Optimization and Algorithms

Optimal Scheduling for Charging and Discharging of Electric Vehicles

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***Abstract*—The optimized problem revolves around intelligent scheduling for charging a set of Electric vehicles (EVs), by minimizing the total electricity cost. In order to make this problem more realistic the influence of battery lifetime reduction in the EVs is also considered. These optimized solutions are then compared with a non optimized scheme in order to predict the real changes that the optimization would bring. Two extra challenges are later proposed and efficiently solved. The overall results are quite favorable to the global optimization, more so than the battery lifetime reduction solution, which is to be expected, although both solutions are optimized when comparing to the non optimized solution.**

I. INTRODUCTION

The following paper addresses a convex problem optimization associated to Electric Vehicles (EVs) charging and discharging scheduling within a period of time. As the main objective, this area of research aims to reduce air pollution and oil consumption hence improving the technology. The problem we approach in this paper refers only to intelligent scheduling for charging and discharging of electric vehicles (EVs). We propose a solution for a global scheduling scheme where the charging powers are optimized by minimizing the total cost of charging and discharging the EVs. We also analyzed an equal scheduling scheme where the charging power for each interval in an EV charging or discharging had a constant absolute value.

Both solutions are supported by an intelligent scheduling approach that revolves around reshaping the load profile of the EVs battery, proceeding to charge when the demand is low and discharge when the demand is high. The global optimal scheduling rests on finding the minimal cost of charging a set of EVs, knowing in advance their arrival times and charging periods.

II. PROBLEM FORMULATION

A. Part I - Global optimal scheduling

To solve the global scheduling optimization we used the same solution in [1] where it's formulated a global scheduling optimization problem which aims to minimize the total cost for charging all EV's within a day.

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In this paper, we divide the time interval into 24 intervals being the interval set denoted by **N** (τ denotes the length of an interval, in this case $\tau = 1$ h). The EVs are divided in two sets, the set in which the EVs perform only charging $\mathbf{M}^{CHG}$ and the set in which the EVs perform both charging and discharging $\mathbf{M}^{V2G}$ during the day. The set that contains all the EVs is denoted by **M** and is the sum of the $\mathbf{M}^{CHG}$ set with the $\mathbf{M}^{V2G}$ set. We chose for **M** the value of 200 so there are 200 EVs in this problem. The charging power x_{mi} of EV $m \in \mathbf{M}$ in the interval $i \in \mathbf{N}$ will have a positive value if the EV is charging and a negative value if the EV is discharging. The optimization problem is formulated in equations (1), (2), (3), (4), (5) and (6) where equation (1) is the cost function and equations (2), (3), (4), (5) and (6) are the constraints.

$$F_{Cost} = \sum_{i \in N} \left(\left(k_0 z_i + \frac{k_1}{2} z_i^2 \right) - \left(k_0 L_i^b + \frac{k_1}{2} (L_i^b)^2 \right) \right) \quad (1)$$

$Minimize_{x,z} \quad F_{Cost}$
$subject\ to$

$$z_i = y_i + L_i^b = \sum_{m \in M} x_{mi} f_{mi} + L_i^b, \quad i \in N \quad (2)$$

$$0 \le x_{mi} \le P^{max}, \quad \forall m \in M^{CHG}, i \in N \quad (3)$$

$$-P^{max} \le x_{mi} \le P^{max}, \quad \forall m \in M^{V2G}, i \in N \quad (4)$$

$$0 \le E_m^{ini} + \sum_{k \in Q^{(i)}} \tau x_{mk} f_{mk} \le E_m^{cap}, \ m \in M, i \in N \quad (5)$$

$$E_m^{ini} + \sum_{i \in N} \tau x_{mi} f_{mi} \ge \gamma_m E_m^{cap}, \quad \forall m \in M \quad (6)$$

The global optimal scheduling scheme determines the optimal charging powers for all EVs for all intervals by optimizing the charging power x_{mi}, of every EV $m \in \mathbf{M}$, for every interval i ($\forall i \in \mathbf{N}$), and by optimizing the total load z_i, for every interval i ($\forall \in \mathbf{N}$, in a single optimization problem. This problem is subjected to the following constrains: (2) defines the total load z_i in the interval i ($\forall i \in \mathbf{N}$) as the sum of the real base load L_i^b in that interval with the sum for every EV $m \in \mathbf{M}$ of the charging power x_{mi} multiplied by the correspondent element of the charging-interval matrix f_{mi} (f_{mi} indicates if the EV m is connected to the grid at interval i). Constraints (3) and (4) specify the upper and lower bounds of the charging power x_{mi} for each EV in the corresponding set. Constraint (5) secures that the energy of any EV at end of an interval won't be negative or greater than the battery capacity E_m^{cap}. Finally, constraint (6) requires the final energy of EV m ($\forall m \in \mathbf{M}$) to be greater than a specific level given by the energy ratio of EV m γ_m times the battery capacity of EV m E_m^{cap}. The constrains that our problem is subjected to are linear because they all describe a linear relationship among the problem variables.

The cost function which we want to minimize is the sum of the EVs charging costs for each interval i, each one of them being the integral of the electricity price model $g(z_i) = k_0 + k_1 z_i$, from the base load L_i^b to the total load z_i. As in [2], this cost function is a quadratic cost function of the form $f(\mathbf{z}) = \frac{1}{2}\mathbf{z}^T\mathbf{A}\mathbf{z} + \mathbf{b}^T\mathbf{z} + \mathbf{c}$ where $\mathbf{A} = k_1\mathbf{I_N}$, $\mathbf{I_N}$ being the identity matrix of the same dimension as the interval set $\mathbf{N}$. Since k_1 is a non-negative number, the matrix $\mathbf{A} \succ 0$, then the cost function is convex. It's important to notice that k_0 k_1 are positive constants, otherwise the problem could be non-convex. These constants represent respectively, the intercept and the slope in the electricity pricing model and we used the values $k_0 = 10^{-4}$ $C\$/kWh/kW$ and $k_1 = 1.2 \times 10^{-4}$ $C\$/kWh/kW$ for these constants.

Knowing that our cost function is convex and all the constrains of the optimization problem are linear we can state that this is a convex optimization problem.

B. Part I - Global optimal scheduling considering the cost of battery lifetime reduction (BLR)

Considering the cost of BLR in the global scheduling optimization, the optimization problem stays the same, except for a different objective function. The difference lies in the sum of an extra term ψ to the cost function of the previous problem, which translates the influence of BLR due to the frequent charging and discharging for all EVs in the set $\mathbf{M}$, during the set of considered intervals $\mathbf{N}$. The extra added term ψ is also the sum of two contributions ψ^A caused by the amount of charging and discharging power in each interval and ψ^F caused by the fluctuation of charging and discharging power between any two consecutive intervals. Thus, the influence of BLR is given by:

$$\begin{aligned}\psi &= \sum_{m\in\mathbf{M}} (\psi_m^A + \psi_m^F) \\ &= \sum_{m\in\mathbf{M}} \Big(\sum_{i\in\mathbf{N}} \beta x_{mi}^2 + \sum_{i=2}^{|\mathbf{N}|} \eta (x_{mi} - x_{m(i-1)})^2 \Big).\end{aligned} \tag{7}$$

$$\begin{aligned}F_{Cost_{BLR}} &= \sum_{i\in\mathbf{N}} \Big(\Big(k_0 z_i + \frac{k_1}{2} z_i^2\Big) - \Big(k_0 L_i^b + \frac{k_1}{2}(L_i^b)^2\Big) \Big) \\ &+ \sum_{m\in\mathbf{M}} \sum_{i\in\mathbf{N}} \beta x_{mi}^2 + \sum_{m\in\mathbf{M}} \sum_{i=2}^{|\mathbf{N}|} \eta \Big(x_{mi} - x_{m(i-1)} \Big)^2.\end{aligned} \tag{8}$$

Both β and η are model parameters, with the same numerical values considered in the paper [1] $\beta = 5 \times 10^{-4}$ $C\$/kWh^2$ and $\eta = 10^{-3}$ $C\$/kWh^2$. The cost function of the global scheduling optimization considering BLR changes to (8) and the optimization variables remain the charging power x_{mi} and the total load z_i.

Analyzing the global scheduling optimization problem which considers the cost of BLR we conclude it's a convex optimization problem, since the cost function is the sum of the convex cost function of the global optimal scheduling of section A. with two other quadratic convex functions ψ^A and ψ^F, as the corresponding matrices $\mathbf{A}$ are positive semi-definite since β and η are positive, and the sum operator preserves the convexity, as seen in [2]. As the constraints are the same as the global scheduling optimization (linear), the optimization problem for the global optimal scheduling when considering BLR is a convex optimization problem.

C. Part II - Challenges

For the second phase of the project we were proposed two extra challenges: adapt and implement an approximate model to the quadratic term of the cost function and make the piecewise constant fitting of the total load vector $\mathbf{z}$.

$$n_i = |z_i - z_{(i-1)}|_1, \forall i \in \mathbf{N}$$

$$F_{Cost} = \sum_{i\in\mathbf{N}} \Big(\Big(k_0 z_i + \frac{k_1}{2} z_i^2\Big) - \Big(k_0 L_i^b + \frac{k_1}{2}(L_i^b)^2\Big) \Big) + \lambda\vec{n} \tag{9}$$

In the first challenge we worked with the quadratic term $k_0 z + \frac{k_1}{2} z^2$ of the cost function, with the objective of adapt it to an approximate linear model by branches. When analyzing the function we can notice that for the lowest values of z the term $k_0 z$ dictates the value of the function, since the term $\frac{k_1}{2} z^2$ has a rather insignificant contribution, but for high values of z, the dominant term is $\frac{k_1}{2} z^2$ due to z^2. The function of our approximate model is defined based on the previous analysis, where it is defined by $k_0 z$ for the lowest values of z, thus $k_0 z > \frac{k_1}{2} z^2$, and by a line with slope $(k_0 + k_1)$ for values of z where $\frac{k_1}{2} z^2 > k_0 z$. The first step to implement this approach was to discover the positive value of z for which $k_0 z = \frac{k_1}{2} z^2$, that will be the separation point between the two lines in our function, which we calculated to be $1.(6)$ by resolving the previous equation. After this step, we needed to plan how we were going to implement this model in the CVX solver, where all the modifications done to the function had to preserve it's convexity. Our solution was to implement it through the $max()$ operation for which we know that the max of a set of convex functions is also convex, as $k_0 z$ and $(k_0 + k_1)z$ are both linear, they are convex functions. Now we need to set the line with slope $(k_0 + k_1)$ to be less than $k_0 z$ for values of z less than $1.(6)$ (branching point), to achieve that we translated $(k_0 + k_1)z$ in the vertical axis defining the function as $(k_0 + k_1)z - (2 \times 10^{-4})$ so that this line would intercept $k_0 z$ for $z = 1.(6)$. The approximated model for the quadratic term $k_0 z + (k_1/2) z^2$

implemented was $\max(k_0 z, (k_0+k_1)z - 2\times 10^{-4})$. The main objective for this challenge was to implement it on CVX, the results of the simulations using this approximated model were not significant.

The second challenge arose from the fact that the total load **z** was described by abrupt changes between each point z_i to z_{i+1} instead of being a piecewise constant vector. One way to solve this problem was to shorten the distance between every two points z_i and z_{i+1} thus making the problem approximately linear. We create a cycle in order to reach every point in **z**, adding a vector $\vec{n}$ with a weight constant λ in F_{Cost} that preforms in every n_i the convex function l_1 norm of two points in **z**. We choose the initial value of $\lambda_i = 0.5$, this prevents the norm vector from becoming the dominant term on the F_{Cost} function. As we increase the λ parameter the total load **z** doesn't change significantly. Knowing that these alterations on the cost function don't change the convexity of the problem, once that $l1$ norm preserves the convexity, we include this term in F_{Cost} minimization. The total load resulting vector for various λ can be seen in Fig. 6 and Fig. 7, in the Attachments section.

III. Approach

The equations stated above in the Problem Formulation Section were implemented in MATLAB computing environment. Because the stated problems revolve around convex functions and constraints, we were able to use CVX solver in MATLAB. In order to compare the results obtained from global optimal solution we implemented the equal allocation scheme. This scheme represents a solution where the scheduling is not optimized, the charging power of an EV in an interval is calculated based on the electricity price on the previous day and the absolute value of the charging power of the EV is equal for each interval. This solution can be implemented with several specifications concerning the discharging intervals for the V2G type. We decided to apply the same approach the authors of paper [1] use. For the EVs that can discharge to the grid, we assume that they will do it always during one interval, when the electricity price (given by the model $g(z_t) = k_0 + k_1 z_t$) is maximum within their charging period.

IV. Numerical results

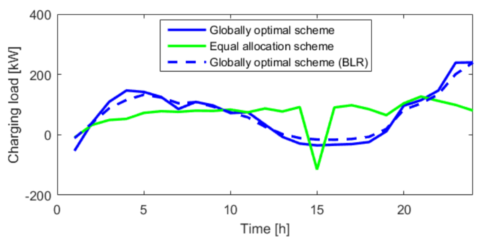

Fig. 1. Variation of charging load in each interval

Fig. 1 shows the variation of the charging load y along the day. The charging load is given by $y_i = \sum_{m\in M} x_{mi} f_{mi}$ so we can conclude that, for any interval of the day, when $y_i > 0$ there are more EVs charging than that are discharging and when $y_i < 0$ there are more EVs discharging than that are charging. Analyzing the graph, we observe that for the intervals where there are more EVs connected to the grid, the grid discharges more EVs than charges EVs to make the electricity price minimum in each interval, minimizing the total cost. We also observe that the globally optimal scheme when considering BLR never reaches the minimum values that the globally optimal scheme reaches when not considering BLR, because the batteries already discharge some of their energy on their own. Since in the equal allocation scheme, for each EV, the discharging is made in only one interval, the graph for this scheme shows an abrupt negative peak around interval 15 where there are more EVs connected to the grid.

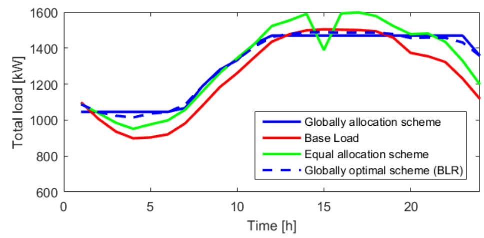

Fig. 2. Variation of total load in each interval

Fig. 2 shows the variation of the total load z during the day. The total load in an interval, given by $z_i = L_b^i + y_i = L_b^i + \sum_{m\in M} x_{mi} f_{mi}$, is the sum of the charging load **y** represented in Fig. 1 with the contribution of the Base Load L_b^i, which is a known variable, that represents the total load of all electricity consumptions in an interval i, except the ones associated with the EV charging and discharging. Analyzing the graph in Fig.2, we observe that both globally optimal scheduling schemes considering and not considering BLR can reshape the total load profile to minimize the total cost.

The globally optimal scheduling scheme flattens the total load profile between 1AM to 7AM and 12PM to 23PM to minimize the total cost. In the globally optimal scheduling scheme when considering BLR graph we can observe that the optimization led to similar reshaping of the total load profile as when not considering BLR, but with small differences that resemble the Base Load graph. Hence, the total cost when considering BLR is slightly higher compared to the total cost when not considering BLR, as it's s going to be analyzed in the Fig. 5.

Comparing the equal allocation scheme to the global schemes, we observe that there isn't an optimization of the total load, thus neither a minimization of the total cost, since the shape of the graph is similar to the Base Load and the abrupt peek in interval 15 seen in Fig. 1 remains unaffected, for the same reasons explained in Fig. 1.

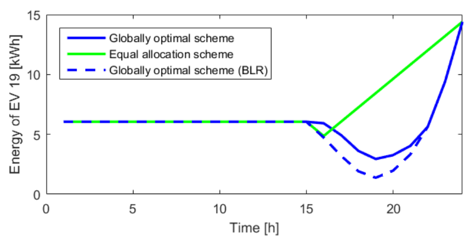

Fig. 3. Variation of energy of EV 19 in each interval

Fig. 3 shows the variation of the energy of a chosen EV (EV 19) along the day. In Fig. 3 we observe that the EV only connects to the grid to charge at 4 PM and that the EV discharges first then charges up to it's final energy. For the global optimal scheme considering BLR we note that, after discharging, the energy of EV 19 reaches a minimum below the minimum reached by the global allocation scheme when not considering BLR after discharging, this is due to the batteries self-discharge that happens when considering BLR. We also observe that in the equal allocation scheme the discharge is done in only one interval and that the same final energy is reached for all the three schemes.

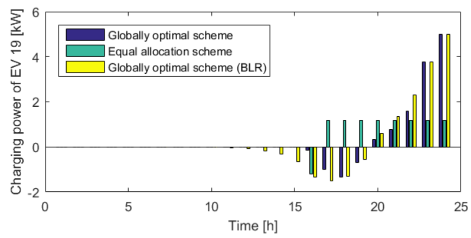

Fig. 4. Variation of charging power of EV 19 in each interval

Fig. 4 shows the variation of the charging power of the same EV of Fig. 3 (EV 19) along the day. In Fig. 4 we observe that the charging power in the global allocation scheme when considering BLR has more abrupt variations than the global allocation scheme when not considering BLR, this is consistent with the bigger variation in energy for the global allocation scheme when considering BLR observed in Fig. 3. We also observe that the charging power in the equal allocation scheme has the same absolute value for all the charging and discharging intervals and that the discharge occurred at 4 PM.

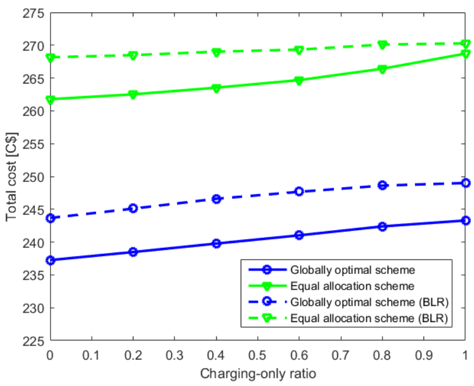

Fig. 5. Variation of total cost with different charging-only ratio

Fig. 5 shows the impact of the charging–only ratio in the total cost. Each EV of the total set M is classified in one of two categories: the charging-only set M^{CHG}, if the EV only performs charging of the battery, and the vehicle-to-grid set M^{V2G} if the EV is willing to discharge the battery to the grid before starting charging. The charging-only ratio is defined as the number of EVs in the charging-only set M^{CHG} divided by the total number of EVs M. As expected, the increasing of the charging-only ratio causes a higher total cost in all four schemes represented in figure 5, since the number of charging-only EVs increases and the number of vehicle-to-grid EVs decreases.

Comparing the globally optimal scheduling scheme with the equal allocation scheme, the total cost reduces by 9.40% with the globally optimal scheduling scheme [1], since the equal allocation scheme is not optimized.

Comparing the two schemes considering battery lifetime reduction with the corresponding schemes without it, we can see that the total cost considering battery lifetime reduction is always higher, in both globally optimal scheduling schemes and equal allocation schemes. This result was expected since the cost function considering battery life time reduction is based on the cost function of the "original" scheme plus the contribution of reducing the lifetime of the battery ψ, as explained in Section II.

Analyzing the graph, we observe that, for both schemes, the increase of the total cost when considering battery lifetime reduction has a higher slope for the lowest values of the charging-only ratio. This can be explained through the contribution of $\psi = \sum_{m \in M}(\psi_m^A + \psi_m^F)$ in the total cost that is higher for a set with more V2G EVs than CHG EVs, since it translates the change of the charging direction in two consecutive intervals (which can only happen in a V2G set) and the amount of discharged energy during the day. Hence, the impact of the cost of battery lifetime reduction ψ in the total cost is reduced with the decreasing of the V2G set, equivalent to the increase of the charging-only ratio.

V. Conclusions

In this paper we studied the scheduling optimization problem for EV charging and discharging. The global scheduling optimization problems with and without BLR were based on the paper [1]. In both problems, the charging powers were optimized to minimize the total cost of all EVs which perform charging and discharging in a day.

The global optimal solution developed provides the globally minimal total cost for a set of EVs, and when considering BLR, the results obtained show that the minimal total cost is slightly higher. Although the globally optimal scheduling schemes are impractical since they assume the arrivals of all EV's and Base Loads are known, the simulations demonstrated that these solutions achieve better results compared to a non-optimized scheduling, reducing the total cost by $9,40\%$ when compared to the equal allocation schemes.

Besides this, we also addressed some challenges proposed to the solution developed where the first challenge was not simulated. However the second challenge was simulated and we conclude that the impact of the parameter λ in the cost function is minimum when compared to the rest of the cost function, so it's laborious and not pratical to make the total load $\mathbf{z}$ a piecewise constant vector.

References

[1] Bala Venkatesh, Yifeng He and Ling Guan, "Optimal Scheduling for Charging and Discharging of Electric Vehicles", in *IEEE Transactions on Smart Grid*, Vol. 3, No. 3, Sep. 2012, pp. 1095-1105. Available: https://fenix.tecnico.ulisboa.pt/downloadFile/1689468335\568774/project5.pdf

[2] Joao Xavier, "Part 2: unconstrained optimization", in *Optimization and Algorithms*, 2016. Available: https://fenix.tecnico.ulisboa.pt/downloadFile/1689468335569337/part2.pdf

VI. ATTACHMENTS

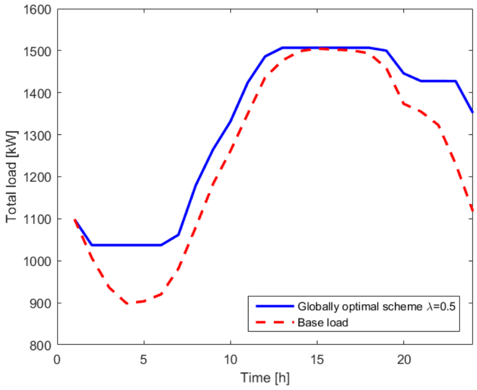

Fig. 6. Piecewise Constant Fitting for $\lambda = 0.5$

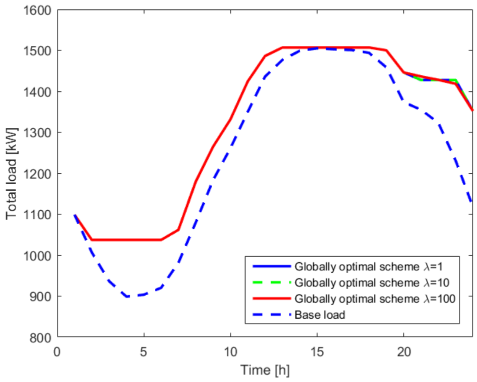

Fig. 7. Piecewise Constant Fitting for various λ