# Optimal Experimental Design: Psychophysics of change point Detection.

Manuel Villarreal and Arturo Bouzas

*Mexico City, Mexico*

**Abstract**

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## 1. Introduction

One of the main challenges in scientific research is the design of an experiment. A good experimental design will make the difference between finding an answer to our research question and wasting valuable resources like time and money.

When designing an experiment, often one start's by making decisions about the number of participants, how many and what values of our independent variable should we test or how many times should we test each of those values. Most of the time, these choices are made on the basis of previous research on the field. However, it might be the case that there is not enough information to make these decisions with confidence, or that the values that are commonly used, do not allow for strong conclusions. Optimal Experimental Design (OED) offers an alternative to solve this kind of problems through the formalization of the design problem.

OED allows us to re-interpret the problem of an experimental design as a decision problem, in which the purpose is to maximize a utility function. This function is a numeric representation of our preference over the possible consequences of running an experiment. Therefore, the optimization of an experimental design requires us to have a formal interpretation of the purpose of the experiment.

The concept of optimizing an experimental design is not new to psychological research. There are already examples in the literature of design optimization (e.g. Myung and Pitt, 2009; Zhang and Lee, 2010). Both of this examples discuss and demonstrate the advantages of OED for model comparison in psychology. In this paper, we will present a different approach where the problem is not to select between two models but to estimate the parameters of a single one.

In particular, we will present an example of OED in the context of Generalized Linear Models. These models are widely used in psychophysics. The problem that will be treated here is fairly new, however, the methodological aspects remain the same even with more straight-forward psychophysical experiments. In order to do this, we will use a particular parametrization of the logistic model which is primarily used in statistical inference.

## 2. Optimal experimental design

In order to apply the concepts of OED to a particular problem, first, we need to define the elements of the design space. This design space is defined by the variables that we can manipulate during the experiment, for example, the values that our independent variable might take or the weight (proportion of observations) assigned to each of those values. These elements are the ones that we can modify in order to optimize the design.

The second step would be to formalise the objective of the experiment. For example, in the case of a logistic model, we might want to find the values of our independent variable that minimize the variance of the model parameters, or we might be interested in the magnitude of the physical stimulus for which the probability of a response takes on a certain value. The formalization of the research question will define a utility function.

The last step is to specify the prior information that we have about the problem at hand. This last step can be carried out in two ways, first, we can try to optimize the experiment for a particular guess about the parameter values of the model of interest, or we could use a probability distribution to account for the uncertainty in the values that we are interested in. This last step is primarily important to the optimization process for generalized linear models, because the optimal design will depend on the values of the parameters.

Once we have defined the design space $\eta$, the objective of the experiment and the prior informartion $p(\theta)$, the expected utility of a design is represented by the following equation:

$$U(\eta) = \int \int U(\eta, y, \theta) p(\theta|y, \eta) p(y|\eta) d\theta dy \tag{1}$$

Therefore, finding the experimental design that is optimal given a utility function reduces to the problem of finding de values for the variables in $\eta$ for which equation 1 takes it's maximum value.

the objective would be to find the values in the design space that bring a higher utility.

Experimental design First we need a research question, then the problem of designing the experiment arises, how many participants should we test, what are the values of the independent variable that we should use, how many times should we present each of those values, etc. The problem is when

Elements:

Design space: what are the elements of the experimental design that we want to optimize Utility Function: function that maps points on the design space to the real numbers, this function should reflect the objective of the experiment, for example if we want to discriminate between two cognitive models, the utility function should assign a greater value to an experimental design for which the models give different predictions that to designs for which the predictions of the models are indistinguishable from one another.

## 3. OUTLINE

## 4. Optimal Experimental Design: Example

Why is is detecting changes important for an organism?

Change detection in probabilistic series.

Arising problems with experimental design.

Research question and its statistichal interpretation

Assumtion about the relationship between a subjects response the dependent variable under study

Design space for this problem and how to reduce the dimensionality of the space by assuming experimental constraints.

Utility function and its relationship with the objective of the experiment

Arising problems with utility function and the proposed response function. Bayesian solution, assigning a prior distribution to the parameters, the less research in a field the more difficult it is to assign an informative prior, however, we could use other cognitive models in order to propose a prior distribution.

### *4.1. Using a model to generate prior distributions*

Using the prior distribution, the utility function and the definition of a design space we can otimize the experimental design in this case we are looking for $\delta\theta^*$ that maximizes the following equation:

$$U(\delta\theta^*) = \max_{\delta\theta} \int_{\beta} log(det(I(\beta|\delta\theta)))\pi(\beta)d\beta \tag{2}$$

The previous integral can be approximated via Monte Carlo sampling

## 5. Results

### *5.1. Consruction of the prior distribution*

Prior over model parameters(Gallistel et al 2014) Results Constructing the prior: we take a multivariate normal distribution with mean and covariance equal to the unbiased estimators for both parameters.

### *5.2. Optimal design*

Aproximating the utility function (integral) throught Monte Carlo simulation Utility aproximation for 2 Design points

the approximation returns a smooth curve over the 2 point design space.

## 6. Discussion

Optimal design for the example Properties of the most useful points (they land on the points of the curve where the steepness changes most dramatically)

Advantages of Optimal Design

Using models to generte prior distributions.

## References

Myung, J., Pitt, M., 2009. Optimal experimental design for model discrimination. Psychological Review 116 (3), 499–518.

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