Calculus exercises

Manuel Lleonart Anguix

Exercise 1. *Of the following say if they are functions and, in this case, define it's Dom and Im.*

- $f(x) = \sin x$

-
$$f(1) = 1, \ f(2) = 1, \ f(3) = 1$$

-
$$f(1) = 1, \ f(1) = 2, \ f(2) = 2, \ f(3) = 3$$

- $f(x) = x$

Exercise 2. *Draw the following functions:*

- $f(x) = \cos x$

- $f(x) = \log x^2$

- $f(x) = \frac{x^2+1}{x^2+1}$

- $f(x) = \frac{1}{x^2+1}$

- $f(x) = \frac{1}{x^2}$

- $f(x) = \frac{\cos x}{\sin x}$

*You **MUST** study its limits when they tend to infinity, study the points where the function may give problems, say when the function is increasing or decreasing and show where the function has a maximum or a minimum.*

Exercise 3. *Are the following functions continuous? Why? Why not?*

- $\frac{1}{x^2}$

- $\frac{1}{x}$

- $\frac{1}{x^2+1}$

- $\log(x)$

- $\frac{\log(x)}{\sin x}$

- $f(x) = \begin{cases} x^2 + 1 & if\, x > 1 \\ 2x & if\, x \le 1 \end{cases}$

Exercise 4. *Find the derivatives of the following functions:*

1. $f(x) = \frac{5}{x^2}$

2. $\frac{x+1}{x-1}$

3. $(5x^2 - 3)(x^2 + x + 4)$

4. $\sqrt{x}$

5. $\frac{1}{\sqrt{x}}$

6. $\frac{1}{x\sqrt{x}}$

7. $\frac{\sqrt[3]{x^2}+\sqrt{x}}{x^2}$

8. $(x^2 + x + 1)^2$

9. $\sqrt{\frac{x^2+1}{2x}}$

10. $\frac{x^2+-1}{x+1}$

11. $e^{\sqrt{x}}$

12. 10^{3-x^2}

13. $f(x) = \frac{e^2 + e^{-x}}{2}$

14. $f(x) = \sin(x^2)$

15. $f(x) = \frac{e^{2x}}{x^2}$

16. $f(x) = \log\left(\frac{x^2+1}{x+2}\right)$

17. $f(x) = \log(2x^4 + x^2)$

18. $f(x) = \log\sqrt{x(1-x)}$

19. $f(x) = \log \sin(x^2)$

20. $f(x) = 0$

21. $f(x) = 27$

22. $f(x) = -\frac{x^3 + x - 1}{2}$

23. $f(x) = -\frac{1}{\sqrt{2^x}}$

24. $f(x) = \log(\sin(x) + 3)$

25. $f(x) = \frac{\log(x)}{\sin(x)}$

26. $f(x) = \frac{1}{\sin(x+1)}$

27. $f(x) = \sin((x^2 + 1)(x^3 - 1))^2$

28. $f(x) = \log(x - 1)e^x$

29. $f(x) = \cos x \tan x$

30. $\arctan(x^2 + 1)$

Exercise 5. *Calculate the Taylor's order three development of:*

1. x

2. e^x

3. $\sin x$

Exercise 6. *Are the functions in exercise 4 $\mathcal{C}^\infty$? And $\mathcal{C}^2$*

Exercise 7. *Solve the following integrals.*

1.
$$\int \frac{1}{x^2 \sqrt[5]{x^2}} dx$$

2.
$$\int (x + 2)^3 dx$$

3.
$$\int (2x + 1)(x^2 + x + 1) dx$$

4.
$$\int \frac{x + 1}{\sqrt[3]{x^2 + 2x + 7}} dx$$

5.
$$\int \sin 2x \cos 2x \, dx$$

6.
$$\int \sin^4 x \cos x \, dx$$

7.
$$\int \tan^2 x \sec^2 x \, dx$$

8.
$$\int \frac{arctanx}{1 + x^2} \, dx$$

9.
$$\int \frac{2x}{1 + x^2} \, dx$$

10.
$$\int \tan x \, dx$$

11.
$$\int \frac{5^{3x}}{5^{3x} + 7} \, dx$$

12.
$$\int \frac{1}{x \ln x} \, dx$$

13.
$$\int \frac{1}{cos^2 x \tan x} \, dx$$

14.
$$\int \frac{x + 1}{x} \, dx$$

15.
$$\int \frac{x + 1}{x - 5} \, dx$$

16.
$$\int \frac{3x^2 + 5x}{x^2 + 1} \, dx$$

17.
$$\int e^{2x+2} \, dx$$

18.
$$\int 5^x \, dx$$

19.
$$\int 2^x 5^x dx$$

20.
$$\int 8^{3x+1} dx$$

21.
$$\int \frac{e^{\ln x}}{x} dx$$

22.
$$\int e^{\sin x} \cos x dx$$

23.
$$\int \frac{e^{\arcsin x}}{\sqrt{1-x^2}} dx$$

24.
$$\int (3 - \sin x) dx$$

Exercise 8. *Do the following integrals whenever you can. (These are the previous integrals but definite, don't do them again).*

1.
$$\int_0^1 \frac{1}{x^2 \sqrt[5]{x^2}} dx$$

2.
$$\int_{-\infty}^{\infty} (x + 2)^3 dx$$

3.
$$\int_0^1 0(2x + 1)(x^2 + x + 1) dx$$

4.
$$\int_0^5 \frac{x + 1}{\sqrt[3]{x^2 + 2x + 7}} dx$$

5.
$$\int_0^{2\pi} \sin 2x \cos 2x dx$$

6.
$$\int_0^{2\pi} \sin^4 x \cos x dx$$

7.
$$\int_0^{2\pi} \tan^2 x \sec^2 x dx$$

8.
$$\int_0^{2\pi} \frac{arctanx}{1+x^2}dx$$

9.
$$\int_0^1 \frac{2x}{1+x^2}dx$$

10.
$$\int_0^{2\pi} \tan x\, dx$$

11.
$$\int_0^{2\pi} \frac{5^{3x}}{5^{3x}+7}dx$$

12.
$$\int_0^e \frac{1}{x\ln x}dx$$

13.
$$\int_0^{2\pi} \frac{1}{cos^2x \tan x}dx$$

14.
$$\int_0^1 \frac{x+1}{x}dx$$

15.
$$\int_{-5}^5 \frac{x+1}{x-5}dx$$

16.
$$\int_0^1 \frac{3x^2+5x}{x^2+1}dx$$

17.
$$\int_0^1 0e^{2x+2}dx$$

18.
$$\int_1^2 5^x dx$$

19.
$$\int_0^3 2^x 5^x dx$$

20.
$$\int_2^4 8^{3x+1}dx$$

21.
$$\int_0^e \frac{e^{\ln x}}{x}dx$$

22.
$$\int_0^{2\pi} e^{\sin x} \cos x \, dx$$

23.
$$\int_0^{2\pi} \frac{e^{\arcsin x}}{\sqrt{1-x^2}} dx$$

24.
$$\int_0^{2\pi} (3 - \sin x) dx$$