Computational Algebra. Lecture 12

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Hensel Lifting

Homework:

1. Implement both algorithms for $R = \mathbb{F}_p[y]$, with $p \in \mathbb{Z}$ prime.

Hensel Lifting

Apply your implementations to solve the following problem: consider

$$\begin{aligned} f \quad = \quad & x^5 + (3y^3 + 39y^2 + 50y + 28)x^4 + (36y^5 + 2y^4 + 47y^3 + 63y^2 + 49y + 58)x^3 \\ & +(91y^6 + 18y^5 + 81y^4 + 37y^3 + 36y^2 + 53y + 64)x^2 \\ & +(74y^7 + 54y^6 + 24y^5 + 39y^4 + 71y^3 + 18y^2 + 93y + 53)x \\ & +(62y^6 + 72y^5 + 87y^4 + 27y^3 + 19y^2 + 61y) \quad \in \mathbb{F}_{97}[x, y]. \end{aligned}$$

1. Compute a factorization $f \equiv gh \pmod{y}$ with coprime non-constant polynomials $s, g, h, t \in \mathbb{F}_{97}[x]$ with $sg + th = 1$.
2. Execute two successive Hensel steps (Hensel lifting with $m = y$ and $m = y^2$) to obtain polynomials $g^*, h^* \in \mathbb{F}_{97}[x, y]$ such that $f \equiv g^* h^* \pmod{y^4}$, with $g \equiv g^* \pmod{y}$ and $h \equiv h^* \pmod{y}$.