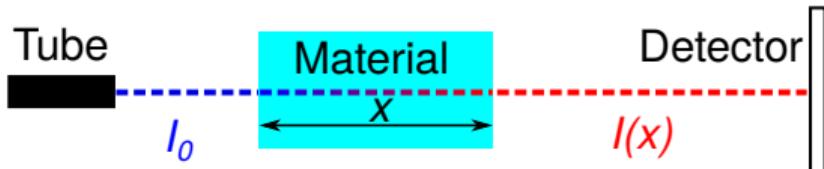


Correction of beam hardening in X-ray radiograms

Attenuation of X-rays

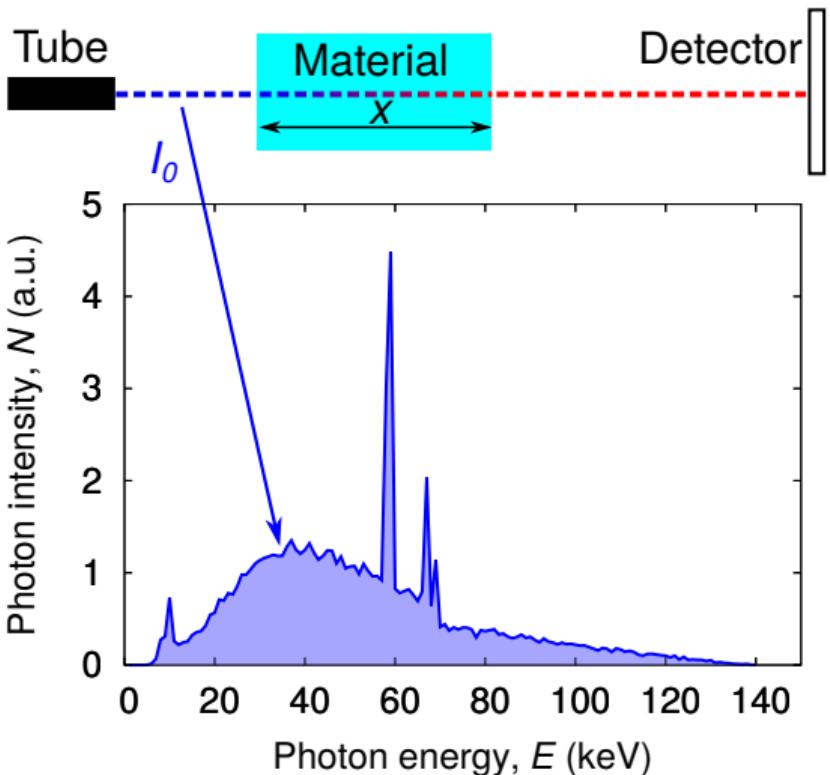


Beer-Lambert's law

$$I(x) = I_0 \exp(-\mu x)$$

$$\text{Thickness: } x = -\frac{1}{\mu} \ln \frac{I(x)}{I_0}$$

Attenuation of X-rays

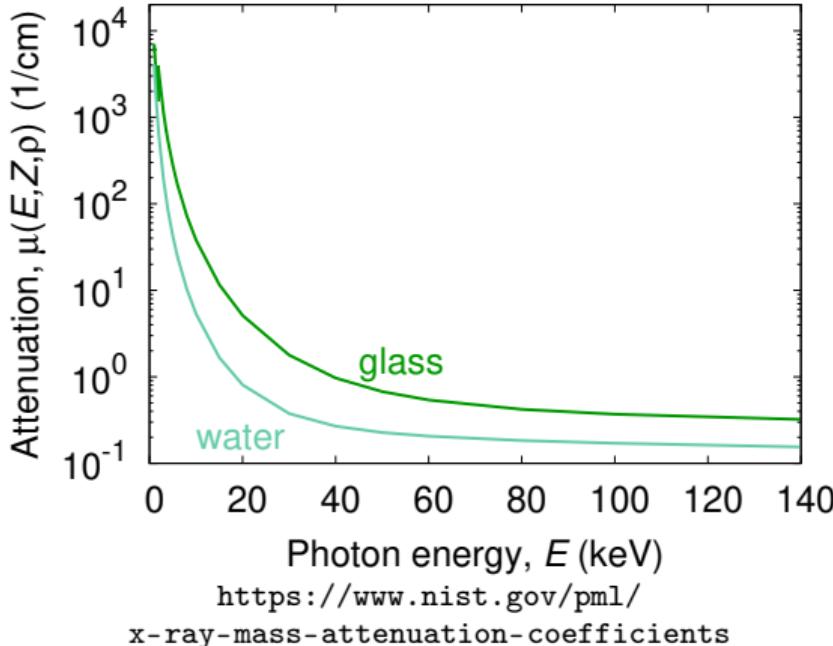


Beer-Lambert's law

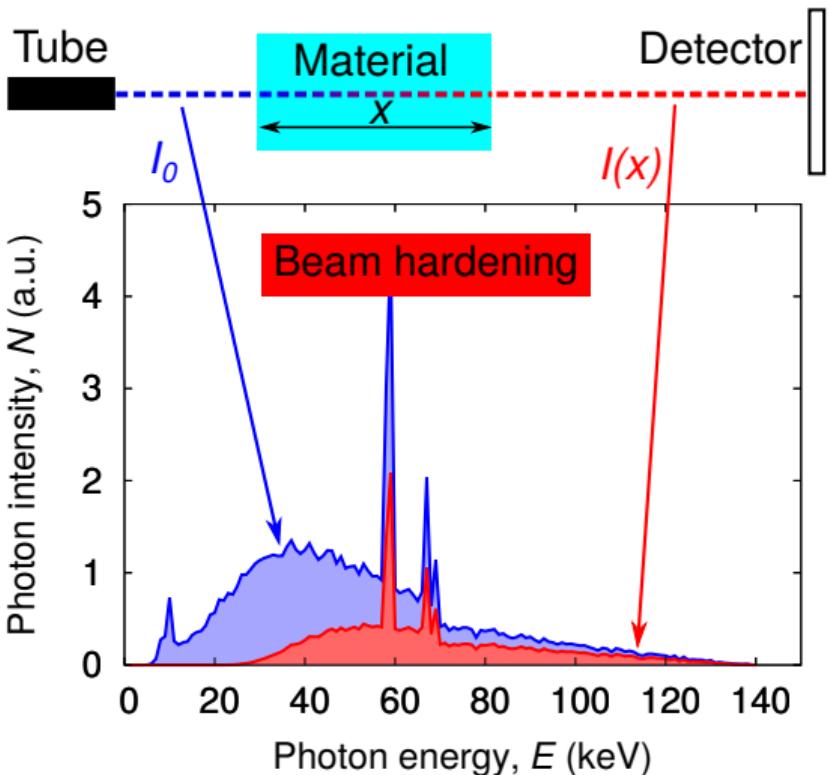
$$I(x) = I_0 \exp(-\mu(E, Z, \rho)x)$$

$\mu \neq \text{const}$

Thickness: $x = ?$



Attenuation of X-rays

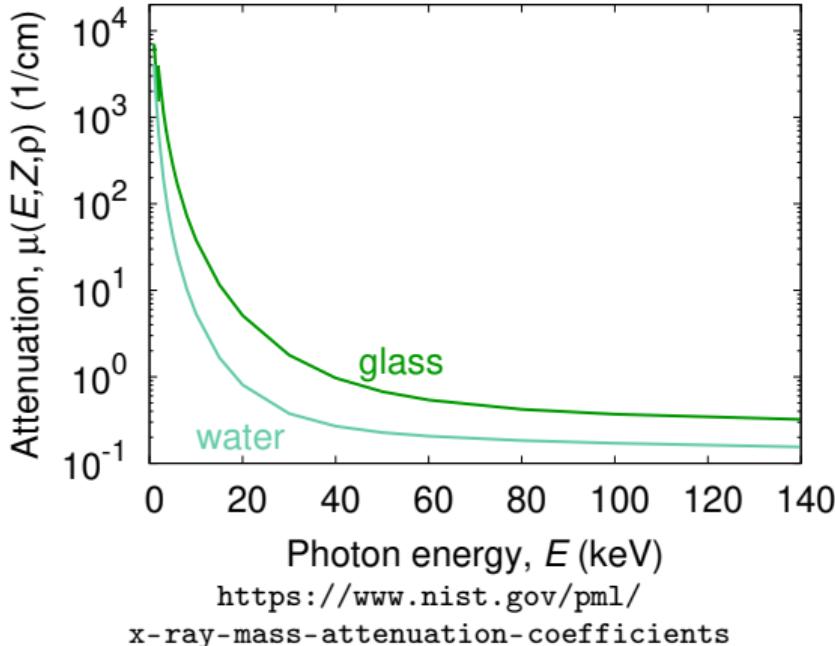


Beer-Lambert's law

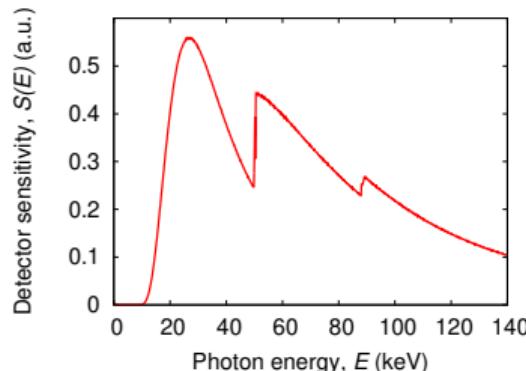
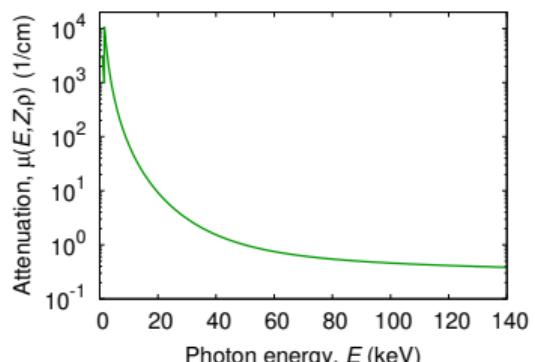
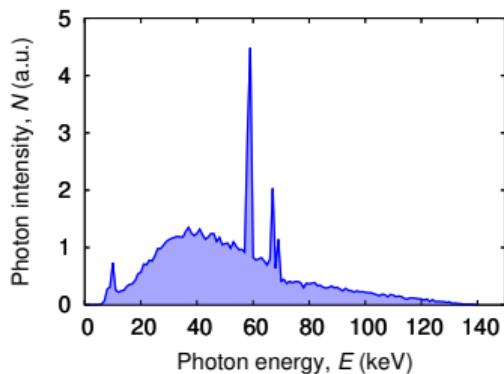
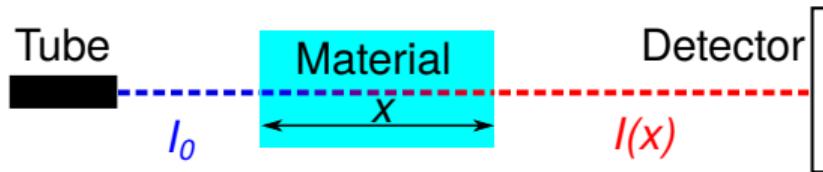
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$\mu \neq \text{const}$

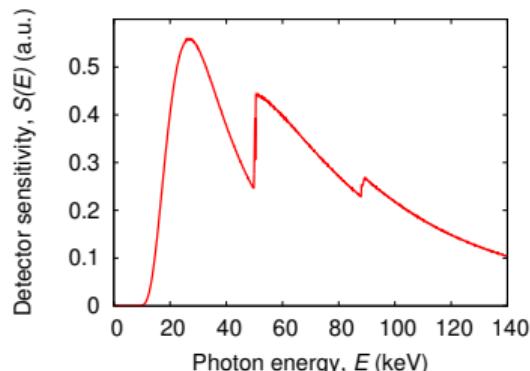
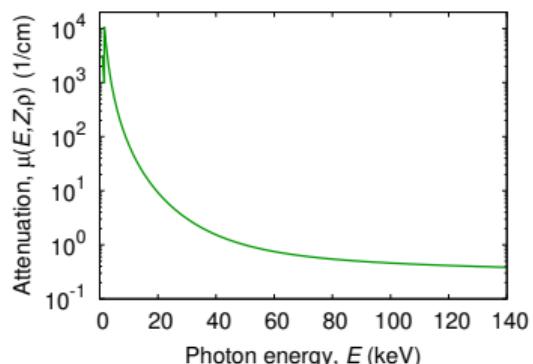
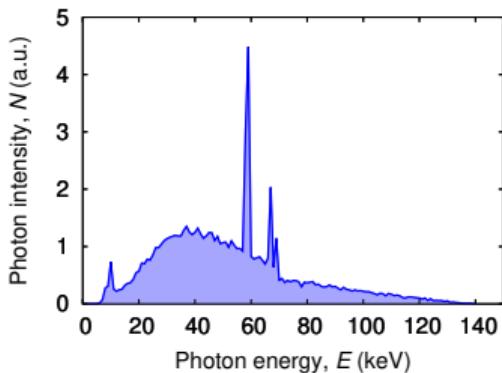
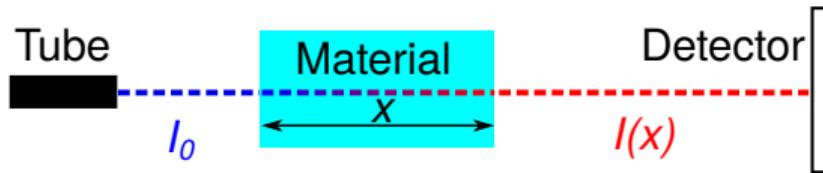
Thickness: $x = ?$



Measurement of X-ray intensities

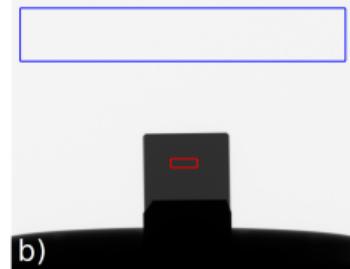
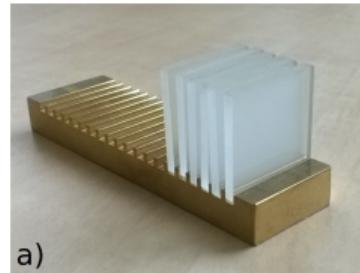
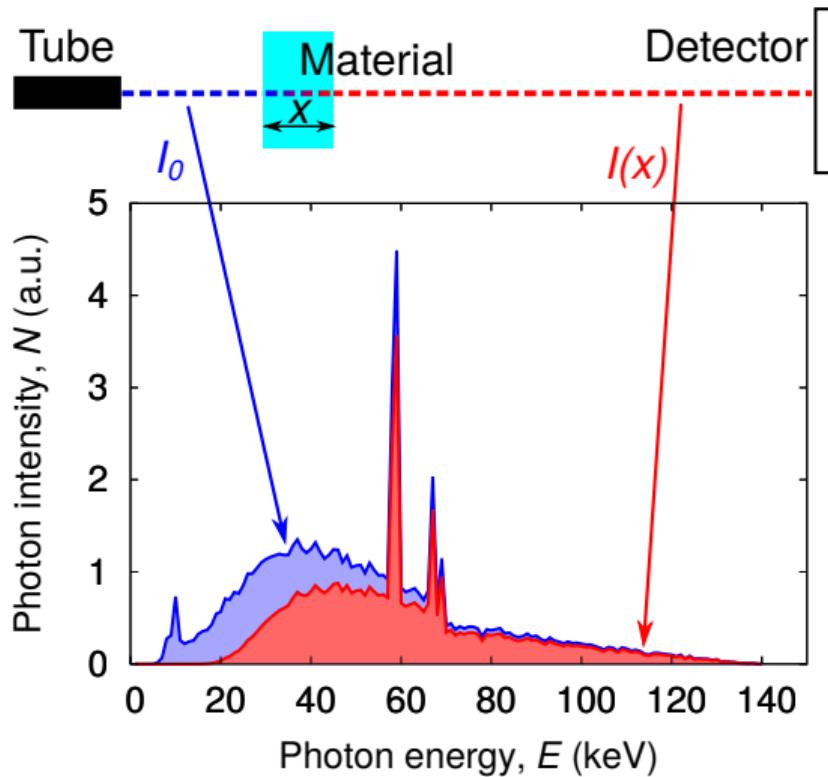


Measurement of X-ray intensities



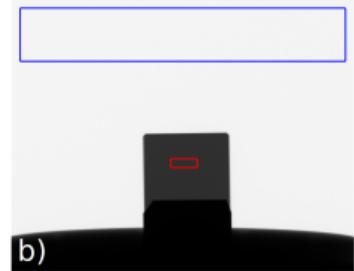
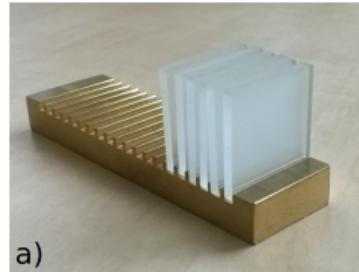
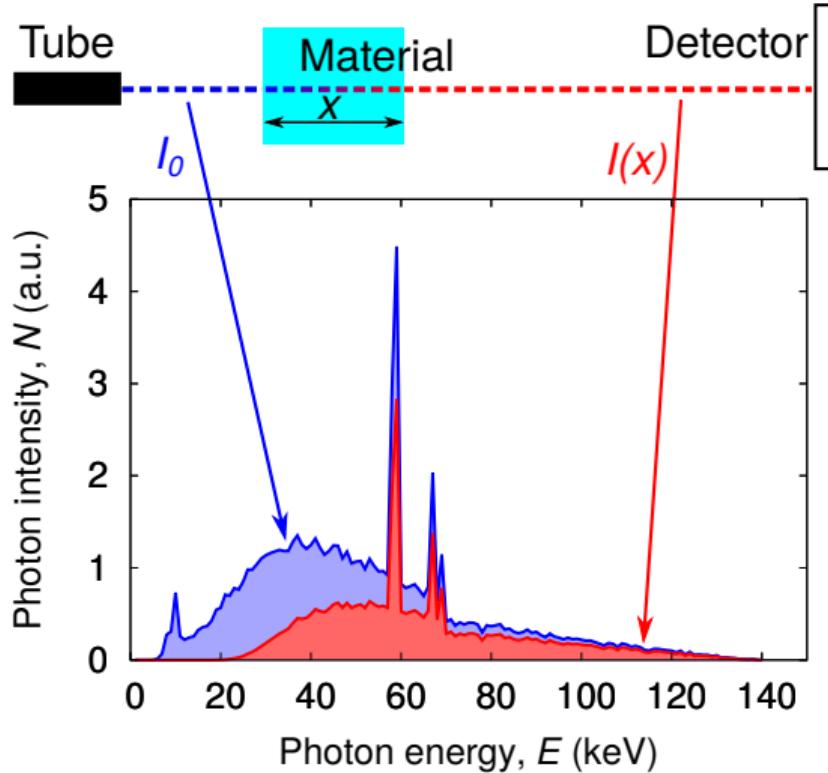
$$I(x) \propto \int N(E) \exp\{-\mu(E, Z, \rho)x\} S(E) dE$$

Energy averaged attenuation coefficient, μ_{eff}



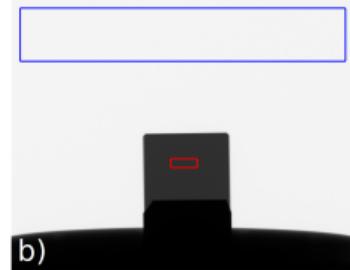
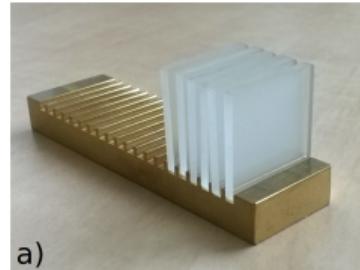
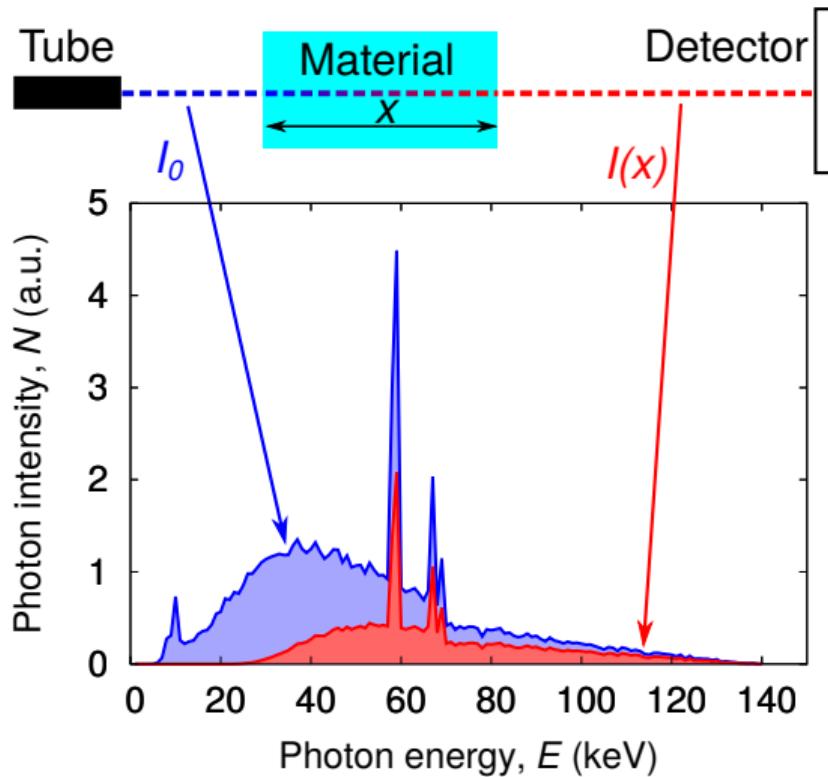
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Energy averaged attenuation coefficient, μ_{eff}



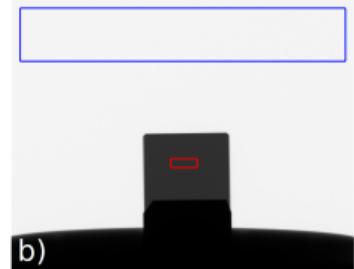
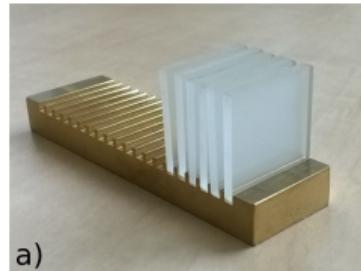
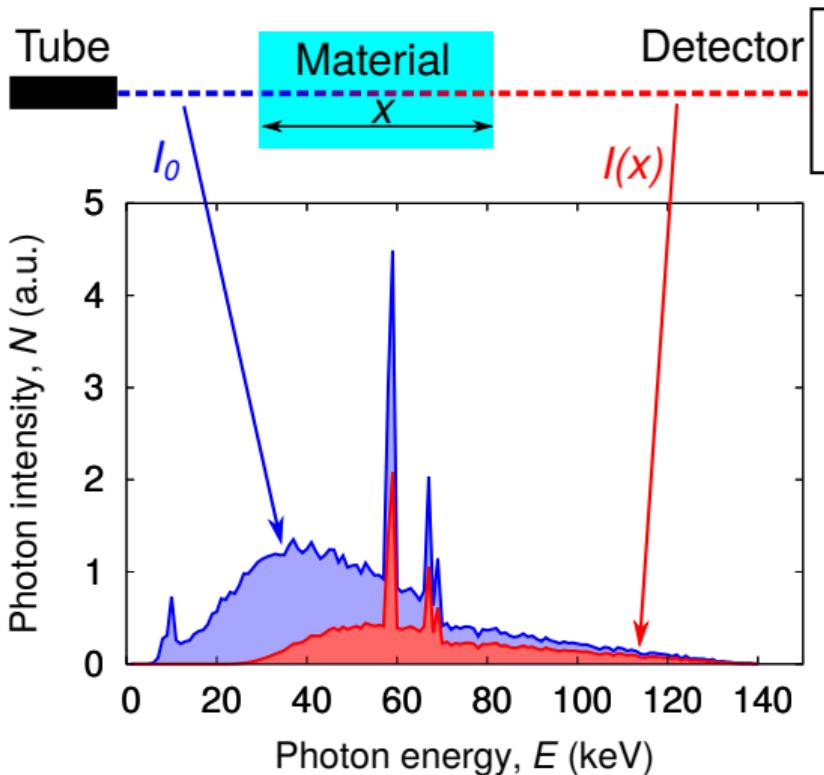
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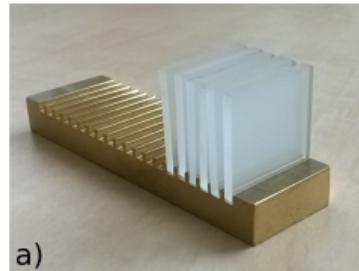
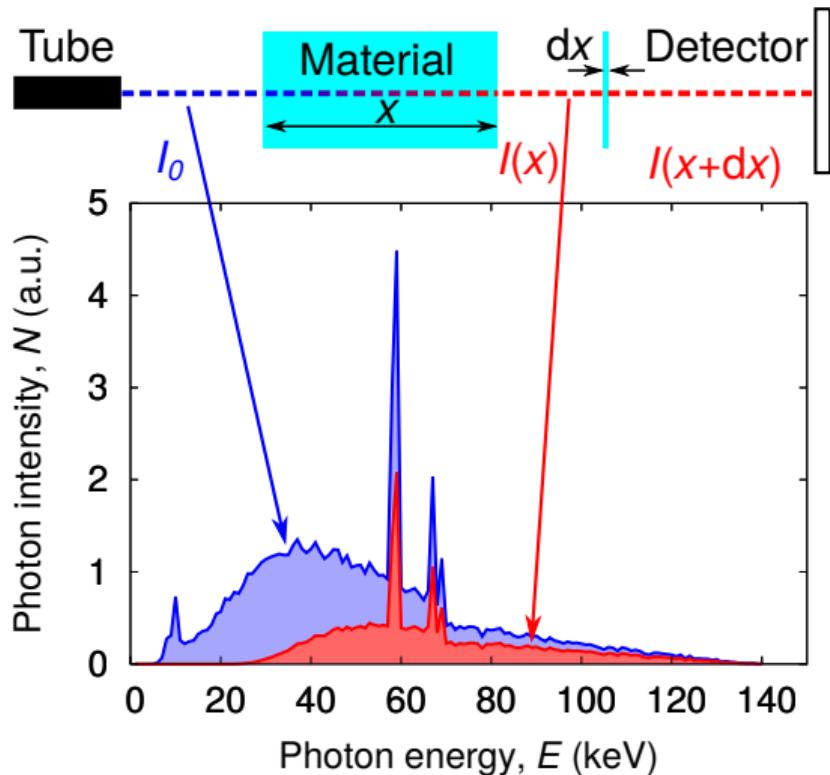


$$I(x) \propto \int N(E) \exp\{-\mu(E, Z, \rho)x\} S(E) dE$$

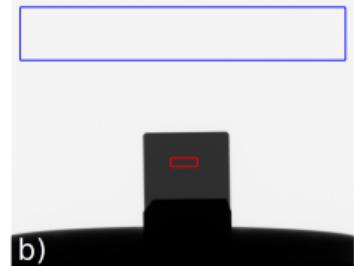
effective attenuation coefficient:

$$I(x) = I_0 \exp(-\mu_{\text{eff}}(x) x)$$

Energy averaged attenuation coefficient, μ_{eff}



a)



b)

$$I(x) \propto \int N(E) \exp\{-\mu(E, Z, \rho)x\} S(E) dE$$

effective attenuation coefficient:

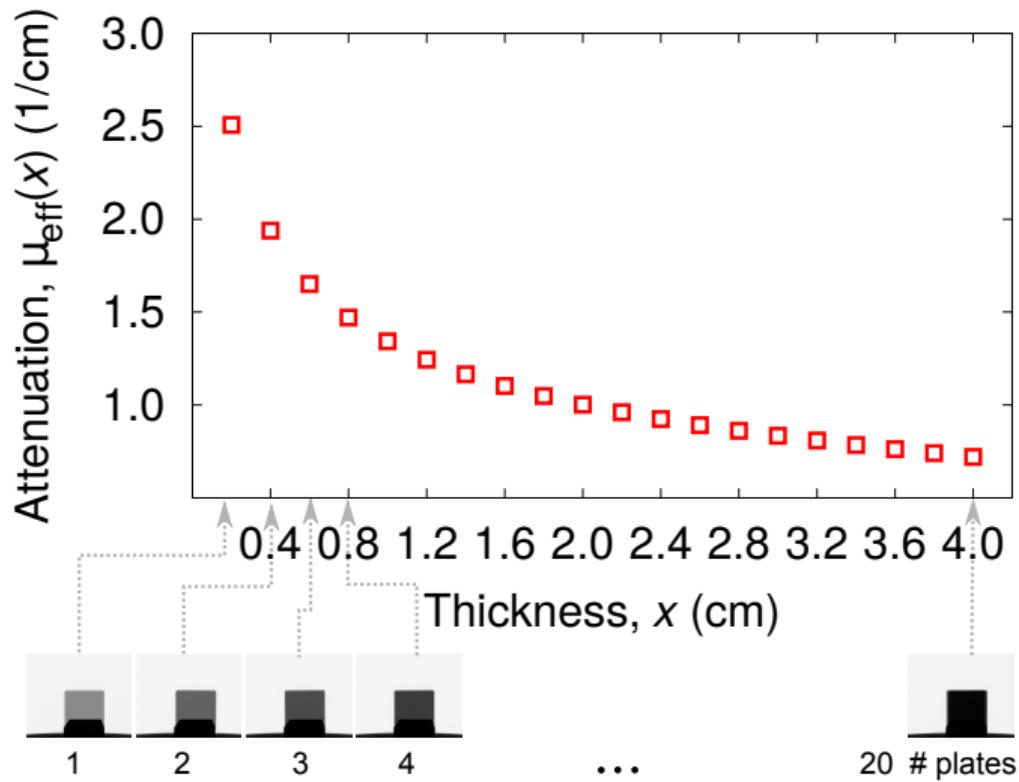
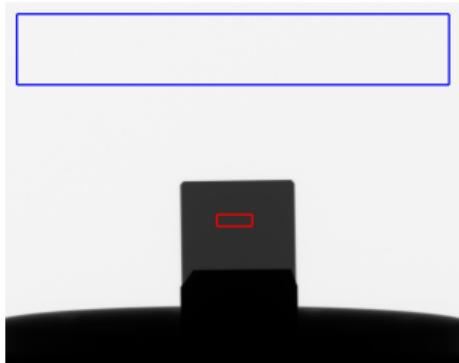
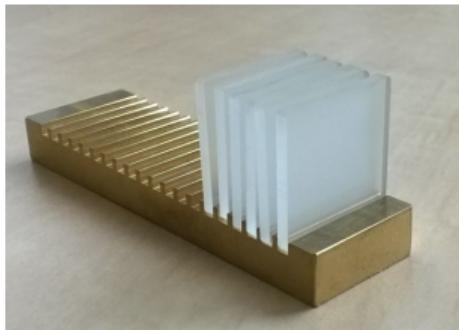
$$I(x) = I_0 \exp(-\mu_{\text{eff}}(x) x)$$

differential attenuation coefficient:

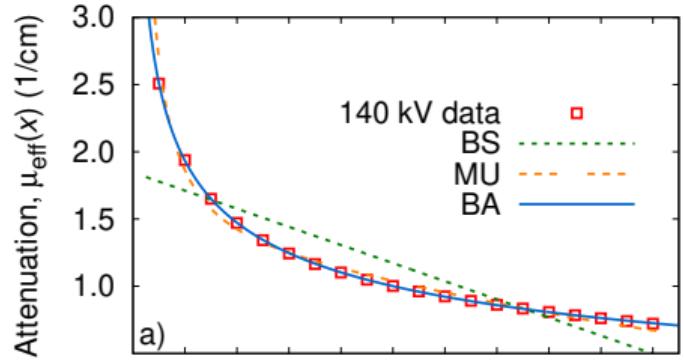
$$\frac{dI(x)}{dx} = \bar{\mu}(x) I(x)$$

Experimental procedure

effective attenuation coefficient: $I(x) = I_0 \exp(-\mu_{\text{eff}}(x) x)$

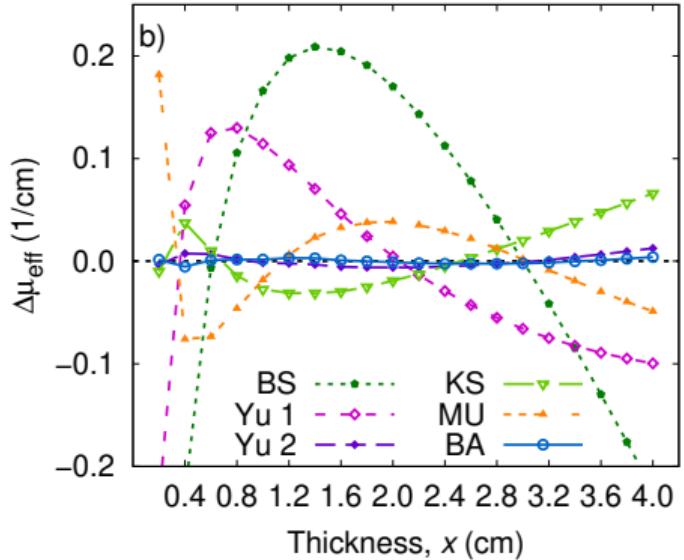


Heuristic model functions



$$\mu_{\text{eff}}(x) = \mu_0 - \lambda x$$

Bjärngard & Shackford
(1994)



$$\mu_{\text{eff}}(x) = \frac{\mu_0}{1+\lambda x}$$

$$\mu_{\text{eff}}(x) = \frac{\mu_0}{(1+\lambda x)^\beta}$$

Yu *et al.* (1997)

$$\mu_{\text{eff}}(x) = \mu(E_{\text{max}}) + \frac{2\mu_1}{x\sqrt{-\lambda_1^2+4\lambda_2}} \times \left[\arctan\left(\frac{\lambda_1+2\lambda_2 x}{\sqrt{-\lambda_1^2+4\lambda_2}}\right) - \arctan\left(\frac{\lambda_1}{\sqrt{-\lambda_1^2+4\lambda_2}}\right) \right]$$

Kleinschmidt (1999)

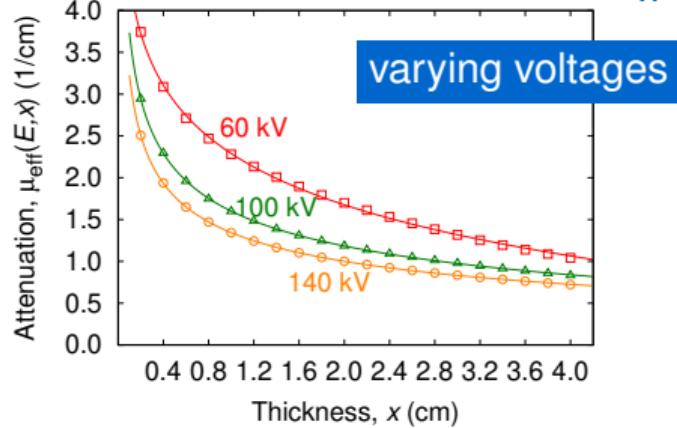
$$\mu_{\text{eff}}(x) = -\frac{1}{x} \ln [A + B \exp(-x/C)]$$

Mudde *et al.* (2008)

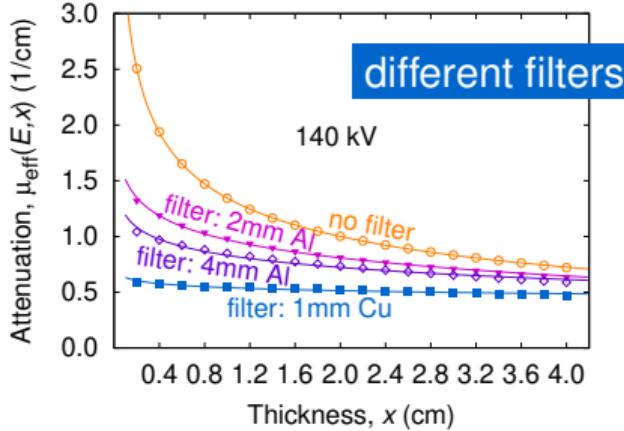
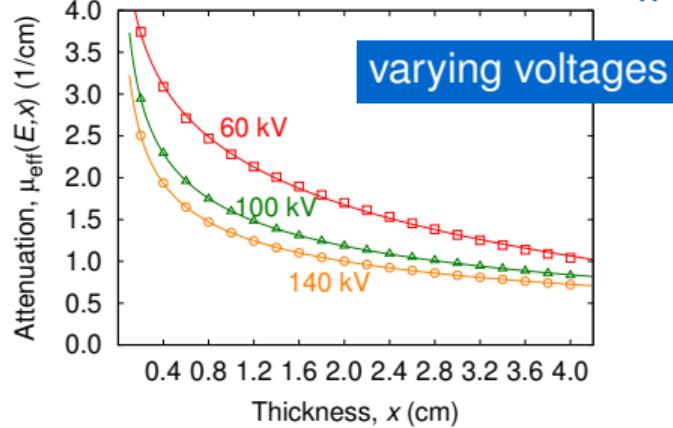
$$\mu_{\text{eff}}(x) = a + \frac{b}{x^\alpha}$$

Baur *et al.* (2019)
(this work)

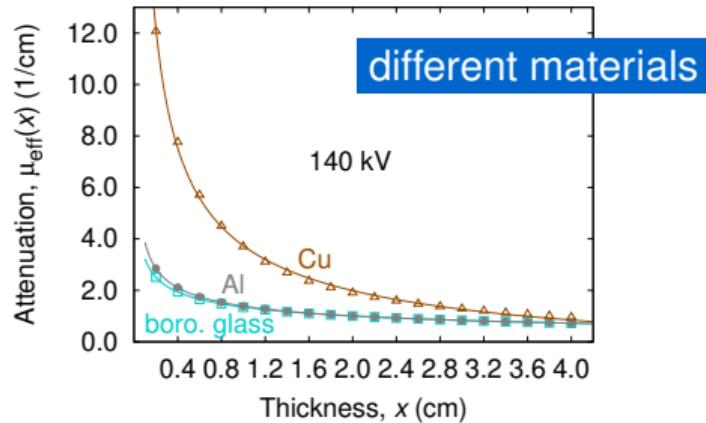
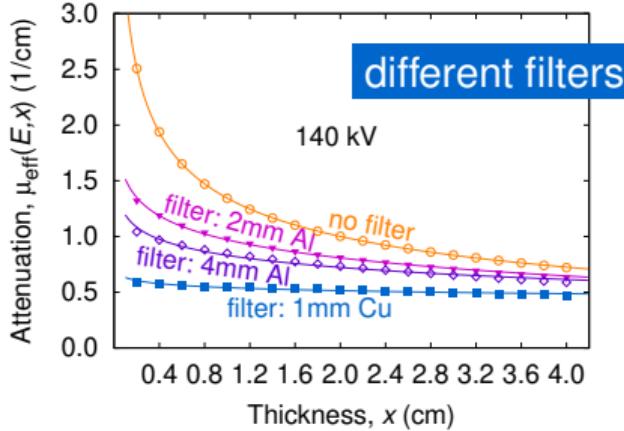
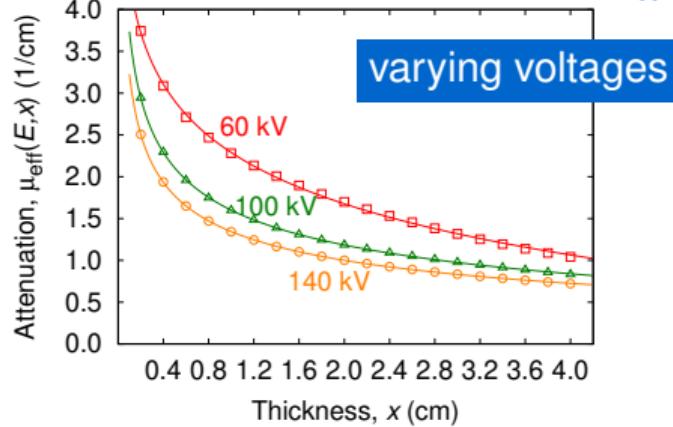
Universality of $\mu_{\text{eff}}(x) = a + \frac{b}{x^\alpha}$



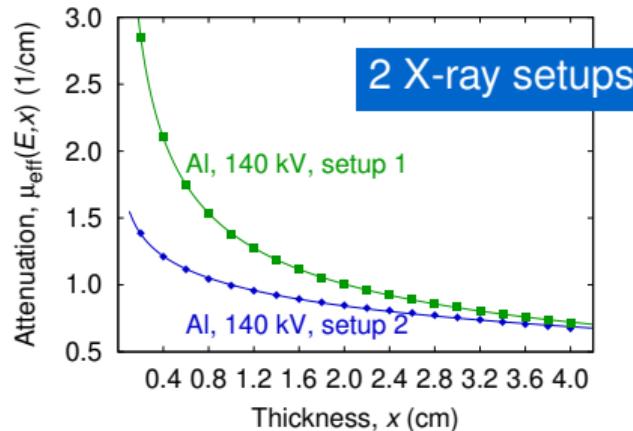
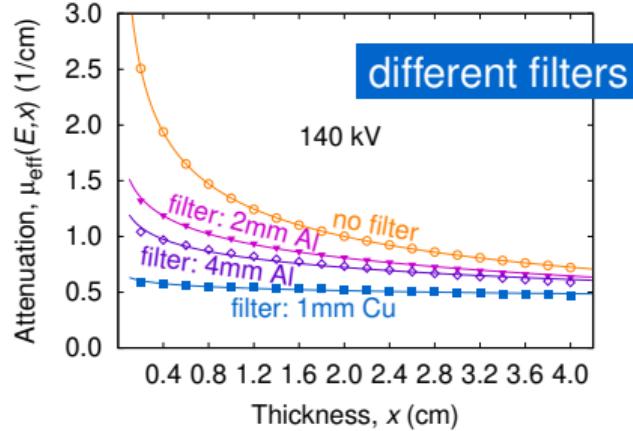
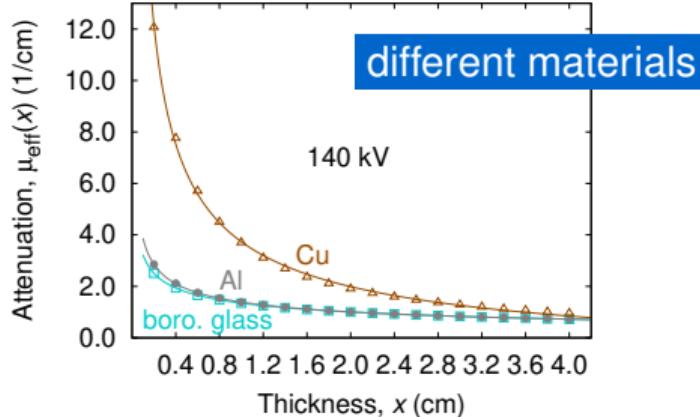
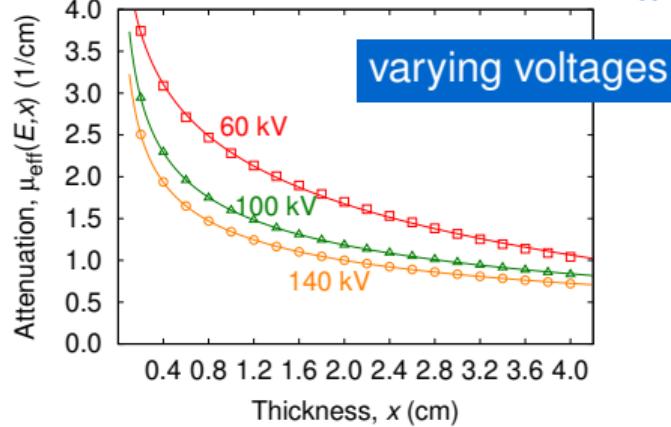
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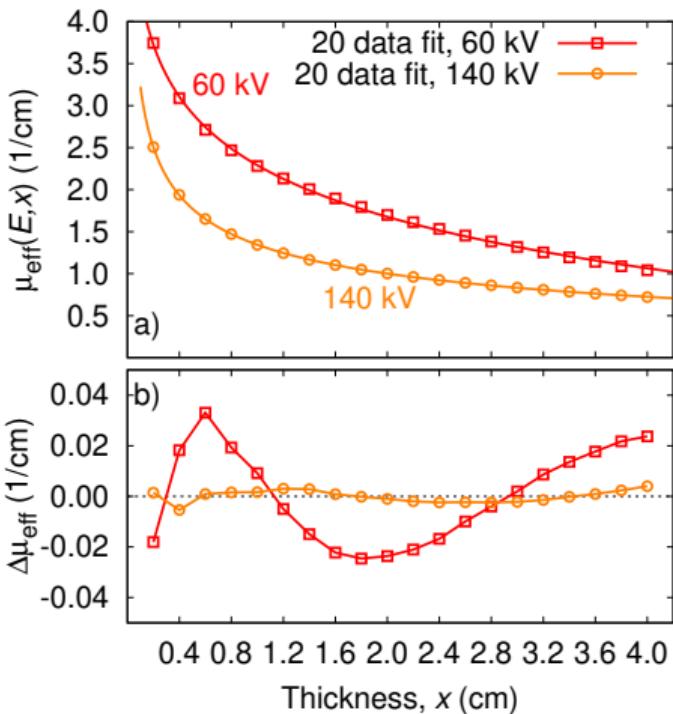
Determining the material thickness x

Generalized Beer-Lambert

$$I(x) = I_0 \exp(-\mu_{\text{eff}}(x) x)$$

Model function

$$\mu_{\text{eff}}(x) = a + \frac{b}{x^\alpha}$$



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Generalized Beer-Lambert

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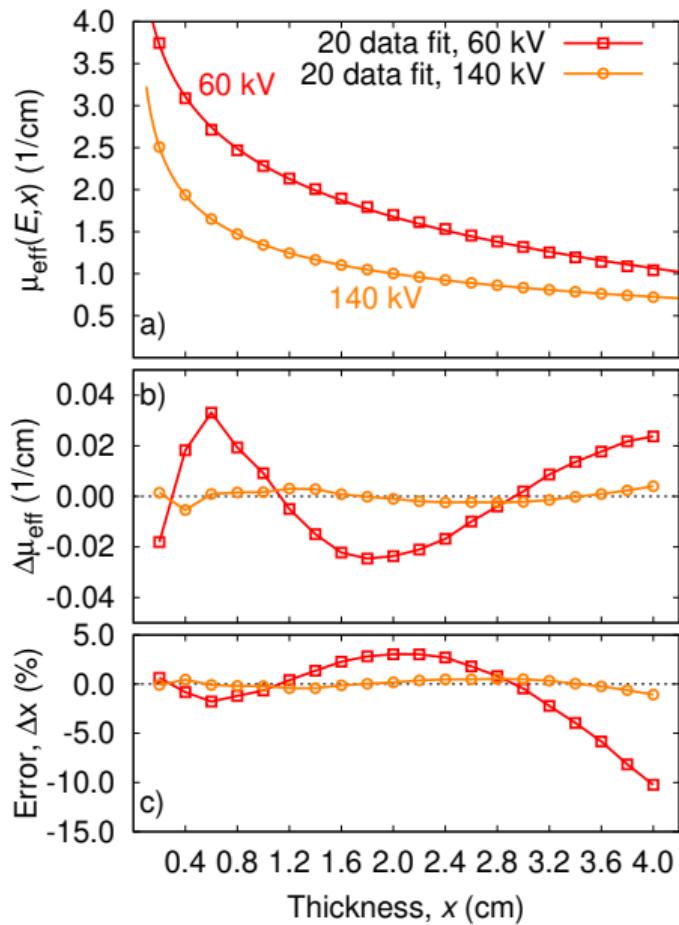
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Solve

$$ax + bx^{1-\alpha} + \ln\left(\frac{I(x)}{I_0}\right) = 0$$

e.g. Newtons method or look-up table



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Generalized Beer-Lambert

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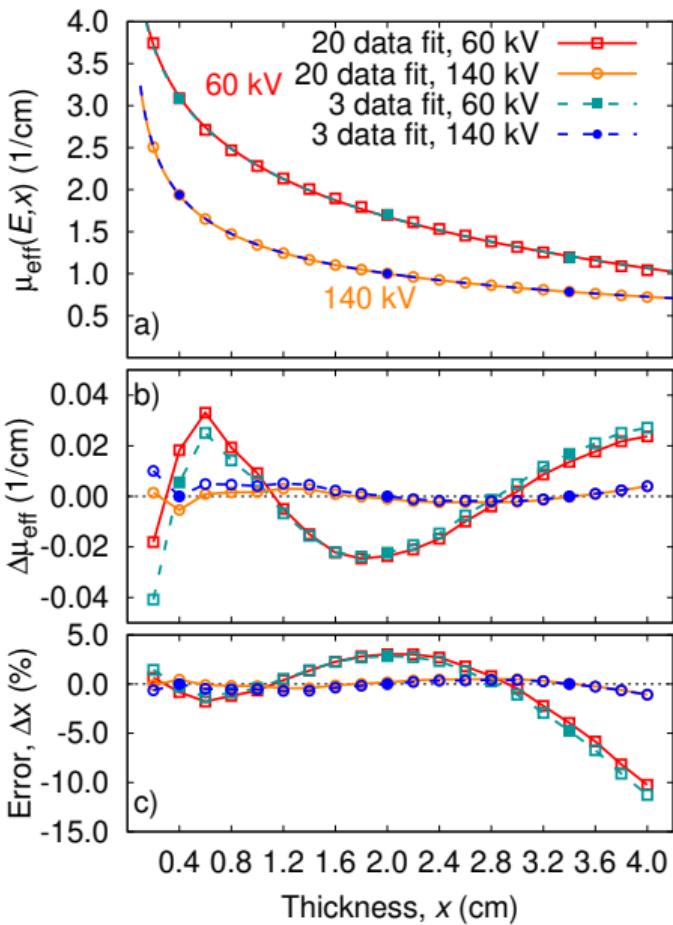
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Applications

