

Homework 1

Simulation and Performance Evaluation – University of Trento

DEADLINE: April 6, 2023

You can solve the following assignments using any programming language. In doing so, make sure to implement the formulas explained in class, and *do not* use functions made available by the languages to achieve the required tasks. For example, do not use built-in functions to draw random variates. However, you are allowed to use built-in utility functions (e.g., for managing data, sorting, plotting, printing messages etc).

Exercise 1

Use simulation to prove the memoriless property of exponential random variables, having PDF $f(x) = \lambda e^{-\lambda x}$. In other words, show that if (i.e., *given that*) an exponential random variate X is such that $X > t$, then the pdf of $X - t$ is also exponential of rate λ , and that this result holds irrespective of t .

Exercise 2

Use simulation to prove the following result. Given that a Poisson process of rate λ yields N arrivals in some interval $[0, T]$, we can draw these arrivals in two ways:

1. By drawing N arrival times uniformly at random in the interval $[0, T]$;
2. By drawing a set of N exponential inter-arrival times of average value $1/\lambda$ in the interval $[0, T]$.

For example, show that the inter-arrival times of method 1 are also exponentially distributed.

(*Hint 1*: start by choosing λ , T and N such that $\lambda T \approx N$ to make things easier. Then see if you observe anything different if you choose other values.)

(*Hint 2*: for method 2, are there extra checks you should implement?)