



# Introduction to the $dq$ Transformation and Field-Oriented Control

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# The Control Problem

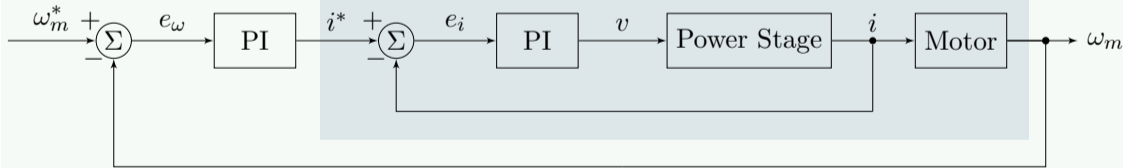
In general,

- Speed control requires torque control for actuation
- Torque control is also current control from proportionality:  $\tau = K_\tau i$
- Bandwidth: Torque  $\gg$  Speed

Cascaded Control Structure:

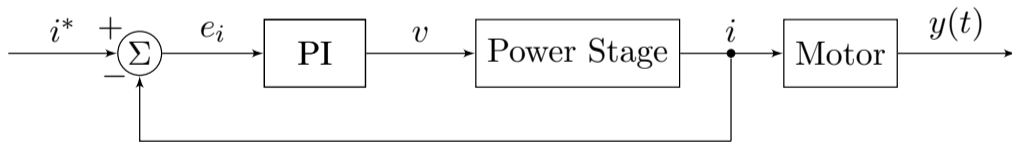
**Speed Loop**

**Torque Loop**



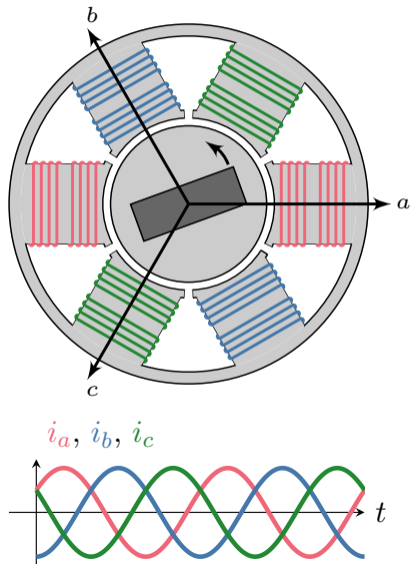
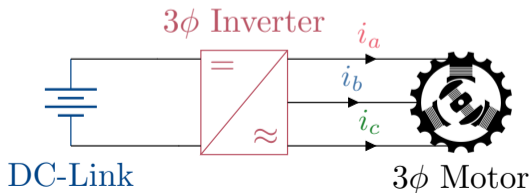
In four steps,

1. Measure the controlled current flowing into the motor.
2. Compare measured current with the desired current, generating error.
3. Amplify the error signal using a PI to generate a correction signal.
4. Modulate the correction voltage and apply to the motor terminals.



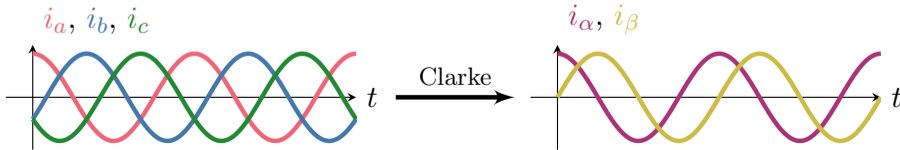
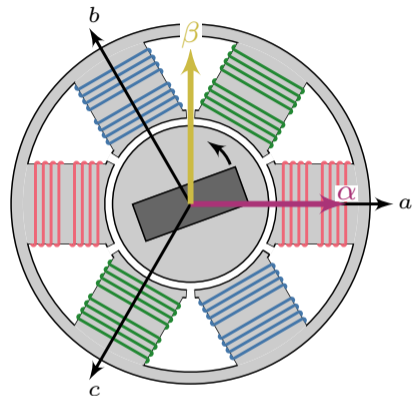
# Three-Phase Motors

- In a Permanent Magnet Synchronous Motor (PMSM), each phase current makes a proportional magnetic flux
- The net flux of all three phases rotates
- The stator flux attracts the rotor flux, causing the rotor to rotate
- Can modulate inverter voltages to control currents



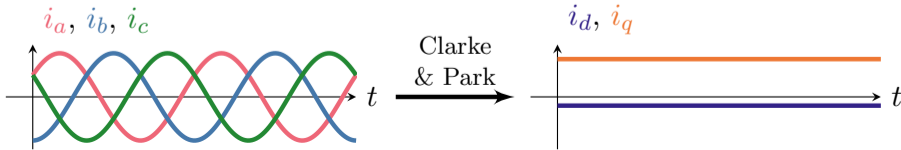
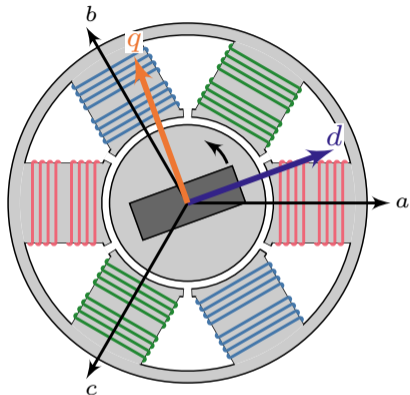
# The Clarke Transformation

- Do we need to control all 3 currents?
- We define the  $\alpha$  axis and  $\beta$  axis
  - ▶ We project  $\vec{i}_{abc}$  onto the  $\alpha\beta$  reference frame
  - ▶ Converts a 3-phase motor into a 2-phase motor
- As space vectors:  $\vec{i}_\alpha + \vec{i}_\beta = \vec{i}_a + \vec{i}_b + \vec{i}_c$ 
  - ▶ Now, we only need to control 2 currents!



# The Park Transformation

- Can we make things even simpler?
- We define the direct axis  $d$  and quadrature axis  $q$ 
  - ▶ Stationary with respect to the rotor
  - ▶ Rotating with respect to the stator
- Our 3 phases are represented as two constants
  - ▶ We can use a PI controller!

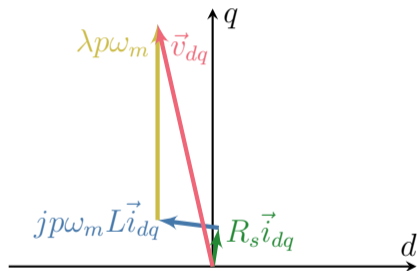


# Modelling the Motor in $dq$

What relationships describe the motor?

1. **Terminal voltage** is applied
2. The system has an **RL response**, except. . .
3. There is **cross-coupling**, where  $i_d$  affects  $i_q$  and vice versa. Cross-coupling is also proportional to the speed  $\omega_m$
4. There is **back electromotive force (EMF)** proportional to the speed  $\omega_m$

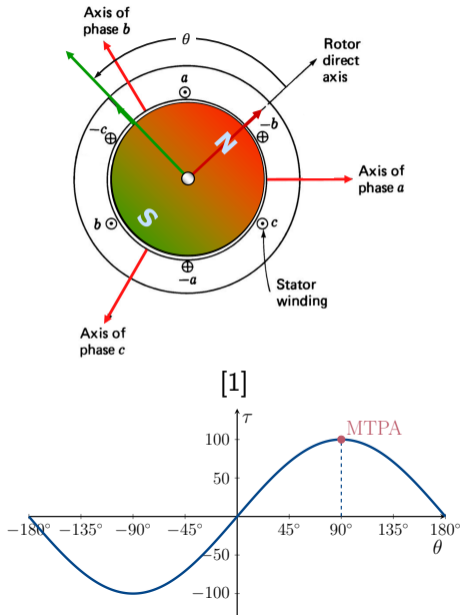
$$\underbrace{\begin{bmatrix} v_d \\ v_q \end{bmatrix}}_{\vec{v}_{dq}} = R_s \underbrace{\begin{bmatrix} i_d \\ i_q \end{bmatrix}}_{\vec{i}_{dq}} + L \frac{d}{dt} \begin{bmatrix} i_d \\ i_q \end{bmatrix} + jp\omega_m L \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \begin{bmatrix} 0 \\ \lambda p\omega_m \end{bmatrix}$$



# Maximum Torque Per Amp (MTPA)

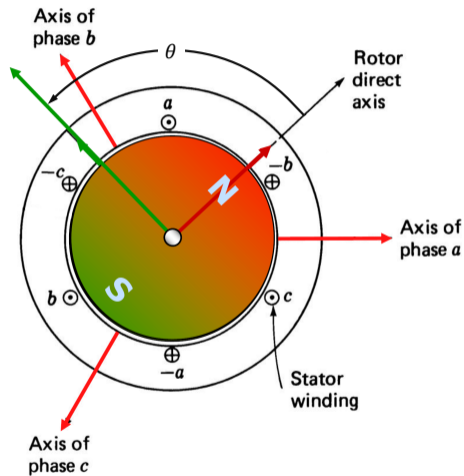
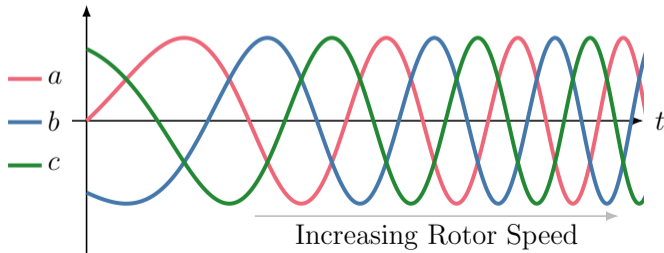
In a PMSM,

- Maximum torque per amp (MTPA) occurs when rotor and stator fluxes are  $90^\circ$  out of phase
- $i_q$  produces a torque;  $i_d$  does not
- Keep balanced three phases, but regulate shared phase for MTPA
- We need to get the rotor angle (e.g., encoder, hall position sensors) for MTPA
- $\tau \propto i_q \propto |\vec{i}_{abc}|$



# Field-Oriented Control

- Induced torque is approximately  $\tau = \frac{3}{2}p\lambda_{dr}i_q$
- Knowing rotor angle and currents, can go to  $dq$  and control for desired  $i_q$  (and thus, torque!)
- Accelerate?  $\rightarrow$  Increase applied torque
- Increase torque?  $\rightarrow$  Increase  $i_q \propto |\vec{i}_{abc}|$
- See this [animation](#)

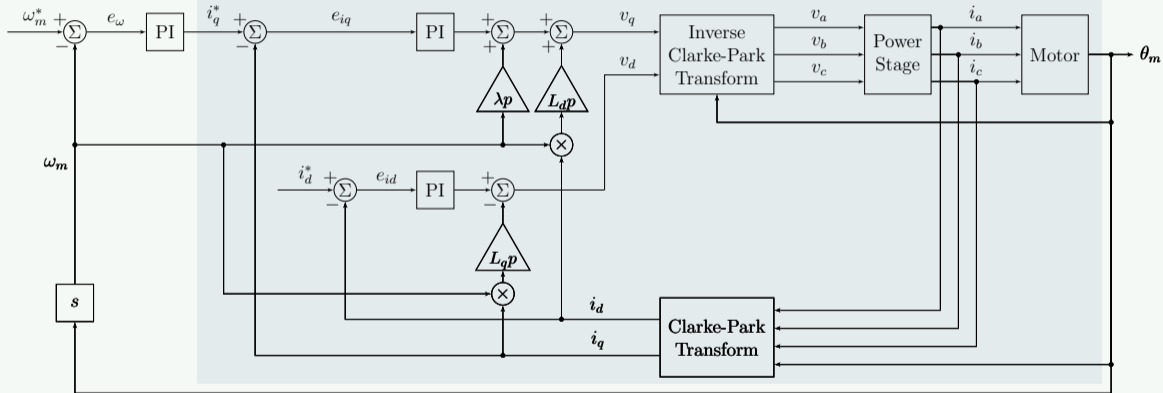


[1]

# Field-Oriented Control Block Diagram

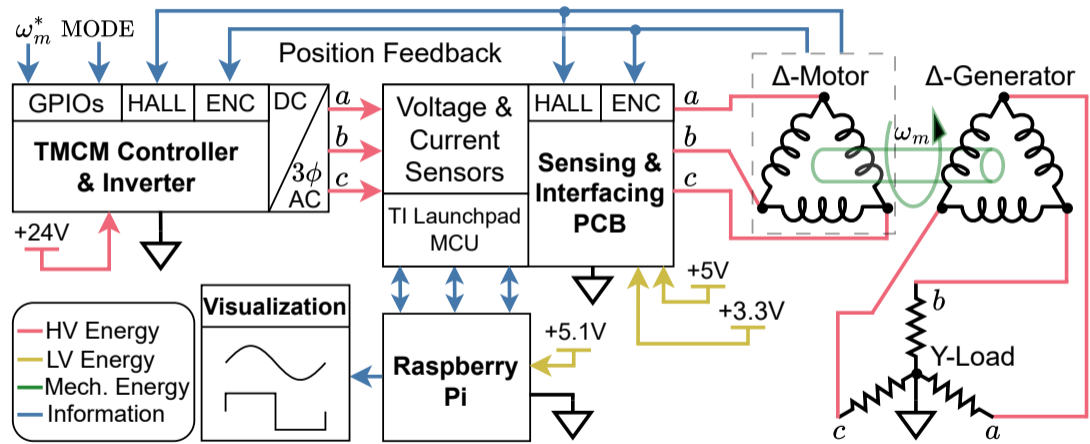
## Speed Loop

## Torque Loop (FOC)



Time for a Demo!

# Motor Control Visualization Tool Block Diagram



- [1] D. Wilson, "Intro to Field Oriented Control," Texas Instruments, 2014. [Online]. Available:  
<https://eggelectricunicycle.bitbucket.io/EmbeddedFiles/26-02%20Intro%20to%20FOC.pdf>
- [2] C. Viana, "Clarke and Park and PMSM Model," Unpublished, 2025.