

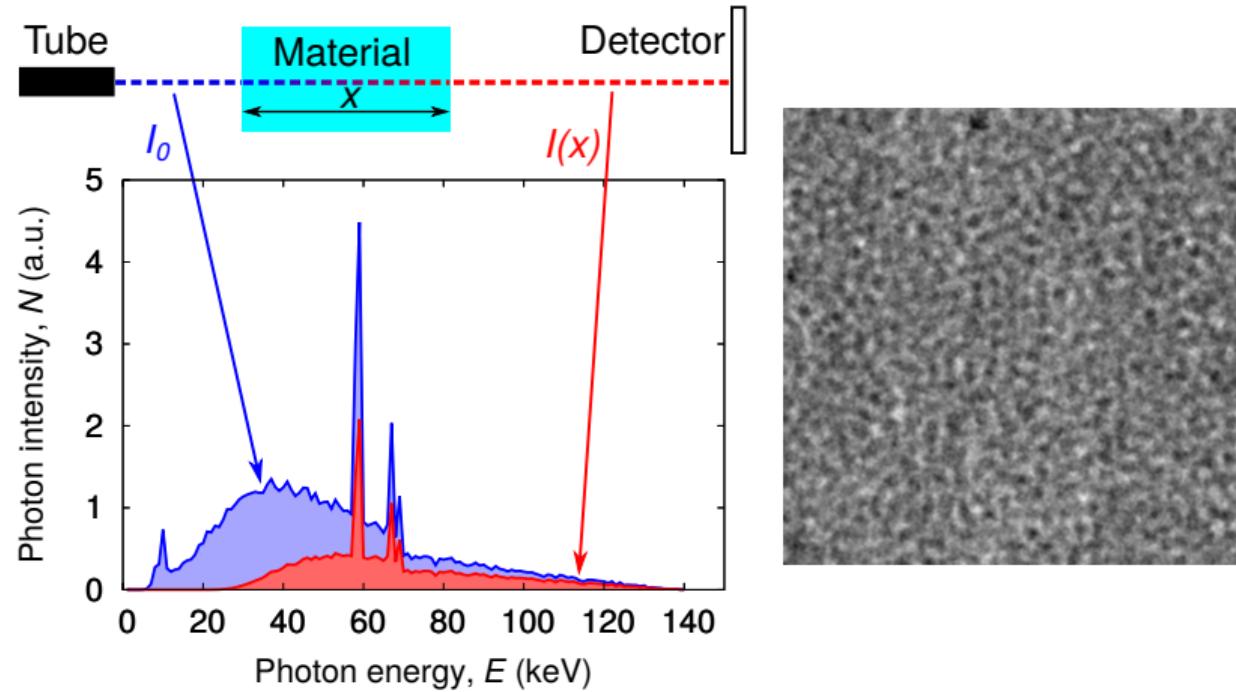


PhD defense
Manuel Baur

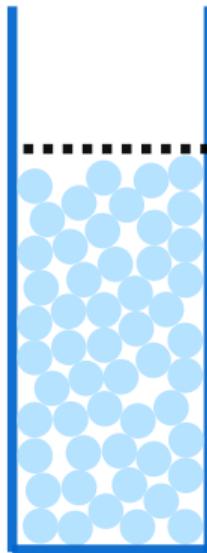
Funded by the German
Federal Ministry for
Economic Affairs and
Energy, grant no. 50WM

1653

X-ray radiography of granular systems – particle densities and dynamics

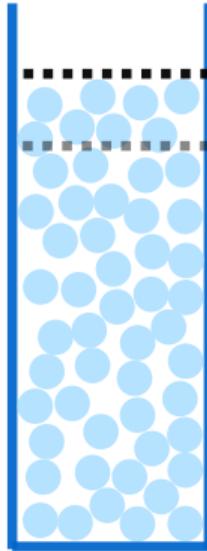


X-ray radiography of granular systems – particle densities and dynamics



$$\Phi = \text{RLP}$$

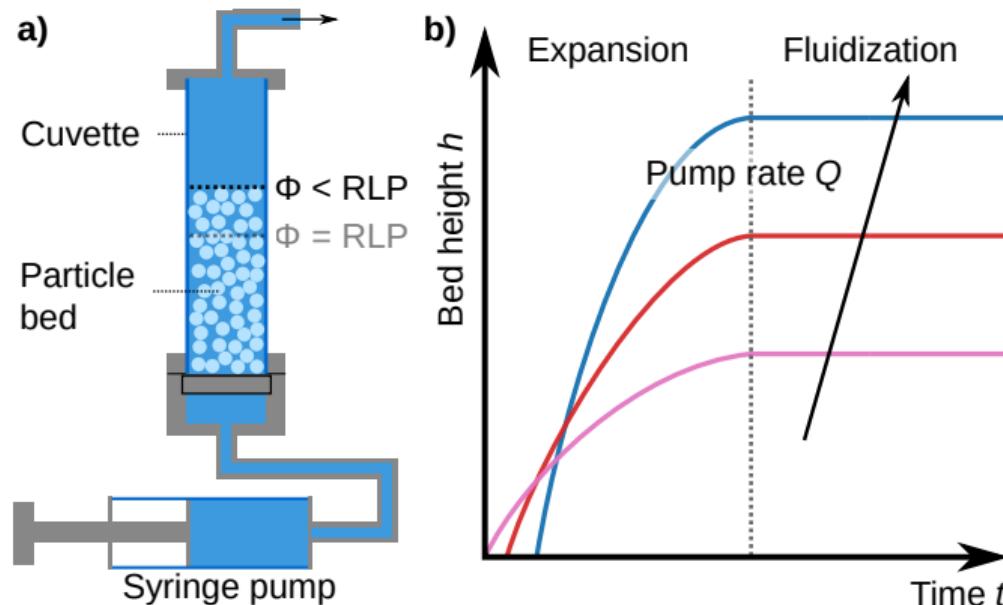
X-ray radiography of granular systems – particle densities and dynamics



$\Phi < \text{RLP}$
 $\Phi = \text{RLP}$

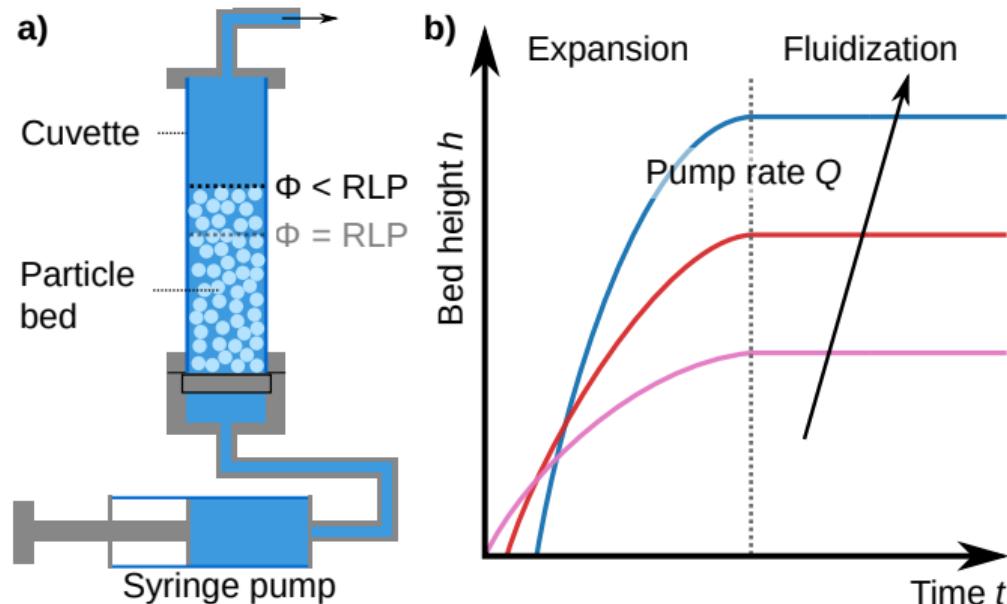
X-ray radiography of granular systems – particle densities and dynamics

Fluidized bed



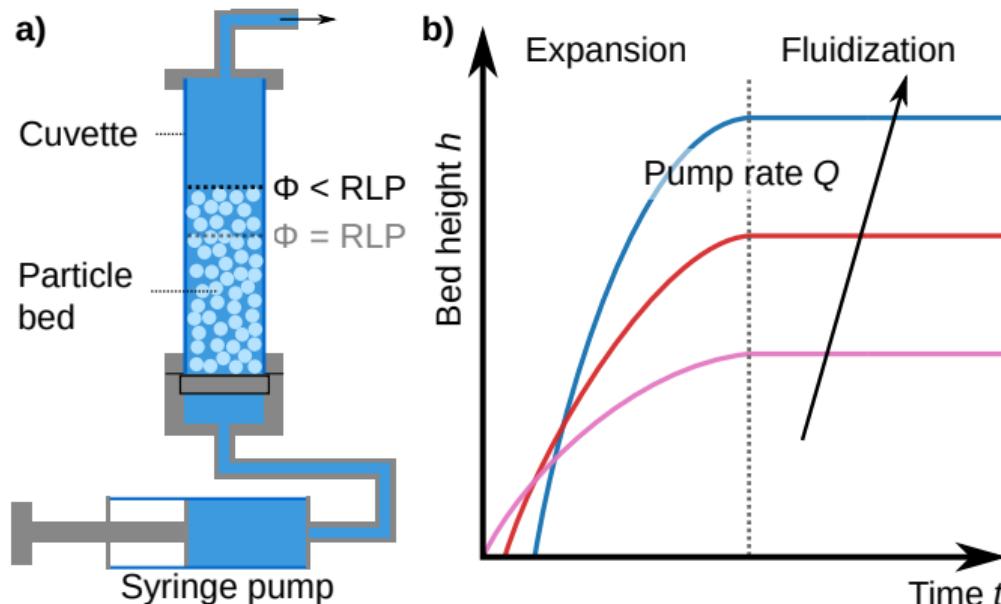
X-ray radiography of granular systems – particle densities and dynamics

Fluidized bed



X-ray radiography of granular systems – particle densities and dynamics

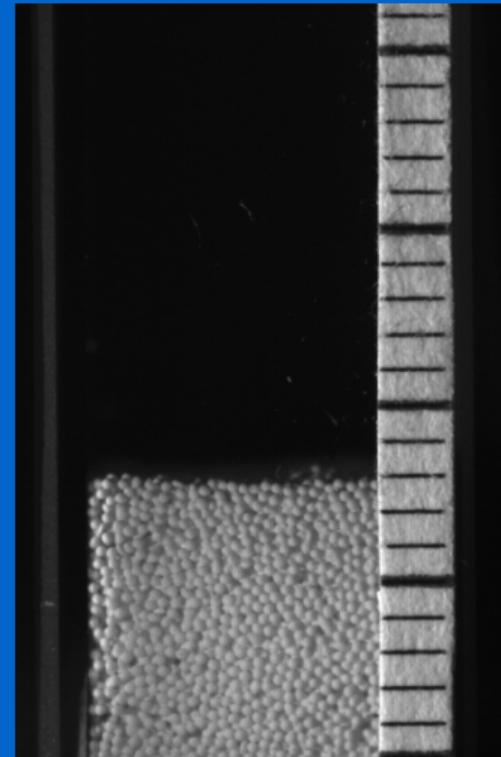
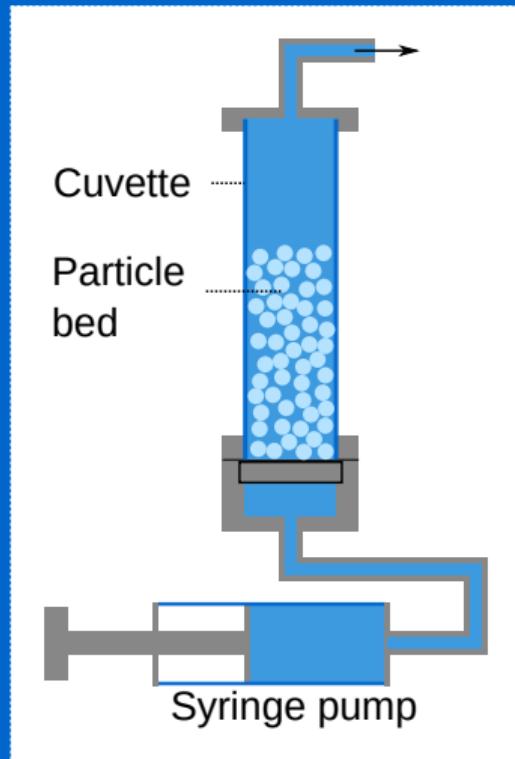
Fluidized bed



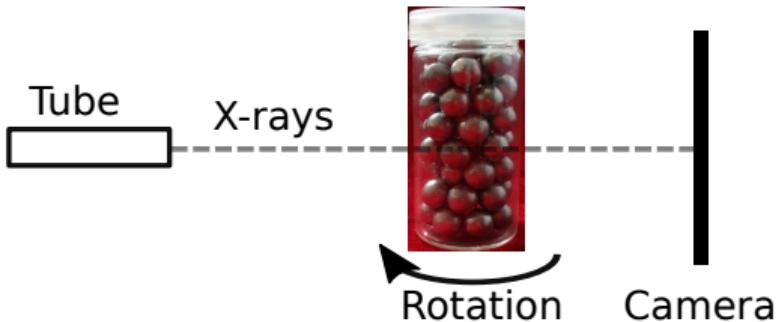
Fluidized bed reactor

"Lack of understanding:
It is very difficult to predict and calculate the **complex mass** [...] **flows** within the bed."

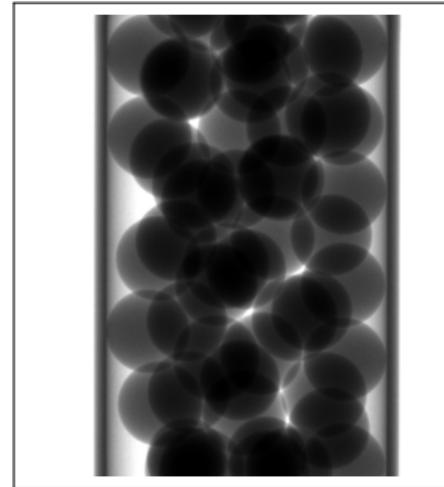
Particulate flows are **opaque**



X-ray radiography

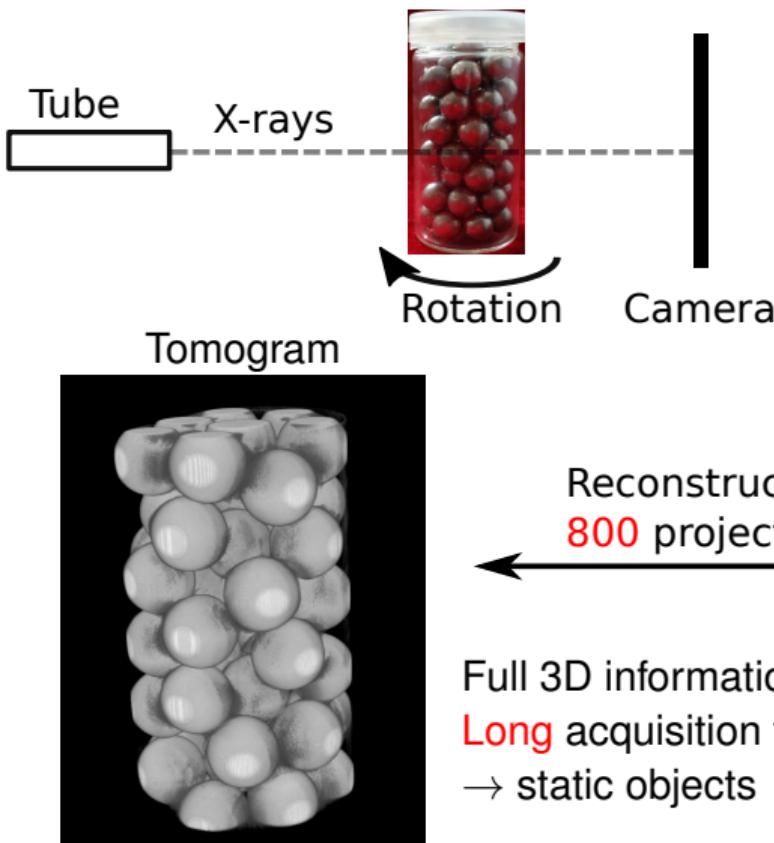


Radiogram

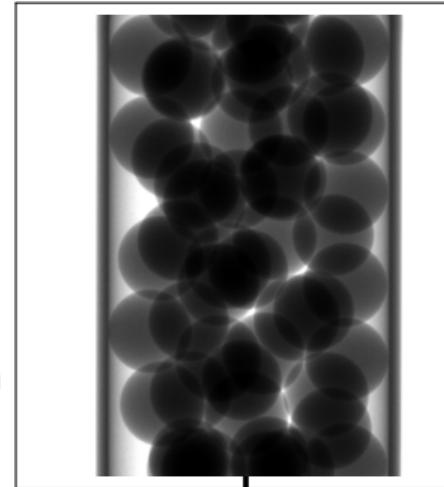


2D projections of 3D object
Short acquisition time

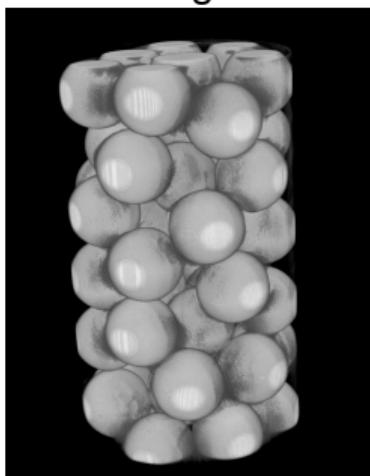
X-ray radiography



Radiogram



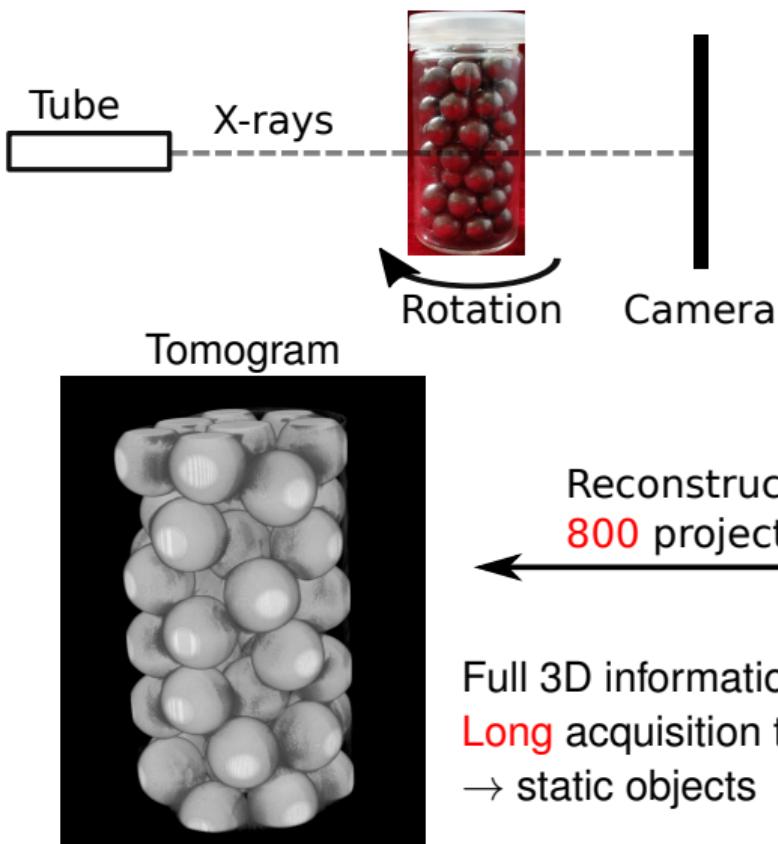
2D projections of 3D object
Short acquisition time



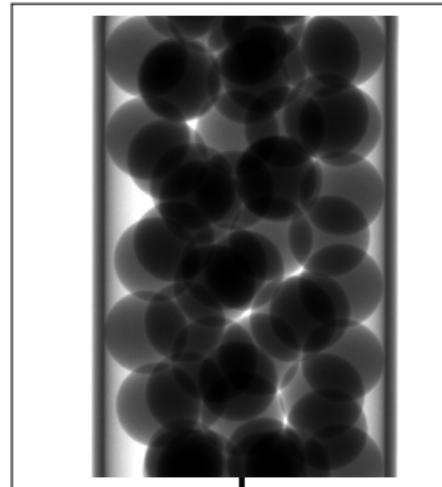
Reconstruction from
800 projections

Full 3D information
Long acquisition time
→ static objects

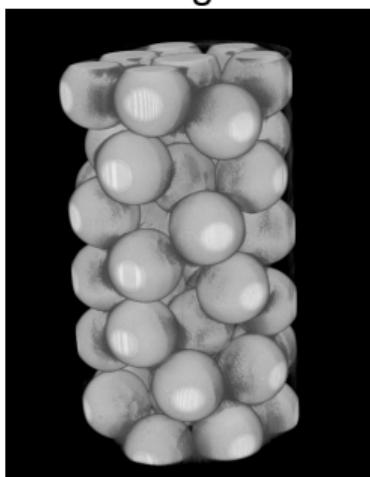
X-ray radiography



Radiogram

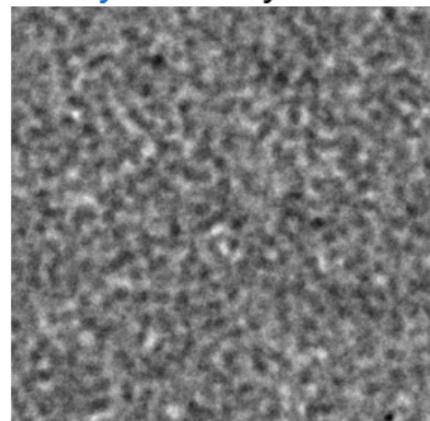


2D projections of 3D object
Short acquisition time

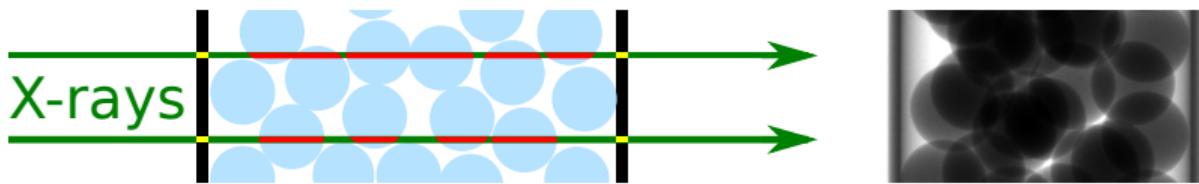


Full 3D information
Long acquisition time
→ static objects

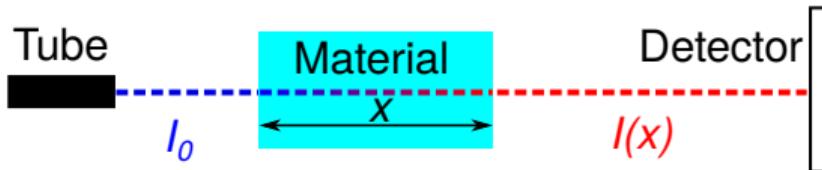
Dynamic system



Microscopy



Attenuation of X-rays

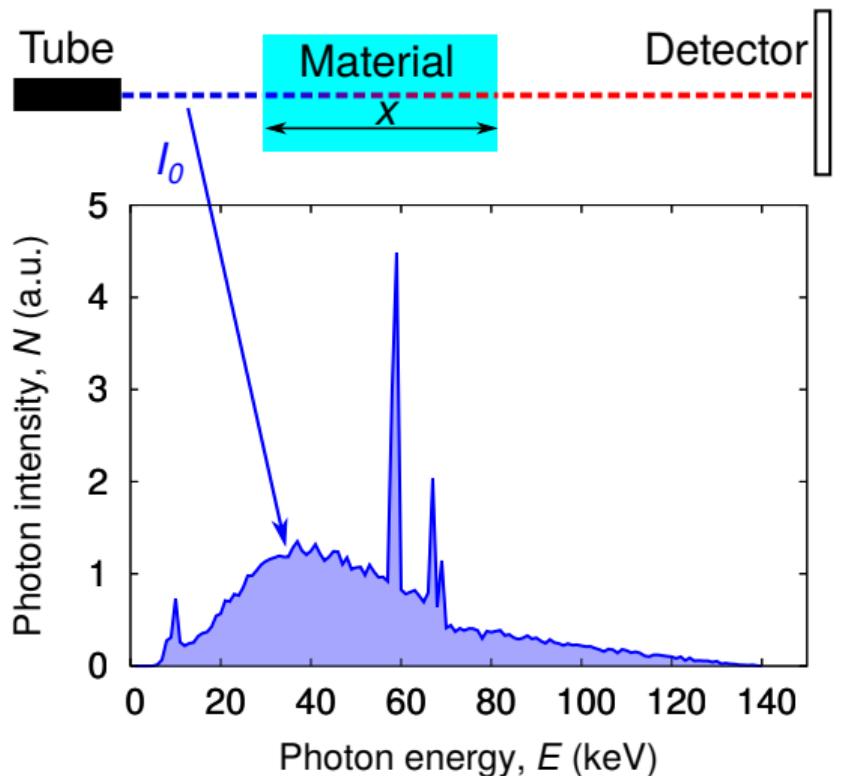


Beer-Lambert's law

$$I(x) = I_0 \exp(-\mu x)$$

$$\text{Thickness: } x = -\frac{1}{\mu} \ln \frac{I(x)}{I_0}$$

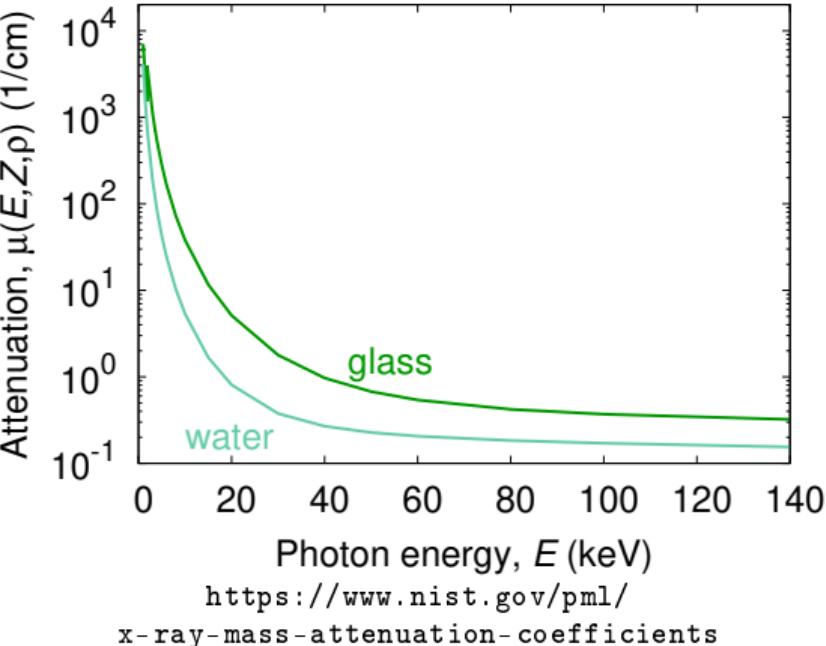
Attenuation of X-rays



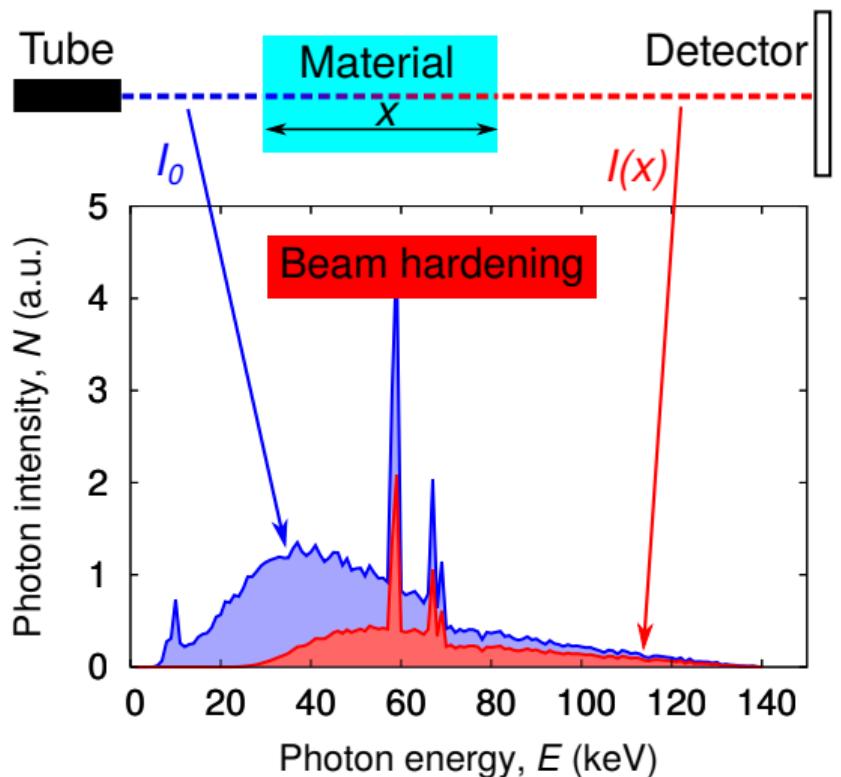
Beer-Lambert's law

$$\mu \neq \text{const}$$
$$I(x) = I_0 \exp(-\mu(E, Z, \rho)x)$$

Thickness: $x = ?$



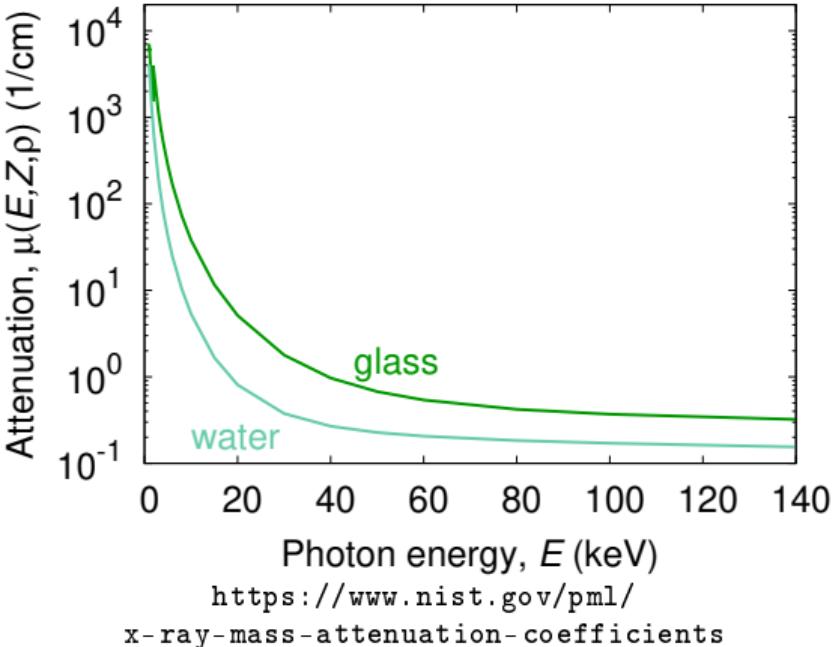
Attenuation of X-rays



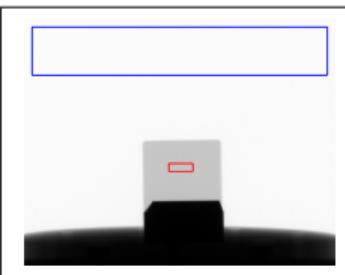
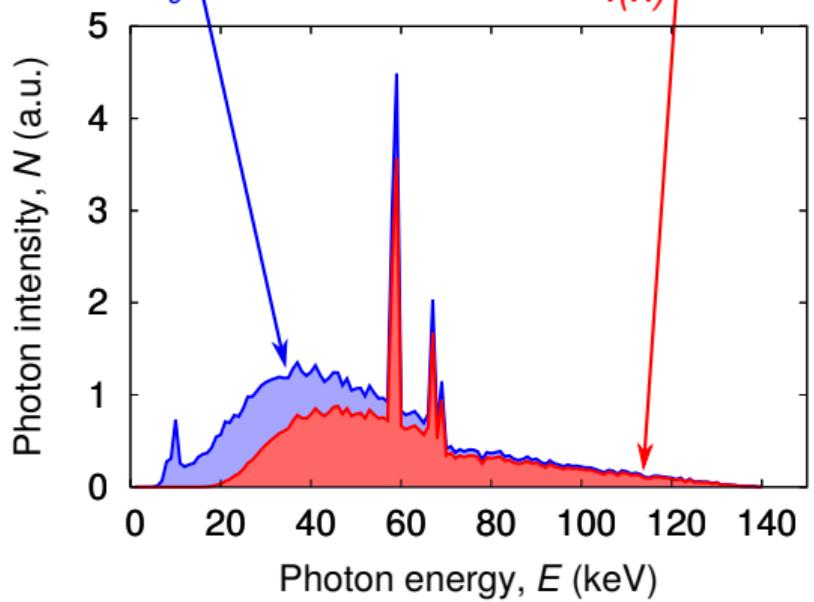
Beer-Lambert's law

$$\mu \neq \text{const}$$
$$I(x) = I_0 \exp(-\mu(E, Z, \rho)x)$$

Thickness: $x = ?$



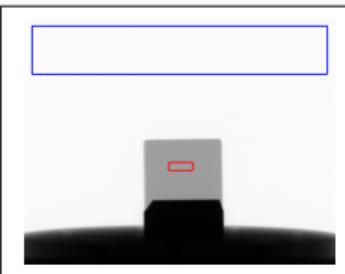
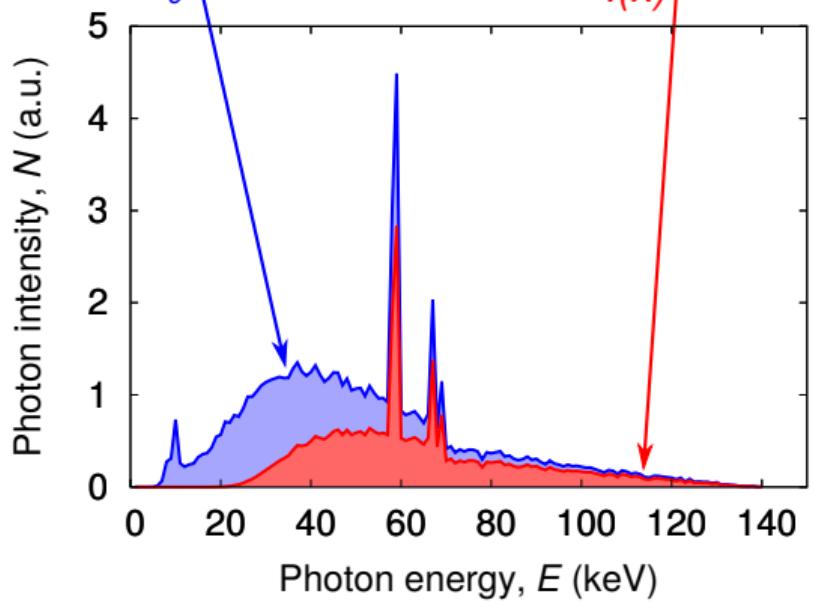
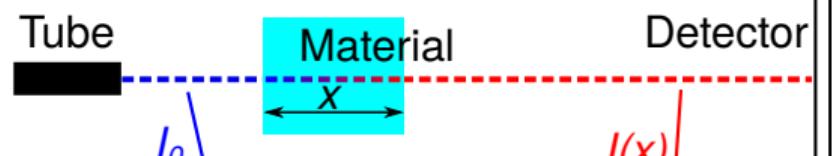
The effective attenuation, $\mu_{\text{eff}}(x)$



effective attenuation:

$$I(x) = I_0 \exp(-\mu_{\text{eff}}(x) x)$$

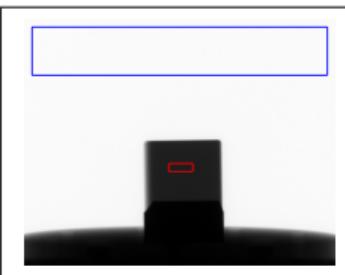
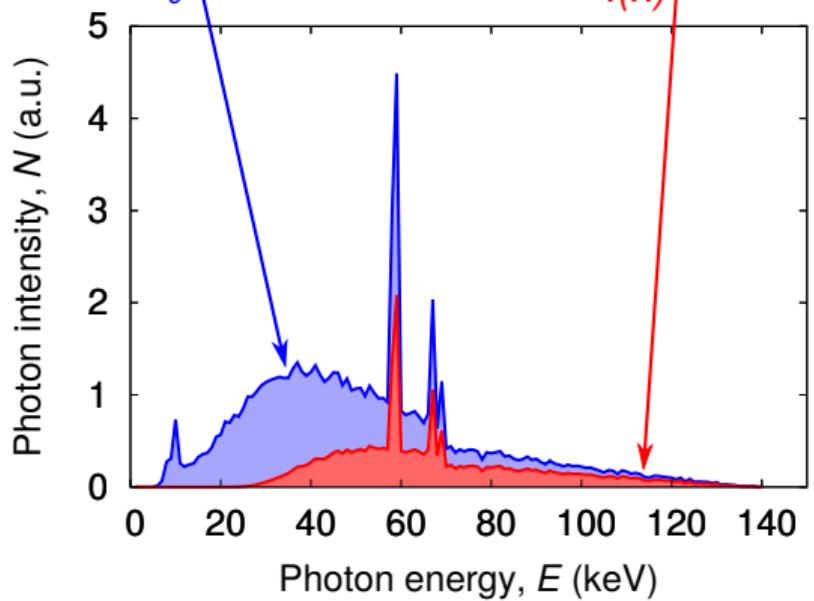
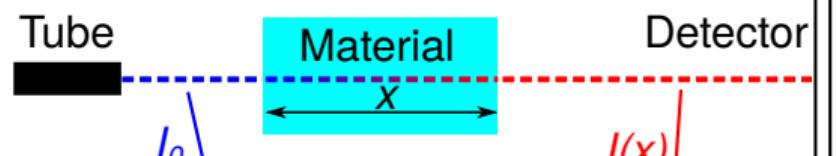
The effective attenuation, $\mu_{\text{eff}}(x)$



effective attenuation:

$$I(x) = I_0 \exp(-\mu_{\text{eff}}(x) x)$$

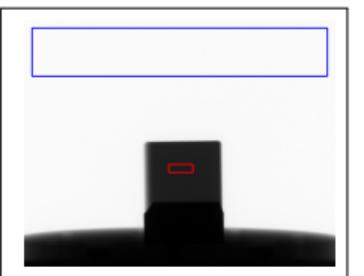
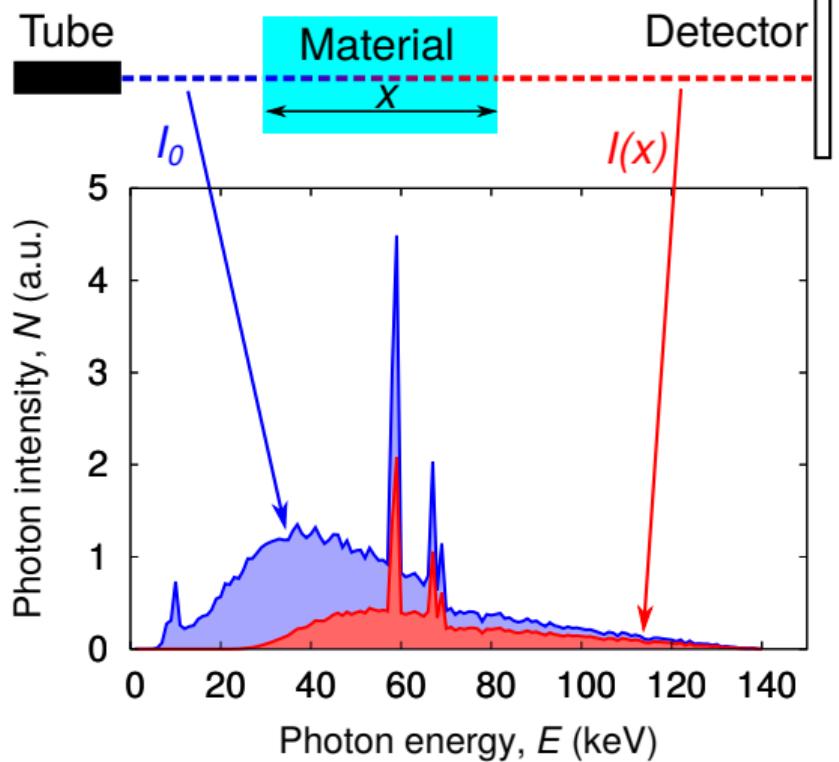
The effective attenuation, $\mu_{\text{eff}}(x)$



effective attenuation:

$$I(x) = I_0 \exp(-\mu_{\text{eff}}(x) x)$$

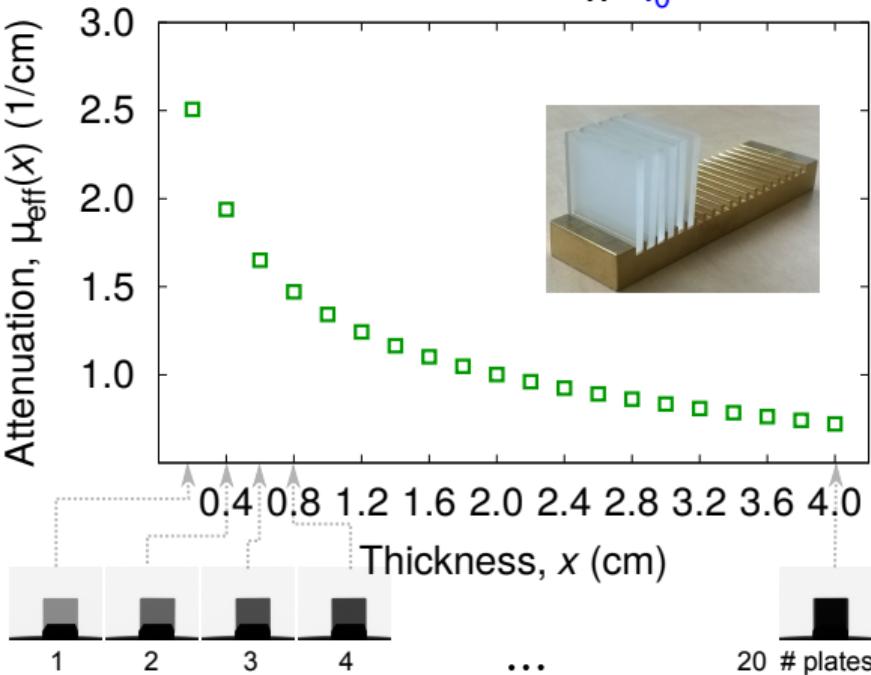
The effective attenuation, $\mu_{\text{eff}}(x)$



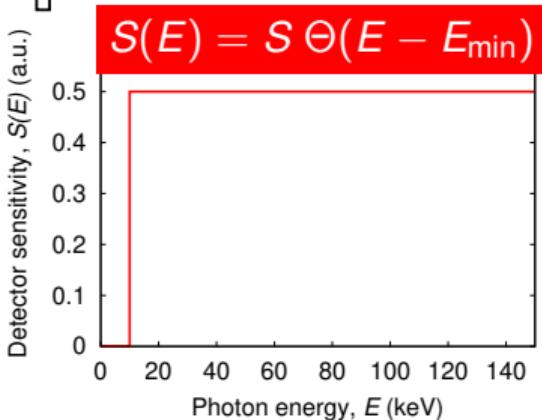
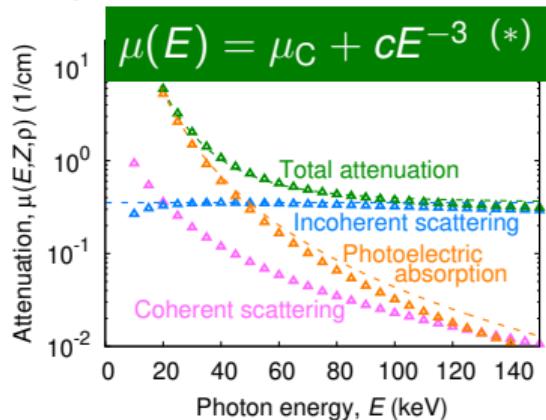
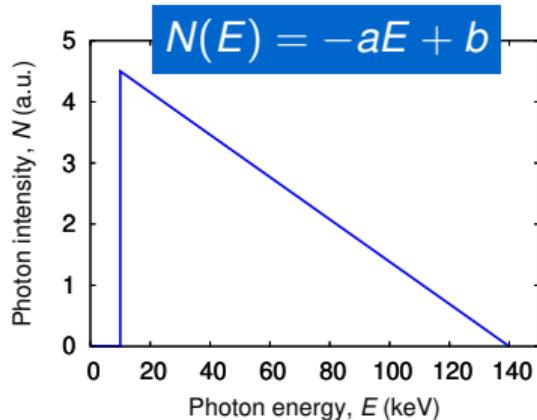
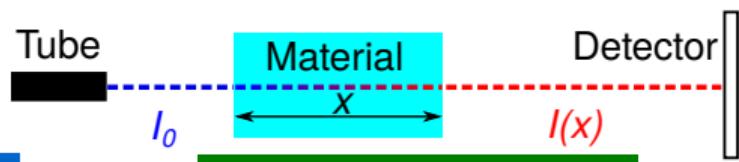
effective attenuation:

$$I(x) = I_0 \exp(-\mu_{\text{eff}}(x) x)$$

$$\mu_{\text{eff}}(x) = -\frac{1}{x} \frac{I(x)}{I_0}$$



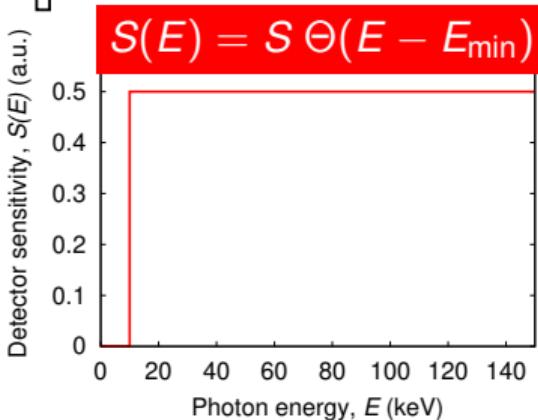
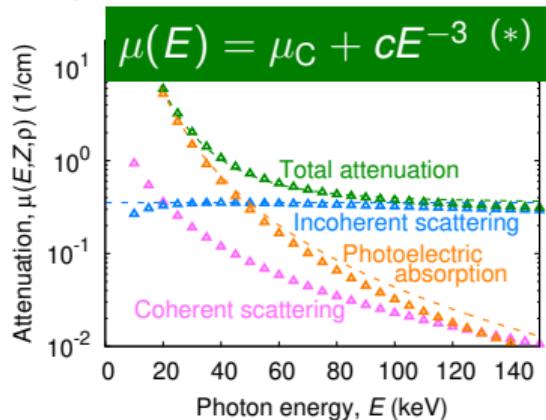
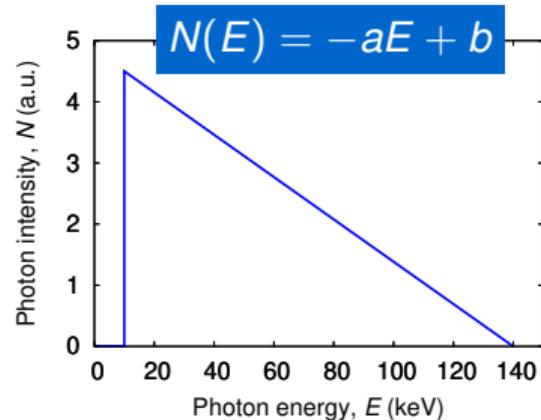
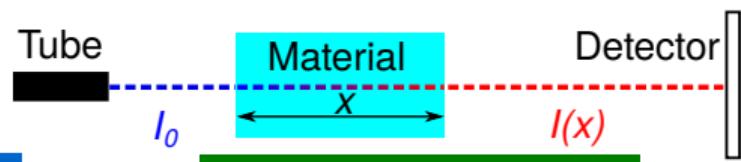
Modeling of $\mu_{\text{eff}}(x)$



$$I(x) \propto \int N(E) \exp\{-\mu(E, Z, \rho)x\} S(E) dE$$

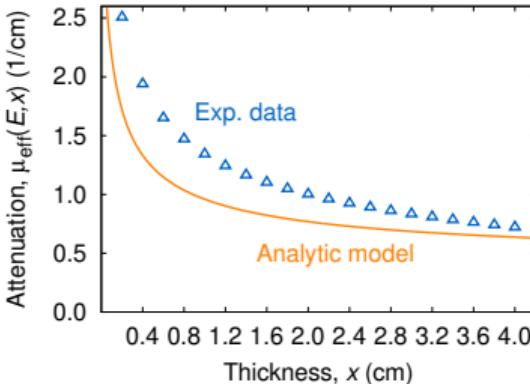
(*) XCOM supplied by NIST

Modeling of $\mu_{\text{eff}}(x)$



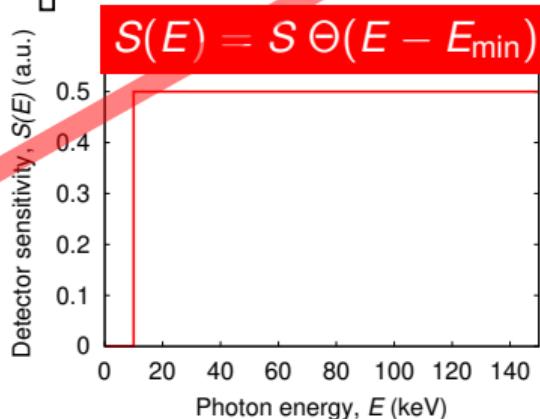
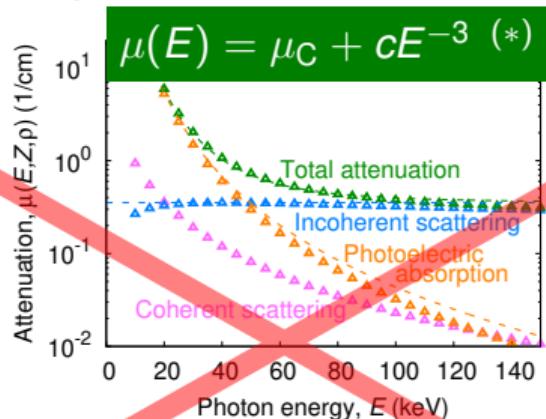
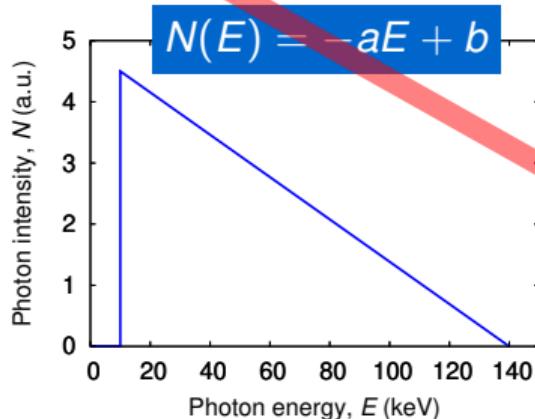
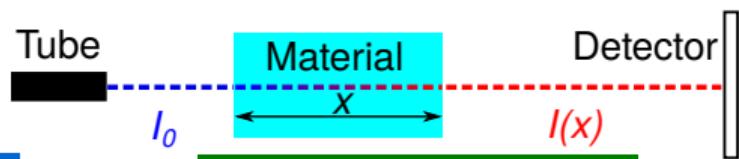
$$I(x) \propto \int N(E) \exp\{-\mu(E, Z, \rho)x\} S(E) dE$$

$$\propto S \int_{E_{\min}}^{E_{\max}} (-aE + b) \exp\{-(\mu_C + cE^{-3})x\} dE$$



(*) XCOM supplied by NIST

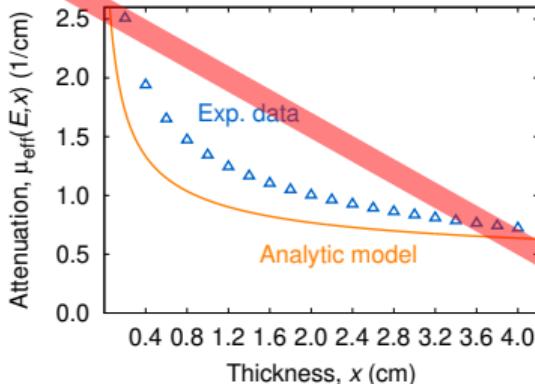
Modeling of $\mu_{\text{eff}}(x)$



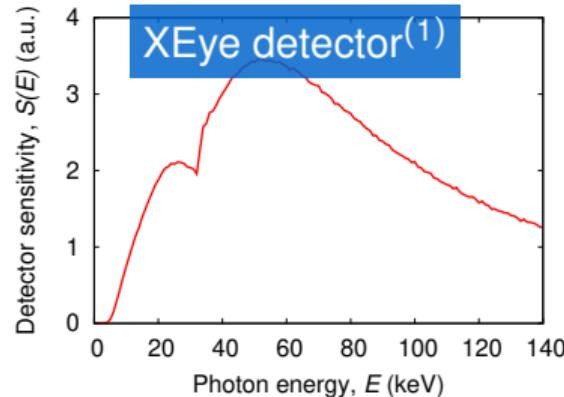
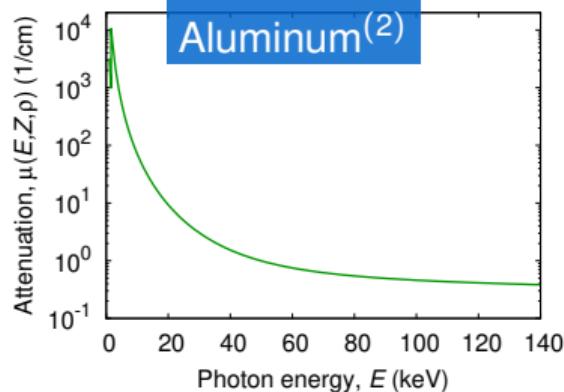
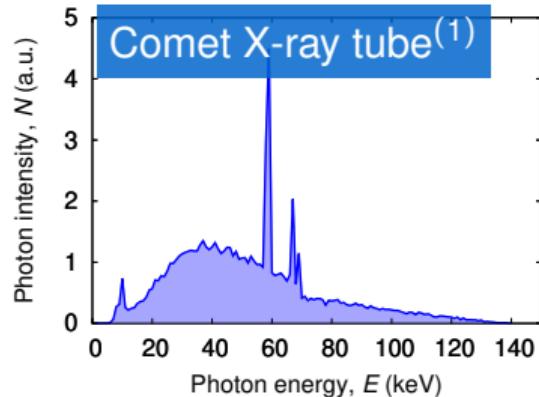
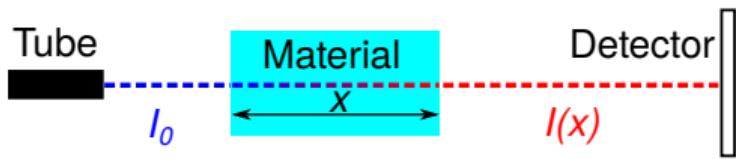
$$I(x) \propto \int N(E) \exp\{-\mu(E, Z, \rho)x\} S(E) dE$$

$$\propto S \int_{E_{\min}}^{E_{\max}} (-aE + b) \exp\{-(\mu_C + cE^{-3})x\} dE$$

(*) XCOM supplied by NIST



Numerical approx. of $\mu_{\text{eff}}(x)$



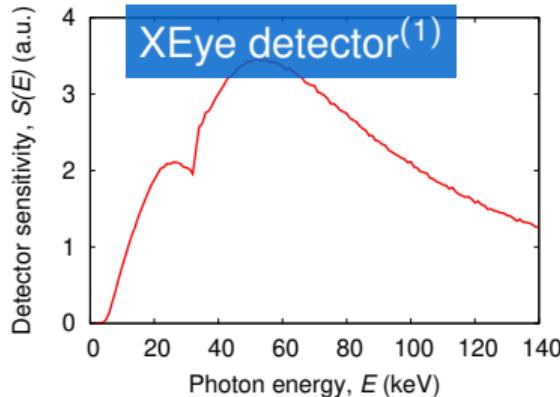
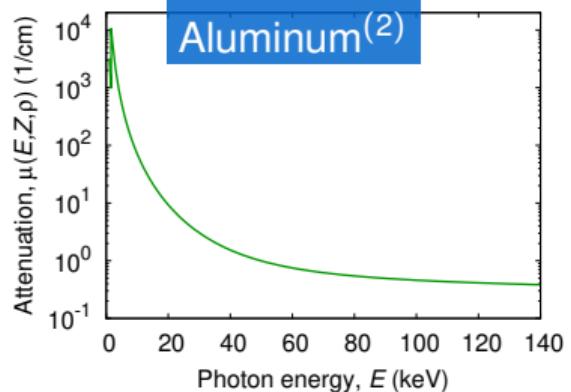
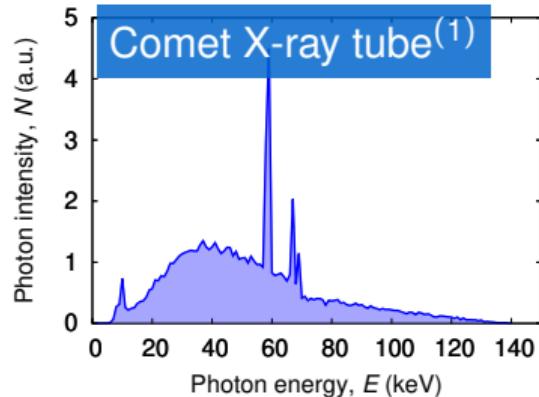
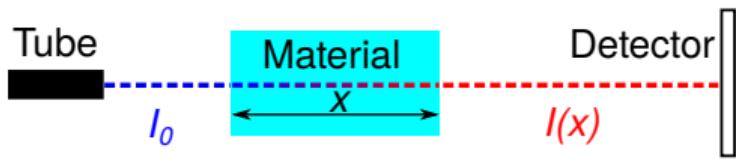
$$I(x) \propto \int N(E) \exp\{-\mu(E, Z, \rho)x\} S(E) dE$$

$$\int \rightarrow \sum$$

(1) Supplied by Norman Uhlmann, Fraunhofer EZRT

(2) XCOM supplied by NIST

Numerical approx. of $\mu_{\text{eff}}(x)$

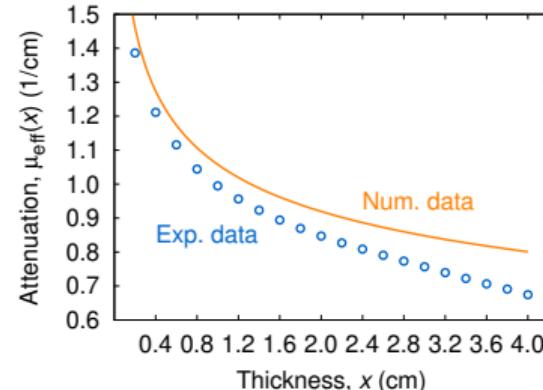


$$I(x) \propto \int N(E) \exp\{-\mu(E, Z, \rho)x\} S(E) dE$$

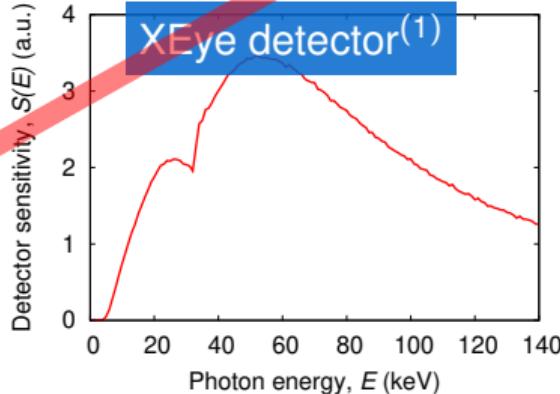
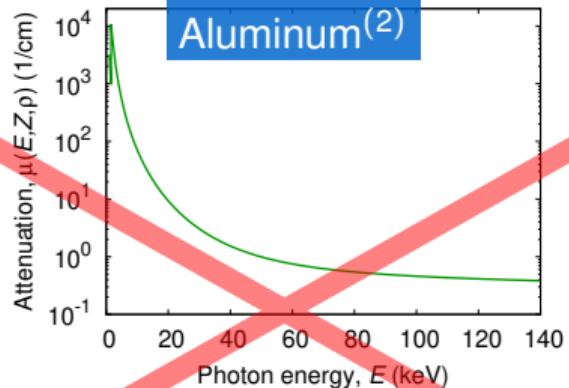
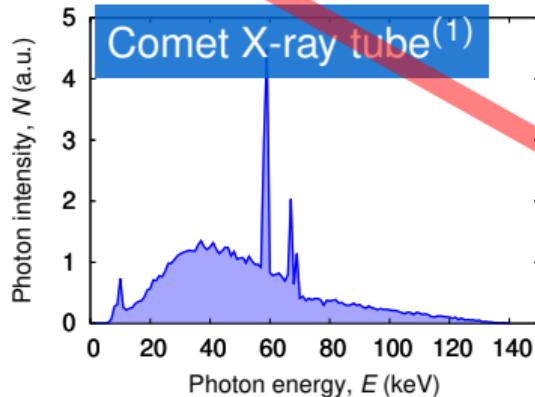
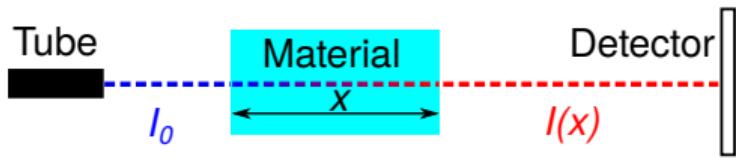
$$\int \rightarrow \sum$$

(1) Supplied by Norman Uhlmann, Fraunhofer EZRT

(2) XCOM supplied by NIST

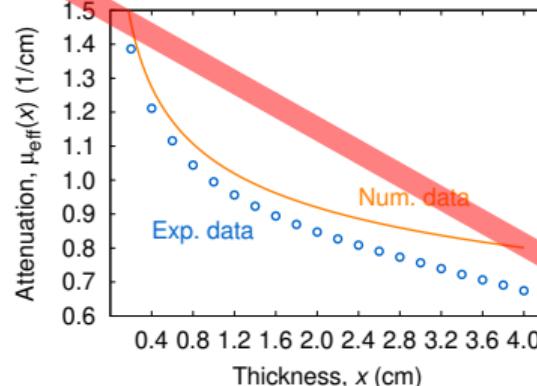


Numerical approx. of $\mu_{\text{eff}}(x)$



$$I(x) \propto \int N(E) \exp\{-\mu(E, Z, \rho)x\} S(E) dE$$

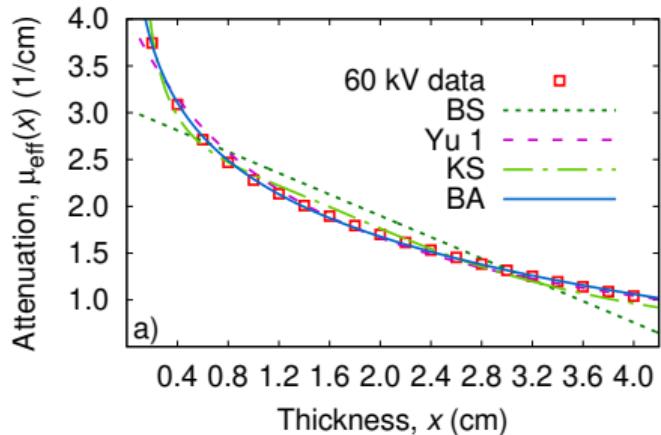
$$\int \rightarrow \sum$$



(1) Supplied by Norman Uhlmann, Fraunhofer EZRT

(2) XCOM supplied by NIST

Heuristic model functions for $\mu_{\text{eff}}(x)$



$$\mu_{\text{eff}}(x) = \mu_0 - \lambda x$$

Bjärngard & Shackford
(1994)

$$\mu_{\text{eff}}(x) = \frac{\mu_0}{1 + \lambda x}$$

Yu *et al.* (1997)

$$\mu_{\text{eff}}(x) = \mu(E_{\text{max}}) + \frac{2\mu_1}{x\sqrt{-\lambda_1^2+4\lambda_2}} \times$$

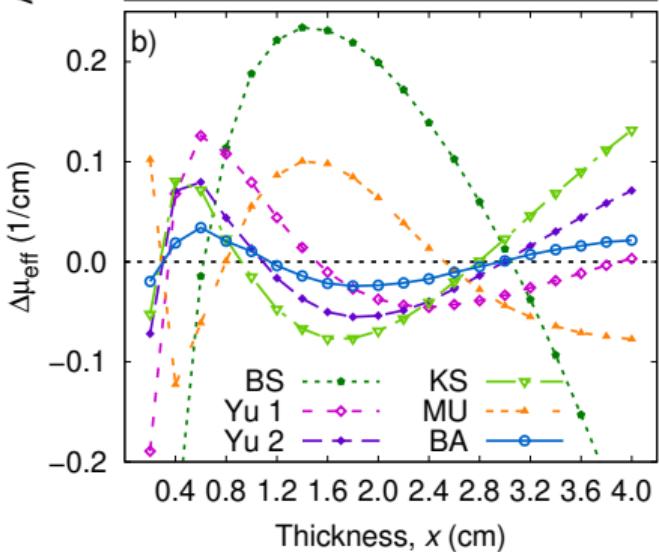
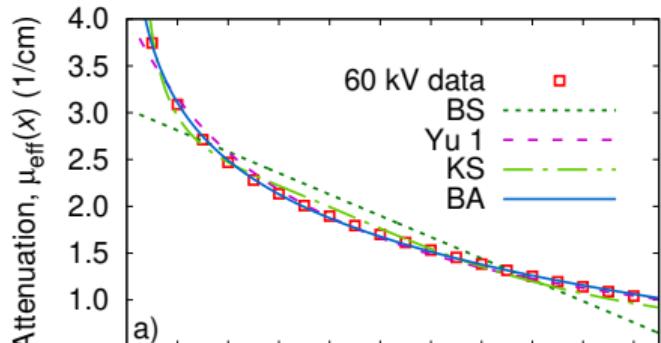
$$\left[\arctan\left(\frac{\lambda_1+2\lambda_2x}{\sqrt{-\lambda_1^2+4\lambda_2}}\right) - \arctan\left(\frac{\lambda_1}{\sqrt{-\lambda_1^2+4\lambda_2}}\right) \right]$$

Kleinschmidt (1999)

$$\mu_{\text{eff}}(x) = a + \frac{b}{x^\alpha}$$

Baur *et al.* (2019)
(this work)

Heuristic model functions for $\mu_{\text{eff}}(x)$



$$\mu_{\text{eff}}(x) = \mu_0 - \lambda x$$

Bjärngard & Shackford
(1994)

$$\mu_{\text{eff}}(x) = \frac{\mu_0}{1 + \lambda x}$$

$$\mu_{\text{eff}}(x) = \frac{\mu_0}{(1 + \lambda x)^\beta}$$

Yu *et al.* (1997)

$$\mu_{\text{eff}}(x) = \mu(E_{\text{max}}) + \frac{2\mu_1}{x\sqrt{-\lambda_1^2 + 4\lambda_2}} \times \left[\arctan\left(\frac{\lambda_1 + 2\lambda_2 x}{\sqrt{-\lambda_1^2 + 4\lambda_2}}\right) - \arctan\left(\frac{\lambda_1}{\sqrt{-\lambda_1^2 + 4\lambda_2}}\right) \right]$$

Kleinschmidt (1999)

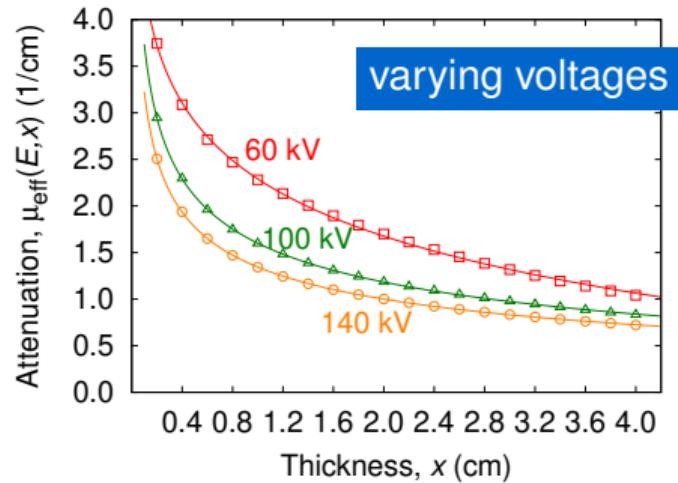
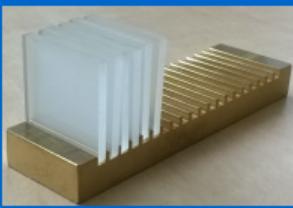
$$\mu_{\text{eff}}(x) = -\frac{1}{x} \ln [A + B \exp(-x/C)]$$

Mudde *et al.* (2008)

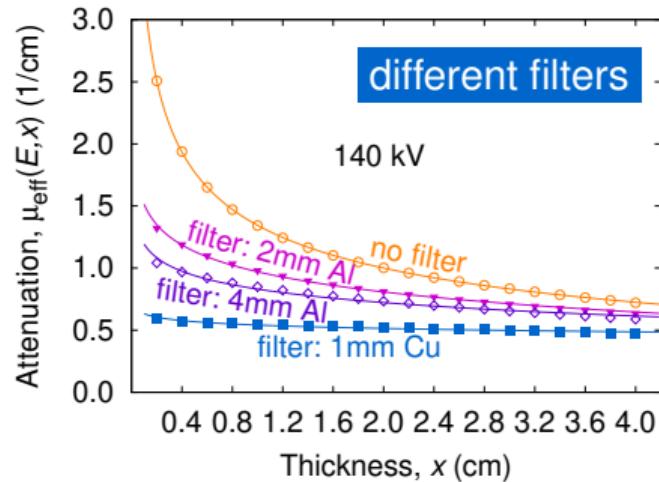
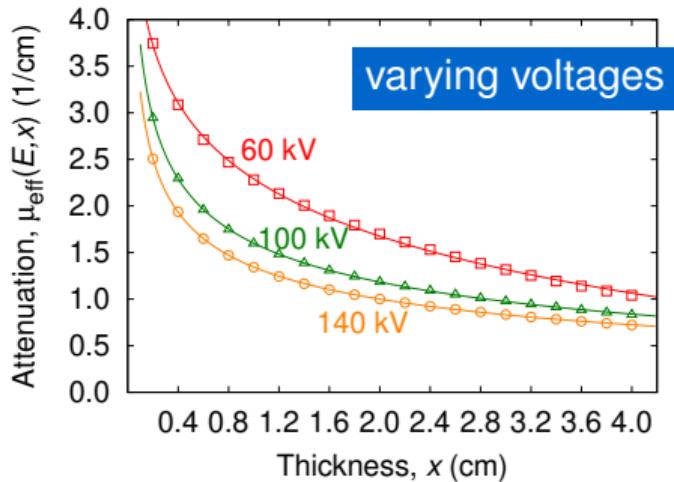
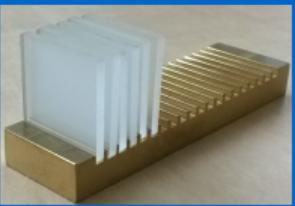
$$\mu_{\text{eff}}(x) = a + \frac{b}{x^\alpha}$$

Baur *et al.* (2019)
(this work)

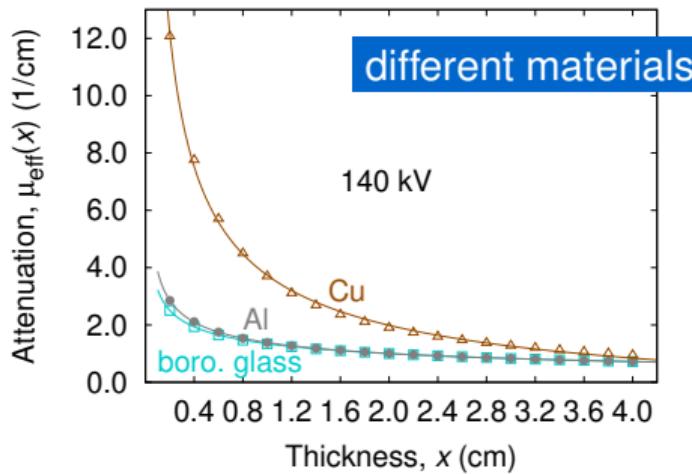
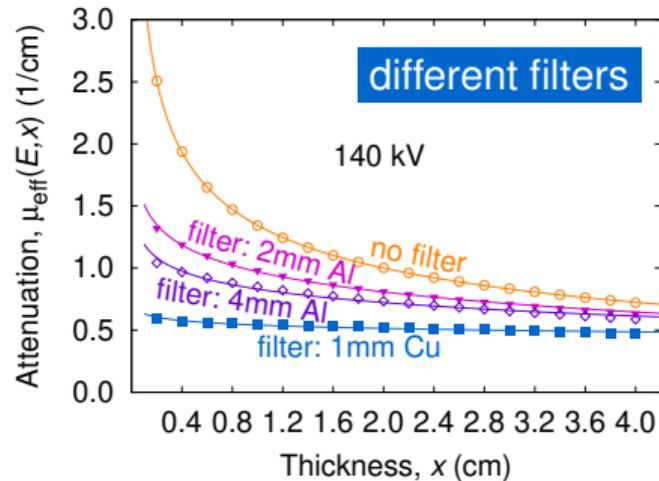
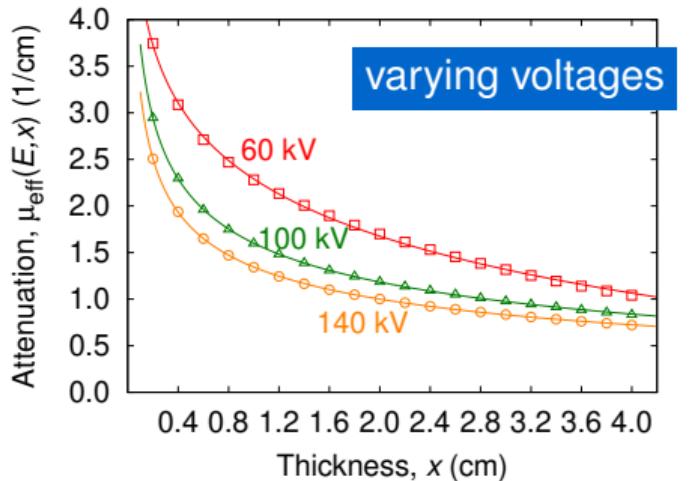
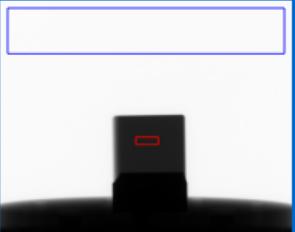
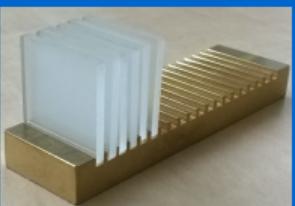
Universality of
 $\mu_{\text{eff}}(x) = a + \frac{b}{x^\alpha}$



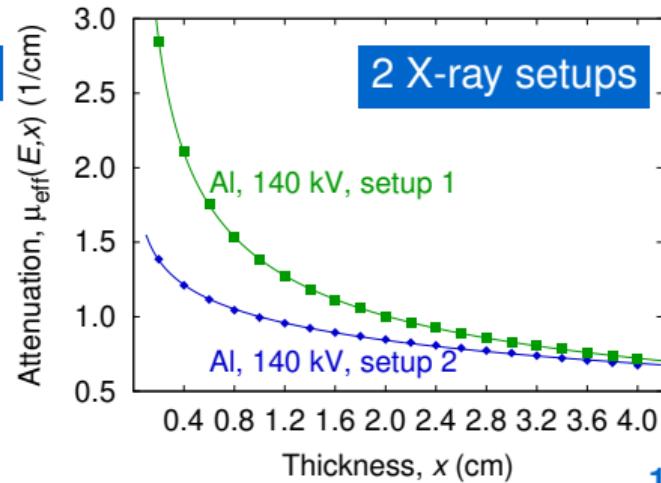
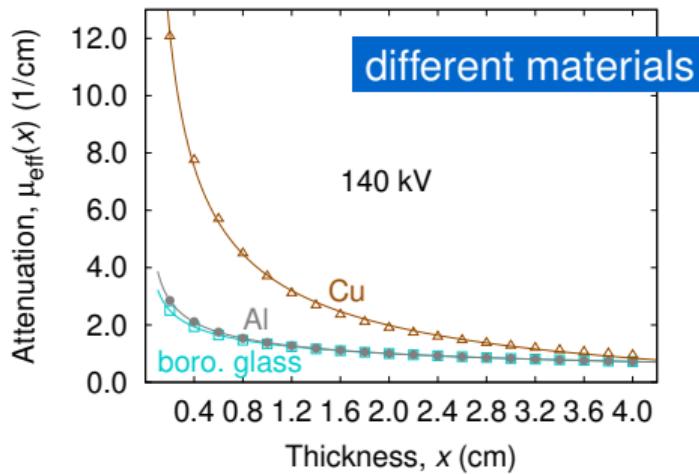
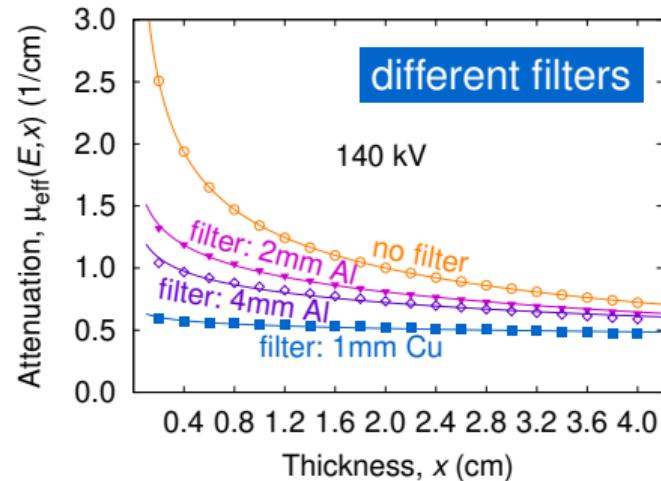
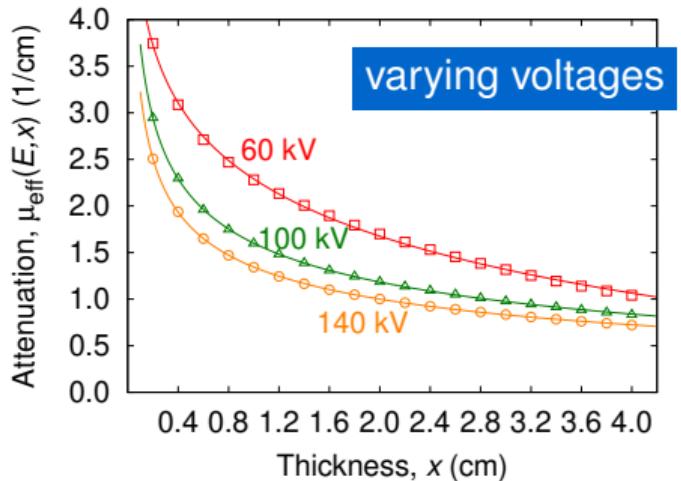
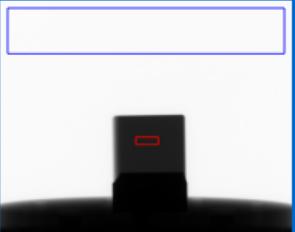
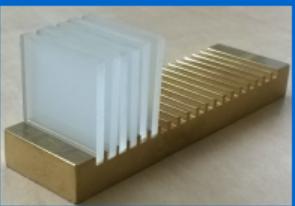
Universality of
 $\mu_{\text{eff}}(x) = a + \frac{b}{x^\alpha}$



Universality of
 $\mu_{\text{eff}}(x) = a + \frac{b}{x^\alpha}$



Universality of
 $\mu_{\text{eff}}(x) = a + \frac{b}{x^\alpha}$



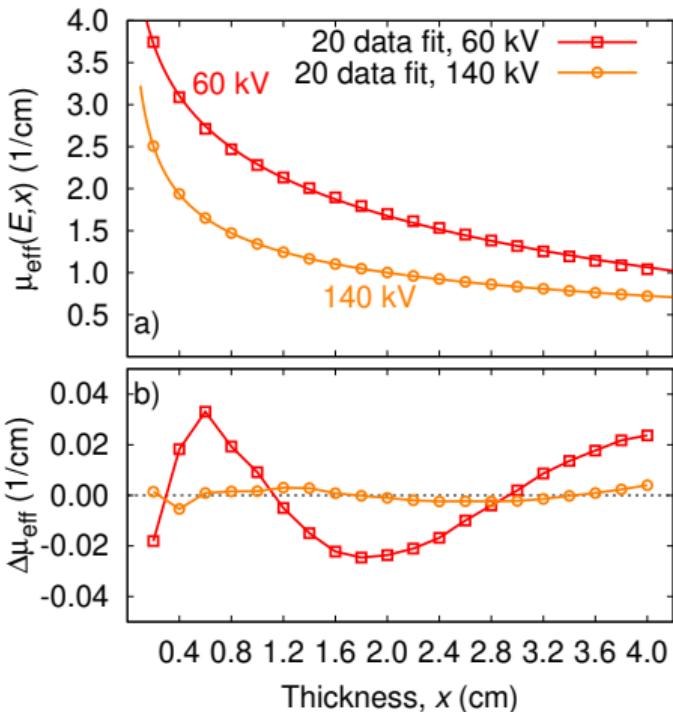
Determining the material thickness x

Generalized Beer-Lambert

$$I(x) = I_0 \exp(-\mu_{\text{eff}}(x) x)$$

Model function

$$\mu_{\text{eff}}(x) = a + \frac{b}{x^\alpha}$$



Determining the material thickness x

Generalized Beer-Lambert

$$I(x) = I_0 \exp(-\mu_{\text{eff}}(x) x)$$

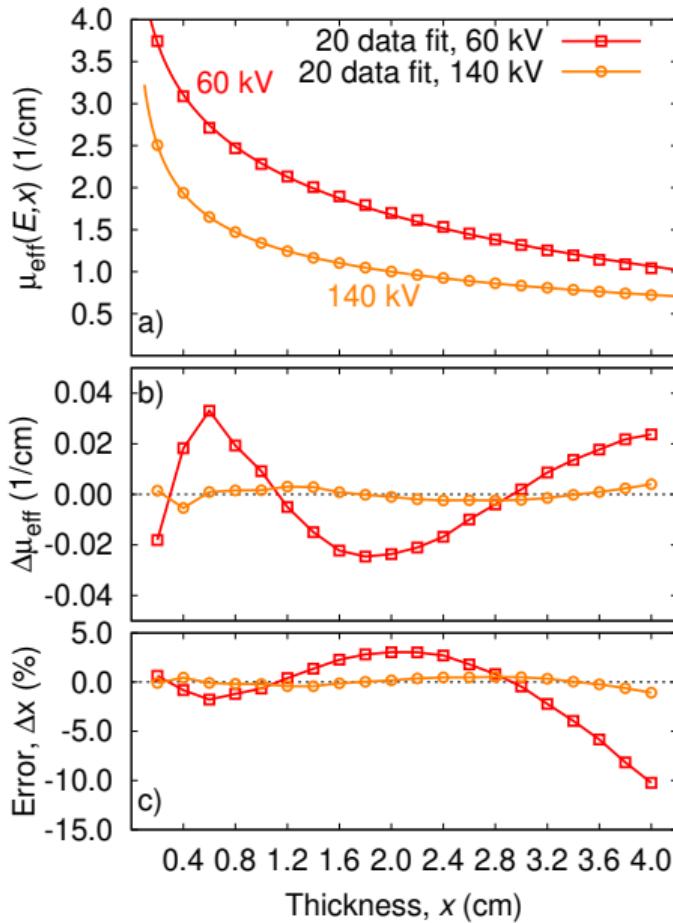
Model function

$$\mu_{\text{eff}}(x) = a + \frac{b}{x^\alpha}$$

Solve

$$ax + bx^{1-\alpha} + \ln\left(\frac{I(x)}{I_0}\right) = 0$$

e.g. Newton's method or look-up table



Determining the material thickness x

Generalized Beer-Lambert

$$I(x) = I_0 \exp(-\mu_{\text{eff}}(x) x)$$

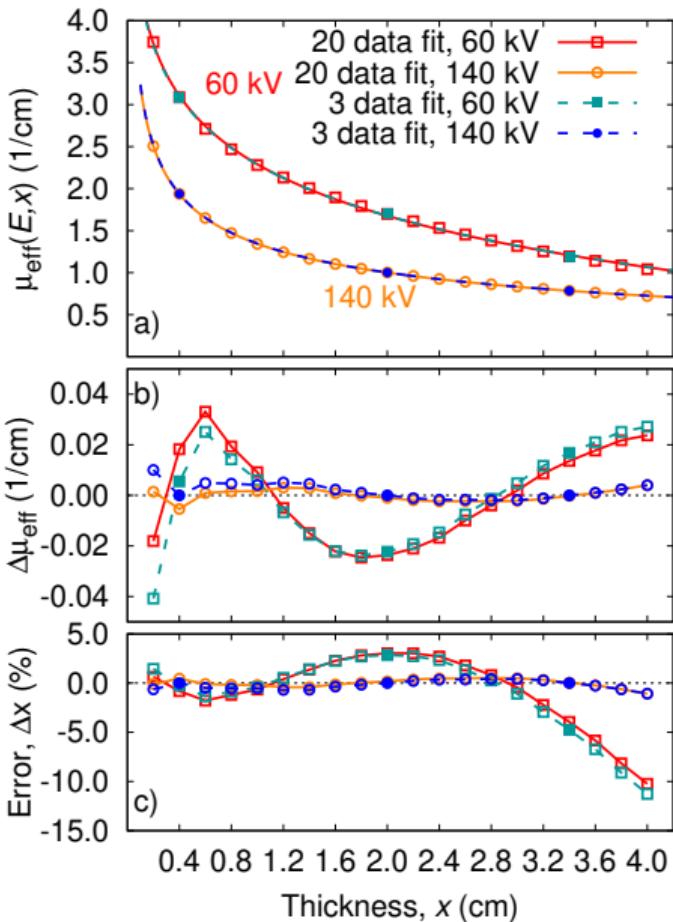
Model function

$$\mu_{\text{eff}}(x) = a + \frac{b}{x^\alpha}$$

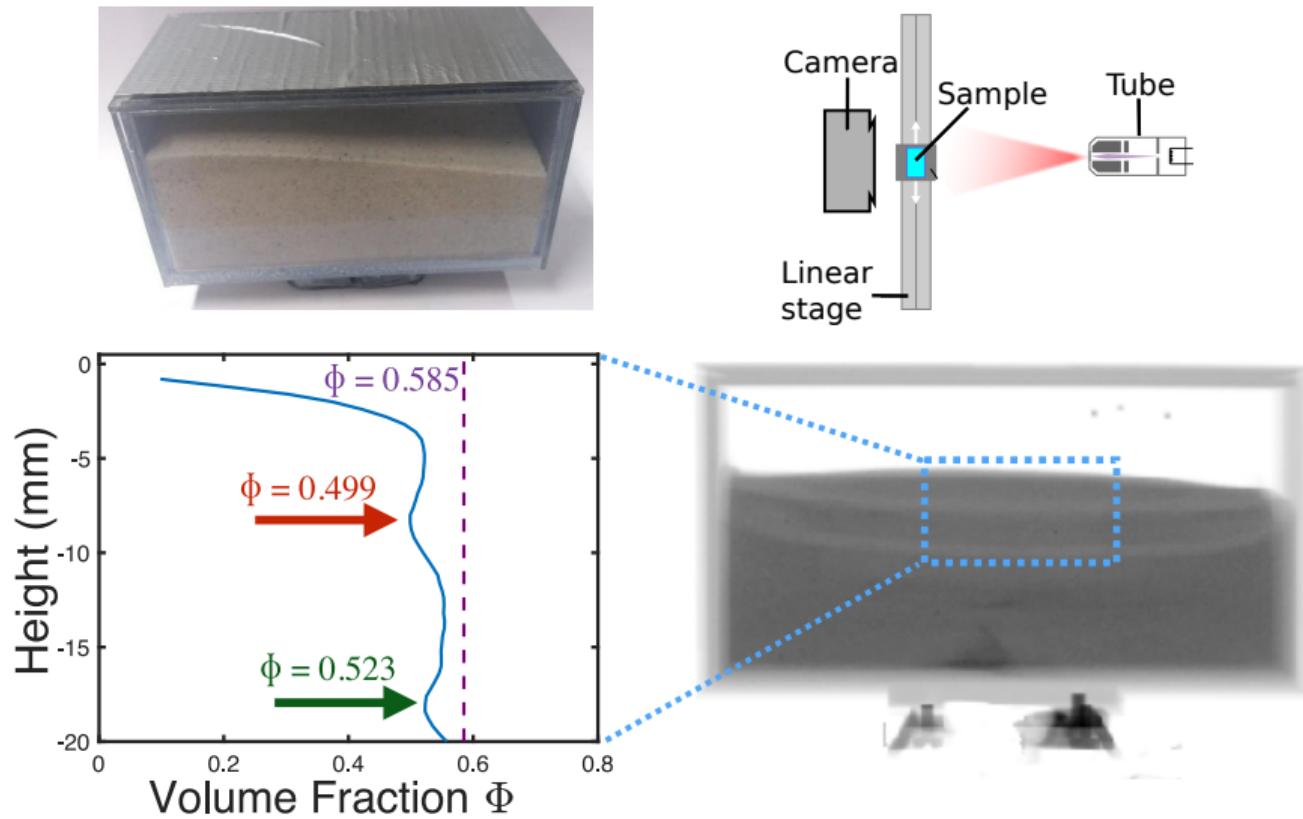
Solve

$$ax + bx^{1-\alpha} + \ln\left(\frac{I(x)}{I_0}\right) = 0$$

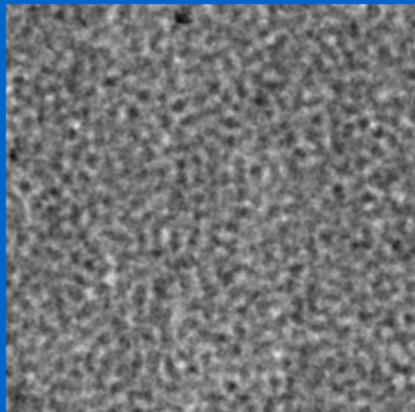
e.g. Newton's method or look-up table



Migrating shear bands in shaken granular matter, Kollmer *et al* (2020)

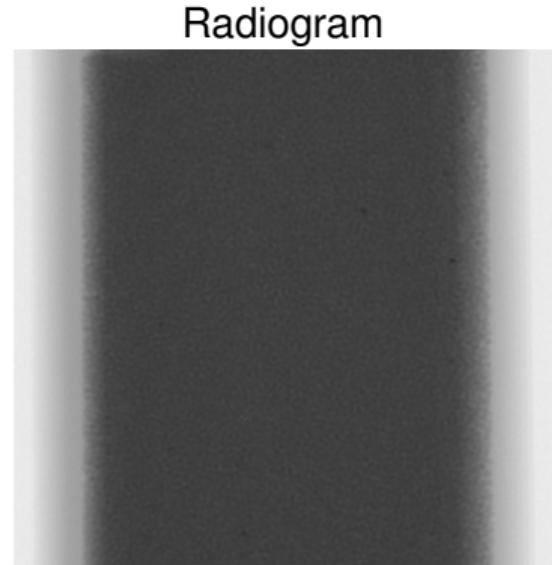
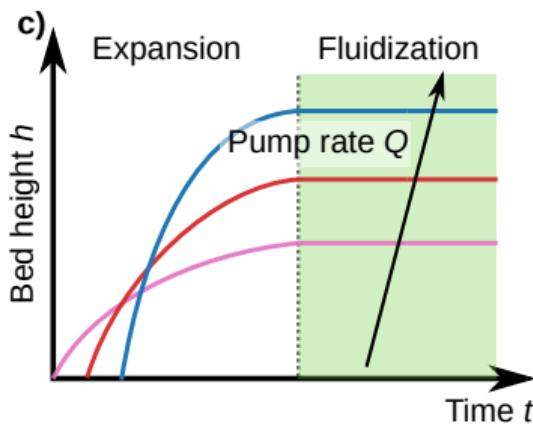
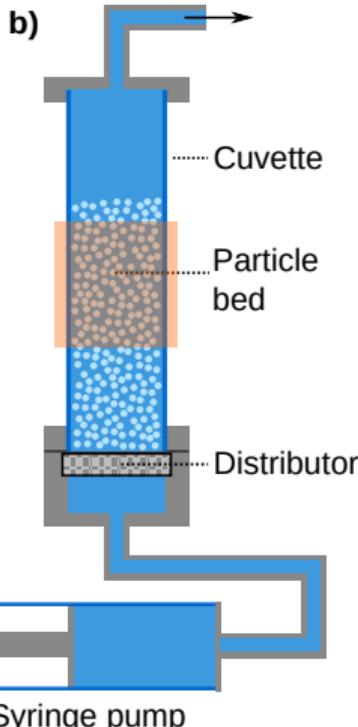
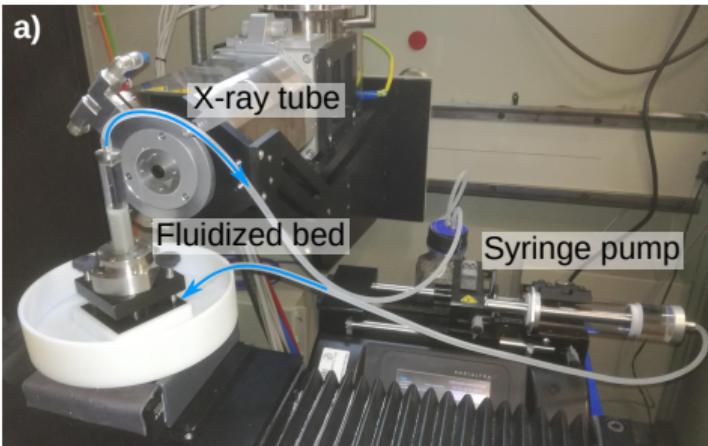


Measuring granular dynamics with X-ray Digital Fourier Analysis (X-DFA)

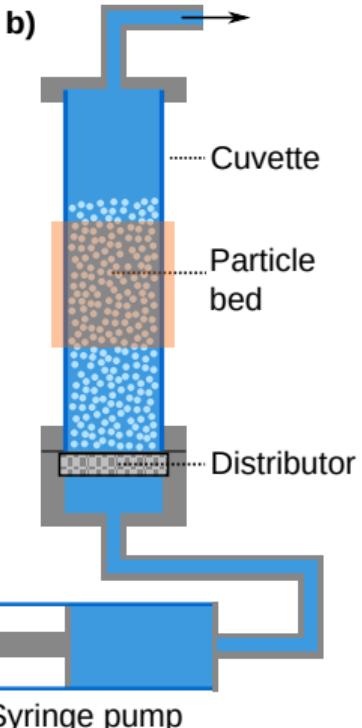
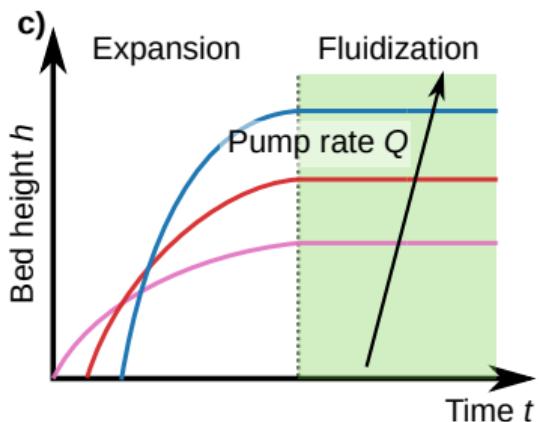
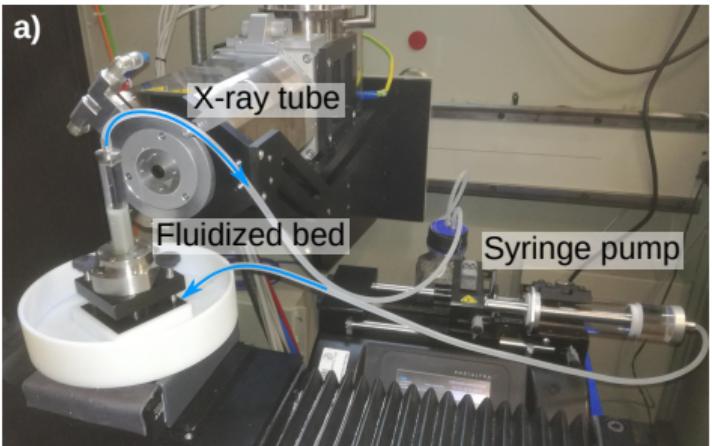


In collaboration with M. Escobedo & S. Egelhaaf, University of Düsseldorf

The system: A liquid fluidized bed



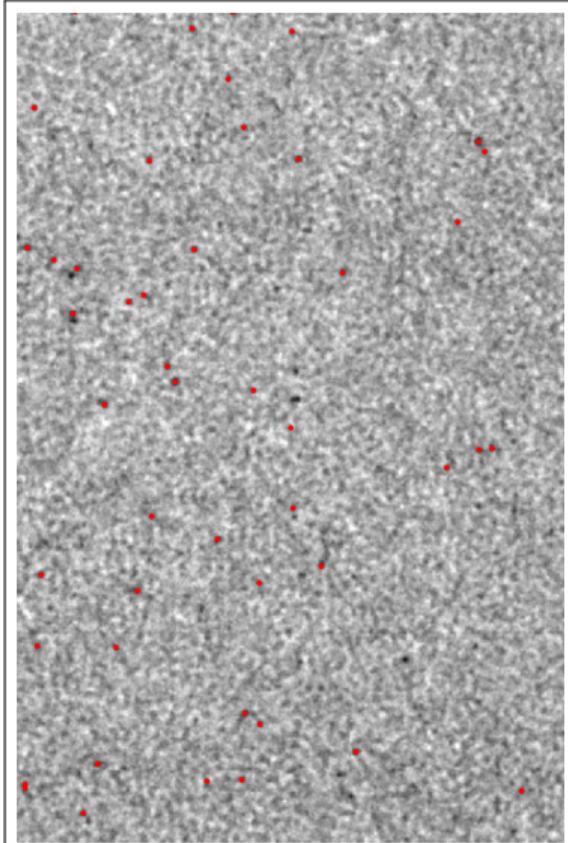
The system: A liquid fluidized bed



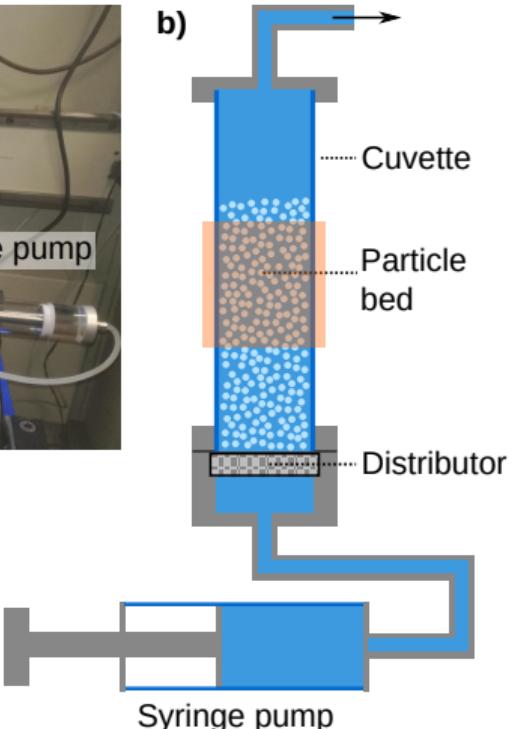
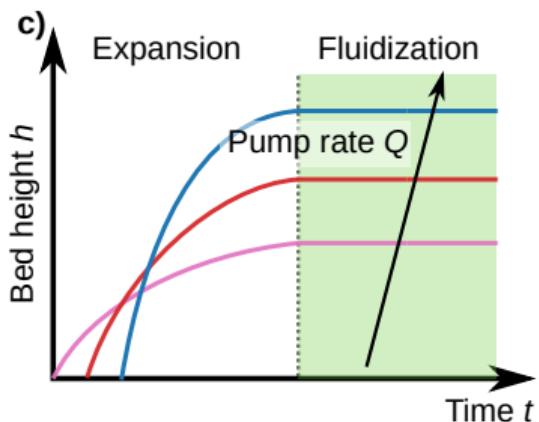
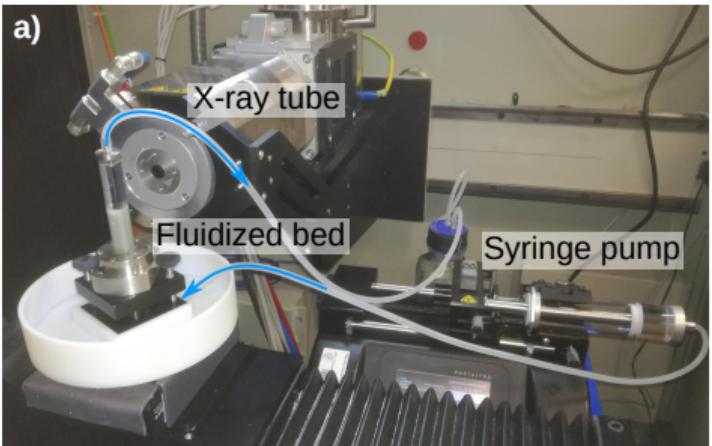
$$0.45 < \phi < 0.56$$

Particle tracking

Contrast: $\rho_{\text{tracer}} > \rho_{\text{bed}}$

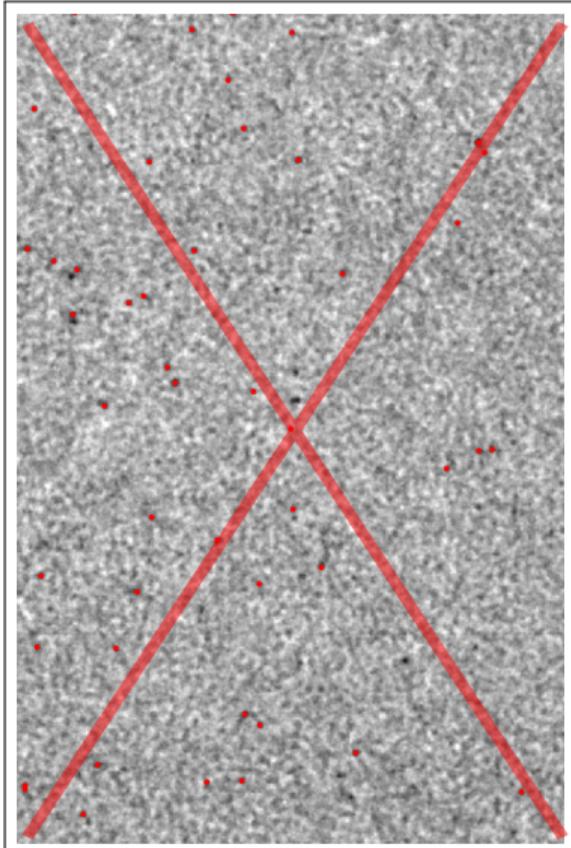


The system: A liquid fluidized bed

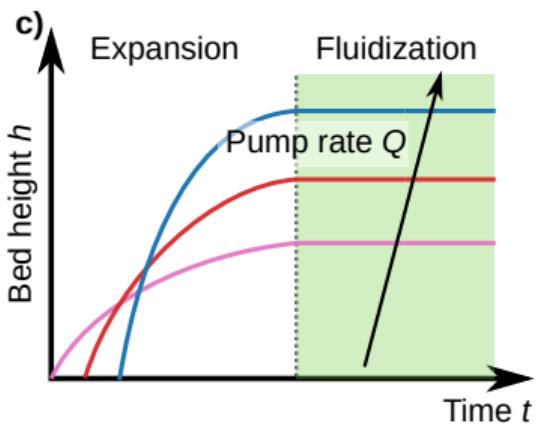
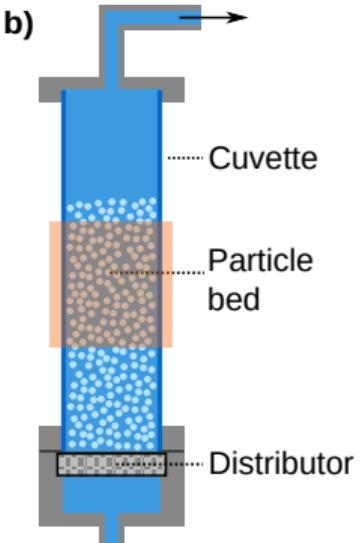
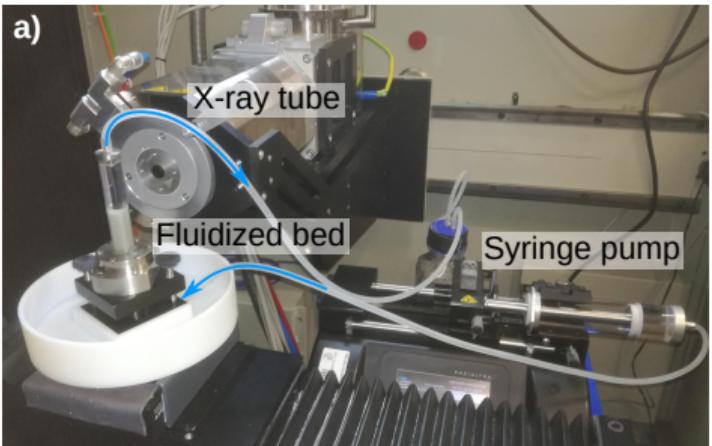


$$0.45 < \phi < 0.56$$

Particle tracking
Contrast: $\rho_{\text{tracer}} > \rho_{\text{bed}}$

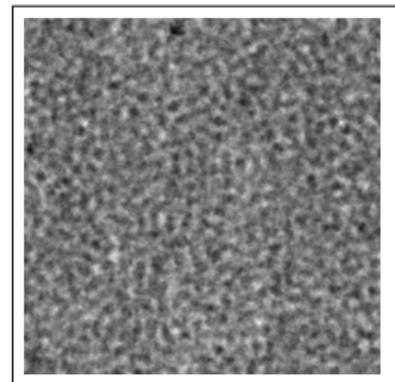
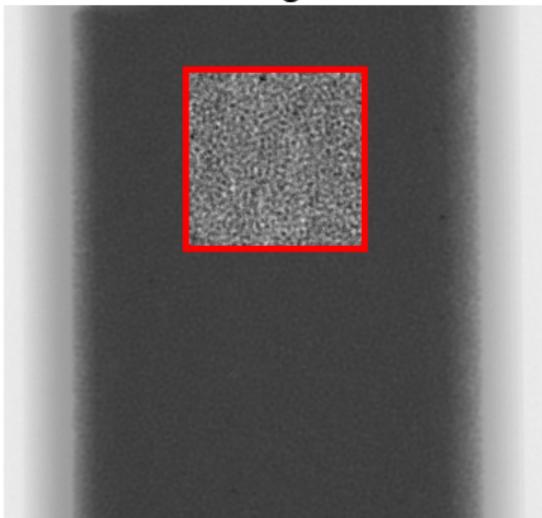


The system: A liquid fluidized bed



$$0.45 < \phi < 0.56$$

Radiogram



Differential Dynamic Microscopy (DDM)

| | Up to now | This work |
|------------------------|----------------------------------------------------|----------------------------------------|
| System | Dispersion, gels | Fluidized bed |
| Particles | Colloids $< 1 \mu\text{m}$ | Granulates ($150 - 180 \mu\text{m}$) |
| Volume fraction | $\Phi \leq 0.33$ | $0.45 < \Phi < 0.56$ |
| imaging | Light microscopy | X-ray radiography |
| Dynamics | Brownian motion, caging, glassy, collective motion | |

Extending Differential Dynamic Microscopy (DDM) to X-ray imaging

| | Up to now | This work |
|------------------------|----------------------------------------------------|--------------------------------------------------------|
| System | Dispersion, gels | Fluidized bed |
| Particles | Colloids $< 1 \mu\text{m}$ | Granulates (150 – 180) μm |
| Volume fraction | $\Phi \leq 0.33$ | $0.45 < \Phi < 0.56$ |
| imaging | Light microscopy | X-ray radiography |
| Dynamics | Brownian motion, caging, glassy, collective motion | |

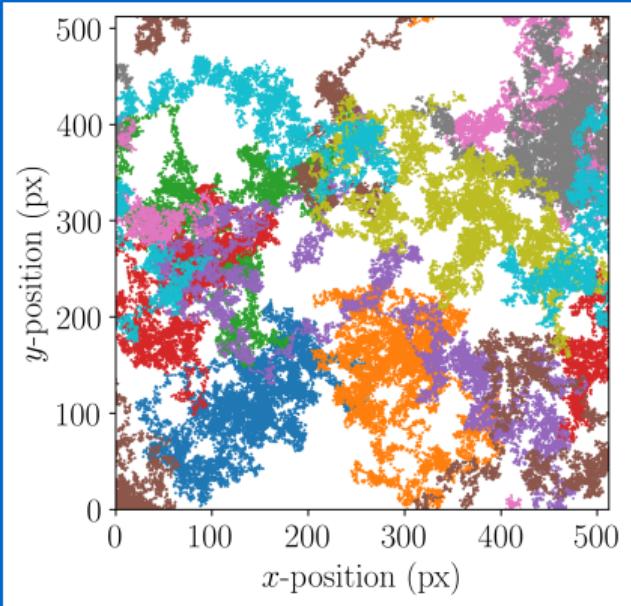
Digital Fourier Analysis of X-Ray Radiograms (X-DFA)

Introduction to Differential Dynamic Microscopy (DDM)

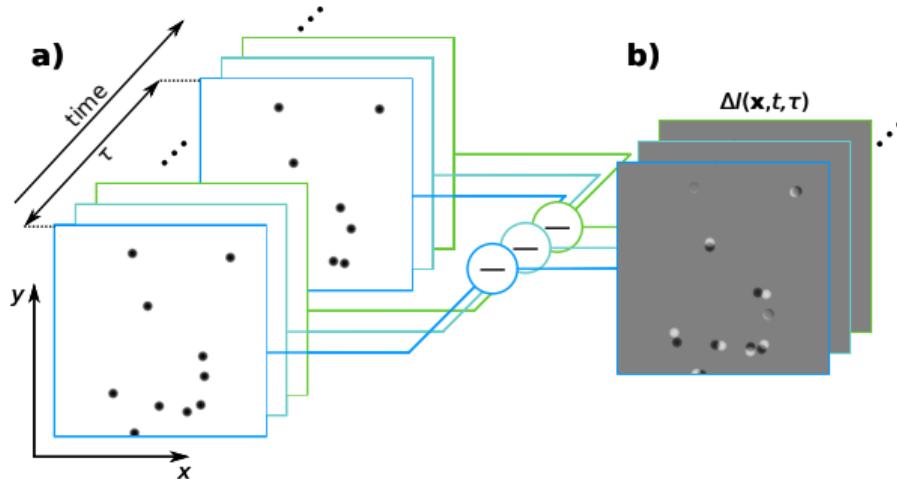
Synthetic radiograms



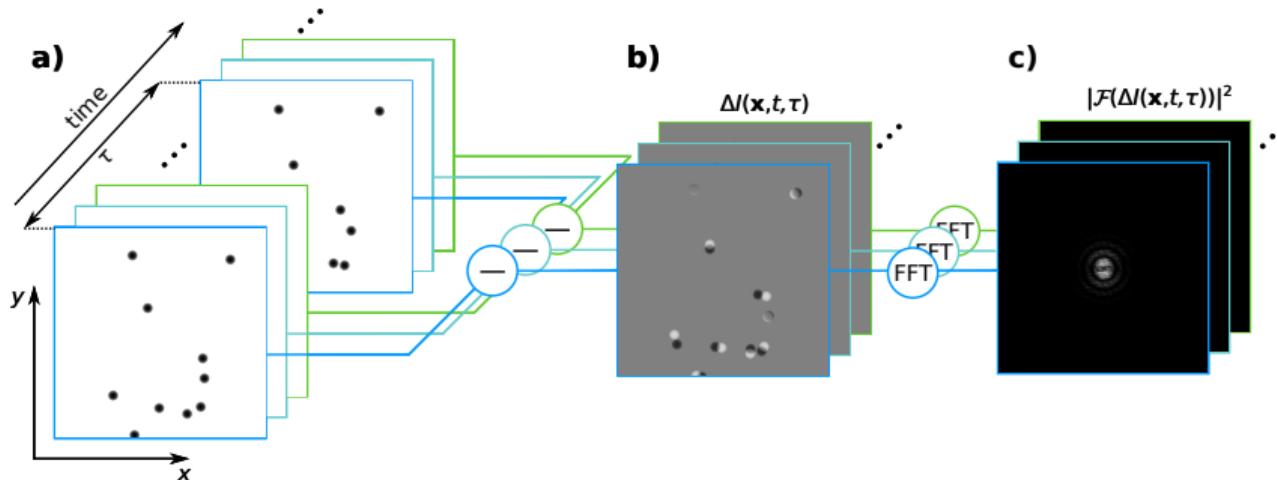
Particle trajectory



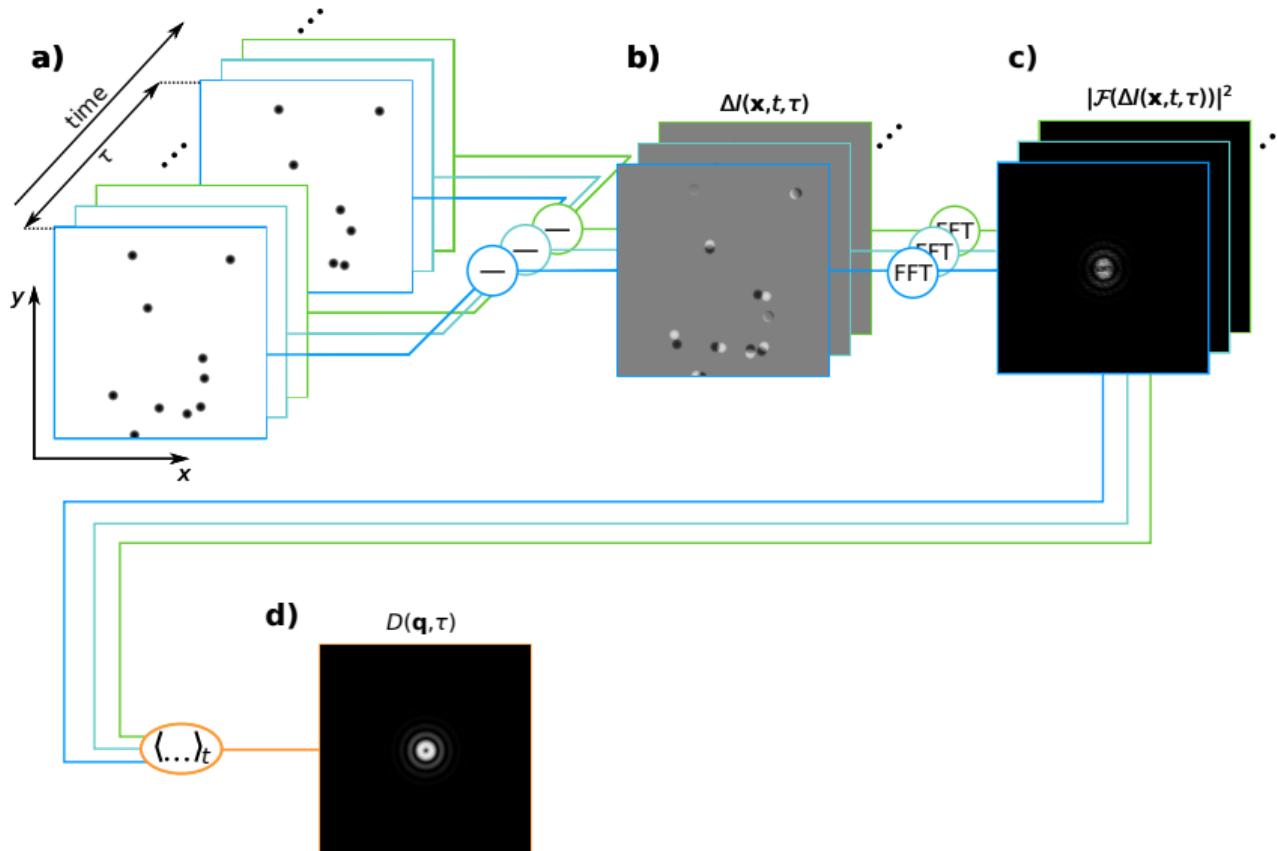
The image structure function $D(\mathbf{q}, \tau)$



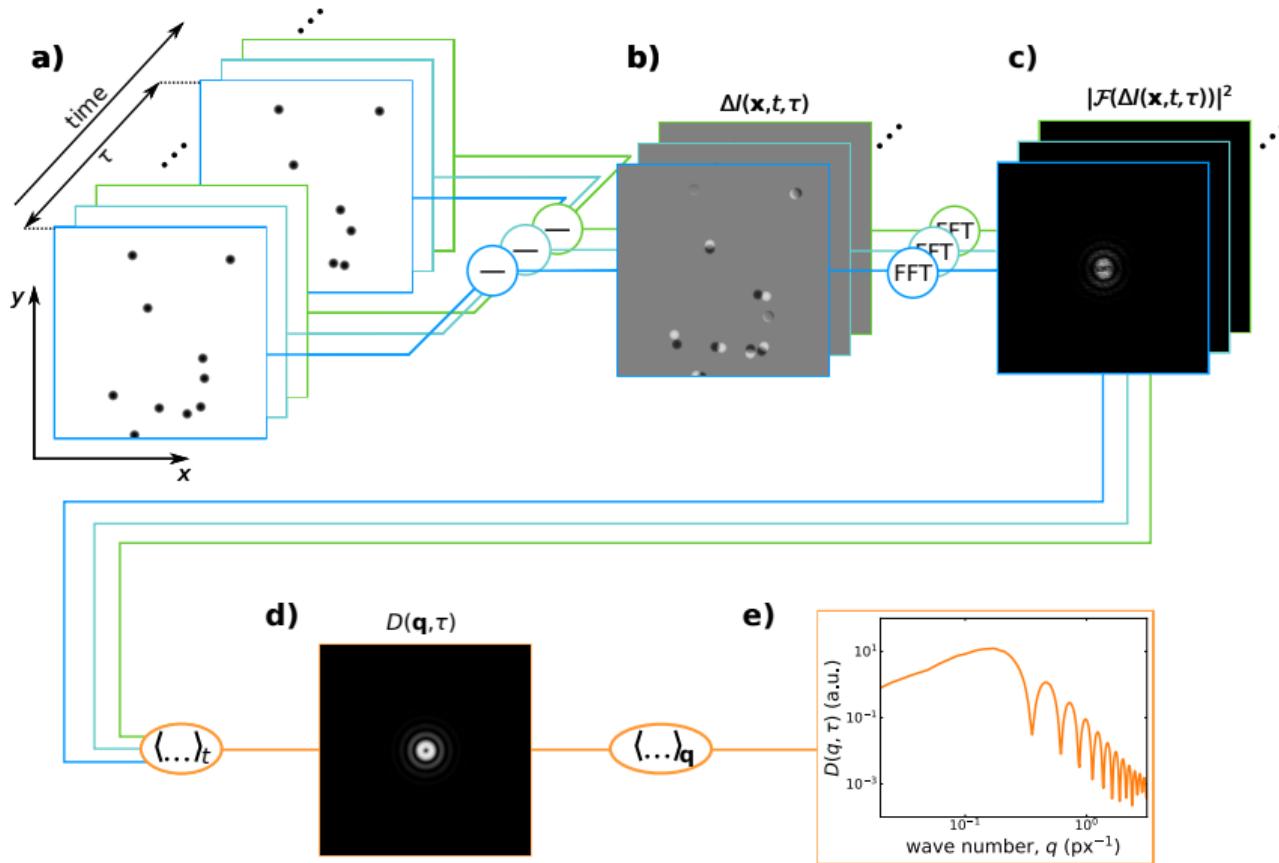
The image structure function $D(\mathbf{q}, \tau)$



The image structure function $D(\mathbf{q}, \tau)$

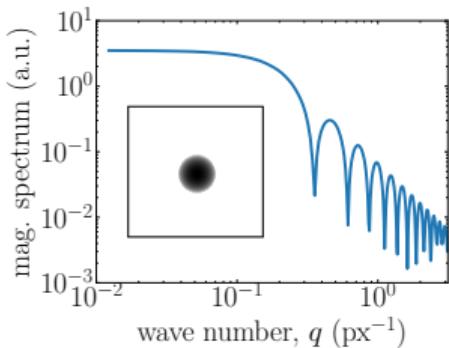
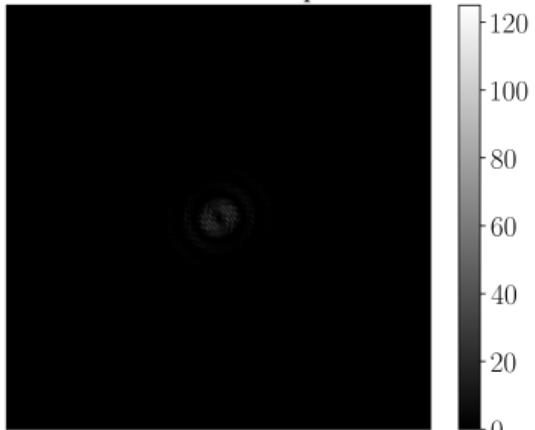


The image structure function $D(\mathbf{q}, \tau)$



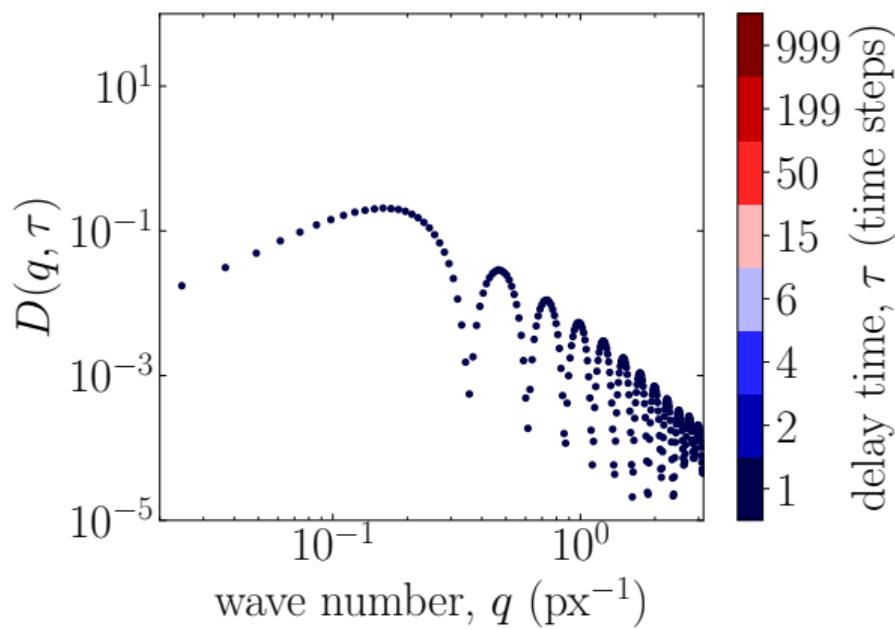
The image structure function $D(q, \tau)$

$\tau = 1$ time steps



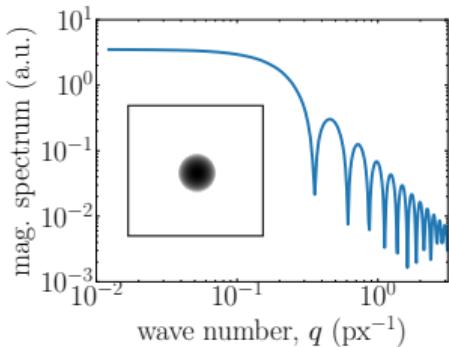
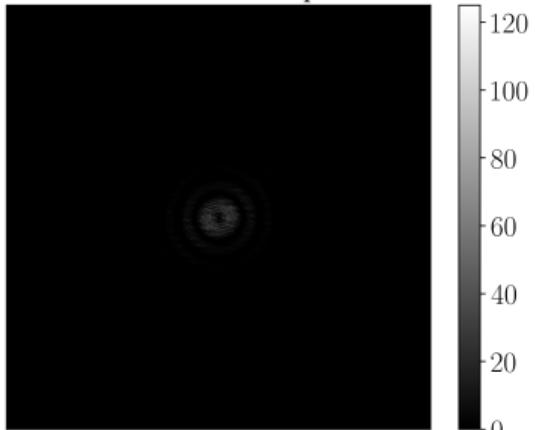
Time averaging: $D(\mathbf{q}, \tau) = \langle |\mathcal{F}(\Delta I)|^2 \rangle_t$

Azimuthal averaging: $D(\mathbf{q}, \tau) \rightarrow D(\mathbf{q}, \tau) \dots$



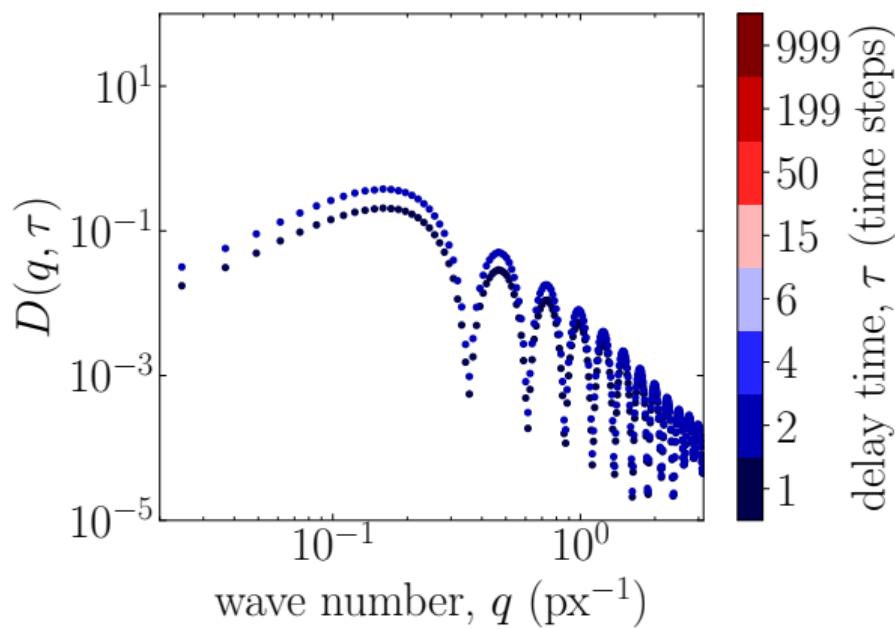
The image structure function $D(q, \tau)$

$\tau = 2$ time steps



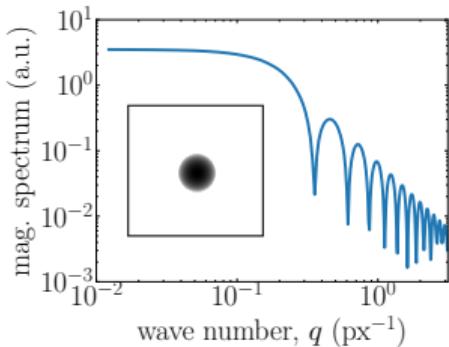
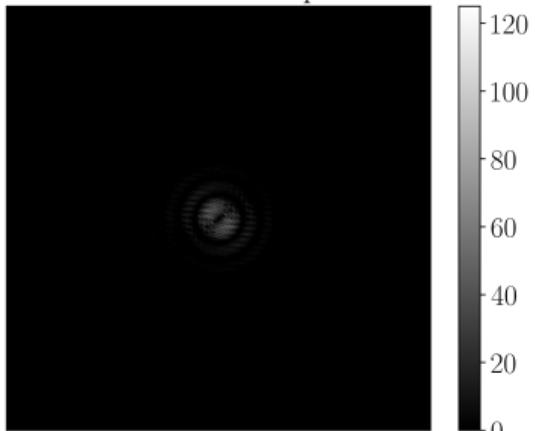
Time averaging: $D(\mathbf{q}, \tau) = \langle |\mathcal{F}(\Delta I)|^2 \rangle_t$

Azimuthal averaging: $D(\mathbf{q}, \tau) \rightarrow D(\mathbf{q}, \tau) \dots$



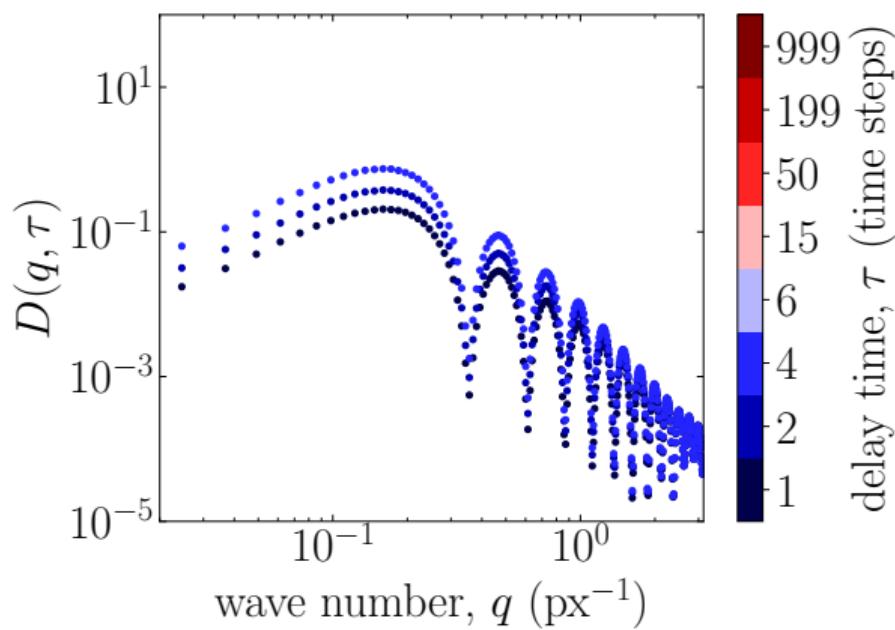
The image structure function $D(q, \tau)$

$\tau = 4$ time steps



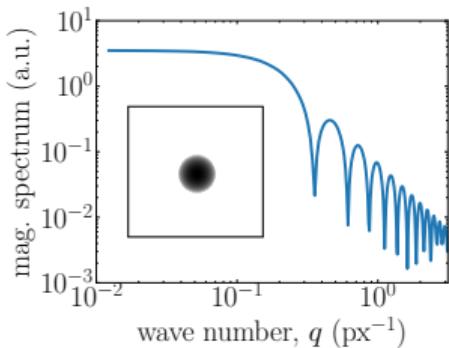
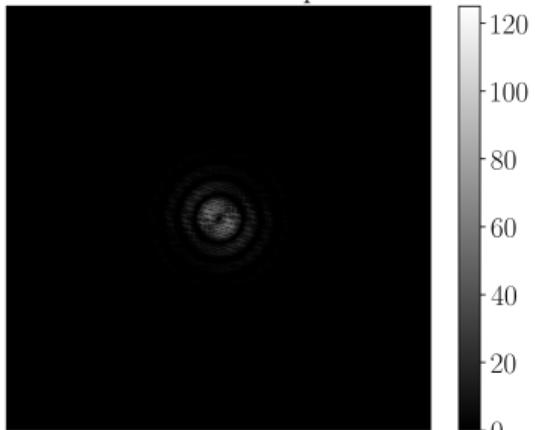
Time averaging: $D(\mathbf{q}, \tau) = \langle |\mathcal{F}(\Delta I)|^2 \rangle_t$

Azimuthal averaging: $D(\mathbf{q}, \tau) \rightarrow D(\mathbf{q}, \tau) \dots$



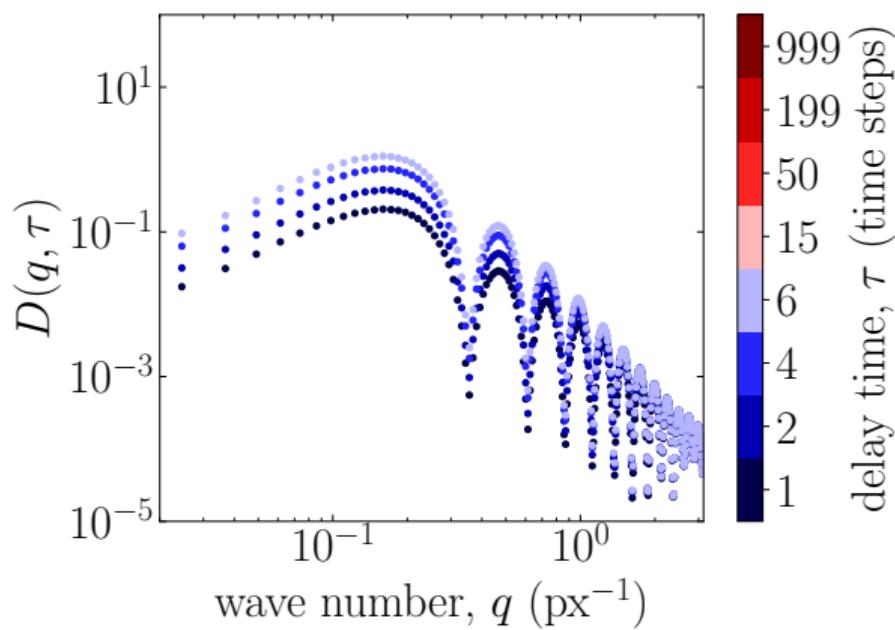
The image structure function $D(q, \tau)$

$\tau = 6$ time steps

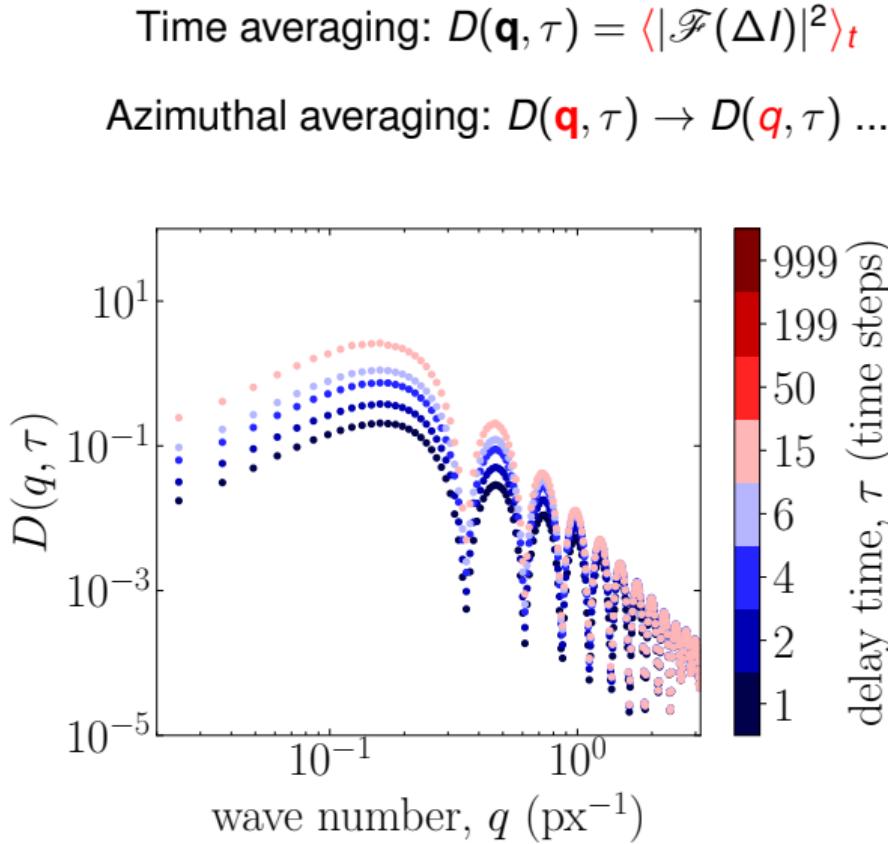
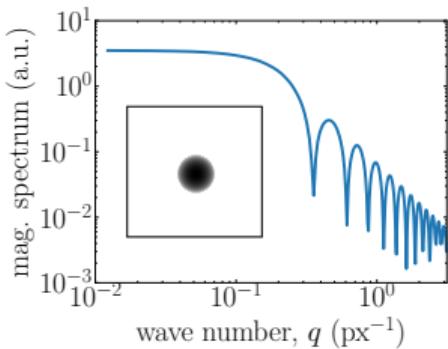
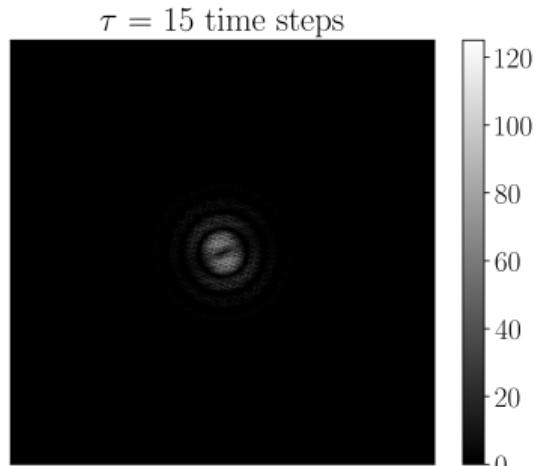


Time averaging: $D(\mathbf{q}, \tau) = \langle |\mathcal{F}(\Delta I)|^2 \rangle_t$

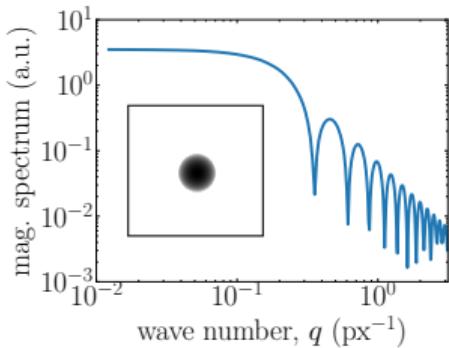
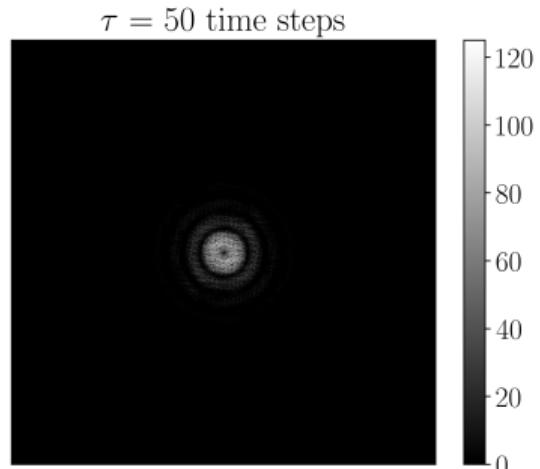
Azimuthal averaging: $D(\mathbf{q}, \tau) \rightarrow D(\mathbf{q}, \tau) \dots$



The image structure function $D(q, \tau)$

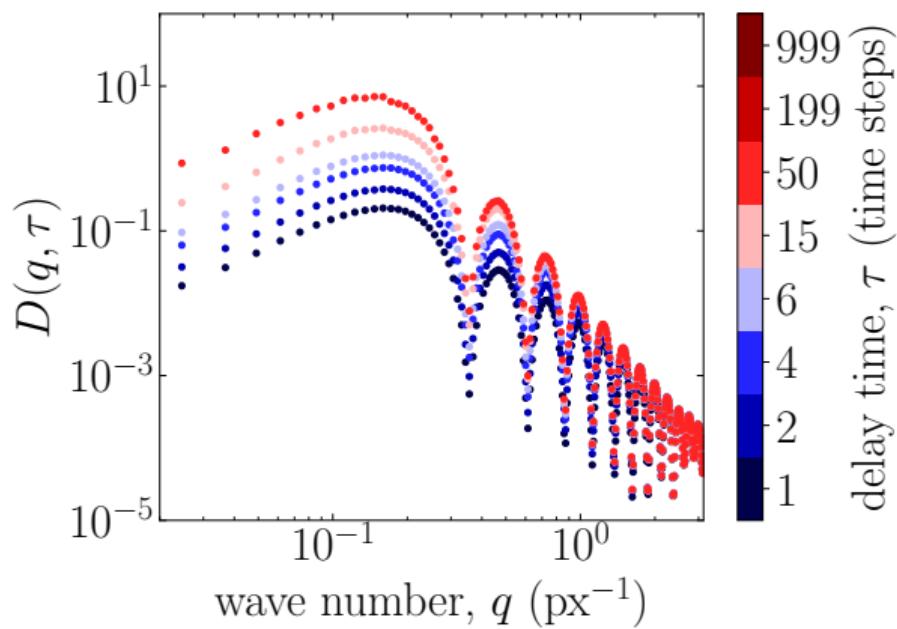


The image structure function $D(q, \tau)$



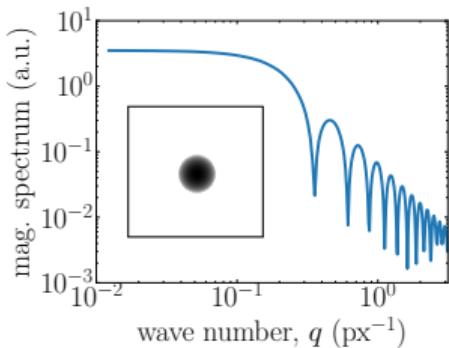
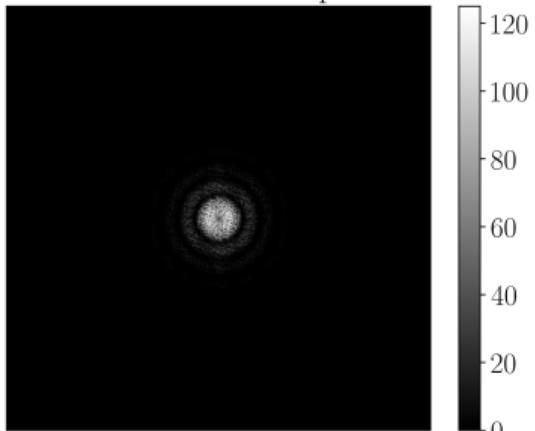
Time averaging: $D(\mathbf{q}, \tau) = \langle |\mathcal{F}(\Delta I)|^2 \rangle_t$

Azimuthal averaging: $D(\mathbf{q}, \tau) \rightarrow D(\mathbf{q}, \tau) \dots$



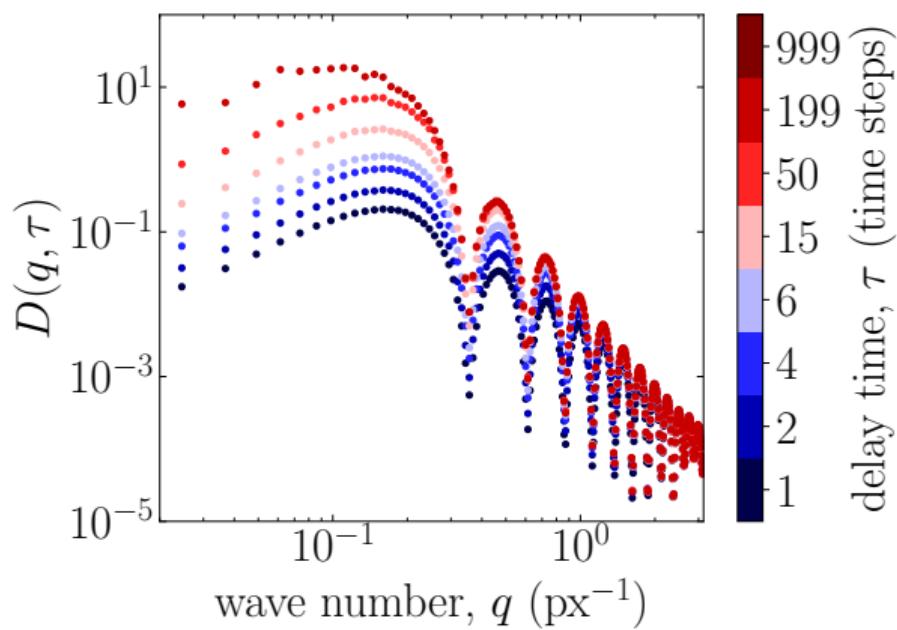
The image structure function $D(q, \tau)$

$\tau = 199$ time steps

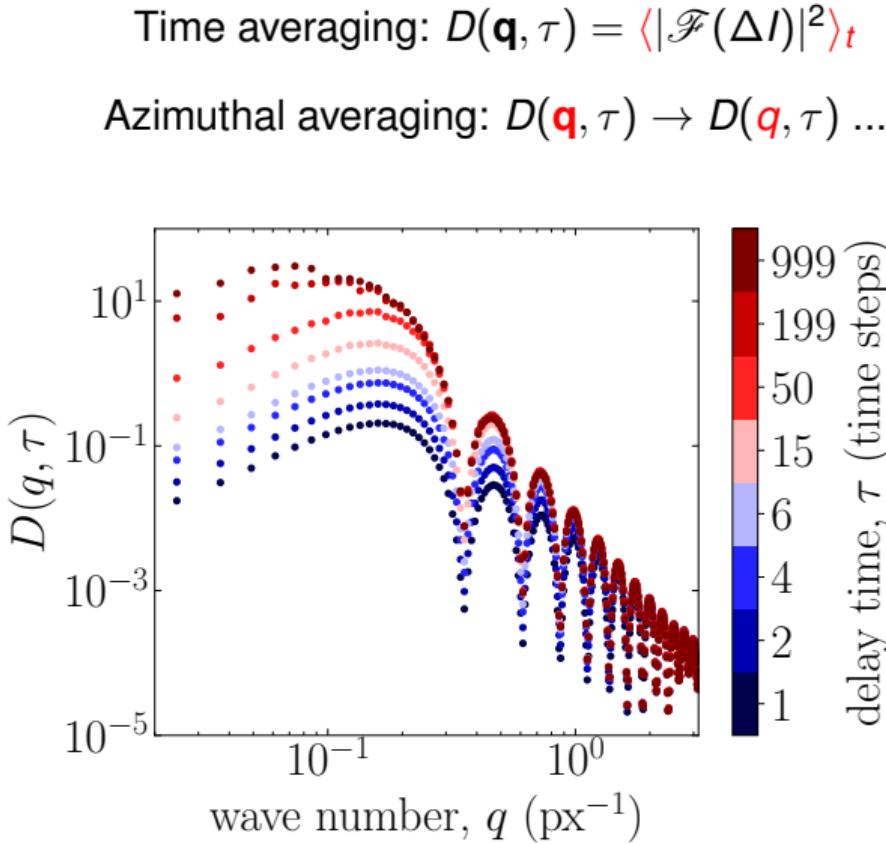
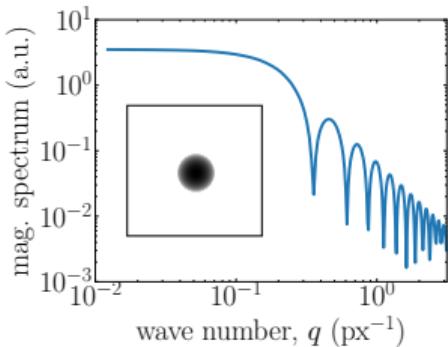
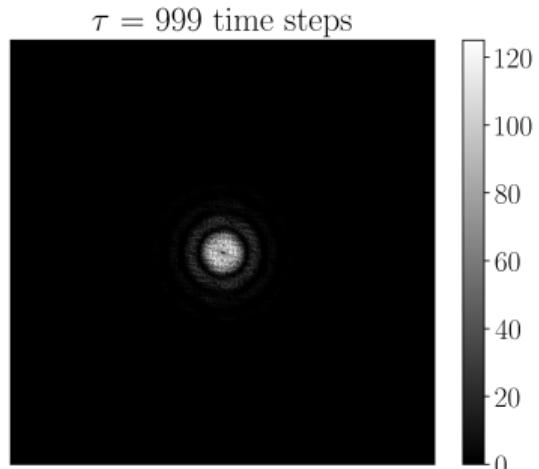


Time averaging: $D(\mathbf{q}, \tau) = \langle |\mathcal{F}(\Delta I)|^2 \rangle_t$

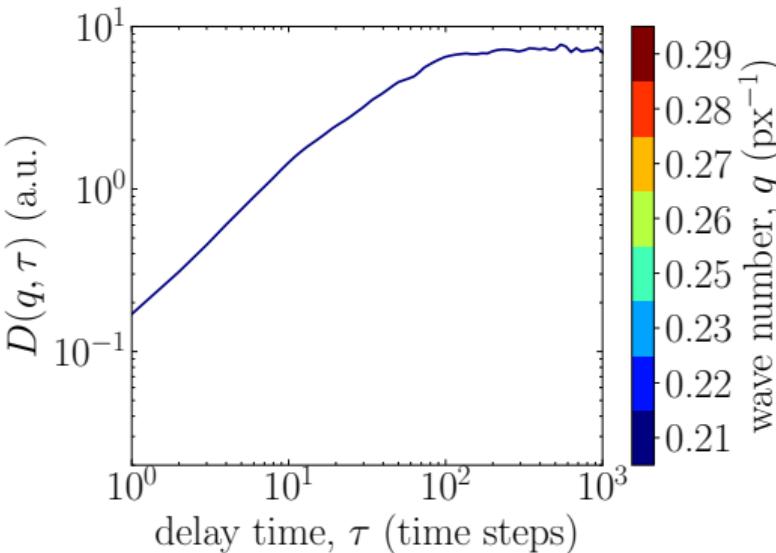
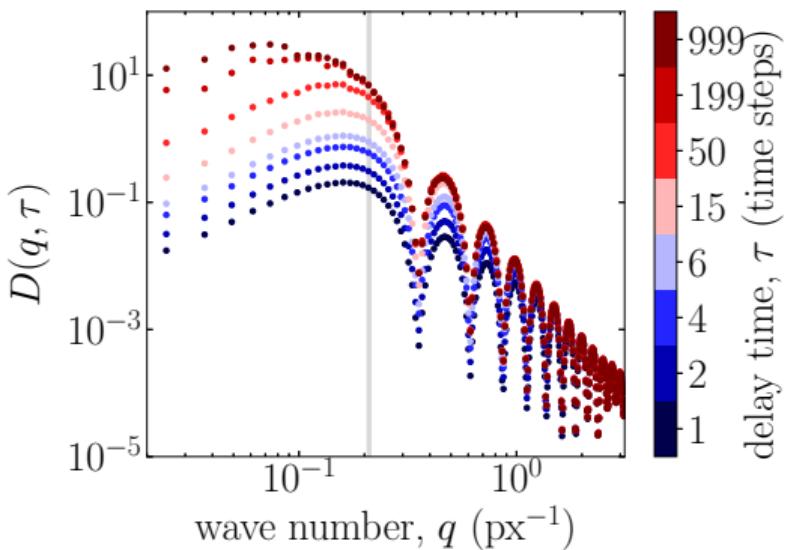
Azimuthal averaging: $D(\mathbf{q}, \tau) \rightarrow D(\mathbf{q}, \tau) \dots$



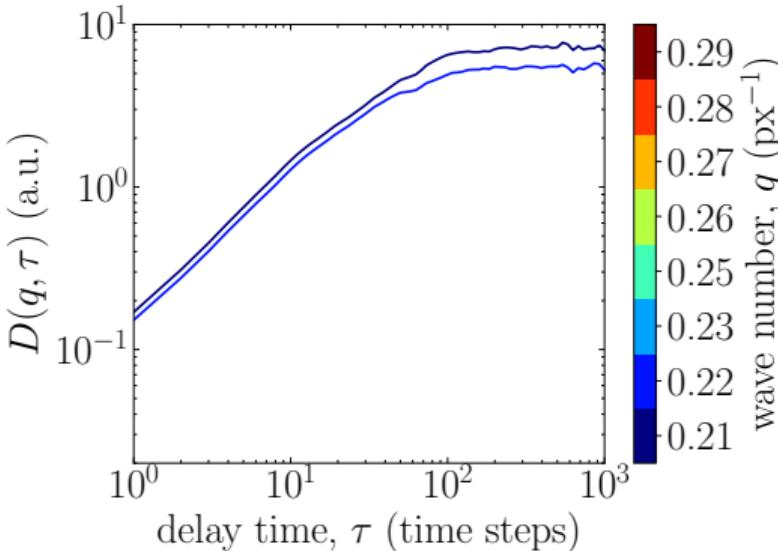
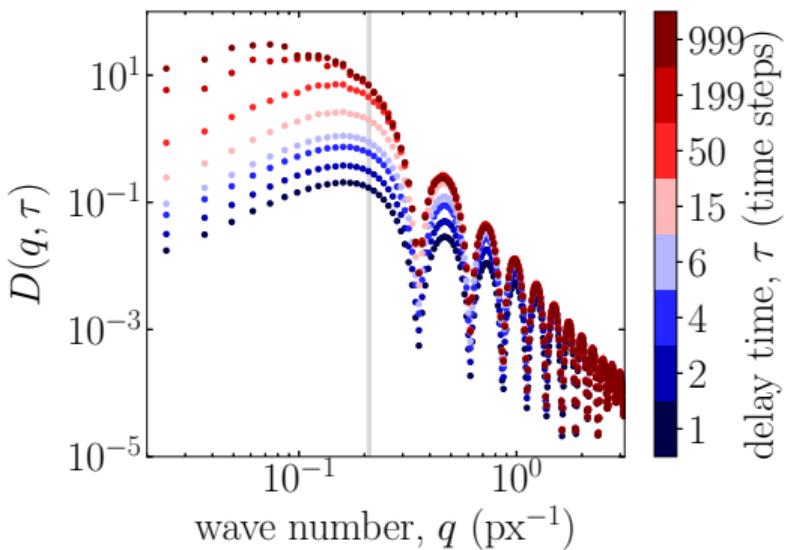
The image structure function $D(q, \tau)$



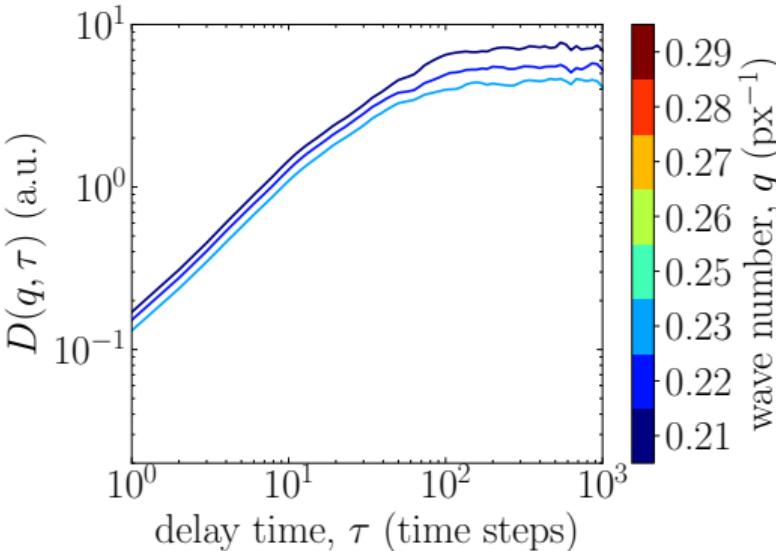
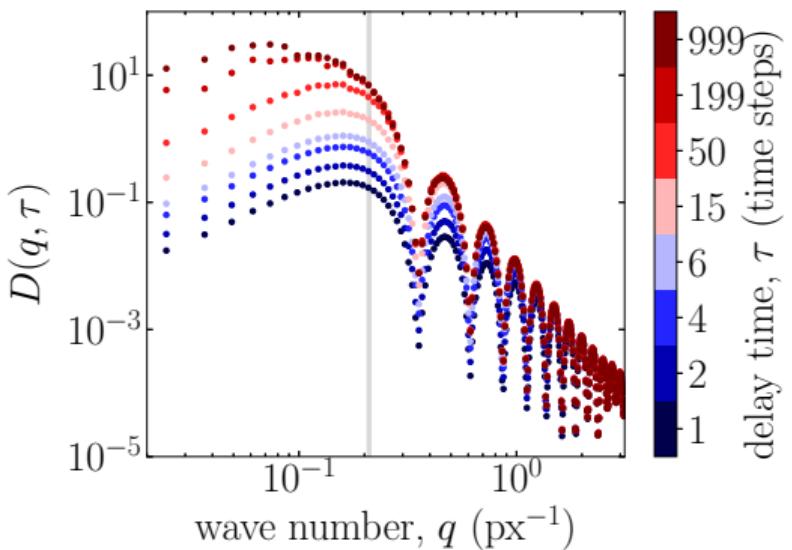
The image structure function $D(q, \tau)$



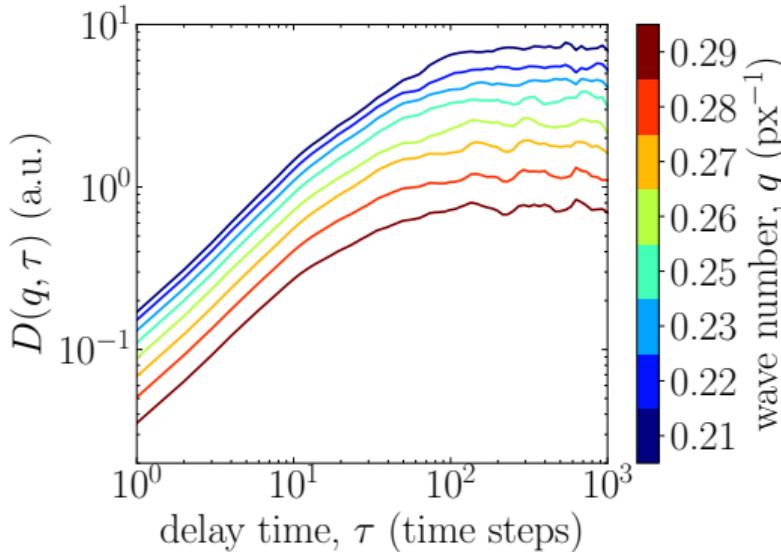
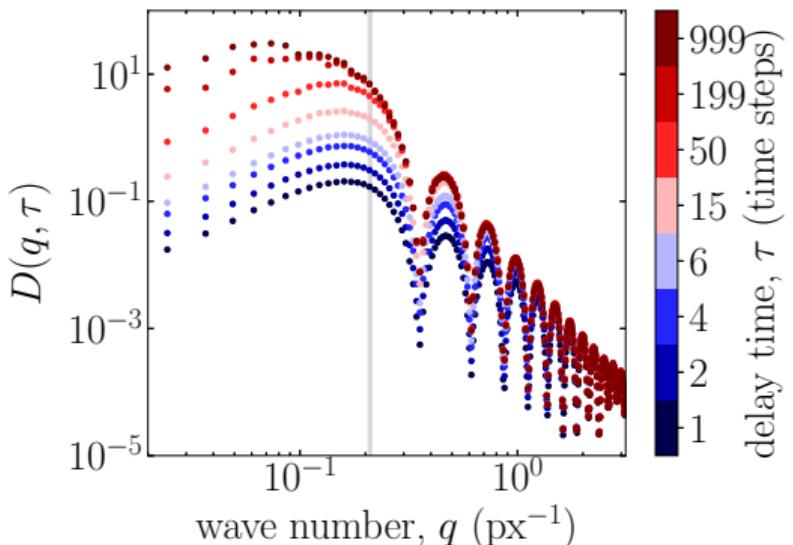
The image structure function $D(q, \tau)$



The image structure function $D(q, \tau)$

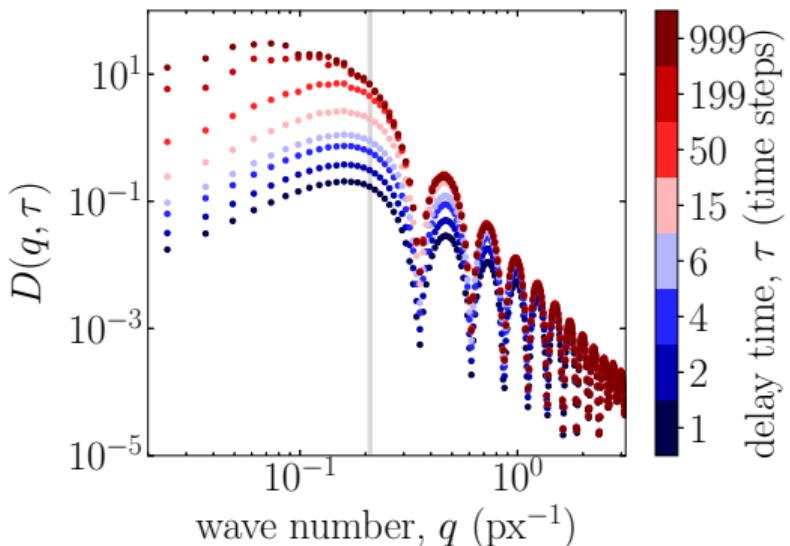


The image structure function $D(q, \tau)$

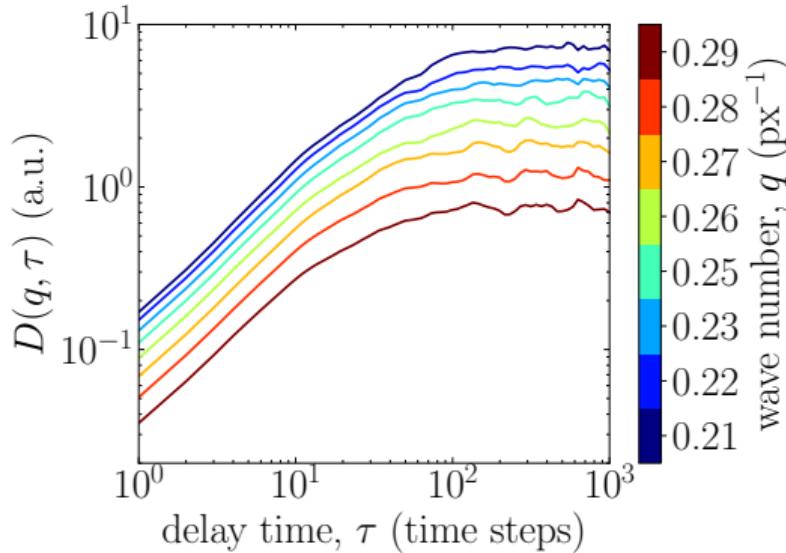


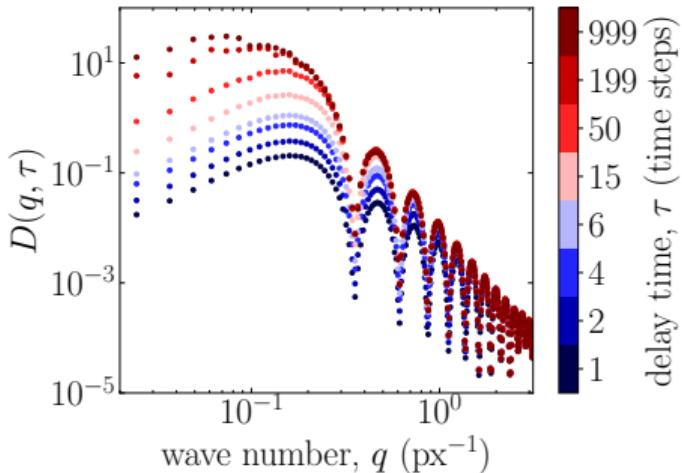
$$\begin{aligned} D(q, \tau) &= \left\langle |I(q, t + \tau) - I(q, t)|^2 \right\rangle_t \\ &= A(q) \left[1 - \frac{\left\langle I^*(q, t) I(q, t + \tau) \right\rangle_t}{\left\langle |I(q, t)|^2 \right\rangle_t} \right] + B(q) \end{aligned}$$

The image structure function $D(q, \tau)$

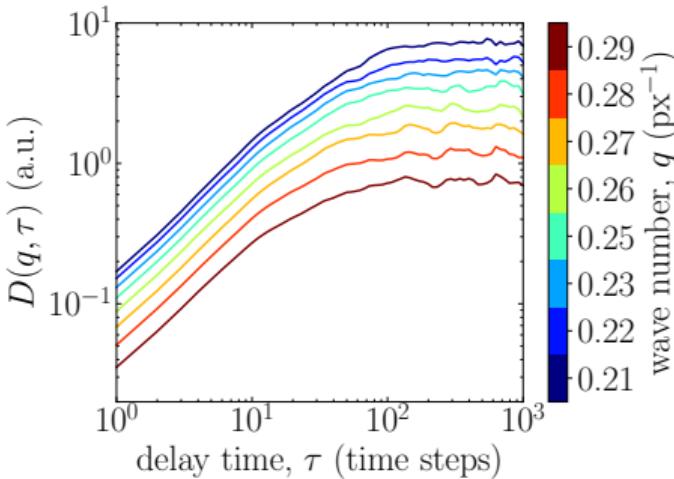


$$\begin{aligned}
 D(q, \tau) &= \left\langle |I(q, t + \tau) - I(q, t)|^2 \right\rangle_t \\
 &= A(q) \underbrace{\left[1 - \frac{\left\langle I^*(q, t) I(q, t + \tau) \right\rangle_t}{\left\langle |I(q, t)|^2 \right\rangle_t} \right]}_{\text{Image correlation function}} + B(q)
 \end{aligned}$$





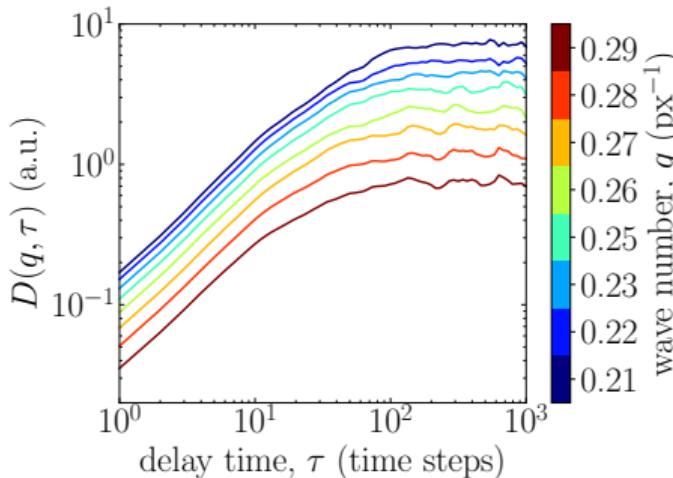
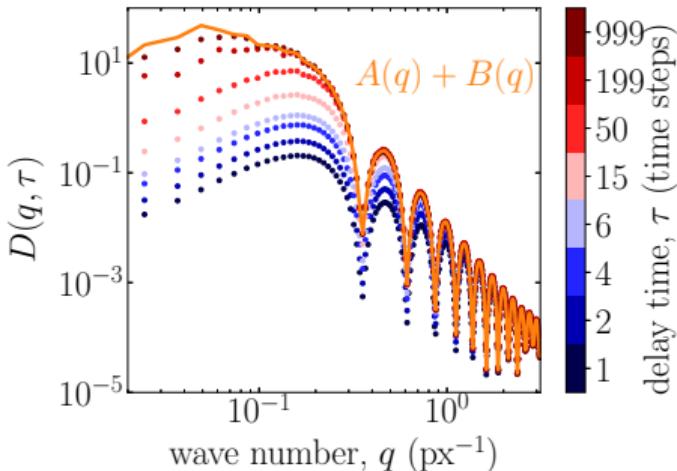
$$\begin{aligned}
 D(q, \tau) &= \left\langle |I(q, t + \tau) - I(q, t)|^2 \right\rangle_t \\
 &= A(q) \underbrace{\left[1 - \frac{\langle I^*(q, t) I(q, t + \tau) \rangle_t}{\langle |I(q, t)|^2 \rangle_t} \right]}_{\text{Image correlation function}} + B(q)
 \end{aligned}$$



Linear space invariant imaging

$$f(q, \tau) = \frac{\langle \rho^*(q, t) \rho(q, t + \tau) \rangle_t}{\langle |\rho(q, t)|^2 \rangle_t}$$

Intermediate scattering function



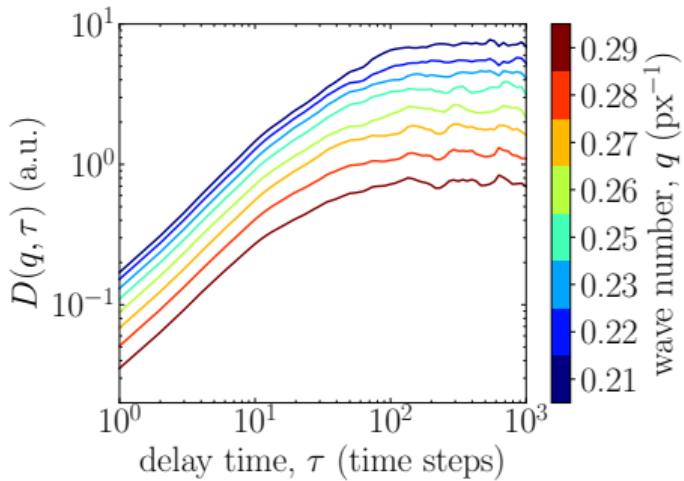
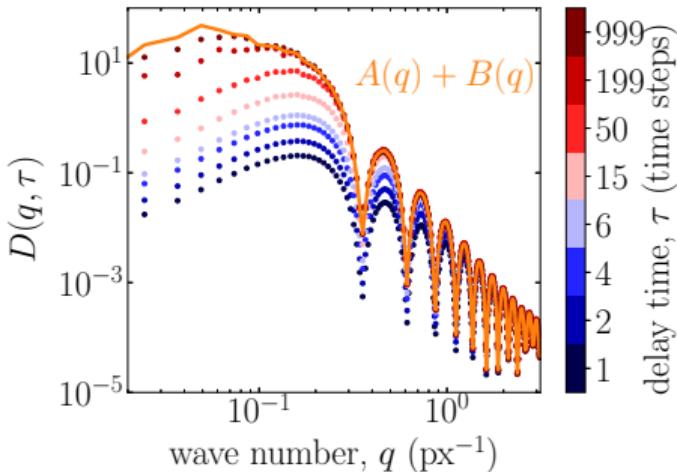
$$\begin{aligned}
 D(q, \tau) &= \left\langle |I(q, t + \tau) - I(q, t)|^2 \right\rangle_t \\
 &= A(q) \left[1 - \frac{\langle I^*(q, t) I(q, t + \tau) \rangle_t}{\langle |I(q, t)|^2 \rangle_t} \right] + B(q)
 \end{aligned}$$

- $D(q, \tau \rightarrow 0) = B(q) = 0$
- $D(q, \tau \rightarrow \infty) = A(q) + B(q)$

Linear space invariant imaging

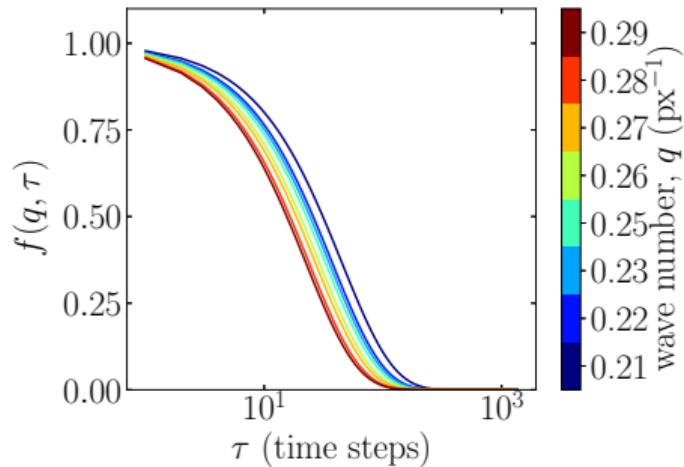
$$f(q, \tau) = \frac{\langle \rho^*(q, t) \rho(q, t + \tau) \rangle_t}{\langle |\rho(\mathbf{q}, t)|^2 \rangle_t}$$

Intermediate scattering function

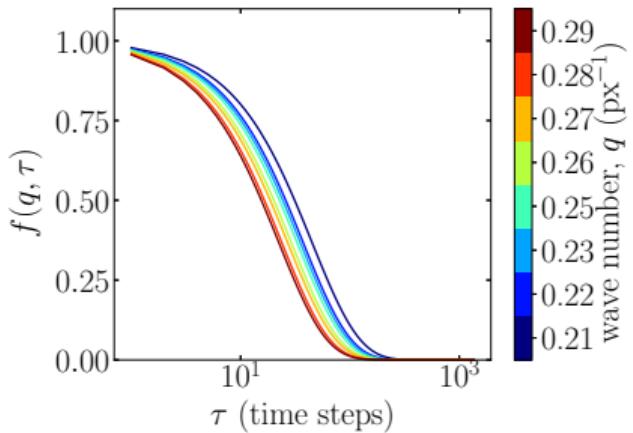


$$\begin{aligned} D(q, \tau) &= \left\langle |I(q, t + \tau) - I(q, t)|^2 \right\rangle_t \\ &= A(q) \left[1 - \frac{\langle I^*(q, t) I(q, t + \tau) \rangle_t}{\langle |I(q, t)|^2 \rangle_t} \right] + B(q) \end{aligned}$$

- $D(q, \tau \rightarrow 0) = B(q) = 0$
- $D(q, \tau \rightarrow \infty) = A(q) + B(q)$

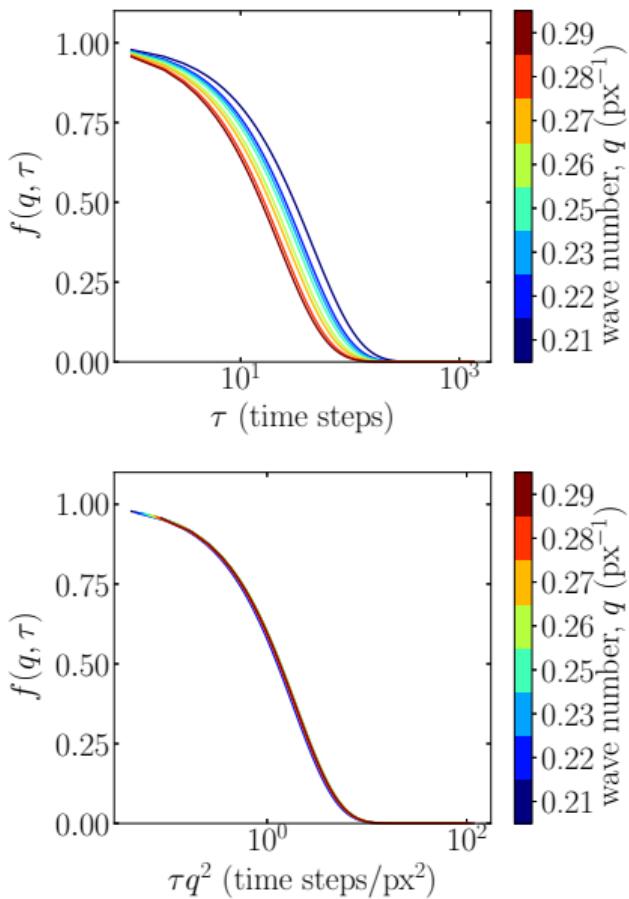


Intermediate scattering function $f(q, \tau)$

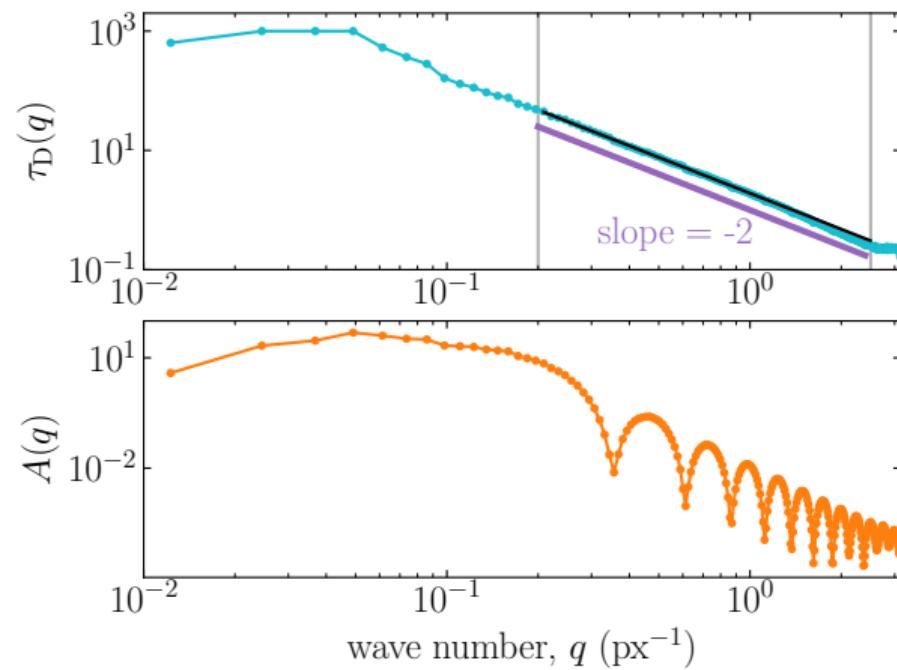


Brownian motion:
 $f(q, \tau) = \exp(-q^2\tau/\tau_D)$

Intermediate scattering function $f(q, \tau)$

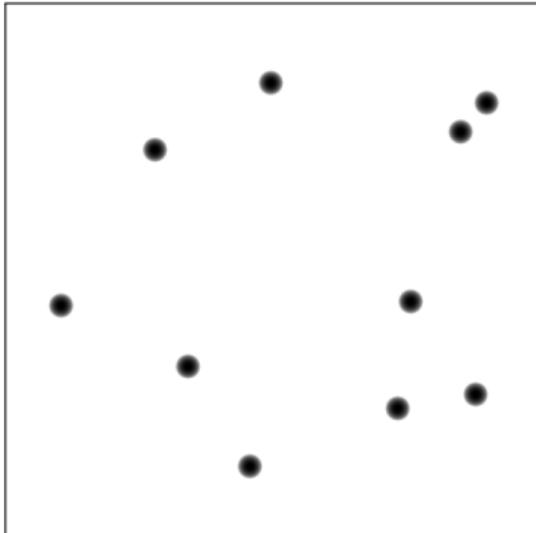


Brownian motion:
 $f(q, \tau) = \exp(-q^2\tau/\tau_D)$

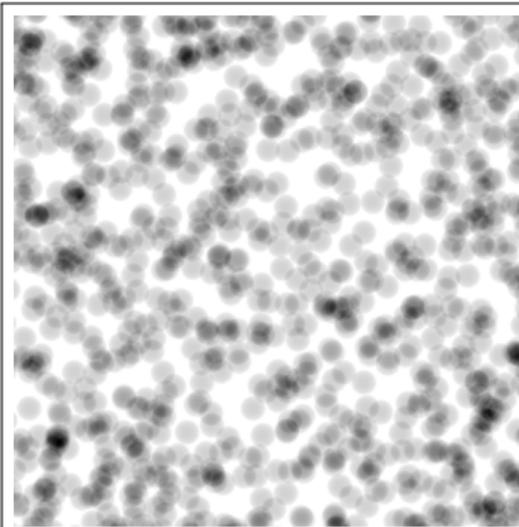


Accuracy of X-DFA: Varying the number of particles

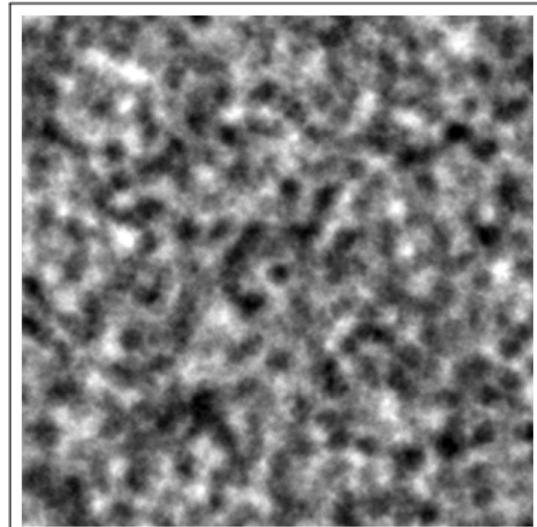
10 particles



1000 particles



100 000 particles



Deviation from the simulation input:

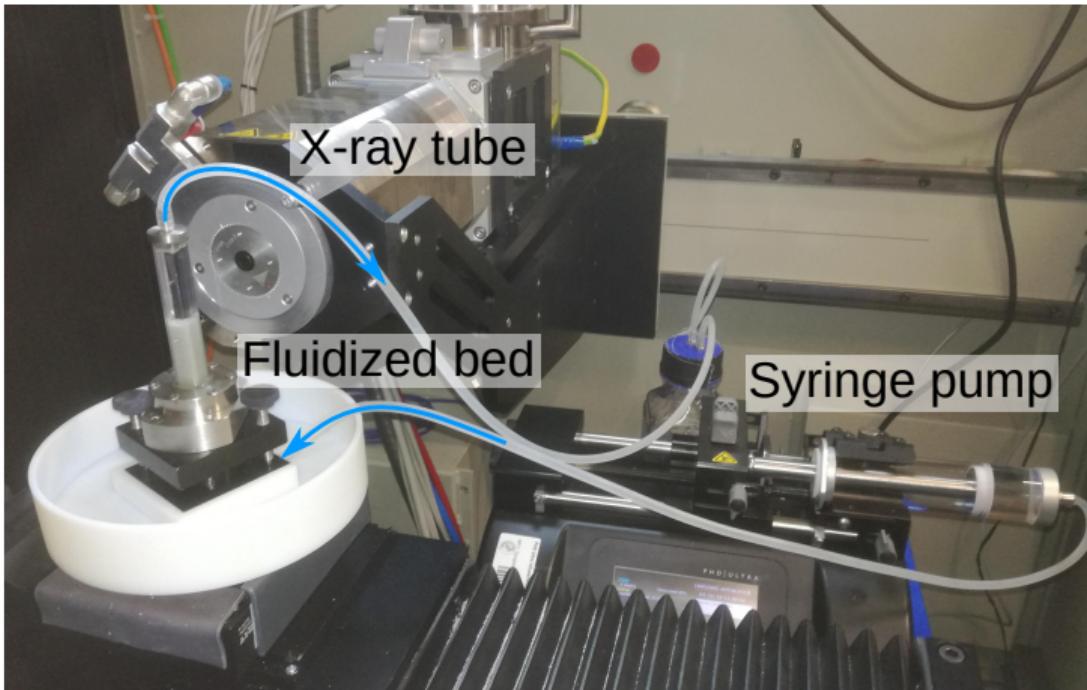
6%

2%

2%

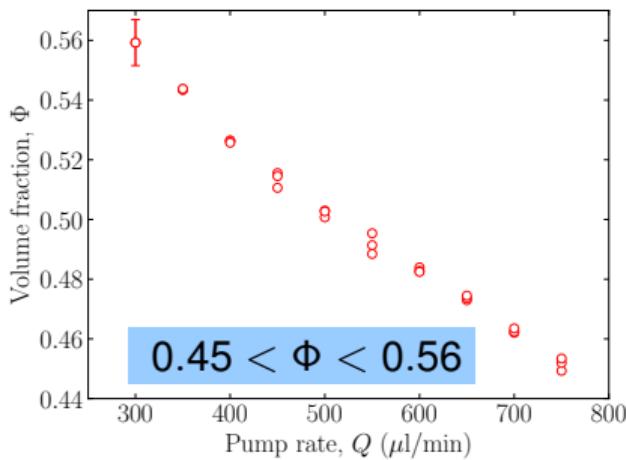
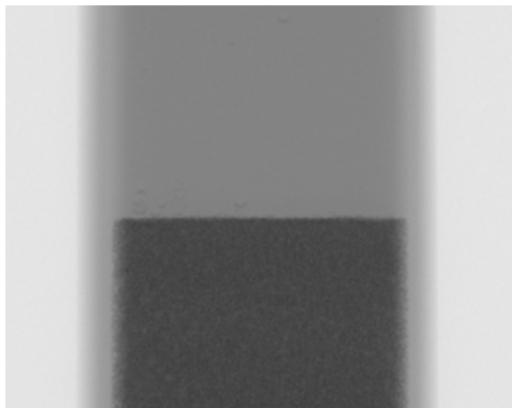
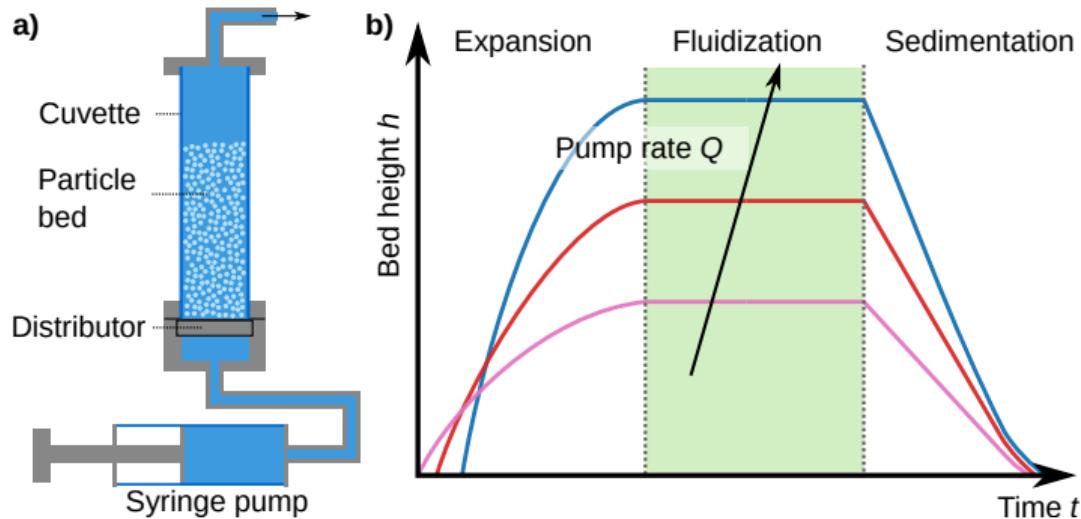
PIV off by $\approx 650\%$

Experimental validation of X-DFA: Sedimenting particle suspension



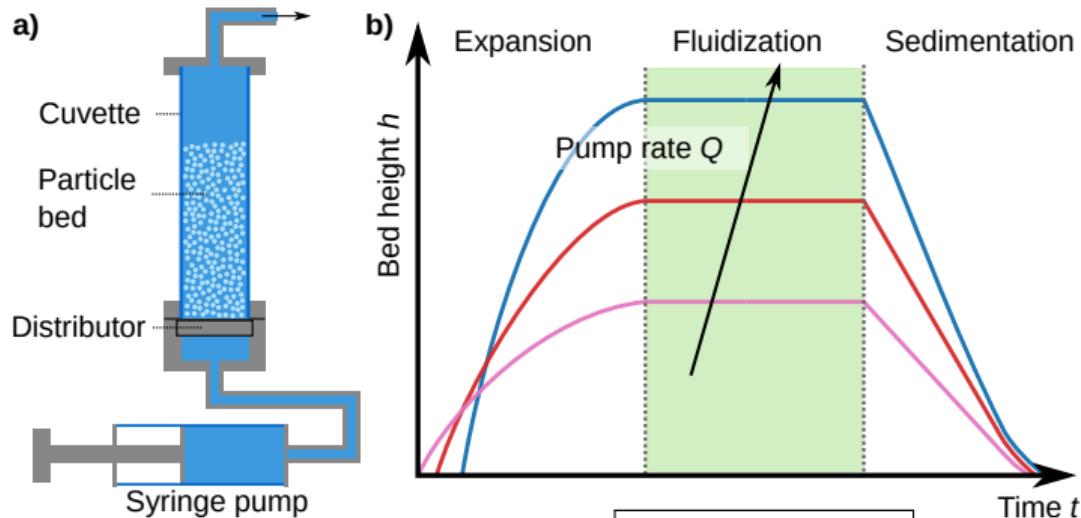
Experimental validation of X-DFA: Sedimenting particle suspension

X-ray radiography

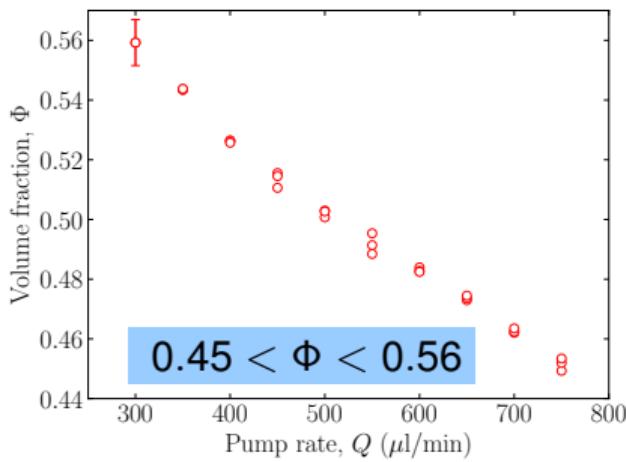
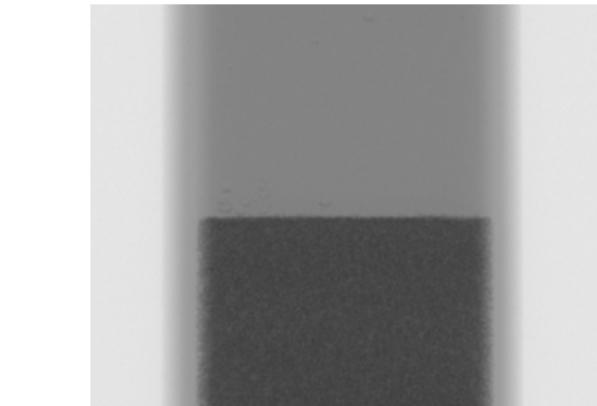
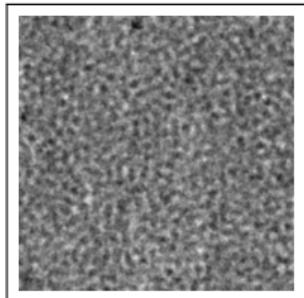


Experimental validation of X-DFA: Sedimenting particle suspension

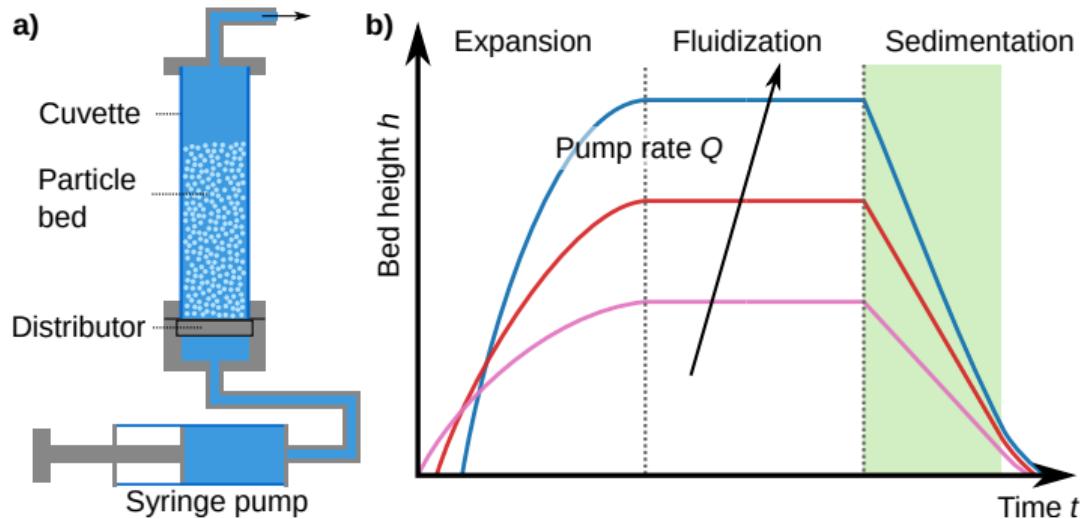
X-ray radiography



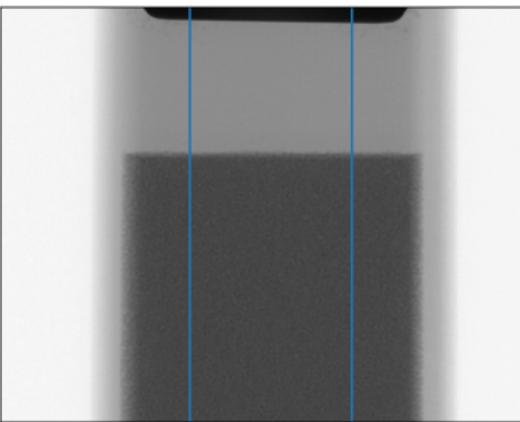
No reliable reference velocity!



Experimental validation of X-DFA: Sedimenting particle suspension

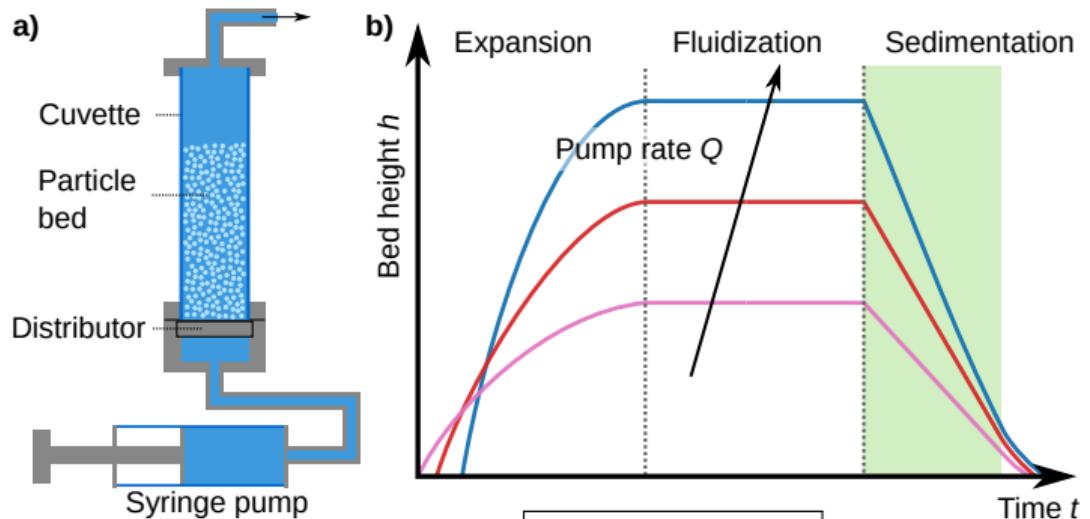


X-ray radiography

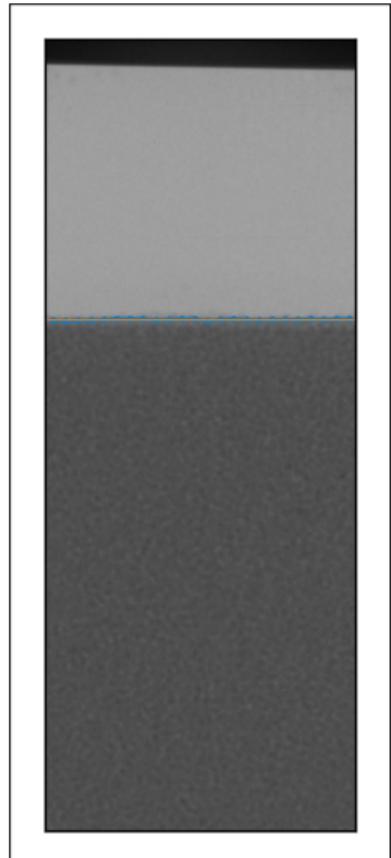
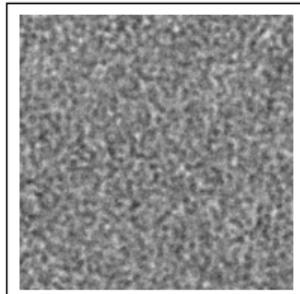


Experimental validation of X-DFA: Sedimenting particle suspension

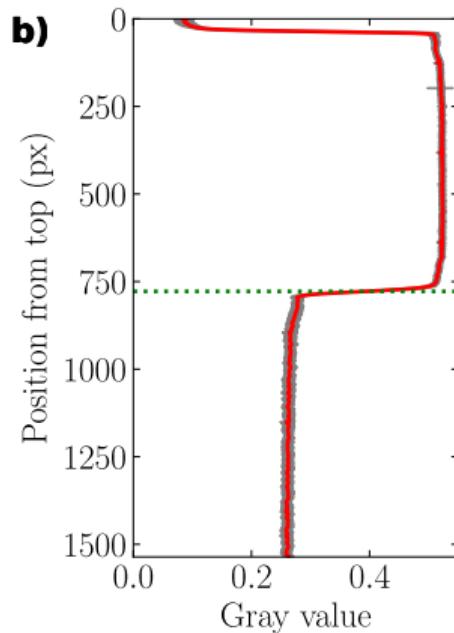
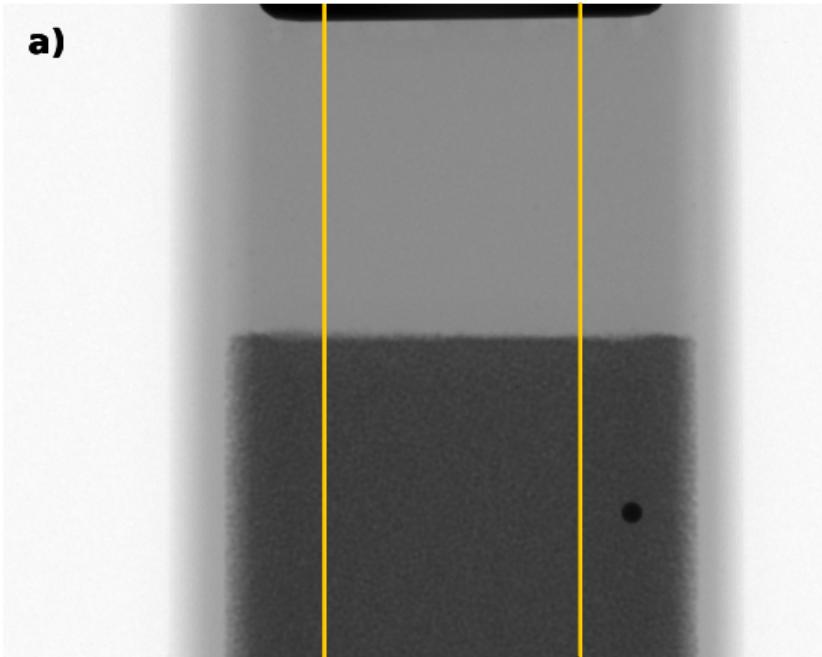
X-ray radiography



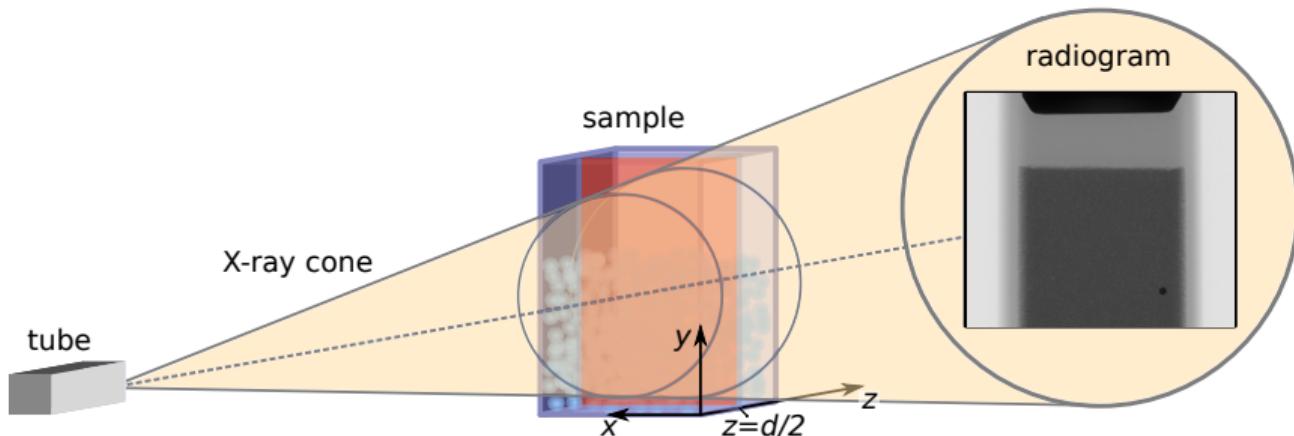
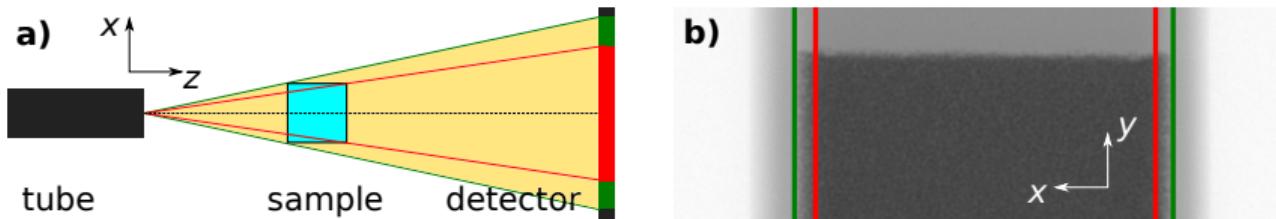
Comparison of
 $\langle v \rangle_{\text{xdfa}}$ and $\langle v \rangle_{\text{front}}$



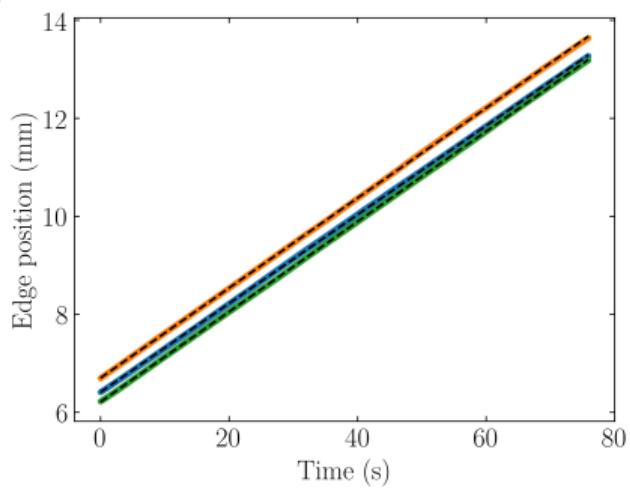
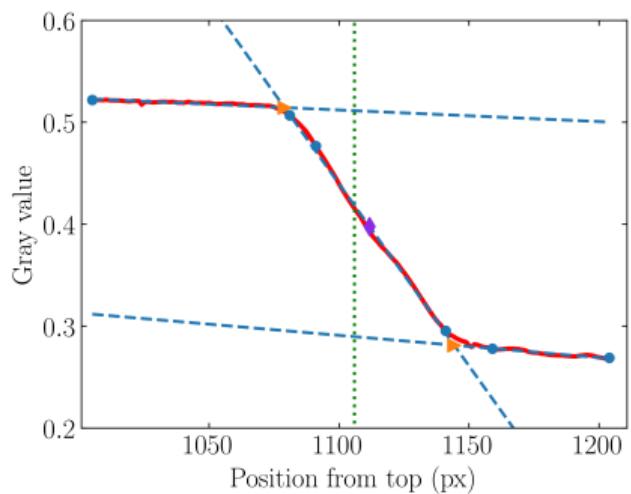
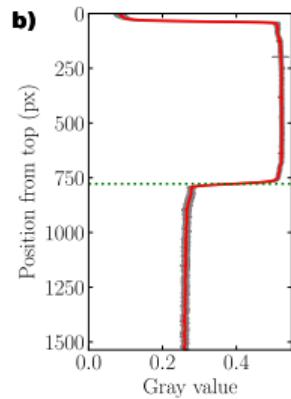
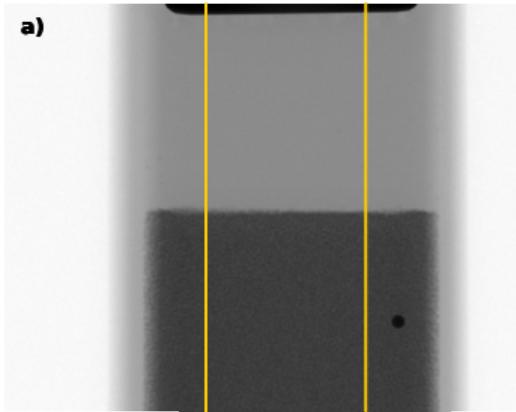
Tracking of particle front - work from here on



Tracking of particle front

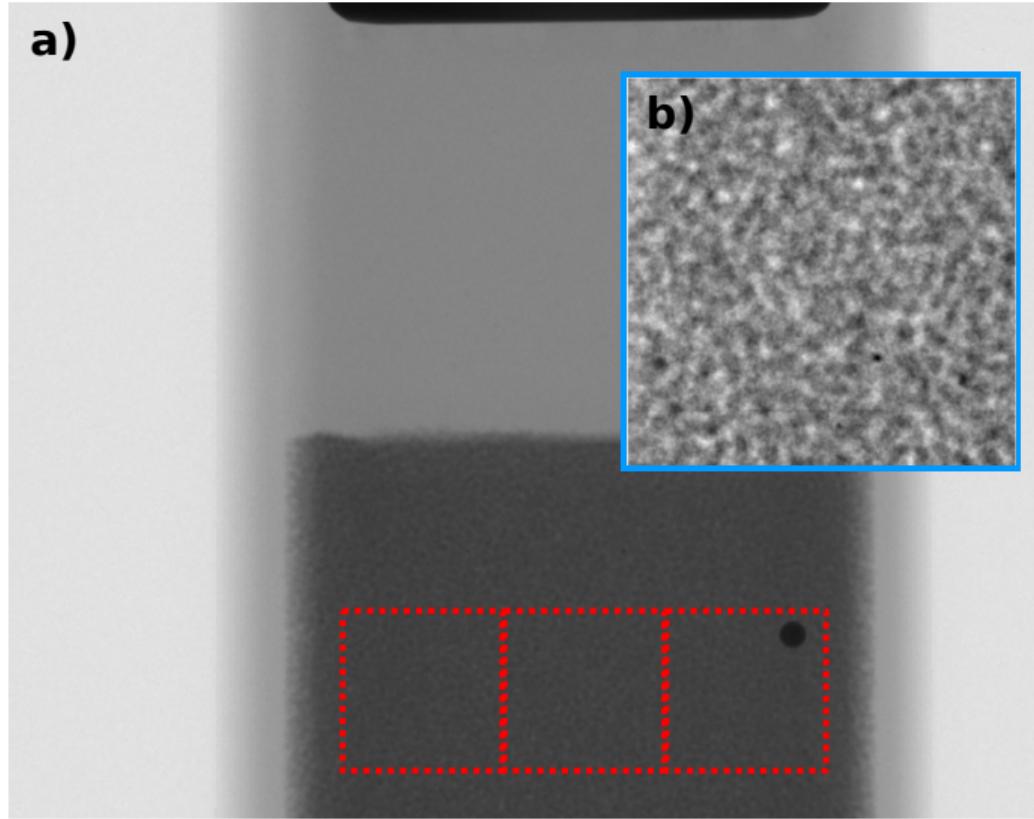


Tracking of particle front

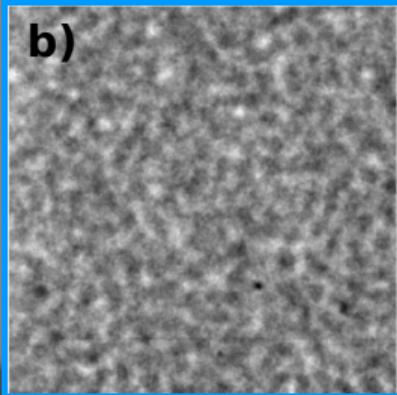


X-DFA for sedimenting particles

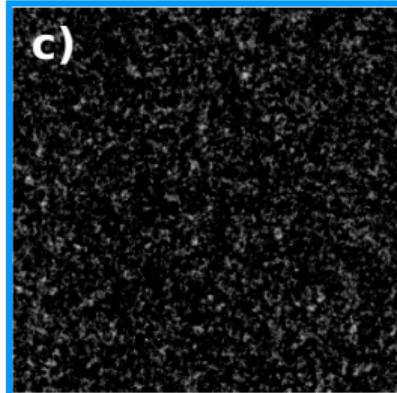
a)



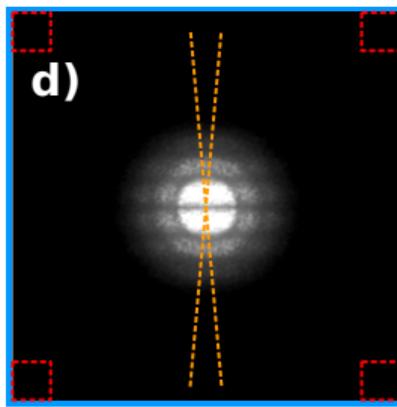
b)



c)

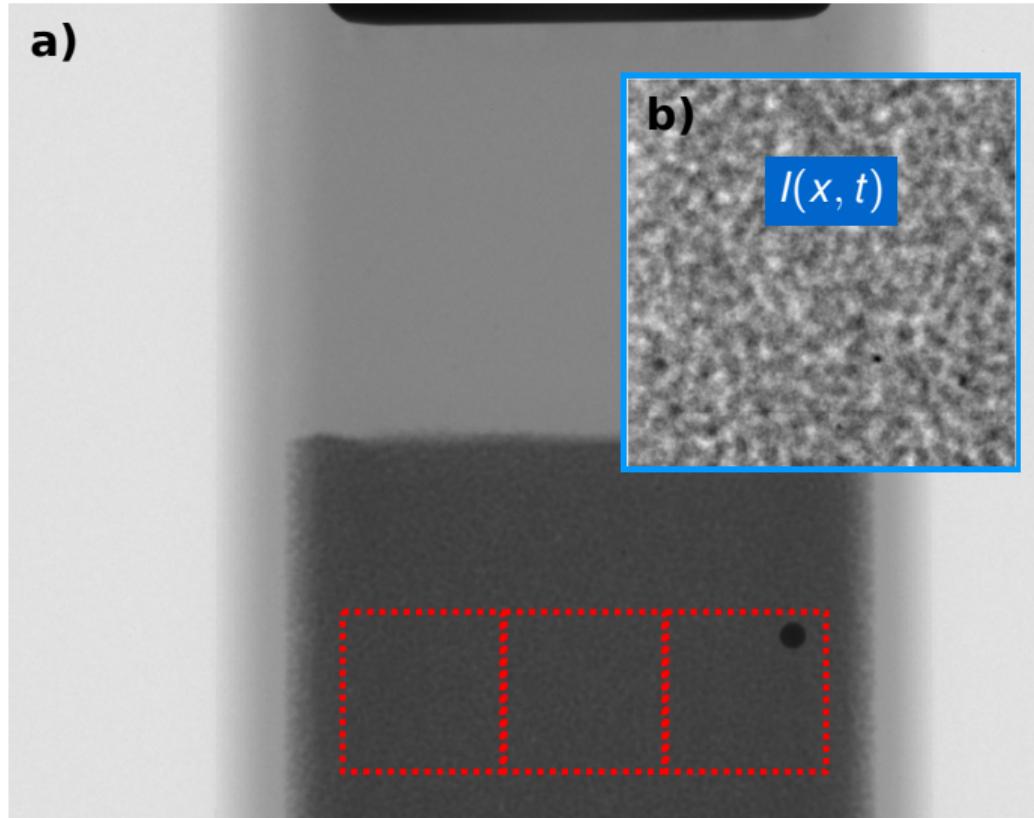


d)



X-DFA for sedimenting particles

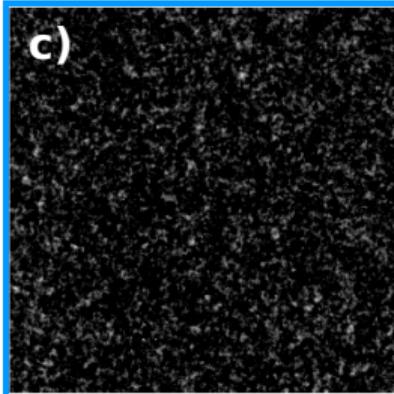
a)



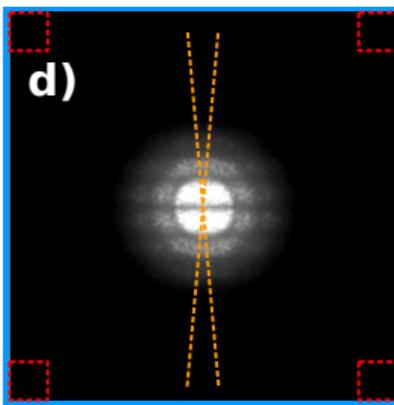
b)

$$I(x, t)$$

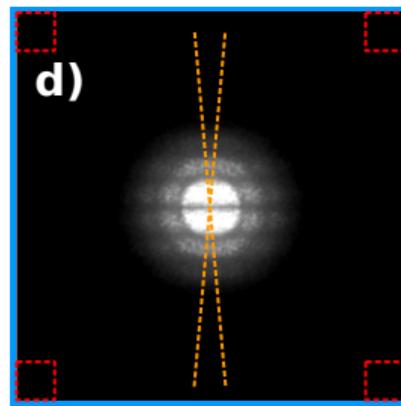
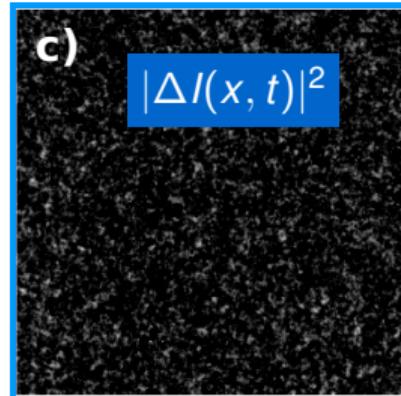
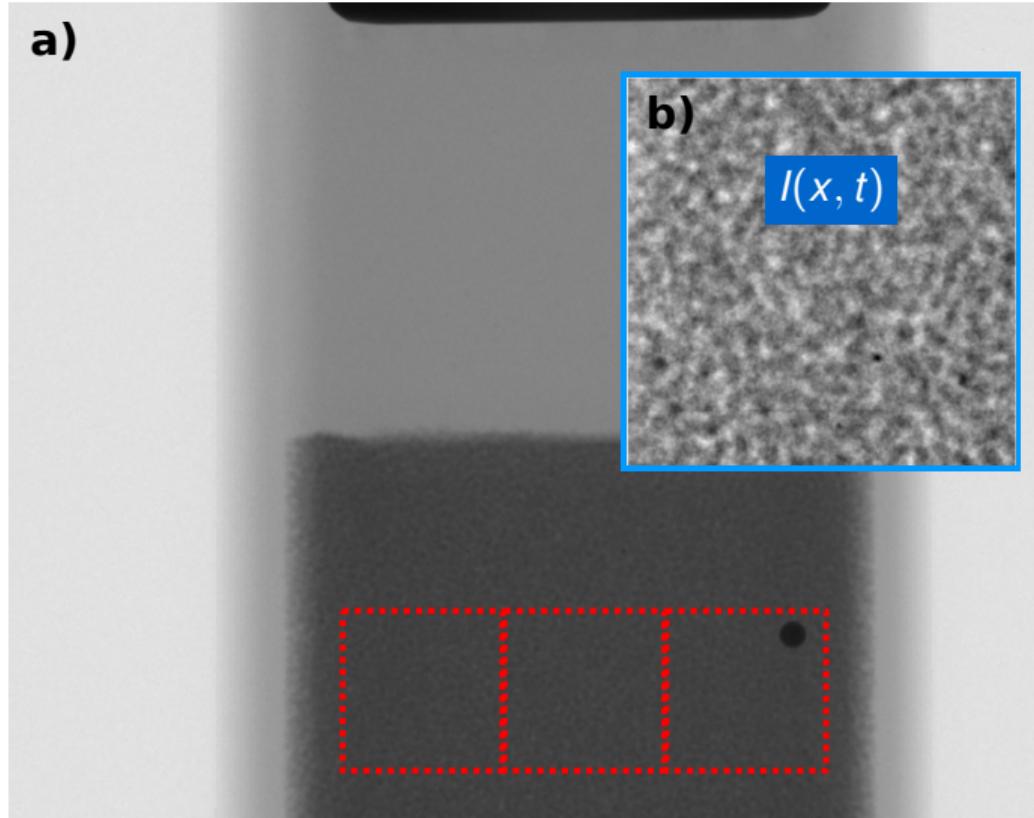
c)



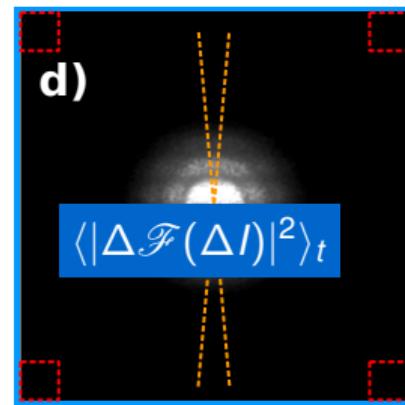
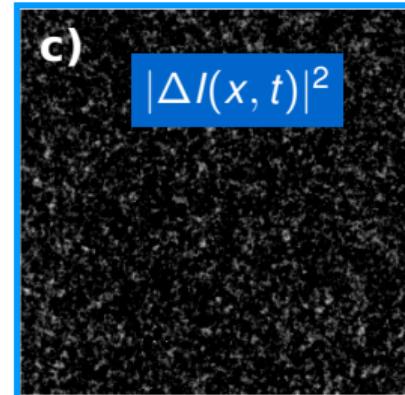
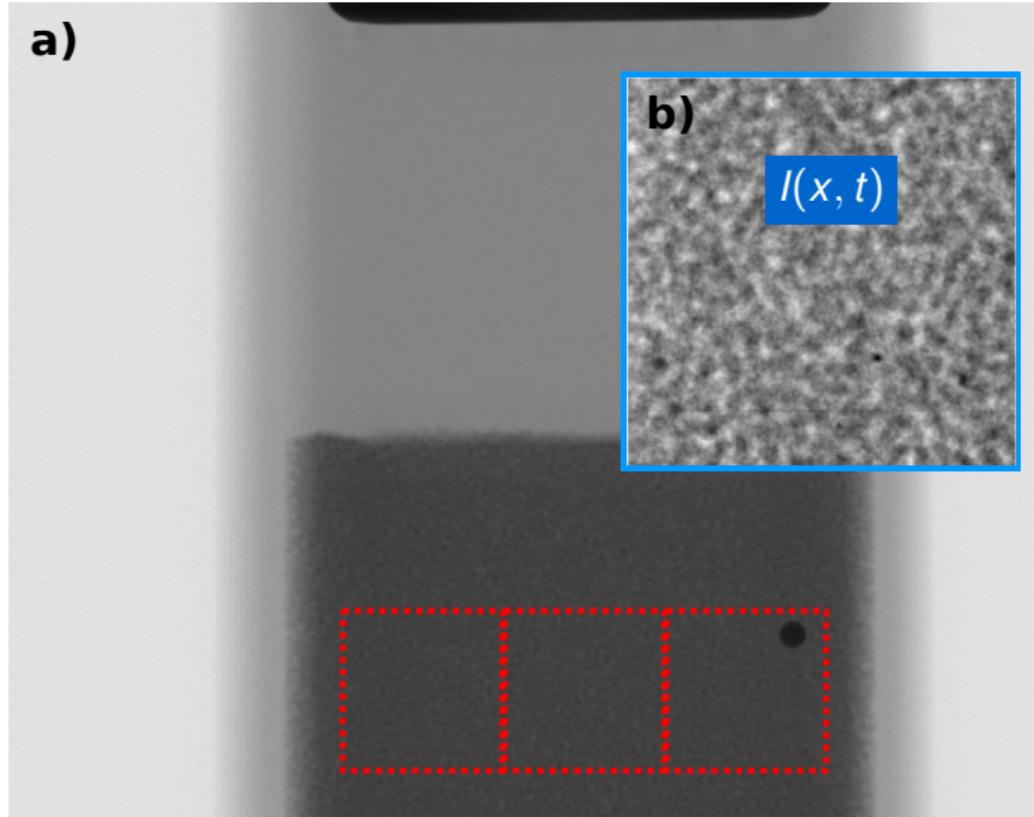
d)



X-DFA for sedimenting particles



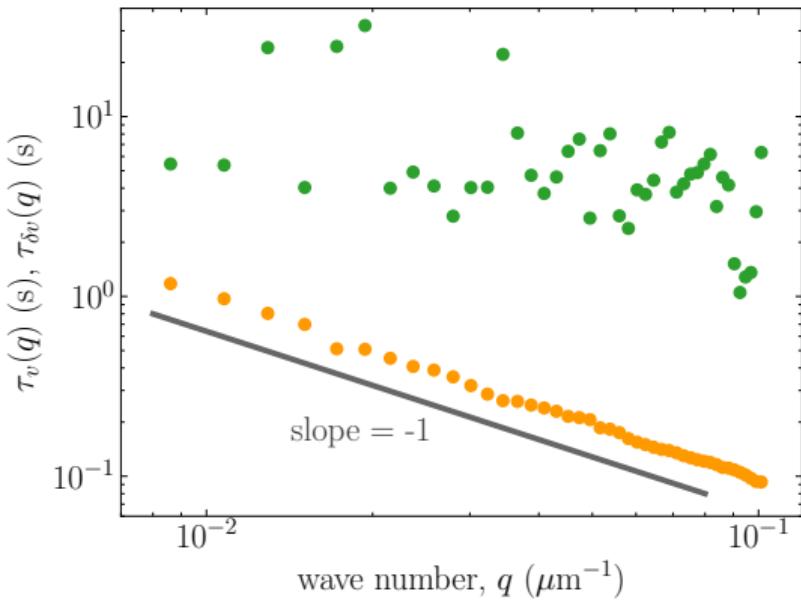
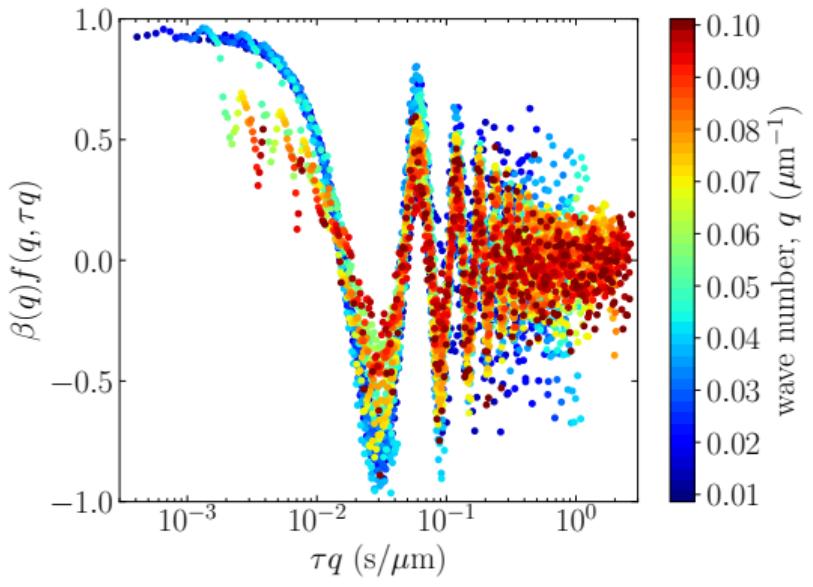
X-DFA for sedimenting particles



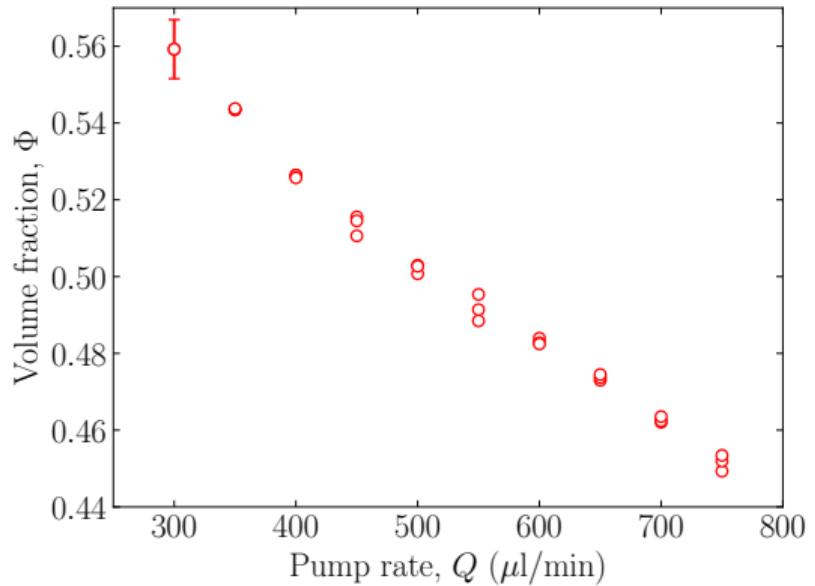
X-DFA for sedimenting particles

$$f(q, \tau) = \cos(q\langle v_s \rangle \tau) \exp\left(-\frac{1}{2}q^2 \delta v^2 \tau^2\right)$$

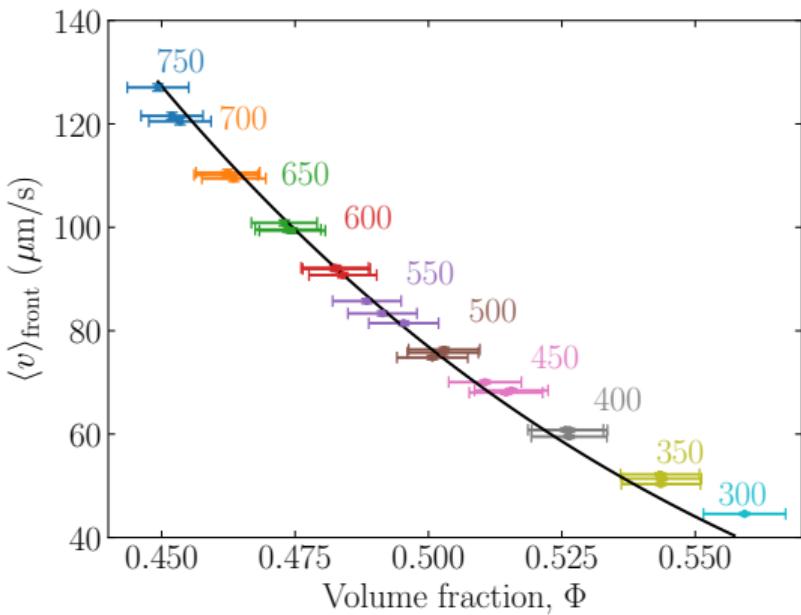
$$\langle v_s \rangle = \langle \Delta r \rangle / \tau_\nu, \langle \delta v \rangle = \langle \delta r \rangle / \tau_{\delta\nu}$$



Richardson-Zaki law

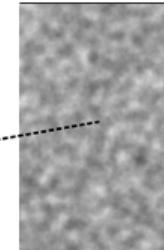
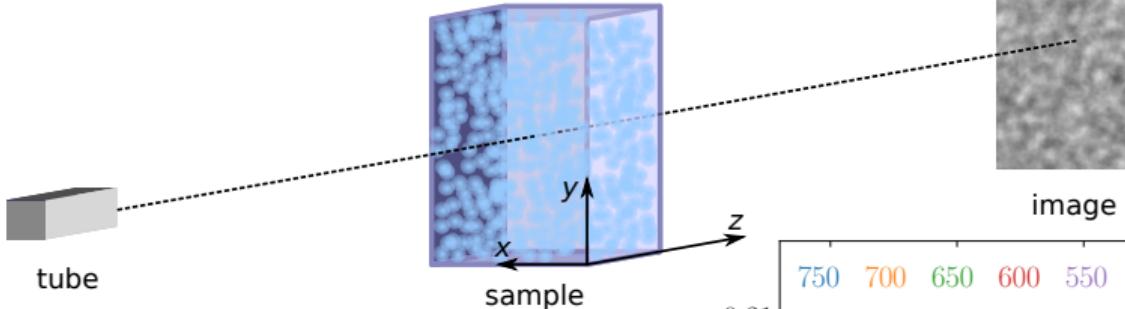


$$\frac{\langle v \rangle_{\text{front}}}{v_{\text{Stokes}}} = (1 - \Phi)^n$$

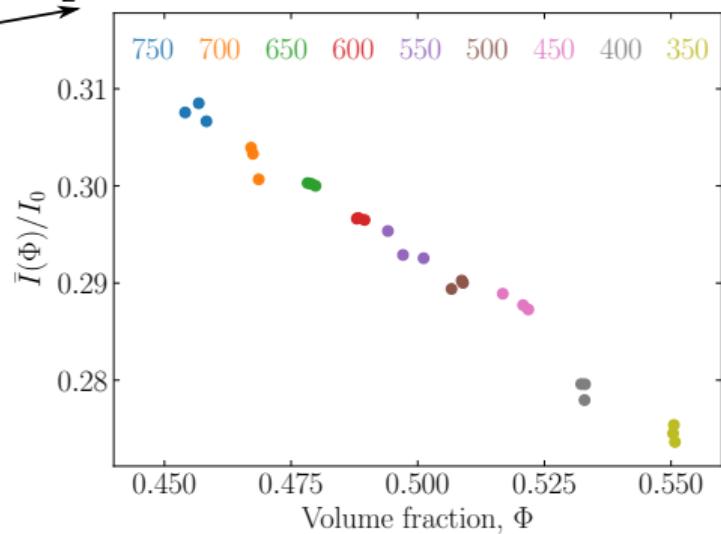


X-DFA: Requirement of linear space invariant imaging

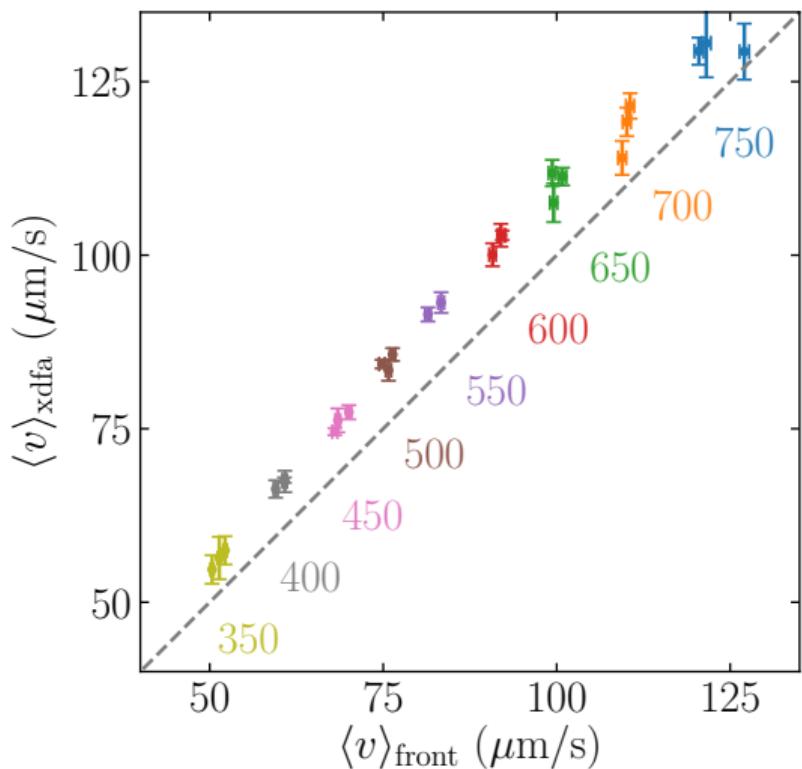
$$I(\mathbf{r}, t) = I_0 + \int d\mathbf{r}' dz' T(\mathbf{r} - \mathbf{r}', -z') c(\mathbf{r}', z', t)$$



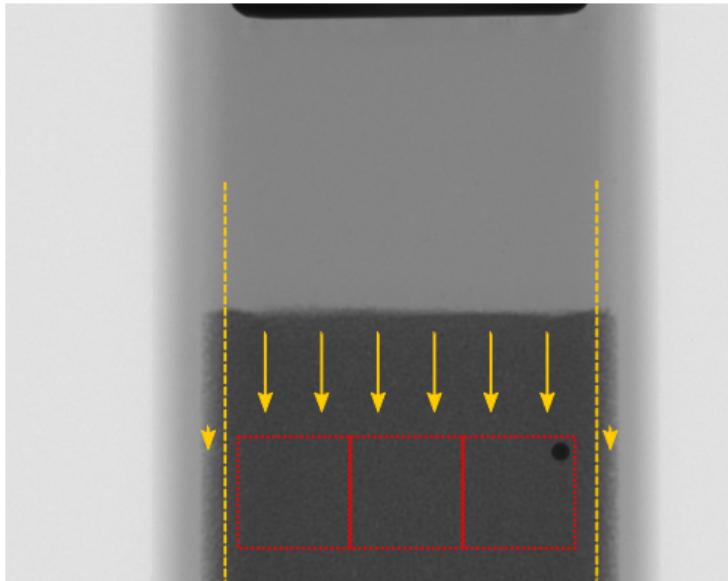
image



Front tracking vs. X-DFA

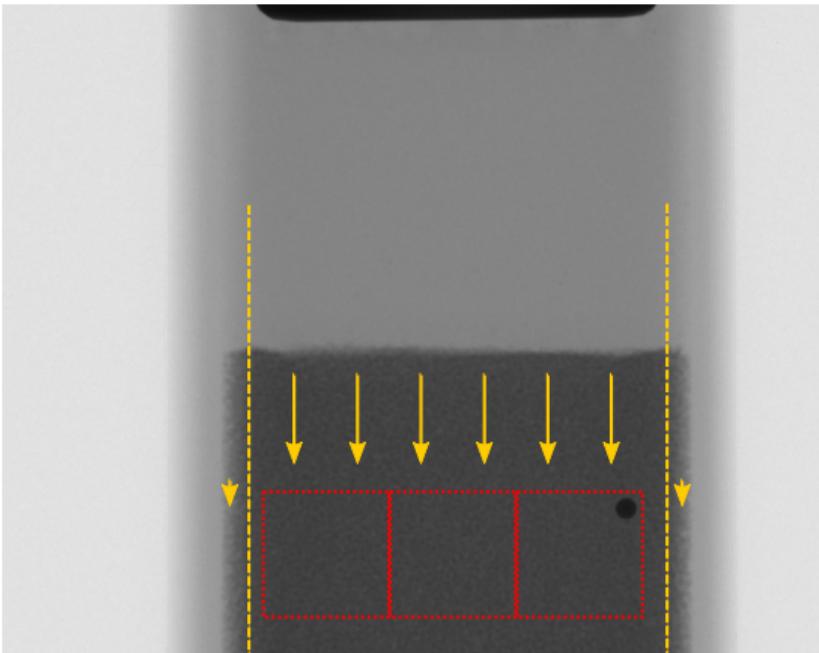


$\langle v \rangle_{\text{xdfa}} > \langle v \rangle_{\text{front}}$ by 9.4%



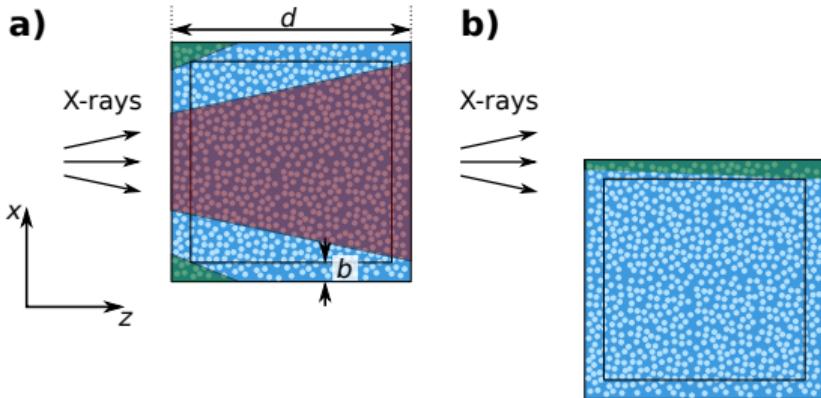
sample video

Estimate width of boundary layer



$\langle v \rangle_{\text{xdfa}} > \langle v \rangle_{\text{front}}$ by 9.4%

$\langle v \rangle_{\text{xdfa}}$ takes two layers into account
 $\langle v \rangle_{\text{front}}$ takes four layers into account



Estimation:

Boundary velocity = 0

Else = const.

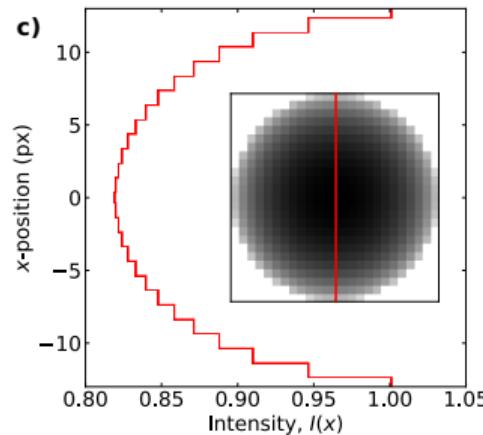
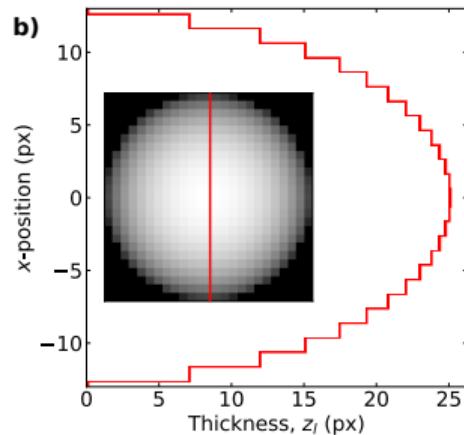
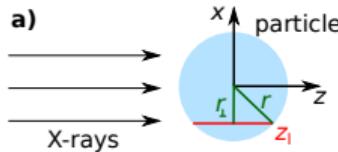
$\rightarrow b \approx 3$ particle diameters

Thank you for your attention!



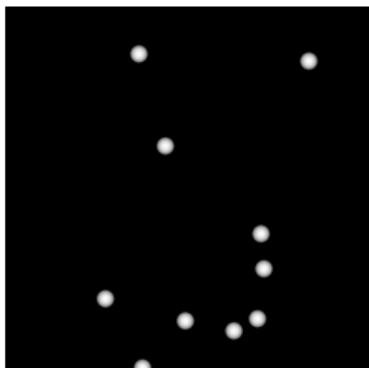
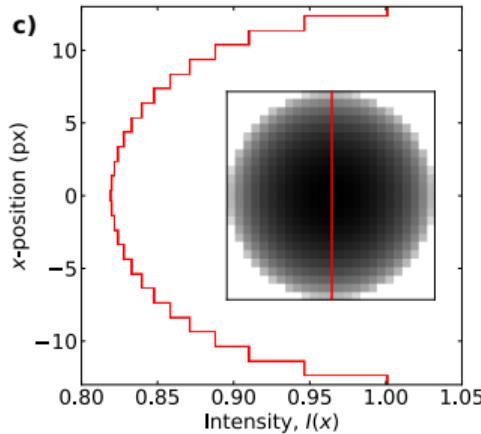
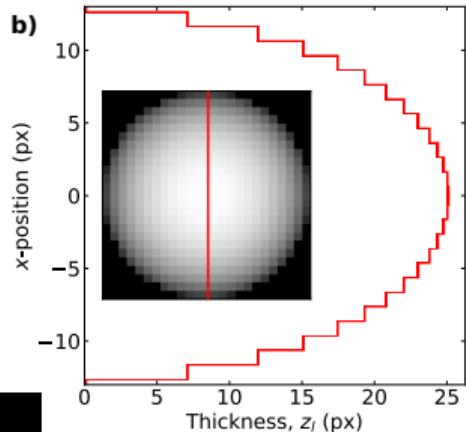
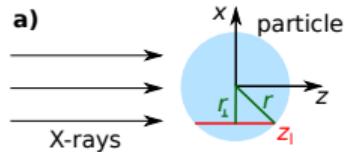
backup slides from here on

Synthetic radiograms

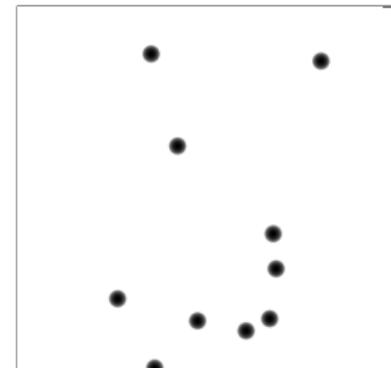


Beer-Lambert
 $I(z_I) = I_0 \exp(-\mu z)$

Synthetic radiograms



Beer-Lambert
 $I(z_l) = I_0 \exp(-\mu z)$



Linear space invariant imaging

Image correlation function

$$g(\mathbf{q}, \tau) = \frac{\langle I^*(\mathbf{q}, t)I(\mathbf{q}, t + \tau) \rangle_t}{\langle |I(\mathbf{q}, t)|^2 \rangle_t}$$

Linear space invariant imaging

Image correlation function

$$g(\mathbf{q}, \tau) = \frac{\langle I^*(\mathbf{q}, t)I(\mathbf{q}, t + \tau) \rangle_t}{\langle |I(\mathbf{q}, t)|^2 \rangle_t}$$

Intermediate scattering function

$$f(\mathbf{q}, \tau) = \frac{\langle \rho^*(\mathbf{q}, t)\rho(\mathbf{q}, t + \tau) \rangle_t}{\langle |\rho(\mathbf{q}, t)|^2 \rangle_t}$$

Linear space-invariant imaging:

$$I(\mathbf{r}, t) = I_0 + \int d\mathbf{r}' dz' T(\mathbf{r} - \mathbf{r}', -z') c(\mathbf{r}', z', t)$$

