CS344: MidTerm (sample)
Thursday Oct 26, 11:45–12:45.

October 26, 2017

1. *(Sorting)*[**10 Pts**]

   Someone claims she can sort n numbers by dividing into $\sqrt{n}$ subproblems of size $\sqrt{n}$ each, solving each subproblem recursively, and then combining those solutions. How much time is she allowed to use to compose the solutions of these subproblems into the solution for the overall problem, if the entire algorithm takes $O(n \log n)$ time?

2. *(Dynamic Programming)* [**10 pts**]

   A tree T consists of a root node and zero or more children, each of which is a tree. The *maximum independent set* of a tree T is a subset I of nodes such that no two of them are children of each other, and such that I is as large as possible. Use dynamic programming[1] solve the maximum independent set problem. What is the best running time you can get?

   Hint: Suppose that we know the size of the largest independent set of all subtrees below a node j. What is the maximum independent set in the subtree hanging from j?

   - j is not in the maximum independent set.

     Then the maximum independent set is simply the union of the maximum independent sets of the subtrees of the children of j.

   - or j is in the maximum independent set

     Then the maximum independent set consists of j, plus the union of the maximum independent sets of the subtrees of the .....

3. *(Rapid Answers)* [**5 pts each**]

   - $f(n) = n$ and $g(n) = 2^{\log_2 n}$. Is $f = o(g(n))$? True or False? Provide reasons.
   - Given n items hashed into m buckets using universal hashing with chaining, what is the expected number of comparisons done in the case of an unsuccessful search?
   - Given strings s and t of length n, we can say s is lexicographically smaller than t if $f(s) < f(t)$ where $f(x)$ is the Karp-Rabin fingerprint for any string x. True or False? Provide reasons. We say s is lexicographically smaller than t if s appears before t in the dictionary of strings.
   - To solve string matching problem with pattern $p[1, m]$ and text $t[1, 2m]$, we used prime $q \in [1..m^5]$ to get probability of false match to be at most $1/m$. Suppose we solve k such problems. Give a bound on the probability that we get a false match.

[1] See some practice dynamic programming problems at `http://people.cs.clemson.edu/~bcdean/dp_practice/`.