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Noorul Islam College of Engineering,Kumaracoil

EE 44 CONTROL SYSTEMS

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Two Marks Questions and answers

1.What is frequency response?

A frequency response is the steady state response of a system when the input to the system is a sinusoidal signal.

2.List out the different frequency domain specifications?

The frequency domain specification are i)Resonant peak. ii)Resonant frequency.

3.Define –resonant Peak (⌠r)?

The maximum value of the magnitude of closed loop transfer function is Called resonant peak.

4.Define –Resonant frequency(⌠f)?

The frequency at which resonant peak occurs is called resonant frequency.

5.What is bandwidth?

The bandwidth is the range of frequencies for which the system gain Is more than 3 dbB.The bandwidth is a measure of the ability of a feedback system to reproduce the input signal ,noise rejection characteristics and rise time.

6.Define Cut-off rate?

The slope of the log-magnitude curve near the cut-off is called cut-off rate. The cut-off rate indicates the ability to distinguish the signal from noise.

7.Define –Gain Margin?

The gain margin,kg is defined as the reciprocal of the magnitude of the open loop transfer function at phase cross over frequency.

Gain margin kg = $1 / | G(j\omega_{pc}) |$.

8.Define Phase cross over?

The frequency at which, the phase of open loop transfer functions is called phase cross over frequency ω_{pc}.

9.What is phase margin?

The phase margin ,γ is the amount of phase lag at the gain cross over Frequency required to bring system to the verge of instability.

10.What is a compensator?

A device inserted into the system for the purpose of satisfying the specifications is called as a compensator.

11.Define Gain cross over?

The gain cross over frequency ω_{gc} is the frequency at which the magnitude of the open loop transfer function is unity.

12.What is Bode plot?

The Bode plot is the frequency response plot of the transfer function of a system. A Bode plot consists of two graphs. One is the plot of magnitude of sinusoidal transfer function versus log ω.The other is a plot of the phase angle of a sinusoidal function versus log ω.

13.What are the main advantages of Bode plot?

The main advantages are:

i) Multiplication of magnitude can be in to addition.

ii) A simple method for sketching an approximate log curve is available.

iii) It is based on asymptotic approximation. Such approximation is sufficient if rough information on the frequency response characteristic is needed.

iv) The phase angle curves can be easily drawn if a template for the phase angle curve of 1+ $j\omega$ is available.

14.Define Corner frequency?

The frequency at which the two asymptotic meet in a magnitude plot is Called corner frequency.

15.Define Phase lag and phase lead?

A negative phase angle is called phase lag.

A positive phase angle is called phase lead.

16.What are M circles?

The magnitude of closed loop transfer function with unit feed back can be shown to be in the for every value if M.These circles are called M circles.

17.What is Nichols chart?

The chart consisting if M & N loci in the log magnitude versus phase diagram is called Nichols chart.

18.What are two contours of Nichols chart?

Nichols chart of M and N contours, superimposed on ordinary graph. The M contours are the magnitude of closed loop system in decibels and the N contours are the phase angle locus of closed loop system.

19.How is the Resonant Peak(M_r), resonant frequency(W_r) , and band Width determined from Nichols chart?

i) The resonant peak is given by the value of ⌠.contour which is tangent to G(jω) locus.

ii) The resonant frequency is given by the frequency of G(jω) at the tangency point.

iii) The bandwidth is given by frequency corresponding to the intersection point of G(jω) and –3dB M-contour.

20.What are the advantages of Nichols chart?

The advantages are:

i) It is used to find the closed loop frequency response from open loop frequency response.

ii) Frequency domain specifications can be determined from Nichols chart.

iii) The gain of the system can be adjusted to satisfy the given specification.

21.What are the two types of compensation?

i. Cascade or series compensation

ii. Feedback compensation or parallel compensation

22.What are the three types of compensators?

i. Lag compensator

ii. Lead compensator

iii. Lag-Lead compensator

23.What are the uses of lead compensator?

speeds up the transient response ,increases the margin of stability of a system, increases the system error constant to a limited extent.

24.What is the use of lag compensator?

Improve the steady state behavior of a system, while nearly preserving its transient response.

25.When is lag lead compensator is required?

The lag lead compensator is required when both the transient and steady State response of a system has to be improved

26.What is nyquist contour

The contour that encloses entire right half of S plane is called nyquist contour.

27.State Nyquist stability criterion.

If the Nyquist plot of the open loop transfer function G(s) corresponding to The nyquist control in the S-plane encircles the critical point –1+j0 in the Counter clockwise direction as many times as the number of right half S-plane poles of G(s),the closed loop system is stable.

28.Define Relative stability

Relative stability is the degree of closeness of the system,it an indication of strength or degree of stability.

29.What are the two segments of Nyquist contour?

i. An finite line segment C1 along the imaginary axis.

ii. An arc C2 of infinite radius.

30.What are root loci?

The path taken by the roots of the open loop transfer function when the loop gain is varied from 0 to _ are called root loci.

31.What is a dominant pole?

The dominant pole is a air of complex conjugate pair which decides the Transient response of the system.

32.What are the main significances of root locus?

i. The main root locus technique is used for stability analysis.

ii. Using root locus technique the range of values of K, for as table system can be determined

33.What are the effect of adding a zero to a system?

Adding a zero to a system increases peak overshoot appreciably.

34.What are N circles?

If the phase of closed loop transfer function with unity feedback is _, then tan will be in the form of circles for every value of _. These circles are called N circles.

35.What is control system?

A system consists of a number of components connected together to perform a specific function . In a system when the output quantity is controlled by varying the input quantity then the system is called control system.

36.What are the two major types of control system?

The two major types of control system are open loop and closed loop

37.Define open loop control system.

The control system in which the output quantity has no effect upon the input quantity are called open loop control system. This means that the output is not feedback to the input for correction.

38.Define closed loop control system.

The control system in which the output has an effect upon the input quantity so as to maintain the desired output value are called closed loop control system.

39.What are the components of feedback control system?

The components of feedback control system are plant , feedback path elements, error detector and controller.

40.Define transfer function.

The T.F of a system is defined as the ratio of the laplace transform of output to laplace transform of input with zero initial conditions.

41.What are the basic elements used for modeling mechanical translational system.

Mass, spring and dashpot

42.What are the basic elements used for modeling mechanical rotational system?

Moment of inertia J,fdashpo with rotational frictional coefficient B and torsional spring with stiffness K

43.Name two types of electrical analogous for mechanical system.

The two types of analogies for the mechanical system are Force voltage and force current analogy

44.What is block diagram?

A block diagram of a system is a pictorial representation of the functions performed by each component of the system and shows the flow of signals. The basic elements of block diagram are blocks, branch point and summing point.

45.What is the basis for framing the rules of block diagram reduction technique?

The rules for block diagram reduction technique are framed such that any modification made on the diagram does not alter the input output relation.

46.What is a signal flow graph?

A signal flow graph is a diagram that represents a set of simultaneousalgebraic equations .By taking L.T the time domain differential equations governing a control system can be transferred to a set of algebraic equations in s-domain.

47. What is transmittance?

The transmittance is the gain acquired by the signal when it travels from one node to another node in signal flow graph.

48.What is sink and source?

Source is the input node in the signal flow graph and it has only outgoing branches.

Sink is a output node in the signal flow graph and it has only incoming branches.

49.Define non touching loop.

The loops are said to be non touching if they do not have common nodes.

50.Write Masons Gain formula.

Masons Gain formula states that the overall gain of the system is

$T = 1/ \Delta \sum P_k \Delta_k$

k-no.of forward paths in the signal flow graph.

P_k- Forward path gain of k^{th} forward path

Δ = 1-[sum of individual loop gains] +[sum of gain products of all possible

combinations of two non touching loops]-[sum of gain products of all

possible combinations of three non touching loops]+…

$\Delta_k = \Delta$ for that part of the graph which is not touching k^{th} forward path.

51.Write the analogous electrical elements in force voltage analogy for the elements

of mechanical translational system.

Force-voltage e

Velocity v-current i

Displacement x-charge q

Frictional coeff B-Resistance R

Mass M- Inductance L

Stiffness K-Inverse of capacitance 1/C

52.Write the analogous electrical elements in force current analogy for the elements of mechanical translational system.

Force-current i

Velocity v-voltage v

Displacement x-flux_

Frictional coeff B-conductance 1/R

Mass M- capacitance C

Stiffness K-Inverse of inductance 1/L

53.Write the force balance equation of an ideal mass element .

$F = M\, d^2x/dt^2$

54. Write the force balance equation of ideal dashpot element .

$F = B\, dx/dt$

55. Write the force balance equation of ideal spring element .

$F = Kx$

56.Distinguish between open loop and closed loop system

open loop system	closed loop system
1.Innaccurate	accurate
2.Simple and economical	Complex and costlier
3.The changes in output due to external disturbance are not corrected	The changes in output due to external disturbances are corrected automatically
4.They are generally stable	Great efforts are needed to design a stable System

57.What is servomechanism?

The servomechanism is a feedback control system in which the output is mechanical

position (or time derivatives of position velocity and acceleration,)

58.Why is negative feedback invariably preferred in closed loop system?

The negative feedback results in better stability in steady state and rejects any disturbance signals.

59.What is transient response?

The transient response is the response of the system when the system changes from one state to another.

60.What is steady state response?

The steady state response is the response of the system when it approaches infinity.

61.What is an order of a system?

The order of a system is the order of the differential equation governing the system. The order of the system can be obtained from the transfer function of the given system.

62.Define Damping ratio.

Damping ratio is defined as the ratio of actual damping to critical damping.

63.List the time domain specifications.

The time domain specifications are

i.Delay time

ii.Rise time

iii.Peak time

iv.Peak overshoot

64.Define Delay time.

The time taken for response to reach 50% of final value for the very first time is delay time.

65.Define Rise time.

The time taken for response to raise from 0% to 100% for the very first time is rise time.

66.Define peak time.

The time taken for the response to reach the peak value for the first time is peak time.

67.Define peak overshoot.

Peak overshoot is defined as the ratio of maximum peak value measured from the Maxmium value to final value

68.Define Settling time.

Settling time is defined as the time taken by the response to reach and stay within specified error.

69.What is the need for a controller?

The controller is provided to modify the error signal for better control action

70.What are the different types of controllers?

Proportional controller, PI controller, PD controller, PID controller

71.What is proportional controller?

It is device that produces a control signal which is proportional to the input error signal.

72.What is PI controller?

It is device that produces a control signal consisting of two terms –one proportional to error signal and the other proportional to the integral of error signal

73.What is the significance of integral controller and derivative controller in a PID controller?

The proportional controller stabilizes the gain but produces a steady state error.

The integral control reduces or eliminates the steady state error.

74.Why derivative controller is not used in control systems?

The derivative controller produces a control action based on the rate of change of error signal and it does not produce corrective measures for any constant error.

75.Define Steady state error.

The steady state error is defined as the value of error as time tends to infinity.

76.What is the drawback of static coefficients?

The main draw back of static coefficient is that it does not show the variation of error with time and input should be standard input.

77. What is step signal?

The step signal is a signal whose value changes from zero to A at t= 0 and remains constant at A for t>0.

78. What is ramp signal?

The ramp signal is a signal whose value increases linearly with time from an initial value of zero at t=0.the ramp signal resembles a constant velocity.

79.What is a parabolic signal?

The parabolic signal is a signal whose value varies as a square of time from an initial value of zero at t=0.This parabolic signal represents constant acceleration input to the signal.

80.What are the three constants associated with a steady state error?

Positional error constant

Velocity error constant

Acceleration error constant

81.What are the main advantages of generalized error co-efficients?

i) Steady state is function of time.

ii) Steady state can be determined from any type of input

82. What are the effects of adding a zero to a system?

Adding a zero to a system results in pronounced early peak to system response thereby the peak overshoot increases appreciably.

83. State-Magnitude criterion.

The magnitude criterion states that s=sa will be a point on root locus if for that value of s ,

$| D(s) | = |G(s)H(s) | =1$

84.State – Angle criterion.

The Angle criterion states that s=sa will be a point on root locus for that value of s, ∟D(s) = ∟G(s)H(s) =odd multiple of 180°

85. What is a dominant pole?

The dominant pole is a pair of complex conjugate pair which decides the transient response of the system.

86.What is stepper motor?

A stepper motor is a device which transforms electrical pulses into equal increments of rotary shaft motion called steps.

87.What is servomotor?

The motors used in automatic control systems or in servomechanism are Called servomotors. They are used to convert electrical signal into angular motion.

88.Name the test signals used in control system

The commonly used test input signals in control system are impulse step ramp acceleration and sinusoidal signals.

89.Define BIBO stability.

A linear relaxed system is said to have BIBIO stability if every bounded Input results in a bounded output.

90.What is the necessary condition for stability.

The necessary condition for stability is that all the coefficients of the characteristic polynomial be positive.

91.What is the necessary and sufficient condition for stability.

The necessary and sufficient condition for stability is that all of the elements in the first column of the routh array should be positive.

92.What is quadrant symmetry?

The symmetry of roots with respect to both real and imaginary axis called quadrant symmetry.

93.What is limitedly stable system?

For a bounded input signal if the output has constant amplitude oscillations Then the system may be stable or unstable under some limited constraints such a system is called limitedly stable system.

94.What is synchros?

A synchros is a device used to convert an angular motion to an electrical signal or vice versa.

95.What is steady state error?

The steady state error is the value of error signal e(t) when t tends to infinity.

96.What are static error constants.

The Kp Kv and Ka are called static error constants.

97.What is the disadvantage in proportional controller?

The disadvantage in proportional controller is that it produces a constant steady state error.

98.What is the effect of PD controller on system performance?

The effect of PD controller is to increase the damping ratio of the system and so the peak overshoot is reduced.

99.Why derivative controller is not used in control system?

The derivative controller produces a control action based on rare of change of error signal and it does not produce corrective measures for any constant error. Hence derivative controller is not used in control system

100.What is the effect of PI controller on the system performance?

The PI controller increases the order of the system by one, which results in reducing the steady state error .But the system becomes less stable than the original system.

101. What is PD controller?

PD controller is a proportional plus derivative controller which produces an output signal consisting of two time -one proportional to error signal and other proportional to the derivative of the signal.

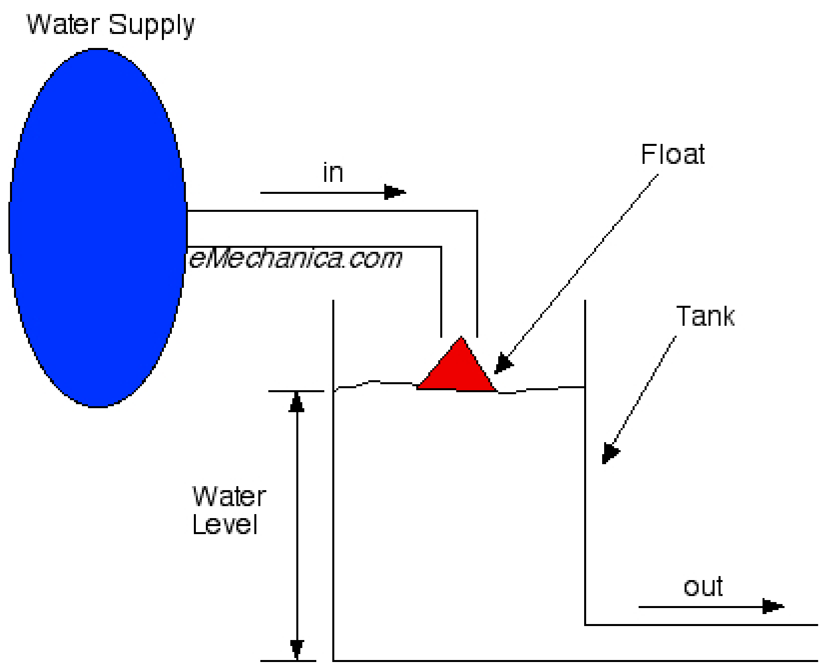
1)Write in detail about the control system

A control system is a dynamical system that affects the behaviour of another system. Examples of control systems can be found all around, and in fact there are very few mechanical or electro-mechanical systems that do not include some kind of a feedback control device. In robotics, control design algorithms are responsible for the motion of the manipulators. In flight applications, control algorithms are designed for stabilization, altitude regulation and disturbance rejection. Cruise control is an interesting application in which the automobile's speed is set at a fixed value. In electronic amplifiers feedback is used to reduce the damaging influence of external noise. In addition, these days control systems can be found in diverse fields ranging from semiconductor manufacturing to environmental regulation.

This course is intended to present you with the basic principles and techniques for the design of feedback control systems. At this point in your study you have mastered the prerequisite topics such as dynamics and the basic mathematical tools that are needed for their analysis. Control system design relies on your knowledge in these fields but also requires additional skills in system interfacing. As you will see from this course, from further electives, or from future experience, the design of feedback control systems depends on

1. Knowledge of basic engineering principles such as dynamics, fluid mechanics, thermal science, electrical and electronic circuits, and materials. These tools are important, as we will soon see, for designing mathematical models of systems. In addition, thorough understanding of the underlying physics is very valuable in determining the most suitable control algorithms and hardware.
2. Knowledge of mathematical tools. In control system design, extensive use is made of matrices and differential equations, and therefore, you should be very comfortable with such concepts. Laplace transforms and complex variables are also used in control applications.
3. Knowledge of simulation techniques. System simulation is essential for verifying the accuracy of the model and for verifying that the final control design meets the desired specifications. In this course we will use the software Matlab to perform control design simulations.
4. Knowledge of control design methodologies, and the basic capabilities and limitations of each control methodology. This aspect of the control design procedure will be the main goal of this course.
5. Knowledge of control hardware such as the different commercially available sensors and actuators. We will cover this part briefly in this course.
6. Knowledge of control software for data acquisition and for the implementation of control algorithms.

   Before we go on discussing the technical aspects of feedback control, we will give a very short outline of its historical beginnings. The use of feedback mechanisms can be traced back to devices that were invented by the Greeks such as liquid level control mechanisms. Early work on the mathematical aspects of feedback and control was initiated by the physicist Maxwell who developed a technique for determining whether or not systems which are governed by linear differential equations are stable. Other prominent mathematicians and physicists, such as Routh and Lyapunov, contributed greatly to the study of stability theory. Their results now form much of the backbone for control design.



Historical Mechanism for Water Level Control.

The study of electronic feedback amplifiers provided the impetus for much of the progress of control design during the first part of the 20th century. The work of Nyquist (1932) and Bode (1945) used mathematical methods based on complex analysis for the analysis of the stability and performance of electronic amplifiers. These techniques are still in use in many technological applications as we will see in this course. Such complex analytic methods are currently called classical control techniques.

During the second world war, advances in control design centered around the use of stochastic analysis techniques to model noise and the development of new methods for filtering and estimation. The MIT mathematician N. Wiener was very influential in this development. Also during that period, research at MIT Radiation Laboratory gave rise to more systematic design methods for servomechanisms.

During the 1950's a different approach to the analysis and design of control systems was developed. This approach concentrated on differential equations and dynamical systems as opposed to complex analytic methods. One advantage of this approach is that it is intimately related to, physical modeling and can be viewed as a continuation to the methods of analytical mechanics. In addition, it provided a computationally attractive methodology for both analysis and design of control systems. Work by Kalman in the USA and Pontryagin in the USSR laid the foundation to what is currently called modern control.

Recently, research aiming at providing reliable and robust control design algorithms resulted in a combination of complex analytic methods and dynamical systems methods. These recent approaches utilize the best features of each method. In this course we will develop techniques that are based on each one of these approaches. At that point it will become clearer what the relative merits are of these two approaches.

2)Explain the modeling of mechanical system

Spring-mass-damper system.

In this example we model the spring-mass-system shown in Figure (a). The mass, *m* is subjected to an external force *f*. Let's suppose that we are interested in controlling the position of *m*. The way to control the position of the mass is by chossing *f*.

We first identify the input and output.
Input: external force, *f*, output: mass position, *x*.

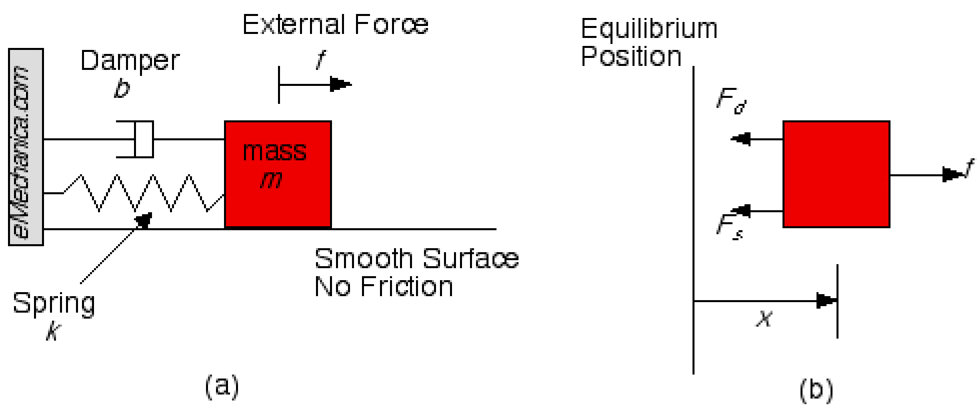


Figure (a) Diagram of the mechanical system components. (b) Free body diagram of the mechanical system.

We apply Newton's second law to obtain the differential equation of this mechanical system. Using the free-body-diagram shown in Figure (b), we have

$$\sum F = ma$$

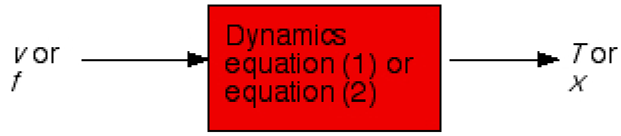
$$f - F_s - F_d = ma$$

$$f - kx - b\frac{dx}{dt} = m\frac{d^2x}{dt^2}$$

Rearranging

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = f, \ldots (2)$$

where, *b* is the damping coefficient and *k* is the spring stiffness . Equation (2) is the differential equation that describes the dynamics of the spring-mass-damper system. Note that the input and output appear in this equation. If we know the input, *f* then we can solve equation (2) for the output, *x*. The mechanical system described can be represented the following block diagram:



Block diagram representation of the mechanical system.

3)Explain with examples the term Stability in detail .

Conceptually, a stable system is one for which the output is small in magnitude whenever the applied input is small in magnitude. In other words, a stable system will not "blow up" when bounded inputs are applied to it. Equivalently, a stable system's output will always decay to zero when no input is applied to it at all. However, we will need to express these ideas in a more precise definition.

Stability (asymptotic stability): A linear system of the form

$$\dot{x} = Ax + Bu, \quad x(0) = x_0$$
$$y = Cx + Du$$

is a stable system if

$$x(t) \to 0 \text{ as } t \to \infty \text{ for any initial condition } x_0 \text{ and } u(t) = 0.$$

Notice that *u(t)* = 0 results in an unforced system:

$$\dot{x} = Ax, \quad x(0) = x_0$$
$$y = Cx$$

By the variation of parameters formula, the state *x(t)* for such an unforced system satisfies

$$x(t) = e^{At} x_0.$$

In this case, the system output *y(t)* = *Cx(t)* is driven only by the initial conditions.

Example 1 Let us analyze the stability of the scalar linear system:

$$\dot{x} = ax + bu, \quad x(0) = x_0.$$

Let *u(t)* = 0 and use the variation of parameters formula. We have

$$x(t) = e^{at} x_0.$$

If a < 0, then the system is stable because *x(t)* goes as to zero as *t* goes to infinity for any initial condition. If a = 0, *x(t)* stays constant at the initial condition and according to our definition of stability the system is unstable. Finally, if a > 0 then *x(t)* goes to infinity ("blows up") as *t* goes to infinity and thus the system is unstable.

Example 2: Let us analyze the stability of the following unforced linear system

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.$$

Using the variation of parameters formula one can see that

$$\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} e^{-t} & 0 \\ 0 & e^{-2t} \end{bmatrix}\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} e^{-t}x_1(0) \\ e^{-2t}x_2(0) \end{bmatrix}.$$

Because both exponentials have a negative number multiplying *t* , for any values of the initial conditions we have

$$x_1(t) \to 0 \text{ and } x_2(t) \to 0 \text{ as } t \to \infty.$$

By the definition of stability given above, this system is stable.

Examples of stable and unstable systems are the spring-mass and spring-mass-damper systems. These two systems are shown in Figure . The spring-mass system (Figure (a)) is unstable since if we pull the mass away from the equilibrium position and release, it is going to oscillate forever (assuming there is no air resistance). Therefore it will not go back to the equilibrium position as time passes. On the other hand, the spring-mass-damper system (Figure (b)) is stable since for any initial position and velocity of the mass, the mass will go to the equilibrium position and stops there when it is left to move on its own.

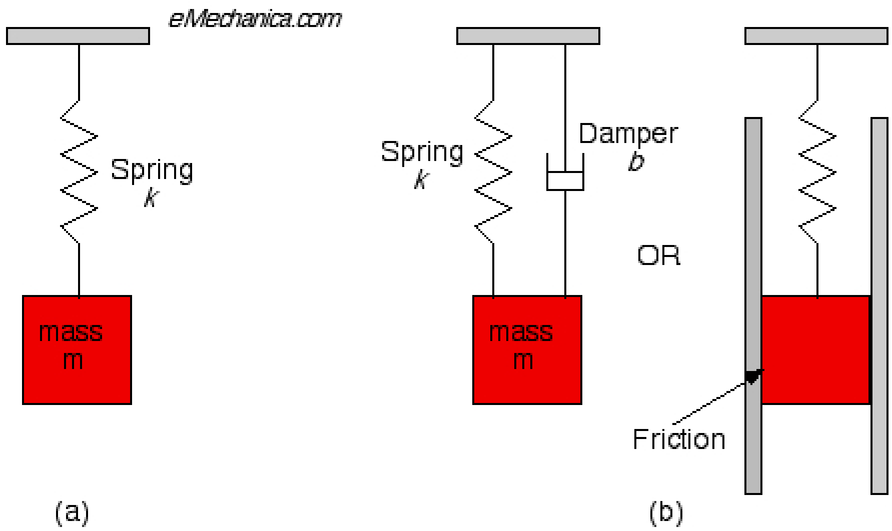


Figure (a) Spring-mass system with no damping. (b) Spring-mass-damper system.

Let us analyze the stability of the spring-mass system using mathematical relations. The equation of motion for the spring-mass system shown in Figure 1(a) is written as

$$m\ddot{x} + kx = 0$$

Note that there is no damping and external force ($b = 0$ and $f = 0$). Rewriting the above differential equation in the state space form with the mass position and velocity as state variables, we get

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.$$

The solution of this state equation is

$$\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = e^{At} \begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix}$$

We now compute the state transition matrix

$$sI - A = \begin{bmatrix} s & -1 \\ \frac{k}{m} & s \end{bmatrix}$$

$$(sI - A)^{-1} = \frac{1}{\det(sI - A)}\begin{bmatrix} s & 1 \\ -\frac{k}{m} & s \end{bmatrix} = \frac{1}{s^2 + \frac{k}{m}}\begin{bmatrix} s & 1 \\ -\frac{k}{m} & s \end{bmatrix} = \begin{bmatrix} \frac{s}{s^2 + \frac{k}{m}} & \frac{1}{s^2 + \frac{k}{m}} \\ \frac{-\frac{k}{m}}{s^2 + \frac{k}{m}} & \frac{s}{s^2 + \frac{k}{m}} \end{bmatrix}$$

$$e^{At} = L^{-1}\left[(sI - A)^{-1}\right] = \begin{bmatrix} \cos\sqrt{\frac{k}{m}}t & \sqrt{\frac{m}{k}}\sin\sqrt{\frac{k}{m}}t \\ -\sqrt{\frac{k}{m}}\sin\sqrt{\frac{k}{m}}t & \cos\sqrt{\frac{k}{m}}t \end{bmatrix}$$

Therefore, the state variables of the system as a function of time are given as

$$\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} \cos\sqrt{\frac{k}{m}}t & \sqrt{\frac{m}{k}}\sin\sqrt{\frac{k}{m}}t \\ -\sqrt{\frac{k}{m}}\sin\sqrt{\frac{k}{m}}t & \cos\sqrt{\frac{k}{m}}t \end{bmatrix}\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} x_1(0)\cos\sqrt{\frac{k}{m}}t + x_2(0)\sqrt{\frac{m}{k}}\sin\sqrt{\frac{k}{m}}t \\ -x_1(0)\sqrt{\frac{k}{m}}\sin\sqrt{\frac{k}{m}}t + x_2(0)\cos\sqrt{\frac{k}{m}}t \end{bmatrix}$$

Note that the two state variables do not go to zero as time goes to infinity for any initial condition. They instead oscillate around the equilibrium point ($x_1 = 0$ and $x_2 = 0$). Therefore, the system is unstable.Another example that demonstrates the concept of stability is the pendulum system which is shown in Figure . If air resistance is negelected the pendulum will oscillate forever and thus the system will be unstable. On the other hand, the mass will always go back to its equilibrum position if air resistance is taken into account and the system will therefore be stable.

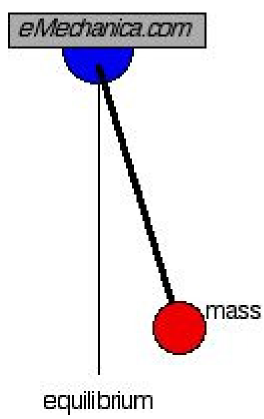


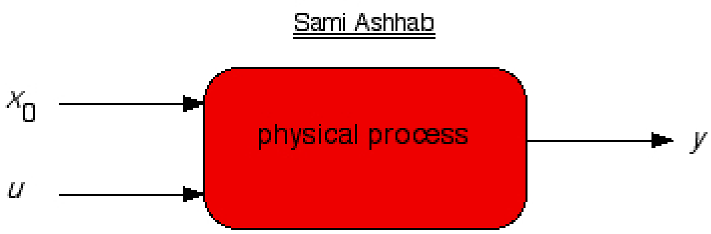
Figure : Pendulum System.

4)Give the Input/Output Description of a Dynamical System

We have studied systems of the form:

$$\dot{x} = Ax + Bu, \quad x(0) = x_0$$
$$y = Cx + Du$$

where *u* is the input, *y* is the output and x_0 is the initial condition. Pictorially, the system can be represented as follows:



From the variation of parameters formula, the state *x(t)* satisfies:

$$x(t) = e^{At}x_0 + \int_0^t e^{A(t-\tau)}Bu(\tau)d\tau,$$

and therefore the output *y(t)* can be written as:

$$\begin{aligned} y(t) &= Cx + Du \\ &= Ce^{At}x_0 + \int_0^t Ce^{A(t-\tau)}Bu(\tau)d\tau + Du(t). \end{aligned}$$

The first term, $Ce^{At}x_0$, is the unforced response resulting only from the initial conditions and the term, $\int_0^t Ce^{A(t-\tau)}Bu(\tau)d\tau + Du(t)$, is the forced response due to the input *u(t)*. A state space description of a system is an internal description because it contains all the information that is available about the internal operation of the system. This information is contained in the state vector. In an external or input/output representation, this internal system information is lost and the effect of the input only on the output is considered. In other words, the output represents only the forced response and the initial conditions are assumed to be equal to zero.

Given the following internal description of a dynamical system

$$\dot{x} = Ax + Bu, \quad x(0) = x_0$$
$$y = Cx + Du,$$

we want to find an input/output relationship in the Laplace domain. We first take the Laplace transform of the above equations with the initial condition set to zero, we have

$$sX(s) = AX(s) + BU(s).$$

Solving for *X(s)* we obtain:

$$(sI - A)X(s) = BU(s)$$
$$X(s) = (sI - A)^{-1}BU(s).$$

Taking the Laplace transform of the output equation *y = Cx+Du* , we find that

$$Y(s) = CX(s) + DU(s) = \left[C(sI - A)^{-1}B + D\right]U(s).$$

An internal or state space system representation describes the evolution of the system in the time domain. However, an external or input/output system description is developed in the Laplace domain. We now consider how an external representation may be obtained from an internal one.

The term $C(sI - A)^{-1}B + D$ is called the transfer function of the system and it determines the output *Y(s)* for any given input *U(s)*. Notice that the equation $Y(s) = \left[C(sI - A)^{-1}B + D\right]U(s)$ does not contain any information about the system state or the initial conditions.

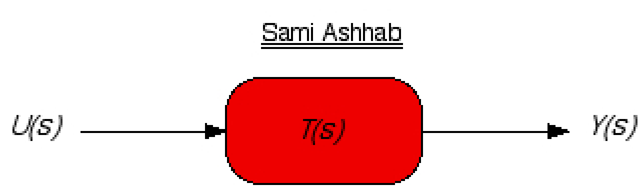
Summary: Given the internal representation of a system:

$$\dot{x} = Ax + Bu, \quad x(0) = x_0$$
$$y = Cx + Du,$$

it straightforward to obtain the transfer function

$$T(s) = \frac{Y(s)}{U(s)} = C(sI - A)^{-1}B + D.$$

A transfer function can be represented by the following block diagram:

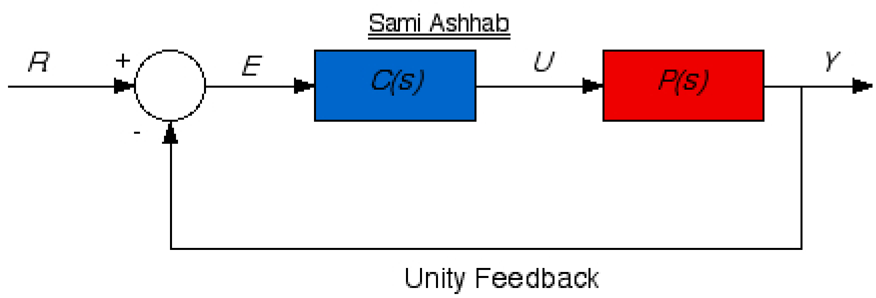


where *Y(s) = T(s)U(s)*.

For a given set of system matrices *A*, *B*, *C*, *D*, there is only one corresponding transfer function T(s). However, one transfer function may correspond to many different state space representations. A state space description obtained from a transfer function is known as a realization and can take on many different forms. We will study a few of these forms such as the controllable canonical form and the observable canonical form later on in the course.Remember that in computing a transfer function, the initial conditions are set to zero. Therefore, a piece of information is lost in the transformation from a state space description to a transfer function.

5)Discuss about the block diagram

We will deal with block diagrams of the form shown below. Comparing this diagram with the one given in the previous lecture we note that the sensor block *S(s)* is missing here. The closed loop system below is called a unity feedback system since *S(s)* = 1. In practice this can be easily achieved by adding an amplifier to the measurement system in order to make the measured and actual outputs equal in value. The units of two outputs will be usually different.



From the previous lecture the transfer function of the above closed loop (feedback) system is (with *S(s)* = 1):

$$T(s) = \frac{Y(s)}{R(s)} = \frac{PC}{1+PC}.$$

The transfer function from the input *R* to the error *E* is equal to

$$J(s) = \frac{E(s)}{R(s)} = \frac{1}{1+PC}.$$

Response of the closed loop system

If we know the input signal *r(t),* then the response of the system due to the input signal can be found as follows

$$y(t) = L^{-1}(Y(s)) = L^{-1}(T(s)R(s)).$$

6)Explain the standard input test signals

1. Unit step, $r(t) = 1 \Rightarrow R(s) = \frac{1}{s}.$

2. Step, $r(t) = \text{constant} \Rightarrow R(s) = \frac{\text{constant}}{s}.$

3. Ramp, $r(t) = \beta t \Rightarrow R(s) = \frac{\beta}{s^2}$ (β is constant).

4. Sine signal, $r(t) = a \sin \omega t \Rightarrow R(s) = a\frac{\omega}{s^2 + \omega^2}.$

Refer note

7) Explain Feedback Control Approach

•Establish control objectives

–Qualitative–don't use too much fuel

–Quantitative

–settling time of step response<3sec

–Typically requires that you understand the process (expected commands and disturbances) and the over all goals (bandwidths).

–Often requires that you have a strong understanding of the physical dynamics of the systems that you do not "fight" them in inappropriate (i.e .,inefficient) ways.

•Selectsensors&actuators

–What aspects of the system are to be sensed and controlled?

–Consider sensor noise and linearity as key discriminators.

–Cost, reliability ,size,...

•Obtain model

–Analytic(FEM) or from measured data (system ID)

–Evaluationmodel→reducesize/complexity→Designmodel

–Accuracy ? Error model ?

•Design controller –Select technique (SISO,MIMO),(classical , state-space)

–Choose parameters (ROT , optimization)

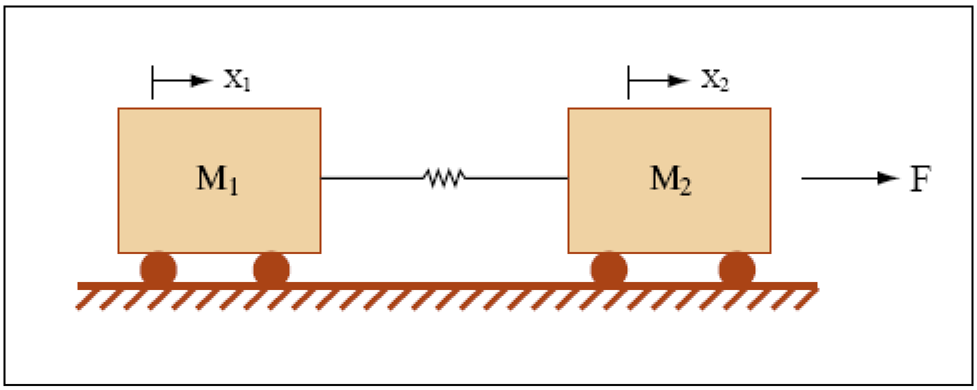
•Analyze closed-loop performance.

Meet objectives ?

–Analysis, simulation, experimentation,...

–Yes- done, No- iterate...

8)Find the transfer function of the given mechanical translational system



1. Start with a free body diagram
2. Develop the 2 equations of motion

$$
\begin{aligned}
m_1\ddot{x}_1 &= k(x_2 - x_1) \\
m_2\ddot{x}_2 &= k(x_1 - x_2) + F
\end{aligned}
$$

3. How determine the relationships between x_1, x_2 and F?
   - Numerical integration - good for simulation, but not analysis
   - Use Laplace transform to get transfer functions
     - ◇ Fast/easy/lots of tables
     - ◇ Provides lots of information (poles and zeros)

- Laplace transform

$$\mathcal{L}\{f(t)\} \equiv \int_{0^-}^{\infty} f(t)e^{-st}dt$$

  – Key point: If $\mathcal{L}\{x(t)\} = X(s)$, then $\mathcal{L}\{\dot{x}(t)\} = sX(s)$ assuming that the initial conditions are zero.

- Apply to the model

$$\mathcal{L}\{m_1\ddot{x}_1 - k(x_2 - x_1)\} = (m_1s^2 + k)X_1(s) - kX_2(s) = 0$$

$$\mathcal{L}\{m_2\ddot{x}_2 - k(x_1 - x_2) - F\} = (m_2s^2 + k)X_2(s) - kX_1(s) - F(s) =$$

$$\begin{bmatrix} m_1s^2 + k & -k \\ -k & m_2s^2 + k \end{bmatrix} \begin{bmatrix} X_1(s) \\ X_2(s) \end{bmatrix} = \begin{bmatrix} 0 \\ F(s) \end{bmatrix}$$

- Perform some algebra to get

$$\frac{X_2(s)}{F(s)} = \frac{m_1s^2 + k}{m_1m_2s^2(s^2 + k(1/m_1 + 1/m_2))} \equiv G_2(s)$$

- $G_2(s)$ is the **transfer function** between the input F and the system response x_2

9) Obtain the Bode plot of the system given by the transfer function G(s)= $\frac{1}{(2s+1)}$

We convert the transfer function in the following format by substituting $s = j\omega$

$$G(j\omega) = \frac{1}{2j\omega + 1}. \tag{1}$$

We call $\omega = \frac{1}{2}$, the break point. So for

$\omega << \frac{1}{2}$, i.e., for small values of ω

$G(j\omega) \approx 1.$

Therefore taking the log magnitude of the transfer function for very small values of ω, we get

$$20\log|G(j\omega)| = 20\log(1) = 0.$$

Hence we see that below the break point the magnitude curve is approximately a constant.

For,

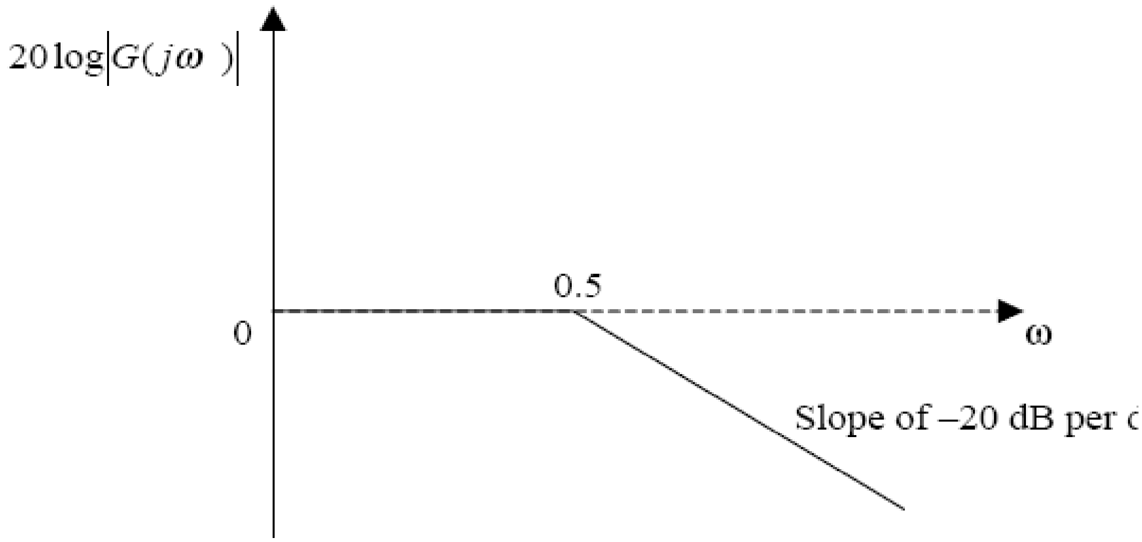
$\omega >> \frac{1}{2}$, i.e., for very large values of ω

$G(j\omega) \approx \frac{1}{2j\omega}.$

Similarly taking the log magnitude of the transfer function for very large values of ω, we have

$$20\log|G(j\omega)| = 20\log\left|\frac{1}{2j\omega}\right| = 20\log\left(\frac{1}{2\omega}\right) = 20\log(1) - 20\log(2\omega) = -20\log(2\omega).$$

So we see that, above the break point the magnitude curve is linear in nature with a slope of –20 dB per decade. The two asymptotes meet at the break point. The asymptotic bode magnitude plot is shown below.



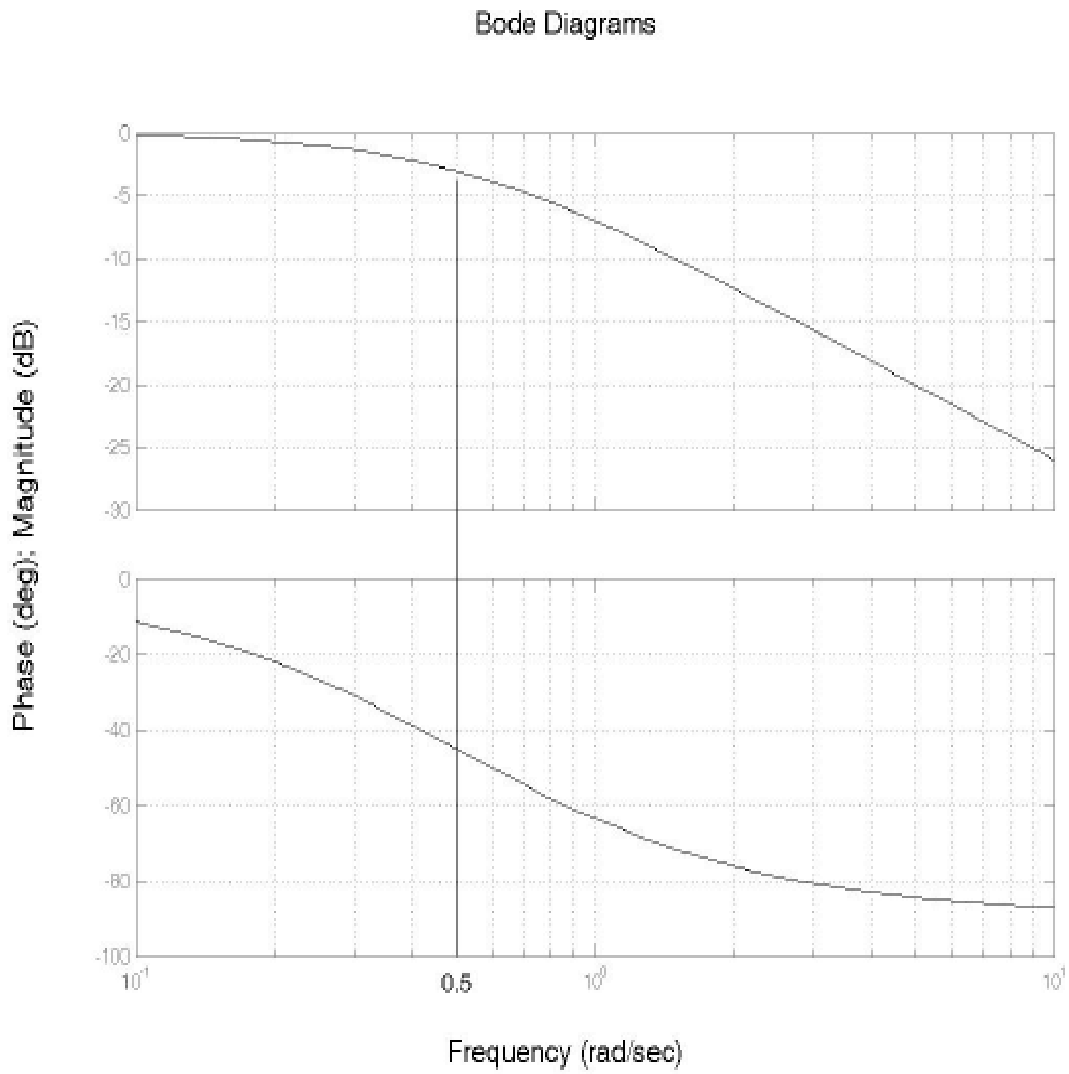
The phase of the transfer function given by equation (1) is given by

$\phi = 0 - \tan^{-1}(2\omega) = -\tan^{-1}(2\omega)$.

So for small values of ω, i.e., $\omega \approx 0$, we get

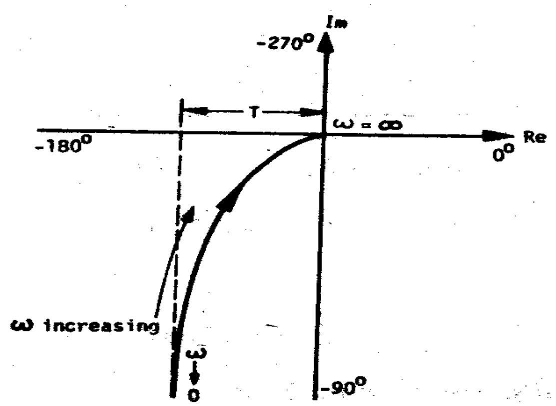
$\phi \approx 0$.

For very large values of ω, i.e., $\omega \to \infty$, the phase tends to –90 degrees.


Bode Diagrams
0
-5
-10
-15
-20
-25
-30
0
-20
-40
-60
-80
-100
Phase (deg); Magnitude (dB)
10^{-1}
0.5
10^{0}
10^{1}
Frequency (rad/sec)

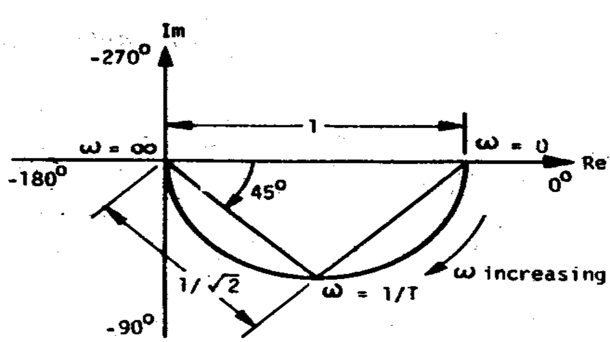
10)Explain Polar Plot.

The polar plot of a sinusoidal transfer function $G(j\omega)$ is a plot of the magnitude of $G(j\omega)$ versus the phase angle of $G(j\omega)$ on polar coordinates as ω is varied from zero to infinity. An advantage of using polar plot is that it depicts the frequency response characteristics of a system over the entire frequency range in a single plot.



Polar plot of 1/jω(1+jωT).

The polar plot of $G(j\omega)=\dfrac{1}{1+j\omega T}=\dfrac{1}{\sqrt{1+\omega^2T^2}}\angle\tan^{-1}\omega T$ is shown in figure below.

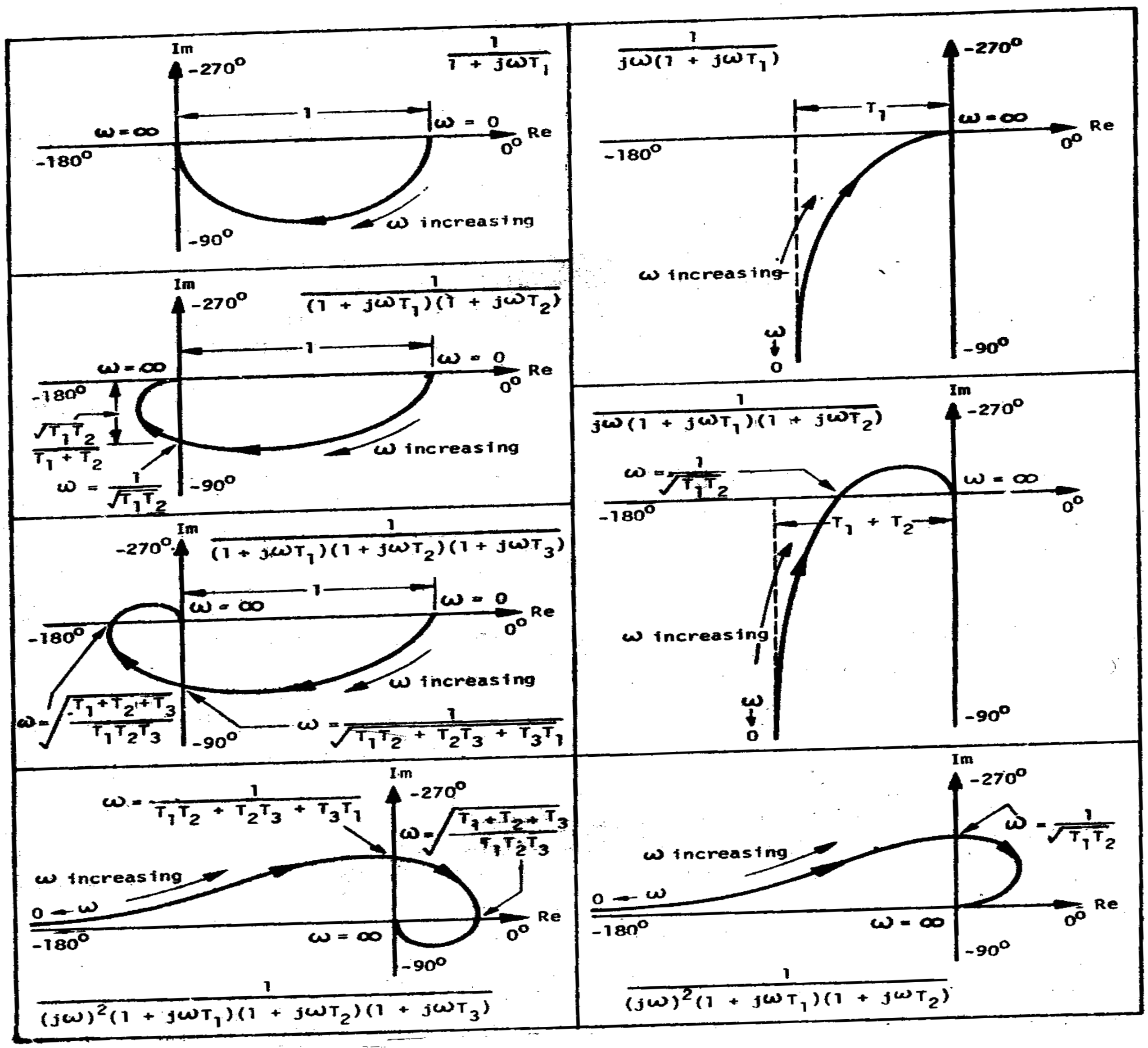


Polar plot of 1/(1+jωT).

The polar plot of the transfer function, $G(j\omega)=\dfrac{1}{j\omega(1+j\omega T)}$ is shown in figure above.

The plot is asymptotic to the vertical line passing through the point *(-T, 0)*.

Polar plots are useful for the stability study of systems. The general shapes of the polar plots of some important transfer functions are given in figure below.



From the plots above, following observations are made,

1. Addition of a nonzero pole to the transfer function results in further rotation of the polar plot through an angle of -90° as $\omega \rightarrow \infty$.
2. Addition of a pole at the origin to the transfer function rotates the polar plot at zero and infinite frequencies by a further angle of -90°.

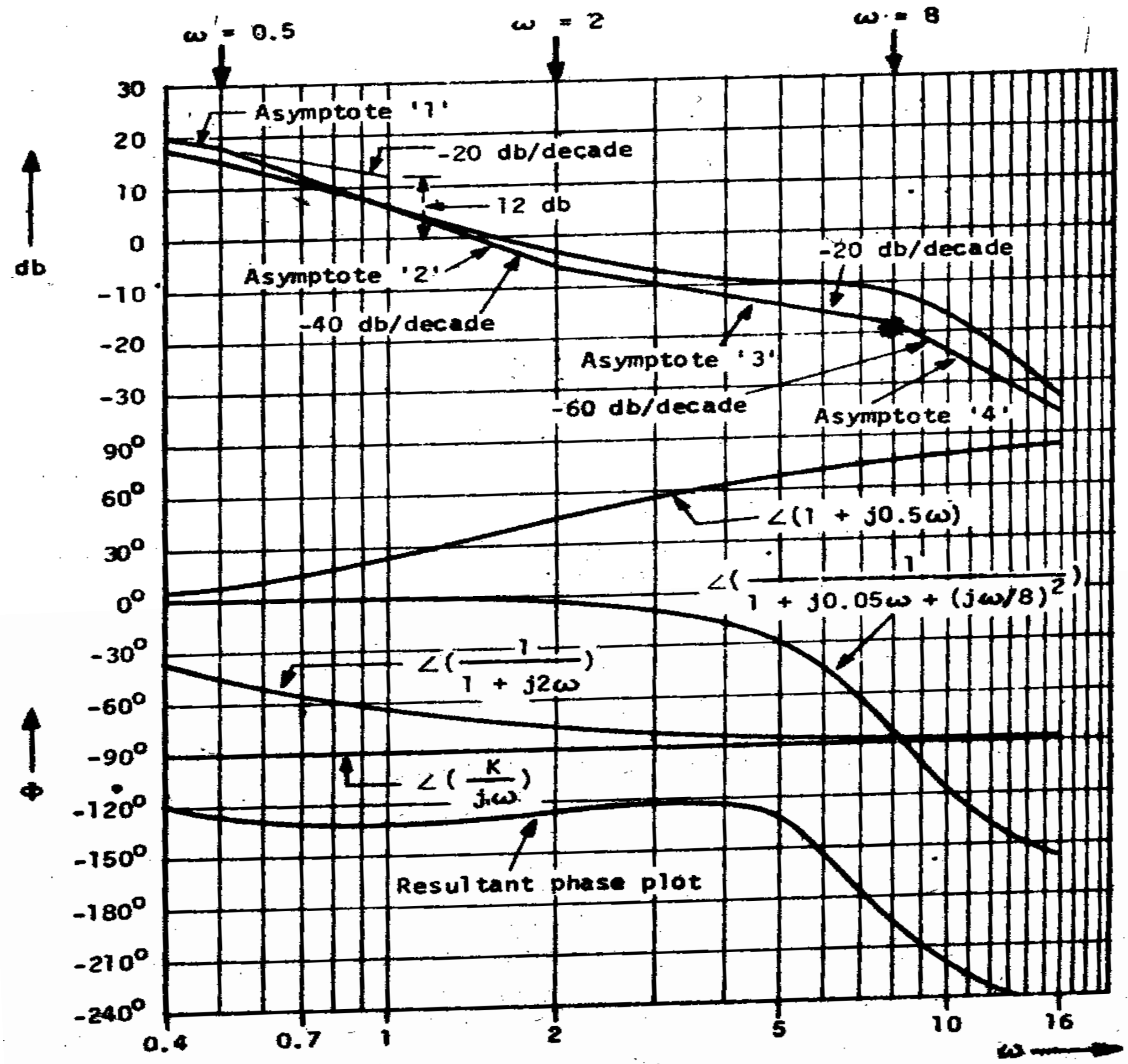
11) Draw the Bode plot of the transfer function G(s)= $\dfrac{64(s+2)}{s(s+0.5)(s^2+3.2s+64)}$

$$G(s)=\frac{64(s+2)}{s(s+0.5)(s^2+3.2s+64)}=\frac{4(1+s/2)}{s(1+2s)(1+0.05s+s^2/64)}.$$

The sinusoidal transfer function in time-constant form is,

$$G(j\omega)=\frac{4(1+j\omega/2)}{j\omega(1+2j\omega)(1+0.4j\left(\frac{\omega}{8}\right)-\left(\frac{\omega}{8}\right)^2}.$$

Factor	f_c	Log-magnitude characteristic	Phase angle characteristic
$4/j\omega$	-	Straight line of slope -20 db/decade, passing through $20\log 4=12$ db point at $\omega=1$.	Constant -90°
$1/1+2j\omega$	$\omega_1=$ 0.5	Straight line of 0 db for $\omega<\omega_1$, straight line of slope -20 db/decade for $\omega>\omega_1$.	0 to -90°, -45° at ω_1.
$1+j0.5\omega$	$\omega_2=$ 2	Straight line of 0 db for $\omega<\omega_2$, straight line of slope +20 db/decade for $\omega>\omega_2$.	0 to $+90^\circ$, 45° at ω_2.
$1+j0.4\left(\frac{\omega}{8}\right)-\left(\frac{\omega}{8}\right)^2$; $\omega_n=8,\ \zeta=0.2$	$\omega_3=$ 8	Straight line of 0 db for $\omega<\omega_3$, straight line of slope -40 db/decade for $\omega>\omega_3$.	0 to -180°, -90° at ω_3.



Bode plot of $\dfrac{4(1+j\omega/2)}{j\omega(1+j2\omega)[1+j0.4(\omega/8)-(\omega/8)^2]}$.

The phase angle curve may be drawn using the following procedure.

1. For the factor $K/(j\omega)^r$, draw a straight line of $-90°r$.

2. The phase angles of the factor $(1+j\omega T)^{\pm 1}$ are

a. $\pm 45^{\circ}$ at ω=1/T

b. $\pm 26.6^{\circ}$ at ω=1/2T

c. $\pm 5.7^{\circ}$ at ω=1/10T

d. $\pm 63.4^{\circ}$ at ω=2/T

e. $\pm 84.3^{\circ}$ at ω=10/T

3. The phase angles for the quadratic factor are

a. -90° at $\omega=\omega_n$

b. A few points of phase angles are read off from the normalized Bode plot for the particular ζ .

12) Construct the root locus for the open-loop transfer function .

$$G_0(s) = \frac{k_0(s+1)}{s(s+2)(s^2+12s+40)} .$$

The degree of the numerator polynomial is $m = 1$. This means that the transfer function has one zero ($s_{Z_1} = -1$). The degree of the denominator polynomial is $n = 4$and we have the four poles ($s_{P_1} = 0$, $s_{P_2} = -2$, $s_{P_3} = -6 + j2$, $s_{P_4} = -6 - j$2). First the poles (x) and the zeros (o) of the open loop are drawn on the $s$plane as shown in Figure 1. According to rule 3 these poles are just

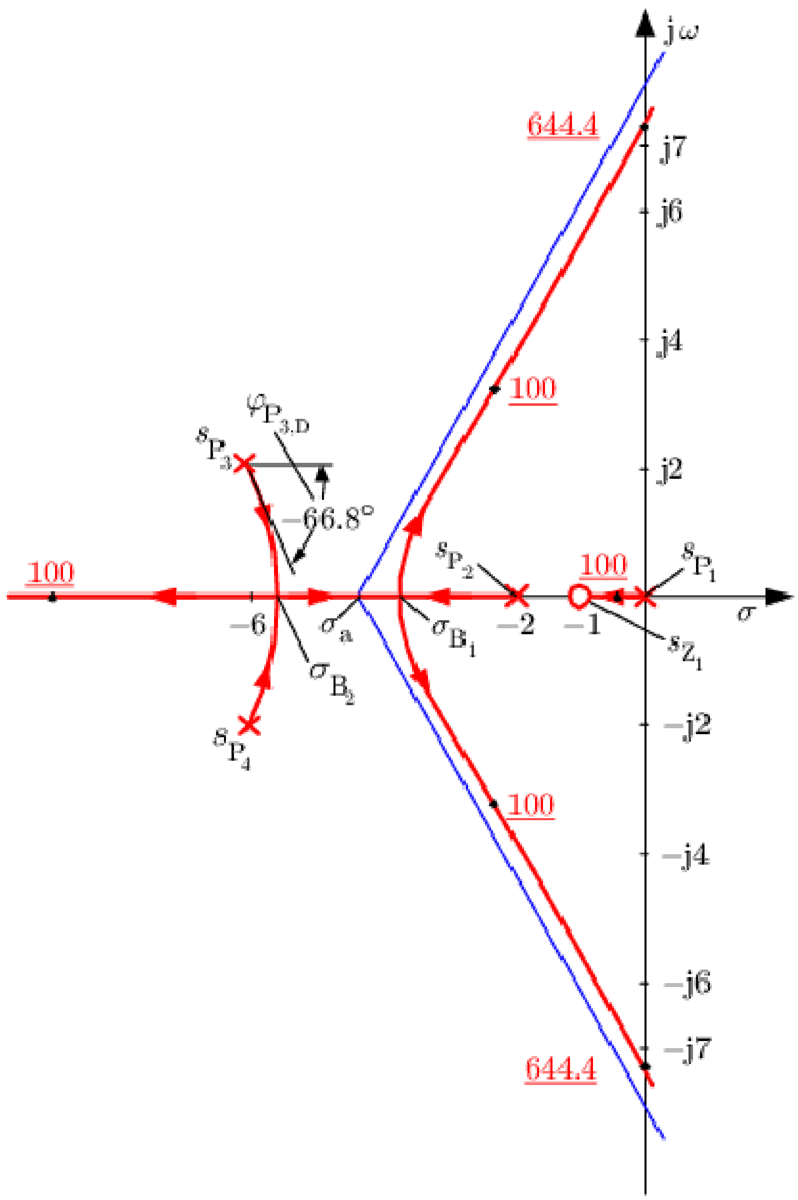


Figure 1: Root locus of $G_0(s) = \frac{k_0(s+1)}{s(s+2)(s+6+j2)(s+6-j2)}$. Values of k_0 are in red and underlined.

those points of the root locus where $k_0 = 0$ and the zeros where $k_0 \to \infty$. We have $(n-m) = 3$ branches that go to infinity and the asymptotes of these three branches are lines which intercept the real axis according to rule 6.The crossing is at

$$\sigma_a = \frac{(0-2-6-6)-(-1)}{3} = -\frac{13}{3} = -4.33$$

and the slopes of the asymptotes are

$$\alpha_k = \frac{\pm 180^\circ(2k+1)}{3} = \pm 60^\circ(2k+1) \quad k = 0, 1, 2, \dots$$

i.e. $\alpha_0 = 60^\circ, \qquad \alpha_1 = +180^\circ, \qquad \alpha_2 = -60^\circ$.

The asymptotes are shown in Figure 1as blue lines. Using Rule 4 it can be checked which points on the real axis are points on the root locus. The points σ with $-1 < \sigma < 0$ and $\sigma < -2$ belong to the root locus, because to the right of them the number of poles and zeros is odd. According to rule 7 breakaway and break-in points can only occur pair wise on the real axis to the left of -2. Here we have

$$\frac{1}{s} + \frac{1}{s+2} + \frac{1}{s+6-\mathrm{j}2} + \frac{1}{s+6+\mathrm{j}2} = \frac{1}{s+1}$$

or

$$3s^4 + 32s^3 + 106s^2 + 128s + 80 = 0 \; .$$

This equation has the solutions $s_{B_1} = -3.68$, $s_{B_2} = -5.47$ and $s_{B_{3,4}} = -0.76 \pm \mathrm{j}0.866$. The real roots $s_{B_1} = -3.68$ and $s_{B_2} = -5.47$ are the positions of the breakaway and the break-in point. The angle of departure $\varphi_{P_3,D}$ of the root locus from the complex pole at $s_{P_3} = -6 + \mathrm{j}2$ can be determined from Figure 2

$$\varphi_{P_3,D} = -90^\circ - 153.4^\circ - 161.6^\circ + 158.2^\circ \pm 180^\circ(2k+1) \tag{6.30}$$

$$= -246.8^\circ + 180^\circ = -66.8^\circ \; .$$

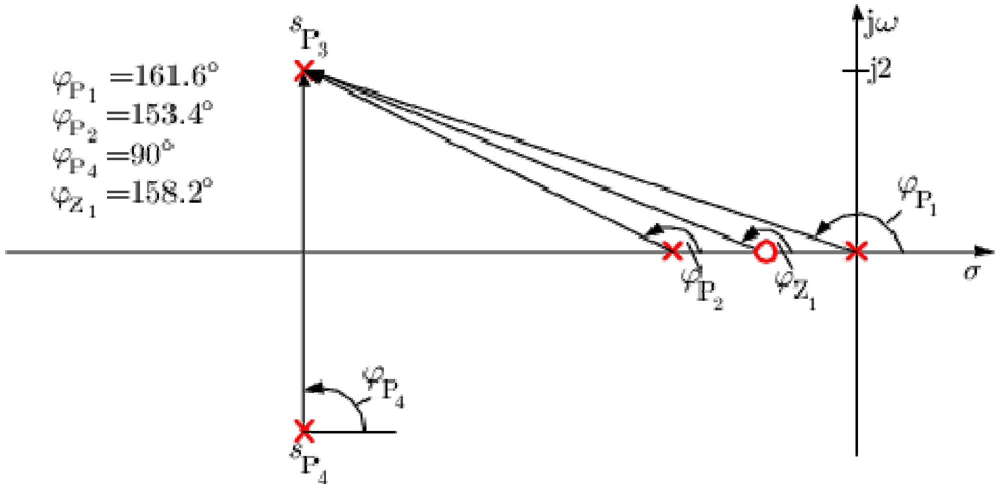


Figure 2: Calculating the angle of departure $\varphi_{P_3,D}$ of the complex pole $s_{P_3} = -6 + j2$

With this specifications the root locus can be sketched. Using rule 9 the value of k_0 can be determined for some selected points. The value at the intersection with the imaginary axis is

$$k_{0,crit} = \frac{7.2 \cdot 7.4 \cdot 7.9 \cdot 11.1}{7.25} = 644.4 \,.$$