



DATA STRUCTURES AND ALGORITHMS

- MANVITH PRABHU



ARRAYS

Syntax: datatype arrayname [size];

Arrays are ordered arrangement of same type of data.

Insertion at a position

- 1> Start
- 2> for ($i = n - 1$; $i \geq pos$; $i--$)
 $A[i + 1] = A[i]$
- 3> $A[pos] = data$
- 4> $n = n + 1$
- 5> Stop

Deletion at a position

- 1> start
- 2> $data = A[pos]$
- 3> for ($i = pos$; $i < n$; $i++$)
 $A[i] = A[i + 1]$
- 4> $n = n - 1$
- 5> Stop

Searching algorithms

1) Linear search:

- 1> Start
- 2> loc = -1
- 3> for (i=0 ; i<n ; i++)
 { if (A[i] == search)
 loc = i }
- 4> if (loc > -1)
 print loc
else print search unsuccessful
- 5> Stop

2) Binary search algorithm (for sorted array)

- 1> Start
- 2> B = lb E = UB loc = -1
- 3> while (B <= E)
 { mid = int (B+E)/2;
 if (ele == a[mid])
 loc = mid
 else if (ele < a[mid])
 E = mid - 1;
 else B = mid + 1; }
- 4> if (loc > -1)
 print loc
else print search unsuccessful
- 5> Stop

Sorting algorithms :

1) Selection sort :

```
1> Start  
2> Input array  
3> for (i=0 ; i<n-1 ; i++)  
4>   min = i;  
5>   for (j=i+1 ; j<n ; j++)  
       if (a[min] > a[j])  
         { t = a[min]  
           a[min] = a[j]  
           a[j] = t }  
6> Output array  
7> Stop.
```

$O(n^2)$

2> Bubble sort

1> Start

$O(n^2)$

2> Input array

```
3> for (i=0; i<n-1; i++)  
4> for (j=0; j<n-i-1; j++)  
{ if (a[j] > a[j+1])  
    { t = a[j];  
     a[j] = a[j+1];  
     a[j+1] = t; }  
}
```

5> Output array

6> Stop.

3> Insertion sort

1> Start

2> Input array

$O(n^2)$

```
3> for (i=1; i<n; i++)  
    for (j=i; j>0; j--)  
    { if (a[j] < a[j-1])  
        { t = a[j];  
         a[j] = a[j-1];  
         a[j-1] = t; }  
    }
```

4> Output array

5> Stop.

4) Quick sort : Divide and conquer

$O(n^2)$

1) Start

2) Partition (A, lb, ub)

3) $pivot = a[lb];$

$start = lb;$

$end = ub;$

4) while ($start < end$)

 while ($a[start] \leq pivot$)

$start ++$

 while ($a[end] > pivot$)

$end --$

5) if ($start < end$)

 swap ($a[start], a[end]$)

 end while

6) if ($start \geq end$)

 swap ($a[lb], a[end]$)

 end if

 return end

7) Quicksort (A, lb, ub)

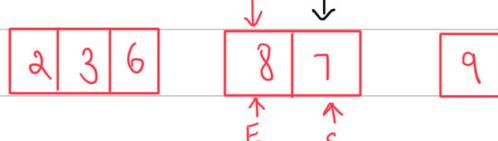
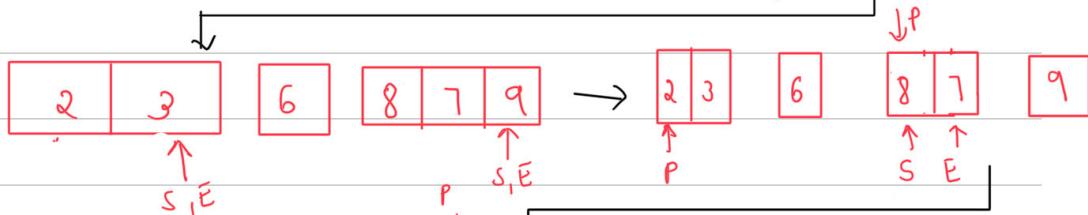
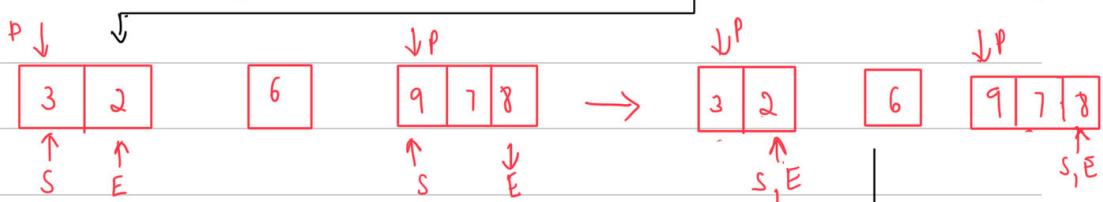
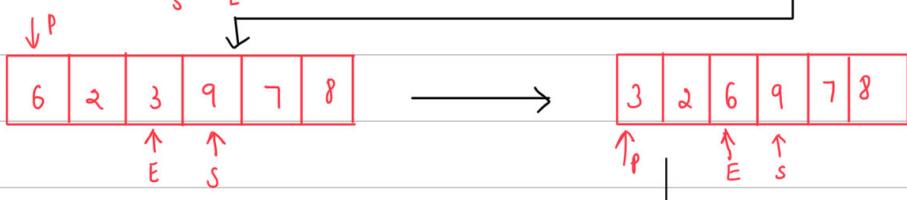
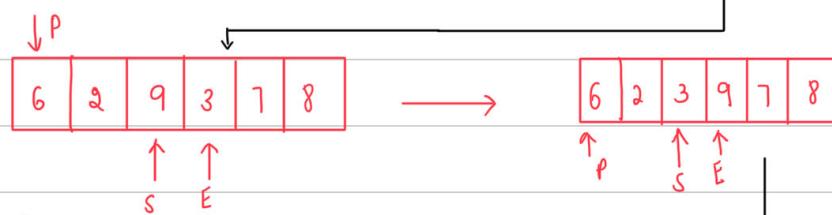
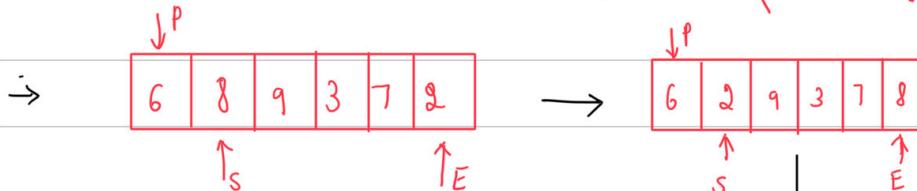
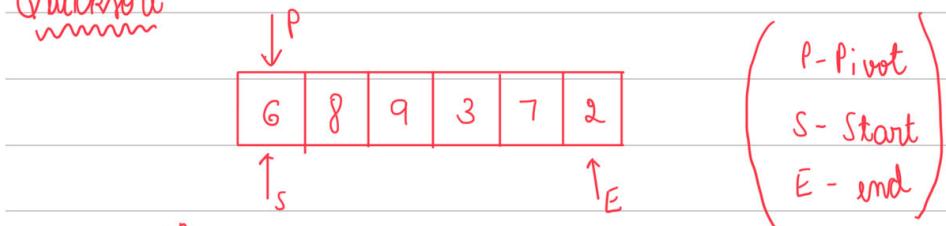
 { if ($lb < ub$)

 { location = Partition (A, lb, ub)

 Quicksort ($A, lb, location - 1$)

 Quicksort ($A, location + 1, ub$) } }

Quicksort



A = [2, 3, 6, 7, 8, 9]

5> Merge sort : Divide & conquer (Find middle element)

1> Start

$O(n \log n)$

2> Mergesort (A , lb , ub)

{ if ($lb < ub$)

{ mid = $(lb + ub) / 2$;

mergesort (A , lb , mid)

mergesort (A , $mid + 1$, ub)

merge (A , lb , mid , ub)

}

3> merge (A , lb , mid , ub)

{ $i = lb$

$n = ub - lb + 1$

$j = mid + 1$

$k = 0$

while ($i \leq mid$ & $j \leq ub$)

{ if ($a[i] \leq a[j]$)

{ $b[k] = a[i]$

$i++$

else { $b[k] = a[j]$

$j++$

$k++$

if ($i > mid$)

{ for (j ; $j \leq ub$; $j++$)

$b[k] = a[j]$; $k++$

if ($j > ub$)

{ for (i ; $i \leq mid$; $i++$)

$b[k] = a[i]$; $k++$

for ($x=0$; $x < n$; $x++$)

$a[lb+x] = b[x]$

Mergesort

A =

| | | | | | |
|---|---|---|---|---|---|
| 6 | 5 | 4 | 3 | 2 | 1 |
| 0 | 1 | 2 | 3 | 4 | 5 |

$$\text{mid} = \frac{0+5}{2} = 2$$

| | | |
|---|---|---|
| 6 | 5 | 4 |
| 0 | 1 | 2 |

| | | |
|---|---|---|
| 3 | 2 | 1 |
| 3 | 4 | 5 |

$$\text{mid} = \frac{0+2}{2} = 1$$

| | |
|---|---|
| 6 | 5 |
| 0 | 1 |

| |
|---|
| 4 |
| 2 |

$$\text{mid} = \frac{3+5}{2} = 4$$

| | |
|---|---|
| 3 | 2 |
| 3 | 4 |

| |
|---|
| 1 |
| 5 |

$$\text{mid} = \frac{0+1}{2} = 0$$

| | | |
|---|---|---|
| 6 | 5 | 4 |
| 0 | 1 | 2 |

| | | |
|---|---|---|
| 3 | 2 | 1 |
| 3 | 4 | 5 |

| | |
|---|---|
| 5 | 6 |
| 2 | 4 |

| |
|---|
| 4 |
| 1 |

| | |
|---|---|
| 2 | 3 |
| 2 | 4 |

| |
|---|
| 1 |
| 5 |

| | | |
|---|---|---|
| 4 | 5 | 6 |
| 1 | 2 | 3 |

| | | |
|---|---|---|
| 1 | 2 | 3 |
| 1 | 2 | 3 |

| | | | | | |
|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|

6>

Counting sort :

 $O(n)$

- 1> no comparison
- 2> check for recurrence
- 3> store in an array
- 4> Find the exact location to be placed.

Ex:

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 2 | 1 | 0 | 1 | 3 | 5 | 4 | 6 | 1 | 3 |
|---|---|---|---|---|---|---|---|---|---|---|

range $k = 0-6$; $\Rightarrow k=6$
 $n=11$

count array with all 0 elements. size = $k+1 = 7$,

count array:

| | | | | | | |
|---|---|---|---|---|---|---|
| 1 | 3 | 2 | 2 | 1 | 1 | 1 |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 |

$\text{count}[i] = \text{count}[i] + \text{count}[i-1]$

count array:

| | | | | | | |
|---|---|---|---|---|----|----|
| 1 | 4 | 6 | 8 | 9 | 10 | 11 |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 |

Sorted array:

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|----|
| 0 | 1 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 5 | 6 |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |

Algorithm:

- 1> start
- 2> for ($i=0$; $i < n$; $i++$)
 $\quad \text{count}[\text{a}[i]] += 1$;
- 3> for ($i=1$; $i < k+1$; $i++$)
 $\quad \text{count}(i) = \text{count}(i) + \text{count}(i-1)$
- 4> for ($i=n-1$; $i > 0$; $i--$)
 $\quad \text{B}[-\text{count}[\text{a}[i]]] = \text{a}[i]$

73 Radix sort - sub routine

Ex:

| | | | | | | | | |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 008 | 326 | 100 | 073 | 169 | 456 | 011 | 079 | 132 |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|

Pass 1: counting sort for LSB
count array

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 0 | 0 | 2 | 0 | 1 | 2 |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | |

$$\text{count}(i) = \text{count}(i) + \text{count}(i+1)$$

count array:

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 4 | 4 | 6 | 6 | 7 | 9 |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

B:

| | | | | | | | | |
|-----|----|-----|----|-----|-----|---|-----|----|
| 100 | 11 | 132 | 73 | 326 | 456 | 8 | 169 | 79 |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |

Pass 2: counting sort for 2nd bit:

count array:

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 2 | 1 | 1 | 1 | 0 | 1 | 1 | 2 | 0 | 0 |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

$$\text{count}(i) = \text{count}(i) + \text{count}(i-1)$$

count array:

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 2 | 3 | 4 | 5 | 5 | 6 | 7 | 9 | 9 | 9 |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

B:

| | | | | | | | | |
|-----|---|----|-----|-----|-----|-----|----|----|
| 100 | 8 | 11 | 326 | 132 | 456 | 169 | 73 | 79 |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |

Pass 3: counting sort for 3rd bit:

count array:

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 4 | 3 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

$$\text{count}(i) = \text{count}(i) + \text{count}(i-1)$$

count array :

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 4 | 7 | 7 | 8 | 9 | 9 | 9 | 9 | 9 | 9 |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

B :

| | | | | | | | | |
|---|----|----|----|-----|-----|-----|-----|-----|
| 8 | 11 | 73 | 79 | 100 | 132 | 169 | 326 | 456 |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |

Algorithm:

```

1> Start
2> radix sort (A, n)
{   max = getmax (A, n)
    for (pos = 1 ; (max / pos) > 0 ; pos * 10)
        { count sort (A, n, pos)
        }
3> count sort (A, n, pos)
{   count [10] = {0} ;
    for (i=0 ; i<n ; i++)
        count [(A[i] / pos) * 10] ++
    for (i=0 ; i<10 ; i++)
        count [i] = count [i] + count [i-1];
    for (i=n-1 ; i>0 ; i--)
        B [ --count [(A[i] / pos) * 10] ] = A[i]
    for (i=0 ; i<n ; i++)
        A[i] = B[i]
}

```

Time complexity = $O(d * (n+b))$

b = base (here 10)
d = no. of digits
n = no. of elements

Conversion of infix to prefix / postfix

Priority: 1. (), [], { }

2. ^ → Right to left
3. / * → Left to right
4. + , - → Left to right

Rules:

- 1> Operands → Print
 - 2> (→ opening bracket → Push into stack.
 - 3> If it is operator:
 - i> If stack is empty push it.
 - ii> If top of stack is opening bracket, push it.
 - iii> If it has higher priority push it, else pop the top of stack and repeat step 3.
 - iv> If it has equal priority check associativity.
 - 4> If it is closing bracket: pop all operators until closing bracket is encountered.
-
- To convert to prefix:
 - 1> Reverse the given infix expression (convert closing braces to open braces and open braces to closing braces)
 - 2> Convert it to postfix by applying the steps above.
 - 3> Reverse the obtained postfix expression to get prefix expression.

- Queues:
- It is a linear abstract data type.
 - It is based on FIFO principle.
 - It can be implemented using stack, linked list, arrays.

Enqueue Algorithm - Array

- 1> Start
- 2> if ($\text{rear} == \text{max} - 1$)
 overflow
- 3> if ($\text{rear} == -1 \&\& \text{front} == -1$)
 { $\text{rear}++$; $\text{front}++$;
 $\text{Queue}[\text{rear}] = \text{data}$ }
 end if
- 4> else { $\text{rear}++$;
 $\text{Queue}[\text{rear}] = \text{data}$; }
- 5> Stop

Dequeue algorithm - Array

- 1> Start
- 2> if ($\text{rear} == -1 \&\& \text{front} == -1$)
 underflow
- 3> if ($\text{front} == \text{rear}$)
 { $\text{data} = \text{Queue}[\text{front}]$;
 $\text{front} = -1$; $\text{rear} = -1$; }
 end if

↳ else { data = Queue[front]
 front++; }

5) Stop

Enqueue algorithm - Linked list

- 1) Start
- 2) if (front == NULL & rear == NULL)
{ n = new node
 front = n rear = n }
end if
- 3) else { n = new node;
 rear->next = n; rear = n; }

↳ Stop

Dequeue algorithm - Linked list

- 1) Start
- 2) if (front == rear)
{ t = front
 front = NULL
 rear = NULL
 delete t } end if

- 3) else
{ t = front
 front = front->next
 delete t }

4) Stop

Circular Queue :

enque - array

```
1> Start  
2> if (Front == (rear + 1) % N)  
    overflow  
3> if (rear == -1 && front == -1)  
    { rear++; front++;  
     Queue[rear] = data}  
    end if  
4> else if (rear == N - 1)  
    { rear = 0; Queue[rear] = data}  
    end if  
5> else { rear++;  
     Queue[rear] = data; }  
6> Stop
```

Dequeue - array

```
1> Start  
2> if (rear == -1 && front == -1)  
    underflow  
3> data = Queue[front]  
    if (front == rear)  
        front = -1  
        rear = -1
```

4> if (front == n-1)

 front = 0;

 use front ++;

5> Stop -

Linked lists

Implementation:

1> Start

2> count = 0 , t = head

3> n = new node

n → data = num

4> while (t != NULL)

 count ++

 t = t → next

end while

5> if (pos == 1)

{ n → next = head

 head = n }

6> if (pos == count + 1)

{ n → next = null

 t → next = n }

7> if (pos <= count)

{ t = head;

 for (i=1 ; i < pos-1; i++)

 { t = t → next }

$n \rightarrow \text{next} = t \rightarrow \text{next}$

$t \rightarrow \text{next} = n \}$

8> else print "invalid position"

9> Stop

Deletion:

1> Start

2> count = 0 $t_1 = \text{head}$

3> while ($t_1 \neq \text{NULL}$)

 count = count + 1

$t_2 = t_1$

$t_1 = t_1 \rightarrow \text{next}$

end while

4> if ($\text{pos} == 1$)

$t_1 = \text{head}$

 print $t_1 \rightarrow \text{data}$

 head = $t_1 \rightarrow \text{next}$

 delete $t_1 \}$

5> if ($\text{pos} == \text{count}$)

 print $t_1 \rightarrow \text{data}$

$t_2 \rightarrow \text{next} = \text{NULL}$

 delete $t_1 \}$

6> if ($\text{pos} < \text{count}$)

$t_1 = \text{head}$

 for (i=1 ; i < pos ; i++)

$t_2 = t_1$

$t_1 = t_1 \rightarrow \text{next} \}$

```
print t1->data  
t2->next = t1->next  
delete t1  
}  
7> else print "invalid position"  
8> Stop.
```

Asymptotic analysis

- 1> $O \rightarrow$ Worst case time complexity
- 2> $\Omega \rightarrow$ Best case time complexity
- 3> $\Theta \rightarrow$ When runtime is same for all cases.

Stacks:

1> Push : - array

1> Start

2> if ($\text{top} == \text{max} - 1$)
 print overflow

3> else { $\text{top} = \text{top} + 1$
 $\text{stack}[\text{top}] = \text{value}$ }

4> Stop

2> Pop : - array

1> Start

2> if ($\text{top} == \text{NULL}$)
 print underflow

3> else $\text{value} = \text{stack}[\text{top}]$

4> $\text{top} = \text{top} - 1$

5> Stop

3> Peek : - array

1> Start

2> if ($\text{top} == \text{NULL}$)
 print underflow

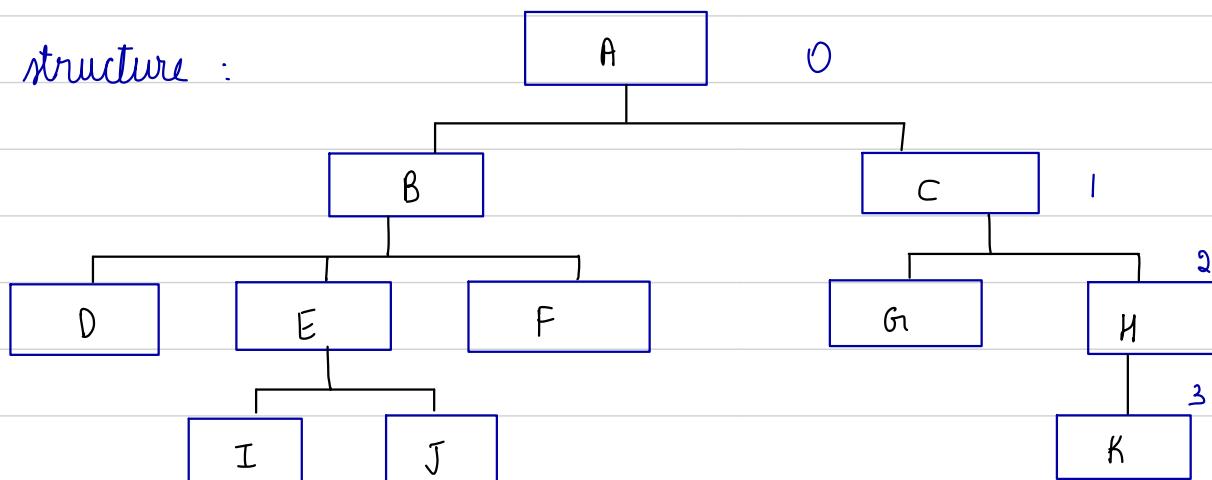
3> else return $\text{stack}[\text{top}]$

4> Stop

Non linear data structures

Trees: (When there is hierarchy, trees are used)

General structure :



Root node: Nodes which do not have any parent (Eg: A)

Intermediate nodes: Nodes which are between root and leaf:
Eg: (B, E, C, H)

Leaf node: Nodes which do not have any child node.

Eg: (D, F, I, J, G, K)

Level / Depth: Distance between root and any node

Ex: Depth of G = 2

Height: Distance between root and last leaf node

Ex: Height of the tree = 3 (max depth)

Degree of a node: Number of child nodes that a node has.

Traversal of tree

- i) ordered tree: Left to right order is maintained in data.

NOTE: Empty trees are possible.

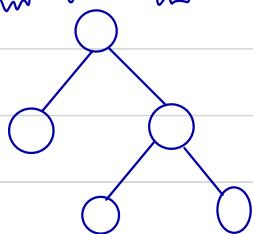
- Binary tree: It is an ordered tree where every node will have atmost 'two' child nodes.

- ii) Unordered tree: No ordering is maintained.

Different types of Binary trees

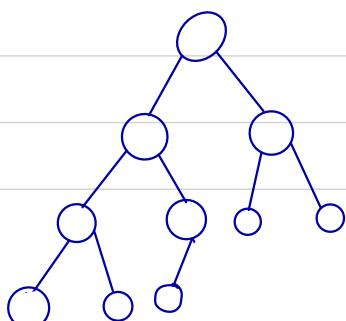
- iii) Full binary tree: Each node will have 0 or 2 child nodes.

Ex:



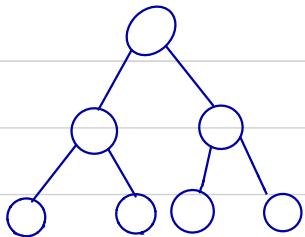
- iv) Complete binary tree: It is a binary tree in which all levels except the last level is completely filled and all nodes in last level are left as possible.

Ex:



iii) Perfect binary tree: All nodes have 2 child nodes and all leaf nodes are at same level.

Ex:



Tree traversal / tree walk

- 1> Preorder traversal
- 2> Inorder traversal
- 3> Postorder traversal

| First | Second | Last |
|-------|--------|-------|
| Root | left | right |
| left | root | right |
| left | right | root |

1> Preorder traversal

Algorithm:

1> Start

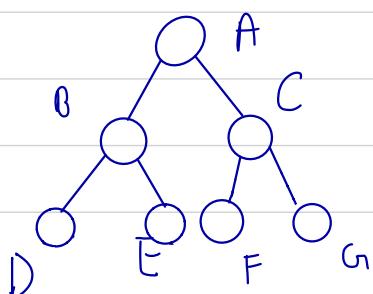
2> Preorder(v)

3> If ($v = \text{null}$) , Return
visit (Root)

Preorder (v. left child [])

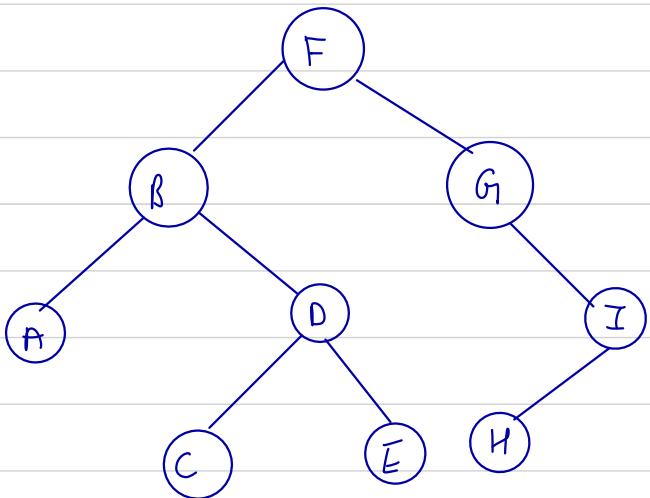
Preorder (v. right child [])

Ex a>



A → B → D → E . → C → F → G

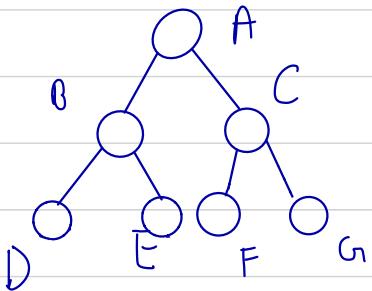
ex b:



Ans: $F \rightarrow B \rightarrow A \rightarrow D \rightarrow C \rightarrow E$
 $\rightarrow G \rightarrow I \rightarrow H$

2) Inorder traversal

ex a:

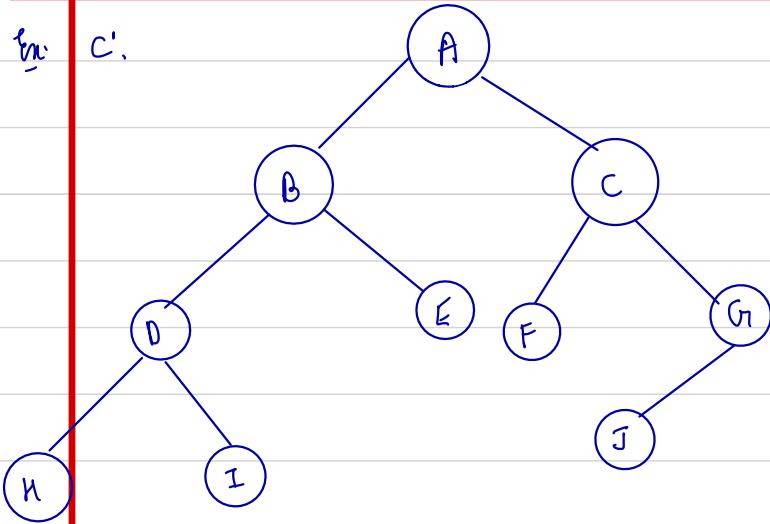


$D \rightarrow B \rightarrow E \rightarrow A \rightarrow F \rightarrow C \rightarrow G$

Algorithm

- 1> Start
- 2> Inorder (v)
- If ($v == \text{null}$) , Return
- 3> Inorder ($v \cdot \text{left}()$)
- visit (root)
- Inorder ($v \cdot \text{right}()$)
- 4> Stop

Ex: C.



Preorder: A, B, D, H, I, E, C, F, G, J

Inorder: H, D, I, B, E, A, F, C, J, G

3) Postorder:
For ex a: D, E, B, F, G, C, A
For ex b: A, C, E, D, B, H, I, G, F
For ex c: H, I, D, E, B, F, J, G, C, A

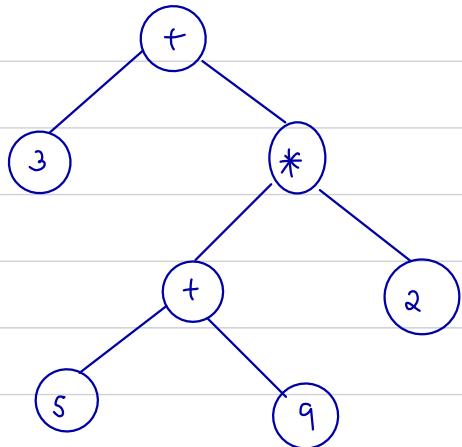
Algorithm:

- 1) Start
- 2) Postorder(v)
If (v == null), return
- 3) Postorder(v.left)
Postorder(v.right)
- 4) visit(root)
- 5) Stop

Binary expression tree

- It is a binary tree which is used to represent mathematical expression, where each internal node and root node represents operator and leaf node represents operand.
- Inorder traversal is used.

Eg: 1)



Inorder

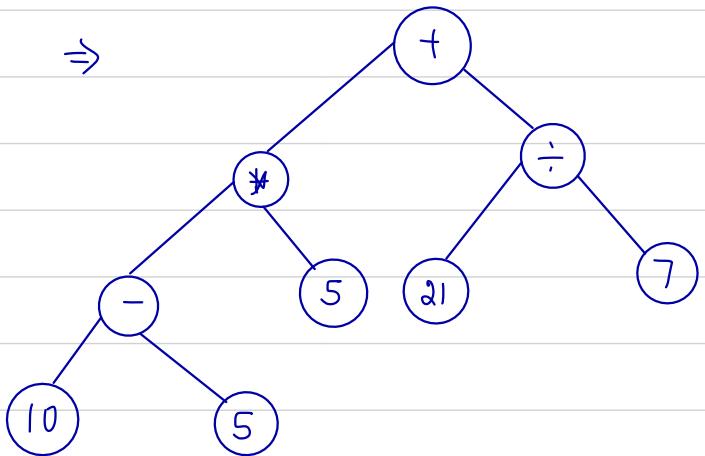
$$\begin{aligned}
 \rightarrow & 3 + (5 + 9) * 2 \\
 = & 3 + 14 * 2 \\
 = & 3 + 28 \\
 = & 31,
 \end{aligned}$$

2) $(10 - 5) * 5 + 21 / 7$

- Rules:
- 1) Root is least priority and associativity operator
 - 2) Divide the expression accordingly
 - 3) Again go to rule 1.

$$(10 - 5) * 5 + 21 / 7$$

\Rightarrow



Eg 3: Write prefix & postfix for eg 1:

Ans: Prefix : + 3 * 5 9 2

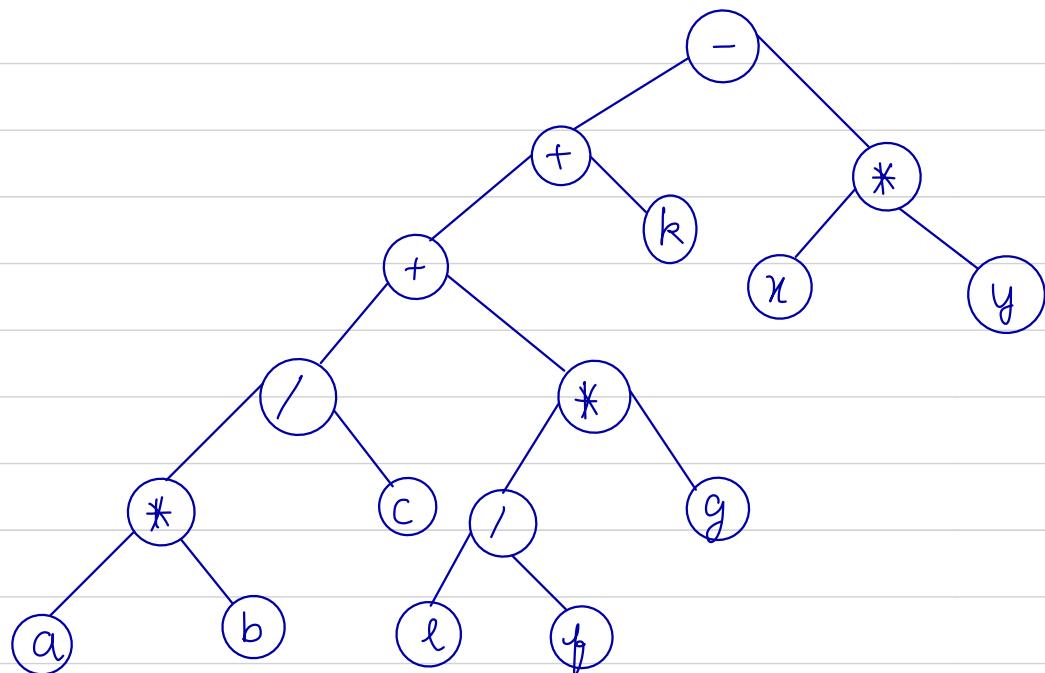
Postfix : 3 5 9 + 2 * +

Eg 4: Write prefix & postfix for eg 2:

Ans : + * - 10 5 5 ÷ 21 7 ← Preorder

Postorder : → 10 5 - 5 * 21 7 ÷ +

Eg 5: $a * b / c + e / f * g + k - x * y$

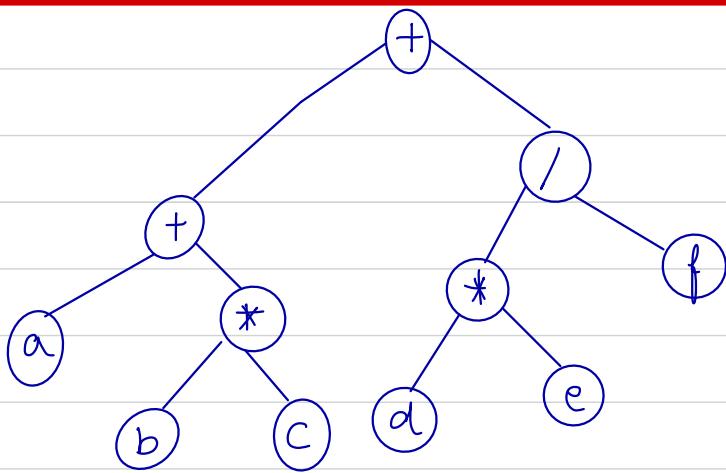


Prefix: - + + / * a b c * / e f g k * x y

Postfix: a b * c / e f / g * + k + x y * -

Eg 6: $a + b * c + d * e / f$

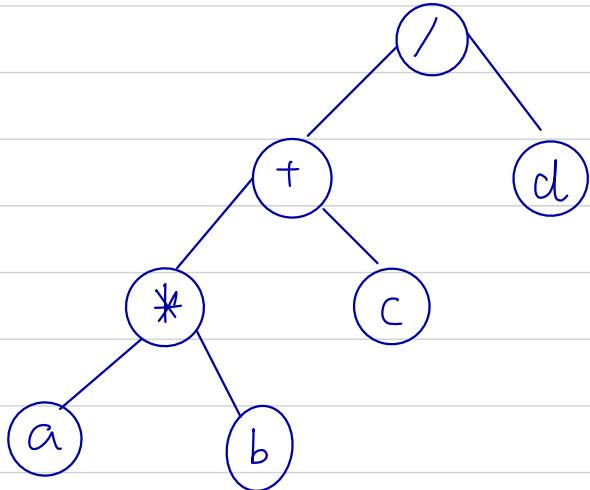
Eg6:



Prefix: + + a * b c / * d e f
Postfix: a b c * + d e * f / +

Eg7

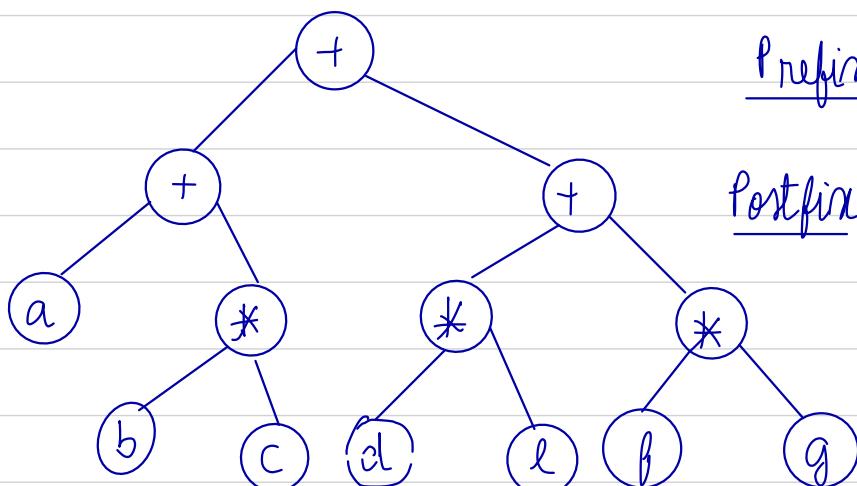
$$((a * b) + c) / d$$



Prefix: / + * a b c d

Postfix: a b * c + d /

$$(a + b * c) + (d * e + f * g)$$



Prefix: + + a * b c + * d e * f g

Postfix: a b c * + d e * f g * + +

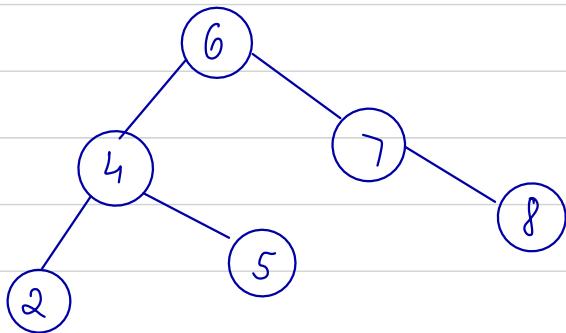
Binary search tree: (BST)

It is a node base tree which has the following property :-

→ Let x be a node in BST. If y is a node in left subtree of x then $y \cdot \text{key} \leq x \cdot \text{key}$ ($\text{key} \rightarrow \text{value of node}$)

→ If y is a node in right subtree then $y \cdot \text{key} \geq x \cdot \text{key}$

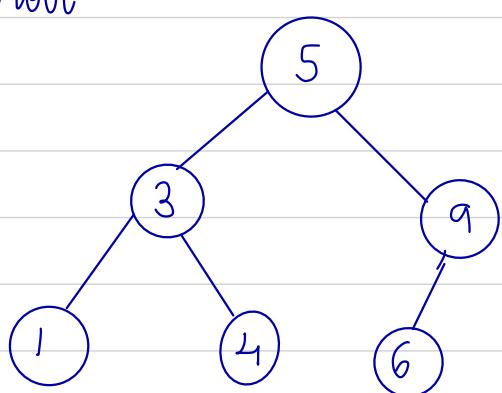
Ex:



1) 5, 3, 1, 9, 6, 4

↓
root

compare $>$ root or $<$ root



Preorder: 5 3 1 4 9 6

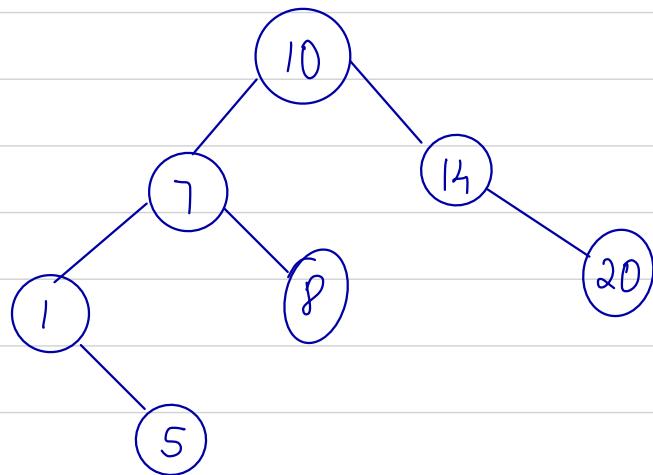
Inorder: 1 3 4 5 6 9

Postorder: 1 4 3 6 9 5

∴ Inorder always gives a sorted order.

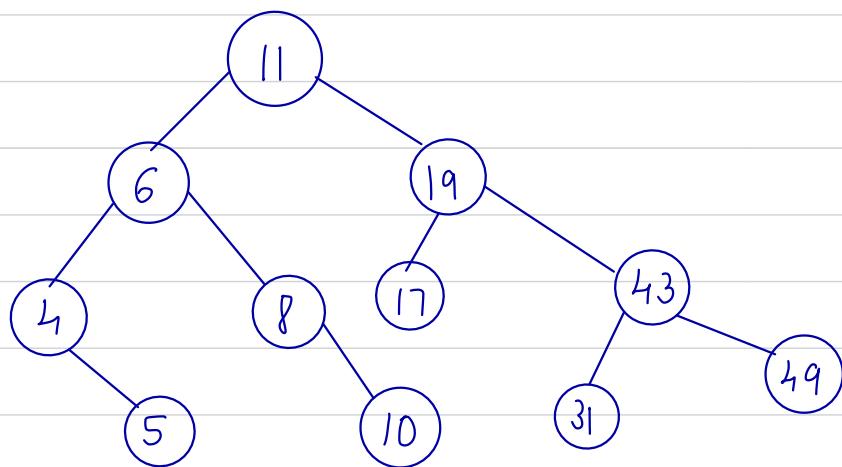
Skew tree: Nodes only towards one side i.e. left skew tree or right skew tree.

2) 10, 7, 14, 20, 1, 5, 8



Inorder: 1 5 7 8 10 14 20

3) 11, 6, 8, 19, 4, 10, 5, 17, 43, 49, 31

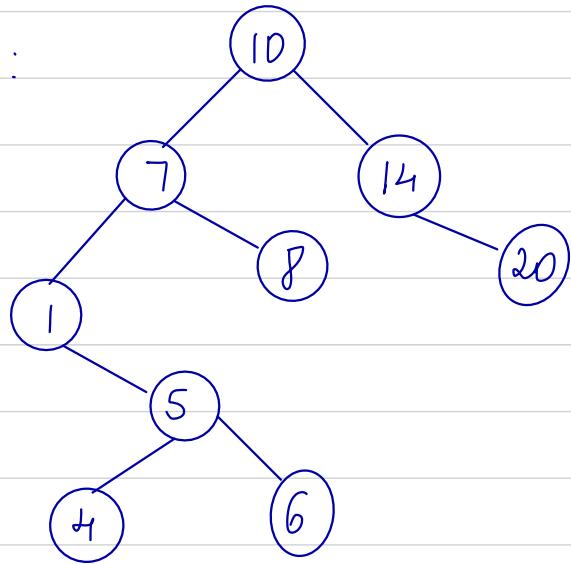


Inorder: 4 5 6 8 10 11 17 19 31 43 49

Insertion: Given a tree, insert a new node as per rules of creation

Deletion: If we want to remove any node with 2 child nodes, we can replace it with either predecessor or successor of inorder travel

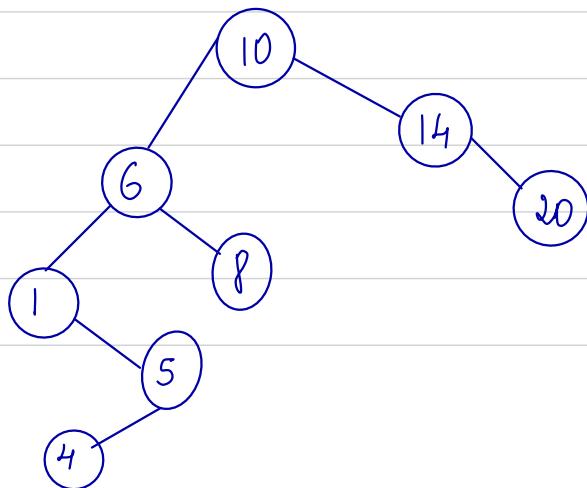
Eg: Remove 7:



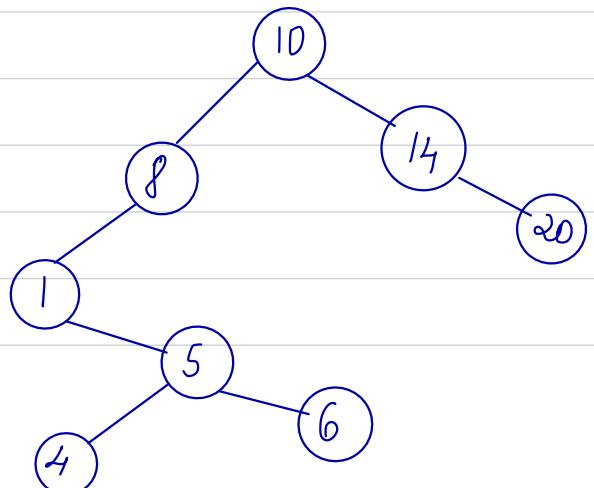
→

Inorder travel: 1 4 5 6 7 8 10 14 20
We can replace it with 6 or 8.

Replace with 6



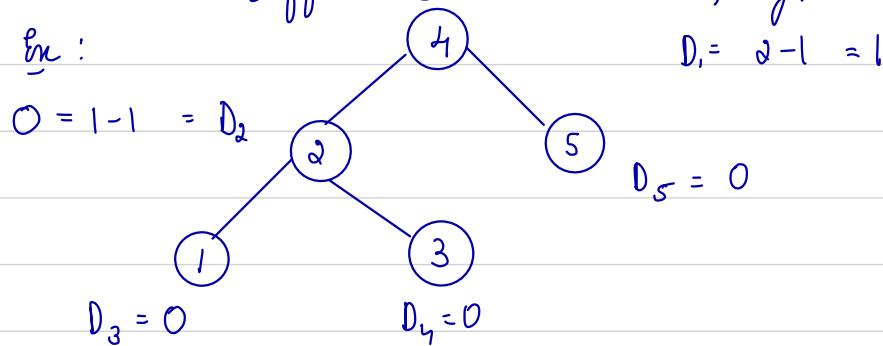
Replace with 8



Search: If element is found then side, level should be shown.

Balanced binary search tree

- Also known as Height balanced BST.
- Max difference in height of subtrees is 1.

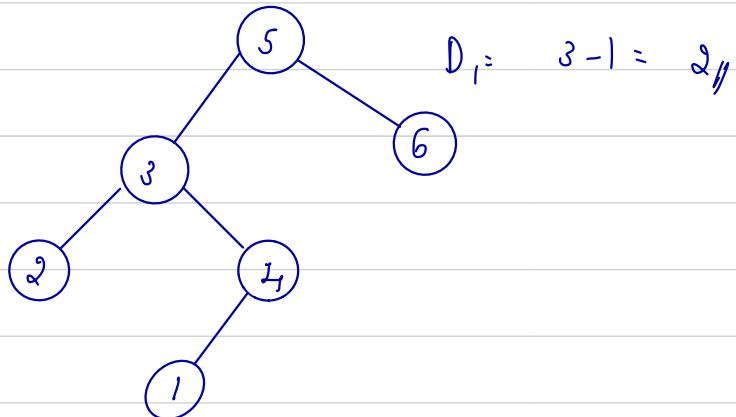


Advantage: Reduces search time

- Balanced binary search tree is a BST in which height of left and right subtree is almost one.

Q> Is it balanced?

⇒ Not balanced

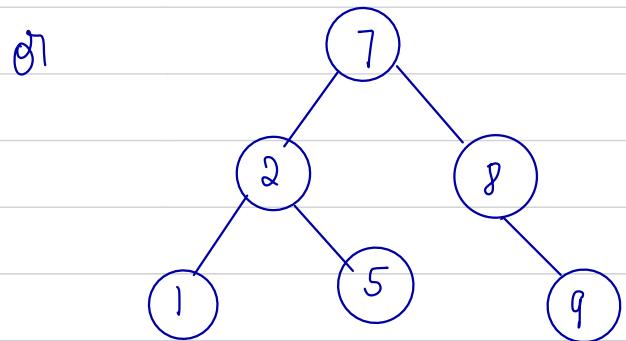
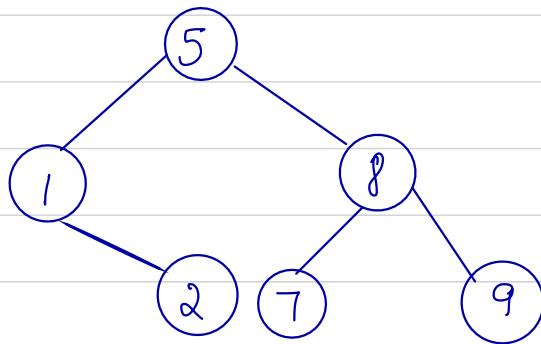


Creation of binary search tree

1> From a list of key values:

- 1> Sort the given values
- 2> Select the mid as root
- 3> Again select the mid of left & right sublists as roots of left and right subtrees and so on.

Ex: 7 5 2 9 8 1
Ans: Sort: 1 2 5 7 8 9



Priority queues & heaps

Operations on Priority queue

1 Insertion

2 Deletion

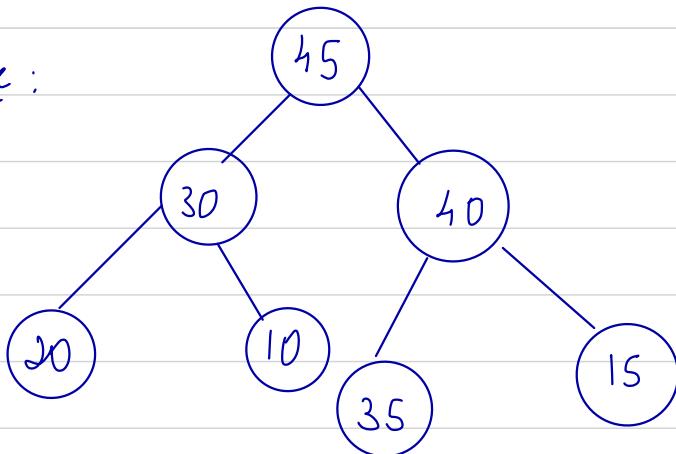
3 Peek

4 Remove

Priority queue is an abstract data type where every element has some priority value associated with it and priority determines the order in which elements are served or processed.

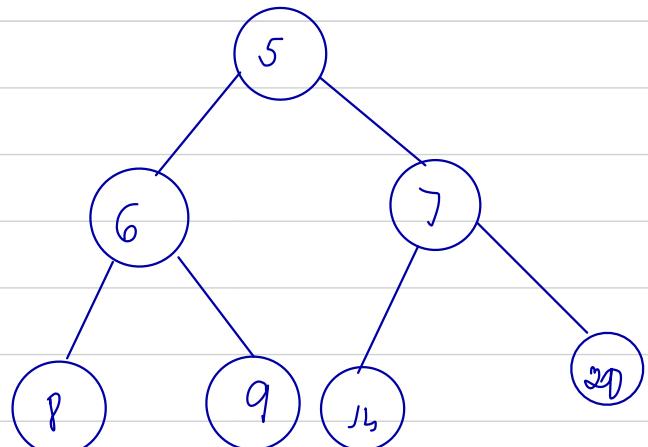
Heap : Heap is a tree type data structure that satisfies the property: if A is a parent node of B then A is ordered with respect to B for all nodes in the heap. Heap should be complete binary tree.

Ex :



Max heap

Parent > leaf

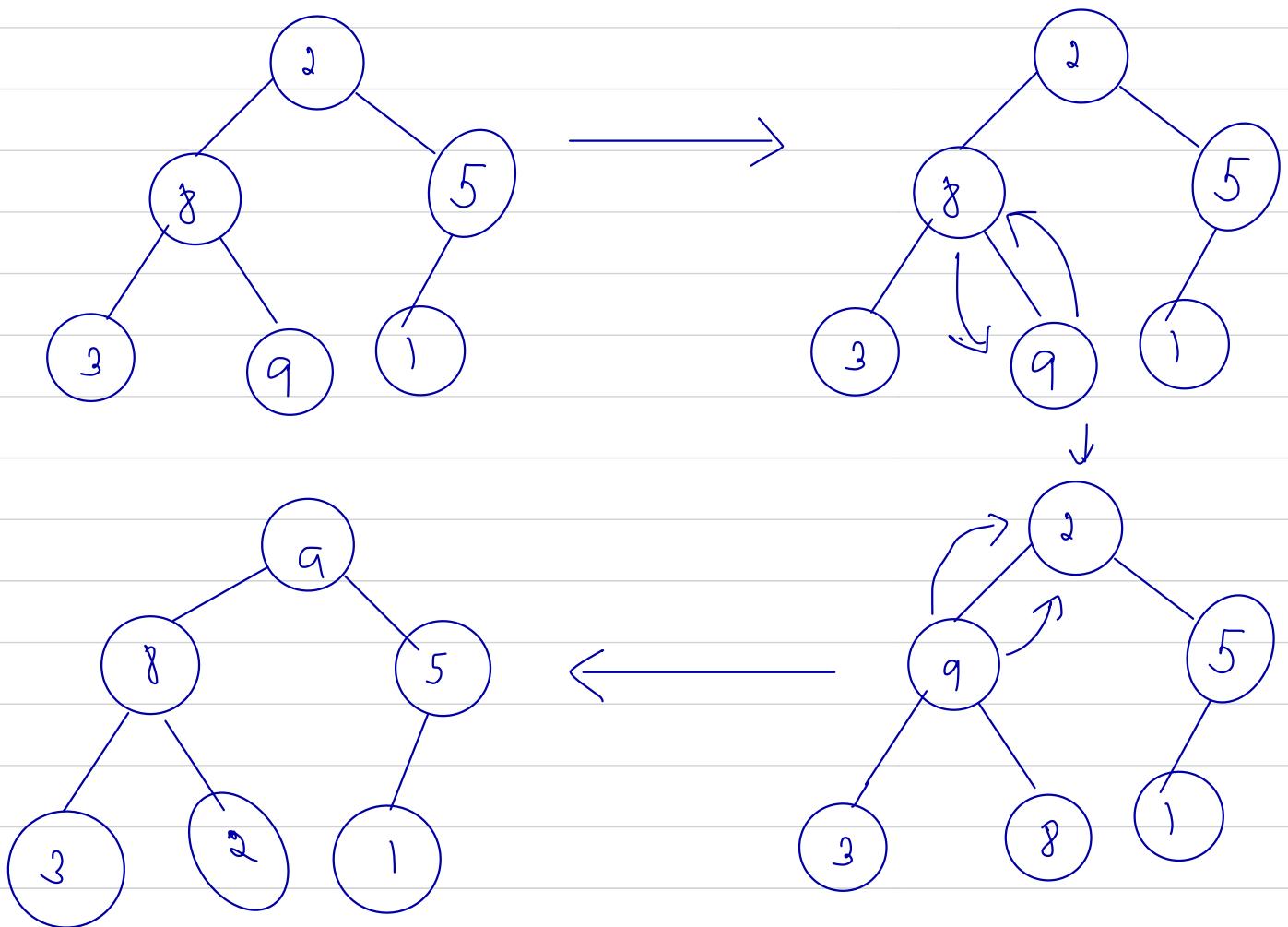


Min heap

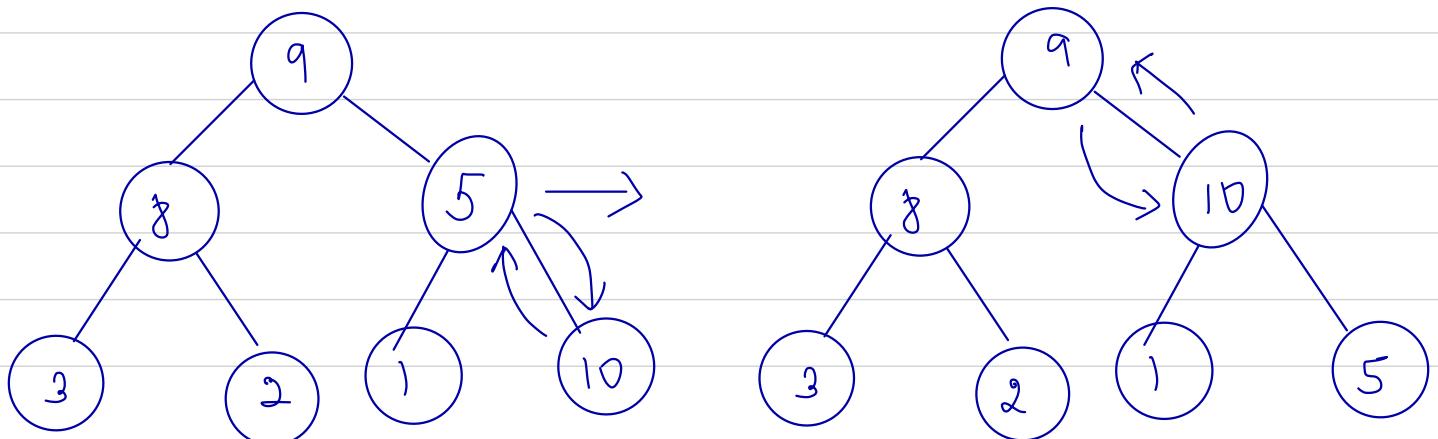
Parent < leaf

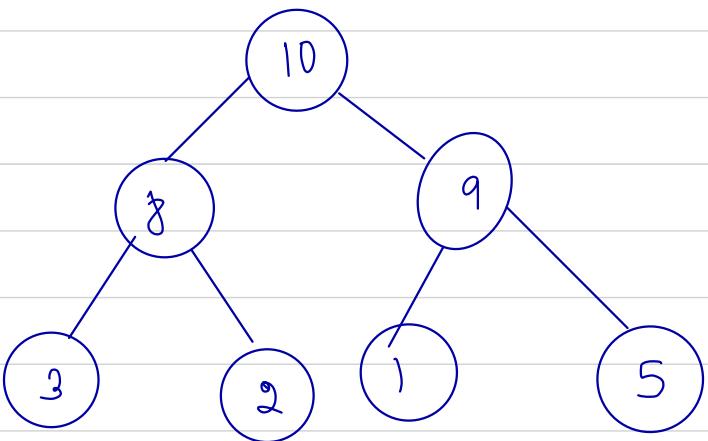
1> Creation of heap

2, 8, 5, 3, 9, 1



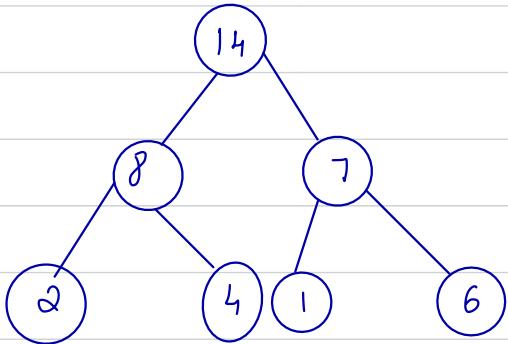
2> Insertion : Add 10 to above heap



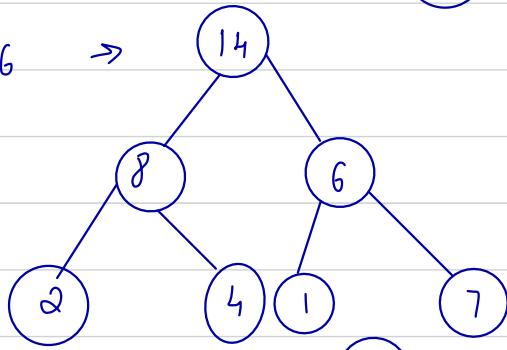


3> Peek: gives value of root

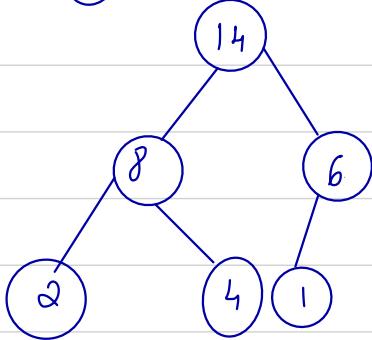
4> Deletion : Remove 7 in
~~~~~



Swap with 6 →



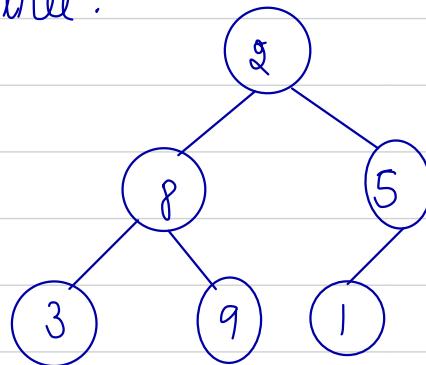
Remove 7 →



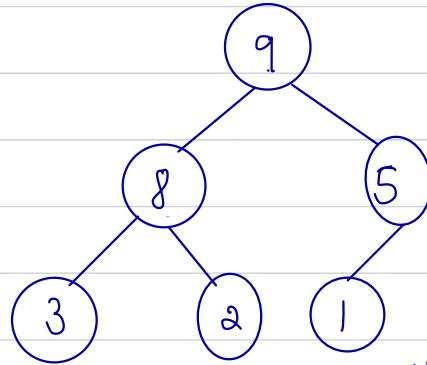
## Heapsort

Ex: 2, 8, 5, 3, 9, 1

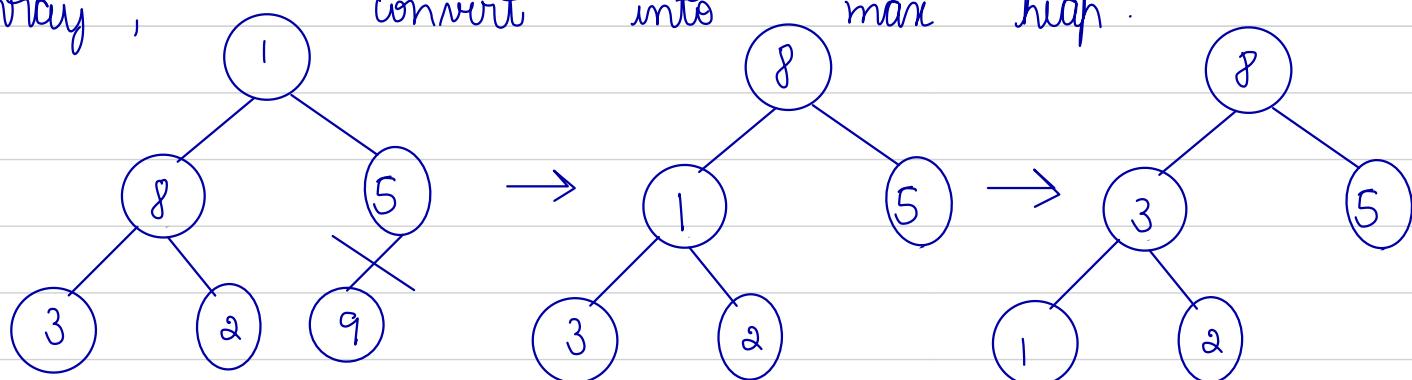
Step 1: Create complete tree:



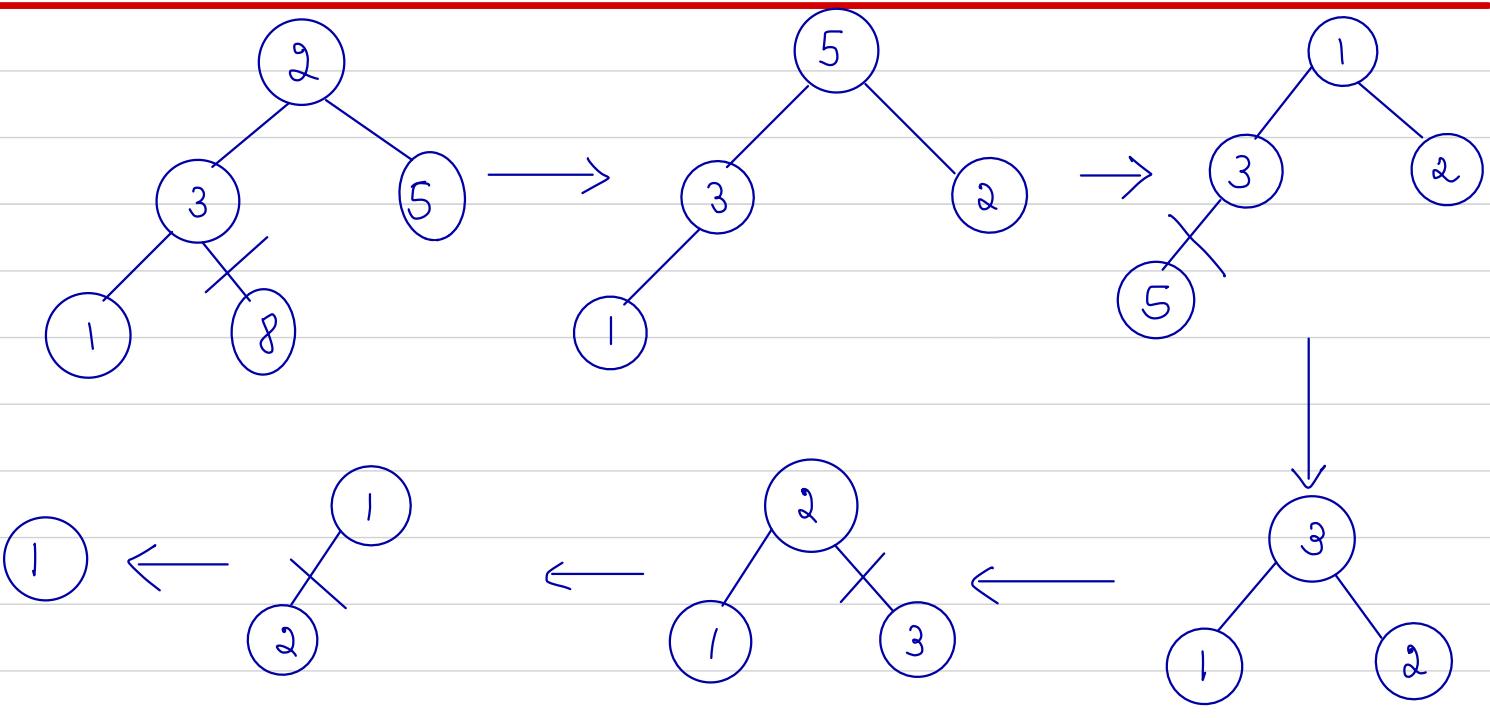
Step 2: Create max heap:



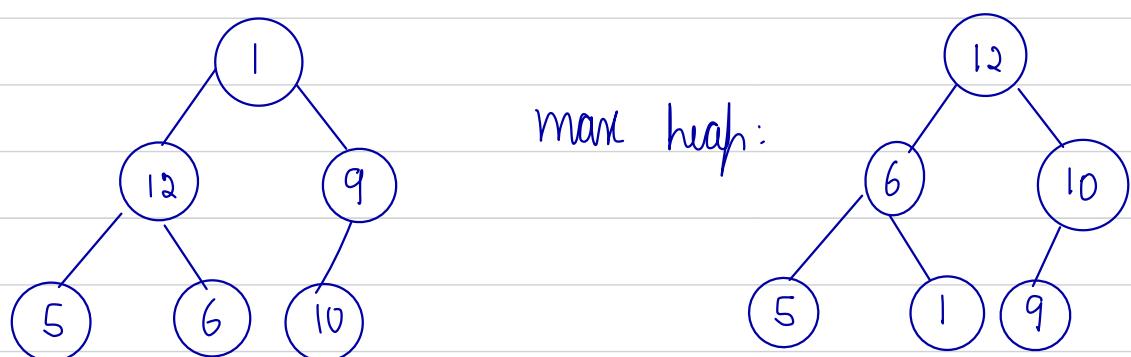
Step 3: Delete root and place it at the end of array, convert into max heap.



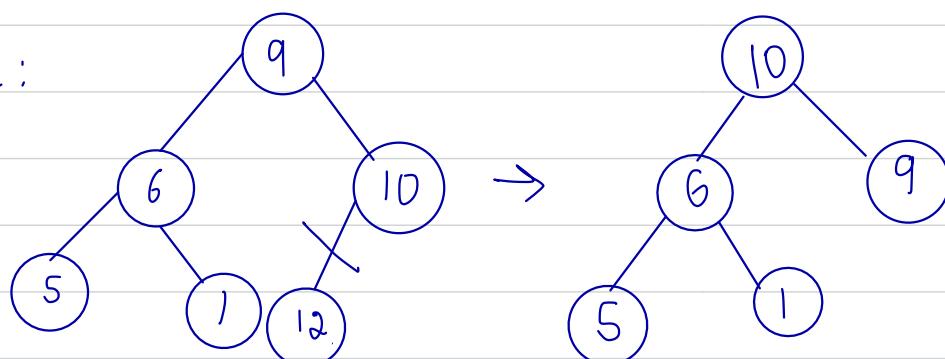
Step 4: Repeat step 3 till only one node is left in tree.

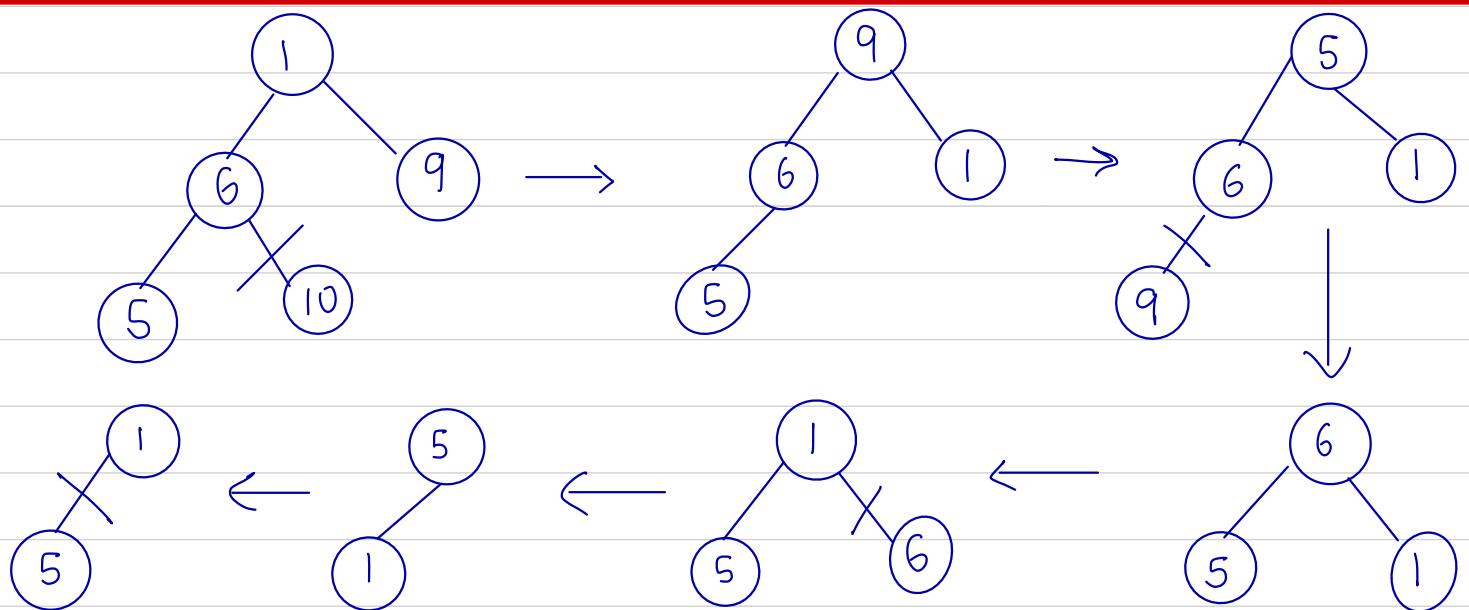


eg 2: 1, 12, 9, 5, 6, 10



Remove root:





Final array :

|   |   |   |   |    |    |
|---|---|---|---|----|----|
| 1 | 5 | 6 | 9 | 10 | 12 |
|---|---|---|---|----|----|

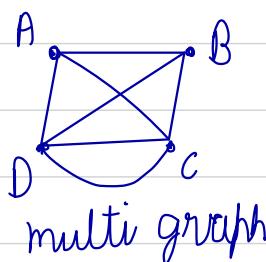
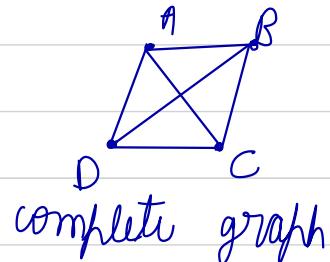
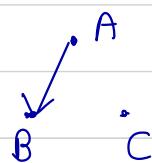
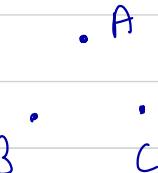
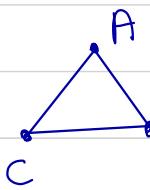
Graphs: It has vertices & edges (atleast one vertex)

A simple graph  $G(V, E)$  consists of a non empty set  $V$  of vertices and possibly empty set  $E$  of edges with each edge being set of 2 vertices from  $V$ . (non directed)

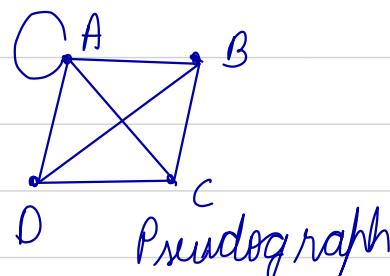
Directed graph: It  $G(V, E)$  consists of a non empty set  $V$  of vertices and possibly empty set  $E$  of edges with each edge being set of 2 vertices from  $V$  such that for each edge,  $\{v_i, v_j\} \neq \{v_j, v_i\}$

NOTE:  $|V|$  = no. of vertices  
 $|E|$  = no. of edges

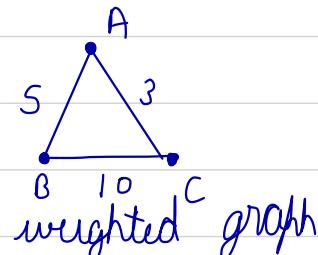
Eg:



multi graph



Pseudograph



complete graph

weighted graph

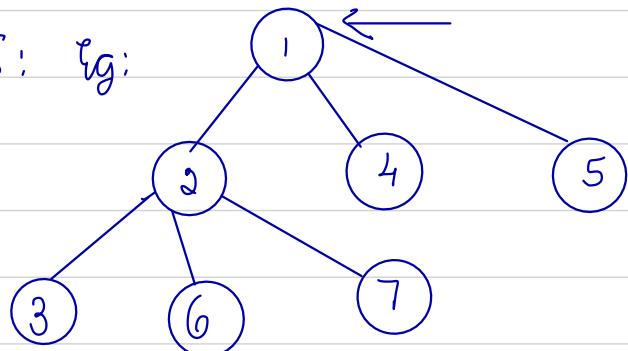
- In a graph if we can reach a node from itself without repetition it is called cyclic graph and if there is repetition it is called path

## Graph traversal

- 1> DFS - Depth first search
- 2> BFS - Breadth first search

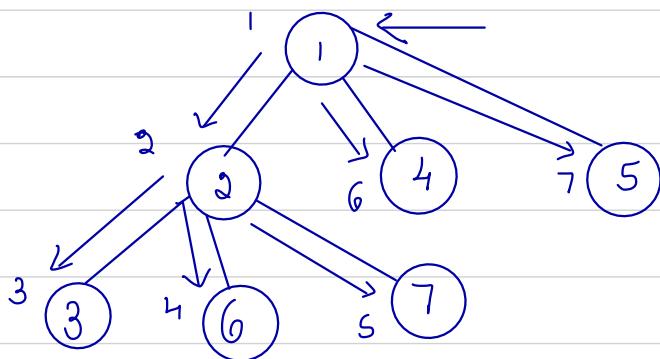
1>

DFS: Eg:

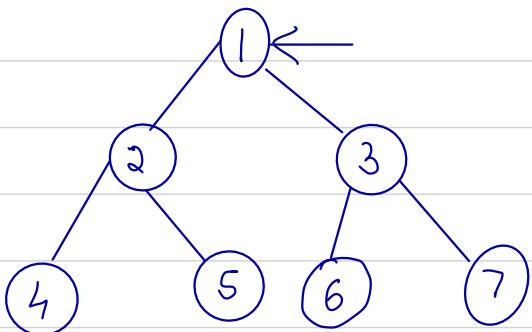


1 → 2 → 3 → 6 → 7 → 4 → 5

1. a



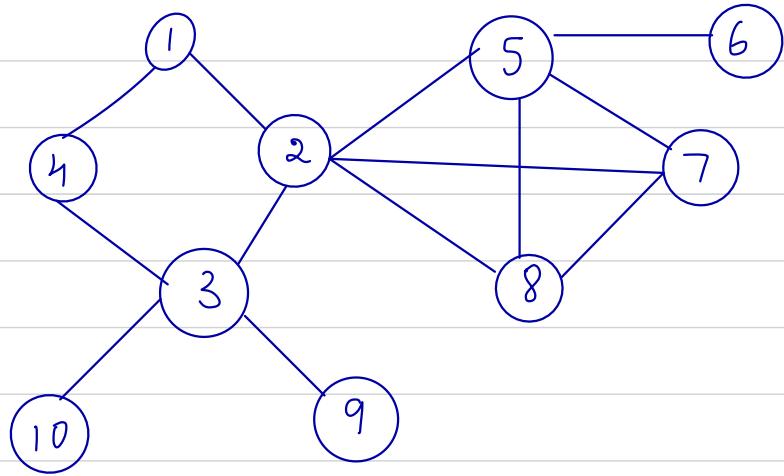
Eg 2:



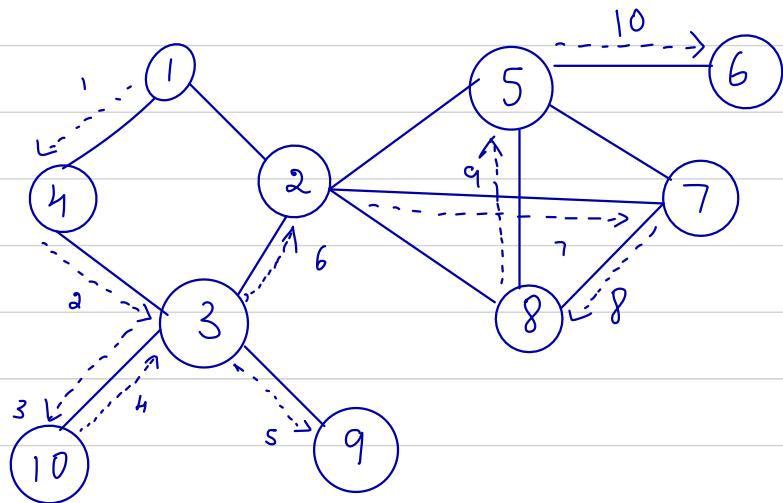
1 → 2 → 4 → 5 → 3 → 6 → 7

NOTE: • In DFS, stacks are used.  
• In BFS, queues are used.

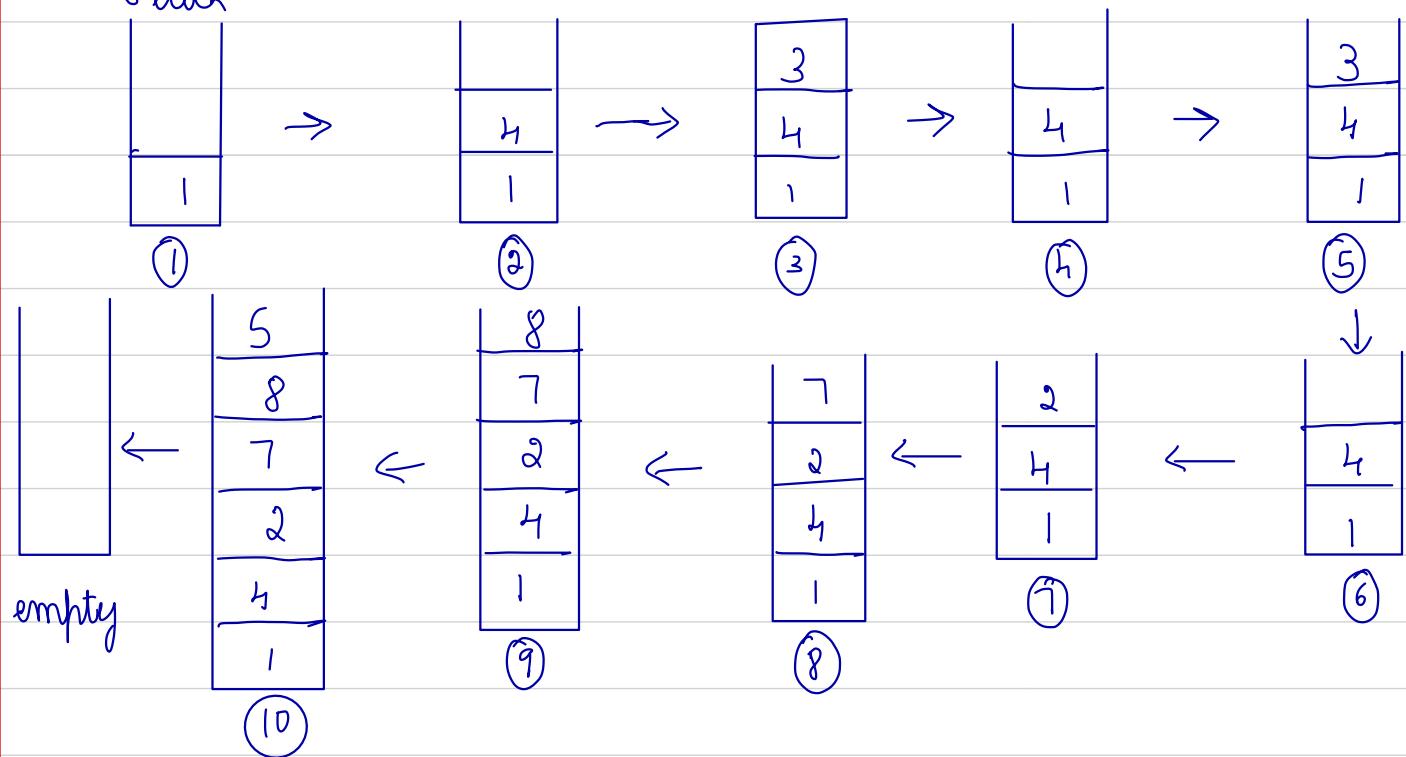
Eg 3:



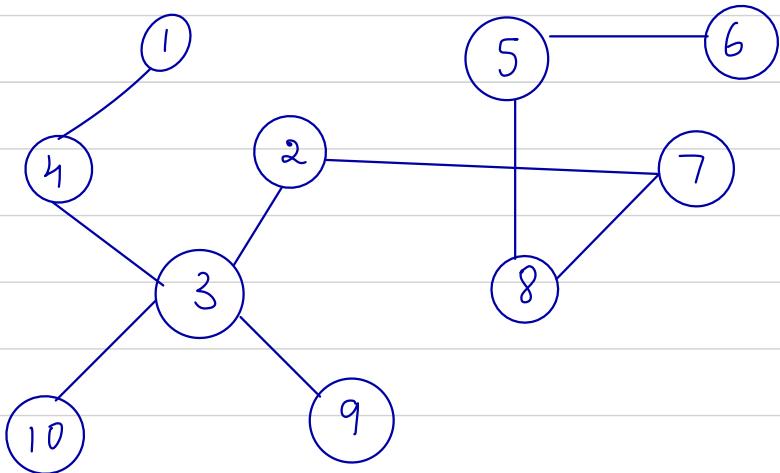
Aro



Stack

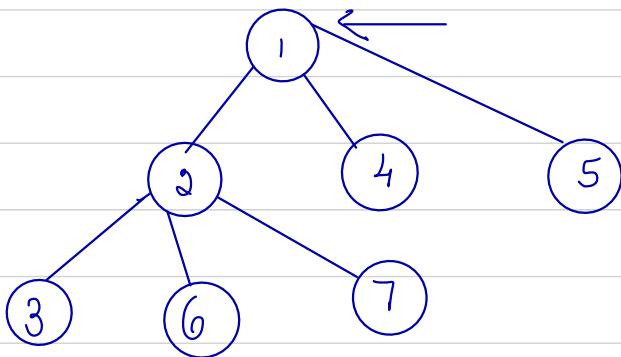


$1 \rightarrow 4 \rightarrow 3 \rightarrow 10 \rightarrow 9 \rightarrow 2 \rightarrow 7 \rightarrow 8 \rightarrow 5 \rightarrow 6$

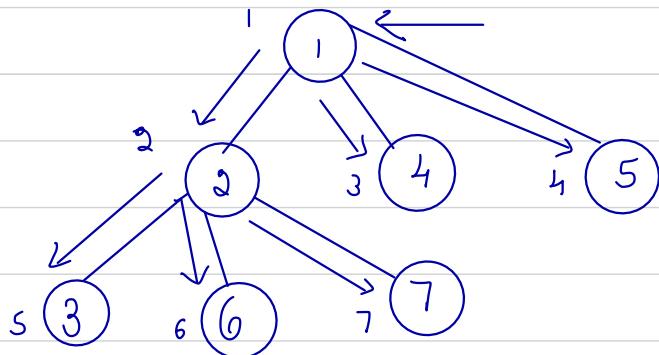


← Spanning tree

Q) BFS: Eq 1:

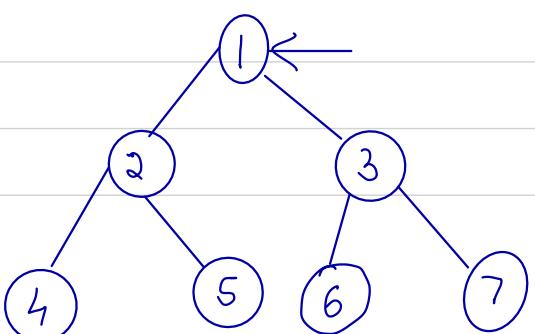


Ans  $1 \rightarrow 2 \rightarrow 4 \rightarrow 5 \rightarrow 3 \rightarrow 6 \rightarrow 7$



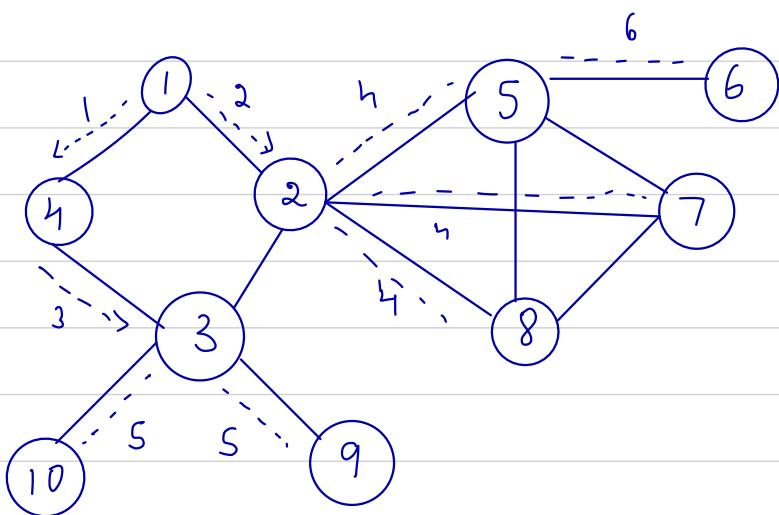
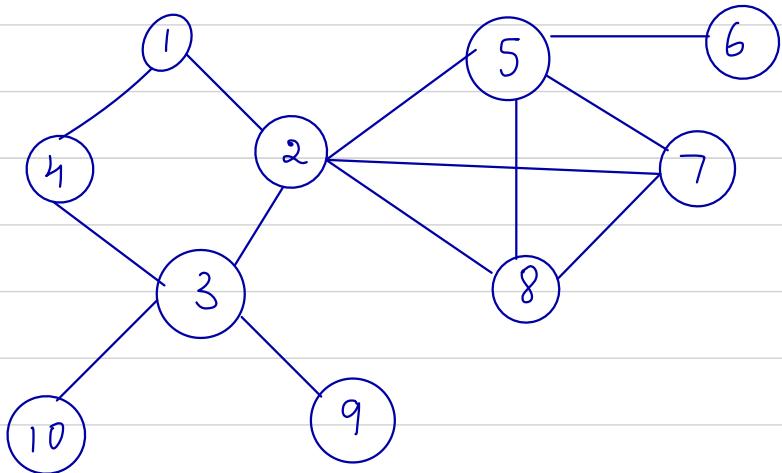
NOTE BFS is also called level ordering in trees.

Eg 2:

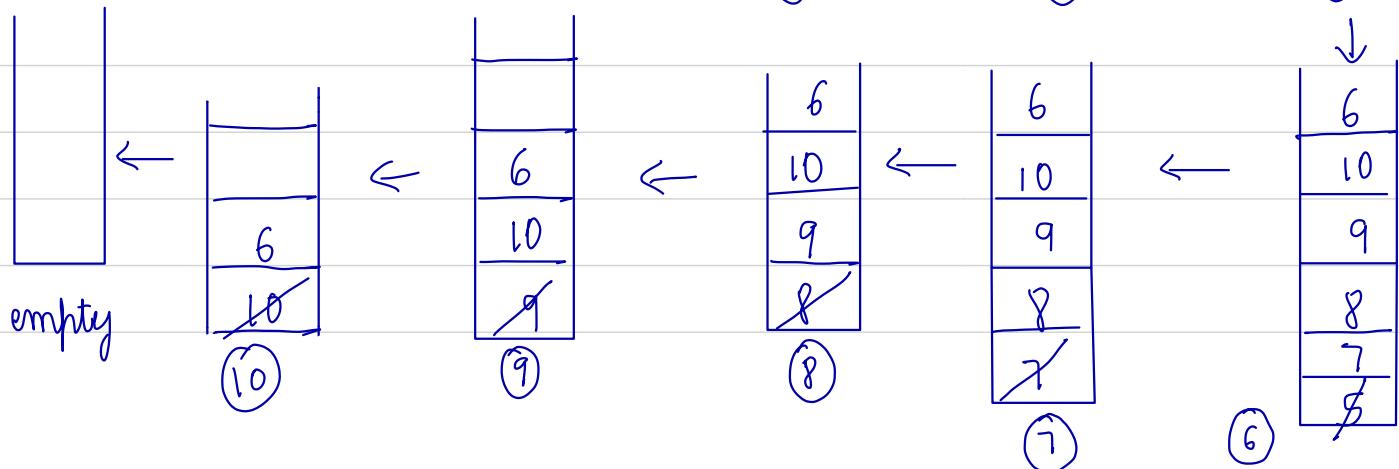
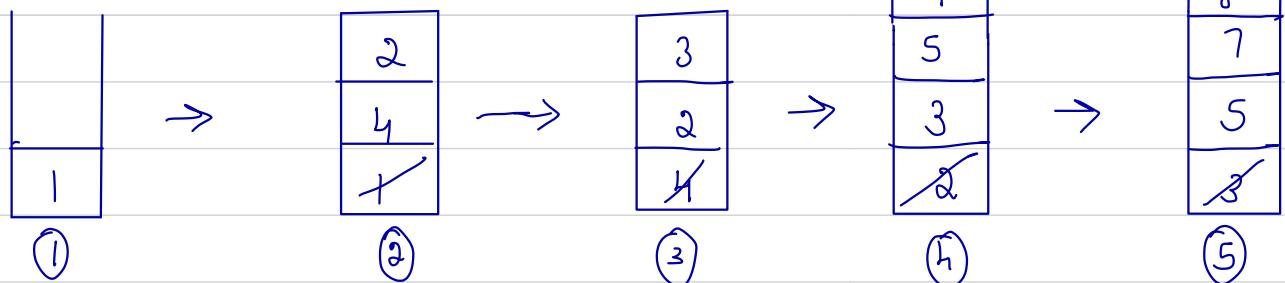


$1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 5 \rightarrow 6 \rightarrow 7$

Eq 3:

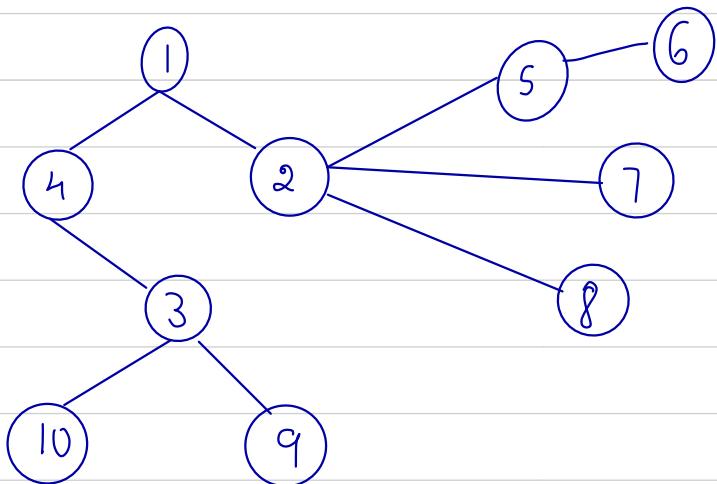


Queue:

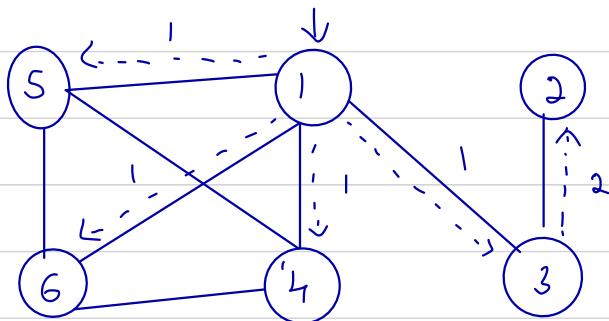


1, 4, 2, 3, 5, 7, 8, 10, 9, 6

Spanning trees:



eg)

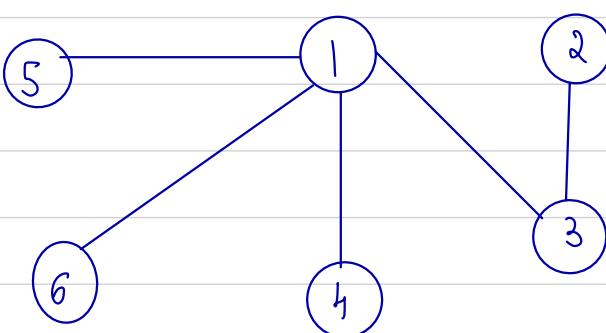


Queue: 

|   |   |   |   |   |   |
|---|---|---|---|---|---|
| X | 3 | 4 | 5 | 6 | 2 |
| 1 | 1 | 1 | 1 | 1 | 2 |

Reached: 1, 3, 4, 5, 6, 2

Tree:



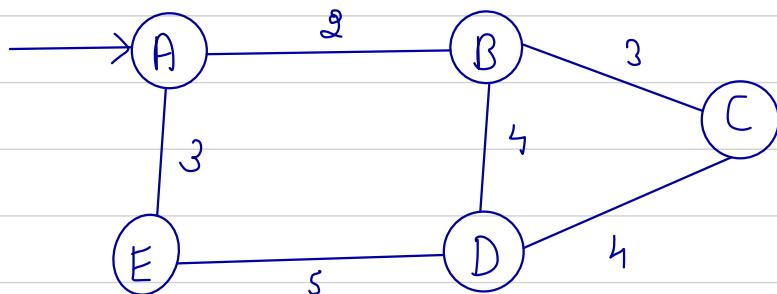
NOTE: Spanning tree satisfies following conditions:

- 1) It is a sub graph
- 2) It should have same number of vertices.  
i.e.  $|V'| = |V|$  ( $V' \rightarrow$  vertex in tree)
- 3) Difference between number of edges & vertices is 1.  
i.e.  $|E'| = |V| - 1$

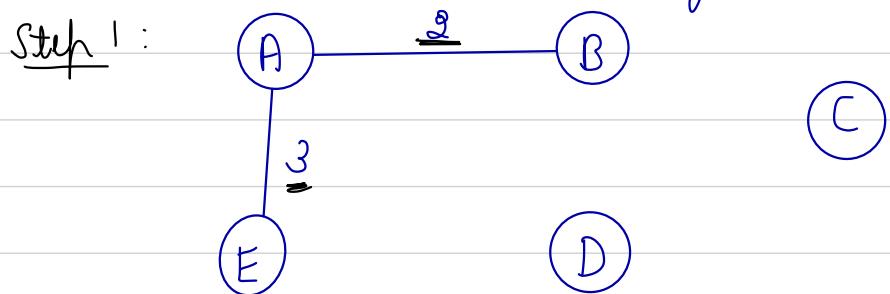
Greedy approach: To find the minimum spanning tree.

1) Prim's approach:

Eg:

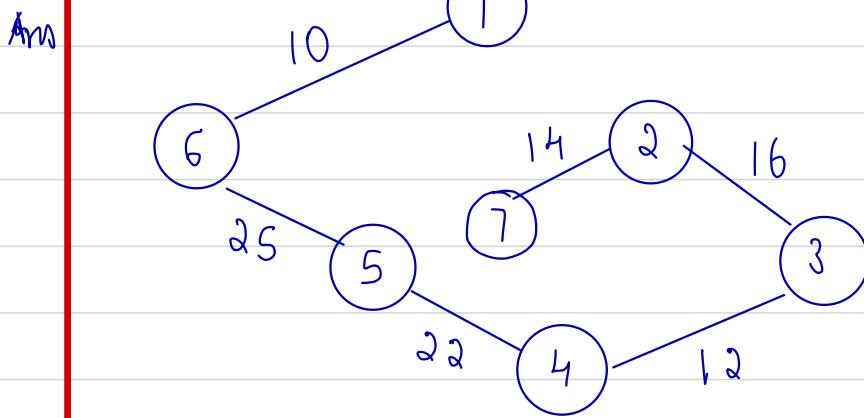
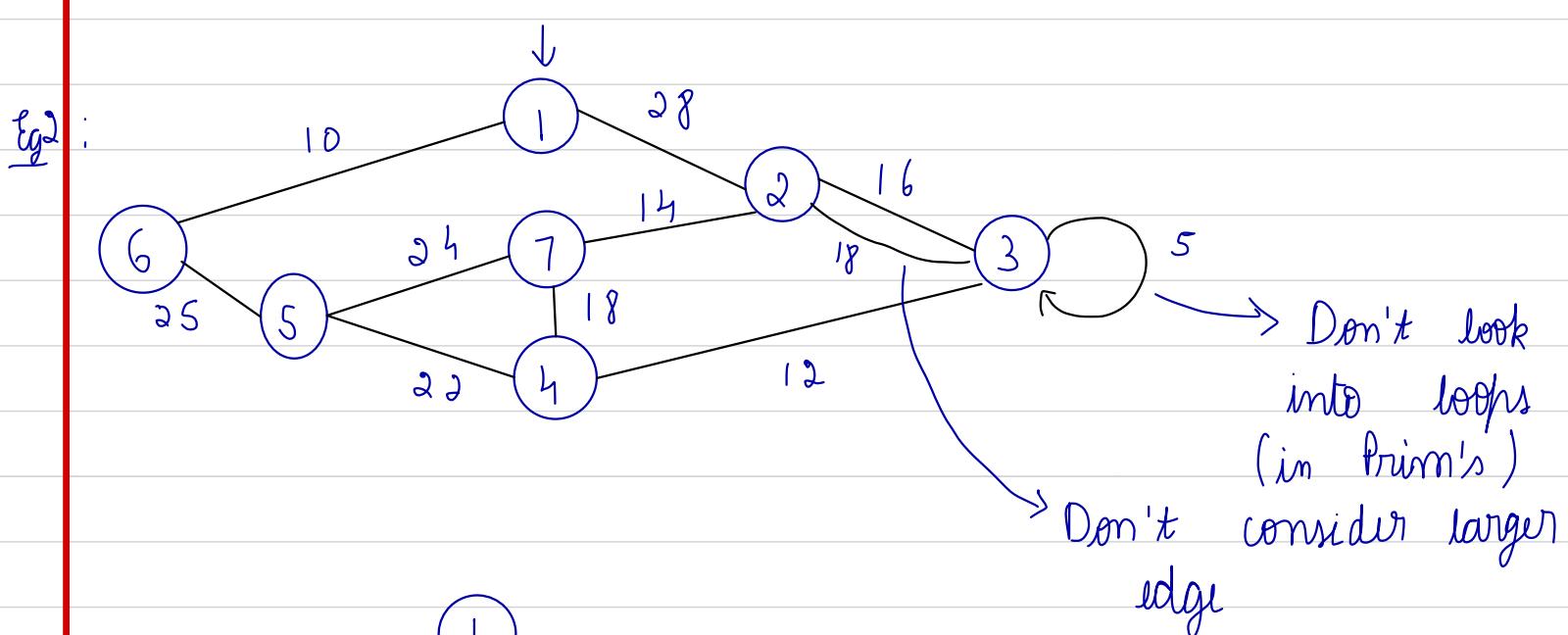
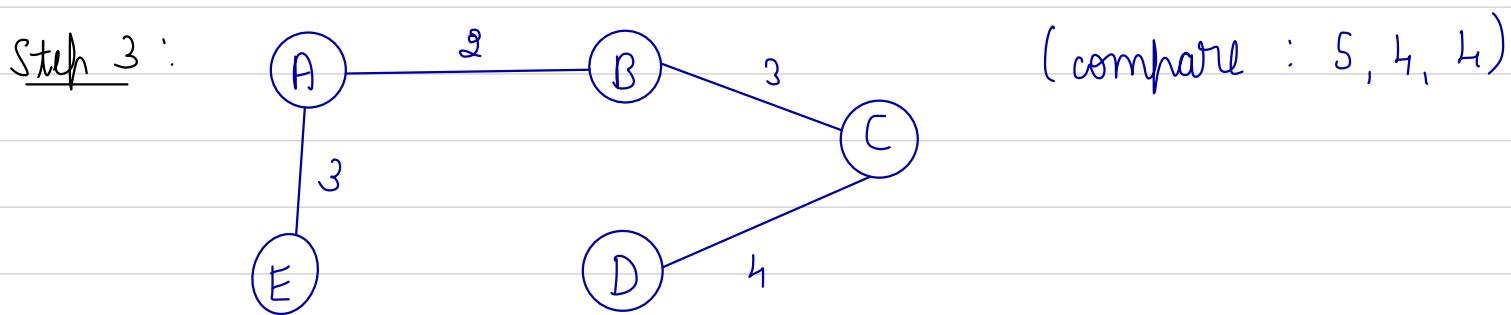
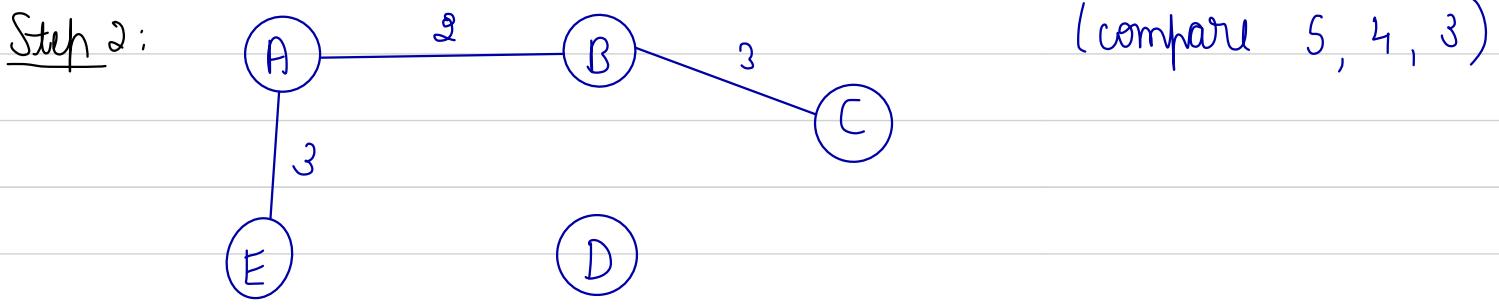


Start from any random node.  
1) Check the weights of connected edges (compare 3, 4, 3)

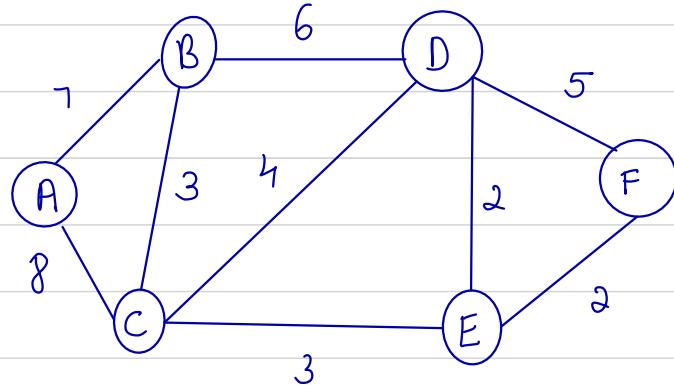


(If weights are same, select any)

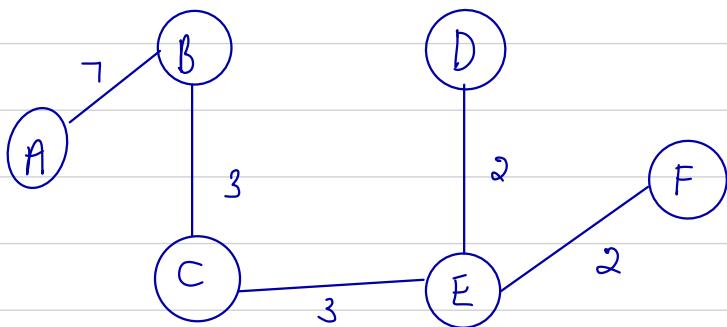
- keep checking weights of all adjacent nodes of the reached nodes



Eg<sup>3</sup>

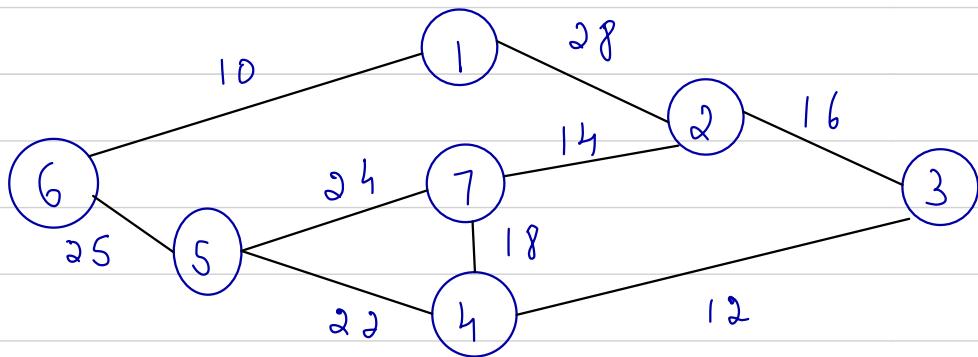


Ans



2> Kruskal's approach

Step 1: Write edges in ascending order of cost.



Edge: (1, 6) (3, 4) (2, 7) (2, 3) (4, 7) (4, 5) (5, 7)

cost: 10 ✓ 12 ✓ 14 ✓ 16 ✓ 18 ✗ 22 ✓ 24 ✗

(5, 6)  
25  
✓

(1, 2)  
28  
✗

Step 2: Connect edges in ascending order and discard the connection if cycle is formed.

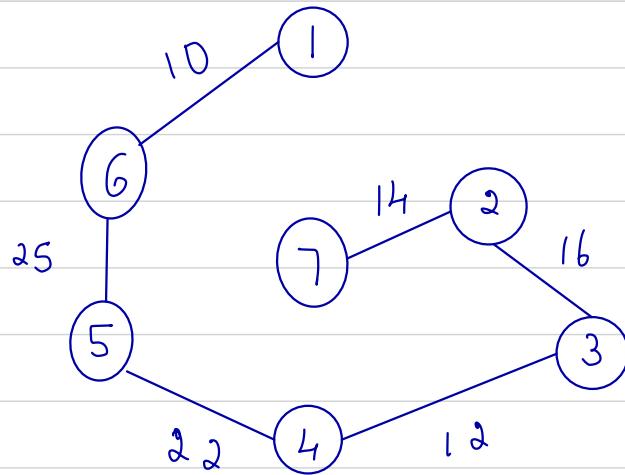
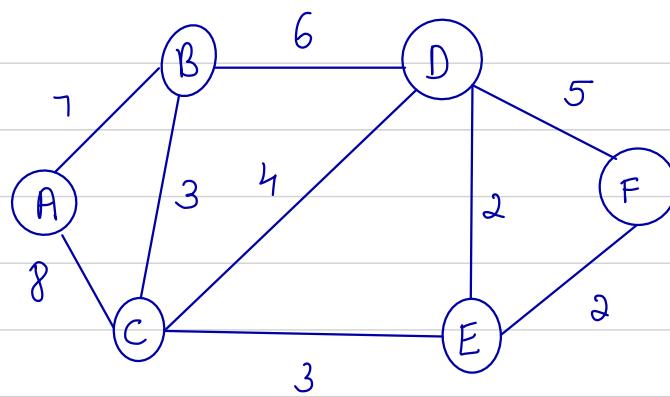
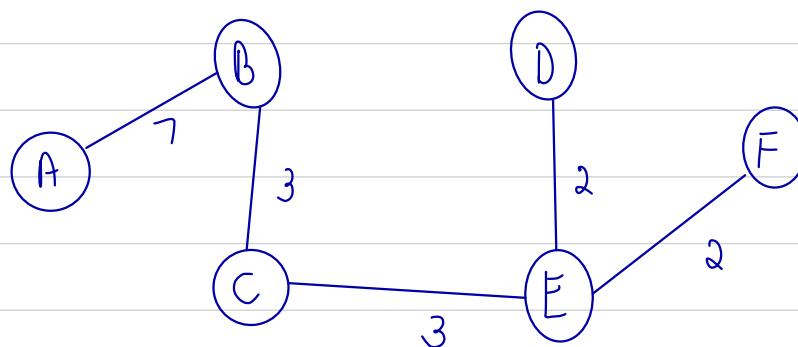


Fig 2:



|       |     |     |     |     |     |     |     |     |     |
|-------|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Edge: | DE  | EF  | BC  | CE  | CD  | DF  | BD  | AB  | AC  |
| cost: | 2 ✓ | 2 ✓ | 3 ✓ | 3 ✓ | 4 ✗ | 5 ✗ | 6 ✗ | 7 ✓ | 8 ✗ |

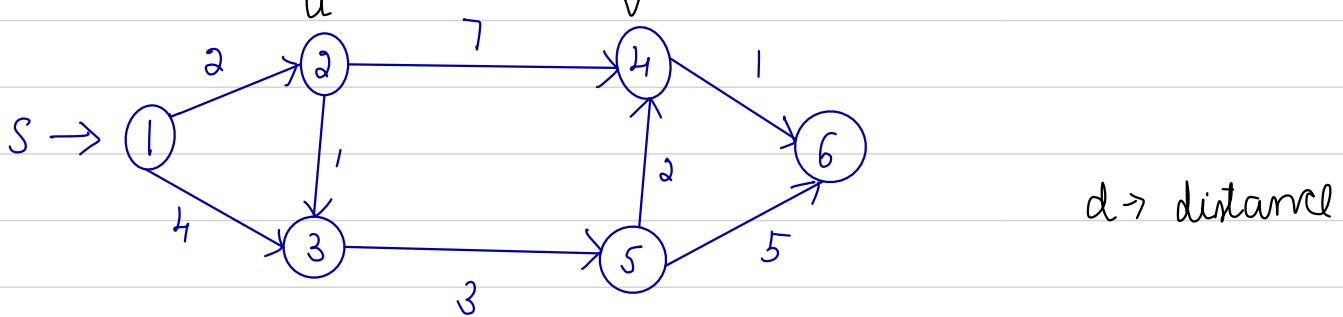


# Shortest Path algorithm

- 1) Dijkstra's algorithm
- 2) Bellman - Ford algorithm

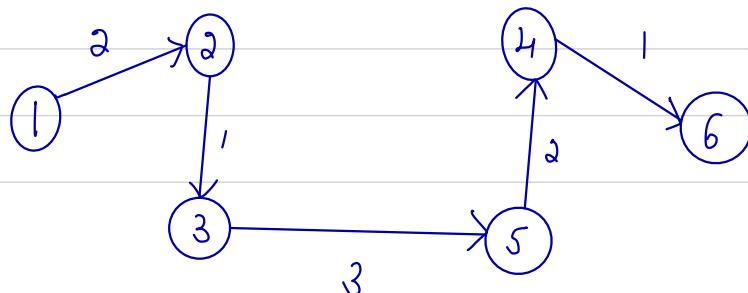
1) Dijkstra's algorithm: (Not applicable for graphs with negative edges)

e.g.:

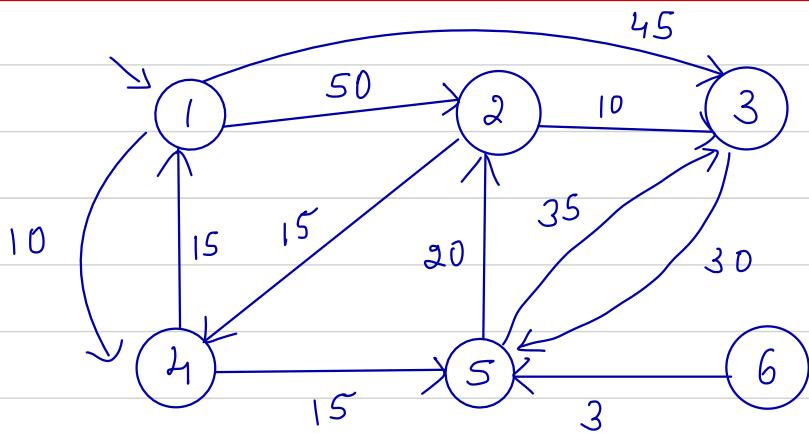


If  $d[u] + \text{cost}(u \rightarrow v) < d[v]$   
then  $d[v] = d[u] + \text{cost}(u \rightarrow v)$

| Selected vertex | 1 | 2 | 3 | 4        | 5        | 6        | ← vertex |
|-----------------|---|---|---|----------|----------|----------|----------|
| (1)             | 0 | 2 | 4 | $\infty$ | $\infty$ | $\infty$ |          |
| (2)             | 0 | 2 | 3 | 9        | $\infty$ | $\infty$ |          |
| (3)             | 0 | 2 | 3 | 9        | 6        | $\infty$ |          |
| (5)             | 0 | 2 | 3 | 8        | 6        | 11       |          |
| (4)             | 0 | 2 | 3 | 8        | 6        | 9        |          |



Eg 2:



| Selected<br>Vertex | 1 | 2        | 3        | 4        | 5        | 6        | ← vertex |
|--------------------|---|----------|----------|----------|----------|----------|----------|
| (1)                | 0 | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ |          |
| (4)                | 0 | 50       | 45       | 10       | $\infty$ | $\infty$ |          |
| (5)                | 0 | 50       | 45       | 10       | 25       | $\infty$ |          |
| (2)                | 0 | 45       | 45       | 10       | 25       | $\infty$ |          |
| (3)                | 0 | 45       | 45       | 10       | 25       | $\infty$ |          |

shortest path:

$$1 \rightarrow 4 \rightarrow 5 \rightarrow 2 = 45$$

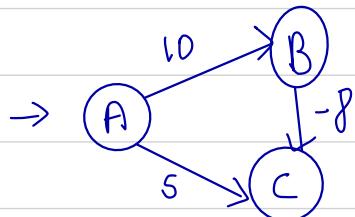
$$1 \rightarrow 3 = 45$$

$$1 \rightarrow 4 = 10$$

$$1 \rightarrow 4 \rightarrow 5 = 25$$

$$1 \rightarrow 6 = \infty$$

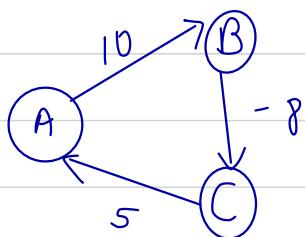
Eg 3:



| Selected vertex | A | B         | C        | vertex |
|-----------------|---|-----------|----------|--------|
| vertex          | 0 | $\infty$  | $\infty$ |        |
| (A)             | 0 | 10        | 5        |        |
| (C)             | 0 | <u>10</u> | 5        |        |
| (B)             | 0 | 10        | 5        |        |

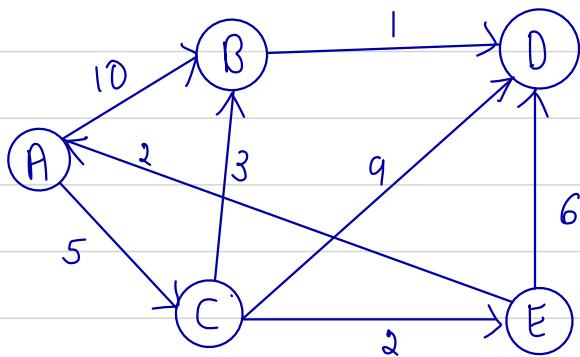
2) Bellman Ford algorithm:

- $V-1$  iterations are carried out, where  $V = \text{no. of vertices}$
- And last iteration will be carried out to check whether there is an negative edged cycle: e.g.



Time complexity (Dijkstra) < Time complexity (Bellman ford)

Eg) Use Dijkstra's method:



| Selected<br>vertex | A | B        | C        | D        | E        | vertex |
|--------------------|---|----------|----------|----------|----------|--------|
| vertex             | 0 | $\infty$ | $\infty$ | $\infty$ | $\infty$ |        |
| (A)                | 0 | 10       | <u>5</u> | $\infty$ | $\infty$ |        |
| (C)                | 0 | 8        | <u>5</u> | 14       | <u>7</u> |        |
| (E)                | 0 | <u>8</u> | 5        | 13       | <u>7</u> |        |
| (B)                | 0 | 8        | 5        | 9        | 7        |        |
| (D)                | 0 | 8        | 5        | 9        | 7        |        |

## Hashing

- 1) Open hashing → chaining
- 2) Closed hashing
  - linear probing
  - Quadratic probing

### 1) Open hashing

Eg: Key space



Hash table

|           |
|-----------|
|           |
|           |
| 12        |
| 3         |
|           |
| 15        |
|           |
| 7         |
| 8, 18, 28 |
| 9         |

0

1

2

3

4

5

6

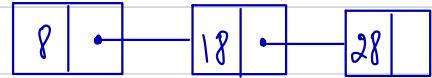
7

8

9

size = 10

→ 8 → collision →



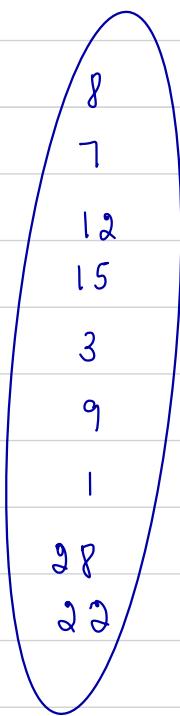
(chaining)

$$h(x) = x \% \text{ size}$$

### 2) Closed hashing

i) linear probing: 
$$h(x) = [h(x) + f(i)] \% \text{ size}$$

key space



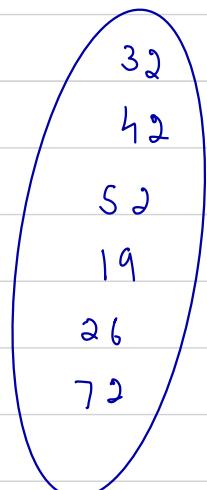
Hash table

|    |   |
|----|---|
| 9  | 0 |
| 1  | 1 |
| 12 | 2 |
| 3  | 3 |
| 22 | 4 |
| 15 | 5 |
|    | 6 |
| 7  | 7 |
| 8  | 8 |
| 28 | 9 |

⇒ Quadratic probing:  $h(x) = [h(x) + f[i]^2] \mod \text{size}$

e.g.:

key space



Hash table

|    |   |   |
|----|---|---|
|    | 0 | 0 |
|    | 1 | 1 |
| 72 | 2 | 2 |
| 32 | 3 | 3 |
| 42 | 4 | 4 |
|    | 5 | 5 |
|    | 6 | 6 |
| 52 | 7 | 7 |
| 26 | 8 | 8 |
|    | 9 | 9 |

quadratic

Linear