Credit Value Adjustment, Bermudan option and Wrong Way Risk

Qian Feng[1]
joint work with Cornelis W. Oosterlee[1,2]

[1]Centrum Wiskunde & Informatica, [2]Delft University of Technology

funded by STW

collaboration with ING Bank

Outline

Centrum Wiskunde & Informatica

Counterparty Credit Risk

Counterparty Credit Risk(CCR)

CCR is the risk to each party of a contract that the counterparty may fail to fulfill its obligations, causing losses to the other party.

Example

Company A agrees to lend Company B a certain amount of money. It is expected that company A will provide the money on time and company B will pay the money back. There is counterparty risk for both that company B may not be able to pay the loan while company A may stop providing the agreed upon funds.

OTC (over-the-counter) derivative contract

- Interest rate swap,
- FX forwards
- ...

Basel and CCR

Basel Committee on Banking Supervision

The Basel Committee on Banking Supervision (BCBS) is a committee of banking supervisory authorities aiming to improve the quality of banking supervision worldwide.

Credit exposure

the replacement cost of the contract, which is equal to the greater of the fair market value of the contract and zero.

Exposure measurements

- Basel II: Expected Exposure(EE), Potential future exposure (PFE)
- Basel III: Credit value adjustment (CVA), after the financial crisis 2008-2009.

Credit Value Adjustment

Credit Value Adjustment (CVA)

CVA is the market value of counterparty credit risk:

$$\text{CVA} = \mathbb{E}^{\mathbb{Q}}\left[\text{LGD}\int_0^T D(0,t)E_t dPD(t)\right], \tag{1}$$

where

- LGD: the percentage of loss given default;
- E: exposure $E_t = \max(V_t, 0)$, where V_t represents the mark-to-market value of the portfolio;
- PD: the default probability, $dPD(t) = PS(t+dt) - PS(t)$ the probability that default happens during time $[t, t+dt]$;
- $\mathbb{Q}$: risk-neutral probability measure.
- D: the discounting factor $D(0,t) = 1/B(t)$, where banking account $B(t) = \exp(-\int_0^t r_s ds)$ with the risk-free short rate r_s;

Intensity approach

Survival probability

$$G(s,t) = \exp\left(-\int_s^t h_u du\right), \tag{2}$$

where h_u represents the positive intensity at time u:

- $h_u du$ defines of probability that default time occurs during period $[u, u+du]$.
- h_u can be a constant, a deterministic function, a stochastic process...

Marginal default/survival probability model

$$\text{Survival: } PS(t) = \mathbb{E}^{\mathbb{Q}}\left[G(0,t)\right]. \tag{3}$$

$$\text{Default: } PD(t) = 1 - PS(t), \tag{4}$$

Bermudan option I

Bermudan option

A Bermudan option is an option that be exercised on a number of dates.

Payoff

Let S_t be the stock price at time t

$$g(S_t) = \begin{cases} \max(0, K - S_t) & \text{put}, \\ \max(0, S_t - K) & \text{call}, \end{cases} \tag{5}$$

where K the fixed strike.

Exercise dates

Let $\mathcal{T} = \{0 < t_1 < \ldots < t_N = T\}$ be the collection of early-exercise dates.

Bermudan option II

Default-free value

$$V_0 = \max_{\tau \in \mathcal{T}} \mathbb{E}^{\mathbb{Q}}\left[D(0,\tau) g(S_\tau) \right], \tag{6}$$

where τ is the exercise time, and let τ^* be the solution which represents the optimal exercise time when maximizing the default-free value.

Default-adjusted value

$$A_0 = \max_{\beta \in \mathcal{T}} \mathbb{E}^{\mathbb{Q}}\left[D(0,\beta) G(0,\beta) g(S_\beta) \right]. \tag{7}$$

where β is the exercise time, and let β^* be the solution which represents the optimal exercise time when maximizing the default-adjusted value.

Optimal exercise boundary at time $t_m \in \mathcal{T}$

Optimal default-free exercise value $x^*(t_m)$:

$$V_t(x^*(t_m)) - g(x^*(t_m)) = 0, \tag{8}$$

where the option value at time t_m conditioned on $S_{t_m} = x$ is given by

$$V_t(x) = \max_{\tau \in \{t_{m+1}, \dots, t_M\}} \mathbb{E}^{\mathbb{Q}} \left[D(t,\tau) g(S_\tau) \middle| S_{t_m} = x \right]. \tag{9}$$

Optimal default-adjusted exercise value $y^*(t_m)$:

$$A_t(y^*(t_m)) - g(y^*(t_m)) = 0, \tag{10}$$

where the option value at time t_m conditioned on $S_{t_m} = x$ is given by

$$A_t(x) = \max_{\beta \in \{t_{m+1}, \dots, t_M\}} \mathbb{E}^{\mathbb{Q}} \left[D(t,\beta) G(t,\beta) g(S_\beta) \middle| S_{t_m} = x \right]. \tag{11}$$

Numerical Schemes

COS method

Fourier-cosine expansion and FFT

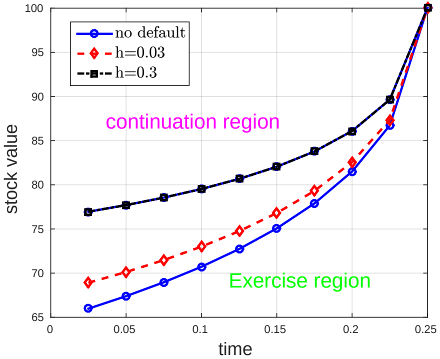
- Fang and Oosterlee , Ruijter and Oosterlee ...

Stochastic Grid Bundling Method (SGBM)

Simulation, regression, bundling, and the relation of the (discounted) characteristic function and the (discounted) moments.

- Jain and Oosterlee ...

Optimal early exercise boundary



Optimal early exercise values of a Bermudan put option with constant intensity $h = \{0.03, 0.3\}$ over period $[0, T]$. Parameters $S_0 = 100$, $r = 0.01$, $\sigma = 0.4$. Expiration $T = 0.25$, early exercise steps $M = 10$, strike $K = 100$.

Bermudan options

β^*: exercised by maximizing default-adjusted value

Intensity	default-adjusted value	default-free value	CVA
$h = 0.03$	7.7943	7.8416	0.0473
$h = 0.3$	7.4077	7.8162	0.4085

τ^*: exercised by maximizing default-free value

Intensity	default-adjusted value	default-free value	CVA
$h = 0.03$	7.7936	7.8422	0.0486
$h = 0.3$	7.3700	7.8422	0.4722

Wrong Way Risk

Wrong Way Risk(WWR)

This type of risk occurs when exposure to a counterparty is adversely correlated with the credit quality of that counterparty.

Example

A put option written by bank A on equity of bank B, and bank A and bank B have similar portfolio:

- Credit quality of bank A and bank B goes worse;
- stock price of bank B decreases;
- market value of the put option increases;
- the likelihood of default of bank A increases;

Stochastic intensity

Hull-White model

$$dx_t = \left(r - \frac{1}{2}\sigma^2\right) dt + \sigma dW_t^1, \tag{12}$$

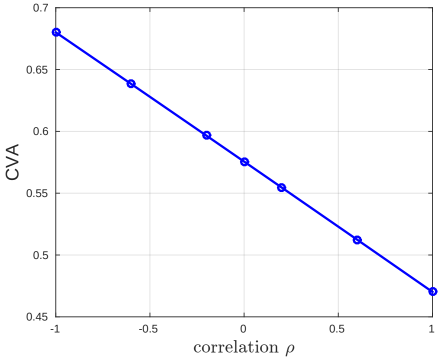
$$dy_t = \gamma(\bar{y} - y_t)dt + \eta dW_t^2, \tag{13}$$

$$h_t = \psi(t) + y_t, \tag{14}$$

where $x_t = \log(S_t)$ is the log-stock value, r represents the risk-free short rate, $\sigma > 0$ the constant implied volatility; $\gamma > 0$ corresponds to reversion speed, $\bar{y}$ to long run reverting level and $\eta > 0$ to the volatility of variable y_t, and dW_t^1 and dW_t^2 are two Wiener processes with correlation $dW_t^1 \cdot dW_t^2 = \rho dt$, and ρ the correlation coefficient; deterministic function $\psi(t)$ satisfies $\psi(0) = h_0 - y_0$.

Drawback: intensity may become negative.

CVA stress testing-European put options



CVA stress testing of w.r.t correlation. Parameters $S_0 = 100$, $r = 0.01$, $\sigma = 0.4$, $\gamma = 0.2$, $\bar{y} = 0.1$, $\eta = 0.2$; marginal survival function $\exp(-0.3t)$ and $y_0 = 0.3$. An European put option with expiration $T = 0.25$ and strike $K = 100$. Worst WWR ratio 1.18; best RWR ratio 0.82.

Conclusion and future work

Conclusion

- Credit risk may change the early exercise strategy for credit-risk-alert investors: the optimal early exercise values are increased for put options and decreased for call options;
- Credit risk and WWR cannot be eliminated by changing the exercise strategy.

Future

- Wrong way risk

CWI Centrum Wiskunde & Informatica

Bibliography

F. Fang and C. W. Oosterlee. A Fourier-Based Valuation Method for Bermudan and Barrier Options under Heston's Model. *SIAM Journal on Financial Mathematics*, 2(1):439–463, 2011.

M. J. Ruijter and C. W. Oosterlee. Two-Dimensional Fourier Cosine Series Expansion Method for Pricing Financial Options. *SIAM Journal on Scientific Computing*, 34(5):B642–B671, 2012. 1994.

T. R. Bielecki and M. Rutkowski. *Credit risk: Modeling, Valuation and Hedging*. Springer Science & Business Media, 2002.

S. Jain and C. W. Oosterlee. The Stochastic Grid Bundling Method: Efficient Pricing of Bermudan Options and Their Greeks . *Applied Mathematics and Computation*, 269:412 – 431, 2015.

Y. Shen, J. van der Weide, and J. Anderluh. A Benchmark Approach of Counterparty Credit Exposure of Bermudan Option under Levy Process: The Monte Carlo-COS Method. *Procedia Computer Science*, 18(0):1163 – 1171, 2013. 2013 International Conference on Computational Science.

I. Ruiz, R. Pachón, and P. Del Boca. Optimal Right and Wrong Way Risk, 2013.

J. Hull and A. White. CVA and Wrong-Way Risk. *Financial Analysts Journal*, 68(5):58–69, 2012.

J. Gregory. *Counterparty Credit Risk: The New Challenge for Global Financial Markets*. The Wiley Finance Series. John Wiley & Sons, 2010.

Feng, Runhuan, and Hans W. Volkmer. An identity of hitting times and its application to the valuation of guaranteed minimum withdrawal benefit. arXiv preprint arXiv:1307.7070 (2013).

CWI Centrum Wiskunde & Informatica