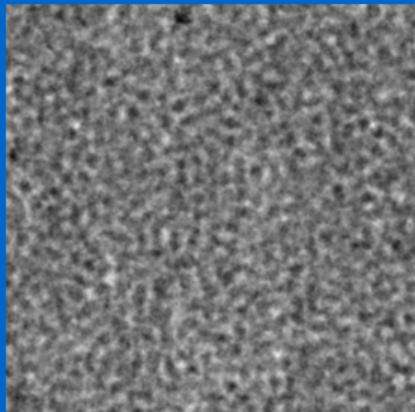
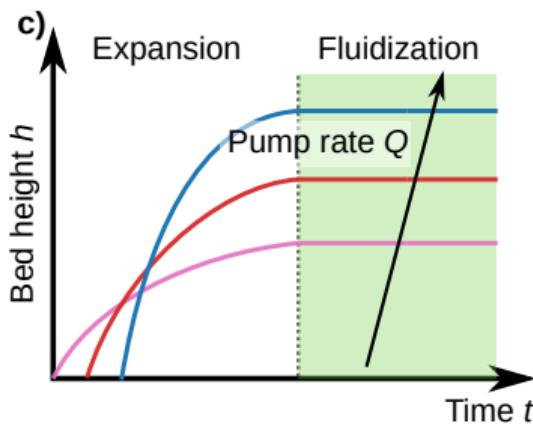
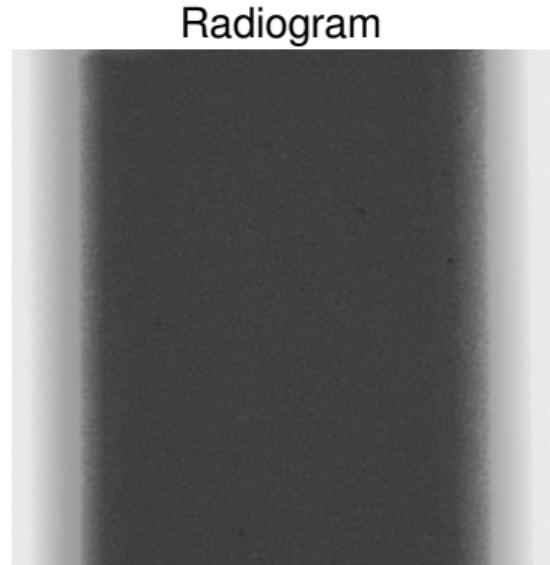
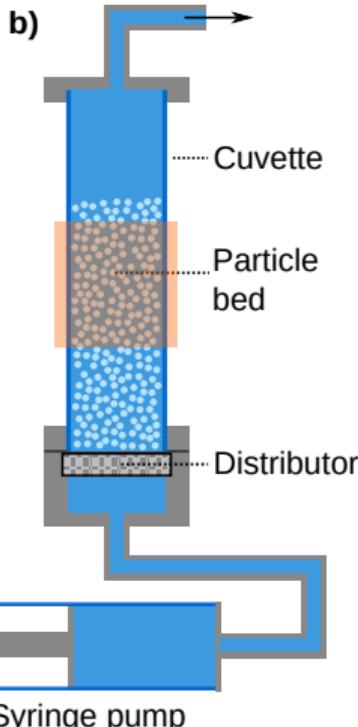
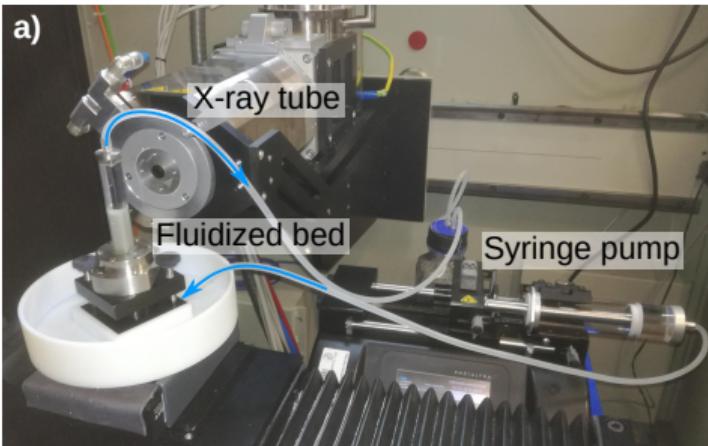


Measuring granular dynamics with X-ray Digital Fourier Analysis (X-DFA)



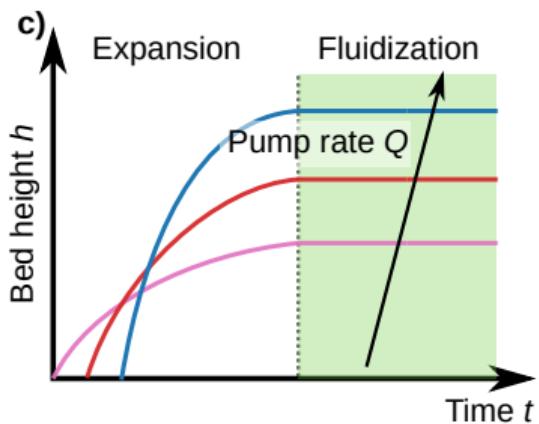
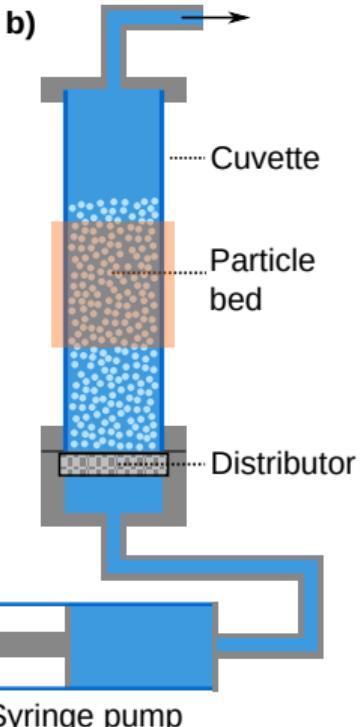
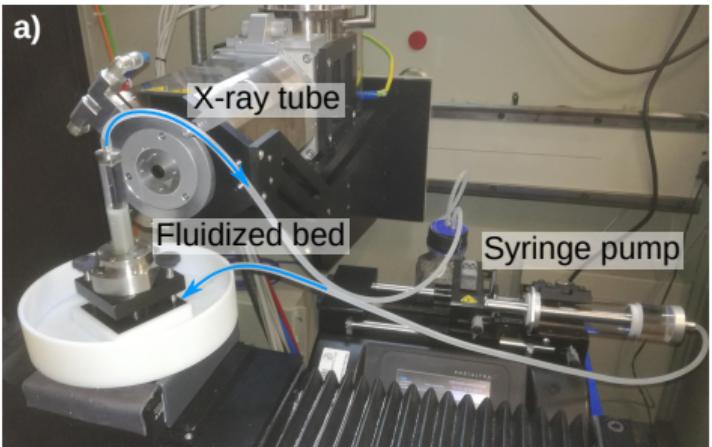
In collaboration with M. Escobedo & S. Egelhaaf, University of Düsseldorf

The system: A liquid fluidized bed



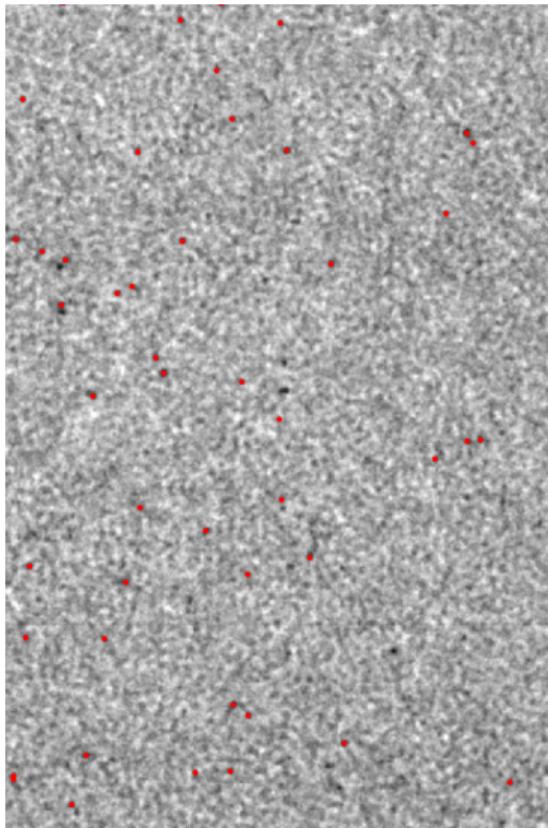
$$0.45 < \phi < 0.56$$

The system: A liquid fluidized bed

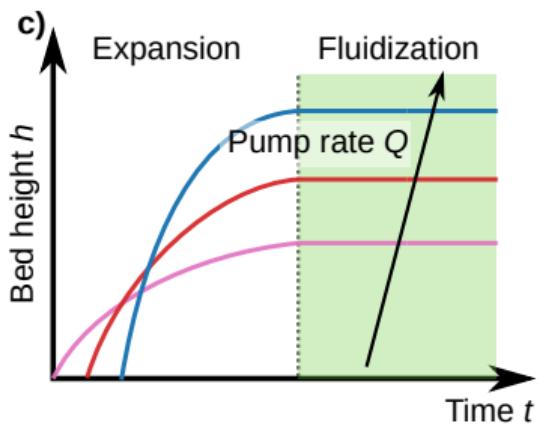
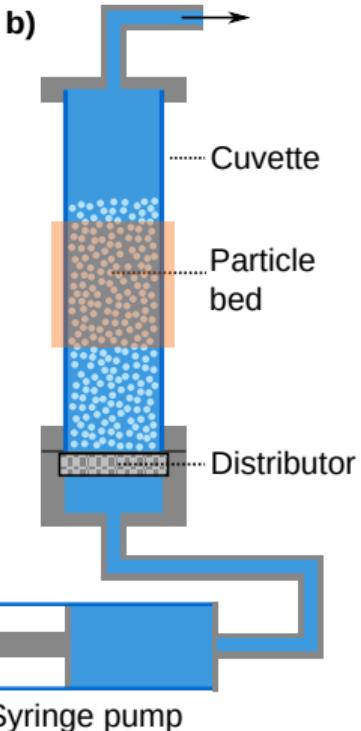
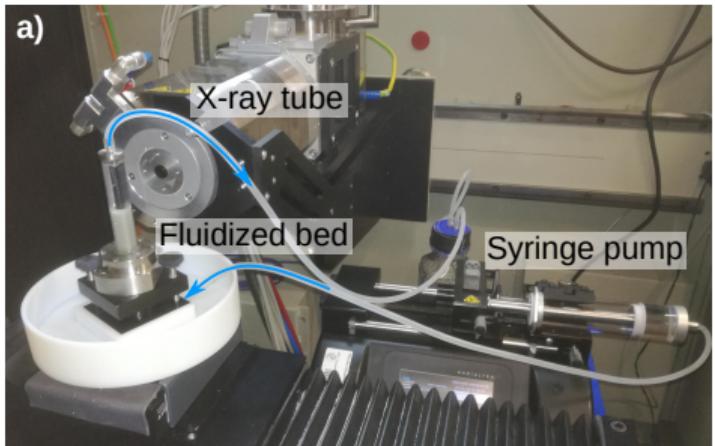


$$0.45 < \phi < 0.56$$

Particle tracking

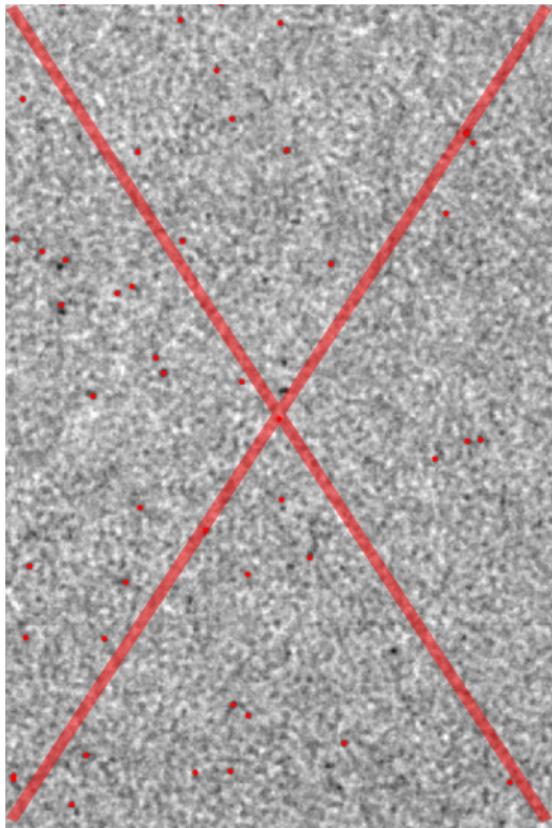


The system: A liquid fluidized bed

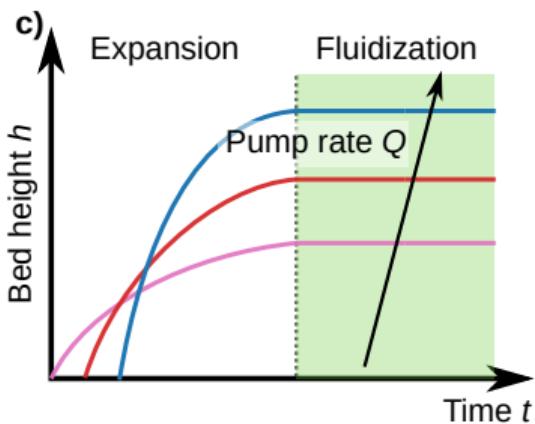
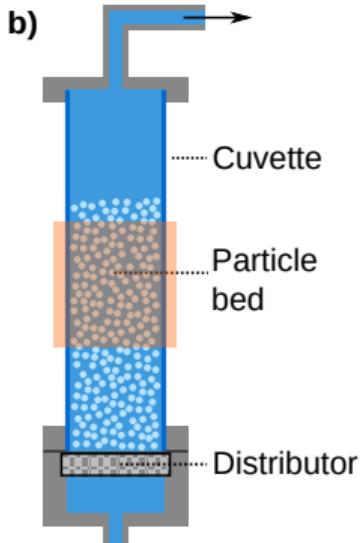
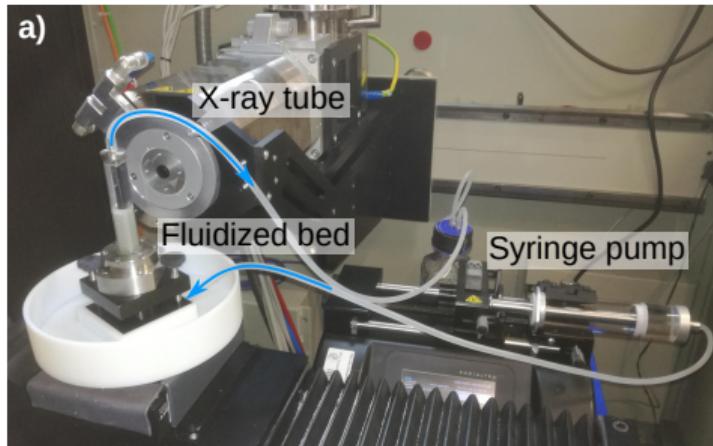


$$0.45 < \phi < 0.56$$

Particle tracking

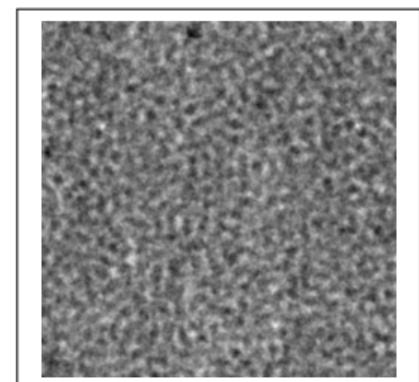
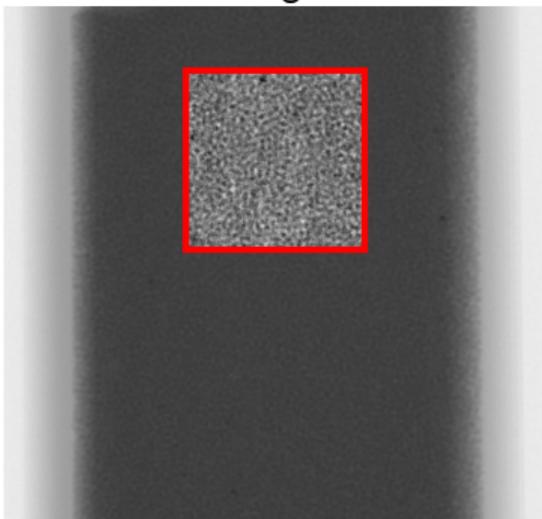


Experiments: A liquid fluidized bed



$$0.45 < \Phi < 0.56$$

Radiogram



Differential Dynamic Microscopy (DDM)

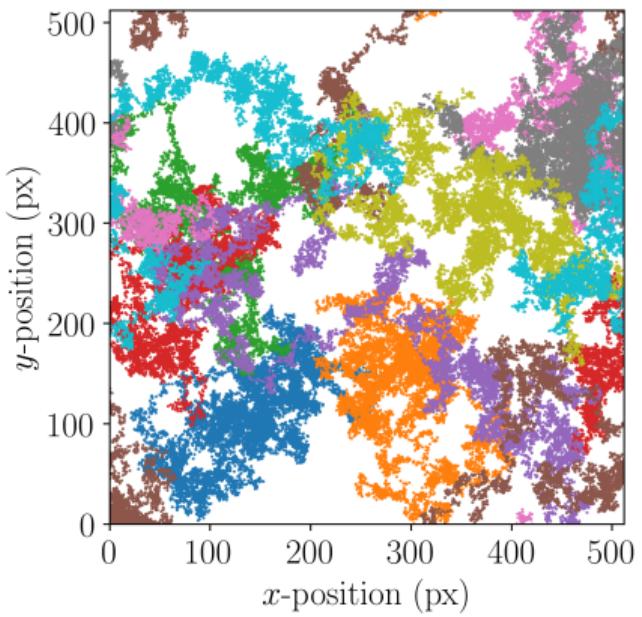
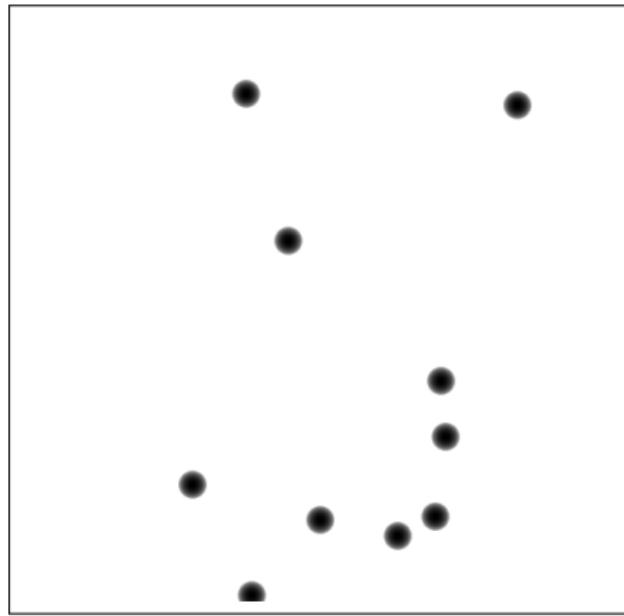
	up to now	this work
system	dispersion, gels	fluidized bed
particles	colloids	granulate
part. diameter	$< 1 \mu\text{m}$	$\approx 200 \mu\text{m}$
volume fraction	$\Phi \leq 0.33$	$0.45 < \Phi < 0.56$
imaging	light microscope	x-ray radiography
dynamics	Brownian motion, caging, glassy, collective motion	

Extending Differential Dynamic Microscopy (DDM) to X-ray imaging

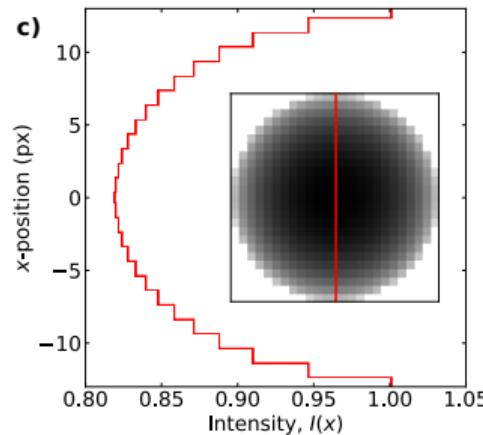
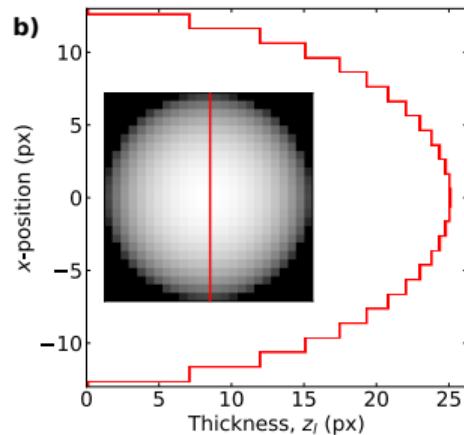
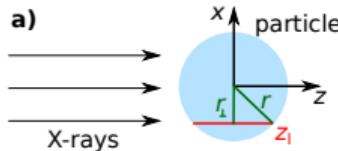
	up to now	this work
system	dispersion, gels	fluidized bed
particles	colloids	granulate
part. diameter	< 1 μm	$\approx 200 \mu\text{m}$
volume fraction	$\Phi \leq 0.33$	$0.45 < \Phi < 0.56$
imaging	light microscope	x-ray radiography
dynamics	Brownian motion, caging, glassy, collective motion	

Digital Fourier Analysis of X-Ray Radiograms (X-DFA)

Synthetic radiograms

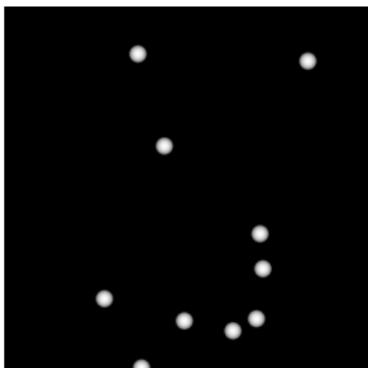
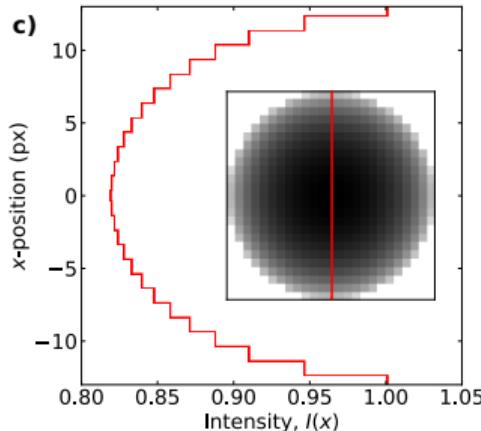
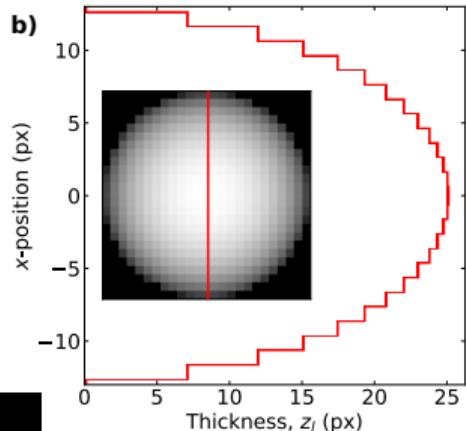
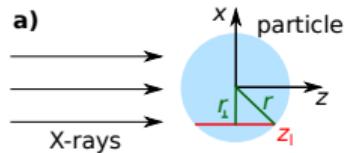


Synthetic radiograms

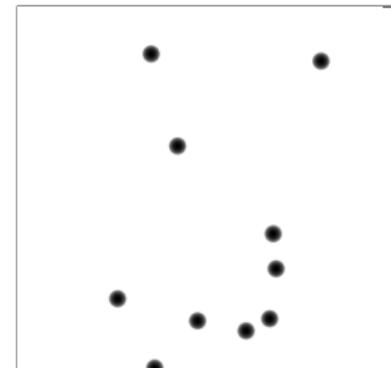


Beer-Lambert
 $I(z_I) = I_0 \exp(-\mu z)$

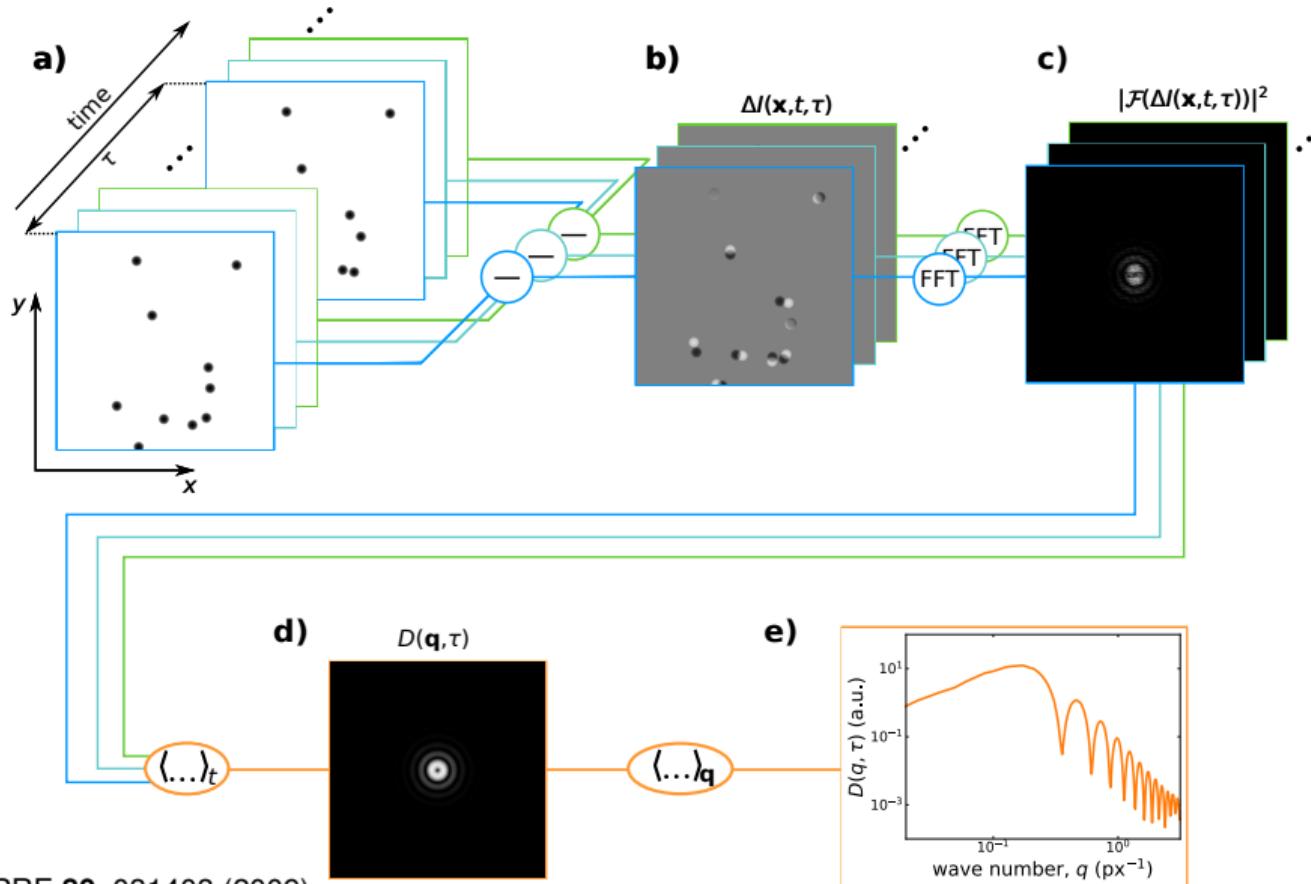
Synthetic radiograms



Beer-Lambert
 $I(z_l) = I_0 \exp(-\mu z)$

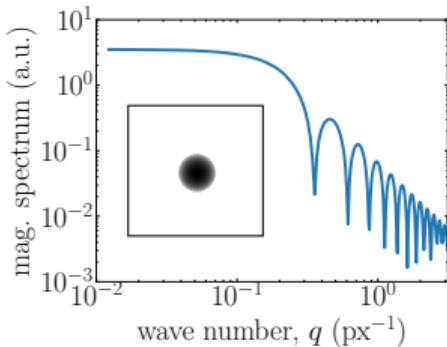
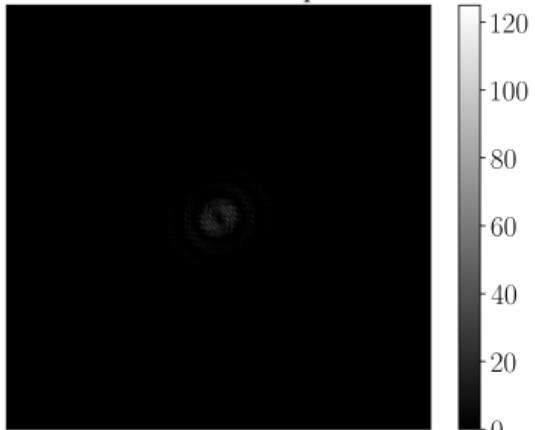


The image structure function $D(\mathbf{q}, \tau)$



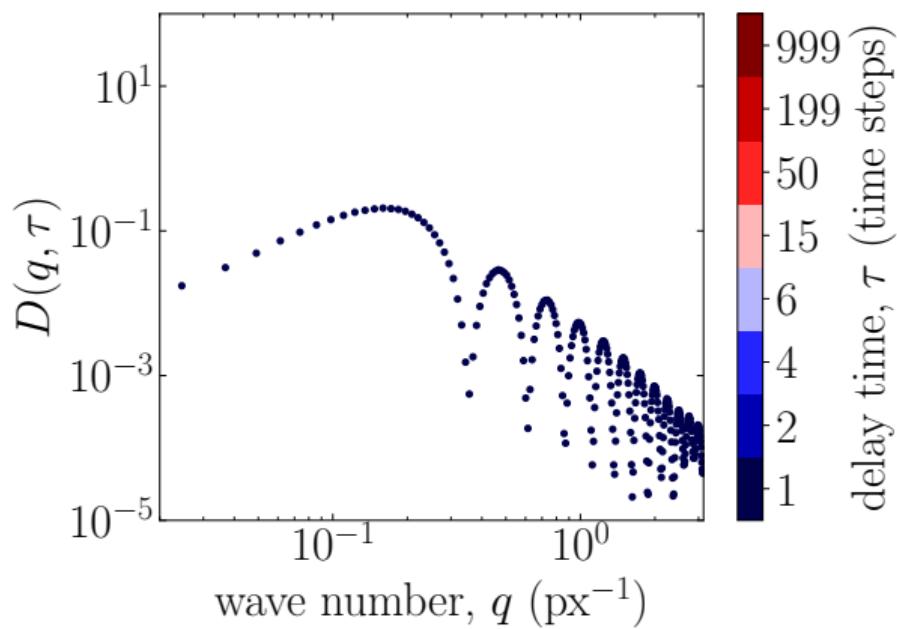
The image structure function $D(q, \tau)$

$\tau = 1$ time steps



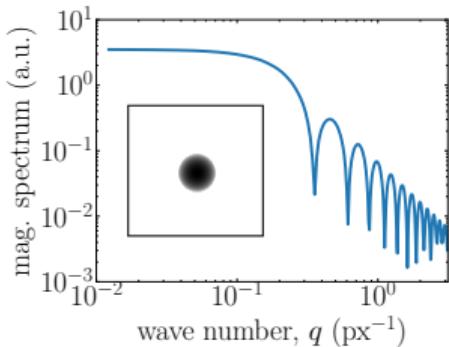
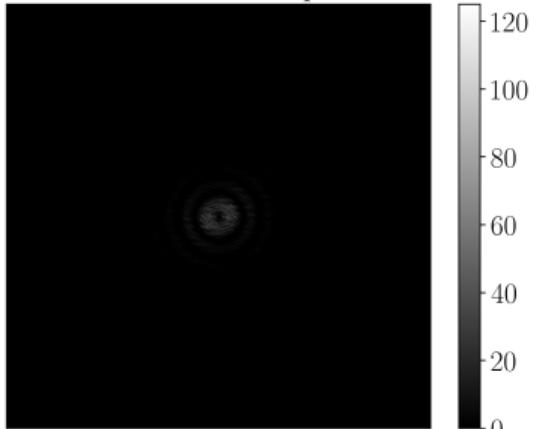
Time averaging: $D(\mathbf{q}, \tau) = \langle |\mathcal{F}(\Delta I)|^2 \rangle_t$

Azimuthal averaging: $D(\mathbf{q}, \tau) \rightarrow D(\mathbf{q}, \tau) \dots$



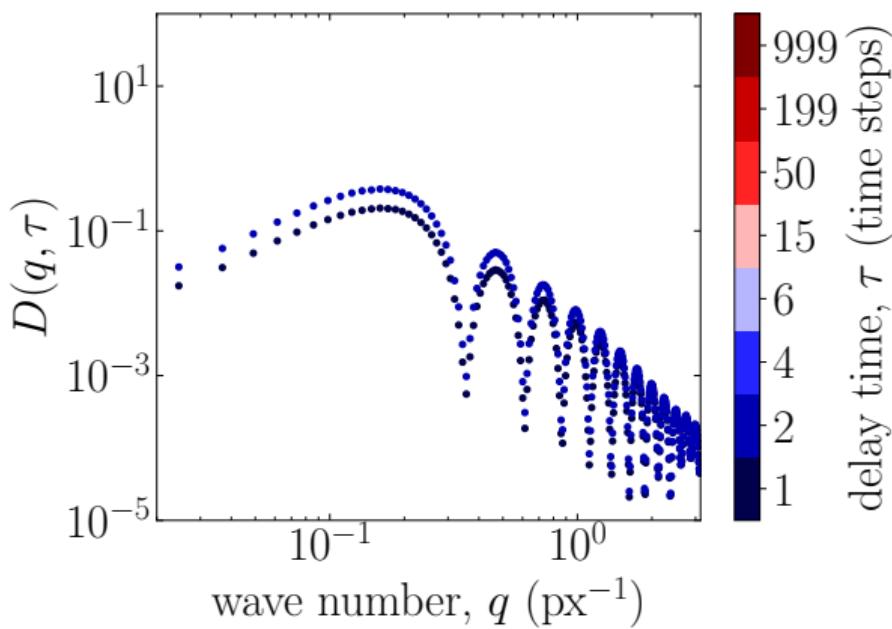
The image structure function $D(q, \tau)$

$\tau = 2$ time steps



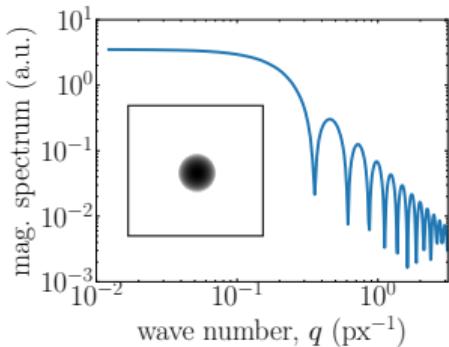
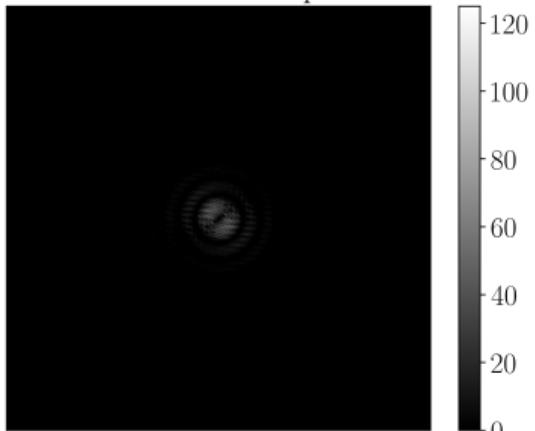
Time averaging: $D(\mathbf{q}, \tau) = \langle |\mathcal{F}(\Delta I)|^2 \rangle_t$

Azimuthal averaging: $D(\mathbf{q}, \tau) \rightarrow D(\mathbf{q}, \tau) \dots$



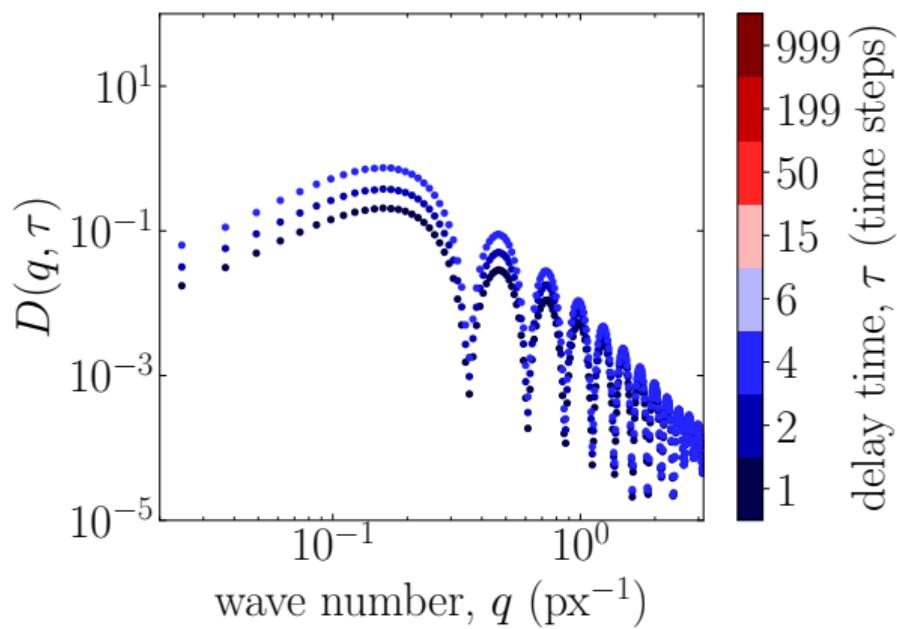
The image structure function $D(q, \tau)$

$\tau = 4$ time steps



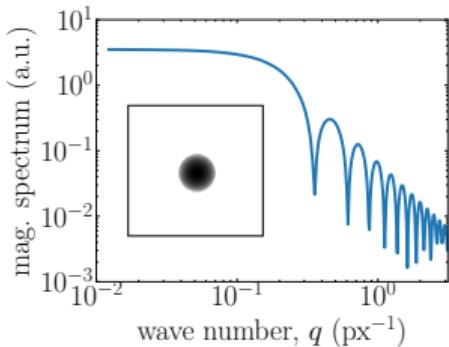
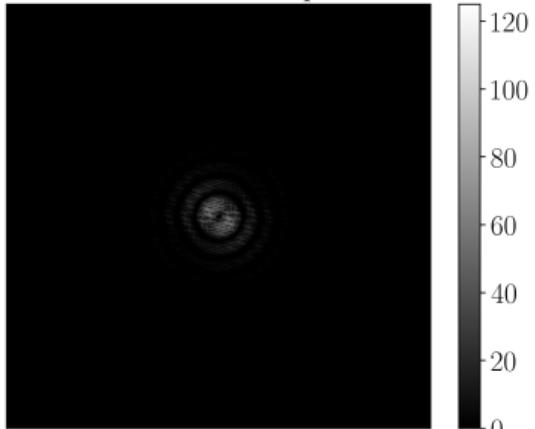
Time averaging: $D(\mathbf{q}, \tau) = \langle |\mathcal{F}(\Delta I)|^2 \rangle_t$

Azimuthal averaging: $D(\mathbf{q}, \tau) \rightarrow D(\mathbf{q}, \tau) \dots$



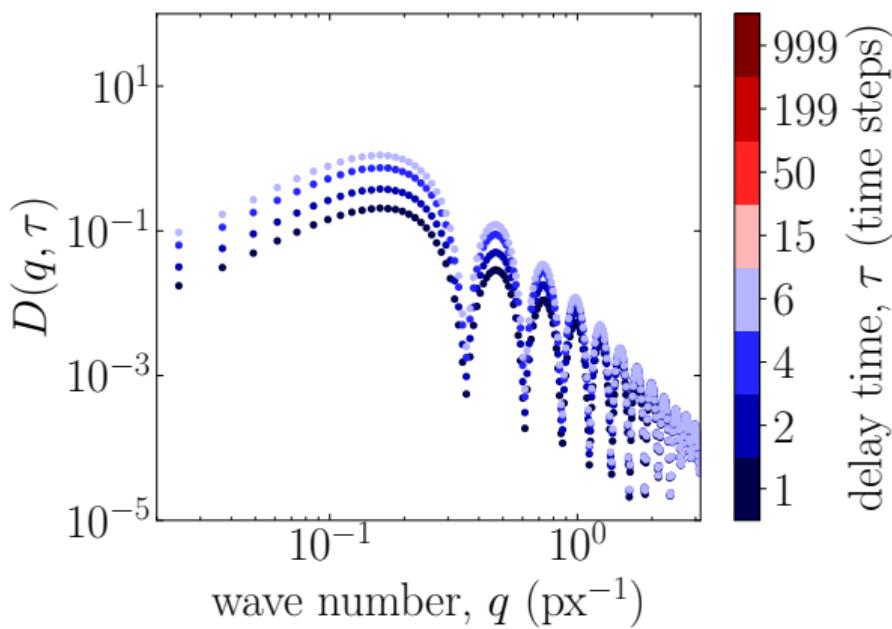
The image structure function $D(q, \tau)$

$\tau = 6$ time steps

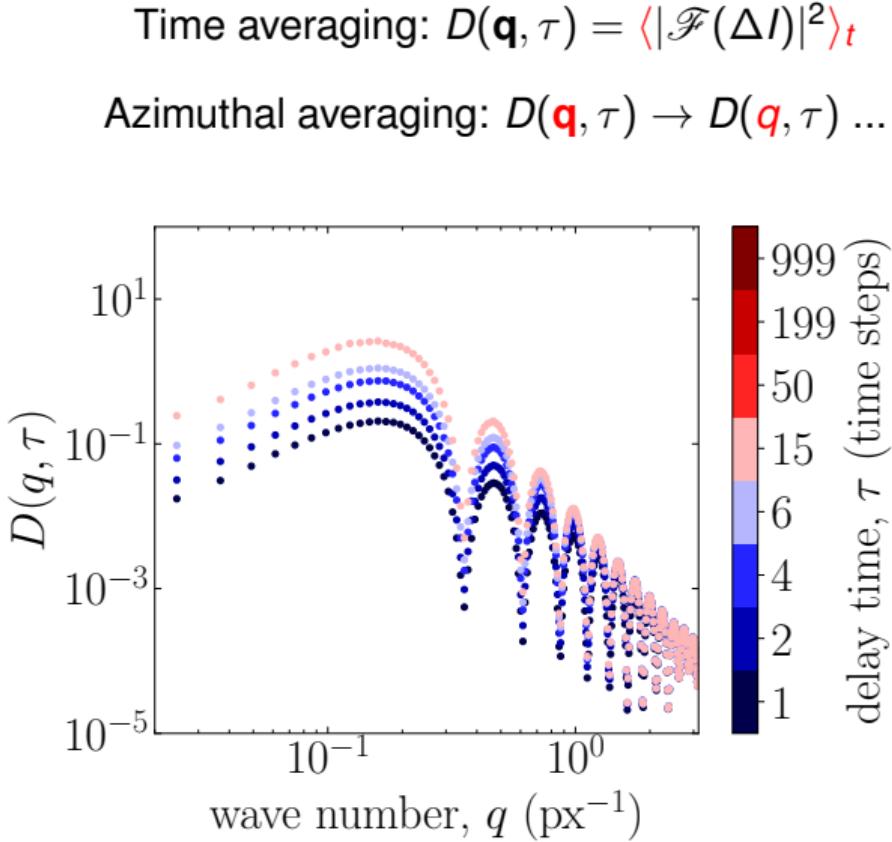
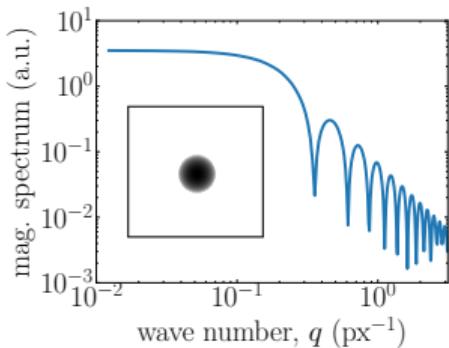
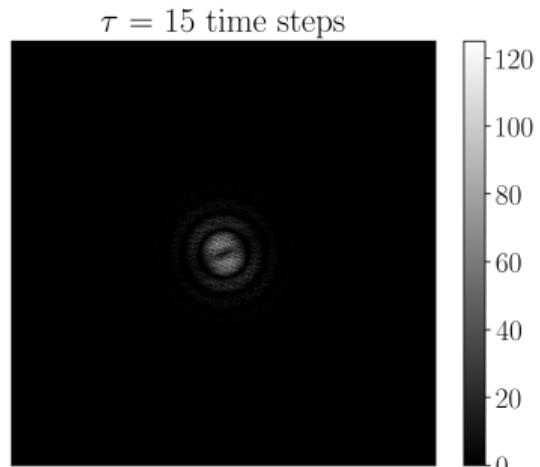


Time averaging: $D(\mathbf{q}, \tau) = \langle |\mathcal{F}(\Delta I)|^2 \rangle_t$

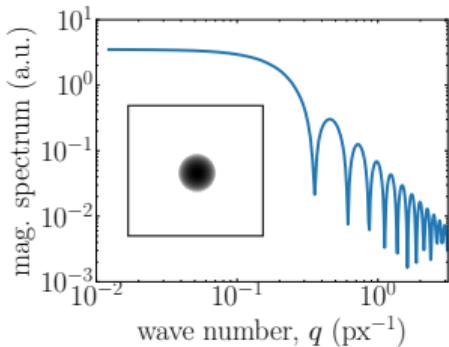
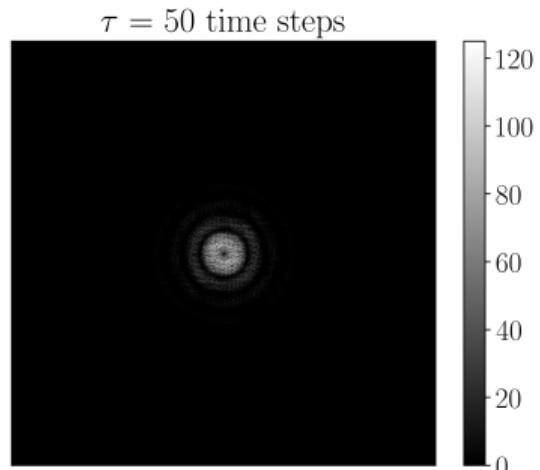
Azimuthal averaging: $D(\mathbf{q}, \tau) \rightarrow D(\mathbf{q}, \tau) \dots$



The image structure function $D(q, \tau)$

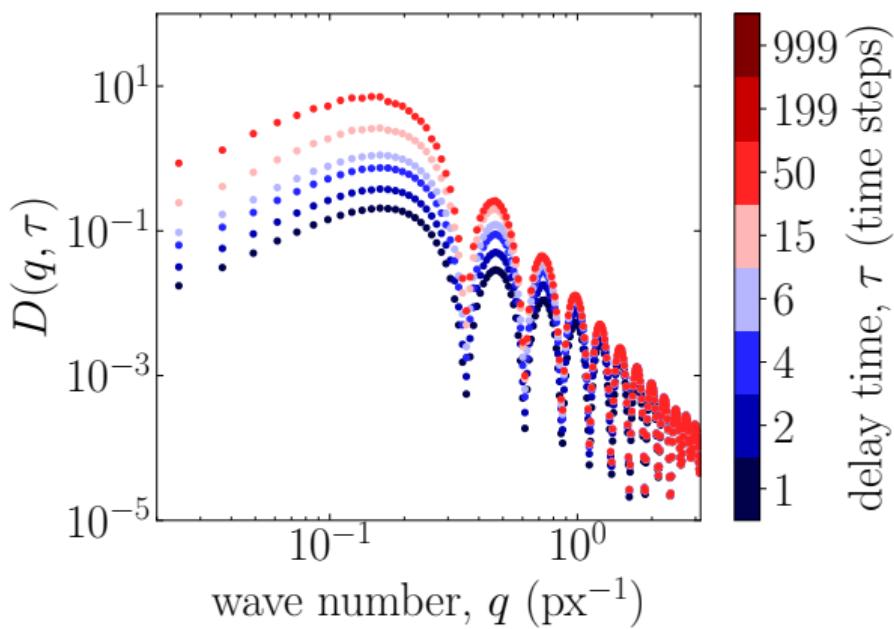


The image structure function $D(q, \tau)$



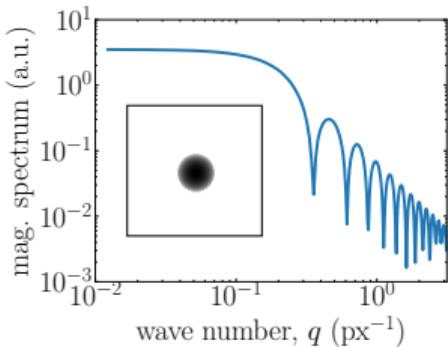
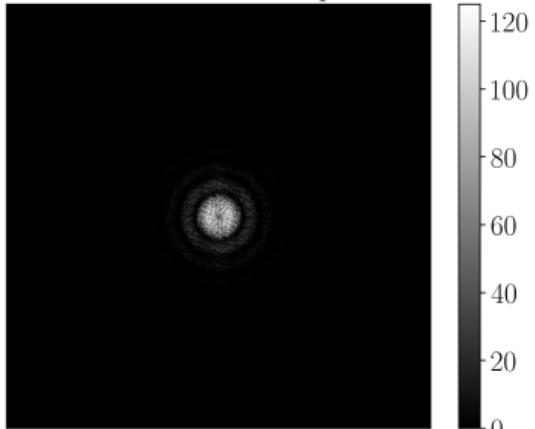
Time averaging: $D(\mathbf{q}, \tau) = \langle |\mathcal{F}(\Delta I)|^2 \rangle_t$

Azimuthal averaging: $D(\mathbf{q}, \tau) \rightarrow D(\mathbf{q}, \tau) \dots$



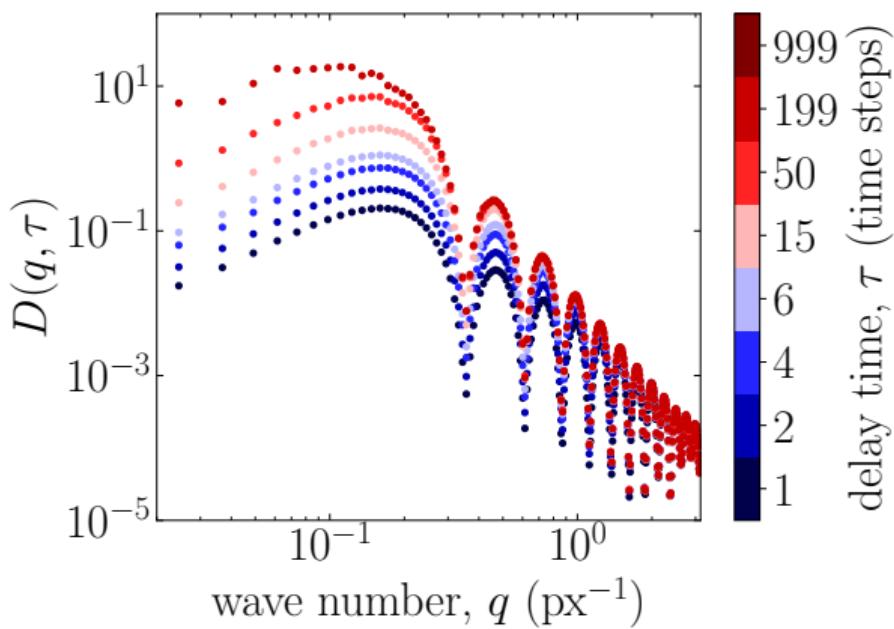
The image structure function $D(q, \tau)$

$\tau = 199$ time steps



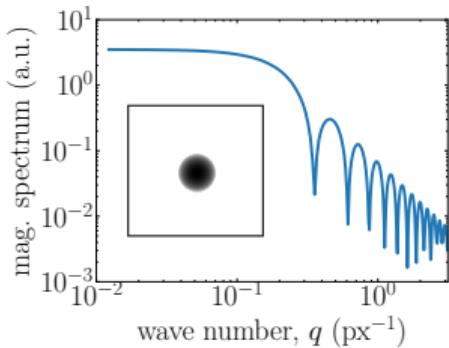
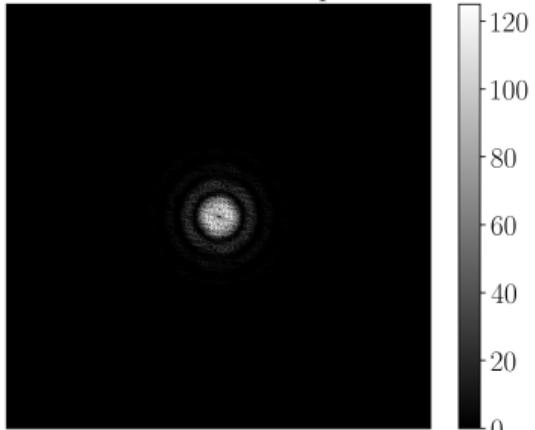
Time averaging: $D(\mathbf{q}, \tau) = \langle |\mathcal{F}(\Delta I)|^2 \rangle_t$

Azimuthal averaging: $D(\mathbf{q}, \tau) \rightarrow D(\mathbf{q}, \tau) \dots$



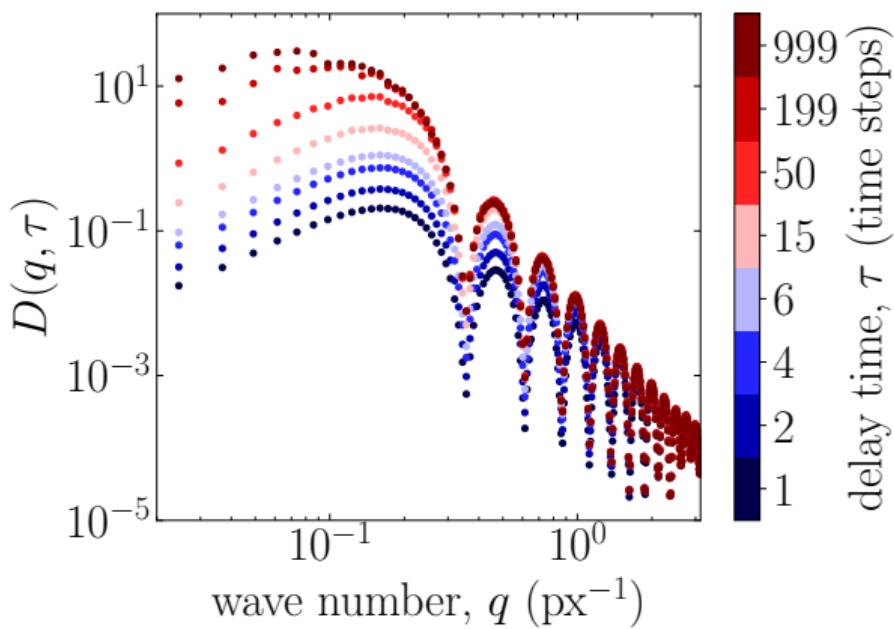
The image structure function $D(q, \tau)$

$\tau = 999$ time steps

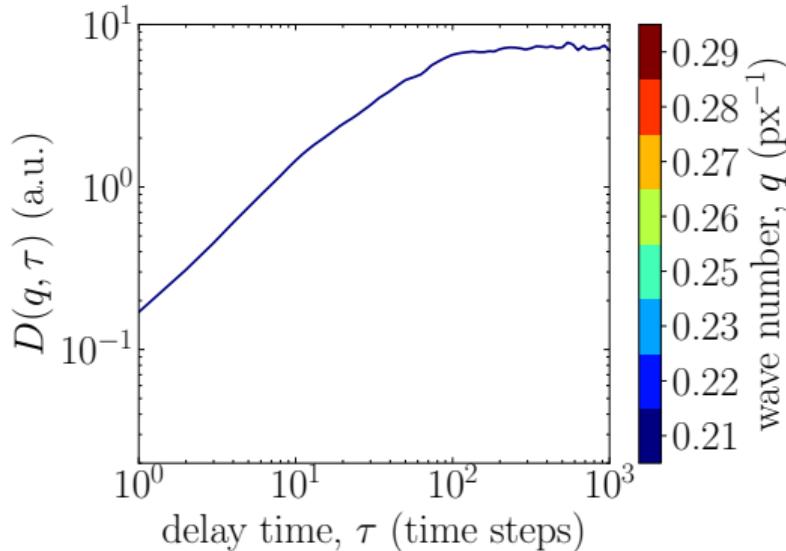
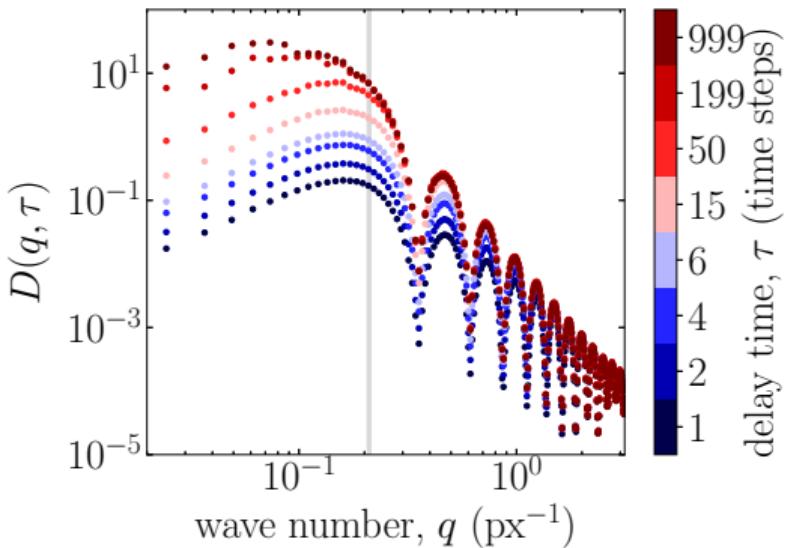


Time averaging: $D(\mathbf{q}, \tau) = \langle |\mathcal{F}(\Delta I)|^2 \rangle_t$

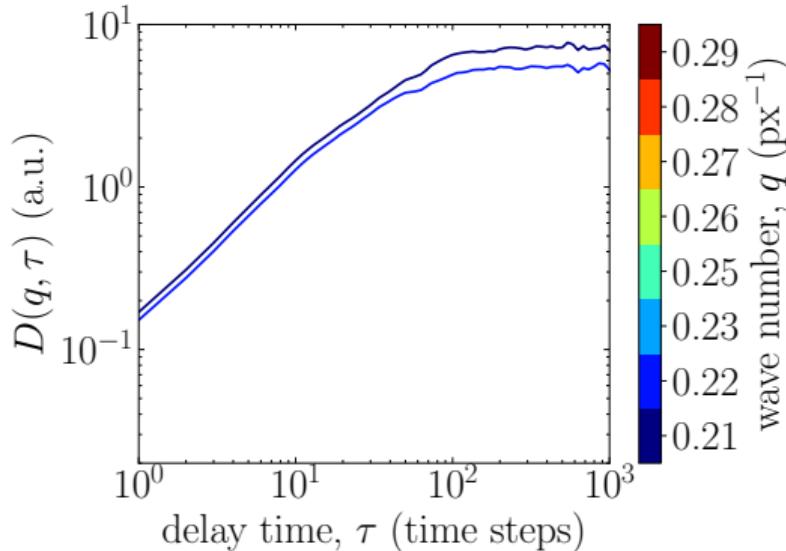
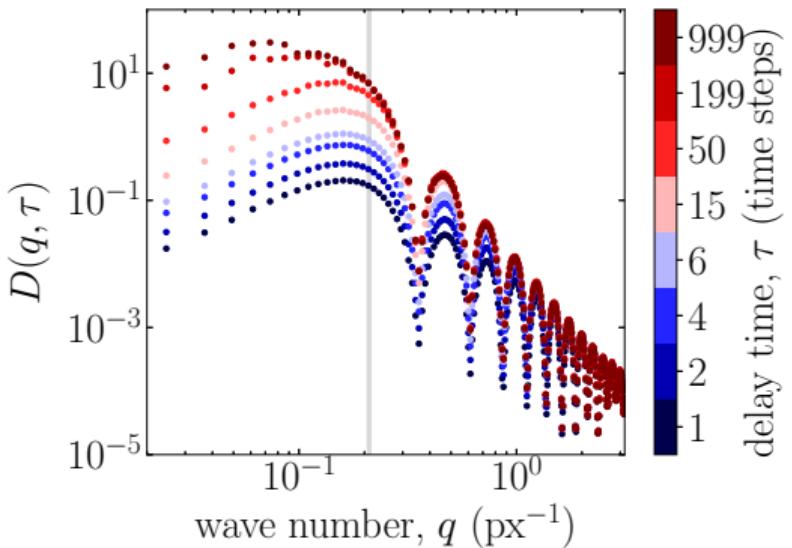
Azimuthal averaging: $D(\mathbf{q}, \tau) \rightarrow D(\mathbf{q}, \tau) \dots$



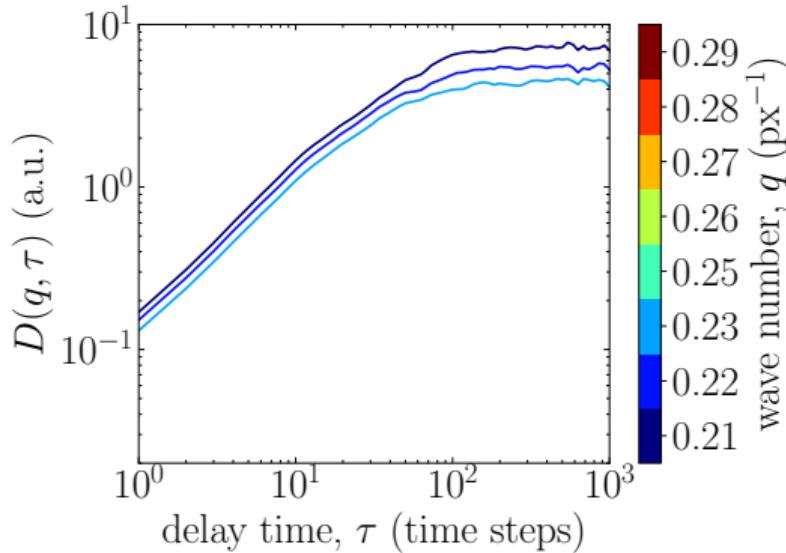
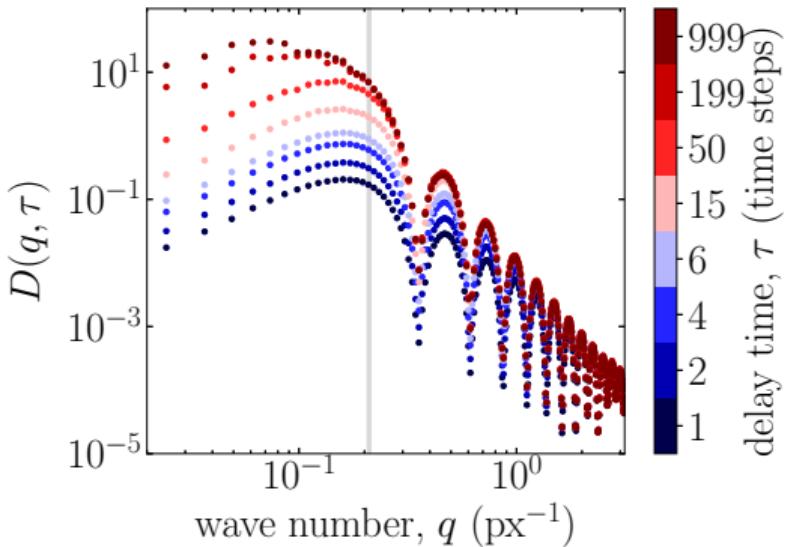
The image structure function $D(q, \tau)$



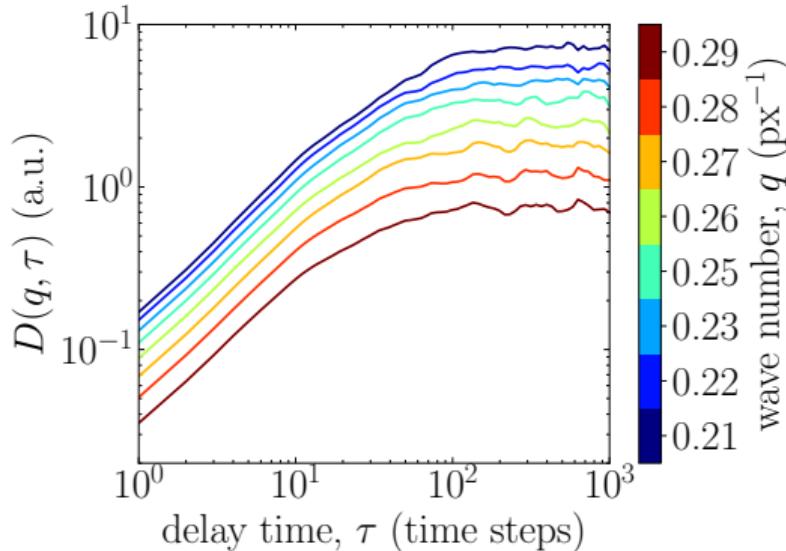
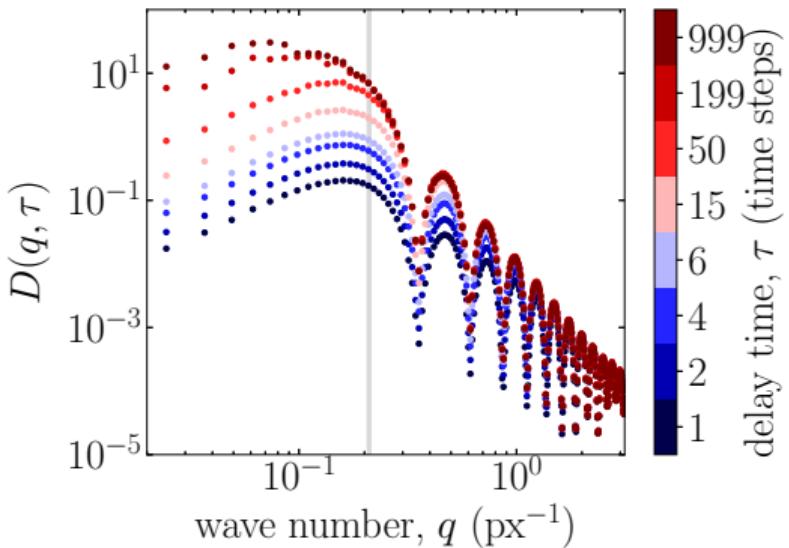
The image structure function $D(q, \tau)$



The image structure function $D(q, \tau)$

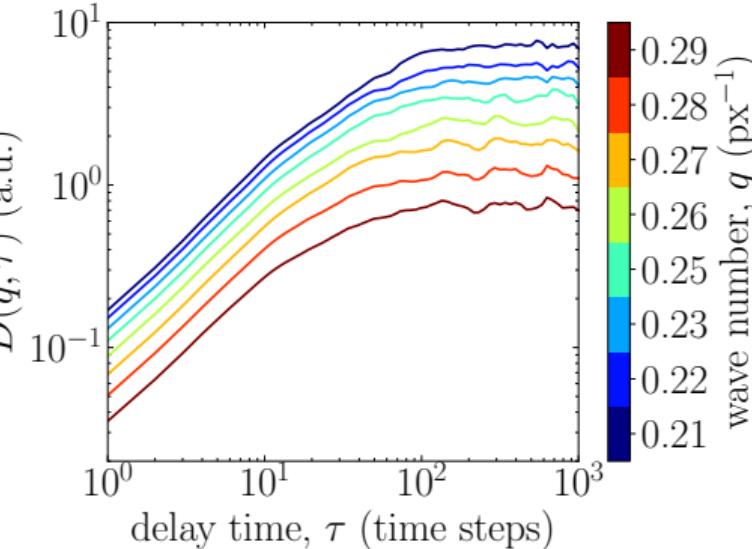
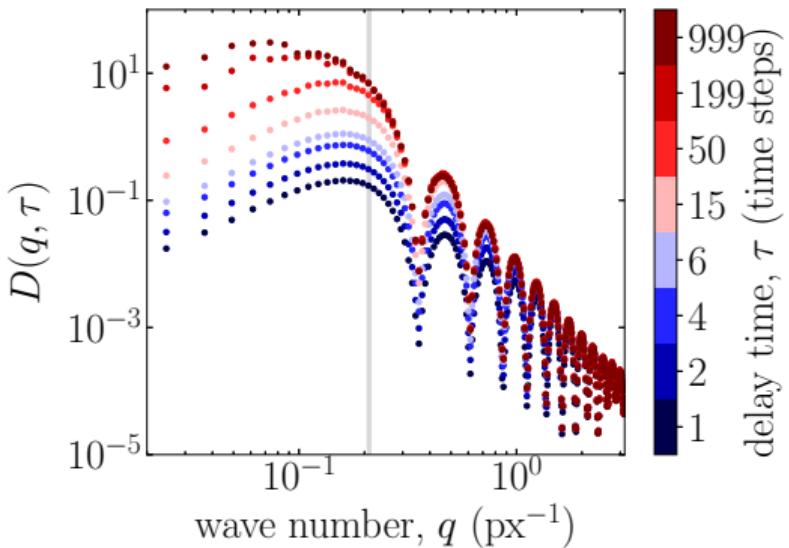


The image structure function $D(q, \tau)$



$$\begin{aligned}
 D(q, \tau) &= \left\langle |I(q, t + \tau) - I(q, t)|^2 \right\rangle_t \\
 &= A(q) \left[1 - \frac{\left\langle I^*(q, t) I(q, t + \tau) \right\rangle_t}{\left\langle |I(q, t)|^2 \right\rangle_t} \right] + B(q)
 \end{aligned}$$

The image structure function $D(q, \tau)$



$$\begin{aligned}
 D(q, \tau) &= \left\langle |I(q, t + \tau) - I(q, t)|^2 \right\rangle_t \\
 &= A(q) \left[1 - \underbrace{\frac{\langle I^*(q, t) I(q, t + \tau) \rangle_t}{\langle |I(q, t)|^2 \rangle_t}}_{\text{Image correlation function}} \right] + B(q)
 \end{aligned}$$

Linear space invariant imaging

Image correlation function

$$g(\mathbf{q}, \tau) = \frac{\langle I^*(\mathbf{q}, t) I(\mathbf{q}, t + \tau) \rangle_t}{\langle |I(\mathbf{q}, t)|^2 \rangle_t}$$

Linear space invariant imaging

Image correlation function

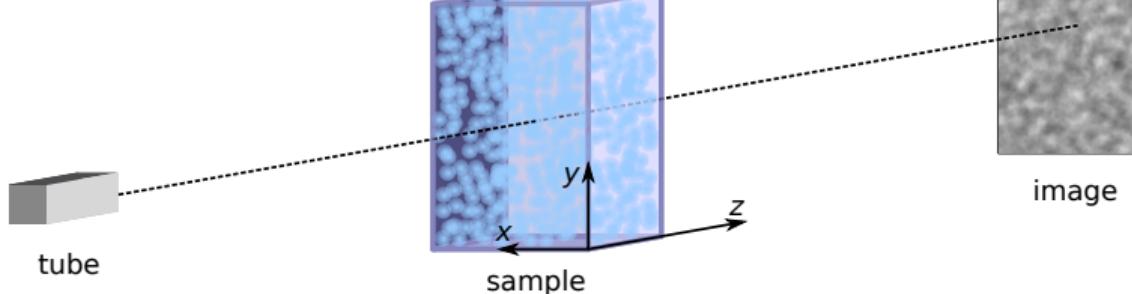
$$g(\mathbf{q}, \tau) = \frac{\langle I^*(\mathbf{q}, t)I(\mathbf{q}, t + \tau) \rangle_t}{\langle |I(\mathbf{q}, t)|^2 \rangle_t}$$

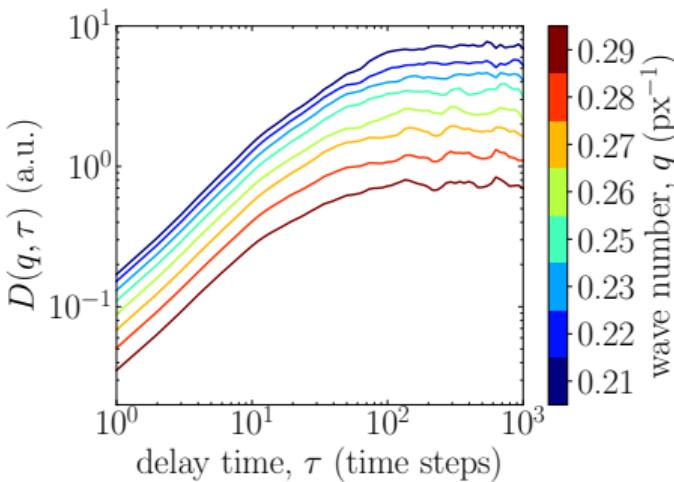
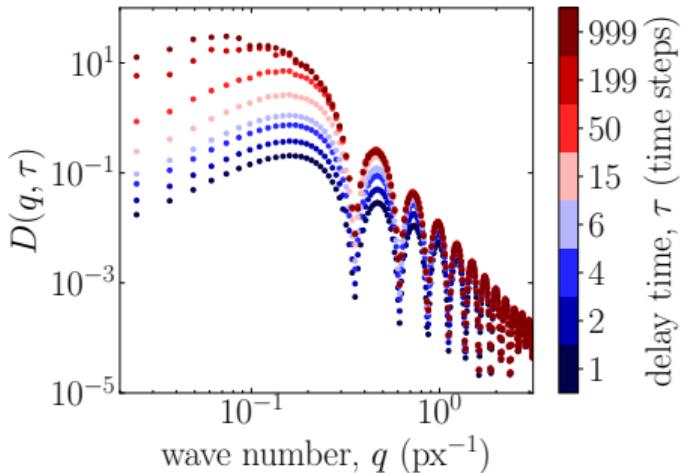
Intermediate scattering function

$$f(\mathbf{q}, \tau) = \frac{\langle \rho^*(\mathbf{q}, t)\rho(\mathbf{q}, t + \tau) \rangle_t}{\langle |\rho(\mathbf{q}, t)|^2 \rangle_t}$$

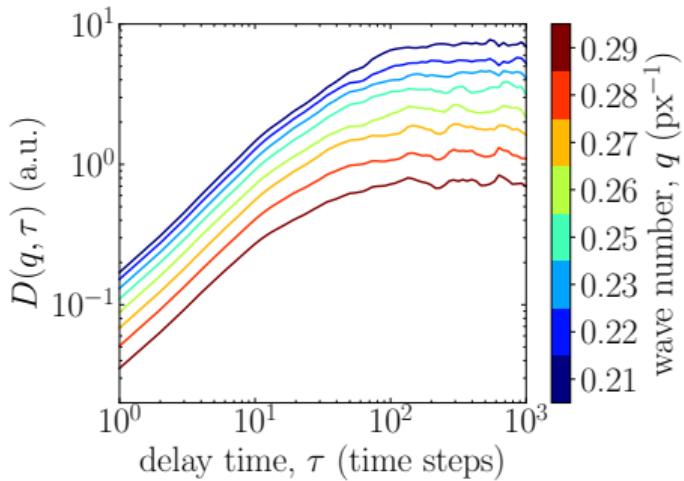
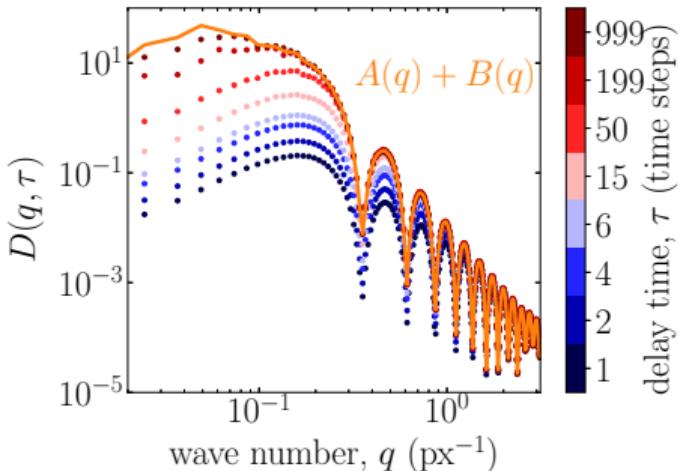
Linear space-invariant imaging:

$$I(\mathbf{r}, t) = I_0 + \int d\mathbf{r}' dz' T(\mathbf{r} - \mathbf{r}', -z') c(\mathbf{r}', z', t)$$



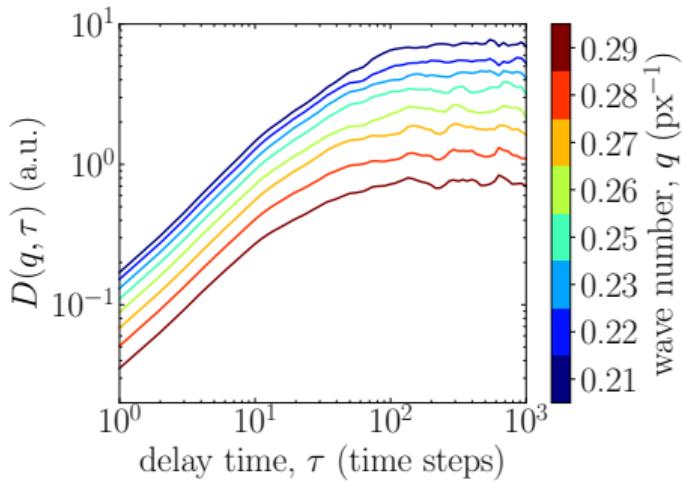
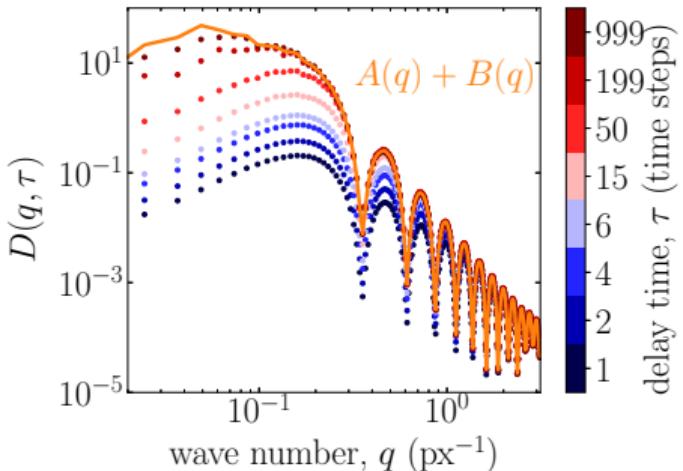


$$\begin{aligned}
 D(q, \tau) &= \left\langle |I(q, t + \tau) - I(q, t)|^2 \right\rangle_t \\
 &= A(q) \underbrace{\left[1 - \frac{\langle I^*(q, t) I(q, t + \tau) \rangle_t}{\langle |I(q, t)|^2 \rangle_t} \right]}_{\text{Image correlation function}} + B(q)
 \end{aligned}$$



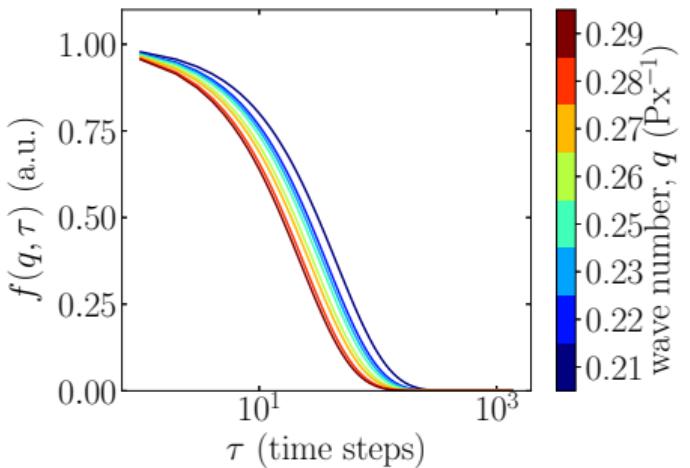
$$\begin{aligned} D(q, \tau) &= \left\langle |I(q, t + \tau) - I(q, t)|^2 \right\rangle_t \\ &= A(q) \left[1 - \frac{\langle I^*(q, t) I(q, t + \tau) \rangle_t}{\langle |I(q, t)|^2 \rangle_t} \right] + B(q) \end{aligned}$$

- $D(q, \tau \rightarrow 0) = B(q) = 0$
- $D(q, \tau \rightarrow \infty) = A(q) + B(q)$

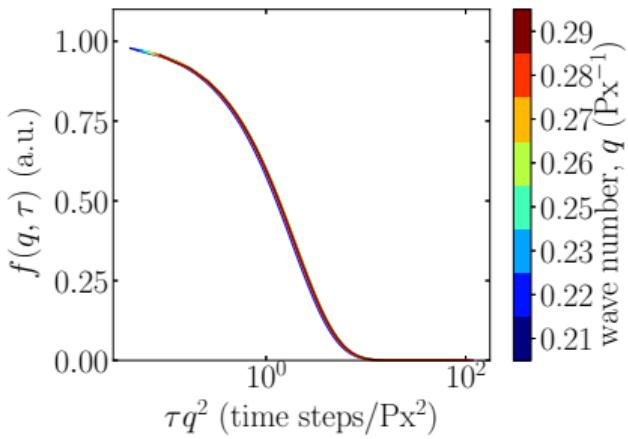
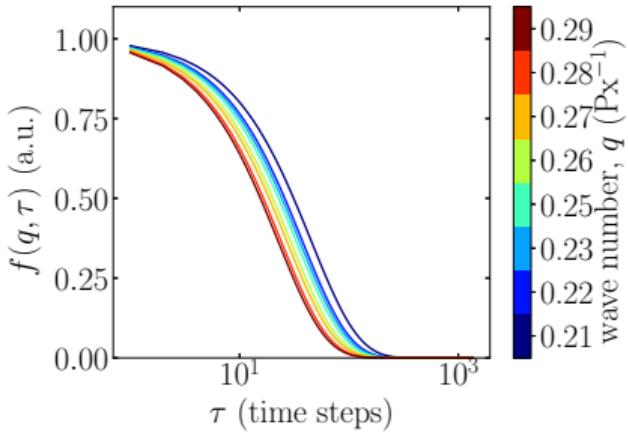


$$\begin{aligned} D(q, \tau) &= \left\langle |I(q, t + \tau) - I(q, t)|^2 \right\rangle_t \\ &= A(q) \left[1 - \frac{\langle I^*(q, t) I(q, t + \tau) \rangle_t}{\langle |I(q, t)|^2 \rangle_t} \right] + B(q) \end{aligned}$$

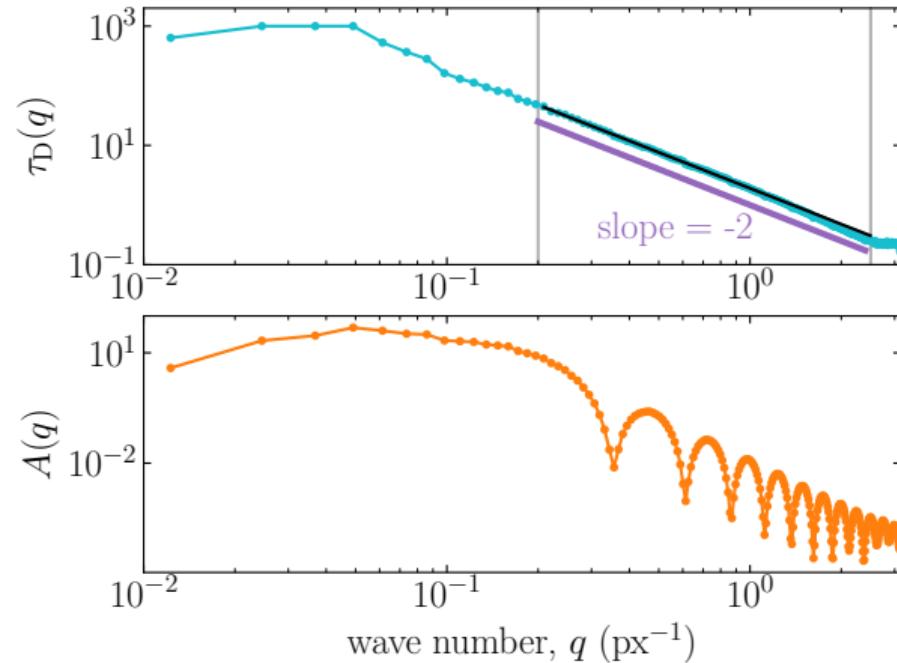
- $D(q, \tau \rightarrow 0) = B(q) = 0$
- $D(q, \tau \rightarrow \infty) = A(q) + B(q)$



Intermediate scattering function $f(q, \tau)$



Brownian motion:
 $f(q, \tau) = \exp(-q^2\tau/\tau_D)$
Accuracy: 2% - 6%



extra slide with videos of 10, 1000, 100000 particles + accuracy, next slide is *Extending DDM to X-DFA*