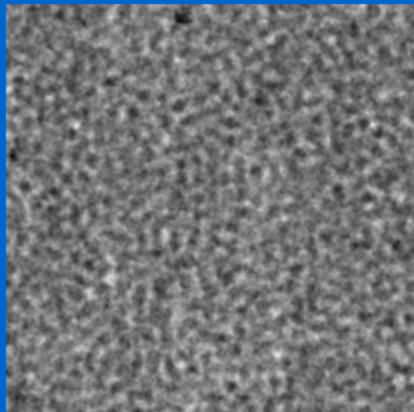
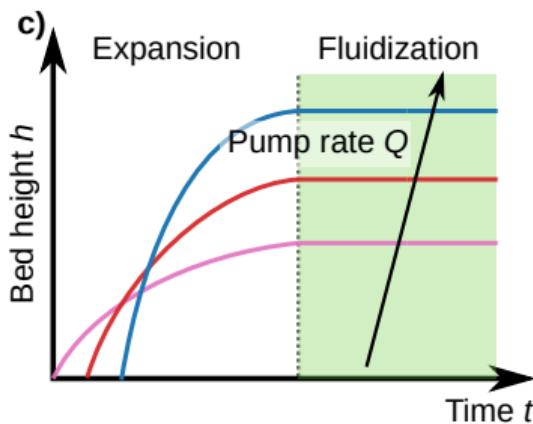
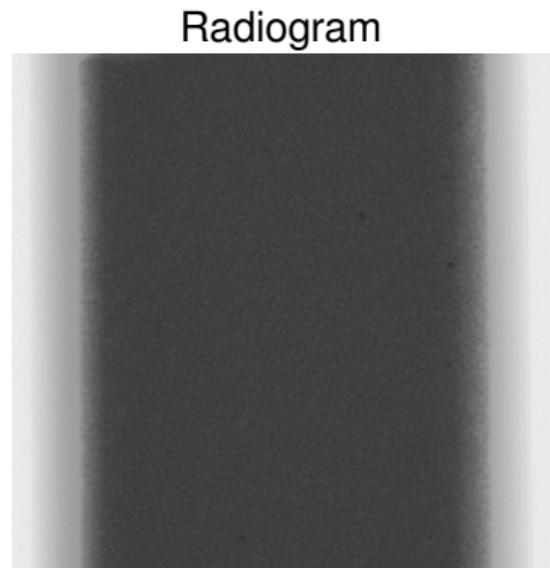
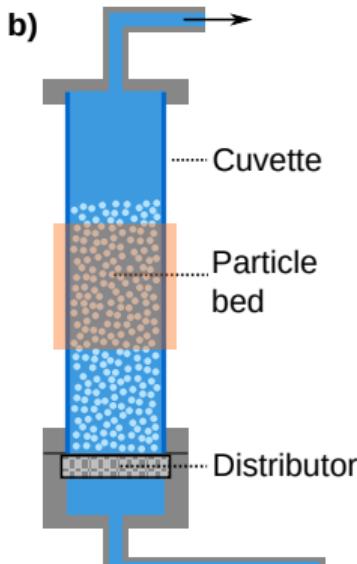
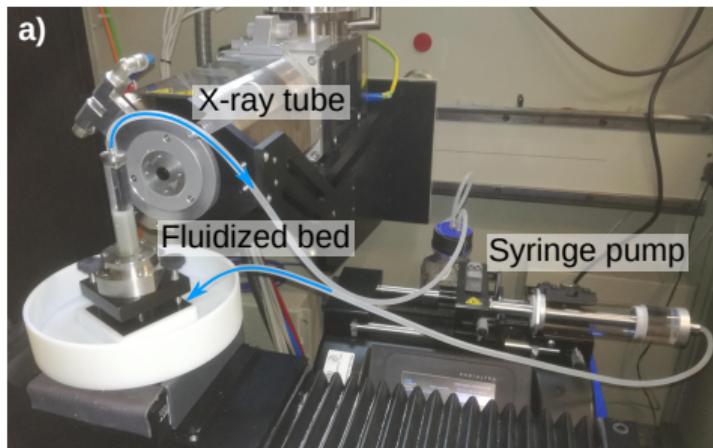


# Measuring granular dynamics with X-ray Digital Fourier Analysis (X-DFA)



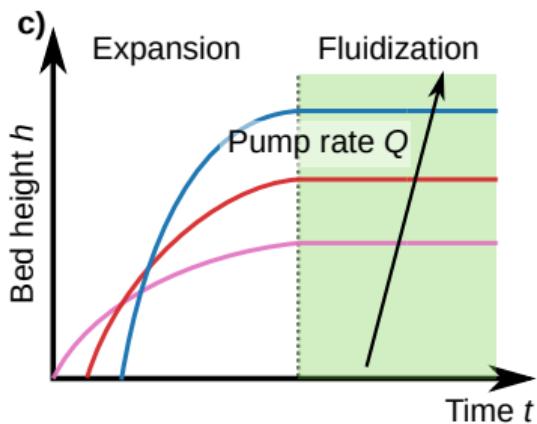
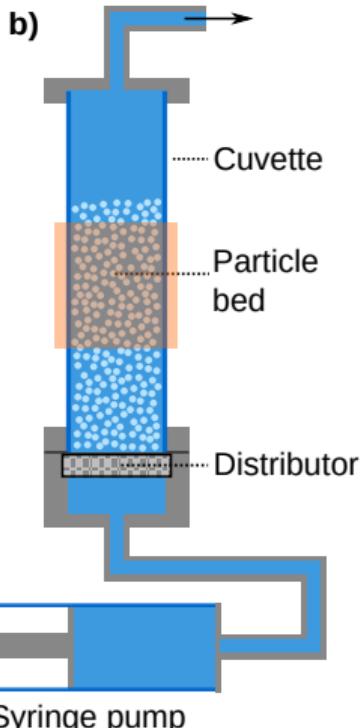
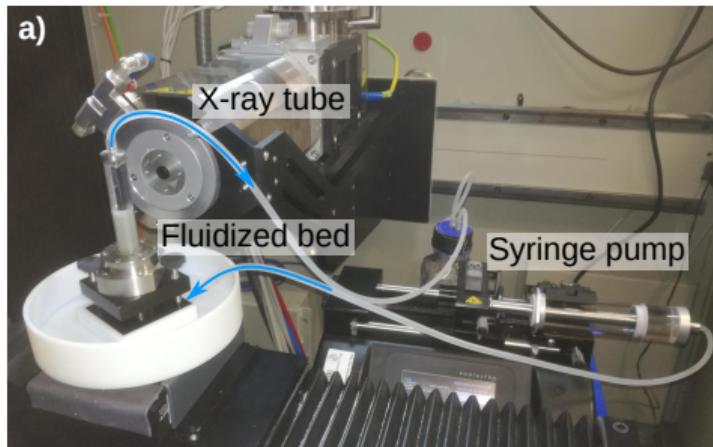
In collaboration with M. Escobedo & S. Egelhaaf, University of Düsseldorf

# Experiments: A liquid fluidized bed



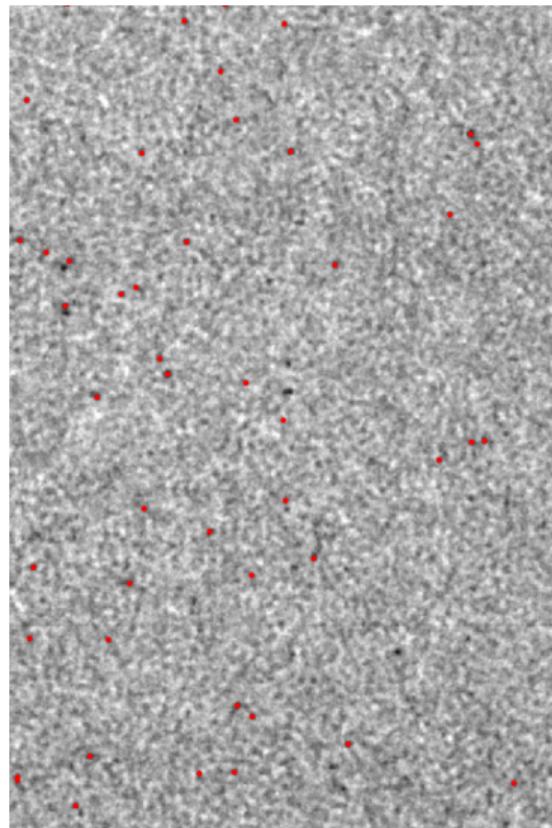
$$0.45 < \phi < 0.56$$

# Experiments: A liquid fluidized bed

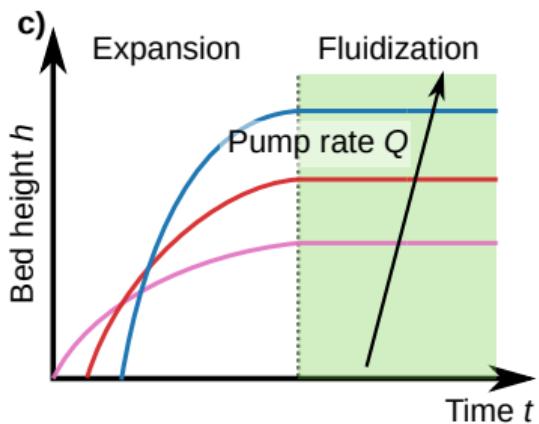
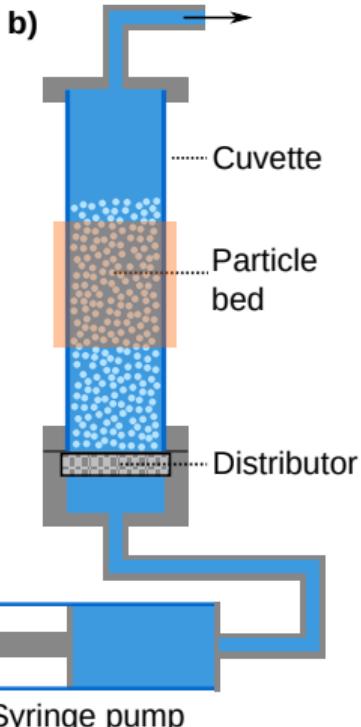
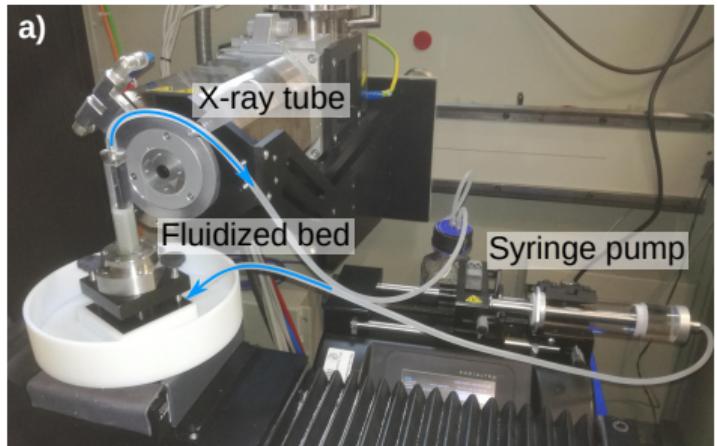


$$0.45 < \phi < 0.56$$

Particle tracking

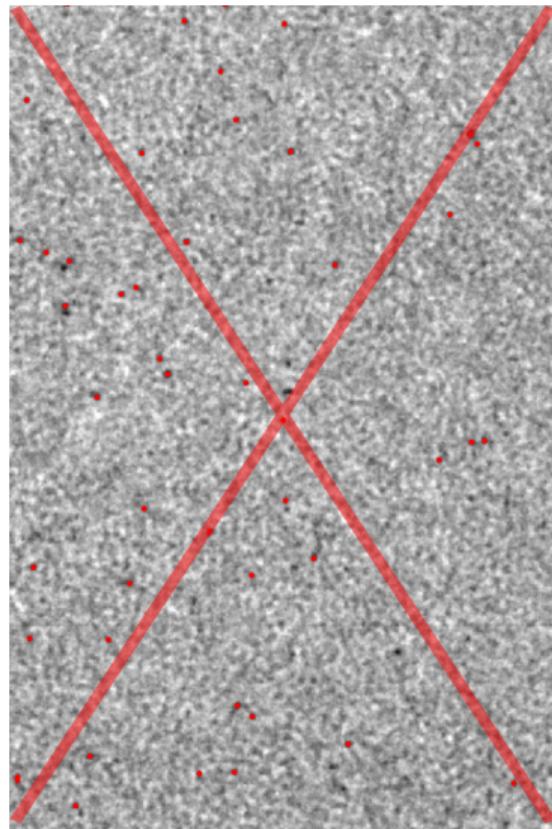


# Experiments: A liquid fluidized bed

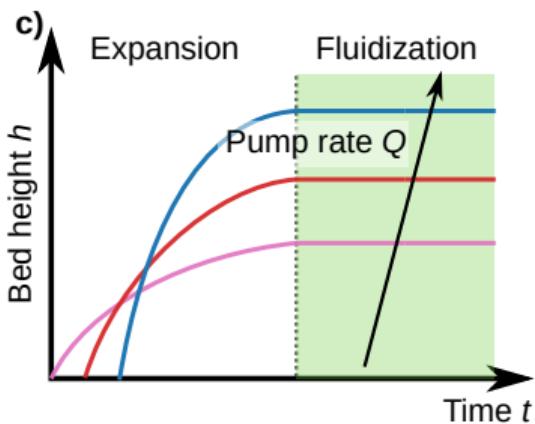
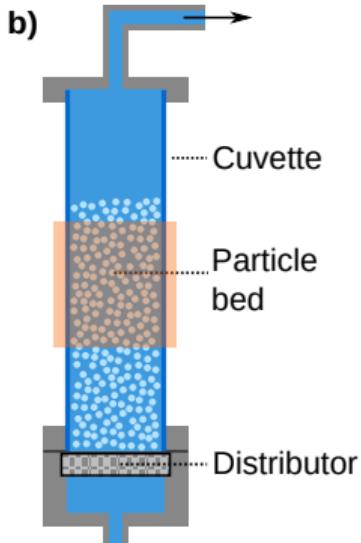
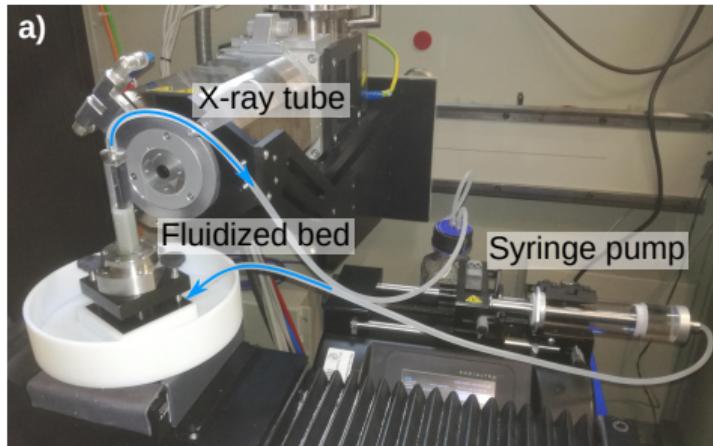


$$0.45 < \phi < 0.56$$

Particle tracking

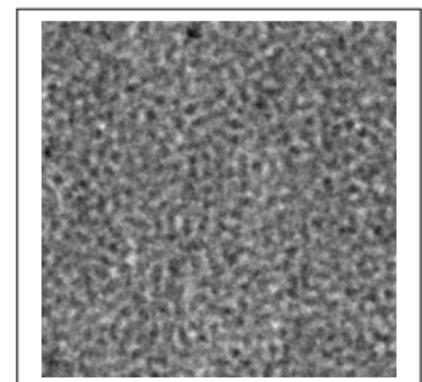
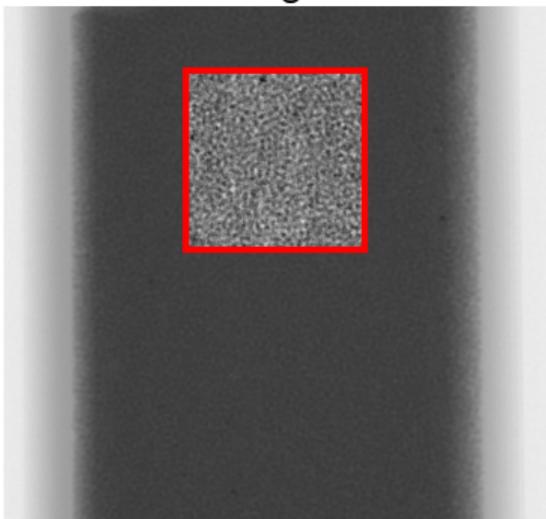


# Experiments: A liquid fluidized bed



$$0.45 < \Phi < 0.56$$

Radiogram



## Differential Dynamic Microscopy (DDM)

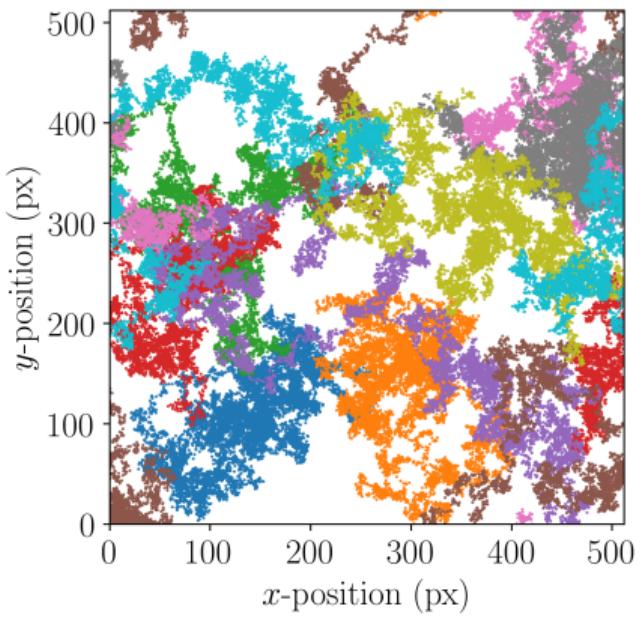
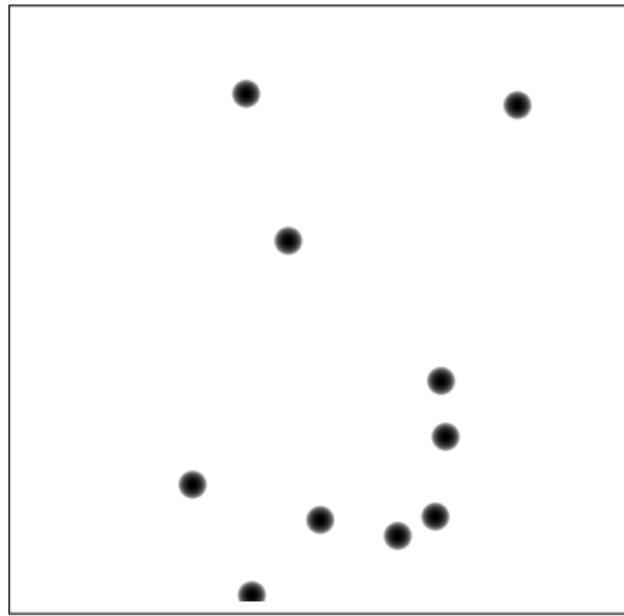
	<b>up to now</b>	<b>this work</b>
<b>system</b>	dispersion, gels	fluidized bed
<b>particles</b>	colloids	granulate
<b>part. diameter</b>	$< 1 \mu\text{m}$	$\approx 200 \mu\text{m}$
<b>volume fraction</b>	$\Phi \leq 0.33$	$0.45 < \Phi < 0.56$
<b>imaging</b>	light microscope	x-ray radiography
<b>dynamics</b>	Brownian motion, caging, glassy, collective motion	

# Extending Differential Dynamic Microscopy (DDM) to X-ray imaging

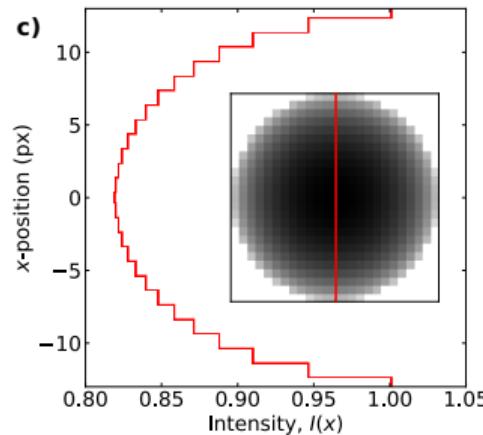
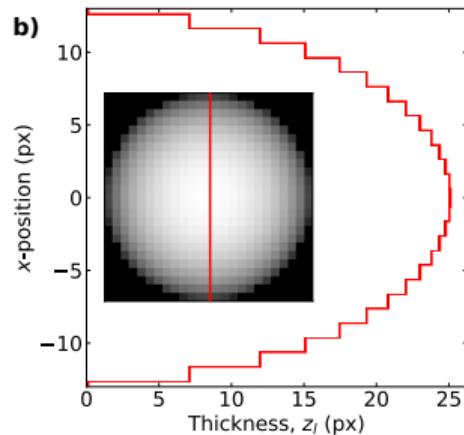
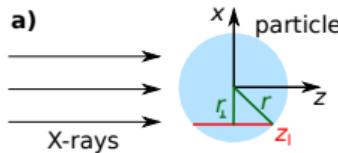
	<b>up to now</b>	<b>this work</b>
<b>system</b>	dispersion, gels	fluidized bed
<b>particles</b>	colloids	granulate
<b>part. diameter</b>	$< 1 \mu\text{m}$	$\approx 200 \mu\text{m}$
<b>volume fraction</b>	$\Phi \leq 0.33$	$0.45 < \Phi < 0.56$
<b>imaging</b>	light microscope	x-ray radiography
<b>dynamics</b>	Brownian motion, caging, glassy, collective motion	

## Digital Fourier Analysis of X-Ray Radiograms (X-DFA)

## Synthetic radiograms

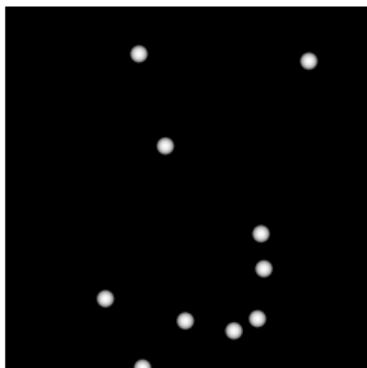
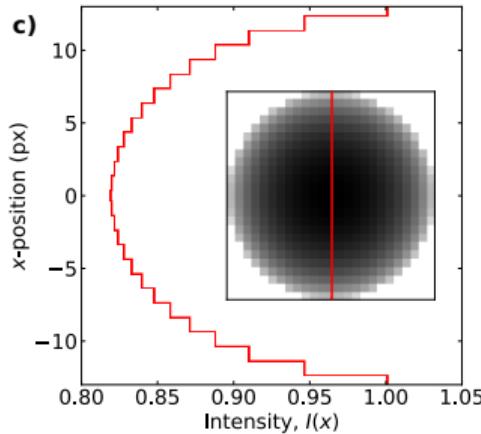
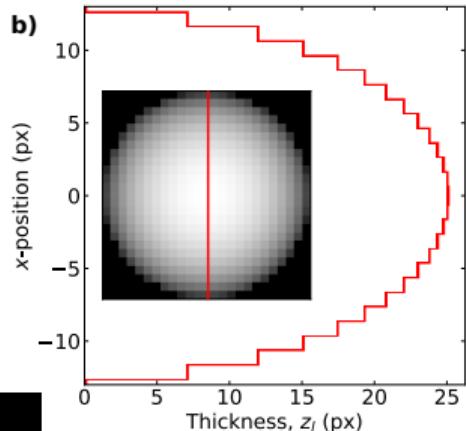
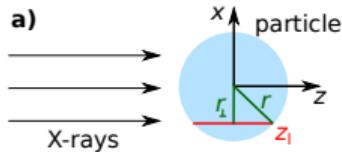


# Synthetic radiograms

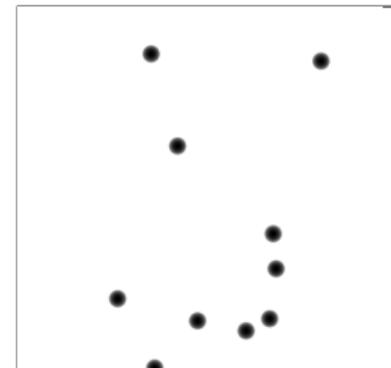


Beer-Lambert  
 $I(z_I) = I_0 \exp(-\mu z)$

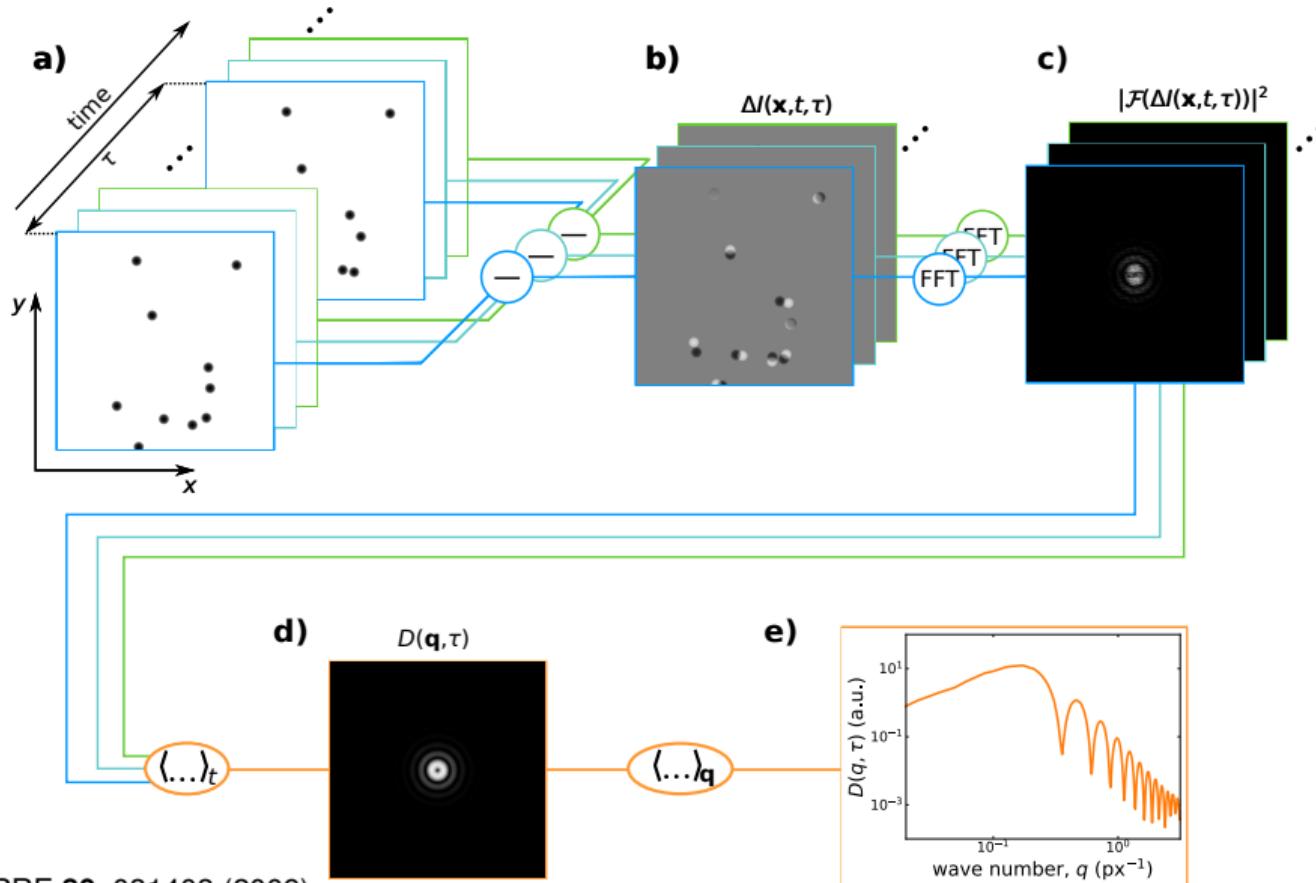
# Synthetic radiograms



Beer-Lambert  
 $I(z_l) = I_0 \exp(-\mu z)$

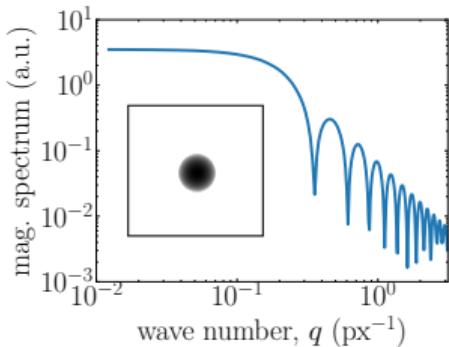
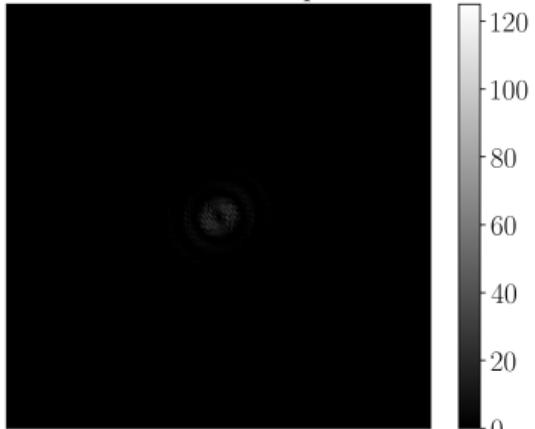


# The image structure function $D(\mathbf{q}, \tau)$



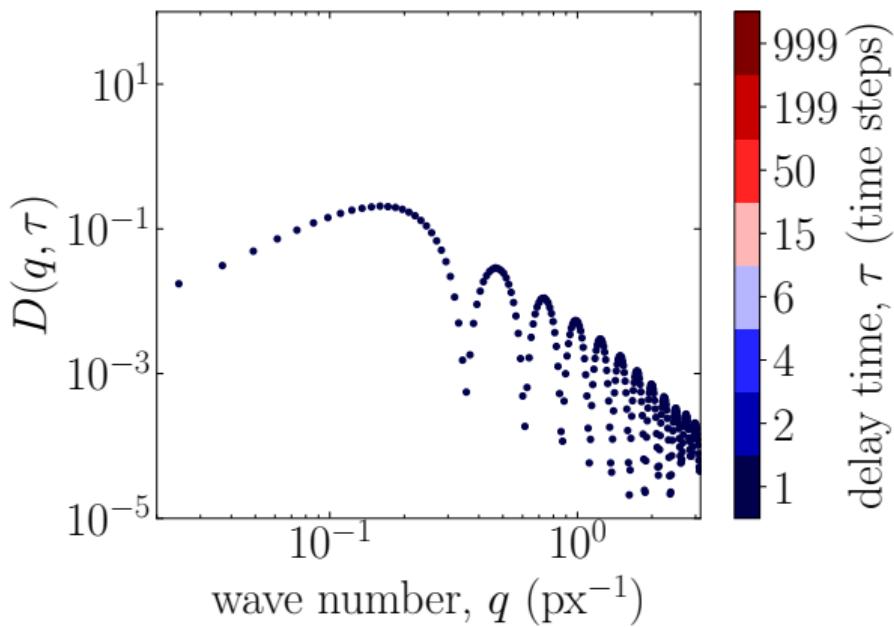
# The image structure function $D(q, \tau)$

$\tau = 1$  time steps



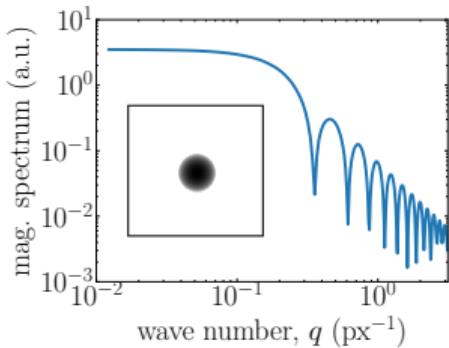
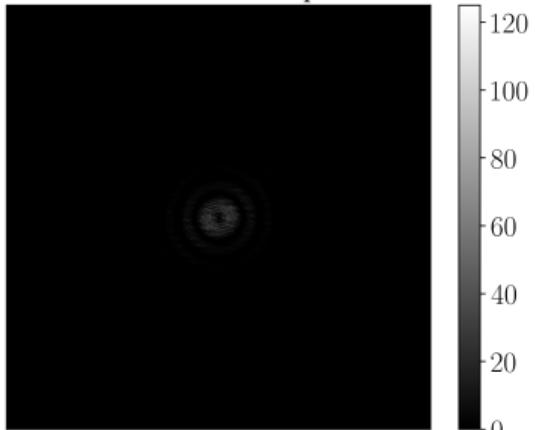
Time averaging:  $D(\mathbf{q}, \tau) = \langle |\mathcal{F}(\Delta I)|^2 \rangle_t$

Azimuthal averaging:  $D(\mathbf{q}, \tau) \rightarrow D(\mathbf{q}, \tau) \dots$



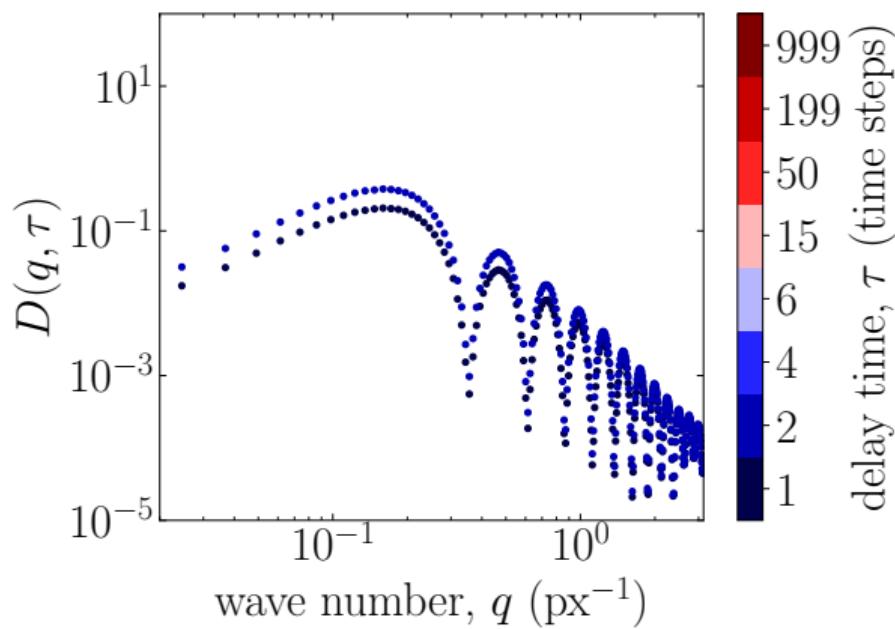
# The image structure function $D(q, \tau)$

$\tau = 2$  time steps



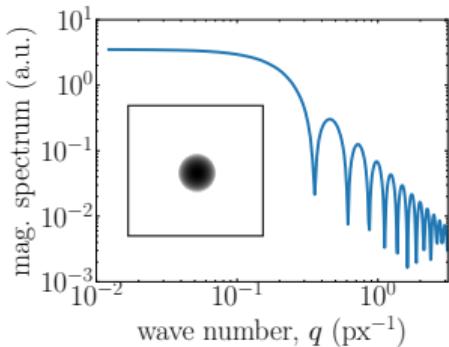
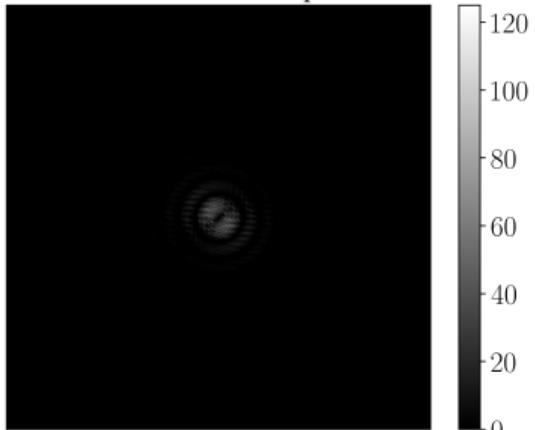
Time averaging:  $D(\mathbf{q}, \tau) = \langle |\mathcal{F}(\Delta I)|^2 \rangle_t$

Azimuthal averaging:  $D(\mathbf{q}, \tau) \rightarrow D(\mathbf{q}, \tau) \dots$



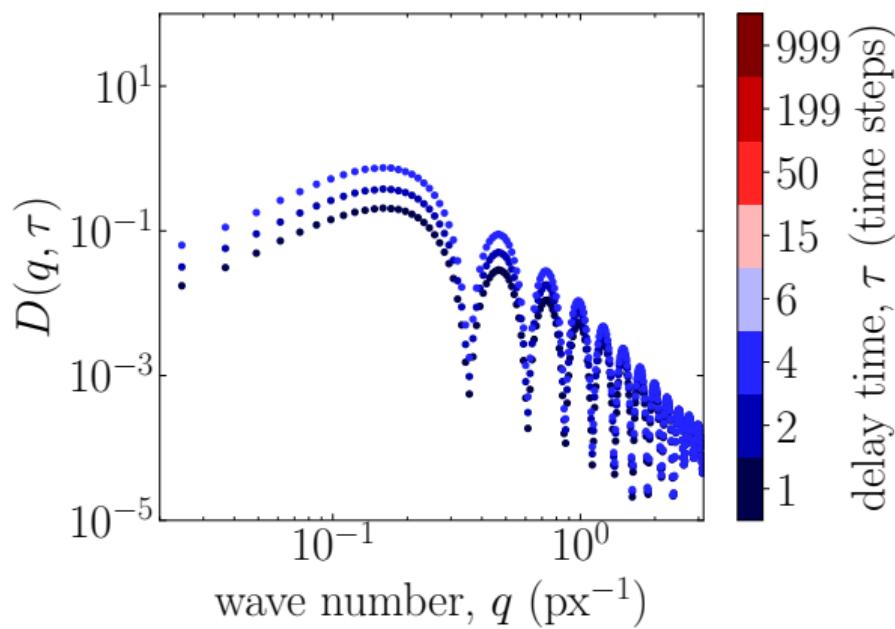
# The image structure function $D(q, \tau)$

$\tau = 4$  time steps



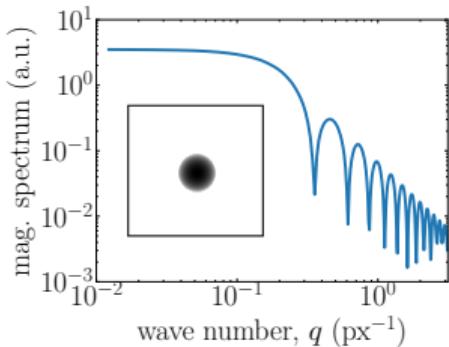
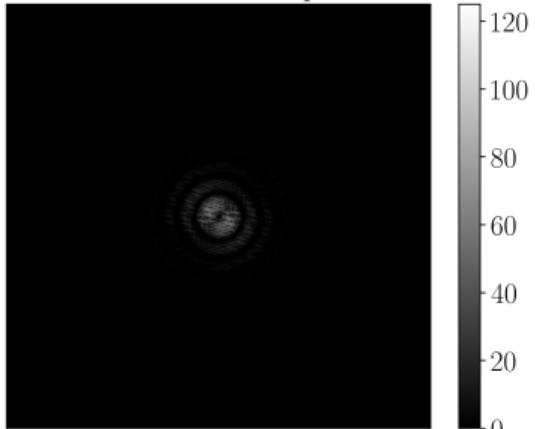
Time averaging:  $D(\mathbf{q}, \tau) = \langle |\mathcal{F}(\Delta I)|^2 \rangle_t$

Azimuthal averaging:  $D(\mathbf{q}, \tau) \rightarrow D(\mathbf{q}, \tau) \dots$



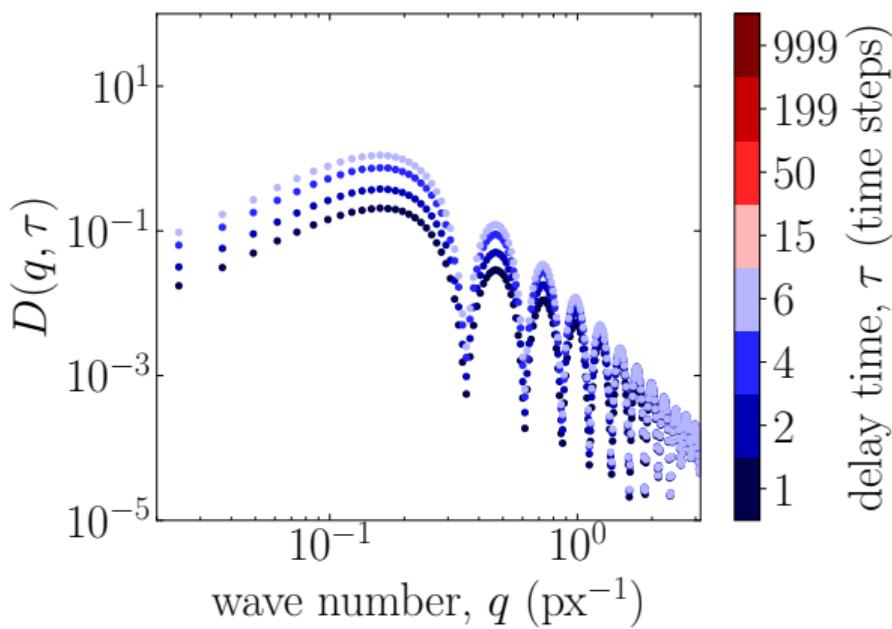
# The image structure function $D(q, \tau)$

$\tau = 6$  time steps

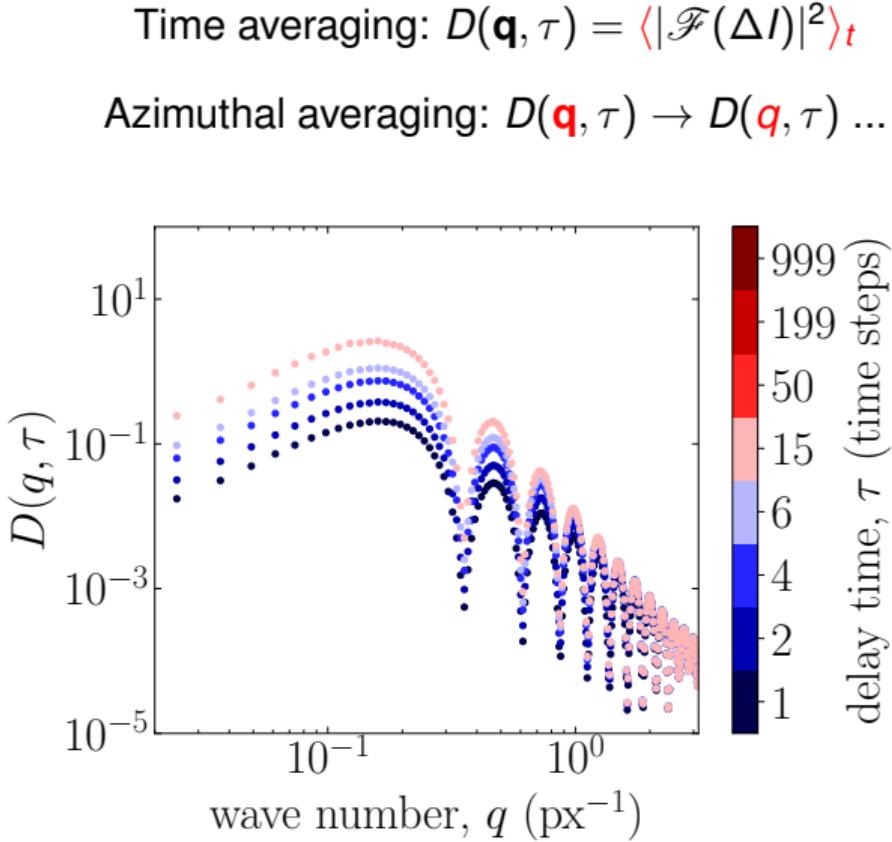
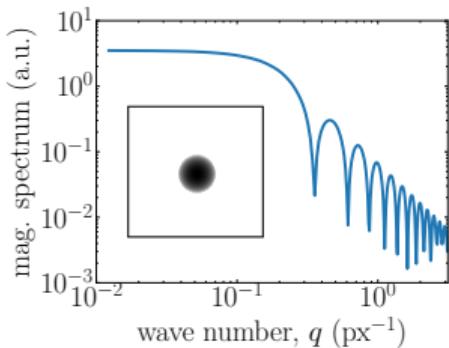
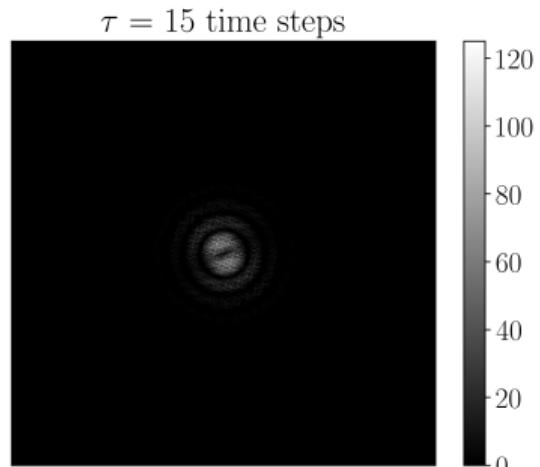


Time averaging:  $D(\mathbf{q}, \tau) = \langle |\mathcal{F}(\Delta I)|^2 \rangle_t$

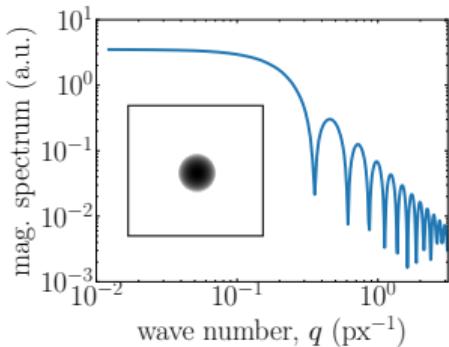
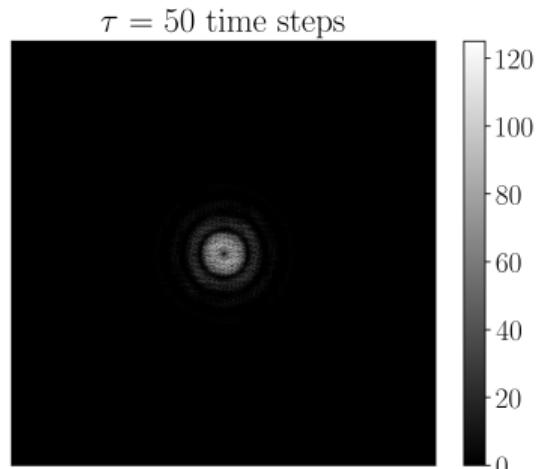
Azimuthal averaging:  $D(\mathbf{q}, \tau) \rightarrow D(\mathbf{q}, \tau) \dots$



# The image structure function $D(q, \tau)$

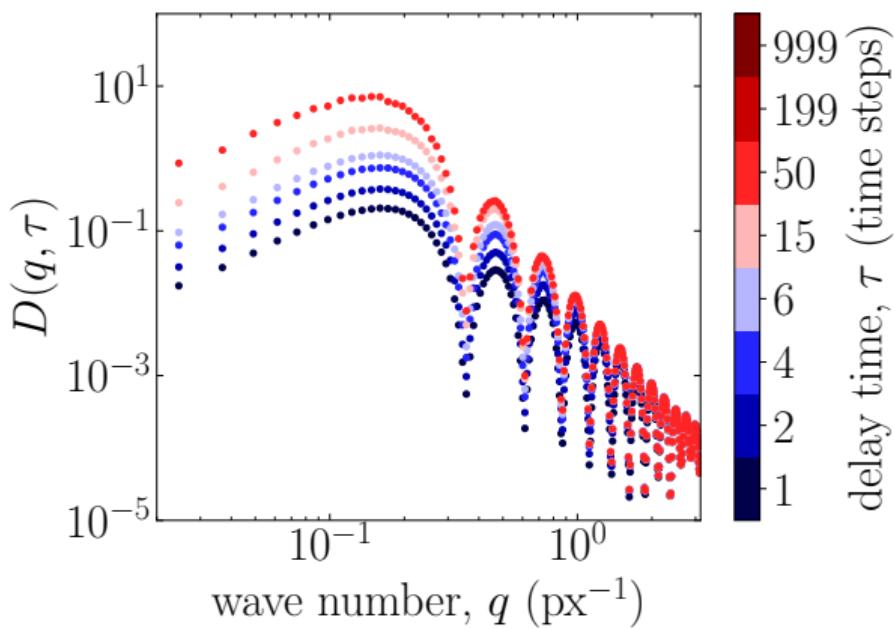


# The image structure function $D(q, \tau)$



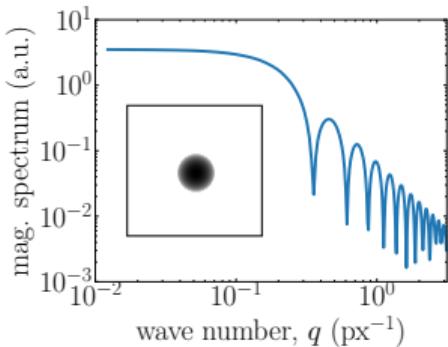
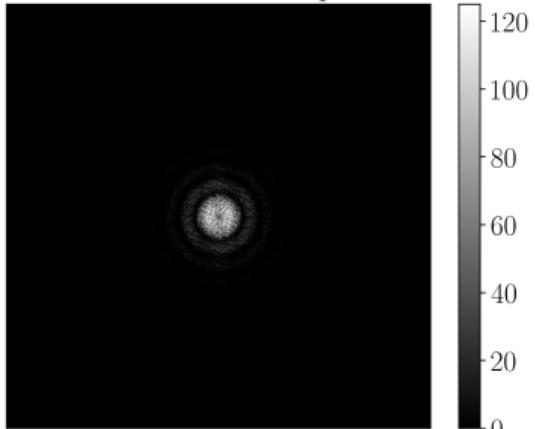
Time averaging:  $D(\mathbf{q}, \tau) = \langle |\mathcal{F}(\Delta I)|^2 \rangle_t$

Azimuthal averaging:  $D(\mathbf{q}, \tau) \rightarrow D(\mathbf{q}, \tau) \dots$



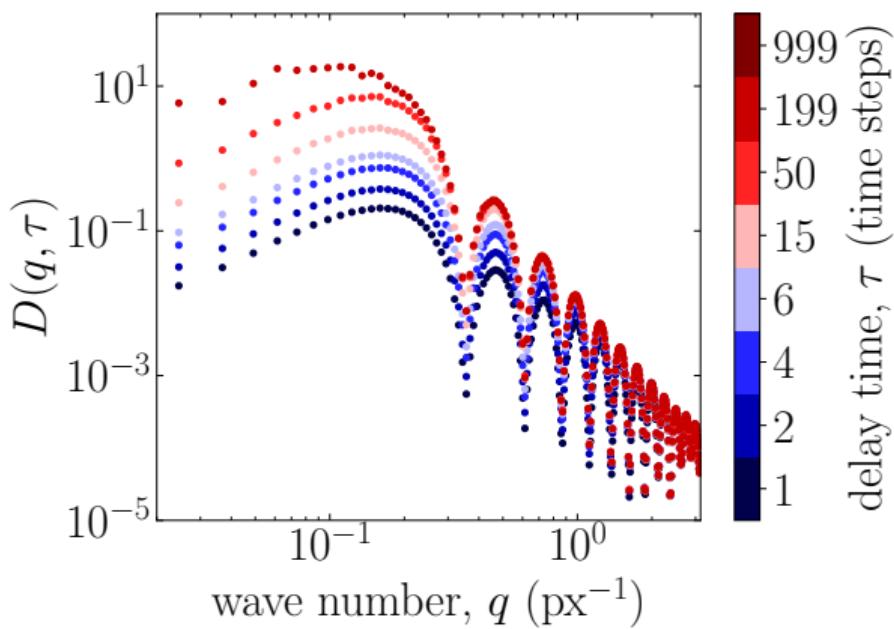
# The image structure function $D(q, \tau)$

$\tau = 199$  time steps



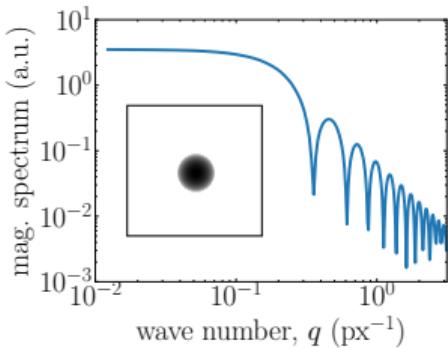
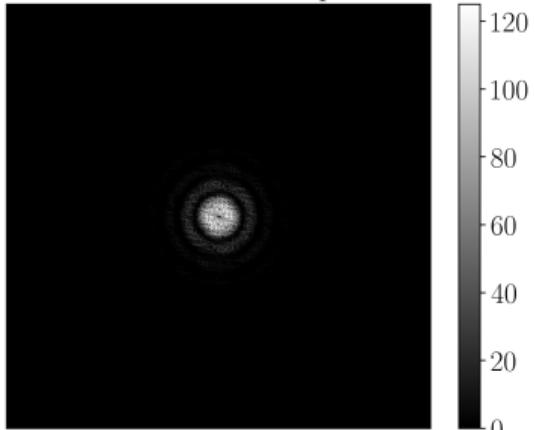
Time averaging:  $D(\mathbf{q}, \tau) = \langle |\mathcal{F}(\Delta I)|^2 \rangle_t$

Azimuthal averaging:  $D(\mathbf{q}, \tau) \rightarrow D(\mathbf{q}, \tau) \dots$



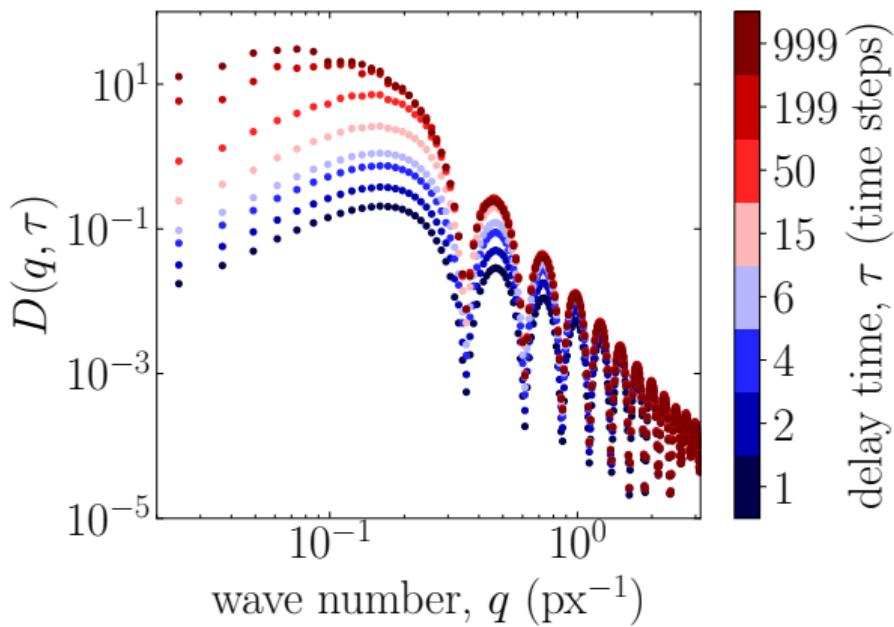
# The image structure function $D(q, \tau)$

$\tau = 999$  time steps

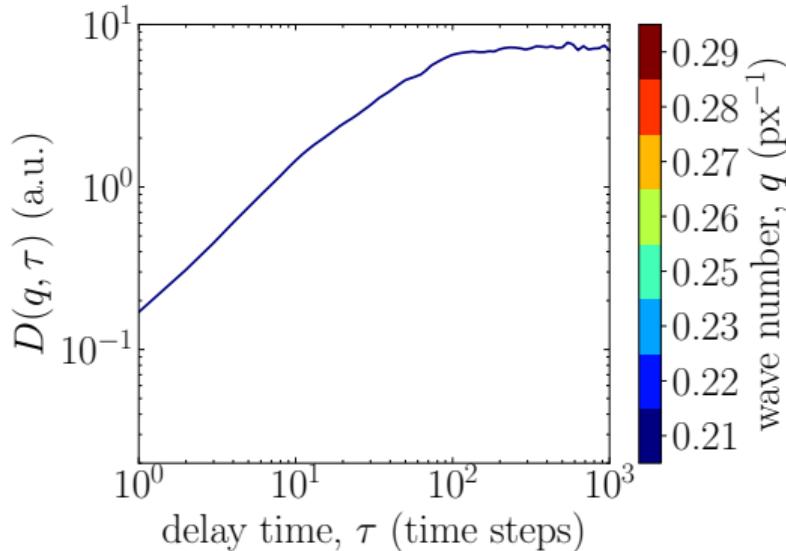
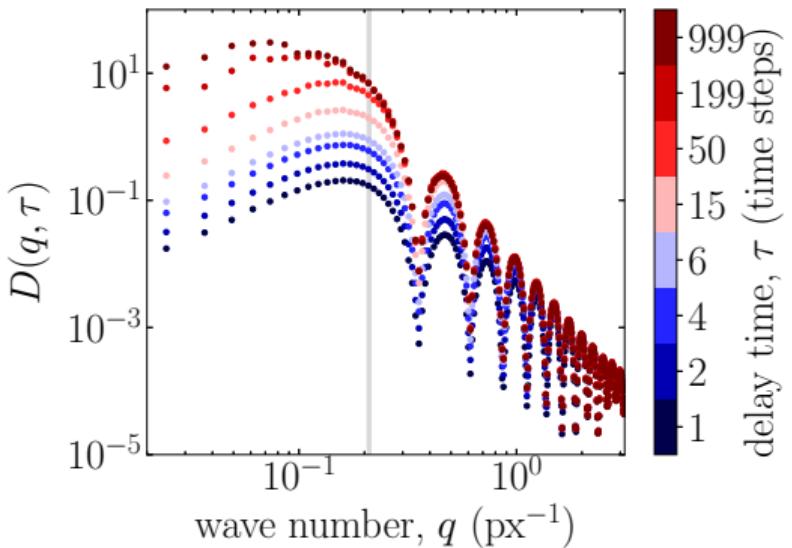


Time averaging:  $D(\mathbf{q}, \tau) = \langle |\mathcal{F}(\Delta I)|^2 \rangle_t$

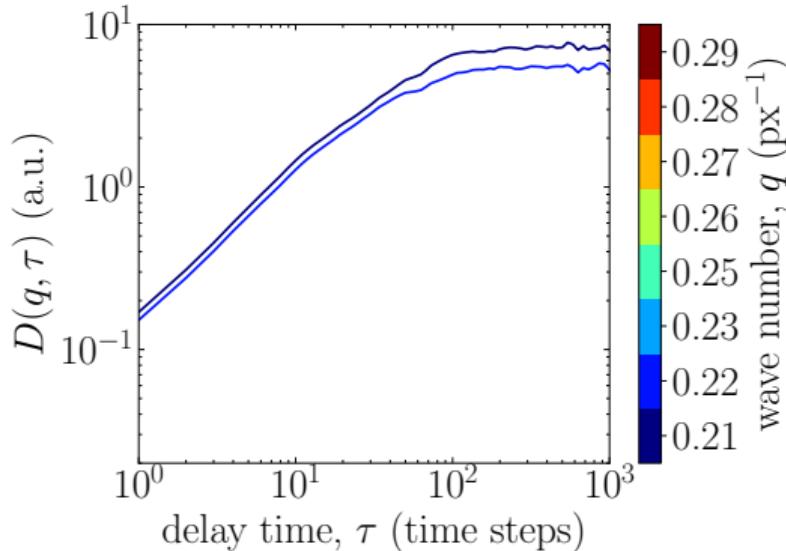
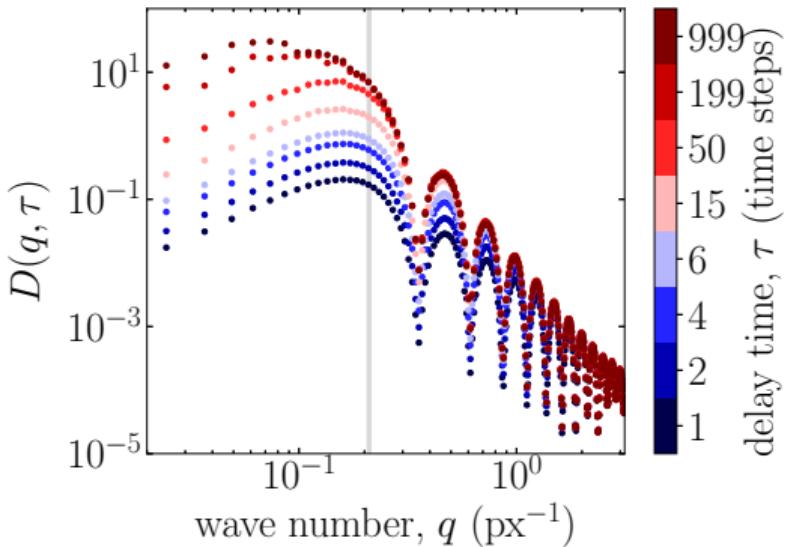
Azimuthal averaging:  $D(\mathbf{q}, \tau) \rightarrow D(\mathbf{q}, \tau) \dots$



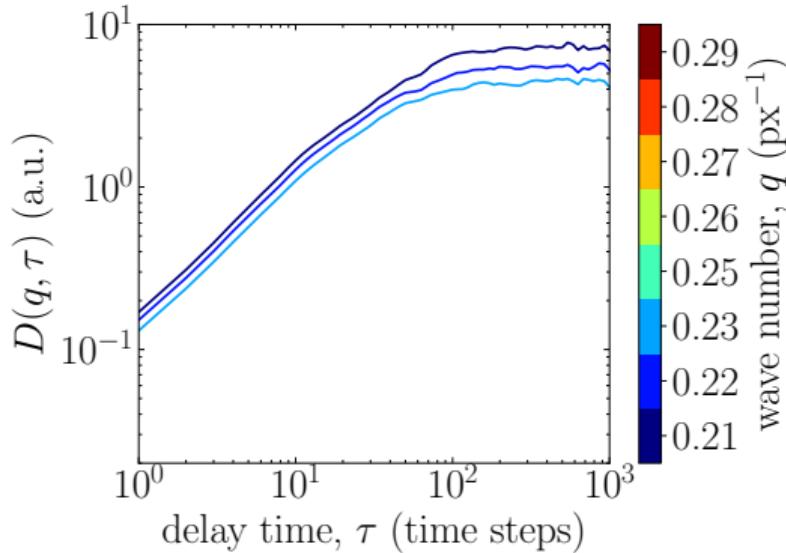
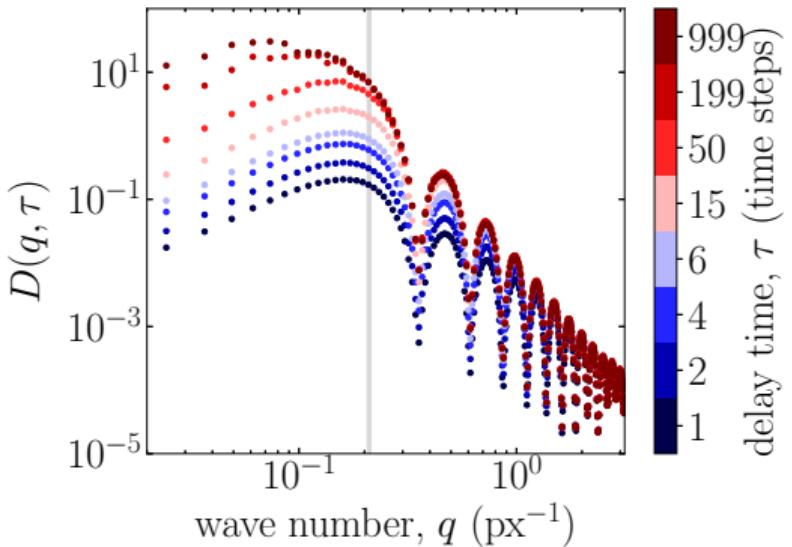
## The image structure function $D(q, \tau)$



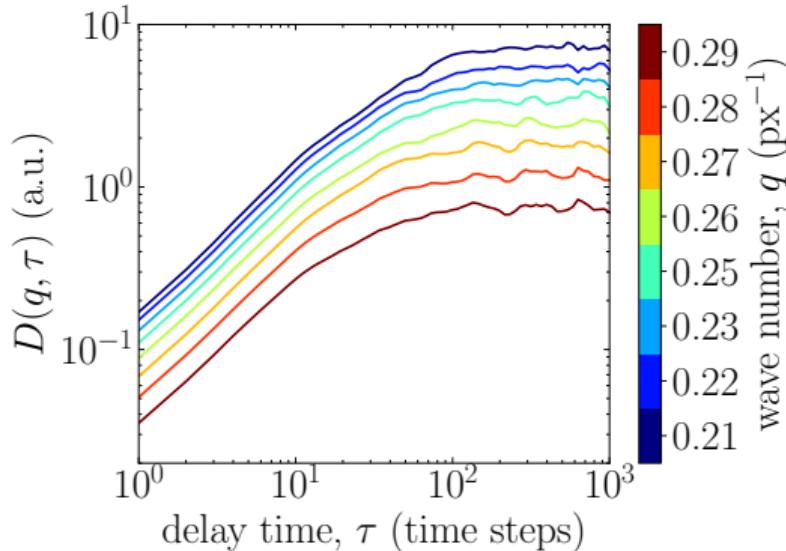
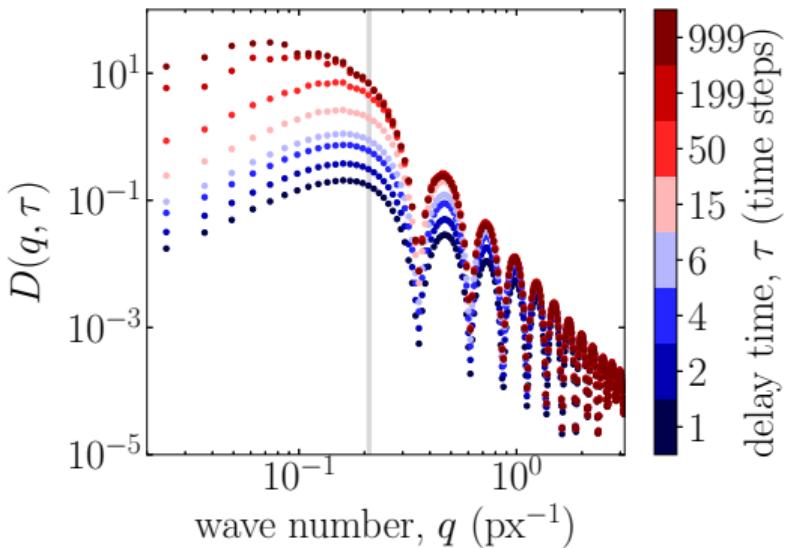
## The image structure function $D(q, \tau)$



## The image structure function $D(q, \tau)$

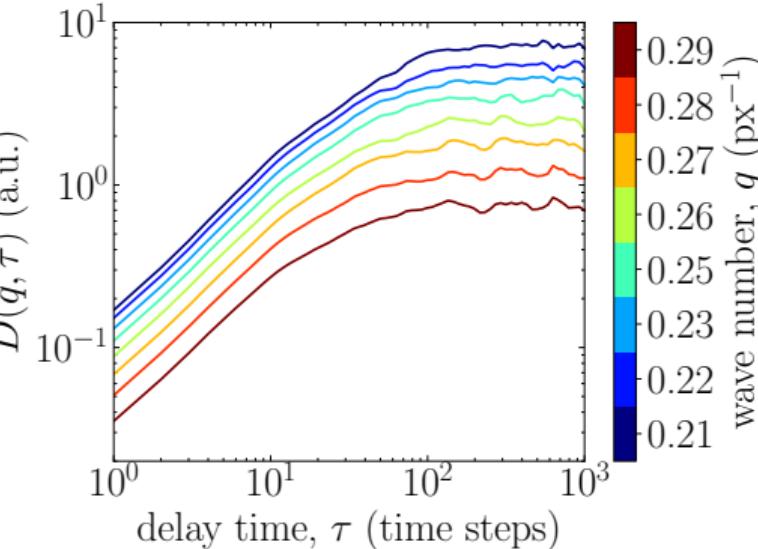
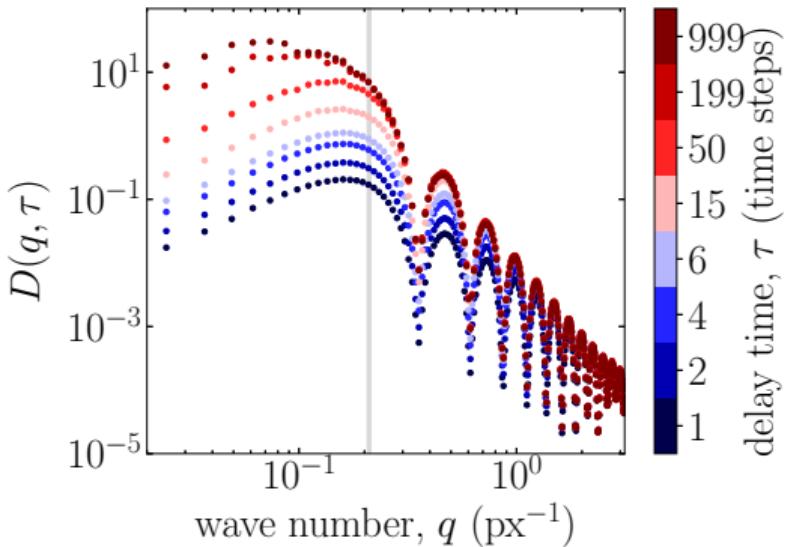


## The image structure function $D(q, \tau)$



$$\begin{aligned} D(q, \tau) &= \left\langle |I(q, t + \tau) - I(q, t)|^2 \right\rangle_t \\ &= A(q) \left[ 1 - \frac{\left\langle I^*(q, t) I(q, t + \tau) \right\rangle_t}{\left\langle |I(q, t)|^2 \right\rangle_t} \right] + B(q) \end{aligned}$$

# The image structure function $D(q, \tau)$



$$\begin{aligned}
 D(q, \tau) &= \left\langle |I(q, t + \tau) - I(q, t)|^2 \right\rangle_t \\
 &= A(q) \left[ 1 - \underbrace{\frac{\langle I^*(q, t) I(q, t + \tau) \rangle_t}{\langle |I(q, t)|^2 \rangle_t}}_{\text{Image correlation function}} \right] + B(q)
 \end{aligned}$$

## Linear space invariant imaging

Image correlation function

$$g(\mathbf{q}, \tau) = \frac{\langle I^*(\mathbf{q}, t)I(\mathbf{q}, t + \tau) \rangle_t}{\langle |I(\mathbf{q}, t)|^2 \rangle_t}$$

# Linear space invariant imaging

Image correlation function

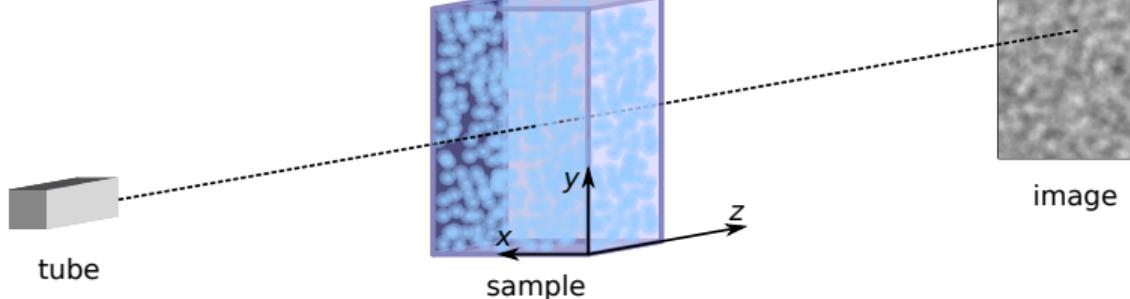
$$g(\mathbf{q}, \tau) = \frac{\langle I^*(\mathbf{q}, t)I(\mathbf{q}, t + \tau) \rangle_t}{\langle |I(\mathbf{q}, t)|^2 \rangle_t}$$

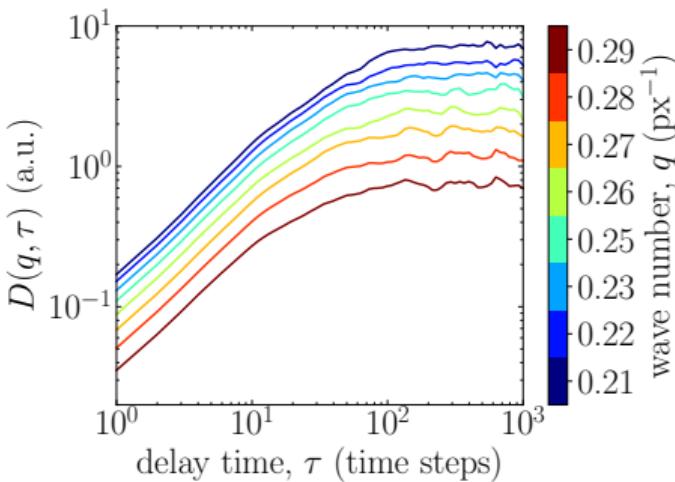
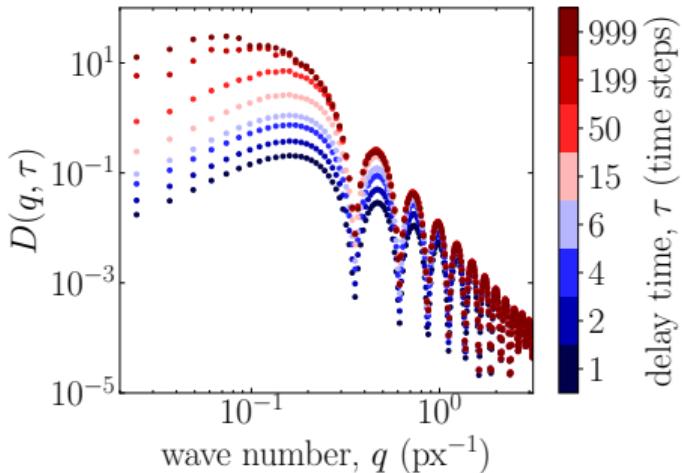
Intermediate scattering function

$$f(\mathbf{q}, \tau) = \frac{\langle \rho^*(\mathbf{q}, t)\rho(\mathbf{q}, t + \tau) \rangle_t}{\langle |\rho(\mathbf{q}, t)|^2 \rangle_t}$$

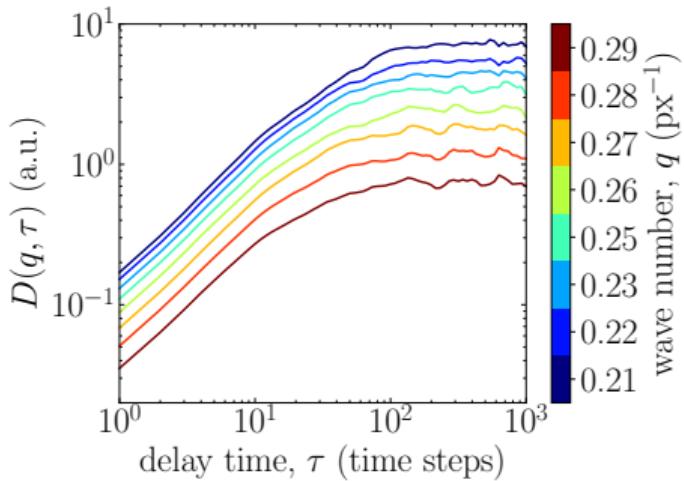
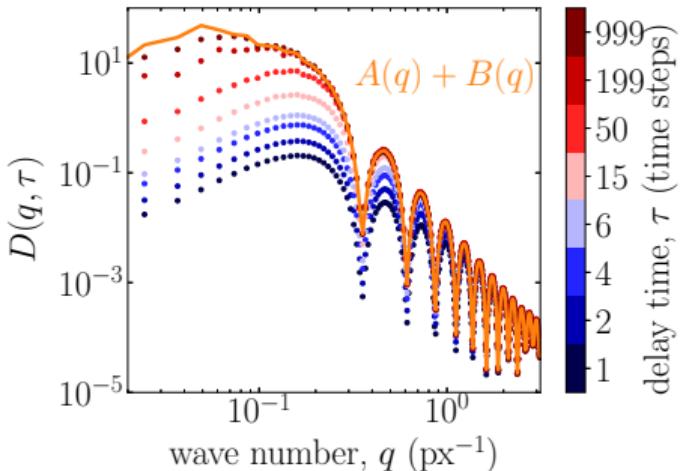
Linear space-invariant imaging:

$$I(\mathbf{r}, t) = I_0 + \int d\mathbf{r}' dz' T(\mathbf{r} - \mathbf{r}', -z') c(\mathbf{r}', z', t)$$



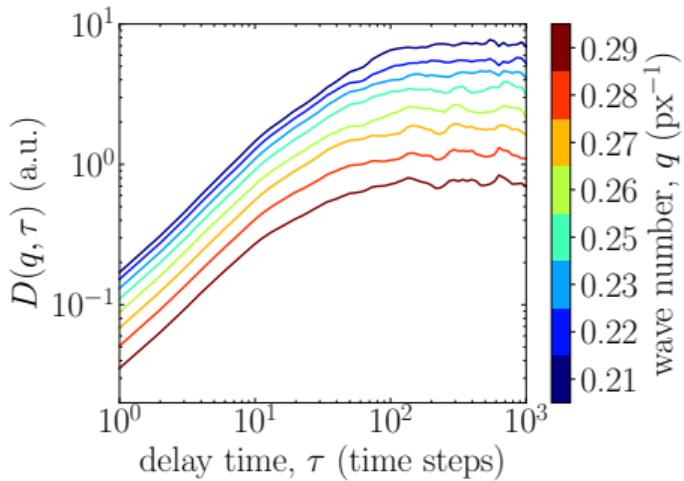
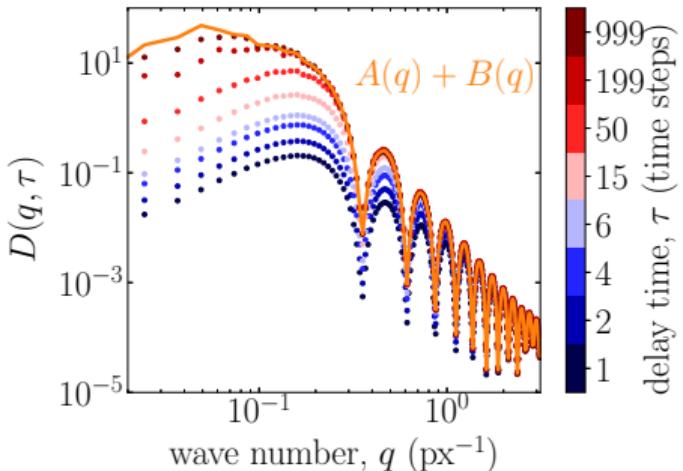


$$\begin{aligned}
 D(q, \tau) &= \left\langle |I(q, t + \tau) - I(q, t)|^2 \right\rangle_t \\
 &= A(q) \underbrace{\left[ 1 - \frac{\langle I^*(q, t) I(q, t + \tau) \rangle_t}{\langle |I(q, t)|^2 \rangle_t} \right]}_{\text{Image correlation function}} + B(q)
 \end{aligned}$$



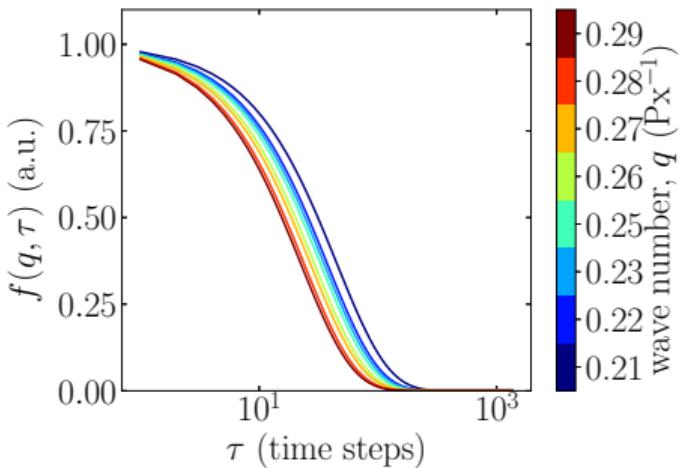
$$\begin{aligned} D(q, \tau) &= \left\langle |I(q, t + \tau) - I(q, t)|^2 \right\rangle_t \\ &= A(q) \left[ 1 - \frac{\langle I^*(q, t) I(q, t + \tau) \rangle_t}{\langle |I(q, t)|^2 \rangle_t} \right] + B(q) \end{aligned}$$

- $D(q, \tau \rightarrow 0) = B(q) = 0$
- $D(q, \tau \rightarrow \infty) = A(q) + B(q)$

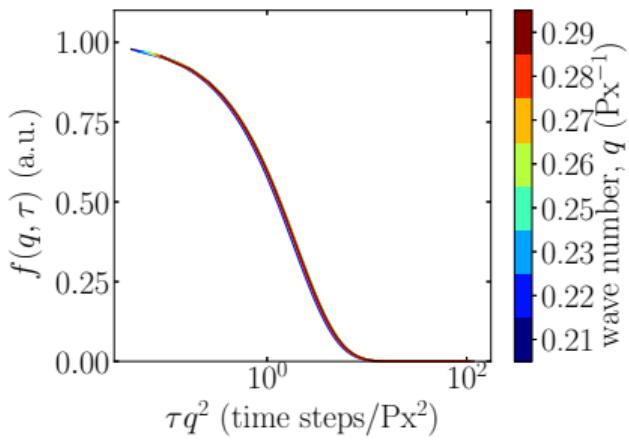
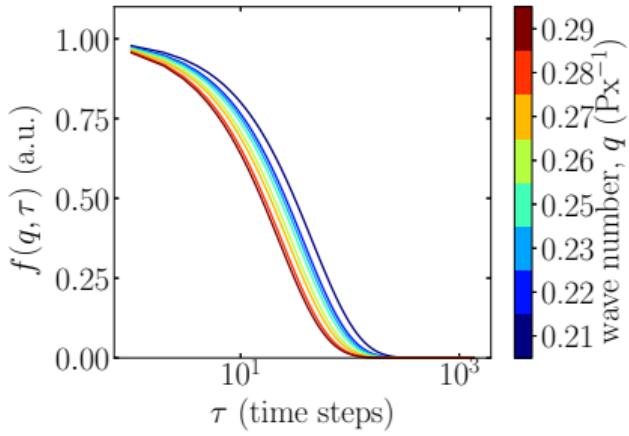


$$\begin{aligned} D(q, \tau) &= \left\langle |I(q, t + \tau) - I(q, t)|^2 \right\rangle_t \\ &= A(q) \left[ 1 - \frac{\langle I^*(q, t) I(q, t + \tau) \rangle_t}{\langle |I(q, t)|^2 \rangle_t} \right] + B(q) \end{aligned}$$

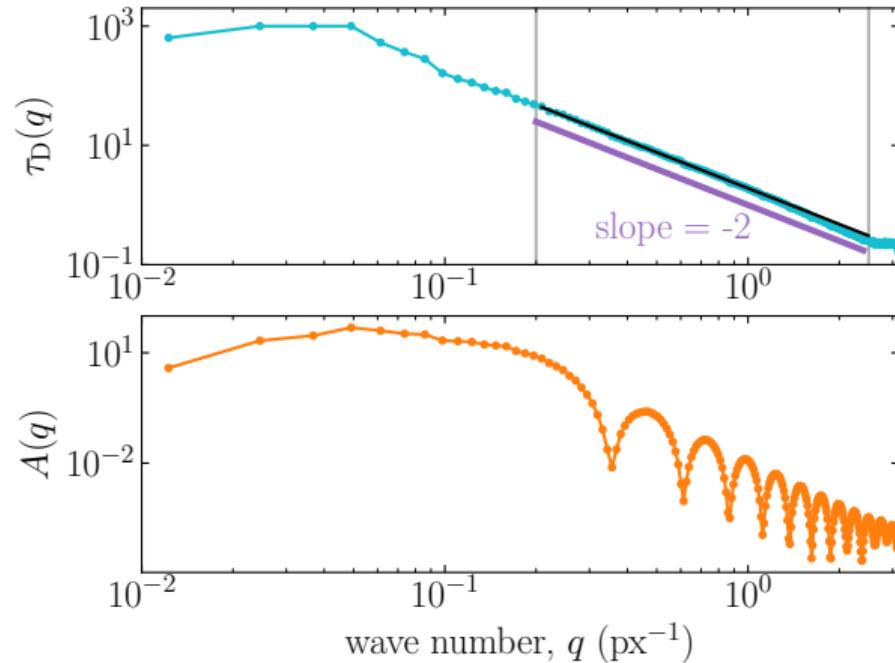
- $D(q, \tau \rightarrow 0) = B(q) = 0$
- $D(q, \tau \rightarrow \infty) = A(q) + B(q)$



## Intermediate scattering function $f(q, \tau)$



Brownian motion:  
 $f(q, \tau) = \exp(-q^2 \tau / \tau_D)$   
Accuracy: 2% - 6%



extra slide with videos of 10, 1000, 100000 particles + accuracy, next slide is *Extending DDM to X-DFA*