FYS4480/9480

Lecture September 11

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$$\overbrace{a_p a_q^\dagger} = \delta_{pq} \quad \left(= \langle 0 | a_p a_q^\dagger | 0 \rangle \right)$$

$$\overbrace{a_p a_q} = \overbrace{a_p^\dagger a_q} = \overbrace{a_p^\dagger a_q^\dagger} = 0$$

Today we will redefine this contraction

Wick's theorem

$$\langle 0 | X Y \cdots W | 0 \rangle$$

$$= \sum_{\left[\frac{M}{2}\right] \text{ all contractions}} N\left[X Y Z \cdots W \right]$$

Wick's generalized theorem

$$\langle c| x y z \cdots w |0\rangle$$

$$= \langle c| N[A_1 A_2 \cdots] \, N[B_1 B_1 \cdots] \, N[C_1 C_2 \cdots] \cdots |0\rangle$$

<u>Example</u>

$$|ij\rangle = a_i^\dagger a_j^\dagger |0\rangle$$

$$|kl\rangle = a_k^\dagger a_l^\dagger |0\rangle$$

$$H_I = \frac{1}{2} \sum_{pqrs} \langle pq|v|rs\rangle a_p^\dagger a_q^\dagger a_s a_r$$

$$\langle ij | H_I | kl \rangle =$$

$$\frac{1}{2} \sum_{pqrs} \langle pq | v | rs \rangle \langle 0 | [a_j a_i] [a_p^\dagger a_q^\dagger a_s a_r] \times [a_k^\dagger a_l^\dagger] | 0 \rangle$$

$$\left\{ a_j a_i a_p^\dagger a_q^\dagger a_s a_r a_k^\dagger a_l^\dagger \right\}$$

$$\langle ij | v | kl \rangle$$

$$- \langle ij | v | lk \rangle$$

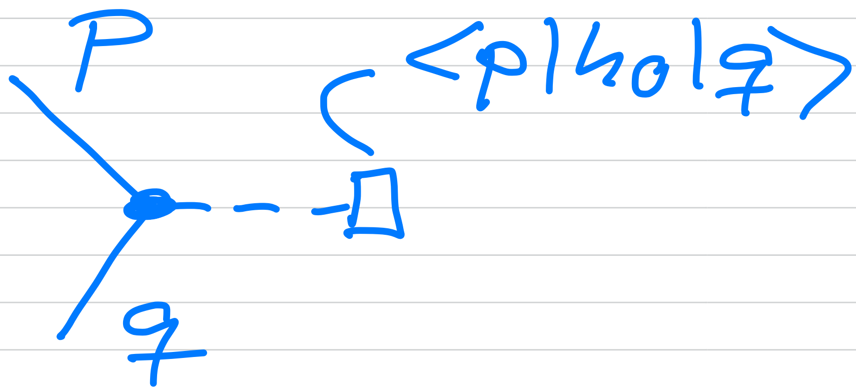
$$\langle ji | v | lk \rangle$$

$$- \langle ji | v | kl \rangle$$

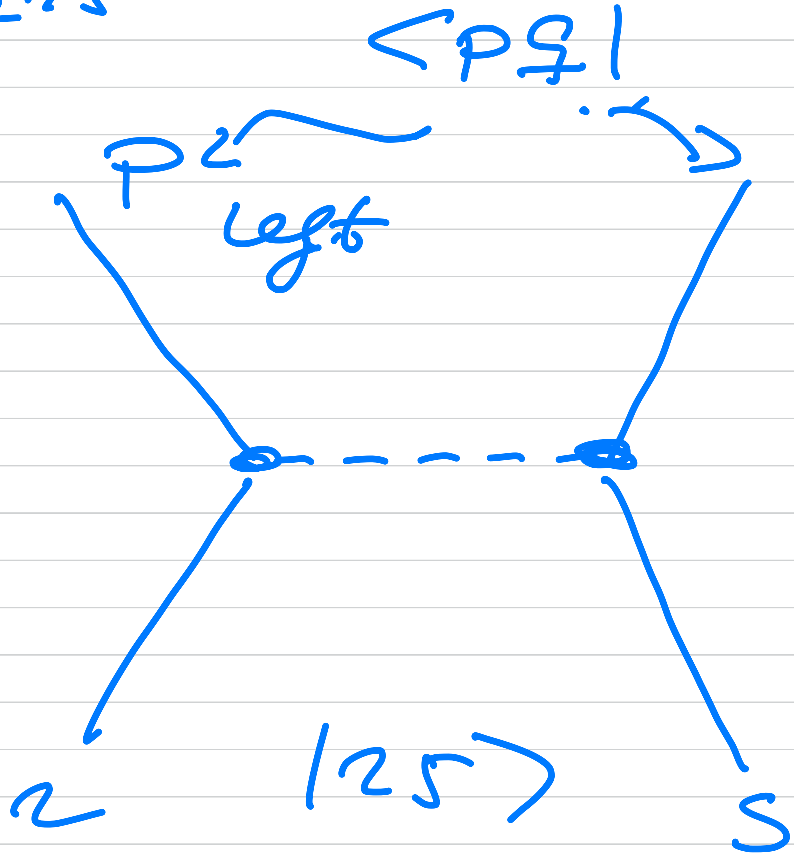
$$= \langle ij|v|ke\rangle - \langle ji|v|ke\rangle$$

Diagrammatic representation

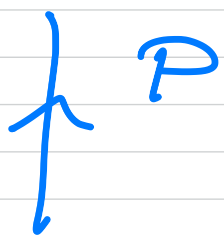
$$\mathcal{H}_0 = \sum_{pq} \langle p|h_0|q\rangle \, a_p^\dagger a_q$$

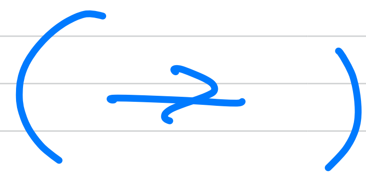


$$\mathcal{H}_I = \frac{1}{2} \sum_{pqrs} \langle pq | v | rs \rangle \, a_p^\dagger a_q^\dagger a_s a_r$$



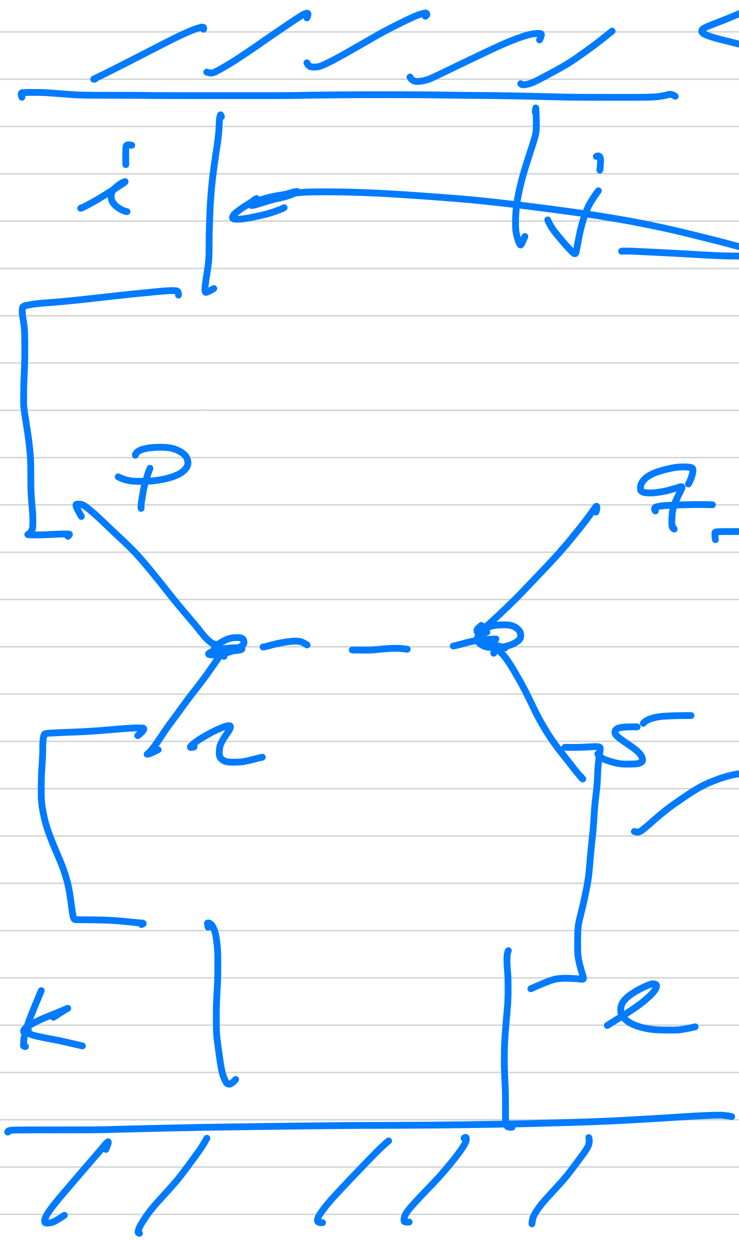
$$\overbrace{a_p \, a_q^\dagger} \qquad \Big[\Big|_{q}^{p} \; \delta_{pq}$$





$\langle ij | v | kl \rangle$

$\langle 0 | a_j a_i$



δ_{pi}

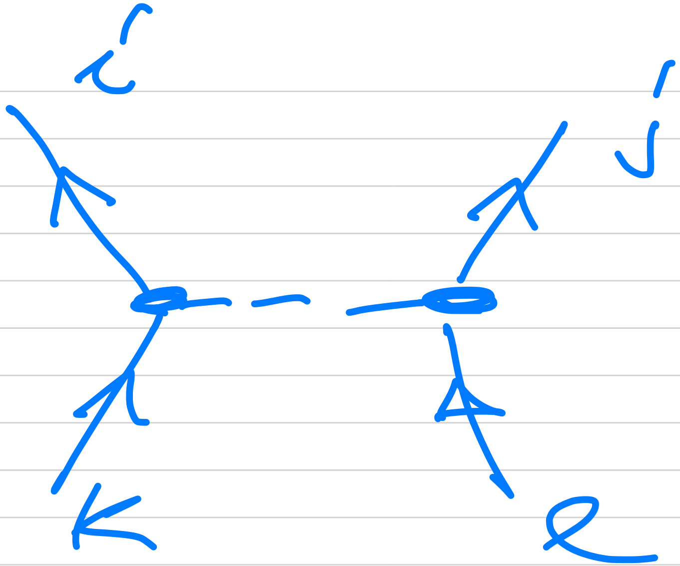
δ_{jq}

δ_{sl}

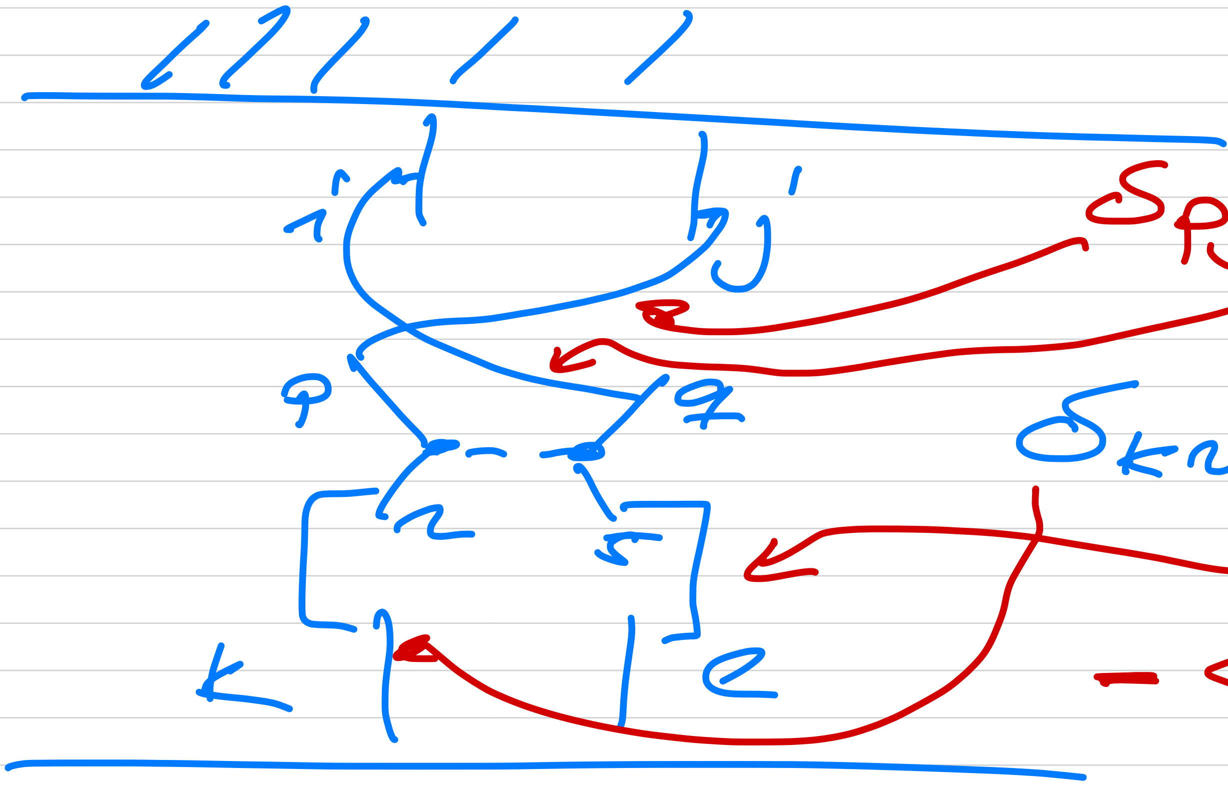
δ_{rk}

$a_k^\dagger a_l^\dagger |0\rangle$

$|0\rangle$



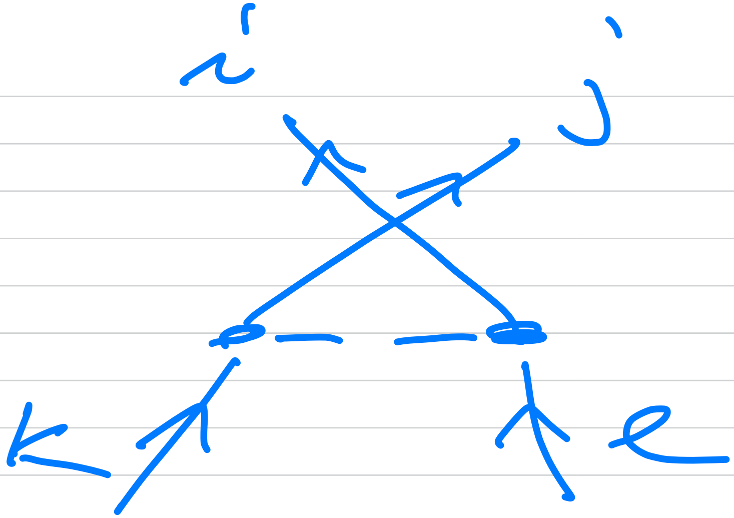
$\langle ij | v | kl \rangle$



$\delta_{pj}\,\delta_{iq}$

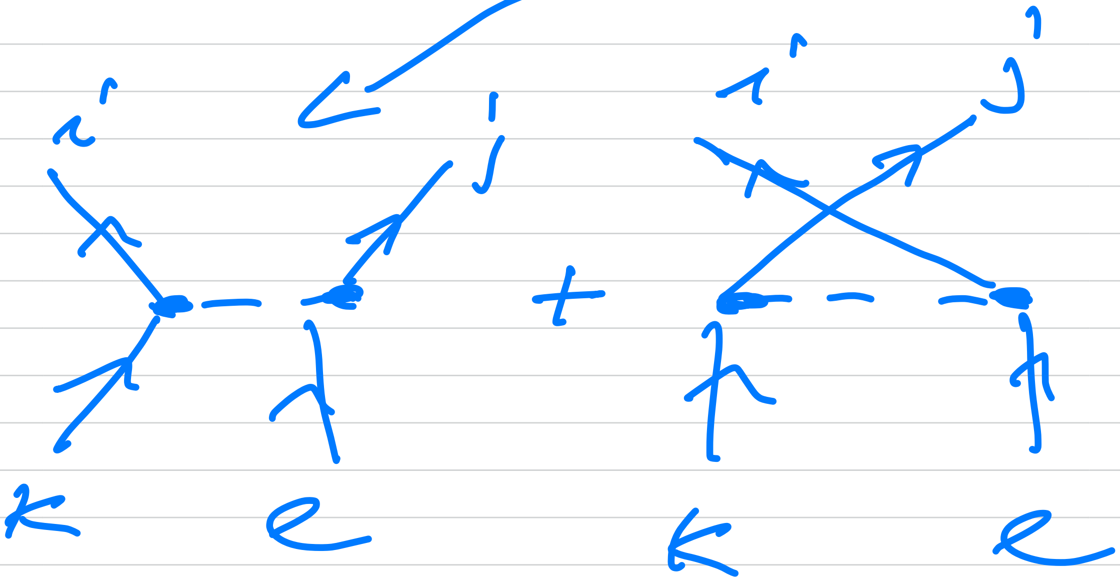
$\delta_{kr}\,\delta_{sl}$

$- \langle ji | v | kl \rangle$



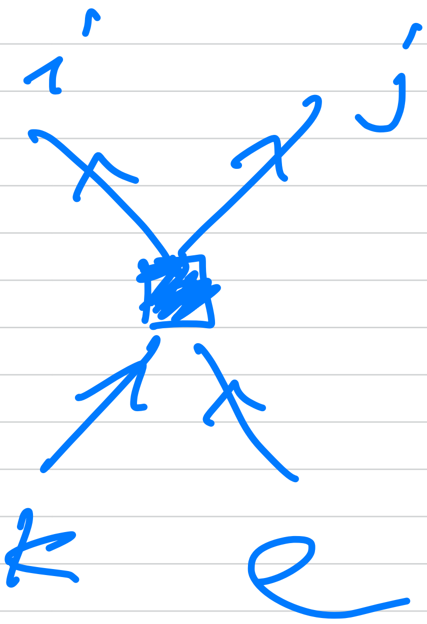
$$\langle ij \,|\, f_{12} \,|\, kl \rangle = \underbrace{\langle ij \,|\, v \,|\, kl \rangle}_{\text{Direct}} - \underbrace{\langle ji \,|\, v \,|\, kl \rangle}_{\text{exchange}}$$

Feynman-Goldstone diagrams



$$\langle ij|v|kl\rangle - \langle ji|v|kl\rangle = \langle ij|v|kl\rangle_{AS}$$

Hugenholtz diagram



Example

$$a_1^\dagger a_2^\dagger a_3^\dagger |0\rangle = |123\rangle$$

$$\langle 123 | \mathcal{H}_I | 123 \rangle$$

$$\overbrace{\langle c | a_3 a_2 a_1 \, a_p^\dagger a_q^\dagger a_s a_r \, a_1^\dagger a_2^\dagger a_3^\dagger |0\rangle}^{\delta_{33}}$$

$\delta_{p1}\delta_{q2}$ $\quad$ $\delta_{s2}\delta_{r1}$ $\quad$ $(12)3$

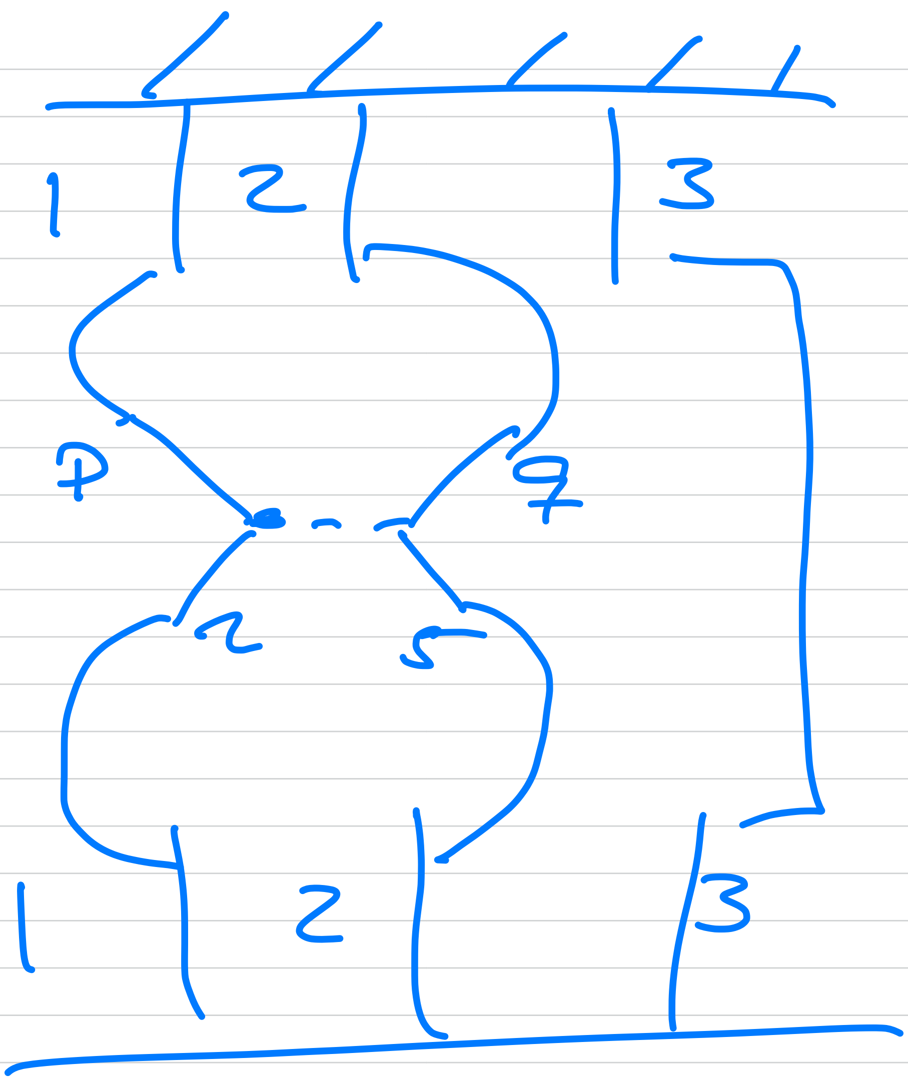
$\delta_{p2}\delta_{q1}$ $\quad$ $\delta_{s1}\delta_{r2}$

$$\delta_{33}\left[\langle 12 | v | 12 \rangle - \langle 21 | v | 12 \rangle\right]$$

$(13)2$ $\qquad$ $(23)1$

$\langle e|$

$\langle 123|$



$|123\rangle$

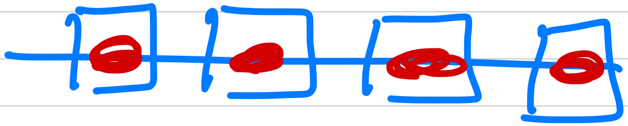
New Reference state

intermezzo : Lipkin model

$N = 4 \qquad n = 8$

P  $\sigma = +1 \ (\uparrow)$

$$|\Phi_c\rangle = a^\dagger_{1\downarrow} a^\dagger_{2\downarrow} a^\dagger_{3\downarrow} a^\dagger_{4\downarrow} |0\rangle$$

P  $\sigma = -1 \ (\downarrow)$

1 2 3 4

single-particle state $|p\sigma\rangle$

$$\mathcal{H}_0 = \frac{1}{2}\,\varepsilon \sum_{p\sigma=\{+1,-1\}} \sigma\, a^{+}_{p\sigma} a_{p\sigma}$$

$$\langle \Phi_0 | \mathcal{H}_0 | \Phi_0 \rangle$$

$$= \frac{1}{2} \sum_{p\sigma} \varepsilon\, \sigma \langle 0 | \boxed{a_{4\downarrow} a_{3\downarrow} a_{2\downarrow} a_{1\downarrow}}\, \boxed{a^{+}_{p\sigma} a_{p\sigma}}$$

$$\times \boxed{a^{+}_{1\downarrow} a^{+}_{2\downarrow} a^{+}_{3\downarrow} a^{+}_{4\downarrow}} | 0 \rangle$$

$$a_{4\downarrow} a_{3\downarrow} a_{2\downarrow} a_{1\downarrow} a^{+}_{p\sigma} a_{p\sigma} a^{+}_{1\downarrow} a^{+}_{2\downarrow} a^{+}_{3\downarrow} a^{+}_{4\downarrow}$$

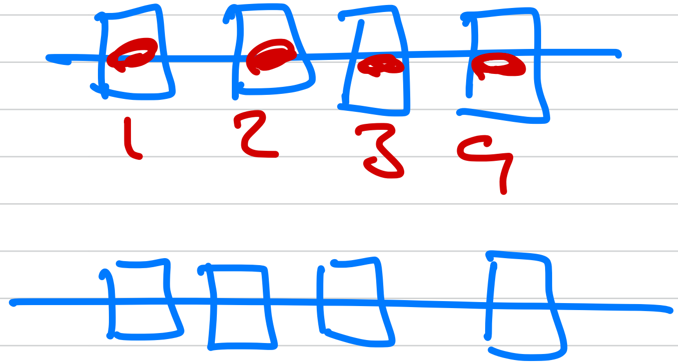
$$\delta_{p1} \delta_{\sigma\downarrow} \quad \delta_{p1} \delta_{\sigma\downarrow}$$

$$\longrightarrow -\frac{1}{2}\varepsilon$$

$$\langle \Phi_0 | H_0 | \Phi_0 \rangle = -2\varepsilon$$

$|\Phi_1\rangle =$

$p =$



1 2 3 4

$\uparrow \quad \sigma = +1$

$\downarrow \quad \sigma = -1$

$p =$ 1 2 3 4

$$= a^\dagger_{1\uparrow} a^\dagger_{2\uparrow} a^\dagger_{3\uparrow} a^\dagger_{4\uparrow} |0\rangle$$

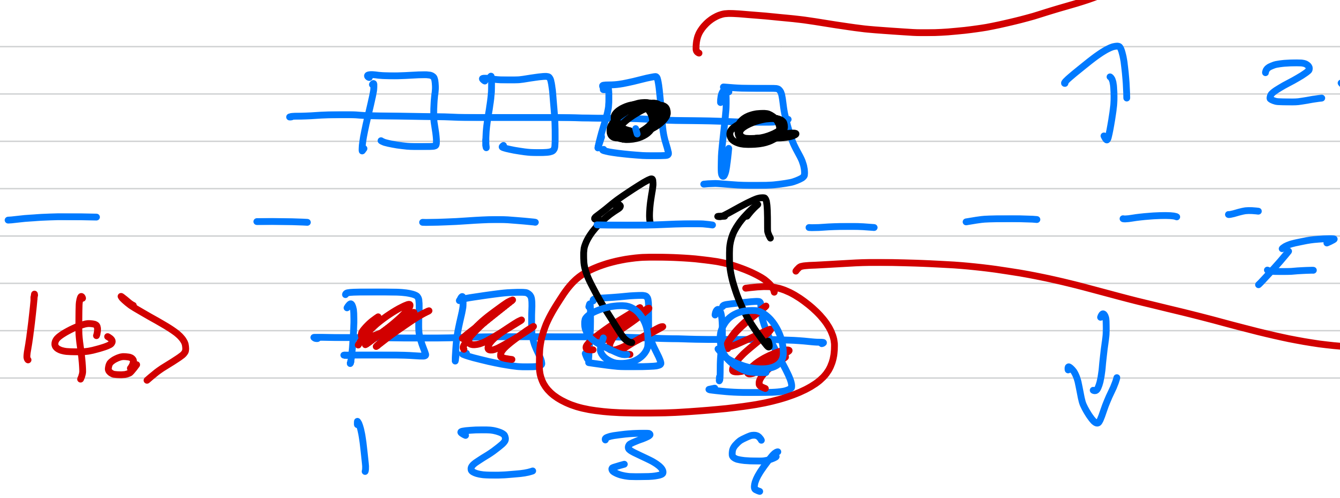
$$\langle \Phi_1 | H_0 | \Phi_1 \rangle = 2\varepsilon$$

$n = 8 \qquad N = 4$

$$\# \text{ configuration } \quad \frac{8!}{4!\,4!} = 70$$

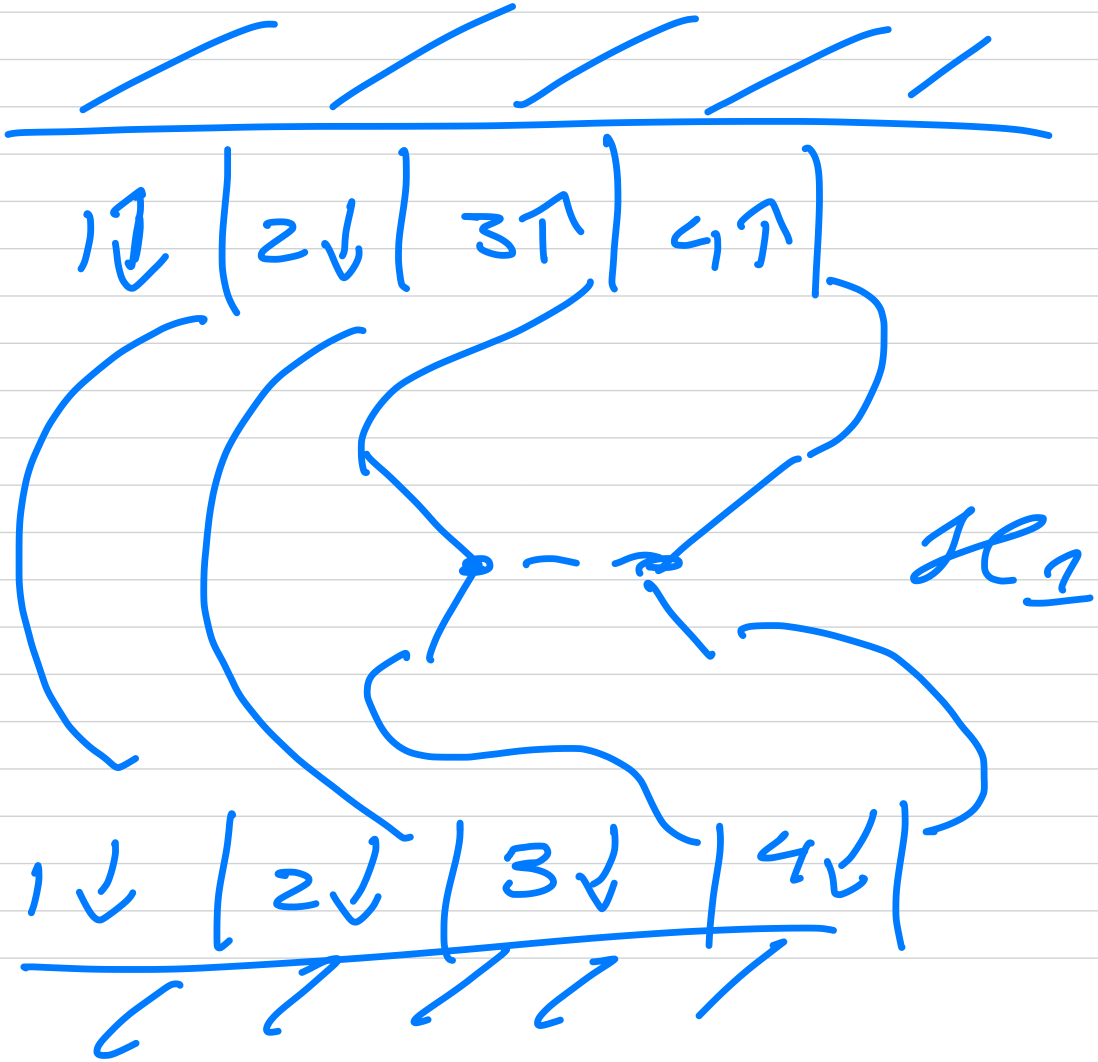
Full Hamiltonian

$$\mathcal{H}_1 = \frac{1}{2} V \sum_{\sigma p p'} \underbrace{a^+_{p\sigma} a^+_{p'\sigma}}\ \underbrace{a_{p'\sigma} a_{p\sigma}}$$

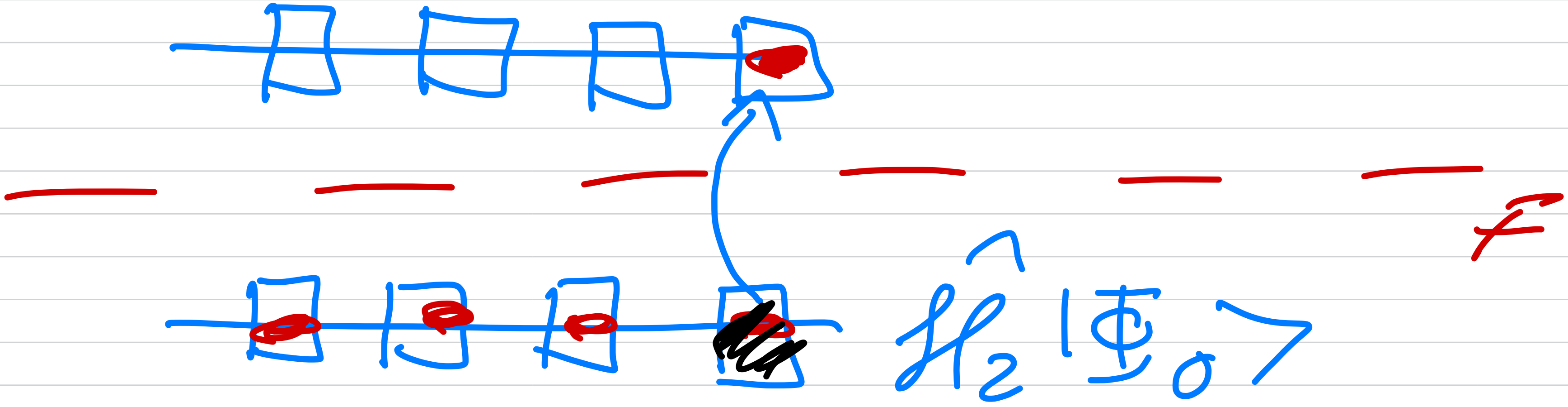


$$\mathcal{H}_1 \, |\Phi_0\rangle \longrightarrow$$

$$\text{constant} \times a^{+}_{1\downarrow}\, a^{+}_{2\downarrow}\, a^{+}_{3\uparrow}\, a^{+}_{4\uparrow}\, |0\rangle$$



$$\hat{H}_2 = \frac{1}{2} w \sum_{\sigma p p'} a^\dagger_{p\sigma} a^\dagger_{p'-\sigma} a_{p'\sigma} a_{p-\sigma}$$



$\hat{H}_2 |\Phi_0\rangle$

$$|\phi_1\rangle = a^\dagger_{1\downarrow} a^\dagger_{2\downarrow} a^\dagger_{3\downarrow} a^\dagger_{4\uparrow}$$

1p1h excitation

$M_\sigma = -2, -1, 0, +1, \quad , +2$

$S = 2$, degeneracy

$2S+1$

commutation relations for angular momentum

$$A_+ = \frac{1}{\sqrt{2}}(A_x + iA_y)$$

$$A_- = \frac{1}{\sqrt{2}}(A_x - iA_y)$$

$J_x + i$

$$[A_\pm, J_z] = \mp A_\pm \qquad (\hbar = 1)$$

$$[A_\pm, J_\pm] = 0$$

$$[A_\pm, J_\mp] = \pm 2 A_z$$

$$[A_z, J_z] = 0$$

$$[A_z, J_\pm] = \pm A_\pm$$

$$J_+ = \sum_p a^+_{p\uparrow} a_{p\downarrow}$$

$$J_- = \sum_p a^\dagger_{p-} a_{p+}$$

$$J_z = \frac{1}{2} \sum_{p\sigma} \sigma\, a^\dagger_{p\sigma} a_{pz}$$

$$(\mathcal{H}_0 = \varepsilon J_z)$$

$$[J_+, J_-] = 2 J_z$$

$$= \sum_{pp'} [a^\dagger_{p+} a_{p-}, a^\dagger_{p'-} a_{p'+}]$$

$$= 2 J_z$$

$$[J_-, J_+] = -[J_+, J_-] = -2 J_z$$

$$\mathcal{H}_0 = \varepsilon J_z$$

$$\mathcal{H}_1 = \frac{1}{2} v \left[J_+^2 + J_-^2 \right]$$

$$\mathcal{H}_2 = \frac{w}{2} \left[J_+ J_- + J_- J_+ - N \right]$$

$$[\mathcal{H}_0, J_z] = 0$$

$$[\mathcal{H}_1 + \mathcal{H}_2 + \mathcal{H}_0, J_z] = 0$$

state $M_J = -2$

$$J_{\pm} |M_J\rangle = C^{\pm}_{M_J} |M_J \pm 1\rangle$$

$$= \sqrt{J(J+1) - M_J(M_J \pm 1)}\ |M_J \pm 1\rangle$$

$J = 2$

$$\langle M_J' | H | M_J \rangle$$