Exercises FYS4480, week 38, September 18-22, 2023

Exercise 1

This exercise is a continuation of the exercises from last week on the so-called Lipkin model. We considered a state with all fermions in the lowest single-particle state

$$|\Phi_{J_z=-2}\rangle = a_{1-}^{\dagger}a_{2-}^{\dagger}a_{3-}^{\dagger}a_{4-}^{\dagger}|0\rangle .$$

This state has $J_z = -2$ and belongs to the set of projections for $J = 2$. We will use the shorthand notation $|J, J_z\rangle$ for states with different spon J and spin projection J_z. The other possible states have $J_z = -1$, $J_z = 0$, $J_z = 1$ and $J_z = 2$.

Use the raising or lowering operators J_+ and J_- in order to construct the states for spin $J_z = -1$ $J_z = 0$, $J_z = 1$ and $J_z = 2$. The action of these two operators on a given state with spin J and projection J_z is given by ($\hbar = 1$) by $J_+|J, J_z\rangle = \sqrt{J(J+1) - J_z(J_z+1)}\,|J, J_z+1\rangle$ and $J_-|J, J_z\rangle = \sqrt{J(J+1) - J_z(J_z-1)}\,|J, J_z-1\rangle$.

Exercise 2

We define the one-particle operator

$$\hat{T} = \sum_{\alpha\beta} \langle\alpha|\,t\,|\beta\rangle\, a_{\alpha}^{\dagger}a_{\beta},$$

and the two-particle operator

$$\hat{V} = \frac{1}{2}\sum_{\alpha\beta\gamma\delta} \langle\alpha\beta|\,v\,|\gamma\delta\rangle\, a_{\alpha}^{\dagger}a_{\beta}^{\dagger}a_{\delta}a_{\gamma}.$$

We have defined a single-particle basis with quantum numbers given by the set of greek letters $\alpha, \beta, \gamma, \dots$

a) Show that the form of these operators remain unchanged under a transformation of the single-particle basis given by

$$|i\rangle = \sum_{\lambda} |\lambda\rangle\langle\lambda|i\rangle ,$$

with $\lambda \in \{\alpha, \beta, \gamma, \dots\}$. Show also that $a_i^{\dagger}a_i$ is the number operator for the orbital $|i\rangle$.

b) Find also the expressions for the operators T and V when T is diagonal in the representation i.