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Predicting the yield curve using forecast combinations

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ABSTRACT

An examination of the statistical accuracy and economic value of modeling and forecasting the term structure of interest rates using forecast combinations is considered. Five alternative methods to combine point forecasts from several univariate and multivariate autoregressive specifications including dynamic factor models, equilibrium term structure models, and forward rate regression models are used. Moreover, a detailed performance evaluation based not only on statistical measures of forecast accuracy, but also on Sharpe ratios of fixed income portfolios is conducted. An empirical application based on a large panel of Brazilian interest rate future contracts with different maturities shows that combined forecasts consistently outperform individual models in several instances, specially when economic criteria are taken into account.

1. Introduction

An interesting question still little-explored in the literature is the implementation and performance evaluation of forecast combinations for the yield curve. Existing evidence has focused on the performance evaluation of individual forecast models. However, combined forecasts have been extensively and successfully applied in many areas; see Granger (1989), Clemen (1989), Granger and Jeon (2004), Timmermann (2006) and Wallis (2011) for reviews. Therefore, a natural question is: can forecast combinations deliver better forecasts for the yield curve? If so, are these forecasts economically relevant?

The motivation to combine forecasts comes from an important result from the methodological literature on forecasting, which shows that a linear combination of two or more forecasts may yield more accurate predictions than using only a single forecast (Granger, 1989; Newbold and Harvey, 2002; Aiolfi and Timmermann, 2006). Moreover, adaptive strategies for combining forecasts might also mitigate structural breaks, model uncertainty and model misspecification, and thus lead to more accurate forecasts (Newbold and Harvey, 2002; Pesaran and Timmermann, 2007). In particular, there is recent evidence that combining forecast of nested models can significantly improve forecasting precision upon forecasts obtained from single model specifications (Clark and McCracken, 2009).

Hendry and Clements (2004) point out a number of potential explanations for the good performance of combined forecasts vis-a-vis individual forecast models. First, if two models provide partial, but incompletely overlapping explanations, then some combination of the two might do better than either alone. Specifically, if two forecasts were differentially biased (one upwards, one downwards), then combining could be an improvement over either. Similarly, if all explanatory variables

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were orthogonal, and models contained subsets of these, an appropriately weighted combination could better reflect all the information. Second, averaging forecasts reduces variance to the extent that separate sources of information are used. Third, forecast combination can also alleviate the problem of model uncertainty. Finally, Granger and Jeon (2004) point out that the benefits of pooling forecasts can be related to the portfolio selection problem, since a portfolio of assets is usually better than investing in a single asset.

A large number of approaches to combine prediction models exist in the literature, ranging from simple averaging schemes to more sophisticated adaptive combinations. Various studies have demonstrated that simple averaging of a multitude of forecasts works well in relation to more sophisticated weighting schemes (Newbold and Harvey, 2002; Clark and McCracken, 2009). Timmermann (2006) argues that equal weights are optimal in situations with an arbitrary number of forecasts when the individual forecast errors have the same variance and identical pairwise correlations. More recently, Geweke and Amisano (2011) study the properties of weighted linear combinations of prediction models ("linear pools") by using a predictive scoring rule. Among other results, the authors find that model weights depend on the set of models analyzed, and models with positive weight in a larger pool may have zero weight if some other models are deleted from that pool, thus indicating the importance of model complementarities for forecasts.

The ability to forecast the behavior of the term structure of interest rates is important for macroeconomists, financial economists and fixed income managers. More specifically, bond portfolio optimization, pricing of financial assets and their derivatives, as well as risk management, rely heavily on interest rate forecasts. Moreover, these forecasts are widely used by financial institution, regulators, and investors to develop macroeconomic scenarios. However, until the seminal work of Diebold and Li (2006), little attention was given to yield curve forecasting, and previous theoretical developments were mainly focused on in-sample fit (see, for example, de Jong, 2000; Dai and Singleton, 2000). Diebold and Li (2006) have taken an out-of-sample perspective based on a dynamic version of the static approach proposed by Nelson and Siegel (1987) and have shown that this model produces accurate forecasts. The seminal work of Diebold and Li (2006) on yield curve forecasting has been followed by a large number of studies that investigate the performance of alternative forecasting models; see, for instance, Diebold and Rudebusch (2013) for a text book review of these improvements.

The paper also provides a comprehensive evaluation of the performance of forecast combinations vis-a-vis individual models in terms of both statistical accuracy and economic relevance, since the final goal of interest rate forecasts is to improve economic and financial decision making. The existing literature, however, has focused mainly on statistical measures of forecast accuracy, taking forecasts out of the context in which they are ultimately used; see, for instance, de Pooter et al. (2010), Diebold and Rudebusch (2013), and Caldeira et al. (2010), amongst others. As pointed out by Granger and Pesaran (2000), Pesaran and Skouras (2002) and Granger and Machina (2006), when forecasts are used in decision making, it is important to consider the decision process in the *ex post* evaluation of these forecasts, maintaining the interaction between the forecasting model and the decision making task. Therefore, it is of main concern to both academics and market practitioners the extent to which forecasts of interest rates are useful to support economic decisions.

More specifically, we propose to evaluate the accuracy and economic relevance of yield curve forecasts based on mean–variance optimal portfolios as introduced by Markowitz (1952). As a first step, we follow Christoffersen and Diebold (1998), Hördahl et al. (2006), and de Pooter et al. (2010) and carry out a traditional evaluation based on statistical measures of forecasting performance such as root mean squared forecast error (RMSFE) and trace root mean squared forecast error (TRMSFE). In a second step, different forecasts are used to construct mean–variance portfolios, which are then compared based on their Sharpe ratios. In order to solve the mean–variance optimization problem, we use the different forecasts to derive estimates of expected bond returns, and use these as inputs to obtain mean–variance portfolios for alternative levels of risk tolerance.

We obtain forecasts of the yield curve based on a broad set of alternative models usually considered in the literature. First, we consider a class of purely statistical models such as the random walk, the univariate autoregressive model, the vector autoregressive and the Bayesian vector autoregressive model. Second, we consider the class of yield curve factor models such as the dynamic versions of the Nelson–Siegel and Svensson specifications. Third, we implement three alternative specifications that exploit the predictive power of the forward rates such as the slope regression suggested by Diebold and Li (2006), and the forward rate regressions of Fama and Bliss (1987) and Cochrane and Piazzesi (2005). Finally, we consider the three-factor equilibrium term structure model proposed by Cox et al. (1985b). More importantly, we combine models from different classes via five alternative combination schemes. First, we consider the case of equally weighted forecasts. Second, we consider the thick modeling approach proposed by Granger and Jeon (2004) which consists of selecting the best forecasting models in the sub-sample period for model evaluation, according to the root mean square error (RMSE) criterion. In this case, we use the selection process of Granger and Jeon and subsequently compute weights by means of OLS regressions. Third, we use the rank-weighted combinations suggested by Aiolfi and Timmermann (2006). Finally, we follow Morales-Arias and Moura (2013) and implement the thick modeling approach with MSE-frequency and RMSE-frequency weights, which consists of selecting models by means of the thick-modeling approach and assigning to each individual forecast a weight equal to a model's empirical frequency of minimizing the MSE and RMSE, respectively, over realized forecasts.

Our paper adds to the existing literature on yield curve forecasting in at least two aspects. First, de Pooter et al. (2010) consider the problem of forecast combination for the yield curve using only equal weights and MSFE-based weighting, and focus on the importance of macro variables in forecasting the yield curve. We, on the other hand, consider a richer set of forecast combination schemes. Second, Carriero et al. (2012) also consider the problem of economic evaluation of the term structure forecasts. Our paper, however, takes this evaluation criterion and extends to the case of forecast combination involving several individual models. Moreover, our paper differs in several important aspects with respect to the few previous studies that address the question regarding the economic evaluation of yield curve forecasts. First, the data set used in this paper carries interesting characteristics, since it refers to a different marketplace, is sampled on a daily basis, and consists of high-liquidity fixed income future contracts that resemble zero-coupon bonds. Second, Carriero et al. (2012) consider trading strategies based only on one-step-ahead forecasts, whereas in our paper we consider the Sharpe ratios from optimal portfolios based on multi-step-ahead forecasts including 1-week-, 1-month-, 2-month-, and 3-month-ahead forecasts. Third, Carriero et al. (2012) obtain optimal mean–variance portfolio weights considering a single value for the risk aversion coefficient, whereas we provide results considering alternative levels of risk tolerance. Fourth, and most importantly, neither Carriero et al. (2012) nor Xiang and Zhu (forthcoming) provide results regarding the statistical differences in the Sharpe ratios of the proposed approach with respect to the benchmark. We, on the other hand, employ a robust test for the Sharpe ratio based on the bootstrap procedure of Ledoit and Wolf (2008), which allows us to formally compare models in terms of an economic criterion. Fifth, none of the existing references focus on the performance of forecast combinations. In this paper, however, we provide a comprehensive evaluation by implementing five alternative forecast combination schemes, and check their performance in terms of statistical accuracy and economic relevance.

Our empirical application is based on a large data set of constant-maturity future contracts of the Brazilian Interbank Deposit (DI-futuro) which is equivalent to a zero-coupon bond and is highly liquid (341 million contracts worth US$17.5 billion traded in 2012). The market for DI-futuro contracts is one of the most liquid interest rate markets in the world. Many banks, insurance companies, and investors use DI-futuro contracts as investment and hedging instruments. The data set considered in the paper contains daily observations of DI-futuro contracts traded on the Brazilian Mercantile and Futures Exchange (BM&FBovespa) with fixed maturities of 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 42 and 48 months. To obtain 1-week-, 1-month-, 2-month-, and 3-month-ahead forecasts for each of the maturities available in the data set, we use individual forecasting models as well as alternative forecast combination schemes. The results show that combined forecasts consistently outperform individual models in terms of lower forecasting errors, and that this outperformance is more evident for shorter forecasting horizons. Moreover, we also observe that the Sharpe ratios of the mean–variance portfolios built upon combined forecasts are substantially and statistically higher than those obtained with the benchmark models. This result is also robust to the level of risk tolerance and to the portfolio re-balancing frequency. Finally, our results also suggest that, as long as yield curve forecasts are concerned, the differences in forecasting performance among candidate models based on statistical criteria are also economic meaningful as they generate optimal fixed income portfolios with improved risk-adjusted returns.

The paper is organized as follows. In Section 2 we describe the methods used to obtain forecasts for the yield rates, including both individual forecast models as well as forecast combinations. Next, in Section 3 we discuss the methodology used to evaluate forecast, both in terms of statistical criteria and in terms of economic relevance. Section 4 brings an empirical application. Section 5 concludes.

2. Methods used to forecast the yield curve

In this section, we describe the methods used to forecast the yield curve. These methods are based on individual forecast models as well as alternative forecast combinations.

2.1. Random walk model

The main benchmark model adopted in the paper is the random walk (RW), whose $t+h$-step-ahead forecasts for an yield of maturity τ are given by:

$$y_{t+h}(\tau) = y_t(\tau) + \varepsilon_t(\tau), \quad \varepsilon_t(\tau) \sim \mathcal{N}(0, \sigma^2(\tau)). \tag{1}$$

In the RW, a h-step-ahead forecast, denoted $\widehat{y}_{t+h}(\tau)$, is simply equal to the most recently observed value $y_t(\tau)$. This model is a good benchmark for judging the relative prediction power of other models, since yields are usually nonstationary or nearly nonstationary. Thus, in practice, it is difficult to beat the RW in terms of out-of-sample forecasting accuracy. Many other studies that consider interest rate forecasting have shown that consistently outperforming the random walk is difficult (see, for example, Duffee, 2002; Ang and Piazzesi, 2003; Diebold and Li, 2006; Hördahl et al., 2006; Möench, 2008).

2.2. Univariate autoregressive model

It is possible to generalize the RW model and forecast the maturity-τ yield using a first-order univariate autoregressive model (AR) that is estimated on the available data for that maturity:

$$y_t(\tau) = \alpha + \beta y_{t-1}(\tau) + \varepsilon_t. \tag{2}$$

The 1-step ahead forecast is produced as $\hat{y}_{t+1}(\tau) = \hat{\alpha} + \hat{\beta} y_{t-1}(\tau)$. The forecasts for h-step ahead horizon are obtained as:

$$\hat{y}_{t+h|t}(\tau) = (1 + \hat{\beta} + \hat{\beta}^2 + \cdots + \hat{\beta}^{h-1})\hat{\alpha} + \hat{\beta}^h y_t(\tau).$$

2.3. Vector autoregressive model

The fact that the yield curve can be considered a vector process composed of yields of different maturities indicates that the cross-section information might be important in understanding yield curve movements. However, neither the RW nor the AR models exploit this information to build their forecasts. Thus, a first-order unrestricted vector autoregressive model (VAR) for yield levels is a natural extension of the univariate AR model. The estimated model is:

$$y_t = A + By_{t-1} + \varepsilon_t, \tag{3}$$

where $y_t = (y_t(\tau_1), y_t(\tau_2), \ldots, y_t(\tau_N))'$. The 1-step ahead forecast is produced as $\hat{y}_t = \hat{A} + \hat{B}y_{t-1}$, while the h-step ahead forecasts are obtained as:

$$\hat{y}_{t+h|t} = (I + \hat{B} + \hat{B}^2 + \cdots + \hat{B}^{h-1})\hat{A} + \hat{B}^h y_t. \tag{4}$$

As argued by de Pooter et al. (2010), a well-known drawback of using an unrestricted VAR model for yields is the large number of parameters that needs to be estimated. Since our data set contains thirteen different maturities, a substantial number of parameters needs to be estimated.

2.4. Bayesian vector autoregressive model

To alleviate the overparameterization of VARs without sacrificing completely the cross-section information as univariate models do, we follow Carriero et al. (2012) and use Bayesian VARs with a Minnesota prior distribution (see Doan et al., 1986; Litterman, 1986). Considering the vector autoregressive model in (3), this a priori belief implies that the expected value of matrix B is $\mathbb{E}[B] = \delta \times I$. We also need to determine how certain we are about our priori understanding, i.e. we need to set a variance around the prior mean. It is assumed that B is conditionally normal, with first and second moments given by:

$$\mathbb{E}\left[B^{ij}\right] = \begin{cases} \delta_i & \text{if } i = j \\ 0 & \text{if } i \neq j, \end{cases} \qquad \mathbb{V}\text{ar}\left[B^{ij}\right] = \theta \frac{\sigma_i^2}{\sigma_j^2}, \tag{5}$$

where $\left(B^{ij}\right)$ denotes the element in position (i, j) in matrix B, and where the covariance among the coefficients in B are zero. Banbura et al. (2010) suggest to set $\delta_i = 1$ for all i, reflecting the belief that all the variables are characterized by high persistence. We believe it is reasonable to think a priori that each of the yields obeys a univariate AR with high persistence, or equivalently, $\mathbb{E}[B] = 0.99 \times I$. The hyperparameter θ controls the tightness of the prior, and the factor σ_i^2/σ_j^2 is a scaling parameter which accounts for the different scale and variability of different maturities. Following Carriero et al. (2012), we set the scale parameters σ_i^2 equal to the variance of the residuals from a univariate autoregressive model for the variables.

The prior specification is completed by assuming a diffuse normal prior on A and an inverted Wishart prior for the matrix of disturbances $\Sigma \sim iW(\nu_0, S_0)$, where ν_0 and S_0 are the prior scale and shape parameters, and are set such that the prior expectation of Σ is equal to a fixed diagonal residual variance $\Sigma = \text{diag}\left(\sigma_1^2, \ldots, \sigma_N^2\right)$.

We can re-write the VAR more compactly as:

$$Y = X\Psi + E, \tag{6}$$

where $Y = [y_2, \ldots, y_T]$ is a $(T-1) \times N$ matrix containing all the data points in y_t, $X = [1, Y_{-1}]$ is a $(T-1) \times M$ matrix containing a vector of ones (1) in the first columns and one lag of Y in the remaining columns, $\Psi = [A, B]'$ is an $M \times N$ matrix, and $E = [\varepsilon_1, \ldots, \varepsilon_T]'$ is a $(T-1) \times N$ matrix of disturbances. As only one lag is considered we have $M = N + 1$. The Normal-Inverted Wishart prior has the form:

$$\text{vec}(\Psi)\,|\Sigma \sim N(\text{vec}(\Psi_0), \Sigma \otimes \Omega_0), \quad \text{and} \quad \Sigma \sim iW(\nu_0, S_0), \tag{7}$$

where the prior parameters Ψ_0 Ω_0, S_0 and ν_0 are chosen so that prior expectations and variances of Ψ coincide with those implied by Eq. (5), $\mathbb{E}[\Psi] = \Psi_0$ and $\text{var}[\Psi] = \Sigma \otimes \Omega_0$, where Σ is the variance matrix of the disturbances and the elements of Ω_0 are given by var $\left[B^{ij}\right]$ in (5). The conditional posterior distribution of this model is also Normal-Inverted Wishart:

$$\text{vec}(\Psi)\,|\Sigma, Y \sim N\left(\overline{\Psi}, \Sigma \otimes \overline{\Omega}\right), \qquad \Sigma|Y \sim \text{IW}(\overline{\nu}, \overline{S}), \tag{8}$$

where the bar denotes parameters from the posterior distribution. Defining $\hat{\Psi}$ as the OLS estimates, we have that $\overline{\Psi} = \overline{\Omega}\left(\Omega_0^{-1}\Psi_0 + X'Y\right)$, $\overline{\Omega} = \left(\Omega_0^{-1} + X'X\right)^{-1}$, $\overline{\nu} = \nu_0 + T$ and $\overline{S} = S_0 + Y'Y + \Psi_0'\Omega_0^{-1}\Psi_0 - \overline{\Psi}\,\overline{\Omega}^{-1}\overline{\Psi}$. Since we are interested only in linear functions of the posterior moments of Ψ, Appendix A of Carriero et al. (2012) shows that it is possible to integrate Σ out of (8), yielding a matrix-variate t marginal posterior distribution for Ψ:

$$\Psi|Y \sim Mt(\overline{\Psi}, \overline{\Omega}^{-1}, \overline{S}, \overline{nu}), \tag{9}$$

whose expected value is given by $\overline{\Psi} = \overline{\Omega}\left(\Omega_0^{-1}\Psi_0 + X'Y\right)$. To make explicit the link between $\overline{\Psi}$ and the OLS estimates $\widehat{\Psi}$, we can use the normal equations to get:

$$\overline{\Psi} = \overline{\Omega}\left(\Omega_0^{-1}\Psi_0 + X'X\widehat{\Psi}\right), \tag{10}$$

which shows that $\overline{\Psi}$ is a weighted average of the prior mean and of the OLS estimates, where weights are inversely proportional to their respective variances. Carriero et al. (2012) provide derivations of these posterior moments in their Appendix A and Bauwens et al. (1999) present a text book treatment of Bayesian VARs in their Section 9.2 and deal explicitly with Minnesota priors in their Section 9.2.3.

After computing $\overline{\Psi}$, we can recover $\hat{A}$ and $\hat{B}$, which allow us to obtain h-step ahead forecasts by iteration:

$$\hat{Y}_{t+h} = \hat{A} + \hat{B} \cdot \hat{Y}_{t+h-1}, \tag{11}$$

where $\hat{Y}_{t+h-1}$ is computed as in (4).

2.5. Models that exploit the predictive power of forward rates

We denote prices and log prices of a discount bond that promises to pay \$1 at time $t + \tau$ by $P_t(\tau) = \exp(-\tau \cdot y_t(\tau))$ and $p_t(\tau)$, respectively, and the yield of a τ-year bond by

$$y_t(\tau) = -\frac{1}{\tau} \cdot p_t(\tau). \tag{12}$$

Once we have the yield curve, we can easily use it to derive the forward rates (see Piazzesi and Schneider, 2009). The forward rate contracted at time t for loans from time $t + h$ to time $t + h + \tau$ can be expressed as a linear function of yields with maturities $\tau + h$ and h:

$$\begin{aligned} f_t^h(\tau) &= \frac{\tau + h}{\tau} \cdot y_t(\tau + h) - \frac{h}{\tau} \cdot y_t(h) \\ &= y_t(h) + \frac{\tau + h}{\tau} \cdot (y_t(\tau + h) - y_t(h)). \end{aligned} \tag{13}$$

2.5.1. Fama–Bliss (FB) forward rate regression

The seminal work of Fama and Bliss (1987) examined the forecasting power in forward rates on the same maturity excess returns and provided an evidence against the expectations hypothesis in long-term bonds. FB estimate a linear regression of the change in the τ-year bond yield on the forward-spot spread,

$$\hat{y}_{t+h}(\tau) - y_t(\tau) = \hat{\alpha}_h(\tau) + \hat{\beta}_h(\tau) \left(f_t^h(\tau) - y_t(\tau) \right), \tag{14}$$

where $f_t^h(\tau)$ is the forward rate contracted at time t for loans from time $t + h$ to time $t + h + \tau$. Hence, the h-step ahead forecasts $\hat{y}_{t+h}(\tau)$ are produced by projecting the changes in yields from time t to time $t + h$ on the forward-spot at time t.

2.5.2. Cochrane–Piazzesi (CP) forward curve regression

Cochrane and Piazzesi (2005) find that the term structure of forward rates explains between 30% and 35% of the variation of bond excess returns over the same bond maturity spectrum investigated by FB. Although the natural extension of this approach would imply putting all available forward rates on the right side of the regression, we use only four forward rates ($y_t(12), f_t^{12}(12), f_t^{24}(12)$, and $f_t^{36}(12)$) to avoid problems of multicollinearity, as suggested by Carriero et al. (2012). Then, we estimate a modified version of Eq. (14), and forecasts are computed as follows:

$$\hat{y}_{t+h}(\tau) - y_t(\tau) = \hat{\alpha}^h + \hat{\beta}_1^h y_t(12) + \hat{\beta}_2^h f_t^{12}(12) + \hat{\beta}_3^h f_t^{24}(12) + \hat{\beta}_3^h f_t^{36}(12). \tag{15}$$

2.5.3. Slope regression

This model was used by Diebold and Li (2006) and is based on the historic spread between an yield on a longer maturity bond and the yield on the shortest maturity bond. This difference is used to forecast the change in the yield of the longer maturity bond. Specifically,

$$\hat{y}_{t+h}(\tau) - y_t(\tau) = \hat{\alpha}_h(\tau) + \hat{\beta}_h(\tau) (y_t(\tau) - y_t(1)), \tag{16}$$

where $y_t(1)$ represents the 3 months yield, the shortest in our data set. The idea that the slope of the yield curve reflects future movements in yields is, of course, not a new one and has its roots firmly in the properties of the Expectations Hypothesis, wherein an upward sloping yield curve implies a future rise in yields. This slope regression formulation is also closely related to, and uses almost the same information, as the forward rate regressions of Fama and Bliss (1987), their generalization by Cochrane and Piazzesi (2005), and the spread regressions of Campbell and Shiller (1991), which represent alternative tests of the Expectations Hypothesis.

2.6. The 3-factor Cox, Ingersoll and Ross Model (CIR) equilibrium model of the term structure

Cox et al. (1985b) developed one of the first general equilibrium theories of the term structure of interest rates. Out of that theory came a model for pricing zero coupon bonds and derivatives. The instantaneous nominal interest rate is assumed to be the sum of $K = 3$ state variables,

$$r = \sum_{j=1}^{K} y_j,$$

and the state variable are assumed to be independent and generated as square root diffusion processes:

$$dy_j = \kappa_j \left(\theta_j - y_j\right) + \sigma_j \sqrt{y_j} dz_j, \quad \text{for } j = 1, \ldots, K. \tag{17}$$

The solution for the nominal price at time t of a nominally risk-free bond that pays \$1 at time s is determined as follows:

$$P_t(\tau) = A_1(\tau) \ldots A_K(\tau) \cdot \exp\left\{-B_1(\tau) y_{1t} - \cdots - B_K(\tau) y_{tK}\right\}, \tag{18}$$

where $A_j(\tau)$ and $B_j(\tau)$ have the form given in CIR:

$$A_j(\tau) = \left[\frac{2\gamma \mathrm{e}^{\frac{1}{2}(\kappa_j+\lambda_j-\gamma_j)\tau}}{2\gamma \mathrm{e}^{-\gamma_j\tau} + \left(\kappa_j + \lambda_j + \gamma_j\right)\left(1 - \mathrm{e}^{-\gamma_j\tau}\right)}\right]^{\frac{2\kappa_j\theta_j}{\sigma_j^2}}, \quad \text{and} \tag{19}$$

$$B_j(\tau) = \frac{2\left(1 - \mathrm{e}^{-\gamma_j\tau}\right)}{2\gamma \mathrm{e}^{-\gamma_j\tau} + \left(\kappa_j + \lambda_j + \gamma_j\right)\left(1 - \mathrm{e}^{-\gamma_j\tau}\right)}, \tag{20}$$

and $\gamma_j = \sqrt{(\kappa_j + \lambda_j)^2 + 2\sigma_j^2}$. Each state variable has a risk premium, $\lambda_j y_j$, and each λ_j is treated as a fixed parameter. The continuously compounded yield for a discount bond is defined as follows:

$$R_t(\tau) = -\frac{\log P_t(\tau)}{\tau}, \tag{21}$$

which is a linear function of the unobservable state variables. Given a set of yields on K discount bonds, one can conceptually invert to infer values for the state variables. This inversion of bond rates to infer values for the state variables has been used in Chen and Scott (1993), Duffie and Kan (1996), and Pearson and Sun (1994). This model can be derived by applying arbitrage methods or by using the utility based model as in Cox et al. (1985a). The risk premia are determined endogenously in a utility based model by the co-variability of the state variables with marginal utility of wealth. The form for the risk premium used here is consistent with a log utility model. To estimate the model we use the modified version of the Kalman filter developed by Chen and Scott (2003).

2.7. Nelson–Siegel model class

Nelson and Siegel (1987) have shown that the term structure can be surprisingly well fitted at a particular point in time by a linear combination of three smooth functions. The Nelson–Siegel model is given by:

$$y(\tau) = \beta_1 + \beta_2 \left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau}\right) + \beta_3 \left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right) + \epsilon_\tau, \tag{22}$$

where β_1 can be interpreted as the level of the yield curve, β_2 as its slope, and β_3 as its curvature. The parameter λ determines the exponential decay of β_2 and of β_3.

Svensson (1994) proposed an extension of the original Nelson–Siegel model by adding an extra smooth function to improve the flexibility and fit of the model. The model proposed by Svensson (1994) is

$$y(\tau) = \beta_1 + \beta_2 \left(\frac{1 - e^{-\lambda_1\tau}}{\lambda_1\tau}\right) + \beta_3 \left(\frac{1 - e^{-\lambda_1\tau}}{\lambda_1\tau} - e^{-\lambda_1\tau}\right) + \beta_4 \left(\frac{1 - e^{-\lambda_2\tau}}{\lambda_2\tau} - e^{-\lambda_2\tau}\right) + \epsilon_\tau. \tag{23}$$

Diebold and Li (2006) have introduced dynamics into the original Nelson–Siegel model, and showed that its dynamic version has good forecasting power. The dynamic Nelson–Siegel model (henceforth DNS) is given by:

$$y_t(\tau) = \beta_{1t} + \beta_{2t} \left(\frac{1 - e^{-\lambda\tau}}{\lambda_t\tau}\right) + \beta_{3t} \left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right) + \epsilon_t(\tau), \tag{24}$$

where the vector of time-varying coefficients β_t follows a VAR process.

Similarly to what Diebold and Li (2006) have done for the Nelson–Siegel model, a dynamic version of the Svensson model (hereafter DSV) can be written as:

$$y_t(\tau) = \beta_{1t} + \beta_{2t}\left(\frac{1-e^{-\lambda_1\tau}}{\lambda_1\tau}\right) + \beta_{3t}\left(\frac{1-e^{-\lambda_1\tau}}{\lambda_1\tau} - e^{-\lambda_1\tau}\right) + \beta_{4t}\left(\frac{1-e^{-\lambda_2\tau}}{\lambda_2\tau} - e^{-\lambda_2\tau}\right) + \epsilon_t(\tau). \tag{25}$$

The fourth factor in the dynamic version of the Svensson model can be interpreted as a second curvature. Svensson (1994) argues that the additional factor provides a better in-sample fit, especially for a richer structure of yields, and therefore provides better estimations of forward rates.

The dynamic versions of both the Nelson–Siegel and Svensson models can be interpreted as dynamic factor models (see, for example, Diebold et al., 2006). More specifically, consider an $N \times T$ matrix of observable yields. The observation at time t is denoted by $y_t = (y_{1t}, \ldots, y_{Nt})'$, for $t = 1, \ldots, T$, and y_{it} is ith variable in the vector y_t at time t. The dynamic factor models considered are of the form

$$y_t = \Lambda(\lambda)f_t + \varepsilon_t, \quad \varepsilon_t \sim \text{NID}\,(0, \Sigma)\,,\ t = 1, \ldots, T, \tag{26}$$

where $\Lambda(\lambda)$ is the $N \times K$ matrix of factor loadings that depends on the decaying parameter λ, f_t is a K-dimensional vector containing the coefficients $\beta_{1t}, \ldots, \beta_{Kt}$ for $K = \{3, 4\}$, ε_t is the $N \times 1$ vector of disturbances and Σ is an $N \times N$ diagonal covariance matrix of the disturbances. The dynamic factors f_t are modeled by the following stochastic process:

$$f_t = A + Bf_{t-1} + \eta_t, \quad \eta_t \sim \text{NID}\,(0, \Omega)\,,\ t = 1, \ldots, T, \tag{27}$$

where A is a $K \times 1$ vector of constants, B is the $K \times K$ transition matrix, and Ω is the conditional covariance matrix of disturbance vector η_t, which are independent of the residuals ε_t $\forall t$. Note that Eqs. (26) and (27) characterize a linear and Gaussian state space model and the Kalman filter can be used to obtain the likelihood function via the prediction error decomposition. Here we follow Jungbacker and Koopman (2008), who developed a simple transformation that generates significant computational gains when the number of factors is smaller than the number of observed series (see Table 1 in Jungbacker and Koopman, 2008, for an example of possible computational gains).

2.8. Combined forecasts

Assuming we are combining forecasts from $\mathcal{M}$ different forecast models, a combined forecast for a h-month horizon for the yield with maturity τ is given by

$$\hat{y}_{t+h|t}(\tau) = \sum_{m=1}^{\mathcal{M}} w_{t+h|t,m}(\tau)\hat{y}_{t+h|t,m}(\tau),$$

where $w_{t+h|t,m}(\tau)$ denotes the weight assigned to the time-t forecast from the mth model, $\hat{y}_{t+h|t,m}(\tau)$. Most of the forecast combination schemes considered are adaptive, meaning that the forecasts included in $\mathcal{M}\ :\ \{\hat{y}_{t+h|t,m}(\tau)\}$ and/or corresponding weights mth are based on alternative selection criteria within a sub-sample of realized observations.

It is worth noting that since a forecaster would only have information available up to the forecast origin ω, the sub-sample for forecast selection and computation of weights must contain data on or before that period. Thus, we start by setting equal weights to all forecasts until the selection of forecasts and weighting schemes could be based on the evaluation of realized forecast errors. This procedure guarantees that we use only information available up to a particular period ω to set weights of forecasts for period $\omega + h$. The following 5 alternative combination strategies $\mathcal{M} = \{$FC-EW, FC-OLS, FC-RANK, FC-MSE, FC-RMSE$\} = \{1, 2, \ldots, 5\}$ are considered:

1. *Equally weighted forecasts (FC-EW)*: Various studies have demonstrated that simple averaging of a multitude of forecasts works well in relation to more sophisticated weighting schemes (Newbold and Harvey, 2002; Clark and McCracken, 2009). Therefore, the first forecast combination method we consider assigns equal weights to the forecasts from all individual models, i.e. $w_{t+h|t,m}(\tau) = 1/\mathcal{M}$ for $m = 1, \ldots, \mathcal{M}$. We denote the resulting combined forecast as Forecast Combination—Equally Weighted (FC-EW). As explained in Timmermann (2006), this approach is likely to work well if forecast errors from different models have similar variances and are highly correlated.
2. *Thick modeling approach with OLS weights (FC-OLS)*: A study by Granger and Jeon (2004) proposes the so-called thick modeling approach (TMA) which consists of selecting the z-percent of the best forecasting models in the sub-sample period for model evaluation, according to the root mean square error (RMSE) criterion. We use the selection process of Granger and Jeon and subsequently compute weights by means of OLS regressions along with the constraint that the weights are all positive and sum up to one. The z-percent of top forecasts selected is set to 30%, which means that we select the best 3 models out of the 10 available.
3. *Rank-weighted combinations (FC-RANK)*: The FC-RANK scheme, suggested by Aiolfi and Timmermann (2006), consists of first computing the RMSE of all models in the sub-sample period for evaluation. Defining $\text{RANK}^{-1}_{t+h|t,m}$ as the rank of the

mth model based on its historical RMSE performance up to time t for horizon h, the weight for the mth forecast is then calculated as:

$$\widehat{w}_{t+h|t,m}(\tau) = \mathrm{RANK}^{-1}_{t+h|t,m} / \sum_{m=1}^{\mathcal{M}} \mathrm{RANK}^{-1}_{t+h|t,m}.$$

4. *Thick modeling approach with MSE-Frequency weights (FC-MSE)*: This scheme consists of selecting models by means of the thick modeling approach and assigning to each mth forecast a weight equal to a model's empirical frequency of minimizing the squared forecast error over realized forecasts. The weight for model m is computed as:

$$\widehat{w}_{t+h|t,m}(\tau) = \frac{1/\mathrm{MSE}_{t+h|t,m}(\tau)}{\sum_{m=1}^{\mathcal{M}} 1/\mathrm{MSE}_{t+h|t,m}(\tau)}.$$

5. *Thick modeling approach with RMSE-weights (FC-RMSE)*: This scheme consists of selecting models by means of the thick modeling approach, then computing the RMSE of all selected models m and setting:

$$\widehat{w}_{t+h|t,m}(\tau) = \frac{1/\mathrm{RMSE}_{t+h|t,m}(\tau)}{\sum_{m=1}^{\mathcal{M}} 1/\mathrm{RMSE}_{t+h|t,m}(\tau)}.$$

3. Forecast evaluation

We describe in this section the methodology used to evaluate the yield curve forecasts obtained from the econometric specifications discussed in Section 2. We first describe the statistical-based evaluation and then the assessment of the economic value of the forecasts based on the Sharpe ratios of the fixed income portfolios.

3.1. Statistical performance measures

In order to evaluate out-of-sample forecasts, we compute popular error metrics. Given a sample of P out-of-sample forecasts for a h-period-ahead forecast horizon, we compute the root mean squared forecast error (RMSFE) for maturity τ and for model m as follows:

$$\mathrm{RMSFE}_m(\tau) = \sqrt{\frac{1}{P}\sum_{t=1}^{P}\left(\hat{y}_{t+h|t,m}(\tau) - y_{t+h}(\tau)\right)^2}, \tag{28}$$

where $y_{t+h}(\tau)$ is the yield for the maturity τ observed at time $t+h$, and $\hat{y}_{t+h|t,m}(\tau)$ is the corresponding forecasting made at time t.

Following Christoffersen and Diebold (1998), and Hördahl et al. (2006), we also summarize the forecasting performance of each model over the entire maturity spectrum by computing the trace root mean squared forecast error (TRMSFE). For each forecast horizon, we compute the trace of the covariance matrix of the forecast errors across all N maturities. Hence, a lower TRMSFE indicates more accurate forecasts. The TRMSFE can be computed as

$$\mathrm{TRMSFE}_m(\tau) = \sqrt{\frac{1}{N}\frac{1}{P}\sum_{i=1}^{N}\sum_{t=1}^{P}\left(\hat{y}_{t+h|t}(\tau) - y_{t+h}(\tau)\right)^2}. \tag{29}$$

The drawback of using RMSFE and TRMSFE is that these are single statistics summarizing individual forecasting errors over an entire sample. Although often used, they do not give any insight as to where in the sample a particular model makes its largest and smallest forecast errors. Therefore, we also graphically analyze the cumulative squared forecast errors (CSFE) proposed by Welch and Goyal (2008). These cumulative prediction errors series clearly depict when a model outperforms or underperforms a given benchmark and could motivate the use of adaptive forecast combination schemes. The CSFE is given by

$$\mathrm{CSFE}_{m,T}(\tau) = \sum_{t=1}^{T}\left[\left(\hat{y}_{t+h|t,\mathrm{benchmark}}(\tau) - y_{t+h}(\tau)\right)^2 - \left(\hat{y}_{t+h|t,m}(\tau) - y_{t+h}(\tau)\right)^2\right]. \tag{30}$$

In the case a model outperforms the benchmark, the $\mathrm{CSFE}_{m,T}$ will be an increasing series. If the benchmark produces more accurate forecasts, then $\mathrm{CSFE}_{m,T}$ will tend to be decreasing.

Finally, we use the Giacomini and White (2006) test to assess whether the forecasts of two competing models are statistically different. The Giacomini–White (GW) test is a test of conditional forecasting ability and is constructed under the assumption that forecasts are generated using a rolling data window. This is a test of equal forecasting accuracy that can handle forecasts based on both nested and non-nested models, regardless of the estimation procedures used in the construction of the forecasts. The test is based on the loss differential $d_{m,t} = \left(e_{rw,t}\right)^2 - \left(e_{m,t}\right)^2$, where e_{mt} is the forecast error of model m at time t. We assume that the loss function is quadratic but it can be replaced by other loss functions depending on the goal of the forecast. The null hypothesis of equal forecasting accuracy can be written as

$$\mathbb{H}_0 : E\left[d_{m,t+h}|\gamma_{m,t}\right] = 0, \tag{31}$$

where $\gamma_{m,t}$ is a $p \times 1$ vector of test functions or instruments and h is the forecast horizon. If a constant is used as instrument, the test can be interpreted as an unconditional test of equal forecasting accuracy. The GW test statistic $\mathrm{GW}_{m,t}$ can be computed as the Wald statistic:

$$\mathrm{GW}_{m,P} = P\left(P^{-1}\sum_{t=\omega+1}^{P-h}\gamma_{m,t}d_{m,t+h}\right)'\hat{\Omega}_P^{-1}\left(P^{-1}\sum_{t=\omega+1}^{P-h}\gamma_{m,t}d_{m,t+h}\right)\xrightarrow{d}\chi^2_{\dim(\gamma)}, \tag{32}$$

where $\hat{\Omega}_n$ is a consistent *HAC* estimator for the asymptotic variance of $\gamma_{m,t}d_{m,t+h}$, and $P = (T - \omega)$ the number of forecasts produced. Under the null hypothesis given in (31), the test statistic $\mathrm{GW}_{i,t}$ is asymptotically distributed as χ^2_p. By requiring that the forecasts be constructed using a small, finite, rolling window of observations, Giacomini and White (2006) are able to substantially weaken many of the most important assumptions needed for the results in West (1996), McCracken (2000) and McCracken (2007).

3.2. Assessing the economic value of forecasts

We follow Carriero et al. (2012) and Xiang and Zhu (forthcoming) and analyze how useful the different forecasting models are when used as a basis for optimal fixed income portfolio allocation. There are alternative answers to the question of what an optimal portfolio means. The approach adopted in this paper is the mean–variance method proposed by Markowitz (1952), which is one of the milestones of modern finance theory. In this framework, individuals choose their allocations in risky assets based on the trade-off between expected returns and risk.

It is worth noting that in the vast majority of the cases the computation of optimal mean–variance portfolio is restricted to the construction of equity portfolios. One explanation is that the relative stability and low historical volatility of fixed income securities ended up discouraging the use of sophisticated methods to exploit the risk-return trade-off in this asset class. However, this situation has been changing rapidly in recent years, even in markets where these assets have low default probability (see, for instance, Korn and Koziol, 2006). The recurrence of turbulent episodes in global markets usually brings high volatility to bond prices, which increase the importance of adopting portfolio selection approaches that take into account the risk-return trade-off in bond returns. Therefore, the problem of optimal fixed income portfolio selection becomes economically relevant, since fixed income securities play a fundamental role in the composition of diversified portfolios held by the vast majority of institutional investors.

We consider the case of an investor who has a h-period investment horizon and re-balances her portfolio also on a h-period basis. The formulation of the mean–variance optimization problem is given by

$$\begin{aligned}&\underset{w_t}{\text{minimize}}\;\; w_t\Sigma_r w_t - \frac{1}{\delta}w_t'\mu_{r_{t|t-h}}\\&\text{subject to } w_t'\iota = 1,\end{aligned} \tag{33}$$

where $\mu_{r_{t|t-h}}$ is the h-period-ahead vector of expected bond returns, Σ_r is the variance–covariance matrix of bond returns, w_t is the vector of portfolio weights at time t chosen at time $t-h$, ι is an $N \times 1$ vector of ones, and δ is the coefficient of risk aversion. To improve the robustness of our empirical study, we solve the mean–variance optimization problem considering alternative values for the risk aversion coefficient δ. In particular, we solve the optimization problem in (33) for $\delta = \{0.1, 0.5, 1.0, 2.0\}$. Finally, we focus on the case in which short-sales are restricted by adding to (33) a constraint to avoid negative weights, i.e. $w_t \geq 0$. Previous works show that adding such a restriction can substantially improve performance, especially reducing the turnover of the portfolio, see Jagannathan and Ma (2003), among others. In this case, the optimization problem in (33) is solved using numerical methods.

In order to perform fixed income portfolio optimization according to Markowitz's mean–variance framework, one needs estimates of the vector of expected bond returns, $\mu_{r_{t|t-h}}$, and of the covariance matrix of bond returns, Σ_r; see (33). However, the forecasting models discussed in Section 2 are designed to model only bond *yields*. Nevertheless, Caldeira et al. (2012) show that it is possible to compute expected bond returns based on yield curve forecasts in a straightforward way. Taking into account that the price of a bond at time t, $P_t(\tau)$, is the present value at time t of \$1 receivable τ periods ahead, the vector of bond prices P_t for all maturities is given by:

$$P_t = \exp\left(-\underline{\tau} \otimes y_t\right), \tag{34}$$

where $\otimes$ is the Hadamard (elementwise) multiplication and $\underline{\tau}$ is the vector of observed maturities. Using the log-return expression, we obtain

$$r_{t|t-h} = \log\left(\frac{P_t}{P_{t-h}}\right) = \log P_t - \log P_{t-h} = -\underline{\tau} \otimes (y_t - y_{t-h}). \tag{35}$$

Finally, taking expectation of (35) conditional on the information set available at time $t-h$, the vector of h-period-ahead expected bond returns, $\mu_{r_{t|t-h}}$, is given by

$$\mu_{r_{t|t-h}} = -\underline{\tau} \otimes y_{t|t-h} + \underline{\tau} \otimes y_{t-h}, \tag{36}$$

where $y_{t|t-h}$ denotes the vector of h-step-ahead forecast of yield of all observed maturities. We further assume that the model does not specify conditional variance dynamics, so that the conditional variance of $r_{t|t-h}$ simply equals the unconditional variance and, thus, can be estimated using the sample variance–covariance matrix of the N bond returns.

The performance of optimal mean–variance portfolios is evaluated in terms of Sharpe ratio (SR), which is defined as the ratio of the realized portfolio returns over its standard deviation. The Sharpe ratios are computed using excess returns based on the Brazilian interbank deposit rate. In order to assess the relative performance of the optimal mean–variance portfolios, we consider as benchmark policy the mean–variance portfolios obtained with estimates of expected bond returns based on the random walk specification discussed in Section 2. Even though Carriero et al. (2012) and Xiang and Zhu (forthcoming) assessed the economic value of yield curve forecasts, neither of them measured the statistical significance of these differences. In order to determine if differences between Sharpe ratios are statistically different, we use the test proposed by Ledoit and Wolf (2008) to compute the resulting bootstrap p-values. We implement the test by using $B = 1000$ bootstrap samples. Moreover, we employed the procedure to determine the optimal block length for the circular block bootstrap approach.

4. Empirical application

4.1. Data

The data set analyzed consists of yields of Brazilian Interbank Deposit Future Contract (DI-futuro), which is one of the largest fixed-income markets among emerging economies, collected on a daily basis. The Brazilian Mercantile and Futures Exchange (BM&FBovespa) is the entity that offers the DI-futuro contract and determines the number of maturities with authorized contracts. We use time series of daily closing yields of the DI-futuro contracts. In practice, contracts with all maturities are not observed on a daily basis. Therefore, based on the observed rates for the available maturities, the data were converted into fixed maturities of 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 42 and 48 months, using a spline based method. More specifically, we fit the curve of zero-coupon yields with a cubic spline interpolation $\hat{y}(\tau, \Psi)$, where Ψ is the vector of spline coefficients, τ is the maturity, and $\tau_{\min} \le \tau \le \tau_{\max}$, with $\tau_{\min}$ and $\tau_{\max}$ denoting the nearest and farthest maturities, respectively. We define $P_i(\tau),\ i = 1, \dots, N$, to be the time-$t$ market price of a bond with maturity τ and define $\hat{P}_i(\tau)$ to be the price of the same bond computed by discounting its coupon and principal payments at the discount rate $\hat{y}$. Next, we choose Ψ to solve the problem

$$\min_{\Psi}\left(\sum_{i=1}^{N}\left(P_i(\tau) - \hat{P}_i(\tau)\right)^2 + \int_{\tau_{\min}}^{\tau_{\max}} \lambda(\tau)\hat{y}''(\tau, \Psi)^2\, d\tau\right). \tag{37}$$

This approach is also employed by Dai et al. (2007) and Chyruk et al. (2012), except that we fit the smoothed cubic spline directly on the zero-coupon yields curve, thus similar to McCulloch (1975) and Chyruk et al. (2012), while Dai et al. (2007) fit the smoothed spline on the forward rates curve. As shown in Chyruk et al. (2012), the penalty function $\lambda(\tau)$ determines the tradeoff between fit and smoothness and is called the roughness penalty. If λ were zero, then we would be in the regression spline case, and as λ increases, the cubic spline function tends to a linear function. The flexibility of the spline is determined by both the spacing of the nodes and the magnitude of λ, but as λ increases, the spacing of the nodes becomes less important. Thus for large values of λ, the flexibility of the spline is approximately the same across all regions (see Chyruk et al., 2012, for more details).

The DI-futuro contract with maturity τ is a zero-coupon future contract in which the underlying asset is the DI-futuro interest rate accrued on a daily basis, capitalized between trading period t and τ. The DI-futuro rate is the average daily rate of Brazilian interbank deposits (borrowing/lending), calculated by the Clearinghouse for Custody and Settlements (CETIP) for all business days. The DI-futuro rate, which is published on a daily basis, is expressed in annually compounded terms, based on 252 business days. When buying a DI-futuro future contract for the price at time t and keeping it until maturity τ, the gain or loss is given by:

$$100.000\left(\frac{\prod_{i=1}^{\zeta(t,\tau)}(1+y_i)^{\frac{1}{252}}}{(1+\mathrm{ID}^*)^{\frac{\zeta(t,\tau)}{252}}} - 1\right),$$

Table 1
Descriptive statistics for the term structure of interest rates. The table reports summary statistics for DI-futuro yields over the period the sample period is January 2007–December 2012 (1488 daily observations). We examine daily data, constructed using the cubic-spline method. Maturity is measured in months. We show for each maturity mean, standard deviation, minimum, maximum and a selection of autocorrelation (Acf, $\widehat{\rho}(1)$, $\widehat{\rho}(5)$, and $\widehat{\rho}(21)$, respectively) and partial autocorrelation (Pacf, $\widehat{\alpha}(2)$ and $\widehat{\alpha}(5)$) coefficients.

Maturity	τ	Mean	Std dev	Min	Max	Skew	Kurt	Acf			Pacf	
								$\widehat{\rho}(1)$	$\widehat{\rho}(5)$	$\widehat{\rho}(21)$	$\widehat{\alpha}(2)$	$\widehat{\alpha}(5)$
Month	1	10.66	1.75	6.97	14.13	−0.337	2.219	0.998	0.987	0.936	−0.016	−0.012
	3	10.66	1.79	7.02	14.52	−0.286	2.296	0.998	0.989	0.940	−0.017	−0.025
	6	10.73	1.86	6.91	15.32	−0.201	2.472	0.998	0.989	0.939	−0.012	−0.023
	9	10.85	1.90	6.86	16.04	−0.132	2.650	0.998	0.988	0.934	0.004	−0.010
	12	11.00	1.92	6.87	16.40	−0.091	2.784	0.997	0.987	0.929	0.019	0.000
	15	11.16	1.92	6.90	16.91	−0.053	2.907	0.997	0.985	0.923	0.024	0.003
	18	11.31	1.88	7.00	17.12	−0.046	3.002	0.997	0.984	0.917	0.020	0.002
	21	11.42	1.85	7.14	17.26	−0.042	3.118	0.997	0.983	0.912	0.023	−0.002
	24	11.52	1.81	7.32	17.44	−0.032	3.243	0.996	0.982	0.907	0.020	−0.001
	27	11.60	1.77	7.49	17.62	−0.004	3.375	0.996	0.981	0.901	0.015	−0.002
	30	11.66	1.74	7.65	17.78	0.028	3.504	0.996	0.980	0.895	0.010	−0.006
	36	11.75	1.66	7.91	17.83	0.080	3.731	0.995	0.978	0.887	0.015	−0.013
	42	11.83	1.59	8.13	17.93	0.104	3.937	0.995	0.976	0.880	0.017	−0.006
	48	11.89	1.56	8.29	18.00	0.181	4.156	0.995	0.975	0.873	0.018	−0.011
	Level	12.41	1.42	9.34	18.99	0.673	5.185	0.992	0.957	0.800	−0.064	−0.014
	Slope	−1.92	1.95	−6.37	2.84	−0.267	2.814	0.995	0.974	0.866	−0.039	−0.036
	Curvature	−1.00	3.94	−9.48	8.60	0.120	2.192	0.997	0.978	0.903	−0.160	−0.011

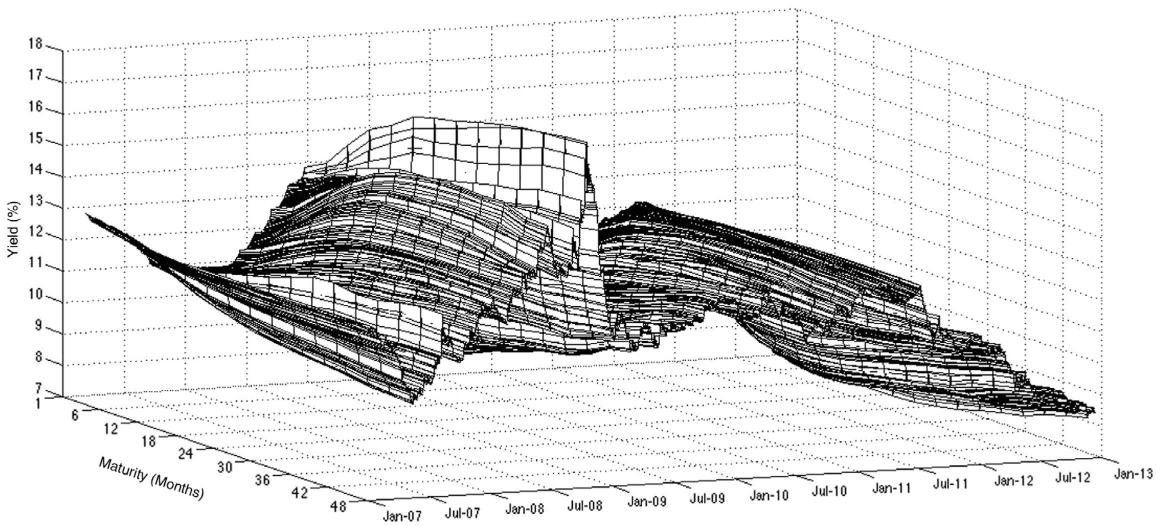


Fig. 1. Evolution of the yield curve. Note: the figure plots the evolution of term structure of interest rates (based on DI-futuro contracts) for the time horizon of 2006:01–2012:12. The sample consisted of the daily yields for the maturities of 1, 3, 4, 6, 9, 12, 15, 18, 24, 27, 30, 36, 42 and 48 months.

where y_i denotes the DI-futuro rate, $(i - 1)$ days after the trading day. The function $\zeta(t, \tau)$ represents the number of working days between t and τ. The contract value is set by its value at maturity, R\$100,000.00, discounted according to the accrued interest rate negotiated between the seller and the buyer. In 2012 the DI-futuro market traded a total of 341 million contracts corresponding to US\$17.5 billion. The DI-futuro contract is very similar to the zero-coupon bond, except for the daily payment of margin adjustments. The data set contains maturities with highest liquidity for January 2006–December 2012, yielding a total of $T = 1488$ daily observations. The data source is the Brazilian Mercantile and Futures Exchange (BM&FBovespa).

Table 1 reports descriptive statistics for the Brazilian interest rate yield curve based on the DI-futuro market. For each time series we report the mean, standard deviation, minimum, maximum and the lag-1 sample autocorrelation. The summary statistics confirms some stylized facts common to yield curve data: the sample average curve is upward sloping and concave, volatility is decreasing with maturity, and autocorrelations are very high. We also present estimates of the level, slope, and curvature of yield curve based on the dynamic Nelson–Siegel model.

Fig. 1 displays a three-dimensional plot of the data set and illustrates how yield levels and spreads vary substantially throughout the sample. The plot also suggests the presence of an underlying factor structure. Although the yield series vary heavily over time for each of the maturities, a strong common pattern in the 14 series is apparent. For most months, the yield curve is an upward sloping function of time to maturity. For example, last year of the sample is characterized by rising interest rates, especially for the shorter maturities, which respond faster to the contractionary monetary policy implemented by the Brazilian Central Bank in the first half of 2010. It is clear from Fig. 1 that not only the level of the term structure fluctuates over time but also its slope and curvature. The curve takes on various forms ranging from nearly flat to (inverted) S-type shapes.

4.1.1. Implementation details

The forecasting exercise is performed in pseudo real time, i.e. we never use information which is not available at the time the forecast is made. For computing our results we use a rolling estimation window of 500 daily observations (2 years). We have also estimated the models using an expanding window. However, the RMSE results obtained were qualitatively similar to those presented here. These results are available upon request. We produce forecasts for 1-week, 1-month, 2-month, and 3-month ahead. The choice of a rolling scheme is suggested by two reasons. First, it is a natural way to avoid problems of instability, see e.g. Pesaran et al. (2011). Second, having fixed the number of observations used to compute the forecasts and therefore the resulting time series of the forecast errors allows the use of the Giacomini and White (2006) test for comparing forecast accuracy. Such a test is valid provided that the size of the estimation window is fixed.

We use iterated forecasts instead of direct forecasts for the multi-period ahead predictions. Marcellino et al. (2006) compare empirical iterated and direct forecasts from linear univariate and bivariate models by applying simulated out-of-sample methods and conclude that iterated forecasts typically outperform the direct forecasts, and that the relative performance of the iterated forecasts improves with the forecast horizon.

4.2. Results

4.2.1. Statistical evaluation

Table 2 reports statistical measures of the out-of-sample forecasting performance of ten alternative individual models and five combination schemes for four forecast horizons. The first line in each panel of the table reports the value of TRMSFE and RMSFE (expressed in basis points) for the random walk model (RW), while all other lines report statistics relative to the RW. Bold values indicate the model with best performance in each maturity. In order to assess the statistical significance of these differences in forecast, we use the test of conditional predictive ability proposed by Giacomini and White (2006).

Panels (a) and (b) of Table 2 show results of 1-day-ahead and 1-week-ahead forecasts, respectively. As documented in the literature, it is very difficult to outperform the RW for short horizons, since the near unit root behavior of the yields seems to dominate and model-based information add little (Diebold and Li, 2006; de Pooter et al., 2010; Nyholm and Vidova-Koleva, 2012; Xiang and Zhu, forthcoming). Nevertheless, we observe that some individual models and combination schemes are able to outperform the RW mainly for short-term maturities. For instance, in the case of the 1-day-ahead forecasts we find that the FB and SR specifications deliver lower TRMSFE in comparison to the RW. When looking at specific maturities, we also find that some individual specifications outperformed the RW but only in the case of short term maturities. Forecast combinations, on the other hand, also outperform the benchmark since they achieve lower TRMSFE and lower RMSFE in the vast majority of the cases, for both short and long term maturities. In the case of 1-week-ahead forecasts we observe that all forecast combination schemes outperform the RW in terms of TRMSFE, and that the FC-OLS achieved the lowest TRMSFE among all individual models and combination schemes. Moreover, forecast performance of both individual models and combination schemes deteriorate as maturity grows longer.

The results for the 1-month-ahead forecasts are shown in panel (c) of Table 2. We observe that for short-term maturities several individual models and all combination schemes outperform the RW in terms of lower RMSFE. The best overall performance is achieved by the DSV model, as it outperforms the RW for all maturities. All forecast combinations considered also consistently outperform the RW as they deliver lower TRMSFE and lower RMSFE in the vast majority of the cases.

Panel (d) of Table 2 reports the results for the 2-month-ahead forecasts. Even tough the DSV model achieved the best overall performance in terms of TRMSFE, this model fails to outperform the RW for maturities longer than 18 months. The same applies to the remaining individual models, with an exception to the VAR and CP specifications. Forecast combination, in contrast, outperforms the RW for both short and long term rates. Similar results are observed in the panel (e) of Table 2 which reports the results for the 3-month-ahead forecasts. In this case, forecast combination also seems to deliver a more consistent outperformance in terms of lower RMSFE with respect to the RW across all maturities.

To explore the accuracy of the forecasts in different time intervals, we follow Welch and Goyal (2008) and plot the difference in cumulative squared forecast errors between each of the prediction models and the RW along the out-of-sample evaluation period. Figs. 2 and 3 display CSFE's for the 1-week and 3-month forecast horizons. The results for the remaining forecast horizons are similar and are available upon request. Each line in the graph represents a different model and shows how a particular model performs relative to the random walk benchmark. In particular, an increasing CSFE indicates outperformance whereas a decreasing CSFE indicates underperformance with respect to the RW.

During the financial turmoil of the 2008–2009 period, interest rates initially went up and then declined sharply from roughly 14% to a level of 8.5% for the short rate accompanied by a substantial widening of spreads between long and short rates (see Fig. 1). The CSFE graphs allow us to examine in detail how different models perform during this period on a day-to-day basis. The CSFE graphs in panel (a) of Fig. 2 reveal that for the 1-week forecast horizons most individual models perform poorly with respect to the RW, and that only the CP specification is able to maintain a weak outperformance even in the

Table 2
Relative (trace)-root mean squared forecast errors. The table reports relative root mean squared forecast errors (RMSFE) and trace RMSFE (TRMSFE) relative to the random walk model obtained by using individual yield models and different forecast combination methods, for the 1-day, 1-week, 1-month, 2-month, and 3-month forecast horizons. The evaluation sample is 2009:1 to 2012:12 (988 out-of-sample forecasts). The first line in each panel of the table reports the value of RMSFE and TRMSFE (expressed in basis points) for the random walk model (RW), while all other lines report statistics relative to the RW. The following model abbreviations are used in the table: AR(1) for the first-order univariate autoregressive model, VAR(1) for the first-order vector autoregressive model, BVAR refers to Bayesian VAR, DNS for the dynamic Nelson–Siegel model, DSV for the dynamic Svensson model. CP refers to the Cochrane and Piazzesi model, FB for the Fama–Bliss model, SR for slope regression, and CIR refers to the Cox–Ingersoll–Ross three factor model. FC-EW, FC-OLS and FC-RANK stand for forecast combinations based on equal weights, OLS-based weights, and rank-weighted combinations, respectively. FC-RMSE and FC-MRMSE refer to forecast combinations based on the thick modeling approach with RMSE-weights and MSE-Frequency weights, respectively. Numbers smaller than one indicate that models outperform the RW, whereas numbers larger than one indicate underperformance. Numbers in *bold* indicate outperformance in the maturity. Stars indicate the level at which the Giacomini and White (2006) test rejects the null of equal forecasting accuracy.

Models	TRMSFE	1-month	3-month	6-month	9-month	12-month	15-month	18-month	21-month	24-month	27-month	30-month	36-month	42-month	48-month
Panel A: Horizon = 1-day ahead															
RW	0.074	0.038	0.040	0.052	0.060	0.066	0.071	0.076	0.079	0.081	0.084	0.086	0.088	0.091	0.094
AR(1)	1.002	0.992	0.994	0.998	1.000	1.001	1.001	1.002*	1.002*	1.003*	1.003	1.003	1.003	1.003	1.003
VAR(1)	1.043	0.905**	0.931**	0.993	1.020*	1.030*	1.046**	1.048**	1.056**	1.057**	1.054**	1.052**	1.052**	1.054**	1.051**
BVAR	0.999	0.907**	0.927**	0.973**	0.990	0.998	1.002	1.004	1.005	1.004	1.004	1.004	1.005	1.005	1.005
DNS	1.638	2.633**	0.917**	1.978**	2.046**	1.841**	1.569**	1.112**	1.021**	1.065**	1.082**	1.056**	1.021	1.016	1.102**
DSV	1.196	2.266**	0.913**	1.234**	1.033**	1.076**	1.086**	1.056**	1.031**	1.056**	1.074**	1.043**	1.034*	1.007	1.067**
CP	1.012	**0.878****	**0.904****	0.964**	0.994	1.001	1.012	1.017	1.024**	1.026**	1.022**	1.021**	1.020*	1.024**	1.017
FB	**0.992**	0.990	0.918	**0.960**	**0.975**	**0.983**	0.988	0.992	0.995	0.997	0.997	0.997	**0.997**	0.999	1.000
SR	**0.991**	0.990	0.918	0.961	0.976	0.983	0.988	**0.991**	**0.993**	**0.995**	0.996	0.997	0.998	0.998	**0.998**
CIR	1.410	0.997	0.997	0.999	1.001	1.002	0.999	1.550	1.371	1.272	1.596	1.741	1.821	1.495	1.196
FC-EW	1.003	1.142**	0.923**	1.001**	1.021**	1.012	1.002	1.012**	0.998	0.996	0.997	**0.995**	0.997	**0.997**	1.006
FC-OLS	**0.990**	0.884**	0.911**	0.961**	0.976**	0.985**	**0.988****	0.994**	0.994**	0.996**	**0.997****	0.998**	0.998**	0.999**	0.999
FC-RANK	**0.994**	0.892**	0.905**	0.967**	0.983**	0.993**	0.993**	0.998	0.999	0.999	1.002	1.000	0.999	1.001	1.001
FC-RMSE	0.991	0.891**	0.906**	0.963**	0.978**	0.986**	0.990**	0.994**	0.996**	0.997**	0.997**	0.998**	0.998**	0.999	0.999
FC-MRMSE	0.991	0.890**	0.905**	0.964**	0.980**	0.987**	0.991**	0.994**	0.996**	0.997**	0.998**	0.998**	0.999	0.999	0.999
Panel B: Horizon = 1-week ahead															
RW	0.171	0.121	0.120	0.141	0.154	0.164	0.174	0.181	0.183	0.184	0.187	0.188	0.188	0.189	0.192
AR(1)	1.009	0.985**	0.986	0.994	0.999	1.003	1.006	1.008	1.011	1.013	1.013	1.014	1.016	1.018*	1.019*
VAR(1)	1.105	**0.725****	0.764**	0.928	1.015	1.062	1.105**	1.130**	1.152**	1.162**	1.159**	1.160**	1.162**	1.169**	1.166**
BVAR	0.995	0.757**	0.784**	0.907**	0.964	0.994	1.011	1.022	1.024	1.022	1.022	1.023	1.023	1.023	1.025*
DNS	1.082	1.730**	0.754**	1.291**	1.340**	1.174**	1.029	0.988*	1.003	1.012	1.008	1.001	0.999	1.003	1.009
DSV	0.985	1.362**	**0.742****	0.906**	0.908**	0.934**	**0.962**	**0.972**	**0.980**	0.991	0.991	**0.977**	0.988	1.003	1.022
CP	0.972	0.889**	0.902**	0.946**	0.961**	0.968**	0.977**	0.979**	0.982**	**0.979****	0.979**	0.98**	**0.980****	**0.984****	**0.984***
FB	0.989	0.993	0.922	0.954	0.971	0.979	0.985	0.991	0.995	0.996	0.997	0.997	0.997	0.998	0.998
SR	0.990	0.994	0.924	0.956	0.973	0.981	0.987	0.991	0.994	0.996	0.997	0.998	0.999	1.001	1.000
CIR	1.420	1.942	2.281	2.043	2.213	1.935	1.503	1.149	1.136	1.012	1.048	1.060	1.084	1.029	1.106
FC-EW	0.972	0.738**	0.805**	0.957**	0.992**	0.996	0.993	0.988	0.987*	0.979	**0.976****	0.978**	0.984**	0.989**	0.999
FC-OLS	**0.957**	0.752**	0.745**	0.892**	**0.907****	**0.931****	0.981**	0.983**	0.985**	0.983*	0.983**	0.984**	0.984**	0.986**	0.987
FC-RANK	0.969	0.748**	0.747**	0.886**	0.919**	0.937**	0.99**	0.993*	0.995	0.996	0.997	0.999	1.001	1.005	1.007
FC-RMSE	0.961	0.739**	0.746**	**0.879****	0.912**	0.933**	0.984**	0.987**	0.988**	0.988*	0.988*	0.990	0.991	0.995	0.996
FC-MRMSE	0.960	0.739**	0.746**	**0.879****	0.909**	0.933**	0.982**	0.986**	0.988**	0.986*	0.987*	0.988*	0.99	0.995	0.994
Panel C: Horizon = 1-month ahead															
RW	0.439	0.405	0.409	0.430	0.442	0.445	0.455	0.459	0.458	0.453	0.453	0.452	0.437	0.420	0.417
AR(1)	1.022	0.995	0.994	0.997	1.001	1.006	1.012	1.017	1.023	1.029	1.031	1.036	1.045*	1.056**	1.061**
VAR(1)	1.038	**0.451****	0.573**	0.749*	0.858	0.963	1.040*	1.101**	1.139**	1.162**	1.158**	1.159**	1.198**	1.230**	1.230**
BVAR	0.954	0.585**	0.665**	0.818	0.911	0.963	0.995	1.019**	1.024**	1.020**	1.019*	1.019*	1.017	1.024	1.034*
DNS	0.894	0.604**	0.580**	0.735**	0.804**	0.850**	0.894**	0.939*	0.970	0.979	0.973	0.972	0.991	0.998	0.980**
DSV	**0.850**	0.536**	**0.561****	**0.683****	**0.746****	**0.803****	**0.848****	**0.882****	**0.905****	**0.923****	**0.923****	**0.919****	**0.954****	0.981**	0.989*
CP	0.976	0.958**	0.962**	0.970**	0.972**	0.976**	0.978**	0.978**	0.979**	0.978*	0.978*	0.977*	0.979**	0.983*	0.981
FB	0.992	0.996	0.970	0.978	0.983	0.986	0.990	0.994	0.996	0.997	0.997	0.998	0.998	0.998	0.998
SR	0.993	0.998	0.971	0.979	0.985	0.989	0.992	0.994	0.996	0.997	0.998	0.999	1.000	0.999	1.002
CIR	1.059	0.486	0.940	1.293	1.358	1.295	1.168	1.063	1.001	0.965	0.943	0.944	0.975	0.987	1.044
FC-EW	0.909	0.677**	0.743**	0.83**	0.878**	0.909**	0.93**	0.943**	0.948**	0.946**	0.944**	0.947**	0.960**	**0.969***	**0.976***
FC-OLS	0.879	0.481**	0.563**	0.771**	0.763**	0.809**	0.856**	0.894**	0.927**	0.979**	0.981	0.973	0.993	1.007	1.015
FC-RANK	0.892	0.511**	0.565**	0.758**	0.771**	0.820**	0.929**	0.942**	0.949**	0.981**	0.968	0.970**	0.999**	1.014	1.021
FC-RMSE	0.870	0.497**	0.564**	0.711**	0.767**	0.817**	0.868**	0.892**	0.905**	0.978**	0.953	0.954	0.988	0.998	1.001
FC-MRMSE	0.871	0.494**	0.565**	0.716**	0.763**	0.816**	0.863**	0.886**	0.908**	0.979**	0.958	0.959*	0.993*	1.003	1.006
Panel D: Horizon = 2-month ahead															
RW	0.805	0.762	0.778	0.815	0.838	0.846	0.847	0.842	0.834	0.820	0.817	0.810	0.777	0.744	0.730
AR(1)	1.036	1.024	1.026	1.026	1.020	1.018	1.017	1.021	1.027	1.035	1.042	1.049	1.058	1.074	1.091*

(continued on next page)

Table 2 (*continued*)

Models	**TRMSFE**	1-month	3-month	6-month	9-month	12-month	15-month	18-month	21-month	24-month	27-month	30-month	36-month	42-month	48-month
VAR(1)	1.026	**0.432****	0.561**	0.726*	0.845	0.954	1.036	1.096**	1.130**	1.152**	1.154**	1.165**	1.217**	1.242**	1.241**
BVAR	0.968	0.600**	0.710**	0.863	0.946	0.984	1.001	1.023	1.032	1.032	1.036	1.035	1.028	1.041	1.060
DNS	0.873	0.502**	0.587**	0.716**	0.791**	0.851**	0.900	0.937	0.956	0.960	0.952	0.954	0.979	0.982	0.958
DSV	**0.786**	0.477**	**0.550****	**0.643****	**0.696****	**0.743****	**0.782***	**0.809**	**0.829**	**0.844**	**0.844**	**0.853**	0.907	**0.941**	**0.950**
CP	0.981	0.977**	0.979**	0.981**	0.980**	0.981**	0.983**	0.982**	0.982**	0.981**	0.980**	0.980**	0.981**	0.981**	0.980**
FB	0.995	0.998	0.985	0.989	0.991	0.993	0.994	0.996	0.997	0.998	0.998	0.998	0.998	0.998	0.996
SR	0.997	0.999	0.986	0.990	0.993	0.994	0.996	0.997	0.998	0.999	0.999	1.001	1.000	1.002	1.001
CIR	1.002	0.684	0.908	1.075	1.113	1.100	1.064	1.029	1.000	0.979	0.958	0.958	0.988	1.001	1.028
FC-EW	0.898	0.702**	0.762**	0.831**	0.870**	0.896**	0.913**	0.924**	0.93**	0.932**	0.932**	0.937**	0.953**	**0.965***	**0.971**
FC-OLS	0.811	0.451**	0.552**	0.660**	0.716**	0.761**	**0.782****	**0.809****	0.833**	0.857**	0.855**	0.866**	0.984**	1.020*	1.043
FC-RANK	0.868	0.450**	0.559**	0.672**	0.733**	0.782**	0.922**	0.931**	0.940**	0.948**	0.953**	0.962**	0.979**	1.029	1.042
FC-RMSE	0.814	0.448**	0.556**	0.666**	0.725**	0.773**	0.826**	0.845**	0.857*	0.865*	0.866	0.874	**0.905**	1.006	1.013
FC-MRMSE	0.820	0.452**	0.556**	0.666**	0.725**	0.769**	0.801**	0.830**	0.867**	0.885**	0.885**	0.892**	0.931**	1.018**	1.029**
Panel E: Horizon = 3-month ahead															
RW	1.183	1.104	1.134	1.192	1.234	1.248	1.252	1.236	1.221	1.200	1.196	1.190	1.154	1.106	1.084
AR(1)	1.099	1.062	1.077	1.085	1.078	1.071	1.067	1.066	1.071	1.085	1.105	1.124	1.144	1.176*	1.205*
VAR(1)	1.048	0.548**	0.727**	0.950	1.119	1.254	1.358*	1.437**	1.495**	1.540**	1.566**	1.597**	1.669**	1.709**	1.724**
BVAR	1.057	0.759**	0.865*	0.957	1.020	1.043	1.061	1.080*	1.095*	1.102*	1.116*	1.126*	1.133	1.157	1.182
DNS	0.876	0.523**	0.629**	0.742**	0.811*	0.863	0.902	0.935	0.951	0.955	0.948	0.946	0.959	0.962	0.945
DSV	**0.752**	0.494**	**0.559****	**0.620****	**0.660****	**0.697****	**0.729***	**0.759**	**0.782**	**0.799**	**0.803**	**0.816**	**0.867**	0.906	0.925
CP	0.991	0.985**	0.986**	0.988**	0.989**	0.990**	0.991**	0.991**	0.992**	0.992**	0.991**	0.991**	0.991**	0.991**	0.992**
FB	0.997	0.998	0.991	0.993	0.995	0.996	0.996	0.997	0.998	0.998	0.998	0.998	0.999	0.997	0.996
SR	0.999	0.999	0.991	0.994	0.996	0.997	0.998	0.999	0.999	1.001	1.002	1.000	1.001	1.001	1.001
CIR	0.997	0.797	0.934	1.032	1.054	1.051	1.034	1.021	1.006	0.993	0.976	0.972	0.988	1.001	1.023
FC-EW	0.913	0.743**	0.801**	0.856**	0.888**	0.907**	0.919**	0.930**	0.936*	0.940**	0.943**	0.950*	0.965*	0.977*	0.985
FC-OLS	0.771	**0.477****	0.565**	0.643**	0.689**	0.727**	0.758**	0.788**	0.784**	0.800**	0.804**	0.820**	0.877*	0.946*	0.980*
FC-RANK	0.804	0.479**	0.579**	0.668**	0.720**	0.762**	0.795**	0.826**	0.871**	0.876*	0.878	0.894	0.906	0.918	0.923*
FC-RMSE	0.785	**0.477****	0.574**	0.657**	0.705**	0.744**	0.777**	0.808**	0.837*	0.846**	0.849**	0.858**	0.889*	**0.906***	**0.914***
FC-MRMSE	0.789	0.479**	0.571**	0.649**	0.695**	0.730**	0.760**	0.791**	0.840**	0.859**	0.865**	0.867**	0.920**	0.932**	0.938**

* Mean rejection at 10% level.
** Mean rejection at 5% level.

crisis period. As for the forecast combinations, panel (b) of Fig. 2 tells a different story. The CSFE graphs indicate that the forecast combination schemes consistently outperform the RW in the vast majority of the cases.

We also observe in the graphs that the outperformance of the combined forecasts is consistent and stable throughout the entire out-of-sample period. This result suggests that the combination of forecasts results in an improvement in forecast accuracy with respect to the benchmark model, and also with respect to individual models. More specifically, it is often the case that the CSFE obtained with a combination strategy for a given maturity and horizon is larger than the CSFEs obtained for single models, highlighting the effects of model complementarities in producing precise forecasts.

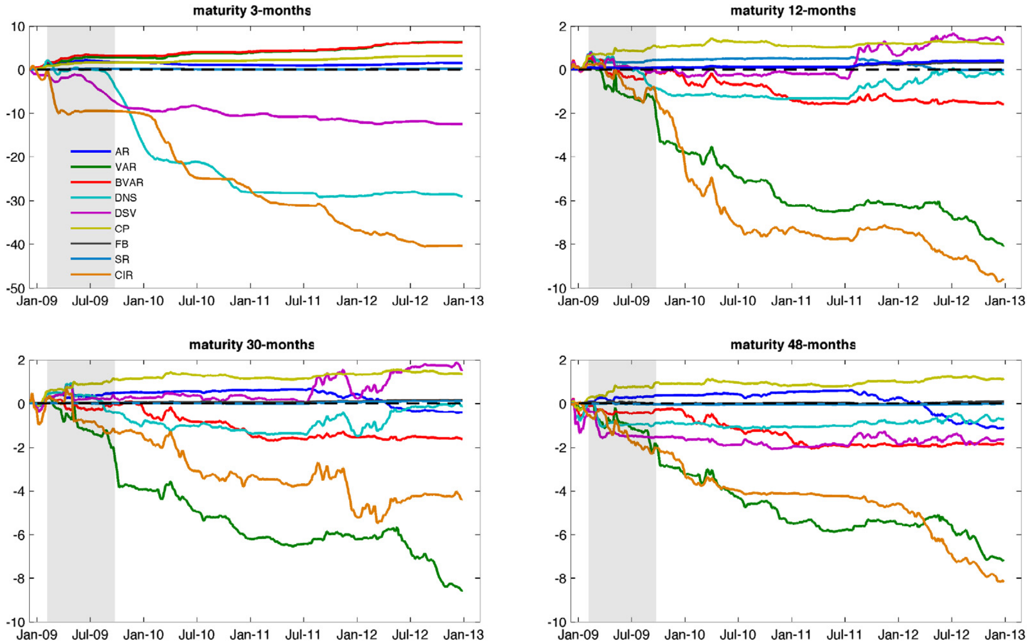
Assessing the relative importance of individual models on the forecast combination schemes

A question so far unaddressed in the previous discussion is the relative importance of individual models on the forecast combination schemes considered in the paper. In order to address this issue, we report in Table 3 information regarding the average weights in each of the individual models across all forecast combination schemes. The table reports for each model the frequency of usage (i.e. the relative number of times the model is selected) and the average weight (along with the 25% and 75% percentiles) across all forecast combination schemes for the 1-week and 3-month forecast horizons and for the 3-, 12-, 30-, and 48-month rates. The results for the remaining forecast horizons and remaining maturity rates are available upon requests.

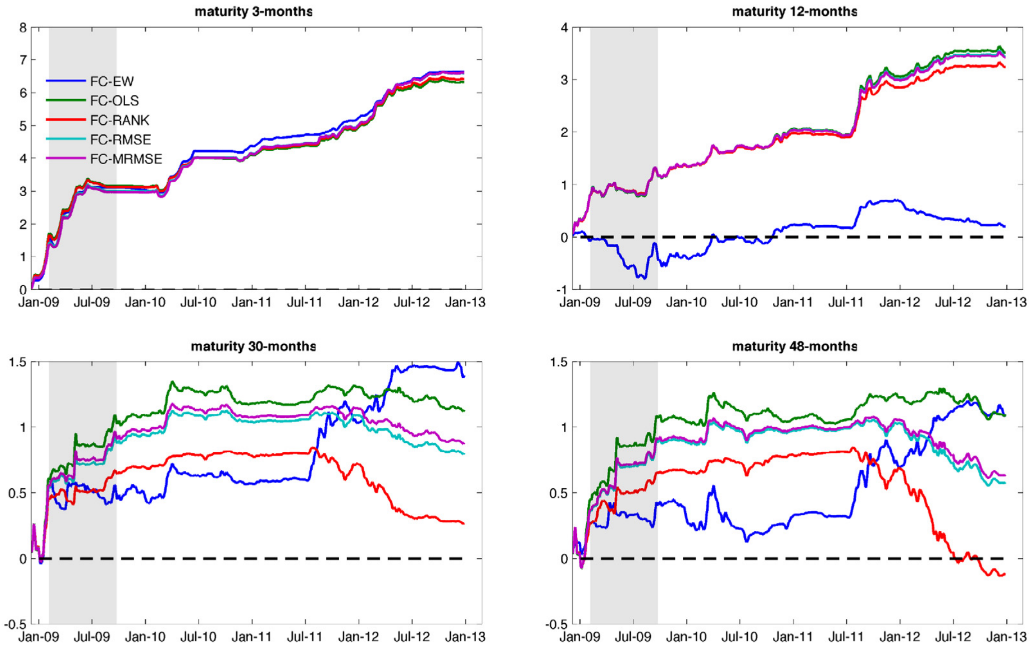
Table 3 reveals that the allocation in individual models changes substantially across forecast horizons and maturity rates. For instance, in the case of the 1-week forecast horizon, forecast combination for the 3-month rate tends to select the AR, DNS and DSV specifications whereas for the 48-month rate the mostly selected specifications are the RW, BVAR, CP and FB. As for the 3-month forecast horizon, forecast combinations select mostly the DNS, DSV and CIR specifications, whereas for the 48-month rate the selected specifications are BVAR, DNS, DSV, and CP.

4.2.2. Economic value of forecasts

In the previous subsection, we showed that alternative individual prediction models as well as forecast combination schemes are able to deliver more accurate forecasts with respect to the benchmark when considering statistical criteria. We observe, however, that in some instances the improvement in forecasting performance (as indicated by lower forecasting errors) is small in magnitude. Therefore, a question that remains unanswered is whether or not this statistical gain is also economically meaningful.



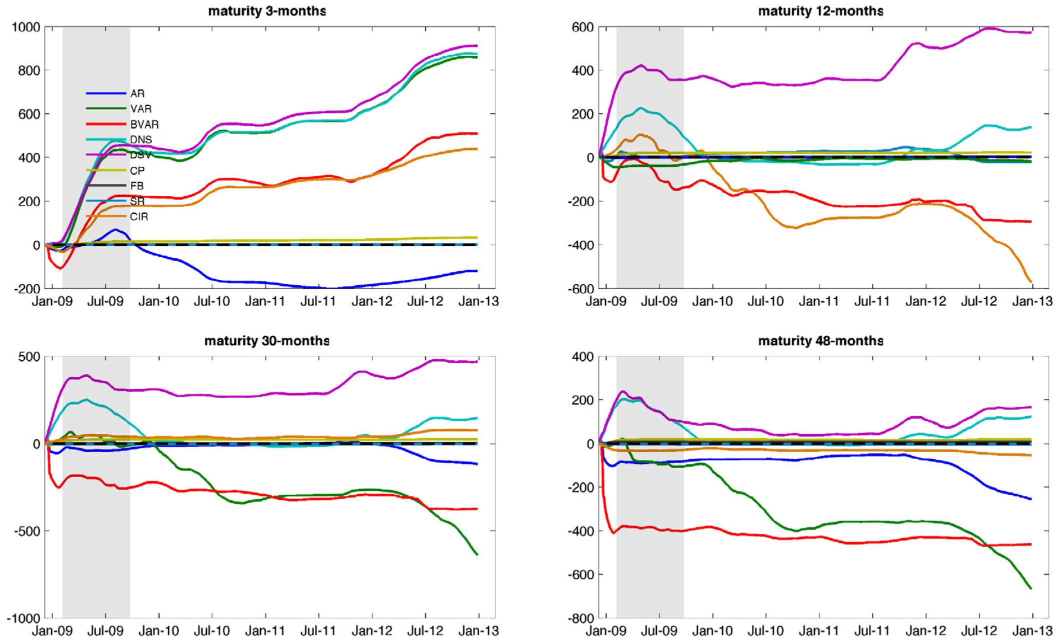
(a) Individual models.



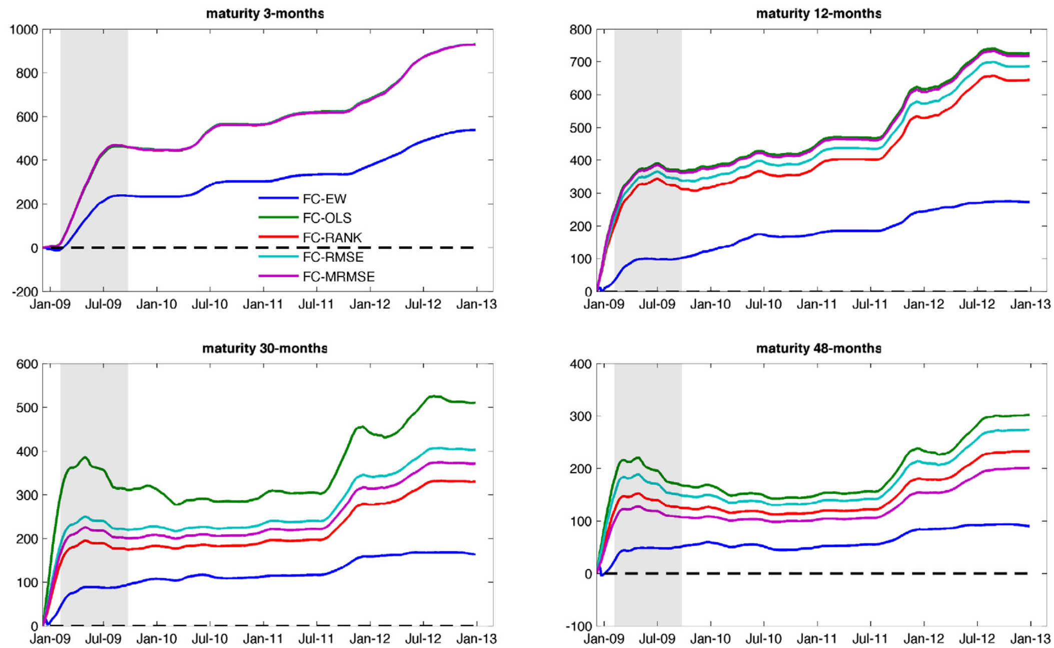
(b) Forecast combinations.

Fig. 2. Cumulative squared forecast errors (1-week forecast horizon). Note: Figures (a) and (b) show the cumulative squared forecast errors (CSFE), relative to the random walk, of individual yield-only models in Panel (a) and of forecast combinations schemes in Panel (b). Figures show CSFEs for a 3-month forecast horizon. The evaluation sample is 2009:1–2012:12 (988 out-of-sample forecasts). Gray bars highlight recession periods.

Table 4 reports the annualized Sharpe ratios of the mean–variance portfolios composed of Brazilian DI-futuro contracts. We observe that in the vast majority of the instances, the mean–variance portfolios obtained with individual models and with forecast combinations achieve statistically higher Sharpe ratios in comparison to the mean–variance portfolios obtained with the random walk model. Moreover, this result is robust to the value of the risk aversion coefficient and to the portfolio re-balancing frequency. Small gains in forecasting performance can lead to Sharpe ratios that are substantially and statistically higher than those obtained by the benchmark model. For instance, the 1-day ahead forecast errors of the AR(1) model are not significantly better than the RW ones and, for some maturities, are significantly worst. Nevertheless, all Sharpe



(a) Individual models.



(b) Forecast combinations.

Fig. 3. Cumulative squared forecast errors (3-month forecast horizon). Note: Figures (a) and (b) show the cumulative squared forecast error (CSFE), relative to the random walk, of individual yield-only models in Panel (a) and of forecast combinations schemes in Panel (b). Figures show CSFEs for a 3-month forecast horizon. The evaluation sample is 2009:1–2012:12 (988 out-of-sample forecasts). Gray bars highlight recession periods.

ratios of daily re-balanced portfolios based on 1-day ahead forecasts from AR(1) are larger than the Sharpe ratios of portfolios based on RW, and 3 out of 5 Sharpe ratios are significantly larger. Additionally, the Sharpe ratio of the mean–variance portfolios obtained with the RW specification with $\delta = 0.01$ and daily re-balancing is 0.493 whereas the same figures for the BVAR and FC-RMSE are, respectively, 1.021 and 1.483.

We also observe that the forecast combinations deliver higher Sharpe ratios in comparison to those obtained with individual models in most of the cases. For instance, the highest Sharpe ratio obtained under daily re-balancing is achieved by the FC-RMSE (1.483), whereas for the weekly, bimonthly, and quarterly re-balancing the highest Sharpe ratio is achieved by

Table 3
Combination weights. The table reports for each model the frequency of usage and the average weight (along with the 25% and 75% percentiles) across all forecast combination schemes (excluding the equally-weighted scheme) for the 1-week and 3-month forecast horizons and for the 3-, 12-, 30-, and 48-month rates. The following model abbreviations are used in the table: RW for the random walk model, AR for the first-order univariate autoregressive model, VAR for the first-order vector autoregressive model, BVAR refers to Bayesian VAR, DNS for the dynamic Nelson–Siegel model with a VAR specification for the factors, DSV for the dynamic Svensson model with a VAR specification for the factors. CP refers to the Cochrane and Piazzesi model, FB for the Fama–Bliss model, SR for slope regression, and CIR refers to the Cox–Ingersoll–Ross three factor model.

	Frequency of usage	Average weight	25% perc.	75% perc.	Frequency of usage	Average weight	25% perc.	75% perc.	Frequency of usage	Average weight	25% perc.	75% perc.	Frequency of usage	Average weight	25% perc.	75% perc.
	1-**week forecast horizon**															
	3-month				12-month				30-month				48-month			
RW	0%	0%	0%	0%	21%	41%	30%	51%	79%	39%	28%	54%	78%	39%	27%	54%
AR	100%	33%	18%	52%	1%	34%	28%	39%	0%	0%	0%	0%	0%	0%	0%	0%
VAR	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
BVAR	0%	0%	0%	0%	100%	36%	27%	45%	100%	39%	31%	45%	100%	37%	34%	45%
DNS	100%	31%	23%	35%	0%	0%	0%	0%	6%	24%	14%	34%	0%	0%	0%	0%
DSV	100%	36%	20%	44%	100%	32%	23%	39%	93%	25%	12%	34%	0%	0%	0%	0%
CP	0%	0%	0%	0%	79%	30%	26%	33%	22%	27%	24%	33%	61%	27%	25%	33%
FB	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	61%	27%	22%	33%
SR	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
CIR	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
	3-**month forecast horizon**															
	3-month				12-month				30-month				48-month			
RW	0%	0%	0%	0%	0%	37%	23%	51%	0%	0%	0%	0%	0%	0%	0%	0%
AR	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
VAR	0%	0%	0%	0%	0%	0%	0%	0%	98%	37%	31%	45%	0%	0%	0%	0%
BVAR	0%	0%	0%	0%	85%	24%	16%	41%	51%	35%	32%	38%	100%	40%	33%	45%
DNS	100%	33%	31%	42%	100%	30%	24%	32%	49%	30%	28%	35%	56%	31%	28%	34%
DSV	100%	43%	36%	42%	100%	46%	34%	58%	100%	31%	22%	39%	100%	27%	26%	34%
CP	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	44%	35%	26%	43%
FB	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
SR	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
CIR	100%	24%	21%	30%	15%	22%	17%	28%	2%	20%	14%	27%	0%	0%	0%	0%

the FC-MSE (1.115), FC-OLS (1.032), FC-OLS (1.146), respectively. These values are substantially higher than those obtained by the individual models, including the benchmark specification. The only exception to this result is found in the monthly re-balancing frequency, since the highest Sharpe ratio is achieved by the DSV specification (1.199).

One of the most important lessons from the economic evaluation of forecasts is that the differences in performance in terms of Sharpe ratios are much more pronounced than those based on statistical measures. In this sense, it seems to be much easier to distinguish between "good" and "bad" predictions when looking at economic criteria. As we noted earlier, differences in statistical performance measures are usually small in magnitude. In contrast, differences in Sharpe ratios tend to be much more evident. In particular, we observe that in several cases the Sharpe ratios obtained by some forecast combination schemes are twice as the ones obtained by some individual models.

4.2.3. *Discussion*

The results discussed in the previous sections reveal that the statistical and economic evaluation of forecast performance might be related, at least to some extent. We observe that those specifications that deliver lower forecasting errors (measured in terms of RMSFE and TRMSFE) also tend to deliver mean–variance portfolio with higher Sharpe ratios in comparison to the benchmark. However, how strong (or weak) is this relation? In order to address this question, we plot in Fig. 4 a two-dimensional graph with the average value across all maturities of the RMSFE in the x-axis and the Sharpe ratios in the y-axis for each forecast horizon. Moreover, we also report in the graph the value of the correlation coefficient between both quantities.

Fig. 4 indicates that the correlation coefficient between RMSFE and Sharpe ratios is negative for all forecasting horizons. This result corroborates our previous findings and suggests that lower forecasting errors is in fact associated to higher Sharpe ratios. In some instances, however, this relation is weaker since the correlation coefficient between RMSFE and Sharpe ratios is closer to zero. For example, the correlation coefficient for the 1-month-ahead is −0.43 whereas for the 3-month-ahead is −0.09. This finding suggests that in some cases statistical and economic evaluation can provide different answers about which candidate model is better. In fact, the results discussed in Tables 2 and 4 seem to corroborate this finding. We observe in these tables that the ranking of specifications based on statistical performance can differ from the ranking based on an economic criterion. As we discussed previously, the statistical evaluation in Table 2 and in the CSFE graphs in Figs. 2 and 3 indicate that the forecast combinations outperformed the remaining specification for shorter horizons, while for the longer horizons some individual specifications such as the DSV model delivers more accurate forecasts. However, the results for the economic evaluation in Table 2 reveal that forecast combinations tend to outperform the remaining specification in the vast

Table 4
Sharpe ratios of mean–variance portfolios. The table reports annualized Sharpe ratios of the optimal mean–variance portfolios using DI-futuro contracts with maturities equal to 1, 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 36, 42 and 48 months, where the risk-free rate used is the CDI, which is the Brazilian overnight rate. The following model abbreviations are used in the table: AR(1) for the first-order univariate autoregressive model, VAR(1) for the first-order vector autoregressive model, BVAR refers to Bayesian VAR, DNS for the dynamic Nelson–Siegel model, DSV for the dynamic Svensson model. CP refers to the Cochrane and Piazzesi model, FB for the Fama–Bliss model, SR for slope regression, and CIR refers to the Cox–Ingersoll–Ross three factor model. FC-EW, FC-OLS and FC-RANK stand for forecast combinations based on equal weights, OLS-based weights, and rank-weighted combinations, respectively. FC-RMSE and FC-MRMSE refer to forecast combinations based on the thick modeling approach with RMSE-weights and MSE-Frequency weights, respectively. The optimal portfolios are re-balanced on a weekly, monthly, bimonthly, and quarterly basis.

Model		Individual models										Forecast combination				
		RW	AR(1)	VAR(1)	BVAR	DNS	DSV	CP	FB	SR	CIR	FC-EW	FC-OLS	FC-RANK	FC-MSE	FC-RMSE
	Daily re-balancing															
Risk aversion	$\delta = 0.01$	0.493	0.542	0.693	1.021*	1.035*	0.691	0.430	0.445	0.649	0.217	0.779	0.676	0.641	1.185*	1.483*
	$\delta = 0.10$	0.557	0.932*	0.595	0.912*	0.836	0.646	0.744	0.673	0.522	0.642	0.733	0.776	0.977*	1.230*	0.676
	$\delta = 0.50$	0.657	1.079*	0.784	1.153*	0.555	0.547	0.252	0.257	0.277	0.576	0.829*	0.891*	0.773	0.842*	0.838*
	$\delta = 1.00$	0.705	0.853	0.831	1.298*	0.757	0.671	0.545	0.559	0.459	0.568	0.823*	0.915*	0.800	0.840*	0.945*
	$\delta = 2.00$	0.728	1.143*	1.013*	1.300*	0.867	0.801	0.582	0.582	0.512	0.607	0.893*	0.983*	0.863	0.912*	1.105*
	Weekly re-balancing															
Risk aversion	$\delta = 0.01$	0.553	0.574	0.592	0.667	0.867*	0.898*	0.755	0.656	0.322	0.219	1.100*	1.164*	0.989*	1.115*	0.838*
	$\delta = 0.10$	0.744	0.694	0.796	0.687	0.878	0.899	0.661	0.490	0.439	0.64	0.704	0.735	0.756	0.717	0.891*
	$\delta = 0.50$	0.576	0.579	0.680	0.678	0.844*	0.802*	0.252	0.386	0.379	0.751	0.752	0.765	0.741	0.763	0.708
	$\delta = 1.00$	0.618	0.694	0.830*	0.644	0.594	0.612	0.545	0.553	0.550	0.568	0.919*	0.999*	1.067*	0.939*	0.789
	$\delta = 2.00$	0.620	0.635	0.605	0.890*	0.943*	0.787	0.582	0.498	0.582	0.678	0.939*	0.972*	0.950*	0.958*	1.050*
	Monthly re-balancing															
Risk aversion	$\delta = 0.01$	0.644	0.732	0.766	0.687	0.864*	0.872*	1.104*	0.783	0.573	0.254	0.829*	0.891*	0.773	0.842*	0.838
	$\delta = 0.10$	0.658	0.750	0.718	0.667	0.881*	0.876*	0.714	0.508	0.521	0.790	0.710	0.802	0.636	0.702	0.893*
	$\delta = 0.50$	0.634	0.741	0.690	0.678	0.995*	0.983*	0.470	0.447	0.444	0.725	0.969*	1.017*	1.018*	0.992*	1.009*
	$\delta = 1.00$	0.788	0.860	0.820	0.850	1.049*	1.030*	0.507	0.528	0.526	0.555	0.937*	1.032*	0.912*	0.940*	0.687
	$\delta = 2.00$	0.577	0.741	0.697	0.632	1.149*	1.199*	0.711	0.744	0.731	0.659	0.960*	1.028*	0.958*	0.970*	0.911*
	Bimonthly re-balancing															
Risk aversion	$\delta = 0.01$	0.533	0.535	0.521	0.702	0.795*	0.775*	0.645	0.967*	0.784	0.628*	0.823*	0.915*	0.800*	0.840*	0.945*
	$\delta = 0.10$	0.543	0.543	0.521	0.702	0.805*	0.764*	0.726	0.593	0.595	0.985*	0.806	0.815*	0.758	0.793	0.931*
	$\delta = 0.50$	0.562	0.554	0.521	0.718	0.865*	0.730	0.510	0.491	0.493	0.724	0.937*	1.032*	0.912*	0.940*	0.687
	$\delta = 1.00$	0.762	0.756	0.736	0.600	0.817	0.717	0.485	0.444	0.467	0.506	0.956*	0.972*	0.988*	0.856	0.665
	$\delta = 2.00$	0.551	0.554	0.618	0.625	0.794*	0.779*	0.767	0.706	0.704	0.793	0.948*	1.029*	0.886*	0.961*	0.831*
	Quarterly re-balancing															
Risk aversion	$\delta = 0.01$	0.647	0.662	0.934*	0.670	0.815	0.727	0.956*	0.841	1.070*	0.760	0.893*	0.983*	0.863*	0.912*	1.105*
	$\delta = 0.10$	0.686	0.667	0.843	0.670	0.815	0.727	0.766	0.702	0.714	0.948*	0.940*	0.952*	0.929*	0.932*	0.928*
	$\delta = 0.50$	0.827	0.670	0.707	0.670	0.825	0.727	0.651	0.657	0.662	0.819	0.937	1.032*	0.912	0.940*	0.687*
	$\delta = 1.00$	0.915	1.019	0.403	0.694	0.870	0.707	0.5	0.504	0.499	0.524	0.981	0.840	1.034	0.743	0.708
	$\delta = 2.00$	0.694	0.598	0.688	0.672	0.883*	0.728	0.845	0.818	0.799	0.885	1.056*	1.146*	0.850	0.905*	0.848

* Indicates that the Sharpe ratio is statistically different with respect to that of the mean–variance portfolio obtained with the random walk specification at a significance level of 10%.

majority of the cases. In this sense, our evidence corroborates the previous results in the literature, such as those reported in Leitch and Tanner (1991), Carriero et al. (2012), and Cenesizoglu and Timmermann (2012).

5. Concluding remarks

This paper examines the statistical accuracy and economic value of modeling and forecasting the term structure of interest rates using forecast combinations. Combined forecasts have been extensively and successfully applied to forecast many economic time series (see, for example, Granger, 1989; Clemen, 1989; Granger and Jeon, 2004; Timmermann, 2006; Wallis, 2011). However, the literature on yield curve forecasting has focused on performance evaluation of individual forecast models. Moreover, the majority of yield curve studies consider only statistical evaluation of point forecasts, while the final goal of interest rate forecasts is not to minimize mean squared prediction error but to improve financial decision making.

Thus, we conduct a detailed forecast performance evaluation of several univariate and multivariate models based not only on statistical measures of accuracy, but also an economic evaluation using Sharpe ratios of optimal mean–variance fixed income portfolios constructed based upon forecasts from individual models and their alternative combinations. Differently from other yield curve studies, we use bootstrap methods to determine if differences between Sharpe ratios are statistically different.

The results reveal that the statistical and economic evaluation of forecast performance might be related. However, the correlations between RMSFE and Sharpe ratios are often very low and close to zero in some cases. Moreover, the ranking

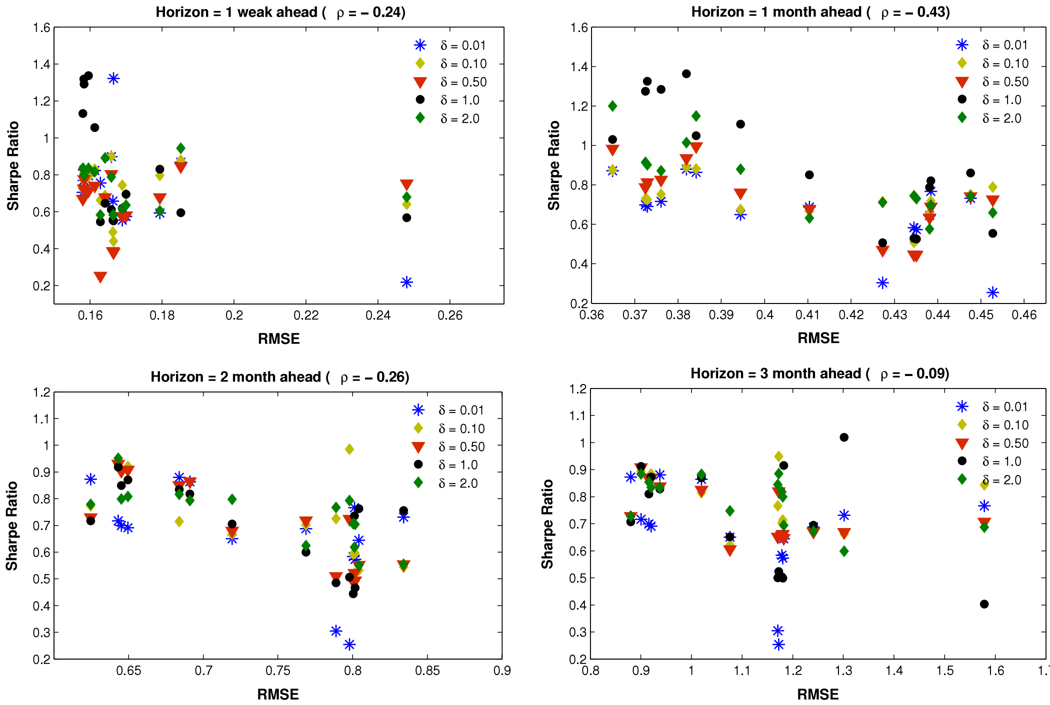


Fig. 4. Relation between RMSFE and Sharpe ratios. Note: this figure presents scatter plots of out-of-sample economic performance measures against RMSFE (root mean squared forecast error). The evaluation sample is 2009:1–2012:12 (988 out-of-sample forecasts).

of specifications based on statistical performance can differ from the ranking based on economic criteria. More specifically, while the forecast combination schemes outperformed in terms of RMSFE the remaining specification mostly for shorter forecast horizons, they outperform individual models in the vast majority of cases when the Sharpe ratio was considered.

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