ANALOG

ELECTRONICS

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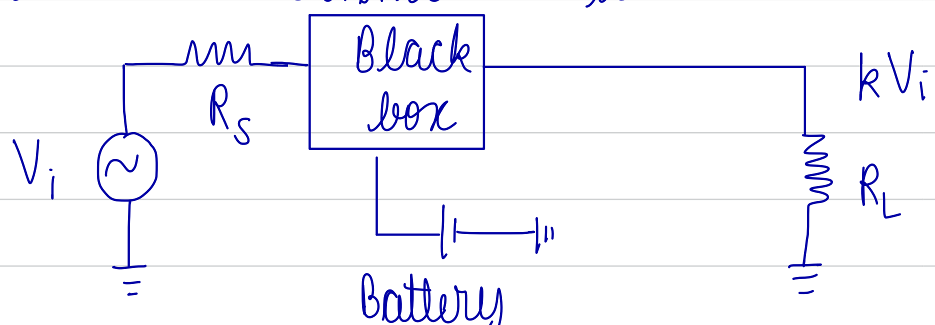
Introduction:

Analog signal → sensor $\xrightarrow{\text{voltage}}$ Pre processor → ADC → DSP → DAC → Amplifier + filter

ADC → Analog to digital convertor.
DSP → Digital signal processing
DAC → Digital to analog convertor.

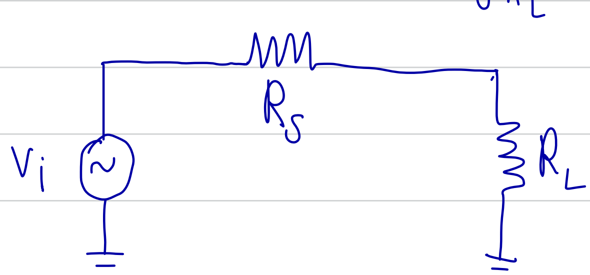
AMPLIFIERS:

- Amplifiers amplify the power of a signal.
- Transformers cannot increase the power of a signal, (as it is a passive device) so it cannot be an amplifier.



$$\text{Power} = \frac{k^2 V_i^2}{R_L}$$

According to maximum power transfer theorem: $P_{max} = \frac{V_s^2}{8R_L}$ when $R_S = R_L$ for



i.e $P = \left(\frac{V_i}{\sqrt{2} \times 2R_S}\right)^2 \times R_S$

Linear and non linear networks

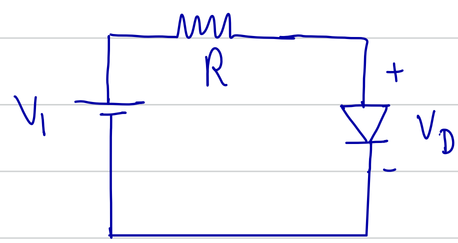
- A linear system obeys superposition principle
- Transfer functions, transforms and convolution integrals are used to study linear networks.
- Incremental linearity is used to study non linear networks.

Superposition principle: $x_1 \longrightarrow y_1$, $x_2 \longrightarrow y_2$

$$\Rightarrow \quad \alpha x_1 + \beta x_2 \longrightarrow \alpha y_1 + \beta y_2$$

- Knowing impulse response of a linear system we can understand everything about the system. However it is not true for non linear system.

Eg for non linear system:



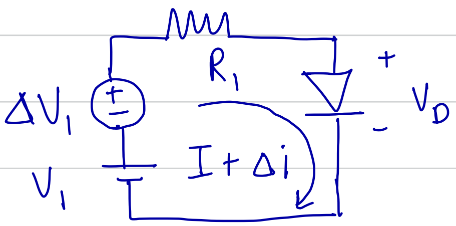
$$V_1 = V_R + V_D$$
$$= IR_1 + V_D$$

But $\quad I = I_s\, e^{\left(\frac{V_D}{V_T} - 1\right)}$

$V_T = \dfrac{kT}{q} = 25\text{ mV}$ at $T = 25°C$

when $\quad V_D \ggg V_T$, $\quad I = I_s \exp \dfrac{V_D}{V_T}$

$\Rightarrow \quad V_D = V_T \ln \dfrac{I}{I_s}$ $\qquad$ So $\quad V_1 = IR_1 + V_T \ln \dfrac{I}{I_s} \rightarrow (1)$



$$(V + \Delta V_1) = (I + \Delta i) R_1 + V_T \ln \left(\frac{I + \Delta i}{I_s} \right) \rightarrow (2)$$

$$(2) - (1) \Rightarrow \Delta V_1 = \Delta i R_1 + V_T \left[\ln \frac{I + \Delta i}{I_s} - \ln \frac{I}{I_s} \right]$$

$$\Delta V_1 = \Delta i R_1 + V_T \left[f(I + \Delta i) - f(I) \right]$$

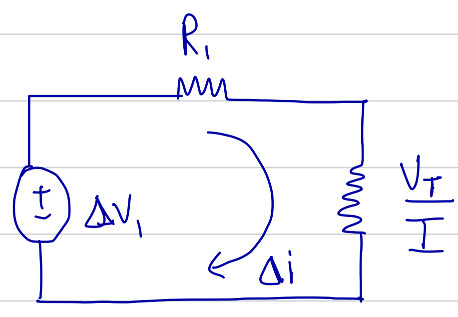
where $f(I) = \ln \frac{I}{I_s}$

But $f(I + \Delta i) = f(I) + \left. \frac{df}{dx} \right|_{x=I} \times \Delta i$

$$\Delta V_1 = \Delta i R_1 + V_T \left[f(I) + \left. \frac{df}{dx} \right|_{x=I} \times \Delta i - f(I) \right]$$

$$= \Delta i R_1 + \frac{V_T \Delta i}{I}$$

$$\Rightarrow \Delta i = \frac{\Delta V_1}{R_1 + \frac{V_T}{I}} \Rightarrow$$



This technique is called linear increment technique.

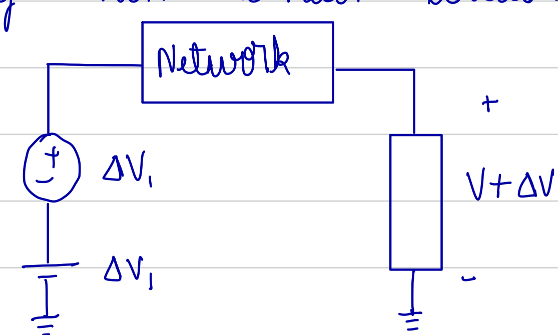
This can be used for any non linear circuit.

$V = f(I)$

$V + \Delta V = f(I + \Delta i)$

$= f(I) + f'(I) \cdot \Delta i$

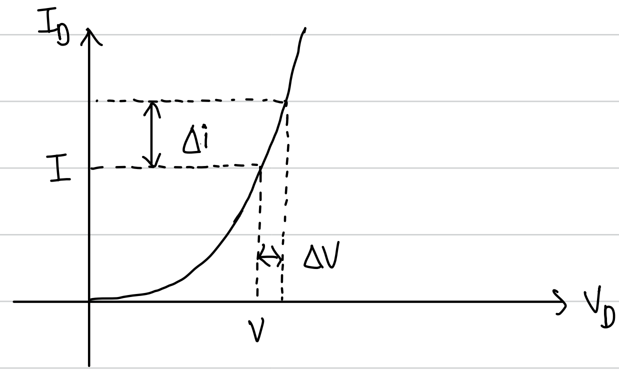
$\Delta V = f'(I) \Delta i \Rightarrow f'(I) = \frac{\Delta V}{\Delta i} \neq \frac{V}{I}$



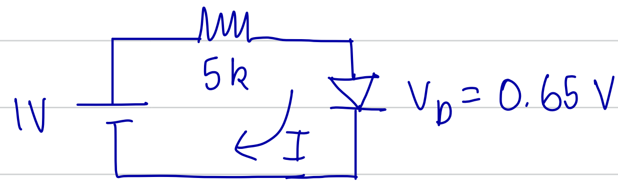
I - Quiscent current　　V - Quiscent voltage

Δi - Incremental current　　ΔV - Incremental voltage

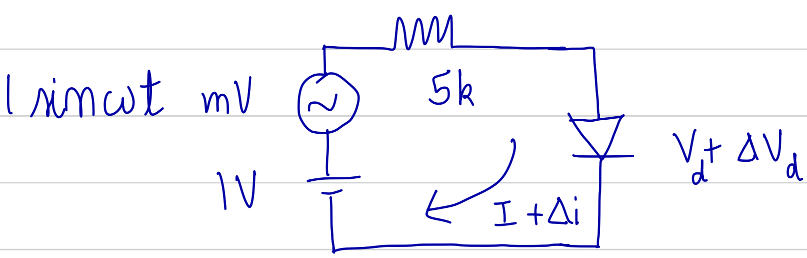
For diode :



Q1) What is the quiscent current in the loop.



Ans



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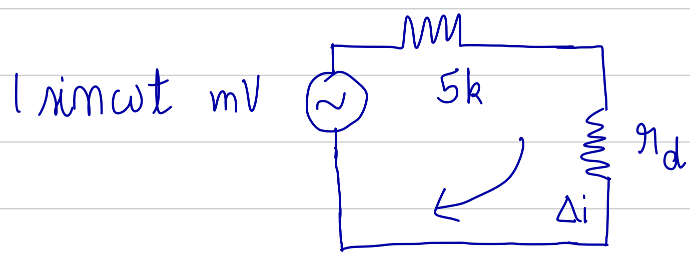


$$I = \frac{1-0.65}{5\times10^3} = 70\ \mu A$$

$$r_d = \frac{V_T}{I} = \frac{25\,mV}{70\ \mu A} = 357\,\Omega$$

$$\Delta V_d = \frac{r_d}{r_d + 5k}\Delta V = \frac{357}{5357} \times \sin\omega t\ mV = 0.067 \sin\omega t\ mV$$

$$= 67 \sin\omega t\ \mu V$$

$\therefore \; V_D + \Delta V_d = (0.65 + 67 \sin\omega t \times 10^{-6})\ V$

$$\Delta i = \frac{1 \sin\omega t \ \text{mV}}{357 + 5k} = 0.18 \sin\omega t \ \mu A$$

$$I + \Delta i = (70 + 0.18 \sin\omega t)\ \mu A$$

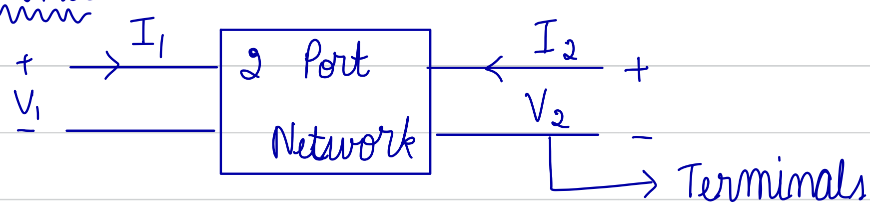
Steps: 1> Find the Operating points (quiscent values)
2> Draw the incremental circuit.
3> Total node voltage / branch current

=

Quiscent voltage / current + Incremental voltage / current

TWO PORT NETWORKS

Port v/s terminal



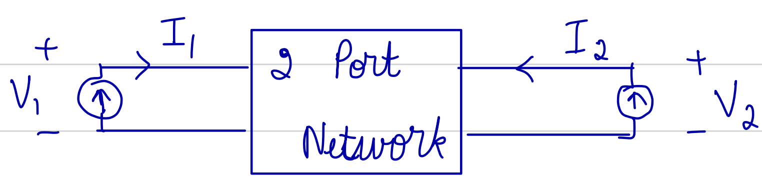
Dependent variables (response)	Independent variables (Excitation)	Parameters
V_1 , V_2	I_1 , I_2	Z - Parameters
I_1 , I_2	V_1 , V_2	Y - Parameters
V_1 , I_2	I_1 , V_2	h - Parameters
I_1 , V_2	V_1 , I_2	g - Parameters
V_1 , I_1	V_2 , I_2	ABCD or transmission
V_2 , I_2	V_1 , I_1	Inverse transmission or script ABCD

1> Z Parameters :

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

$$\Rightarrow \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

i.e

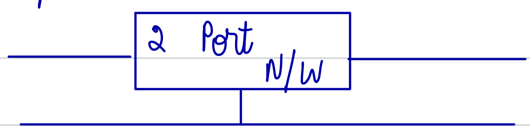


$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \qquad Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} \qquad Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} \qquad Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

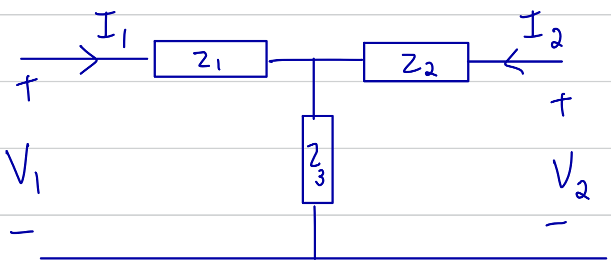
$Z_{11}, Z_{22} \rightarrow$ Driving point impedances
$Z_{21} \rightarrow$ Forward transimpedance.
$Z_{12} \rightarrow$ Reverse transimpedance.

- Reciprocal network $\Rightarrow Z_{12} = Z_{21}$

NOTE: 2 port networks can be 3 terminal as well



Ex1

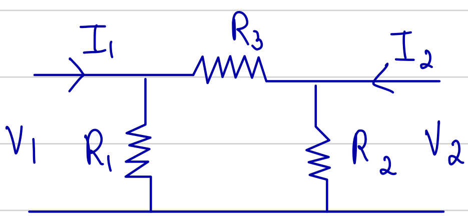


Find Z_{11}, Z_{21}, Z_{22}, Z_{12}

Ans $Z_{11} = \left.\dfrac{V_1}{I_1}\right|_{I_2=0} = Z_1 + Z_3$ $\qquad Z_{22} = \left.\dfrac{V_2}{I_2}\right|_{I_1=0} = Z_2 + Z_3$

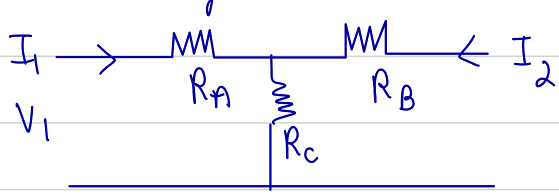
$Z_{12} = \left.\dfrac{V_1}{I_2}\right|_{I_1=0} = Z_3$ $\qquad Z_{21} = \left.\dfrac{V_2}{I_1}\right|_{I_2=0} = Z_3$

Ex2



Find Z_{11}, Z_{12}, Z_{21}, Z_{22}

Ans The given circuit is equivalent to:



$R_A = \dfrac{R_1 R_3}{R_1 + R_2 + R_3}$ $\qquad R_B = \dfrac{R_2 R_3}{R_1 + R_2 + R_3}$

$R_C = \dfrac{R_1 R_2}{R_1 + R_2 + R_3}$

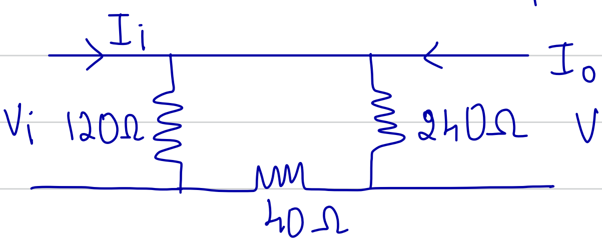
$\Rightarrow$ From last question

$$Z_{11} = R_A + R_C = \frac{R_1(R_2+R_3)}{R_1+R_2+R_3}$$

$$Z_{12} = R_C = \frac{R_1 R_2}{R_1+R_2+R_3} = Z_{21}$$

$$Z_{22} = R_B + R_C = \frac{R_2(R_1+R_3)}{R_1+R_2+R_3}$$

Ex 3

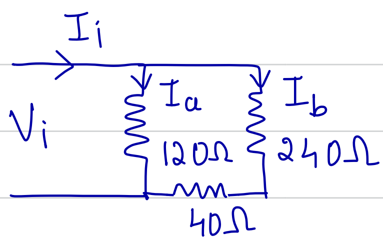


Find $Z_{11}, Z_{12}, Z_{21}, Z_{22}$

Ans

$$Z_{11} = \left.\frac{V_i}{I_i}\right|_{I_o=0}$$

$\Rightarrow$

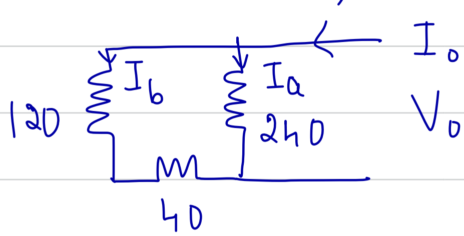


$V_i = 120 I_a$

$I_a = \frac{280}{400} \cdot I_i$

$V_i = 84\, I_i$ $\Rightarrow$ $Z_{11} = 84\Omega$ //

$$Z_{22} = \left.\frac{V_o}{I_o}\right|_{I_1=0}$$



$V_o = 240\, I_a$

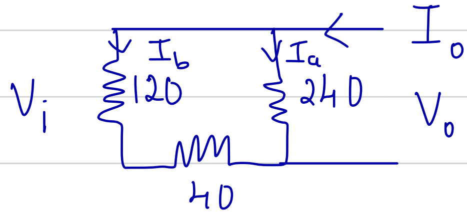
$I_a = \frac{160}{400} = \frac{2}{5} I_o$

$V_o = 240 \times \frac{4}{10} I_o$

$V_o = 96\, I_o$

$\therefore Z_{22} = 96\Omega$ //

$$Z_{12} = \left.\frac{V_i}{I_o}\right|_{I_1=0} \Rightarrow$$



$$V_i = I_b \times 120$$

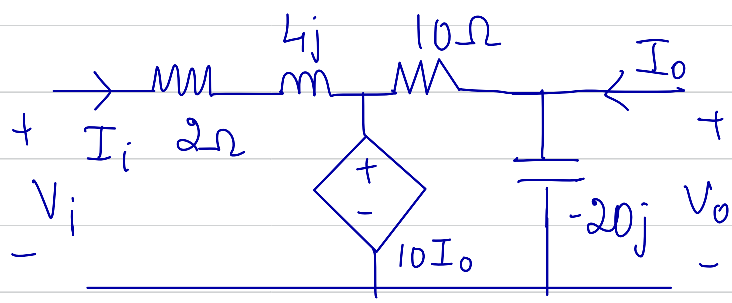
$$V_i = \frac{6}{10} \times 120\, I_o \qquad I_b = \frac{240}{400} I_o = \frac{6}{10} I_o$$

$$V_i = 72\, I_o \Rightarrow Z_{12} = 72\,\Omega$$

$|||^{ly}$ $Z_{21} = 72\,\Omega$

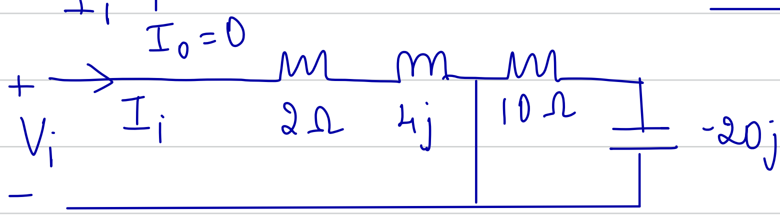
$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} 84 & 72 \\ 72 & 96 \end{bmatrix}$$

4> Find $Z_{11}, Z_{12}, Z_{21}, Z_{22}$



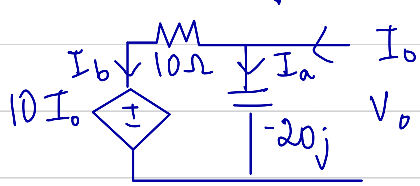
Ans $Z_{11} = \left.\frac{V_i}{I_i}\right|_{I_o=0}$

$\Rightarrow$



$$\frac{V_i}{I_i} = 2 + 4j \Rightarrow Z_{11} = 2 + 4j$$

$$Z_{22} = \left.\frac{V_o}{I_o}\right|_{I_i=0} \Rightarrow$$



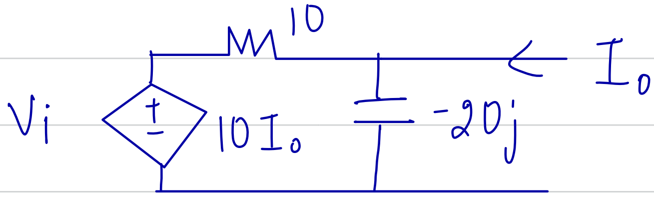
$$I_o = \frac{V_o}{-20j} + \frac{V_o - 10 I_o}{10} = \frac{V_o}{10}\left(1 + \frac{j}{2}\right) - I_o$$

$$2 I_o = V_o\left(\frac{2+j}{20}\right)$$

$$\frac{V_o}{I_o} = \frac{40}{(2+j)(2-j)}(2-j) = \frac{40}{5}(2-j) = 8(2-j) = 16-8j$$

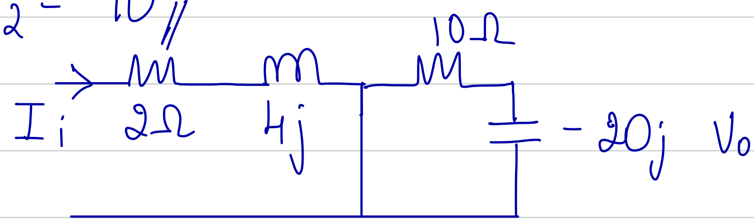
$$Z_{22} = 16-8j$$

$$Z_{12} = \left.\frac{V_i}{I_o}\right|_{I_i=0} \Rightarrow$$



$V_i = 10I_o$ $\quad \therefore Z_{12} = 10$

$$Z_{21} = \left.\frac{V_o}{I_i}\right|_{I_o=0} \Rightarrow$$



$V_o = 0$ ($\because$ No current flows through $10-20j$)

$\therefore Z_{21} = 0$

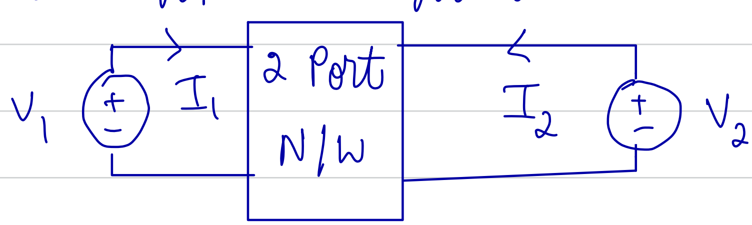
$$\therefore \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} 2+4j & 10 \\ 0 & 16-8j \end{bmatrix}$$

Y parameters

$I_1 = y_{11}V_1 + y_{12}V_2$

$I_2 = y_{21}V_1 + y_{22}V_2$

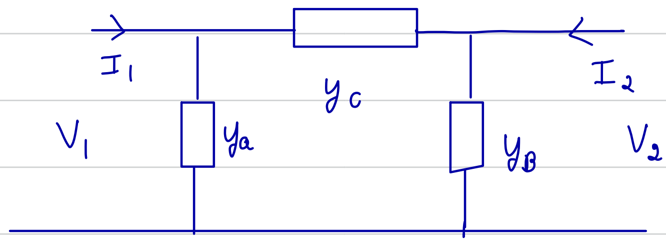
i.e. $\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$



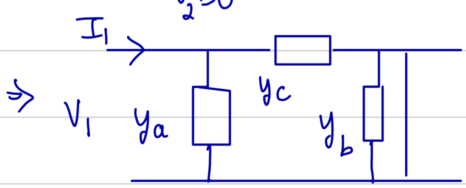
$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}$ $\quad y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$ $\quad y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$ $\quad y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$

$Y_{11}, Y_{22} \rightarrow$ Driving point input, output admittances respectively
$Y_{12} \rightarrow$ Reverse Transadmittance
$Y_{21} \rightarrow$ Forward transadmittance

Ex1: Find out Y-parameters.



Ans $Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}$



$\therefore Y_{11} = y_a + y_c$

$Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} = -y_c$

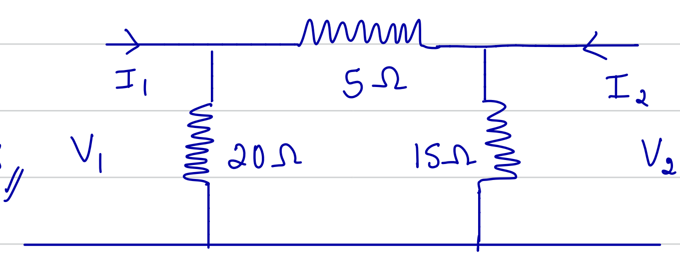
$Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = y_B + y_c$

$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = -y_c$

If y_a, y_b, y_c are replaced by R_1, R_2, R_3 (resistors)

$Y_{11} = \frac{1}{R_1} + \frac{1}{R_3}$ $Y_{12} = Y_{21} = \frac{-1}{R_3}$ $Y_{22} = \frac{1}{R_2} + \frac{1}{R_3}$ //

Ex2: Find out Y-parameters

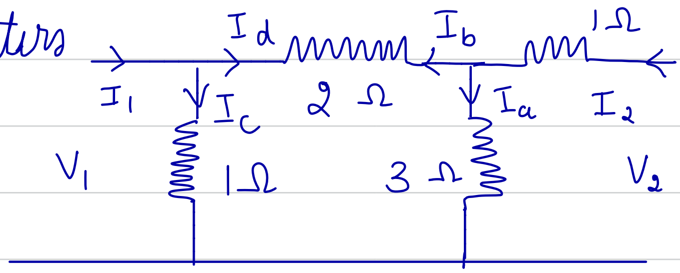


Ans $Y_{11} = \frac{1}{20} + \frac{1}{5} = \frac{4+1}{25} = \frac{1}{5}$ S //

$Y_{12} = Y_{21} = \frac{-1}{5}$ S //

$Y_{22} = \frac{1}{5} + \frac{1}{15} = \frac{4}{15}$ S //

3> Find Z and y parameters



Ans $Z_{11} = \left.\dfrac{V_1}{I_1}\right|_{I_2=0}$

$= \dfrac{5}{6}\,\Omega$ //

$Z_{12} = \left.\dfrac{V_1}{I_2}\right|_{I_1=0}$

$V_1 = I_b \times 1$ $\qquad$ $I_b = \dfrac{I_2}{2} = V_1$

$\therefore Z_{12} = 0.5\,\Omega$ //

$Z_{22} = \left.\dfrac{V_2}{I_2}\right|_{I_1=0}$

$V_2 = I_2 \times (2.5)$ $\qquad$ $\therefore Z_{22} = 2.5\,\Omega$ //

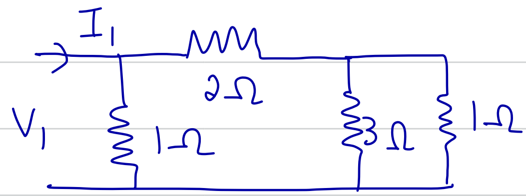
$Z_{21} = \left.\dfrac{V_2}{I_1}\right|_{I_2=0}$

$V_2 = I_d \times 3$ $\qquad$ $I_d = \dfrac{1}{6} \times I_1$

$\therefore Z_{21} = 0.5\,\Omega$ //

Z parameters $= \begin{bmatrix} 5/6 & 0.5 \\ 0.5 & 2.5 \end{bmatrix}$ //

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} \Rightarrow$$



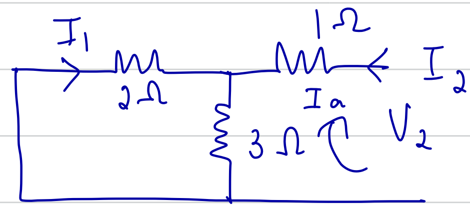
$$1 \parallel \left(2 + (3 \parallel 1)\right)$$

$$(3 \parallel 1) = \frac{3}{4}\,\Omega$$

$$1 \parallel \frac{11}{4} = \frac{11}{15} = R$$

$$\therefore\ Y_{11} = (15/11)\ S$$

$$Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$$



$$-2I_1 - 3(I_1 - I_a) = 0 \Rightarrow 5I_1 = 3I_a$$

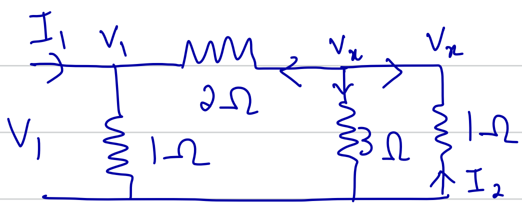
$$-3(I_a - I_1) - I_a = V_2 \Rightarrow -4\left(\frac{5}{3}\right)I_1 + 3I_1 = V_2$$

$$V_2 = \frac{-11}{3}I_1 \Rightarrow \frac{V_2}{I_1} = Y_{12} = -\frac{3}{11}\ S$$

$$Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$

$$R = (2 \parallel 3) + 1 = \frac{6}{5} + 1 = \frac{11}{5} \qquad \therefore\ Y_{22} = \frac{5}{11}\ S$$

$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$$



$$\frac{V_x - V_1}{2} + \frac{V_x}{3} + \frac{V_x}{1} = 0$$

$$\frac{V_x}{2} + \frac{V_x}{1} + \frac{V_x}{3} = \frac{V_1}{2}$$

$$\frac{11}{6} V_x = \frac{V_1}{2} \quad \Rightarrow \quad V_1 = \frac{11}{3} V_x$$

$$V_x = -I_2$$

$$V_1 = -\frac{11}{3} I_2$$

$$\therefore \; Y_{21} = \frac{-3}{11} \, S$$

$$Y \text{ parameters} = \begin{bmatrix} 15/11 & -3/11 \\ -3/11 & 5/11 \end{bmatrix}$$

Hybrid (h) parameters

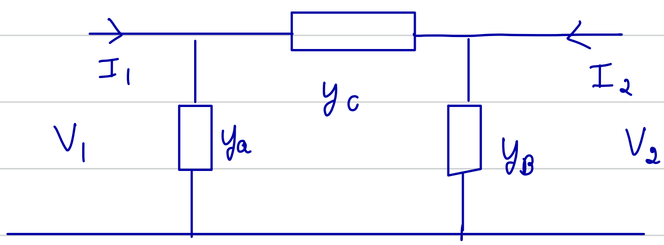
$V_1, I_2 \rightarrow$ Dependent variables (Responses)
$I_1, V_2 \rightarrow$ Independent variables (Excitations)

$$V_1 = h_{11} I_1 + h_{12} V_2$$
$$I_2 = h_{21} I_1 + h_{22} V_2$$

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} \qquad h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} \qquad h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} \qquad h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$

$h_{11} \rightarrow$ Short circuit input impedance
$h_{12} \rightarrow$ Open circuit voltage transfer function
$h_{21} \rightarrow$ Short circuit current transfer function
$h_{22} \rightarrow$ Open circuit output admittance

Q1> Find h parameters



Ans $h_{11} = \left.\dfrac{V_1}{I_1}\right|_{V_2=0}$

$$= \frac{1}{y_a + y_c}$$

$h_{12} = \left.\dfrac{V_1}{V_2}\right|_{I_1=0}$ But $V_1 = V_2 \times \dfrac{1/y_a}{1/y_a + 1/y_c}$

$\Rightarrow$ $h_{12} = \dfrac{y_c}{y_a + y_c}$

$h_{21} = \left.\dfrac{I_2}{I_1}\right|_{V_2=0}$ $I_2 = -I_1 \times \dfrac{1/y_a}{1/y_a + 1/y_c}$

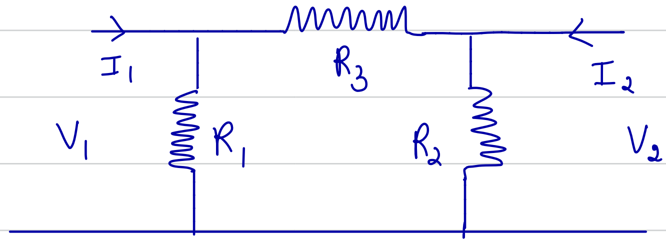
$\therefore h_{21} = \dfrac{-y_c}{y_a + y_c}$

$h_{22} = \left.\dfrac{I_2}{V_2}\right|_{I_1=0} = 1 \Big/ \left(\dfrac{1}{y_a} + \dfrac{1}{y_c}\right) \parallel \dfrac{1}{y_B}$

$\left(\dfrac{1}{y_a} + \dfrac{1}{y_c}\right) \parallel \dfrac{1}{y_B} = \dfrac{\dfrac{1}{y_B} \cdot \left(\dfrac{1}{y_a} + \dfrac{1}{y_c}\right)}{\dfrac{1}{y_a} + \dfrac{1}{y_B} + \dfrac{1}{y_c}} = \dfrac{y_c + y_a}{y_a y_b + y_b y_c + y_c y_a}$

$$h_{22} = \frac{y_a y_b + y_b y_c + y_c y_a}{y_a + y_c}$$

Q2> Find h parameters



Ans

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} = \frac{R_1 R_3}{R_1+R_3}$$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} = \frac{R_1}{R_1+R_3}$$

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} = \frac{-R_1}{R_1+R_3}$$

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0} = \frac{1}{R_2 \parallel (R_1+R_3)}$$

$$R_2 \parallel (R_1+R_3) = \frac{R_2R_1 + R_2R_3}{R_1+R_2+R_3}$$

$$h_{22} = \frac{R_1+R_2+R_3}{R_1R_2+R_2R_3}$$

g parameters

$I_1, V_2 \rightarrow$ Dependent variables
$V_1, I_2 \rightarrow$ Independent variables

$$I_1 = g_{11}V_1 + g_{12}I_2$$
$$V_2 = g_{21}V_1 + g_{22}I_2$$

$$g_{11} = \left.\frac{I_1}{V_1}\right|_{I_2=0} \quad g_{12} = \left.\frac{I_1}{I_2}\right|_{V_1=0} \quad g_{21} = \left.\frac{V_2}{V_1}\right|_{I_2=0} \quad g_{22} = \left.\frac{V_2}{I_2}\right|_{V_1=0}$$

ABCD parameters (T - parameters)

$V_2, I_2 \rightarrow$ Independent $\qquad V_1, I_1 \rightarrow$ Dependent

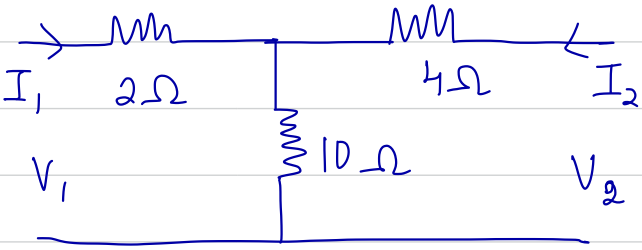
$V_1 = AV_2 - BI_2$

$I_1 = CV_2 - DI_2$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$A = \left.\dfrac{V_1}{V_2}\right|_{I_2=0} \qquad B = \left.\dfrac{V_1}{-I_2}\right|_{V_2=0} \qquad C = \left.\dfrac{I_1}{V_2}\right|_{I_2=0} \qquad D = \left.\dfrac{I_1}{-I_2}\right|_{V_2=0}$

Q1) Find ABCD parameters:



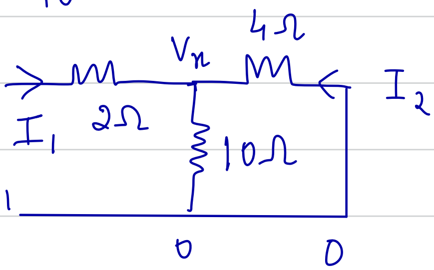
Ans $A = \left.\dfrac{V_1}{V_2}\right|_{I_2=0}$

$= 6/5$ //

$C = \left.\dfrac{I_1}{V_2}\right|_{I_2=0} = \dfrac{1}{10}$ S //

$D = \left.\dfrac{I_1}{-I_2}\right|_{V_2=0} = \dfrac{7}{5}$ //

$B = \left.\dfrac{V_1}{I_2}\right|_{V_2=0}$



$$\frac{V_1 - V_x}{2} = \frac{V_x}{4} + \frac{V_x}{10}$$

$$V_1 - V_x = \frac{V_x}{5} + \frac{V_x}{2} \Rightarrow V_1 = V_x\left(\frac{3}{2} + \frac{1}{5}\right) = \frac{17V_x}{10}$$

$V_x = -4I_2$

$\therefore V_1 = \frac{-17 \times 4 \times I_2}{10} = \frac{-34 I_2}{5}$

$\therefore B = \frac{34}{5} \Omega$ //

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1.2 & 6.8 \\ 0.1 & 1.4 \end{bmatrix}$$

Relationship between ABCD and Z & y parameters

$V_1 = AV_2 - BI_2$ | $V_1 = Z_{11} I_1 + Z_{12} I_2$

$I_1 = CV_2 - DI_2$ | $V_2 = Z_{21} I_1 + Z_{22} I_2$

$A = \left.\frac{V_1}{V_2}\right|_{I_2=0}$ Also $\left.\frac{V_1}{V_2}\right|_{I_2=0} = \frac{Z_{11}}{Z_{21}}$

$\therefore A = \frac{Z_{11}}{Z_{21}}$ //

$C = \left.\frac{I_1}{V_2}\right|_{I_2=0}$ Also $\left.\frac{I_1}{V_2}\right|_{I_2=0} = \frac{1}{Z_{21}}$

$\therefore C = \frac{1}{Z_{21}}$

Y parameters: $I_1 = y_{11} V_1 + y_{12} V_2$

$I_2 = y_{21} V_1 + y_{22} V_2$

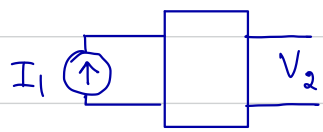
$B = \left.\frac{V_1}{-I_2}\right|_{V_2=0} = \frac{-1}{y_{21}}$ |||ly $D = \left.\frac{I_1}{-I_2}\right|_{V_2=0} = -\frac{y_{11}}{y_{21}}$

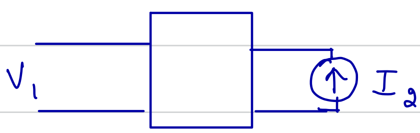
$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \dfrac{z_{11}}{z_{21}} & -\dfrac{1}{y_{21}} \\ \dfrac{1}{z_{21}} & -\dfrac{y_{11}}{y_{21}} \end{bmatrix}$$

Condition for Reciprocity

$V_1 = AV_2 - BI_2$ $\qquad\qquad$ $I_1 = CV_2 - DI_2$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0}$$





For the circuit to be reciprocal, $\left.\dfrac{V_2}{I_1}\right|_{I_2=0} = \left.\dfrac{V_1}{I_2}\right|_{I_1=0}$

when $I_1 = 0$

$$CV_2 = DI_2$$

$$\frac{V_2}{I_2} = \frac{D}{C}$$

$$V_1 = \frac{AD}{C}I_2 - BI_2 \quad \Rightarrow \quad \frac{V_1}{I_2} = \frac{AD}{C} - B$$

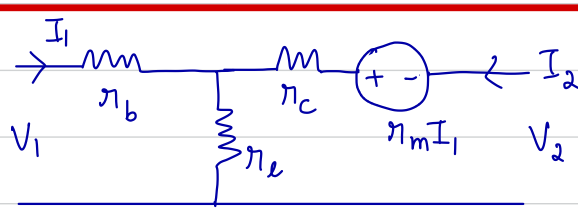
$$\therefore \quad \frac{1}{C} = \frac{AD - BC}{C} \qquad \therefore \quad AD - BC = 1$$

Ex1: Find Z & h parameters of CE transistor configuration represented by T circuit model.

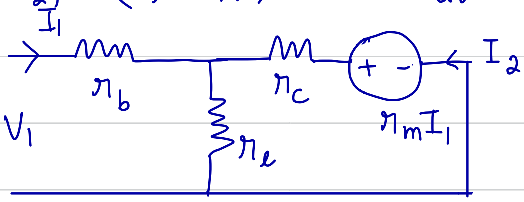
Ans $V_1 = I_1(r_b + r_e) + I_2 r_e$

$V_2 = I_1(r_e) + I_2(r_e + r_c) - r_m I_1$

$V_2 = I_1(r_e - r_m) + I_2(r_e + r_c)$



$\therefore Z_{11} = r_b + r_e \qquad Z_{12} = r_e \qquad Z_{21} = (r_e - r_m) \qquad Z_{22} = r_e + r_c$ //



H parameters : $h_{11} = \left.\dfrac{V_1}{I_1}\right|_{V_2=0}$

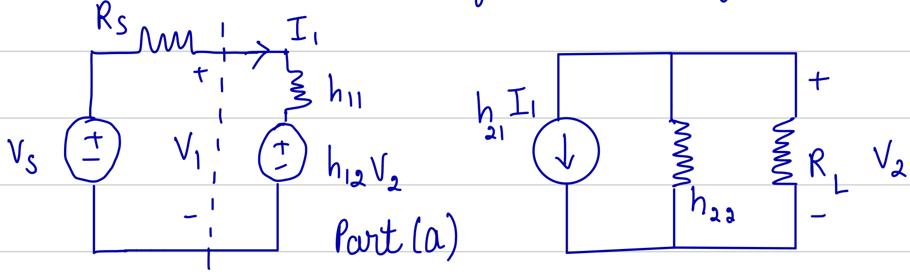
$V_1 = I_1(r_b + r_e) + I_2 r_e$

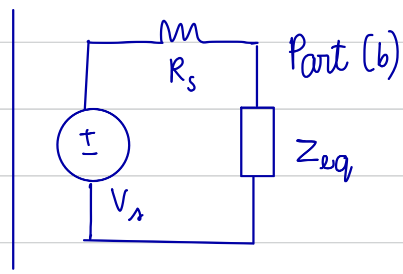
$0\,(V_2) = (r_e - r_m) I_1 + I_2(r_e + r_c)$

$\therefore \quad h_{11} = \dfrac{(r_b + r_e) r_c + r_b r_e + r_e r_m}{r_c + r_e}$

$|||^{ly}$ we can find other h parameters.

Ex2 The circuit in part (a) of figure is to be described by an equivalent input shown in part (b). Determine Z_{eq} in (b) as a function of elements & voltages in part (a)





Ans $Z_{eq} = \dfrac{V_1}{I_1}$

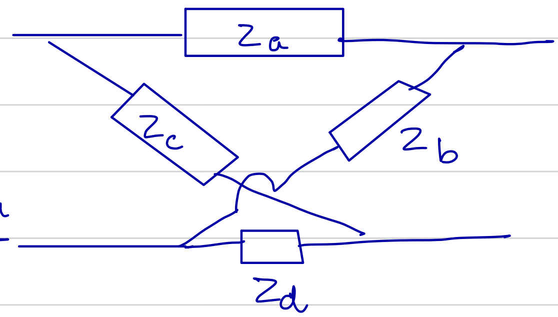
$V_1 = h_{11} I_1 + h_{12} V_2$

$V_2 = -h_{21} I_1 \times \dfrac{1/h_{22}}{\dfrac{1}{h_{22}} + R_L} = \dfrac{-h_{21} I_1}{1 + R_L h_{22}}$

$$V_1 = h_{11} I_1 - \frac{h_{12} h_{21} I_1}{1 + R_L h_{22}}$$

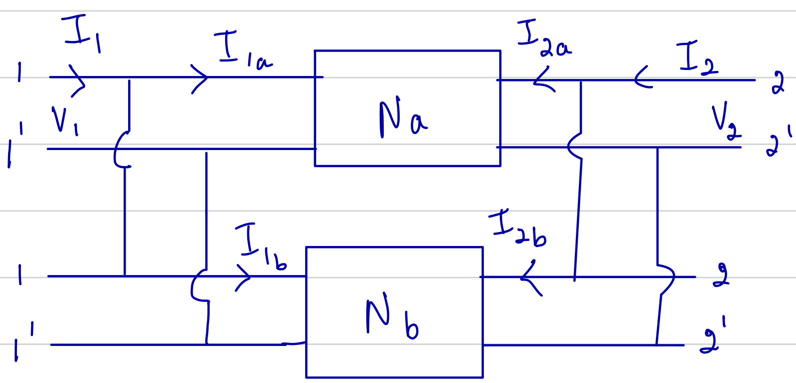
$$\therefore Z_{eq} = \frac{V_1}{I_1} = h_{11} - \frac{h_{12} h_{21}}{1 + R_L h_{22}} //$$

HW Find Z parameters if $Z_a = Z_d$ & $Z_b = Z_c$ //



$$Z_{11} = Z_{22} = \frac{Z_a + Z_b}{2} \qquad Z_{12} = Z_{21} = \frac{Z_b - Z_a}{2}$$

Interconnection of 2-Port Networks



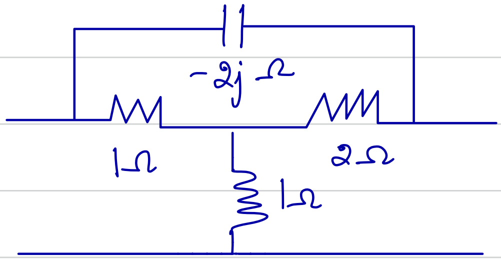
$V = ZI \qquad I = YV$

$$\begin{bmatrix} I_{1a} \\ I_{2a} \end{bmatrix} = \begin{bmatrix} y_{11a} & y_{12a} \\ y_{21a} & y_{22a} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

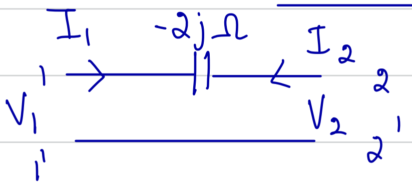
$$\begin{bmatrix} I_{1b} \\ I_{2b} \end{bmatrix} = \begin{bmatrix} y_{11b} & y_{12b} \\ y_{21b} & y_{22b} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11a} + y_{11b} & y_{12a} + y_{12b} \\ y_{21a} + y_{21b} & y_{22a} + y_{22b} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

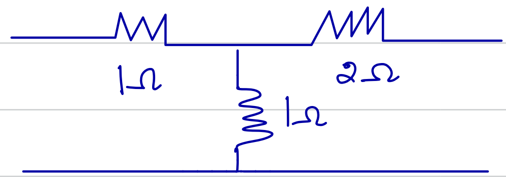
Q1) Solve :



Ans



+



$$V_1 - V_2 = -2j \times (-I_1)$$
$$= 2j\, I_1$$

Also $I_1 = -I_2$

$$y_{11a} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = \frac{j}{2}$$

|||ly
$$\begin{bmatrix} y_{11a} & y_{12a} \\ y_{21a} & y_{22a} \end{bmatrix} = \begin{bmatrix} j/2 & -j/2 \\ -j/2 & j/2 \end{bmatrix}$$

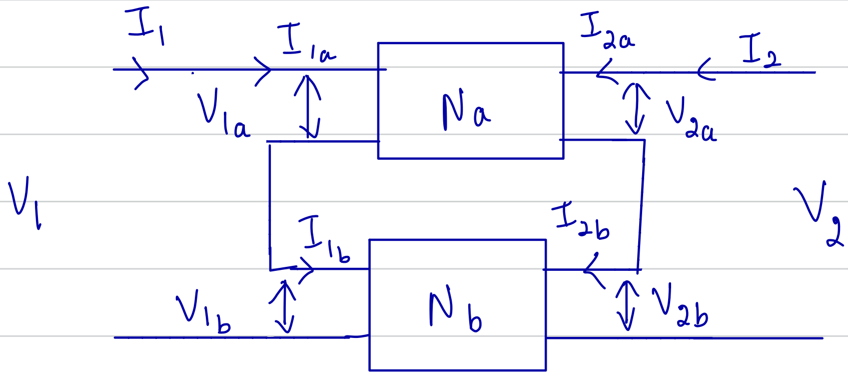
Also
$$Z_b = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$$

NOTE: $[Z]^{-1} = [Y]$

$$\therefore \quad Y_b = \frac{1}{5}\begin{bmatrix} 3 & -1 \\ -1 & 2 \end{bmatrix}$$

$$\therefore \quad y = y_a + y_b$$
$$= \begin{bmatrix} j/2 + 3/5 & -j/2 - 1/5 \\ -j/2 - 1/5 & j/2 + 2/5 \end{bmatrix}$$

Series interconnections



$V = ZI$

$I_1 = I_{1a} = I_{1b}$ $\quad$ $I_2 = I_{2a} = I_{2b}$

$V_1 = V_{1a} + V_{1b}$ $\quad$ $V_2 = V_{2a} + V_{2b}$

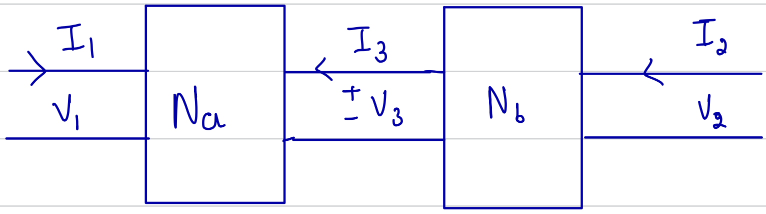
Z parameters

$$\begin{bmatrix} V_{1a} \\ V_{2a} \end{bmatrix} = \begin{bmatrix} Z_{11a} & Z_{12a} \\ Z_{21a} & Z_{22a} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\begin{bmatrix} V_{1b} \\ V_{2b} \end{bmatrix} = \begin{bmatrix} Z_{11b} & Z_{12b} \\ Z_{21b} & Z_{22b} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\begin{bmatrix} V_{1a} + V_{1b} = V_1 \\ V_{2a} + V_{2b} = V_2 \end{bmatrix} = \begin{bmatrix} Z_{11a} + Z_{11b} & Z_{12a} + Z_{12b} \\ Z_{21a} + Z_{21b} & Z_{22a} + Z_{22b} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Cascade Interconnections

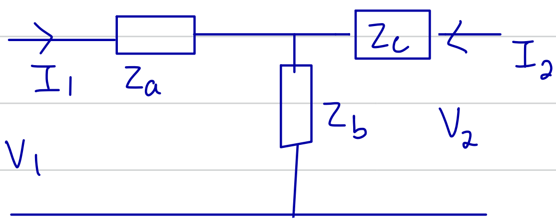


$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} V_3 \\ -I_3 \end{bmatrix}$$

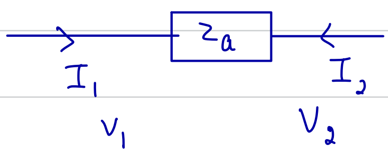
$$\begin{bmatrix} V_3 \\ -I_3 \end{bmatrix} = \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$$\therefore \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

Q> Find ABCD parameters:

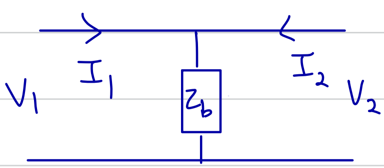


Ans



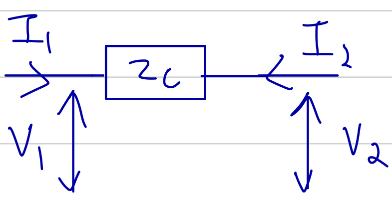
$V_1 - I_1 Z_a = V_2$ But $I_1 = -I_2$

$$\therefore \text{ABCD parameters} = \begin{bmatrix} 1 & Z_a \\ 0 & 1 \end{bmatrix}$$



$V_1 = V_2$ $\quad$ $I_1 + I_2 = \dfrac{V_2}{Z_b}$

$I_1 = \dfrac{V_2}{Z_b} - I_2$ $\quad$ $\therefore$ ABCD parameters $= \begin{bmatrix} 1 & 0 \\ 1/Z_b & 1 \end{bmatrix}$



ABCD parameters $= \begin{bmatrix} 1 & Z_c \\ 0 & 1 \end{bmatrix}$

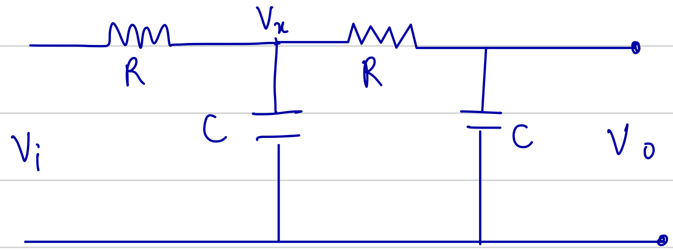
$\therefore$ Total ABCD parameters: $\begin{bmatrix} 1 & Z_a \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1/Z_b & 1 \end{bmatrix} \begin{bmatrix} 1 & Z_c \\ 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 1 + Z_a/Z_b & Z_a \\ 1/Z_b & 1 \end{bmatrix} \begin{bmatrix} 1 & Z_c \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 + \frac{Z_a}{Z_b} & Z_a + Z_c + \frac{Z_a Z_c}{Z_b} \\ \frac{1}{Z_b} & \frac{Z_c}{Z_b} + 1 \end{bmatrix}$$

Here $AD - BC = 1$ //

Q>



Find the transfer function $\frac{V_o}{V_i}$

Ans Transfer function:

$$\left(\frac{1}{Cs}\right) \parallel \left(R + \frac{1}{Cs}\right) \Rightarrow \frac{1}{Z} = Cs + \frac{1}{R + \frac{1}{Cs}}$$

$$= Cs\left(1 + \frac{1}{RCs+1}\right)$$

$$= Cs\left(\frac{2 + RCs}{1 + RCs}\right)$$

$$\Rightarrow Z = \frac{1 + RCs}{Cs(2 + RCs)}$$

Also $V_x = \frac{V_i Z}{Z + R}$

$$V_o = \frac{V_x \times 1/Cs}{R + \frac{1}{Cs}} = \frac{V_x}{1 + RCs}$$

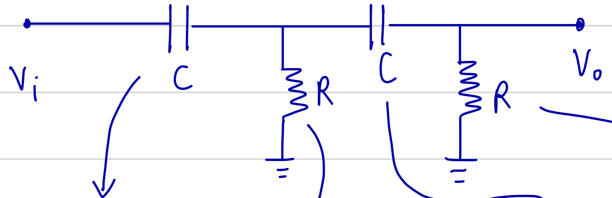
$$\Rightarrow V_o = \frac{V_i Z}{(Z+R)(1+RCs)} = \frac{V_i}{(1+RCs)} \times \frac{(1+RCs)}{(2+RCs)Cs} \times \frac{1}{\frac{(1+RCs)}{(2+RCs)Cs} + R}$$

$$\Rightarrow V_o = \frac{V_i}{\cancel{(1+RCs)}} \times \frac{\cancel{(1+RCs)}}{\cancel{(2+RCs)Cs}} \times \frac{1}{\frac{(1+RCs) + R(2+RCs)Cs}{\cancel{(2+RCs)Cs}}}$$

$$V_o = \frac{V_i}{1 + RCS + 2RCS + (RCS)^2} = \frac{V_i}{(RCS)^2 + 3RCS + 1}$$

$$\therefore \frac{V_o}{V_i} = \frac{1}{(RCS)^2 + 3RCS + 1}$$

Q>



Find Transfer function =

Ans
$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 1/CS \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1/R & 1 \end{bmatrix} \begin{bmatrix} 1 & 1/CS \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1/R & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 + \frac{1}{RCS} & \frac{1}{CS} \\ \frac{1}{R} & 1 \end{bmatrix} \begin{bmatrix} 1 + \frac{1}{RCS} & \frac{1}{CS} \\ \frac{1}{R} & 1 \end{bmatrix}$$

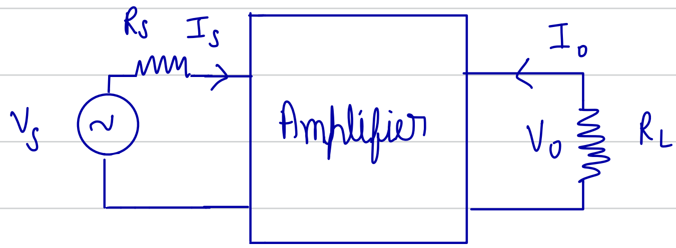
$$= \begin{bmatrix} 1 + \frac{3}{RCS} + \frac{1}{(RCS)^2} & \frac{2}{CS} + \frac{1}{C^2S^2R} \\ \frac{2}{R} + \frac{1}{R^2CS} & 1 + \frac{1}{RCS} \end{bmatrix}$$

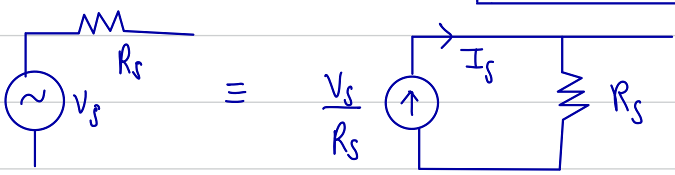
Transfer function $\frac{V_o}{V_i} = \frac{1}{A}$

$$A = \frac{(RCS)^2 + 3RCS + 1}{(RCS)^2}$$

$$\frac{1}{A} = \frac{(RCS)^2}{(RCS)^2 + 3RCS + 1}$$

Amplifiers



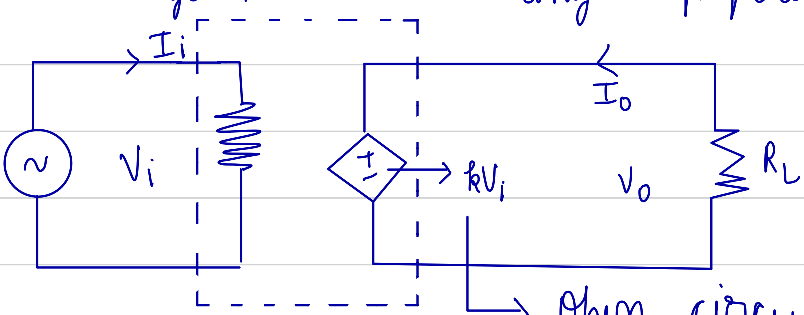


If R_s is very large, It acts as current source.
If R_s is very small, It acts as voltage source.

Type of Amplifiers

1) voltage amplifiers
2) Current amplifiers
3) Transadmittance amplifiers
4) Transimpedance amplifiers
5) Transconductance amplifiers

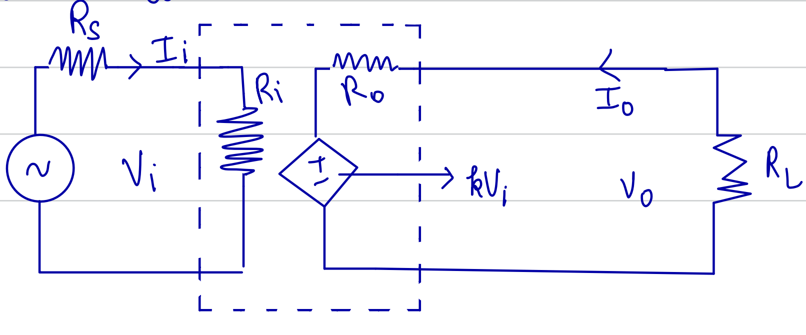
Output = gain x Input
where gain is any proportionality constant.



Ideal case

Open circuit voltage gain

Actual case:



R_i is ideally ∞. Practically $R_i >> R_s$. If $R_i >>> R_s$ amplifier is efficient. $K \rightarrow$ multiplication factor.

$R_o > 0$, $V_o < kV_i$, $\therefore$ To minimise the difference between kV_i & V_o, $R_L >>> R_o$

Transducer: It converts real world signal to electrical signal. It contains sensors.

$$V_i = \frac{R_i \times V_s}{R_i + R_s} \qquad V_o = \frac{A_v V_i \cdot R_L}{R_L + R_o}$$

$$\therefore \frac{V_o}{V_s} = \frac{A_v R_i R_L}{(R_i + R_s)(R_L + R_o)}$$

$$\therefore \frac{V_o}{V_s} = A_v \times \text{loading} \quad \text{where} \quad \text{loading} = \frac{R_i R_L}{(R_i + R_s)(R_L + R_o)}$$

Here there is no feedback. It is unilateral.

$$\frac{P_o}{P_i} = \frac{V_o I_o}{V_i I_i} \Rightarrow P_{dB} = 10 \log \frac{P_o}{P_i}$$

Also $A_{V\,dB} = 20 \log \frac{V_o}{V_i}$

Ex) $R_i = 100\,k\Omega$ $\quad R_o = 1\,k\Omega$ $\quad R_s = 10\,k\Omega$ $\quad R_L = 10\,k\Omega$

$A_v = 10^3$

Ans
$$\frac{V_o}{V_s} = A_v \times \frac{R_i R_L}{(R_i + R_s)(R_L + R_o)} = 10^3 \times \frac{1000 \times 10^6}{(110)(11) \times 10^6}$$

$$= 826.44$$

in dB $= 20 \log 826.44 = 58.34$ dB

$$I_0 = \frac{V_0}{R_L} \qquad I_i = \frac{V_i}{R_i}$$

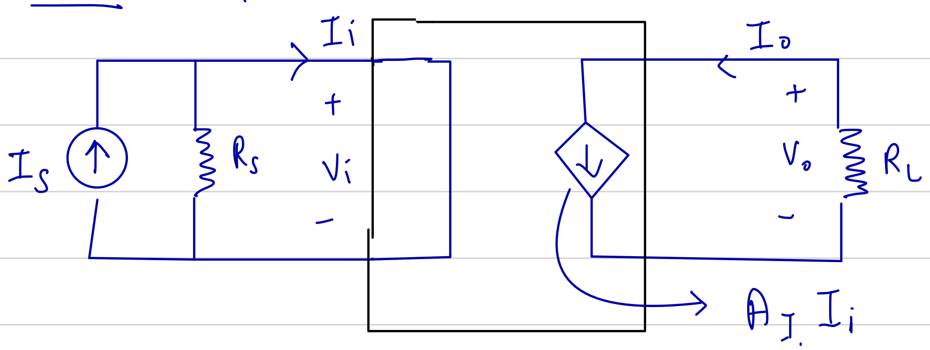
$$\frac{I_o}{I_i} = \frac{V_o}{V_i} \times \frac{R_i}{R_L} = 9090.9$$

$$\frac{P_o}{P_i} = \frac{V_o I_o}{V_i I_i} = 8.26 \times 10^6$$

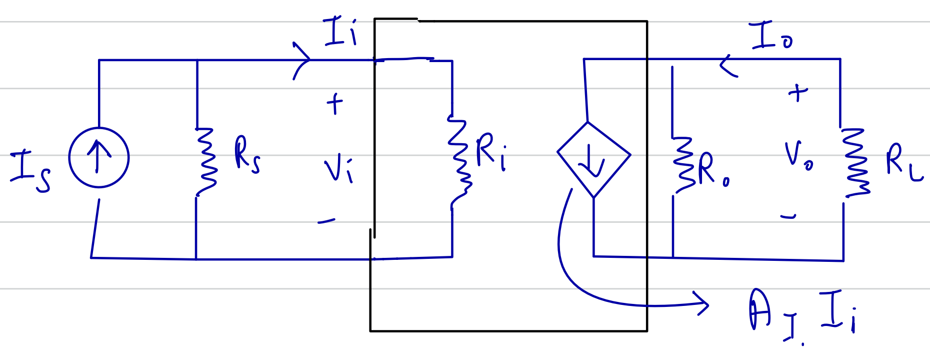
$$\therefore P_{dB} = 10 \log (8.26 \times 10^6)$$
$$= 10 (6 + 0.917)$$
$$= 69.17 \text{ dB}$$

2) Current amplifiers: Current controled current source.

Ideal: $R_i = 0$ $\qquad R_o = \infty$



Practical;



$R_i \ll R_s$ $\qquad R_o \gg R_L$

$$I_i = \frac{R_S I_S}{R_S + R_i} \qquad\qquad I_o = \frac{R_o \times A_I I_i}{R_o + R_L}$$

$$\therefore \frac{I_o}{I_i} = \frac{R_o A_I R_S}{(R_o + R_L)(R_S + R_i)}$$

$$\therefore \frac{I_o}{I_S} = A_i \times \text{loading}$$

Q) An amplifier with R_i of $1\,k\Omega$ is fed by independent current source with $R_S = 10\,k\Omega$, $A_I = 100$, R_o for dependent I source is $100\,k\Omega$. Determine I_o / I_S when $R_L = 10\,k\Omega$, also find the power gain

Ans
$$\frac{I_o}{I_S} = A_I \frac{R_o R_S}{(R_o + R_L)(R_i + R_S)} = 100 \times \frac{100 \times 10 \times 10^6}{(110)(11) \times 10^6}$$

$$= 82.64$$

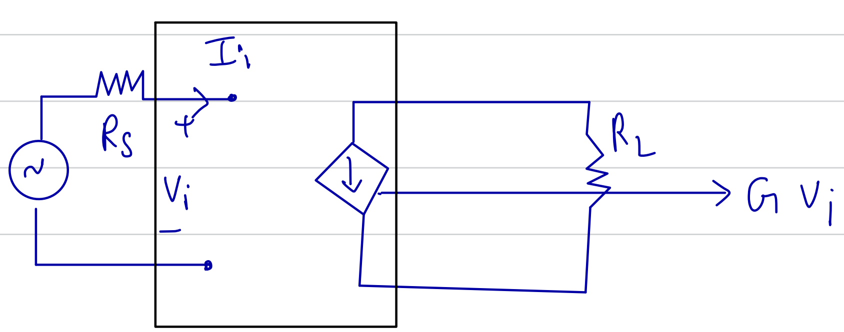
$$\therefore \text{ in dB} = 20 \log (82.64) = 38.34 \text{ dB}$$

$$\frac{P_o}{P_i} = \frac{V_o I_o}{V_i I_i} = \frac{R_L}{R_i} \left(\frac{I_o}{I_i} \right)^2$$

$$= 10\,(82.64)^2 = 68293.7$$

$$P_{gain} = 10 \log (68293.7) = 48.34 \text{ dB}$$

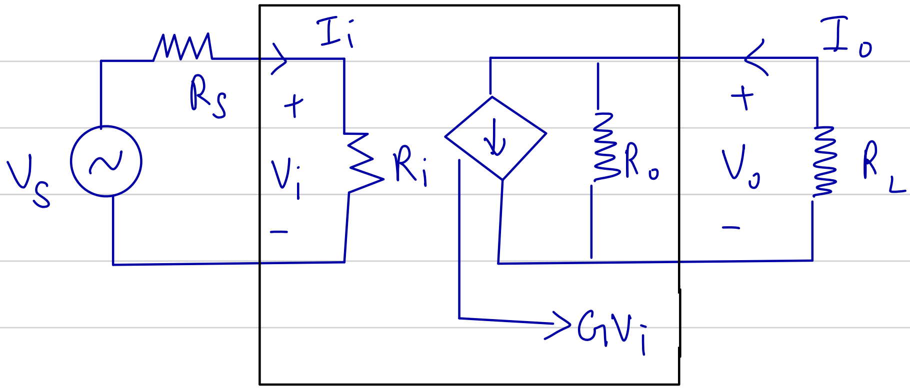
3) Transconductance Amplifier



$R_i = \infty$

$R_o = \infty$

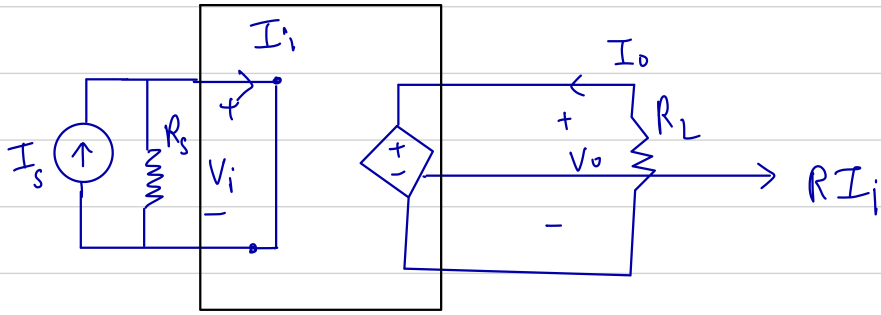
Ideal case.



$R_i >> R_s$
$R_o >> R_L$

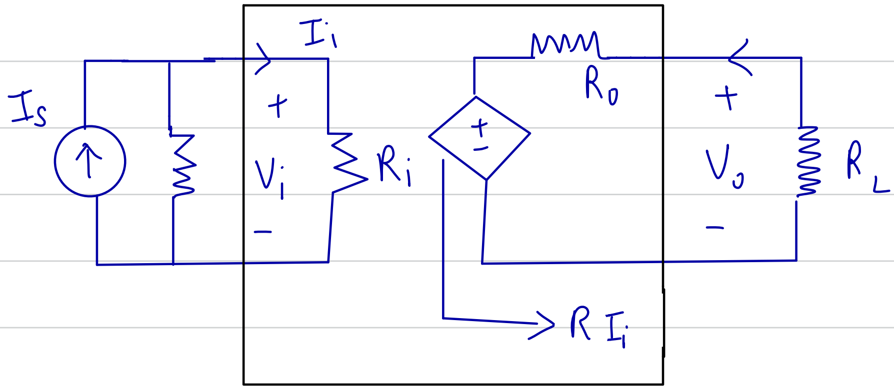
Actual case

h> <u>Transresistance</u> Amplifier



$R_i = 0$
$R_o = 0$

Ideal case



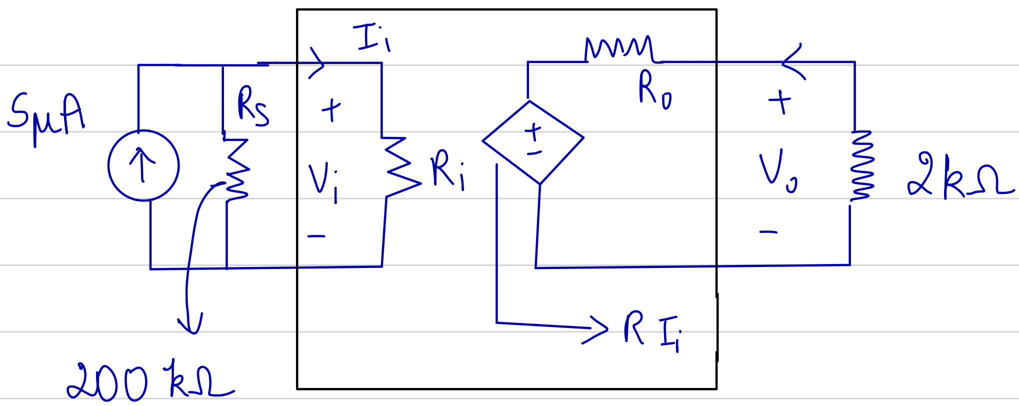
$R_i << R_s$
$R_o << R_L$

Amplifier types

Amplifier	R_i		R_o	
	Ideal	Practical	Ideal	Practical
1> Voltage Amplifier (VCVS)	∞	$R_i \gg R_S$	0	$R_o \ll R_L$
2> Current Amplifier (CCCS)	0	$R_i \ll R_s$	∞	$R_o \gg R_L$
3> Transconductance amplifier (VCCS)	∞	$R_i \gg R_S$	∞	$R_o \gg R_L$
4> Transresistance amplifier (CCVS)	0	$R_i \ll R_S$	0	$R_o \ll R_L$

Q> A source of 5μA with internal resistance 200 kΩ drives a transresistance amplifier V at input port is 100 mV. Open circuit voltage and short circuit current are 14V & 30 mA. Find R_i, R_o, R, V_o/I_s. Given that amplifier is terminated by 2kΩ resistance.

Ans $V_i = 100\ mV$



$$V_i = 100\ mV = I_i R_i = \frac{I_s R_s R_i}{R_i + R_s}$$

$\therefore R_i = 22.22\ k\Omega$

$$30\ mA = \frac{O.C\ voltage}{R_o} = \frac{R I_i}{R_o}$$

$R_o = 466.66\ \Omega$

$$I_i = \frac{V_i}{R_i} = \frac{100\ mV}{22.22\ k\Omega} = 4.5\ \mu A$$

$I_s = 0.5\ \mu A$ $\quad (5 - I_i)$

$$R = \frac{14}{I_i} = \frac{14}{4.5} \quad 3.11 \times 10^6\ V/A$$

$$V_o = \frac{R_L R I_i}{R_L + R_o} \qquad I_i = \frac{I_s R_s}{R_s + R_i}$$

$$\frac{V_o}{I_s} = \frac{R_s \times R \times R_L}{(R_s + R_i)(R_L + R_o)} = 2.27 \ V/\mu A$$

Q> A transresistance amplifier is designed with R_i of 50 $k\Omega$, $R = 0.5\ V/mA$ which is decided by the input current I_i and $R_o = 100\,\Omega$. If it is driven by a source I_s with parallel internal resistance of 300 $k\Omega$ and resistance of 500 Ω

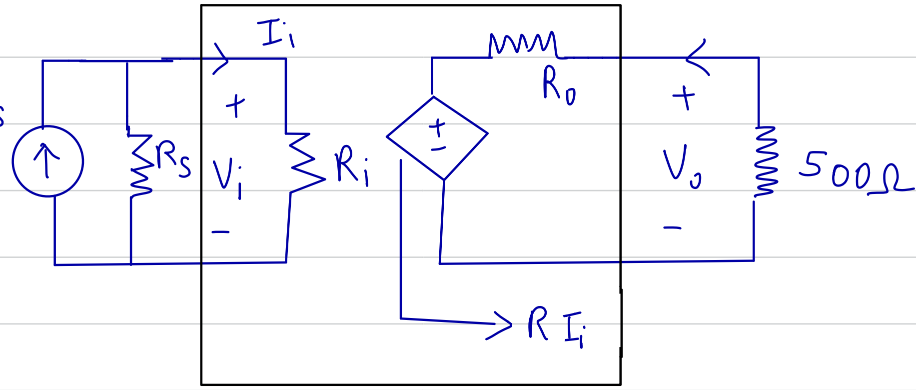
a> What is the transresistance V_o/V_s. Also find Power gain $\frac{P_i}{P_s}$ where P_i is the power absorbed by source & P_s is power delivered by the source.

b> What should be the gain of the amplifier if we want to achieve a V_o/I_s of 1V/mA. And what is the power gain in this case

Ans $R_s = 300\,k\Omega$

$R_i = 50\,k\Omega$

$R_o = 100\,\Omega$



$$V_o = \frac{R_L R I_i}{R_L + R_o}$$

$$\frac{V_o}{I_s} = \frac{R_L \cdot R \cdot R_s}{(R_L + R_o)(R_i + R_s)}$$

$$= 0.357 \ V/mA$$

$$P_S = I_S^2 (R_S \parallel R_i) = I_S^2 \frac{R_S R_i}{R_i + R_S} = 42.857 \ k\Omega \times I_s^2$$

$$P_L = \frac{V_o^2}{R_L} = \frac{V_o^2}{500}$$

$$\frac{P_L}{P_S} = \frac{V_o^2}{500 \times 42.857\ k \times I_S^2} = 5.9 \ mW/W$$

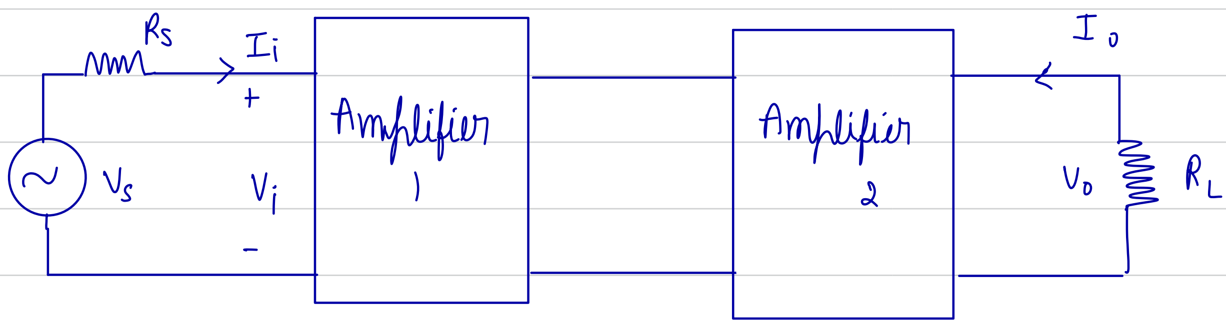
b) $\frac{V_o}{I_S} = 1 \ V/mA$

$$\frac{V_o}{I_S} = \frac{R_L \times R \times R_S}{(R_L + R_o)(R_i + R_S)}$$

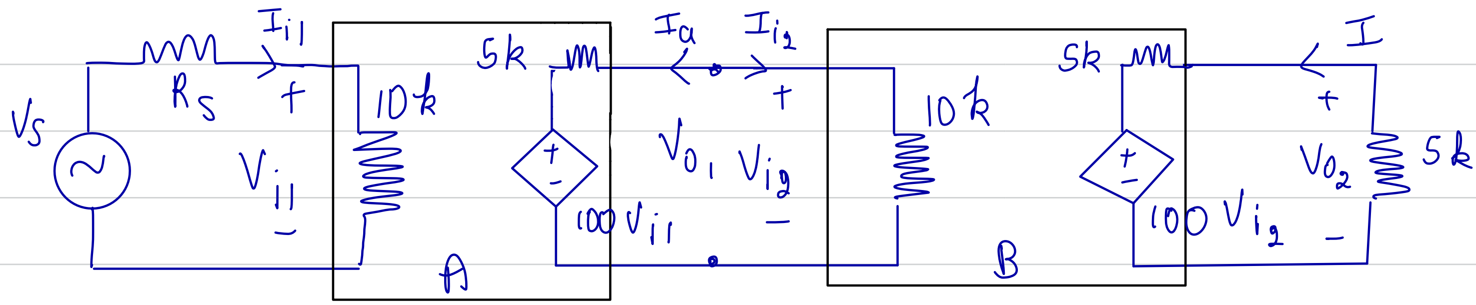
$\therefore R = 1.4 \ V/mA$

Also $\frac{P_L}{P_S} = 46.6 \ mW/W$

Cascading of amplifiers



Q) An amplifier A with R_i of $10\ k\Omega$ & $R_o = 5\ k\Omega$ has open loop gain 100. It is cascaded with another identical amplifier B. The source has resistance of $4\ k\Omega$. The load resistance for amplifier B is $5\ k\Omega$. Determine the overall voltage gain, current gain, power gain.



Ans $\frac{V_{i1}}{V_S} = \frac{10}{14}$; $\frac{V_{01}}{V_{i1}} \Rightarrow \frac{100 V_{i1} \times 10}{15} = V_{01} = V_{i2}$

$\frac{V_{01}}{V_{i1}} = \frac{100 \times 10}{15} = \frac{V_{i2}}{V_{i1}}$

$\frac{100 V_{i2} \times 5}{10} = V_{02}$

$\frac{V_{02}}{V_{i2}} = \frac{100 \times 5}{10} = 50$

$\frac{V_{02}}{V_{i1}} = \frac{V_{02}}{V_{i2}} \times \frac{V_{i2}}{V_{i1}} = 50 \times \frac{1000}{15} = 3333.33$ //

i> $\frac{V_{02}}{V_s} = \frac{V_{02}}{V_{i1}} \times \frac{V_{i1}}{V_S} = 3333.33 \times \frac{5}{7} = 2380.95$ //

ii> $V_{i1} = I_{i1} \times 10k$ $V_{02} = -I_{02} \times 5k$

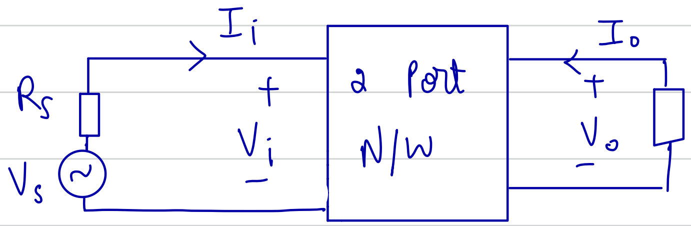
$\therefore \frac{I_{02}}{I_{i1}} = -\frac{V_{02}}{5k} \times \frac{10k}{I_{i1}} = -2 \times 3333.33 = -6666.66$

iii> Power gain = $\left| \frac{V_{02} \cdot I_{02}}{V_{i1} \times I_{i1}} \right| = (3333.33) \cdot (6666.66)$

$= 22.22 \times 10^6$ //

$P_{dB} = 10 \log (22.22 \times 10^6) = 10 (6 + 1.346) = \underline{\underline{73.46 \text{ dB}}}$

Link between 2 port parameters and Amplifiers



Z parameters

$$V_i = z_i I_i + z_r I_o$$
$$V_o = z_f I_i + z_o I_o$$

$$\Rightarrow [z] = \begin{bmatrix} z_i & z_r \\ z_f & z_o \end{bmatrix}$$

For ideal amplifier, $z_r = 0$

If z_i, z_o are also zeroes then it becomes ideal CCVS (transresistance) amplifier.

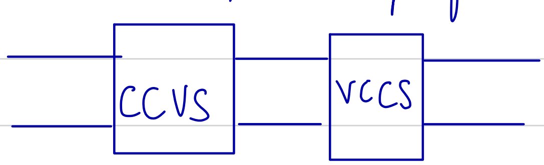
Y parameters

$$I_i = y_i V_i + y_r V_o$$
$$I_o = y_f V_i + y_o V_o$$

$$[y] = \begin{bmatrix} y_i & y_r \\ y_f & y_o \end{bmatrix}$$

If y_i, y_r, $y_o = 0$ then we get ideal VCCS (transconductance) amplifier.

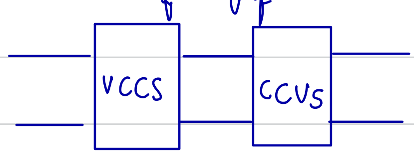
Now



CCCS

current gain $= z_f' \cdot y_f$

Similarly



VCVS

voltage gain $= z_f' y_f$

h parameters

$$V_i = h_i I_i + h_r V_o$$
$$I_o = h_f I_i + h_o V_o$$

$$[h] = \begin{bmatrix} h_i & h_r \\ h_f & h_o \end{bmatrix}$$

If $h_i, h_r, h_o = 0$ then we get ideal CCCS amplifier i.e current amplifier.

g parameters

$$I_i = g_i V_i + g_r I_o$$
$$V_o = g_f V_i + g_o I_o$$

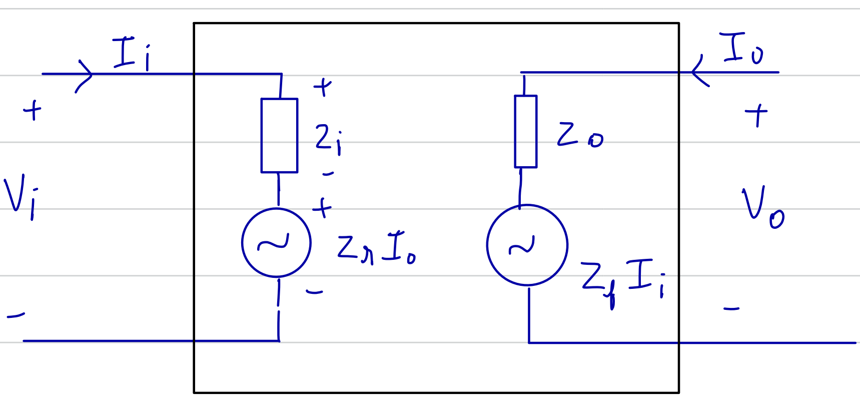
$$[g] = \begin{bmatrix} g_i & g_r \\ g_f & g_o \end{bmatrix}$$

If $g_i, g_r, g_o = 0$ then we get ideal VCVS amplifier i.e voltage amplifier.

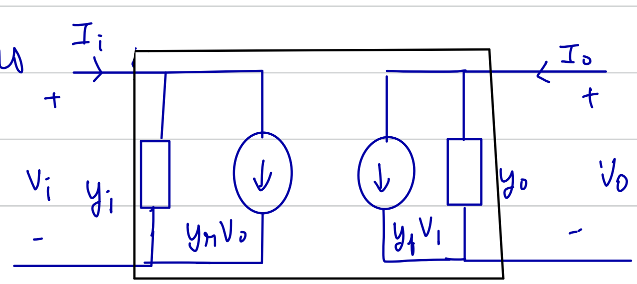
Equivalent circuit

z parameters

$$V_i = z_i I_i + z_r I_o$$
$$V_o = z_f I_i + z_o I_o$$



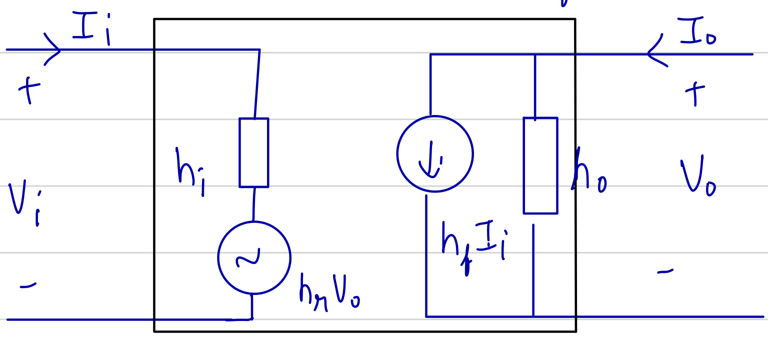
y - parameters



$$I_i = y_i V_i + y_r V_o$$
$$I_o = y_f V_i + y_o V_o$$

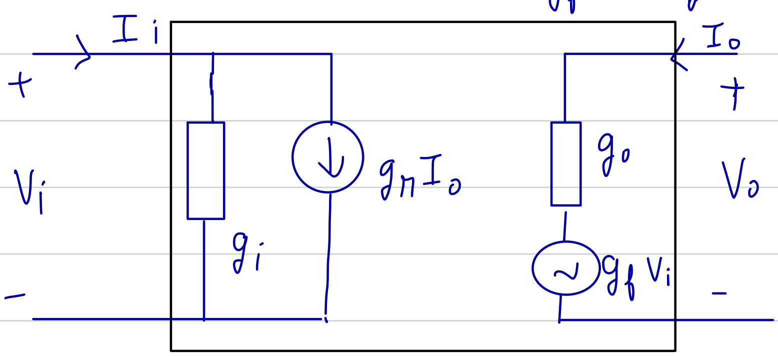
h parameters :

$$V_i = h_i I_i + h_r V_o$$
$$I_o = h_f I_i + h_o V_o$$



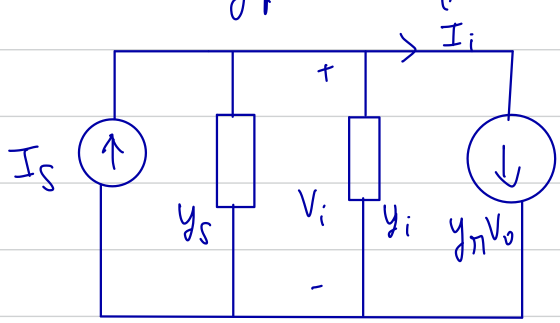
g parameter :

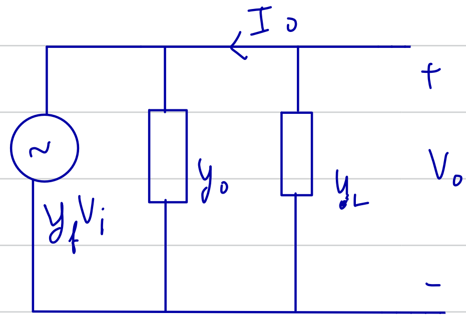
$$I_i = g_i V_i + g_r I_o$$
$$V_o = g_f V_i + g_o I_o$$



y parameters : (Actual amplifier)

$$I_i = y_i V_i + y_r V_o$$
$$I_o = y_f V_i + y_o V_o$$





$y_i = \frac{I_i}{V_i}$ (ideal) $\quad V_o = -y_f V_i \times \frac{1}{y_o + y_L} = -\frac{y_f V_i}{y_o + y_L}$

$$I_i = y_i V_i - \frac{y_r y_f V_i}{y_o + y_L}$$

$$\therefore \frac{I_i}{V_i} = y_i - \frac{y_r y_f}{y_o + y_L} = y_{in}$$

$$\boxed{\text{voltage gain } \frac{V_o}{V_i} = \frac{-y_f}{y_o + y_L}}$$

$V_o = \frac{-I_o}{y_L} \qquad V_i = \frac{I_i}{y_{in}} \qquad \therefore \frac{V_o}{V_i} = \frac{-I_o y_{in}}{I_i y_L}$

$$\therefore \text{current gain} = \boxed{\frac{I_o}{I_i} = \frac{-V_o y_L}{V_i y_{in}}}$$

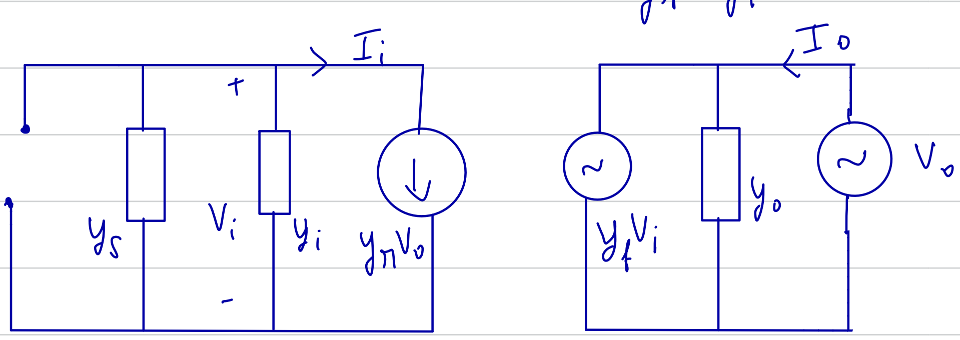
Make $I_s = 0$.

Then, $V_i = \frac{-y_r V_o}{y_s + y_i}$

$$I_o = y_f V_i + y_o V_o$$

$$I_o = \left(-\frac{y_f y_r}{y_s + y_i} + y_o \right) V_o$$

$$\therefore \frac{I_o}{V_o} = y_{out} = y_o - \frac{y_r y_f}{y_s + y_i}$$

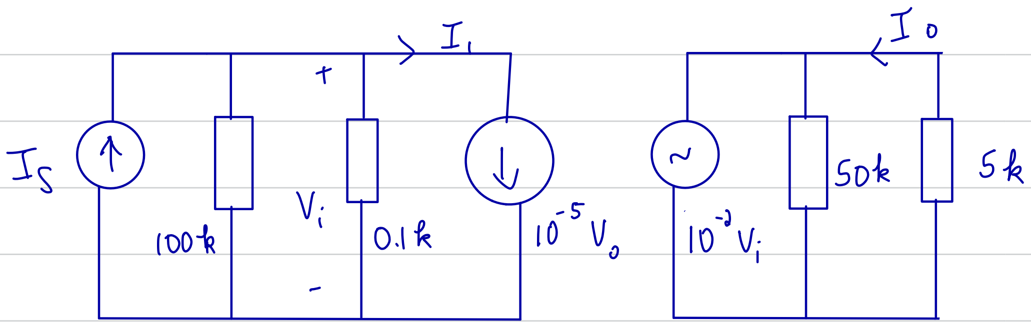


Power gain $\left|\frac{P_o}{P_i}\right| = \left|\frac{V_o I_o}{V_i I_i}\right|$

$|||^{ly}$ $Z_{in} = Z_i - \frac{Z_r Z_f}{Z_o + Z_L}$ $\quad Z_{out} = Z_o - \frac{Z_r Z_f}{Z_s + Z_i}$

Immitance: $p_{in} = p_i - \frac{p_r p_f}{p_o + p_L}$ $\quad p_{out} = p_o - \frac{p_r p_f}{p_s + p_i}$

Q) For the 2 port network shown, determine $\frac{V_o}{V_i}$, $\frac{I_o}{I_i}$, R_{in}, R_{out}, $\frac{P_o}{P_i}$



Ans $\begin{bmatrix} y_i & y_r \\ y_f & y_o \end{bmatrix} = \begin{bmatrix} 10^{-2} & 10^{-5} \\ 10^{-2} & 2\times10^{-5} \end{bmatrix}$

$$\frac{V_o}{V_i} = \frac{-y_f}{y_o + y_L} = \frac{-10^{-2}}{2\times10^{-5} + 20\times10^{-5}} = -10^3 \times \frac{1}{22} = -45.45$$

$$y_{in} = y_i - \frac{y_r y_f}{y_o + y_L} = 10^{-2} + 10^{-5}(-45.45)$$

$$= 9.54 \times 10^{-3}\ S$$

$$R_{in} = \frac{1}{y_{in}} = 104.8\ \Omega$$

$$\frac{I_o}{I_i} = -\frac{V_o}{V_i} \frac{R_{in}}{R_L}$$

$$= 45.45 \times \frac{104.82}{5\times10^3} = 0.952$$

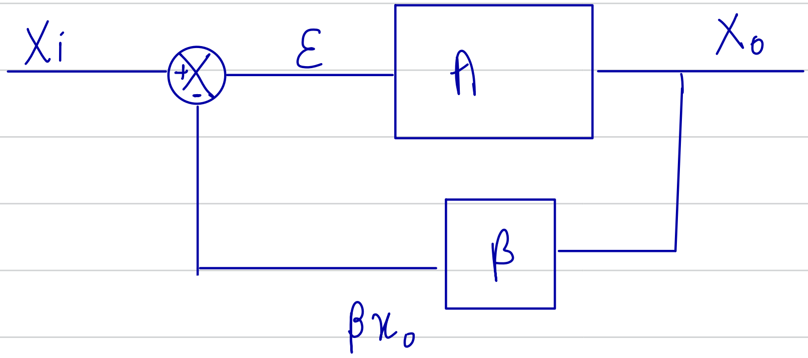
$$\frac{P_o}{P_i} = \left|\frac{V_o I_o}{V_i I_i}\right| = 45.45 \times 0.952 = 43.03$$

$$Y_{out} = Y_o - \frac{Y_r Y_f}{Y_s + Y_i} = 2\times10^{-5} - \frac{10^{-5} \times 10^{-2}}{10^{-5}+10^{-2}} = 10^{-5}$$

$R_{out} = 100\ k\Omega$

Feedback concept

Negative feedback :



$$\varepsilon = x_i - \beta x_o$$

$$x_o = A(x_i - \beta x_o)$$

$$\Rightarrow x_o(1+A\beta) = Ax_i$$

$$x_o = \left(\frac{A}{1+A\beta}\right) x_i$$

$\therefore A \Rightarrow \frac{A}{1+A\beta}$

$\therefore$ gain reduces.

If $A\beta$ is very large, $\frac{A}{1+A\beta} \cong \frac{A}{A\beta} = \frac{1}{\beta}$

$$\frac{A}{1+A\beta} = A_f$$

Advantages of negative feedback

1) Sensitivity or Reduction in active parameter sensitivity.

$$S_A^{A_f} = \frac{\partial A_f / A_f}{\partial A / A} = \left(\frac{\partial A_f}{\partial A}\right) \cdot \frac{A}{A_f} = \frac{1}{(1+A\beta)^2} \cdot \frac{A}{A} (1+A\beta)$$

$$= \frac{1}{1+A\beta} \cong \frac{1}{A\beta} \quad (\because A\beta \gg 1)$$

2) Linearity Improvment :

• A non linear circuit will have

$$V_{out} = V_{offset} + AV_i + k_1 V_i^2 + k_2 V_i^3 + \ldots$$

For negative feedback:

$$V_{out} = V_{offset} + A(V_i - \beta V_o) + N \qquad \text{where } N = k_1 V_i^2 + k_2 V_i^3 + \ldots$$

$$V_{out}[1 + A\beta] = V_{offset} + AV_i + N$$

$$\therefore V_{out} = \frac{V_{offset}}{1+A\beta} + \frac{AV_i}{1+A\beta} + \frac{N}{1+A\beta}$$

$$\Rightarrow \frac{V_{out}}{V_i} \cong \frac{1}{\beta}$$

3) Bandwidth Improvement

Let $A = \dfrac{A_0}{1 + \frac{s}{\omega_0}}$ (forward gain)

$$A_f = \frac{A}{1+A\beta} = \frac{\frac{A_0}{1+S/\omega_0}}{1+\frac{\beta A_0}{1+S/\omega_0}} = \frac{A_0}{1+\frac{S}{\omega_0}+A_0\beta} = \frac{\frac{A_0}{1+A_0\beta}}{1+\frac{S}{\omega_0(1+A_0\beta)}}$$

So $A_0 \rightarrow \frac{A_0}{1+A_0\beta}$ and $\omega_0 \rightarrow \omega_0(1+A_0\beta)$

Hence bandwidth increases.

But $A_{initial} \cdot W_{initial} = A_{final} \cdot W_{final}$

Q> A feedback amplifier consists of 2 Amplifier blocks in series. Each has a gain of 100. a> If overall gain is 100, then calculate the gain of feedback circuit.

b> If gain of each block reduces by 50% then calculate the overall gain of feedback.

Ans a> Total gain = $A_1 A_2 = 10^4$

$$A_f = \frac{A}{1+A\beta} = 100$$

$$\Rightarrow \frac{10^4}{1+\beta \cdot 10^4} = 100 \qquad \therefore \beta = 0.0099$$

b> Total gain = $A_1' A_2' = 2500$

$$A_f = \frac{2500}{1+0.0099 \times 2500} = 97.08$$

Q> An amplifier with open loop gain of $A = 2000 \pm 150$ is available. It is necessary to have an amplifier whose gain varies not more than 0.2%. Compute A_f & β.

Ans $\frac{dA_f}{A_f} = 0.2\%$

$$\boxed{\frac{dA_f}{A_f} = \frac{dA}{A} \times \frac{1}{1+A\beta}}$$

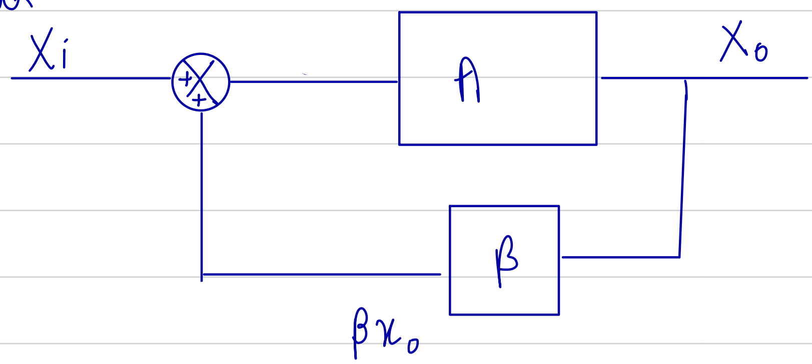
$$\frac{0.2}{1\cancel{00}} = \frac{15\cancel{0}}{2\cancel{000}} \frac{1}{1+2000\beta}$$

$$1+2000\beta = 37.5$$

$$2000\beta = 36.5 \qquad \therefore \beta = 0.01825$$

$$A_f = \frac{A}{1+A\beta} = \frac{2000}{37.5} = 53.33$$

Positive feedback



$$A_f = \frac{A}{1-A\beta} \qquad \text{i.e} \quad A_f > A$$

$$S_A^{A_f} = \frac{1}{1-A\beta}$$

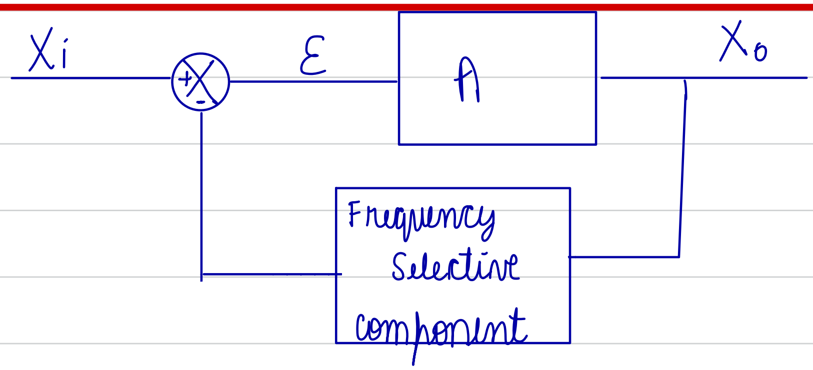
Oscillation criterion

Barkhausen oscillation criterion :

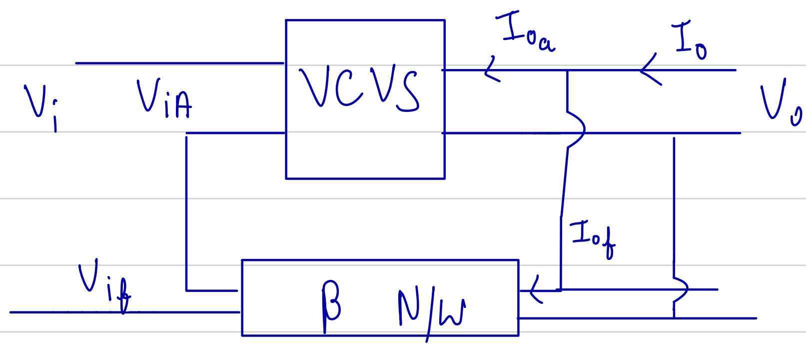
$A\beta = 1$ i.e $|A\beta| = 1$ $\angle A\beta = 0°$ or $360°$

$$L(s) = A(s)\ B(s)$$

$$L(j\omega) = A(j\omega)\ B(j\omega)$$



VCVS with -ve feedback



(h - feedback)

$$[h] = \begin{bmatrix} h_{iA} + h_{if} & h_{rA} + h_{rf} \\ h_{fA} + h_{ff} & h_{oA} + h_{of} \end{bmatrix} \qquad [g] = [h]^{-1}$$

i.e $$g_i = \frac{h_{oA} + h_{of}}{\Delta h} \qquad g_r = \frac{-(h_{rA} + h_{rf})}{\Delta h}$$

$$g_o = \frac{-(h_{iA} + h_{if})}{\Delta h} \qquad g_f = \frac{-(h_{fA} + h_{ff})}{\Delta h}$$

$$\Delta h = (h_{iA} + h_{if}) \cdot (h_{oA} + h_{of}) - (h_{rA} + h_{rf}) \cdot (h_{fA} + h_{ff})$$

$$= \left[1 - \frac{h_r h_f}{h_i h_o}\right] h_i h_o \qquad \text{but} \quad \frac{h_r h_f}{h_i h_o} = A\beta$$

$$= (1 - A\beta)\, h_i h_o$$

If $A\beta$ is -ve it is negative feedback. If $A\beta$ is +ve it is positive feedback.

$$g_i = \frac{h_o}{(1 - A\beta)\, h_o h_i} = \frac{1}{(1 - A\beta)\, h_i} \Rightarrow g_i \rightarrow 0$$

$$g_f = \frac{-h_f}{\frac{(h_i h_o - h_r h_f)}{\cancel{h_i h_o}} \cancel{h_i h_o}} \cong \frac{1}{h_r} = \frac{1}{h_{ra} + h_{rf}} \cong \frac{1}{h_{rf}}$$

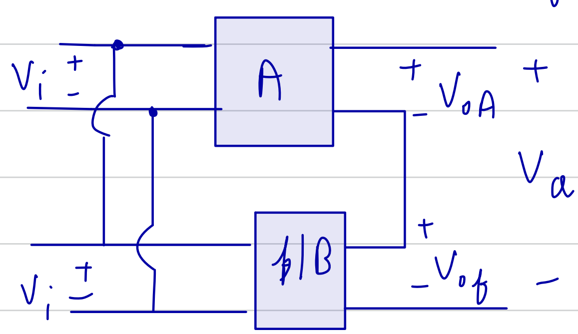
$$g_r = \frac{-h_r}{\left(1 - \frac{h_r h_f}{h_i h_o}\right) h_i h_o} \cong \frac{1}{h_f} \Rightarrow g_r \downarrow$$

$$g_o = \frac{h_i}{(1-A\beta) h_i h_o} = \frac{1}{(1-A\beta) h_o} \Rightarrow g_o \to 0$$

CCCS:

$$\text{ideal} = \begin{bmatrix} 0 & 0 \\ h_f & 0 \end{bmatrix}$$

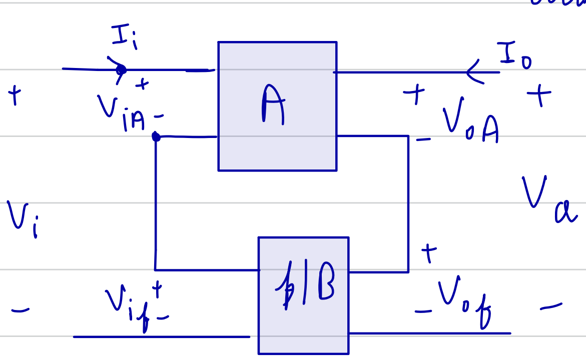
$$I_i = g_i V_i + g_r I_o$$
$$V_o = g_f V_i + g_o I_o$$



Improve h parameters using g parameters

VCCS:

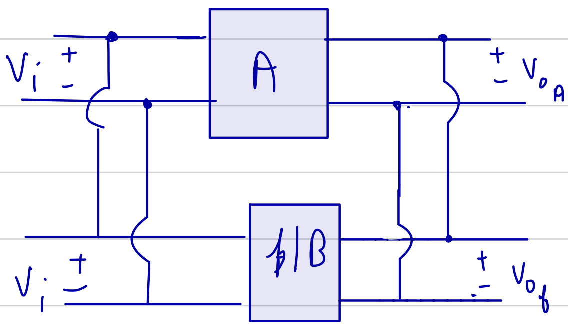
ideal = input impedance = ∞
output impedance = ∞



Improve y parameters using z parameters

$$V_i = Z_i I_i + Z_r I_o$$
$$V_o = Z_f I_1 + Z_o I_o$$

CCVS : Ideal → input impedance = 0
output impedance = 0

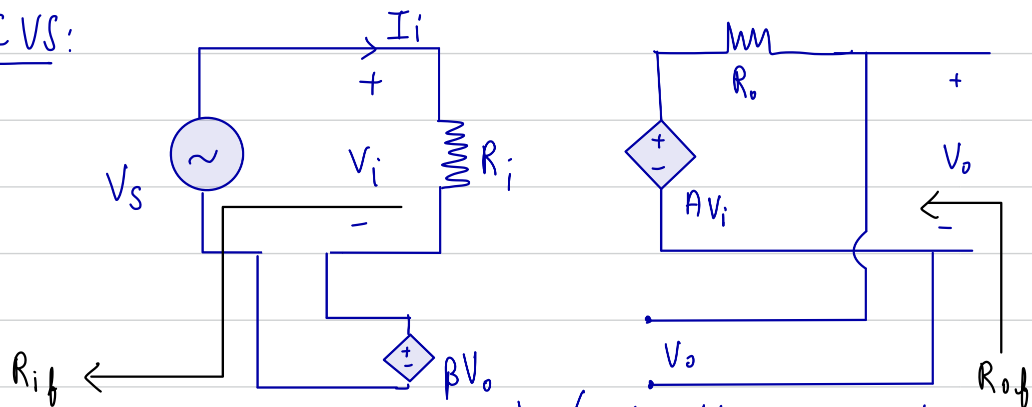


Improve z parameters using y parameters

- In general $A_\beta = \dfrac{P_\pi P_f}{P_i P_o}$ (immittance)

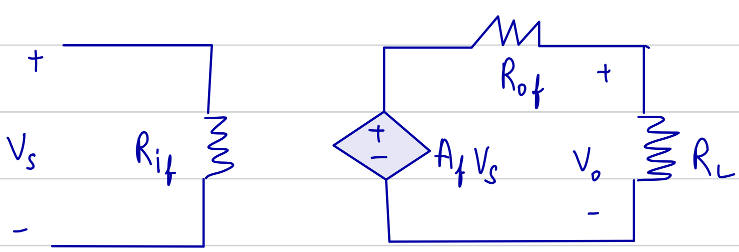
Effect of -ve feedback on gain and I/p, o/p impedances

VCVS:



(series - shunt feedback) / (voltage - voltage feedback)
o/p i/p

Equivalent circuit:



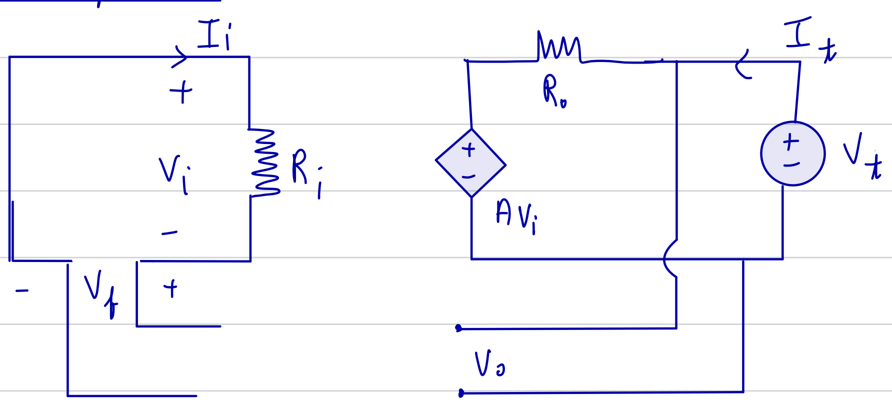
Gain: $A_f \rightarrow \dfrac{A}{1+A\beta}$

$$V_o = AV_i$$
$$= A(V_s - V_f)$$
$$= A(V_s - \beta V_o)$$
$$V_o(1+A\beta) = AV_s$$
$$\frac{V_o}{V_s} = \frac{A}{1+A\beta}$$

<u>Input impedance</u>, $R_{if} = \frac{V_s}{I_i} = \frac{V_s}{V_i/R_i}$

$$R_{if} = \frac{R_i(V_i+V_f)}{V_i} = \frac{R_i(V_i+\beta V_o)}{V_i} = \frac{R_i(V_i + A\beta V_i)}{V_i} = R_i(1+A\beta)$$

<u>output impedance</u>:



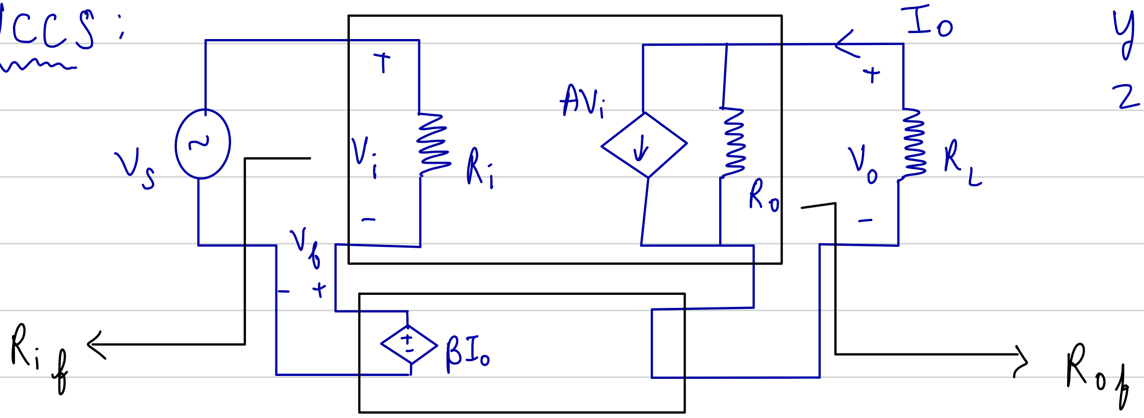
$$R_{of} = \frac{V_t}{I_t} \qquad I_t = \frac{V_t - AV_i}{R_o}$$

$V_s = 0$ $\qquad V_i = -V_f = -\beta V_o = -\beta V_t$

$$I_t = \frac{V_t + A\beta V_t}{R_o}$$

$$\frac{V_t}{I_t} = R_{of} = \frac{R_o}{1+A\beta}$$

VCCS:



y N/w
2 f B

i/p o/p
Series - Series Feedback ($V_s - V_f$ are in series)

current - series feedback
o/p i/p

current - volltage feedback
o/p i/p

Gain: $A_f = \dfrac{I_o}{V_s}$

$I_o = AV_i = A(V_s - V_f)$

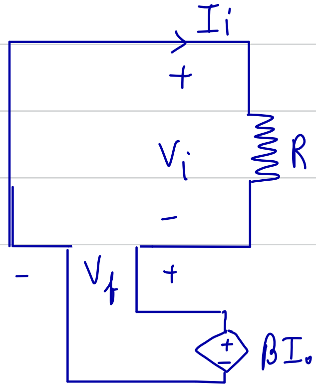
$I_o = A(V_s - \beta I_o) \Rightarrow I_o(1 + A\beta) = V_s A$

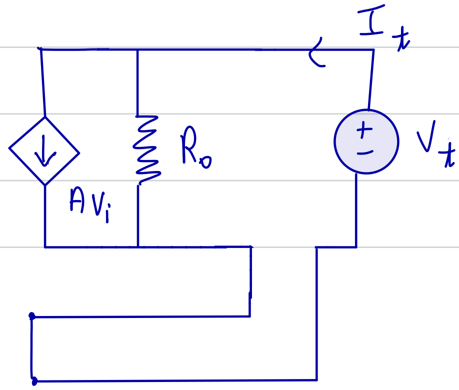
$\therefore A_f = \dfrac{A}{1 + A\beta}$

Input impedance: $R_{if} = \dfrac{V_s}{I_i} = \dfrac{V_i + V_f}{I_i} = \dfrac{V_i + \beta I_o}{I_i} = \dfrac{V_i + A\beta V_i}{I_i}$

$R_{if} = R_i(1 + A\beta)$ //

Output impedance:





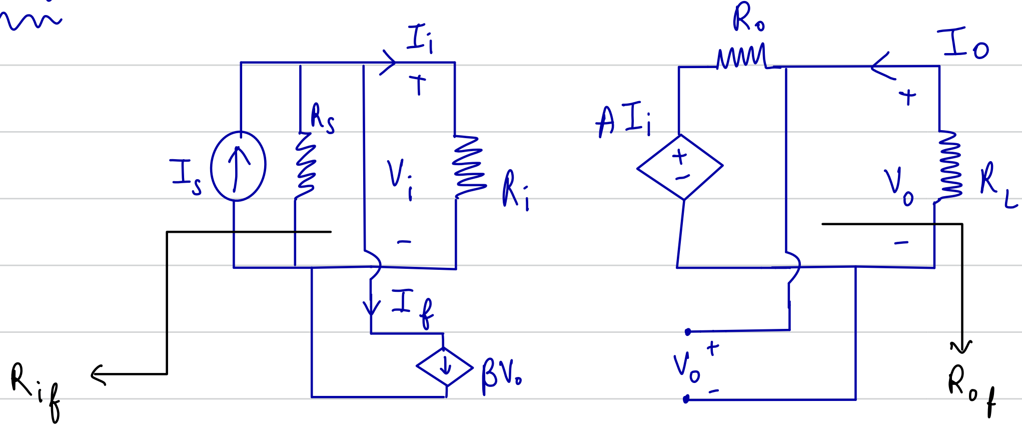
$$R_{of} = \frac{V_t}{I_t} \qquad V_i = -V_f = -\beta I_o = -\beta I_t$$

$$V_t = (I_t - AV_i)R_o$$

$$V_t = (I_t + A\beta I_t)R_o \qquad \therefore \frac{V_t}{I_t} = R_{of} = R_o(1+A\beta)$$

CCVS:

2 N/w
y fb



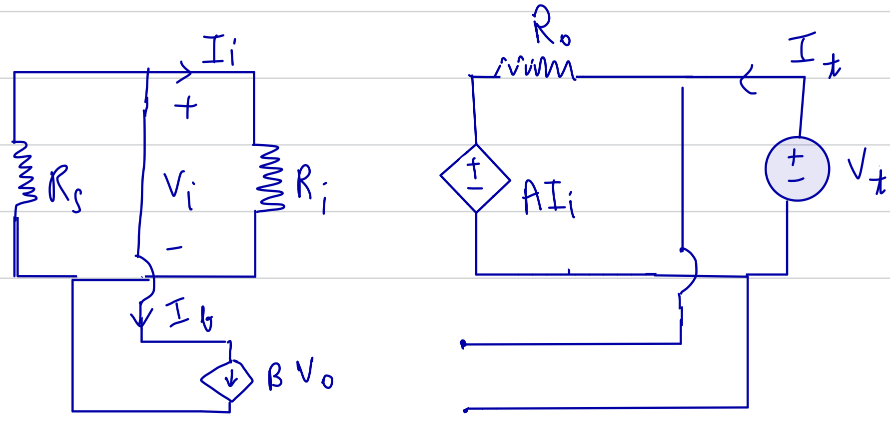
Gain: $A_f = \frac{V_o}{I_s} = \frac{V_o}{I_i + I_f} = \frac{V_o}{I_i + \beta V_o} = \frac{V_o}{I_i + A\beta I_i}$

$$A_f = \frac{A}{1+A\beta}$$

Input impedance: $R_{if} = \frac{V_i}{I_s} = \frac{V_i}{I_f + I_i} = \frac{V_i}{I_i + \beta V_o} = \frac{V_i}{I_i(1+A\beta)}$

$$R_{if} = \frac{R_i}{1+A\beta}$$

Output impedance:



$$R_{of} = \frac{V_t}{I_t}$$

$$I_t = \frac{V_t - AI_i}{R_o}$$

$I_s = 0 \Rightarrow I_i + I_f = 0 \Rightarrow I_i = -I_f \Rightarrow I_i = -\beta V_o = -\beta V_T$

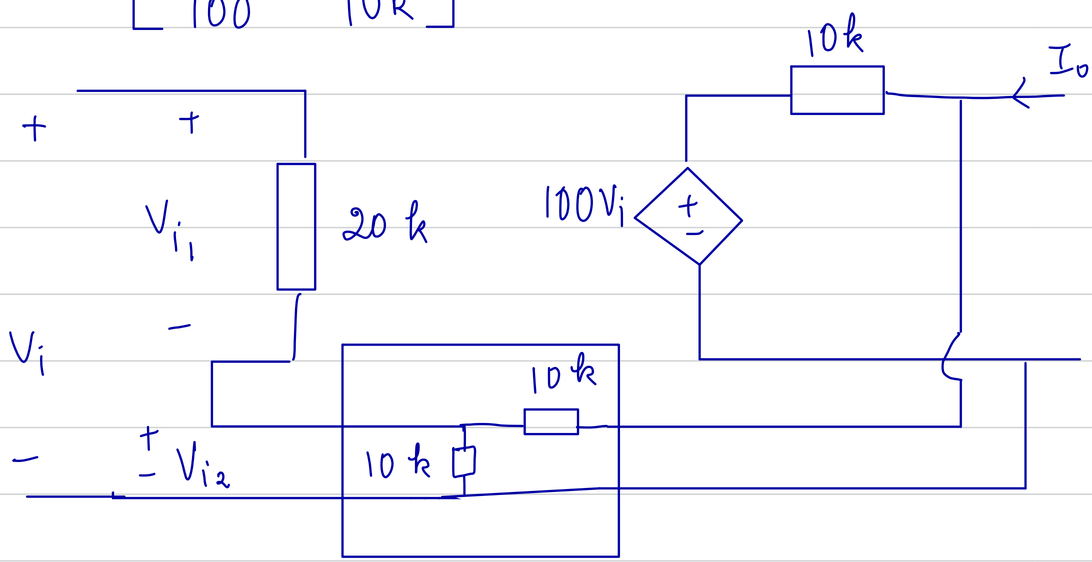
$$I_t = \frac{V_t(1+A\beta)}{R_o}$$

$$\therefore R_{of} = \frac{R_o}{1+A\beta}$$

Q VCVS of a stable gain of 2 should be designed. Design a suitable feedback amplifier if VCVS is represented by following matrix.

$$g = \begin{bmatrix} \frac{1}{20k} & 0 \\ 100 & 10k \end{bmatrix}$$

Ans



Feedback network

$$\frac{A}{1+A\beta} \simeq \frac{1}{\beta} = 2 \qquad \beta = 0.5$$

$$h_i = \left.\frac{V_i}{I_i}\right|_{V_o=0} = 20k + (10k \parallel 10k) = 25\,k\Omega$$

$$h_f = \left.\frac{I_{out}}{I_i}\right|_{V_o=0}$$

$$I_{out} = \frac{100\,V_i}{10k} + \frac{V_i/25k}{2}$$

$$I_i = \frac{V_i}{25k}$$

$$\frac{I_{out}}{I_i} = -250.5 = h_f$$

$$h_r = \left.\frac{V_o}{V_i}\right|_{I_i=0} = \frac{1}{2} = 0.5$$

$$h_o = \left.\frac{I_o}{V_o}\right|_{I_i=0} = \frac{1}{20 \parallel 10} = 1.5 \times 10^{-4}\ S$$

$$[h] = \begin{bmatrix} 25k & 0.5 \\ -250.5 & 1.5 \times 10^{-4} \end{bmatrix}$$

$\frac{h_r h_f}{h_i h_o}$ should be -ve for negative feedback.

$$\frac{h_r h_f}{h_i h_o} = -33.4$$

composite g parameters $[g] = [h]^{-1}$

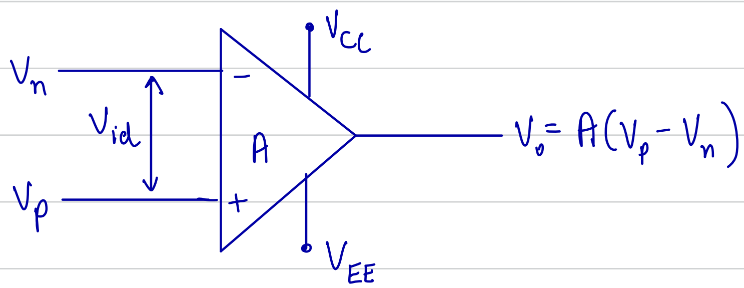
$$g = \begin{bmatrix} \frac{1.5 \times 10^{-4}}{129} & \frac{-0.5}{129} \\ \frac{250.5}{129} & \frac{25k}{129} \end{bmatrix}$$

I/p impedance = $R_{if} = \frac{129}{1.5 \times 10^{-4}} = 860\ k\Omega$

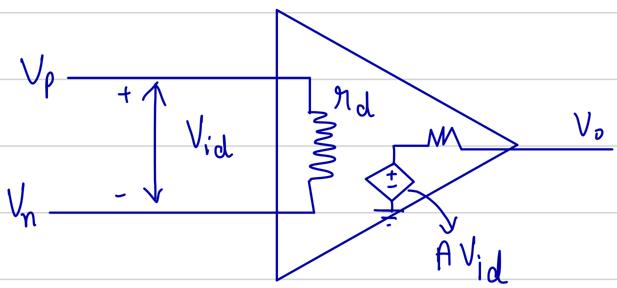
o/p impedance = $R_{of} = \frac{25k}{129} = 193.8\ \Omega$ //

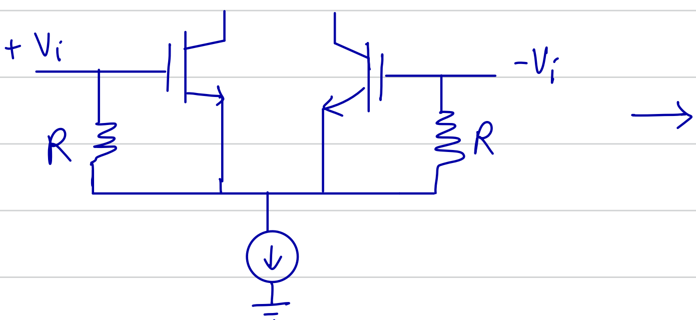
$g_f = 1.94$ //

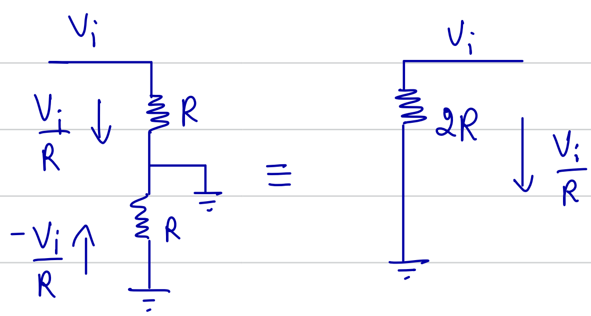
OPERATIONAL AMPLIFIERS



Equivalent circuit





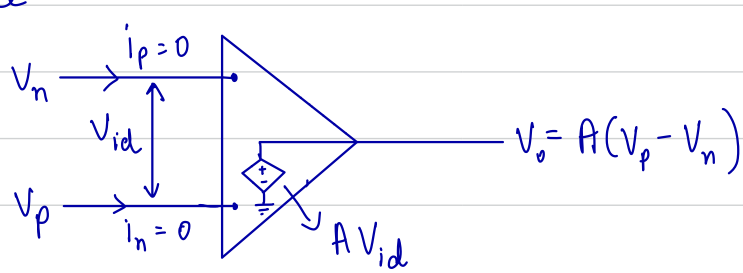


Generally $\quad r_d = R_i = 2M\Omega$

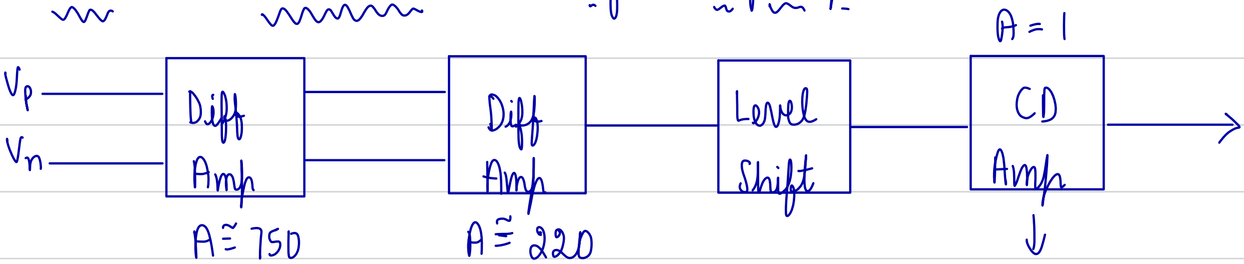
$A = 200 \ V/mV$

$r_o \cong 75\,\Omega \qquad\qquad I_L = \pm 25 \ mA$

Ideal opamp model



Basic architecture of Op-amp



or Emitter follower
Source follower
Push-pull Amp

Requirement of OP-Amp

1> Infinite open loop gain $(10^5 - 10^6)$
2> Infinite input impedance
3> zero output impedance
4> Infinite bandwidth
5> Zero offset voltage
6> Zero bias & offset currents

Input offset current, $I_{io} = |I_{Bp} - I_{Bn}|$

I_B (Bias current) $= \dfrac{I_{Bp} + I_{BN}}{2}$

7> Slew rate must be ∞

slew rate $= \left.\dfrac{dV_{out}}{dt}\right|_{max}$ (For IC741, $SR = 0.5\,V/\mu s$)

8> Common mode rejection ratio (CMRR) must be very high

$$V_{out} = A(V_p - V_n) + A_{cm}V_{cm}$$

Q) Determine the output voltage of an opamp for input voltages, $V_{i1} = 300\,\mu V$, $V_{i2} = 240\,\mu V$. Amplifier has differential gain of 5000. Value of CMRR are: a) 100 b) 10^5

Ans $CMRR = \dfrac{A_{dm}}{A_{cm}}$ $\qquad A_{dm} = 5000$

$V_{i1} - V_{i2} = V_{dm}$ $\qquad V_{cm} = \dfrac{V_{i1} + V_{i2}}{2}$

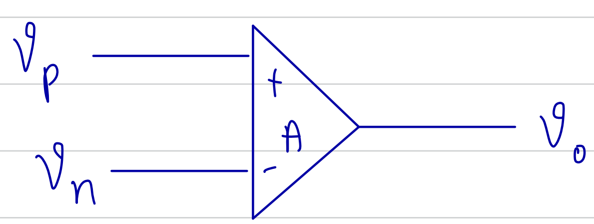
$V_{dm} = 60\,\mu V$ $\qquad V_{cm} = 270\,\mu V$

a) $A_{cm} = \dfrac{A_{dm}}{CMRR} = 50$

$V_o = V_{dm} A_{dm} + V_{cm} A_{cm}$
$= 0.316\ V$

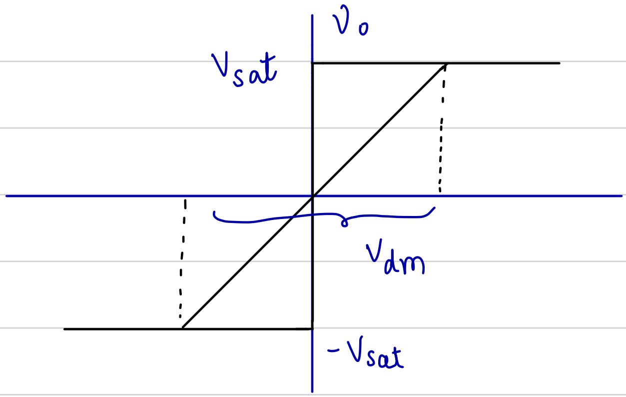
b) $A_{cm} = 0.5$
$V_o = V_{dm} A_{dm} + V_{cm} A_{cm}$
$= 0.3\ V$ //

Transfer characteristic of OPAMP



$V_o = A(v_p - v_n)$
$= A\, v_{dm}$

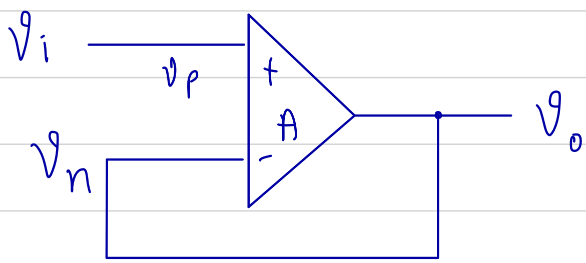
as $A \to \infty$, v_o is finite $\therefore$ $v_p - v_n = 0$ i.e $v_{dm} = 0$



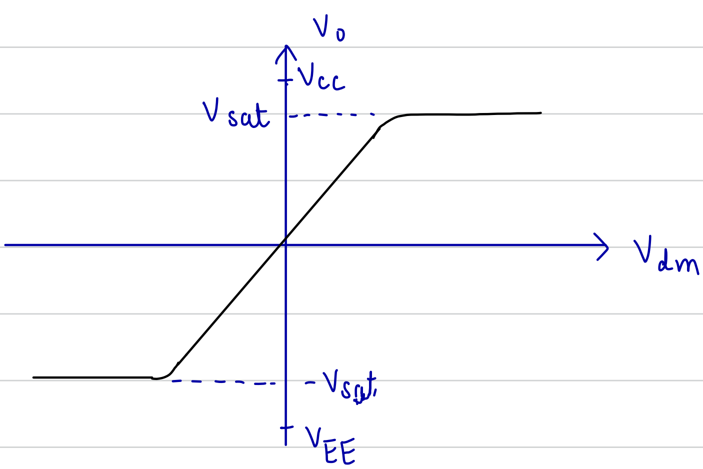
It means when v_o is finite & A is infinite v_{dm} is zero $\Rightarrow$ $v_p = v_n$. $\therefore$ v_p & v_n are virtually short.

v_o can be finite only in negative feedback.

In negative feedback: (voltage follower)

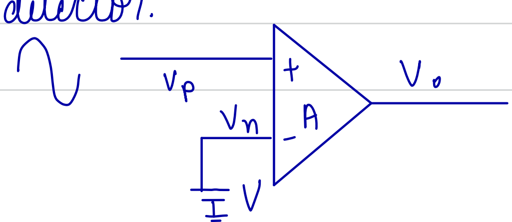


$$v_o = A(v_p - v_n)$$
$$v_o = A(v_i - v_o)$$
$$\frac{v_o}{v_i} = \frac{A}{1+A} \approx 1$$

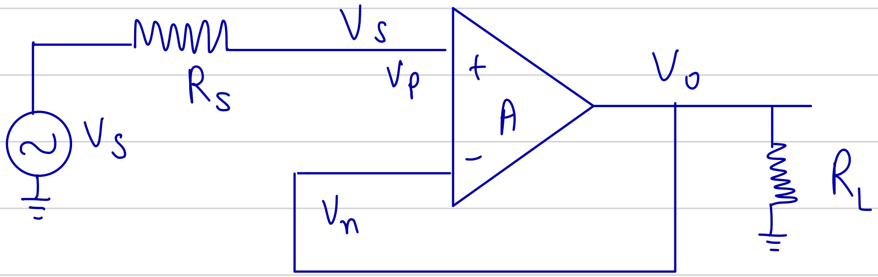


It acts as a buffer

Voltage crossover detector:

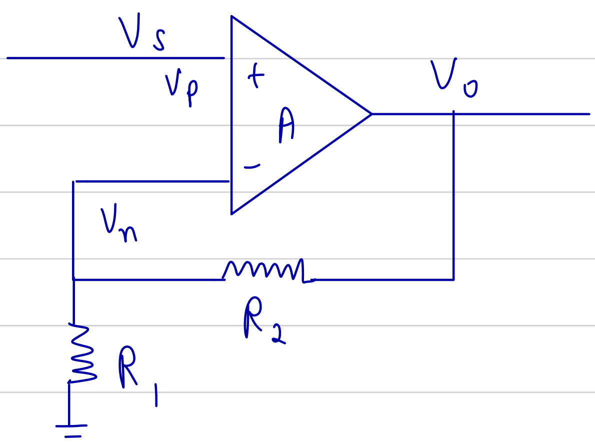


$$V_o = \begin{cases} V_{sat}, & V_p > V \\ -V_{sat}, & V_p < V \end{cases}$$



$V_o \simeq V_s$

- If V_o is +ve the OPAMP is current source with max current = ± 25 mA
- If V_o is -ve the OPAMP absorbs current with max current = ± 25 mA.
- $V_o = \dfrac{R_L}{R_s + R_L} V_s$



$$V_n = \frac{R_1}{R_1 + R_2} V_o$$

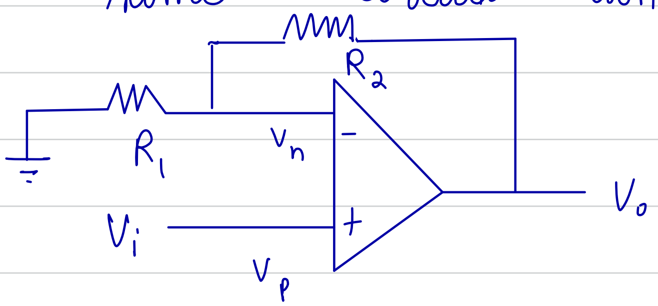
$$V_o = A[V_p - V_n] \Rightarrow V_o = A\left[V_i - \frac{R_1}{R_1+R_2} V_o\right]$$

$$\Rightarrow \frac{V_o}{V_i} = \frac{A}{1 + \frac{AR_1}{R_1+R_2}}$$

If $V_p = V_n \Rightarrow$ $V_i = V_o \dfrac{R_1}{R_1+R_2}$ then $\dfrac{V_o}{V_i} = 1 + \dfrac{R_2}{R_1}$

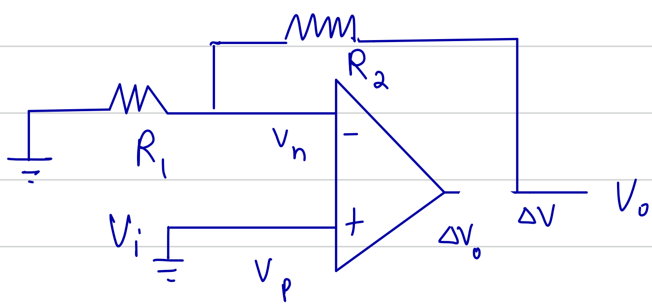
i.e $(A \to \infty)$

The same circuit can be written as:

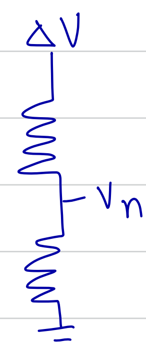


$$v_o = \left(1 + \frac{R_2}{R_1}\right) v_1$$

How to check if we have negative feedback



$\Rightarrow$



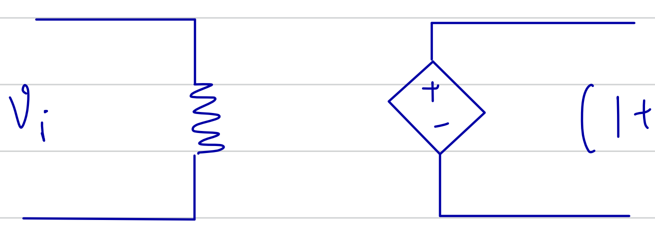
$$V_n = \frac{R_1 \Delta V}{R_1 + R_2}$$

$\Delta V_o = A(0 - V_n)$

So as $\Delta V \uparrow$ $\Delta V_o \downarrow$ in negative feedback

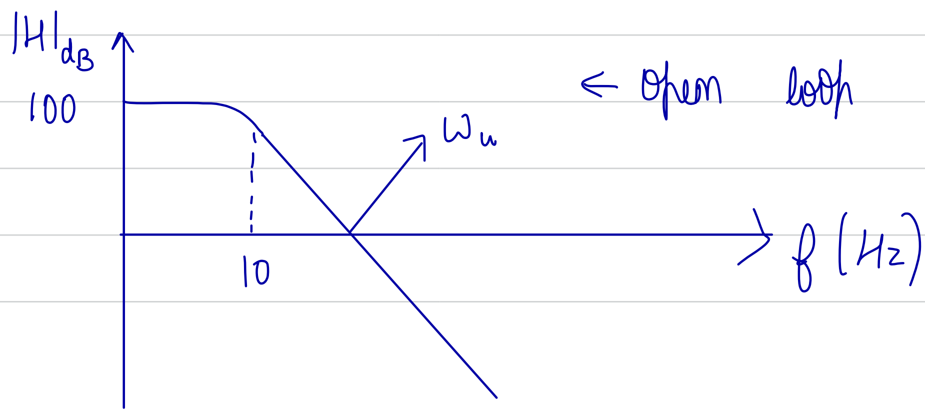
In positive feedback as $\Delta V \uparrow$, ΔV_o also increases.

Equivalent circuit



$$\left(1 + R_2/R_1\right) v_i = v_o$$

Frequency response :



← Open loop

$20 \log(10^5) = 100$ dB

$$\frac{A_0}{1+\frac{j\omega}{\omega_0}} = H$$

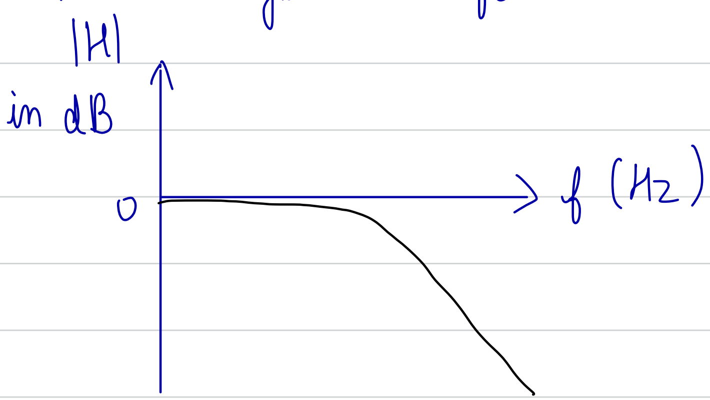
$$|H| = 1 \quad \Rightarrow \quad \frac{A_0^2}{1+\left(\frac{\omega}{\omega_0}\right)^2} = 1$$

$$A_0^2 - 1 = \left(\frac{\omega}{\omega_0}\right)^2 \quad \Rightarrow \quad \omega = \omega_0\sqrt{A_0^2 - 1}$$

$$\omega \cong A\omega_0$$

ω_u (unit gain frequency) $\cong A\omega_0$ //

For negative feedback:



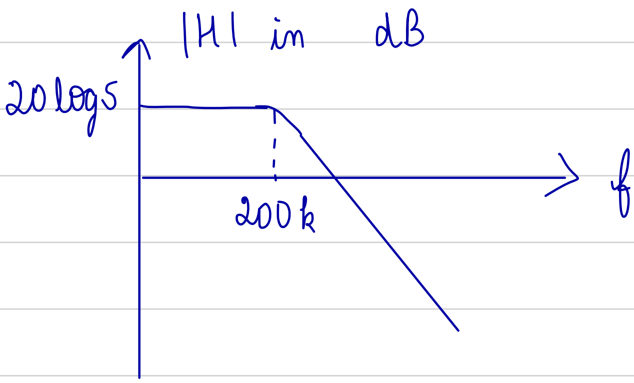
$$A_f = \frac{\frac{A_0}{1+\frac{s}{\omega_0}}}{1+\frac{A_0\beta}{1+\frac{s}{\omega_0}}} = \frac{A_0}{1+A_0\beta+\frac{s}{\omega_0}} = \frac{\frac{A_0}{1+A_0\beta}}{1+\frac{s}{\omega_0(1+A\beta)}}$$

$$\therefore \; A_0 \rightarrow \frac{A_0}{1+A_0\beta} \; ; \qquad \omega_0 \rightarrow \omega_0(1+A\beta)$$

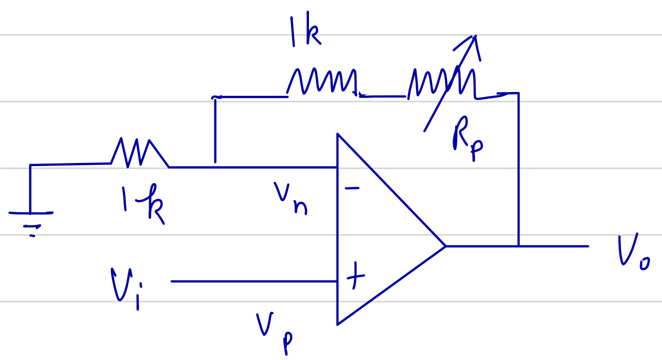
Q> What is bandwidth / Draw frequency response for an non-inverting amplifier of gain 5 & bandwidth of 10 Hz.

Ans Gain = 5 $= A'$ $\beta = \frac{1}{5}$

$\omega' A' = \omega_o A_o$ $\omega' = \frac{10^6}{5} = 200 \text{ kHz}$ //



Expt 6



$A = \left(1 + \frac{(1+R_p)}{1}\right) = 2 + R_p$

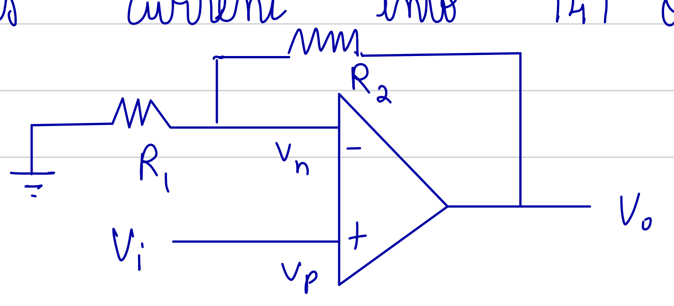
Gain $\Rightarrow$ 2 to 10 $\Rightarrow$ $2 < A < 10$

when gain $= 2$ $R_p = 0$

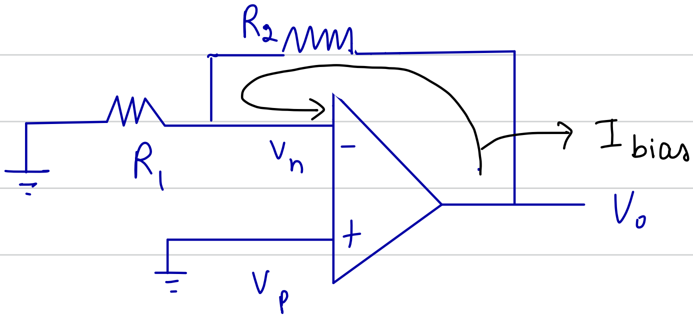
$= 6$ $R_p = 4 \text{ k}\Omega$

$= 10$ $R_p = 8 \text{ k}\Omega$

Bias current into 741 opamp input $\simeq 80 \text{ nA}$



$V_o = \left(1 + \frac{R_2}{R_1}\right) v_i + I_{Bias} R_2$



To observe effect of I_{bias} we connect V_p to ground.

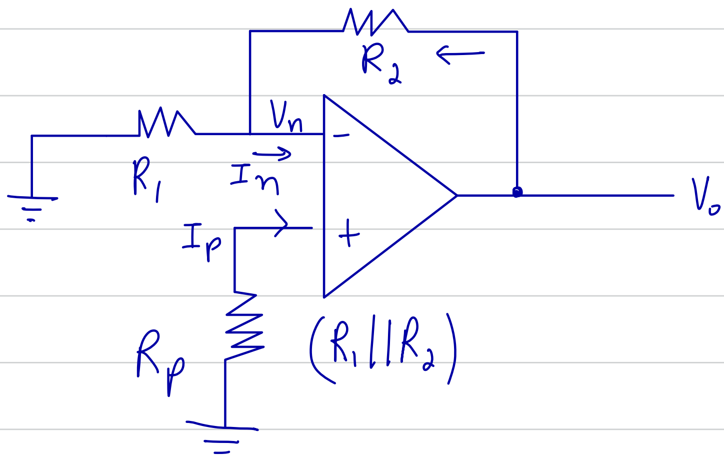
$\Rightarrow$ V_n becomes virtually ground $\Rightarrow$ $V_n = 0$

Observe DC shift for $R_2 = 10\,k\Omega$, $1\,M\Omega$, $10\,M\Omega$

$$\therefore I_{bias} = \frac{\text{DC shift}}{10\,M\Omega} \cong 80\,nA$$

$$= \frac{\text{DC shift} - 2 \times (\text{DC of } V_{in})}{10\,M\Omega} \cong 80\,nA$$

To remove the effect of I_{bias}:



$$\frac{0 - v_p}{R_p} = I_p$$

$$v_p = -I_p R_p$$

$$\frac{0 - v_n}{R_1} + \frac{E_o - v_n}{R_2} = I_n$$

where E_o is the voltage due to bias current

$$E_o = I_n R_2 + \frac{v_n R_2}{R_1} + v_n$$

$$E_o = I_n R_2 - \frac{I_p R_p R_2}{R_1} - I_p R_p$$

$$= I_n R_2 + \left(1 + \frac{R_2}{R_1}\right)(-I_p R_p)$$

$$= \left(1 + \frac{R_2}{R_1}\right)\left(-I_p R_p + \frac{I_n R_2 R_1}{R_1 + R_2}\right)$$

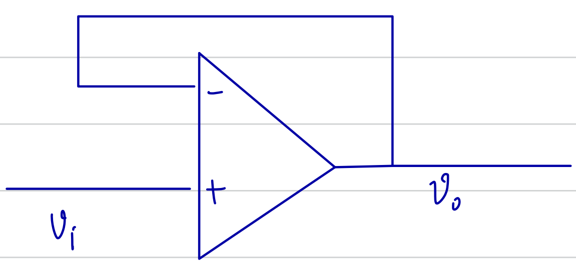
$$= \left(1 + \frac{R_2}{R_1}\right)\left(-I_p R_p + I_n (R_1 || R_2)\right)$$

$\Rightarrow R_p = R_1 || R_2$

Slew rate limitation

$$\text{Slew rate} = \left.\frac{dV_o}{dt}\right|_{max}$$

For voltage buffer



$v_i = V_m \sin\omega t$ $\quad$ $v_o = V_m \sin\omega t$

$$\frac{dv_o}{dt} = V_m \omega \cos\omega t \qquad \left.\frac{dv_o}{dt}\right|_{max} = V_m \omega$$

$$\text{Slew rate} = 2\pi f V_m \ V/s = \frac{2\pi f V_m}{10^6} \ V/\mu s$$

Q> An amplifier is to be designed for a gain of 5 using opamp. Input has an amplitude of 200 mV and bandwidth of 20 kHz. Find slew rate required for the opamp

Ans $SR = \frac{2\pi f \cdot V_m}{10^6}$ $V_m = 5 \times 200 \ mV = 1V$

$\Rightarrow SR = \frac{2\pi \times 20 \ kHz \times 1}{10^6} = 0.1256 \ V/\mu s$

Q) A voltage buffer is to be designed using opamp. It is expected to operate for frequencies in the range 10 kHz to 100 KHz. The swing in voltage is in the range of 1V pk-pk to 20 V pk-pk. The error between i/p & o/p in amplification / buffering has to be less than 0.01%. What are the op-amp specifications required to get desired performance (Find A_o, ω_o, ω_u, SR)

Ans $\frac{|V_o - v_{in}|}{v_{in}} \leqslant \frac{0.01}{100}$

$\frac{|V_{in} - V_o|}{V_{in}} = 1 - \frac{|V_o|}{V_{in}} = 1 - \frac{A_o}{1+A_o} = \frac{1}{1+A_o} \leqslant \frac{0.01}{100}$

$100 \leqslant 0.01 + 0.01 A_o$

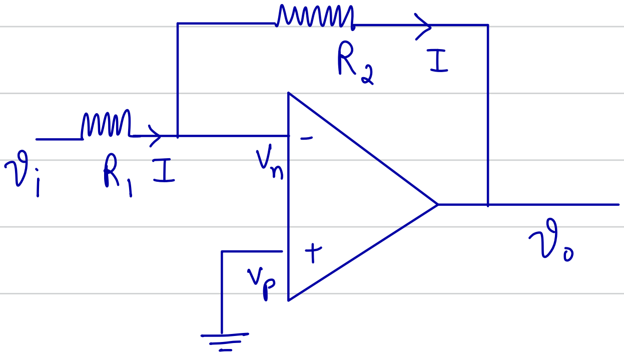
$\Rightarrow A_o \geqslant 10^4 - 1$

$\omega_u > (2\pi \times 100k) \ rad/sec$

$\omega_o > \frac{\omega_u}{A_o}$ $\Rightarrow \omega_o > \frac{2\pi \ 100k}{10^4 - 1} = 62.8 \ rad/sec$

$SR > \frac{2\pi f V_m}{10^6} = \frac{2\pi \times 100k \times 10}{10^6} = 2\pi \ V/\mu s$

Inverting amplifier:

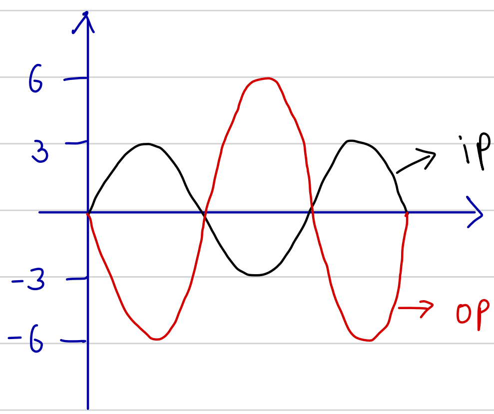


$$\frac{v_1 - V_n}{R_1} = \frac{V_n - v_o}{R_2} \qquad V_n \cong 0$$

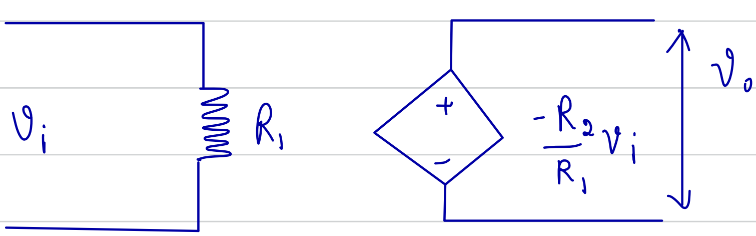
$$\Rightarrow \frac{v_i}{R_1} = \frac{-v_o}{R_2} \qquad \Rightarrow \frac{v_o}{v_i} = \frac{-R_2}{R_1}$$

If $v_i = 6$ V p-p and $\frac{R_2}{R_1} = 2$

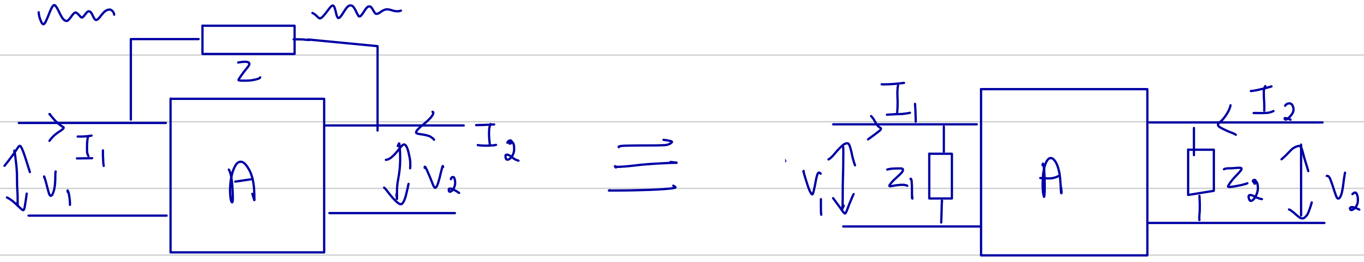
then :



Equivalent circuit



Miller's theorem



i) $\frac{V_1 - V_2}{Z} = I_1 = \frac{V_1}{Z_1}$

$$\frac{\cancel{V_1}}{Z}\left[1 - \frac{V_2}{V_1}\right] = \frac{\cancel{V_1}}{Z_1}$$

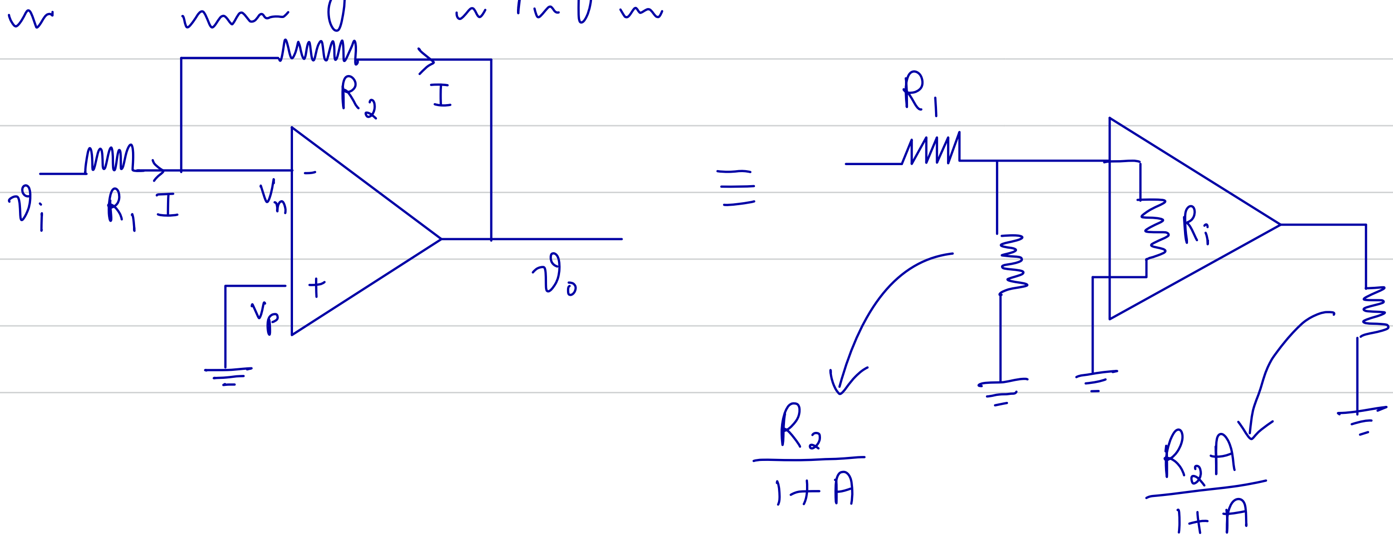
$$\frac{1}{Z}[1 - A] = \frac{1}{Z_1} \Rightarrow \boxed{Z_1 = \frac{Z}{1-A}}$$

ii) $\frac{V_2 - V_1}{Z} = I_2 = \frac{V_2}{Z_2}$

$$\frac{\cancel{V_2}}{Z}\left[1 - \frac{V_1}{V_2}\right] = \frac{\cancel{V_2}}{Z_2}$$

$$\frac{1}{Z}\left[1 - \frac{1}{A}\right] = \frac{1}{Z_2} \Rightarrow \boxed{Z_2 = \frac{AZ}{A-1}}$$

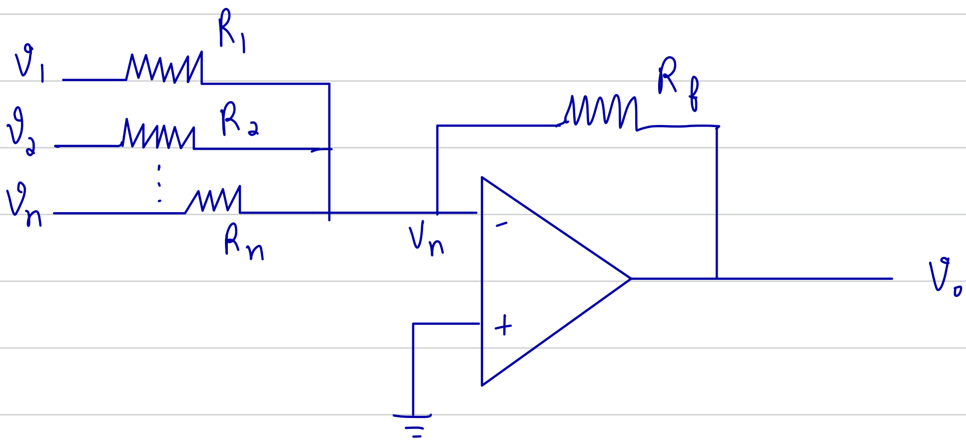
In inverting amplifiers:



$$R_{if} = R_1 + \left(\frac{R_2}{1+A} \parallel R_i\right)$$

$$R_{if} \cong R_1$$

Summing Amplifiers (Inverting adder)



$$\frac{v_1}{R_1} + \frac{v_2}{R_2} + \dots + \frac{v_n}{R_n} = -\frac{v_o}{R_f}$$

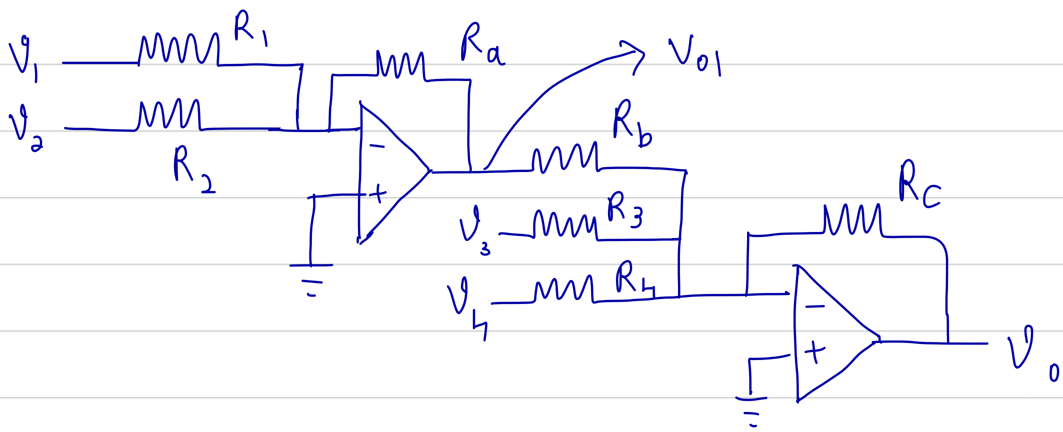
$$v_o = -R_f\left[\frac{v_1}{R_1} + \frac{v_2}{R_2} + \frac{v_3}{R_3} + \dots + \frac{v_n}{R_n}\right]$$

If $R_1 = R_2 = \dots = R_n = R$

$$v_o = \frac{-R_f}{R}\left[v_1 + v_2 + \dots + v_n\right]$$

To make it positive pass v_o to another inverting amplifier.

Ex: Find v_o



Ans $v_{01} = -R_a\left[\dfrac{V_1}{R_1} + \dfrac{V_2}{R_2}\right]$

$$v_0 = -R_c\left[-\frac{R_a}{R_b}\left[\frac{V_1}{R_1} + \frac{V_2}{R_2}\right] + \frac{V_3}{R_3} + \frac{V_4}{R_4}\right]$$

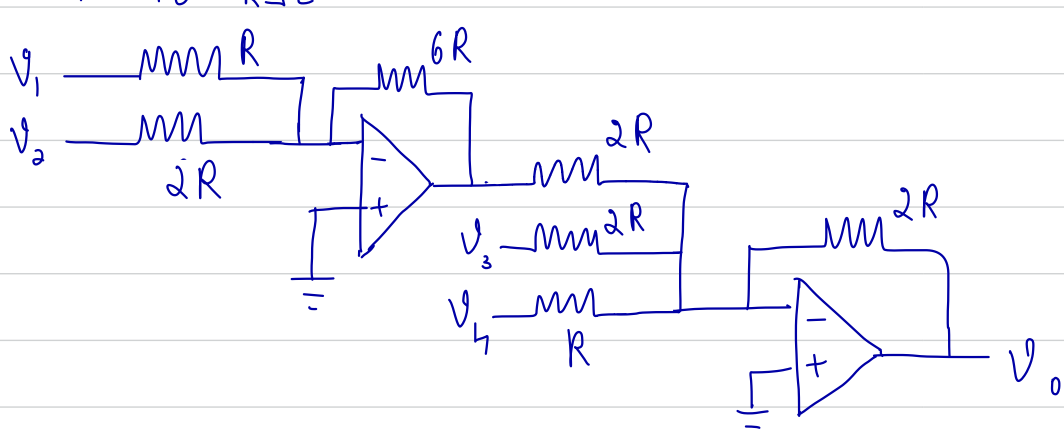
$$v_0 = V_1\left(\frac{R_a}{R_1}\right)\left(\frac{R_c}{R_b}\right) + V_2\left(\frac{R_a}{R_2}\right)\left(\frac{R_c}{R_b}\right) - V_3\left(\frac{R_c}{R_3}\right) - V_4\left(\frac{R_c}{R_4}\right)$$

Exp Design an opamp based circuit to implement the following function:

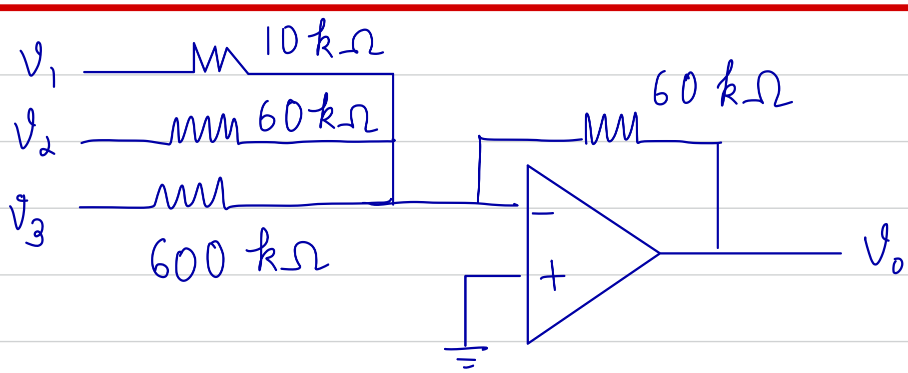
$$v_0 = 6v_1 + 3v_2 - v_3 - 2v_4.$$

Assume that only $10\ k\Omega$ resistors are available.

Ans Let $R = 10\ k\Omega$



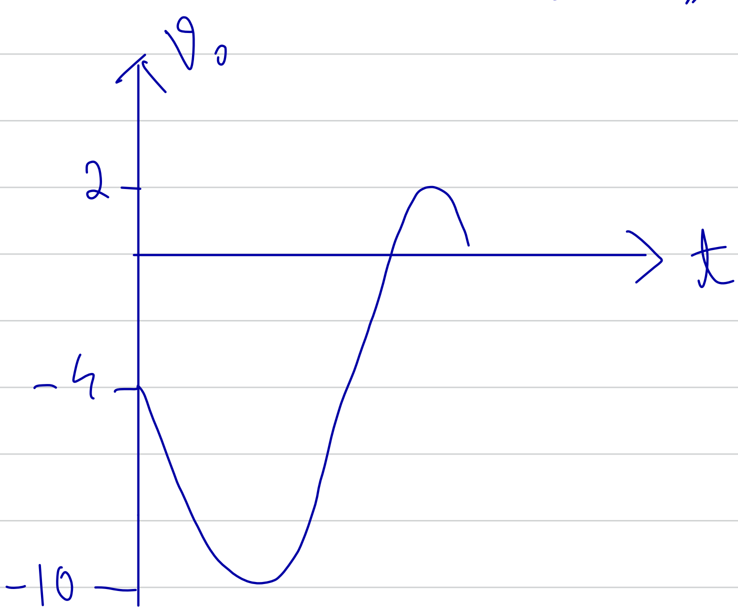
3>



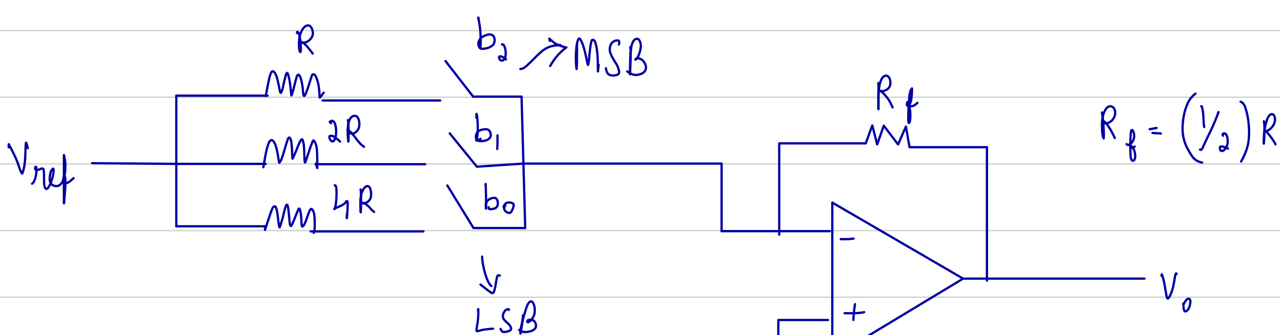
$v_1 = \sin\omega t$ $\quad V_2 = 5V \quad V_3 = -10V$. Find v_0.

Ans $V_{01} = \frac{-60}{10}\sin\omega t = -6\sin\omega t$

$V_{02} = -5V \qquad V_{03} = 1V$



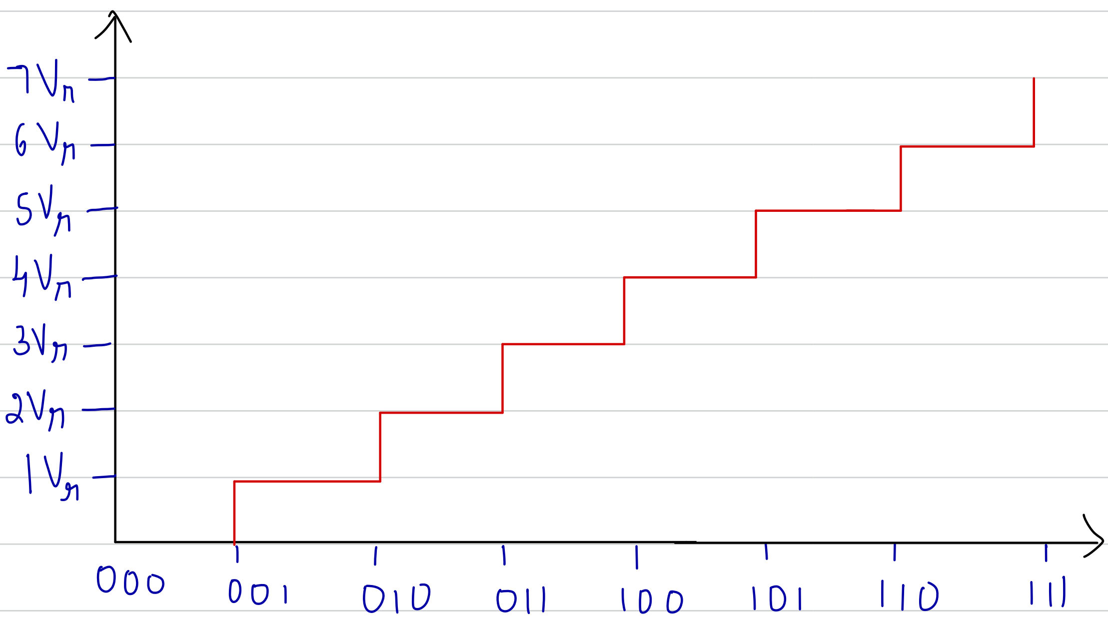
Digital to analog convertor (DAC)



→ Binary weighted DAC

$$V_0 = -R_f V_{ref}\left(\frac{b_2}{R} + \frac{b_1}{2R} + \frac{b_0}{4R}\right) = -V_{ref}\frac{R_f}{R}\left(\frac{b_2}{1} + \frac{b_1}{2} + \frac{b_0}{4}\right)$$

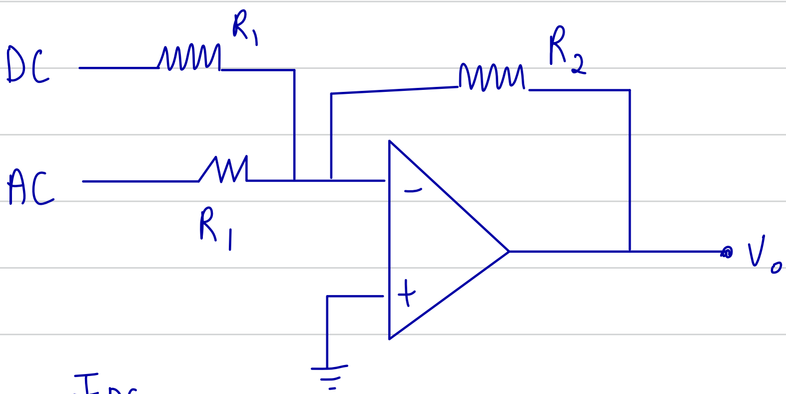
b_2	b_1	b_0	$V_0(V)$
0	0	0	0
0	0	1	$(1/8)\cdot V_{ref}$
0	1	0	$(2/8)\cdot V_{ref}$
0	1	1	$(3/8)\cdot V_{ref}$
1	0	0	$(4/8)\cdot V_{ref}$
1	0	1	$(5/8)\cdot V_{ref}$
1	1	0	$(6/8)\cdot V_{ref}$
1	1	1	$(7/8)\cdot V_{ref}$



where $V_r = \frac{V_{ref}}{8}$

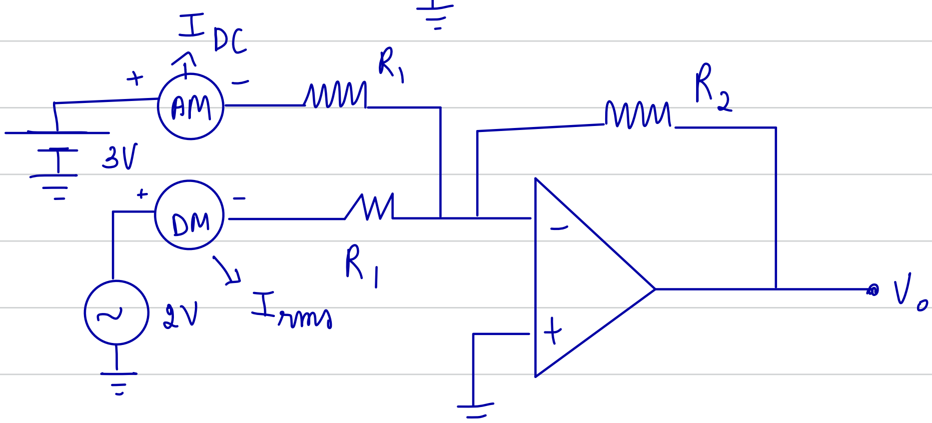
Max voltage $= V_{ref}\left(1 - \frac{1}{2^n}\right)$

Experiment 7 – Inverting adder



$R_1 = R_2 = 10\ k\Omega$

i>



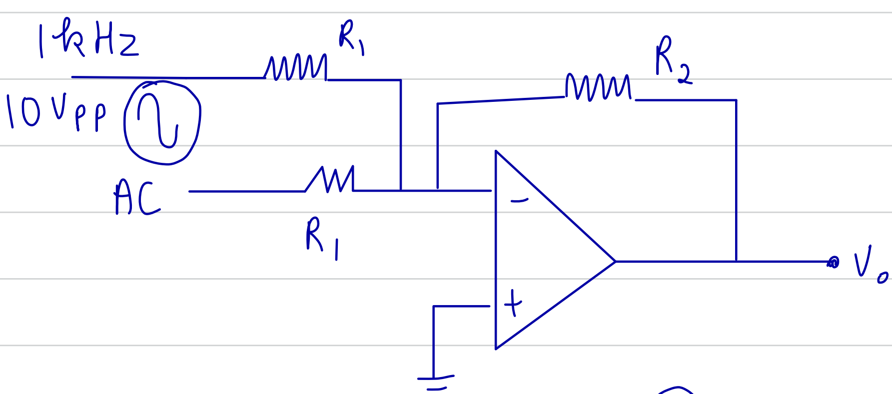
AM - Analog multimeter

DM - Digital multimeter

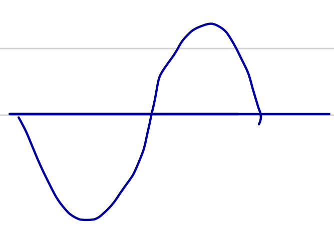
$$V_{rms} = \frac{2}{\sqrt{2}} = \sqrt{2}\ V$$

Impedance seen by source = $\dfrac{\text{Voltage supplied to source}}{\text{current drawn from source}}$ (measured by multimeter)

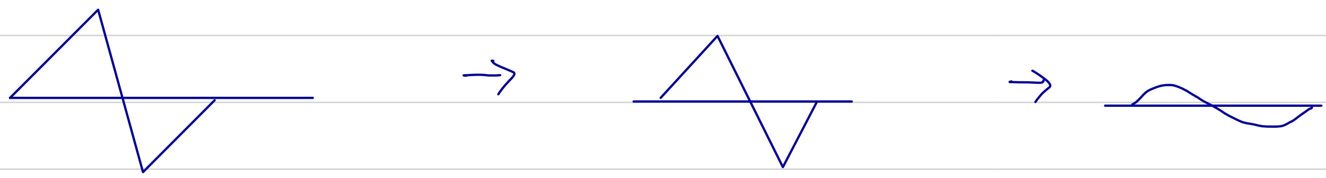
ii>



graph :



as frequency increases

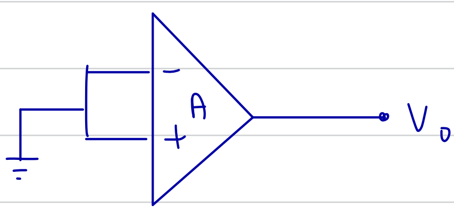


Slew rate limitation

$0.5\ V/\mu s = \left(\frac{2\pi f V_o}{10^6}\right)$ find f

Both f & V_o contribute to o/p of distortionless system

iii>

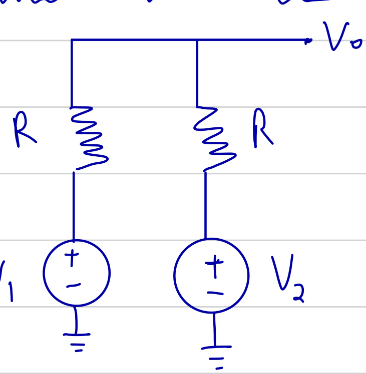


Ideally $V_o = 0$

i.e $A(V_+ - V_-) = 0$

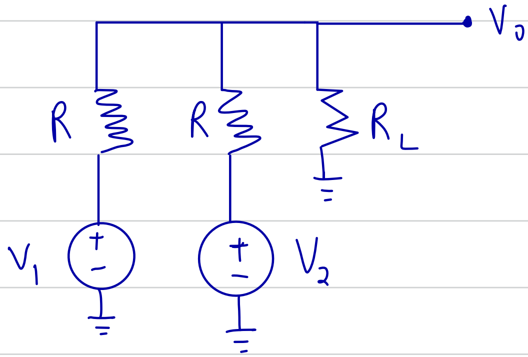
But $V_o = \pm V_{sat}$ due to small differences in V_+ & V_-

Non inverting adder



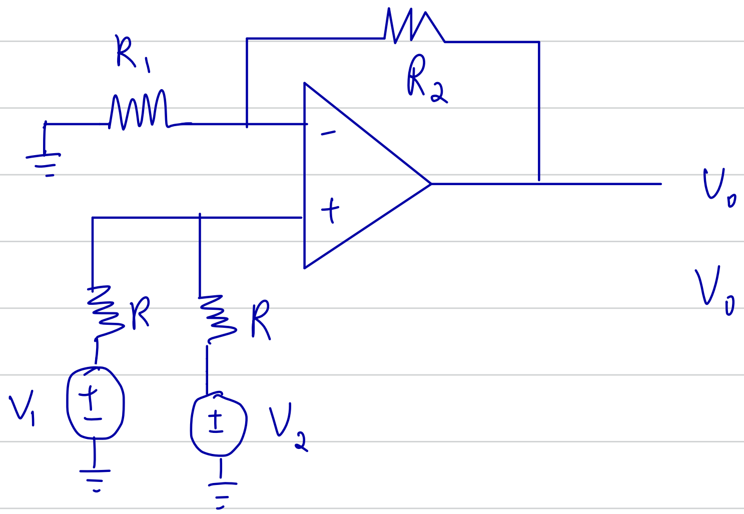
$$V_o = V_1 \frac{R}{R+R} + V_2 \frac{R}{R+R}$$

$$= \frac{V_1 + V_2}{2}$$



$$V_o = \left(\frac{R \| R_L}{R + (R \| R_L)}\right) V_1 + \left(\frac{R \| R_L}{R + (R \| R_L)}\right) V_2$$

Using opamp:



$$V_o = \left(1 + \frac{R_2}{R_1}\right) V_p$$

$$= \left(1 + \frac{R_1}{R_1}\right)\left(\frac{V_1 + V_2}{2}\right)$$

Q) For 741 opamp, calculate typical output voltage that would result from a 80 mV common input voltage

Ans $A_d = 2 \times 10^5$ $\quad SR = 0.5\ V/\mu s$ $\quad CMRR = 90\ dB$

$V_{icm} = 80\,mV$ $\quad V_{ocm} = ?$

$$CMRR = \frac{A_d}{A_{cm}} = \frac{A_d}{\left(\frac{V_{ocm}}{V_{icm}}\right)} = \frac{2 \times 10^5 \times 80 \times 10^{-3}}{V_{ocm}}$$

$$\therefore 90 = 20 \log\left(\frac{16D}{V_o}\right) \qquad V_o = \frac{16 \times 10^3}{10^{4.5}} = 0.5059$$

Q> An opamp has $A = 10^5$ V/V $R_i \cong \infty$ $R_o \cong \infty$
Find the following:

a> V_o if $V_1 = 870\mu V$, $V_2 = 975.2\mu V$

b> V_1 if $V_2 = V_o = 6V$

c> V_2 if $V_1 = 0V$, $V_o = -7V$

d> V_2 if $V_1 = -V_o = 2V$

Ans $A(V_1 - V_2) = V_o$

a> $V_o = 10^5(870 - 975.2) \times 10^{-6} = -10.52\,V$

b> $6 = 10^5(V_1 - 6)$

$\Rightarrow V_1 - 6 = 6 \times 10^{-5}$

$\therefore V_1 = 6 + 6\times10^{-5} = 6.00006\,V$

c> $-7 = 10^5(0 - V_2)$

$V_2 = 7\times10^{-5} = 70\,\mu V$

d> $-2 = 10^5(2 - V_2)$

$V_2 - 2 = 2\times10^{-5}$

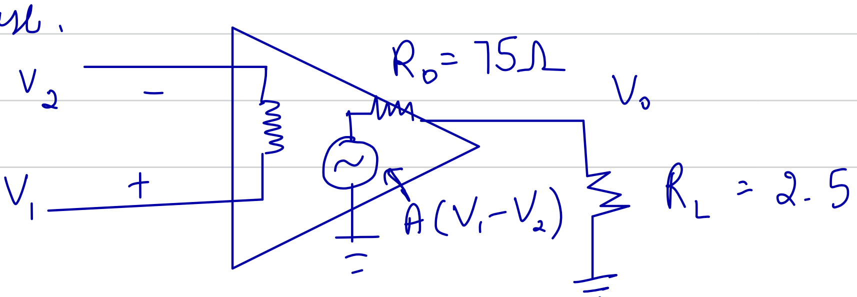
$V_2 = 2 + 2\times10^{-5} = 2.00002\,V$

Q> An opamp with open loop gain 2×10^5, R_i of $1\,M\Omega$, and R_o of $75\,\Omega$; drives load resistance of $2.5\,k\Omega$. Write the equivalent circuit indicating all these values. Also find the current through & voltage across R_i & R_o.

If: i> $V_1 = 1V$ $V_o = 5V$

ii> $V_1 = 5mV$ $V_o = 6V$ also find V_2 in each case.

Ans



$$i_{R_o} = \frac{V_o}{R_L} = \frac{5}{2.5k} = 2\,mA$$

$$v_{R_o} = 75 \times 2m = 150\,mV$$

$$A(v_1 - v_2) = 5 + 150\,mV = 5.15\,V$$

$$v_{R_i} = v_1 - v_2 = \frac{5.15}{2 \times 10^5} = 25.75\,\mu V$$

$$i_{R_i} = \frac{v_{R_i}}{R_i} = 25.75\,\mu V$$

$$v_2 = v_1 - v_{R_i} = 1 - 25.75\,\mu V = 0.9997\,V$$

Eg2: $R_i = 1\,k\Omega$ $\quad R_f = 9\,k\Omega$ $\quad v_{in} = 1V$ $\quad v_o = ?$ if

a) $A = 10^2$ $\quad$ b) 10^4 $\quad$ c) $A = 10^6$

Ans $1 + \frac{R_f}{R_i} = 1 + \frac{9}{1} = 10$

$$\beta = \frac{1}{10}$$

$$v_{oa} = \left(\frac{A}{1+A\beta}\right) \times v_{in} = \frac{100}{1+10} = 9.0909\,V$$

$$v_{oB} = \left(\frac{A}{1+A\beta}\right) \times v_{in} = \frac{10^4}{1+10^3} = 9.9900\,V$$

$$v_{oc} = \left(\frac{A}{1+A\beta}\right) \times v_{in} = \frac{10^6}{1+10^6} = 9.9999\,V$$

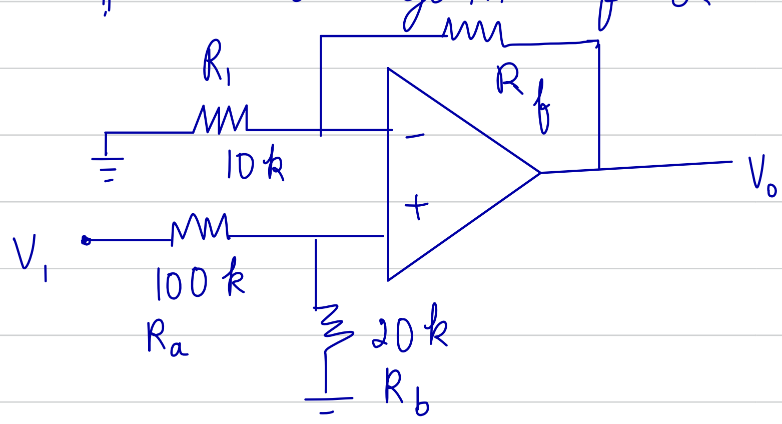
→ Better the open loop gain better is the performance.

Q A non inverting amplifier is fed by voltage source of $V_s = 8V$. Divider is implemented by R_a of $100\,\Omega$ & R_b of $20\,k\Omega$. The voltage across R_b is fed to amplifier. Amplifier circuit is designed to offer gain of 6 to input applied to non inverting pin. Let $R_1 = 10\,k\Omega$.

a> Sketch the amplifier circuit & find amplified output voltage

b> Repeat the above for an inverting amplifier for a gain of 6.

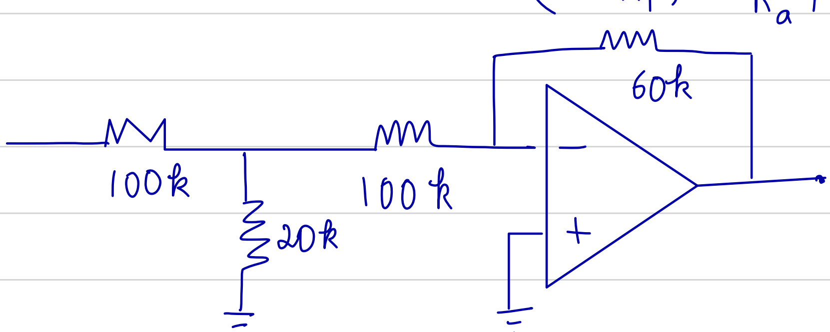
Ans a>



$R_f = 50\,k\Omega$

$$v_o = \left(1 + \frac{R_f}{R_1}\right)\frac{R_b\, v_1}{R_a + R_b} = 8V$$

b>



$$v_x = \frac{(20 \| 10)\, v_i}{(20 \| 10) + 100} = 0.5\,V$$

$$V_0 = \left(-\frac{60}{10}\right) 0.5 = -3V$$

Q) The gain of an opamp has a magnitude of 90 dB at 10 Hz. It exhibits a phase angle of -50° at 300 Hz. Estimate A, f_0, UGB

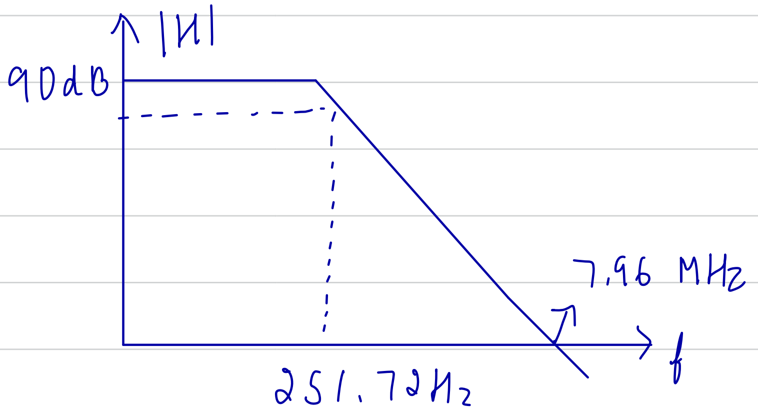
Ans
$$A_{OL}(f) = \frac{A_0}{1 + j\frac{f}{f_0}}$$

$$-\tan^{-1}\frac{f}{f_0} = -50^\circ \quad \text{at} \quad f = 300\,Hz$$

$$\tan 50^\circ = \frac{f}{f_0} \Rightarrow f_0 = \frac{300}{\tan 50^\circ} = 251.72\,Hz$$

$f_0 \ggg 10\,Hz$

$$90\,dB = 20 \log A_0$$
$$A_0 = 31.622\,kV/V$$
$$UGB = A \times f_0$$
$$= 7.96\,MHz$$



Q) A 741 opamp with ±15 V power supply is used to build non-inverting amplifier with closed loop gain of 8. Find:

a) Max frequency before the output distorts in the AC amplitude is 1V.

b) Max AC input amplitude before the input distorts if the input frequency is 12 kHz.

c) Useful frequency range of operation if max AC input amplitude is 20 mV.

d) Input amplitude range if frequency is 5 kHz

Ans Slew rate = 0.5 V/μs

$$0.5\ V/\mu s = \frac{2\pi f V_o}{10^6}$$

$$V_{o\,max} = 8\ V_{i\,max} = 8\ V$$

a) $$f_{max} = \frac{SR}{2\pi V_{o\,max}} = \frac{0.5 \times 10^6}{2\pi \times 8} = 9.952\ kHz$$

b) $$V_{max} = \frac{SR}{2\pi f} = \frac{0.5 \times 10^6}{2\pi \times 12 \times 10^3} = 6.63\ V$$

$$V_{i\,max} = \frac{V_{o\,max}}{A_f} = \frac{6.63}{8} = 0.829\ V$$

c) $V_{i\,max} = 20\ mV$

$$V_{o\,max} = 8 \times 20\ mV = 160\ mV$$

$$f_{max} = \frac{SR}{2\pi V_{o\,max}} = \frac{0.5 \times 10^6}{2\pi \times 160 \times 10^{-3}} = 497.3\ kHz$$

$$UGB = \frac{10^6}{8} = 125\ kHz$$

$\because$ UGB < f_{max}

$\therefore$ Useful frequency range = 125 kHz

d) $$V_{o\,max} = \frac{SR}{2\pi f} = \frac{0.5 \times 10^6}{2\pi \times 5\ kHz} = 15.923V > 15$$

$V_{sat} = \pm 13V$

$$V_{i\,max} = \frac{13}{8} = 1.625\ V$$ ← useful input

output offset voltage (V_{oo})

Due to:

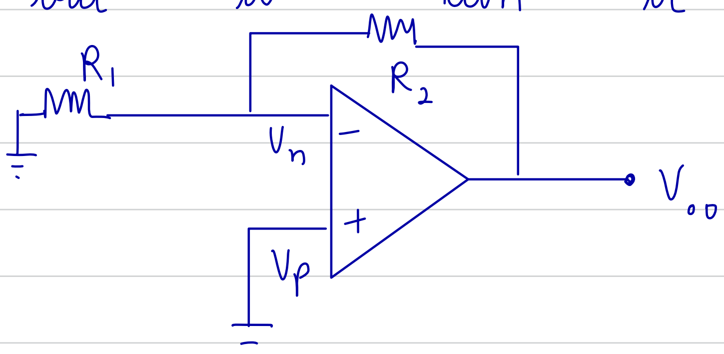
→ Input offset voltage V_{io}

→ Bias current → I_B → $\left(\frac{I_{BP} + I_{BN}}{2}\right)$

→ Input offset current → I_{io} → $\left(|I_{B_P} - I_{B_N}|\right)$

To nullify the o/p voltage we need V_{io} (opposite to o/p) & this is the i/p offset voltage.

• V_{io} need not be only due to I_{bias} but it can be because of V_{cm}



$$V_{io} = |V_P - V_n|$$

$$V_P = 0$$

$$V_n = \frac{R_1}{R_1 + R_2} V_{oo}$$

$$V_{io} = |0 - V_n| = V_n$$

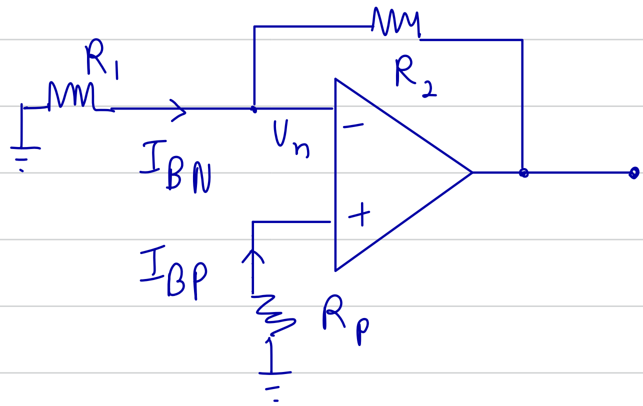
$$\Rightarrow V_{oo} = \left(1 + \frac{R_2}{R_1}\right) V_n = \left(1 + \frac{R_2}{R_1}\right) V_{io}$$

• For both configurations (inverting & non inverting) the V_{oo} & V_{io} relation is same.

$$V_{oo} = \left(1 + \frac{R_2}{R_1}\right) V_p$$

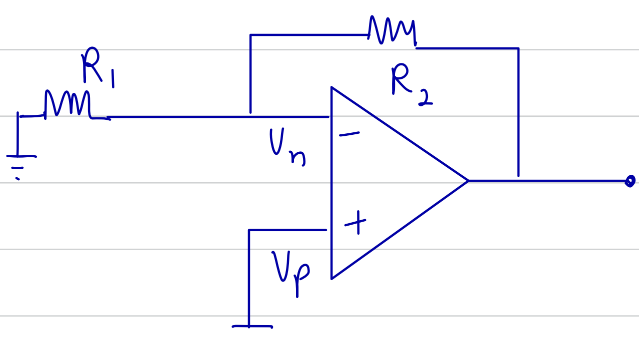
$$= \left(1 + \frac{R_2}{R_1}\right) R_p \, I_{B_p}$$

$$= \left(1 + \frac{R_2}{R_1}\right) \frac{R_2 R_1}{R_1 + R_1} \times I_{BP}$$

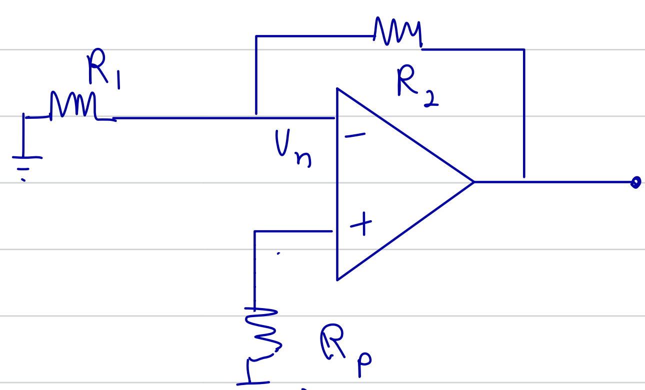


$$\Rightarrow V_{oo} = R_2 I_{BP}$$

$$V_{oo} = R_2 \left| I_{Bp} - I_{BN} \right| = R_2 I_{io}$$



$$V_{oo} = \left(1 + \frac{R_2}{R_1}\right) V_{io} + R_2 I_B$$



$$V_{oo} = \left(1 + \frac{R_2}{R_1}\right) V_{io} + R_2 I_{io}$$

contribution due to i/p offset voltage is much more than $I_B R_2$.

Q) Compute the max o/p offset voltages in the amplifier circuits a) without R_p & b) with R_p. The opamp has following specifications: $V_{io} = 7.5$ mV max

$I_{io} = 50\ nA$ max $\quad I_B = 250\ nA$ max at $25°C$

let $R_2 = 10\ k\Omega$ & $R_1 = 1\ k\Omega$

Ans a> without R_p

$$V_{oo} = \left(1 + \frac{R_2}{R_1}\right) V_{io} + I_B R_2$$

$$= (1+10)\ 7.5 + 250 \times 10^{-9} \times 10 \times 10^3$$

$$= 82.5 + 2.5 = 85\ mV$$

b> with R_p :

$$V_{oo} = \left(1 + \frac{R_2}{R_1}\right) V_{io} + I_{io} \times R_2$$

$$= 82.5 + 50 \times 10^{-9} \times 10 \times 10^3$$

$$= 82.5 + 0.5 = 83\ mV$$

Power supply rejection ratio (PSRR) or supply voltage rejection ratio (SVRR)

It is defined by, change in i/p offset voltage / change in power supply i.e

$$SVRR = \frac{dV_{io}}{dV_{cc}}$$

- It should be as less as possible i.e as less as $30\ \mu V/V$

$$20 \log \frac{dV_{cc}}{dV_{io}} = 90\ dB$$

reciprocal of PSRR.

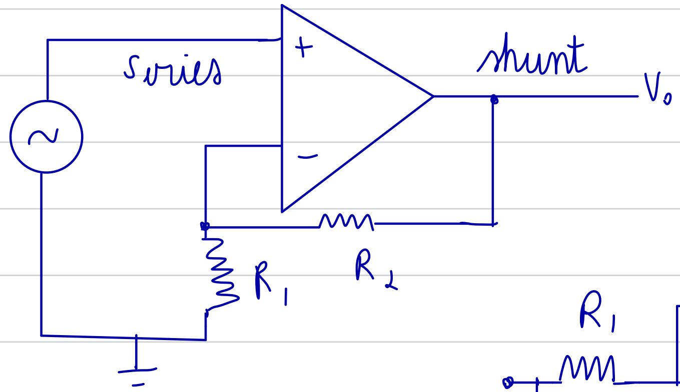
Q) Consider the circuit given below, showing the i/p offset voltage due to changes in the power supply. The opamp used is μA741, let R1 be 100 Ω, R2 be 100 kΩ. Determine the maximum ripple at the o/p for a power supply of 0.15 V_{pp} at 120 Hz.

Ans
$$20 \log \frac{\Delta V_{CC}}{\Delta V_{io}} = 90 \text{ dB}$$

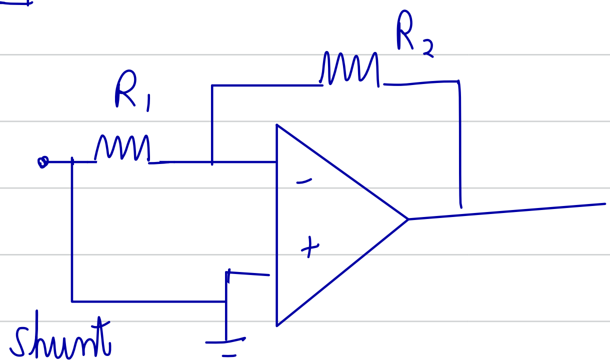
$$\Delta V_{io} = \frac{0.15}{31622.77} = 4.74 \ \mu V$$

$$V_{oo} = \left(1 + \frac{R_2}{R_1}\right) V_{io} = \left(1 + 10^3\right) 4.74 \ \mu V$$
$$= 4.74 \text{ mV}$$

Non inverting amplifier gain with non idealities

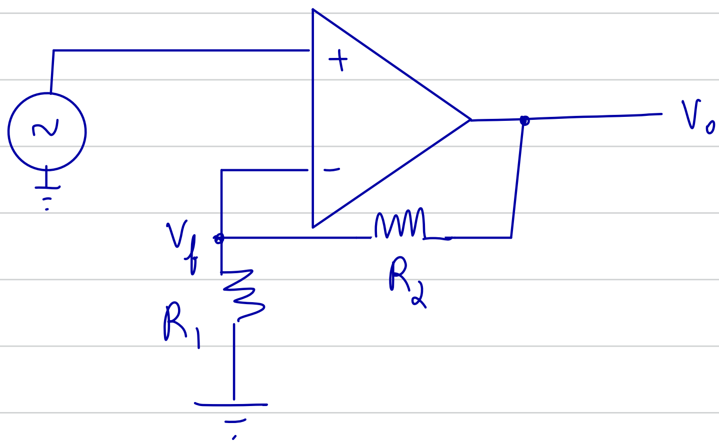


h- feedback



y - feedback

Non - inverting amplifier (voltage - series feedback)



CL gain : $A_f = \dfrac{V_o}{V_{in}}$

OL gain : $A = \dfrac{V_o}{V_{id}}$

gain of feedback circuit : $\dfrac{V_f}{V_o}$

$V_o = A V_{id}$

$V_o = A \left(V_{in} - \dfrac{R_1 V_o}{R_1 + R_2} \right)$

$\therefore \quad V_o \left(1 + \dfrac{A R_1}{R_1 + R_2} \right) = A V_{in}$

$\therefore \quad A_f = \dfrac{A}{1 + \dfrac{A R_1}{R_1 + R_2}}$

$A_f = 1 + \dfrac{R_1}{R_2} \rightarrow$ ideal

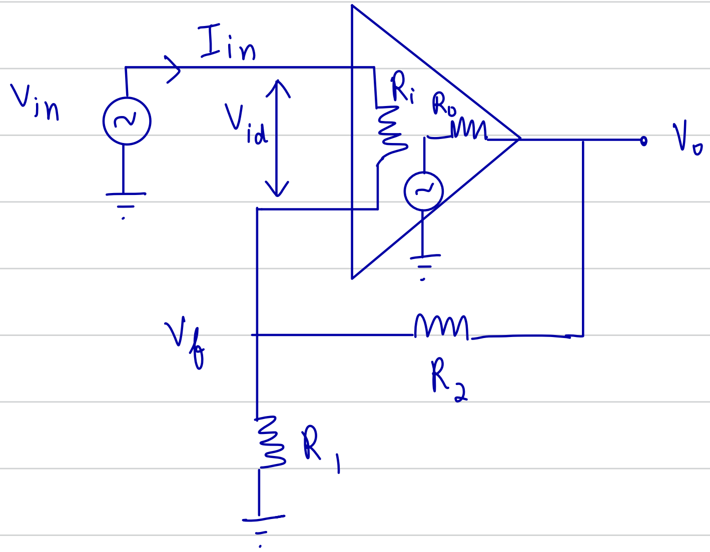
$\beta = \dfrac{R_1}{R_1 + R_2} \Rightarrow A_f = \dfrac{1}{\beta} \rightarrow$ ideal

$$A_f = \frac{A\left(\frac{R_1 R_2}{R_1+R_2}\right)}{\frac{R_1+R_2}{R_1+R_2} + \frac{AR_1}{R_1+R_2}} = \frac{A}{1+\frac{AR_1}{R_1+R_2}}$$

$$= \frac{A}{\frac{AR_1}{R_1+R_2}\left(1+\frac{R_1+R_2}{AR_1}\right)} = \left(1+\frac{R_2}{R_1}\right)\frac{1}{\left(1+\frac{1}{A\beta}\right)}$$

$$\approx \left(1+\frac{R_2}{R_1}\right)\left(1-\frac{1}{A\beta}\right) \quad \left[\because \frac{1}{A\beta} << 1\right]$$

Input impedance



$$R_{if} = \frac{V_{in}}{I_{in}} = \frac{V_{in}}{V_{id}/R_i}$$

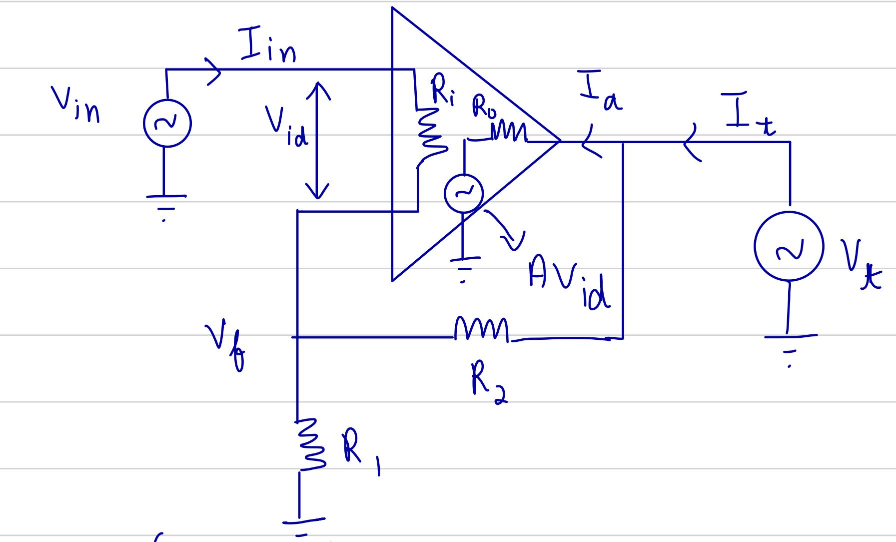
$$V_{id} = \frac{V_o}{A}$$

$$V_o = \frac{A}{1+A\beta}V_{in}$$

$$R_{if} = \frac{V_{in}}{\frac{V_{in}}{(1+A\beta)R_i}}$$

$$R_{if} = R_i(1+A\beta)$$

Output impedance:



$$V_n = \frac{V_t R_1}{R_1 + R_2}$$

$$V_{id} = 0 - V_n$$
$$= -\frac{V_t R_1}{R_1 + R_2}$$

$$R_2 + (R_1 \| R_2) \gg R_0 \qquad I_t \cong I_a$$

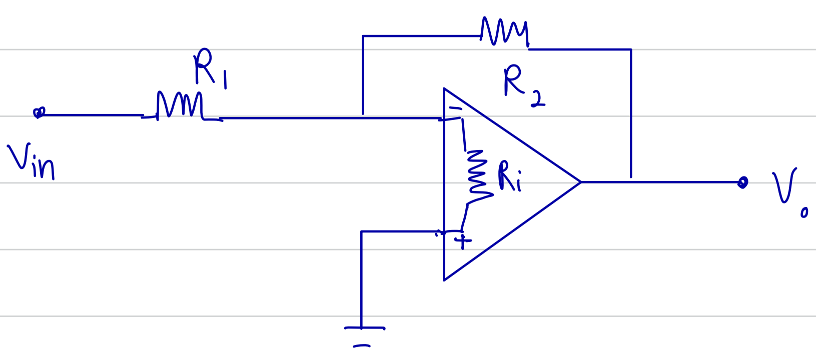
$$\therefore I_t = \frac{V_t - A V_{id}}{R_0}$$

$$V_{id} = -\frac{V_t R_1}{R_1 + R_2}$$

$$I_t = \left(V_t + \frac{A V_t R_1}{R_1 + R_2} \right) R_0^{-1}$$

$$\frac{R_0}{1 + A\beta} = \frac{V_t}{I_t} \qquad \therefore R_{0f} = \frac{R_0}{1 + A\beta}$$

Inverting amplifier (voltage-shunt feedback)



$$R_{if} = R_1 + \left(\frac{R_2}{1+A} \,\|\, R_i \right) \approx R_1$$

op impedance with feedback:

- Same model & analysis as that of non-inverting case.

C.L gain:

$$V_o = A(0 - V_n) = -AV_n$$

$$V_n = \frac{V_o R_1}{R_1+R_2} + \frac{V_{in} R_2}{R_1+R_2} \qquad \text{(superposition principle)}$$

$$V_o = \frac{-A}{R_1+R_2}\left(V_o R_1 + V_{in} R_2\right)$$

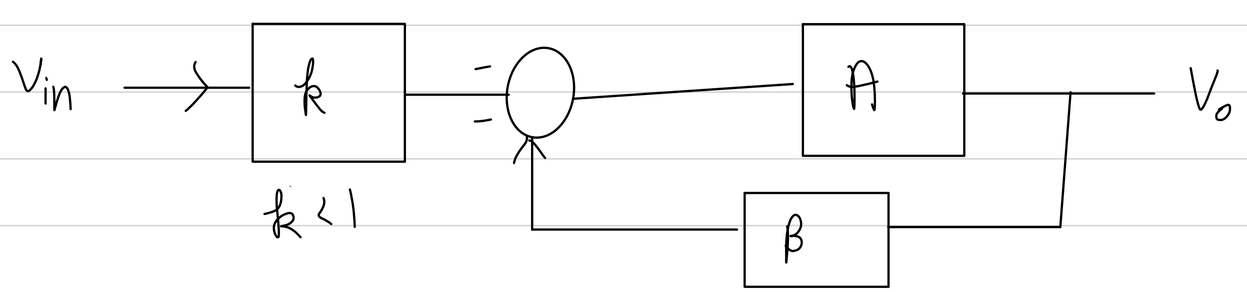
$$\therefore \quad V_o\left(1 + \frac{AR_1}{R_1+R_2}\right) = -\frac{A V_{in} R_2}{R_1+R_2}$$

$$\therefore \quad \frac{V_o}{V_{in}} = \frac{-\frac{A R_2}{R_1+R_2}}{1 + \frac{AR_1}{R_1+R_2}} = \frac{-\frac{AR_2}{R_1+R_2}}{\frac{AR_1}{R_1+R_2}\left(1 + \frac{R_1+R_2}{AR_1}\right)}$$

$$= \frac{-R_2}{R_1\left(1+\frac{1}{A\beta}\right)} = -\frac{R_2}{R_1}\left(1 - \frac{1}{A\beta}\right) \quad \because A\beta << 1$$

Now take, $k = \dfrac{R_2}{R_1+R_2}$ $\qquad k < 1$

$$A_f = \frac{V_o}{V_{in}} = \frac{-Ak}{1+A\beta} \simeq \frac{k}{\beta}$$



Q> Find the minimum value of A for an opamp

a> The gain of voltage follower built using it does not deviate more than 0.02%.

b> The gain of the inverting amplifier with $R_1 = R_2$ does not deviate more than 0.02%.

Ans a> $$\text{gain} = \frac{A}{1+A} = \frac{1}{1+\frac{1}{A}} \cong 1 - \frac{1}{A}$$

$$\frac{1}{A} \leq \frac{0.02}{100} \qquad \therefore A \geq \frac{10^4}{2}$$

$$\therefore A \geq 5000 \qquad \therefore A_{min} = 5000$$

b> $$\text{gain} = \frac{-Ak}{1+A\beta} \qquad k = 0.5 \qquad \beta = 0.5$$

$$= \frac{-A/2}{1+A/2} = \frac{-1}{1+\frac{2}{A}} \cong -1\left(1 - \frac{2}{A}\right)$$

$$= \frac{2}{A} - 1$$

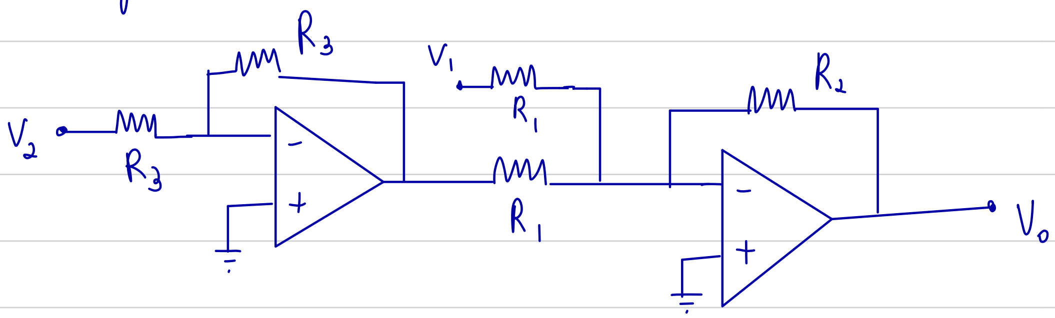
$$\therefore \frac{2}{A} \leq \frac{0.02}{100} \qquad \therefore A \geq 10^4 \qquad \therefore A_{min} = 10^4$$

Difference amplifier:

$$V_o = (V_2 - V_1) \times \text{some gain}$$
$$= (V_2 - V_1)\frac{R_2}{R_1}$$

(This can be done using 2 opamps easily but then it occupies more silicon area. So we need to construct a single opamp for this purpose)

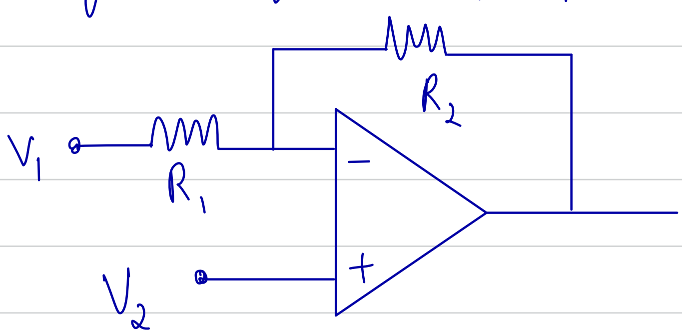
Using 2 opamps:



$$\frac{V_1 - V_2}{R_1} = \frac{V_2 - V_o}{R_2}$$

$$\Rightarrow \frac{V_o}{R_2} = \frac{V_2}{R_2} + \frac{V_2}{R_1} - \frac{V_1}{R_1}$$

Using single opamp:



Using KCL : $\dfrac{V_1 - V_2}{R_1} = \dfrac{V_2 - V_o}{R_2}$

$$\therefore \frac{V_o}{R_2} = V_2\left(\frac{1}{R_1} + \frac{1}{R_2}\right) - \frac{V_1}{R_1}$$

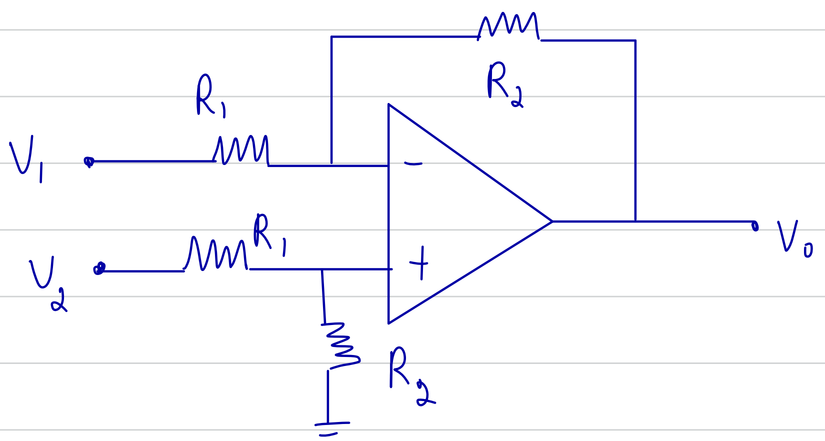
$$V_0 = V_2\left(1+\frac{R_2}{R_1}\right) - V_1\frac{R_2}{R_1}$$

$$\therefore V_0 = \frac{R_2}{R_1}\left(V_2 - V_1\right) + V_2$$

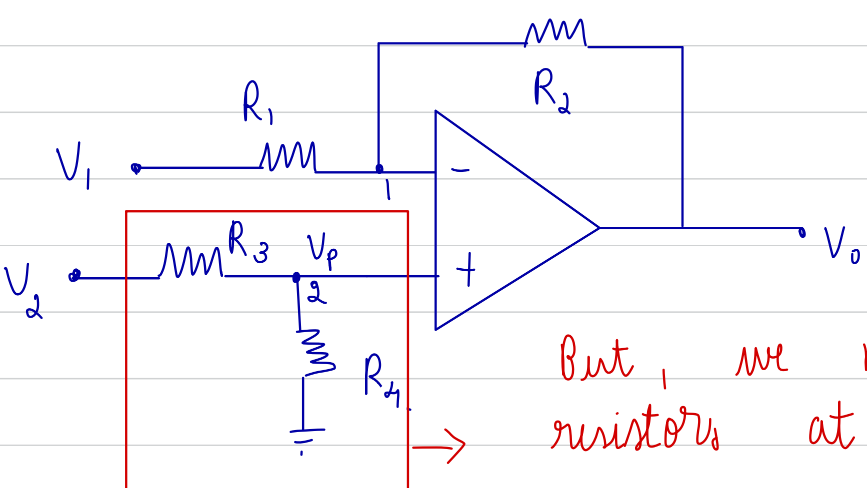
Superposition:

$$V_0 = \left(1+\frac{R_2}{R_1}\right)V_2 - \frac{R_2 V_1}{R_1} = \frac{R_2}{R_1}\left(V_2 - V_1\right) + V_2$$

if $V_2 = \dfrac{V_2}{\left(1+\frac{R_2}{R_1}\right)}\dfrac{R_2}{R_1}$ then $V_0 = \dfrac{R_2}{R_1}\left(V_2 - V_1\right)$



$$V_p = \frac{V_2 R_2}{R_1 + R_2}$$



$$\frac{R_3}{R_4} = \frac{R_1}{R_2}$$

But, we need to change 2 resistors at a time hence, this causes a lot of problem.

Applying KCL :

Node 1 :

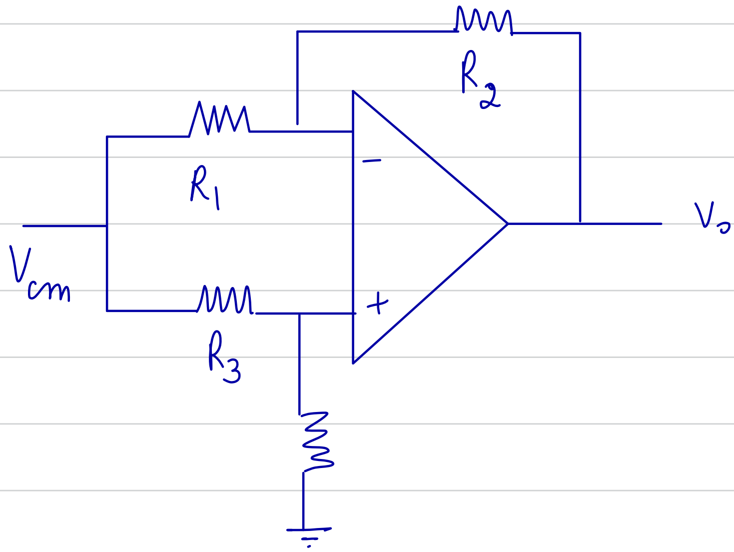
$$V_p = \frac{R_4}{R_3+R_4} V_2 = \frac{V_2}{1+\frac{R_3}{R_4}} = \frac{V_2}{1+\frac{R_3}{R_4}} = \frac{V_2(R_2/R_1)}{1+\frac{R_2}{R_1}}$$

Node 2 :

$$\frac{V_1 - V_p}{R_1} = \frac{V_p - V_0}{R_2}$$

$$\therefore V_0 = -\frac{R_2}{R_1} V_1 + \left(1+\frac{R_2}{R_1}\right)\frac{R_4}{R_3+R_4} V_2$$

With common mode signal :



$$\frac{R_3}{R_4} = \frac{R_1}{R_2}$$

$$V_0 = (V_2 - V_1)\frac{R_2}{R_1}$$

$$= 0 \text{ (ideally)}$$

$$\frac{V_{cm} - \frac{R_4 V_{cm}}{R_3+R_4}}{R_1} = \frac{\frac{R_4 V_{cm}}{R_3+R_4} - V_0}{R_2}$$

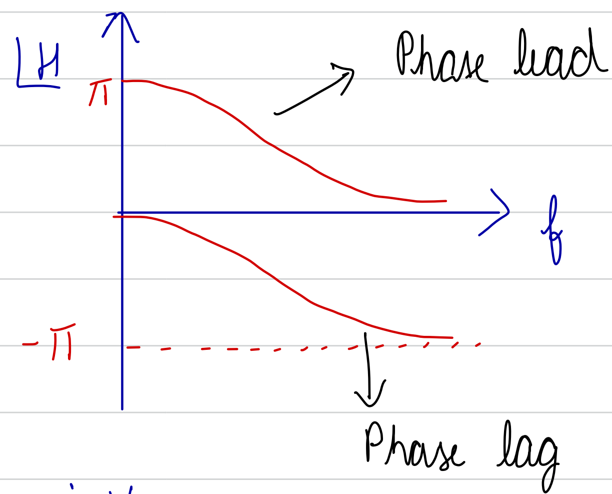
$V_1 = V_{cm} - \frac{V_{dm}}{2}$ $\qquad$ $V_2 = V_{cm} + \frac{V_{dm}}{2}$

$\Rightarrow \quad V_{dm} = V_1 - V_2$ $\qquad$ $V_{cm} = \frac{V_1 + V_2}{2}$

$\frac{V_0}{V_{cm}} = A_{cm} = 0$ $\qquad$ CMRR $= \infty$ (ideally)

Experiment 8

All pass filter: (Phase shift) or (Delay equalizer)

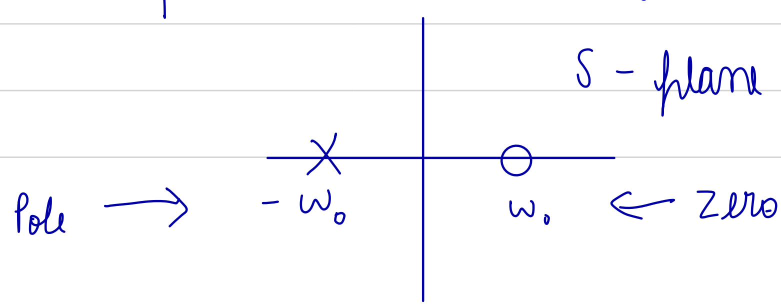


$H(s) = \frac{s - \omega_0}{s + \omega_0}$ $\qquad$ $H(j\omega) = \frac{j\omega/\omega_0 - 1}{j\omega/\omega_0 + 1}$

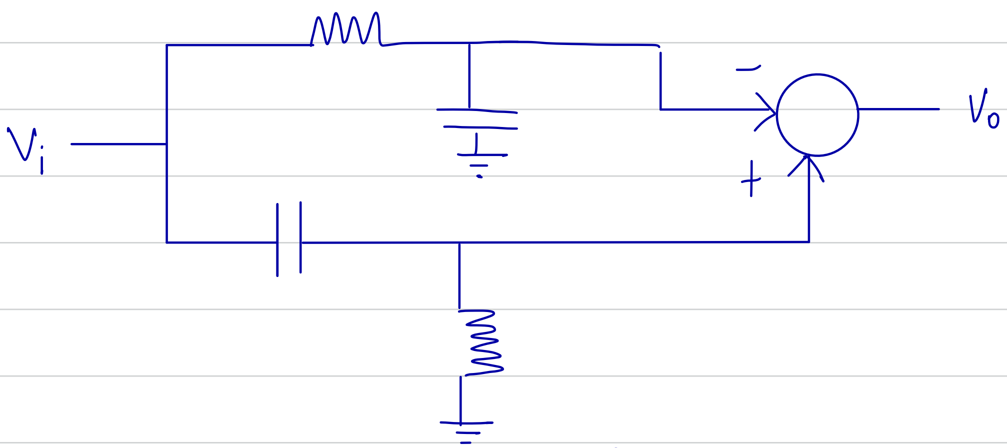
$\Rightarrow \quad |H| = 1$

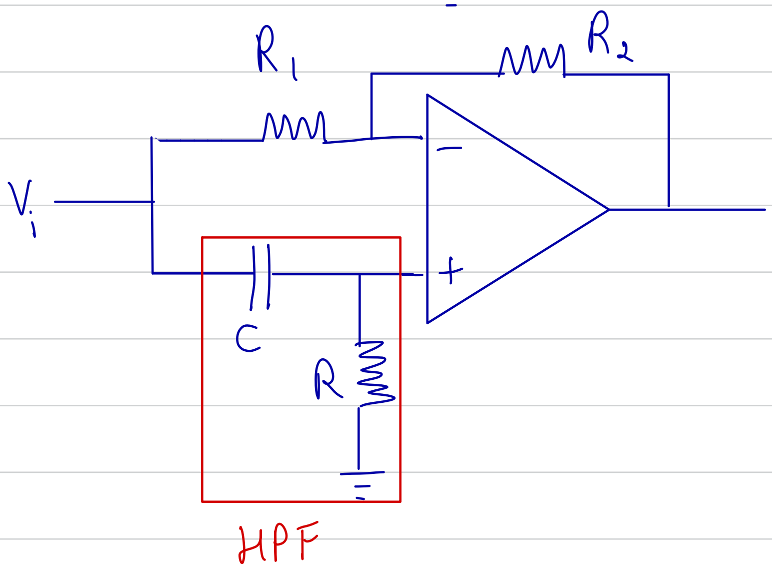
$\angle H = \tan^{-1}\left(\frac{(\omega/\omega_0)}{-1}\right) - \tan^{-1}\frac{(\omega/\omega_0)}{1} = -2\tan^{-1}\frac{\omega}{\omega_0}$

$= 180 - 2\tan^{-1}\frac{\omega}{\omega_0}$



$$H(s) = \underbrace{\frac{s/\omega_0}{s/\omega_0 + 1}}_{\text{HPF}} - \underbrace{\frac{1}{s/\omega_0 + 1}}_{\text{LPF}}$$





→ All pass filter with phase lead configuration.

$$V_o = \left(1 + \frac{R_2}{R_1}\right) \frac{s/\omega_0}{(s/\omega_0) + 1} V_i$$

$$V_o = \frac{(s/\omega_0) V_i}{\left(\frac{s}{\omega_0}\right) + 1} + \frac{(s/\omega_0) \frac{R_2}{R_1} V_i}{\frac{s}{\omega_0} + 1} - \frac{R_2}{R_1} V_i$$

$$= \frac{(s/\omega_0) V_i}{\frac{s}{\omega_0} + 1} + V_i \frac{R_2}{R_1} \left[\frac{s/\omega_0}{\frac{s}{\omega_0} + 1} - 1 \right]$$

Now choose $R_1 = R_2$

$$V_o = \frac{(s/\omega_0) V_i}{\frac{s}{\omega_0} + 1} - \frac{V_i}{\frac{s}{\omega_0} + 1}$$

- For phase lag interchange position of R & C.

$$H(j\omega) = \frac{\frac{j\omega}{\omega_0} - 1}{\frac{j\omega}{\omega_0} + 1} = \frac{j\omega RC - 1}{j\omega RC + 1}$$

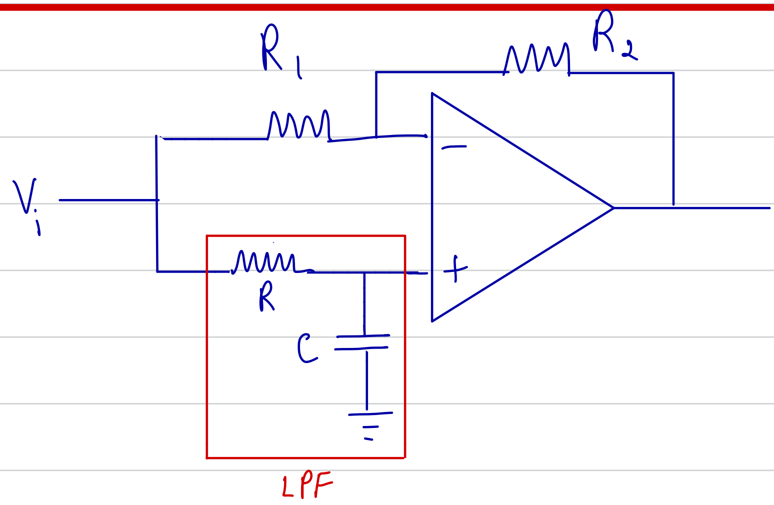
$$|H| = \frac{\sqrt{1 + (\omega RC)^2}}{\sqrt{1 + (\omega RC)^2}} = 1$$

$$\angle H = 180° - 2\tan^{-1} \omega RC$$

At $\omega = 0$, $\angle H = 180°$

$\omega = \infty$ $\angle H = 0°$

All pass filter with lagging phase:



$$V_o = -\frac{R_2}{R_1}V_i + \left(1+\frac{R_2}{R_1}\right)\frac{V_i}{1+RCS}$$

take $R_1 = R_2$

$$V_o = -V_i + \frac{2V_i}{RCS+1}$$

$$= \frac{V_i}{1+RCS} + V_i\left(\frac{1}{1+RCS} - 1\right)$$

$$H(j\omega) = \frac{1-j\omega RC}{1+j\omega RC}$$

$|H(j\omega)| = 1$ $\qquad\qquad \angle H = -2\tan^{-1}\omega RC$

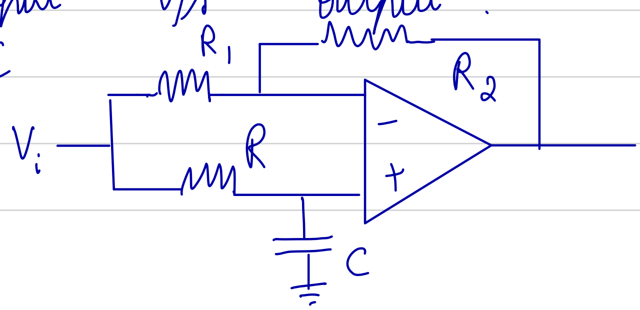
Q) Design a phase shifter that offers phase lag of 45° for an input of 500 Hz. Plot the time domain response of the system and compare with that of input. Also plot a graph of input v/s output.

Ans $-45° = 2\tan^{-1}(2\pi\times 500)RC$

$RC = 1.319\times 10^{-4}$

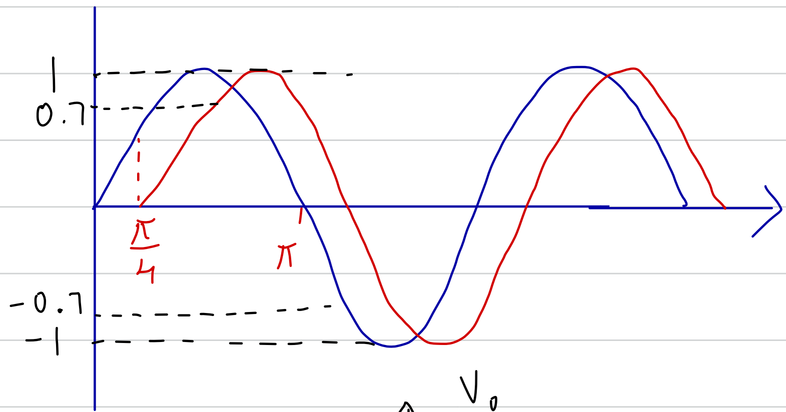
let $C = 0.01\,\mu F$

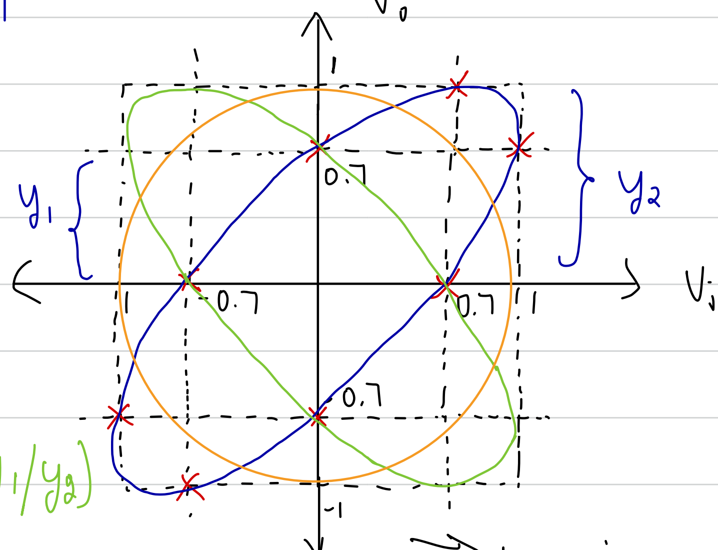
$R_1 = R_2 = 10\,k\Omega$



$R = 13.19\ k\Omega$

Input	output
0.7	0
1	0.7
0.7	1
0	0.7
-0.7	0
-1	-0.7
-0.7	1
0	-0.7





→ Lissajous figure

$\phi < 90^\circ \qquad \phi = \sin^{-1}(y_1/y_2)$

$\phi > 90^\circ \qquad \phi = 180 - \sin^{-1}(y_1/y_2)$

$\phi = 90^\circ \qquad \phi = \sin^{-1}(1)$

Experiment a> $V_i = 2\sin 4000\pi t$

$V_o = 2\sin(4000\pi t - \pi/3)$

$-60^\circ = -2\tan^{-1}(2\pi \times 2kHz)RC$

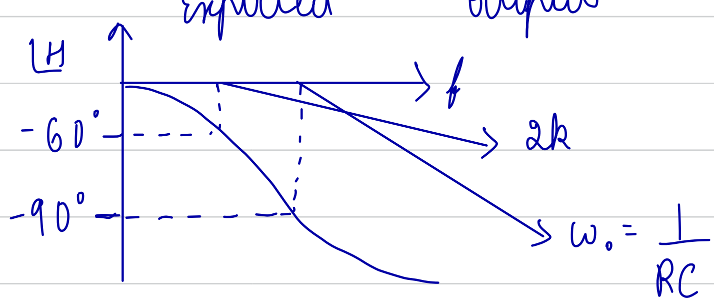
$\tan 30^\circ = 4000\pi RC$

$RC = 4.59 \times 10^{-5}$

let $C = 0.01\ \mu F$

$R = 4.59\ k\Omega$ //

Expected output



$\phi = \Delta t \times f \times 360^\circ$

Interchange R & C for phase lead

ACTIVE FILTERS

→ Active filters v/s Passive filters

• Active filters have active devices such as opamp, operational transconductance amplifiers (OTA) etc.

• Passive filters have only passive devices such as R, L, C.

→ Analog filters v/s digital filters

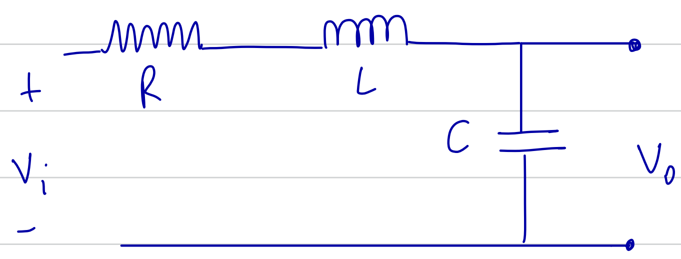
• Analog filters filter analog signals.
• Digital filters also filter analog signal but using digital techniques.

→ Audio frequency v/s radio frequency

• Audio frequencies are smaller → RC filter
• Radio frequencies are larger → LC filter and crystal oscillators.

Type of filters:
1> Low pass filter
2> High pass filter
3> Band pass filter
4> Band reject filter
5> All pass filter.

Second order RLC circuit



$$\frac{V_0}{V_i} = \frac{1/Cs}{R + Ls + \frac{1}{Cs}} = \frac{1}{RCs + LCs^2 + 1}$$

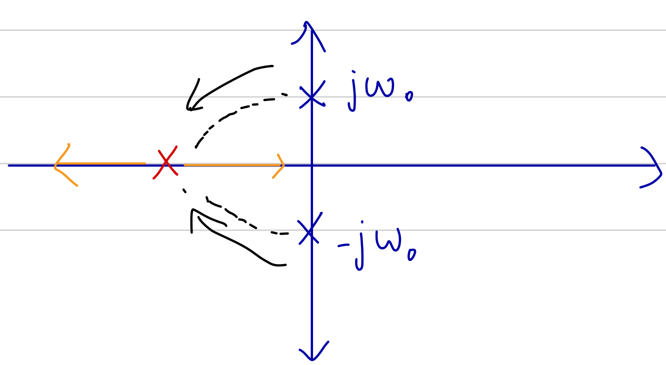
$$\frac{V_0}{V_i} = H = \frac{1/LC}{s^2 + \frac{R}{L}s + \frac{1}{LC}} = \frac{\omega_0^2}{s^2 + 2\xi\omega_0 s + \omega_0^2}$$

$\omega_0 = \frac{1}{\sqrt{LC}}$ = undamped natural frequency

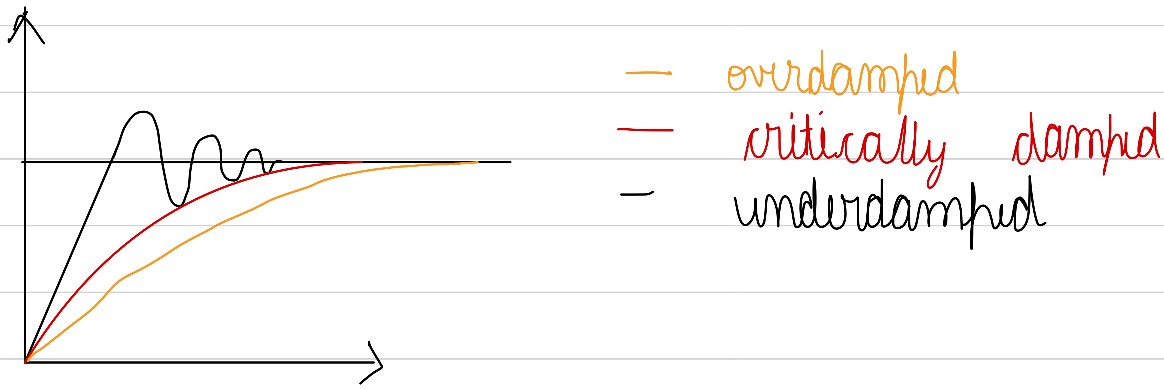
$$s_1, s_2 = -\xi\omega_0 \pm j\omega_0\sqrt{1-\xi^2}$$

$$\xi = \frac{R}{2}\sqrt{\frac{C}{L}}$$

$\xi = 0 \rightarrow$ undamped
$\xi = 1 \rightarrow$ Critically damped
$\xi > 1 \rightarrow$ overdamped
$0 < \xi < 1 \rightarrow$ underdamped



$X \rightarrow \xi = 0$
$X \rightarrow \xi = 1$
$X \rightarrow 0 < \xi < 1$
$X \rightarrow \xi > 1$



$$Q = \frac{1}{2\xi} = \frac{1}{2} \times \frac{2}{R}\sqrt{\frac{L}{C}} = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{\omega_0 L}{R} \quad \text{(series)}$$

$Q \rightarrow$ Quality factor

For parallel circuit, $Q = R\sqrt{\frac{C}{L}}$

Q is the ratio of stored energy to dissipated energy over one radian of oscillation.

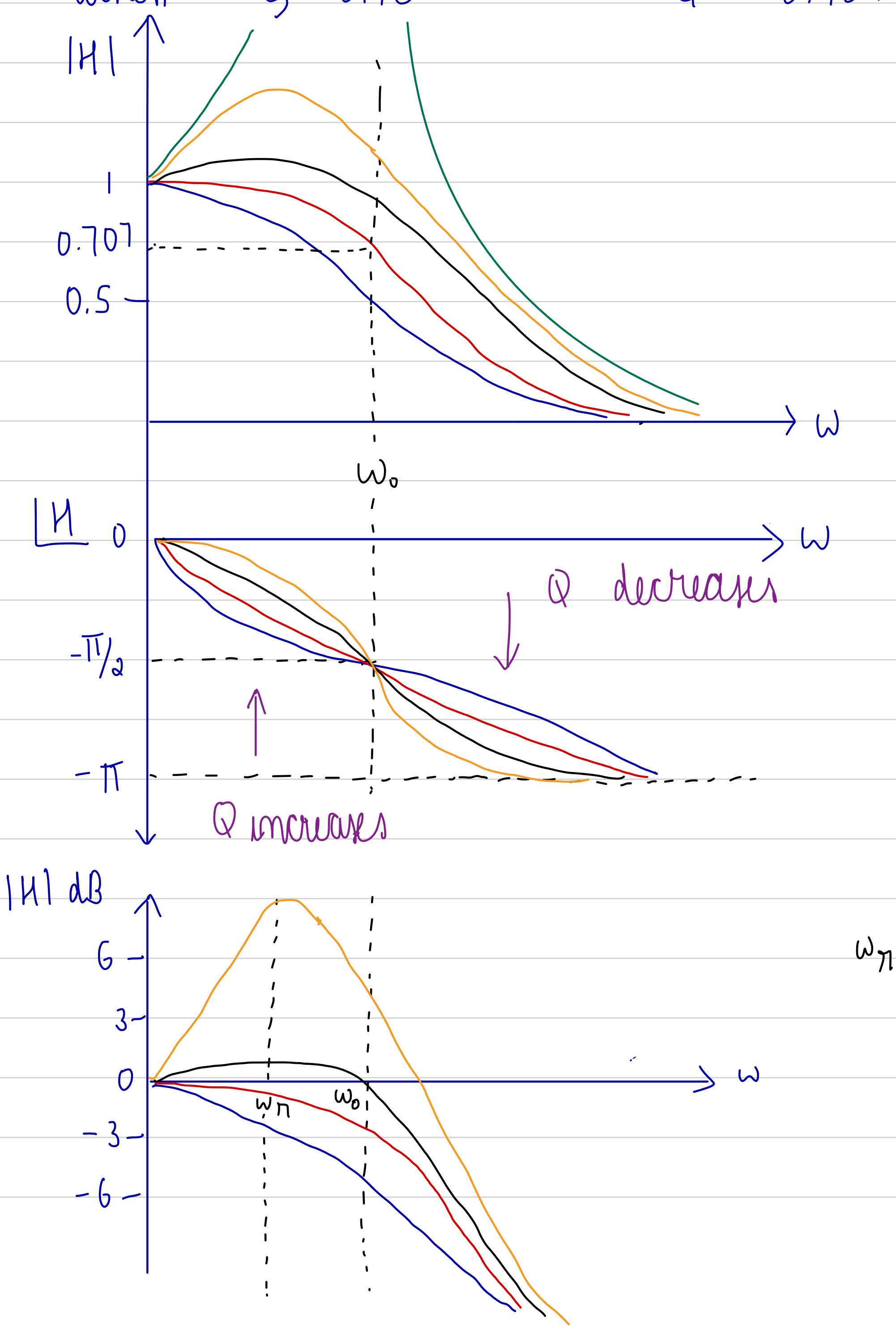
$$\frac{V_o}{V_i} = H(s) = \frac{\omega_0^2}{s^2 + \frac{\omega_0 s}{Q} + \omega_0^2} = \frac{1}{\left(\frac{s}{\omega_0}\right)^2 + \frac{s}{\omega_0 Q} + 1}$$

$$H(j\omega) = \frac{1}{1 - \left(\frac{\omega}{\omega_0}\right)^2 + j\frac{\omega}{Q\omega_0}}$$

$$|H(j\omega)| = \frac{1}{\sqrt{\left[1 - \left(\frac{\omega}{\omega_0}\right)^2\right]^2 + \left[\frac{\omega}{Q\omega_0}\right]^2}} \qquad \angle H(j\omega) = -\tan^{-1}\frac{(\omega/Q\omega_0)}{\left(1 - \left(\frac{\omega}{\omega_0}\right)^2\right)}$$

ω	$\lvert H\rvert$	$\angle H$
0	1	0
ω_0	Q	-90°
$\omega \gg \omega_0$	0	$\approx -180^\circ$

when $\xi = 0.707$ $\quad Q \cong 0.707$



$Q = \infty$

$Q = 2$

$Q = 1$

$Q = 0.707$

$Q = 0.5$

$\omega_r =$ resonant frequency

NOTE: Maximally flat response or butterworth response occurs at $Q = 0.707$

To find $|H(j\omega)|_{max}$,

Find $\frac{d|H(j\omega)|}{d\omega} = 0 \rightarrow \omega_r$

$$\boxed{\omega_r = \omega_0 \sqrt{1 - \frac{1}{2Q^2}}} \qquad Q \geq 0.707$$

$$\boxed{|H(j\omega)|_{max} = \frac{Q}{\sqrt{1 - \frac{1}{4Q^2}}}} \qquad Q \geq 0.707$$

Proof:

$$\frac{d|H|}{d\omega} = \frac{1}{\sqrt{\left(1 - \left(\frac{\omega}{\omega_0}\right)^2\right)^2 + \left(\frac{\omega}{Q\omega_0}\right)^2}} \times \frac{1}{2}\left(2\left(1 - \left(\frac{\omega}{\omega_0}\right)^2\right)\left(\frac{-2\omega}{\omega_0}\right) + \frac{2\omega}{Q^2\omega_0}\right)\left(\frac{1}{\omega_0}\right) = 0$$

$$\Rightarrow \quad \frac{\omega}{Q^2\omega_0} = \left(\frac{\omega}{\omega_0}\right) 2\left(1 - \left(\frac{\omega}{\omega_0}\right)^2\right)$$

$$\Rightarrow \quad \omega = 0 \quad \text{or} \quad 1 - \left(\frac{\omega}{\omega_0}\right)^2 = \frac{1}{2Q^2}$$

$$\Rightarrow \quad \left(\frac{\omega}{\omega_0}\right)^2 = 1 - \frac{1}{2Q^2} \qquad \therefore \quad \omega_r = \omega_0\sqrt{1 - \frac{1}{2Q^2}} \; //$$

At $\omega = \omega_r$, $|H(j\omega)|_{max}$

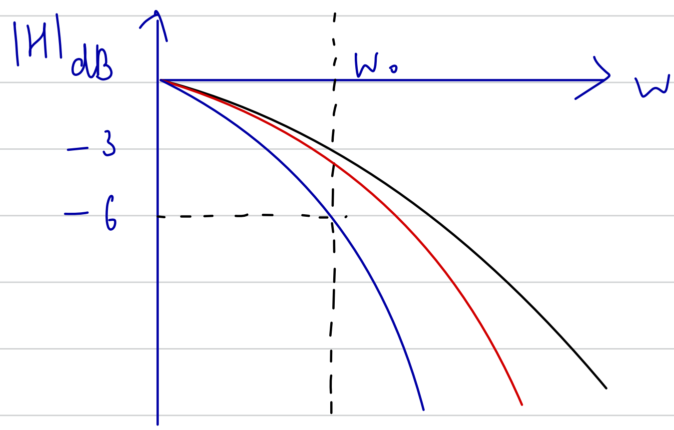
$$= \frac{1}{\sqrt{\frac{1}{4Q^4} + \left(1 - \frac{1}{2Q^2}\right)\frac{1}{Q^2}}} = \frac{Q}{\sqrt{\frac{1}{4Q^2} + 1 - \frac{1}{2Q^2}}} = \frac{Q}{\sqrt{1 - \frac{1}{4Q^2}}} \; //$$

A good filter will have steeper curve in $|H|_{dB}$ v/s f in transition region. So higher value of Q is desired.

A higher order active filter can be created using : 1> Direct synthesis
2> Cascading of lower order filters

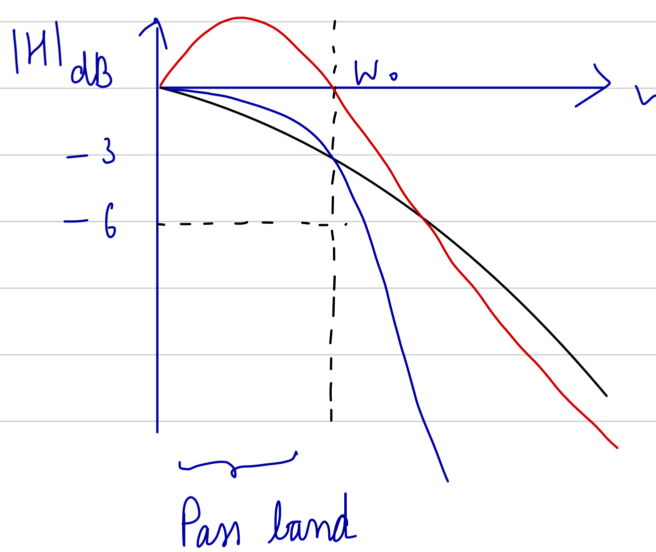
Ex:

$$H_1(S) = \frac{1}{\left(\frac{s}{\omega_0}\right)^2 + \frac{s}{Q\omega_0} + 1} \qquad H_2(S) = \frac{1}{1 + \frac{s}{\omega_0}}$$



$H_2(S) = -20$ dB/dec
$H_1(S) = -40$ dB/dec
$H_1(S) \cdot H_2(S) = -60$ dB/dec

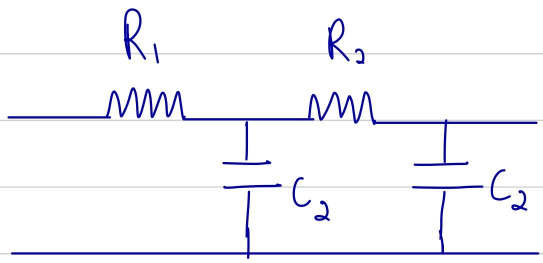
with $Q = 0.707$



$H_2(S) = -20$ dB/dec
$H_1(S) = -40$ dB/dec
$H_1(S) \cdot H_2(S) = -60$ dB/dec

with $Q = 1$

Experiment 9 – Sallen-key filter (KRC filter)



$$\begin{bmatrix} 1 & R_1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ C_1 s & 1 \end{bmatrix} \begin{bmatrix} 1 & R_2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ C_2 s & 1 \end{bmatrix}$$

$$H(s) = \frac{1}{R_1 R_1 C_1 C_2 s^2 + (R_1 C_1 + R_2 C_2 + R_1 C_2) s + 1}$$

$$= \frac{\dfrac{1}{R_1 R_2 C_1 C_2}}{s^2 + \dfrac{R_1 C_1 + R_2 C_2 + R_1 C_2}{R_1 R_2 C_1 C_2} s + \dfrac{1}{R_1 R_2 C_1 C_2}}$$

$$= \frac{\omega_0^2}{s^2 + 2\xi \omega_0 s + \omega_0^2} = \frac{1}{\left(\dfrac{s}{\omega_0}\right)^2 + \dfrac{s}{Q\omega_0} + 1}$$

where $\omega_0 = \dfrac{1}{\sqrt{R_1 R_2 C_1 C_2}}$ $\qquad$ $Q = \dfrac{\sqrt{R_1 R_2 C_1 C_2}}{R_1 C_1 + R_2 C_2 + R_1 C_2}$

let $R_1 = R$ $\qquad R_2 = mR$ $\qquad C_1 = C$ $\qquad C_2 = nC$

then $$Q = \frac{RC\sqrt{mn}}{RC(1 + mn + n)}$$

$$= \frac{\sqrt{mn}}{1 + mn + n}$$

let $m = 1$

$$Q = \frac{\sqrt{n}}{1+2n}$$

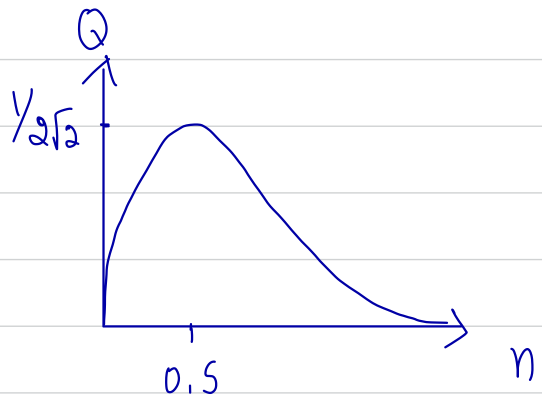
$$\frac{dQ}{dn} = \frac{\left(\frac{1}{2\sqrt{n}}\right)(1+2n) - 2\sqrt{n}}{(1+2n)^2} = 0$$

$$\Rightarrow \quad 1+2n = (2\sqrt{n})^2$$

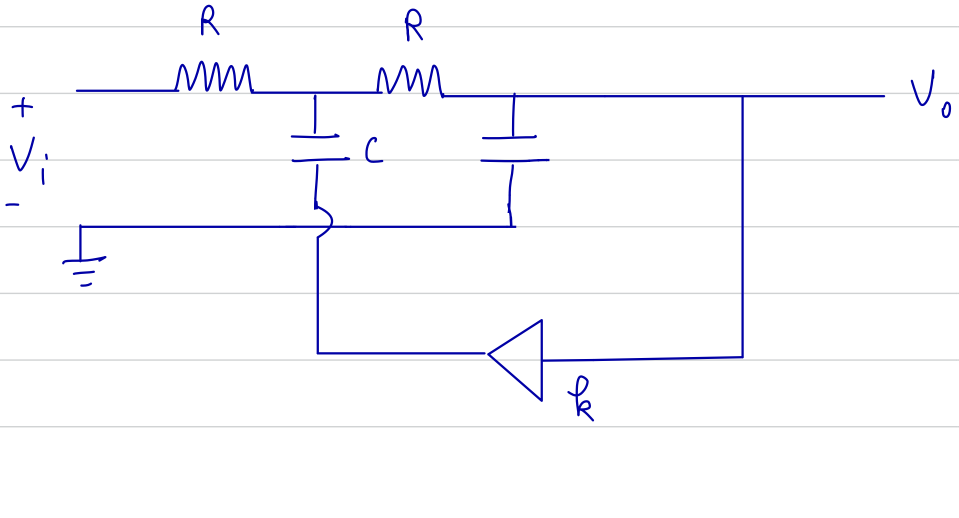
$$1+2n = 4n$$

$$\therefore \quad n = 0.5$$

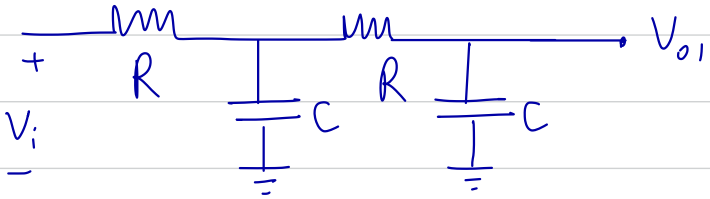
$\therefore$ when $n = 0.5$, $Q_{max} = \dfrac{1}{2\sqrt{2}} = 0.3535$



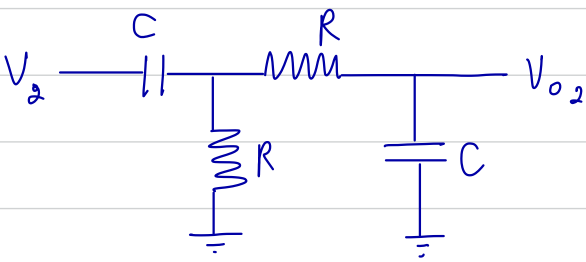
Also considering all combinations of m, n the max value of Q is obtained at m=1 and n = 0.5.



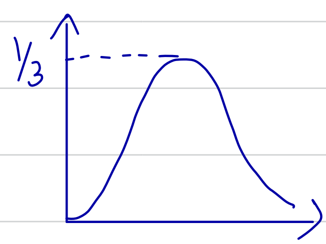
$$V_o = V_{o1} + V_{o2}$$



$$\frac{V_{o1}}{V_i} = \frac{1}{(RCs)^2 + 3RCs + 1}$$



$$\frac{V_{o2}}{V_2} = \frac{RCs}{(RCs)^2 + 3RCs + 1}$$



$$V_o = \frac{V_i + RCs \cdot kV_o}{(RCs)^2 + 3RCs + 1}$$

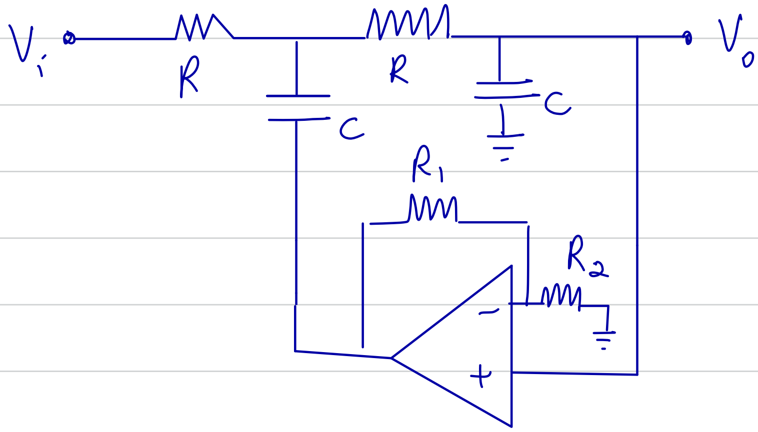
$$\frac{V_o}{V_i} = \frac{1}{(RCs)^2 + 3RCs + 1} + \frac{kRCs}{(RCs)^2 + 3RCs + 1}\left(\frac{V_o}{V_i}\right)$$

$$\frac{V_o}{V_i} = \frac{\dfrac{1}{(RCs)^2 + 3RCs + 1}}{\dfrac{(RCs)^2 + (3-k)RCs + 1}{(RCs)^2 + 3RCs + 1}} = \frac{1}{(RCs)^2 + (3-k)RCs + 1}$$

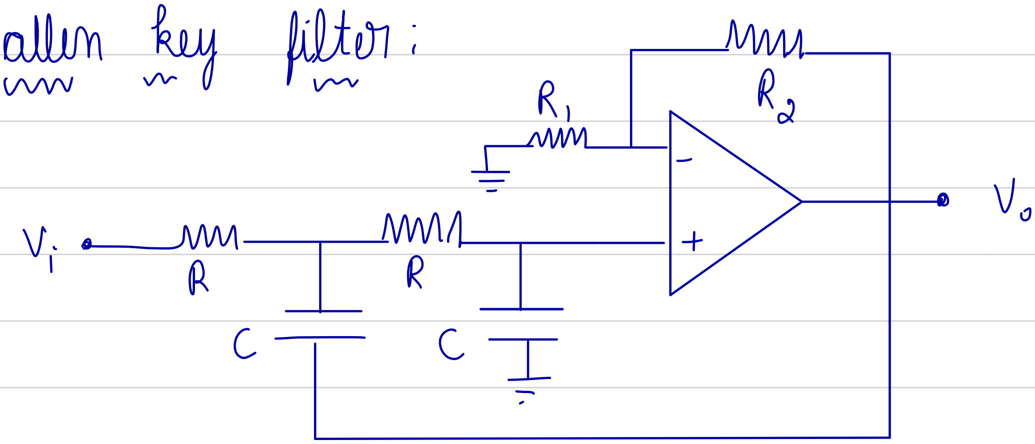
$$= \frac{\omega^2}{s^2 + \frac{(3-k)}{\omega}s + \omega^2}$$

where $\omega = \dfrac{1}{RC}$

$Q = \dfrac{1}{3-k}$

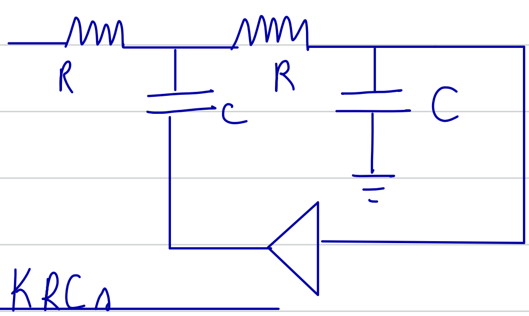


Sallen key filter:



$$\frac{V_o}{V_i} = \frac{k}{(RCs)^2 + (3-k)RCs + 1}$$

Negative feedback is dominant here. Because -ve feedback is instantaneous. In case of positive feedback there is delay due to capacitor connected.

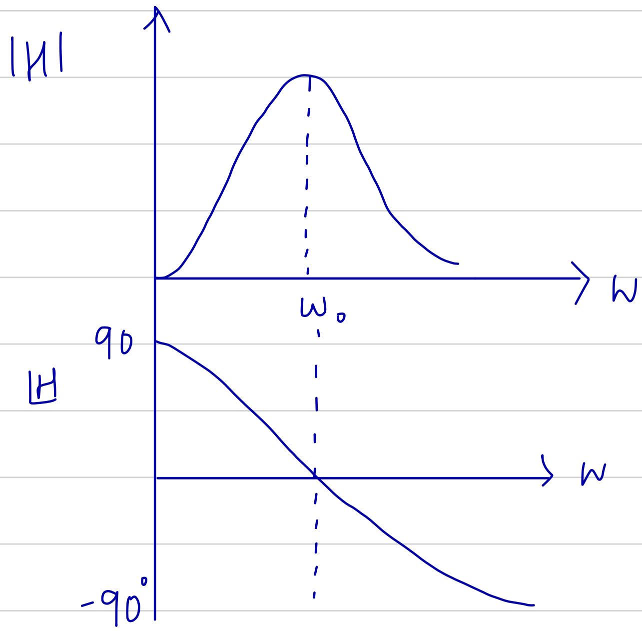


$|A\beta| = 1$ at phase $= 0°$

$$H = \frac{KRCs}{(RCs)^2 + 3RCs + 1}$$

when $K = 3$, gain $= 1$

$$1 + \frac{R_1}{R_2} \leq 3 \qquad k = 3$$



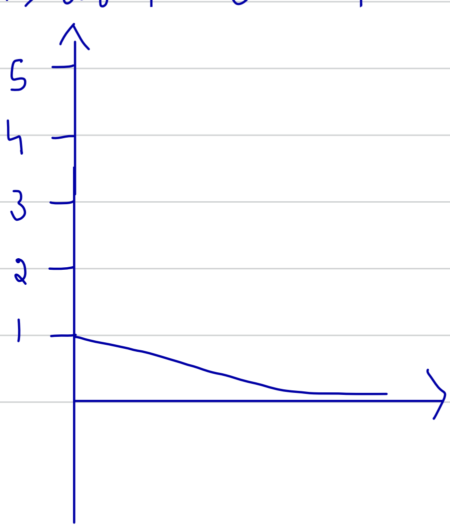
$$H(j\omega) = \frac{k}{1 - \left(\frac{\omega}{\omega_0}\right)^2 + \frac{j\omega}{Q\omega_0}}$$

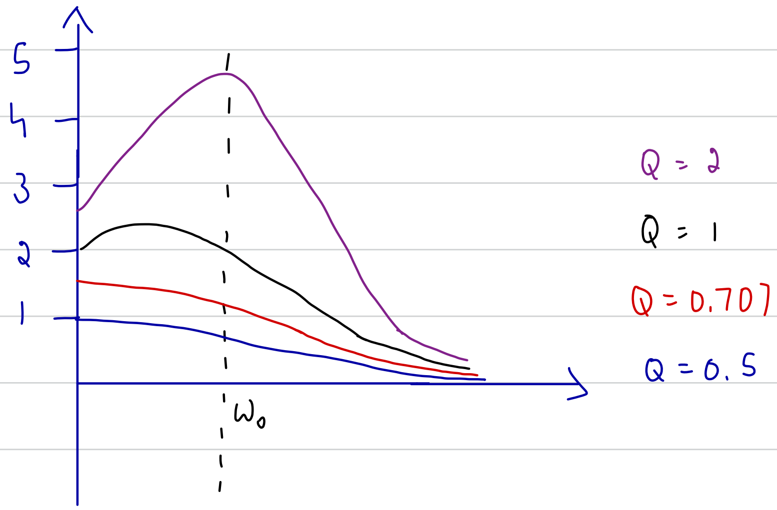
$$|H| = \frac{k}{\sqrt{\left(1 - \left(\frac{\omega}{\omega_0}\right)^2\right)^2 + \left(\frac{\omega}{Q\omega_0}\right)^2}}$$

$$\angle H = -\tan^{-1} \frac{\omega/Q\omega_0}{1 - \left(\frac{\omega}{\omega_0}\right)^2}$$

ω	$\lvert H \rvert$	$\angle H$
0	k	0°
ω_0	kQ	$-\pi/2$
$\omega \gg \omega_0$	0	$-\pi$

Q	K	$\lvert H \rvert$ at $\omega = 0$	$\lvert H \rvert$ at $\omega = \omega_0$
0.5	1	1	0.5
0.707	1.586	1.586	1.213
1	2	2	2
2	2.5	2.5	5

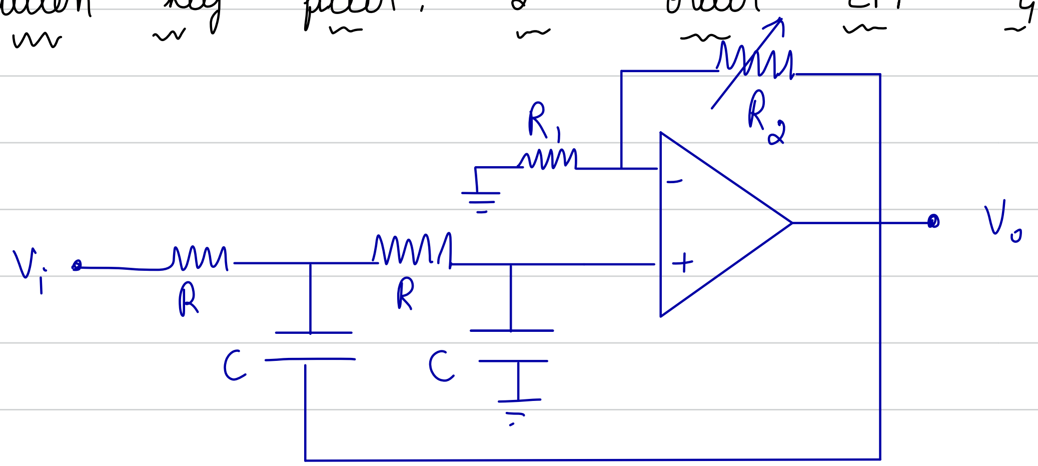




$$\omega_r = \omega_0 \sqrt{1 - \frac{1}{2Q^2}}$$

At ω_r, $$|H|_{max} = \frac{Q}{\sqrt{1 - \frac{1}{4Q^2}}}$$

Sallen key filter : 2nd order LPF & HPF



$R_1 = 1\,k\Omega$

$$H(s) = \frac{k}{(RCs)^2 + (3-k)RCs + 1}$$

where $k = 1 + \frac{R_2}{R_1}$, $Q = \frac{1}{3-k}$

$V_{ipp} = 10\,V$, $R_2 \rightarrow$ Potentiometer is used

Q	$R_2(\Omega)$	k	f_0 (Hz)	$\lvert H\rvert_p$	f_p (Hz)	Stop band roll off (−40 dB/dec)
0.707	586	1.586	1k			
0.5	0	1	1k			
2	1.5 k	2.5	1k			

↓
decrease the i/p to 4 V_{pp} for some values beyond ω_0.

$$-40 = 20 \log\left(\frac{V_0}{V_i}\right)$$

$$V_i = 100 \times V_0 = 100 \times 100 \times 10^{-3} = 10\ V_{pp}$$

Phase for $Q = 0.707$

→ cntl + right click on R_2
→ change value as {x}
→ click on spice directive (.op)
→ step param x list 1 586 500

- Interchange R & C to get HPF

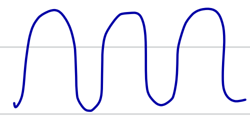
→ Magnitude & phase response for Q = 0.707, .5, 2 for LPF

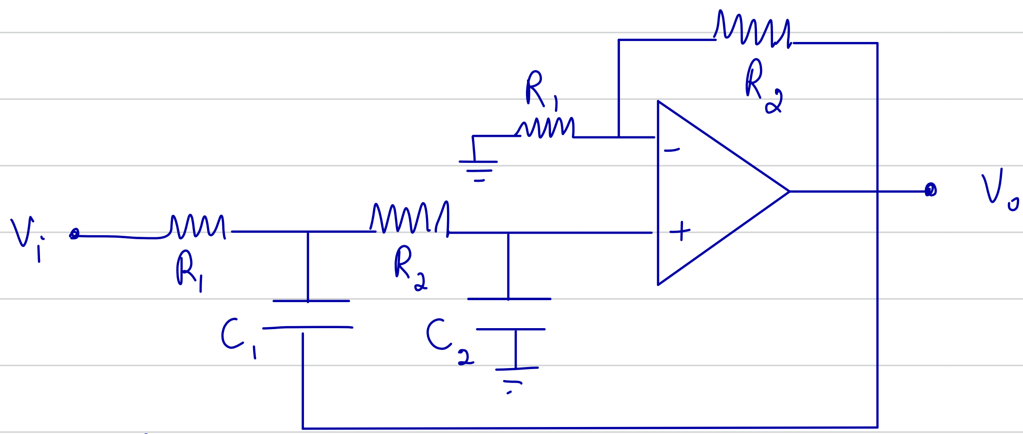
→ Magnitude & phase for HPF with Q = 0.707

$\omega_0 = \dfrac{1}{RC}$, when $C = 0.01\,\mu F$, $R = 15.9\ k\Omega$

$$|H_p|_{max} = \frac{Qk}{\sqrt{1 - \frac{1}{4Q^2}}} \qquad \omega_p = \omega_0 \sqrt{1 - \frac{1}{2Q^2}}$$

Ground or disconnect the signal generator, then ground it. This gives sustained oscillations at the output (keep $k > 3$).



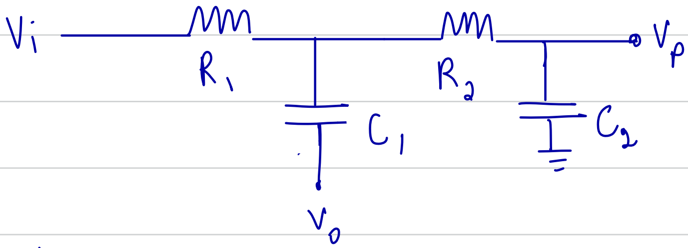


$$V_o = \left(1 + \frac{R_2}{R_1}\right) V_p$$

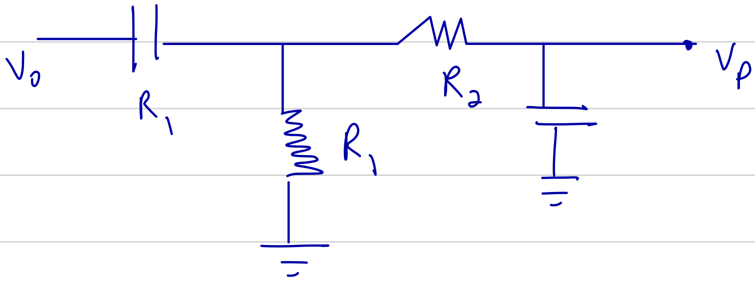
$$V_o = k V_p$$

$$V_p = \frac{V_1}{C_2 s} \left(\frac{1}{R_2 + 1/C_2 s}\right) = \frac{V_1}{1 + R_2 C_2 s}$$

$$V_i = (1 + R_2 C_2 s) V_p$$



$$V_p = \frac{V_i}{R_1 R_2 C_1 C_2 s^2 + (R_1 C_1 + R_2 C_2 + R_1 C_2) s + 1}$$



At V_1

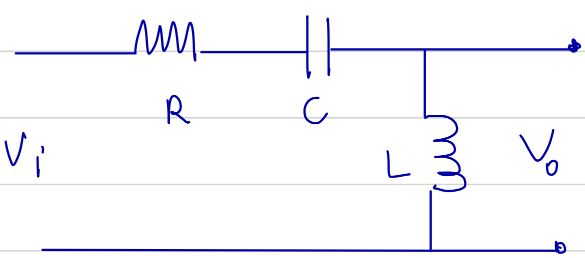
$$\frac{V_i - V_1}{R_1} = \frac{V_1 - V_p}{R_2} + \frac{V_1 - V_o}{1/C_1 s}$$

$$\frac{V_i}{R_1} - \frac{V_1}{R_1} = V_1\left(\frac{1}{R_2} + C_1 s\right) - V_o\left[\frac{1}{kR_2} + C_1 s\right]$$

$$V_1 = \frac{V_o}{k}\left(1 + R_2 C_2 s\right) \qquad \frac{V_i}{R_i} = V_1\left(\frac{1}{R_1} + \frac{1}{R_2} + C_1 s\right) - V_o\left[\frac{1}{kR_2} + C_1 s\right]$$

2nd order HPF series RLC circuit

Characteristic equation must remain same.
(LPF ; 2 poles → all pole filter)



$$\frac{V_o}{V_i} = \frac{Ls}{Ls + R + 1/Cs} = \frac{LCs^2}{LCs^2 + RCs + 1}$$

$$\Rightarrow H(s) = \frac{(s/\omega_o)^2}{\left(\frac{s}{\omega_o}\right)^2 + \frac{s}{\omega_o} + 1} = \frac{s^2}{s^2 + 2\xi\omega_o s + \omega_o^2}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \qquad Q\omega_0 = \frac{1}{RC}$$

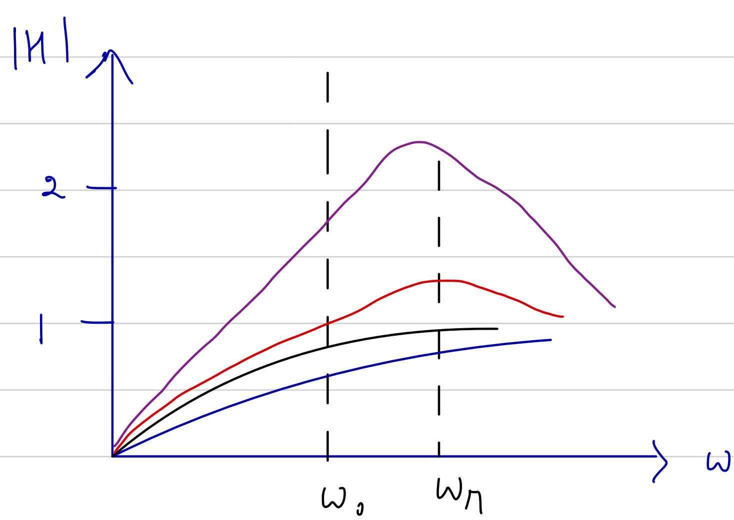
$$Q = \frac{1}{\sqrt{RC}} \times \frac{\sqrt{LC}}{RC} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$H(j\omega) = \frac{-(\omega/\omega_0)^2}{1 - \left(\frac{\omega}{\omega_0}\right)^2 + \frac{j\omega}{\omega_0}}$$

$$|H(j\omega)| = \frac{(\omega/\omega_0)}{\sqrt{\left(1 - \left(\frac{\omega}{\omega_0}\right)^2\right)^2 + \left(\frac{\omega}{Q\omega_0}\right)^2}}$$

$$\angle H(j\omega) = 180^\circ - \tan^{-1}\frac{(\omega/Q\omega_0)}{1 - \left(\frac{\omega}{\omega_0}\right)^2}$$

ω	$\lvert H \rvert$	$\angle H$
0	0	π
ω_0	Q	$\pi/2$
$\omega \gg \omega_0$	1	0



$Q = 0.5$

$Q = 0.707$

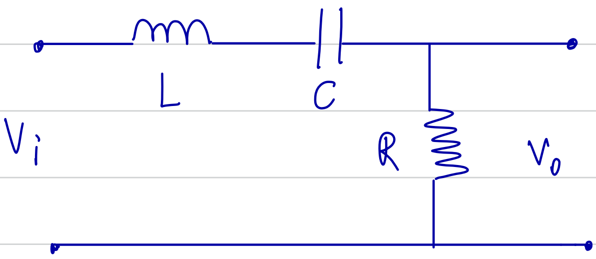
$Q = 1$ $\qquad$ $Q = 2$

ω_r - resonance frequency

at $Q = 1$

$$\omega_r = \frac{\omega_0}{\sqrt{1 - \frac{1}{2Q^2}}} \qquad |H|_{max} = \frac{Q}{\sqrt{1 - \frac{1}{4Q^2}}}$$

2nd order Band pass filter - (Prototype)



$$\frac{V_o}{V_i} = H(s) = \frac{R}{Ls + \frac{1}{Cs} + R} = \frac{RCs}{LCs^2 + RCs + 1} = \frac{(R/L)s}{s^2 + \left(\frac{R}{L}\right)s + \frac{1}{LC}}$$

$$H(s) = \frac{\frac{s}{Q\omega_o}}{\left(\frac{s}{\omega_o}\right)^2 + \frac{s}{Q\omega_o} + 1}$$

where $Q = \frac{1}{R}\sqrt{\frac{L}{C}}$

$\omega_o = \frac{1}{\sqrt{LC}}$

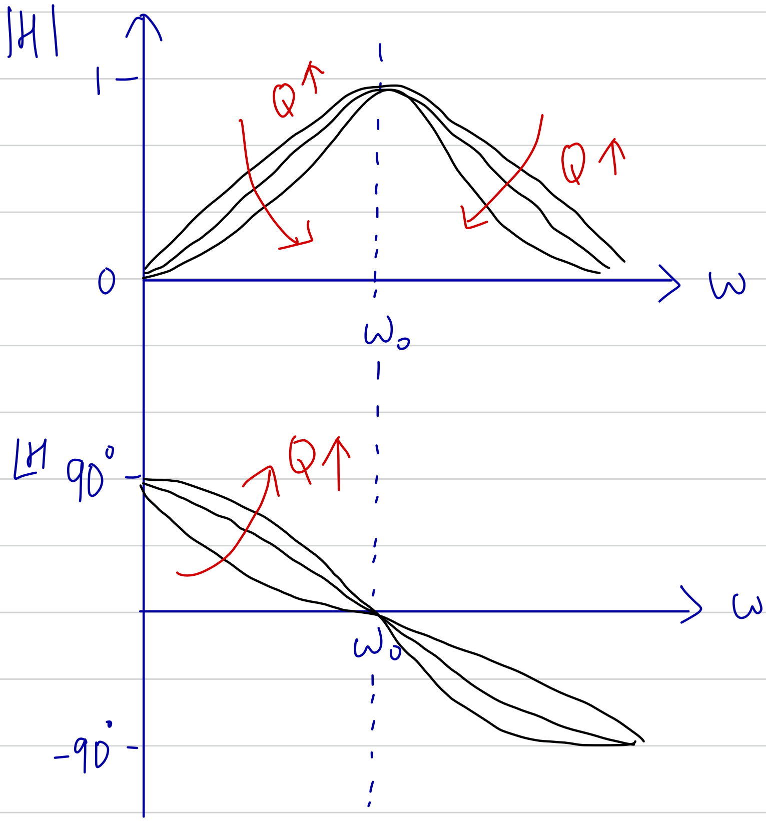
$$H(j\omega) = \frac{\frac{j\omega}{Q\omega_o}}{1 - \left(\frac{\omega}{\omega_o}\right)^2 + j\frac{\omega}{Q\omega_o}}$$

$$|H(j\omega)| = \frac{\frac{\omega}{Q\omega_o}}{\sqrt{\left(1 - \left(\frac{\omega}{\omega_o}\right)^2\right)^2 + \left(\frac{\omega}{Q\omega_o}\right)^2}}$$

$$\underline{|H(j\omega)} = \tan^{-1}\left(\frac{\omega/Q\omega_o}{0}\right) - \tan^{-1}\left(\frac{\omega/Q\omega_o}{1 - (\omega/\omega_o)^2}\right)$$

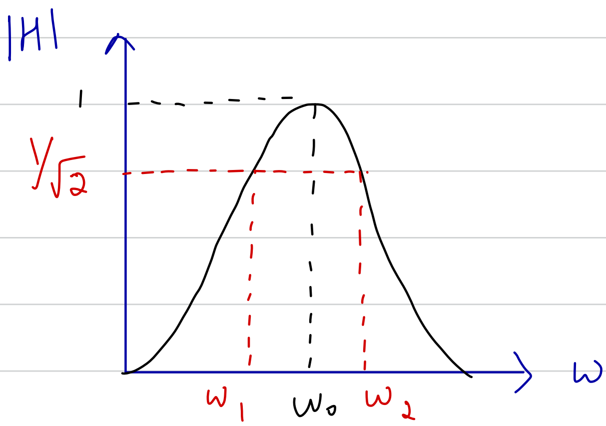
$$= 90^\circ - \tan^{-1}\left(\frac{\omega/Q\omega_o}{1 - (\omega/\omega_o)^2}\right)$$

ω	$\lvert H \rvert$	$\angle H$
0	0	90°
ω_0	1	0°
$\omega \gg \omega_0$	0	-90°



$$\frac{d|H|}{d\omega} = 0 \quad \text{at} \quad \omega_r = \omega_0$$

with $|H|_{max} = 1$



Bandwidth $= \omega_2 - \omega_1$ also $\omega_2 - \omega_1 = \dfrac{\omega_0}{Q}$

So as Q increases bandwidth decreases.

Proof: At ω_1 & ω_2 $|H| = 1/\sqrt{2}$

$$|H|^2 = \frac{1}{2}$$

$$|H|^2 = \frac{(\omega/Q\omega_0)^2}{\left(1-\left(\frac{\omega}{\omega_0}\right)^2\right)^2 + \left(\frac{\omega}{Q\omega_0}\right)^2} = \frac{1}{2}$$

$$\Rightarrow 2\left(\frac{\omega}{Q\omega_0}\right)^2 = \left(1-\left(\frac{\omega}{\omega_0}\right)^2\right)^2 + \left(\frac{\omega}{Q\omega_0}\right)^2$$

$$\Rightarrow \left(\frac{\omega}{Q\omega_0}\right)^2 = \left(1-\left(\frac{\omega}{\omega_0}\right)^2\right)^2$$

Let $\frac{\omega}{\omega_0} = \alpha$

$$\frac{\alpha^2}{Q^2} = 1 - 2\alpha^2 + \alpha^4$$

$$\Rightarrow \alpha^4 - \alpha^2\left(2 + \frac{1}{Q^2}\right) + 1 = 0$$

$$\alpha^2 = \frac{\left(2+\frac{1}{Q^2}\right) \pm \sqrt{\left(2+\frac{1}{Q^2}\right)^2 - 4}}{2}$$

$$\alpha^2 = \frac{2+\frac{1}{Q^2} \pm \sqrt{\frac{1}{Q^4} + 4 + \frac{4}{Q^2} - 4}}{2}$$

$$\alpha^2 = \frac{1}{2}\left(2+\frac{1}{Q^2} \pm \frac{1}{Q}\sqrt{\frac{1}{Q^2}+4}\right)$$

$$\omega^2 = \omega_0^2\alpha^2$$

$$= \frac{\omega_0^2}{2}\left(\left(\frac{1}{Q^2}+2\right) \pm \frac{1}{Q}\sqrt{4+\frac{1}{Q^2}}\right)$$

Let $\left(\frac{1}{Q^2} + 2\right) = a$ & $\left(\frac{1}{Q}\sqrt{4 + \frac{1}{Q^2}}\right) = b$

$$\omega_2^2 = \frac{\omega_0^2}{2}(a+b)$$

$$\omega_1^2 = \frac{\omega_0^2}{2}(a-b)$$

$$\omega_2^2\omega_1^2 = \frac{\omega_0^4}{4}\left(a^2 - b^2\right) = \frac{\omega_0^4}{4}\left(4 + \frac{1}{Q^4} + \frac{4}{Q^2} - \frac{1}{Q^4} - \frac{4}{Q^2}\right) = \omega_0^4$$

$$\therefore \boxed{\omega_0 = \sqrt{\omega_1\omega_2}}$$

$$(\omega_2 - \omega_1)^2 = \frac{\omega_0^2}{2}\left(\sqrt{a+b} - \sqrt{a-b}\right)^2$$

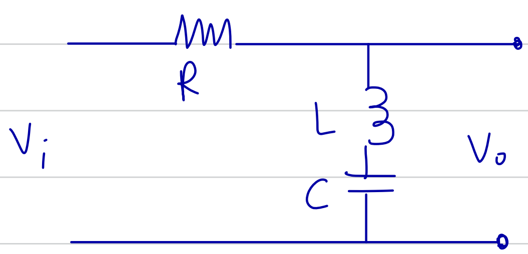
$$= \frac{\omega_0^2}{2}\left(a+b+a-b - 2\sqrt{a^2-b^2}\right)$$

$$= \omega_0^2\left(a - \sqrt{a^2-b^2}\right)$$

$$= \omega_0^2\left(2 + \frac{1}{Q^2} - \sqrt{4}\right) = \frac{\omega_0^2}{Q^2}$$

$$\therefore \boxed{\omega_2 - \omega_1 = \frac{\omega_0}{Q}}$$

Second order band reject filter - (Prototype) (notch - filter)



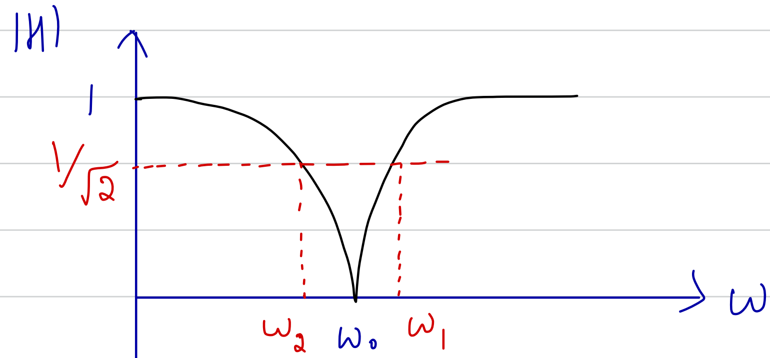
$$\frac{V_o}{V_i} = H(s) = \frac{1 + LCs^2}{1 + RCs + LCs^2} = \frac{(s/\omega_o)^2 + 1}{\left(\frac{s}{\omega_o}\right)^2 + \frac{s}{Q\omega_o} + 1}, \quad \omega_o = \frac{1}{\sqrt{LC}}$$

$$H(j\omega) = \frac{1 - \frac{\omega^2}{\omega_o^2}}{1 - \left(\frac{\omega}{\omega_o}\right)^2 + \frac{j\omega}{Q\omega_o}}$$

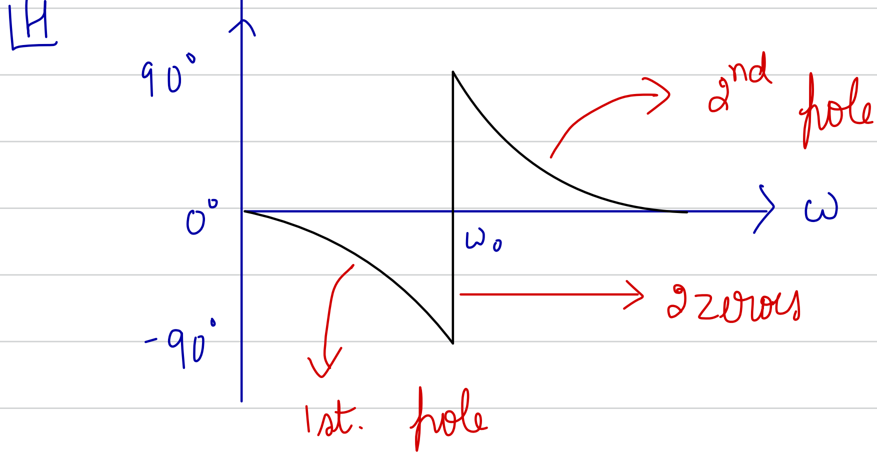
$$|H(j\omega)| = \frac{1 - \left(\frac{\omega}{\omega_o}\right)^2}{\sqrt{\left(1 - \left(\frac{\omega}{\omega_o}\right)^2\right)^2 + \left(\frac{\omega}{Q\omega_o}\right)^2}}$$

$$\angle H(j\omega) = -\tan^{-1}\left(\frac{(\omega/Q\omega_o)}{1 - (\omega/\omega_o)^2}\right)$$

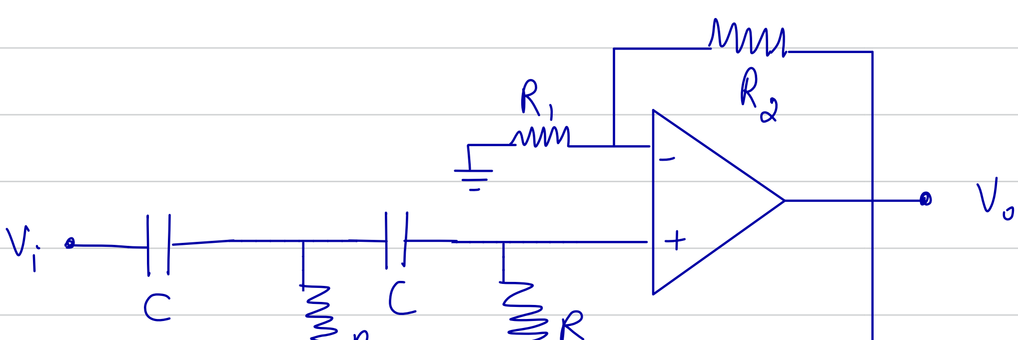
ω	$\|H\|$	$\angle H$
0	1	0°
ω_o	0	-90° (at $\omega = \omega_o^-$) & 90° (at $\omega = \omega_o^+$)
$\omega >>> \omega_o$	1	0°



$$\omega_1 - \omega_2 = \frac{\omega_0}{Q}$$



Second order Sallen-key HPF



$$V_o = \frac{k\,(RCs)^2}{(RCs)^2 + (3-k)RCs + 1}$$

$$\begin{bmatrix} 1 & 1/sC \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1/R & 1 \end{bmatrix} \begin{bmatrix} 1 & 1/sC \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1/R & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 + \frac{1}{RCs} & \frac{1}{sC} \\ \frac{1}{R} & 1 \end{bmatrix} \begin{bmatrix} 1 + \frac{1}{RCs} & \frac{1}{sC} \\ \frac{1}{R} & 1 \end{bmatrix}$$

$$\frac{V_p}{V_o} = \frac{1}{A} = \frac{1}{\left(1 + \frac{1}{RCs}\right)^2 + \frac{1}{RCs}}$$

$$V_p = \frac{R^2 C^2 s^2}{(RCs)^2 + 3RCs + 1} V_o$$

$$\therefore \; V_o = \left(1 + \frac{R_2}{R_1}\right) V_p = k V_p = \frac{k R^2 C^2 s^2}{(RCs)^2 + 3RCs + 1}$$

$$V_o = k V_p$$

$$\frac{V_i - V_1}{1/sC} = \frac{V_1 - V_o}{R} + \frac{V_1 - V_p}{1/sC}$$

$$\therefore \; V_i - V_1 = \frac{V_1 - V_o}{RCs} + V_1 - V_p$$

$$\frac{V_1 - V_p}{1/sC} = \frac{V_p}{R}$$

$$V_1 = V_p + \frac{V_p}{RCs}$$

$$\therefore \; V_i = \frac{-V_o}{RCs} + 2V_1 - \frac{V_1}{RCs} - V_p$$

$$= \frac{-V_o}{RCs} - V_p + 2V_p + \frac{2V_p}{RCs} + \frac{V_p}{RCs} + \frac{V_p}{(RCs)^2}$$

$$\therefore \; V_i = \frac{(3-k)}{RCs} V_p + \frac{V_p}{(RCs)^2} + V_p$$

$$V_i = \frac{V_0}{R}\left[\frac{(RCs)^2 + (3-k)RCs + 1}{(RCs)^2}\right]$$

$$\therefore \frac{V_0}{V_i} = H(s) = \frac{k(RCs)^2}{(RCs)^2 + (3-k)RCs + 1}$$

$$H(j\omega) = \frac{k(s/\omega_0)^2}{\left(\frac{s}{\omega_0}\right)^2 + \frac{s}{Q\omega_0} + 1}$$

$$|H(j\omega)| = \frac{k\left(\frac{\omega}{\omega_0}\right)^2}{\sqrt{\left(1-\left(\frac{\omega}{\omega_0}\right)^2\right)^2 + \left(\frac{\omega}{Q\omega_0}\right)^2}}$$

$$\angle H = 180^\circ - \tan^{-1}\left(\frac{(\omega/Q\omega_0)}{1-\left(\frac{\omega}{\omega_0}\right)^2}\right)$$

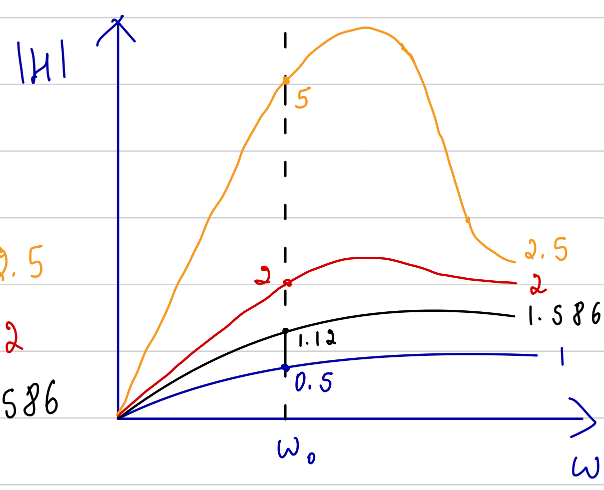
ω	$\lvert H\rvert$	$\angle H$
0	0	180°
ω_0	kQ	90°
$\omega \gg \omega_0$	k	0°

$Q = 2$, $K = 2.5$

$Q = 1$, $K = 2$

$Q = 0.707$, $K = 1.586$

$Q = 0.5$, $K = 1$



Butterworth response:

Pole frequency = cutoff frequency. , $Q = 0.707 = \frac{1}{\sqrt{2}}$

Consider the general equation for LPF:

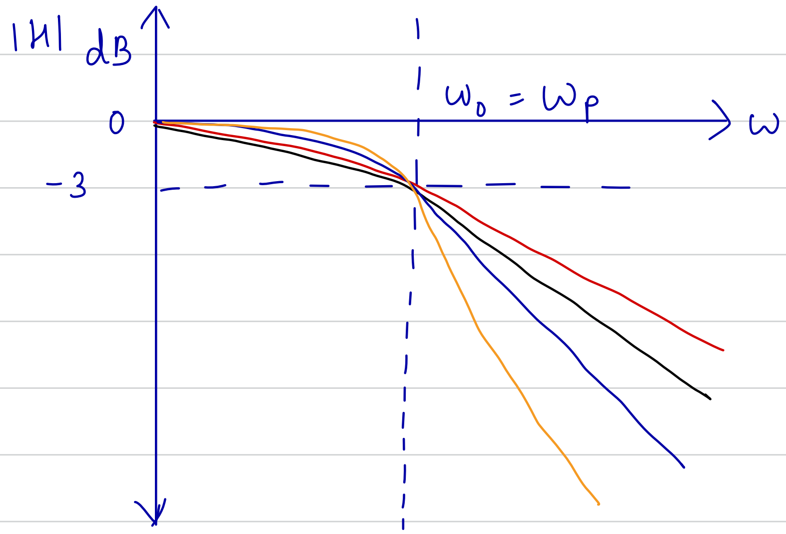
$$|H(j\omega)| = \frac{1}{\sqrt{\left(1-\left(\frac{\omega}{\omega_0}\right)^2\right)^2 + \left(\frac{\omega}{Q\omega_0}\right)^2}}$$

$$|H(j\omega)|_{Q=0.707} = \frac{1}{\sqrt{1+\left(\frac{\omega}{\omega_0}\right)^4}}$$

For n^{th} order butterworth response:

$$|H(j\omega)| = \frac{1}{\sqrt{1+\left(\frac{\omega}{\omega_0}\right)^{2n}}}$$

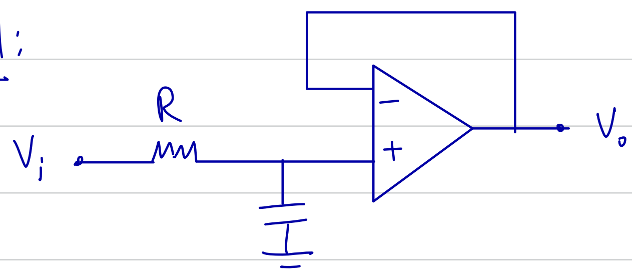
n	$\|H\|$ at		
	$\omega = 0.9\omega_0$	$\omega = \omega_0$	$\omega = 1.1\omega_0$
1	0.746 (-2.545 dB)	0.71 (-2.97 dB)	0.67 (-3.478 dB)
2	0.78 (-2.15 dB)	0.71 (-2.97 dB)	0.64 (-3.876 dB)
3	0.81 (-1.83 dB)	0.71 (-2.97 dB)	0.6 (-4.43 dB)
4	0.84 (-1.51 dB)	0.71 (-2.97 dB)	0.56 (-5.036 dB)



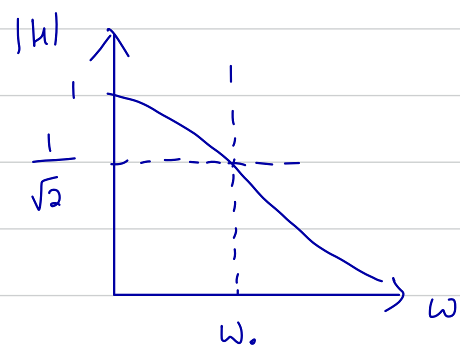
$n = 4$, -80 dB/dec
$n = 3$, -60 dB/dec
$n = 2$, -40 dB/dec
$n = 1$, -20 dB/dec

Consider Sallen key stages:

$n = 1$:

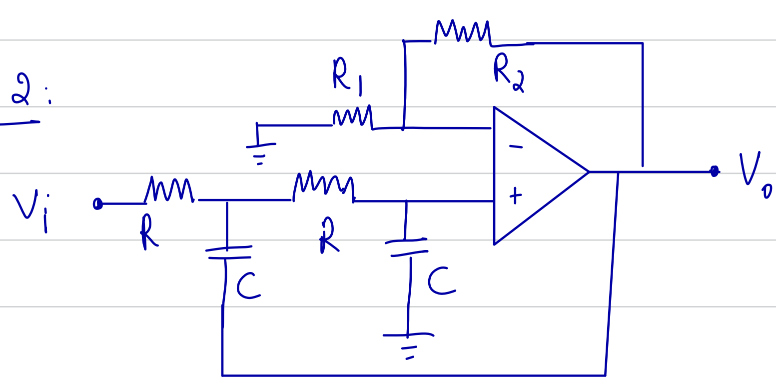


$$H(s) = \frac{1}{1 + RCs} = \frac{1}{1 + \frac{s}{\omega_0}}$$



$$|H(j\omega)| = \frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_0}\right)^2}}$$

$n = 2$:



$$Q = \frac{1}{3 - k}$$

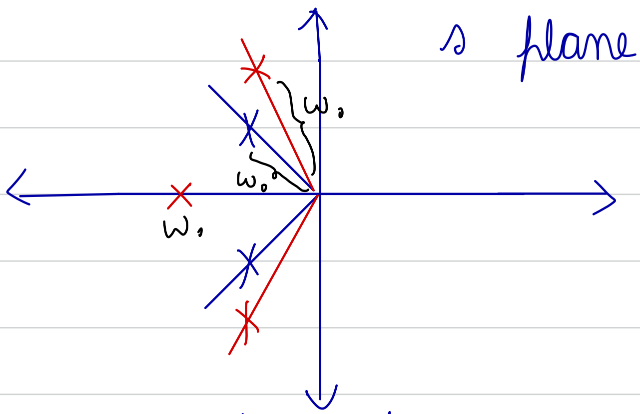
$$k = 1 + \frac{R_2}{R_1}$$

At $\omega = \omega_0$, $|H| = k\,Q$

$|H(j\omega)|$ at $\omega = 0$ $= k$

$$|H(j\omega_0)| = \frac{|H(j0^{\circ})|}{\sqrt{2}} \qquad \therefore\ Q = \frac{1}{\sqrt{2}}$$

Pole locations



$n=2$

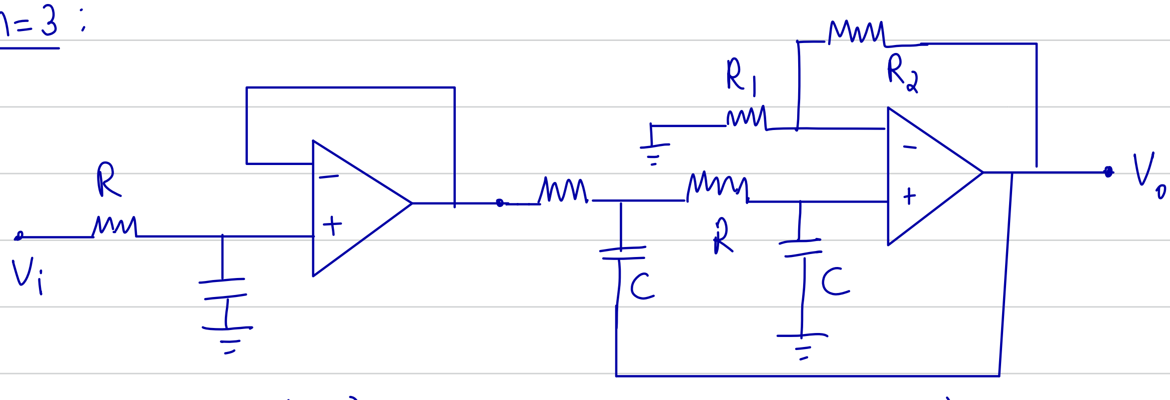
$n=3$

$\xi = \cos\theta$

Angle subtended between any 2 poles $= 180^{\circ}/n$

$Q = \frac{1}{\sqrt{2}} = \frac{1}{2\xi}$ where $\xi = 0.707$

$n=3$:



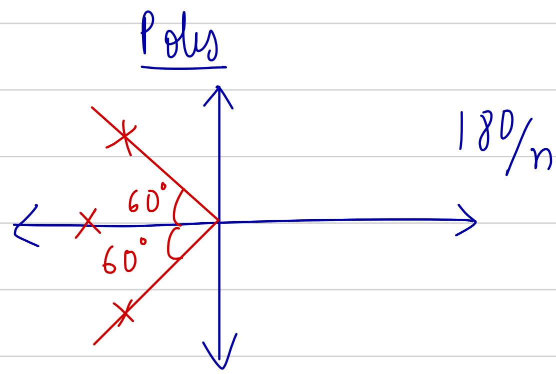
$H_1(j\omega)$	$H_2(j\omega)$
k_1	$k_2 Q_2$
$H_1(j\omega)\big\vert_{\omega=\omega_0} = 1/\sqrt{2}$	$H_2(j\omega)\big\vert_{\omega=\omega_0} = k_2 Q_2$
DC gain $= K_1 = 1$	DC gain $= k_2$

At $\omega = \omega_0$ $\quad |H(j\omega)| = \dfrac{|H(j0)|}{\sqrt{2}}$ for butterworth response

$$|H(j\omega)| = |H_1(j\omega)|\,|H_2(j\omega)|$$

$$|H(j\omega)|_{\omega=0^\circ} = |H_1(j\omega)|_{\omega=0^\circ}\,|H_2(j\omega)|_{\omega=0^\circ} = k_1 k_2 = k_2$$

$$|H(j\omega)|_{\omega=\omega_0} = \frac{k_2}{\sqrt{2}}$$



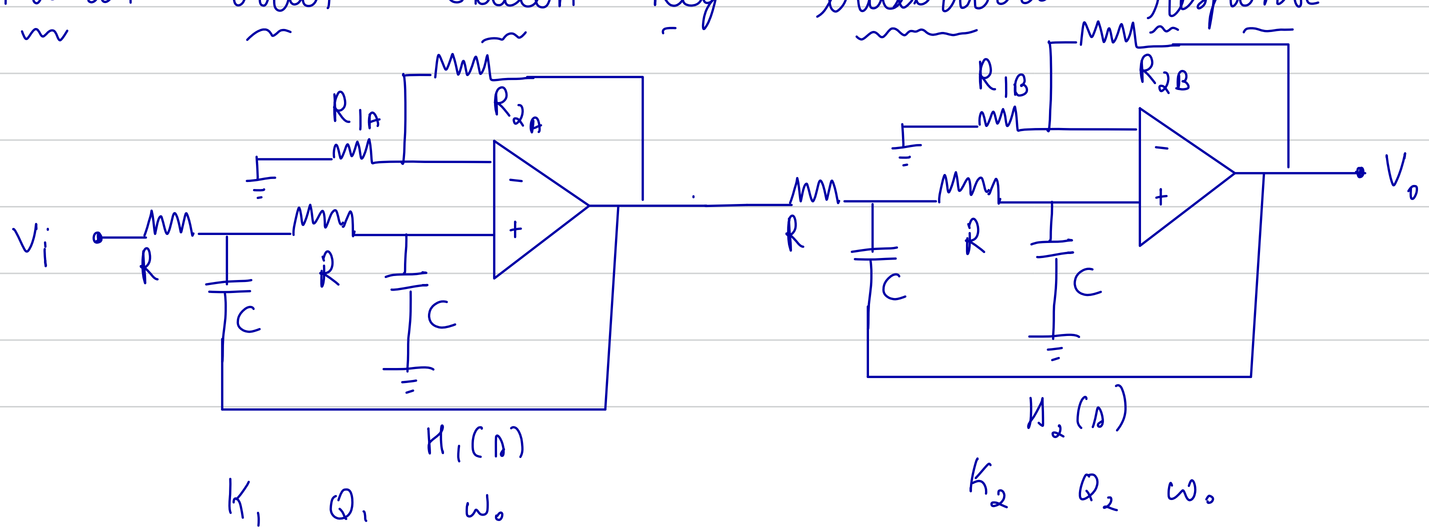
$$\frac{k_2 Q_2}{\sqrt{2}} = \frac{k_2}{\sqrt{2}}$$

$\Rightarrow \quad Q_2 = 1 \quad , \quad k_2 = 2$

$$H(s) = \frac{k_2}{(1+s)(s^2+s+1)}$$ with $\omega_0 = 1$ rad/s

$$|H(j\omega)| = \frac{1}{\sqrt{1+\left(\frac{\omega}{\omega_0}\right)^6}}$$

Fourth order Sallen-key butterworth response



$$|H(j0^\circ)| = |H_1(j0^\circ)|\,|H_2(j0^\circ)|$$
$$= k_1 k_2$$

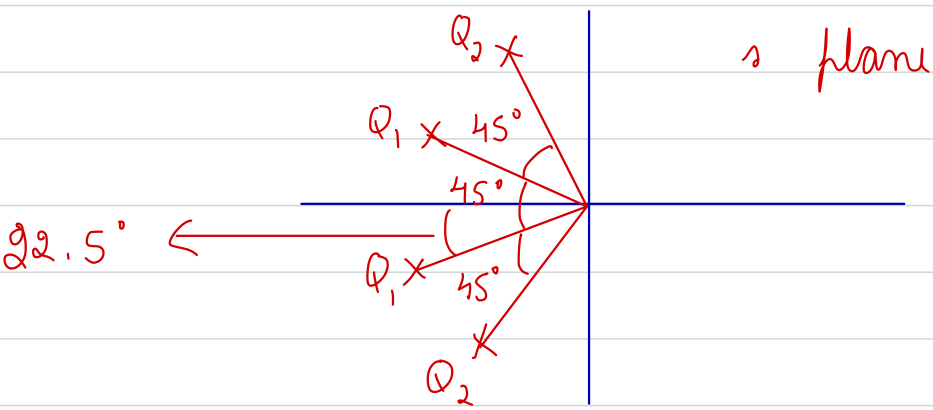
$$|H(j\omega_0)| = |H_1(j\omega_0)|\,|H_2(j\omega_0)|$$
$$= k_1 Q_1\, k_2 Q_2$$

$$|H(j\omega_0)| = \frac{|H(j0^\circ)|}{\sqrt{2}}$$

$$\frac{k_1 k_2}{\sqrt{2}} = k_1 k_2\, Q_1 Q_2$$

$$Q_1 Q_2 = \frac{1}{\sqrt{2}}$$

4 poles : $\frac{180}{4} = 45^\circ$



$$Q_1 = \frac{1}{2\xi_1} = \frac{1}{2\cos\theta_1} = \frac{1}{2\cos(22.5^\circ)} = 0.5414$$

$$Q_2 = \frac{1}{2\xi_2} = \frac{1}{2\cos\theta_2} = \frac{1}{2\cos(45^\circ + 22.5^\circ)} = 1.306$$

$$Q_1 Q_2 = \frac{1}{\sqrt{2}}$$

$K_1 K_2 \Rightarrow K_1 = 3 - \frac{1}{Q_1} = 1.153 = 1 + \frac{R_{2A}}{R_{1A}}$

$K_2 = 3 - \frac{1}{Q_2} = 2.234 = 1 + \frac{R_{2B}}{R_{1B}}$

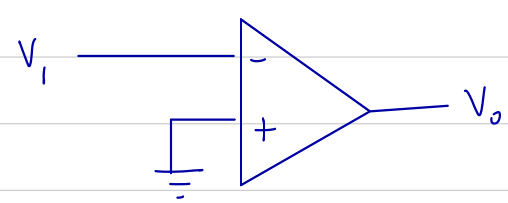
$K_1 K_2 = |H(j0^\circ)| = 2.5758$

$$H(j\omega) = \frac{k_1 k_2}{\left[\left(\frac{s}{\omega_0}\right)^2 + \frac{s}{Q_1 \omega_0} + 1\right]\left[\left(\frac{s}{\omega_0}\right)^2 + \frac{s}{Q_2 \omega_0} + 1\right]}$$

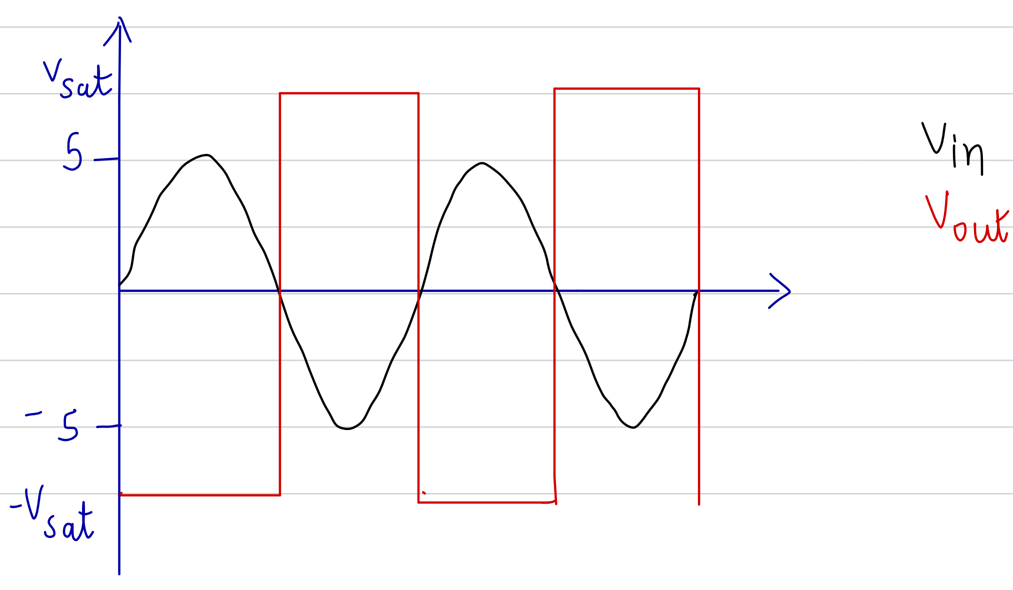
$$|H(j\omega)| = \frac{k_1 k_2}{\sqrt{1 + \left(\frac{\omega}{\omega_0}\right)^8}}$$

Experiment 10: Comparator & Schmitt triggers

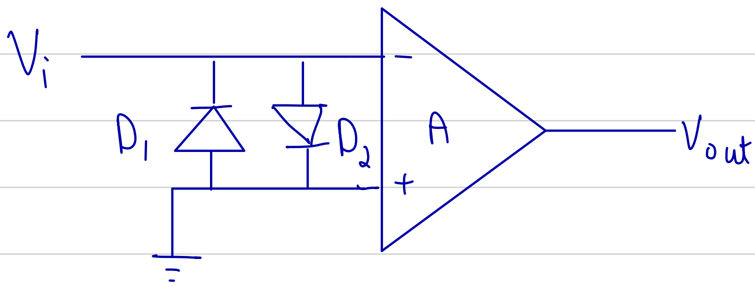
Comparator:



→ Zero crossing Detector (ZCD)

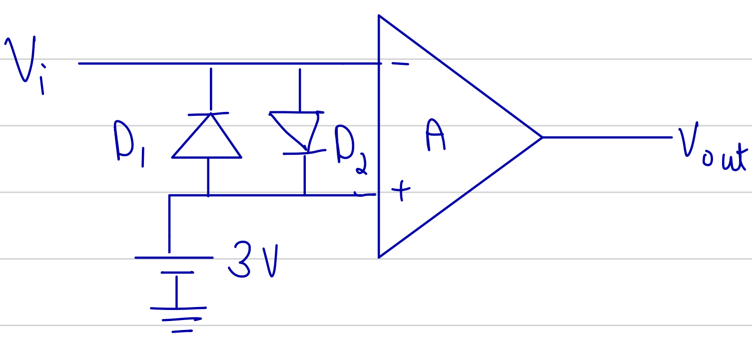


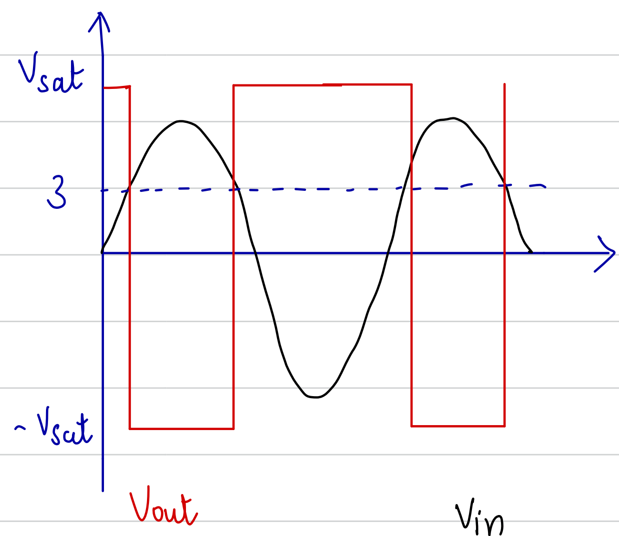
In lab:



D_1 & D_2 are connected to maintain small differential voltage.

To detect 3 V crossing:

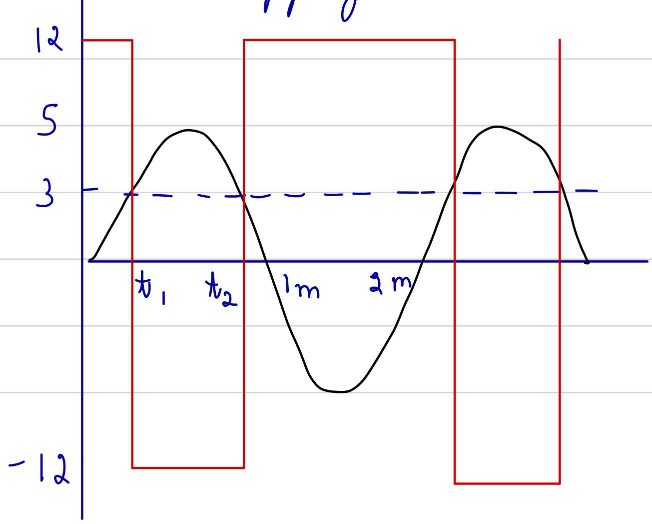




Q> For $V_{in} = 5 \sin 1000\pi t$. Indicate all the salient features assuming ideal opamp comparator with power supply ± 12 V with $V_{ref} = 3$ V

Ans

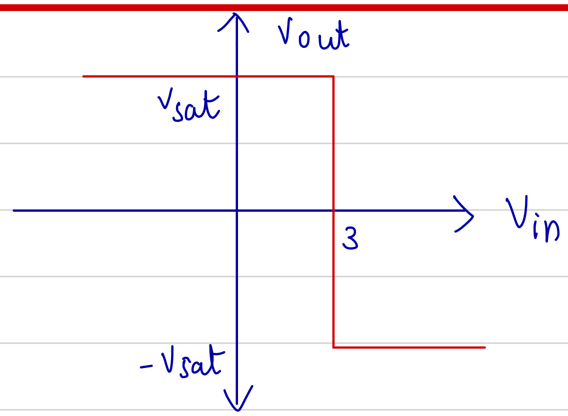
V_{out}
V_{in}



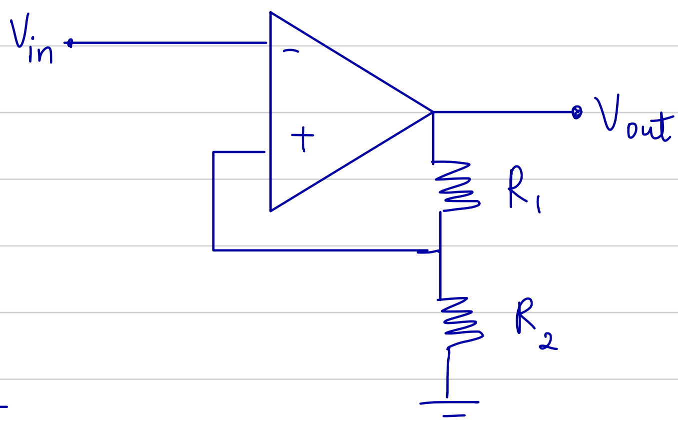
$t_1 = 0.2$ ms

$t_2 = \frac{T}{2} - t_1$

$= 0.8$ ms



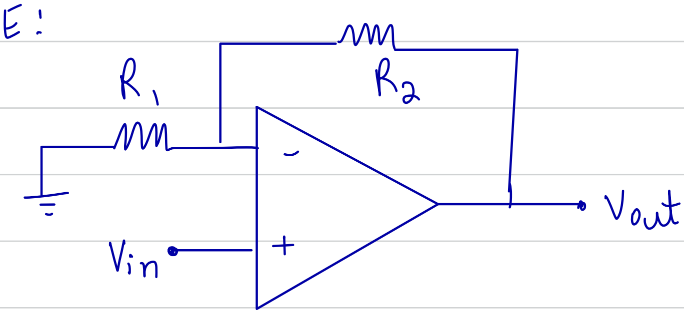
i> Inverting Schmitt trigger:



reinforce Perturbation (noise/ disturbance)

→ Regenerative effect

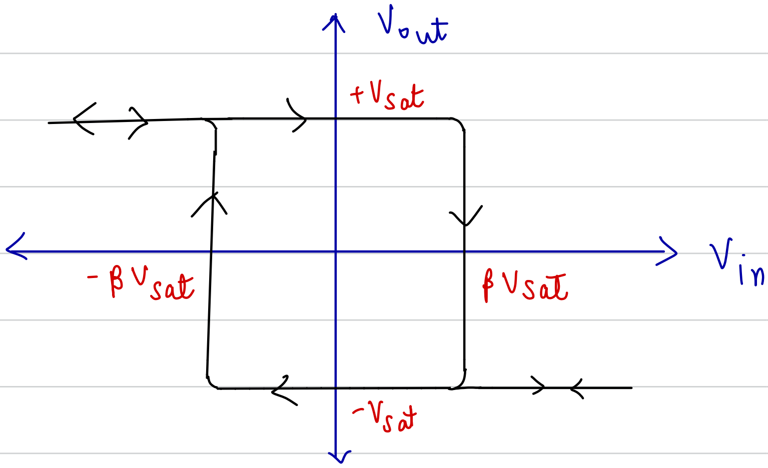
NOTE:

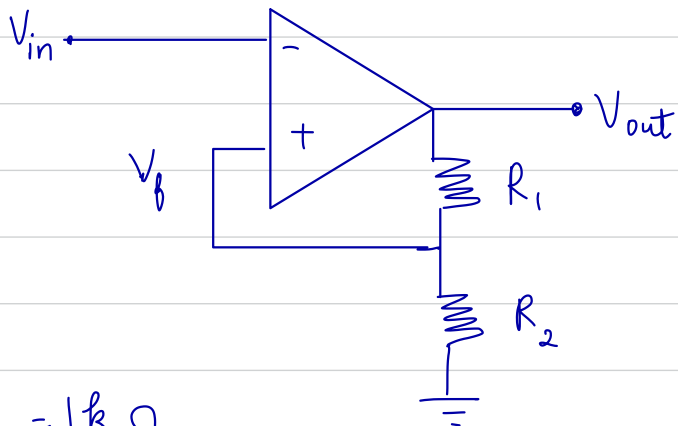


→ Negative feedback

→ Neutralises Perturbation

$$V_f = \frac{R_2 V_{sat}}{R_1+R_2} = \beta V_{sat}$$





If $V_{sat} = 12$

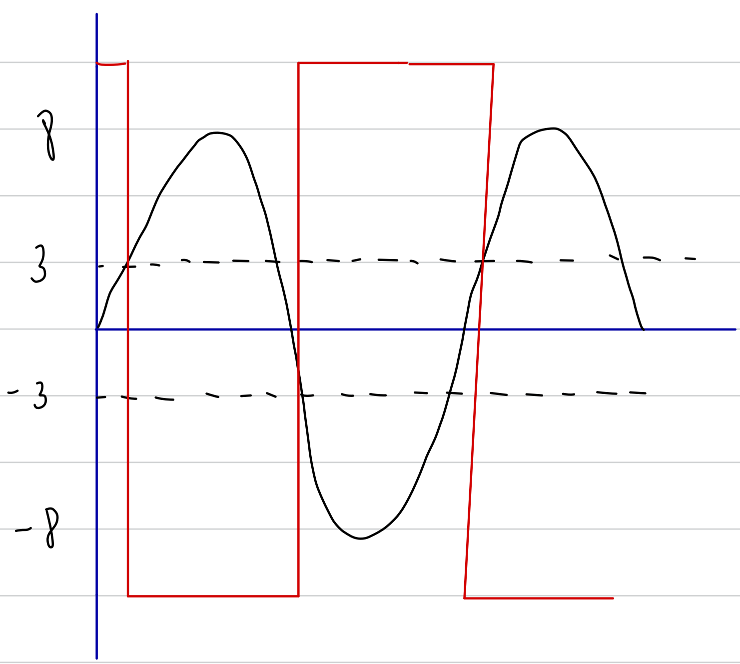
$$\beta = \frac{R_2}{R_1 + R_2}$$

$$\beta = \frac{1}{4}$$

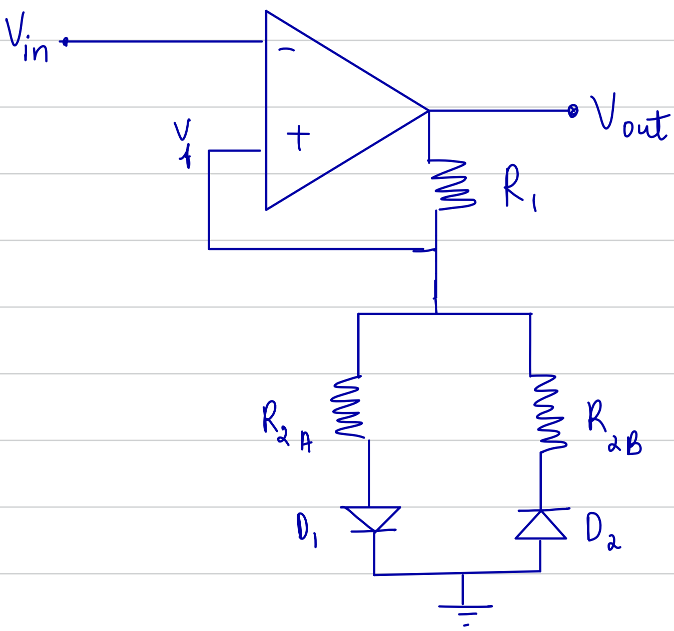
$R_2 = 1k\Omega$

$R_1 = 3k\Omega$

$\therefore V_f = 3V$



a>



If $V_o = V_{sat}$

$$V_f = \frac{R_{2A}}{R_{2A} + R_1} V_{sat} \quad \text{(Ideal)}$$

If $V_o = -V_{sat}$

$$V_f = \frac{-R_{2B}}{R_{2B} + R_1} (V_{sat}) \quad \text{(Ideal)}$$

Practical :

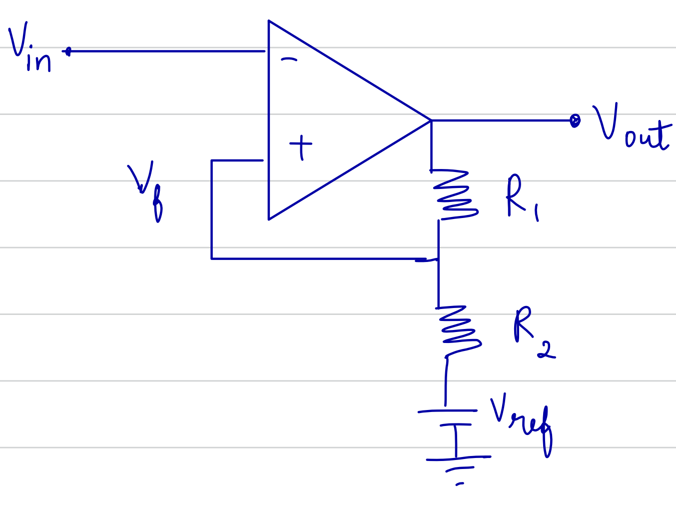
$V_o = V_{sat}$

$$V_f = \frac{R_{2A}}{R_{2A} + R_1}\left(V_{sat} - V_{D_{on}}\right)$$

$V_o = -V_{sat}$

$$V_f = \frac{-R_{2B}}{R_{2B} + R_1}\left(V_{sat} - V_{D_{on}}\right)$$

b>



$V_f > V_i \longrightarrow V_o = V_{sat}$

$V_f < V_i \longrightarrow V_o = -V_{sat}$

$$V_f = \frac{R_2 V_{sat}}{R_1 + R_2} + \frac{R_1 V_{ref}}{R_1 + R_2} = V_{UT} \quad \text{at } V_o = V_{sat}$$

UT → upper threshold LT - lower threshold at $V_0 = -V_{sat}$, $V_f = \frac{-R_2 V_{sat}}{R_1 + R_2} + \frac{R_1 V_{ref}}{R_1 + R_2} = V_{LT}$

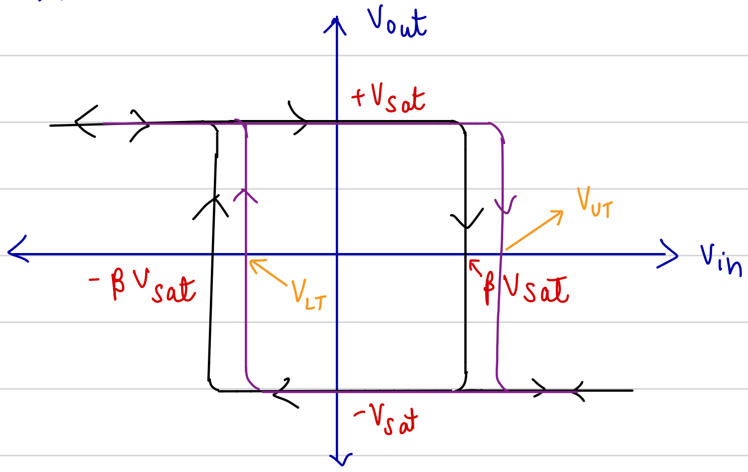
Also $V_{UT} = \beta V_{sat} + \frac{R_1}{R_2} \beta V_{ref}$ at $V_0 = V_{sat}$

$V_{LT} = -\beta V_{sat} + \frac{R_1}{R_2} \beta V_{ref}$ at $V_0 = -V_{sat}$

$V_{UT} - V_{LT} = 2\beta V_{sat}$ → width of hysteresis curve

$V_{UT} + V_{LT} = 2 \frac{R_1}{R_2} \beta V_{ref}$

$V_{UT} - V_{LT}$ is also known as Deadzone or deadband



Q> Design a schmitt trigger for $V_{UT} = 5\,V$ $V_{LT} = -3V$

$R_2 = 1\,k\Omega$

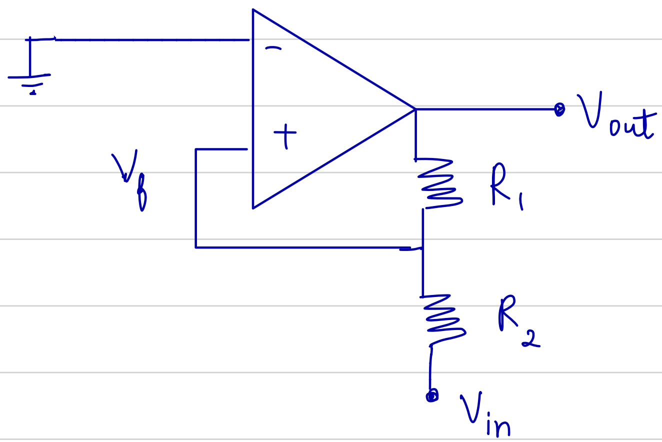
$R_1 = 2\,k\Omega$

Ans $V_{UT} = \beta V_{sat} + \frac{R_1}{R_2} \beta V_{ref}$

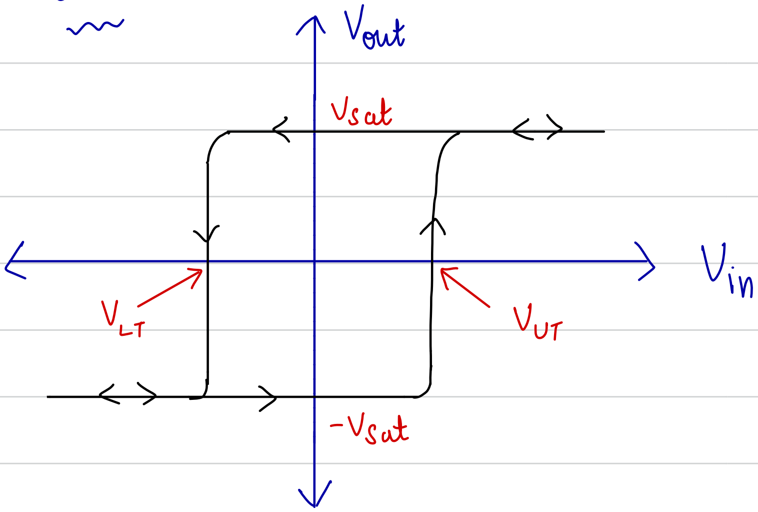
$\beta = \frac{R_2}{R_1 + R_2} = \frac{1}{3}$ $V_{sat} = 12$

$\therefore 5 = 4 + \frac{2}{3} V_{ref}$ $\therefore V_{ref} = 1.5\,V$

ii) Non Inverting Schmitt trigger



Hysteresis curve:



Non inverting Schmitt trigger = Inverting schmitt trigger + ZCD.

$$V_f = \frac{R_2 V_{sat}}{R_1 + R_2} + \frac{R_1 V_i}{R_1 + R_2} \qquad \text{at } V_o = V_{sat}$$

$V_f > 0 \longrightarrow V_o = +V_{sat}$

$V_f < 0 \longrightarrow V_o = -V_{sat}$

$$\frac{R_2 V_{sat}}{R_1 + R_2} > \frac{-R_1 V_i}{R_1 + R_2} \qquad \text{at} \quad V_o = V_{sat}$$

$$\Rightarrow \quad V_i > \frac{-R_2}{R_1} V_{sat}$$

$$\therefore \quad V_{LT} = -\frac{R_2}{R_1} V_{sat}$$

$$V_f = \frac{-R_2 V_{sat}}{R_1 + R_2} + \frac{R_1 V_i}{R_1 + R_2} \qquad \text{at} \quad V_o = -V_{sat}$$

$$\frac{R_1 V_i}{R_1 + R_2} < \frac{R_2 V_{sat}}{R_1 + R_2}$$

$$V_i < \frac{R_1}{R_1} V_{sat}$$

$$\Rightarrow \quad V_{UT} = \frac{R_2}{R_1} V_{sat}$$

If V_{ref} is used.

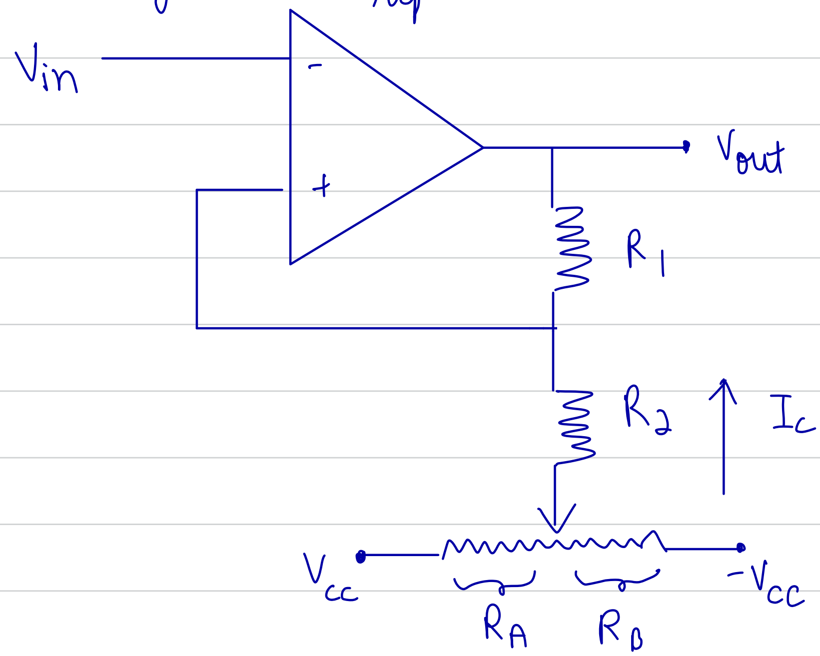
at $V_o = V_{sat}$, $V_f > V_{ref}$

at $V = -V_{sat}$ $\qquad V_f < V_{ref}$

$$\therefore \quad V_{UT} = \frac{R_1 + R_2}{R_1} V_{ref} + \frac{R_2}{R_1} V_{sat}$$

$$\therefore \quad V_{LT} = \frac{R_1 + R_2}{R_1} V_{ref} - \frac{R_2}{R_1} V_{sat}$$

To get V_{ref} we connect in following manner

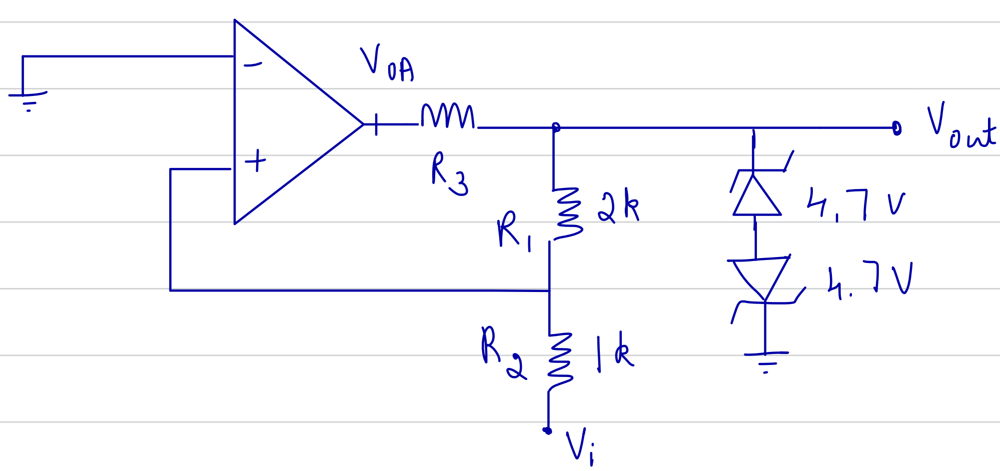


$$V_{ref} = \frac{R_A}{R}(-V_{cc}) + \frac{R_B}{R} V_{cc}$$ ($R \to$ Total resistance of potentiometer)

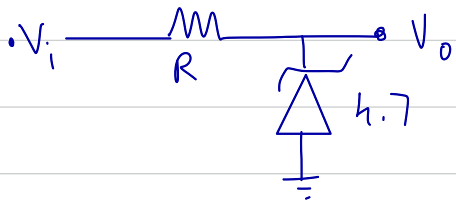
To avoid damage of opamp

$R_1, R_2 >> R$ so that $I_A \approx I_B$ if $I_C = 0$

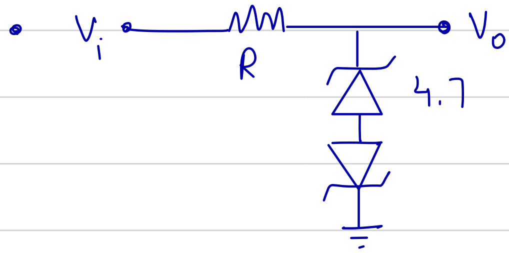
Ex: Plot V_o v/s V_i. Find R_3 min. $V_i = 8 \sin \omega t$
Power supply = 12V



NOTE



$V_0 = 4.7V$



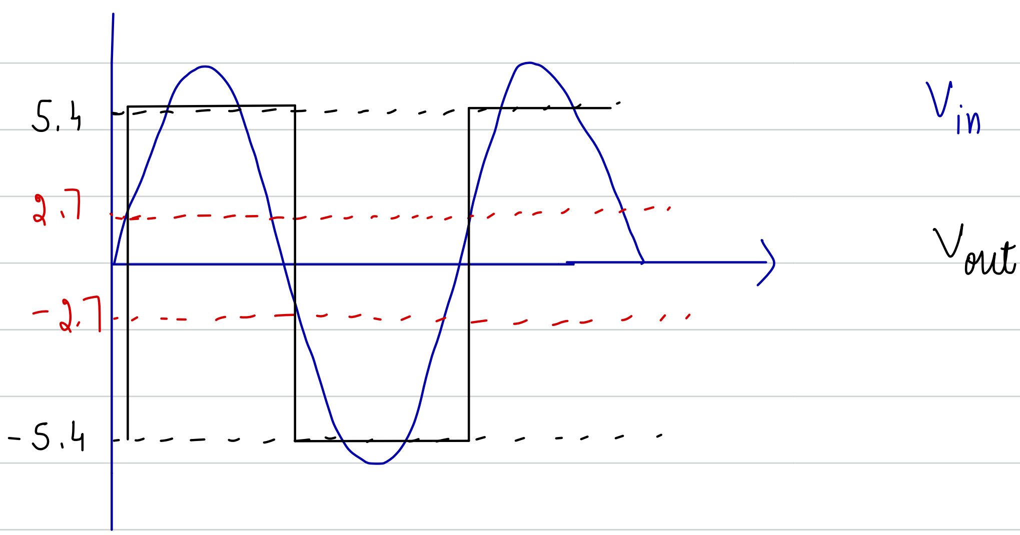
$V_0 = 4.7 + 0.7 = 5.4V$

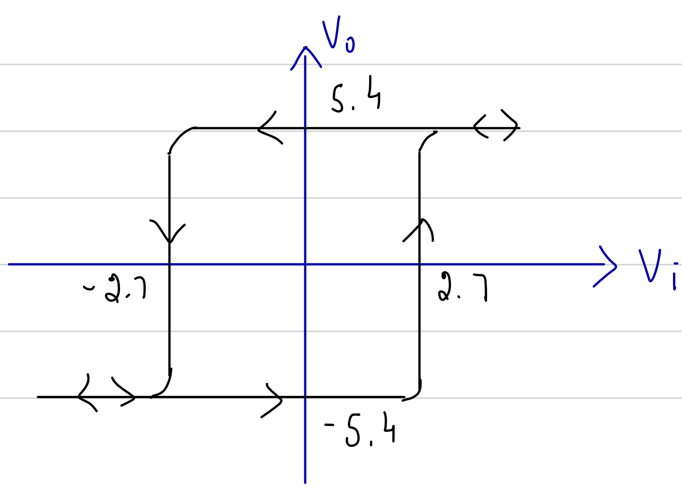
- R is connected to prevent zener from drawing very large current.

Ans $\therefore \quad V_{clamp} = \pm 5.4 \ V$

$V_{UT} = \frac{R_2}{R_1} = \frac{1}{2} \times 5.4 = 2.7V$ $\quad$ $|||^{ly}$ $\quad V_{LT} = -2.7V$

$V_{out} = \pm 5.4V$

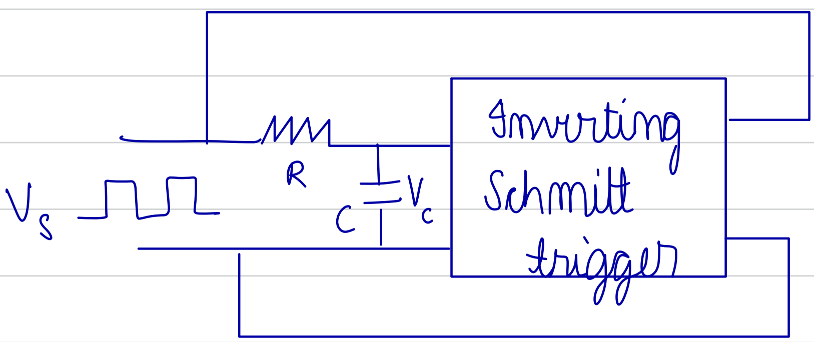


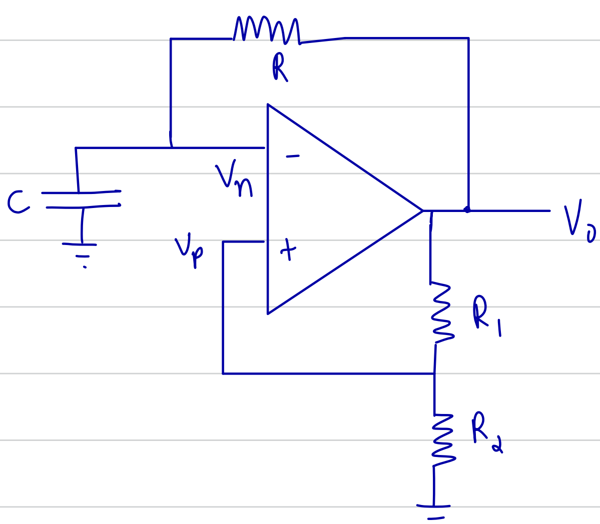


$$\frac{12-5.4}{R_3} < 25\,mA$$

$R_3 > 264\,\Omega$ $\therefore$ $R_3\ min = 264\,\Omega$

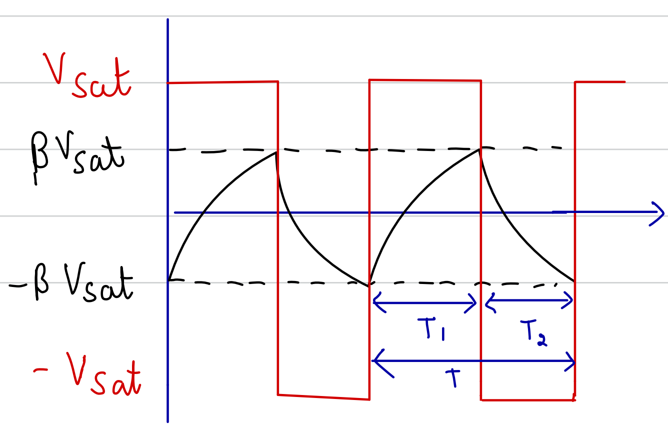
Astable multivibrator





Positive feedback is dominant as there is a capacitor in negative feedback which takes time to change

$$V_p = \frac{R_2}{R_1+R_2} V_{sat} = \beta V_{sat}$$



Duty cycle = 50%

$T_1 = T_2$

$$V_c = V_f + (V_i - V_f)\, e^{-t/\tau}$$

$$\beta V_{sat} = V_{sat} + (-\beta V_{sat} - V_{sat})\, e^{-T_2/RC}$$

$$\beta = 1 - (1+\beta)\, e^{-T_2/RC}$$

$$e^{-T_2/RC} = \frac{1+\beta}{1-\beta}$$

$$\Rightarrow T_2 = RC \ln\left(\frac{1+\beta}{1-\beta}\right) = T_1$$

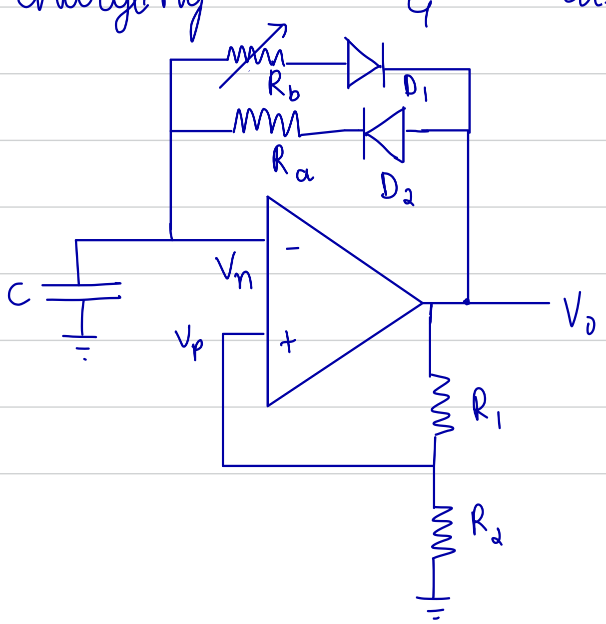
$$T_2 = T_1 = RC \ln\left(\frac{2R_2 + R_1}{R_1}\right)$$

$$\therefore \boxed{T = 2T_2 = 2RC \ln\left(\frac{2R_2 + R_1}{R_1}\right)}$$

If $R_1 = R_2$ $\qquad T = 2RC \ln 3$

$$T = 2.2RC$$

To change duty cycle we can either connect V_{ref} or we can have 2 different paths for charging & discharging.

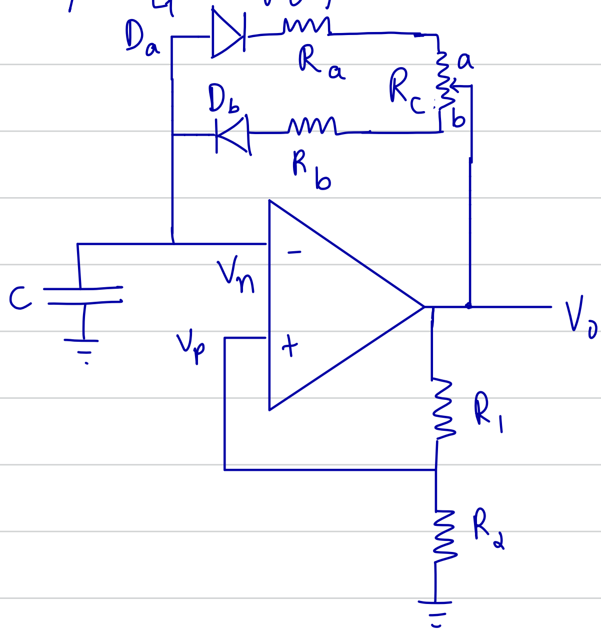


$T_{ON} = R_a C \ln 3$

$T_{OFF} = R_b C \ln 3$

$\therefore \boxed{\text{Duty cycle} = \frac{T_{ON}}{T} = \frac{R_a}{R_a + R_b}}$

Q Design an Astable multivibrator using an opamp having 1kHz frequency with duty cycle varying between 20% & 80%.



Ans $T = T_{on} + T_{off} = 1ms$

For 20% duty cycle $T_{on} = 0.2$ ms

$= 1.1 RC$

$= 1.1 R_b C$

Let $C = 0.1 \mu F$

$\therefore R_b = 1.81 \ k\Omega$

$T_{off} = 0.8ms = 1.1 RC = 1.1 (R_a + R_c) C$

$R_a = R_b = 1.81 \ k\Omega$

$R_c = 5.46 \ k\Omega$ (Potentiometer)

Eg 2: $V_1 = 5\cos 2\pi 100t \ V - 2\cos 2\pi 10^3 t \ mV$.

$V_2 = 5\cos 2\pi 100t \ V - 2\cos 2\pi 10^3 t \ mV$.

Find A_{cm}, A_{dm}, CMRR (in dB).

$V_0 = 25\cos 2\pi 100t \ mV + \cos 2\pi 10^3 t \ V$

Ans

$$V_{cm} = \frac{V_1 + V_2}{2} = 5\cos 2\pi 100 t \ V$$

$$V_{dm} = V_2 - V_1 = 4\cos 2\pi \times 10^3 t \ mV$$

$$A_{cm} = \frac{25 \ mV}{5} = 5 \times 10^{-3} = 0.005$$

$$A_{dm} = \frac{1}{4} \times 10^3 = 250$$

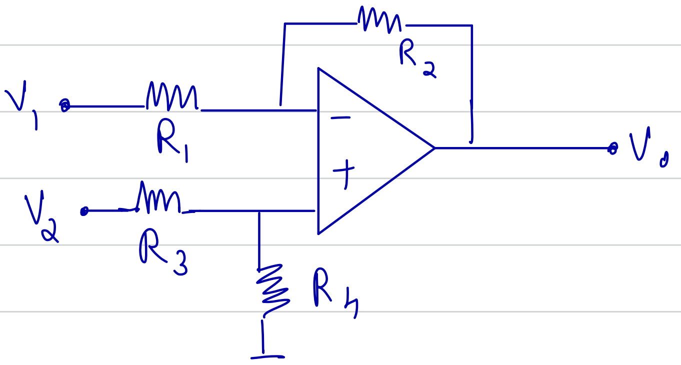
$$CMRR = \frac{A_{dm}}{A_{cm}} = \frac{250}{0.005} = 50 \times 10^3$$

$$CMRR_{dB} = 20 \log \frac{A_{dm}}{A_{cm}} = 20 \log (50 \times 10^3) = 93.97 \ dB$$

Eg 3. In the difference amplifier circuits values of resistors are $R_1 = 1.05 \ k\Omega$, $R_2 = 99 \ k\Omega$, $R_3 = 0.998 \ k\Omega$, $R_4 = 100.2 \ k\Omega$.

Find A_{cm}, A_{dm}, $CMRR_{dB}$.

Ans



$$V_0 = -\frac{R_2}{R_1}V_1 + \left(1+\frac{R_2}{R_1}\right)\left(\frac{R_4}{R_3+R_4}\right)V_0$$

$$= \frac{-99}{1.05}V_1 + \left(1+\frac{99}{1.05}\right)\frac{100.2\ V_2}{(100.2+0.998)}$$

$$= -94.285V_1 + 95.285(0.990)V_2$$

$$V_0 \simeq 94.3(V_2 - V_1)$$

$$\Rightarrow \quad A_{dm} \simeq 94.3 = 39.49\ dB$$

when V_{cm} is connected

$$V_0 = (-94.285)V_{cm} + 94.33\ V_{cm}$$

$$V_0 = 0.04715\ V_{cm}$$

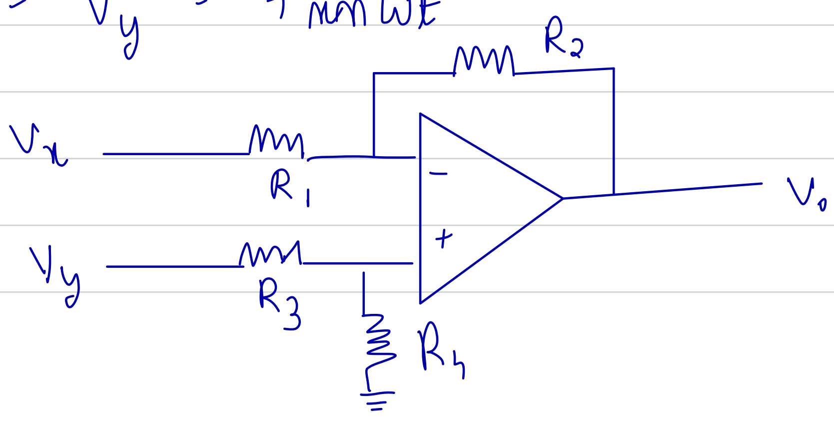
$$\therefore A_{cm} = -24.392$$

$$\therefore CMRR = 39.49 - (-24.392) = 63.8836\ dB$$

Eg 4: Single op amp based circuit where $V_0 = V_y - 2V_x$. Where V_x & V_y are 2 +ve inputs.

Also, sketch V_0 for: a> $V_x = 2\sin\omega t$, $V_y = 2V$

b> $V_x = V_y = 4\sin\omega t$



$$V_0 = -\frac{R_2}{R_1} V_x + \left(1 + \frac{R_2}{R_1}\right) \frac{R_4 V_y}{R_3 + R_4} V_y$$

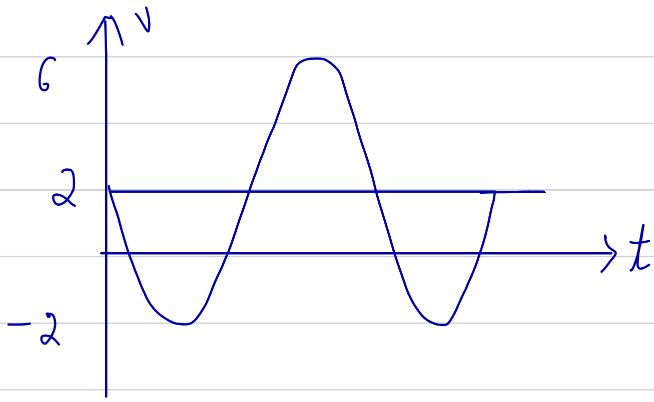
$$= -2V_x + V_y$$

$$\frac{R_2}{R_1} = 2 \Rightarrow \text{If } R_1 = 10\,k\Omega \qquad R_2 = 20\,k\Omega$$

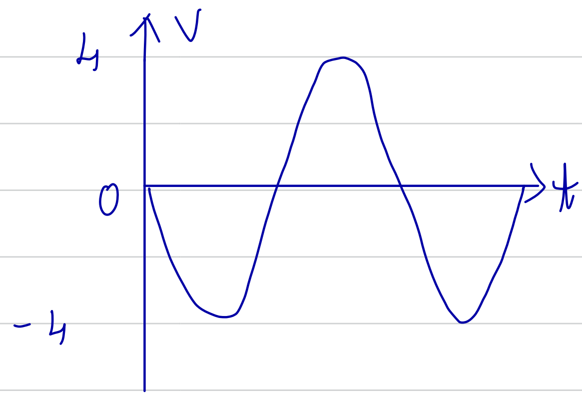
$$\left(1 + \frac{R_2}{R_1}\right) \frac{R_4}{R_3 + R_4} = 1$$

$$\frac{3R_4}{R_3 + R_4} = 1 \Rightarrow R_4 = 5\,k\Omega \qquad R_3 = 10\,k\Omega$$

a> $V_0 = 2 - 4 \sin \omega t$

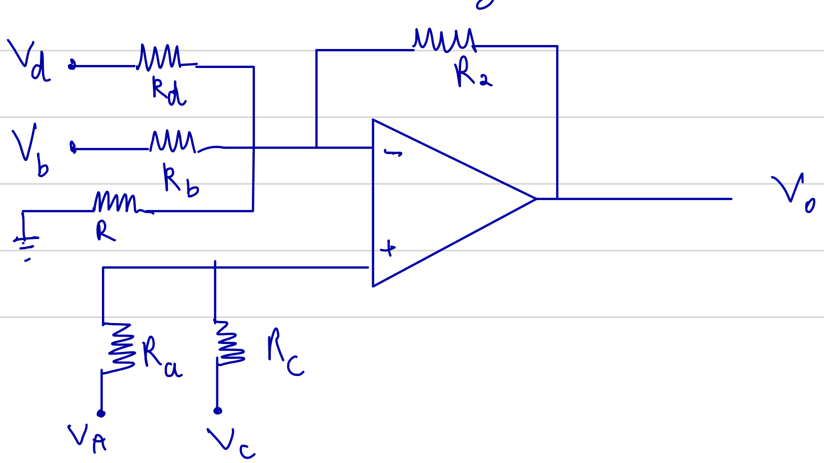


b> $V_0 = -4 \sin \omega t$



Eg5: Using a single opamp and minimum number of resistors design a 4 input amplifier circuit to realize $V_0 = V_A - 4V_B + 10V_C - 2V_D$

Ans



$V_0 = V_A - 4V_b + 10V_c - 2V_D$ $\quad (R_p = R \| R_b \| R_d)$

But $V_0 = \left(1 + \frac{R_2}{R_p}\right)\frac{R_c}{R_A + R_c} V_A - \frac{R_b}{R_2} V_b + \left(1 + \frac{R_2}{R_p}\right)\left(\frac{R_a}{R_c + R_a}\right) V_c - \frac{R_D}{R_2} V_D$

$\frac{R_2}{R_b} = 4 \Rightarrow$ If $R_B = 10\,k\Omega \quad R_2 = 40\,k\Omega$

$\frac{R_2}{R_d} = 2 \quad \therefore R_d = 20\,k\Omega$

$\left(1 + \frac{R_2}{R_p}\right)\frac{R_c}{R_A + R_c} = 1 \rightarrow (1)$ & $\left(1 + \frac{R_2}{R_p}\right)\left(\frac{R_a}{R_c + R_a}\right) = 10 \rightarrow (2)$

$(2) \div (1) \Rightarrow R_a = 10R_c$ $\quad$ If $R_c = 10\,k\Omega \Rightarrow R_a = 100\,k\Omega$

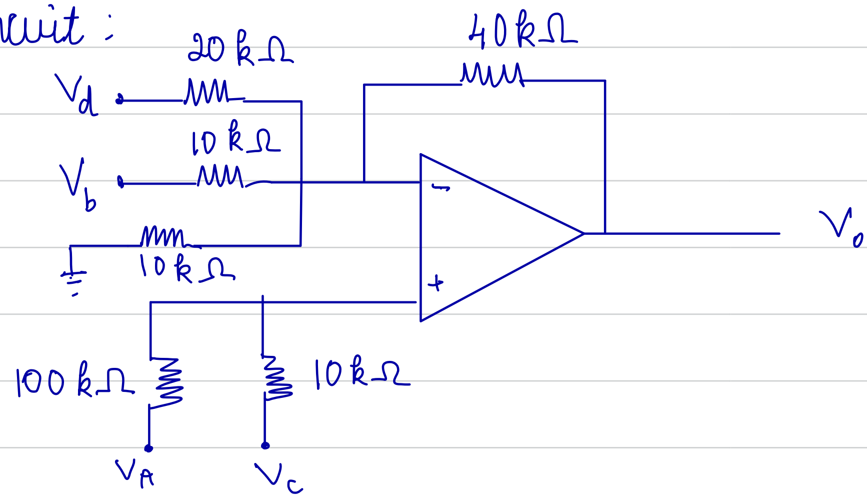
From (1) $\quad \left(1 + \frac{40}{R_p}\right) \times \frac{1}{11} = 1 \Rightarrow \frac{40}{R_p} = 10$

$R_p = 4\,k\Omega$

$\Rightarrow \frac{1}{4} = \frac{1}{R} + \frac{1}{10} + \frac{1}{20} \Rightarrow \frac{1}{R} = \frac{1}{4} - \frac{1}{10} - \frac{1}{20} = \frac{5-2-1}{20} = \frac{2}{20}$

$\therefore R = 10\,k\Omega$

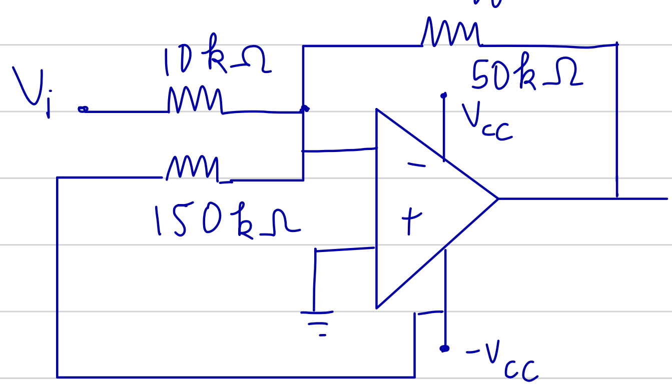
Final circuit:



Q1) In some applications such as function generators there is a need to "offset" the amplifier output voltage such that $V_o = V_o + kV_i$ where V_o is offset desired at output. Design a circuit using summing amplifiers to realize $V_o = 4 - 5V_i$. You may use the power supply $\pm 12V$ as one of the inputs to provide the desired output offset voltage

Ans

$V_{cc} = 12V$



$$V_o = -\frac{50}{10}V_i - \left(\frac{50}{150}\right)(-12)$$

$$V_o = 4 - 5V_i$$ //

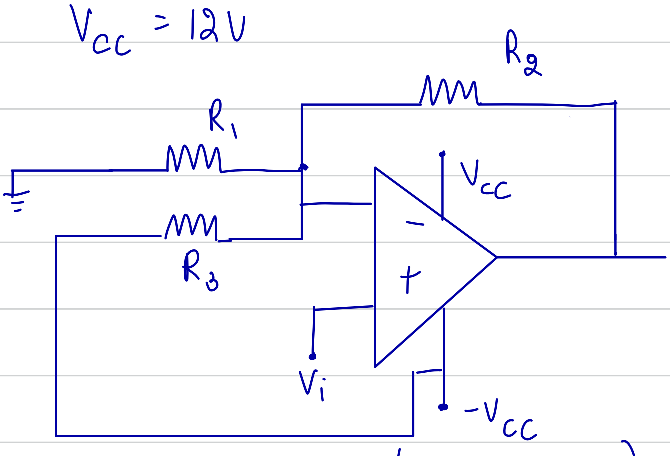
Q2) Design a single opamp based circuit to realize using

a) $V_o = 12V_i + 6V$

b) $V_o = 8(V_2 - V_1) - 4\ V$

$V_{cc} = 12V$

Ans



a) $V_o = \left(1 + \frac{R_2}{R_1 || R_3}\right)V_i + \left(\frac{-R_2}{R_3}\right)(-12)$

$$\frac{R_2}{R_3} \times 12 = 6$$

$$\frac{R_2}{R_3} = \frac{1}{2}$$ If $R_2 = 10\,k\Omega$ $R_3 = 20\,k\Omega$

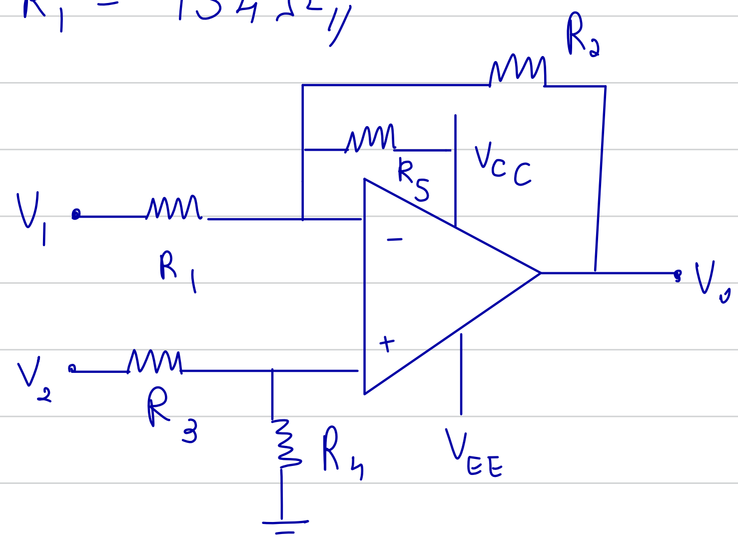
$$\Rightarrow \quad 1 + 10\left(\frac{1}{R_1} + \frac{1}{20}\right) = 12$$

$$\frac{11}{10} = \frac{1}{R_1} + \frac{1}{20}$$

$$\frac{1}{R_1} = \frac{220-10}{200}$$

$$\therefore R_1 = 954\,\Omega$$

b)



$$V_o = \left(1 + \frac{R_2}{R_1 \| R_5}\right) \cdot \frac{R_4}{R_3 + R_4} V_2 - \frac{R_2}{R_1} V_1 - \frac{R_2}{R_5} 12$$

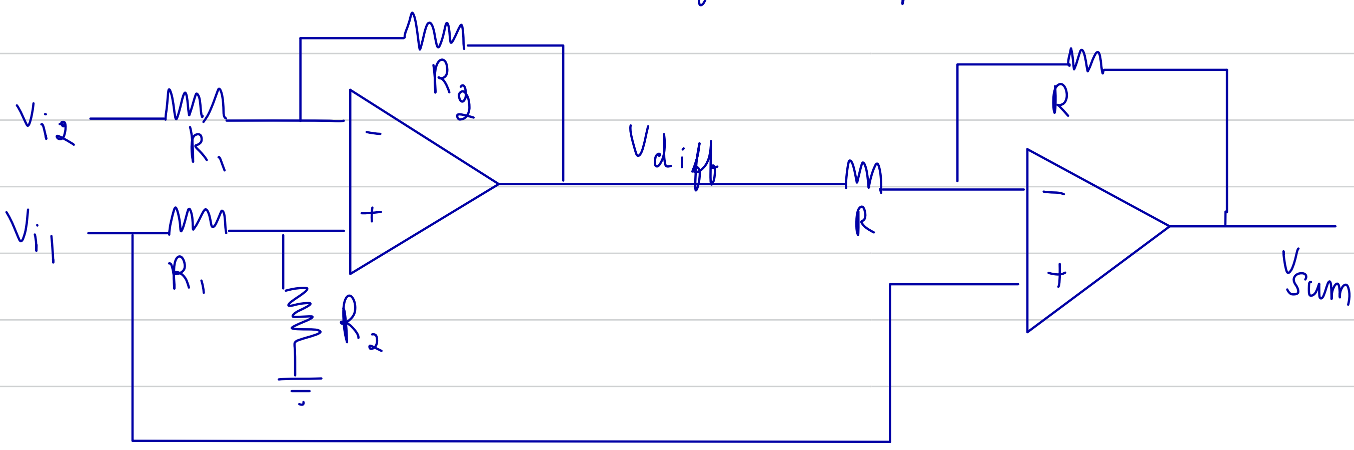
Let $R_1 = 10\,k\Omega \Rightarrow R_2 = 80\,k\Omega$ $R_5 = 240\,k\Omega$

Let $R_3 = 10\,k\Omega \Rightarrow R_4 = 60.1\,k\Omega$

Q3) Design a 2 input, 2 output circuit that yields sum & difference of inputs.

Use minimum number of components.

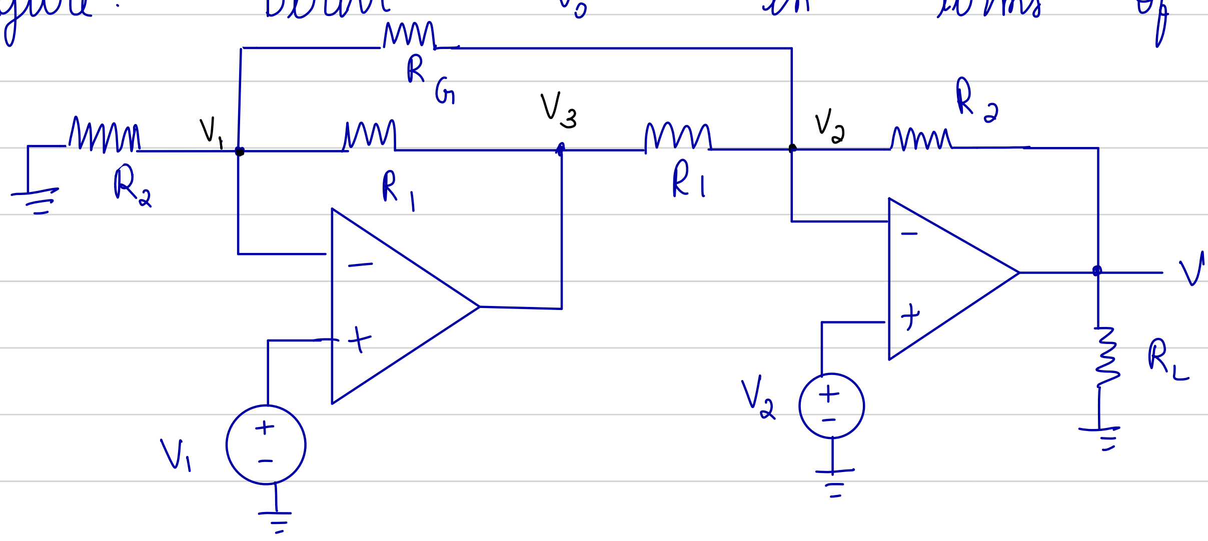
Ans



$V_{diff} = V_{i2} - V_{i1}$

$V_{sum} = 2V_{i1} - V_{diff} = 2V_{i1} - V_{i1} + V_{i2} = V_{i1} + V_{i2}$

Q4) Consider the circuit shown in following figure. Derive V_o in terms of inputs.



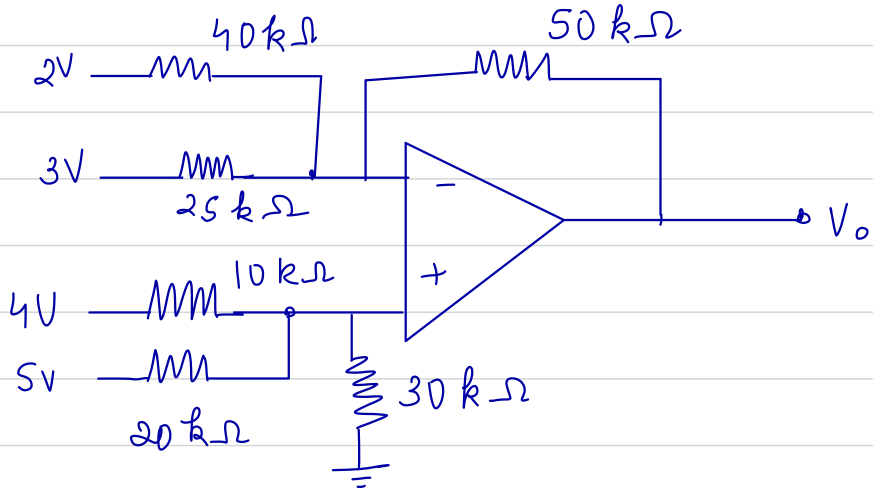
Ans

1) $\dfrac{0 - V_1}{R_2} = \dfrac{V_1 - V_2}{R_G} + \dfrac{V_1 - V_3}{R_1}$

2) $\dfrac{V_1 - V_2}{R_G} + \dfrac{V_3 - V_2}{R_1} = \dfrac{V_2 - V_o}{R_2}$

$$\therefore \quad V_o = (V_2 - V_1)\left[1 + \frac{R_2}{R_1} + \frac{2R_2}{R_G}\right]$$

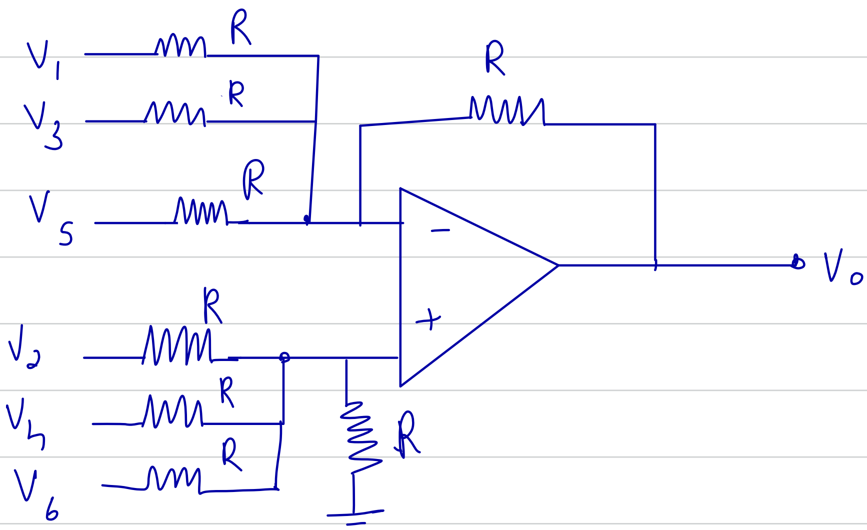
Q5) Find V_o



Ans $$V_o = -\frac{50}{40}\times 2 - \frac{50}{25}\times 3 + 4\times\left(1+\frac{50}{15.38}\right)\times\frac{12}{22} + 5\left(1+\frac{50}{15.38}\right)\left(\frac{7.5}{27.5}\right)$$

$$= -2.25 - 6 + 9.2727 + 5.79 = 6.56\ V$$

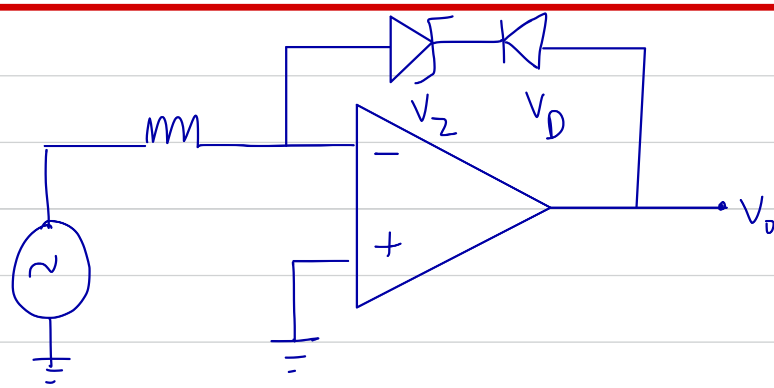
Q6)



Find V_o.

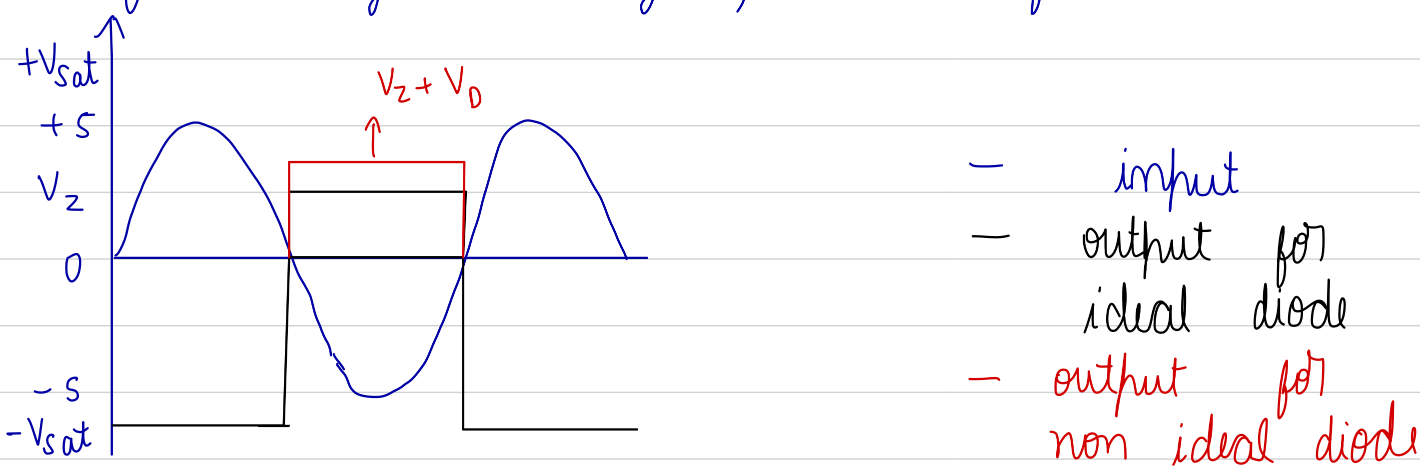
Ans $V_o = V_2 + V_4 + V_6 - V_1 - V_3 - V_5$

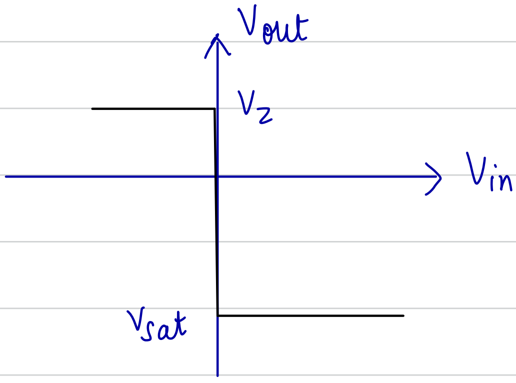
Q7)



Draw output waveform w.r.t input waveform and find transfer characteristics

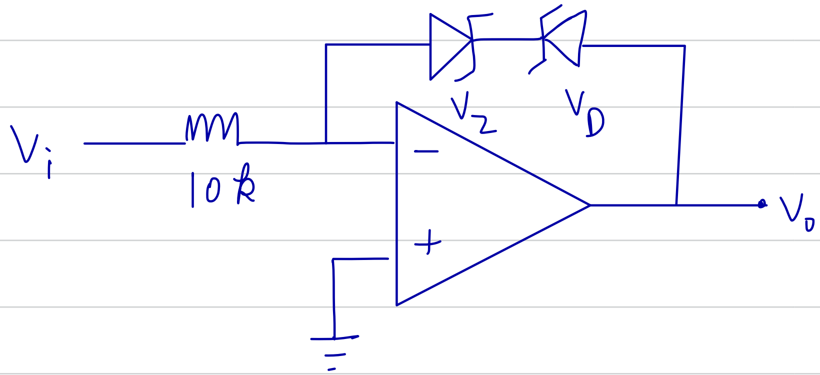
Ans During positive cycle, +ve feedback.
During negative cycle, -ve feedback



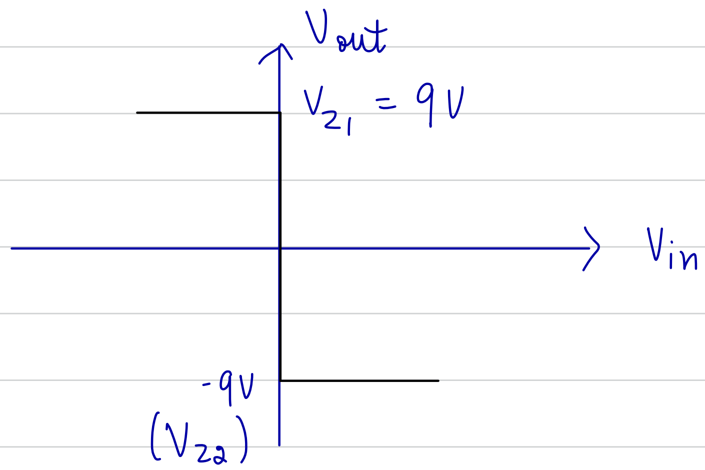


Q8) For the circuit shown, plot the transfer characteristics if opamp & zener diodes are ideal with a) $V_{Z1} = V_{Z2} = 9\,V$

b) repeat (a) if open loop gain is 10^4

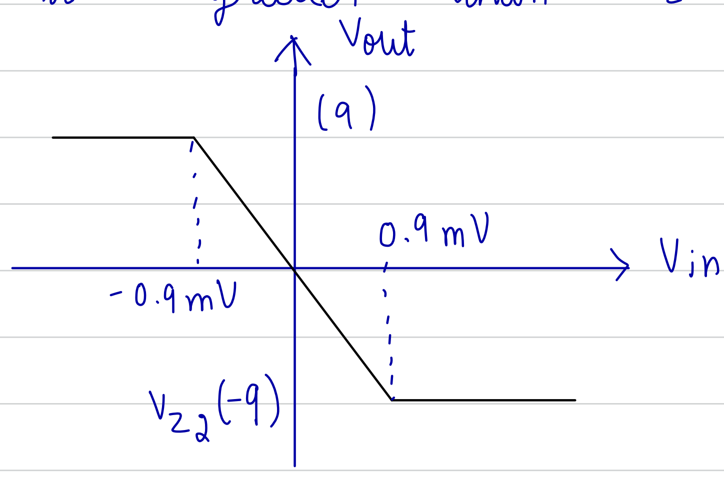


Ans a)



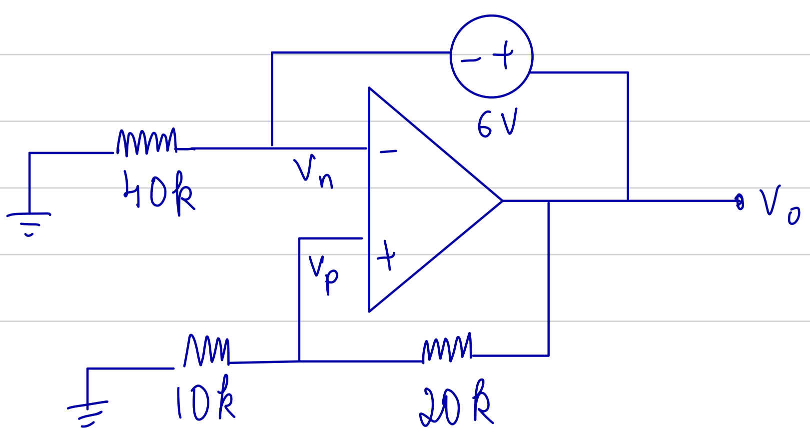
V_o must be greater than V_z

b)



$$V_{id} = \frac{9}{10^4} = 0.9\,mV$$

Q8)



Find V_p, V_n & V_o

Ans -ve feedback is dominant

$\Rightarrow V_p = V_n$

$$\frac{V_o - V_p}{20k} = \frac{V_p}{10k}$$

$$\Rightarrow V_o - V_p = 2V_p$$

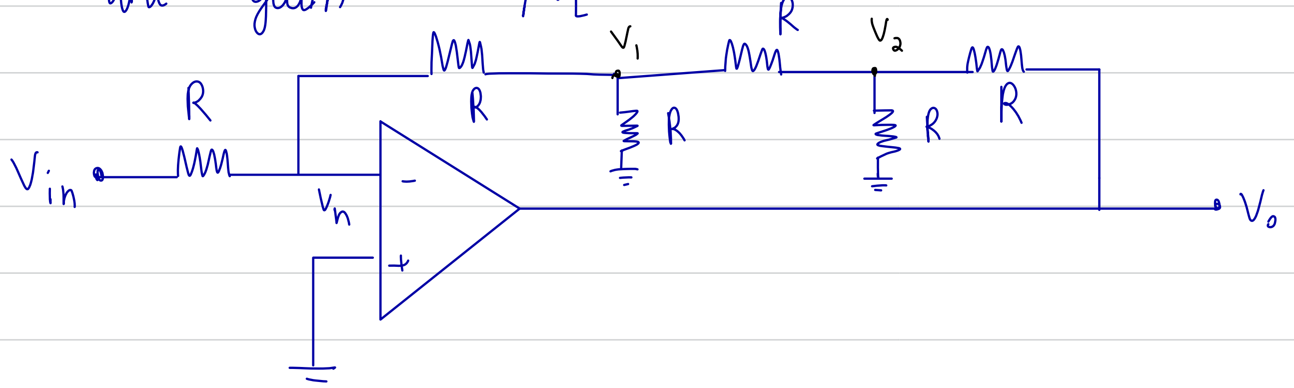
$$V_o = 3V_p$$

$$V_n + 6 = V_o$$

$$\Rightarrow V_p + 6 = 3V_p$$

$\therefore V_p = 3V$ // $\qquad \therefore V_n = 3V \qquad \therefore V_o = 9V$ //

Q9) Find the gain V_o / V_L



Ans $V_p = V_n = 0V$

$$\frac{V_{in} - 0}{R} = \frac{0 - V_1}{R} \qquad \therefore V_1 = -V_{in}$$

$$\frac{0 - V_1}{R} = \frac{V_1}{R} + \frac{V_1 - V_2}{R}$$

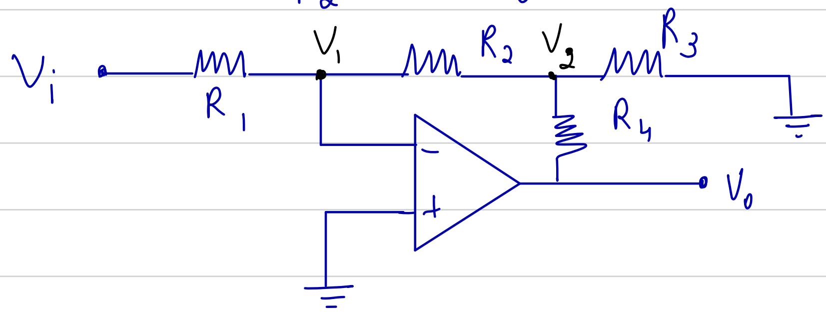
$$\frac{-V_1}{R} = \frac{2V_1 - V_2}{R} \Rightarrow \therefore V_2 = 3V_1$$

$$\frac{V_1 - V_2}{R} + \frac{V_2}{R} = \frac{V_2 - V_o}{R} \Rightarrow \therefore V_1 = 3V_2 - V_o$$

$$\Rightarrow V_o = 3V_2 - V_1 = 9V_1 - V_1 = 8V_1 = -8V_{in}$$

$$\therefore \frac{V_o}{V_{in}} = -8 \text{ //}$$

Q10) Show that the circuit has $A = \frac{V_0}{V_i} = -k\frac{R_2}{R_1}$ with $k = 1 + \frac{R_4}{R_2} + \frac{R_4}{R_3}$



Ans $V_1 = 0$

$$\frac{-V_2}{R_2} = \frac{V_2}{R_3} + \frac{V_2 - V_0}{R_4}$$

$$\frac{V_0}{R_4} = V_2\left[\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_2}\right]$$

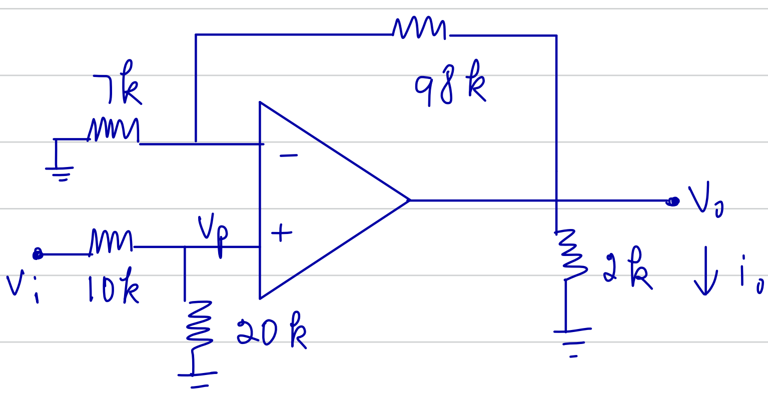
$$\frac{V_i}{R_1} = \frac{-V_2}{R_2}$$

$$V_2 = -\frac{R_2 V_i}{R_1}$$

$$V_0 = -\frac{R_2}{R_1}V_i\left[1 + \frac{R_4}{R_2} + \frac{R_4}{R_3}\right]$$

$$\therefore \quad k = \left[1 + \frac{R_4}{R_2} + \frac{R_4}{R_3}\right]$$

Q11)



Find V_0 & i_0 if $V_i = 0.5 \sin\omega t$

Ans $V_0 = \left(1 + \frac{98}{7}\right)V_p$

$$v_p = \frac{20}{30}V_i = \frac{2}{3} \times 0.5 \sin\omega t$$

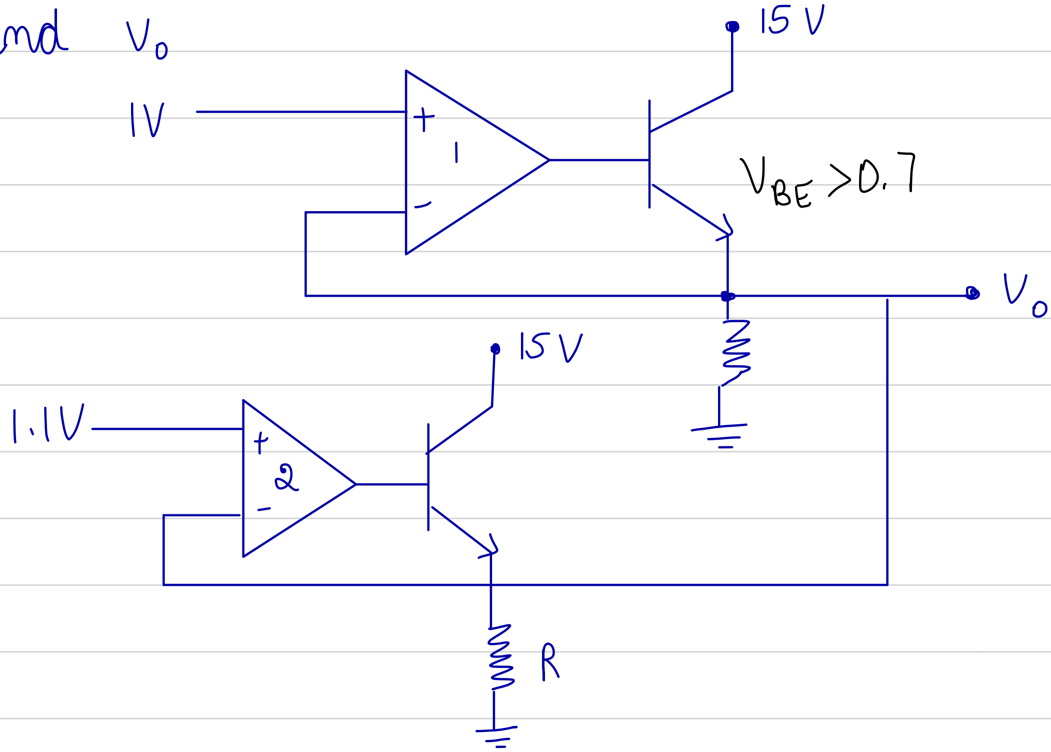
$= 15\, V_p$

$= 15 \times \frac{2}{3} \times 0.5 \sin\omega t$

$\therefore \quad V_0 = 5 \sin\omega t$

$i_o = \frac{V_o}{2k\Omega} = \frac{5 \sin\omega t}{2 \times 10^3} = 2.5 \sin\omega t \ mA$

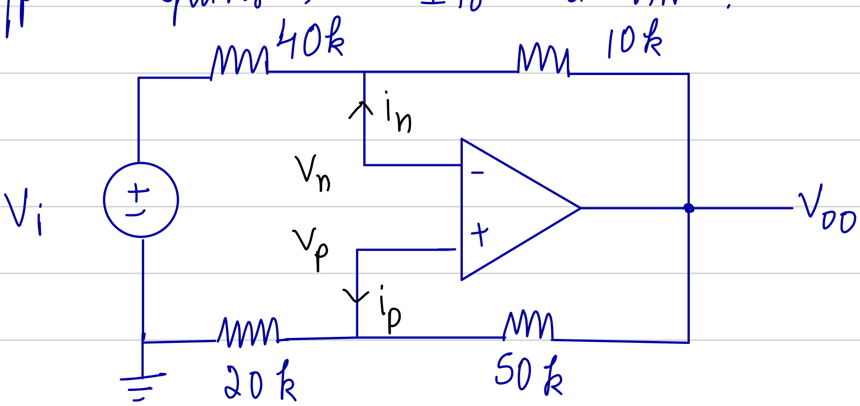
Q12) Find V_o



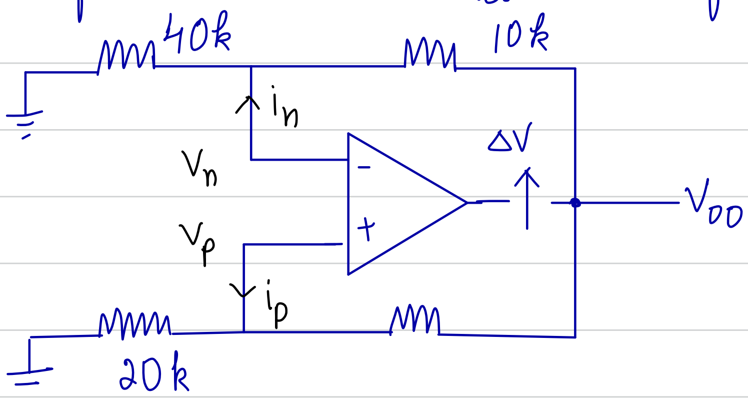
Ans Here both are voltage buffer :

$\therefore \ V_o = 1.1 \ V$

Q13) Consider the following circuit, what is the min & max o/p offset voltage If the bias current $I_B = 100$ nA flowing out of the opamps i/p pins. $I_{io} = 20$ nA.



Ans To findout dominant feedback response :



$$\frac{40}{50}\Delta V_0 = V_n \qquad \frac{20}{70}\Delta V_0 = V_p$$

$\because$ $V_n > V_p$, negative feedback is dominant.

$$I_B = \frac{i_n + i_p}{2} \quad , \quad I_{i0} = |i_n - i_p|$$

$$\frac{V_{00} - V_n}{10k} + i_n = \frac{V_n}{40k}$$

$$\frac{V_{00} - V_p}{50k} + i_p = \frac{V_p}{20k}$$

Also $V_n = V_p$

$$\frac{V_{00}}{10k} + i_n = V_p\left[\frac{5}{40k}\right]$$

$$\frac{V_{00}}{50k} + i_p = V_p\left[\frac{7}{100k}\right]$$

$$\Rightarrow \frac{V_{00}}{10k} + i_n = \left(\frac{V_{00}}{50k} + i_p\right) \cdot \frac{100k}{7} \times \frac{5}{40k}$$

$$\frac{V_{00}}{10k} + i_n = \frac{25}{14}\left(\frac{V_{00}}{50k} + i_p\right)$$

$$\therefore \left(5 - \frac{25}{14}\right) \frac{V_{00}}{50k} = \frac{25}{14} i_p - i_n$$

$$\therefore \frac{9}{140k} V_{00} = \frac{25}{14} i_p - i_n$$

$$\therefore V_{00} = \frac{250k}{9} i_p - \frac{140k}{9} i_n$$

$$V_{00} = 27.77k \, i_p - 15.55k \, i_n$$

When, $i_p = 90\ nA$ $\quad i_n = 110\ nA$

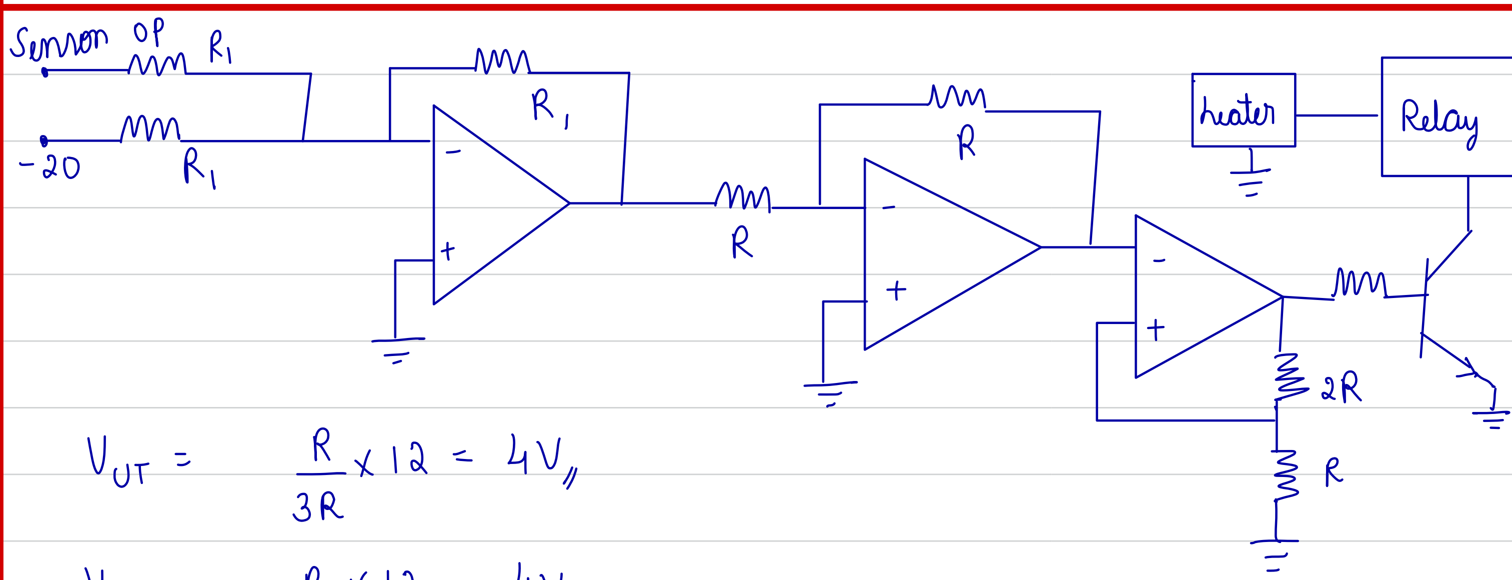
$$\therefore (V_{00})_{min} = 27.77k \times 90\ nA - 15.55k \times 110\ nA$$
$$= 788.8\ \mu V = 0.788\ mV$$

When $i_p = 110\ nA$ $\quad i_n = 90\ nA$

$$\therefore (V_{00})_{max} = 27.77k \times 110\ nA - 15.5k \times 90\ nA$$
$$= 1655.2\ \mu V = 1.6552\ mV$$

Q14> An Opamp based control system for room heater with following specification. When room temperature goes below 16°C, the heater is turned on and it is turned off if temperature is above 24°C. Assume that a thermister based sensor gives voltage output proportional to room temperature. Suggest a scheme to control heater with a circuit diagram.

Ans



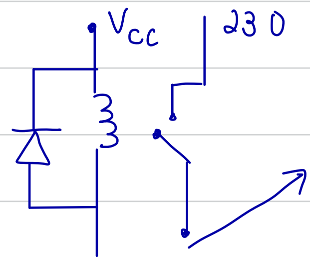
$V_{UT} = \frac{R}{3R} \times 12 = 4V$

$V_{LT} = \frac{-R}{3R} \times 12 = 4V$

So at 16 V $\rightarrow$ $V_{out} = V_{sat}$

at 24 V $\rightarrow$ $V_{out} = -V_{sat}$

Relay internal circuit



connected to heater

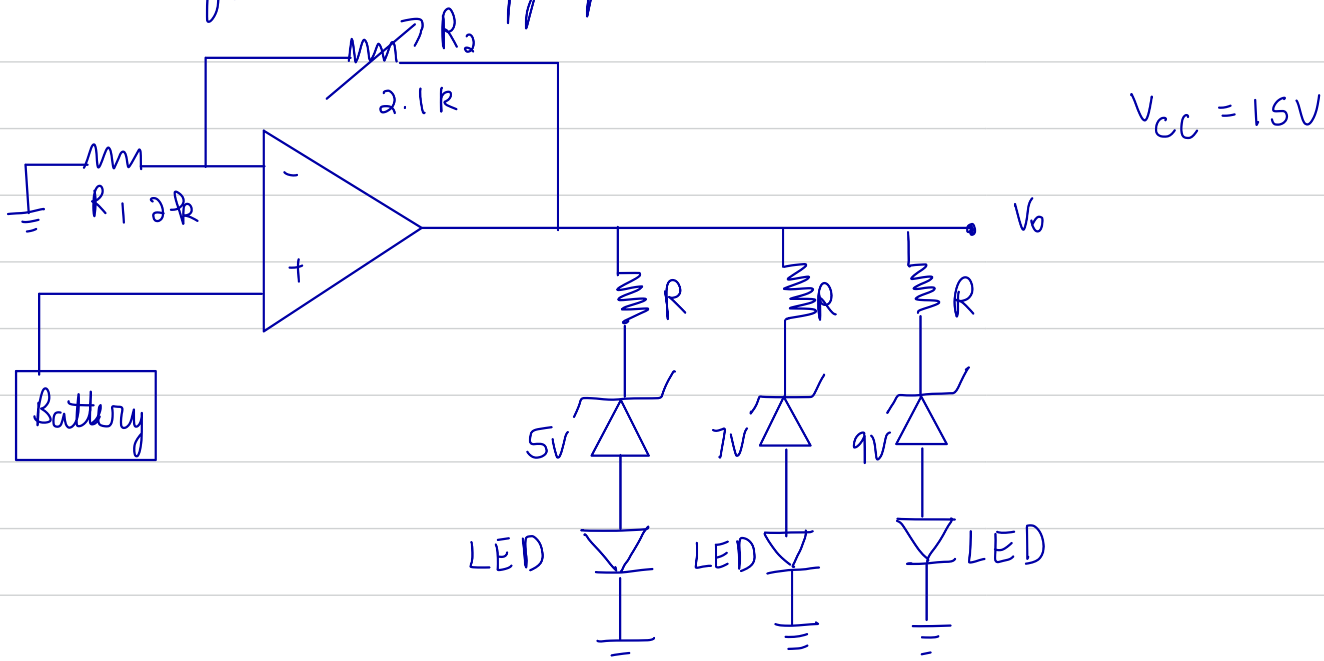
Q15) A 6V battery charge indicator has to be designed with following specifications:

Battery voltage	Indication
Below 3V	No LED should glow
3-4 V	1 LED should glow
4-5 V	2 LED should glow
Above 5V	3 LED should glow

You are provided with 5, 7, 9 V zener diodes, opamps, LEDs (which will drop 1V when they conduct), resistors.

Using these components design a battery indicator circuit. Design the circuit diagram and find appropriate values.

Ans



R_2 should be slightly greater than R_1

$R_t = R \parallel R \parallel R \parallel (R_1 + R_2)$

$R = 3.3\ k$ then $R_t \cong 0.8\ k$

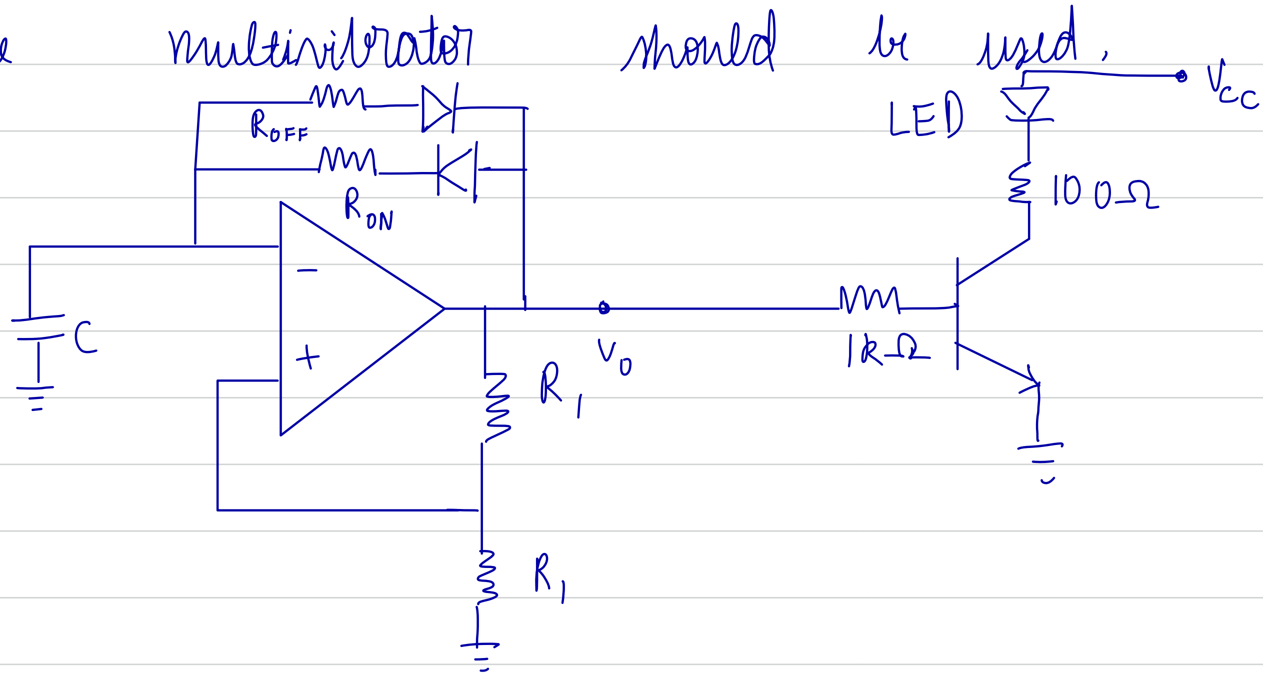
$\therefore \dfrac{12}{0.8\ k} = 15\ mA$ $< 25\ mA$

Q16) A flasher has to produce 12 flashes of light per minute, the duration of each flash is 1 sec. Design a driver circuit. Flasher bulb has to be driven by a npn transistor.

Ans $T = \dfrac{60}{12} = 5s$ $T_{on} = 1\ s$

$T_{off} = 4s$

Astable multivibrator should be used.

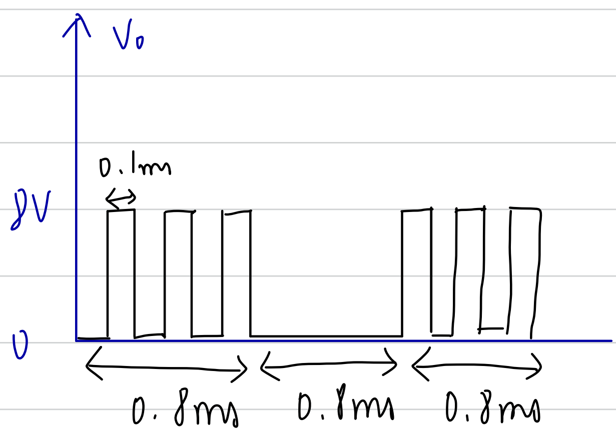


Let $C = 0.01\ mF$

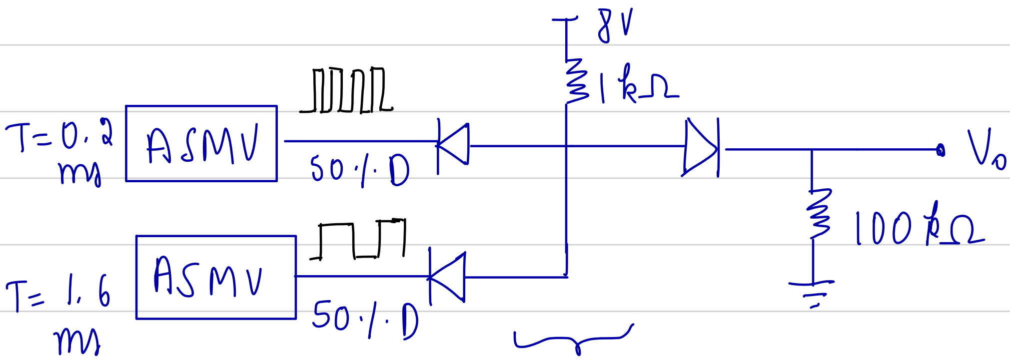
$1.1\ R_{ON} C = 1s$ $\qquad$ $1.1\ R_{off} C = 4s$

$R_{ON} = 90.90\ k\Omega$ $\qquad$ $R_{off} = 363.64\ k\Omega$

Q17) Generate the following waveform



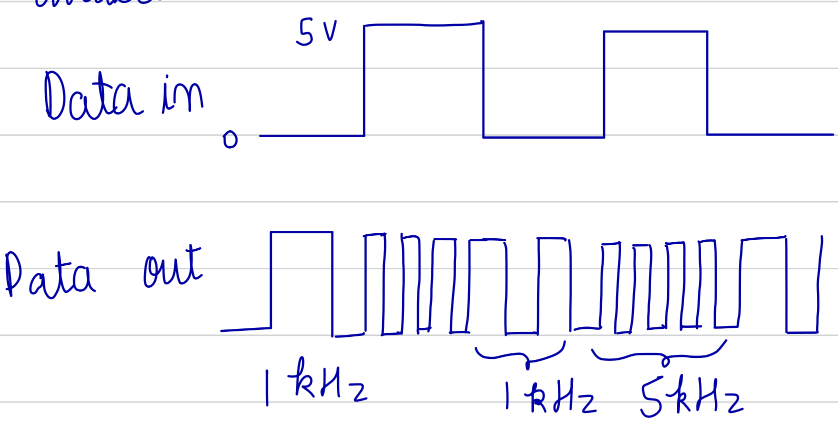
Ans

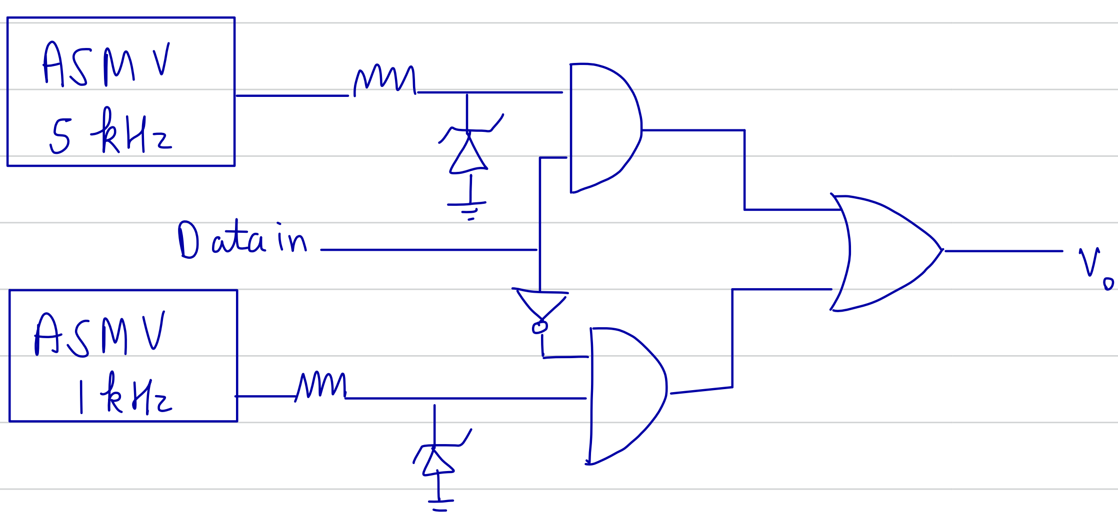


Diode AND

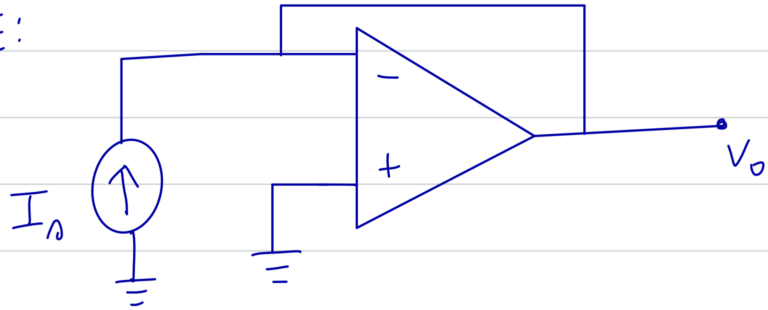
NOTE: ASMV + Integrator + Triangular wave generator are included for AE lab exam along with 10 experiments.

Q18) In digital communication system, the logic 1 & logic 0 are transmitted as FSK symbols with frequencies of 5 kHz & 1 kHz respectively. Design a circuit for such a FSK (frequency shift keying), assume opamp, digital ICs & zener diodes are available.

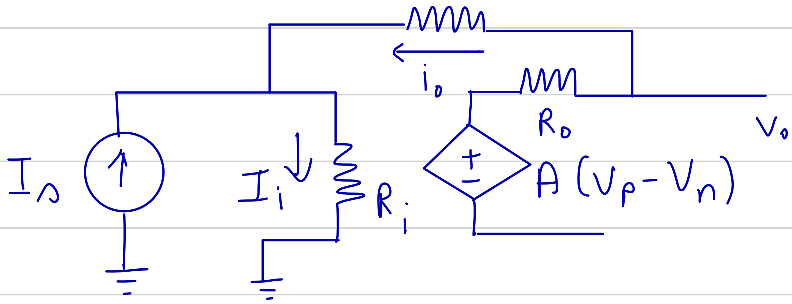




NOTE:



⇒



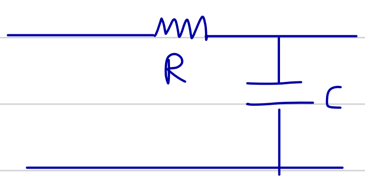
$$I_s + i_o = I_i$$

$$I_s = -I_i$$

$$Z_{if} = \frac{V_n}{I_s} = 0$$

A should be as large as possible.

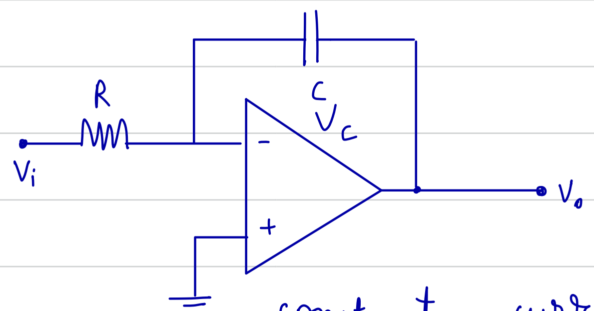
Integrator



constant input voltage integrator

$$i : \frac{V_i - 0}{R} \rightarrow \frac{V_i - V_c}{R}$$

$$V_t = V_f + (V_{in} - V_f)e^{-t/RC}$$



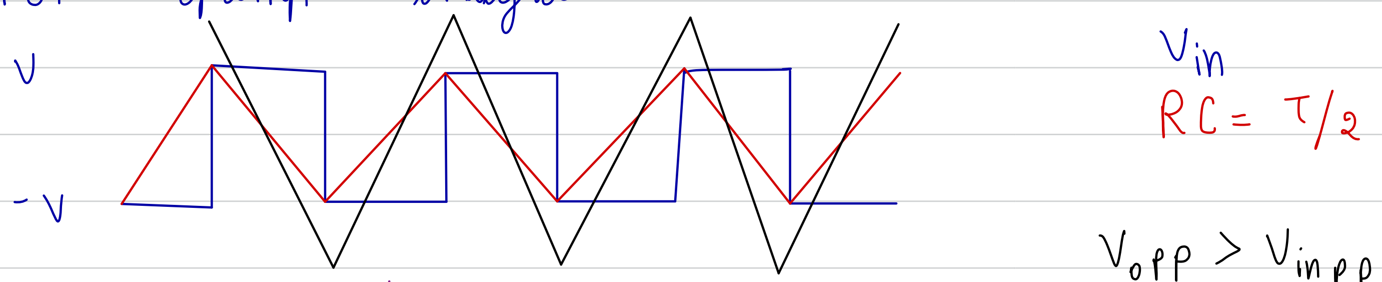
constant current integrator

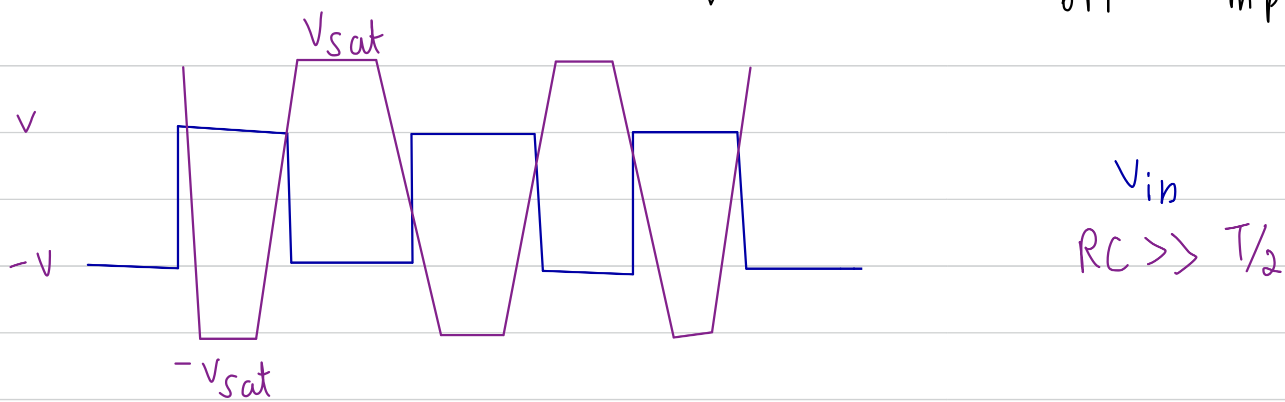
$$C \Delta V = I_c t$$

$$I_c = C \frac{dV}{dt}$$

ΔV = peak to peak voltage

For opamp integrator:





$$V_c = \frac{1}{C}\int i_c \, dt$$

$$V_c = \frac{1}{RC}\int V_i \, dt$$

$$V_o = V_p - V_c = -V_c$$

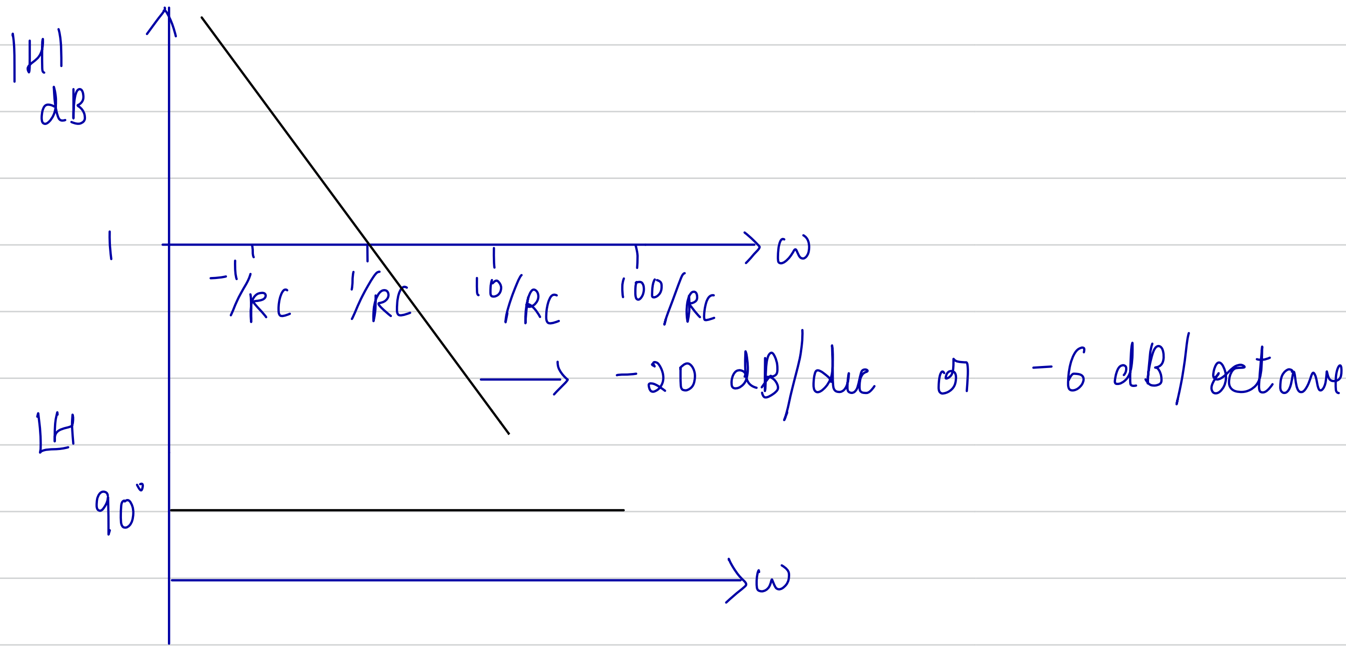
$$\therefore \quad V_o = \frac{-1}{RC}\int V_i \, dt$$

$$\frac{V_o}{V_i} = -\frac{1}{RCs} \Rightarrow H(s) = \frac{-1}{RCs}$$

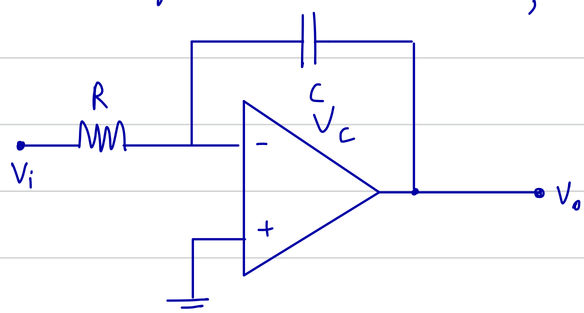
$$H(j\omega) = \frac{-1}{RCj\omega} \qquad \therefore \ |H| = \frac{1}{RC\omega}$$

$$\angle H(j\omega) = 90^\circ$$

Pole at $s = 0$

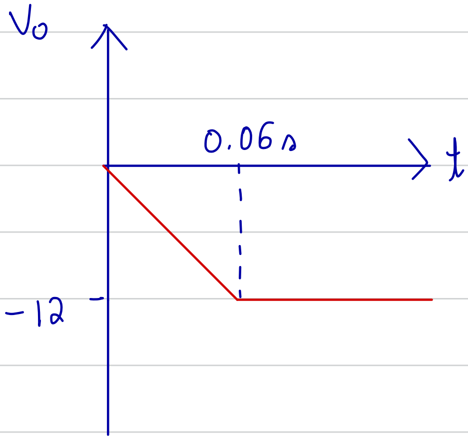


Q19) Plot V_o if $R = 1\,k\Omega$, $C = 10\,\mu F$ & $V_i = 2u(t)$ in



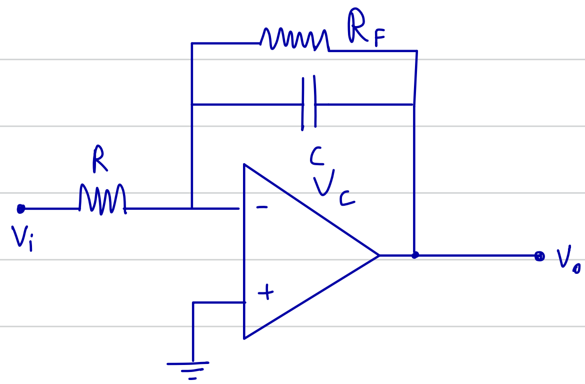
Ans $V_o = \dfrac{-1}{RC}\int V_i\,dt = \dfrac{-1}{10^4 \times 10^{-6}}\int 2u(t)\,dt$

$= -200t$



$-12 = -200t$

$t = \dfrac{6}{100} = 0.06\,s$



⟵ Practical integrator circuit.

A large resistance of R_F is connected parallel to capacitor so that V_o does not get saturated when capacitor acts as a open circuit.

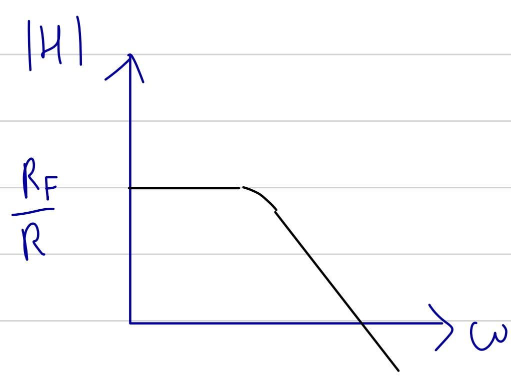
For DC, $Z_f \approx R_f$ | $Z_F \approx \frac{1}{\omega C}$, for AC

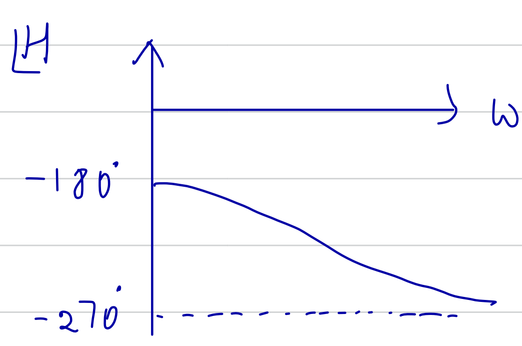
$$\frac{V_o}{V_i} = \frac{R_F \parallel 1/Cs}{R}$$

$$H(s) = \frac{-(R_F/R)}{1 + \frac{s}{\omega_o}} \qquad \text{where} \quad \omega_o = \frac{1}{R_F C}$$

$$H(s) = \frac{-(R_F/R)}{1 + R_F C s} \qquad \text{Pole at } s = -\frac{1}{R_F C}$$

$\therefore$ cutoff frequency $= \frac{-1}{R_F C}$





$$|H| = \frac{R_F/R}{\sqrt{1+\omega^2 C^2 R_F}}$$

$$\angle H = \pm 180° - \tan^{-1} \omega C R_F$$

Q20) Design a voltage integrator for a sinusoidal signal $V_i(t) = 2\sin 4000\pi t$ with gain equal to unity.

a) Design a practical integrator with $f_{-3dB} = \frac{f_{in}}{15}$

b) If a square wave input of $4V_{p-p}$ is applied with $f_{in} = 2\,kHz$. Plot V_0 w.r.t V_i

c) If a square wave input of $4V_{p-p}$ is applied with $f_{in} = 130\,Hz$. Plot V_0 w.r.t V_i

Ans

$$V_0 = \frac{-1}{RC}\int 2\sin 4000\pi t \, dt$$

$$= \frac{1}{RC} \frac{2\cos 4000\pi t}{4000\pi}$$

Gain = unity $\Rightarrow$ $V_0 = 2\cos 4000\pi t$

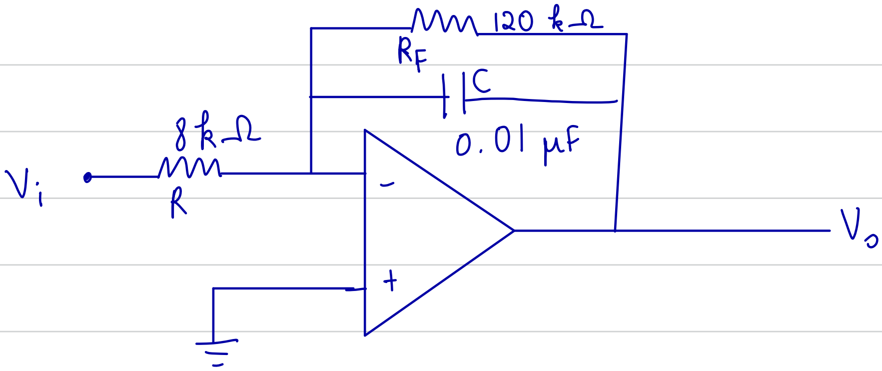
$\Rightarrow$ $RC = \frac{1}{4000\pi}$

Let $C = 0.01\,\mu F$

then $R = 7.96\,k\Omega \cong 8\,k\Omega$

a) $f_0 = \frac{f_{in}}{15} = \frac{2k}{15} = 133.33\,Hz$

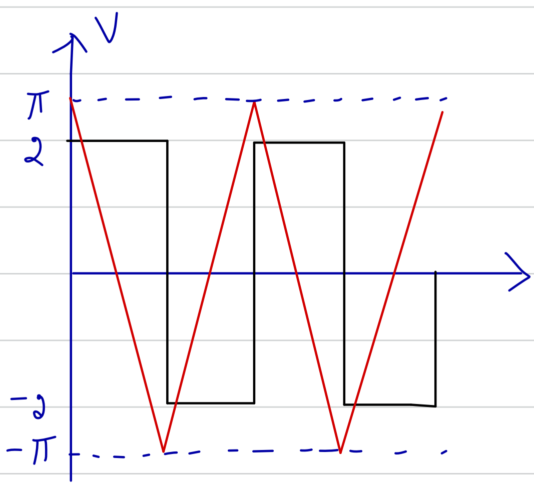
$\frac{1}{R_F C} = \omega_0 = 2\pi \times 133.33$ $\quad \therefore R_F = 119.36\,k\Omega \cong 120\,k\Omega$



b) $C \Delta V = I \Delta t$

$$\Delta V = \frac{2}{RC} \times \frac{T}{2}$$

$$\Delta V = 4000\pi \times \frac{1}{2k} = 2\pi$$



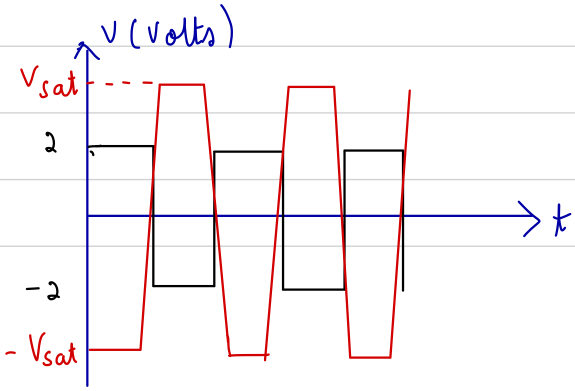
— input
— output

c) $C \Delta V = I \Delta T$

$$\Delta V = \frac{2}{RC} \times \frac{T}{2}$$

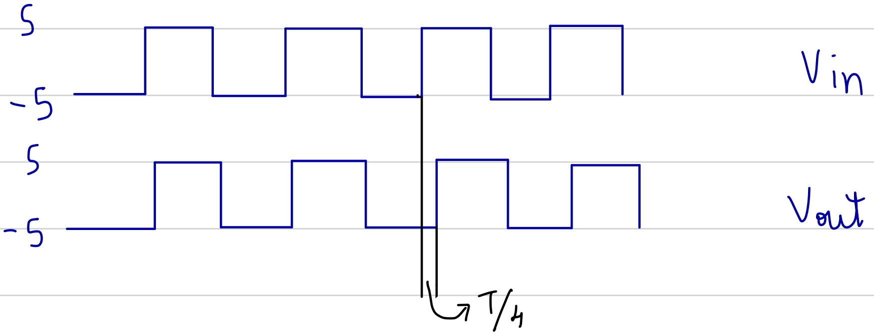
$$\Delta V = \frac{4000\pi}{130} = 96.6 \text{ V} > V_{sat}$$

∴

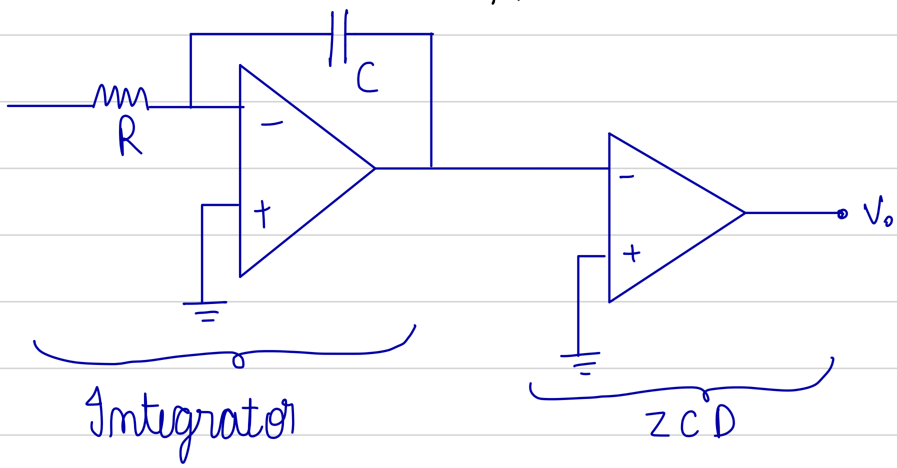


— input
— output

Q21) Design an opamp based circuit for the following waveform. The output must be delayed exactly by T/4 seconds from the input. As frequency of input varies, so does the output. But duty cycle remains the same.



Ans

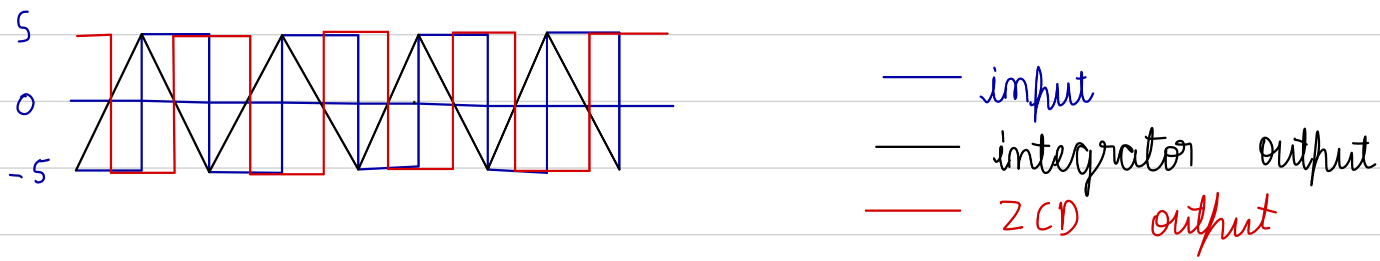


$$C \Delta V = I \Delta t$$

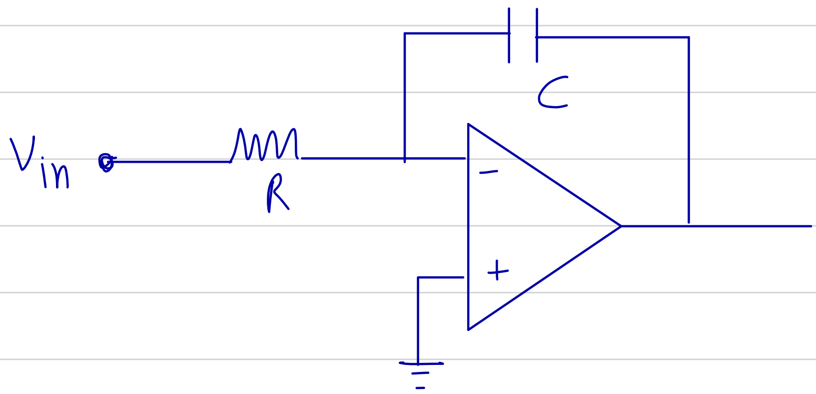
$$C \times 10 = \frac{5}{R} \times \frac{T}{2}$$

$$\Rightarrow T = 4RC$$

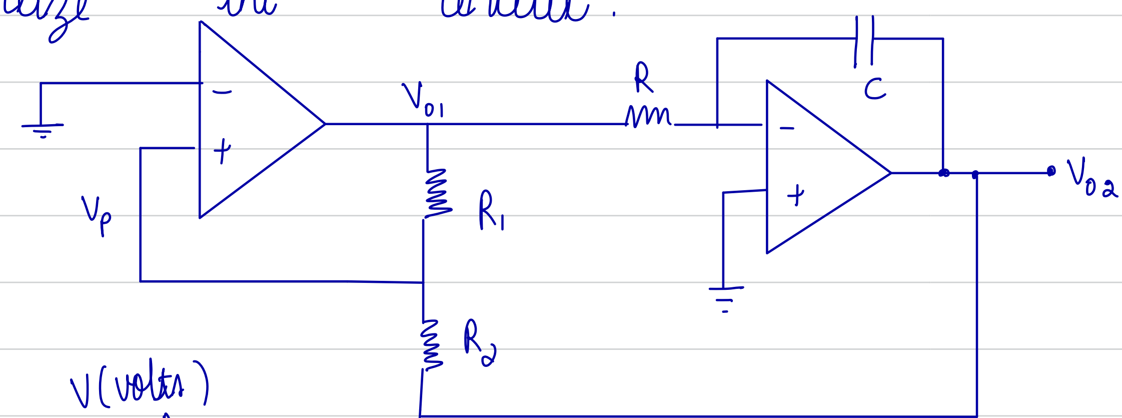
i.e

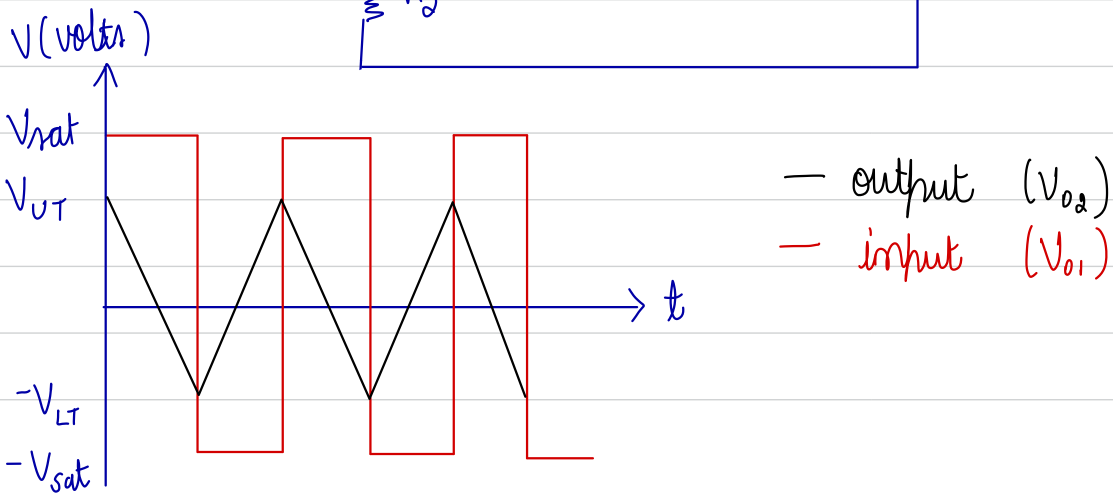


Triangular waveform generator



- Integrator also generates triangular waveform.
- But a small DC in input is enough to make output go to saturation.
- So we have to increase input level if it is going to $-V_{sat}$ & decrease level if it is going to $+V_{sat}$.
- Therefore Schmitt trigger is connected to stabilize the circuit.





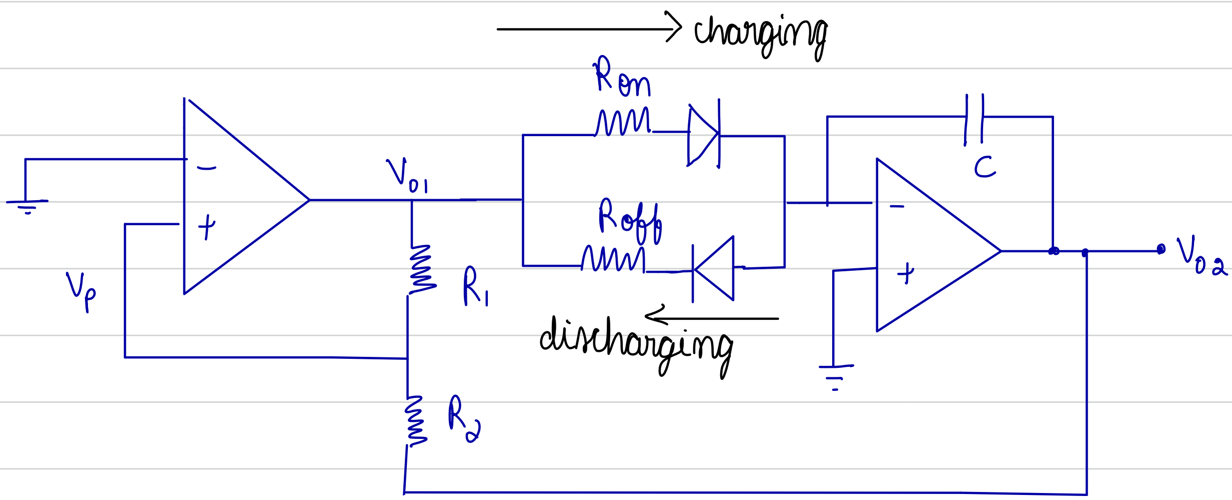
$V_{UT} = \frac{R_2}{R_1} V_{sat}$ $\qquad$ $V_{LT} = \frac{-R_2}{R_1} V_{sat}$

$C \Delta V = I \Delta t$

$C \times \frac{2R_2 V_{sat}}{R_1} = \frac{V_{sat}}{R} \times \frac{T}{2}$

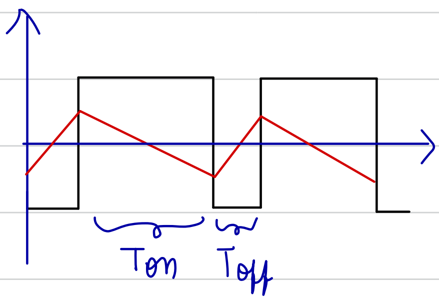
$T = \frac{4R_2 RC}{R_1}$ $\qquad$ $f = \frac{R_1}{4R_2 RC}$

To get duty cycle other than 50% :



If $R_{on} > R_{off}$

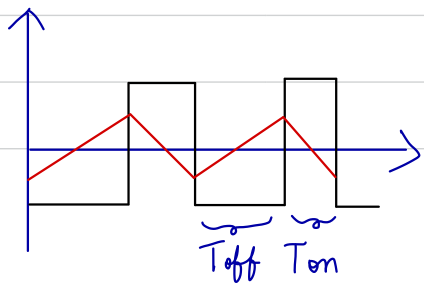
$\Rightarrow R_{on} C > R_{off} C$ $\qquad \Rightarrow$



D > 50%

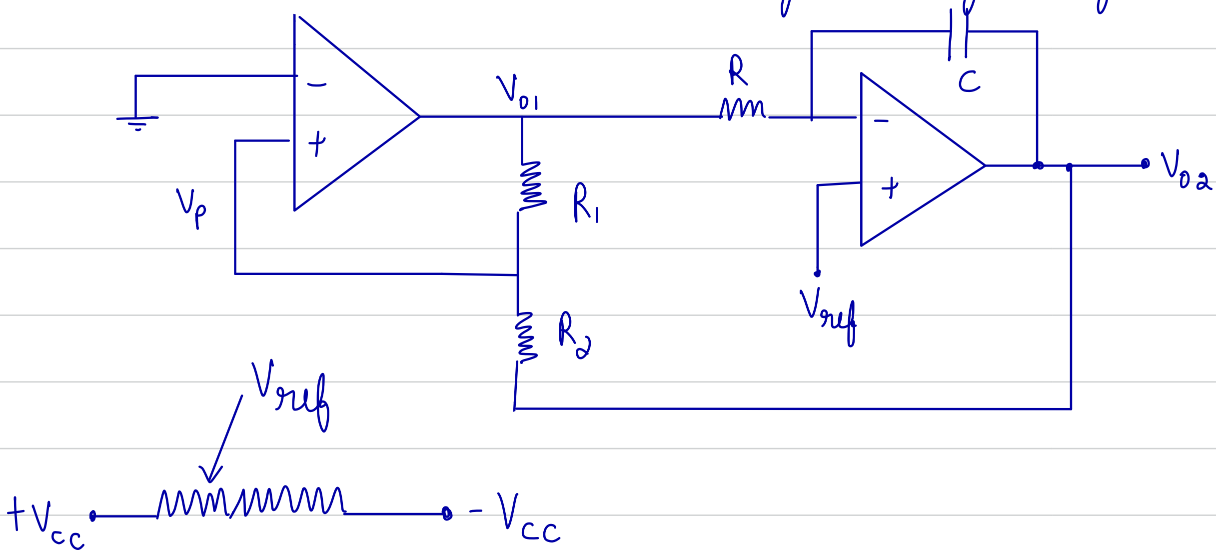
If $R_{off} > R_{on}$

$\Rightarrow R_{off} C > R_{on} C$ $\qquad \Rightarrow$



D < 50%

2nd method to vary duty cycle:



charging current $= \dfrac{V_{sat} - V_{ref}}{R}$

discharging current $= \dfrac{-V_{sat} - V_{ref}}{R}$

If $V_{ref} > 0$, then $|I_{on}| < |I_{off}| \rightarrow T_{on} > T_{off}$
$\Rightarrow D > 50\%$

If $V_{ref} < 0$, then $|I_{on}| > |I_{off}| \rightarrow T_{off} < T_{on}$
$\Rightarrow D < 50\%$

$$C\Delta V = I\Delta t$$

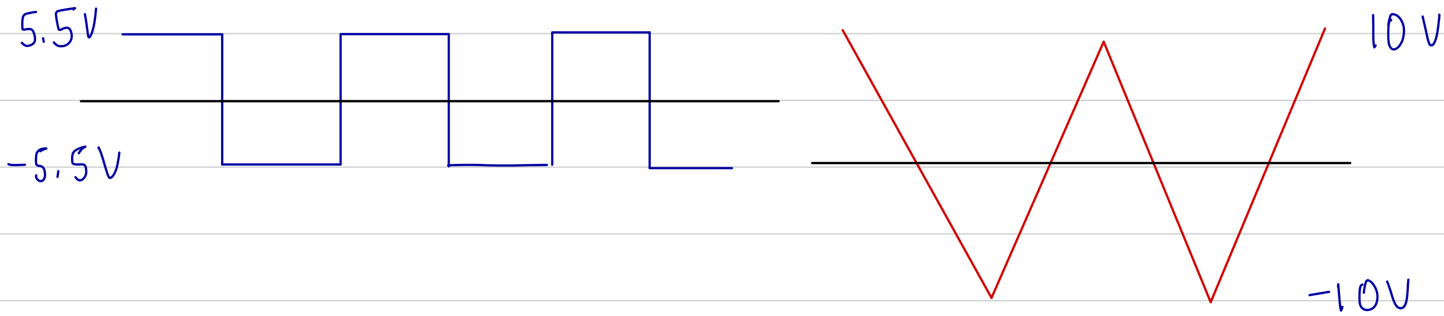
$$\Rightarrow C \times \frac{2R_2 V_{sat}}{R_1} = \frac{V_{sat} - V_{ref}}{R} \times T_{on}$$

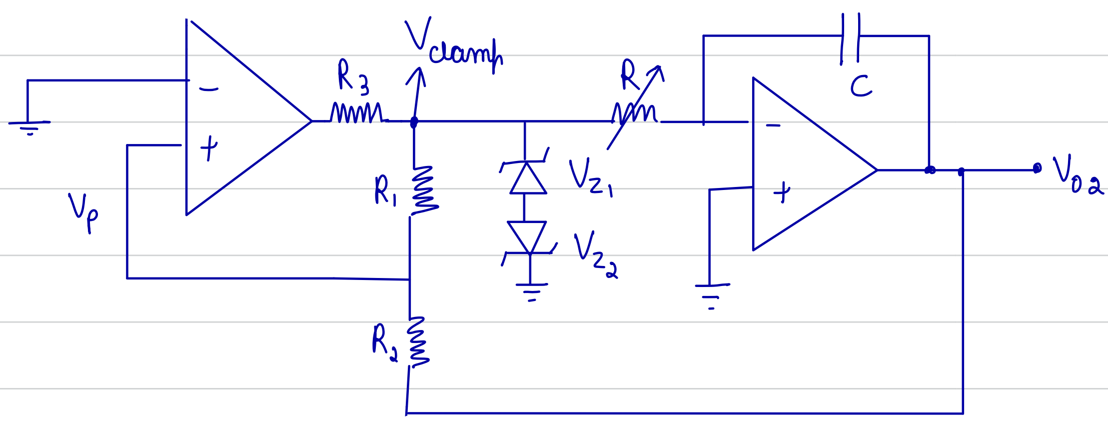
$$T_{on} = \frac{2R_2RCV_{sat}}{R_1[V_{sat} - V_{ref}]} = \frac{2R_2RC}{R_1\left[1 - \frac{V_{ref}}{V_{sat}}\right]}$$

|||ly $$T_{off} = \frac{2R_2RC}{R_1\left[1 + \frac{V_{ref}}{V_{sat}}\right]}$$

Q1> Design a triangular waveform generator for approximately 5.5V peak amplitude & a 10V peak amplitude triangular wave. Frequency is variable between 1 kHz & 2 kHz.

Ans





$V_{clamp} = \pm 5.4V$ $\qquad V_{Z_1} = V_{Z_2} = 4.7V,$

$V_{tri} = \pm 10V \quad \Rightarrow V_{UT} = 10V \qquad V_{LT} = -10V$

$$\Rightarrow \frac{R_2}{R_1} V_{clamp} = 10$$

$$\frac{R_2}{R_1} = \frac{10}{5.4} = 1.85 \quad \Rightarrow \quad R_2 = 1.85 R_1$$

Let $R_2 = 10 \text{ k}\Omega \quad \Rightarrow \quad R_1 = 5.4 \text{ k}\Omega$

$$f = \frac{R_1}{4R_2RC}$$

Let $0.1 \mu F$. $\quad \Rightarrow \quad 1k \leq \dfrac{R_1}{4R_2RC} \leq 2k$

$$R_{max} \text{ (at } f = 1\,kHz) = \frac{5.4\,k}{0.1 \times 10^{-6} \times 4 \times 10^{3} \times 10^{4}}$$

$$\therefore R_{max} = 1.35\ k\Omega$$

$$R_{min} \text{ (at } f = 2\,kHz) = 675\ \Omega$$

$$I_{max} = \frac{5.4}{675} = 8\,mA$$

$\Rightarrow$ current through zener $= 6mA$

current through R_1 & R_2 $= \dfrac{5.4-(-10)}{R_1+R_2} = 1\,mA$

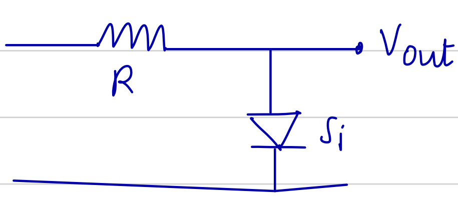
$\therefore$ Total current $= 8mA + 6mA + 1mA$

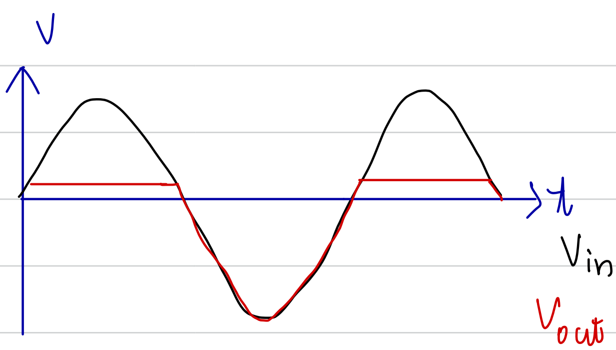
$= 15\ mA$

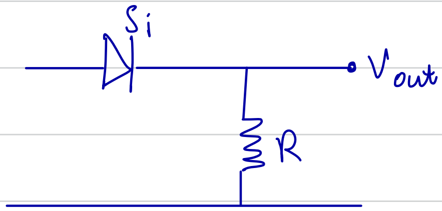
$$\therefore R_3 = \frac{12 - 5.4}{15 \times 10^{-3}} = 433\ \Omega$$

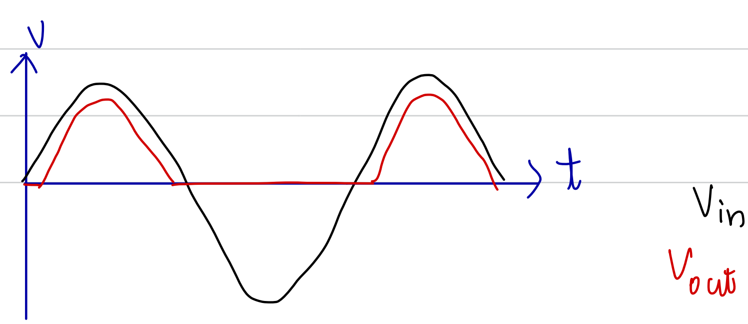
Precision rectifier

Halfwave rectifier

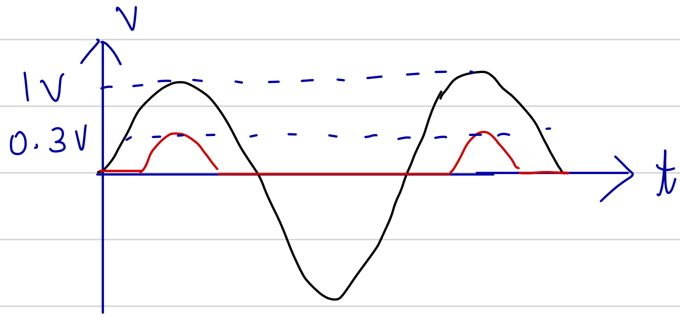




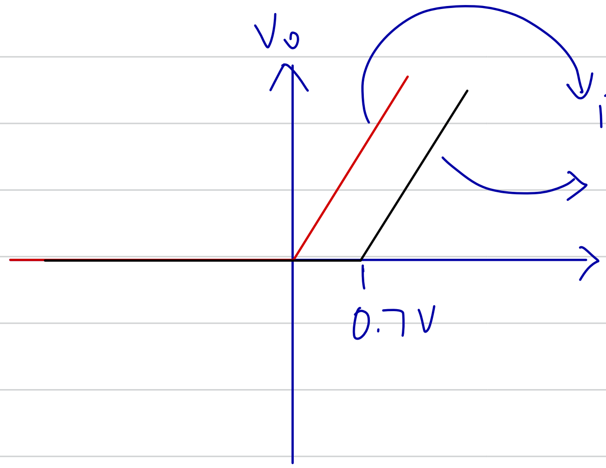




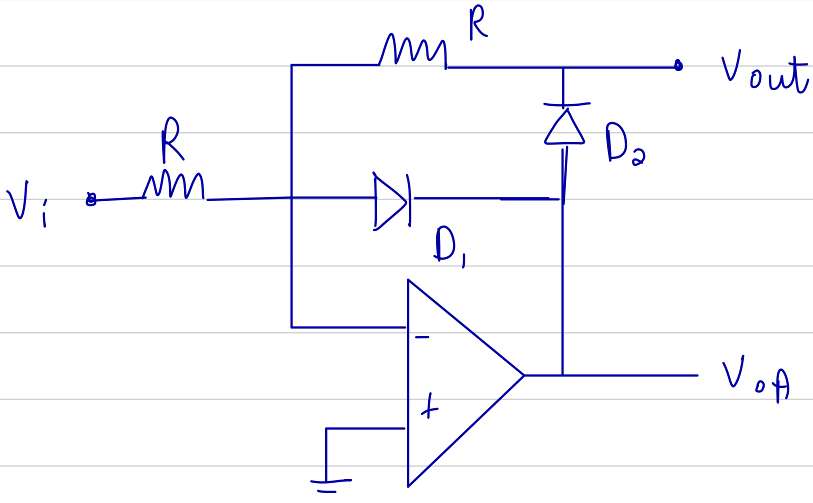
Suppose input to circuit 2 is $1V_{pp}$ sinusoid



V_o v/s V_i →



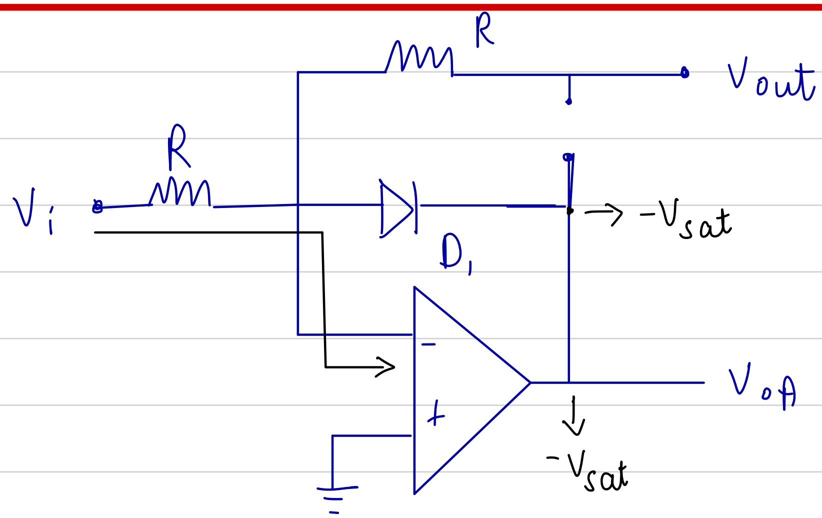
Halfwave precision rectifier



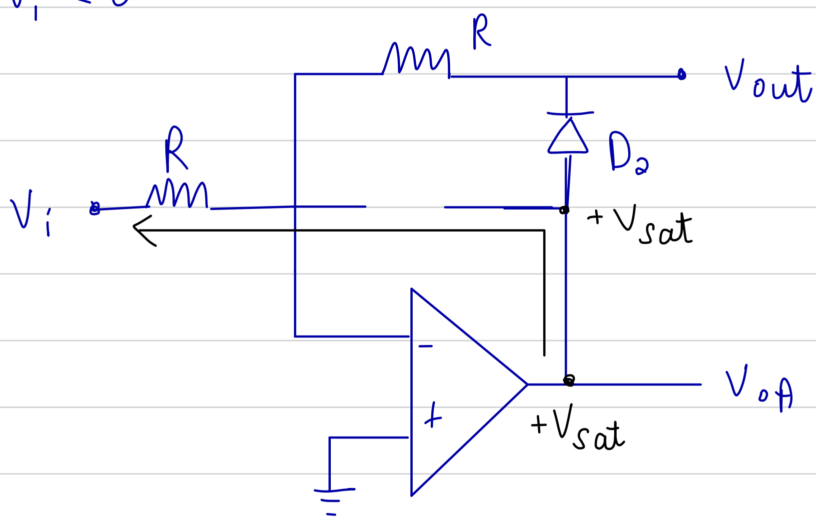
When $V_i > 0$, V_{D_2} is reverse bias

$\Rightarrow V_{out} = 0$

$V_{oA} \simeq -V_{D_1}$



When $V_i < 0$

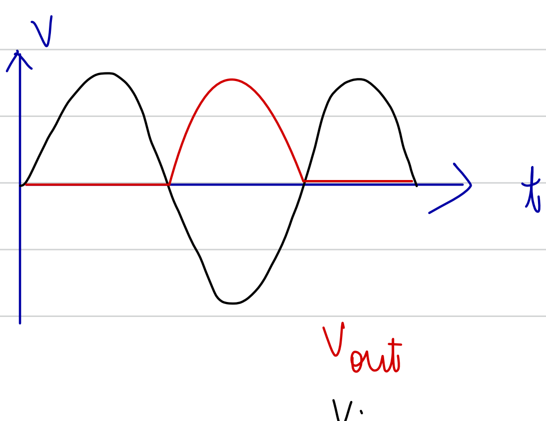


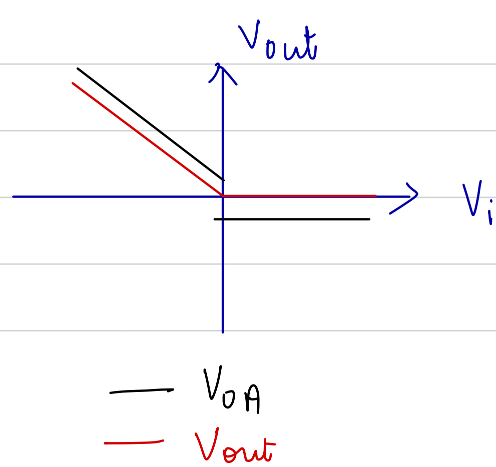
$\Rightarrow \quad V_{out} = -V_i$

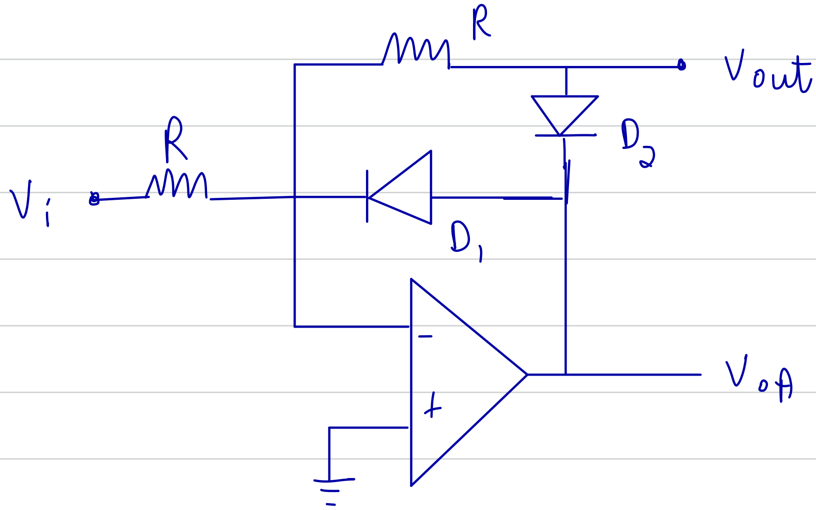
$V_{oA} = V_{out} + V_{D_2}$

$$\frac{V_{out} - 0}{R} = \frac{0 - V_i}{R} \qquad \Rightarrow V_{out} = -V_i$$

$\therefore$







(Reversed diodes)

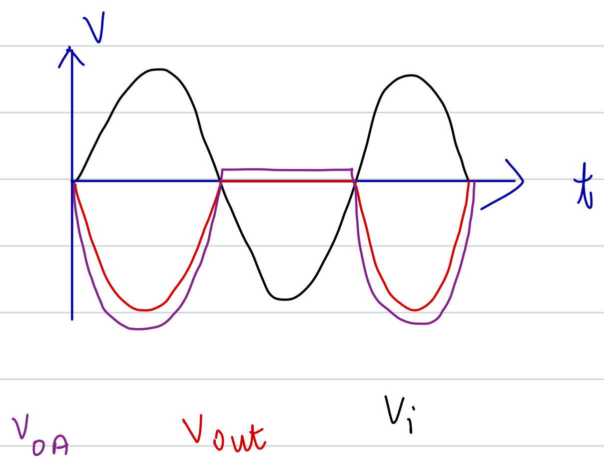
$V_i > 0$

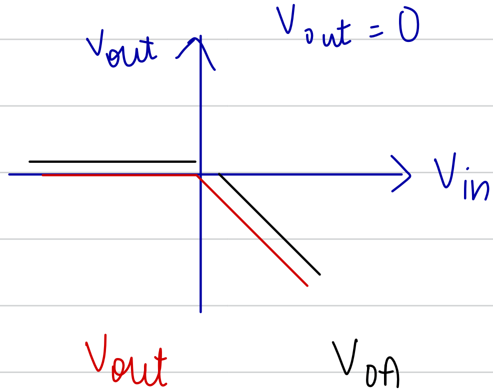
$V_{out} = -V_i$

$V_{oA} = V_{out} - V_{D_2}$

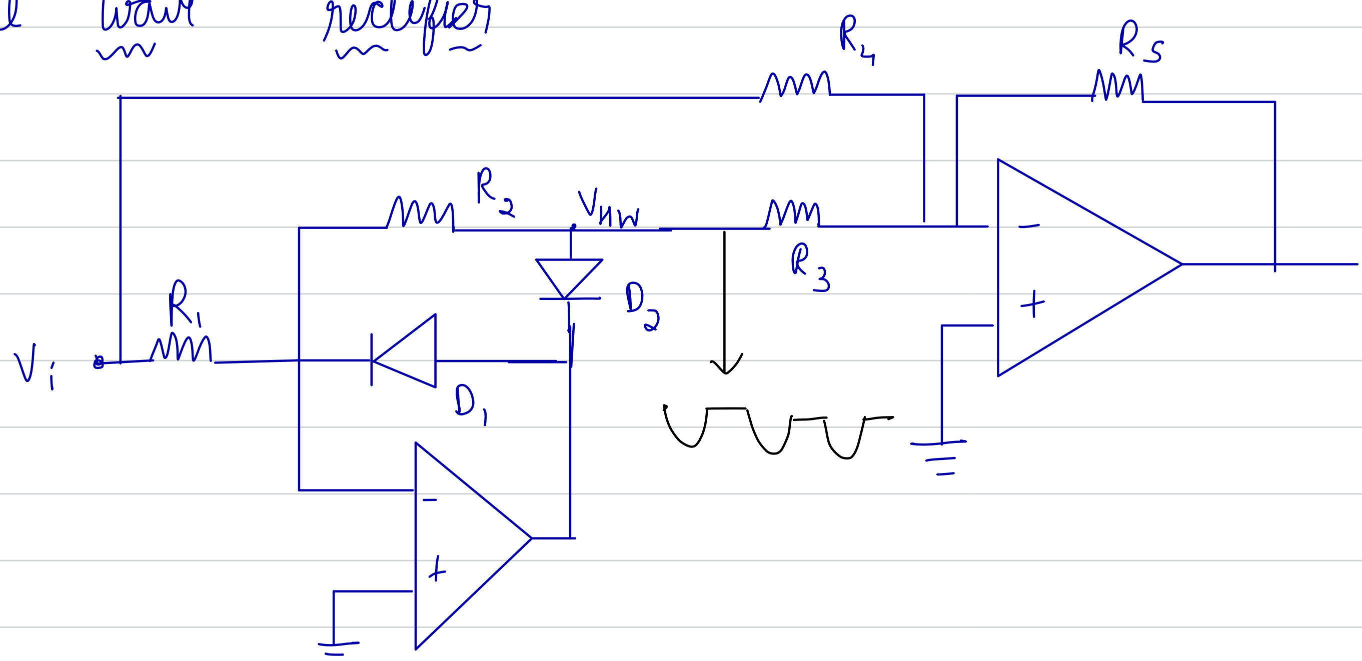
$V_i < 0$

$V_{out} = 0 \qquad V_{oA} = V_{D_1}$





Full wave rectifier



$$V_0 = -\frac{R_4}{R_5} V_i - \frac{R_5}{R_3} V_{HW}$$ (during +ve half cycle)

$$= -\frac{R_5}{R_4} V_i - \left(\frac{R_5}{R_3}\right)\left(-\frac{R_2}{R_1}\right) V_i'$$

$$\Rightarrow \left[\frac{R_5 R_2}{R_3 R_1} - \frac{R_5}{R_4}\right] V_i$$

During -ve half cycle,

$$V_0 = -\frac{R_5}{R_4} V_i = -A_m V_i$$

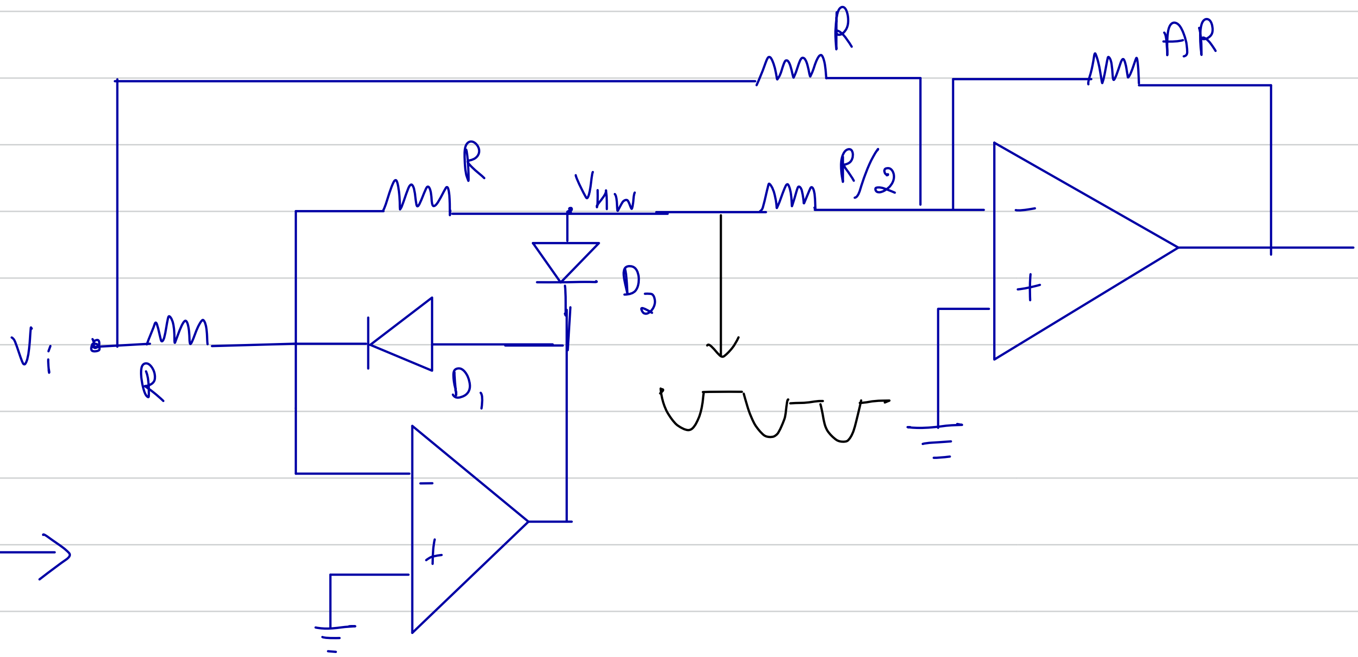
$$\Rightarrow \frac{R_5 R_2}{R_3 R_1} - \frac{R_5}{R_4} = \frac{R_5}{R_4}$$

$$\Rightarrow \frac{R_5 R_2}{R_3 R_1} = \frac{2R_5}{R_4}$$

Then $R_1 = R_2 = R_4 = R$

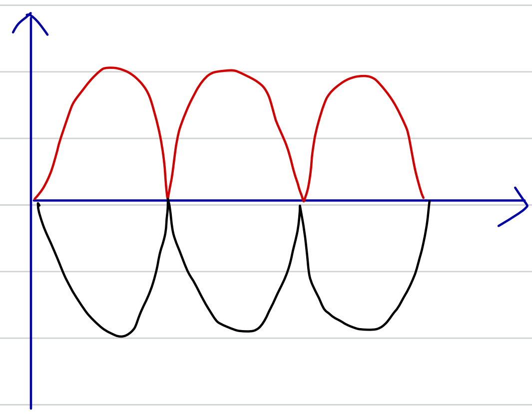
$R_3 = \frac{R}{2}$ & $R_5 = AR$

Then final circuit is :



Where $A =$ amplification factor for output.

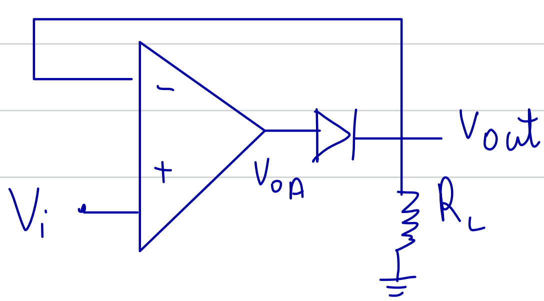
Eg: If $V = V_m \sin\omega t$ & $(V_{out})_{max} = 2V_m$ then $A = 2$

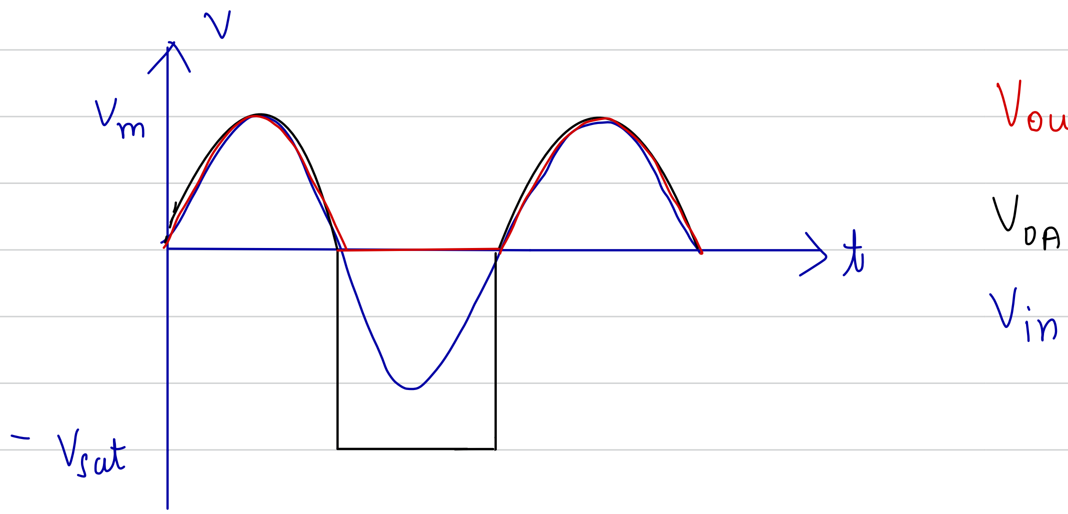


V_{out} in above case

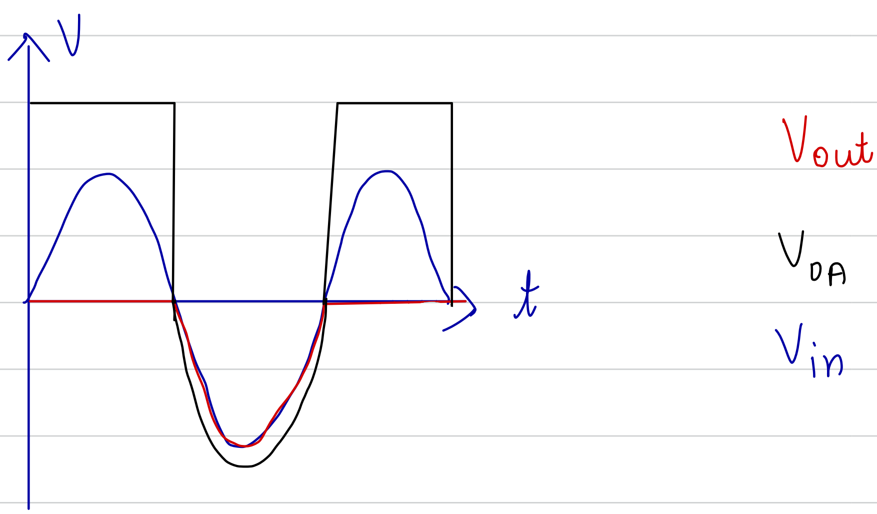
V_{out} with diodes reversed.

Q1> For given circuit draw output waveform if $V_{in} = V_m \sin\omega t$

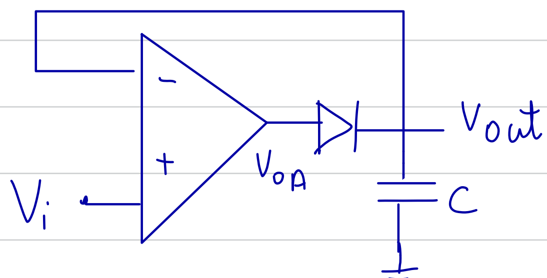


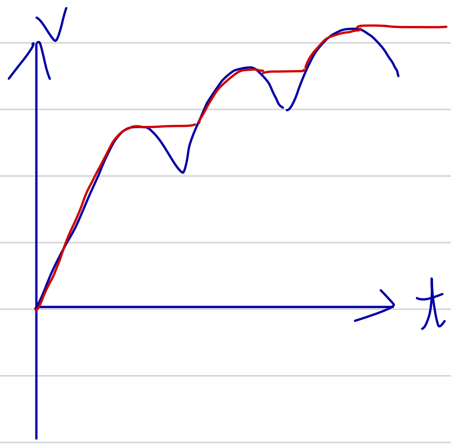


When direction of diode is reversed :



Q2> For given circuit draw output waveform





V_{out}

V_{in}