Lecture 1

1 Main Definitions

Definition 1: A <u>time series</u> is a collection of <u>random variables</u>

$$\{X_t\}_{t \in T}$$

that are indexed by time.

A <u>time series data set</u> usually refers to a realization of a time series. In practice, time series and time series data are used interchangeably.

Definition 2: A <u>time series model</u> is a collection of **joint** distributions for a time series $\{X_1, X_2, \ldots, X_T\}$.

EXAMPLE 1 (based on section 1.1.4 in the book, p. 3). For $t = 1 \ldots 200$, let $(\varepsilon_1, \ldots, \varepsilon_{200})$ be a collection of i.i.d. $\mathcal{N}(0, .25)$. An example of a time series model for $(X_1, \ldots, X_{200})$ is:

$$X_t = \cos(t/10) + \varepsilon_t, \quad t = 1, 200.$$

There are two things that we will usually do with a time series model (or any other statistical model): *analyze the marginal distributions* and *model simulation.*

1. Analysis of Marginal Distributions: The time series model gives the distribution of the full collection $(X_1, X_2, \ldots, X_t)$ (this is a multivariate distribution). In particular, it gives the distribution of each of the X_t's individually. In this example:

$$X_t \sim \mathcal{N}(\cos(t/10), .25)$$

This means that each X_t is normal centered at $\cos(t/10)$ (thus, the mean depends on time) and variance .25.

2. Model Simulation: One thing that we will do throughout the course is use the model to *simulate* data. Simulation will refer to act as if we were nature (more of this on Lecture 3).