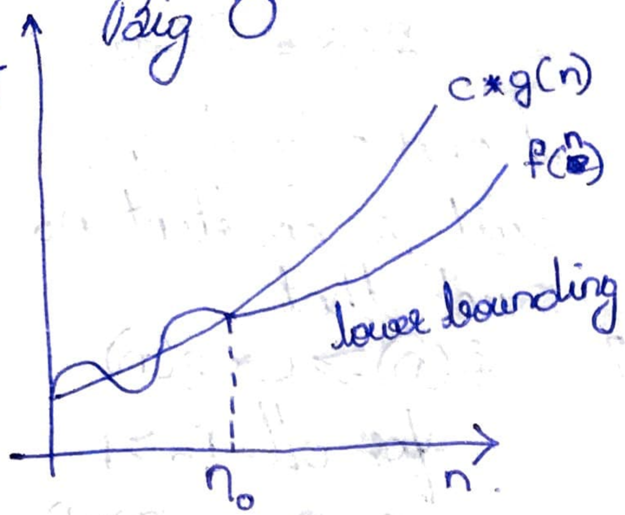
★ Big O



$f(n) = O(g(n))$
if and only if their exist a constant n_0 and C such that
$$0 \le f(n) \le C*g(n)$$
for all $n \ge n_0$

$n \geq n_0$

Let $\boxed{C=10}$

Example: $2n^2+n+3 \le 10n^2$ for all $n \ge n_0$.

$\div$ by n^2 $\quad 2+\frac{1}{n}+\frac{3}{n^2} \le 10$ for all $n \ge 1$

if $n=1 \Rightarrow 2+1+3 \le 10$ ✓

$n=2 \Rightarrow 2+\frac{1}{2}+\frac{3}{4} \le 10$ ✓

$n=3 \Rightarrow 2+\frac{1}{3}+\frac{3}{9} \le 10$ ✓

$n=4 \Rightarrow 2+\frac{1}{4}+\frac{3}{16} \le 10$ ✓

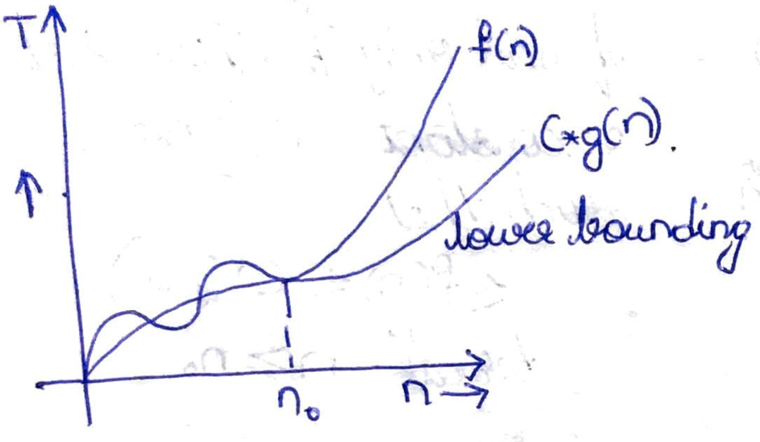
so $f(x) \le 10n^2$ for all $n \ge 1$ so $f(n) = O(n^2)$

★ Big Theta $\Theta()$

$f(n) = \Theta(g(n))$.
if and only if there exists n_0, C_1 & C_2 such that.
$$0 \le C_1*g(n) \le f(n) \le C_2*g(n)$$
for all $n \ge n_0$

It is lower bound and upper bound or tightly bound.

★ Big Ω (Omega)



$f(n) = \Omega(g(n))$

if and only if their exist a constant no. C such that

$f(n) \geq C * g(n)$.

for all $n_0 \geq 1$

$C > 0$ $\quad n \geq n_0$

★ Small-Oh and Small-Omega.

O, $\boxed{\Theta}$, Ω ← big

Theta

O ↓ small-Oh, little-Oh

Ω ↓ small-omega.

small-oh

$f(x) = o(g(n))$ iff $\forall\ \underline{C > 0}$

$\exists\ n_0 > 0$

such that $0 \leq f(n) < C.g(n)$

$\forall\ n \geq n_0$.

iff = if and only if

$\forall$ = for all.

$\exists$ = there exist

Eg:: $2n = O(n) \Rightarrow 2n < C.n$ for some $\frac{C=3}{C>0}$

$2n \neq o(n)$

$\Rightarrow 2n < C.n \quad \forall\ C > 0$.

$\times .C_1$

$2n = o(n^2)$

so $2n < C.n^2 \quad \forall\ C$.

$2 < Cn$ $\quad$ Let $C = 1$ $\quad$ so $\underline{2 < 3}$

$2 < (1)n$. $\quad \forall\ n \geq 3$

so. $2n < Cn^2$. $\quad \forall\ C > 0 \quad n_0 \leq n$

$2 < Cn$. $\quad C = 0.1 \rightarrow$ Let

$2 < 0.1(n) \quad \forall\ n \geq n_0 = 21$

$2 < 2.1$

so finally $\begin{cases} 2n = O(n) \\ 2n \neq o(n) \\ 2n = o(n^2) \end{cases}$

so $2n^2 \neq o(n^2)$

$2n^2 = o(n^3)$

Alternate Defination

$f(n) = o(g(n))$ iff $\lim_{n\to\infty} \frac{f(n)}{g(n)} = 0$.

$2n = o(n^2)$

$\lim_{n\to\infty} \frac{2n}{n^2} \Rightarrow 0$ ✓

So if $2n \neq o(n)$ because

$\lim_{n\to\infty} \frac{2n}{n} \Rightarrow 2$ ✗

★ Small- Omega (ω).

① $f(n) = \omega(g(n))$ iff $g(n) = o(f(n))$.

$2n = o(n^2) \Rightarrow n^2 = \omega(n)$ $\qquad \omega(2n) = \omega(n)$

② $f(n) = \omega(g(n))$ iff $\forall\ c>0\ \exists\ n_0>0$ such that $0 \leq c.g(n) < f(n)$ $\forall\ n \geq n_0$.

③ $f(n) = \omega(g(n))$ iff $\lim_{n\to\infty} \frac{f(n)}{g(n)} = \infty$.

$\frac{n^2}{2} = \omega(n)$.

$\frac{n^2}{2} \neq \omega(n^2)$

$\Rightarrow \frac{n^2}{2} = f(n)$; $g(n) = n^2$

So $\lim_{n\to\infty} = \frac{n^2/2}{n^2} \Rightarrow 1/2$ ✗

$\Rightarrow f(n) = \frac{n^2}{2}$; $g(n) = n$.

So $\lim_{n\to\infty} = \frac{n^2/2}{n} \Rightarrow \infty$ ✓

★ So $\underbrace{O,\ \Theta,\ \Omega,\ o,\ \omega}$

① How are they related ?

order of function → ② $n,\ n^2, \log(n),\ n^3,\ 2^n,\ n!,\ \log(\log(n))$.

$f(n)$ & $g(n)$.

★ Relationships

$f(n) = O(g(n)) \rightarrow f \leq g$ (up) asymptotic Notation

$f(n) = \Omega(g(n)) \Rightarrow f \geq g$ (Low)

$f(n) = \Theta(g(n)) \rightarrow f = g$ (Tight)

$f(n) = o(g(n)) \rightarrow f < g$

$f(n) = \omega(g(n)) \rightarrow f > g$

⇒ If $f(n) = \Theta(g(n))$ and $g(n) = \Theta(h(n))$.
then $f(n) = \Theta(h(n))$.
If $f = g$ and $g = h$ then $f = h$.

Transitive relationship

⇒ If $f(n) = \omega(g(n))$ and $g(n) = \omega(h(n))$.
then $f(n) = \omega(h(n))$.
If, $f > g$ and $g > h$ then $f > h$.

★ So Transitive relation is obeyed by $O, \Theta, \Omega, o, \omega$

★ Reflexive:-

$f(n) = \Theta(f(n)) \rightarrow f = f$ ✓

$f(n) = O(f(n)) \rightarrow f \leq f$ ✓

$f(n) = \Omega(f(n)) \rightarrow f \geq f$ ✓

$f(n) \neq o(f(n)) \rightarrow f < f$ X

$f(n) \neq \omega(f(n)) \rightarrow f > f$ X

★ Symmetric Relationship

→ $f(n) = \Theta(g(n))$ iff $g(n) = \Theta(f(n))$.

$f = g$ iff $g = f$. ✓

⇒ $O \rightarrow f \leq g$ iff $g \leq f$ X

→ $\Omega \rightarrow f \geq g$ iff $g \geq f$ X

⊗

* **Transpose Symmetery**

$\Rightarrow$ $f(n) = O(g(n))$ iff $g(n) = \Omega(f(n))$.

$f \leq g$ iff $g \geq f$. ✓

$\Rightarrow$ $f(n) = o(g(n))$. iff $g(n) = \omega(f(n))$.

$f < g$. iff $g > f$. ✓

★ Trichotomy

So Let $\Rightarrow a, b$.

$a > b, a < b, a = b$. $\longleftarrow$ One of these should be true.

$O, \Theta, \Omega, o, \omega \longrightarrow$ Trichotomy doesn't holds.