Exercises FYS4480, week 45, November 7-11, 2022

Exercise 1

We consider a one-particle system with the following Hamiltonian $H = H_0 + H_1$ where

$$H_0 = \sum_{i=1,2} \varepsilon_i a_i^\dagger a_i$$

$$H_1 = \lambda \sum_{i \neq j=1,2} a_i^\dagger a_j$$

a) Find the ground state energy to third order in perturbation theory using both Brillouin-Wigner and Rayleigh-Schödinger perturbation theory.

b) Write down the corresponding diagrams in the particle picture (using the true vacuum).

c) Find the exact energy and expand the exact results in terms of the parameter λ and compare with the results obtained with the above two expansions. Discuss the eventual differences.

d) Rewrite the unperturbed ground state in the particle-hole representation

$$|c\rangle = |\Phi_1\rangle = a_1^\dagger |0\rangle ,$$

and write down the corresponding diagrams

e) To fourth order in perturbation theory we have unlinked diagrams. Give examples of these and show how they can be cancelled.

Exercise 2, time-ordered product

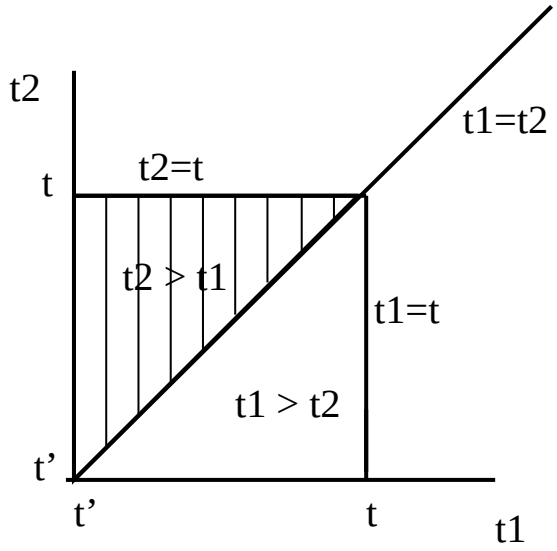
Show that

$$\int_{t'}^{t} dt_1 \int_{t'}^{t_1} dt_2 H_1(t_1) H_1(t_2) = \frac{1}{2} \int_{t'}^{t} dt_1 \int_{t'}^{t} dt_2 T\left[H_1(t_1) H_1(t_2)\right]$$

Hint: Use the definition of T in order to distinguish between $t_1 > t_2$ and $t_1 < t_2$;

$$\int_{t'}^{t} dt_1 \int_{t'}^{t} dt_2 T\left[H_1(t_1) H_1(t_2)\right] = \int_{t'}^{t} dt_1 \left\{ \int_{t'}^{t_1} dt_2 H_1(t_1) H_1(t_2) + \int_{t_1}^{t} dt_2 H_1(t_2) H_1(t_1) \right\}$$

Show that the last term on the right-hand side equals the first term (change the order of the integrations and thereafter integration variables). The area of integration for the first term is shown in the figure below.


t2
t1=t2
t2=t
t
t2 > t1
t1=t
t1 > t2
t'
t'
t
t1