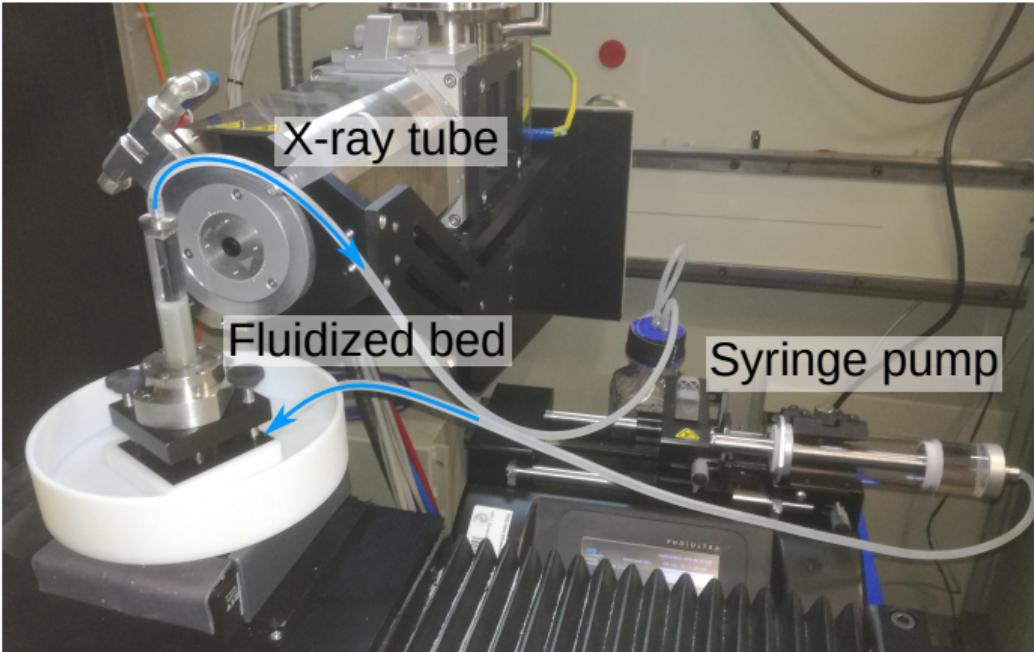
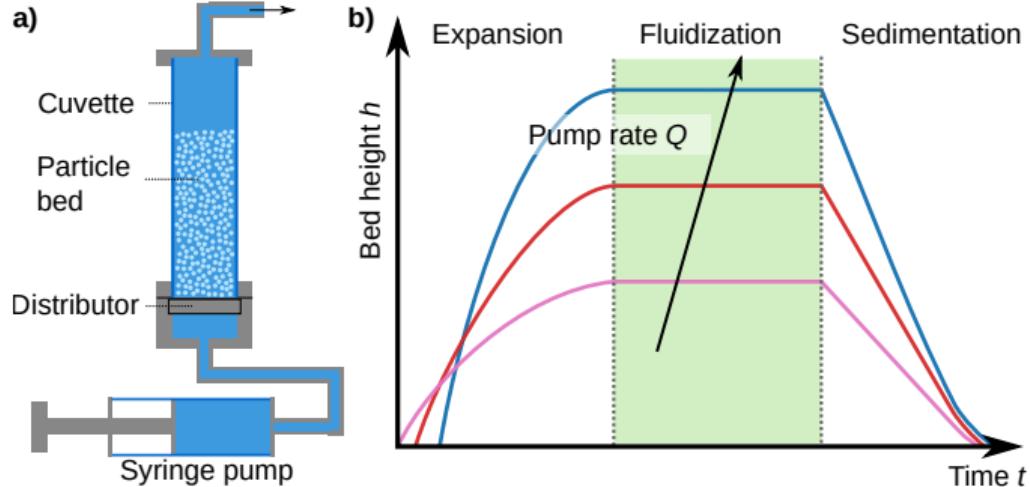


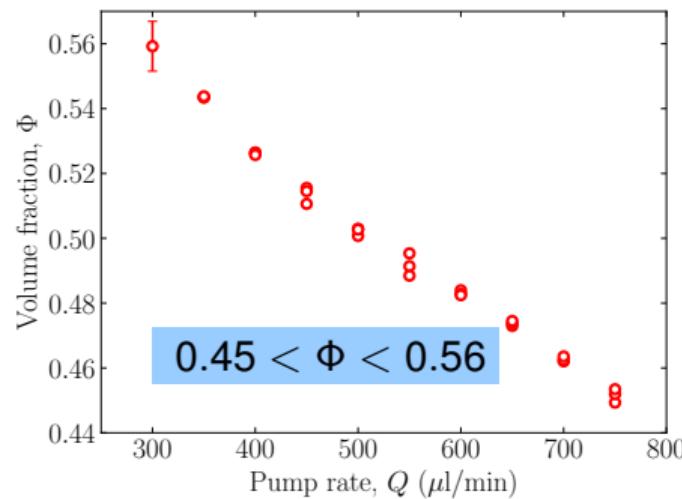
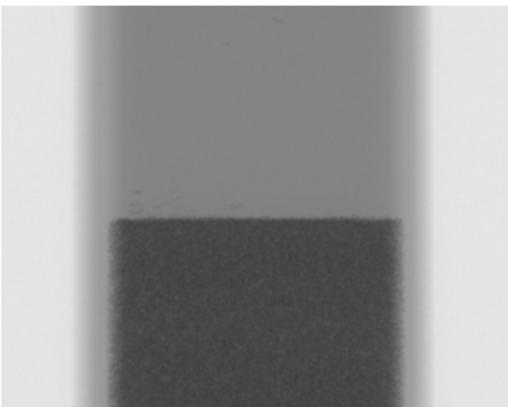
# Experimental validation of X-DFA: A suspension of sedimenting particles



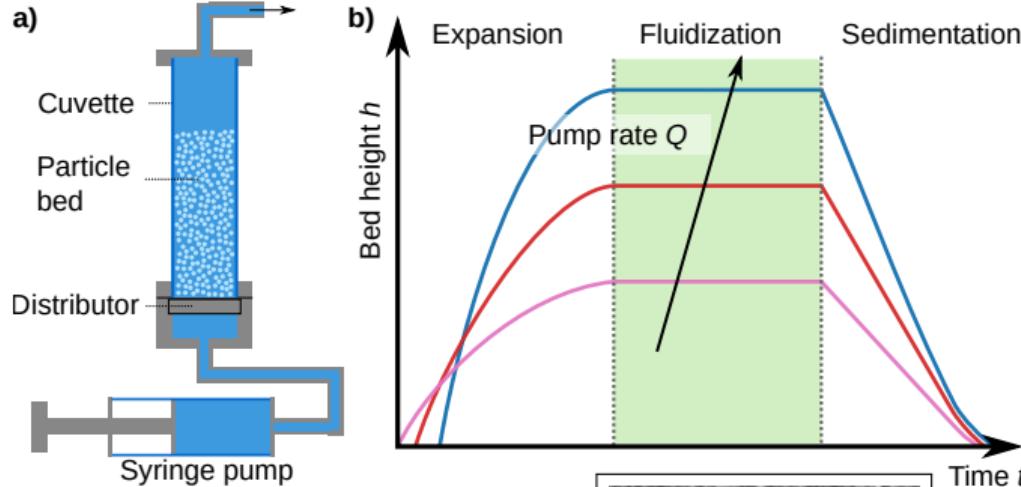
# Experimental validation of X-DFA: A suspension of sedimenting particles



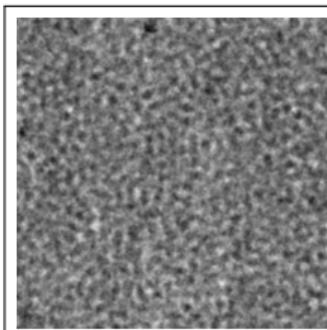
X-ray radiography



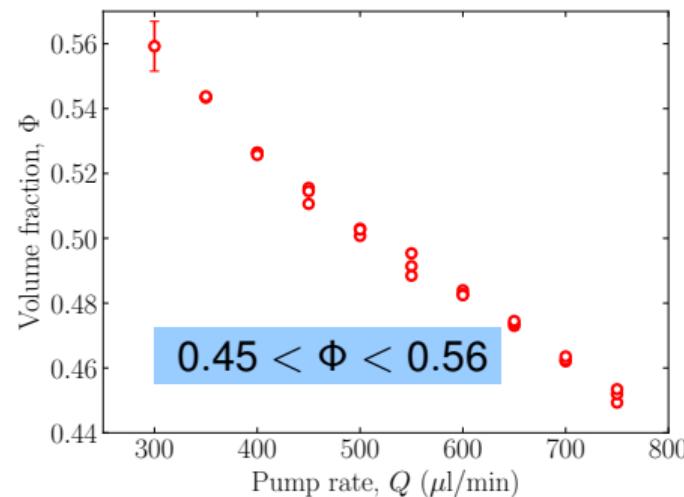
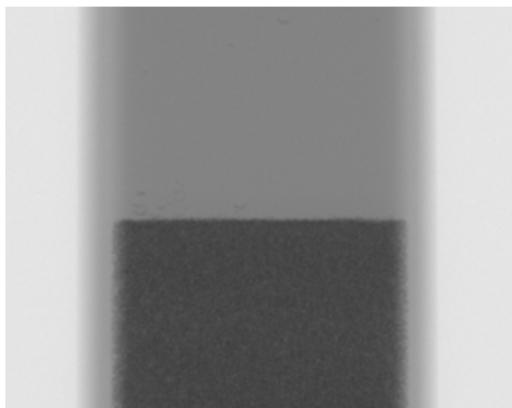
# Experimental validation of X-DFA: A suspension of sedimenting particles



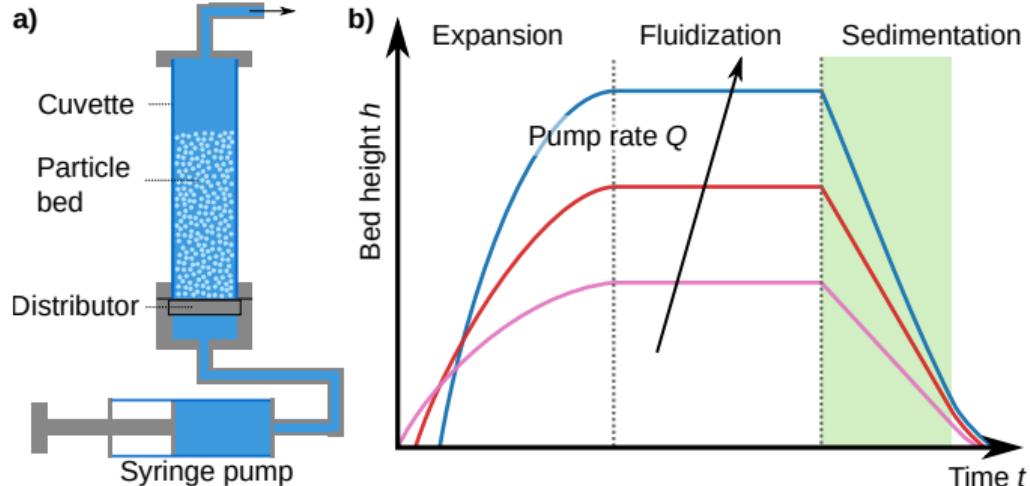
No reliable  
reference  
velocity!



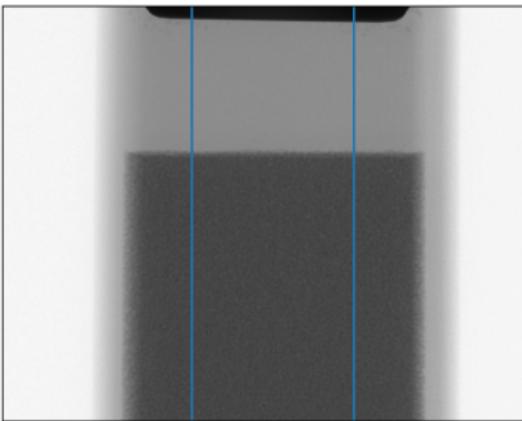
X-ray radiography



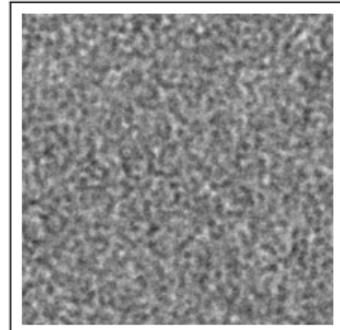
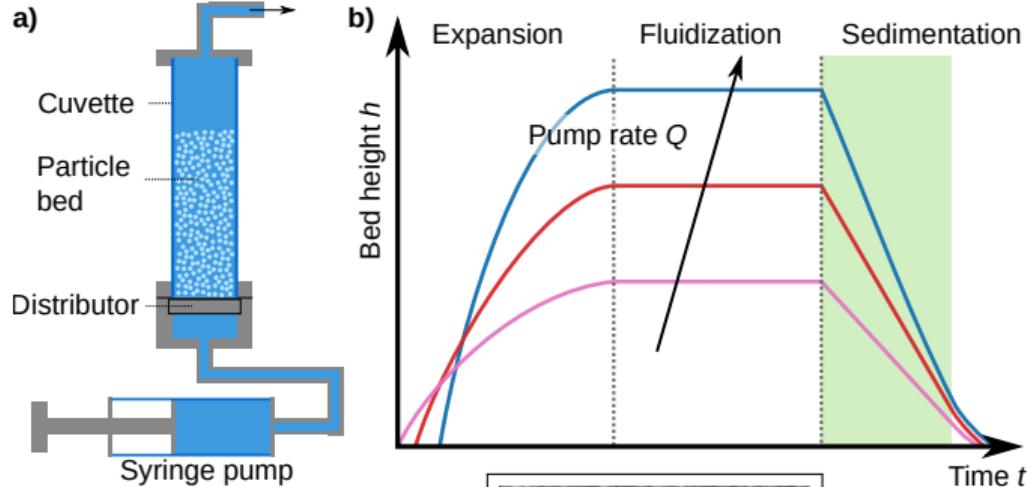
# Experimental validation of X-DFA: A suspension of sedimenting particles



X-ray radiography

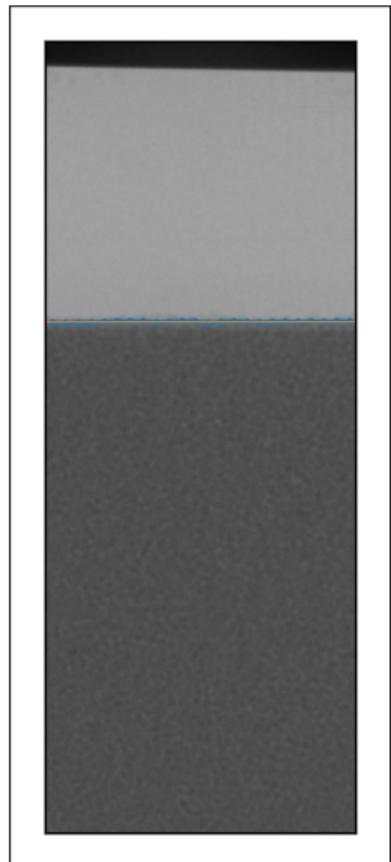


# Experimental validation of X-DFA: A suspension of sedimenting particles

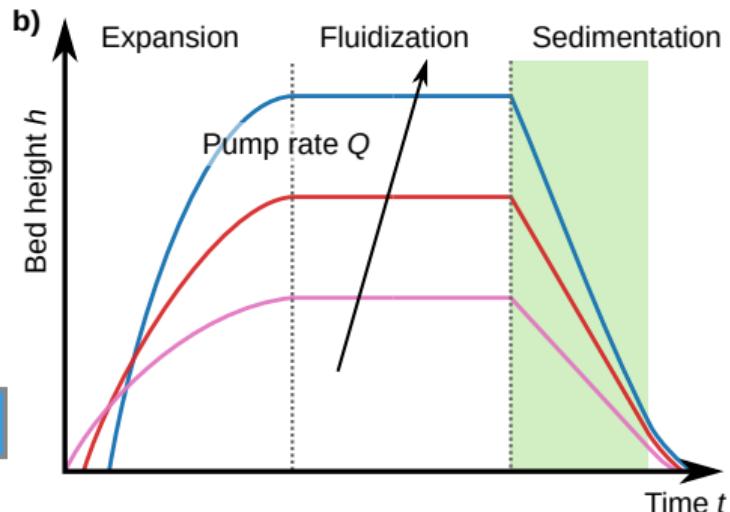
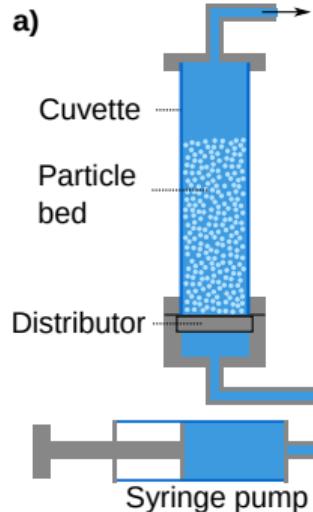


Comparison of  
 $\langle v \rangle_{\text{dfa}}$  and  $\langle v \rangle_{\text{front}}$

X-ray radiography

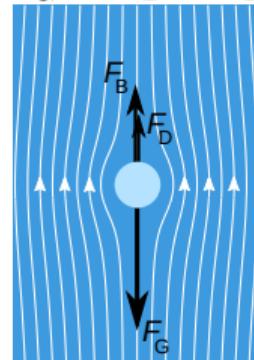


# Liquid fluidized bed: Richardson-Zaki law



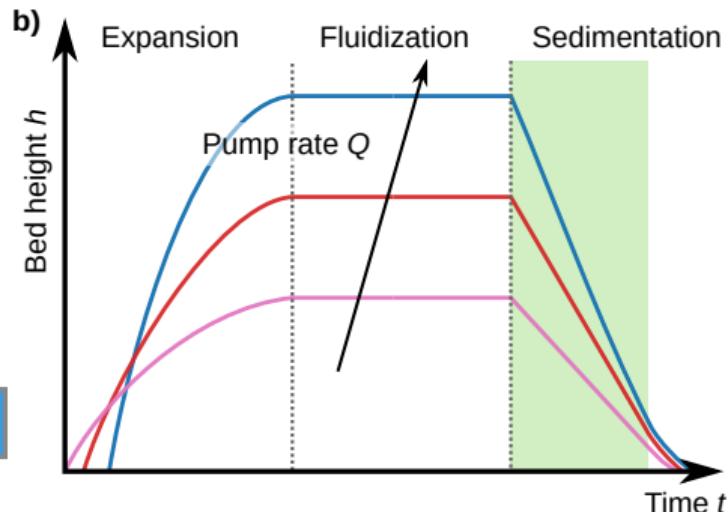
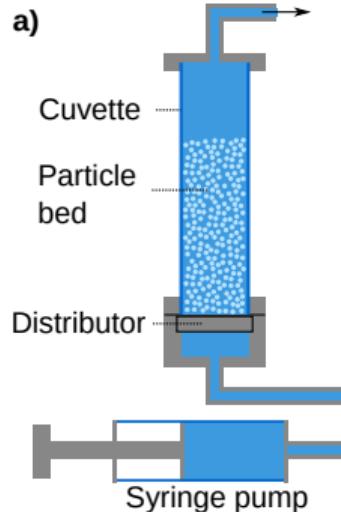
Gravitation      Buoyancy      Drag

$$F_G = F_B + F_D$$



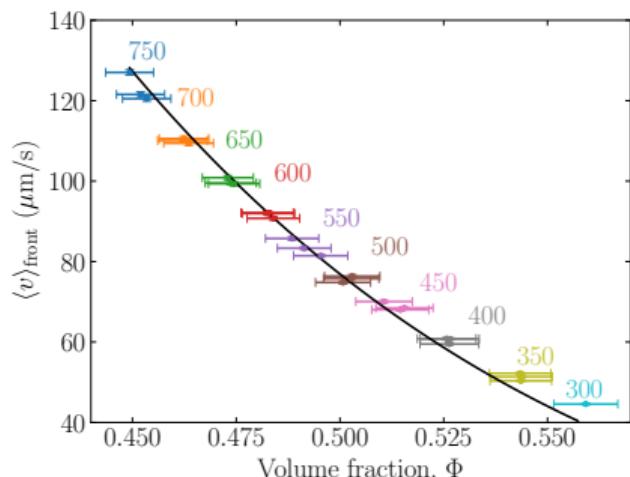
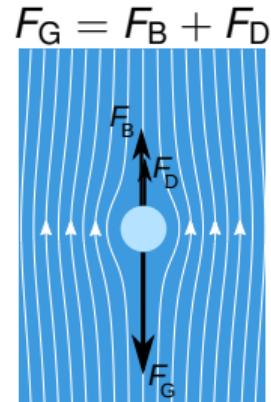
$$\frac{\langle v \rangle_{\text{fluid}}}{v_{\text{Stokes}}} = (1 - \Phi)^n$$

# Liquid fluidized bed: Richardson-Zaki law

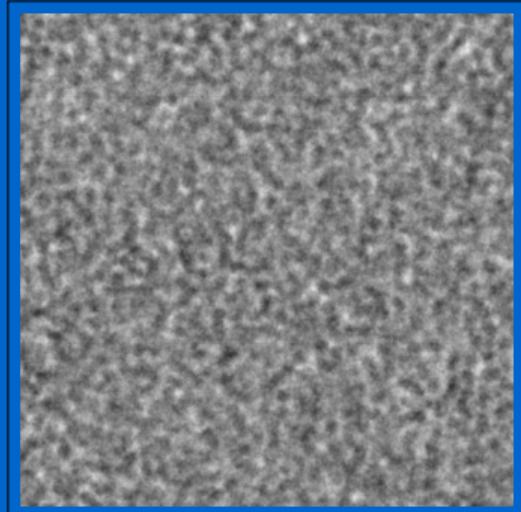
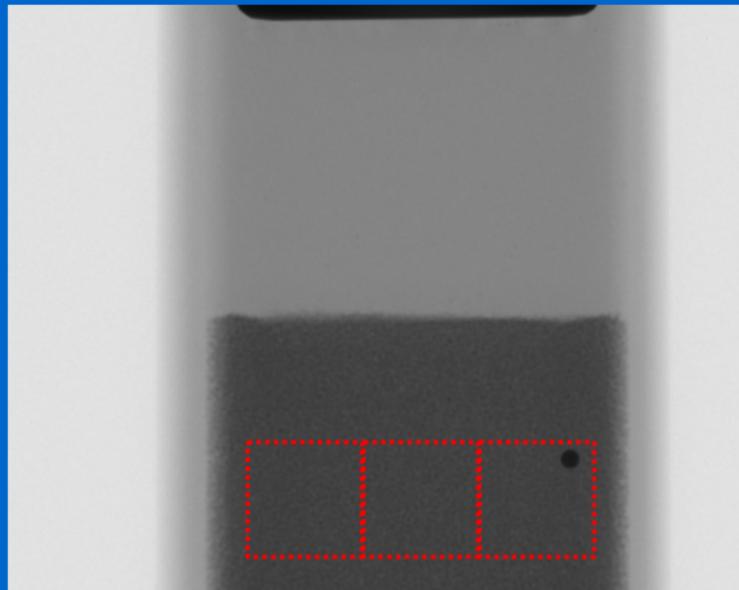


$$\frac{\langle v \rangle_{\text{front}}}{v_{\text{Stokes}}} = (1 - \Phi)^n$$

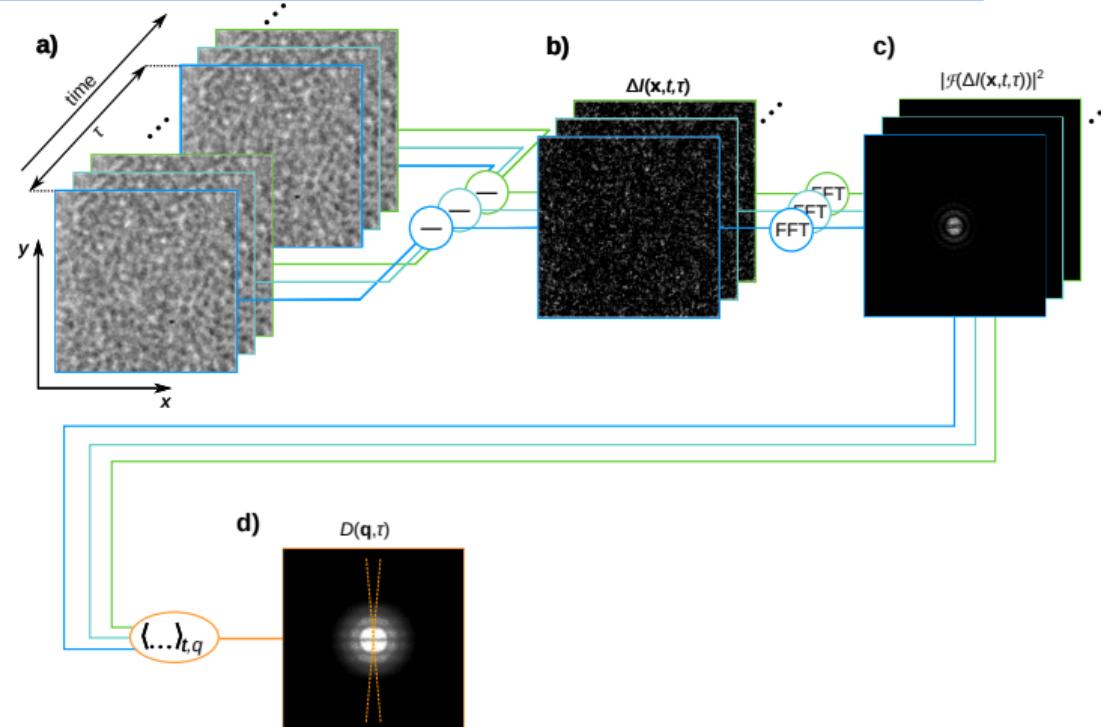
Gravitation      Buoyancy      Drag



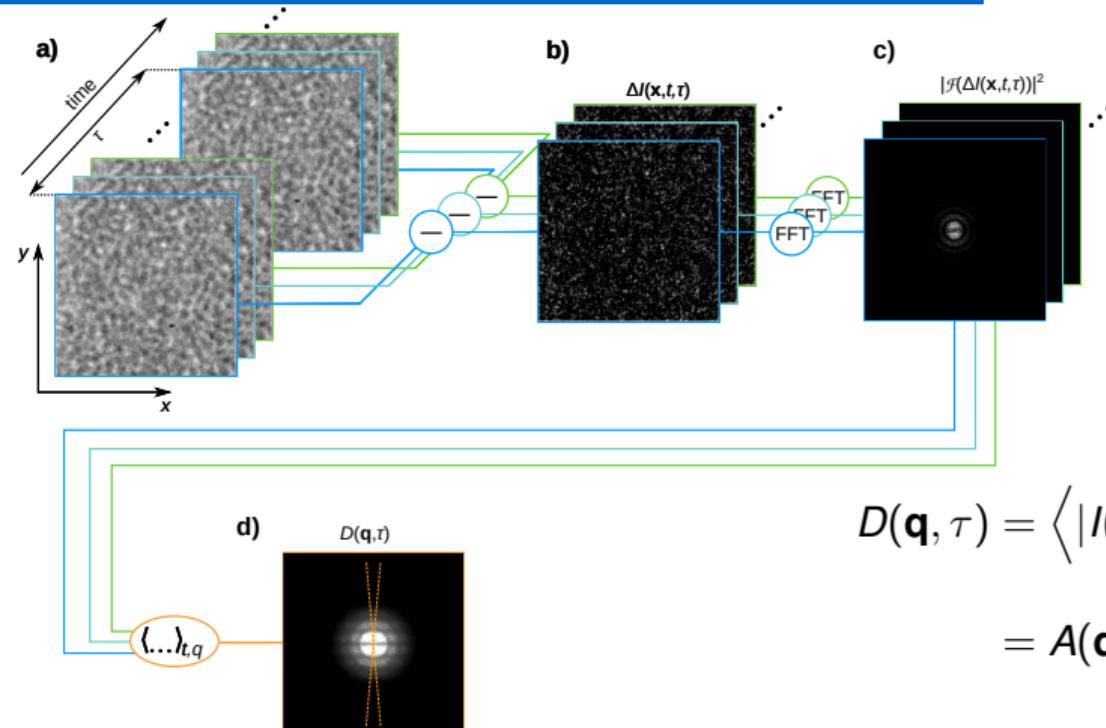
# X-ray Digital Fourier Analysis of a suspension of sedimenting particles



# The image structure function $D(\mathbf{q}, \tau)$



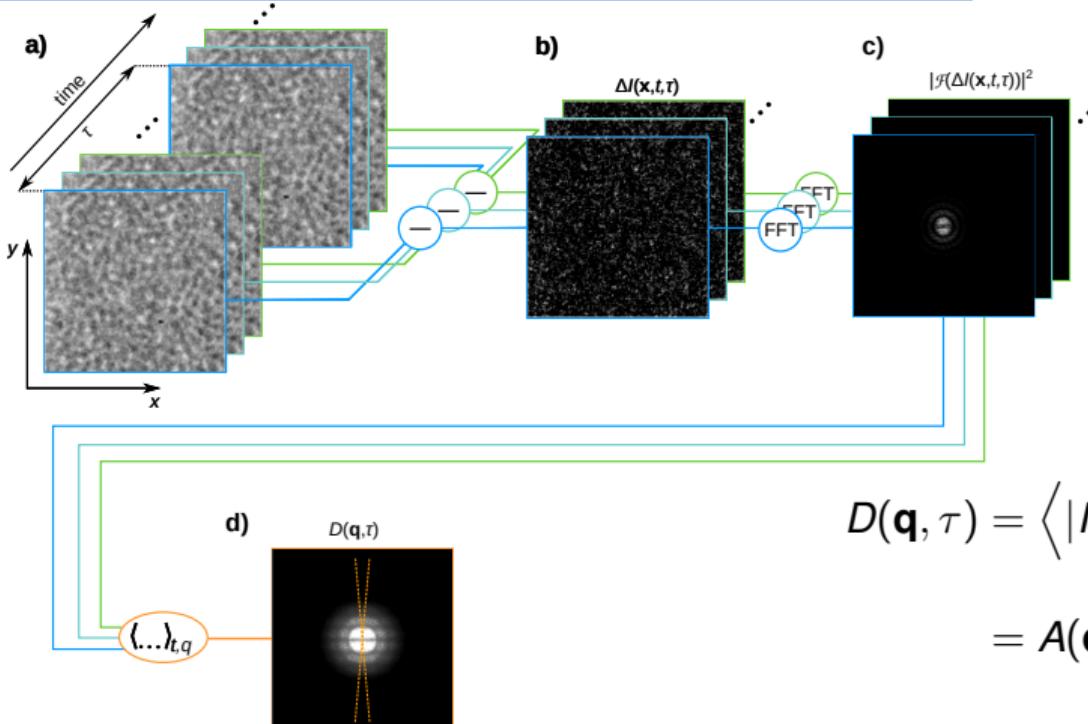
# The image structure function $D(\mathbf{q}, \tau)$



$$\begin{aligned} D(\mathbf{q}, \tau) &= \left\langle |I(\mathbf{q}, t + \tau) - I(\mathbf{q}, t)|^2 \right\rangle_t \\ &= A(\mathbf{q}) \left[ 1 - \frac{\langle I^*(\mathbf{q}, t) I(\mathbf{q}, t + \tau) \rangle_t}{\langle |I(\mathbf{q}, t)|^2 \rangle_t} \right] + B(\mathbf{q}) \end{aligned}$$

- $D(\mathbf{q}, \tau \rightarrow 0) = B(\mathbf{q})$
- $D(\mathbf{q}, \tau \rightarrow \infty) = A(\mathbf{q}) + B(\mathbf{q})$

# The image structure function $D(\mathbf{q}, \tau)$



Linear space invariant imaging

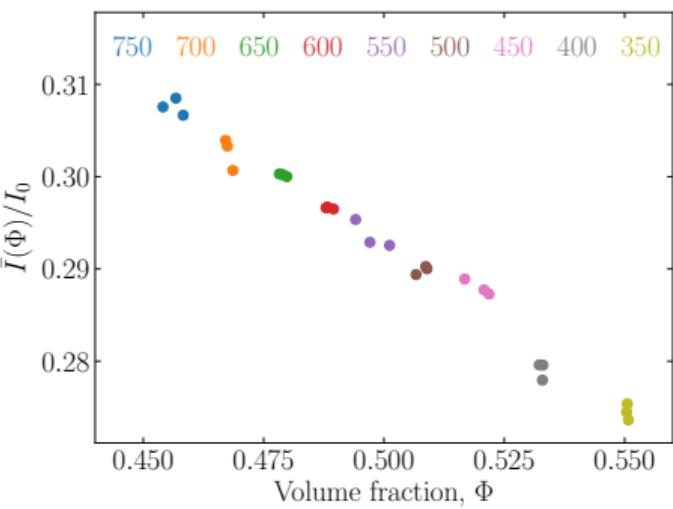
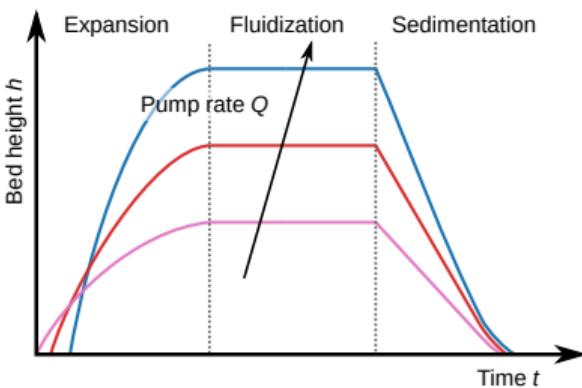
$$f(\mathbf{q}, \tau) = \frac{\langle \rho^*(\mathbf{q}, t)\rho(\mathbf{q}, t + \tau) \rangle_t}{\langle |\rho(\mathbf{q}, t)|^2 \rangle_t}$$

Intermediate scattering function

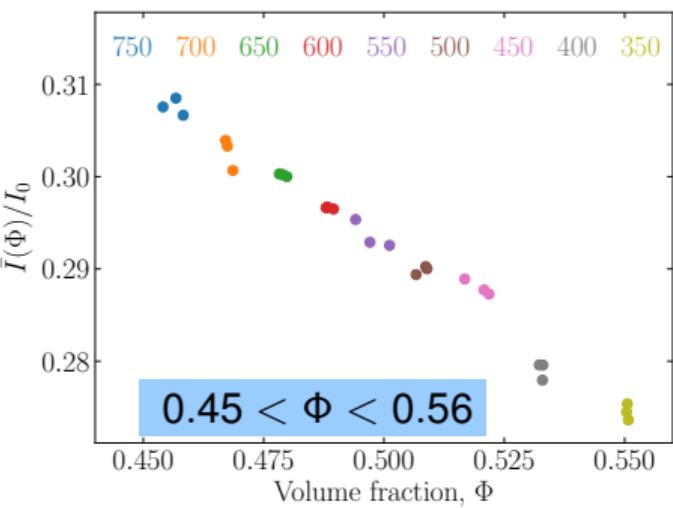
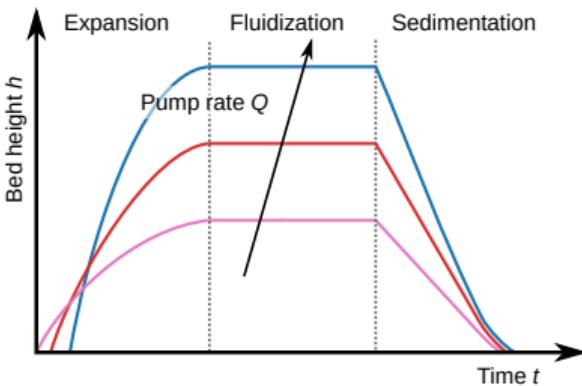
$$\begin{aligned} D(\mathbf{q}, \tau) &= \left\langle |I(\mathbf{q}, t + \tau) - I(\mathbf{q}, t)|^2 \right\rangle_t \\ &= A(\mathbf{q}) \left[ 1 - \frac{\langle I^*(\mathbf{q}, t)I(\mathbf{q}, t + \tau) \rangle_t}{\langle |I(\mathbf{q}, t)|^2 \rangle_t} \right] + B(\mathbf{q}) \end{aligned}$$

- $D(\mathbf{q}, \tau \rightarrow 0) = B(\mathbf{q})$
- $D(\mathbf{q}, \tau \rightarrow \infty) = A(\mathbf{q}) + B(\mathbf{q})$

# X-ray imaging – Linear space invariant?



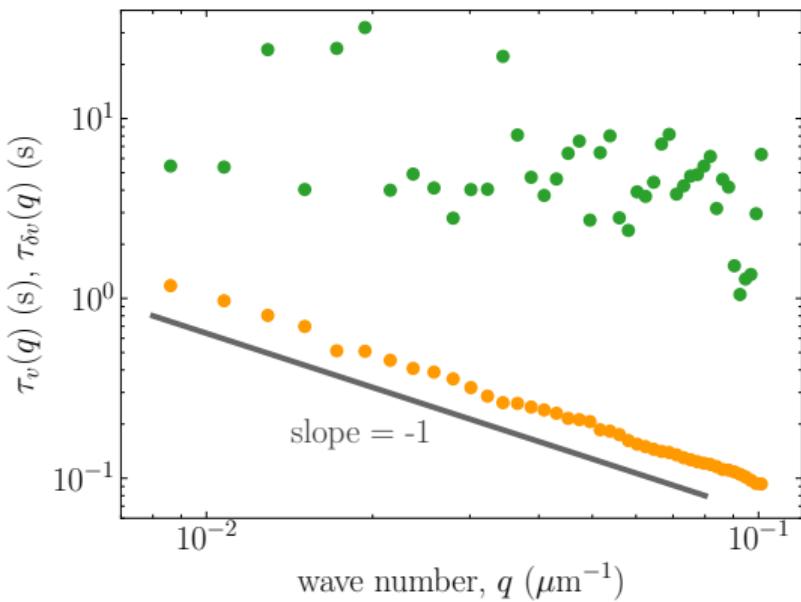
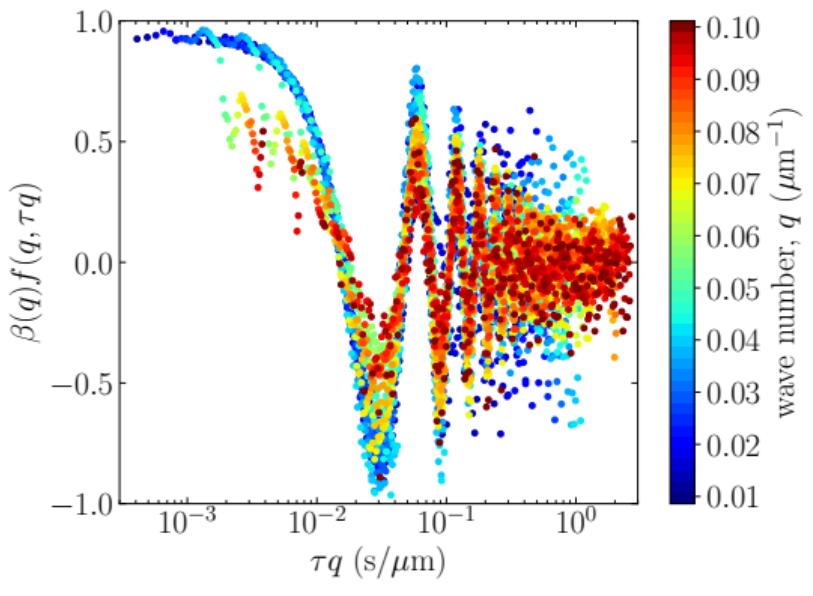
# X-ray imaging – Linear space invariant?



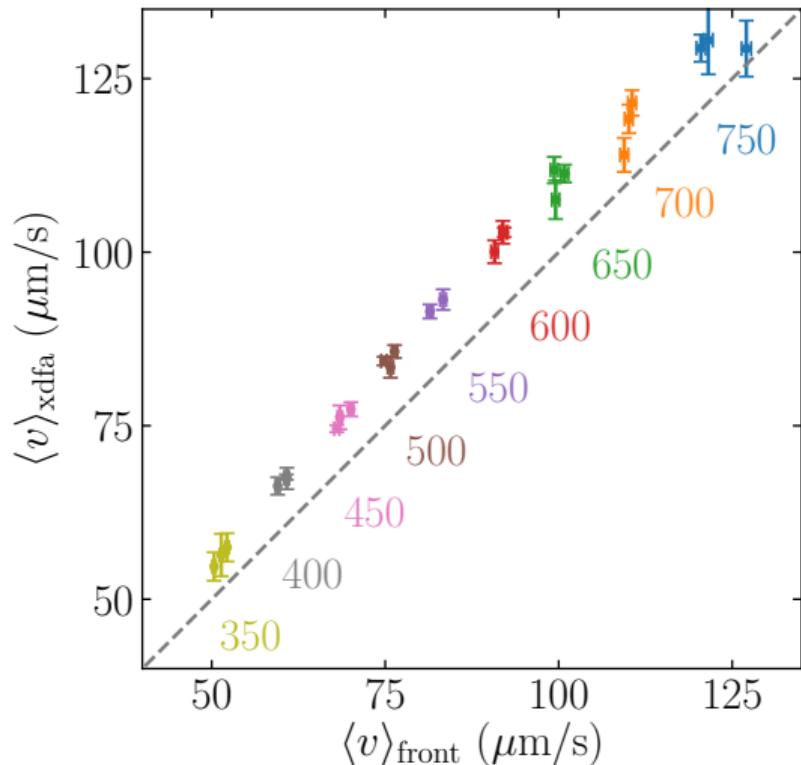
# X-DFA for a suspension of sedimenting particles

$$f(q, \tau) = \cos(q\langle v_s \rangle \tau) \exp\left(-\frac{1}{2}q^2 \delta v^2 \tau^2\right)$$

$$\langle v_s \rangle = \langle \Delta r \rangle / \tau_\nu, \langle \delta v \rangle = \langle \delta r \rangle / \tau_{\delta\nu}$$

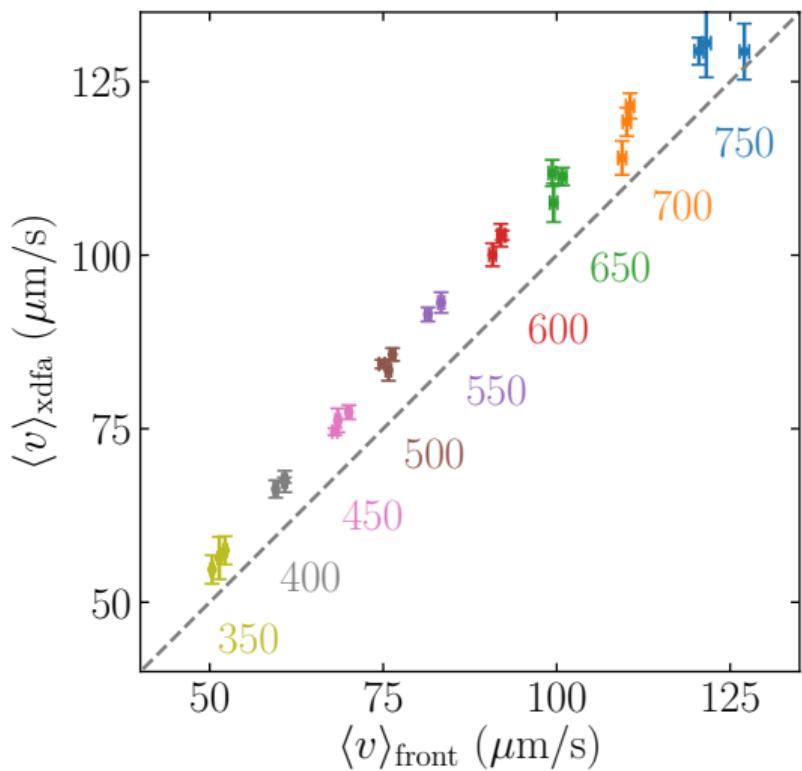


## Front tracking vs. X-DFA

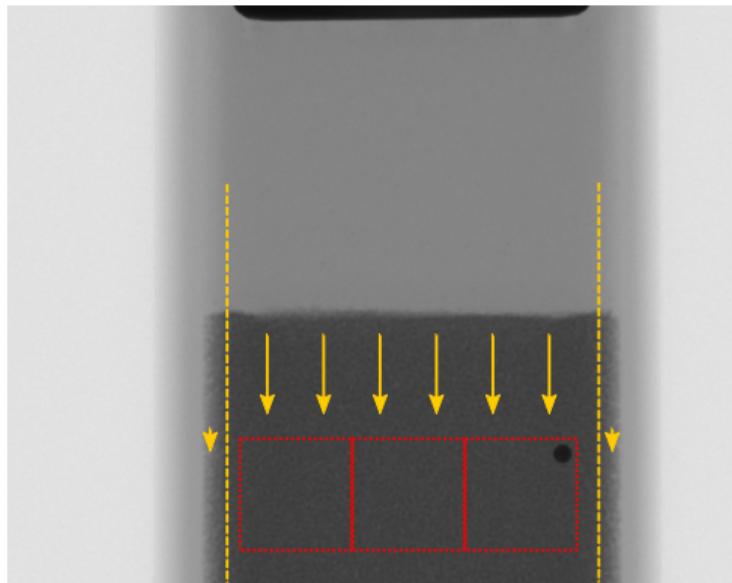


$\langle v \rangle_{\text{xdfa}} > \langle v \rangle_{\text{front}}$  by 9.4%

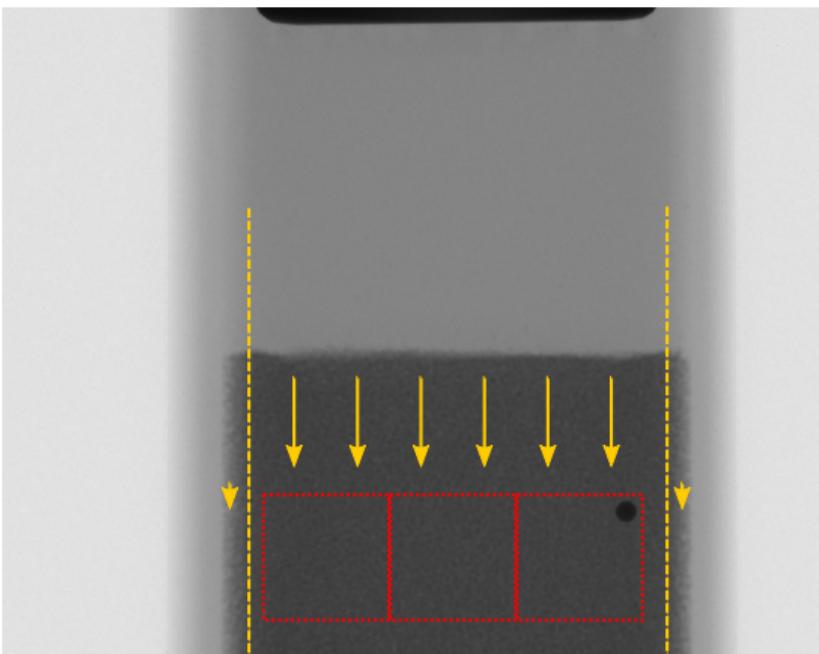
# Front tracking vs. X-DFA



$\langle v \rangle_{\text{xdfa}} > \langle v \rangle_{\text{front}}$  by 9.4%

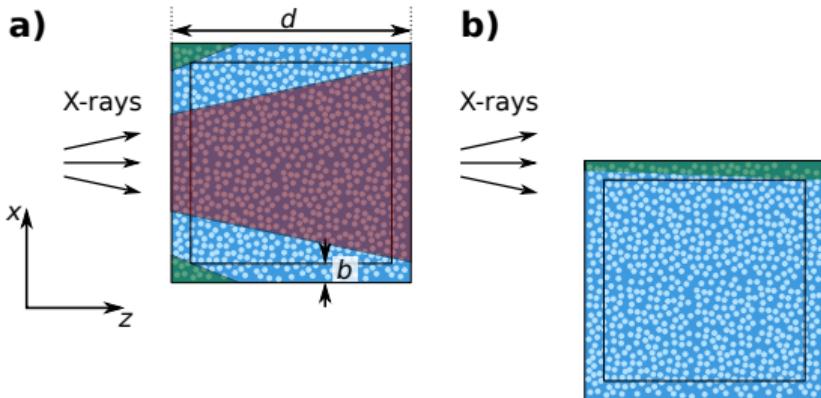


# Estimate width of boundary layer



$\langle v \rangle_{\text{xdfa}} > \langle v \rangle_{\text{front}}$  by 9.4%

$\langle v \rangle_{\text{xdfa}}$  takes two layers into account  
 $\langle v \rangle_{\text{front}}$  takes four layers into account



## Estimation:

Boundary velocity = 0

Else = const.

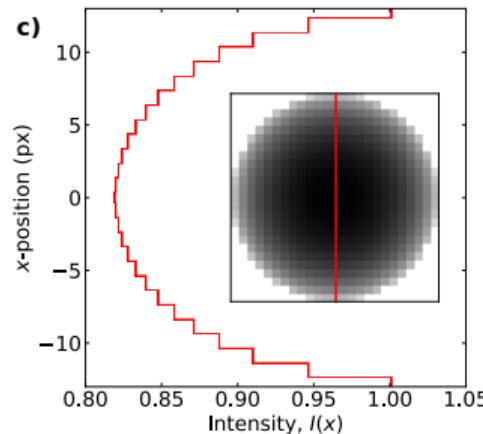
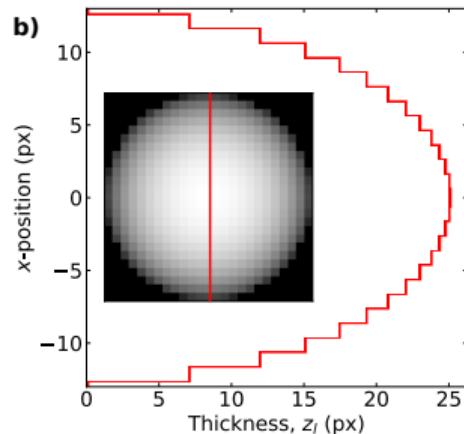
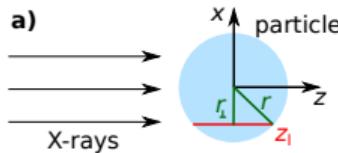
→  $b \approx 3$  particle diameters

Thank you for your attention!



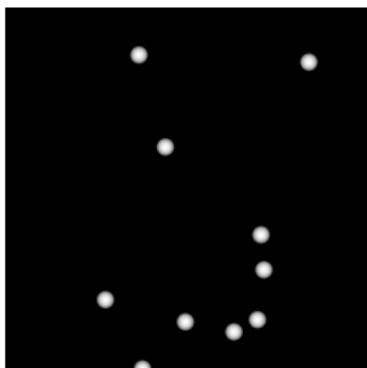
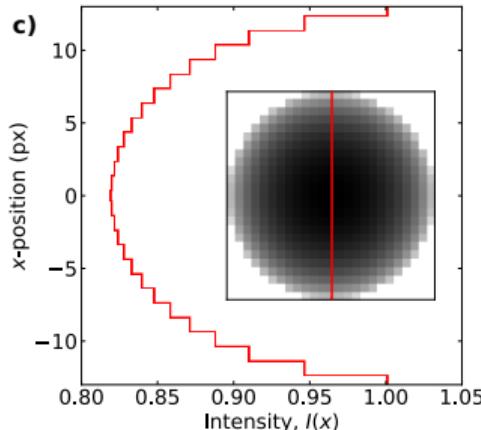
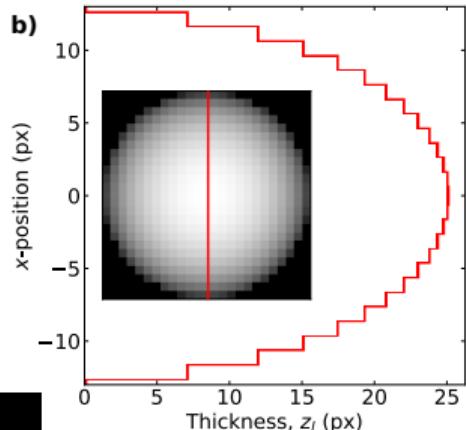
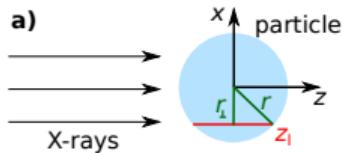
# Backup slides

# Synthetic radiograms

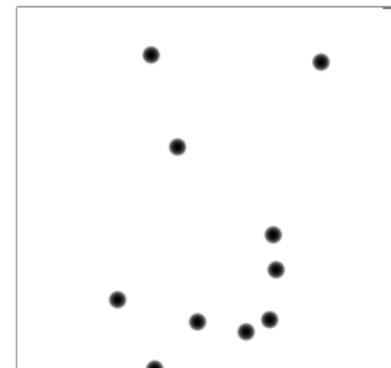


Beer-Lambert  
 $I(z_I) = I_0 \exp(-\mu z)$

# Synthetic radiograms



Beer-Lambert  
 $I(z_l) = I_0 \exp(-\mu z_l)$



## Linear space invariant imaging

Image correlation function

$$g(\mathbf{q}, \tau) = \frac{\langle I^*(\mathbf{q}, t) I(\mathbf{q}, t + \tau) \rangle_t}{\langle |I(\mathbf{q}, t)|^2 \rangle_t}$$

# Linear space invariant imaging

Image correlation function

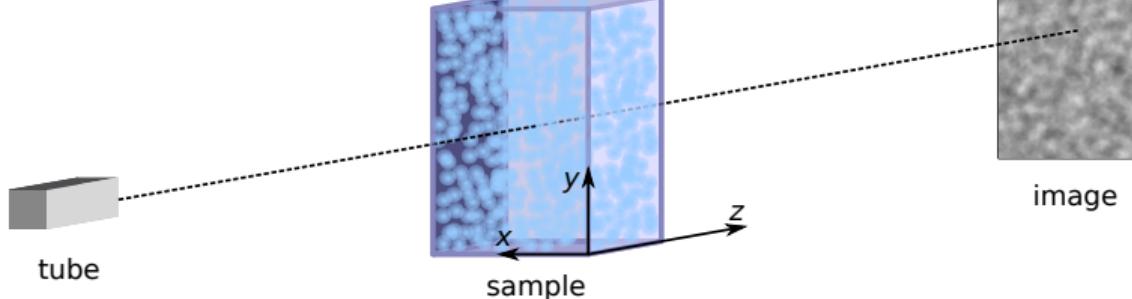
$$g(\mathbf{q}, \tau) = \frac{\langle I^*(\mathbf{q}, t)I(\mathbf{q}, t + \tau) \rangle_t}{\langle |I(\mathbf{q}, t)|^2 \rangle_t}$$

Intermediate scattering function

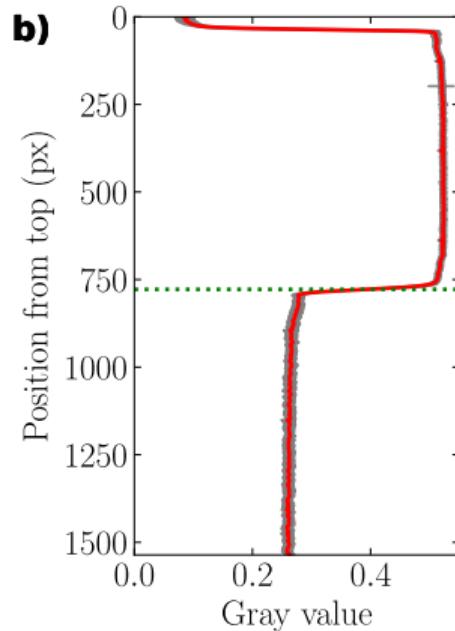
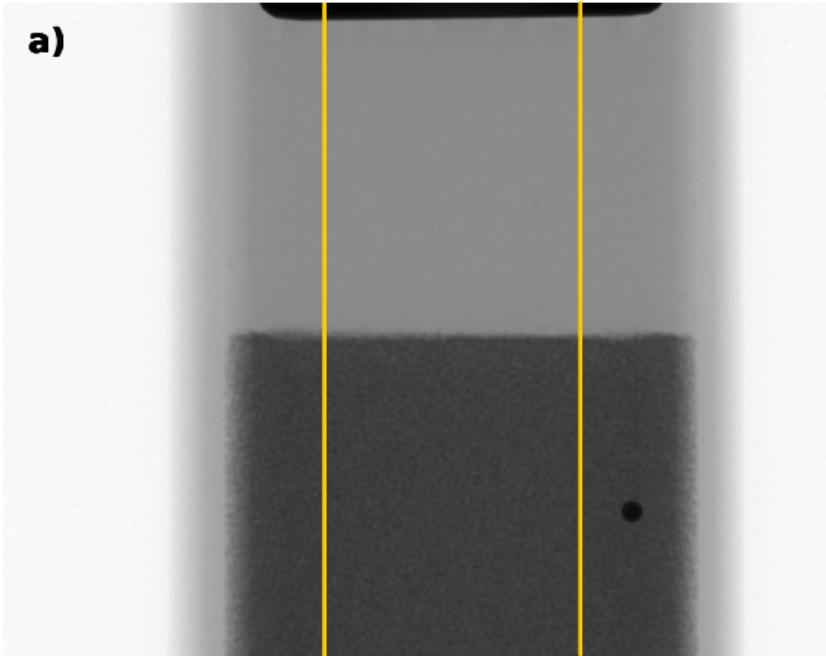
$$f(\mathbf{q}, \tau) = \frac{\langle \rho^*(\mathbf{q}, t)\rho(\mathbf{q}, t + \tau) \rangle_t}{\langle |\rho(\mathbf{q}, t)|^2 \rangle_t}$$

Linear space-invariant imaging:

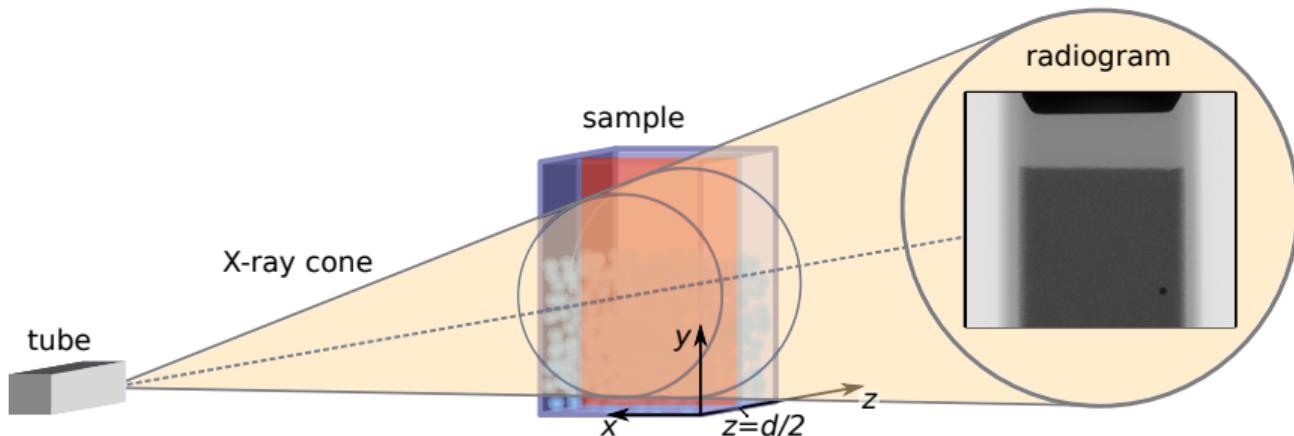
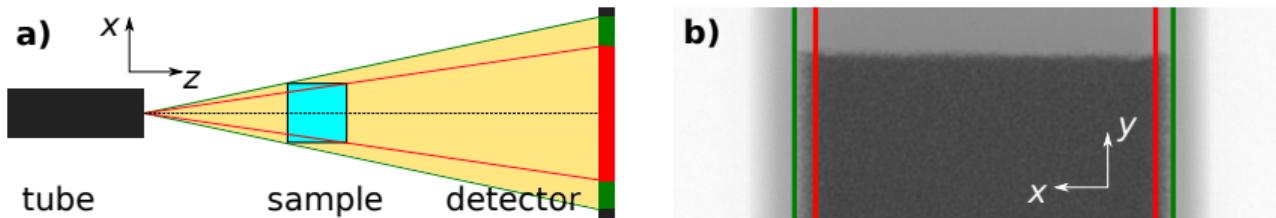
$$I(\mathbf{r}, t) = I_0 + \int d\mathbf{r}' dz' T(\mathbf{r} - \mathbf{r}', -z') c(\mathbf{r}', z', t)$$



## Tracking of particle front



## Tracking of particle front



# Tracking of particle front

