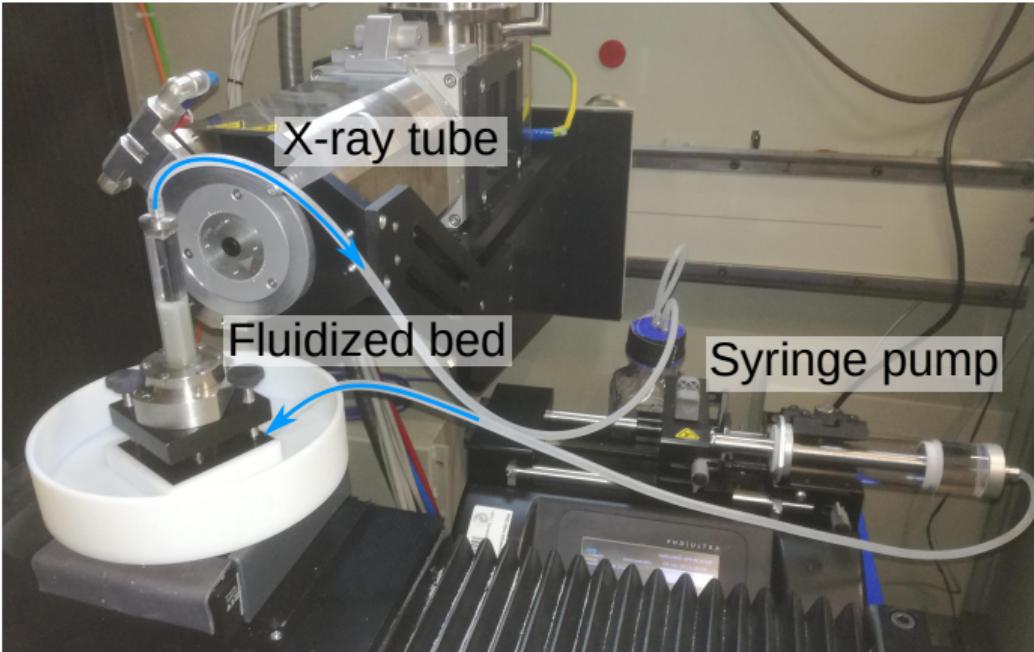
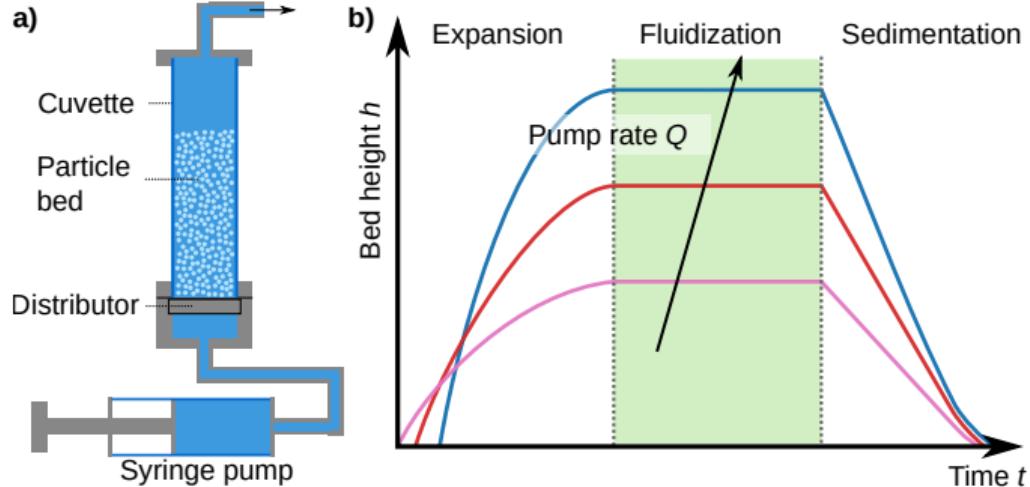


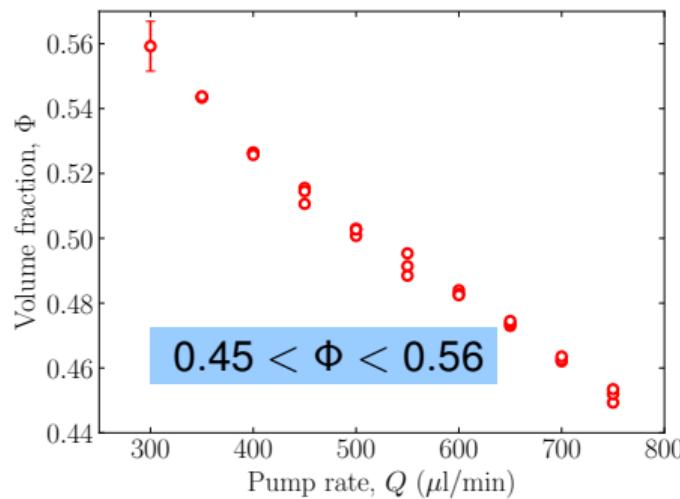
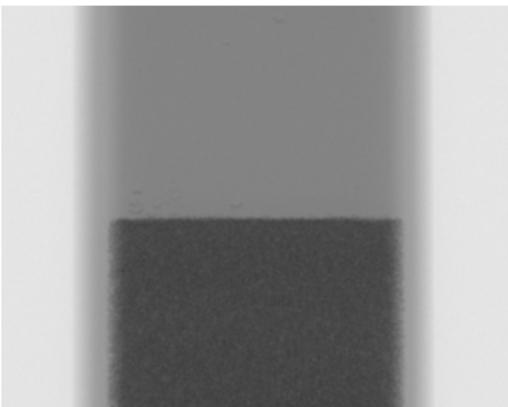
Experimental validation of X-DFA: A suspension of sedimenting particles



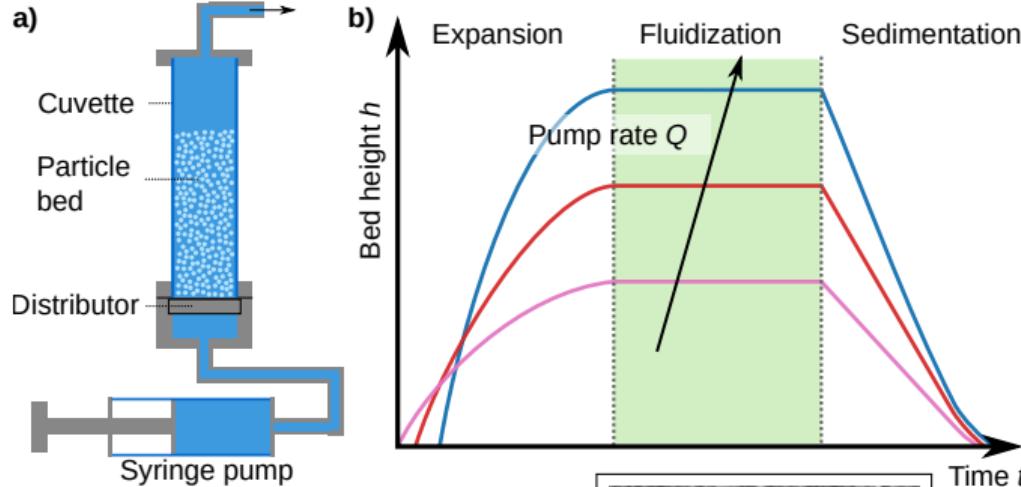
Experimental validation of X-DFA: A suspension of sedimenting particles



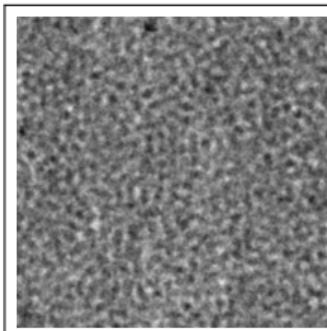
X-ray radiography



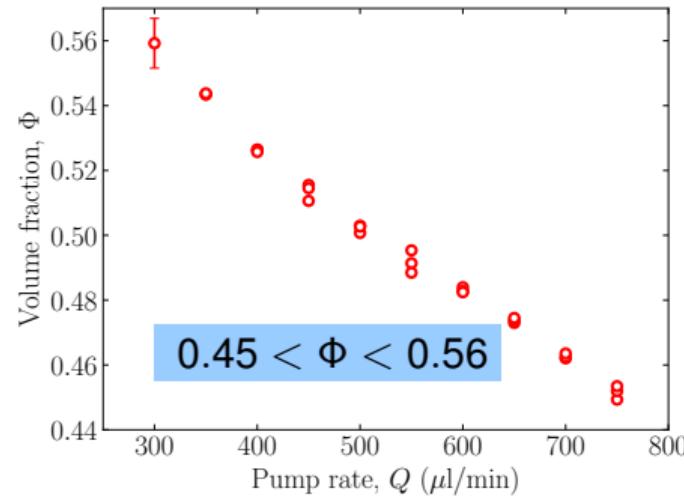
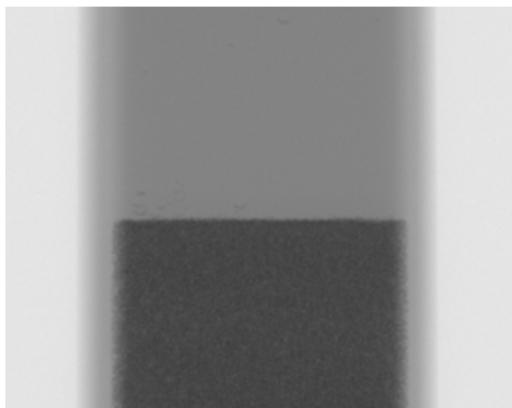
Experimental validation of X-DFA: A suspension of sedimenting particles



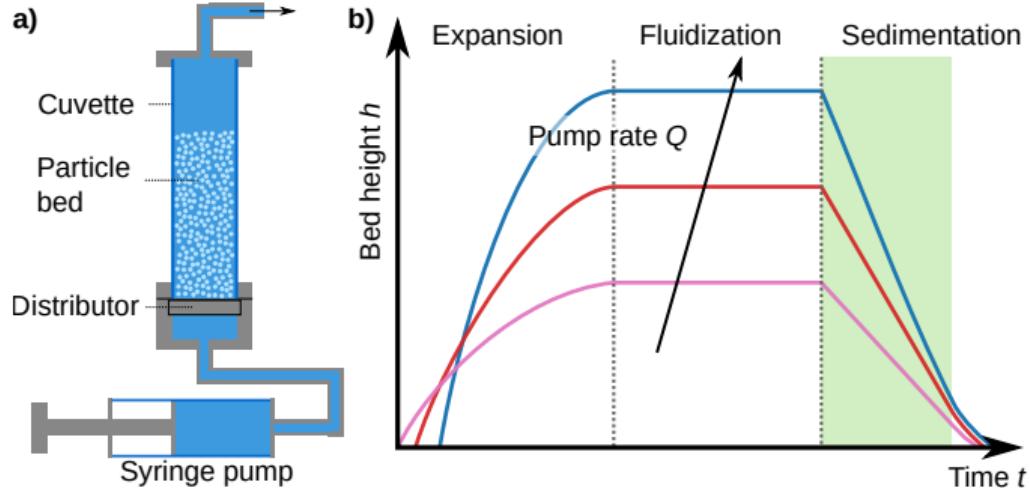
No reliable
reference
velocity!



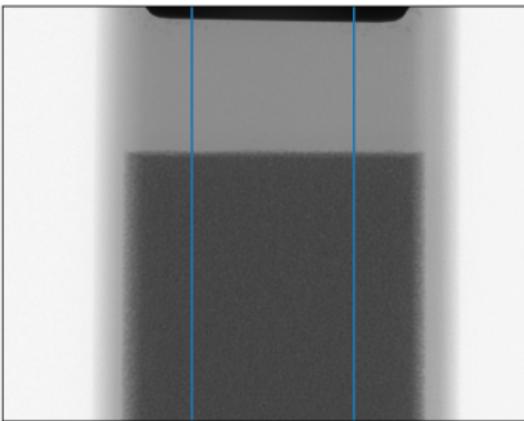
X-ray radiography



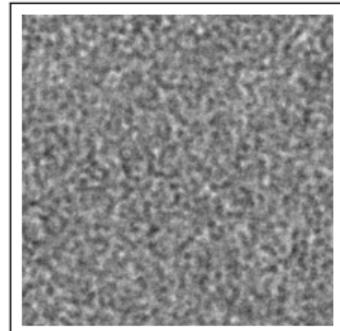
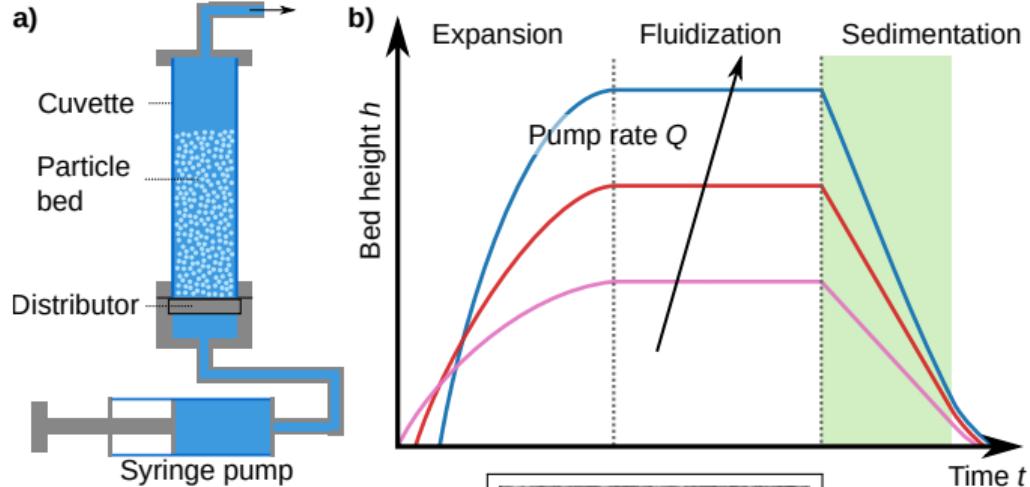
Experimental validation of X-DFA: A suspension of sedimenting particles



X-ray radiography

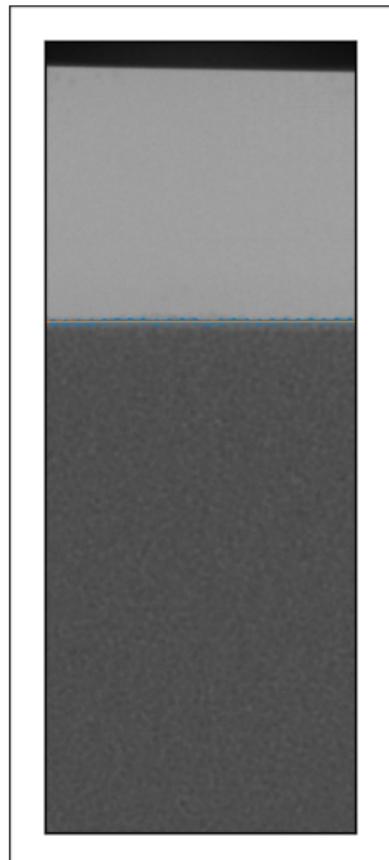


Experimental validation of X-DFA: A suspension of sedimenting particles

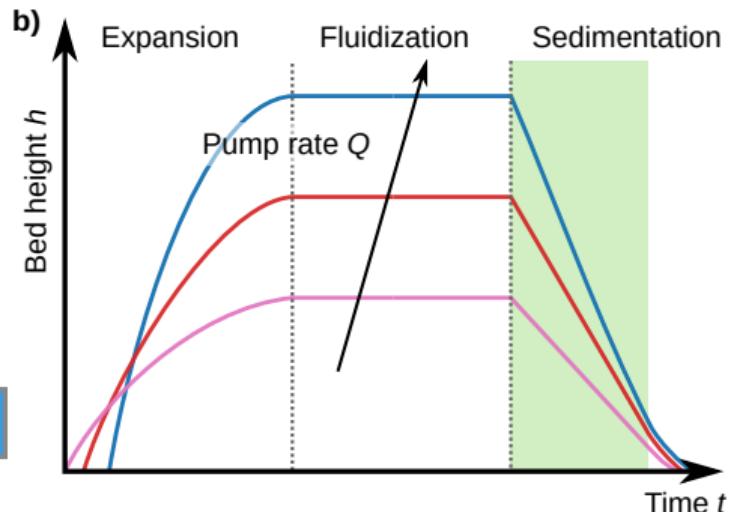
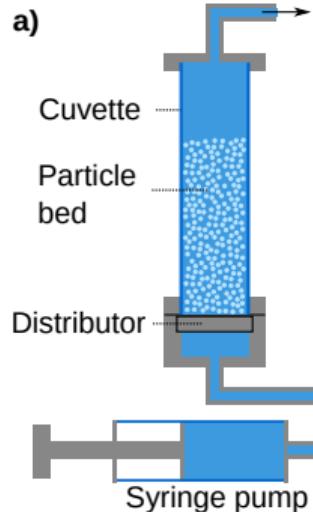


Comparison of
 $\langle v \rangle_{\text{dfa}}$ and $\langle v \rangle_{\text{front}}$

X-ray radiography

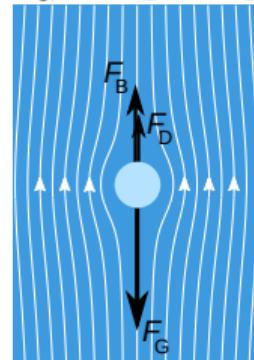


Liquid fluidized bed: Richardson-Zaki law



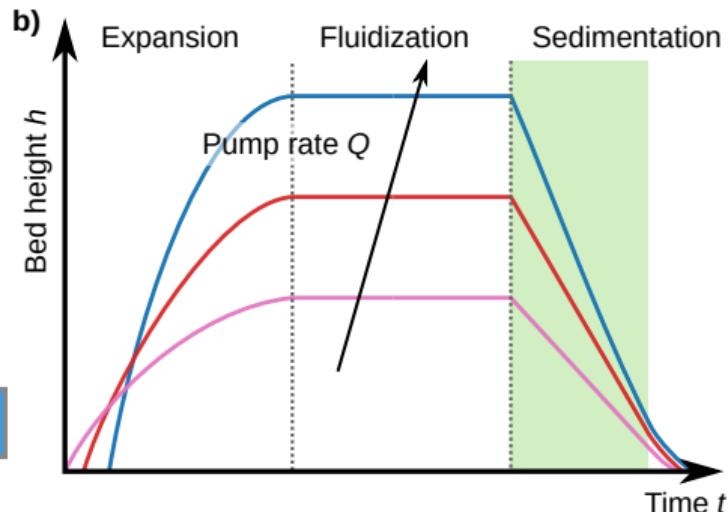
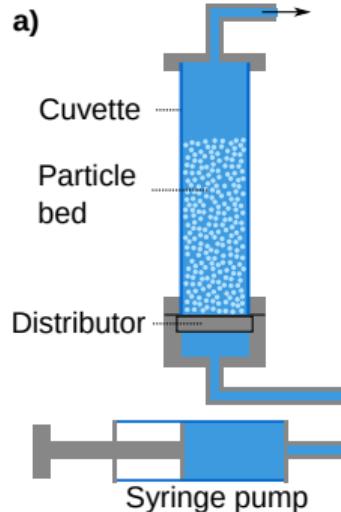
Gravitation Buoyancy Drag

$$F_G = F_B + F_D$$



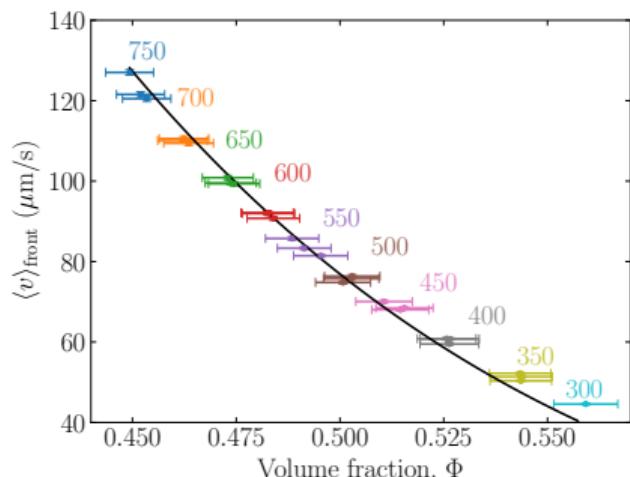
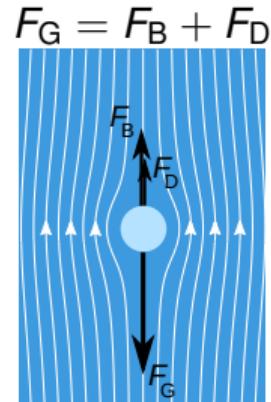
$$\frac{\langle v \rangle_{\text{fluid}}}{v_{\text{Stokes}}} = (1 - \Phi)^n$$

Liquid fluidized bed: Richardson-Zaki law

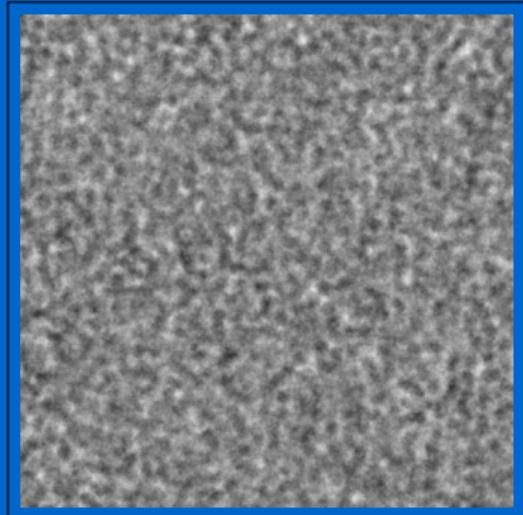
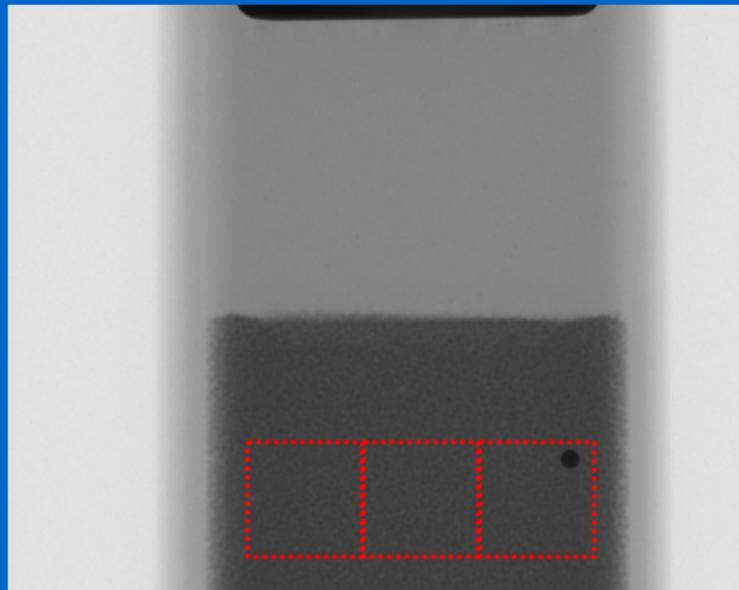


$$\frac{\langle v \rangle_{\text{front}}}{v_{\text{Stokes}}} = (1 - \Phi)^n$$

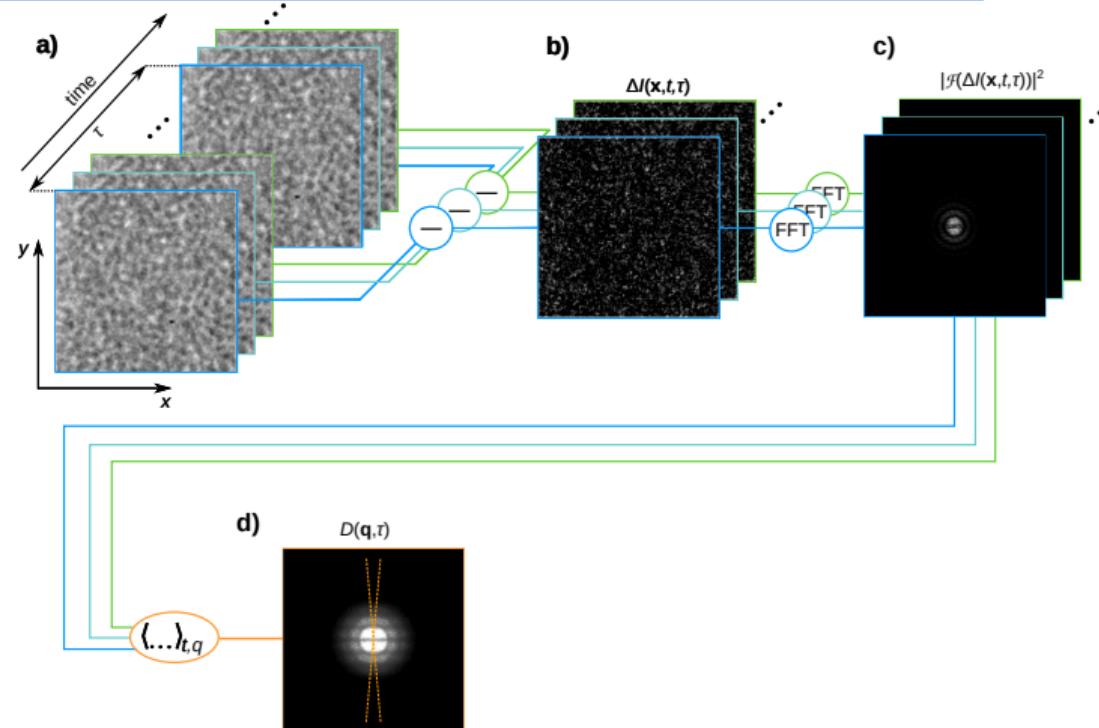
Gravitation Buoyancy Drag



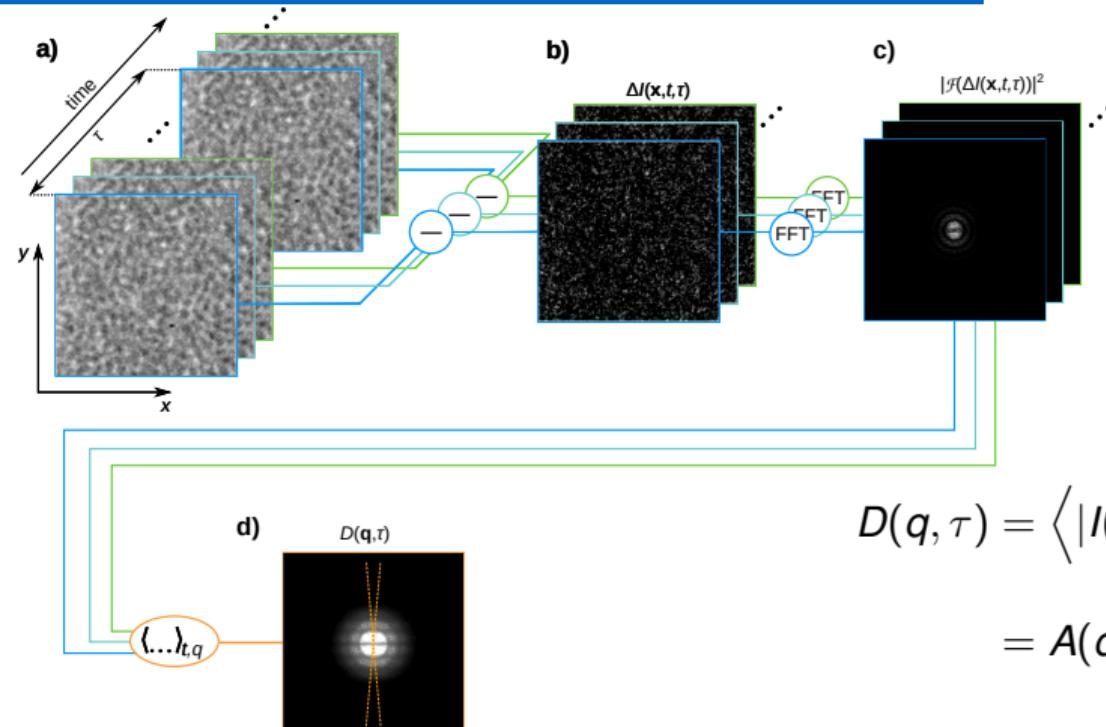
X-ray Digital Fourier Analysis of a suspension of sedimenting particles



The image structure function $D(\mathbf{q}, \tau)$



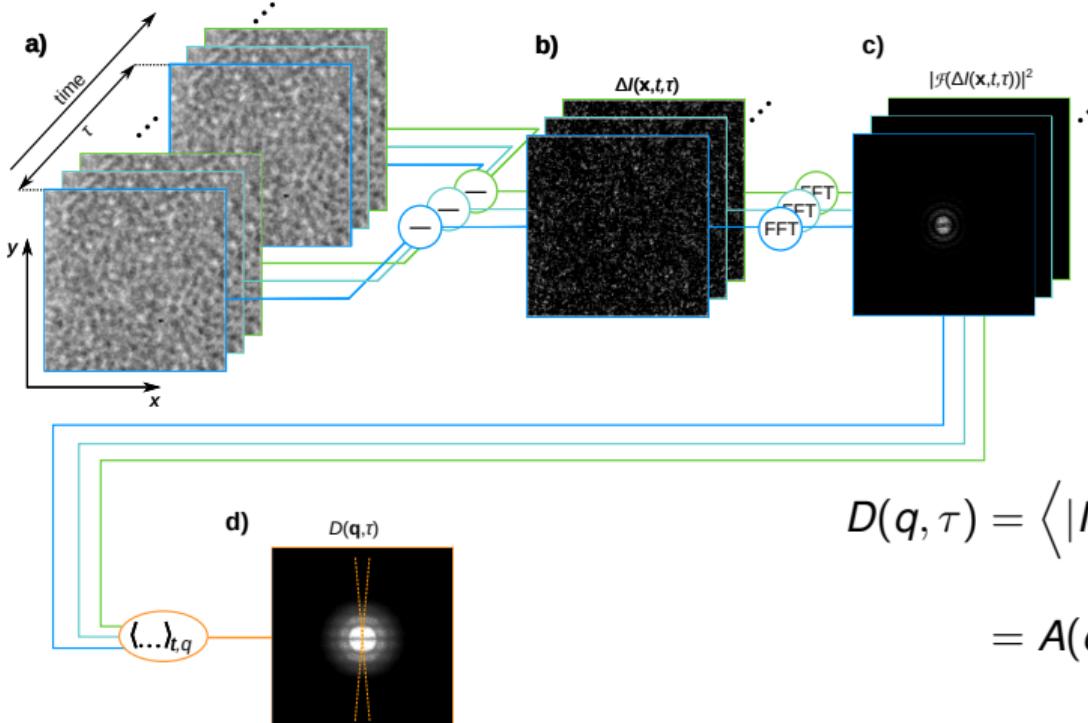
The image structure function $D(\mathbf{q}, \tau)$



$$\begin{aligned} D(\mathbf{q}, \tau) &= \left\langle |I(\mathbf{q}, t + \tau) - I(\mathbf{q}, t)|^2 \right\rangle_t \\ &= A(\mathbf{q}) \left[1 - \frac{\langle I^*(\mathbf{q}, t) I(\mathbf{q}, t + \tau) \rangle_t}{\langle |I(\mathbf{q}, t)|^2 \rangle_t} \right] + B(\mathbf{q}) \end{aligned}$$

- $D(\mathbf{q}, \tau \rightarrow 0) = B(\mathbf{q})$
- $D(\mathbf{q}, \tau \rightarrow \infty) = A(\mathbf{q}) + B(\mathbf{q})$

The image structure function $D(\mathbf{q}, \tau)$



Linear space invariant imaging

$$f(q, \tau) = \frac{\langle \rho^*(\mathbf{q}, t)\rho(\mathbf{q}, t + \tau) \rangle_t}{\langle |\rho(\mathbf{q}, t)|^2 \rangle_t}$$

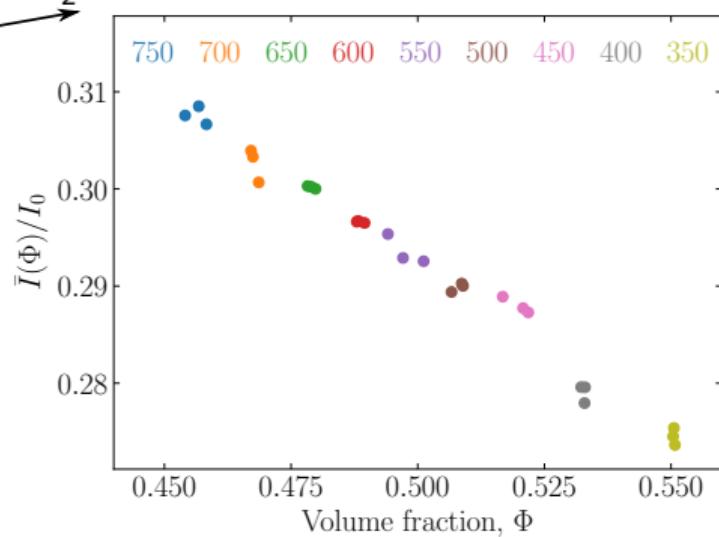
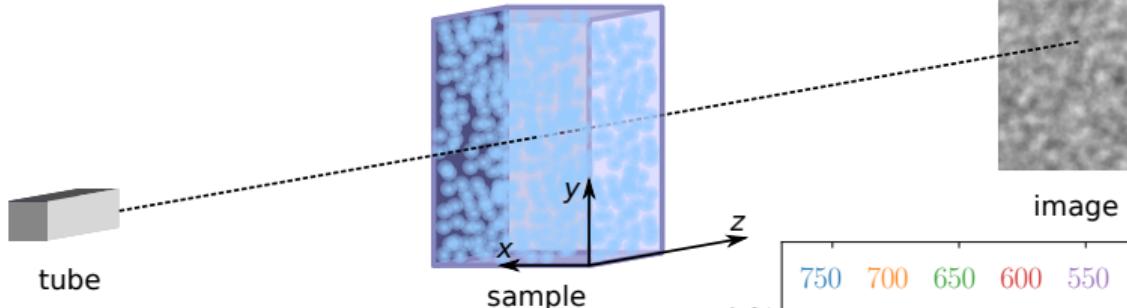
Intermediate scattering function

$$\begin{aligned} D(\mathbf{q}, \tau) &= \left\langle |I(\mathbf{q}, t + \tau) - I(\mathbf{q}, t)|^2 \right\rangle_t \\ &= A(\mathbf{q}) \left[1 - \frac{\langle I^*(\mathbf{q}, t)I(\mathbf{q}, t + \tau) \rangle_t}{\langle |I(\mathbf{q}, t)|^2 \rangle_t} \right] + B(\mathbf{q}) \end{aligned}$$

- $D(\mathbf{q}, \tau \rightarrow 0) = B(\mathbf{q})$
- $D(\mathbf{q}, \tau \rightarrow \infty) = A(\mathbf{q}) + B(\mathbf{q})$

X-ray radiography: Linear space invariant?

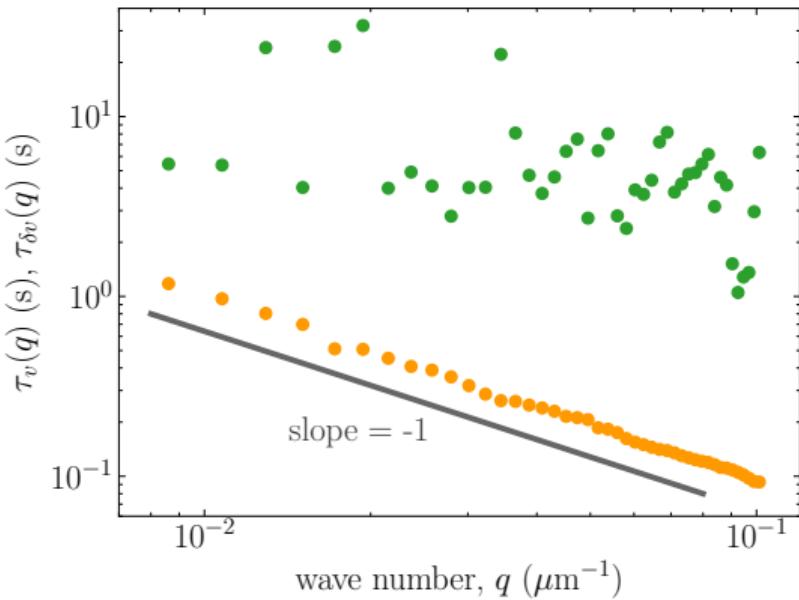
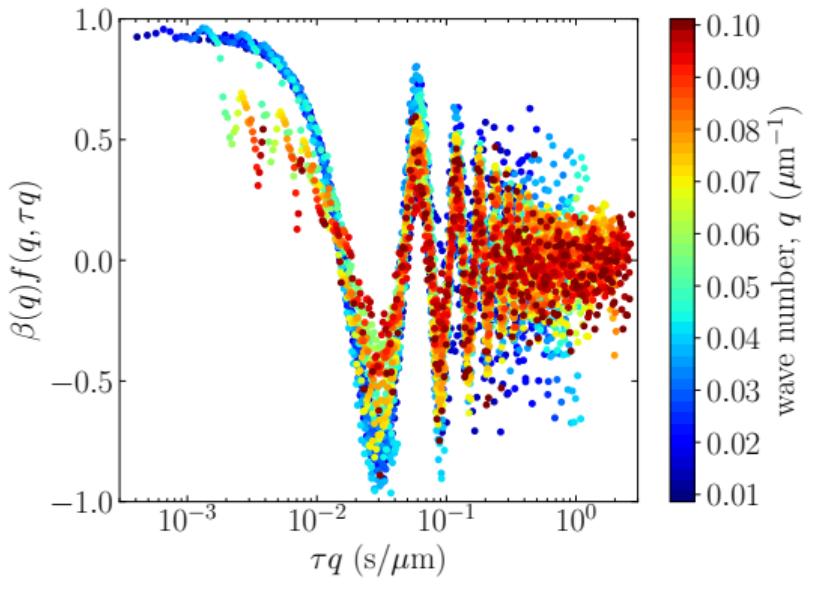
$$I(\mathbf{r}, t) = I_0 + \int d\mathbf{r}' dz' T(\mathbf{r} - \mathbf{r}', -z') c(\mathbf{r}', z', t)$$



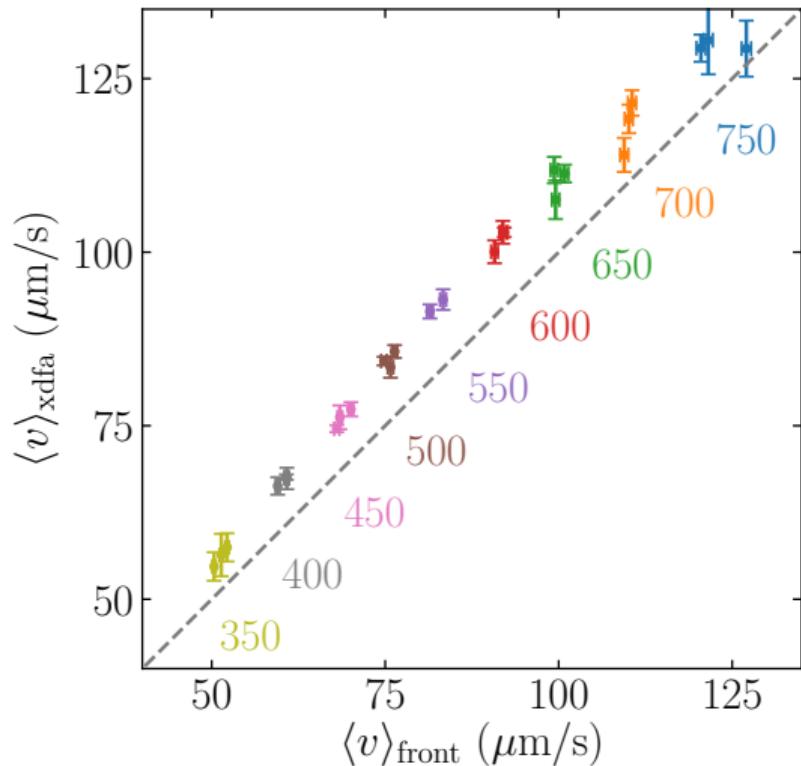
X-DFA for a suspension of sedimenting particles

$$f(q, \tau) = \cos(q\langle v_s \rangle \tau) \exp\left(-\frac{1}{2}q^2 \delta v^2 \tau^2\right)$$

$$\langle v_s \rangle = \langle \Delta r \rangle / \tau_\nu, \langle \delta v \rangle = \langle \delta r \rangle / \tau_{\delta\nu}$$

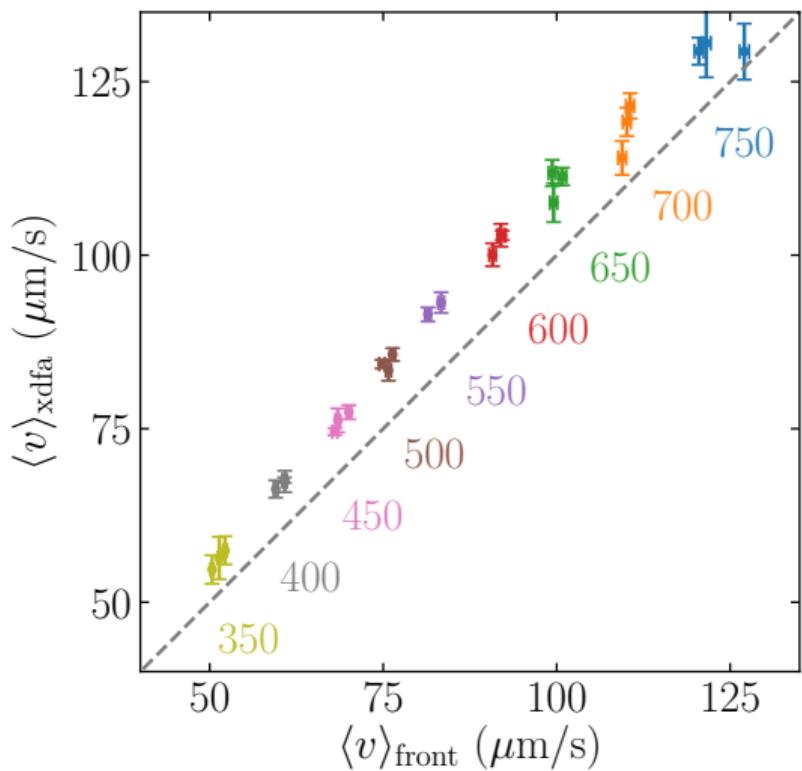


Front tracking vs. X-DFA

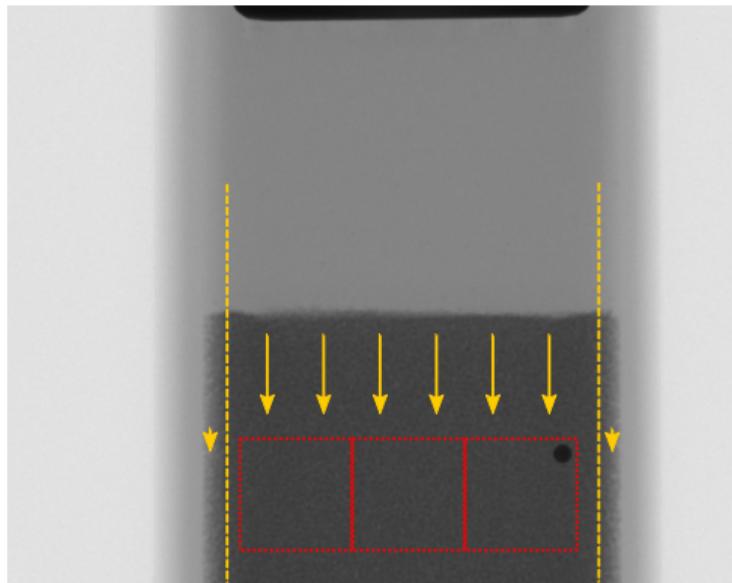


$\langle v \rangle_{\text{xdfa}} > \langle v \rangle_{\text{front}}$ by 9.4%

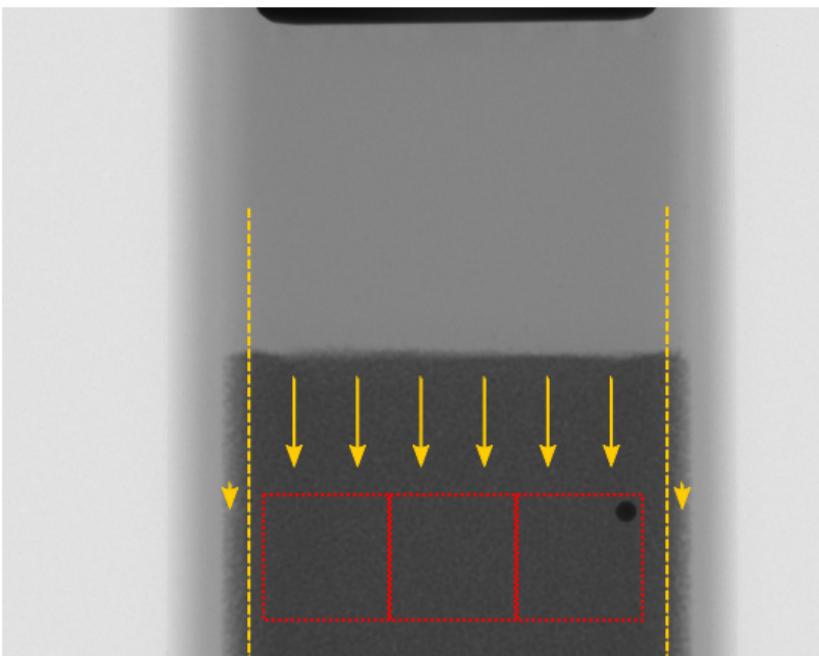
Front tracking vs. X-DFA



$\langle v \rangle_{\text{xdfa}} > \langle v \rangle_{\text{front}}$ by 9.4%

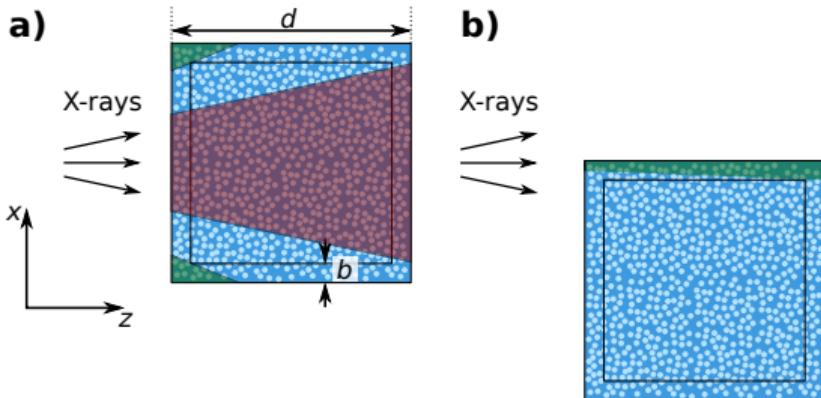


Estimate width of boundary layer



$\langle v \rangle_{\text{xdfa}} > \langle v \rangle_{\text{front}}$ by 9.4%

$\langle v \rangle_{\text{xdfa}}$ takes two layers into account
 $\langle v \rangle_{\text{front}}$ takes four layers into account



Estimation:

Boundary velocity = 0

Else = const.

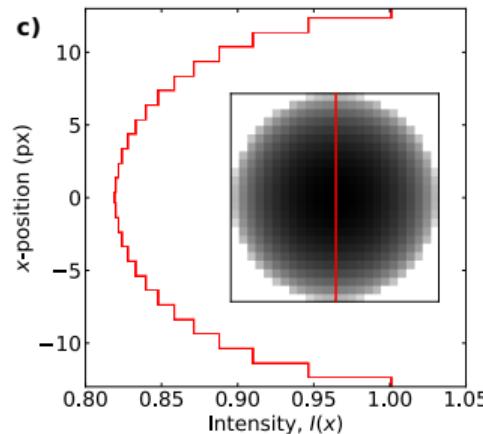
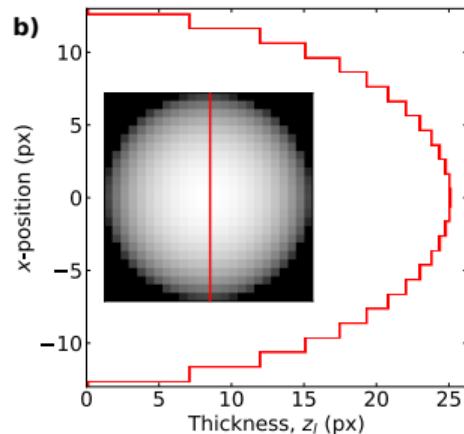
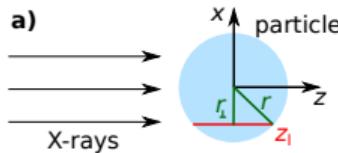
→ $b \approx 3$ particle diameters

Thank you for your attention!



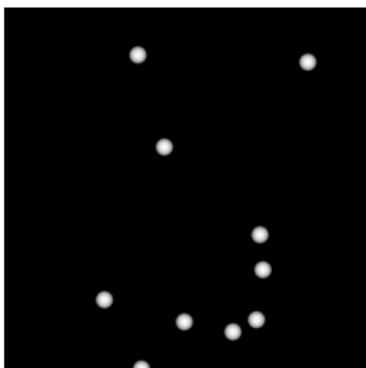
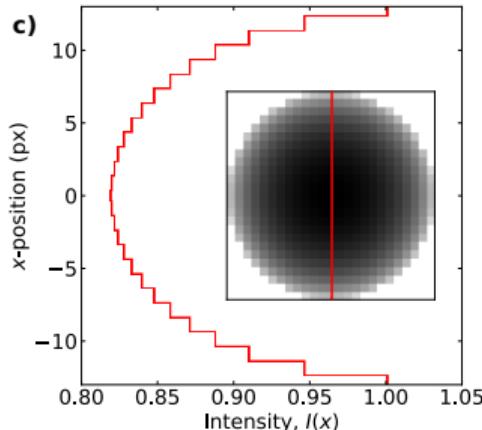
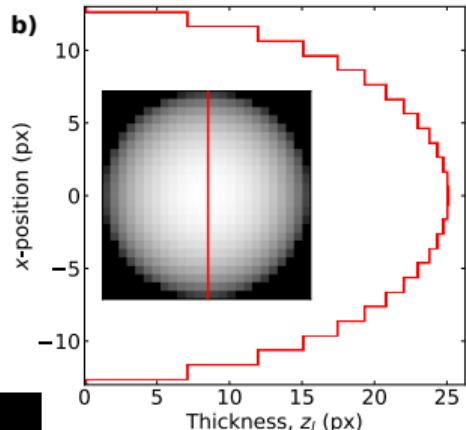
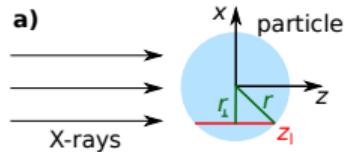
Backup slides

Synthetic radiograms

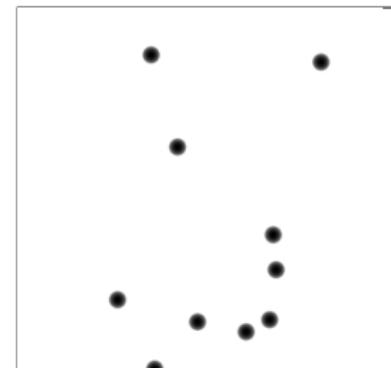


Beer-Lambert
 $I(z_I) = I_0 \exp(-\mu z)$

Synthetic radiograms



Beer-Lambert
 $I(z_I) = I_0 \exp(-\mu z)$



Linear space invariant imaging

Image correlation function

$$g(\mathbf{q}, \tau) = \frac{\langle I^*(\mathbf{q}, t)I(\mathbf{q}, t + \tau) \rangle_t}{\langle |I(\mathbf{q}, t)|^2 \rangle_t}$$

Linear space invariant imaging

Image correlation function

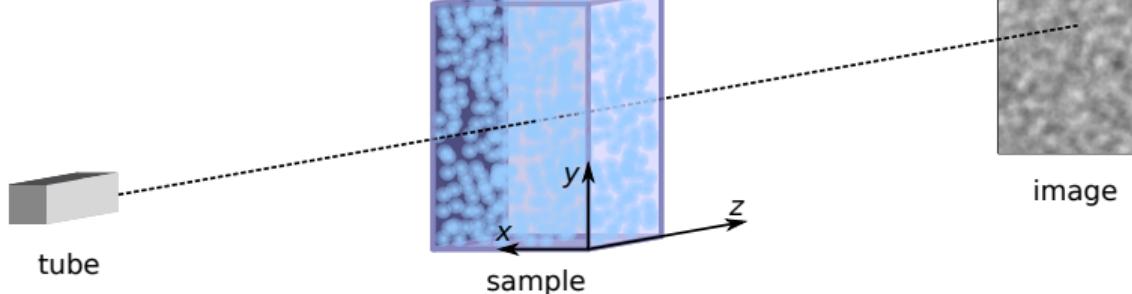
$$g(\mathbf{q}, \tau) = \frac{\langle I^*(\mathbf{q}, t)I(\mathbf{q}, t + \tau) \rangle_t}{\langle |I(\mathbf{q}, t)|^2 \rangle_t}$$

Intermediate scattering function

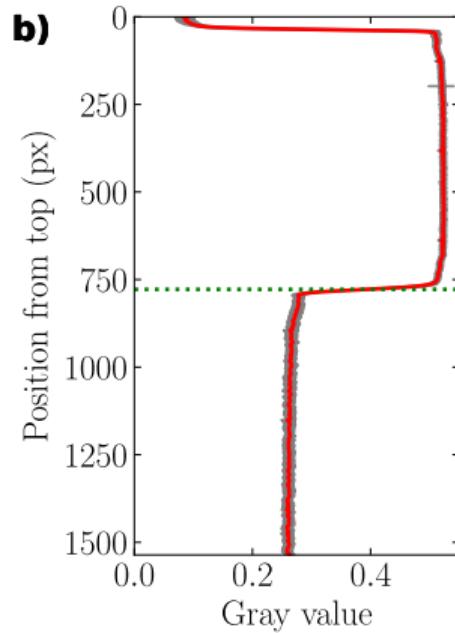
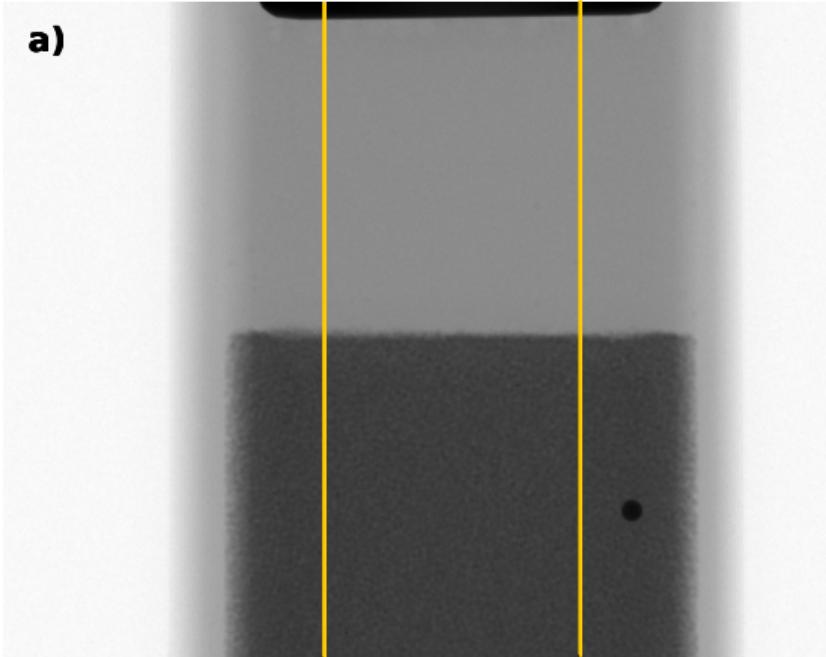
$$f(\mathbf{q}, \tau) = \frac{\langle \rho^*(\mathbf{q}, t)\rho(\mathbf{q}, t + \tau) \rangle_t}{\langle |\rho(\mathbf{q}, t)|^2 \rangle_t}$$

Linear space-invariant imaging:

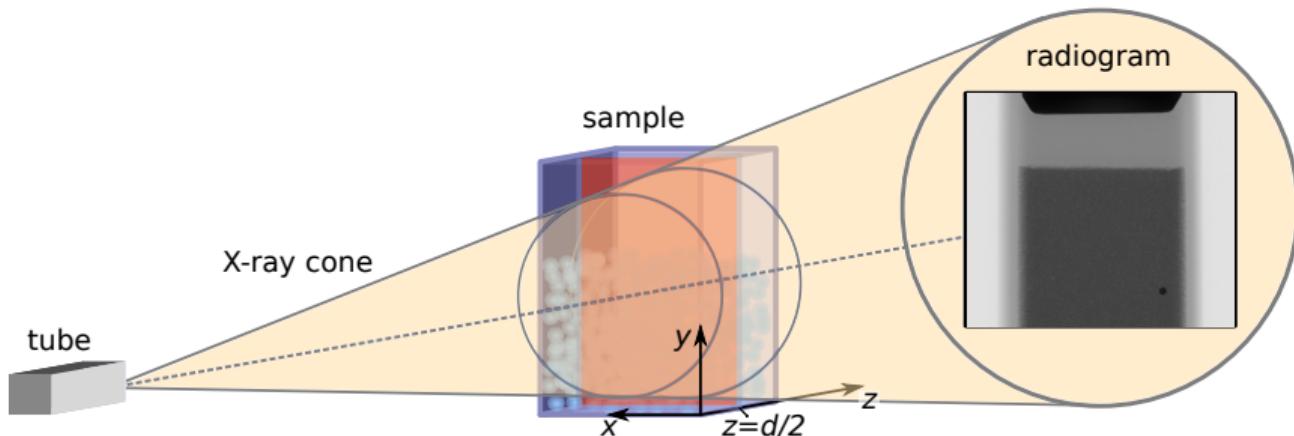
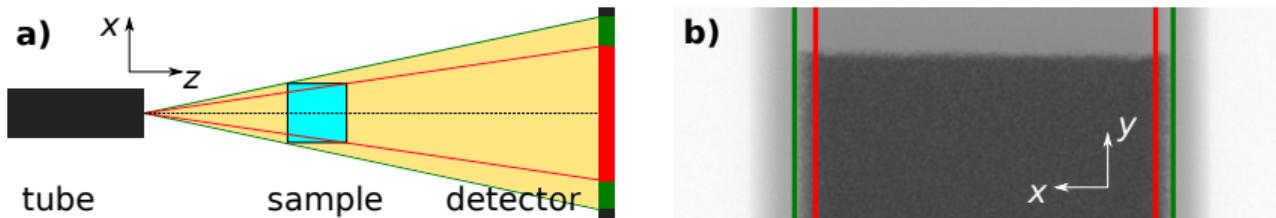
$$I(\mathbf{r}, t) = I_0 + \int d\mathbf{r}' dz' T(\mathbf{r} - \mathbf{r}', -z') c(\mathbf{r}', z', t)$$



Tracking of particle front



Tracking of particle front



Tracking of particle front

