HW 2 Hints Part 2

1) Goal: Compute

$$\mathbb{E}[Q(u_h)] = \mathbb{E}[\tilde{F}_1(Y_1, \dots, Y_N)]$$
$$= \mathbb{E}[\tilde{F}_2(U_1, \dots, U_N)]$$

setup:
- use Sobol' sequence
- randomize
- calculate the expectation

error estimation:
- statistical error using $\left\{ \begin{array}{l} \text{RQMC} \\ + \text{CLT} \end{array} \right.$
- bias error using Richardson extrapolation:
  - $\mathbb{E}[Q(u_h)] - \mathbb{E}[Q(u_{2h})] \simeq \left(1 - \frac{1}{2^p}\right) C h^p + \dots$
  - determine p

relative error: $\varepsilon < TOL$

ε_{bias} $\quad$ ε_{CLT}^{RQMC}

$\rightarrow$ adaptively
- for the optimal number of samples M
- for the optimal meshsize h.

2) Starting from:

$$\mathbb{E}[Q(u_h)] = \mathbb{E}[Q(u_{2h})] + \mathbb{E}[Q(u_h) - Q(u_{2h})]$$

where

$$Q(u_{2h}) = \tilde{F}_1^{2h}(Y_1, \dots, Y_N)$$
$$Q(u_h) = \tilde{F}_1^{h}(Y_1, \dots, Y_N).$$

Solve the minimization problem

$$\underset{M_1, M_2, S}{\operatorname{argmin}} \ \overset{\text{costs}}{\tilde{C}(M_1, M_2, S)}$$

$$\text{s.t.} \quad \varepsilon_{stat} \leq TOL$$

using Lagrange multiplier and comment on its worthiness

3) Compute the rare event probability

$$\mathbb{P}(Q(u_n) < K)$$

using Importance sampling the same way as in HW1.

- use RQMC to estimate the expectation
- use RQMC + CLT to estimate the error.

4) self-explanatory

5) 1. We know that

$$|Q(u)| \leq \tilde{q}_0 \exp\left(\sum_{n \geq 1} \|b_n\|_\infty |Y_n| \right)$$

with

Norm on the dual of H_0^1

$$\tilde{q}_0 = \frac{\|Q\|_{H^{-1}(D)} \, C_D \, \|f\|_{L^2(D)}}{\exp(\min_{x \in D} b_0(x))}$$

2. approximate $I_{-Q(u)}(-K)$ numerically.