

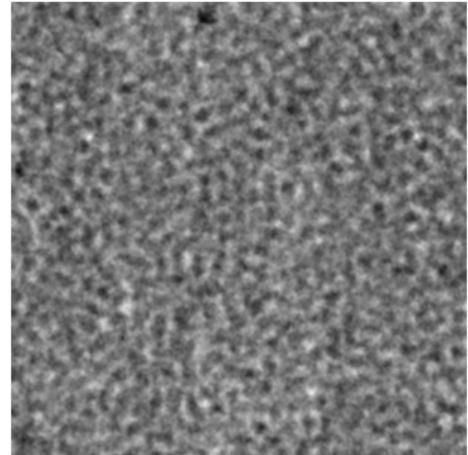
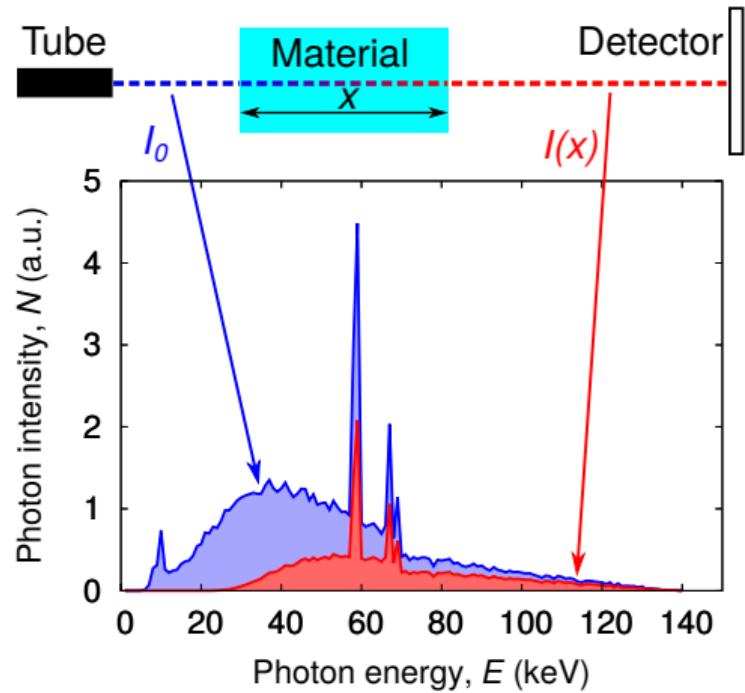


PhD defense
Manuel Baur

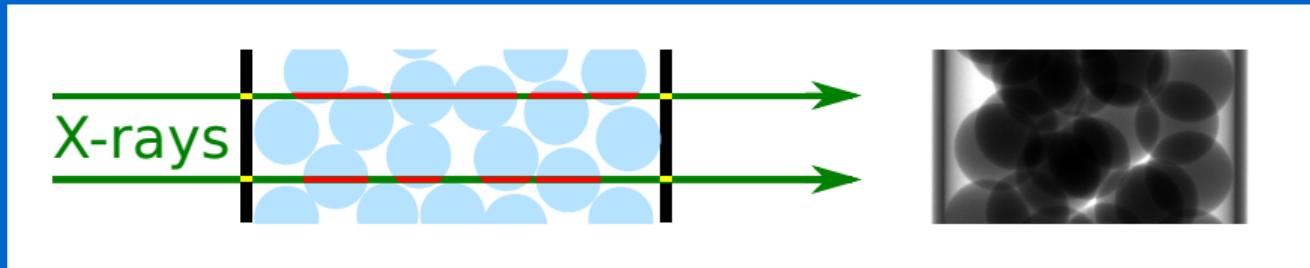
Funded by the German
Federal Ministry for
Economic Affairs and
Energy, grant no. 50WM

1653

X-ray radiography of granular systems – particle densities and dynamics

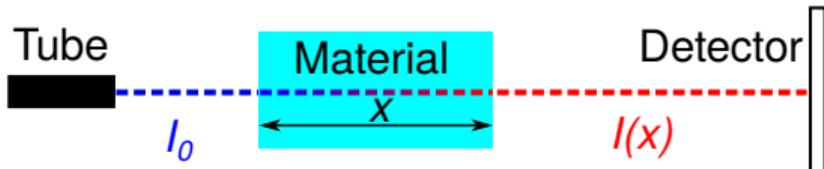


Measuring the volume fraction of granular packings



Correction of beam hardening
in X-ray radiograms
Collaboration Norman Uhlmann

Attenuation of X-rays

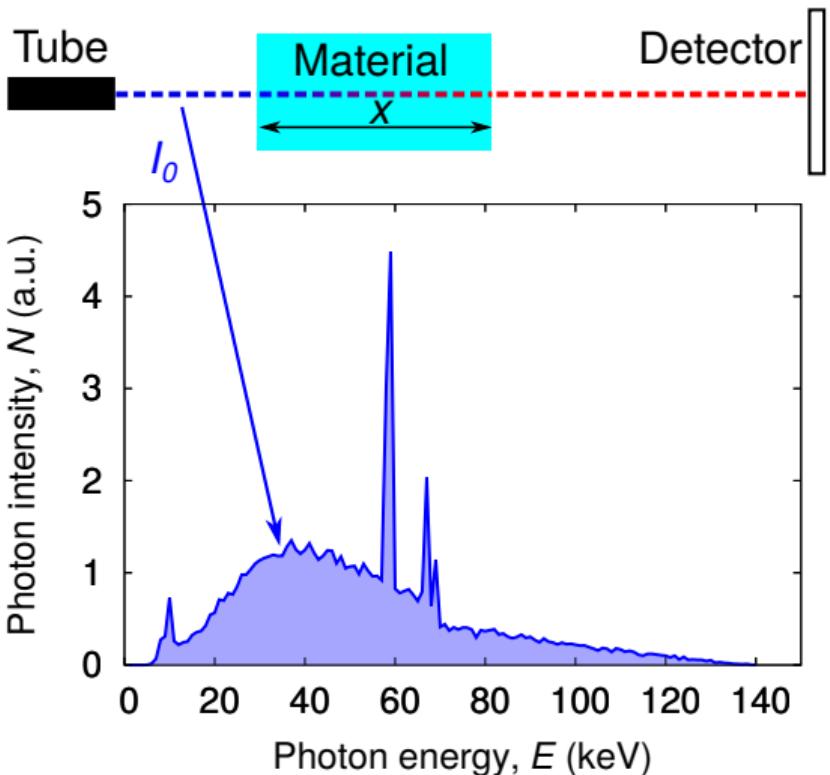


Beer-Lambert's law

$$I(x) = I_0 \exp(-\mu x)$$

$$\text{Thickness: } x = -\frac{1}{\mu} \ln \frac{I(x)}{I_0}$$

Attenuation of X-rays

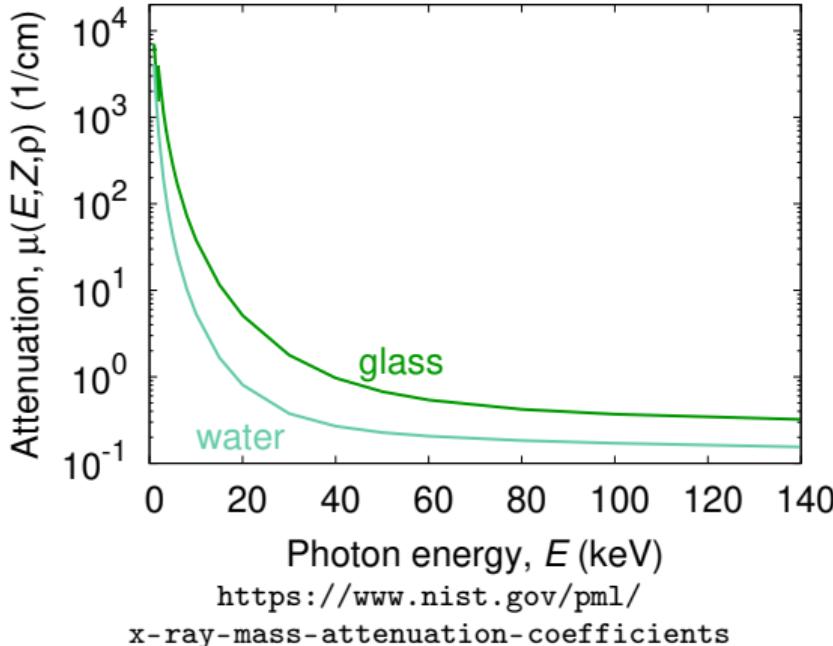


Beer-Lambert's law

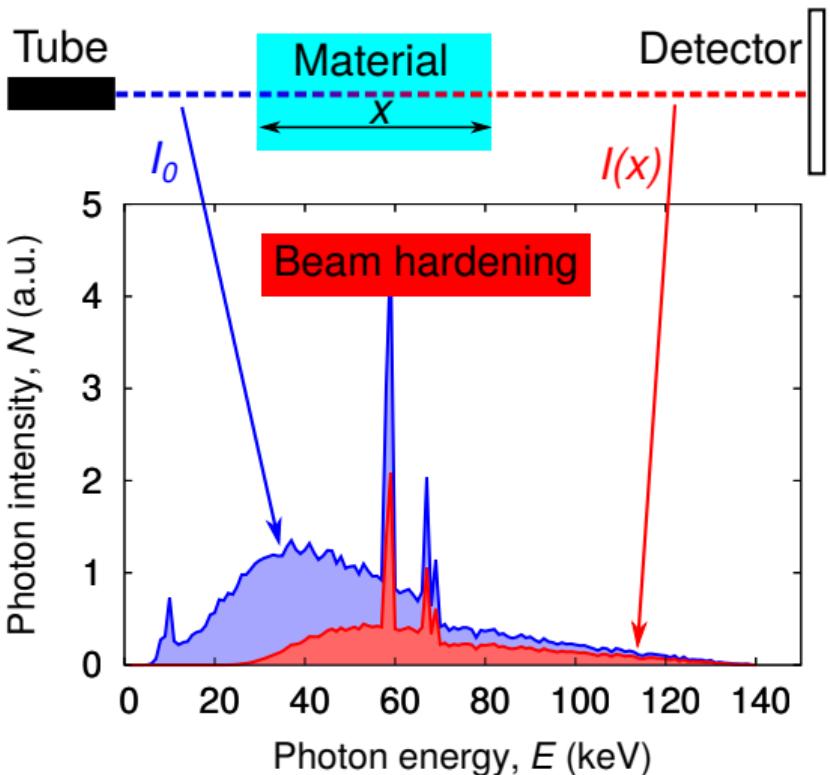
$$I(x) = I_0 \exp(-\mu(E, Z, \rho)x)$$

$\mu \neq \text{const}$

Thickness: $x = ?$



Attenuation of X-rays

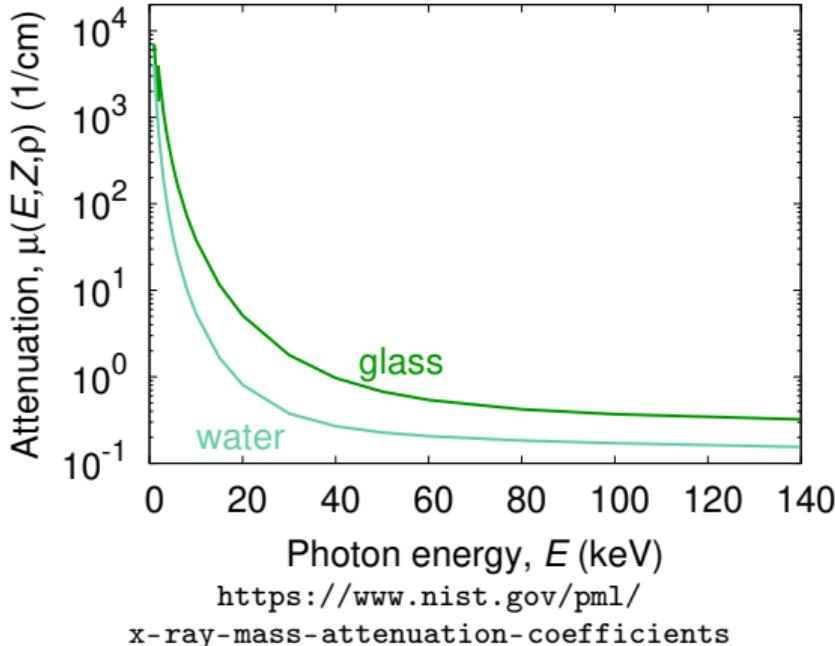


Beer-Lambert's law

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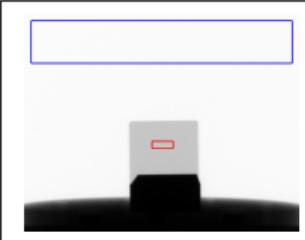
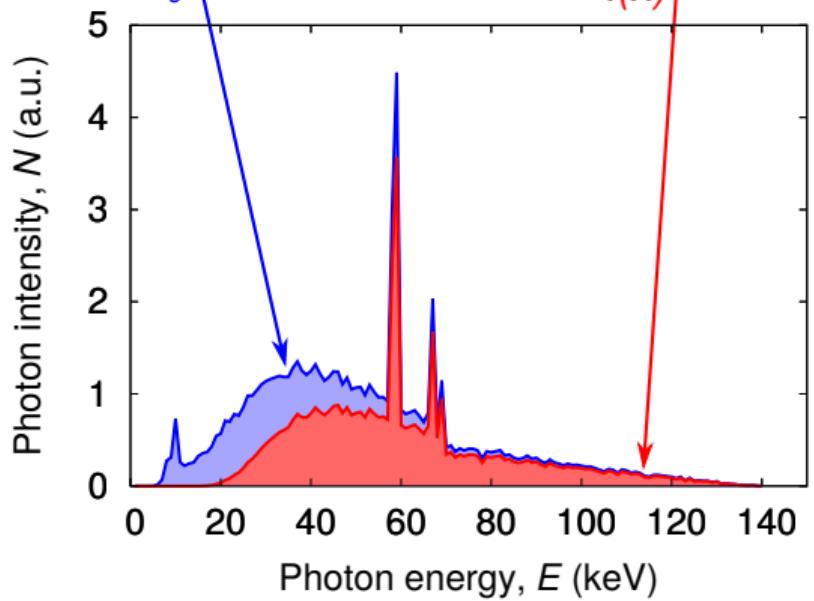
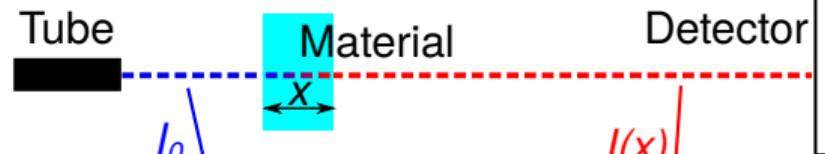
Thickness: $x = ?$



<https://www.nist.gov/pml/>

x-ray-mass-attenuation-coefficients

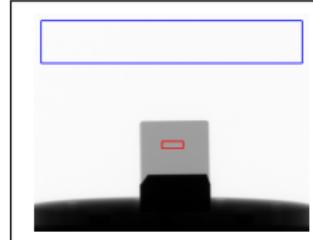
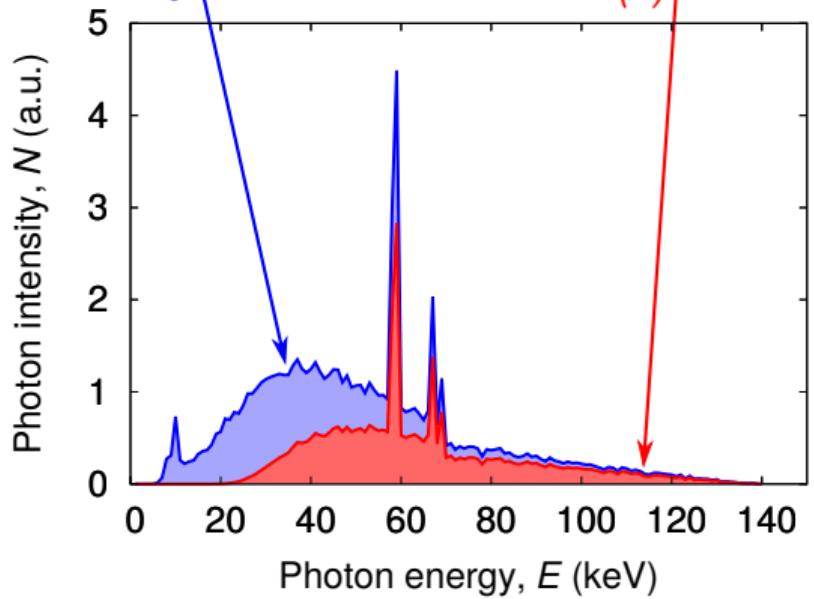
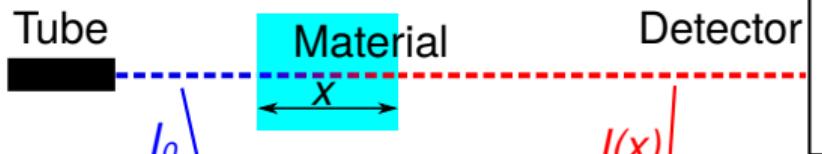
The effective attenuation, μ_{eff}



effective attenuation:

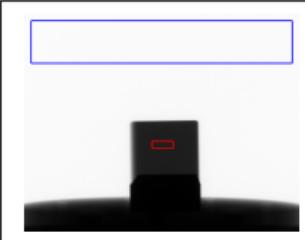
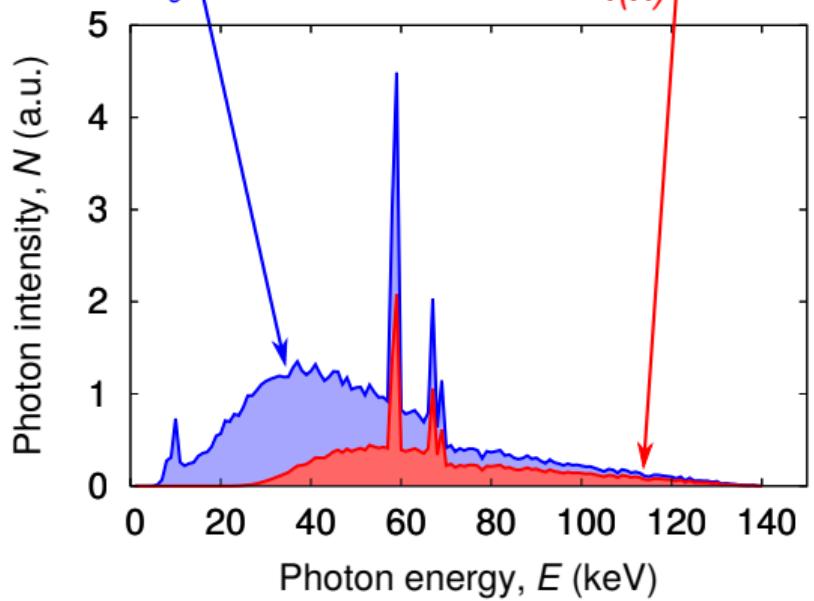
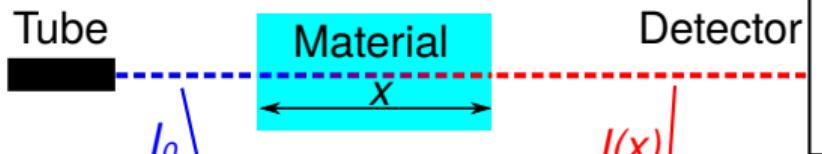
$$I(x) = I_0 \exp(-\mu_{\text{eff}}(x) x)$$

The effective attenuation, μ_{eff}



effective attenuation:
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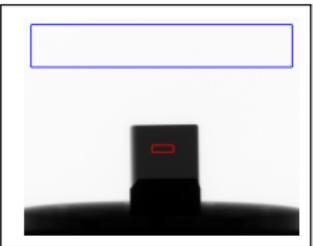
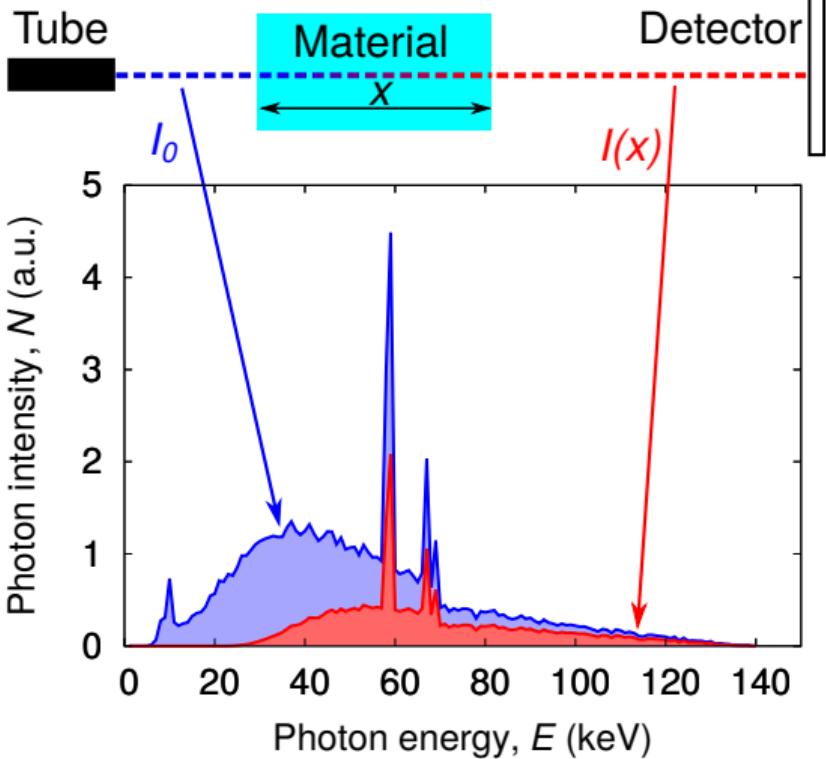
The effective attenuation, μ_{eff}



effective attenuation:

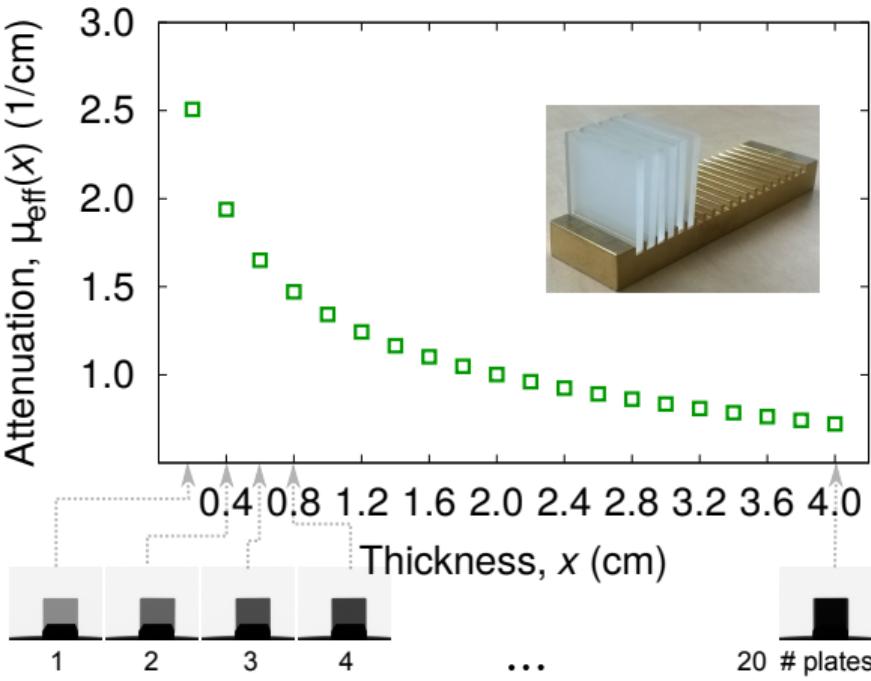
$$I(x) = I_0 \exp(-\mu_{\text{eff}}(x) x)$$

The effective attenuation, μ_{eff}

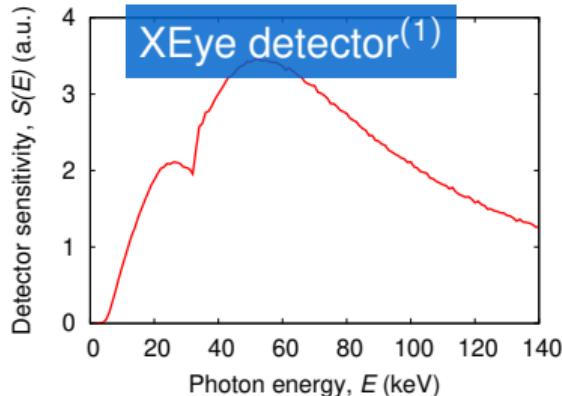
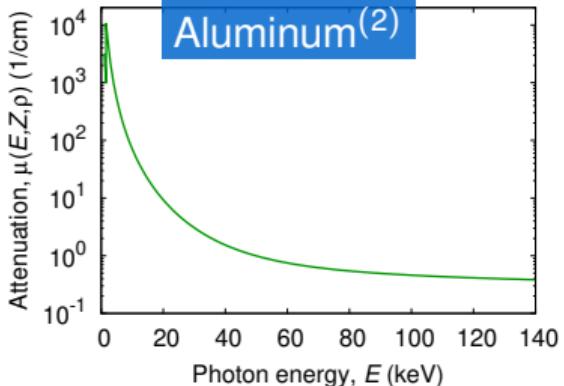
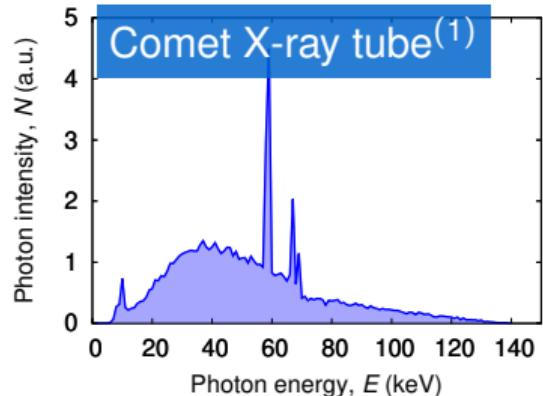
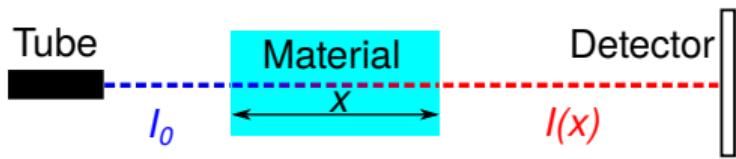


effective attenuation:
 $I(x) = I_0 \exp(-\mu_{\text{eff}}(x)x)$

$$\mu_{\text{eff}}(x) = -\frac{1}{x} \frac{I(x)}{I_0}$$



Modeling of μ_{eff}

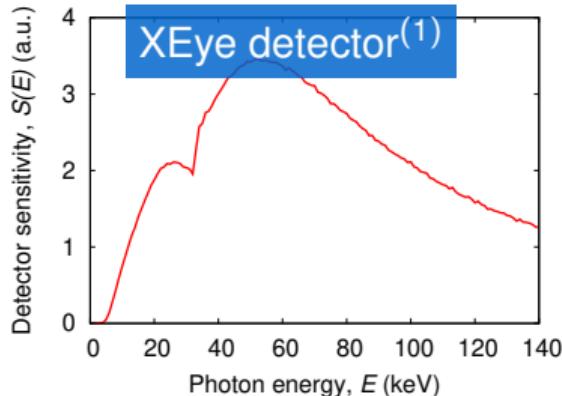
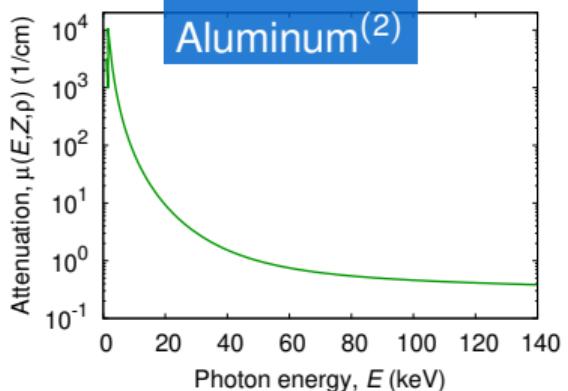
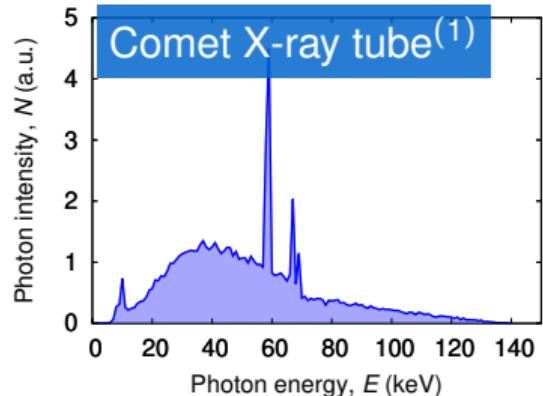
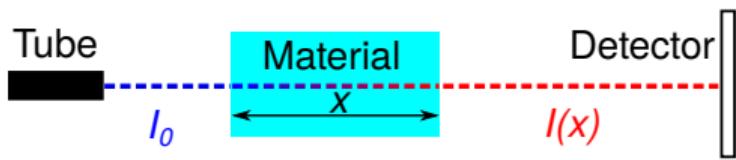


$$I(x) \propto \int N(E) \exp\{-\mu(E, Z, \rho)x\} S(E) dE$$

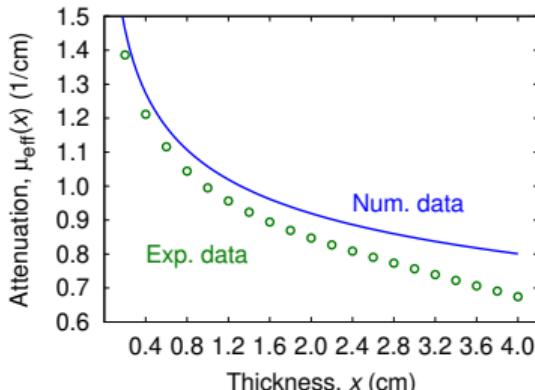
(1) Supplied by Norman Uhlmann, Fraunhofer EZRT

(2) XCOM supplied by NIST

Modeling of μ_{eff}



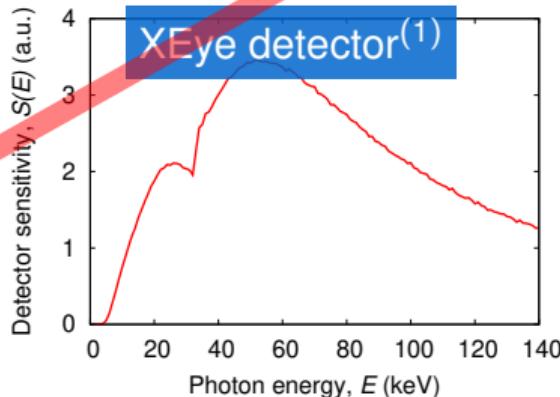
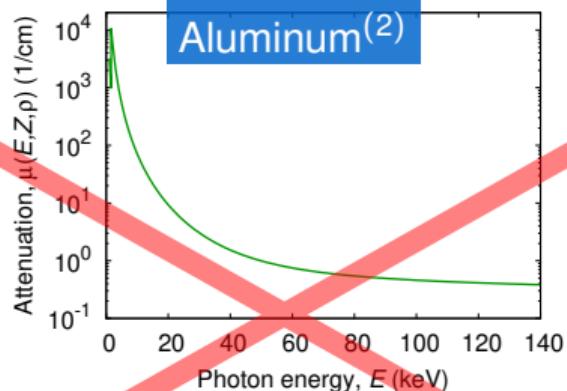
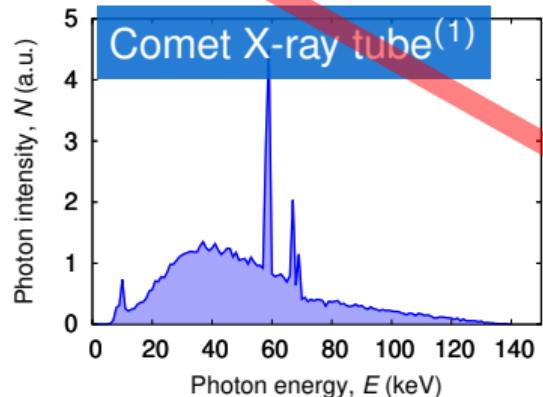
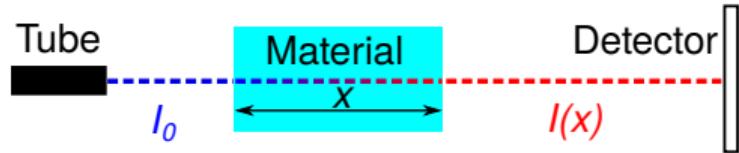
$$I(x) \propto \int N(E) \exp\{-\mu(E, Z, \rho)x\} S(E) dE$$



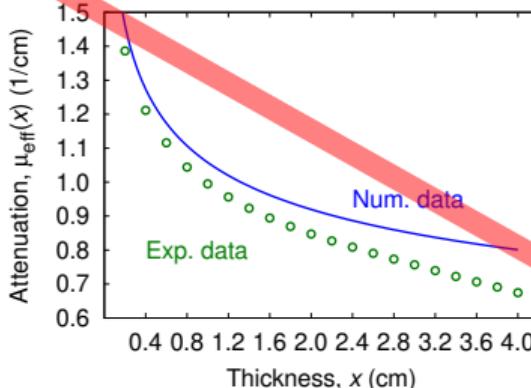
(1) Supplied by Norman Uhlmann, Fraunhofer EZRT

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Modeling of μ_{eff}



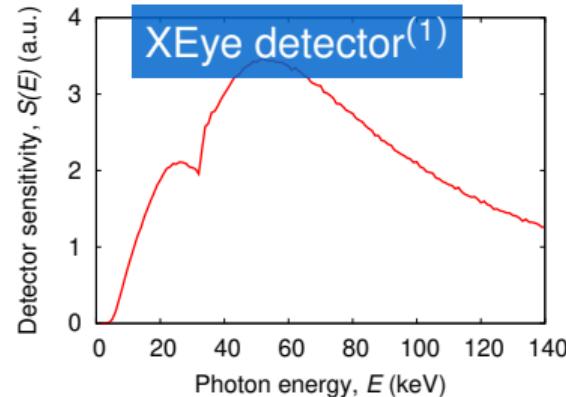
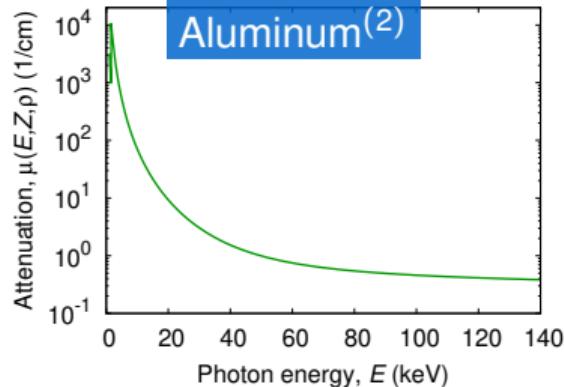
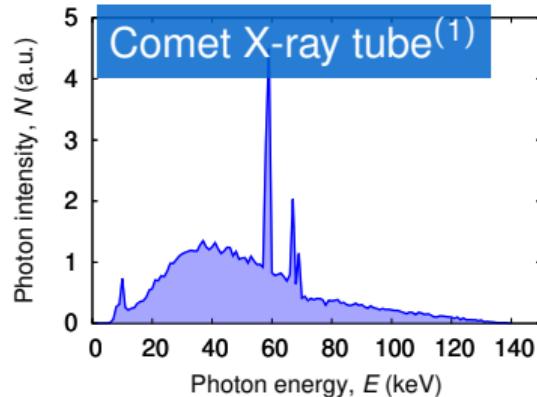
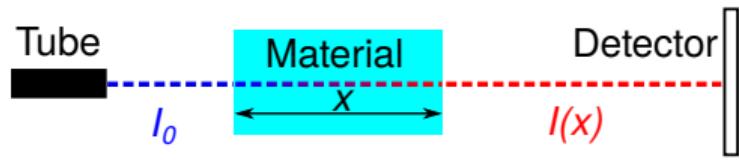
$$I(x) \propto \int N(E) \exp\{-\mu(E, Z, \rho)x\} S(E) dE$$



(1) Supplied by Norman Uhlmann, Fraunhofer EZRT

(2) XCOM supplied by NIST

Numerical approx. of μ_{eff}

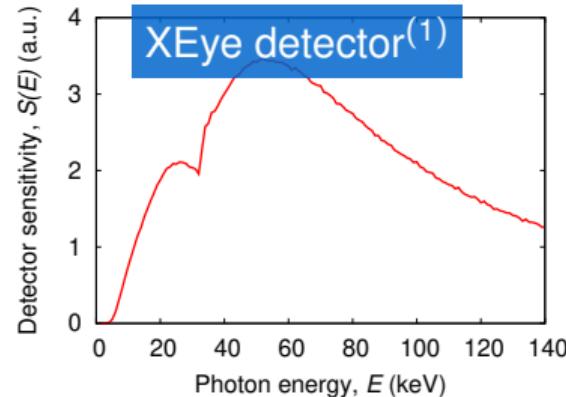
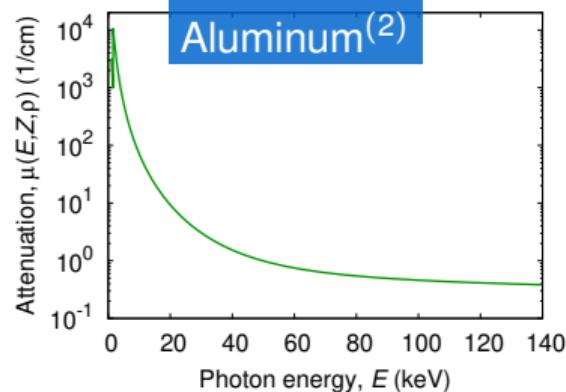
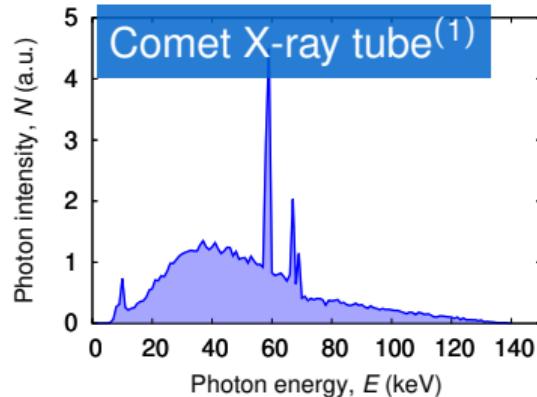
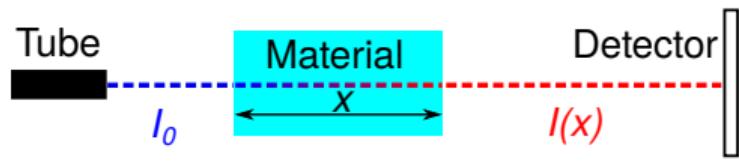


$$I(x) \propto \int N(E) \exp\{-\mu(E, Z, \rho)x\} S(E) dE$$

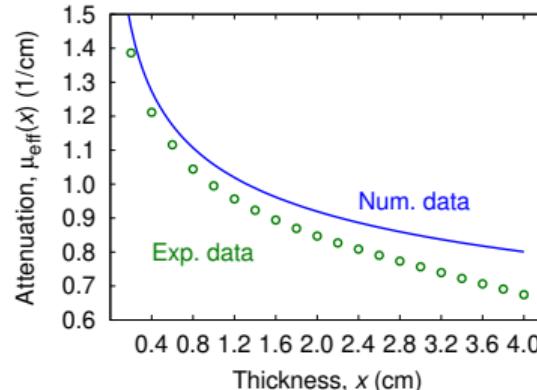
(1) Supplied by Norman Uhlmann, Fraunhofer EZRT

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Numerical approx. of μ_{eff}



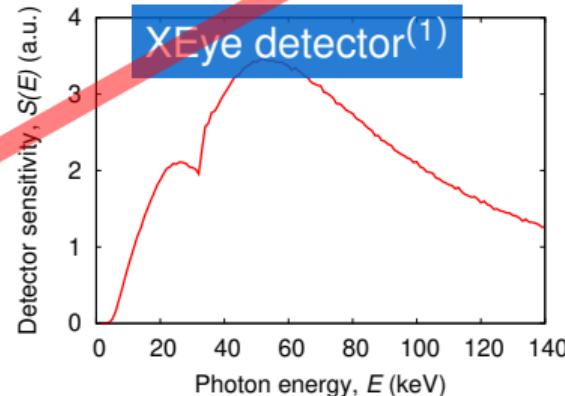
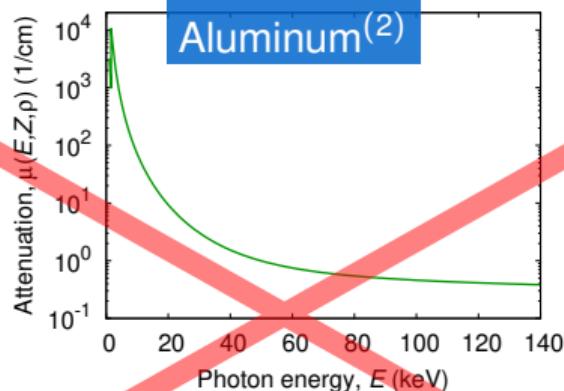
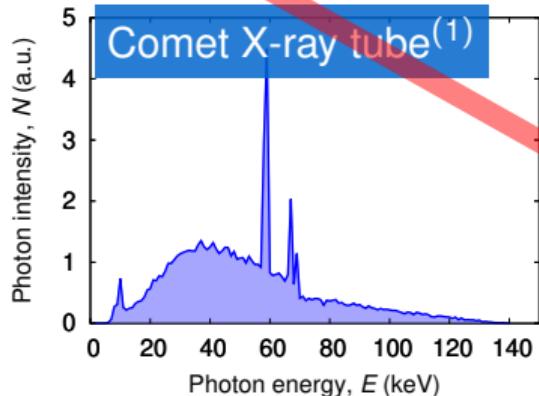
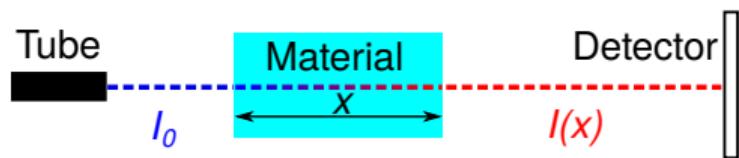
$$I(x) \propto \int N(E) \exp\{-\mu(E, Z, \rho)x\} S(E) dE$$



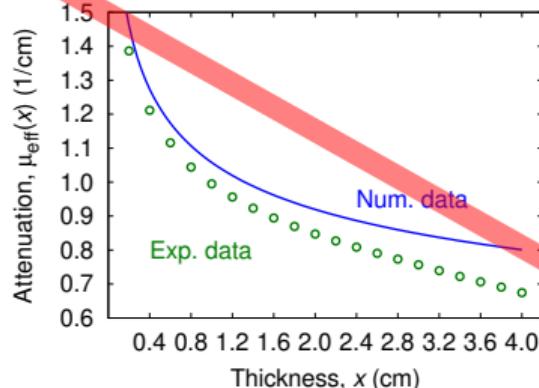
(1) Supplied by Norman Uhlmann, Fraunhofer EZRT

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Numerical approx. of μ_{eff}



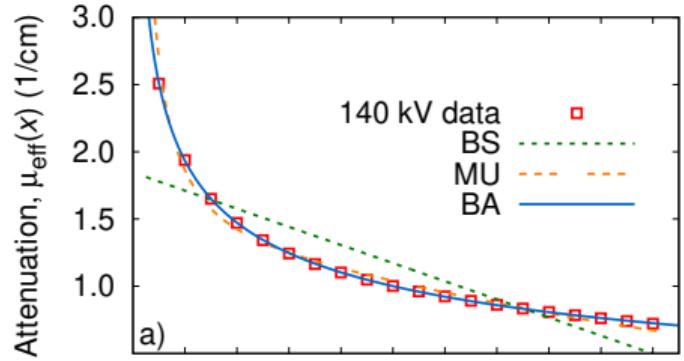
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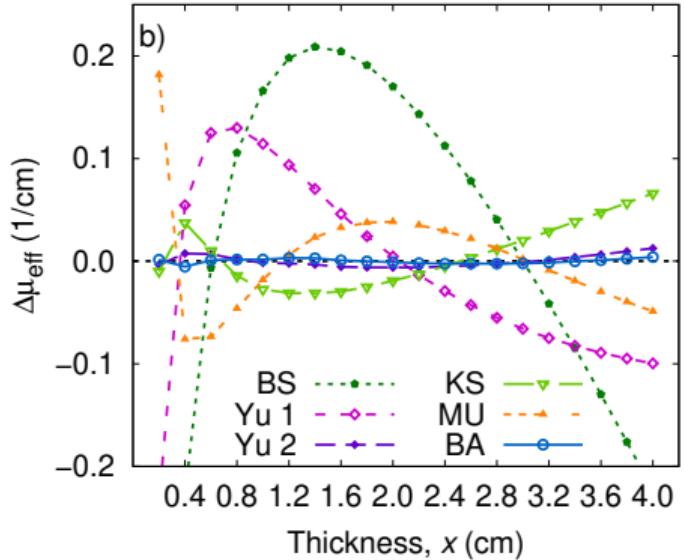
(2) XCOM supplied by NIST

Heuristic model functions



$$\mu_{\text{eff}}(x) = \mu_0 - \lambda x$$

Bjärngard & Shackford
(1994)



$$\mu_{\text{eff}}(x) = \frac{\mu_0}{1+\lambda x}$$

$$\mu_{\text{eff}}(x) = \frac{\mu_0}{(1+\lambda x)^\beta}$$

Yu *et al.* (1997)

$$\mu_{\text{eff}}(x) = \mu(E_{\text{max}}) + \frac{2\mu_1}{x\sqrt{-\lambda_1^2+4\lambda_2}} \times \left[\arctan\left(\frac{\lambda_1+2\lambda_2 x}{\sqrt{-\lambda_1^2+4\lambda_2}}\right) - \arctan\left(\frac{\lambda_1}{\sqrt{-\lambda_1^2+4\lambda_2}}\right) \right]$$

Kleinschmidt (1999)

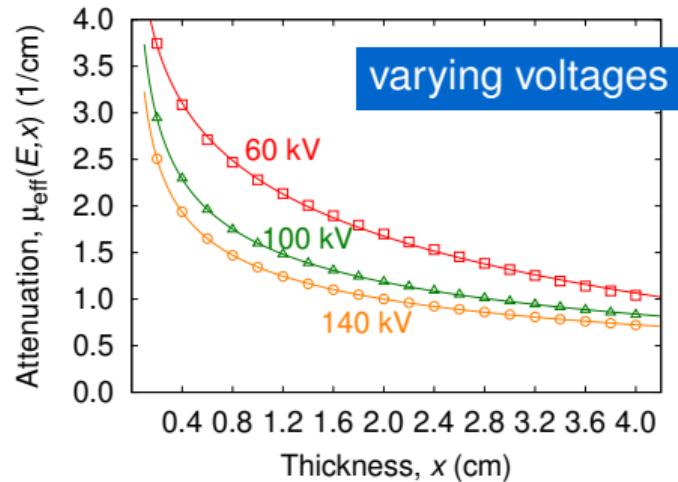
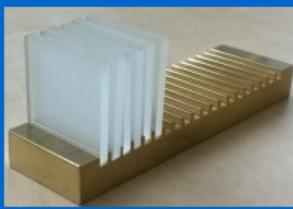
$$\mu_{\text{eff}}(x) = -\frac{1}{x} \ln [A + B \exp(-x/C)]$$

Mudde *et al.* (2008)

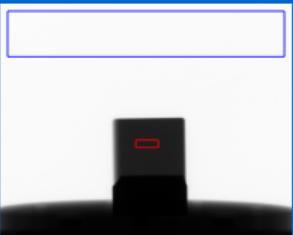
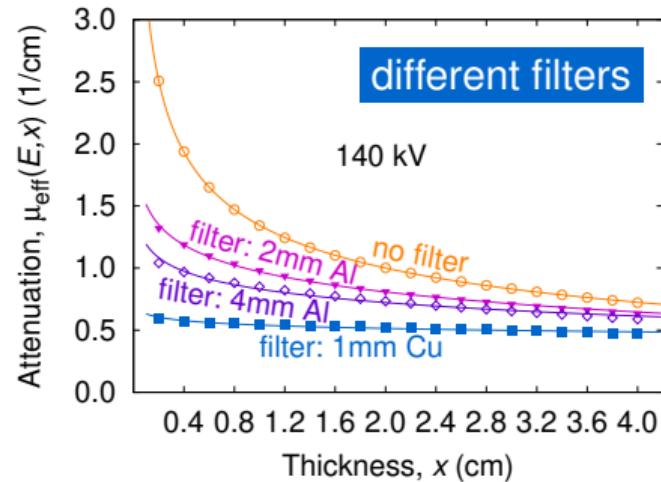
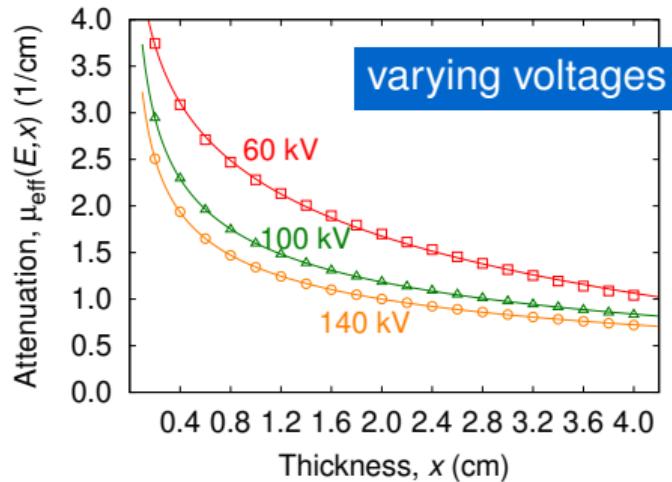
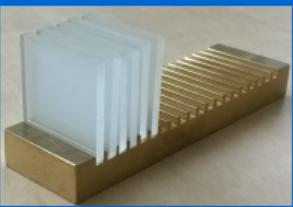
$$\mu_{\text{eff}}(x) = a + \frac{b}{x^\alpha}$$

Baur *et al.* (2019)
(this work)

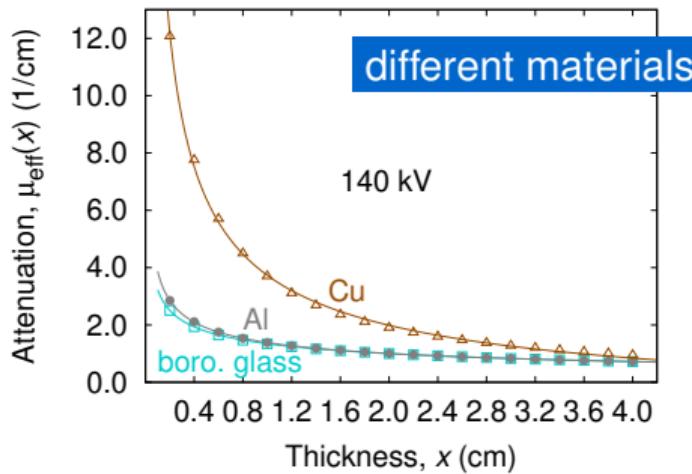
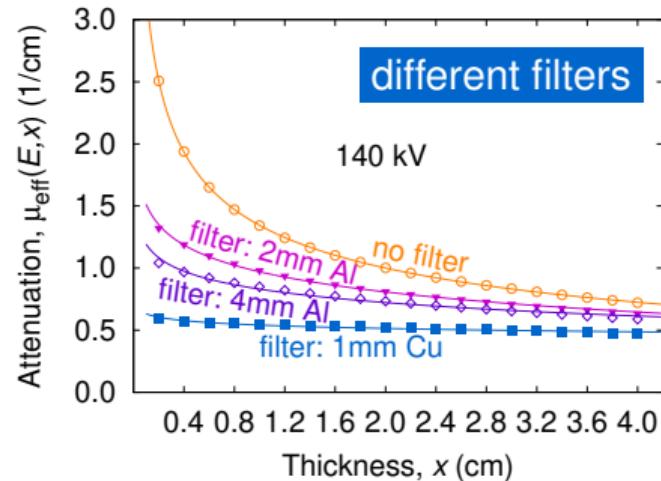
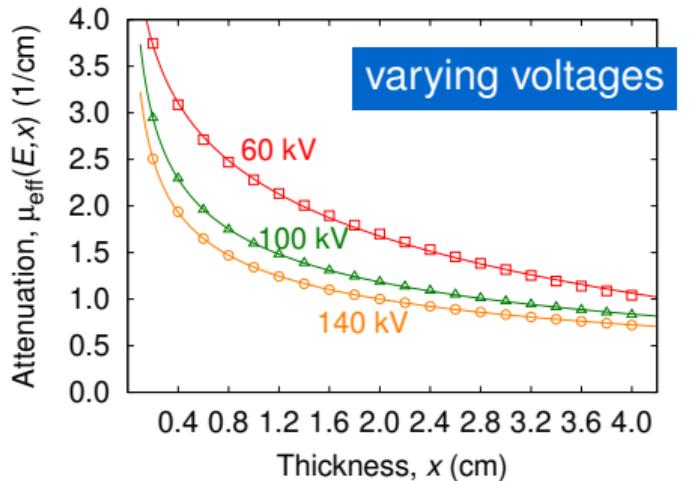
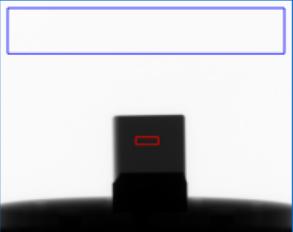
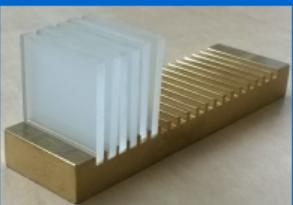
Universality of
 $\mu_{\text{eff}}(x) = a + \frac{b}{x^\alpha}$



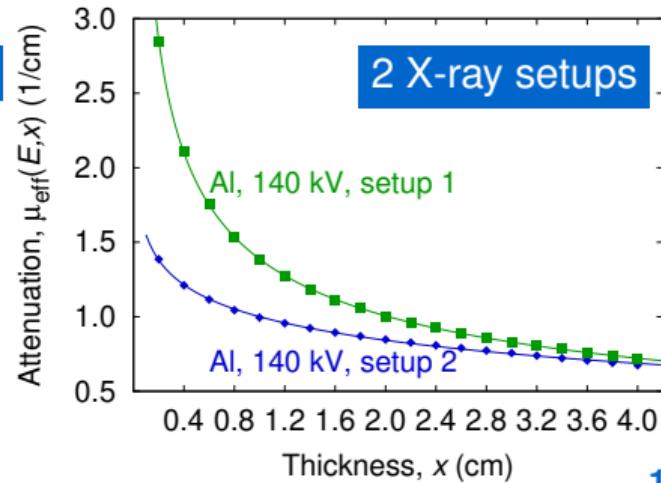
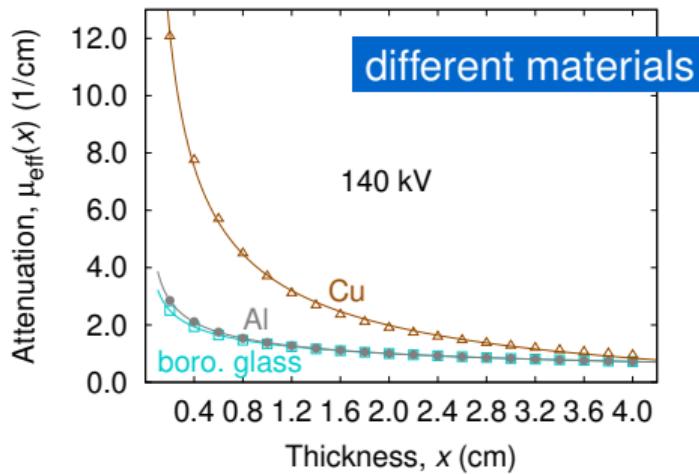
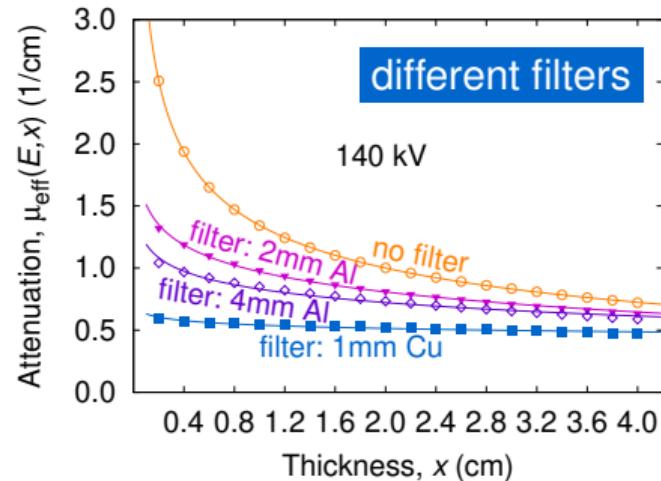
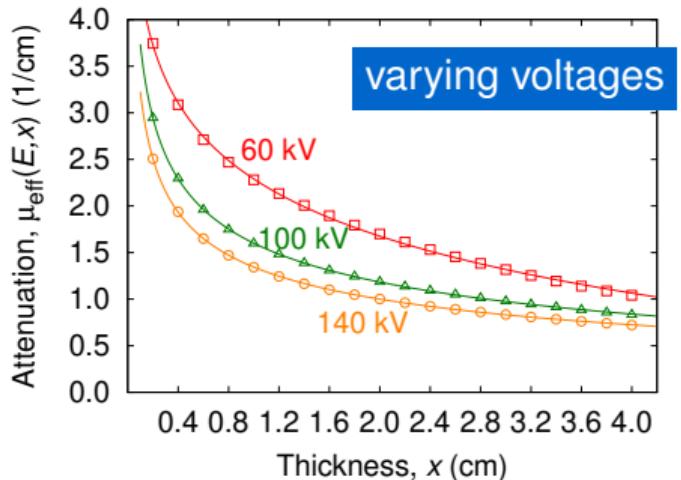
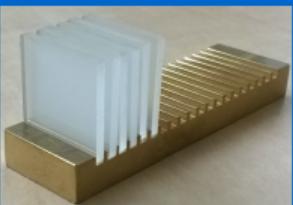
Universality of
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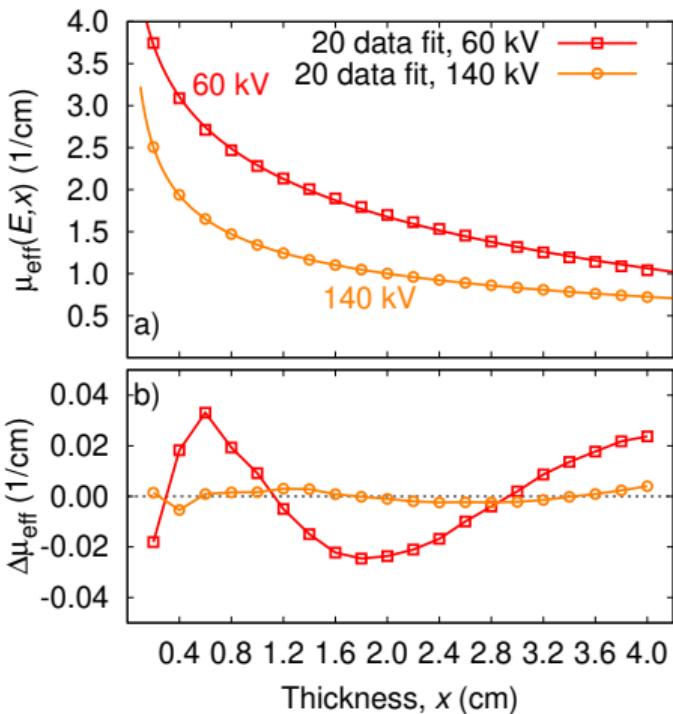
Determining the material thickness x

Generalized Beer-Lambert

$$I(x) = I_0 \exp(-\mu_{\text{eff}}(x) x)$$

Model function

$$\mu_{\text{eff}}(x) = a + \frac{b}{x^\alpha}$$



Determining the material thickness x

Generalized Beer-Lambert

$$I(x) = I_0 \exp(-\mu_{\text{eff}}(x) x)$$

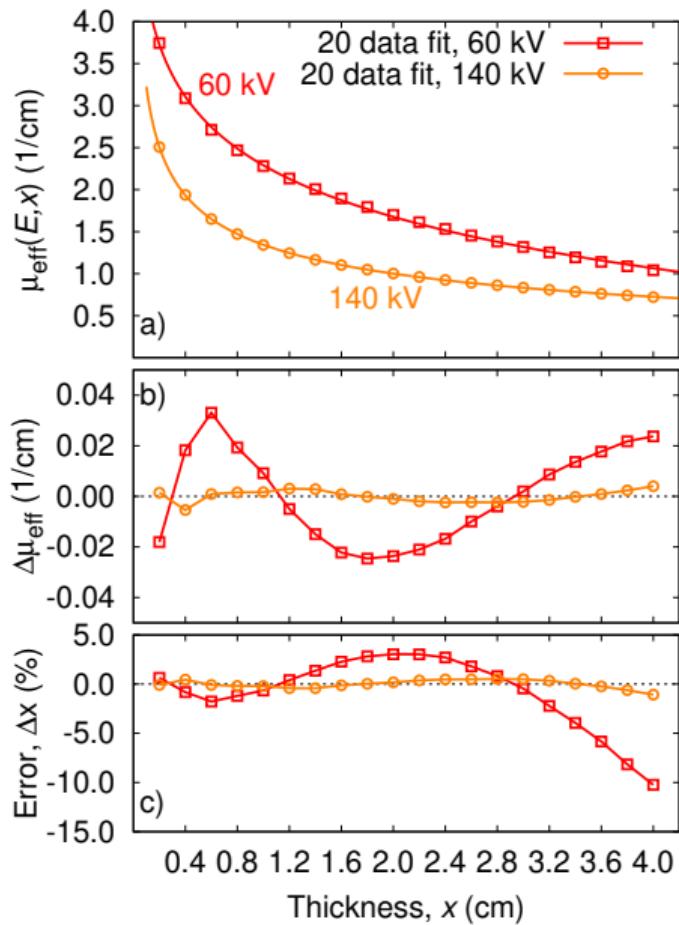
Model function

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Solve

$$ax + bx^{1-\alpha} + \ln\left(\frac{I(x)}{I_0}\right) = 0$$

e.g. Newtons method or look-up table



Determining the material thickness x

Generalized Beer-Lambert

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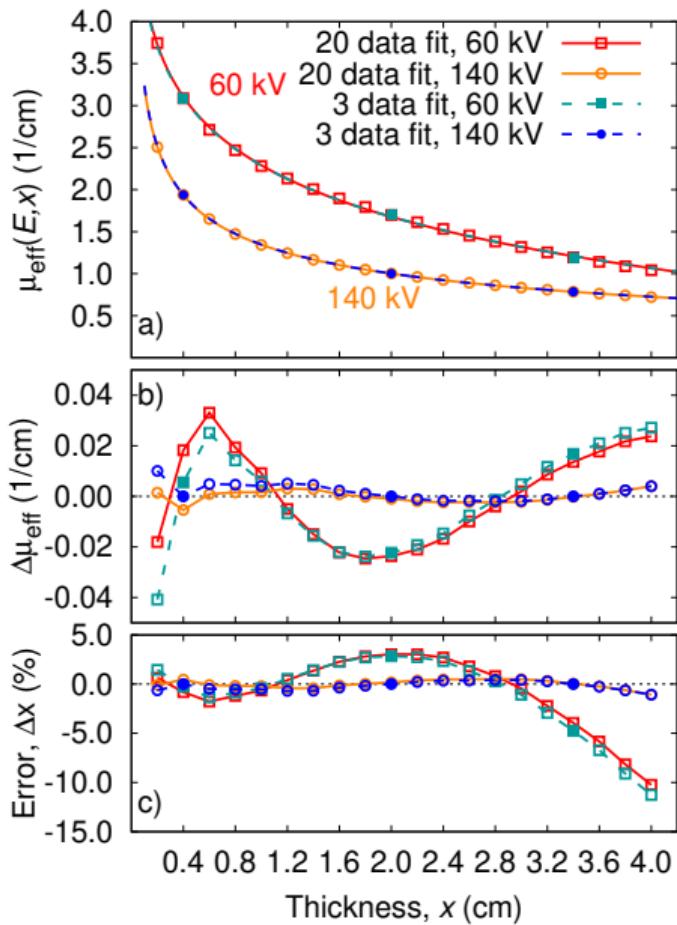
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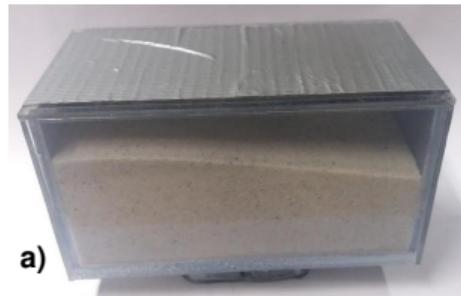
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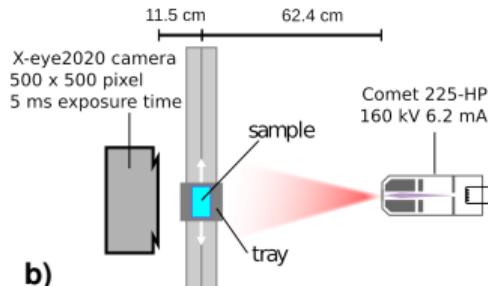
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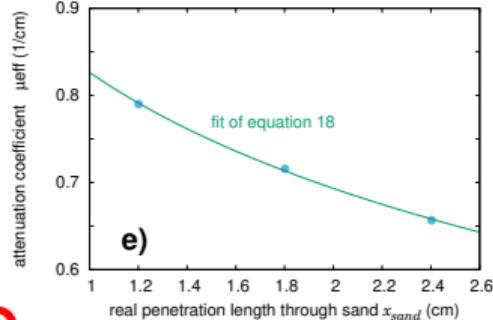
Applications



a)

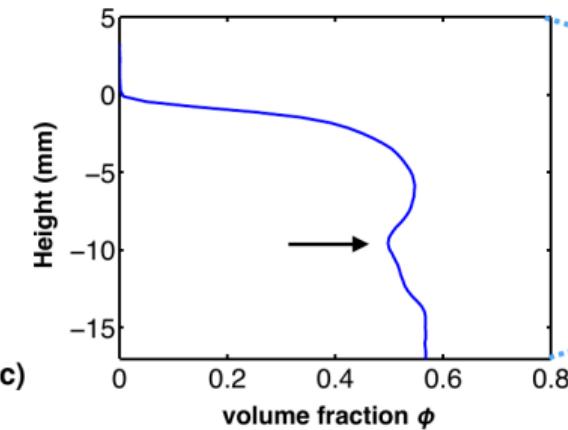


b)

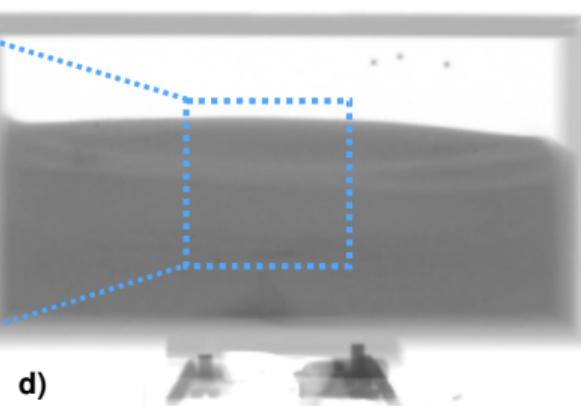


e)

reduce to useful illustrative



c)



d)