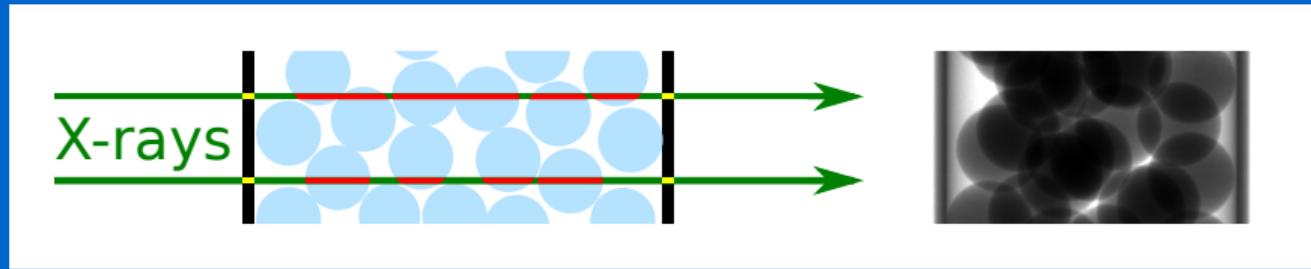


Measuring the volume fraction of **dynamic** granular systems

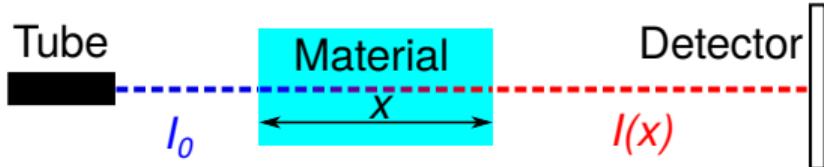


Correction of beam hardening
in X-ray radiograms

Baur *et al*, *Rev. Sci. Instrum.* (2019)

In collaboration with Norman Uhlmann, Fraunhofer EZRT

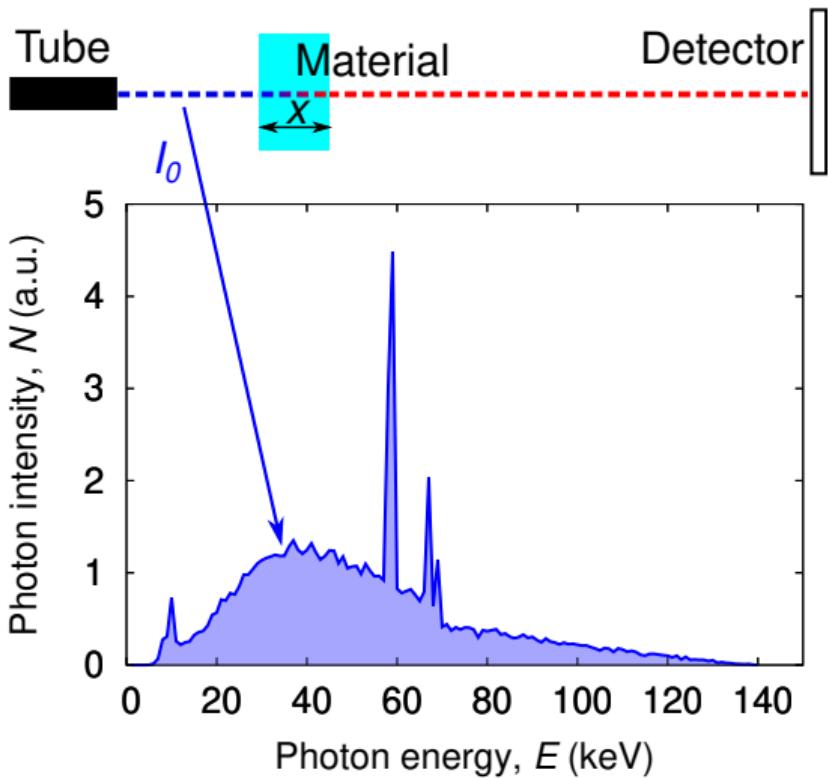
Attenuation of X-rays



Beer-Lambert's law

$$I(x) = I_0 \exp(-\mu x)$$

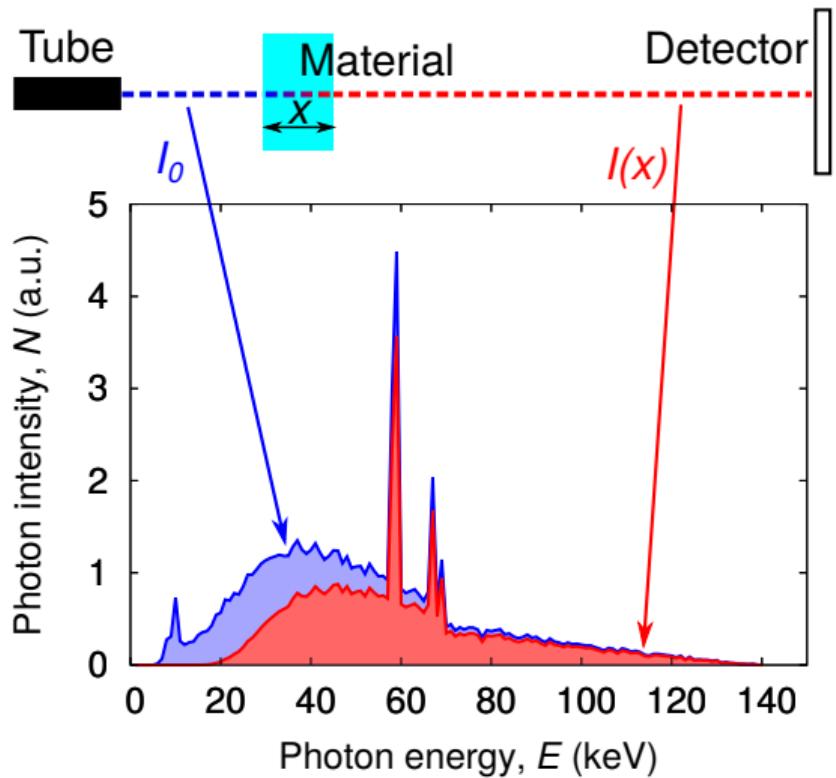
Attenuation of X-rays



Beer-Lambert's law

$$I(x) = I_0 \exp(-\mu x)$$

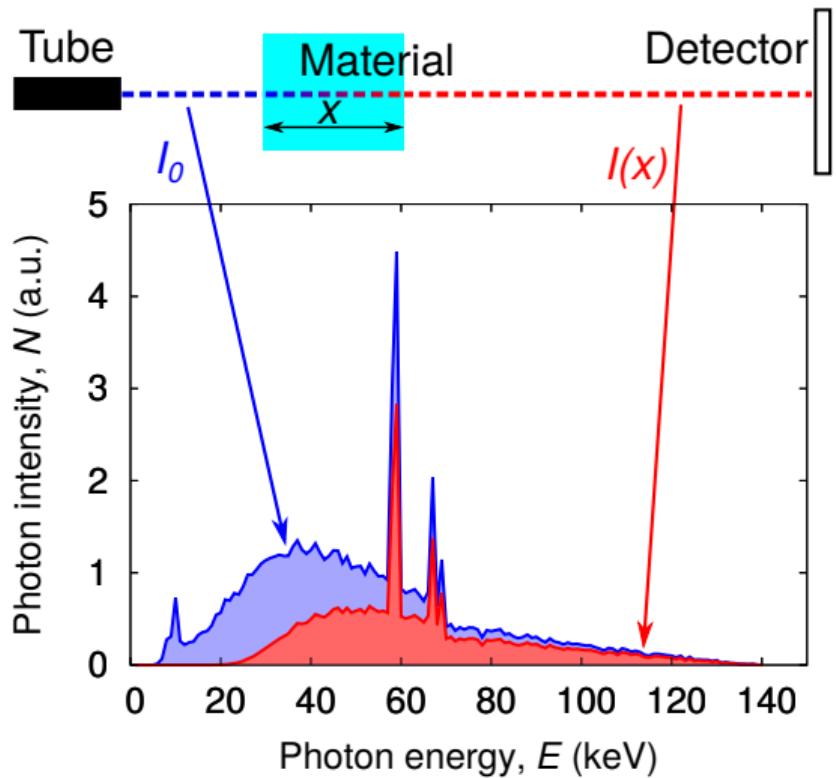
Attenuation of X-rays



Beer-Lambert's law

$$I(x) = I_0 \exp(-\mu x)$$

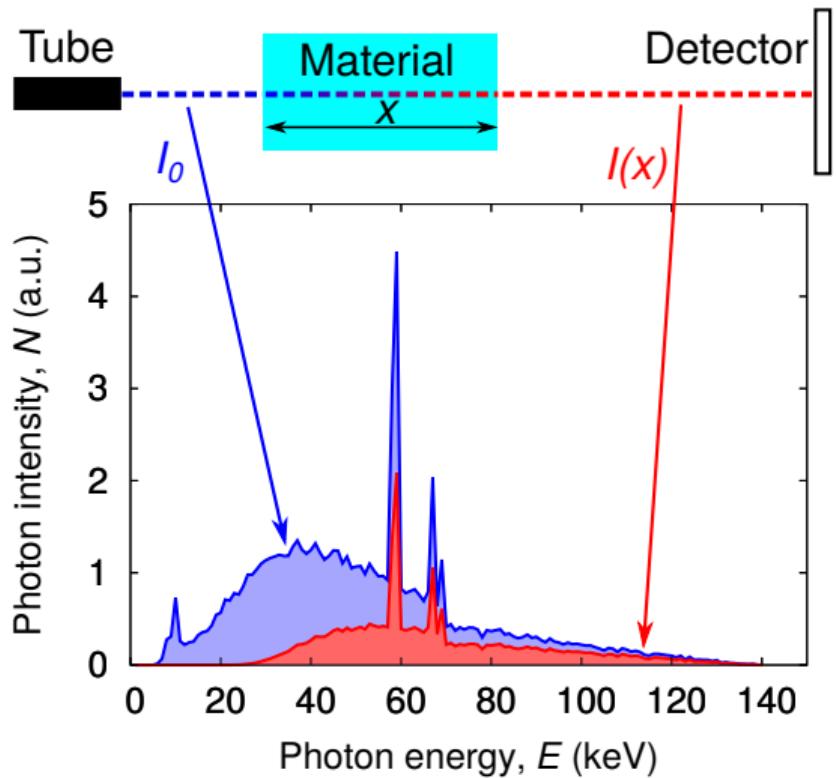
Attenuation of X-rays



Beer-Lambert's law

$$I(x) = I_0 \exp(-\mu x)$$

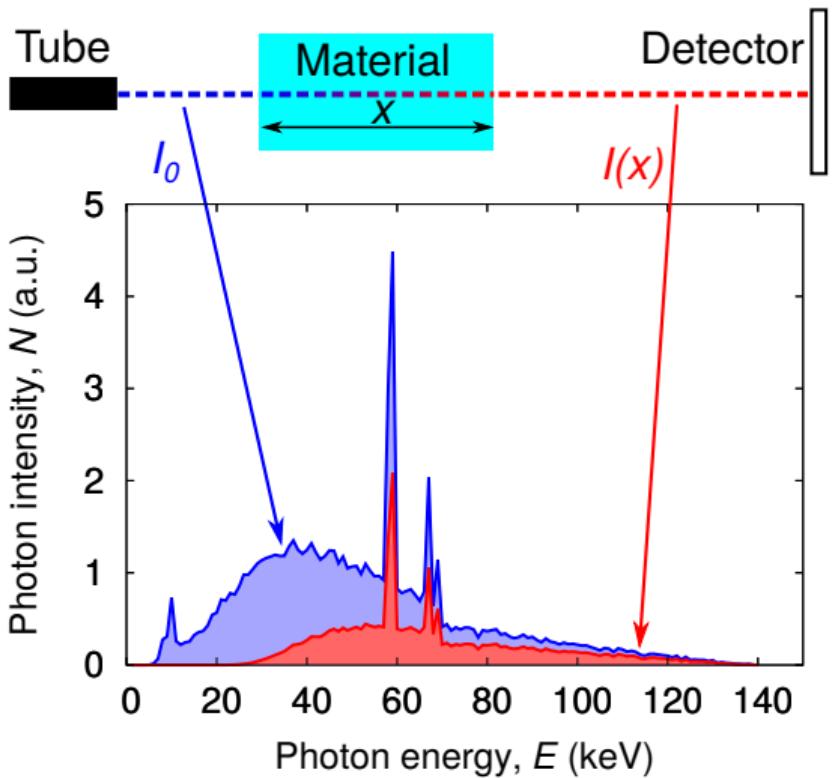
Attenuation of X-rays



Beer-Lambert's law

$$I(x) = I_0 \exp(-\mu x)$$

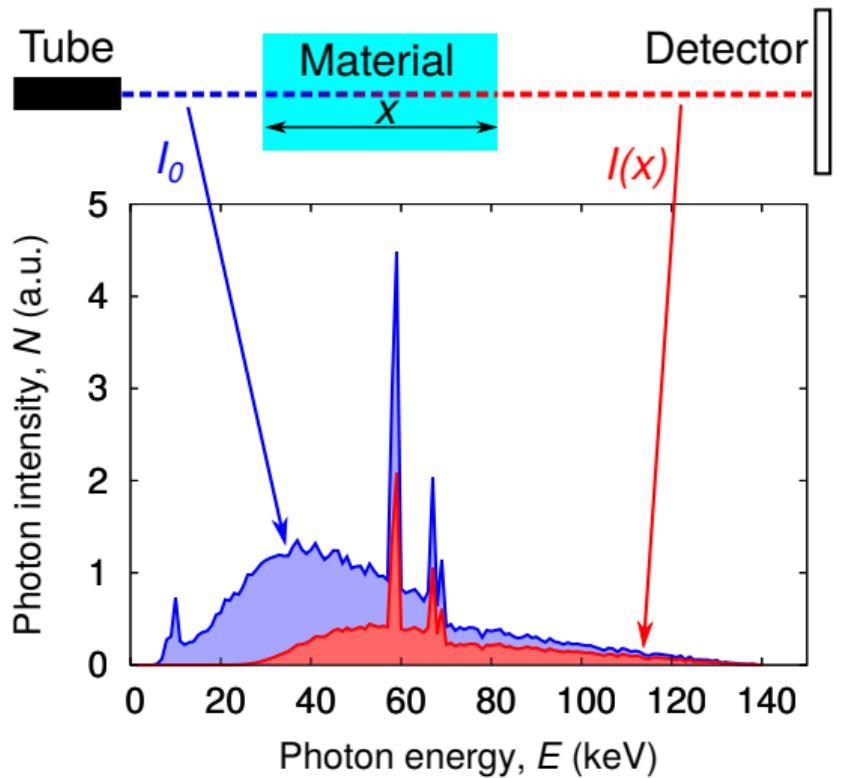
Attenuation of X-rays



Beer-Lambert's law

$$\cancel{I(x) \equiv I_0 \exp(-\mu x)}$$
$$I(x) = I_0 \exp(-\mu_{\text{eff}}(x) x)$$

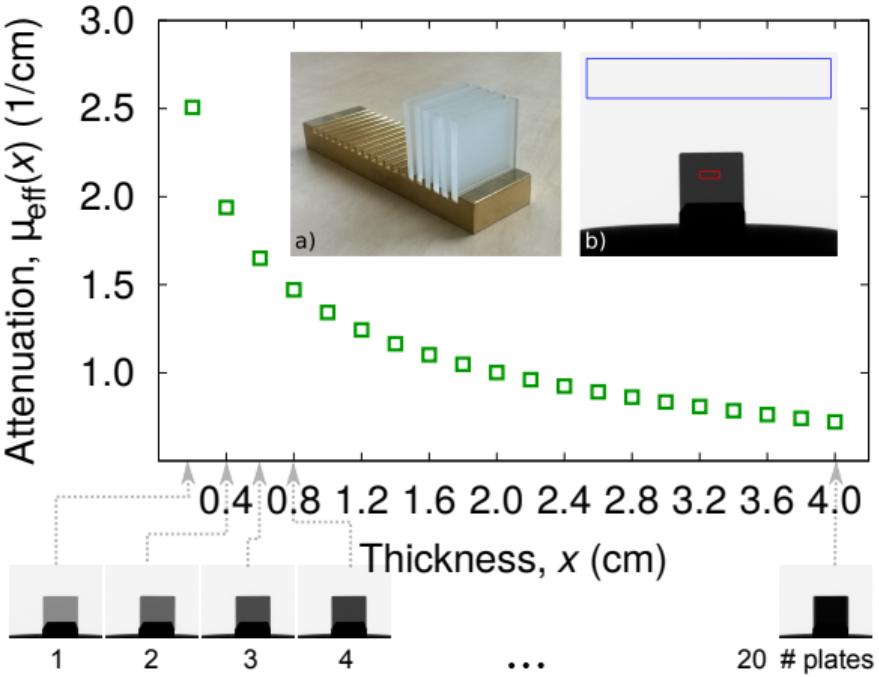
Attenuation of X-rays



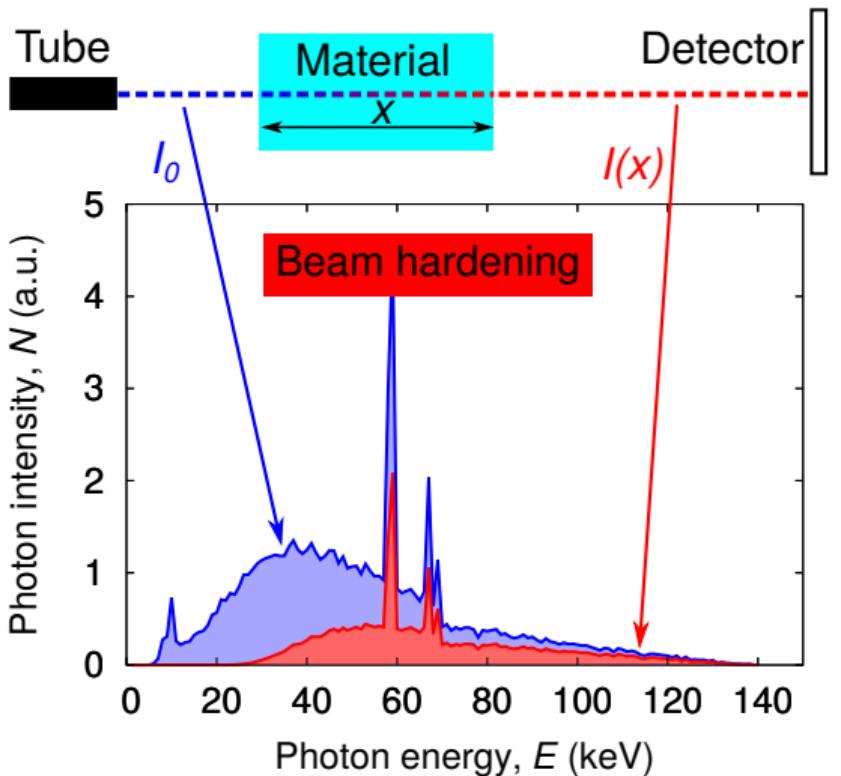
Beer-Lambert's law

$$I(x) \equiv I_0 \exp(-\mu x)$$

$$I(x) = I_0 \exp(-\mu_{\text{eff}}(x) x)$$

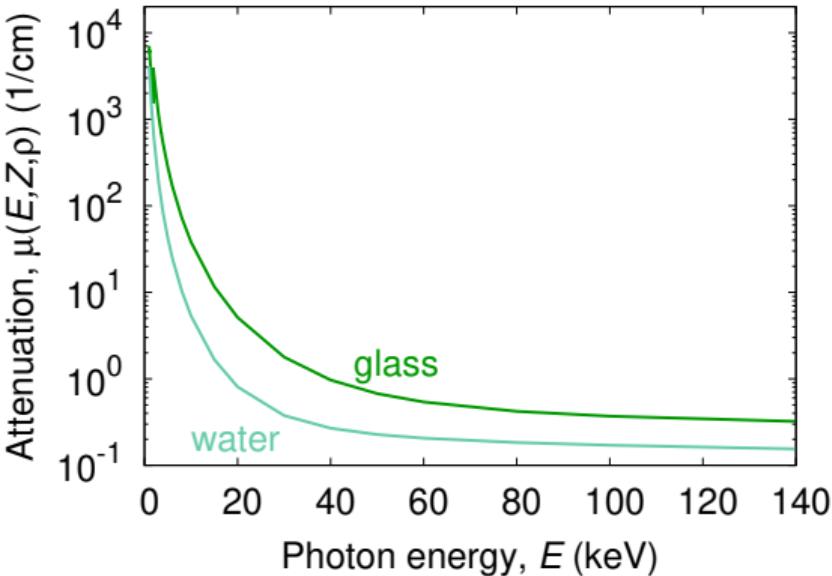


Attenuation of X-rays



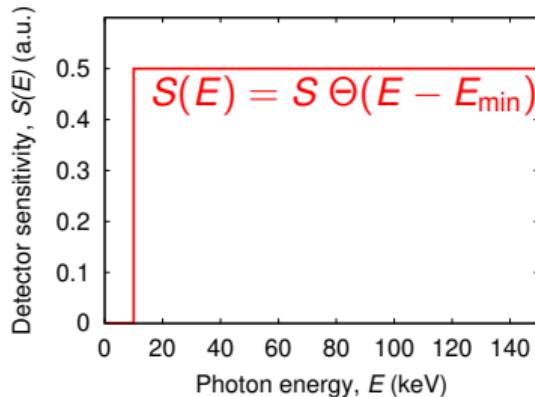
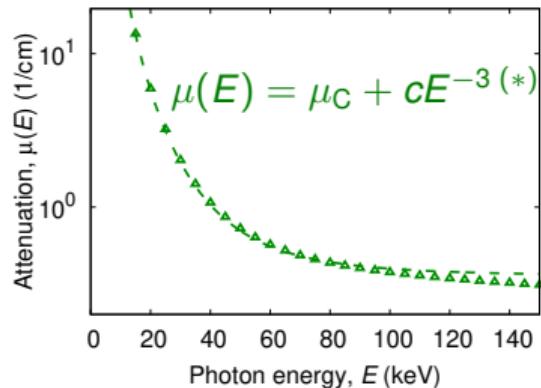
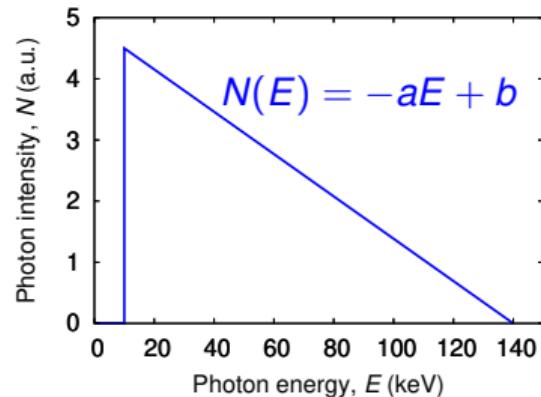
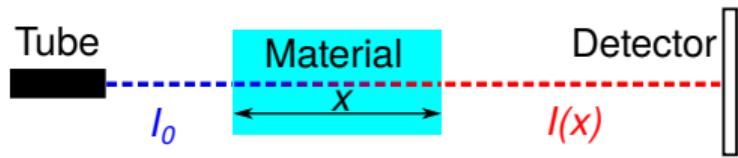
Beer-Lambert's law

$$I(x) = I_0 \exp(-\mu(E, Z, \rho)x)$$



Tabulated attenuation coefficients from NIST.gov

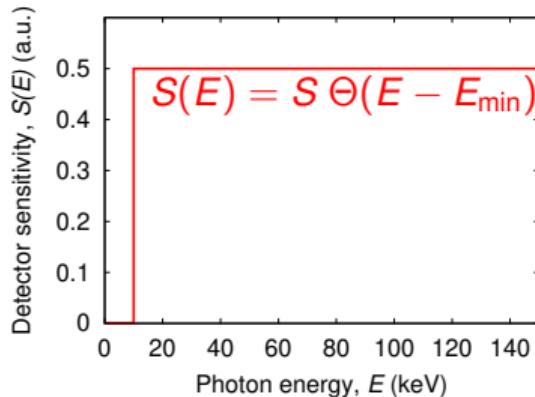
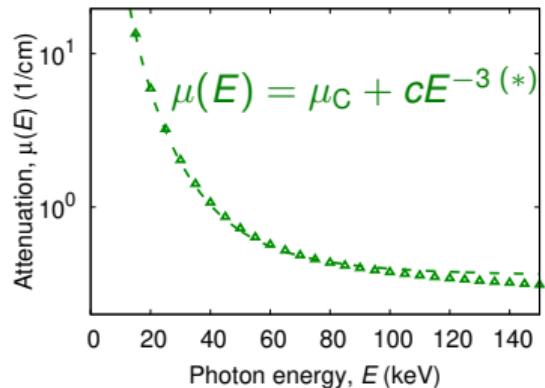
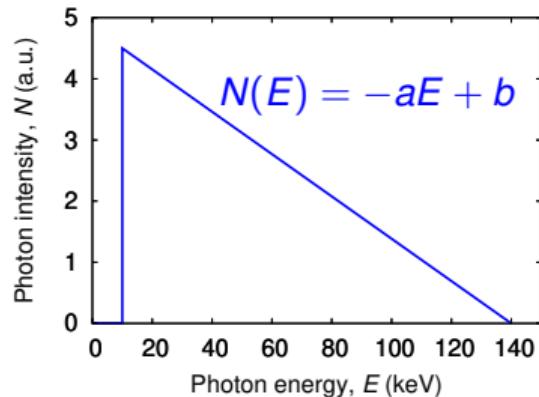
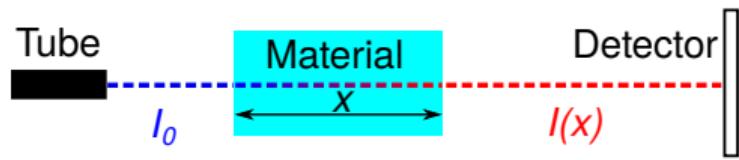
Modeling of $\mu_{\text{eff}}(x)$



$$I(x) \propto \int N(E) \exp\{-\mu(E)x\} S(E) dE$$

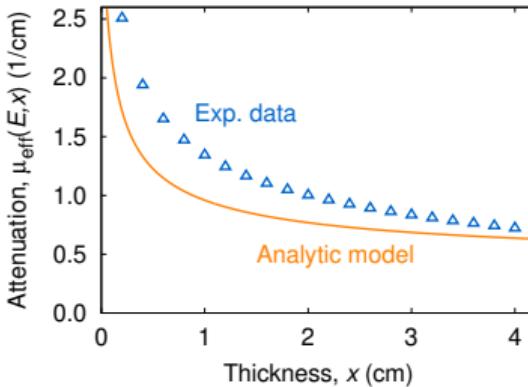
(*) XCOM supplied by NIST

Modeling of $\mu_{\text{eff}}(x)$



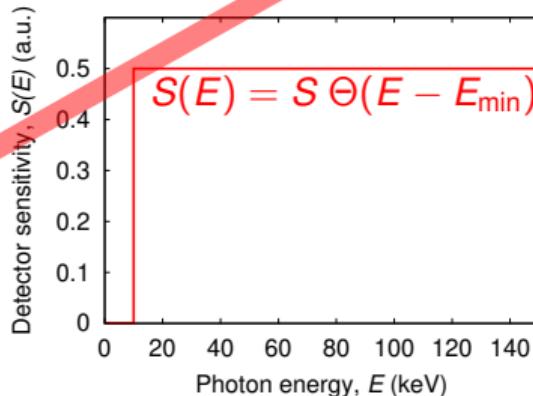
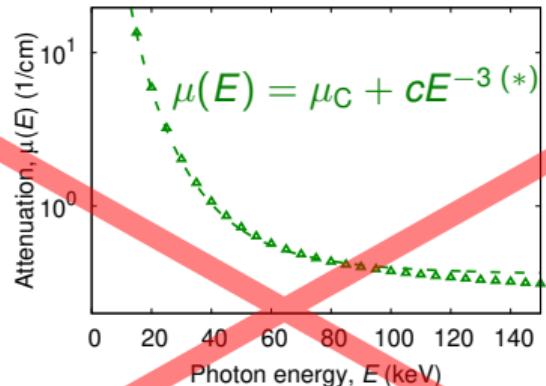
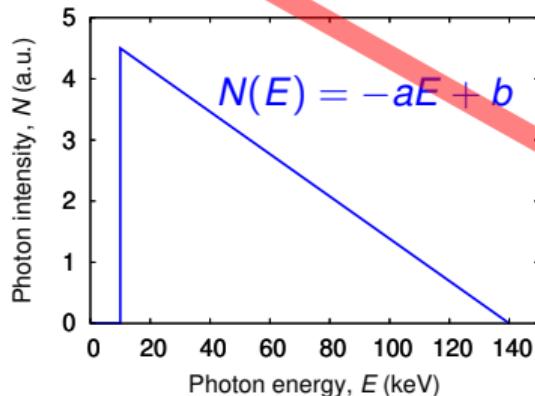
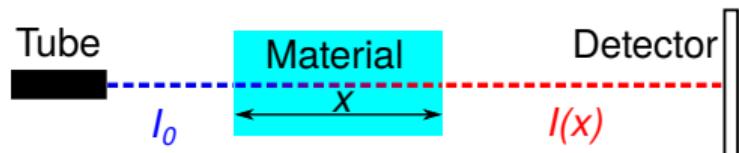
$$I(x) \propto \int N(E) \exp\{-\mu(E)x\} S(E) dE$$

$$\propto S \int_{E_{\min}}^{E_{\max}} (-aE + b) \exp\{-(\mu_C + cE^{-3})x\} dE$$



(*) XCOM supplied by NIST

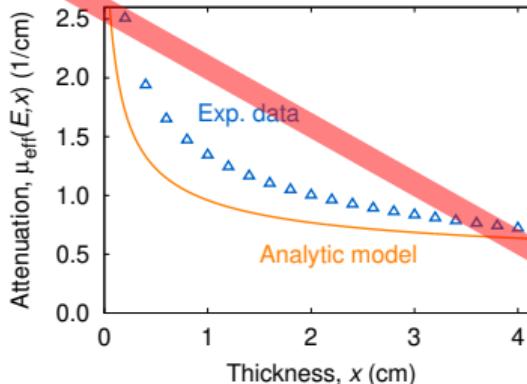
Modeling of $\mu_{\text{eff}}(x)$



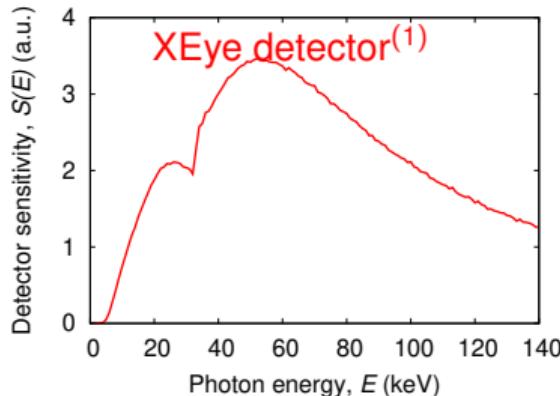
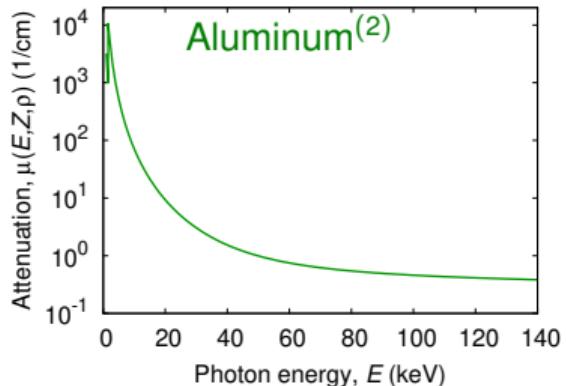
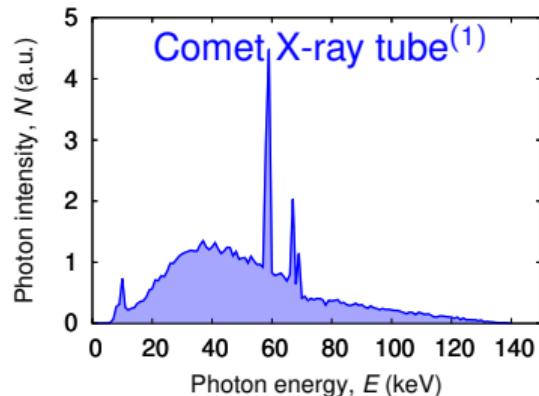
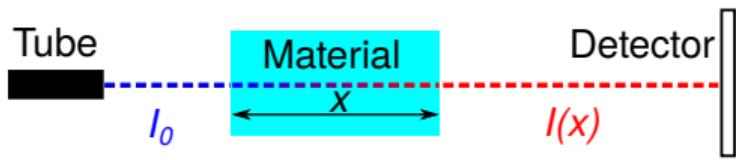
$$I(x) \propto \int N(E) \exp\{-\mu(E)x\} S(E) dE$$

$$\propto S \int_{E_{\min}}^{E_{\max}} (-aE + b) \exp\{-(\mu_C + cE^{-3})x\} dE$$

(*) XCOM supplied by NIST



Numerical approx. of $\mu_{\text{eff}}(x)$



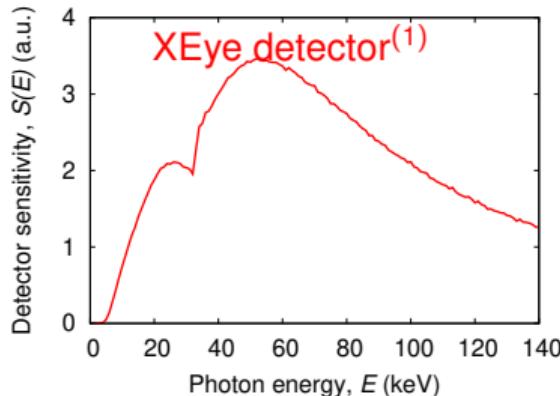
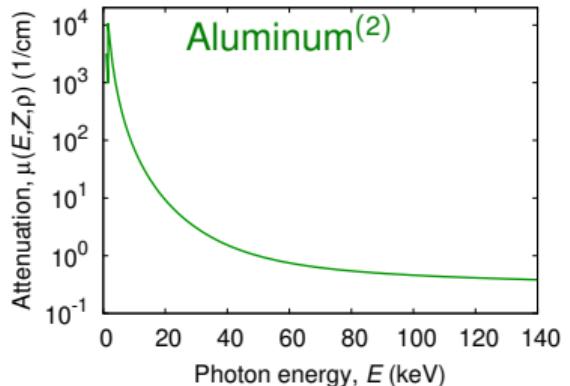
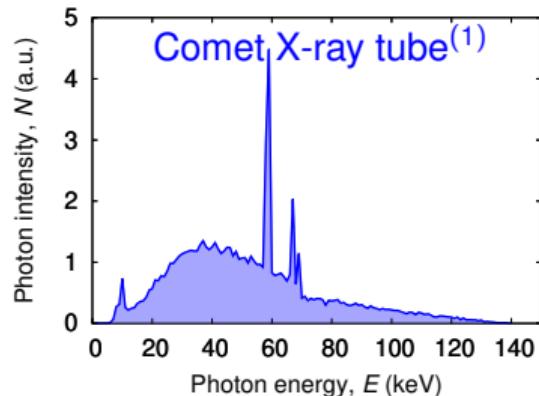
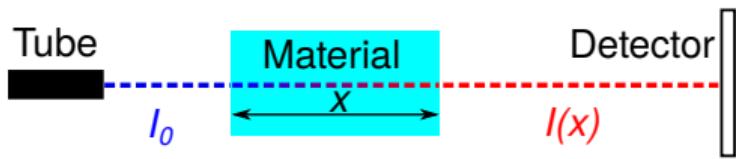
$$I(x) \propto \int \mathbf{N}(E) \exp\{-\mu(E)x\} \mathbf{S}(E) dE$$

$$\int \rightarrow \sum$$

(1) Supplied by Norman Uhlmann, Fraunhofer EZRT

(2) XCOM supplied by NIST

Numerical approx. of $\mu_{\text{eff}}(x)$

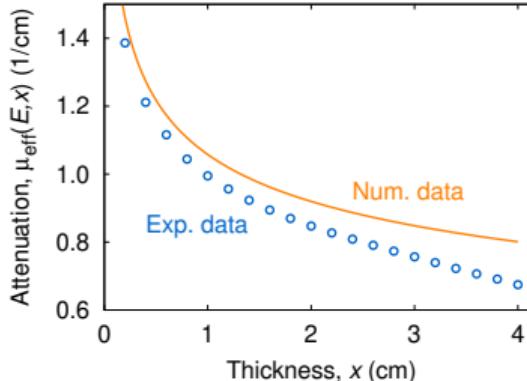


$$I(x) \propto \int \mathbf{N}(E) \exp\{-\mu(E)x\} \mathbf{S}(E) dE$$

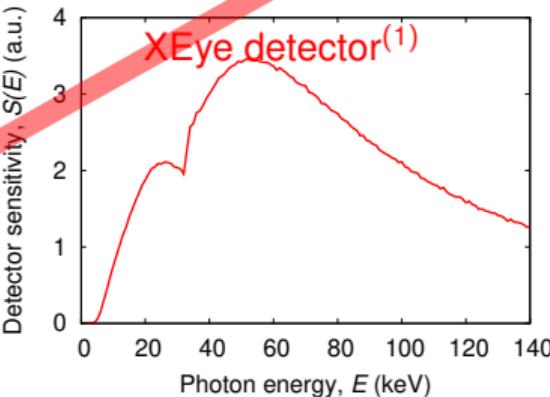
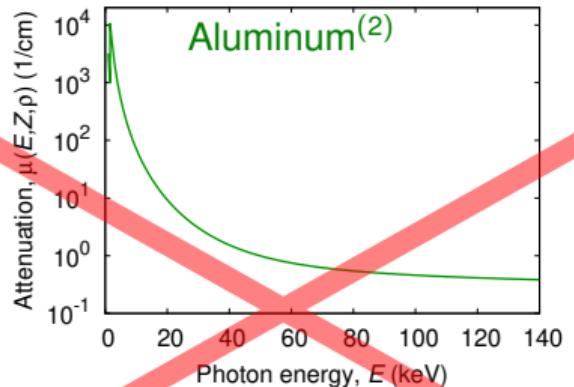
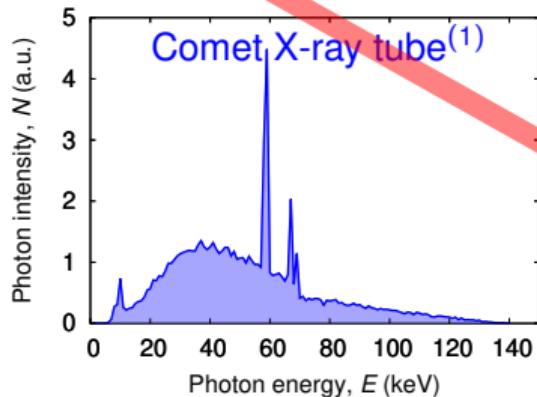
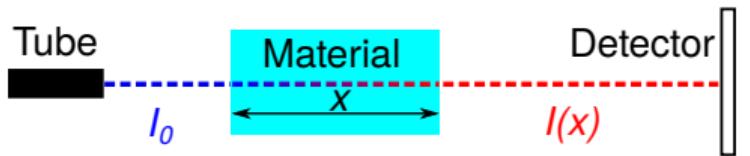
$\int \rightarrow \sum$

(1) Supplied by Norman Uhlmann, Fraunhofer EZRT

(2) XCOM supplied by NIST



Numerical approx. of $\mu_{\text{eff}}(x)$

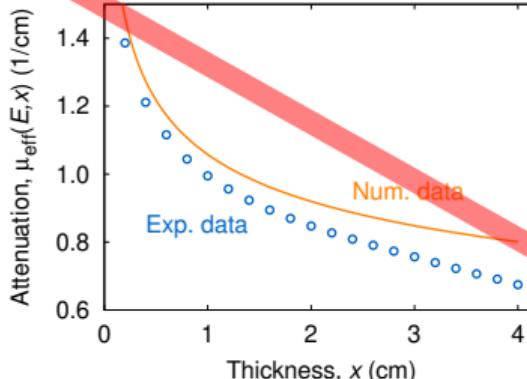


$$I(x) \propto \int N(E) \exp\{-\mu(E)x\} S(E) dE$$

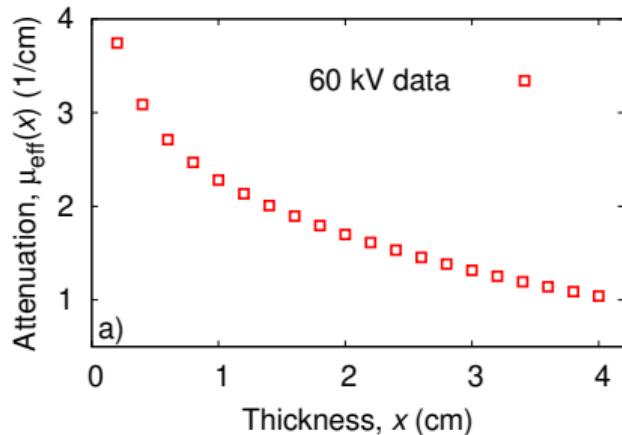
$\int \rightarrow \sum$

(1) Supplied by Norman Uhlmann, Fraunhofer EZRT

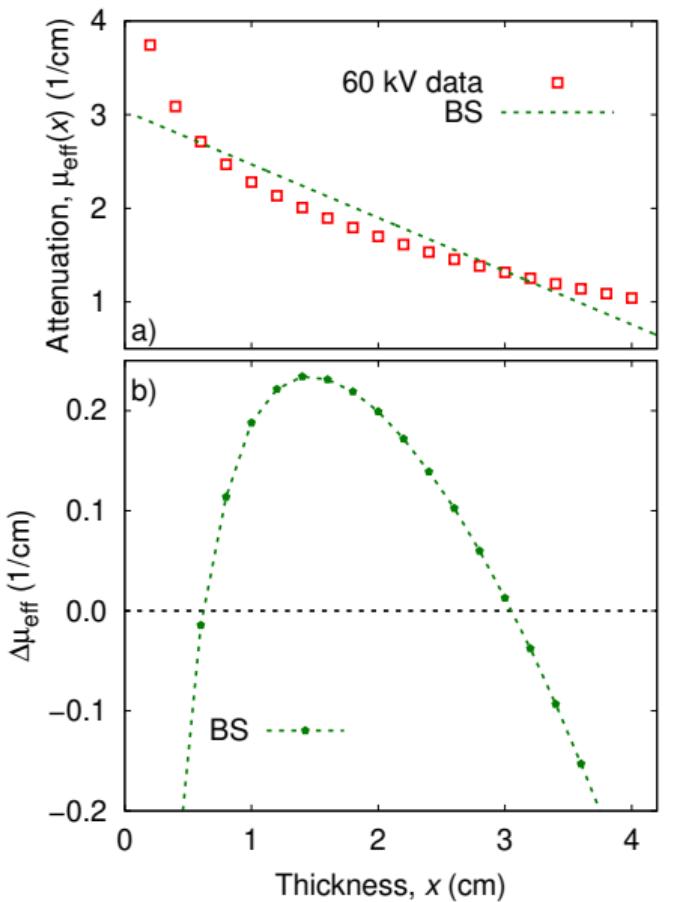
(2) XCOM supplied by NIST



Heuristic model functions for $\mu_{\text{eff}}(x)$



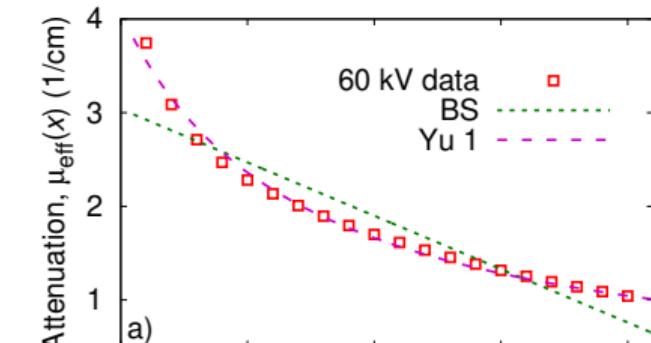
Heuristic model functions for $\mu_{\text{eff}}(x)$



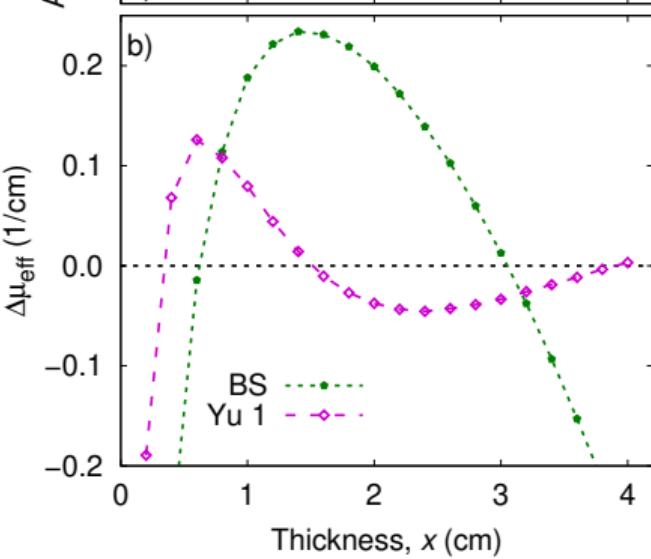
$$\mu_{\text{eff}}(x) = \mu_0 - \lambda x$$

Bjärngard & Shackford
(1994)

Heuristic model functions for $\mu_{\text{eff}}(x)$



$$\mu_{\text{eff}}(x) = \mu_0 - \lambda x$$

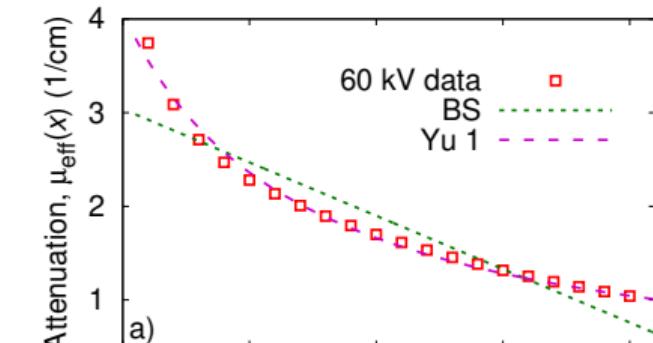


$$\mu_{\text{eff}}(x) = \frac{\mu_0}{1 + \lambda x}$$

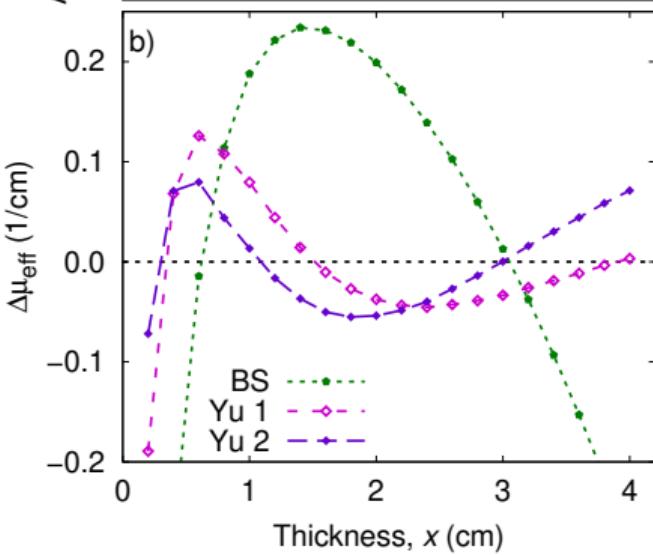
Bjärngard & Shackford
(1994)

Yu *et al.* (1997)

Heuristic model functions for $\mu_{\text{eff}}(x)$



$$\mu_{\text{eff}}(x) = \mu_0 - \lambda x$$



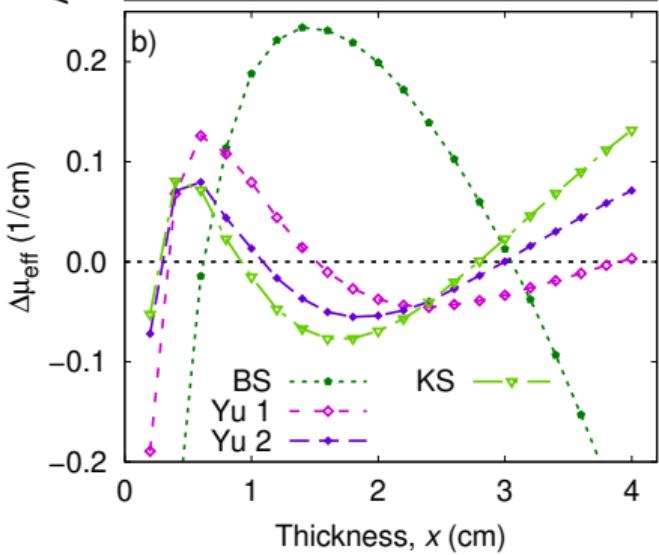
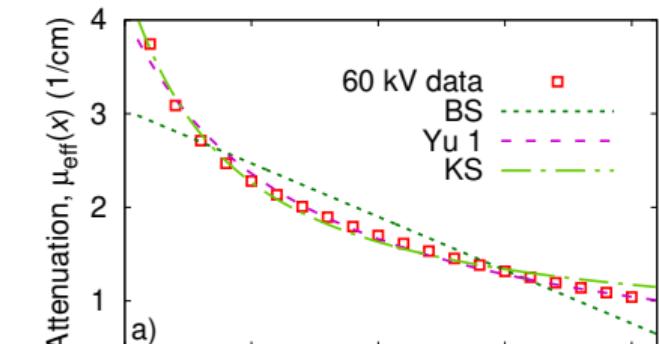
$$\mu_{\text{eff}}(x) = \frac{\mu_0}{1+\lambda x}$$

$$\mu_{\text{eff}}(x) = \frac{\mu_0}{(1+\lambda x)^\beta}$$

Bjärngard & Shackford
(1994)

Yu *et al.* (1997)

Heuristic model functions for $\mu_{\text{eff}}(x)$



$$\mu_{\text{eff}}(x) = \mu_0 - \lambda x$$

Bjärngard & Shackford
(1994)

$$\mu_{\text{eff}}(x) = \frac{\mu_0}{1 + \lambda x}$$

$$\mu_{\text{eff}}(x) = \frac{\mu_0}{(1 + \lambda x)^\beta}$$

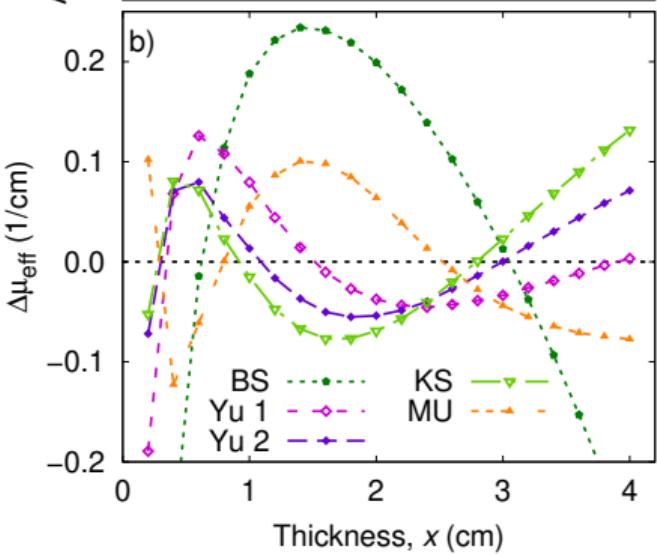
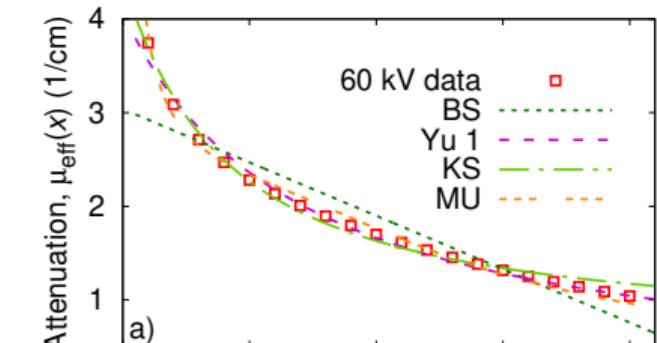
Yu *et al.* (1997)

$$\mu_{\text{eff}}(x) = \mu(E_{\max}) + \frac{2\mu_1}{x\sqrt{-\lambda_1^2 + 4\lambda_2}} \times$$

$$\left[\arctan\left(\frac{\lambda_1 + 2\lambda_2 x}{\sqrt{-\lambda_1^2 + 4\lambda_2}}\right) - \arctan\left(\frac{\lambda_1}{\sqrt{-\lambda_1^2 + 4\lambda_2}}\right) \right]$$

Kleinschmidt (1999)

Heuristic model functions for $\mu_{\text{eff}}(x)$



$$\mu_{\text{eff}}(x) = \mu_0 - \lambda x$$

Bjärngard & Shackford
(1994)

$$\mu_{\text{eff}}(x) = \frac{\mu_0}{1 + \lambda x}$$

$$\mu_{\text{eff}}(x) = \frac{\mu_0}{(1 + \lambda x)^\beta}$$

Yu *et al.* (1997)

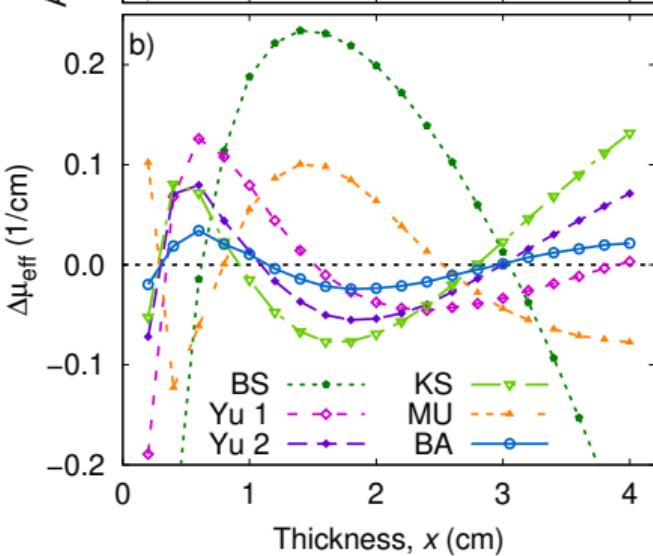
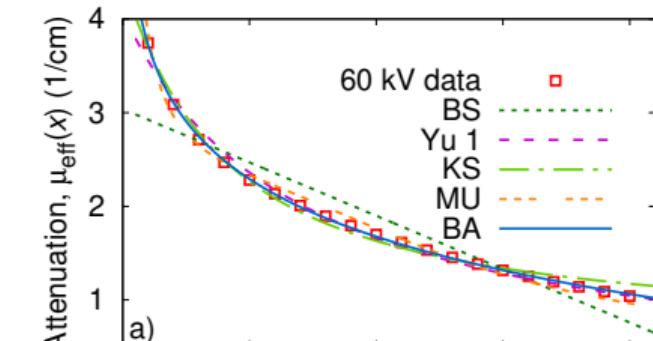
$$\mu_{\text{eff}}(x) = \mu(E_{\text{max}}) + \frac{2\mu_1}{x\sqrt{-\lambda_1^2 + 4\lambda_2}} \times \left[\arctan\left(\frac{\lambda_1 + 2\lambda_2 x}{\sqrt{-\lambda_1^2 + 4\lambda_2}}\right) - \arctan\left(\frac{\lambda_1}{\sqrt{-\lambda_1^2 + 4\lambda_2}}\right) \right]$$

Kleinschmidt (1999)

$$\mu_{\text{eff}}(x) = -\frac{1}{x} \ln [A + B \exp(-x/C)]$$

Mudde *et al.* (2008)

Heuristic model functions for $\mu_{\text{eff}}(x)$



$$\mu_{\text{eff}}(x) = \mu_0 - \lambda x$$

Bjärngard & Shackford
(1994)

$$\mu_{\text{eff}}(x) = \frac{\mu_0}{1 + \lambda x}$$

$$\mu_{\text{eff}}(x) = \frac{\mu_0}{(1 + \lambda x)^\beta}$$

Yu *et al.* (1997)

$$\mu_{\text{eff}}(x) = \mu(E_{\text{max}}) + \frac{2\mu_1}{x\sqrt{-\lambda_1^2 + 4\lambda_2}} \times \left[\arctan\left(\frac{\lambda_1 + 2\lambda_2 x}{\sqrt{-\lambda_1^2 + 4\lambda_2}}\right) - \arctan\left(\frac{\lambda_1}{\sqrt{-\lambda_1^2 + 4\lambda_2}}\right) \right]$$

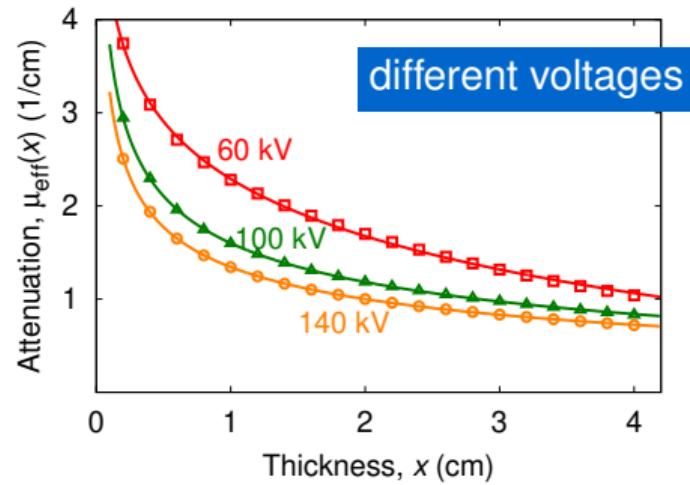
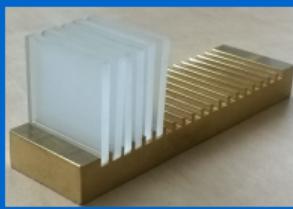
$$\mu_{\text{eff}}(x) = -\frac{1}{x} \ln [A + B \exp(-x/C)]$$

Kleinschmidt (1999)
Mudde *et al.* (2008)

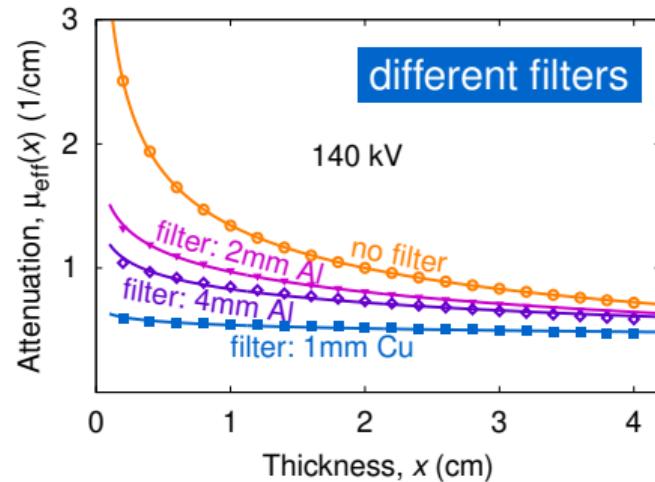
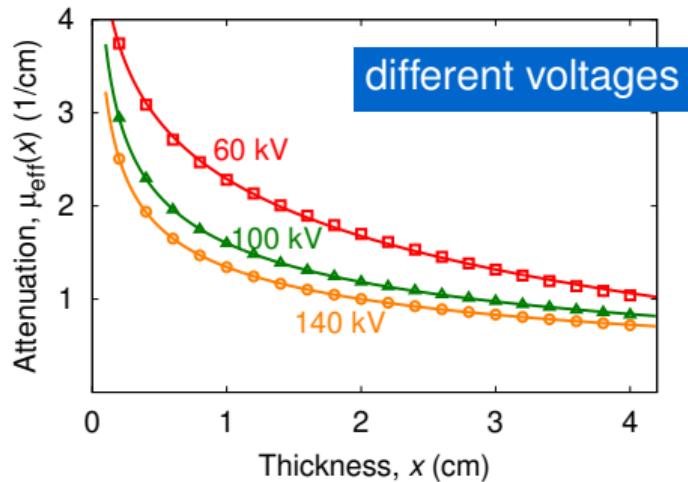
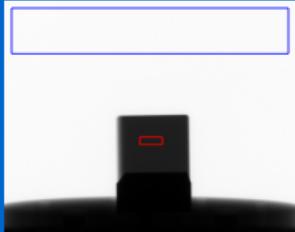
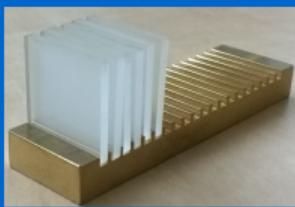
$$\mu_{\text{eff}}(x) = a + \frac{b}{x^\alpha}$$

Baur *et al.* (2019)
(this work)

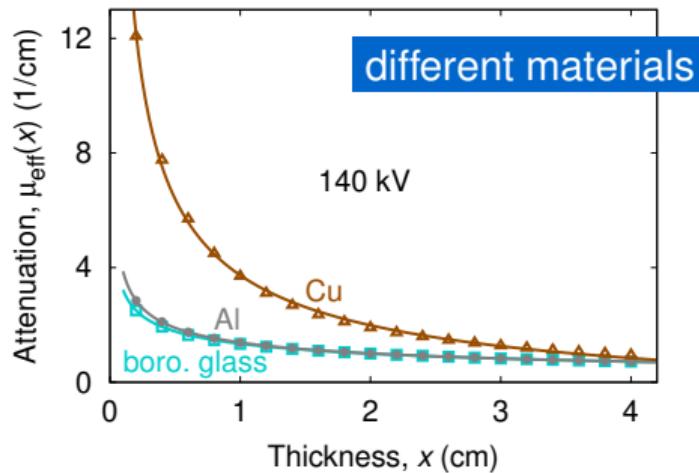
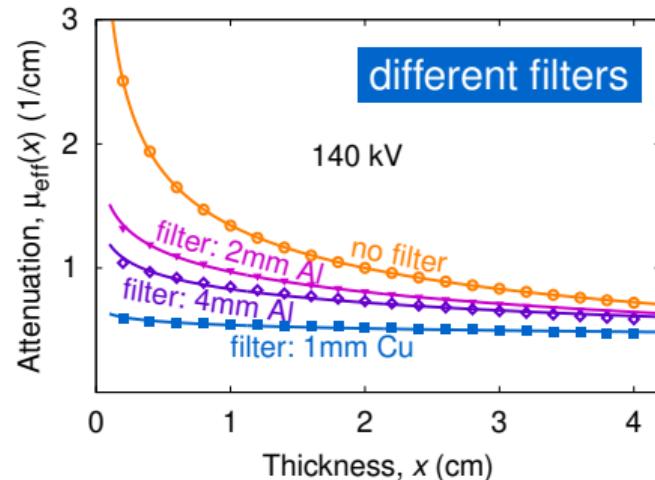
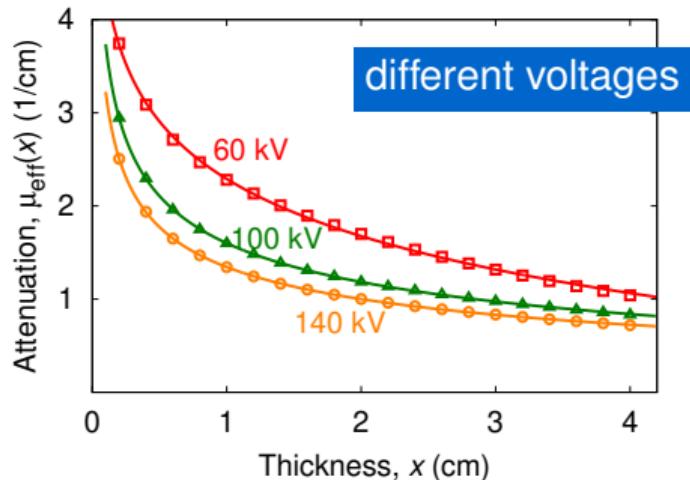
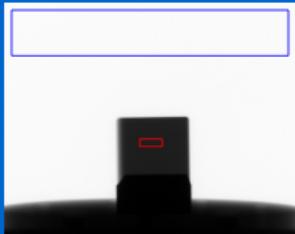
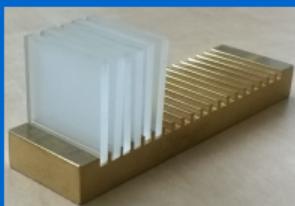
Applicability of
 $\mu_{\text{eff}}(x) = a + \frac{b}{x^\alpha}$



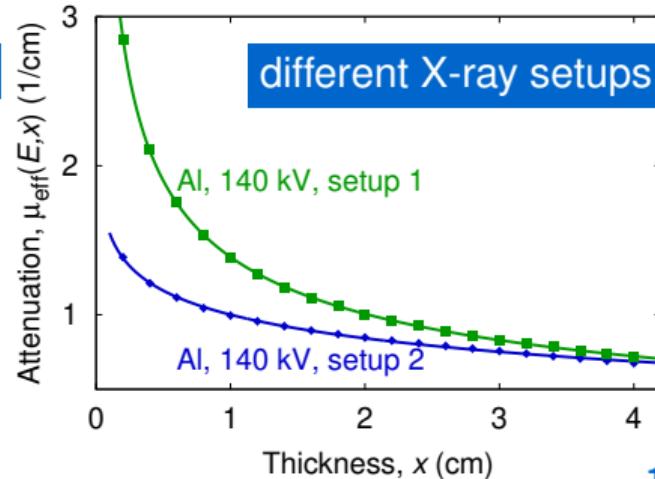
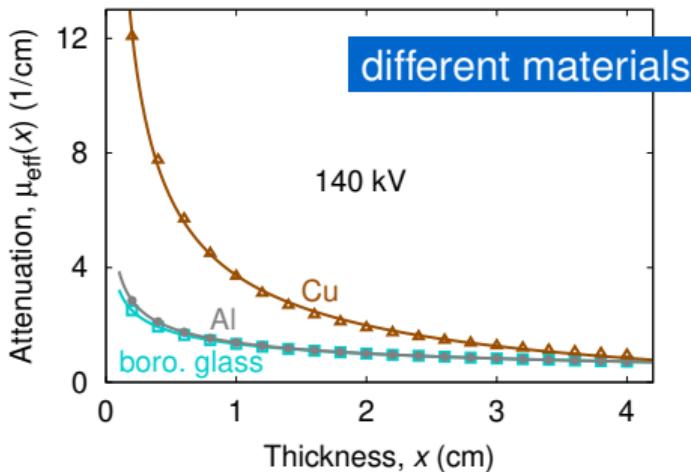
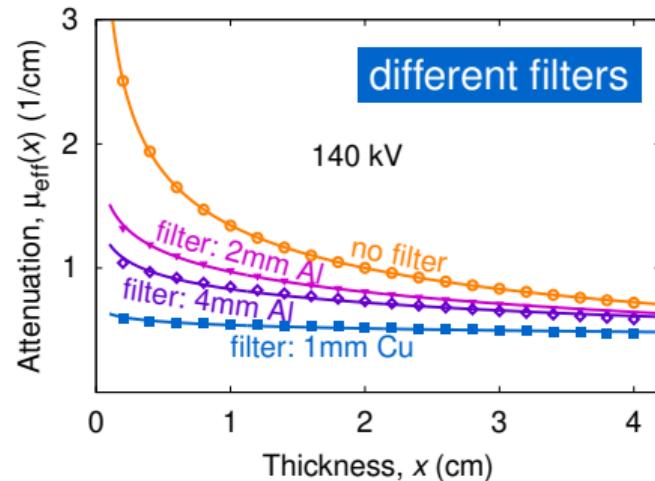
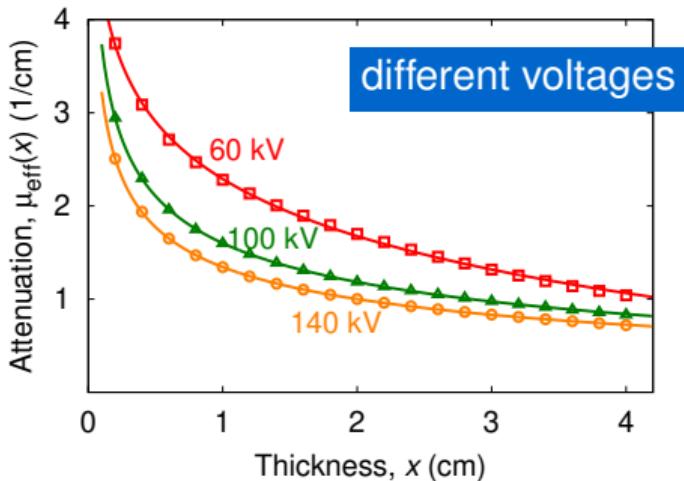
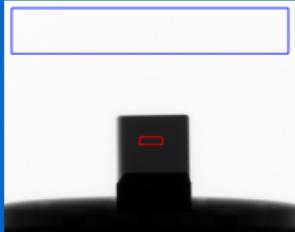
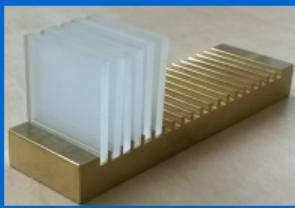
Applicability of
 $\mu_{\text{eff}}(x) = a + \frac{b}{x^\alpha}$



Applicability of
 $\mu_{\text{eff}}(x) = a + \frac{b}{x^\alpha}$



Applicability of
 $\mu_{\text{eff}}(x) = a + \frac{b}{x^\alpha}$



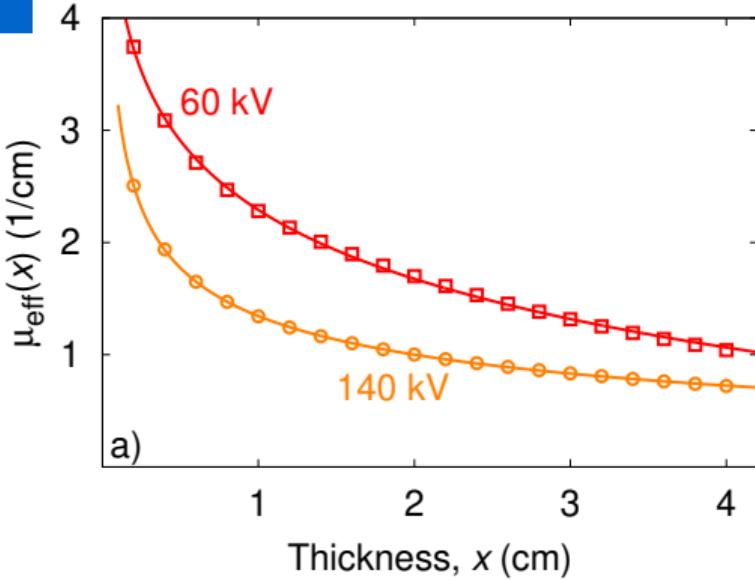
Determining the material thickness x

Generalized Beer-Lambert

$$I(x) = I_0 \exp(-\mu_{\text{eff}}(x) x)$$

Model function

$$\mu_{\text{eff}}(x) = a + \frac{b}{x^\alpha}$$



Determining the material thickness x

Generalized Beer-Lambert

$$I(x) = I_0 \exp(-\mu_{\text{eff}}(x)x)$$

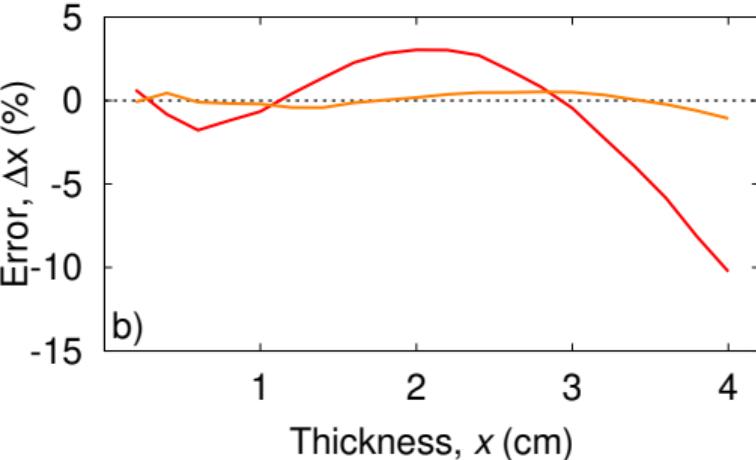
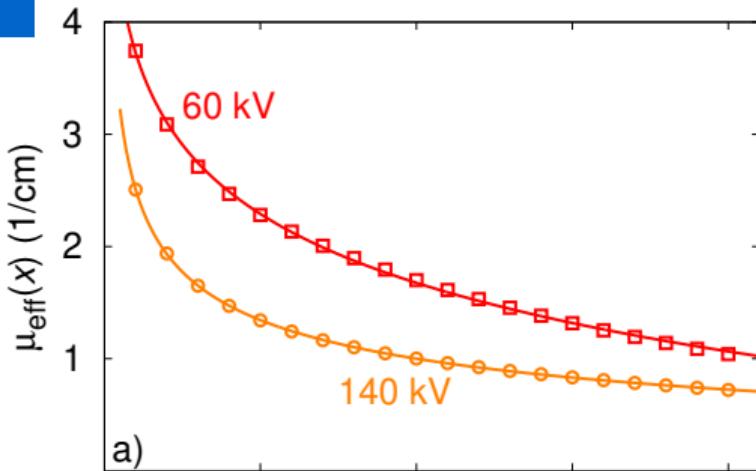
Model function

$$\mu_{\text{eff}}(x) = a + \frac{b}{x^\alpha}$$

Solve

$$ax + bx^{1-\alpha} + \ln\left(\frac{I(x)}{I_0}\right) = 0$$

e.g. Newton's method or look-up table



Determining the material thickness x

Generalized Beer-Lambert

$$I(x) = I_0 \exp(-\mu_{\text{eff}}(x)x)$$

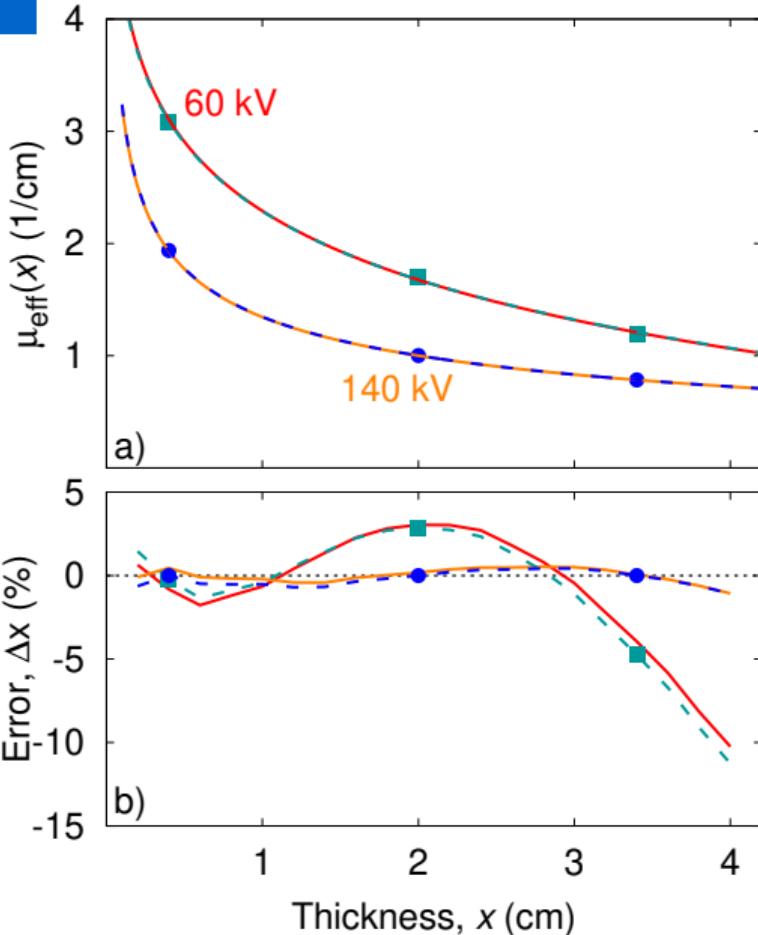
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Migrating shear bands in shaken granular matter, Kollmer *et al* (2020)

