

# TUM ModSim, SoSe 2023

Mitschriften basierend auf der Vorlesung von Dr. Hans-Joachim Bungartz

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# Introduction

## About

Hier sind die wichtigsten Konzepte der ModSim Vorlesung von Dr. Hans-Joachim Bungartz im Sommersemester 2023 zusammengefasst.

Die Mitschriften selbst sind in Markdown geschrieben und werden mithilfe einer GitHub-Action nach jedem Push mithilfe von [Pandoc](#) zu einem PDF konvertiert.

Eine stets aktuelle Version der PDFs kann über [modsim\\_SS23\\_IN2010\\_merge.pdf](#) heruntergeladen werden.

## Implementation

Außerdem befindet sich eine Implementation von verschiedenen Algorithmen im Ordner `/algorithms` auf [GitHub](#). Diese sind in Python und unter der Verwendung von [NumPy](#) geschrieben.

## How to Contribute

1. Fork this Repository
2. Commit and push your changes to **your** forked repository
3. Open a Pull Request to this repository
4. Wait until the changes are merged

## Contributors

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# Focus Analysis / Calculus

## Foundations

### Functions and their representations

- One-Dimensional

$$f : D \subseteq \mathbb{R}^n \rightarrow \mathbb{R}^m, x \mapsto f(x)$$

- Multidimensional

$$f : D \subseteq \mathbb{R}^n \rightarrow \mathbb{R}^m, x = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} \mapsto f(x) = \begin{pmatrix} f_1(x_1, \dots, x_n) \\ \vdots \\ f_m(x_1, \dots, x_n) \end{pmatrix}$$

### Names for special types of functions

- Curves:  $n = 1$  and  $m \in \mathbb{N}$ 
  - plane curves (2D):  $n = 1$  and  $m = 2$
  - space curves (3D):  $n = 1$  and  $m = 3$
- Surfaces:  $n = 2$  and  $m = 3$
- Scalar fields:  $n \in \mathbb{N}$  and  $m = 1$
- Vector fields:  $n = m$

### Topology concepts in higher dimensions

There is an analogous concept to open and closed intervals in multi-dimensional spaces.

Given a domain  $D \subseteq \mathbb{R}^n$  and its complement  $D^c = \mathbb{R}^n \setminus D$  - A point  $x$  is called *inner point* if there exists an arbitrarily small ball around this point that fully lies inside  $D$ . - The set of all inner points of  $D$  is called the *interior* of  $D$  and is denoted as  $\mathring{D}$ . - The domain is called open if  $D = \mathring{D}$ . - A point  $x_0 \in \mathbb{R}^n$  is called *boundary point* if any arbitrarily small ball around this point intersects with both  $D$  and its complement  $D^c$ . - The set of all boundary points of  $D$  is called the *boundary* of  $D$ , denoted  $\partial D$ . - The set  $\bar{D} = D \cup \partial D$  is called the *closure* of  $D$ .

Using these definitions there are multiple attributes assignable to domains.

A domain  $D$  is called: - *closed* if  $\partial D \subseteq D$ , i.e.  $\bar{D} = D$  - *bounded* if  $\exists K \in \mathbb{R} : \|x\| < K, \forall x \in D$  - *compact* if it is closed and bounded - *convex* if all points on a straight line between two points in  $D$  are themselves element of  $D$

### Continuity

We define continuity in multi-dimensional spaces using converging vector sequences.

A sequence  $(x^{(k)})$  converges to the limit  $x$  if

$$\lim_{k \rightarrow \infty} \|x^{(k)} - x\| = 0$$

Convergence of a vector sequence is also equivalent to the convergence of all components.

A vector function is then called continuous at  $a \in D$  if for all sequences  $(x^{(k)})_{k \in \mathbb{N}_0}$  in  $D$  converging to  $a$  the corresponding sequence  $(f(x^{(k)}))_{k \in \mathbb{N}_0}$  in  $\mathbb{R}^m$  converges to  $f(a)$  and continuous on  $D$  if this holds for all points  $a \in D$