

Vector Arithmetic (Section Formula)

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Question:

Show that the points **A**(-6, 10), **B**(-4, 6) and **C**(3, -8) are collinear and prove that $AB = \frac{2}{9}AC$.

Solution:

If this determinant is zero, $\begin{vmatrix} 1 & 1 & 1 \\ -6 & -4 & 3 \\ 10 & 6 & -8 \end{vmatrix}$ then A , B , and C are collinear.

$$\begin{aligned} \text{Det} &= \begin{vmatrix} 1 & 1 & 1 \\ -6 & -4 & 3 \\ 10 & 6 & -8 \end{vmatrix} = 1 \cdot ((-4)(-8) - (3)(6)) - 1 \cdot ((-6)(-8) - (3)(10)) + 1 \cdot ((-6)(6) - (-4)(10)) \\ &= 1 \cdot (32 - 18) - 1 \cdot (48 - 30) + 1 \cdot (-36 + 40) \\ &= 1 \cdot 14 - 1 \cdot 18 + 1 \cdot 4 = 14 - 18 + 4 = 0 \end{aligned}$$

$\therefore A$, B , and C are collinear.

Using the section formula, if B divides AC in the ratio $m : n$, then:

$$B\left(\frac{m \cdot 3 + n \cdot (-6)}{m + n}, \frac{m \cdot (-8) + n \cdot 10}{m + n}\right) = (-4, 6)$$

Equating the coordinates, we get

$$\frac{m}{n} = \frac{2}{7}$$

\therefore It confirms that B divides AC in the ratio $2 : 7$. Since B divides AC in the ratio $2 : 7$, we have:

$$\frac{AB}{AC} = \frac{2}{2+7} = \frac{2}{9}$$

Hence, $AB = \frac{2}{9}AC$, as required.

