PhD Econometrics (ECON50580)
Problem Set 3: IV and MTE

This problem set will be graded. Rules:

- You have to work in groups of 3-5 students
- Submit your solutions in 1 pdf
- The code should be in the appendix
- Results should be presented graphically or in tables. No screenshots from statistical software!
- Use soft-coding in your code.
- For empirical exercises, show evidence that you used version control
- Submit via Brighspace

Submission deadline: Wednesday, February 24

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1 Theory

Consider a potential outcomes model with

$$Y^1 = X\beta^1 + U^1, \quad Y^0 = X\beta^0 + U^0, \tag{1}$$

whereby the superscripts stand for treated (1) and untreated (0) states. The observed treatment decision is denoted by $D(Z) \in \{0,1\}$. Treatment is assigned based on the rule

$$D(Z) = 1[Z\theta + U^D \geq 0]. \tag{2}$$

(U^1, U^0, U^D) is assumed to be independent of Z and X, and jointly normally distributed with

$$\begin{bmatrix} U^D \\ U^1 \\ U^0 \end{bmatrix} \sim N\left(0, \begin{bmatrix} 1 & \sigma_{1D} & \sigma_{0D} \\ \sigma_{1D} & \sigma_1^2 & \sigma_{10} \\ \sigma_{0D} & \sigma_{10} & \sigma_0^2 \end{bmatrix}\right) \tag{3}$$

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1) Show that for $x = X$ and $z = Z$ the Average Treatment Effect on the Treated (ATT) is

$$\text{ATT}(x, z, D[z] = 1) = x\left(\beta^1 - \beta^0\right) + \left(\rho_1\sigma_1 - \rho_0\sigma_0\right)\frac{\phi(z\theta)}{\Phi(z\theta)}. \tag{4}$$

2) Explain how the ATT and Average Treatment Effect (ATE) are related if people self-select into treatment based on their gains from treatment ($cov(U^1 - U^0, U^D) > 0$).

3) a) Show that the LATE for $z' > z > 0$ is

$$\text{LATE}\left(x, D[z] = 0, D\left[z'\right] = 1\right) = x\left(\beta^1 - \beta^0\right) + \left(\rho_1\sigma_1 - \rho_0\sigma_0\right)\frac{\phi\left(z'\theta\right) - \phi(z\theta)}{\Phi\left(z'\theta\right) - \Phi(z\theta)}. \tag{5}$$

b) Suppose the instrument is a compulsory schooling reform that forces everyone to take the treatment. Explain how in this case the LATE is related to the Average Treatment Effect on the Untreated (ATU).

4) Show that the Marginal Treatment Effect (MTE) at $x = X$ and $u^D = U^D$ is

$$MTE(x, u^D) = x\left(\beta^1 - \beta^0\right) + \left(\rho_1\sigma_1 - \rho_0\sigma_0\right)u^D. \tag{6}$$

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2 Simulation Exercise

An important property of the IV estimator is that it is biased in small samples but consistent. For this reason one should never write that an IV estimator is used to obtain unbiased estimates. We want to better understand the small sample properties through simulations of the sampling distribution of IV and OLS estimators. In all simulations, let x, y, z, u and ε be random variables and assume the data-generating process

$$\begin{aligned} y &= \alpha + \beta x + \varepsilon, \\ x &= \gamma_0 + \gamma_1 z + u \end{aligned}$$

Set the parameter $\beta = 1$ and, unless required otherwise, $\alpha = \gamma_0 = 0$. Moreover, construct ε such that it's Pearson correlation with x is 0.4.

a) Sampling distribution under strong instruments Construct the instrument z such that its Pearson correlation with x is 0.5 while its correlation with ε is zero. Consider four different sample sizes: $N = 50$, $N = 100$, $N = 250$ and $N = 1000$. For each sample size, run at least 10,000 simulations whereby you estimate β^{OLS} and β^{IV} (choose fewer replications if you have problems with computing power). In the same graph, plot the sampling distributions for OLS and IV for all four sample sizes. Discuss the difference in shape of the sampling distributions between OLS and IV.

b) Weaker instruments Now repeat the analysis from a), but instead assume that the corrleation between the instrument z and the regressor x is 0.15. Discuss the difference in sampling distributions between OLS and IV and, in addition, discuss the difference between sampling distributions based on weak and strong IVs.