Introduction to the dq Transformation and Field-Oriented Control

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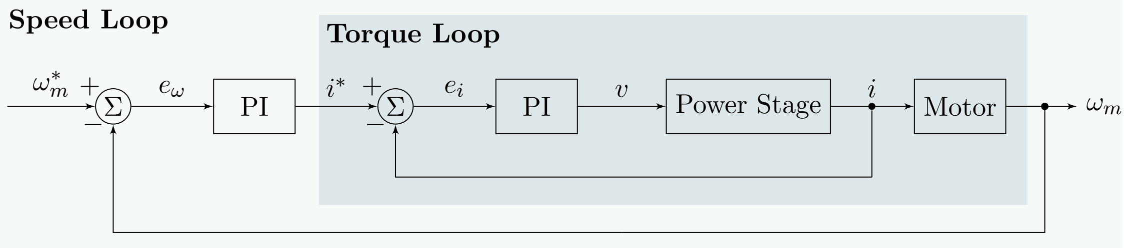
03 November 2025

The Control Problem

In general,

- Speed control requires torque control for actuation
- Torque control is also current control from proportionality: $\tau = K_\tau i$
- Bandwidth: Torque $\gg$ Speed

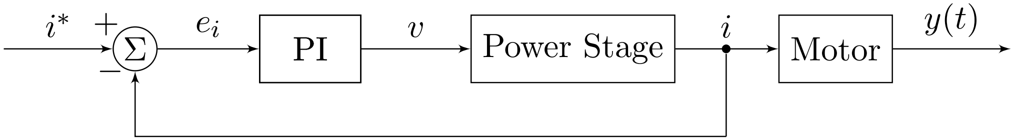
Cascaded Control Structure:



Torque Control in a DC Motor

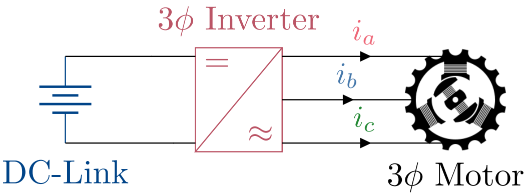
In four steps,

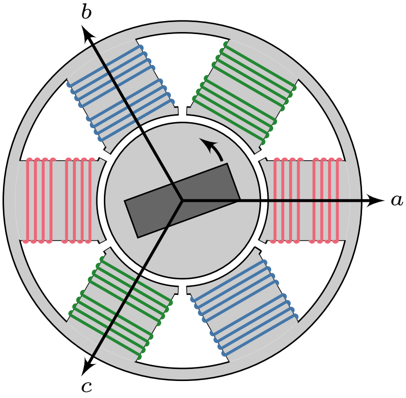
1. Measure the controlled current flowing into the motor.
2. Compare measured current with the desired current, generating error.
3. Amplify the error signal using a PI to generate a correction signal.
4. Modulate the correction voltage and apply to the motor terminals.

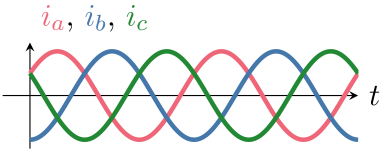


Three-Phase Motors

- In a Permanent Magnet Synchronous Motor (PMSM), each phase current makes a proportional magnetic flux
- The net flux of all three phases rotates
- The stator flux attracts the rotor flux, causing the rotor to rotate
- Can modulate inverter voltages to control currents

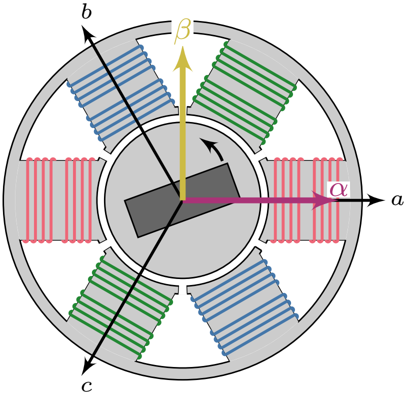


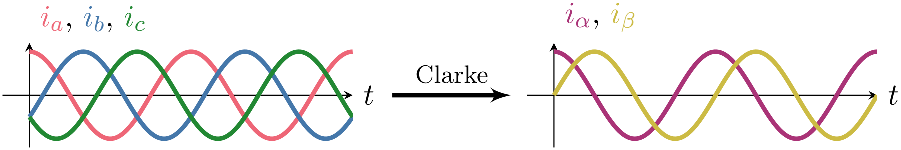




The Clarke Transformation

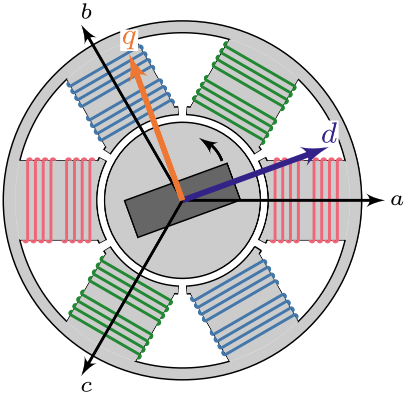
- Do we need to control all 3 currents?
- Define the α **axis** and β **axis**
  - We project $\vec{i}_{abc}$ onto the $\alpha\beta$ reference frame
  - Converts a 3-phase motor into a 2-phase motor
- As space vectors: $\vec{i}_\alpha + \vec{i}_\beta = \vec{i}_a + \vec{i}_b + \vec{i}_c$
  - Now, we only need to control 2 currents!

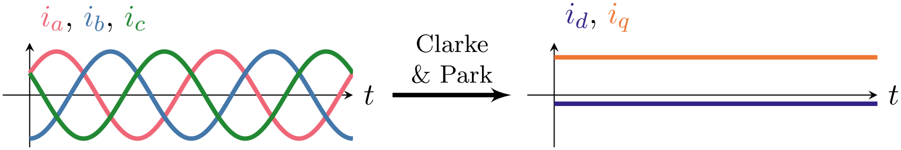




The Park Transformation

- Can we make things even simpler?
- Define the **direct axis** d and **quadrature axis** q
  - Stationary with respect to the rotor
  - Rotating with respect to the stator
- Our 3 phases are represented as two constants
  - We can use a PI controller!

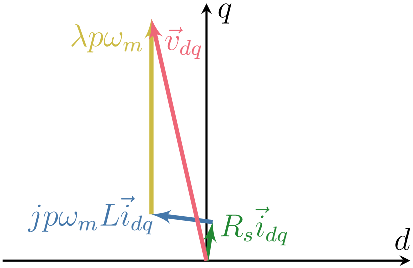




Modelling the Motor in dq

What relationships describe the motor?

1. Terminal voltage is applied
2. The system has an RL response, except...
3. There is cross-coupling , where i_d affects i_q and vice versa. Cross-coupling is also proportional to the speed ω_m
4. There is back electromotive force (EMF) proportional to the speed ω_m

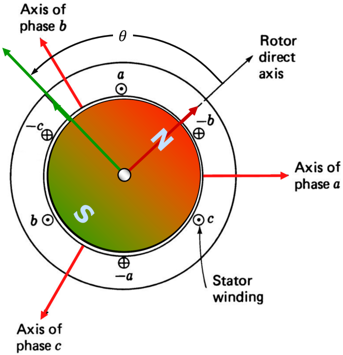


$$\underbrace{\begin{bmatrix} v_d \\ v_q \end{bmatrix}}_{\vec{v}_{dq}} = R_s \underbrace{\begin{bmatrix} i_d \\ i_q \end{bmatrix}}_{\vec{i}_{dq}} + L\frac{d}{dt}\begin{bmatrix} i_d \\ i_q \end{bmatrix} + jp\omega_m L \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \begin{bmatrix} 0 \\ \lambda p \omega_m \end{bmatrix}$$

Maximum Torque Per Amp (MTPA)

In a PMSM,

- Maximum torque per amp (MTPA) occurs when rotor and stator fluxes are 90° out of phase
- i_q produces a torque; i_d does not
- Keep balanced three phases, but regulate shared phase for MTPA
- We need to get the rotor angle (e.g., encoder, hall position sensors) for MTPA
- $\tau \propto i_q \propto |\vec{i}_{abc}|$



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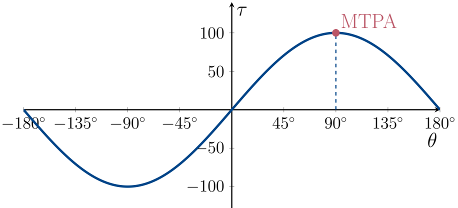
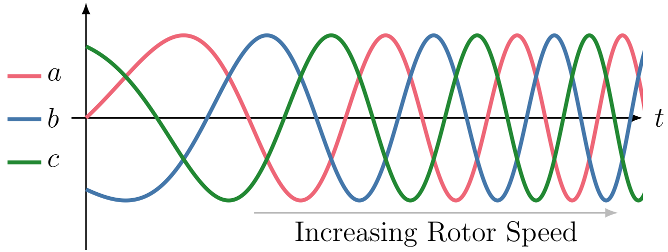
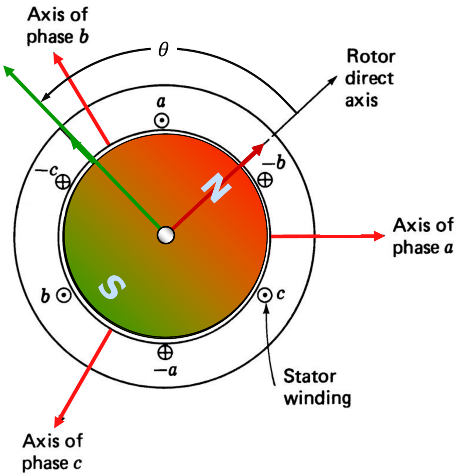


Field-Oriented Control

- Induced torque is approximately $\tau = \frac{3}{2}p\lambda_{dr}i_q$
- Knowing rotor angle and currents, can go to dq and control for desired i_q (and thus, torque!)
- Accelerate? $\rightarrow$ Increase applied torque
- Increase torque? $\rightarrow$ Increase $i_q \propto |\vec{i}_{abc}|$
- See this animation





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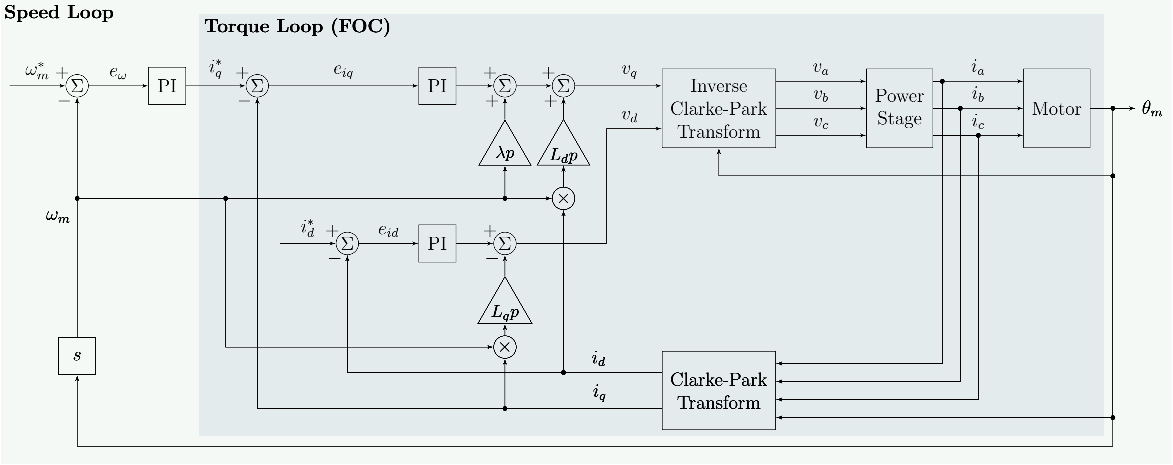
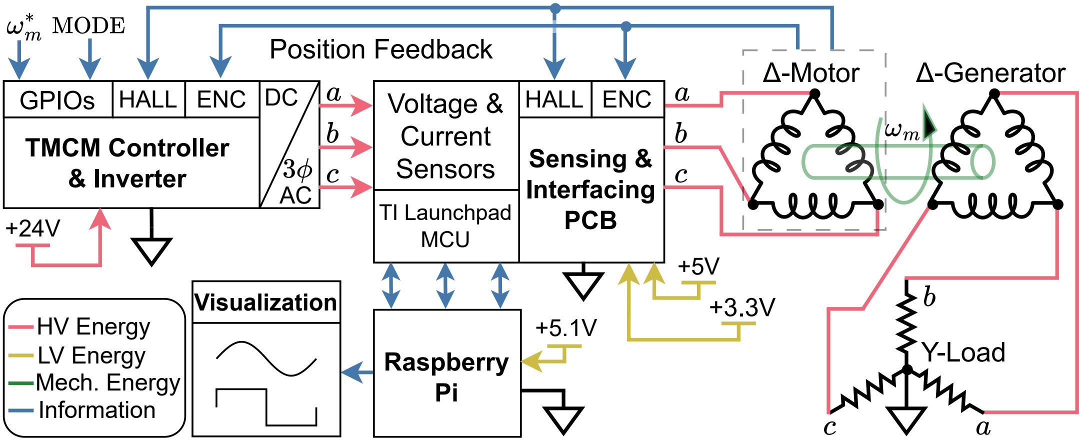
Field-Oriented Control Block Diagram



Time for a Demo!

Motor Control Visualization Tool Block Diagram



References

[1] D. Wilson, "Intro to Field Oriented Control," Texas Instruments, 2014. [Online]. Available: https://eggelectricunicycle.bitbucket.io/EmbeddedFiles/26-02%20Intro%20to%20FOC.pdf

[2] C. Viana, "Clarke and Park and PMSM Model," Unpublished, 2025.