Complex Arithmetic Operator

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***Abstract*—The Complex Number Arithmetic Operator is designed to compute addition, subtraction, multiplication, division, angle and conjugate of complex numbers by gate-level description in modular Verilog. The operators is efficient and capable of supporting both signed and unsigned inputs along with comprehensive testbenches to cover all of the test cases possible for 8-bit input. It has been designed with accuracy of handling8-bit precision with the usage of optimized modules for high reliability and efficiency which will inturn reflect on the suitability of various applications such as robotics, control system and signal processing.**

***Index Terms*—Complex Numbers, Addition, Multiplication, Division, Subtraction, Conjugate, Angle, Gates, Half Adder, Full Adder.**

I. Introduction

The Complex Arithmetic Operator is a versatile system which is efficient and accurate in performing various operations on complex numbers. These operations consist of addition, subtraction, multiplication, division, conjugate and angle calculations. The ALU designed for this operator includes gate-level components such as AND, OR, NAND, XOR, Half Adders and Full Adders which are all written in Verilog. The design is equipped to maintain an 8-bit precision while taking input from both the real and imaginary channels. The outputs shown have been tested and have been proven accurate and reliable. More applications where this operator can be used involves robotics, control systems having chip-level arithmetic operational solution and signal processing.

II. Objectives

Goals of the Project: Complex Arithmetic Operator

The following are the primary objectives of the Complex Arithmetic Operator project:

1. Calculate Complex Math Evaluations: Design an apparatus that is able to perform easily basic arithmetic operations which are addition, subtraction, multiplication, division, conjugate computation, and angle computation especially for complex numbers.

2. Manipulate Unsigned and Signed Complex Numbers: It is imperative that the operator functions proficiently with both unsigned and signed complex numbers, facilitating precise calculations across various numerical representations.

3. Enhance Performance: Design the operator to be computationally efficient, minimizing resource usage while maintaining speed, particularly for applications requiring real-time processing.

4. Simplify Modular Design: Modular architectural design it permits easier extension and potential enhancements, such as adding additional operations or optimizations.

5. Guarantee Accuracy and Precision: Achieve high precision in operations, particularly in multiplication and division, while handling edge cases like division by zero.

6. Give a clear and understandable output:

And the result of each operation should be clearly displayed, both in real and imaginary parts terms, and with appropriate negative flags for subtraction and division processes.

7. Hardware Implementation Support:

It should be designed with hardware implementations in mind, such as on FPGAs or ASICs, and so applicable in embedded systems and digital signal processing.

8. Enable Agility and Scalability: Facilitate the system's smooth adaptation or extension to the performance of more advanced operations, like handling higher-dimensional complex numbers or integration with other activities of signal processing. 9.Graphical Representation of Results: Present graphical presentations of the results of operations, including waveforms that correspond to complex number results, so data may be more readily visualized and interpreted. 10.]Practical Implementations]: Ensure that the operator should be applicable to some real-world problem in the fields of telecommunications, digital signal processing, computational physics and control systems.

III. Pseudo code for Complex Number Operations

A. Algorithmic Pseudo code

IV. Circuit diagram

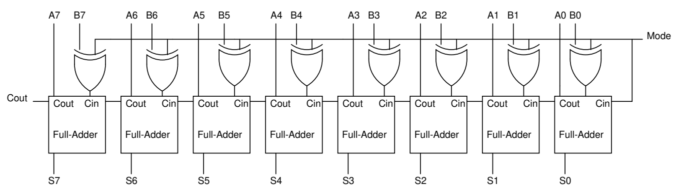


Fig. 1. 8 bit subtractor.

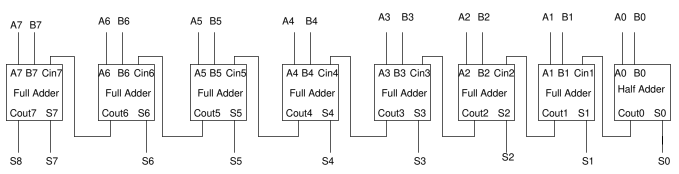


Fig. 2. 8 bit adder

Divider, Conjugate and angle The divider is constructed using the multiplier module in which the conjugate of the

Algorithm 1 Complex Arithmetic Operator

```
procedure COMPLEX ARITHMETIC OPERATOR($A, B$)
    $A_{\text{real}} \leftarrow$ Real part of $A$
    $A_{\text{imag}} \leftarrow$ Imaginary part of $A$
    $B_{\text{real}} \leftarrow$ Real part of $B$
    $B_{\text{imag}} \leftarrow$ Imaginary part of $B$
    if $Operation =$ Addition then
        $R_{\text{real}} \leftarrow A_{\text{real}} + B_{\text{real}}$
        $R_{\text{imag}} \leftarrow A_{\text{imag}} + B_{\text{imag}}$
    else if $Operation =$ Subtraction then
        $R_{\text{real}} \leftarrow A_{\text{real}} - B_{\text{real}}$
        $R_{\text{imag}} \leftarrow A_{\text{imag}} - B_{\text{imag}}$
    else if $Operation =$ Multiplication then
        $R_{\text{real}} \leftarrow (A_{\text{real}} \times B_{\text{real}}) - (A_{\text{imag}} \times B_{\text{imag}})$
        $R_{\text{imag}} \leftarrow (A_{\text{real}} \times B_{\text{imag}}) + (A_{\text{imag}} \times B_{\text{real}})$
    else if $Operation =$ Division then
        $D \leftarrow (B_{\text{real}}^2 + B_{\text{imag}}^2)$
        if $D = 0$ then
            return Error: Division by Zero
        end if
        $R_{\text{real}} \leftarrow \frac{(A_{\text{real}} \times B_{\text{real}}) + (A_{\text{imag}} \times B_{\text{imag}})}{D}$
        $R_{\text{imag}} \leftarrow \frac{(A_{\text{imag}} \times B_{\text{real}}) - (A_{\text{real}} \times B_{\text{imag}})}{D}$
    else if $Operation =$ Conjugate then
        $R_{\text{real}} \leftarrow A_{\text{real}}$
        $R_{\text{imag}} \leftarrow -A_{\text{imag}}$
    else if $Operation =$ Angle then
        if $A_{\text{real}} = 0$ and $A_{\text{imag}} = 0$ then
            return Error: Undefined Angle
        end if
        $R_{\text{angle}} \leftarrow \text{atan2}(A_{\text{imag}}, A_{\text{real}}) \times \frac{180}{\pi}$
        if $R_{\text{angle}} < 0$ then
            $R_{\text{angle}} \leftarrow R_{\text{angle}} + 360$
        end if
    end if
    return Result $R$
end procedure=0
```

denominator is multiplied and divided. To find the conjugate of the complex number we have to change the sign of the imaginary number. To find the angle of the complex number we have to take the tan inverse of imaginary number upon the real number.

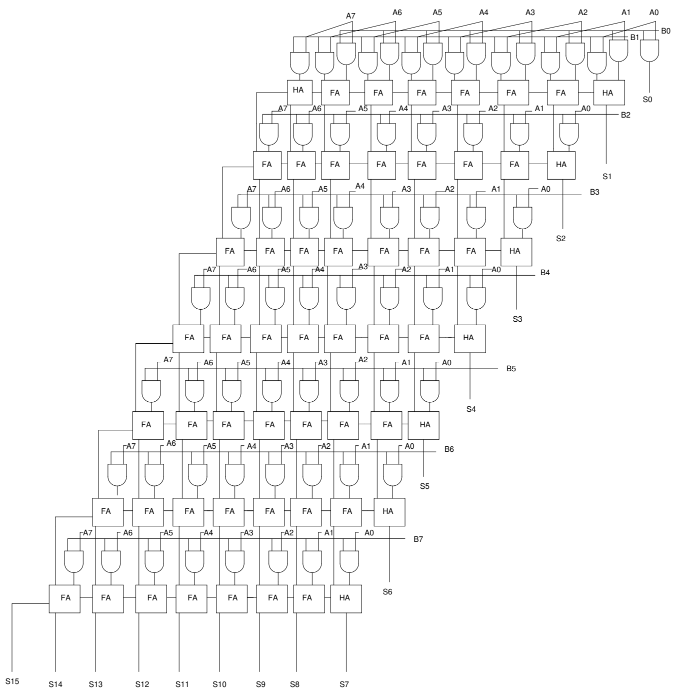


Fig. 3. 8 bit multiplier.

V. Flowchart of Complex Arithmetic Operator

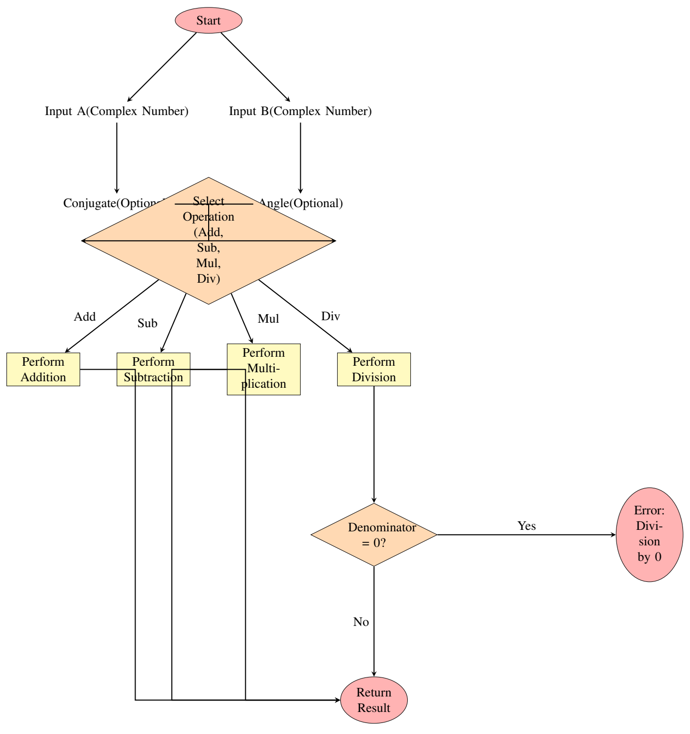


Fig. 4. Compact Flowchart of the Complex Arithmetic Operator

VI. Outputs

A. Output of unsigned bit

Test Case: Complex Number Multiplication, Division, Addition, and Subtraction

Multiplication Results: Input 1: r1 = 11111111 (255), i1 = 11111111 (255) Input 2: r2 = 11111111 (255), i2 = 11111111 (255) Multiplication Output Real Part: 0000000000000000 (0) Multiplication Output Imaginary Part: 1111110000000010 (64514)

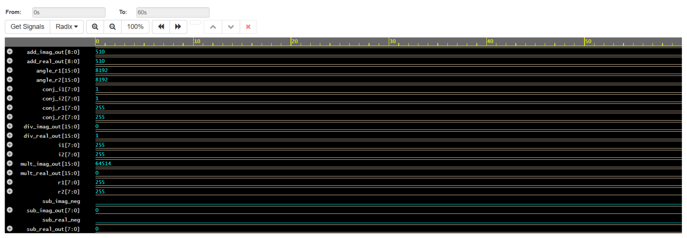
Division Results: Division Output Real Part: 0000000000000001 (1) Division Output Imaginary Part: 0000000000000000 (0)

Addition Results: Addition Output Real Part: 111111110 (510) Addition Output Imaginary Part: 111111110 (510)

Subtraction Results: Subtraction Output Real Part: 00000000 (0), Negative Flag: 0 Subtraction Output Imaginary Part: 00000000 (0), Negative Flag: 0

Conjugate Results: Conjugate 1: Real = 11111111 (255), Imag = 00000001 (1) Conjugate 2: Real = 11111111 (255), Imag = 00000001 (1)

Angle Results: Angle r1 = 45 degrees, Angle r2 = 45 degrees



```
From: 0s        To: 60s
add_imag_out[8:0]    510
add_real_out[8:0]    510
angle_r1[15:0]       8192
angle_r2[15:0]       8192
conj_i1[7:0]         1
conj_i2[7:0]         1
conj_r1[7:0]         255
conj_r2[7:0]         255
div_imag_out[15:0]   0
div_real_out[15:0]   1
i1[7:0]              255
i2[7:0]              255
mult_imag_out[15:0]  64514
mult_real_out[15:0]  0
r1[7:0]              255
r2[7:0]              255
sub_imag_neg
sub_imag_out[7:0]    0
sub_real_neg
sub_real_out[7:0]    0
```

Fig. 5. Output for unsigned bit.

B. Output of signed bit

Addition Results: Real = 000001010 (10), Imag = 000001010 (10)

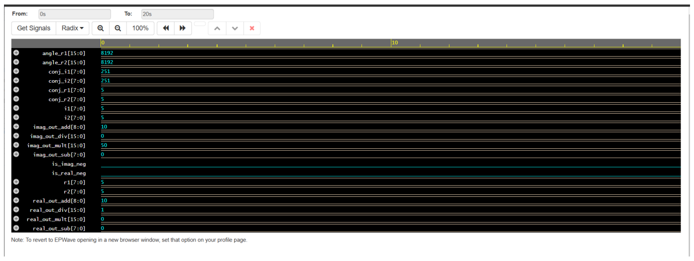
Subtraction Results: Real = 00000000 (0) (Neg=0), Imag = 00000000 (0) (Neg=0)

Multiplication Results: Real = 0000000000000000 (0), Imag = 0000000000110010 (50)

Division Results: Real = 0000000000000001 (1), Imag = 0000000000000000 (0)

Conjugate Results: Conjugate 1: Real = 00000101 (5), Imag = 11111011 (-5) Conjugate 2: Real = 00000101 (5), Imag = 11111011 (-5)

Angle Results: Angle r1 = 45 degrees, Angle r2 = 45 degrees



```
From: 0s        To: 20s
angle_r1[15:0]       8192
angle_r2[15:0]       8192
conj_i1[7:0]         251
conj_i2[7:0]         251
conj_r1[7:0]         5
conj_r2[7:0]         5
i1[7:0]              5
i2[7:0]              5
imag_out_add[8:0]    10
imag_out_div[15:0]   0
imag_out_mult[15:0]  50
imag_out_sub[7:0]    0
is_imag_neg
is_real_neg
r1[7:0]              5
r2[7:0]              5
real_out_add[8:0]    10
real_out_div[15:0]   1
real_out_mult[15:0]  0
real_out_sub[7:0]    0
```

Note: To revert to EPWave opening in a new browser window, set that option on your profile page.

Fig. 6. Output for signed bit.

VII. Comparision Table

Paper Name	Similarities	Differences
Introduction to Complex Binary System	• The operations of addition, subtraction, multiplication, and division are carried out. • No practical hardware usage is shown, only the concept of complex binary number calculations is depicted.	• Arithmetic numbers are calculated by evaluating the imaginary and real parts as a single entity. • Division is performed by multiplying with the conjugate before dividing. • Two additional functionalities: conjugate and angle of complex number.
8-Bit ALU Design using m-GDI Technique	• The concept of full adder and subtractor depicted in the ALU unit is the same for both. • Logical units described here are similar to modules in our code for basic gates (e.g., AND, OR, XOR, NAND).	• Designed using m-GDI technique with modules like comparator, multiplexer, and demultiplexer. • Inputs are treated as standard binary numbers, not by separating real and imaginary parts. • The m-GDI technique reduces power consumption by up to 15%.
The Efficient Implementation of Complex Number Arithmetic	• Focus on multiplication is shared by both papers, as the multiplier is used in both multiplication and division. • No hardware implementation of complex numbers is considered.	• Only multiplication is performed, no addition, subtraction, division, conjugate, or angle calculation. • Different radices (e.g., $2j$ and $-1+j$) are used for multiplication of complex numbers.

TABLE I
Comparison of Related Papers

VIII. Results and Discussion

The Verilog code for both unsigned and signed complex arithmetic operators works successfully. All the outputs of addition, subtraction, multiplication, division, conjugate and angle are displayed. The final outputs shown are accurate and reliable.

IX. Real Life Applications

Complex numbers contain some very distinct features and representations, although they are found to be applied in various domains within the real world. The applications of complex numbers are presented in the following along with detailed explanations of each application.

Complex number is used in the field of Electronics.

Complex number is used for Electromagnetism.

Complex number is used to simplify the unknown roots if roots are not real for quadratic equations.

In computer science engineering, complex numbers have been applied. Use of complex numbers in mechanical and civil engineering

X. Future Scope

Some potential uses for the Complex Arithmetic Operator

The complex arithmetic operator is a multidimensional and mainly basic tool of contemporary engineering and scientific research into many fields. Further development and utilization promise much and offer opportunities in numerous sectors:

1.Hardware Implementation: The future direction of development may be an improvement in speed or power consumption with minimal complexity in hardware realization of complex arithmetic operators, possibly by applying the techniques of CMOS, FinFET, and Quantum-dot Cellular Automata, among others.

2.Integrating FPGA/ASIC: This operand can be embedded in FPGAs or ASICs; it is then, therefore, feasible to incorporate the complex arithmetic operator into circuits for real-time systems with swift prototyping and implementation.

3. Optimized Design Techniques: Further reduction in power while allowing operators to be performed more efficiently in heavy applications can thus be achieved through research into new design methodologies, such as reversible logic and approximate computing.

4.Support for High-precision Arithmetic Operations: An extension to be deployed with or based on higher precision or floating-point arithmetic enables the operator to be used in applications demanding exceptional numerical accuracy, like computational physics and astronomy.

5. Parallel Processing Capability: Improvements in parallelism would speed up the operator on big datasets, thereby increasing its usage in big data analytics, machine learning, and neural network computations involving complex numbers.

6. Use in Emerging Technologies: - Quantum Computing: The operator can also be modified to accommodate quantum algorithms needing complex arithmetic for the manipulations performed on qubits.

- 5G/6G Networks: It may add to the signal processing functionalities in high-speed communication systems.

- Artificial Intelligence: Optimize the use of applications in AI for complex number computations, especially in deep learning and signal processing.

XI. Conclusion

The experiment is successful in demonstrating the design and implementation of complex arithmetic operators by taking two different input channels for real and imaginary part. The system is accurate, reliable and efficient in computing tasks involving complex numbers.

XII. Acknowledgment

We would like to thank Prof. Hitesh Shrimali, our course instructor of CSE213 and the teaching assistants for such great guidance throughout the preparation of this report. Encouraging words from them really helped penetrate our reasoning behind Icarus Verilog.

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