Week 45: Many-body perturbation theory

Morten Hjorth-Jensen[1]

Department of Physics and Center for Computing in Science Education,
University of Oslo, Norway[1]

November 3-7

Week 45, November 3-7, 2025

Topics to be covered

1. Thursday:
   1.1 Time-independent perturbation theory: examples of contributions in perturbation theory and diagrammatic representations
   1.2 Diagram rules and their derivations
2. Friday:
   2.1 Diagram examples
   2.2 Linked diagram theorem
3. Exercises week 45 at https://github.com/ManyBodyPhysics/FYS4480/blob/master/doc/Exercises/2025/Exercisesweek45.pdf
4. Lecture Material: Whiteboard notes (see above) and Shavitt and Bartlett chapters 5-7

Second midterm

The second midterm will be available from Friday November 14 with deadline November 24. We hope this will not interfere too much with other activities.

Diagram rules, topological distinct diagrams

Draw all topologically distinct diagrams by linking up particle and hole lines with various interaction vertices. Two diagrams can be made topologically equivalent by deformation of fermion lines under the restriction that the ordering of the vertices is not changed and particle lines and hole lines remain particle and hole lines.
For the explicit evaluation of a diagram: Sum freely over all internal indices and label all lines.

Diagram rules: matrix elements

Extract matrix elements for the one-body operators (if present) as $\langle \mathrm{out}|\hat{f}|\mathrm{in}\rangle$ and for the two-body operator (if present) as $\langle \mathrm{left\ out}, \mathrm{right\ out}|\hat{v}|\mathrm{left\ in}, \mathrm{right\ in}\rangle$.

Diagram rules: phase factors

Calculate the phase factor: $(-1)^{\text{holelines}+\text{loops}}$

Diagram rules: equivalent pairs

Multiply by a factor of $\frac{1}{2}$ for each equivalent pair of lines (particle lines or hole lines) that begin at the same interaction vertex and end at the same (yet different from the first) interaction vertex.

Diagram rules: energy denominators

For each interval between successive interaction vertices with minimum one single-particle state above the Fermi level with n hole states and m particle stat es there is a factor ! bt

$$\frac{1}{\sum_i^n \epsilon_i - \sum_a^m \epsilon_a}.$$