



To prove that

$$\boxed{\frac{CB}{CD} \times \frac{AD}{AB} = \frac{C'B'}{C'D'} \times \frac{A'D'}{A'B'}}$$

Area

$$A(\triangle COB) = \frac{1}{2} CB \cdot h_1 = \frac{1}{2} OC \cdot OB \cdot \sin \alpha_2$$

$$A(\triangle C'OB') = \frac{1}{2} C'B' \cdot h_2 = \frac{1}{2} OC' \cdot OB' \cdot \sin \alpha_2$$

(Similarly use $\triangle DOC$ & $\triangle D'OC'$) $\therefore \frac{h_2}{h_1} = \frac{OC' \cdot OB'}{OC \cdot OB} \cdot \frac{CB}{C'B'}$

$$\text{III'y } \frac{h_2}{h_1} = \frac{OD' \cdot OC'}{OD \cdot OC} = \frac{OD' \cdot OC'}{OD \cdot OC} \cdot \frac{DC}{D'C'}$$

$$\therefore \frac{OC' \cdot OB'}{OC \cdot OB} \cdot \frac{CB}{C'B'} = \frac{OD' \cdot OC'}{OD \cdot OC} \cdot \frac{DC}{D'C'} \quad \text{--- ①}$$

from

$$A(\triangle AOB) = \frac{1}{2} AB \cdot h_1 = \frac{1}{2} OB \cdot OA \cdot \sin \alpha_3$$

$$A(\triangle A'OB') = \frac{1}{2} A'B' \cdot h_2 = \frac{1}{2} OA' \cdot OB' \sin \alpha_3$$

$$\Rightarrow \frac{h_2}{h_1} = \frac{OA' \cdot OB'}{OB \cdot OA} \cdot \frac{AB}{A'B'}$$

(III'y from $\triangle AOD$ & $\triangle A'OD'$) $\frac{h_2}{h_1} = \frac{OA' \cdot OD'}{OA \cdot OD} \cdot \frac{AD}{A'D'}$

$$\therefore \frac{OA' \cdot OB'}{OB \cdot OA} \cdot \frac{AB}{A'B'} = \frac{OA' \cdot OD'}{OA \cdot OD} \cdot \frac{AD}{A'D'} \quad \text{--- ②}$$

$$\text{①} \div \text{②} \quad \left(\frac{CB}{C'B'} \right) / \left(\frac{AB}{A'B'} \right) = \left(\frac{DC}{D'C'} \right) / \left(\frac{AD}{A'D'} \right)$$

$$\Rightarrow \frac{CB}{C'B'} \cdot \frac{A'B'}{AB} = \frac{DC}{D'C'} \cdot \frac{A'D'}{AD} \Rightarrow \boxed{\frac{CB}{DC} \times \frac{AD}{AB} = \frac{C'B'}{C'D'} \times \frac{A'D'}{A'B'}}$$

thus proved.