

# Transport Properties and Segregation Phenomena in Vibrating Granular Beds<sup>†</sup>

Chun-Chung Liao<sup>1</sup> and Shu-San Hsiau<sup>2\*</sup>

<sup>1</sup> Department of Mold and Die Engineering, National Kaohsiung University of Applied Sciences, Taiwan

<sup>2</sup> Department of Mechanical Engineering, National Central University, Taiwan

## Abstract

Granular materials are common in daily life and in many industrial processes. Both fundamental research and industrial application studies are crucial for understanding the transport properties and segregation mechanisms of vibrating granular beds. One major related research topic is granular materials subjected to external vibration; such granular materials exhibit complex movement and Brazil nut segregation. Understanding the transport properties and the rising of an intruder immersed in granular materials is a challenge in granular flow research. This paper presents a review of transport properties and segregation phenomena in a vibrating granular bed, and discusses the relationship between transport properties and granular segregation. Furthermore, how the vibration conditions, liquid bridge force, bed height, surface roughness of granular materials, and a bumpy base of granular beds affect the transport properties, convection, and granular segregation are reported. The results indicate that the transport properties and segregation behavior are significantly influenced by the addition of small amounts of liquids and by the surface roughness and a bumpy base. Diffusive and convective motions are weakened as the base roughness increases, leading to a weaker Brazil nut effect.

**Keywords:** granular flows, vibrating granular beds, transport properties, convection, granular segregation, surface roughness

## 1. Introduction

Granular materials and powders are aggregates of discrete solid particles dispersed in an interstitial fluid. Granular materials (e.g., sand, salt, sugar, metal powders, glass beads, steel balls, coffee beans, pills, wheat, and rice) are commonly encountered in daily life and widely used in many industrial processes such as pharmaceutical manufacturing, gasification, pyrolysis, chemical manufacturing, mineral processing, metal powder injection molding, additive manufacturing, and powder metallurgy. The materials are also used for food transport and storage. Granular flows are also found in nature (e.g., avalanches, landslides, and debris flows) and can cause disasters. Understanding the transport properties and physical mechanism of granular flows is crucial; however, our understanding of granular materials is poor. Random particle motions resulting from interactive collisions between

particles are the dominant mechanism influencing the flow behavior and transport properties of granular materials (Campbell, 1990). These materials do not flow homogeneously like a fluid when the external driving force is small. By contrast, they may behave like a gas if the granular system is relatively diluted and the external force is sufficiently large. Furthermore, the materials may behave like a solid when the external energy is insufficient. These three motion states may occur simultaneously or exist individually in granular systems (Jaeger, 1996). Although the rheological behavior of granular matter is complex, an understanding of granular matter is crucial in numerous industrial applications and for resolving various environmental problems.

Knowledge of granular segregation and transport properties has become crucial in many industrial processes (e.g., pharmaceutical manufacturing, foodstuff production, powder metallurgy, metal powder injection molding, and detergent, chemical, and plastic manufacturing) and in nature research (e.g., landslides, avalanches, and debris flows). In particular, granular segregation is a complex and poorly understood process. Therefore, numerous researchers have studied the transport properties and segregation mechanisms of granular materials (Bose et al., 2007; Breu et al., 2003; Bridgwater, 1976; Brito et al., 2008; Campbell,

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<sup>1</sup> 415 Chien Kung Road, Kaohsiung 807, Taiwan

<sup>2</sup> No.300, Zhongda Rd. Zhongli City, Taoyuan County 32001, Taiwan

\* Corresponding author: Shu-San Hsiau;

E-mail: sshsiau@cc.ncu.edu.tw

TEL: +886-3-426-7341 FAX: +886-3-425-4501

1997; Halsey and Levine, 1998; Herminghaus, 2005; Hsiau and Shieh, 1999; Hsiau and Yang, 2002; Hsiau and Yang, 2003; Hsiau et al., 2013; Khakhar et al., 1997; Kudrolli, 2004, 2008; Liao and Hsiau, 2009, 2010; Liao et al., 2010a, 2015; Liffman et al., 2011; Lozano et al., 2015; Lu and Hsiau, 2008; Natarajan et al., 1995; Saez et al., 2005; Shi et al., 2007; Tai and Hsiau, 2004; van der Vaart et al., 2015; Williams, 1976; Windows-Yule et al., 2015a). The segregation phenomenon in granular materials can be influenced by external driving conditions (Ciamarra et al., 2006; Hsiau and Yu, 1997; Hsiau et al., 2002), the interstitial fluid (Clement et al., 2010; Klein et al., 2006), the container geometry (Hsiau et al., 2002), and particle properties such as size (Cooke et al., 1996; Knight et al., 1993; Mobius et al., 2004; Zamankhan, 2013), density (Klein et al., 2006; Tai et al., 2010; Windows-Yule and Parker, 2015; Zeilstra et al., 2008), restitution coefficient (Brito and Soto, 2009), shape (Kudrolli, 2004; Williams, 1976), and friction coefficient (Kondic et al., 2006; Liao et al., 2012; Plantard et al., 2006). The segregation of granular materials also occurs in vibrating granular beds. The well-known Brazil nut problem associated with large particles (the so-called intruders) immersed in smaller granular materials has been studied extensively by using experiments, computer simulations, and theoretical models. Segregation has been demonstrated as occurring when grains vary in size, density, friction coefficient, and restitution coefficient. Segregation has also been observed in several configurations when an external driving force, in the form of a vibrating bed, shear cell, rotating drum, or chute flow, is applied to the system.

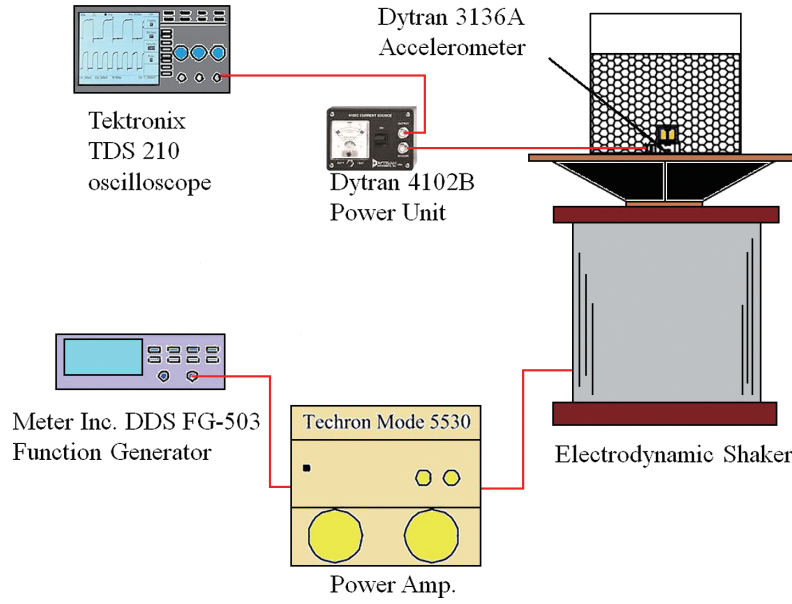
Granular temperature is defined as the specific fluctuation kinetic energy of particles, analogous to the thermodynamic temperature of a gas, and is a key parameter for describing the flow behavior of granular materials (Campbell, 1990; Hsiau et al., 2008; Liao et al., 2015; Ogawa, 1978; Wildman and Huntley, 2000; Zivkovic et al., 2011). Ogawa (1978) proposed the concept of granular temperature to quantify random motions of particles, and this concept has been widely used in related research. In the dense-gas kinetic theory for granular flows, granular temperature is assumed to be isotropically distributed. However, this key assumption is not observed in many practical scenarios, primarily because inelastic collisions and the frictional effect continually dissipate the energy of the granular materials. Thus, external energy must be continually introduced into the system to maintain the granular temperature. The granular temperature depends on the net energy between the energy generated by external vibration and the energy dissipated by the interstitial fluid, inelastic collisions, and frictional effect.

Understanding transport properties and segregation mechanisms is a challenge in granular material research. Studies have shown that granular materials may transition

from a solid-like state to a liquid-like state when sufficient energy is supplied to the granular system (Hsiau and Yu, 1997; Liao et al., 2012). During the expansion of a granular bed, voids are formed between particles and they trigger particle reorganization; consequently, smaller particles fall through the voids to the bottom of the bed, resulting in size segregation (Hsiau and Yu, 1997; Rosato et al., 1987). Duran et al. (1993) showed that large disks rise upward continuously in small steps through the arching effect, in which an intruder is supported by the surrounding network of smaller particles. However, smaller particles may rise intermittently only when the excitation amplitude is sufficiently strong. Cooke et al. (1996) found that the segregation rate increased with the peak acceleration and size ratio. They also observed that convection cells play a crucial role in the segregation mechanism. Liao et al. (2012) demonstrated that the Brazil nut effect was reduced when the intruder surface roughness was high. Furthermore, they found that the penetration length of the intruder increased with a decrease in the intruder surface roughness and that the penetration length scaled exponentially with the vibration frequency. Segregation also occurs in a binary mixture system with various densities because of the buoyancy effect of denser particles sinking to lower levels of the granular bed while lighter particles rise. The granular temperature and convection also influence density-induced segregation in vibrating granular beds appreciably (Tai et al., 2010). When the number of filling layers is small, heavier beads with a low granular temperature migrate to the bottom regions. Thus, the granular temperature plays a significant role in density-induced segregation. For the higher filling layers, bulk convective motion is the main mechanism affecting segregation, and thus heavier beads cluster at the convection center. Tai and Hsiau (2004) determined the transport properties in vibrating granular beds and found that the convection strength increases when the dimensionless vibration velocity exceeds 2. van der Vaart et al., (2015) reported that large and small particles show an underlying asymmetry that is dependent on the local particle concentration, with small particles segregation faster in regions of many large particles and large particles segregation slower in regions of many small particles.

## 2. Vibrating granular bed equipment and transport property measurement technique

Vibrating granular beds are widely used in industrial processes for drying, transporting, mixing, and segregating granular materials. **Fig. 1** shows a schematic of the experimental apparatus used in the current study. A Techron VTS-100 electromagnetic vibration system driven by sinusoidal signals produced by a function gen-



**Fig. 1** Schematic drawing of the vibration granular bed apparatus.

erator (Meter Inc., DDS FG-503) and supplied through a power amplifier (Techron Model 5530) was used to vertically shake the assembly. The vibration frequency  $f$  and vibration acceleration  $a$  were measured using a Dytran 3136A accelerometer attached to the shaker and connected to an oscilloscope (Tektronix TDS 210). Given the radian frequency ( $\omega = 2\pi f$ ), the amplitude  $A$  of the vibration was calculated using the relation  $A = a/\omega^2$ . The dimensionless vibration acceleration  $\Gamma$  is defined as  $\Gamma = a/g$ , where  $g$  is the gravitational acceleration.

The granular bed was vertically shaken. In a two-dimensional granular bed, rougher sidewalls induce stronger particle motions and convection cells (Hsiau et al., 2002). Therefore, a layer of glass beads with identical physical properties or a layer of emery paper was glued to the sidewalls of the container to induce a shear force sufficient for generating motions in the granular material. A high-speed charge-coupled device camera recorded the front view of the intruder motion. Using a particle-tracking method and image-processing system, the position of the intruder could be determined. All images were taken after the system had vibrated for at least 1 minute, to ensure that the flow field was in a steady state. The bed was divided into square bins, and an autocorrelation technique was employed to process the recorded images and determine the shift of each tracer particle between two consecutive images. The local velocities were then calculated from all the tracer velocities in the bin (Hsiau and Shieh, 1999; Hsiau et al., 2002; Liao et al., 2010b; Natarajan et al., 1995).

$$\langle u_i \rangle = \frac{\sum_{k=1}^{N_i} u_{ki}}{N_i} \quad (1)$$

$$\langle v_i \rangle = \frac{\sum_{k=1}^{N_i} v_{ki}}{N_i} \quad (2)$$

Here,  $\langle u_i \rangle$  and  $\langle v_i \rangle$  denote the ensemble average velocities in the horizontal and vertical directions, respectively, in the  $i$ th bin; the average velocities are obtained using the  $N_i$  tracer particles. The subscript  $k$  denotes the  $k$ th tracer particle in the  $i$ th bin.

The fluctuation velocities in the  $i$ th bin were defined as the root mean square value of the deviations between the local velocities and ensemble average velocities:

$$\langle u_i'^2 \rangle^{1/2} = \sqrt{\frac{\sum_{k=1}^{N_i} (u_{ki} - \langle u_i \rangle)^2}{N_i}} \quad (3)$$

$$\langle v_i'^2 \rangle^{1/2} = \sqrt{\frac{\sum_{k=1}^{N_i} (v_{ki} - \langle v_i \rangle)^2}{N_i}} \quad (4)$$

The granular temperature in the  $i$ th bin was then determined as

$$T_i = \frac{\langle u_i'^2 \rangle + \langle v_i'^2 \rangle}{2} \quad (5)$$

The average granular temperature of the entire bed was calculated as

$$T = \frac{\sum_i N_i \times T_i}{\sum_i N_i} \quad (6)$$

The granular temperature quantifies the fluctuation kinetic energy per unit mass associated with the random motions of the particles. The granular temperature was defined as the ensemble average of the fluctuation velocities of all the tracer particles in the vibrating granular bed. A higher granular temperature indicated a more energetic and fluidized vibrating bed.

Particle diffusion results from particle fluctuations. The concept of analyzing diffusive phenomena involving suspended particles undergoing Brownian motion in a liquid was first proposed by Einstein (1956). Campbell (1997) and Savage and Dai (1993) used the technique to investigate the self-diffusive behavior of granular flow systems through computer simulations. The self-diffusion coefficient  $D_{ij}$  is defined as

$$\lim_{t \rightarrow \infty} \langle \Delta x_i(t) \Delta x_j(t) \rangle = 2D_{ij}t \quad (7)$$

where  $\Delta x_i$  and  $\Delta x_j$  are functions of  $t$  and represent the diffusive displacements in directions  $i$  and  $j$ , respectively, relative to their initial positions. Natarajan et al. (1995) and Liao and Hsiao (2009) employed a similar method in measuring self-diffusion in a vertical channel and in sheared granular flows, respectively.

Convection is also a crucial mechanism that affects particle motion and segregation in vibrating granular beds. Hsiao and Chen (2000) suggested that within each convection cell, the mass flow rates in the top, right, bottom, and left portions should be conserved. In the horizontal plane including the center of the convection cells, the horizontal velocity is zero and only a vertical velocity component  $\langle v \rangle_c$  exists. Hence, at this center location, the convection mass flow rate  $\dot{m}$  (at the same horizontal level as the convection centers) can be defined as

$$\dot{m} = \int_{x=0}^{x=W} \frac{\rho_p V \langle v \rangle_c}{2} dx \quad (8)$$

where  $v$  is the solid fraction,  $\rho_p$  is the true bead density, and  $W$  is the half-width of the container measured in the aforementioned horizontal plane including the convection cell center. Assuming that the solid fraction is constant across each horizontal plane, the channel can be divided horizontally into experimental bins. Accordingly,  $J$  can be expressed as

$$J = \frac{\dot{m}}{\rho_p v 2W \sqrt{gd}} = \frac{\sum \langle v \rangle_c \Delta x}{2W \sqrt{gd}} \quad (9)$$

where  $d$  is the bead diameter,  $\Delta x$  is the width of each bin. Hsiao and Chen (2000) detailed the definition and formation of bins.

### 3. Results and discussion

#### 3.1 Transport properties in wet vibrating granular beds

Granular materials are common in many industries. In particular, the existence of a small amount of interstitial liquid in granular materials may complicate their handling or processing. Granular flow behavior changes considerably because of the formation and rupture of liquid bridges in wet granular materials (Grof et al., 2008; Liao et al., 2010c, 2013; Lim, 2014; Mason et al., 1999; Schulz et al., 2003). Some studies have investigated the forces generated by liquid bridges, both theoretically and experimentally (Ennis et al., 1990; Fisher, 1926; Mason and Clark, 1965; Mehrotra and Sastry, 1980; Mikami et al., 1998; Pitois et al., 2000). Fraysse et al. (1999) performed experiments to study how humidity affects the stability of sandpiles. They found that the maximal angle of stability increased with humidity. Hsiao and Yang (2003) used the discrete element method to study the effect of liquid content on the transport properties in a vertically vibrating wet granular bed. They observed that the liquid bridge force is a critical force influencing the particle motions and transport properties of wet granular systems. Liao and Hsiao (2010) found that the effect of the liquid bridge force on the dynamic behavior of wet granular matter was dependent on the kinetic energy of the granular system and liquid viscosity.

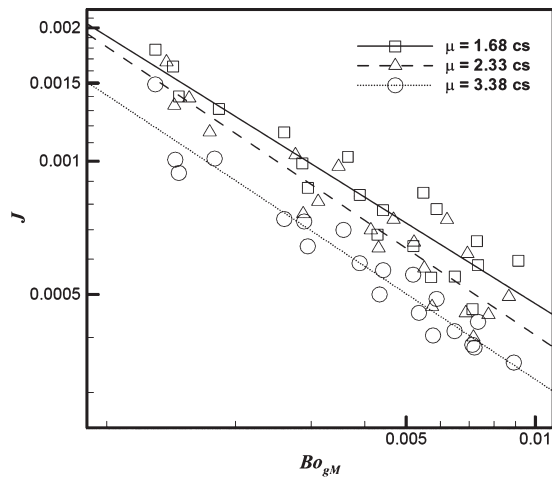
Many researchers have studied the dynamics of dry granular systems (Fortini and Huang, 2015; Hsiao et al., 2011; Knight et al., 1996; Tai and Hsiao, 2004). However, the flow behavior and dynamic properties of wet granular materials have received less attention. Most studies have focused on how liquid addition affects the flow behavior and dynamic properties of wet granular matter. Few experimental studies have examined how liquid surface tension affects the dynamic properties and convective behavior of granular matter.

The granular Bond number is somewhat reminiscent of the Bond number in fluid mechanics. Nase et al. (2001) defined the granular Bond number as follows:

$$Bo_g = \frac{F_c}{W_p} = \frac{2\pi R \gamma}{\frac{4}{3}\pi R^3 \rho_p g} = \frac{3\gamma}{2R^2 \rho_p g} \quad (10)$$

where  $F_c$  is the capillary force,  $W_p$  is the particle weight,  $\gamma$  is the surface tension,  $g$  is the gravity and  $R$  is the sphere radius. The granular material becomes cohesive and the liquid bridge force becomes dominant when  $Bo_g$  exceeds 1. However, Nase et al. (2001) did not consider the effect of liquid content in their calculations. Some studies have reported that the number of liquid bridges and liquid bridge volume increase with liquid content, enhancing the



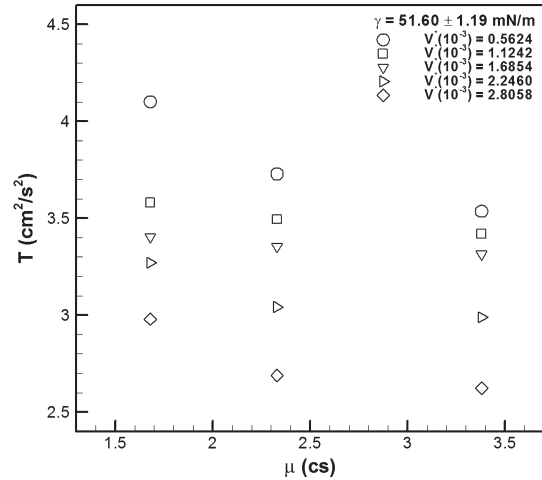


**Fig. 2** The dimensionless convection flow rate plotted against the dimensionless modified granular Bond number ( $Bo_{gM}$ ) in logarithmic scale (Hsiau S.S. et al., 2013).

strength of existing liquid bridges (Herminghaus, 2005; Kohonen et al., 2004; Liao et al., 2010c; Scheel et al., 2008). Herminghaus' research group (Herminghaus, 2005; Scheel et al., 2008) calculated the liquid content threshold for bridge formation to be approximately  $7 \times 10^{-4}$  and that capillary bridges were predominant when the liquid content was below  $2.4 \times 10^{-2}$ . Notably, the dimensionless liquid content  $V^*$  was defined as  $V^* = V_w / (V_p + V_w)$ , where  $V_w$  is the total volume of liquid added and  $V_p$  is the total volume of all beads in the bed. The number and strength of liquid bridges increase with the liquid content in the pendular state. Therefore, the granular Bond number should be modified by multiplying it with the dimensionless liquid content. The modified granular Bond number  $Bo_{gM}$  can be defined as

$$Bo_{gM} = Bo_g \times V^* = \frac{3\gamma}{2R^2\rho_p g} \times V^* \quad (11)$$

**Fig. 2** shows the dimensionless convective flow rate plotted as a function of the modified granular Bond number on the logarithmic scale. The dimensionless convective flow rate decreases in a power decay with an increase in the modified granular Bond number for each specific liquid viscosity. The exponents for the different liquid viscosities ( $\mu = 1.68, 2.33$ , and  $3.38$  cs) are  $-0.604$ ,  $-0.651$ , and  $-0.642$ , respectively. This indicates that the convective flow rate decreases as the surface tension and added liquid content increase, resulting in stronger liquid bridge forces. A larger modified granular Bond number implies more and stronger liquid bridges and larger liquid bridge forces between particles; the larger liquid bridge forces result from the higher surface tension and liquid content. A stronger liquid bridge force reduces particle motion and interactive collisions. Energy dissipation caused by the formation and rupture of liquid bridges also increases



**Fig. 3** The average granular temperatures plotted against with the added liquid viscosity  $\mu$  with different liquid content at specific liquid surface tension (Hsiau S.S. et al., 2013).

with the liquid bridge force. Thus, because of the stronger liquid bridge forces, the convective flow rate decreases as the modified granular Bond number increases.

**Fig. 3** shows the average granular temperature plotted as a function of the liquid viscosity for different  $V^*$  values at specific surface tensions. Clearly, the granular temperature decreases as the liquid viscosity increases (at a given liquid content). Adding more viscous liquid reduces particle motion and interactive collisions, thereby increasing the liquid bridge forces. A higher liquid viscosity leads to higher energy dissipation because of the formation and rupture of the liquid bridges between particles and the generation of a larger viscous force. Hence, an increase in the liquid viscosity reduces the strength of particle motion and interactive collisions, lowering the granular temperature. Sheared cell studies by Liao and Hsiau (2010) and Yang and Hsiau (2005) showed similar results, specifically that the presence of viscous and frictional force within a granular system causes high dissipation of the kinetic energy of particles. Therefore, the granular temperature decreases as the liquid viscosity and liquid content increase.

### 3.2 Segregation in wet vibrated granular beds

Adding liquid to granular materials creates liquid bridges among particles. The creation and disintegration of such bridges causes the behavior in a wet granular system to change substantially (Schulz et al., 2003). In recent years, although dry granular systems' segregation mechanism has been researched thoroughly, considerably few studies have researched the effect of adding small amounts of liquid on the segregation behavior of granular materials (Chou et al., 2010; Geromichalos et al., 2003; Li and McCarthy, 2003, 2006; McCarthy, 2009; Samadani

and Kudrolli, 2000, 2001). Geromichalos et al. (2003) found segregation behavior to increase as the liquid content was increased but began to drop after reaching a critical amount of liquid content. Hsiao and Yang (2003) researched the mixing and diffusion processes of a wet vibrating granular bed, finding that liquid content changed both processes. Samadani and Kudrolli (2000) discovered that segregation behavior decreased as more liquid was added and then reached a steady state once reaching a specific amount of liquid. The results of Chou et al. (2010) showed that the segregation condition can be determined from the repose angle of a granular mixture for wet granular systems, notwithstanding the addition of a liquid or viscosity.

The index used to calculate segregation degree is described below:

$$I = \sqrt{\frac{\sum_{i=1}^n (C_i - \bar{C})^2}{n-1}} \quad (12)$$

in which  $C_i$  represents the concentration of nylon beads in the  $i$ th cell,  $\bar{C}$  represents the average concentration of nylon beads in the entire bed ( $\bar{C} = 0.5$ ), and  $n$  represents the number of sub-regions. The segregation index spans from 0.5 to 0; 0.5 signifies a totally segregated state while 0 signifies a consistently mixed state (Chou et al., 2010; Khakhar, 1997; Liao et al., 2010a).

In order to understand the effect of liquid bridges on the segregation of wet granular matter, some of the primary forces involved must first be discussed. The quasi-static force of the liquid bridge among particles can be estimated as  $2\pi R\gamma$ , in which  $\gamma$  is the surface tension of the liquid and  $R$  is the radius of the particle. When considering only capillary force, the viscosity of the added liquid is generally not considered in the assessment of the static properties of wet granular matter. However, the viscosity of the liquid present in the dynamic interactions among particles influences the bonding force. With regards to lubrication theory, the Reynolds Equation provides a measurement method for the viscous force, which is derived in this equation from two rigid spherical surfaces (Pitois et al., 2000; Samadani and Kudrolli, 2001). The equation suggests that the pressure  $P$  created within the liquid relies on the relative displacement of the two solid particles; the equation is written below:

$$\frac{d}{dr} \left[ r H_d^3(r) \frac{dP(r)}{dr} \right] = 12\eta r \frac{dS}{dt} \quad (13)$$

in which  $H_d(r) = S + r^2/R$  represents the distance between the two spherical surfaces at radial distance  $r$  from the center of the neck,  $S$  represents the distance between particles, and  $\eta$  represents liquid dynamic viscosity. Assuming that the particles are totally submerged into the liquid,

the viscous force working on the particles can be expressed through integration of Eq. (13) two times:

$$F_v = -\frac{3}{2} \pi \eta R^2 \frac{1}{S} \frac{dS}{dt} \quad (14)$$

A liquid bridge requires a correction coefficient to be added to Eq. (14), which creates the following equation:

$$F_v = -\frac{3}{2} \pi \eta R^2 \left[ 1 - \frac{D_s}{H(b)} \right]^2 \frac{1}{D_s} \frac{dD_s}{dt} \quad (15)$$

in which  $b$  represents the radius of the wet area, and  $D_s$  represents the length of the liquid bridge. The total dynamic force of the liquid bridge is found using the superposition of the capillary force and the viscous force. Samadani and Kudrolli (2001) applied Reynolds Equation and discovered that the viscous force is proportional to the liquid viscosity.

The segregation index was applied to measure the degree of granular segregation within a granular system. Fig. 4 depicts variations to the final segregation index once a steady state is reached due to adding liquids of varying viscosities. In most cases, the segregation index decreased as the liquid content and viscosity increase. Increasing the quantity and viscosity of the liquid added to the granular materials results in both more and stronger liquid bridges created among the particles. Consequently, the energy that dissipates as a result of the creation and disintegration of said liquid bridges also increases, thus decreasing particle motion within the wet system. As has been found in prior studies, the particles' kinetic energy probably dissipates due to the creation of liquid bridge forces among the particles in the wet system (Geromichalos et al., 2003; Schulz et al., 2003). The liquid bridges generally pushed the small glass beads together to clump and form larger clusters, thereby decreasing the percolation

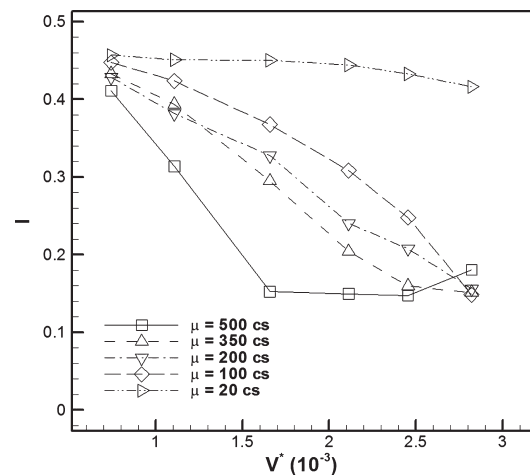


Fig. 4 The segregation index of the final state in the shaker with  $V^*$  with different viscosities of added silicon oil (Liao C.C. et al., 2010c).

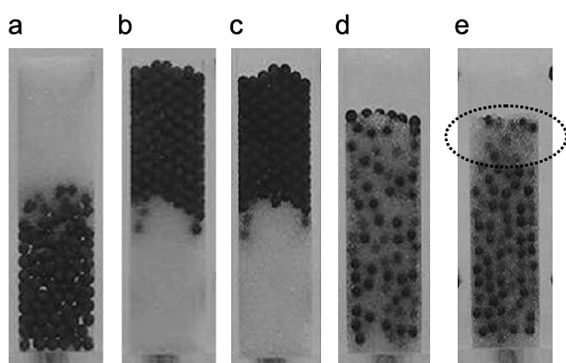
effect on the segregation amount (Samadani and Kudrolli, 2000, 2001). The amount of segregation did not change much even when increasing the liquid content with a low liquid viscosity of 20 cs, as shown in **Fig. 4**, which suggests that the liquid bridge force is insignificant with no substantial effect on either the particles' motion or segregation. Furthermore, **Fig. 4** shows that the addition of a highly viscous liquid (500 cs) to the granular materials significantly decreases the final segregation index and increases the value of  $V^*$ . Nevertheless, the final segregation index remained constant as  $V^* \geq 1.655 \times 10^{-3}$ , which is similar to that the findings of Samadani and Kudrolli (2000). Remarkably, the final segregation index showed a small increase in the highest liquid content and viscosity ( $V^* = 2.8137 \times 10^{-3}$  and  $\mu = 500$  cs). This is the liquid content and viscosity that creates the strongest liquid bridge force, which leads to the most feeble particle motion, which subsequently results in a minor increase in the final segregation index (**Fig. 4**).

**Figs. 5(a)–5(e)** are photographs taken of the amount of segregation in the binary mixtures for varying liquid contents and viscosities. **Fig. 5(a)** depicts the initial amount of segregation, where the smaller glass beads are positioned above the larger nylon beads. Meanwhile, the last segregation amount of the dry system is revealed in **Fig. 5(b)**, which shows that complete segregation was attained once the larger particles moved to the upper bed. As presented in the aforementioned material, the liquid that was added to the granular materials can be assumed to restrict the movement of particles, reducing segregation in many cases. However, the addition of extremely little liquid with a very low viscosity did not influence the final segregation amounts ( $V^* = 0.7428 \times 10^{-3}$ ,  $\mu = 20$  cs). This find-

ing is depicted in **Fig. 5(c)** and suggests that the liquid bridge force was too weak to lessen particle motion or significantly affect the segregation index. **Fig. 5(d)** depicts the final condition (the consistently mixed state) of the binary mixtures for  $V^* = 1.6550 \times 10^{-3}$  and  $\mu = 500$  cs. The adequate liquid content and viscosity reduced the driving segregation force due to the creation of the liquid bridge force. As a result, the amount of segregation was converted into the final homogeneous mixing state illustrated in **Fig. 5(d)**. **Fig. 5(e)** shows the final nonhomogeneous mixing state for  $V^* = 2.8137 \times 10^{-3}$  and  $\mu = 500$  cs. A small portion of the upper part of the container, which is circled in **Fig. 5(e)**, clearly depicts the nonhomogeneous mixing state. The particle motions were not sufficiently adequate for obtaining the homogeneous mixing state with the addition of the largest liquid content and viscosity to the granular materials ( $V^* = 2.8137 \times 10^{-3}$  and  $\mu = 500$  cs). Therefore, this study has demonstrated that the amount of segregation can be managed by altering the liquid content and viscosity.

### 3.3 Influence of bed height on convection cell formation

The convection of granular materials is vital to granular flow. Vibration acceleration, once it surpasses a specific threshold, causes convective flow patterns. In addition, convection is important to granular segregation behavior (Hsiau et al., 2002; Huerta and Ruiz-Suarez, 2004; Liao et al., 2014; Saez et al., 2005). The granular materials within a bed move in a circular pattern, from the top of the bed downward along the sidewalls toward the bottom and then toward the top through the center of the container. The occurrence of vertical vibration causes the granular system to become fluid, which thus transforms the granular bed from a consolidated state to a fluid-like state. In certain situations, the flow of granular materials between the top and bottom occurs as convection cells. In recent years, such convection has attracted many researchers (Chung et al., 2013; Gallas et al., 1992; Hsiau and Chen, 2000; Huerta and Ruiz-Suarez, 2004; Saez et al., 2005; Viswanathan, 2011). Inelastic collisions cause energy dissipation in a vibrofluidized granular system, which results in larger densities and temperature gradients. Once the temperature gradient becomes big enough, convection occurs (Wildman et al., 2001). Elperin and Golshtein (1997) applied a particle dynamics method to evaluate the influence of convection and friction on the level of segregation within a two-dimensional vibrating bed and discovered that the segregation rate significantly increased with convective roll. Furthermore, Majid and Walzel (2009) realized that convection cell patterns are contingent on the operating conditions. Hsiau et al. (2002) researched the effect of container geometry, wall friction, and vibration conditions on



**Fig. 5** Snapshots of the level of segregation of binary granular mixtures: (a) initial segregation level of the dry system, (b) steady segregation level of the dry system, (c) steady segregation level of the wet system,  $\mu = 20$  cs and  $V^* = 0.7428 \times 10^{-3}$ , (d) steady homogeneous and mixed level of the wet system,  $\mu = 500$  cs and  $V^* = 1.6550 \times 10^{-3}$ , (e) steady non-homogeneous mixing level of the wet system  $\mu = 500$  cs and  $V^* = 2.8137 \times 10^{-3}$ , with the circled part depicting the non-homogeneous mixing region (Liao C.C. et al., 2010c).

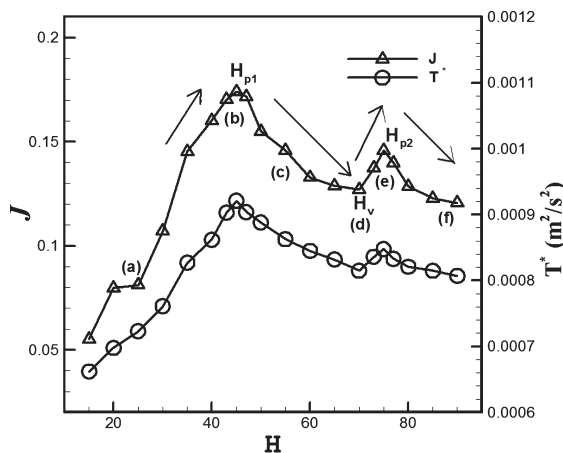
convection cells in a two-dimensional vibrating bed and determined that reverse convection cells were created when the two sidewalls had an  $8^\circ$  incline. Klongboonjit and Campbell (2008) completed a soft-particle recreation, in which convection strength was usually augmented with particle stiffness, as well as upon reaching a maximum value, the convective strength then decreases even with the further increase of particle stiffness. Fortini and Huang (2015) discovered that wall-induced convection happens within the bouncing bed region of the parameter's space, within which the granular bed acts similarly to a bouncing ball.

The original bed height can be non-dimensionalized by dividing it by bead diameter  $d$ , resulting in  $H = h/d$ .  $H$  spans from 15 to 90 with intervals of 5, which is consistent with a range of  $h$  from 3 to 18 cm. **Fig. 6** depicts the variations of dimensionless convection strength  $J$  and overall average granular temperature  $T^*$  with the initial dimensionless bed height  $H$  for the fixed vibration conditions of  $f = 40$  Hz and  $\Gamma = 16$ . The value of  $J$  has two distinct peak values, which are  $H = H_{p1}$  ( $= 45$ ) and  $H_{p2}$  ( $= 70$ ), and a single minimum value, which is  $H = H_v$  ( $= 75$ ). Augmenting  $H$  from 15 to 45 causes the dimensionless convection strength to significantly increase (in an almost linear manner). A smaller amount of material (smaller  $H$ ) in the bed fluidizes the bed materials more violently than if they had been in a thicker bed. When height of the initial bed surpasses the first critical (maximum) value  $H_{p1}$ , the convection strength starts decreasing as the bed height increases. Once the bed height surpasses a specific value, a “solid-like” flow region is created within the material at the bottom of the bed due to gravity. The beads in this solid-like region show weak relative movement and inter-active collisions, which both dissipate the energy that

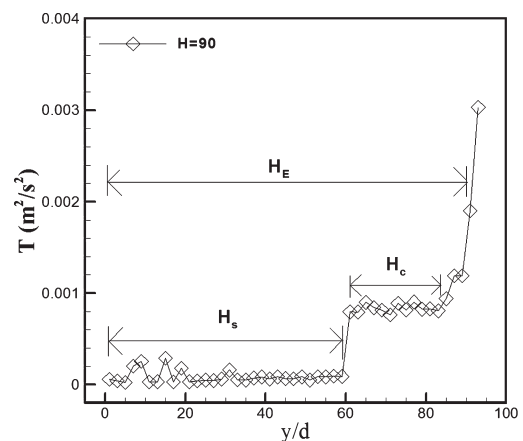
comes from the shaker, thus resulting in less energy being transferred to the convection cells in the upper bed layers (the fluid-like region).

Increased convection strength as a result of increased bed height stops once  $H$  surpasses the second peak  $H_{p2}$ , at which point the whole solid-like region acts similar to a solid body. An increase in  $H$  signifies a thicker “solid.” Said thicker solid can absorb more energy, thus transferring less energy to then be received by the upper convection region, ultimately resulting in weaker convection. **Fig. 6** shows the two peaks and one valley of the curve for  $T^*$ . The aforementioned variation of the initial bed height with  $T^*$  resembles the variations in the  $J$  values. A larger  $J$  value denotes that the granular bed becomes more active when the  $T^*$  value is higher.

The granular temperature of each individual horizontal layer can be determined by taking the average of the root-mean-square fluctuation velocity of each tracer particle that moves through the layer. **Fig. 7** depicts variations of the granular temperature with the dimensionless height  $y/d$  within a fixed vibration environment of  $f = 40$  Hz,  $\Gamma = 16$ , and  $H = 90$ . The curve can best be described categorized into three regions. The granular temperatures of the lower bed have small values near 0 and can vary ( $y/d < 60$ ), which suggests that the beads in this section have extremely weak motion, resulting in the lower bed acting like a solid.  $H_s$  indicates the thickness of this solid-like region. The beads located above the solid-like region have substantial convective motion ( $60 < y/d < 85$ ) and significantly larger granular temperatures than those in the solid-like region. The so-called “liquid-like” region features convective rolling.  $H_c$  indicates the thickness of the convection rolls. The beads in certain layers (free surface,  $y/d > 85$ ) have substantially higher granular temperatures.



**Fig. 6** Variations of the dimensionless convection strength  $J$  and the overall average granular temperature  $T^*$ , with changes to the initial dimensionless bed height  $H$ , with a fixed vibration environment of  $f = 40$  Hz and  $\Gamma = 16$  (Hsiau S.S. et al., 2011).

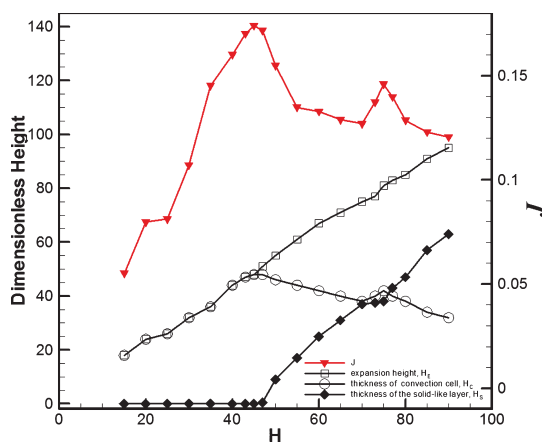


**Fig. 7** The granular temperature varying with the dimensionless height with a fixed vibration environment of  $f = 40$  Hz,  $\Gamma = 16$ , and  $H = 90$ . The inset plot depicts the total bed thickness in complete expansion state  $H_E$ , the thicknesses of “solid-like” region  $H_s$  and the thickness of convection region  $H_c$  (Hsiau S.S. et al., 2011).



The total bed thickness throughout the state of maximal expansion is written as  $H_E$ . The inset depicts the total bed thickness throughout complete expansion state  $H_E$ , the thickness of solid-like region  $H_S$ , and the thickness of convection region  $H_C$ .

**Fig. 8** depicts the various values of  $H_E$ ,  $H_C$ , and  $H_S$ , and of  $J$  with  $H$  in the fixed vibration environment of  $f=40$  Hz and  $\Gamma=16$ . These three thicknesses' data were found through the analysis of the experimental plots of the granular temperature versus initial bed thickness, as shown in **Fig. 7**. The expansion height (open square symbols) goes up practically linearly as the initial bed height is increased, and  $H_E > H$ . The bed has two different flow regimes: a convection region with thickness  $H_C$  and a solid-like region with thickness  $H_S$  ( $H_E \cong H_C + H_S$ ). With  $H$  values between 25 and 45, all of the bed material moves to create two symmetric convection cells, and the bed has no solid-like region ( $H_S \cong 0$ ,  $H_C \cong H$ ). Furthermore, convection cell height  $H_C$  maintains its increasing behavior as the initial bed height becomes larger. Once the initial bed height surpasses 47, the mass of the bed becomes too thick to be completely fluidized, and the solid-like region is forced to expand. The solid-like region's height begins to increase with an increasing  $H$ , as shown in **Fig. 8**. A thicker solid-like region shows that the beads within it utilize more input energy, thus leaving less energy for the upper material, which results in decreased  $H_C$ . This tendency reverses once  $H=70$ . For  $70 < H < 75$ , the solid-like region appears to be a rigid, solid body, in which the "solid" beads dissipate less energy, which causes more energy to be transferred into the upper layers. While the thicknesses of the solid-like region remain similar in both situations, the thickness of the convection cells increases with a growing  $H$ . As the bed mass ( $H$ ) further increases,  $H_C$  starts to decrease.



**Fig. 8** The thicknesses of total expansion height  $H_E$ , convection cells  $H_C$ , "solid-like" region  $H_S$ , and dimensionless convection strength  $J$ , against initial bed height  $H$ , in a fixed vibration environment of  $f=40$  Hz,  $\Gamma=16$  (Hsiao S.S. et al., 2011).

The relationship between  $H_C$  and the initial bed height closely resembles the variation of  $J$  values with the initial bed height, both of which feature two peaks ( $H_{p1}$  and  $H_{p2}$ ) and a valley ( $H_v$ ). Although the initial bed height influences convection strength, these two parameters were not found to be monotonically related. The thickness of the convection rolls substantially influences dimensionless convection strength, and the thicker the convection roll is, then the more fluidized the granular bed in the container is. Therefore,  $J$  and  $H_C$  ought to present similar tendencies versus initial bed height. **Fig. 8** depicts the relationship between  $J$  and  $H_C$ .

### 3.4 Influence of intruder surface roughness on granular segregation in a vertically vibrated bed

Much research has already investigated friction-induced segregation, which is a serious problem in many industries (Kondic et al., 2006; Pohlman et al., 2006; Plantard et al., 2006; Liao et al., 2014). Srebro and Levine (2003) evaluated the effect of friction with regard to the segregation of binary granular mixtures by combining Edwards' thermodynamic hypothesis using a simple mechanical model and found that segregation took place with specific degrees of compactness within the binary mixtures of grains with varying frictional properties. Furthermore, Kondic et al. (2006) found that friction led to granular segregation in a horizontally shaken container with a mound centered at the bottom. Ulrich et al. (2007) performed experiments that resulted in a transition from the reverse Brazil nut effect to the Brazil nut effect when the particle friction coefficient became higher due to long periods of shaking, which was explained as the result of sidewall-driven convection and buoyancy. Liao et al. (2014) discovered that the internal friction did not significantly affect Brazil nut granular segregation when dealing with a small filling bed height. However, internal friction was vital for the rise dynamics of intruders when dealing with a large bed height in a granular system. Unac et al. (2014) further discovered that isolated intruders may sometimes rise when tapped based not only on the size ratio but also on the environment's degree of ordering.

Recently, research regarding the rise of an intruder in a granular fluid has garnered considerable attention. However, the influence of surface roughness on a rising intruder has not been studied as much but is worthy of further investigation. Understanding the influence of friction on segregation in vibrating granular systems may provide insights into unknown physical mechanisms.

**Fig. 9** shows the time dependence curves of the intruder height of three degrees of surface roughness of intruders and three  $\Gamma$  values (3, 4.2, and 4.8) with a vibration frequency of 35 Hz. The slow rise of the intruder from just

above the bottom to the free surface occurs in all cases because of the Brazil nut effect. The rougher the intruder is, the longer the rise time is, and vice versa. Energy dissipation happens during the binary collision of particles due to friction and inelastic collisions. The friction that exists between the intruder and the beads is vital for the rise dynamics of the intruder. Furthermore, the restitution coefficient values of each intruder may affect the rise dynamics. In a considerably densely packed bed, the drag between the intruder and beads typically results in the dissipation of the rising intruder's kinetic energy. Rougher intruders experience greater degrees of dissipation due to their larger friction coefficient and smaller restitution coefficient, thus creating weaker upward motion and a longer rise time, as is shown in the curves in **Fig. 9**. Moreover, small  $\Gamma$  values cause the intruder dynamics to be divided into two stages during the rise of the intruder. The rise experienced is slower in the lower part of the granular bed and quicker in the upper part (convection region). In such a condition, the system has less external energy, and bead compactness is higher, which creates weaker motion in the bed's lower part. Therefore, the intruder rises more slowly as a consequence of the high friction between said intruder and the beads located in the lower part of the bed during the vibration cycle, thus causing weaker percolation. On the other hand, the convection cell increases in size and strength as the value of  $\Gamma$  increases. Thus, the intruder rises more easily and quickly and demonstrates a generally linear evolution with larger  $\Gamma$  values.

Furthermore, the slopes of the intruders' curves with three varying degrees of surface roughness all resembled one another in the bed's upper part. The convection mechanism in the bed's upper part controlled the rise dynamics, and convection strength was found to be similar within the studied vibration conditions. Moreover, intruders with a smooth surface had reduced friction, and the rise time

variation was not significant with higher  $\Gamma$  values.

Nahmad-Molinari et al. (2003) defined penetration length  $P_1$  and discovered that the intruder penetrates the granular bed due to its inertia over a small distance during each cycle. This study further found that the intruder's kinetic energy dissipates because of friction as it enters the granular bed during each cycle. Based on the aforementioned findings, the researchers developed a model based on a simple energy balance during each cycle:

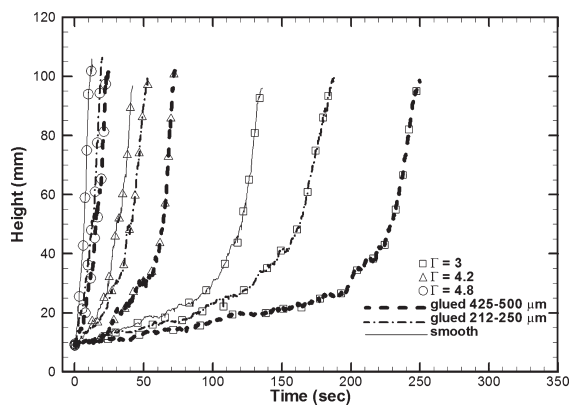
$$\frac{1}{2} m_i v_t^2 = \beta P_1 \quad (16)$$

in which  $m_i$  signifies the intruder mass,  $v_t$  signifies the intruder take-off velocity upon the system reaching a negative acceleration,  $a_i = -g$ ,  $\beta$  signifies the drag between the beads and intruder, and  $P_1$  signifies the penetration length. The parameter  $v_t$  is the value of  $\dot{z}(t)$  when  $\dot{z}(t) = -g$ , and  $z(t) = A \sin(\omega t)$ . Therefore, the following can be derived:

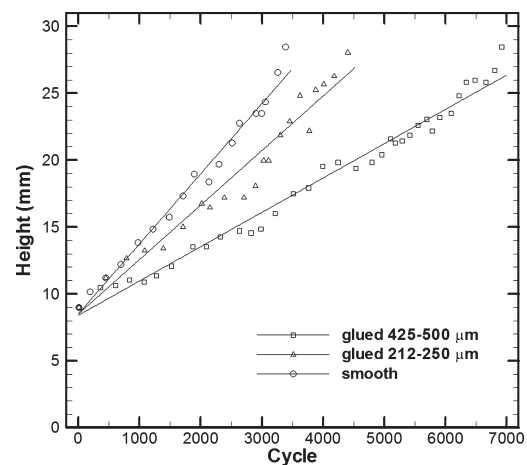
$$v_t = [A^2 \omega^2 - g^2 / \omega^2]^{1/2} \quad (17)$$

The position of the intruder is graphed as a function of the vibration cycle ( $t\omega/2\pi$ ) with the condition of  $\Gamma = 3$  and  $f = 35$  Hz (**Fig. 10**). Each curve's slope is the exact same as the penetration length of the corresponding intruder (Nahmad-Molinari et al., 2003). The lower parts of the curves have linear fits as a result of the convection's decrease. The linear fit of the data reveals the penetration length of each vibration cycle. The penetration length is graphed as a function of the vibration amplitude for varying intruders (**Fig. 11**), which shows that the penetration length decreases as the surface roughness of the intruder increases under the given vibration conditions, as well as that the penetration length increases as the vibration amplitude increases.

Eq. (16) shows that the penetration length is inversely proportional to drag  $\beta$ . Rougher intruders with larger  $\beta$

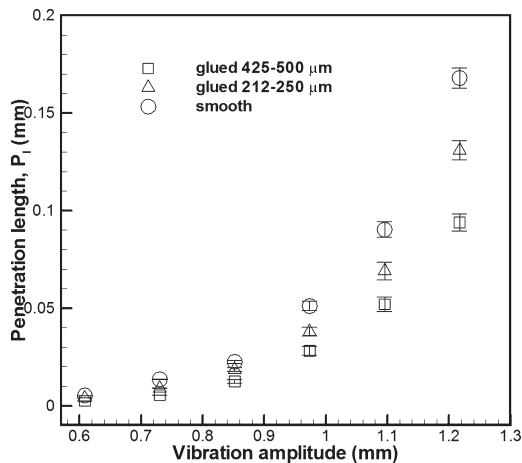


**Fig. 9** Intruder height as a function of vibrating time with the conditions of  $\Gamma = 3, 4.2, 4.8$  and  $f = 35$  Hz (Liao C.C. et al., 2012).

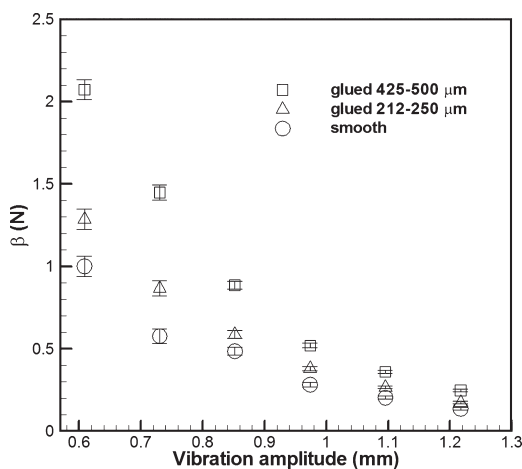


**Fig. 10** Intruder height as a function of vibrating cycle ( $t\omega/2\pi$ ) (Liao C.C. et al., 2012).

dissipate more kinetic energy and thus have shorter penetration lengths. To test this hypothesis, we calculated  $\beta$  using Eq. (16). By plotting  $\beta$  against the vibration amplitude (Fig. 12), the  $\beta$  value increased with the intruder friction coefficient under the condition of a given vibration amplitude. Higher drag causes higher intruder friction coefficients due to the increased friction between the beads and the intruder during its rise. The curves in Fig. 12 show that  $\beta$  is reduced as the vibration amplitude becomes bigger. Both convection and fluidization increase along with the vibration amplitude within the system. The flow behavior of the granular materials further transforms from a quasistatic state into a dynamic state. The strength of both the particle motions and the collisions is enhanced when in the dynamic state. Consequently, friction has a less serious effect on the intruder's rise dynamics. Even with varying intruder friction coefficient values, increasing the vibration amplitude reduces both  $\beta$  and its variations.



**Fig. 11** Penetration length as a function of vibration amplitude for intruders with varying degrees of surface roughness (Liao C.C. et al., 2012).

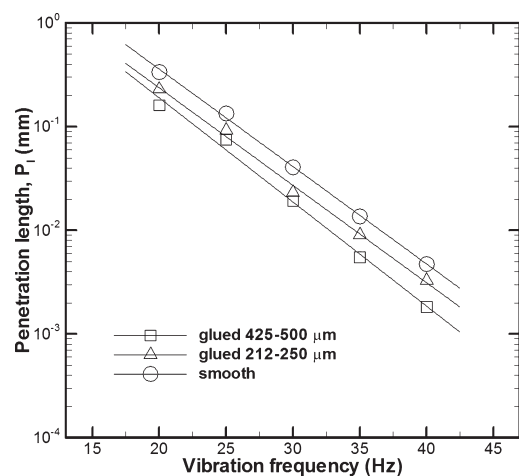


**Fig. 12** Drag  $\beta$  as a function of vibration amplitude for intruders of varying degrees of surface roughness (Liao C.C. et al., 2012).

In Fig. 13, the penetration length is plotted as a function of vibration frequency on the logarithmic-normal scale for intruders with varying degrees of surface roughness. The straight lines, which are found with the least-squares method, reveal the exponential decay of the penetration length with the frequency. Vanel et al. (1997) found that the dimensionless rise time (the number of vibration cycles) is scaled exponentially when  $f < 15$ . Similarly, the current study's dimensionless rise time was proportional to  $1/P_i$ , even though the vibration frequency range does not match completely. The discrepancy is most likely due to the use of different experimental containers, in which would have undoubtedly affected particle motion and flow behavior. A two-dimensional container was used in our experiments, while a three-dimensional one was employed by Vanel et al. (1997). Nevertheless, both studies found convection to be vital to the rise dynamics of the intruder.

### 3.5 Effect of a bumpy base on granular segregation and transport properties for vertical vibration

In a real granular system, the wall conditions may also be bumpy or uneven. The effect of a bumpy base on the rise dynamics of an intruder and transport properties has not been previously examined, and it offers insights into the complexities of segregation. Farkas et al. (1999) performed experiments and simulations to study the horizontal transport of granular particles in a vertically vibrating bed with a sawtooth-shaped base. Levanon and Rapaport (2000) investigated the horizontal flow of granular materials in a vertically vibrating sawtooth-shaped base. Their simulation results showed that the induced flow rate varies with the bed height and that counterflow could occur



**Fig. 13** Penetration length plotted against vibration frequency in logarithmic-normal scale for intruders with varying degrees of surface roughness. The straight lines represent fit data obtained using the least-square method (Liao C.C. et al., 2012).

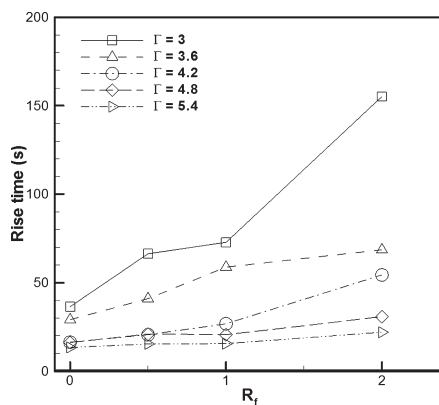
at different levels. Hsiau et al. (2002) indicated that convection and segregation rates increase with the sidewall friction coefficient in a vertical shaker. Hsiau and Yang (2002) reported that for sheared granular flow, rougher wall conditions enhance the transport properties of granular materials. Hsiau et al. (2006) reported that the stress and self-diffusion coefficients of particles are inversely proportional to the internal friction coefficient. Lu and Hsiau (2008) investigated the mixing behavior in vibrating granular beds by using a discrete element method. They demonstrated that the mixing rates increase exponentially with the convection and vertical self-diffusion coefficient. Ma et al. (2014) found that even with the same size and density ratio of the intruder to the background particles, the intruder exhibits a distinct behavior at the given vibrational conditions. Windows-Yule et al. (2015b) demonstrated that even for a fixed input energy from the wall, energy conveyed to the granular system under excitation could vary significantly dependent on the amplitude and frequency of the driving oscillations.

In the current study, a layer of densely packed glass beads was glued to the base of a container to form a bumpy base surface. To quantify the relative size of the particles on the base, we defined a base factor  $R_f$ :

$$R_f = \frac{P_d}{d_b} \quad (18)$$

where  $P_d$  is the diameter of the glued beads and  $d_b$  is the diameter of immersed glass beads ( $d_b = 2$  mm in this study). Four different base factors ( $R_f = 0, 0.5, 1, 2$ ) were used;  $R_f = 0$  corresponded to a flat base surface, and  $R_f = 2$  represented the largest wall protrusions.

**Fig. 14** shows the rise time plotted as a function of  $R_f$  for different  $\Gamma$  at  $f = 25$  Hz. The rise time increases with  $R_f$  for each  $\Gamma$  because the strength of the particle motions is reduced for a large base factor. The rise times also decrease as the  $\Gamma$  for each container base condition increases. Previous research has shown that for higher  $\Gamma$ ,

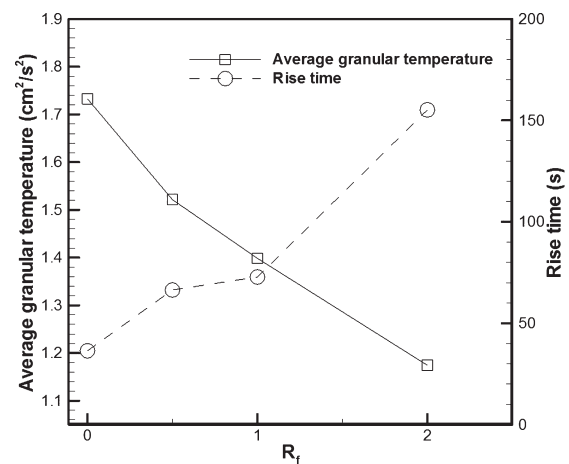


**Fig. 14** Rise time as a function of  $R_f$  with different  $\Gamma$  for fixed  $f = 25$  Hz (Liao C.C. et al., 2014).

the granular bed becomes more fluidized, enhancing the percolation effect and resulting in a faster rise time. Additionally, the influence of a bumpy base on the rise time is significant at smaller  $\Gamma$  and becomes less pronounced at higher  $\Gamma$  because more energy can be introduced in the system at higher  $\Gamma$ . Previous studies have shown that the rise time is similar if the granular bed is fluidized. Increased fluidization could be achieved with higher  $\Gamma$ . Hence, the rise time increases slightly at greater  $\Gamma$  in bumpy surface instances. By contrast, the particle motions are weakened and the granular bed becomes less fluidized for smaller  $\Gamma$ . Under these conditions, the base roughness has significantly influences the dynamic properties of the granular material and leads to a larger variation of the rise time for different base conditions.

The transport properties of the immersed glass beads play a crucial role in the Brazil nut mechanism. Here, we discuss the Brazil nut problem from the viewpoint of micromechanics by determining the transport properties of immersed glass beads. **Fig. 15** shows the average granular temperature and rise time as a function of  $R_f$  for  $f = 25$  Hz and  $\Gamma = 3$ . The base condition significantly influences the granular temperature. The average granular temperature decreases and the rise time increases with an increase in the base factor. The total external energy is constant for fixed vibration conditions. More energy could be introduced into the bed to increase the granular temperature for a small base factor. The particle motions and interactive collisions become stronger and the bed shows greater fluidization as the granular temperature increases. Hence, the intruder experiences less resistance during the rise process, which leads to a shorter rise time for a larger granular temperature.

Particle diffusion is a crucial parameter in studying the mixing and segregation of granular materials. The self-diffusion coefficients are determined from the rela-



**Fig. 15** Average granular temperature and rise time as a function of  $R_f$  for fixed  $f = 25$  Hz and  $\Gamma = 3$  (Liao C.C. et al., 2014).



tionship between mean-square diffusive displacements and time, as shown in Eq. (7), by conducting experimental procedures similar to those of previous studies (Hsiau and Yang, 2002; Liao and Hsiau, 2009; Tai and Hsiau, 2004). Self-diffusion coefficients and rise times are plotted as a function of  $R_f$  for  $f = 25$  Hz and  $\Gamma = 3$ , as shown in Fig. 16. The self-diffusion coefficients are inversely proportional to the rise time of the base factor. Particle diffusion behavior results from particle fluctuations. Previous studies (Hsiau and Yang, 2002; Savage and Dai, 1993) have indicated that the self-diffusion coefficient is proportional to the square root of granular temperature. For a higher base factor, external energy could not be introduced into the system effectively, leading to weaker interactive collisions and a smaller granular temperature. Therefore, a decreasing self-diffusion coefficient increases the rise time for a large base factor, as shown in Fig. 16.

Fig. 17 shows the dimensionless convective flow rate  $J$  and rise time plotted as a function of  $R_f$  for the fixed conditions  $f = 25$  Hz and  $\Gamma = 3$ . As the base factor decreases,  $J$  increases and the rise time decreases. The flow field is more fluidized, which strengthens the convection for a smaller base factor. Consequently,  $J$  increases and the rise time decreases as the base factor decreases. This result supports those of previous studies (Hsiau, 2002; Liao et al., 2012).

#### 4. Conclusions

Granular materials are widely used in many industries and commonly encountered in daily life. To handle and control them, understanding the transport properties and segregation mechanisms in dry and wet granular systems is essential. Furthermore, controlling granular segregation is difficult. This paper presents the main results obtained

by our research group; the results pertain to transport properties and segregation in a vibrating granular bed. The addition of a small amount of liquid to granular materials generally reduced the fluidization of the granular bed because liquid bridges formed between particles. The convective flow rate and granular temperature decreased as the added liquid content, surface tension, and viscosity increased. Increasing the added liquid content, surface tension, and viscosity also caused higher energy dissipation in a wet vibrating granular bed. Thus, higher liquid content, surface tension, and viscosity reduce the granular temperature and convection strength. The convective flow rate decreases according to a power law as the modified granular Bond number increases. The granular segregation can also be reduced by adding a small amount of liquid to the granular materials. Both the liquid content and viscosity considerably influence granular segregation. We demonstrated that the initial segregation state may change to a homogeneous mixing state when an appropriate amount of liquid with sufficient viscosity is added to the granular matter.

This paper also discusses how the bed height influences convection cells. The results of our study show that the convection strength and overall average granular temperature lead to a two-peak phenomenon with the increasing bed height. We demonstrated that this two-peak pattern occurs because a solid-like region forms in the granular bed.

The current results also indicated that intruder surface roughness has a significantly influence on the rise dynamics of intruder. The Brazil nut segregation effect is mitigated as the intruder surface roughness increases. The rise time for a rough intruder is longer than that for a smooth intruder because higher surface roughness results in more energy dissipation. The penetration length is enhanced as the intruder surface roughness is reduced, and

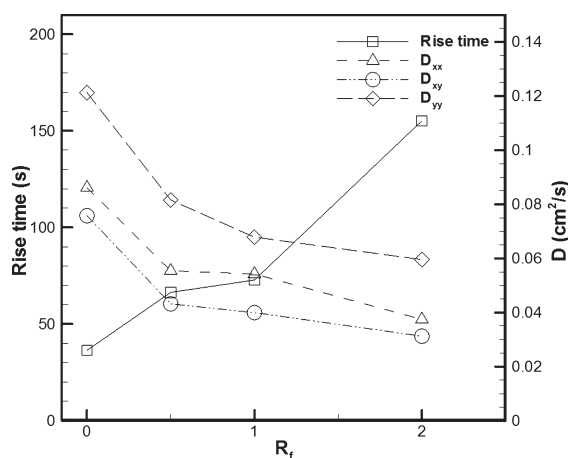


Fig. 16 Self-diffusion coefficients  $D$  and rise time as a function of  $R_f$  for fixed  $f = 25$  Hz and  $\Gamma = 3$  (Liao C.C. et al., 2014).

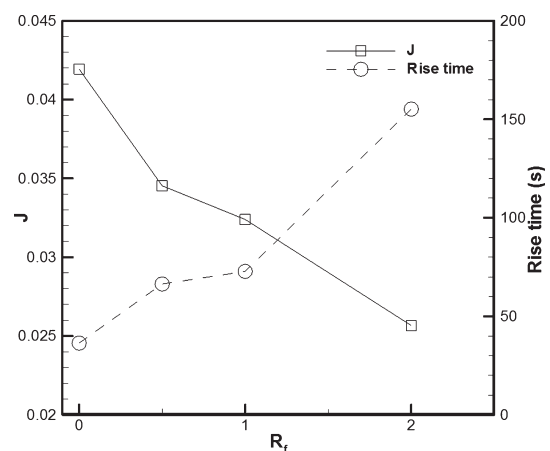


Fig. 17 Dimensionless convection flow rate  $J$  and rise time as a function of  $R_f$  for fixed  $f = 25$  Hz and  $\Gamma = 3$  (Liao C.C. et al., 2014).

is scaled exponentially with the vibration frequency. Additionally, the drag increases with the intruder surface roughness. We also observed that the intruder rise dynamics and transport properties of the immersed glass beads are influenced significantly by the base roughness. The penetration length is enhanced as the base factor is reduced. Additionally, the drag force increases with the base factor. The granular temperature, diffusion coefficients, and dimensionless convective flow rate decrease as the base factor increases and are inversely proportional to the rise time.

## Acknowledgments

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## Nomenclature

$A$	vibration amplitude, mm	$H_C$	convection region
$a$	vibration acceleration, $\text{m/s}^2$	$H_{p1}$	first critical (maximum) value
$a_i$	negative gravitational acceleration	$H_{p2}$	second peak
$Bo_g$	granular Bond number	$H_v$	the valley
$Bo_{gM}$	modified granular Bond number	$I$	segregation index
$b$	radius of the wetted area, mm	$J$	dimensionless convection strength
$C_i$	concentration of nylon beads in the $i$ th cell	$\dot{m}$	convection mass flow rate
$\bar{C}$	average concentration of nylon beads in the entire bed	$m_i$	intruder mass
$D_s$	separation distance of liquid bridge, mm	$n$	number of subregions
$D_{ij}$	self-diffusion coefficient	$N_i$	number of tracer particles in the $i$ th bin
$d$	bead diameter	$P$	pressure generated in the liquid, $\text{N/m}^2$
$d_b$	diameter of immersed glass beads	$P_d$	diameter of glued beads
$F_c$	capillary force, N	$P_1$	penetration length
$F_v$	viscous force, N	$r$	radial distance
$f$	vibration frequency, Hz	$R$	particle radius, mm
$g$	gravitational acceleration, $\text{m/s}^2$	$R_f$	base factor
$H$	dimensionless initial bed height	$S$	distance between particles, mm
$h$	initial bed height, cm	$t$	time
$H(b)$	distance between the two spherical surfaces at radius of the wetted area, mm	$T$	average granular temperature
$H_d(r)$	distance between the two spherical surfaces, mm	$T_i$	granular temperature in the $i$ th bin
$H_E$	total bed thickness in the full expansion state	$T^*$	overall average granular temperature
$H_S$	solid-like region	$\langle u_i \rangle$	the ensemble average velocity in the horizontal direction in the $i$ th bin
		$\langle u_i^2 \rangle^{1/2}$	fluctuation velocity in the horizontal direction in the $i$ th bin
		$u_{ki}$	the velocity of the $k$ th tracer particle in the horizontal direction in the $i$ th bin
		$V^*$	dimensionless liquid content
		$V_w$	total volume of silicon oil added, $\text{cm}^3$
		$V_p$	total volume of whole beads in the bed, $\text{cm}^3$
		$\langle v \rangle_c$	vertical velocity component
		$\langle v_i \rangle$	ensemble average velocity in the vertical direction in the $i$ th bin
		$\langle v_i^2 \rangle^{1/2}$	fluctuation velocity in the vertical direction in the $i$ th bin
		$v_{ki}$	the velocity of the $k$ th tracer particle in the vertical direction in the $i$ th bin
		$v_t$	intruder take-off velocity
		$W$	the half-width of the container
		$W_p$	particle weight
		$\Delta x$	the width of each bin
		$\Delta x_i$	the diffusive displacement in direction $i$
		$\Delta x_j$	the diffusive displacement in direction $j$

$y$	bed height
$z(t)$	Sine function
<i>Greek letters</i>	
$\Gamma$	dimensionless vibration acceleration
$\gamma$	surface tension of the liquid, mN/m
$\beta$	drag between the beads and the intruder
$\mu$	liquid kinematic viscosity, cs
$v$	solid fraction
$\eta$	liquid dynamic viscosity, cP
$\rho_p$	the true bead density, g/cm <sup>3</sup>
$\omega$	vibration radian frequency, rad/s

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### Author's short biography



#### Chun-Chung Liao

Dr. Chun-Chung Liao received his PhD degree from National Central University (Taiwan) in 2010, majored in Mechanical Engineering. After receiving the PhD degree, he started his academic career as a postdoctoral fellow at National Central University in Jhongli, Taiwan. He was a visiting scholar in Mechanical Engineering in Caltech from Oct. 2013–Jul. 2014. Currently, he is the assistant professor at Department of Mold and Die engineering in National Kaohsiung University of Applied Sciences from Feb. 2015. His research interests include powder technology, granular flows, fast pyrolysis of biomass.



#### Shu-San Hsiau

Dr. Shu-San Hsiau received his MS and PhD degrees from California Institute of Technology, majored in Mechanical Engineering in 1993, and started his professorship life in National Central University (NCU). Currently he is a Distinguished Professor of Mechanical Engineering, and also Institute of Energy Engineering of NCU. He received the Humboldt award and went to Darmstadt University of Technology (Germany) in 2003. His research area includes Powder Technology, Clean Coal Technology, Biomass (Pyrolysis, Gasification), Modeling and Design of MOCVD, Energy Technology, Hot-Gas Cleanup, Debris Flow and Avalanche, Powder Technology, Hydrogen Energy, Energy Education, Creative Engineering Education, Thermo-Fluids, Pharmaceutical Engineering. He served as the chairman of Mechanical Engineering (2010–2013) and also as the director of Institute of Energy Engineering (2008–2010). Currently he is the deputy dean of R&D office (NCU), and also the director of Clean Coal Research Center.