9.5.11

AI24BTECH11021 - Manvik Muthyapu

Question:
Prove that the curves $y^2 = 4x$ and $x^2 = 4y$ divide the area of the square bounded by sides $x = 0, x = 4, y = 4$, and $y = 0$ into three equal parts. (12, 2018)

Solution: First, to find points of intersection, substitute $x = \frac{y^2}{4}$ into $x^2 = 4y$:

$$\left(\frac{y^2}{4}\right)^2 = 4y \tag{1}$$

$$\frac{y^4}{16} = 4y \tag{2}$$

$$\implies y^4 = 64y \tag{3}$$

$$\implies y(y^3 - 64) = 0 \tag{4}$$

From that we get the $y = 0, 4$ as solutions. So, the points of intersection are (0,0) and (4,4).
Area of the Square $= 4 \times 4 = 16$.
To find area under the curve $y^2 = 4x$ between $y = 0$ and $y = 4$, we integrate the expression for x with respect to y:

$$\text{Area} = \int_0^4 \frac{y^2}{4}\, dy = \frac{1}{4}\int_0^4 y^2\, dy \tag{5}$$

$$= \frac{1}{4}\left[\frac{y^3}{3}\right]_0^4 = \frac{1}{4} \times \frac{64}{3} \tag{6}$$

$$= \frac{16}{3} \tag{7}$$

Now, for area under the $x^2 = 4y$ between $x = 0$ and $x = 4$, we integrate the expression for y with respect to x:

$$\text{Area} = \int_0^4 \frac{x^2}{4}\, dx = \frac{1}{4}\int_0^4 x^2\, dx \tag{8}$$

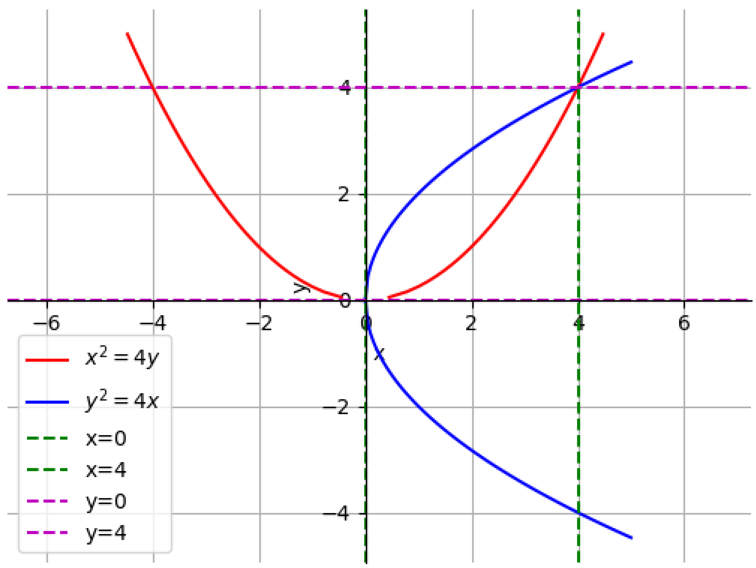
$$= \frac{1}{4}\left[\frac{x^3}{3}\right]_0^4 = \frac{1}{4} \times \frac{64}{3} \tag{9}$$

$$= \frac{16}{3} \tag{10}$$

The area enclosed by both the curves together is the area of square remaining after removing the area enclosed by the curves and the coordinate axes:

$$16 - \frac{16}{3} + \frac{16}{3} = \frac{16}{3} \tag{11}$$

Thus, from the above results we get that the parabolas divide the the square into three regions of equal area, each having an area of $\frac{16}{3}$.



$x^2 = 4y$
$y^2 = 4x$
x=0
x=4
y=0
y=4
x
y
−6
−4
−2
0
2
4
6