Numerical Simulation of Bloodflow in Aneurysms using Lattice Boltzmann Method

- A Master Thesis by Jan Götz at FAU Erlangen -

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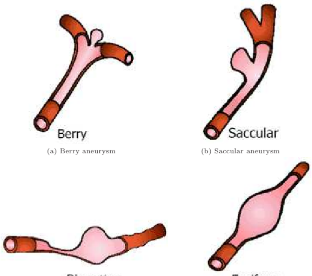
What are aneurysms and what differs blood from water?

- Aneurysms are localized, blood-filled balloon-like bulges in the wall of a blood vessel.
- Currently, no one can predict whether an aneurysm will rupture. $\rightarrow$ **pressure simulation** desirable



(a) Berry aneurysm (b) Saccular aneurysm (c) Dissecting aneurysm (d) Fusiform aneurysm

- Blood is assumed to be a Newtonian, isotropic, incompressible and homogeneous fluid.

Fluids are normally modeled by the

Navier Stokes Equations

$$\frac{\partial \mathbf{U}_j}{\partial t} + \mathbf{U}_i \frac{\partial \mathbf{U}_j}{\partial \mathbf{x}_i} = \mathbf{g}_j - \frac{1}{\rho}\frac{\partial P}{\partial \mathbf{x}_j} + \frac{\eta}{\rho}\frac{\partial \boldsymbol{\sigma}_{ij}}{\partial \mathbf{x}_i}, \quad j \in \{1,2,3\}, \tag{1}$$

$$\frac{\partial \mathbf{U}_i}{\partial \mathbf{x}_i} = 0, \tag{2}$$

where the following physical properties are introduced:

- $\mathbf{U}_j$ are the components of the velocity of the fluid,
- ρ is the density of the fluid,
- P is the pressure of the fluid,
- $\boldsymbol{\sigma}_{ij}$ is the stress tensor.

The Boltzmann equation is based on a **particle model**
$\rightarrow$ small scales

The Boltzmann Equation

$$\frac{\partial f_j}{\partial t} + \xi \cdot \frac{\partial f_j}{\partial \mathbf{x}} + \underbrace{\mathbf{G} \cdot \frac{\partial f_j}{\partial \xi}}_{\approx 0} = Q(f_j, f_j), \quad j \in 0, 1, ..., N-1 \qquad (3)$$

- $f_j(\mathbf{x}, \xi, t)$ is a distribution function,
- ξ is the velocity of a particle,
- $\mathbf{x}$ is the position of a particle,
- $\mathbf{G}$ is the force acting on a particle,
- Q is the quadratic collision operator.

Advantages of the LBM:

- direct calculation of the stress tensor σ_{ij},
- it avoids the nonlinear convection term of the NSE,

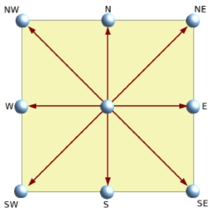
Approximation of the collision operator Q (right-hand side):

$$\frac{\partial f_\alpha}{\partial t} + \xi_\alpha \cdot \nabla f_\alpha = \frac{1}{\lambda}(f_\alpha - f_\alpha^{(eq)})$$

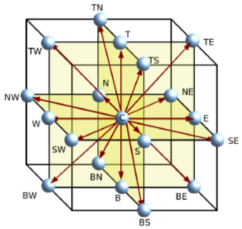
- relaxation time $\lambda \in \mathbb{R}_+$
- equilibrium solution $f_\alpha^{(eq)}$

The Algorithm:

- Introduce the unknown discretized distribution function f_α
- Start with initial particle positions $\mathbf{x}^0$, velocities ξ^0 and f_α^0



(a) D2Q9 model for two dimensions



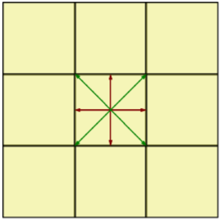
(b) D3Q19 model for three dimensions

- After each time step, update density and velocity:

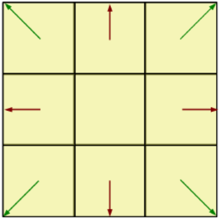
$$\rho = \sum_\alpha f_\alpha, \quad \rho\mathbf{U} = \sum_\alpha f_\alpha \xi_\alpha$$

Stream Step:

- move fluid particles to neighbor cells



(a) Particle distribution before stream step



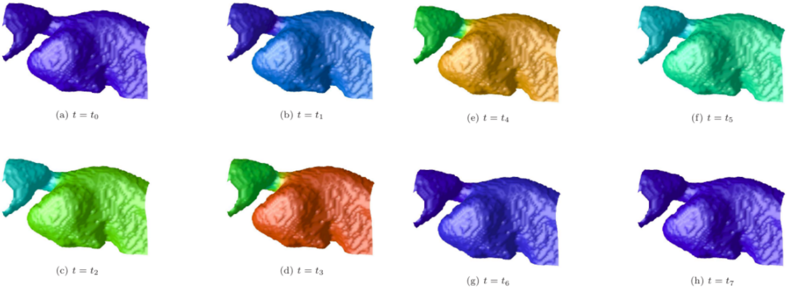
(b) Particle distribution after stream step

Collide Step:

- evaluate collision function

Let's consider some numerical results...

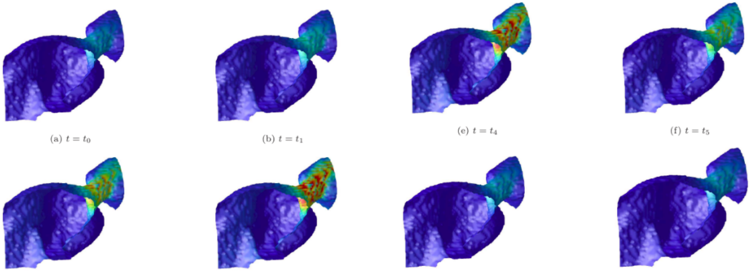
- Pressure on the wall in lattice units in the CCA during pulse:



(a) $t = t_0$ (b) $t = t_1$ (e) $t = t_4$ (f) $t = t_5$

(c) $t = t_2$ (d) $t = t_3$ (g) $t = t_6$ (h) $t = t_7$

- Von Mises stress near wall in lattice units in the CCA during pulse:



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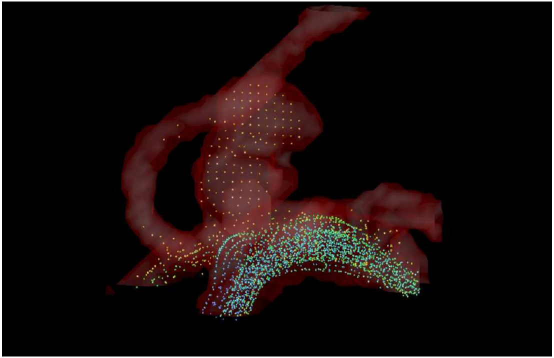
(c) $t = t_2$ (d) $t = t_3$ (g) $t = t_6$ (h) $t = t_7$

- Example of particle tracing
- Red colors show particles with a long time in the geometry, blue with a short duration



- The simulation of the bloodflow in the human brain is possible.
- Simulation runs unstable .
- Simulation time for relevant physical time is too long for clinical use during surgery.
- Outlook: Implementation of moving walls

Thanks for your attention!

Are there any questions / remarks ?