Vector Arithmetic (Section Formula)

AI24BTECH11021 - Manvik Muthyapu

Question:

Show that the points $\mathbf{A}\,(-6,10)\,,\mathbf{B}\,(-4,6)$ and $\mathbf{C}\,(3,-8)$ are collinear and prove that $AB = \frac{2}{9}AC$.

Solution:

If this determinant is zero, $\begin{vmatrix} 1 & 1 & 1 \\ A & B & C \end{vmatrix}$ then A, B, and C are collinear.

$$\text{Det} = \begin{vmatrix} 1 & 1 & 1 \\ -6 & -4 & 3 \\ 10 & 6 & -8 \end{vmatrix} \tag{1}$$

$$= 1 \cdot (32 - 18) - 1 \cdot (48 - 30) + 1 \cdot (-36 + 40) \tag{2}$$

$$= 0 \tag{3}$$

$\therefore$ A, B, and C are collinear.

Using the section formula, if B divides AC in the ratio $m : n$, then:

$$\left(\frac{m \cdot 3 + n \cdot (-6)}{m+n}, \frac{m \cdot (-8) + n \cdot 10}{m+n}\right) = (-4, 6) \tag{4}$$

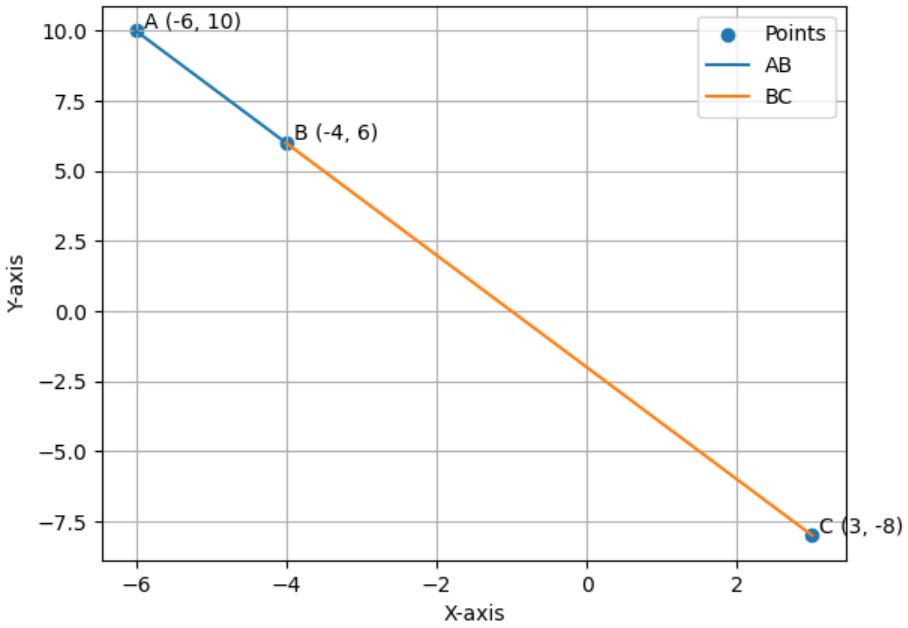
Equating the coordinates, we get

$$\frac{m}{n} = \frac{2}{7} \tag{5}$$

$\therefore$ It confirms that B divides AC in the ratio $2 : 7$. Since B divides AC in the ratio $2 : 7$, we have:

$$\frac{AB}{AC} = \frac{2}{2+7} = \frac{2}{9} \tag{6}$$

$\therefore$ $AB = \frac{2}{9}AC$, Hence proved.


A (-6, 10)
B (-4, 6)
C (3, -8)
Points
AB
BC
10.0
7.5
5.0
2.5
0.0
−2.5
−5.0
−7.5
Y-axis
−6
−4
−2
0
2
X-axis