ESCUELA POLITÉCNICA SUPERIOR DE SEVILLA

MATEMÁTICAS II

Boletín 1 - Integrales Indefinidas

1. $\int \frac{x}{\sqrt{(x^2+3)^3}}dx$
2. $\int \frac{x+1}{\sqrt{x}}dx$
3. $\int \frac{\cos x}{\sqrt{9-4\,\mathrm{sen}^2 x}}dx$
4. $\int \frac{\cos(\ln x)}{x}dx$
5. $\int \frac{1}{\mathrm{tg}\, x}dx$
6. $\int \mathrm{sen}\ (3x+2)dx$
7. $\int x(x^2+1)^8 dx$
8. $\int x\sqrt{9-x^2}dx$
9. $\int xe^{-x^2}dx$
10. $\int x^2 \mathrm{sen}\ x^3 dx$
11. $\int \frac{1}{x\ln x}dx$
12. $\int \mathrm{sen}^5 2x\cos 2x dx$
13. $\int \frac{2x}{1+x^4}dx$
14. $\int \frac{e^x}{1+e^{2x}}dx$
15. $\int \frac{1}{x+x(\ln x)^2}dx$
16. $\int \frac{1+2x}{1+x^2}dx$
17. $\int x^2 \ln x dx$
18. $\int \mathrm{sen}\ x \ln(\cos x)\, dx$
19. $\int x\ \cos x dx$
20. $\int \ln x dx$
21. $\int (\ln x)^2 dx$
22. $\int x^3 e^{x^2} dx$
23. $\int \frac{\ln x}{x^3}dx$
24. $\int x\mathrm{sen}\ x\cos x dx$
25. $\int x^2 \mathrm{arctg}\ x dx$
26. $\int \left(3x^2+1\right)\mathrm{arctg}(2x)\, dx$
27. $\int x^2 \ln\left(x^2+3\right) dx$
28. $\int e^{2x}\cos(5x)\, dx$
29. $\int \frac{1}{x^2(x-1)^2}dx$
30. $\int \frac{1}{x^3-1}dx$
31. $\int \frac{1+x}{1-x}dx$
32. $\int \frac{1+e^x}{e^x-4+4e^{-x}}dx$
33. $\int \frac{x-2}{(x-1)^2(x^2+1)}dx$
34. $\int \frac{x^3+x+1}{x(x^2-1)}dx$
35. $\int \frac{1}{x(x+1)^2}dx$
36. $\int \frac{x^3+x-2}{x^4-x^3-2x^2}dx$
37. $\int \frac{2x^2-8x+1}{2x^2-7x+3}dx$
38. $\int \frac{x^4-x^2-2x-6}{x^2(2+x^2)}dx$
39. $\int \frac{1}{x^4-1}dx$
40. $\int \frac{x^4}{(x+1)^2}dx$
41. $\int \frac{3x+5}{x^3-x^2-x+1}dx$
42. $\int \frac{x^2+1}{(x-1)(x^2+2)}dx$
43. $\int \frac{x+3}{(x^2-6x+5)^2}dx$
44. $\int \mathrm{sen}(9x)\cos(4x)dx$
45. $\int \mathrm{sen}^2(4-x)\, dx$
46. $\int \cos^2(5-3x)\, dx$
47. $\int \mathrm{sen}(8x+3)\,\mathrm{sen}(5x-4)dx$
48. $\int \cos(5x+9)\ \cos(7x-2)dx$
49. $\int \mathrm{sen}\left(x+\frac{\pi}{4}\right)\mathrm{sen}\left(x-\frac{\pi}{4}\right)dx$

50. $\int \frac{\operatorname{sen} x \cos x}{(1+\cos x)^3} dx$
51. $\int \frac{\cos^4 x}{\operatorname{sen}^3 x} dx$
52. $\int \operatorname{sen}^3 x dx$
53. $\int \frac{\operatorname{sen}^2 x + \cos^3 x}{\operatorname{sen} x \cos x} dx$
54. $\int \frac{\cos^3 x \operatorname{sen} x}{1+\operatorname{sen}^2 x} dx$
55. $\int \frac{\operatorname{sen}^3 x}{\cos x} dx$
56. $\int \frac{3\cos x}{\operatorname{sen}^3 x} dx$
57. $\int \frac{\cos^3 x}{(\operatorname{sen}^2 x - 4)^2} dx$
58. $\int \cos^3 3x dx$
59. $\int \frac{1}{5\cos^2 x + 3\operatorname{sen}^2 x} dx$
60. $\int \frac{1}{1+\operatorname{tg} x} dx$
61. $\int \frac{\operatorname{sen} x + \cos x}{\cos^3 x} dx$
62. $\int \frac{1+\cos x}{1-\cos x} dx$
63. $\int \frac{1}{3-2\cos x} dx$
64. $\int \frac{2-\operatorname{sen} x}{2+\operatorname{sen} x} dx$
65. $\int \frac{1+\operatorname{tg} x}{1-\cos x} dx$
66. $\int \frac{1}{3+5\cos x} dx$
67. $\int \frac{x^2-5}{\sqrt{9-x^2}} dx$
68. $\int \frac{7x^3+3}{\sqrt{4-x^2}} dx$

SOLUCIONES:

• Integrales inmediatas

1. $-\frac{1}{\sqrt{x^2+3}} + C$
2. $\frac{2}{3}x^{3/2} + 2x^{1/2} + C$
3. $\frac{1}{2}\operatorname{arcsen}\left(\frac{2\operatorname{sen} x}{3}\right) + C$
4. $\operatorname{sen}(\ln x) + C$
5. $\ln|\operatorname{sen} x| + C$
6. $\frac{-1}{3}\cos(3x+2) + C$
7. $\frac{1}{18}(x^2+1)^9 + C$
8. $-\frac{1}{3}\sqrt{(9-x^2)^3} + C$
9. $-\frac{1}{2}e^{-x^2} + C$
10. $-\frac{1}{3}\cos x^3 + C$
11. $\ln|\ln x| + C$
12. $\frac{1}{12}\operatorname{sen}^6 2x + C$
13. $\operatorname{arctg} x^2 + C$
14. $\operatorname{arctg}(e^x) + C$
15. $\operatorname{arctg}(\ln x) + C$
16. $\operatorname{arctg}\ x + \ln(1+x^2) + C$

• Integrales por partes

17. $u = \ln x$, $\frac{1}{3}x^3 \ln x - \frac{x^3}{9} + C$
18. $-\cos x \ln(\cos x) + \cos x + C$
19. $\cos x + x\operatorname{sen}\ x + C$
20. $x(\ln x - 1) + C$
21. $x(\ln x)^2 - 2x\ln x + 2x + C$
22. $e^{x^2}\left(\frac{x^2-1}{2}\right) + C$
23. $-\frac{\ln x}{2x^2} - \frac{1}{4x^2} + C$
24. $-\frac{1}{4}x\cos 2x + \frac{1}{8}\operatorname{sen} 2x + C$
25. $\frac{1}{3}x^3 \operatorname{arctg} x - \frac{1}{6}x^2 + \frac{1}{6}\ln\left(x^2+1\right) + C$
26. $-\frac{3}{16}\ln(4x^2+1) + (x^3+x)\operatorname{arctg}\ (2x) - \frac{x^2}{4} + C$
27. $\frac{1}{3}\left[(x^3+3)\ln\left|x^3+3\right| - x^3 - 3\right] + C$
28. $e^{2x}\left(\frac{2}{29}\cos 5x + \frac{5}{29}\operatorname{sen}\ 5x\right) + C$

• Integral de una función racional.

29. $-\dfrac{1}{x}-\dfrac{1}{x-1}-2\ln|x-1|+2\ln|x|+C$

30. $\dfrac{1}{3}\ln|x-1|-\dfrac{1}{6}\ln(x^2+x+1)-\dfrac{1}{\sqrt{3}}\text{arctg}\left(\dfrac{2x+1}{\sqrt{3}}\right)+C$

31. $-x+2\ln|1-x|+C$

32. Hacer la sustitución $t=e^x$. $\ln(e^x-2)-\dfrac{3}{e^x-2}+C$

33. $\ln\left|\dfrac{(x-1)}{(x^2+1)^{1/2}}\right|+\dfrac{1}{2}\dfrac{1}{x-1}-\dfrac{1}{2}\text{arctg}x+C$

34. $x-\ln|x|-\dfrac{1}{2}\ln|x+1|+\dfrac{3}{2}\ln|x-1|+C$

35. $\left|\dfrac{x}{x+1}\right|+\dfrac{1}{x+1}+C$

36. $-\dfrac{1}{x}+\ln\left|\dfrac{(x-2)^{2/3}(x+1)^{4/3}}{x}\right|+C$

37. $x-\ln|x-3|+\ln|x-\dfrac{1}{2}|+C$

38. $x+\dfrac{3}{x}-\ln|x|+\dfrac{1}{2}\ln(x^2+2)+C$

39. $\dfrac{1}{4}\ln|x-1|-\dfrac{1}{4}\ln|x+1|-\dfrac{1}{2}\text{arctg }x+C$

40. $\dfrac{x^3}{3}-x^2+3x-4\ln|x+1|-\dfrac{1}{x+1}+C$

41. $-\dfrac{4}{x-1}+\dfrac{1}{2}\ln\left|\dfrac{x+1}{x-1}\right|+C$

42. $\dfrac{2}{3}\ln|x-1|+\dfrac{1}{6}\ln\left|x^2+2\right|+\dfrac{\sqrt{2}}{6}\text{arctg }\left(\dfrac{x}{6}\right)+C$

43. $\dfrac{3}{16}\ln|x-1|-\dfrac{3}{16}\ln|x-5|+\dfrac{1}{4}\dfrac{-3x+7}{x^2-6x+5}+C$

• Integral trigonométrica.

44. $-\dfrac{1}{10}\cos(5x)-\dfrac{1}{26}\cos(13x)+C$

45. $-\dfrac{1}{4}\text{sen}(2x-8)+\dfrac{x}{2}+C$

46. $\dfrac{1}{12}\text{sen}(6x-10)+\dfrac{x}{2}+C$

47. $-\dfrac{1}{26}\text{sen}(13x-1)+\dfrac{1}{6}\text{sen}(3x+7)+C$

48. $\dfrac{1}{24}\text{sen}(12x+7)+\dfrac{1}{4}\text{sen}(2x-11)+C$

49. $\dfrac{-1}{4}\text{sen}2x$

• Integral racional en senos y cosenos

Impar en seno.

50. $\dfrac{\frac{1}{2}+\cos x}{(1+\cos x)^2}+C$

51. $-\dfrac{1}{2}\dfrac{\cos^5 x}{\text{sen}^2 x}-\dfrac{1}{2}\cos^3 x-\dfrac{3}{2}\cos x-\dfrac{3}{2}\ln|\text{cosec }x-\text{cotg }x|+C$

52. $-\cos x+\dfrac{1}{3}\cos^3 x+C$

53. $-\ln|\cos x|+\cos x+\ln|\text{cosec }x-\text{cotg }x|+C$

Impar en cosenos.

54. $-\dfrac{\text{sen}^2 x}{2}+\ln(1+\text{sen}^2 x)+C$

55. $-\ln|\cos x|+\dfrac{\cos^2 x}{2}+C$

56. $-\dfrac{3}{2}\dfrac{1}{\text{sen}^2 x}+C$

57. $\dfrac{1}{3}\text{sen}3x-\dfrac{1}{9}\text{sen}^3 3x+C$

58. $-\dfrac{5}{32}\ln|2-\text{sen }x|+\dfrac{5}{32}\ln|2+\text{sen }x|+\dfrac{3}{8}\dfrac{\text{sen }x}{\text{sen }^2x-4}+C$

Par en seno y coseno.

59. $\dfrac{1}{\sqrt{15}}\text{arctg}\left(\dfrac{\sqrt{3}}{\sqrt{5}}\text{tg}\,x\right)+C$

60. $\dfrac{x}{2}+\dfrac{1}{2}\ln|\text{sen}\,x+\cos x|+C$

61. $\dfrac{1}{2}\dfrac{1}{\cos^2 x}+\dfrac{\text{sen}x}{\cos x}+C$

62. $-x-2\,\text{cotg}\left(\dfrac{x}{2}\right)+C$

Integral racional en senos y cosenos. Cambio universal.

63. $\dfrac{2}{\sqrt{5}}\operatorname{arctg}\left(\sqrt{5}\operatorname{tg}\left(\dfrac{x}{2}\right)\right)+C$

64. $-x-\dfrac{8}{\sqrt{3}}\operatorname{arctg}\left(\dfrac{1}{\sqrt{3}}+\dfrac{2}{\sqrt{3}}\operatorname{tg}\left(\dfrac{x}{2}\right)\right)+C$

65. $-\dfrac{1}{\operatorname{tg}\left(\frac{x}{2}\right)}+2\ln\left|\operatorname{tg}\left(\tfrac{x}{2}\right)\right|-\ln\left|\operatorname{tg}\left(\tfrac{x}{2}\right)-1\right|-\ln\left|\operatorname{tg}\left(\tfrac{x}{2}\right)+1\right|+C$

66. $\dfrac{1}{4}\ln\left|\dfrac{\operatorname{tg}\left(\frac{x}{2}\right)+2}{\operatorname{tg}\left(\frac{x}{2}\right)+-2}\right|+C$

• Integral mediante cambios trigonométricos.

67. $-\dfrac{1}{2}\operatorname{arcsen}\left(\dfrac{1}{3}x\right)-\dfrac{1}{2}x\sqrt{9-x^2}+C$

68. $3\operatorname{arcsen}\left(\dfrac{x}{2}\right)+2\left(\dfrac{-7}{6}x^2-\dfrac{28}{3}\right)\sqrt{4-x^2}+C$