

Vector Arithmetic (Section Formula)

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Question:

Show that the points **A**(-6, 10), **B**(-4, 6) and **C**(3, -8) are collinear such that $AB = \frac{2}{9}AC$.

Solution: To prove that the given points are collinear, we check the determinant of the following matrix:

$$\text{Det} = \begin{vmatrix} -6 & 10 & 1 \\ -4 & 6 & 1 \\ 3 & -8 & 1 \end{vmatrix}$$

If the determinant is zero, then the points A , B , and C are collinear. Therefore, we have:

$$\text{Det} = \begin{vmatrix} -6 & 10 & 1 \\ -4 & 6 & 1 \\ 3 & -8 & 1 \end{vmatrix} = 0$$

Thus, A , B , and C are collinear.

We can prove that $AB = \frac{2}{9}AC$ using the section formula.

We want to show that $B(-4, 6)$ divides AC in the ratio $2 : 7$.

Using the section formula, if B divides AC in the ratio $m : n$, then:

$$B\left(\frac{m \cdot 3 + n \cdot (-6)}{m + n}, \frac{m \cdot (-8) + n \cdot 10}{m + n}\right) = (-4, 6)$$

Equating the coordinates, we get

$$\frac{m}{n} = \frac{2}{7}$$

Thus, it confirms that B divides AC in the ratio $2 : 7$. Since B divides AC in the ratio $2 : 7$, we have:

$$\frac{AB}{AC} = \frac{2}{2 + 7} = \frac{2}{9}$$

Hence, $AB = \frac{2}{9}AC$, as required.

