PHYS 600 : HW 5

Growth of Matter Perturbations - Matter and Radiation

The equation for the growth of a matter perturbation in linear theory above the Jean's scale is given by

$$\ddot{\delta}_m + 2H\dot{\delta}_m = 4\pi G\bar{\rho}_m\delta_m\,.$$

In what follows, we will assume that the Universe is only made of matter and radiation, and is flat. Furthermore, we will ignore any fluctuations in the radiation, although it will contribute to the expansion rate (H).

- Show that this above evolution equation can be written as

$$\frac{d}{da}\left(a^3 H\frac{d\delta_m}{da}\right) = 4\pi G\bar{\rho}_{m,0}\frac{\delta_m}{Ha^2}\,.$$

- Define $y = a/a_{eq}$, where a_{eq} is the scale factor at matter-radiation equality. Show that the Hubble parameter can be written as

$$H(y) = \frac{A}{y^2}\sqrt{1+y}\,.$$

  Work out A in terms of physical constants and the energy density at equality.
- Substitute this into the equation for growth and show that it can be written as

$$\frac{d^2\delta_m}{dy^2} + \frac{2+3y}{2y(1+y)}\frac{d\delta_m}{dy} = \frac{3}{2}\frac{\delta_m}{y(1+y)}\,.$$

- Verify (by eg. direct substitution, you may use Mathematica) that the solutions to this equation are

$$\delta_m \propto 1 + \frac{3}{2}y$$

  and

$$\delta_m \propto \left(1+\frac{3}{2}y\right)\ln\left(\frac{\sqrt{1+y}+1}{\sqrt{1+y}-1}\right) - 3\sqrt{1+y}\,.$$

- Check to see that the growing modes at early and late times match your expectation (for radiation and matter domination respectively).

Spherical Collapse

In class, we used the following parametric solution for a spherical overdensity of mass M and energy E :

$$r(\theta) = A(1 - \cos\theta)$$

and

$$t(\theta) = B(\theta - \sin\theta)$$

with

$$A = \frac{GM}{2|E|}$$

and $A^3 = GMB^2$. Verify (either by solving or substituting) this solution.

Equality Scale

As we saw in class, the equality scale sets the turnover in the matter power spectrum. Show that the equality scale is given by

$$k_{eq} = a_{eq} H_{eq} = H_0 \sqrt{\frac{2\Omega_M}{a_{eq}}}\,.$$

Estimate this scale for a cosmology roughly like the one we live in.

A Study in Simulations

(coming soon...)

This problem has you explore the results of an N-body simulation, and compare it to the various aspects of the class so far. For this, we will use one of the Quijote simulations `https://quijote-simulations.readthedocs.io/en/latest/`.

We will use the fiducial cosmology, which is a flat ΛCDM cosmology with $\Omega_M = 0.3175$, $\Omega_b = 0.049$, $h = 0.6711$, $n_s = 0.9624$, and $\sigma_8 = 0.834$.

On Canvas (and our Github repository), you will find a directory with the following data. All of these are simple text files. There is a header with the column information as well.

- `linear_pk.txt` : The linear theory power spectrum at $z = 0$.
- `pk_z=<z>.txt` : The non-linear power spectrum at different redshifts ($z = 0$, $z = 0.5$, $z = 1$ and $z = 2$)
- `halo_z=<z>.txt` : The halos in the simulation at different redshifts ($z = 0$, and $z = 1$)

Orienting Yourself : The Linear Power Spectrum

- Start by making a plot of the linear power spectrum at $z = 0$. You should plot these on log scales on both axes.
- Verify that at large scales (low k), the power spectrum is given by

$$P(k) \propto k^{n_s}$$

  which is the primordial power spectrum (since $T(k) = 1$ at large scales). Note the slight deviation from the Harrison-Zel'dovich spectrum ($n_s = 1$).
- Estimate k_{eq} from the plot. Compare this to your expectation (see previous problem).
- Compute the transfer function $T(k)$ and plot it. Normalize it to 1 at large scales. Compare the shape at large k (small scales) to what we calculated in class.
- Calculate σ_8 from the linear power spectrum. Compare this to the value given above.

Non-linear Power Spectrum and Structure Growth

- Plot the power spectrum from the simulation at the redshifts given (again on log scales). Compare this to the linear power spectrum.
- Now plot the ratio of the non-linear to linear power spectrum. Explain the shape of this ratio on large scales based on what you know about linear structure growth. At what k does linear structure growth start to break down (should be redshift dependent)?
- From this plot, extract the growth function as a function of redshift. How are you choosing to normalize this?
- Calculate the growth function for the cosmology of the simulation, and compare to your extracted growth function. How well do they agree?

Halo Mass Function