U&UPDE.

Exercise session 5

May 16

2022

Feedback: Homework 2 — Part 1

1] we are interested in calculating:

$$g = \int_{[0,1]^N} f(x)\, dx$$

* **QMC under exact sampling.**

$$g = \int_{[0,1]^N} f(x)\, dx = E[f(U)] \quad U \sim \mathcal{U}([0,1]^N)$$

suppose $v_1 \cdots v_M$ low discrepancy pts

$$\hat{g} \simeq \frac{1}{M} \sum_{i=1}^{M} f(v_i)$$

↳ deterministic

* For errors, we use **Randomized QMC.**

$v_i \sim \tilde{v}_i \qquad \tilde{v}_i \sim \mathcal{U}([0,1]^N)$

$$\tilde{v}_i = \{(v_i + u)\}_1$$

$\rightarrow \tilde{v}_i$ is uniform over $[0,1]^N$.

RQMC estimator:

standard error: $O(m^{-1/2} M^{-1/2-\delta})$

$$\hat{g}_{RQMC} = \frac{1}{m} \sum_{k=1}^{m} \frac{1}{M} \sum_{j=1}^{M} f(\tilde{v}_{j,k}).$$

$$\tilde{v}_{j,k} = (v_j + u_k) \bmod 1 .$$

CLT: estimate the variance.

$$\mathrm{var}(\hat{g}_{RQMC}) = \frac{1}{m-1} \sum_{k=1}^{m} \left(\frac{1}{M} \sum_{j=1}^{M} f(\tilde{v}_{k,j}) - \hat{g} \right)^2$$

$$\rightarrow RQMC : \varepsilon \simeq \frac{C_\alpha}{\sqrt{m}} \sqrt{\mathrm{var}(\hat{g})}$$

Tasks :

choose $m = 10, 20$ ($m << \Pi$).

plot the variance as f^{-} of Π.

compare it to $\Pi^{-1/2}$ (MC).

Do the computation for at least

$N = 2$ and $N = 20$.

$\boxed{3}$

$$I = \frac{1}{2\pi} \int \max(e^{x_1} + e^{x_2} - K, 0)\, e^{-\frac{(x_1^2 + x_2^2)}{2}} \, dx_1 \, dx_2 .$$

$$= E[\max(e^{x_1} + e^{x_2} - K, 0)] \quad *$$

$$x = (x_1, x_2) \sim N(0, I).$$

indep

$$* = E_{\mu_1^*, \mu_2^*}\left[\max(e^{x_1} + e^{x_2} - K, 0)\, L(x_1, x_2)\right]$$

$x_1 \sim N(\mu_1^*, 1)$, $x_2 \sim N(\mu_2^*, 1)$.

Importance sampling (shift).

use a new dist with a new mean.

$(\mu_1^*, \mu_2^*) = \text{argmax}(g \cdot f)$. (use the one from HW1).

$\Rightarrow \mathbb{E}[\max(e^x + e^{-x} - K, 0)] =$

$\mathbb{E}[\max(\quad) * L(x)]$.

$x \sim N((\mu_1^*, \mu_2^*), 1)$.

How to sample:

* For MC, $\{x_u\} \sim U([0,1])$

to sample from Gauss:

$$x_G = \Phi^{-1}(x_u)$$

Φ is the CDF of the standard normal distr.

For RQMC :

* Generate Sobol seq $\tilde{V}_i = \{(V_i + U)\}_1$

* Map it back to Normal (mean μ)

$$V_N = \Phi^{-1}(\tilde{V}_i) + \mu \sim N(\mu, 1)$$

$$\rightarrow \mathbb{E}[\max(\ ) \cdot L(x)]^2$$

$$\frac{1}{m} \sum_{k=1}^{m} \frac{1}{n} \sum_{j=1}^{n} \max(e^{V_{N_1}} + e^{V_{N_2}} - K, 0)$$

$$* \ L(V_N, \mu).$$

tasks :

plot $\varepsilon_{RQMC}|_{IS}$ and compare it to

ε_{MC} and ε_{RQMC}