

## Exercises FYS4480, week 45, October 6-November 10, 2023

### Exercise 1

Let  $H = H_0 + V$  and  $|\phi_n\rangle$  be the eigenstates of  $H_0$  and that  $|\psi_n\rangle$  are the corresponding ones for  $H$ . Assume that the ground states  $|\phi_0\rangle$  and  $|\psi_0\rangle$  are not degenerate. Show that

$$E_0 - \varepsilon_0 = \frac{\langle \phi_0 | V | \psi_0 \rangle}{\langle \phi_0 | \psi_0 \rangle},$$

with  $H |\psi_0\rangle = E |\psi_0\rangle$  and  $H_0 |\phi_0\rangle = \varepsilon_0 |\phi_0\rangle$ .

- a) Define the new operators  $P = |\phi_0\rangle \langle \phi_0|$  and  $Q = 1 - P$ . Show that these operators are idempotent.
- b) Show that for any  $z$  we have

$$|\psi_0\rangle = \langle \phi_0 | \psi_0 \rangle \sum_{n=0}^{\infty} \left( \frac{Q}{z - H_0} (z - E_0 + V) \right)^n |\phi_0\rangle,$$

and

$$E_0 = \varepsilon_0 + \sum_{n=0}^{\infty} \langle \phi_0 | V \left( \frac{Q}{z - H_0} (z - E_0 + V) \right)^n |\phi_0\rangle.$$

- c) Discuss these results for  $z = E_0$  (Brillouin-Wigner perturbation theory) and  $z = \varepsilon_0$  (Rayleigh-Schrödinger perturbation theory). Compare the first few terms in these expansions.

### Exercise 2

Consider a system of two fermions spin  $s = 1/2$  in the pair-orbitals  $|m_0\rangle$  and  $|-m_0\rangle$  in a single shell  $j = l + s$  with  $2j + 1 > 2$ , where  $l$  is the orbital momentum with  $l = 0, 1, 2, \dots$ . Assume that the matrix elements for the interaction between the particles takes the form

$$\langle m, -m | v | m', -m' \rangle = -G.$$

- a) Show that the Brillouin-Wigner expansion from the previous exercise can be used to give

$$E_0 = -(j + 1/2)G.$$

- b) Show thereafter by direct diagonalization of the Hamiltonian matrix that this is the exact energy. Use thereafter Rayleigh-Schrödinger perturbation theory and discuss the differences.