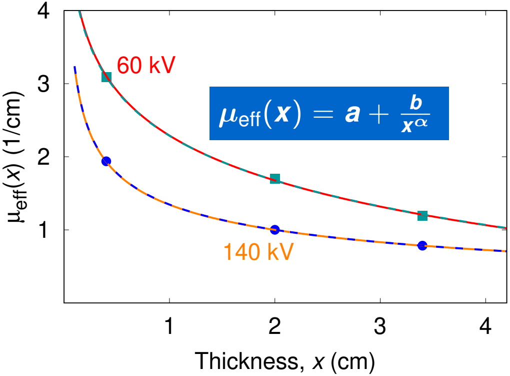


Conclusion

Correction of beam hardening

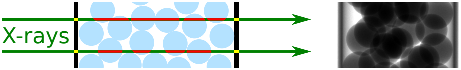


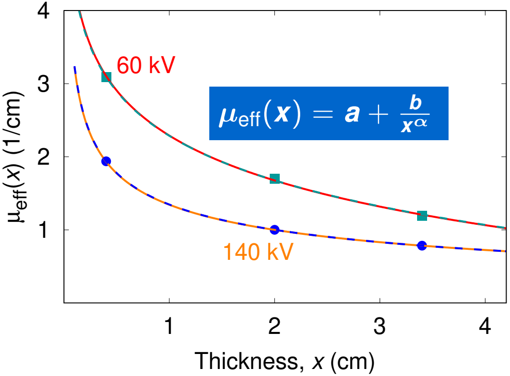




Conclusion

Correction of beam hardening



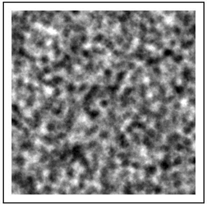


X-ray Digital Fourier Analysis

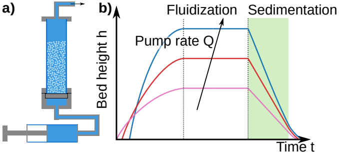
Synthetic radiograms

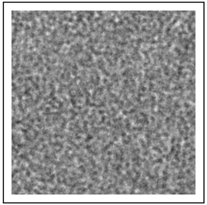
~~Particle Tracking~~

~~PIV~~



Experimental validation



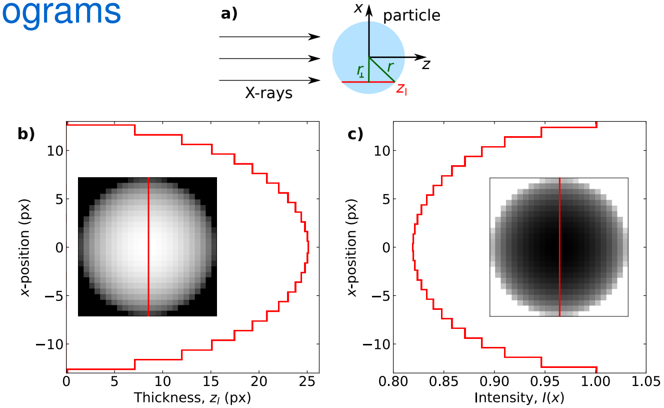


Temporal resolution of X-ray radiography

Thank you for your attention!

Backup slides

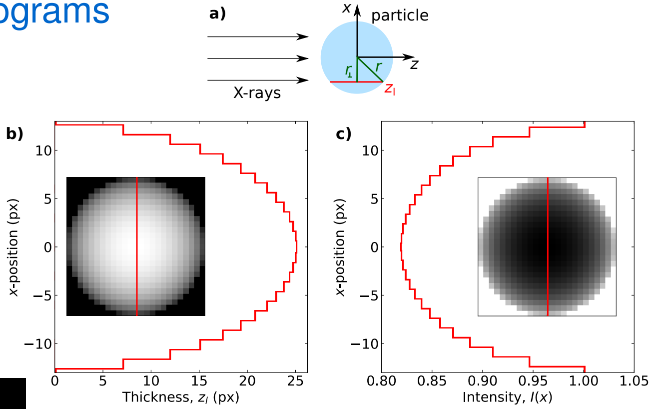
Synthetic radiograms

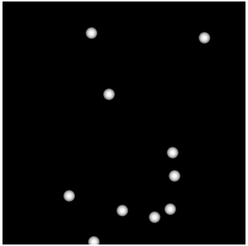


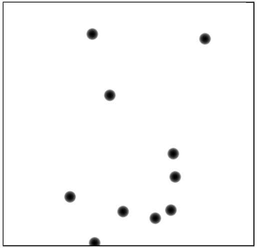
Beer-Lambert

$$I(z_l) = I_0 \exp(-\mu z)$$

Synthetic radiograms







Beer-Lambert

$$I(z_l) = I_0 \exp(-\mu z)$$

Linear space invariant imaging

Image correlation function

$$g(\mathbf{q}, \tau) = \frac{\langle I^*(\mathbf{q}, t) I(\mathbf{q}, t + \tau) \rangle_t}{\langle |I(\mathbf{q}, t)|^2 \rangle_t}$$

Linear space invariant imaging

Image correlation function

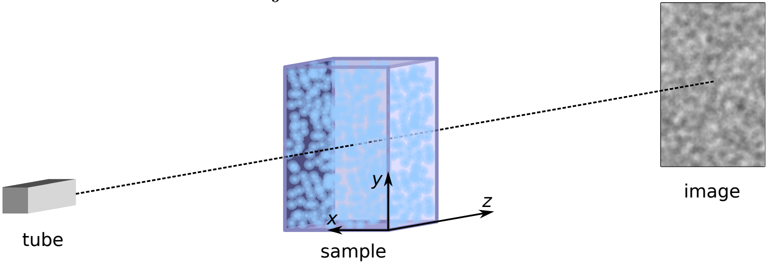
$$g(\mathbf{q},\tau) = \frac{\langle I^*(\mathbf{q},t) I(\mathbf{q},t+\tau)\rangle_t}{\langle |I(\mathbf{q},t)|^2\rangle_t}$$

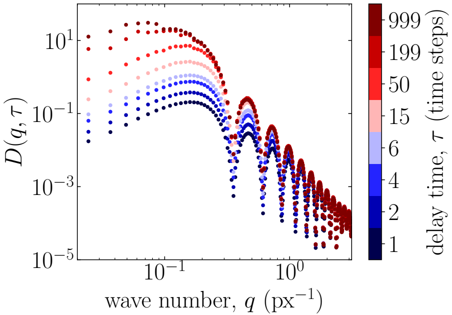
Intermediate scattering function

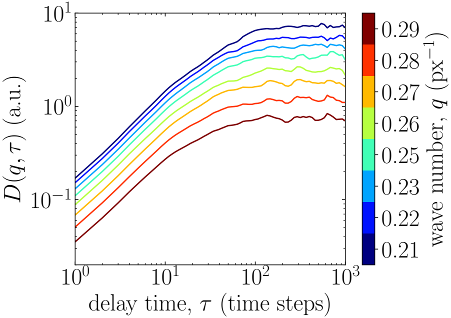
$$f(\mathbf{q},\tau) = \frac{\langle \rho^*(\mathbf{q},t) \rho(\mathbf{q},t+\tau)\rangle_t}{\langle |\rho(\mathbf{q},t)|^2\rangle_t}$$

Linear space-invariant imaging:

$$I(\mathbf{r},t) = I_0 + \int \mathrm{d}\mathbf{r}'\,\mathrm{d}z'\, T(\mathbf{r}-\mathbf{r}', -z') c(\mathbf{r}', z', t)$$







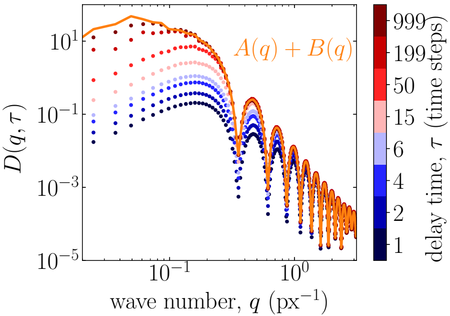
$$D(q,\tau) = \left\langle |I(q,t+\tau) - I(q,t)|^2 \right\rangle_t$$

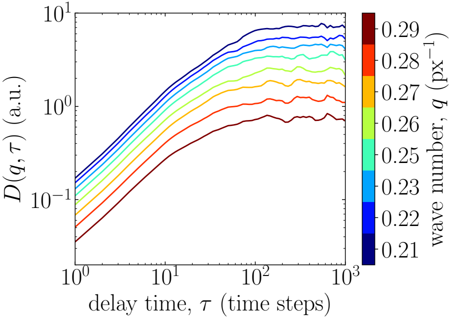
$$= A(q)\left[1 - \underbrace{\frac{\langle I^*(q,t)I(q,t+\tau)\rangle_t}{\langle |I(q,t)|^2\rangle_t}}_{\text{Image correlation function}}\right] + B(q)$$

Linear space invariant imaging

$$f(q,\tau) = \frac{\langle \rho^*(q,t)\rho(q,t+\tau)\rangle_t}{\langle |\rho(q,t)|^2\rangle_t}$$

Intermediate scattering function





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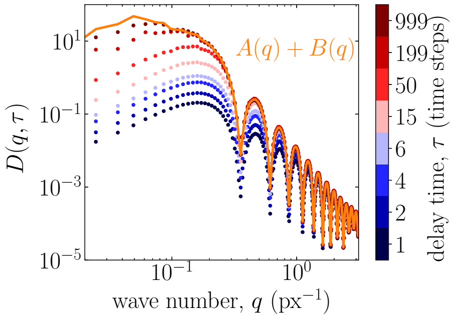
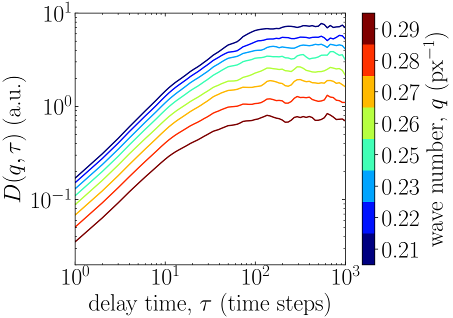
- $D(q,\tau \to 0) = B(q) = 0$
- $D(q,\tau \to \infty) = A(q) + B(q)$

Linear space invariant imaging

$$f(q,\tau) = \frac{\langle \rho^*(q,t)\rho(q,t+\tau)\rangle_t}{\langle |\rho(q,t)|^2\rangle_t}$$

Intermediate scattering function





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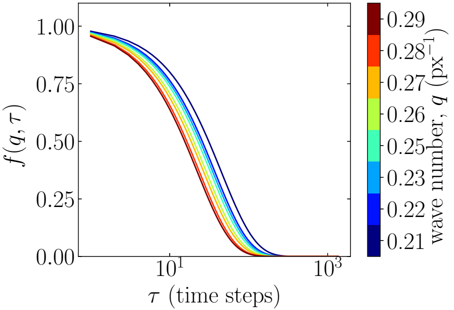
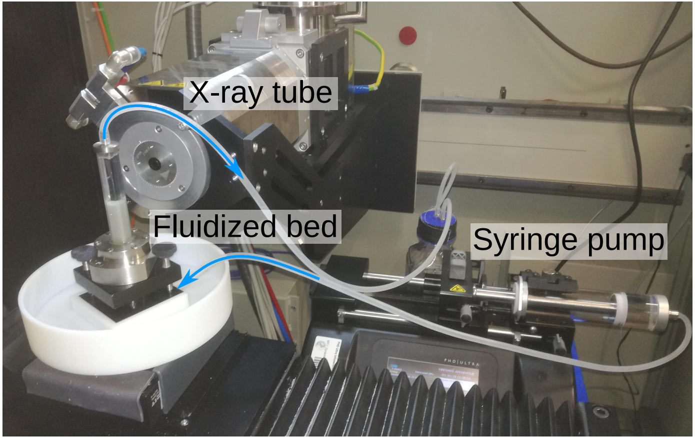
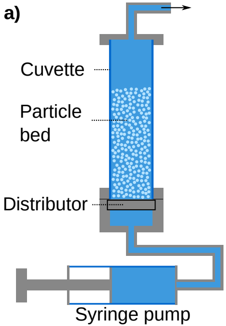
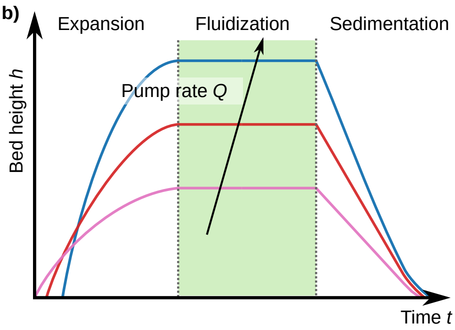


Experimental validation of X-DFA: A suspension of sedimenting particles

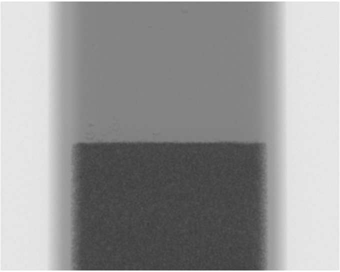


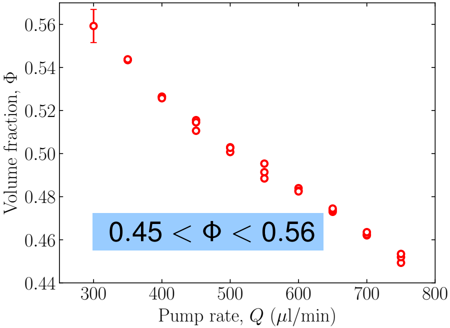
Experimental validation of X-DFA: A suspension of sedimenting particles





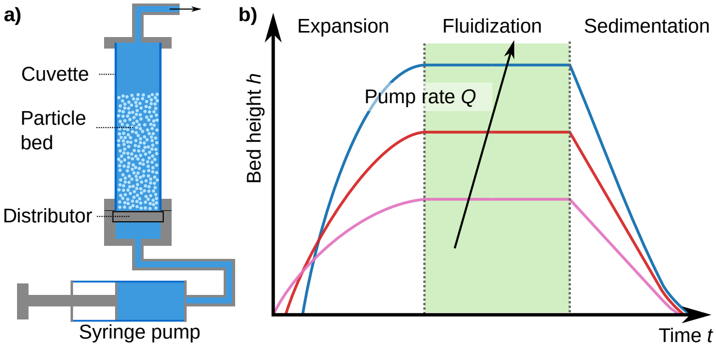
X-ray radiography





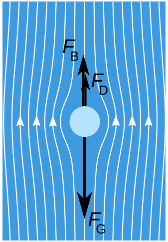
$0.45 < \Phi < 0.56$

Liquid fluidized bed: Richardson-Zaki law



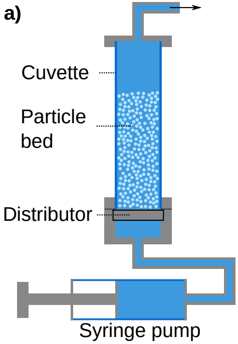
Gravitation Buoyancy Drag

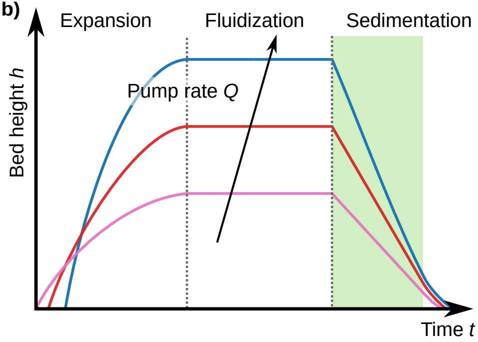
$$F_G = F_B + F_D$$



$$\frac{\langle v \rangle_{\text{fluid}}}{v_{\text{Stokes}}} = (1 - \Phi)^n$$

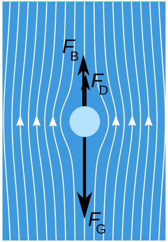
Liquid fluidized bed: Richardson-Zaki law



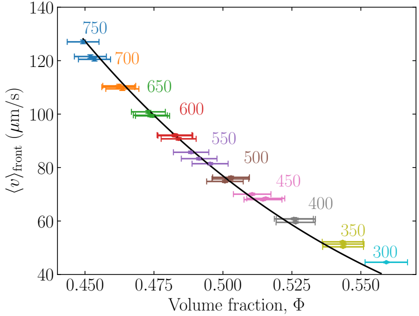


Gravitation Buoyancy Drag

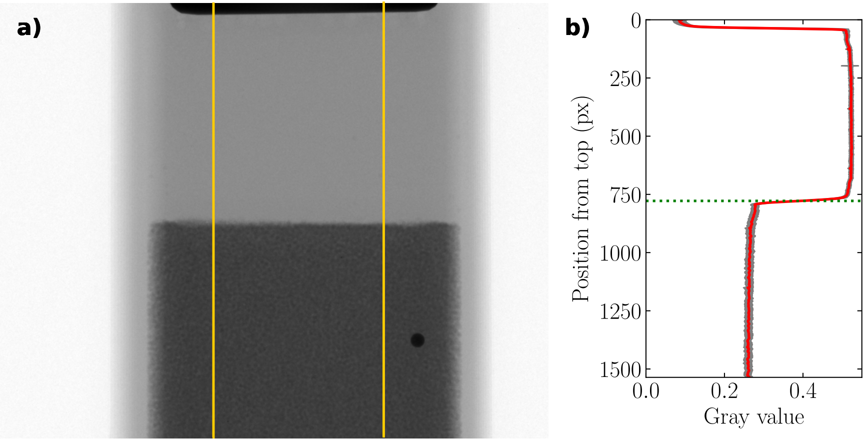
$$F_G = F_B + F_D$$



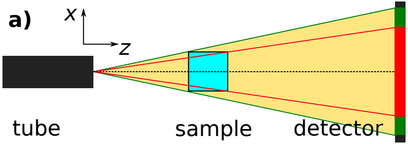
$$\frac{\langle v \rangle_{\mathrm{front}}}{v_{\mathrm{Stokes}}} = (1 - \Phi)^n$$

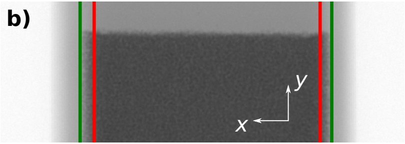


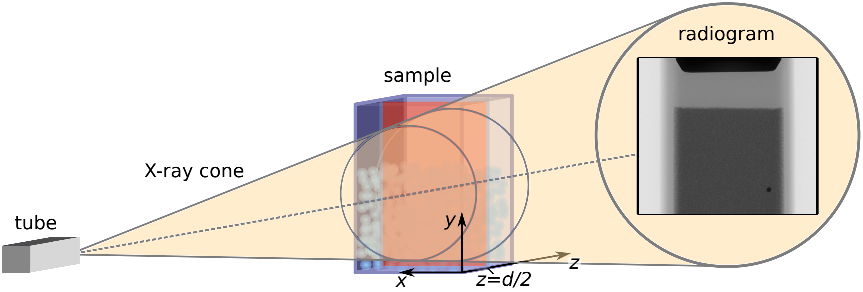
Tracking of particle front



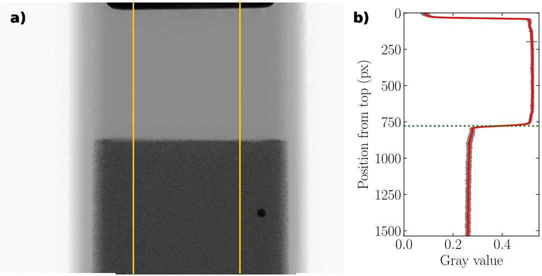
Tracking of particle front

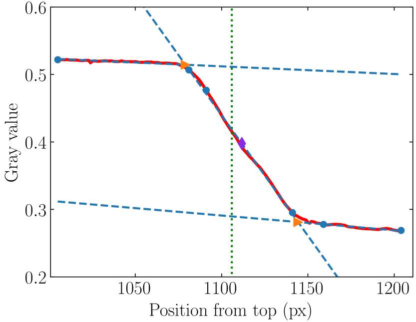


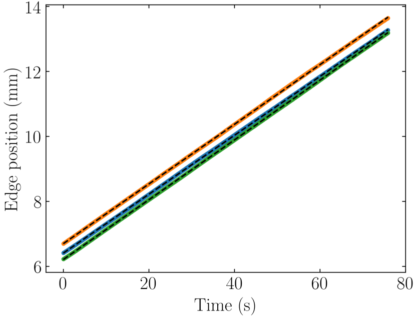




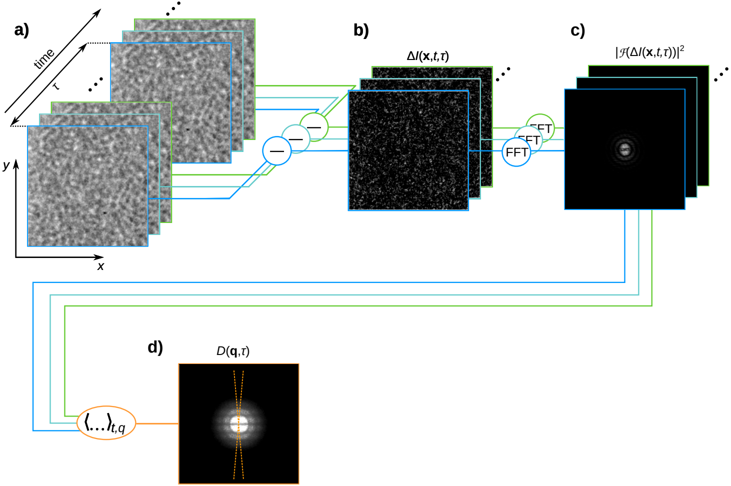
Tracking of particle front





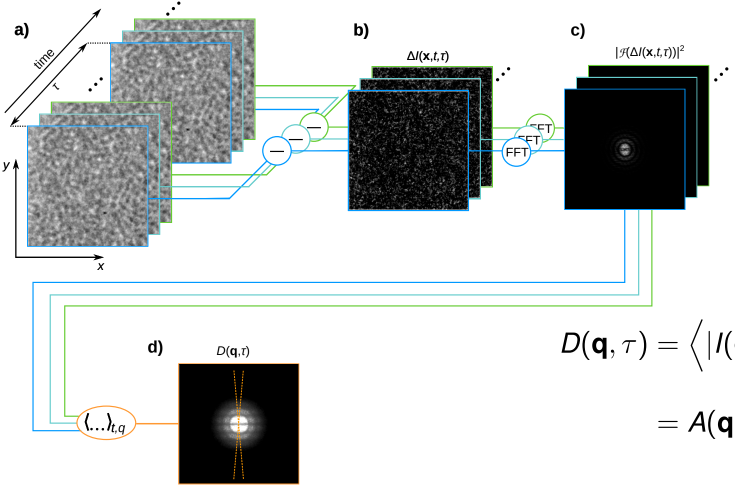


The image structure function $D(\mathbf{q}, \tau)$



In collaboration with Manuel Escobedo, University of Düsseldorf

The image structure function $D(\mathbf{q}, \tau)$



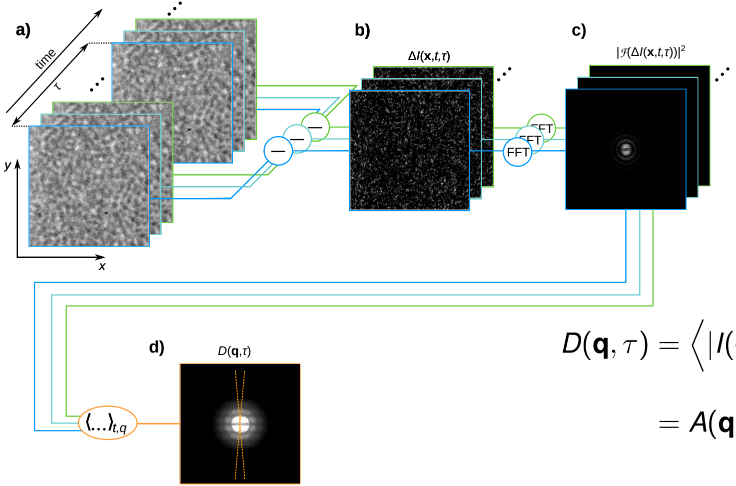
$$D(\mathbf{q}, \tau) = \left\langle |I(\mathbf{q}, t+\tau) - I(\mathbf{q}, t)|^2 \right\rangle_t$$

$$= A(\mathbf{q}) \left[1 - \frac{\left\langle I^*(\mathbf{q}, t) I(\mathbf{q}, t+\tau) \right\rangle_t}{\left\langle |I(\mathbf{q}, t)|^2 \right\rangle_t} \right] + B(\mathbf{q})$$

- $D(\mathbf{q}, \tau \to 0) = B(\mathbf{q})$
- $D(\mathbf{q}, \tau \to \infty) = A(\mathbf{q}) + B(\mathbf{q})$

In collaboration with Manuel Escobedo, University of Düsseldorf

The image structure function $D(\mathbf{q},\tau)$



Linear space invariant imaging

$$f(\mathbf{q},\tau) = \frac{\langle \rho^*(\mathbf{q},t)\rho(\mathbf{q},t+\tau)\rangle_t}{\langle|\rho(\mathbf{q},t)|^2\rangle_t}$$

Intermediate scattering function

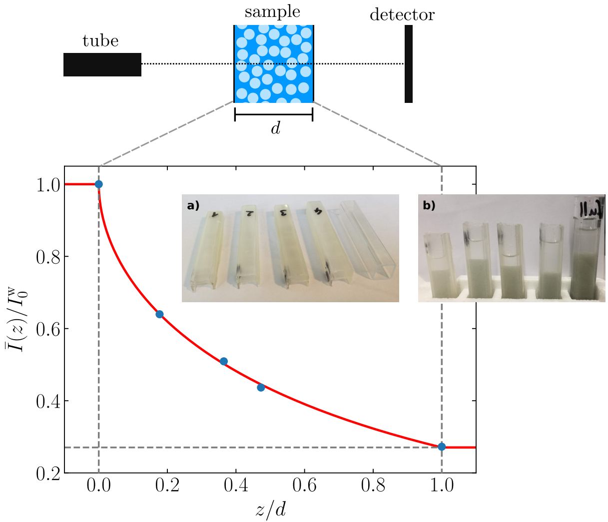
$$D(\mathbf{q},\tau) = \left\langle \left| I(\mathbf{q},t+\tau) - I(\mathbf{q},t)\right|^2 \right\rangle_t$$

$$= A(\mathbf{q})\left[1 - \frac{\left\langle I^*(\mathbf{q},t)I(\mathbf{q},t+\tau)\right\rangle_t}{\left\langle |I(\mathbf{q},t)|^2\right\rangle_t}\right] + B(\mathbf{q})$$

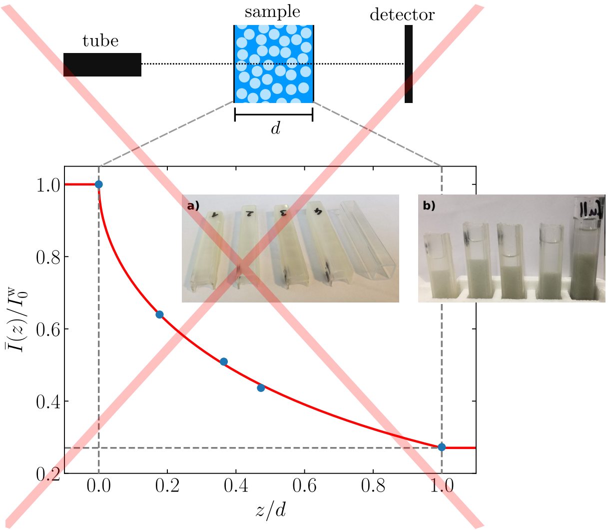
- $D(\mathbf{q},\tau\to 0) = B(\mathbf{q})$
- $D(\mathbf{q},\tau\to\infty) = A(\mathbf{q}) + B(\mathbf{q})$

In collaboration with Manuel Escobedo, University of Düsseldorf

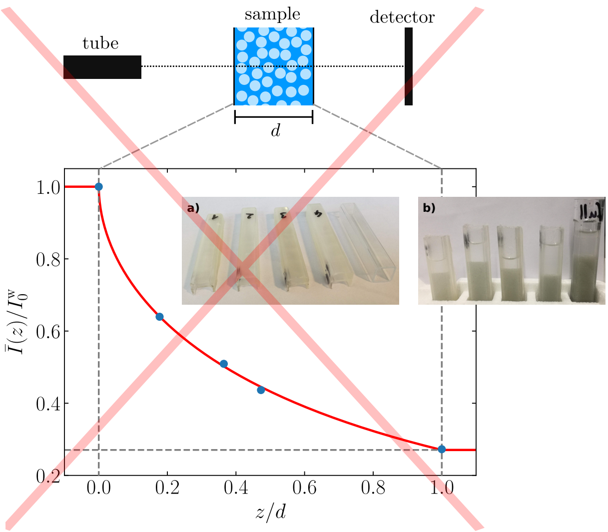
X-ray imaging – Linear space invariant?

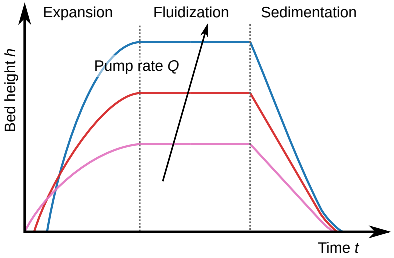


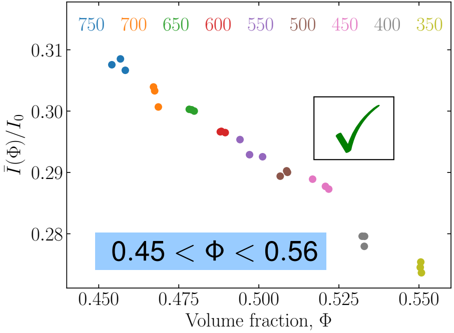
X-ray imaging – Linear space invariant?



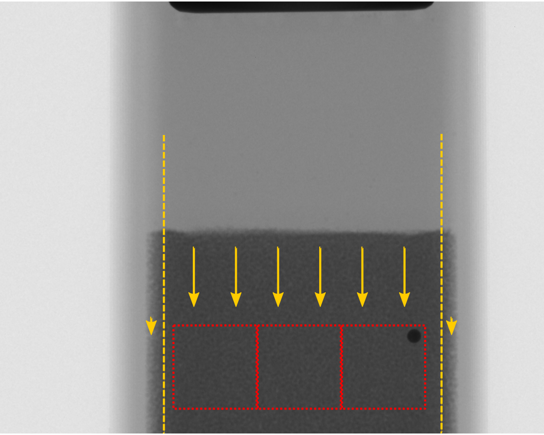
X-ray imaging – Linear space invariant?





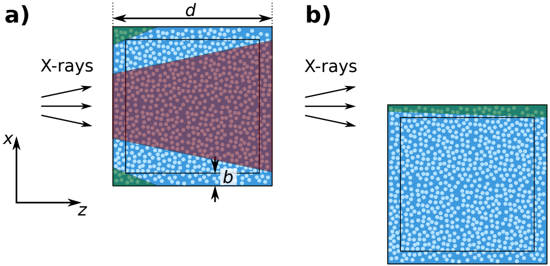


Estimate width of boundary layer



$\langle v \rangle_{\text{xdfa}} > \langle v \rangle_{\text{front}}$ by 9.4%

$\langle v \rangle_{\text{xdfa}}$ takes two layers into account
$\langle v \rangle_{\text{front}}$ takes four layers into account



Estimation:
Boundary velocity = 0
Else = const.
$\rightarrow b \approx 3$ particle diameters