TUM ModSim, SoSe 2023

Mitschriften basierend auf der Vorlesung von Dr. Hans-Joachim Bungartz

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Introduction

About

Hier sind die wichtigsten Konzepte der ModSim Vorlesung von Dr. Hans-Joachim Bungartz im Sommersemester 2023 zusammengefasst.

Die Mitschriften selbst sind in Markdown geschrieben und werden mithilfe einer GitHub-Action nach jedem Push mithilfe von Pandoc zu einem PDF konvertiert.t

Eine stets aktuelle Version der PDFs kann über modsim_SS23_IN2010_merge.pdf heruntergeladen werden.

Implementation

Außerdem befindet sich eine Implementation von verschiedenen Algorithmen im Ordner `/algorithms` auf GitHub. Diese sind in Python und unter der Verwendung von NumPy geschrieben.

How to Contribute

1. Fork this Repository
2. Commit and push your changes to **your** forked repository
3. Open a Pull Request to this repository
4. Wait until the changes are merged

Contributors

Inhaltsverzeichnis

Focus Analysis / Calculus

Foundations

Functions and their representations

- One-Dimensional

$$f : D \subseteq \mathbb{R}^n \to \mathbb{R}^m, x \mapsto f(x)$$

- Multidimensional

$$f : D \subseteq \mathbb{R} \to \mathbb{R}, x = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} \mapsto f(x) = \begin{pmatrix} f_1(x_1, \dots, x_n) \\ \vdots \\ f_m(x_1, \dots, x_n) \end{pmatrix}$$

Names for special types of functions

- Curves: $n = 1$ and $m \in \mathbb{N}$
  - plane curves (2D): $n = 1$ and $m = 2$
  - space curves (3D): $n = 1$ and $m = 3$
- Surfaces: $n = 2$ and $m = 3$
- Scalar fields: $n \in \mathbb{N}$ and $m = 1$
- Vector fields: $n = m$

Topology concepts in higher dimensions

There is an analogous concept to open and closed intervals in multi-dimensional spaces.
Given a domain $D \subseteq \mathbb{R}^n$ and its complement $D^c = \mathbb{R}^n \setminus D$ - A point x is called *inner point* if there exists an arbitrarily small ball around this points that fullly lies inside D. - The set of all inner points of D is called the *interior* of D and is denoted as $\mathring{D}$. - The domain is called open if $D = \mathring{D}$ - A point $x_0 \in \mathbb{R}^n$ is called *boundary point* if any arbitrarily small ball around this point intersects with both D and its complement D^c - The set of all boundary points of D is called the *boundary* of $D, denoted ∂D - The set $\bar{D} = D \cup 0\partial D$ is called the *closure* of D

Using these definitions there are multiple attributes assignable to domains.
A domain D is called: - *closed* if $\partial D \subseteq D$, i.e. $\bar{D} = D$ - *bounded* if $\exists K \in \mathbb{R} : ||x|| < K, \forall x \in D$ - *compact* if it is closed and bounded - *convex* if all points on a straight line between to points in D are themselves element of D

Continuity

We define continuity in multi-dimensional spaces using converging vector sequences.
A sequence $(x^{(k)})$ converges to the limit x if

$$\lim_{k \to \infty} ||x^{(k)} - x|| = 0$$

Converges of a vector sequence is also equivalent to the convergence of all components.
A vector function is then called continuous at $a \in D$ if for all sequences $(x^{(k)})_{k \in \mathbb{N}_0}$ in D converging to a the corresponding sequence $(f(x^{(k)}))_{k \in \mathbb{N}_0}$ in $\mathbb{R}^m$ converges to $f(a)$ and continuous on D if this holds for all points $a \in D$