CS344: Final

Thursday Dec 21, 12.10 – 2:50 (160 min)

$T(n) = 2T(n/3) + n$

Instructions. Answer Problem 1 and **ANY THREE** of Problems 2–5. Total **100 Pts.**

1. Rapid Questions. Answer **ANY FIVE.** 5 points each. − 25 points

(a) We have an algorithm that takes time $T(n)$ in the worst case, given by a recurrence: $T(n) = T(n/3) + T(n/4) + n$. Solve the recurrence to get $T(n)$ asymptotically.

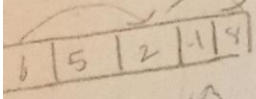
(b) There exists a polynomial time algorithm to determine whether 3-SAT formula with 10 clauses is satisfiable. T/F? Provide reasons.

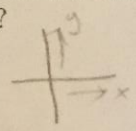
(c) State the problem Count-Min Sketch solves and state the time, space and approximation guarantee provided by the Count-Min Sketch. log 1/6 ? e log

(d) State precisely and clearly one problem from the homeworks you liked to solve, and why. $O(n^2)$

(e) Say P is a shortest path from some vertex s to some other vertex t in a graph. If the weight of each edge in the graph is increased by 1, P will still be a shortest path from s to t. T/F? Provide reasons. True

(f) Given an undirected graph G, show an algorithm that will determine if it is 2-colorable. How much time does your algorithm take?





2. Dynamic Programming. 25 Pts.

You are given a sequence of n numbers (positive or negative): $x_1, x_2, \ldots, x_n$. Your job is to select a subset of these numbers of maximum total sum, subject to the constraint that you can not select two elements that are adjacent (that is, if you pick x_i then you cannot pick either x_{i-1} or x_{i+1}).

n^2

Explain how you can find, in time polynomial in n, the subset of maximum total sum.

bottom up

3. **Network Flow.** 25 Pts. $O(1)$

You have n jobs each taking 1 unit of time, to be scheduled on m processors, $n >> m$. The ith job has a list L_i of the processors on which it can be run. Each job i is to be assigned to exactly one processor on its list L_i. Multiple jobs can be assigned to

bloom filter?

the same processor, but each processor j has a job limit P_j which is the maximum number of jobs that can be assigned to processor j. Describe an efficient algorithm which either finds a legal assignment of all jobs or determines that no such assignment exists. How much time does your algorithm take?

$1 + n/m$

4. `Finding Approximate Maximum. 25 Pts`

You have n integer items and want to return any item from the input with rank $\geq n/16$ (the *rank* of an item is the number of items smaller than it). Design a randomized algorithm with probability of error at most $1/1024$. Precisely state your algorithm and analyze it to prove it has the error probability needed.

5. `Total Recall. 25 Pts`

State any one problem from the homeworks you liked to solve (**different** from Problem 1(d)). State the problem precisely and provide the full solution, with proofs if any.