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Preconditioning the elastic wave equation in 2D and 3D based on inexact MSSS matrix computations

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Problem statement

The discretized **time-harmonic elastic wave equation** yields,

$$(K + i\omega_k C - \omega_k^2 M)x_k = b, \quad k = 1, ..., N, \qquad (1)$$

where $\{\omega_1, ..., \omega_N\}$ are a range of (angular) frequencies.

1. Linearization:

$$(\mathcal{K} - \omega_k \mathcal{M})\mathbf{x}_k := \left\{ \begin{bmatrix} iC & K \\ I & 0 \end{bmatrix} - \omega_k \begin{bmatrix} M & 0 \\ 0 & I \end{bmatrix} \right\} \begin{bmatrix} \omega_k x_k \\ x_k \end{bmatrix} = \begin{bmatrix} b \\ 0 \end{bmatrix}$$

2. Apply shift-and-invert preconditioner:

$$\mathcal{P}(\tau)^{-1} := (\mathcal{K} - \tau\mathcal{M})^{-1} \qquad \text{[cf. Fig. 3 for choice of } \tau \in \mathbb{C}] \qquad (2)$$

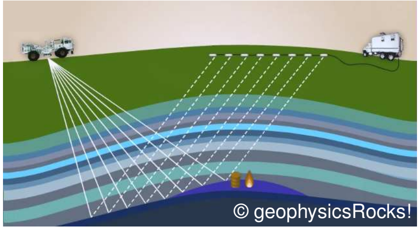

Fig. 1: Full-waveform inversion

Preconditioning the elastic operator in 2D and 3D

For the **2D operator** (red in Fig. 2) we compute a block-LU factorization of the form $\mathcal{P}(\tau) = LSU$, with

$$L_{i,j} = \begin{cases} I & \text{if } i = j \\ P_{i,j}S_j^{-1} & \text{if } i = j+1 \end{cases}, \quad U_{i,j} = \begin{cases} I & \text{if } j = i \\ S_i^{-1}P_{i,j} & \text{if } j = i+1 \end{cases},$$

and Schur complements given by the recursion,

$$S_i = \begin{cases} P_{i,i} & \text{if } i = 1 \\ P_{i,i} - P_{i,i-1}S_{i-1}^{-1}P_{i-1,i} & \text{if } 2 \le i \le n_x. \end{cases}$$

Here, the matrices S_i are SSS matrices; and inverses are computed *inexactly*, cf. [2].

For the **3D operator**, we consider a splitting $\mathcal{P}(\tau) = L + D + U$:

$$\mathcal{P}_h(\tau) = \underbrace{(LD^{-1} + I)D(D^{-1}U + I)}_{\text{"}n_z\text{ times a 2D problem"}} + \underbrace{P\mathcal{P}_H(\tau)^{-1}R}_{\text{"small 3D"}}$$

- The block SSOR preconditioner makes use of efficient 2D computations, cf. [1].
- Additive coarse grid correction yields grid-independent convergence ($H \ll h$).

MSSS matrix structure

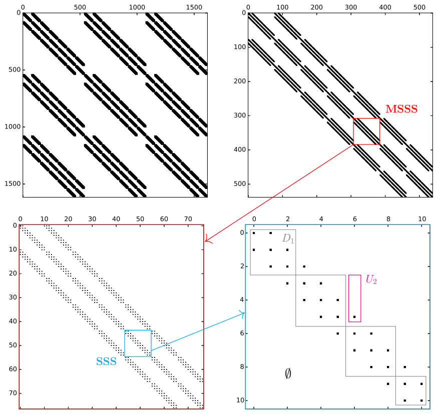

Fig. 2: MSSS structure of the 3D matrix (1)

Simulation results for 2D and 3D

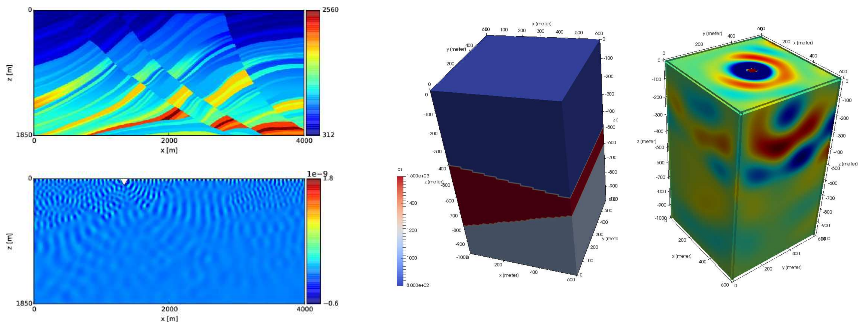

Fig. 4: Simulation results for the 2D `Marmousi-II` problem, and a 3D `wedge` problem

Preconditioned spectra

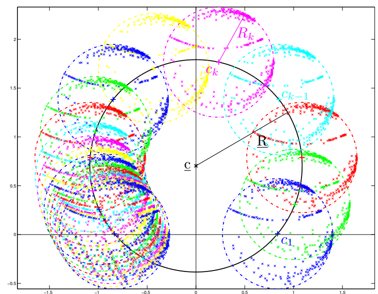

Fig. 3: Spectra after Möbius transformation. Choose τ such that $R_k/\|c_k\| \to \min$

References

[1] M. Baumann, R. Astudillo, Y. Qiu, E.Y.M. Ang, M.B. van Gijzen, and R.-É. Plessix (2017). *An MSSS-Preconditioned Matrix Equation Approach for the Time-Harmonic Elastic Wave Equation at Multiple Frequencies.* Springer Computat. Geosci., DOI: 10.1007/s10596-017-9667-7.

[2] Y. Qiu (2015). *Preconditioning Optimal Flow Control Problems Using Multilevel Sequentially Semiseparable Matrix Computations.* PhD Thesis, Delft University of Technology.