

Lecture
FYS4480/9480,
September 12,
2024

FYS4480/9980 September 12

Wick's theorem, with examples

$$\langle 0 | xyz \dots w | 0 \rangle = N [xyz \dots w]$$

$$+ \sum_{(x)} N [xyz \dots \overbrace{w}]$$

$$+ \sum_{(y)} N [x \overbrace{yz} \dots w]$$

$$+ \dots \sum_{(\frac{M}{z})} N [x \overbrace{yz} \dots \overbrace{w}]$$

$$\langle 0 | \alpha_\alpha \alpha_\beta^\dagger | 0 \rangle = \delta_{\alpha\beta} - \langle 0 | q_\beta^\dagger q_\alpha | 0 \rangle$$

$$= \alpha_\alpha \alpha_\beta^\dagger + N [\alpha_\alpha \alpha_\beta^\dagger]$$

$$\alpha_\alpha \alpha_\beta^\dagger = \delta_{\alpha\beta}$$

$$\alpha_\beta^\dagger \alpha_\alpha = 0 = \alpha_\alpha^\dagger \alpha_\beta^\dagger = \alpha_\alpha \alpha_\beta^\dagger$$

Energy

$$\hat{H}_0 = \sum_{\alpha\beta} \langle \alpha | h_0 | \beta \rangle q_\alpha^+ q_\beta^-$$

$$\hat{H}_I = \frac{1}{2} \sum_{\alpha\beta\gamma\delta} \langle \alpha\beta | v | \gamma\delta \rangle q_\alpha^+ q_\beta^+ q_\delta^- q_\gamma^-$$

$$= \frac{1}{4} \sum_{\alpha\beta\gamma\delta} \langle \alpha\beta | v | \gamma\delta \rangle_{AS} \\ \times q_\alpha^+ q_\beta^+ q_\delta^- q_\gamma^-$$

$$\langle \alpha\beta | v | \gamma\delta \rangle_{AS} = \langle \alpha\beta | v | \gamma\delta \rangle \\ - \langle \alpha\beta | v | \delta\gamma \rangle$$

$$\hat{O}^{(1)} = \sum_{\alpha\beta} \langle \alpha | \hat{o}^{(1)} | \beta \rangle a_\alpha^\dagger a_\beta$$

$$\hat{O}^{(2)} = \frac{1}{4} \sum_{\alpha\beta\gamma\delta} \langle \alpha\beta | \hat{o}^{(2)} | \gamma\delta \rangle_{AS} a_\alpha^\dagger a_\beta^\dagger a_\delta a_\gamma$$

Diagrammatic representation

$$a_\alpha^\dagger a_\beta^\dagger = \begin{bmatrix} \alpha \\ \downarrow \\ \beta \end{bmatrix} \left(\begin{bmatrix} \alpha \\ \downarrow \\ \beta \end{bmatrix} \right)^\dagger$$

$$q_\alpha q_\beta^+ = \downarrow$$

$$\hat{H}_0 = \sum_{\alpha\beta} \begin{array}{c} \nearrow^\alpha \\ \searrow^\beta \end{array} - x \quad \sum \langle \alpha | h_0 | \beta \rangle$$

$q_\beta \times q_\alpha^+ q_\beta$

$$= \begin{array}{c} \nearrow^\alpha \\ \searrow^\beta \end{array} - \langle \alpha | \hat{h}_0 | \beta \rangle$$

$$H_I = \begin{array}{c} \text{Diagram showing two horizontal bars labeled } \alpha \text{ and } \beta \text{ above a dashed oval, and } \alpha \text{ and } \delta \text{ below it. Blue arrows indicate interactions between them.} \\ \text{Top bar: } \alpha \xrightarrow{\quad} \beta \xleftarrow{\quad} \text{ (with a dashed oval in the middle)} \\ \text{Bottom bar: } \alpha \xrightarrow{\quad} \delta \end{array} \sim \langle \alpha \beta \rangle$$

$$\left(\frac{1}{2} \sum_{\alpha \beta \delta} \langle \alpha \beta | \sim | \alpha \delta \rangle a_\alpha^\dagger a_\beta^\dagger \times a_\delta a_\alpha \right)$$

$$\langle d_1, d_2 | H_I | \alpha_1, \alpha_2 \rangle =$$

$$\langle \alpha_1, \alpha_2 | \psi | d_1, d_2 \rangle - \langle d_1, d_2 | \psi | \alpha_1, \alpha_2 \rangle$$

$$| d_1, d_2 \rangle = a_{d_1}^\dagger a_{d_2}^\dagger | 0 \rangle$$

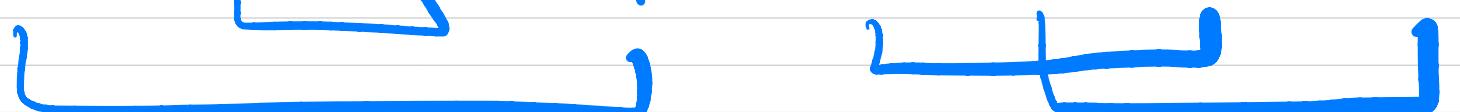
$$\langle \alpha_1, \alpha_2 | H | \alpha_1, \alpha_2 \rangle =$$

$$\frac{1}{2} \sum_{\alpha \beta \gamma \delta} \langle \alpha \beta | \nu | \gamma \delta \rangle$$

$$\times \langle \alpha | \boxed{\alpha_2 \alpha_1} | \boxed{\alpha^+ \alpha_P^+ \alpha_S \alpha_X^+} | \boxed{\alpha^+ \alpha_{d_2}^+} | \bar{\alpha}$$

$$\alpha_2 \alpha_1, \alpha^+ \alpha_P^+ \alpha_S \alpha_X^+ \alpha_1^+ \alpha_{d_2}^+$$


$$\langle \alpha_1, \alpha_2 | \nu | \alpha_1, \alpha_2 \rangle$$

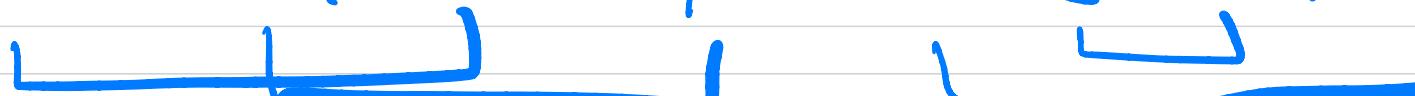
$$\alpha_2 \alpha_1, \alpha^+ \alpha_P^+ \alpha_S \alpha_X^+ \alpha_1^+ \alpha_{d_2}^+$$


$$-\langle d_1 d_2 | v | d_2 q_1 \rangle$$

$$d_2 q_{\alpha_1} d_{\alpha}^+ q_{\beta}^+ q_S q_T q_{\alpha_1}^+ d_{\alpha_2}^+$$


$$= \langle d_2 d_1 | v | d_2 q_1 \rangle$$

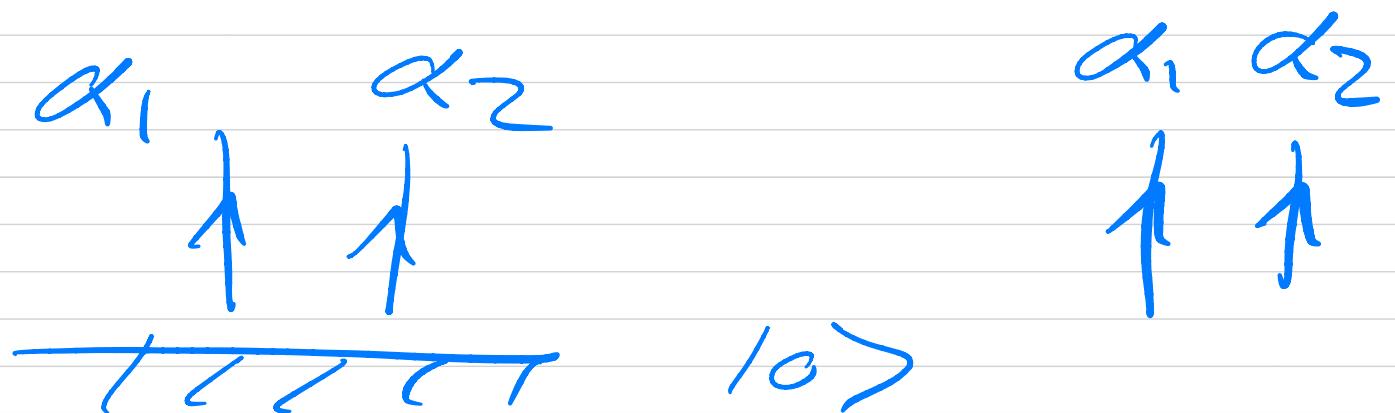
$$= \langle d_1 d_2 | v | d_1 d_2 \rangle$$

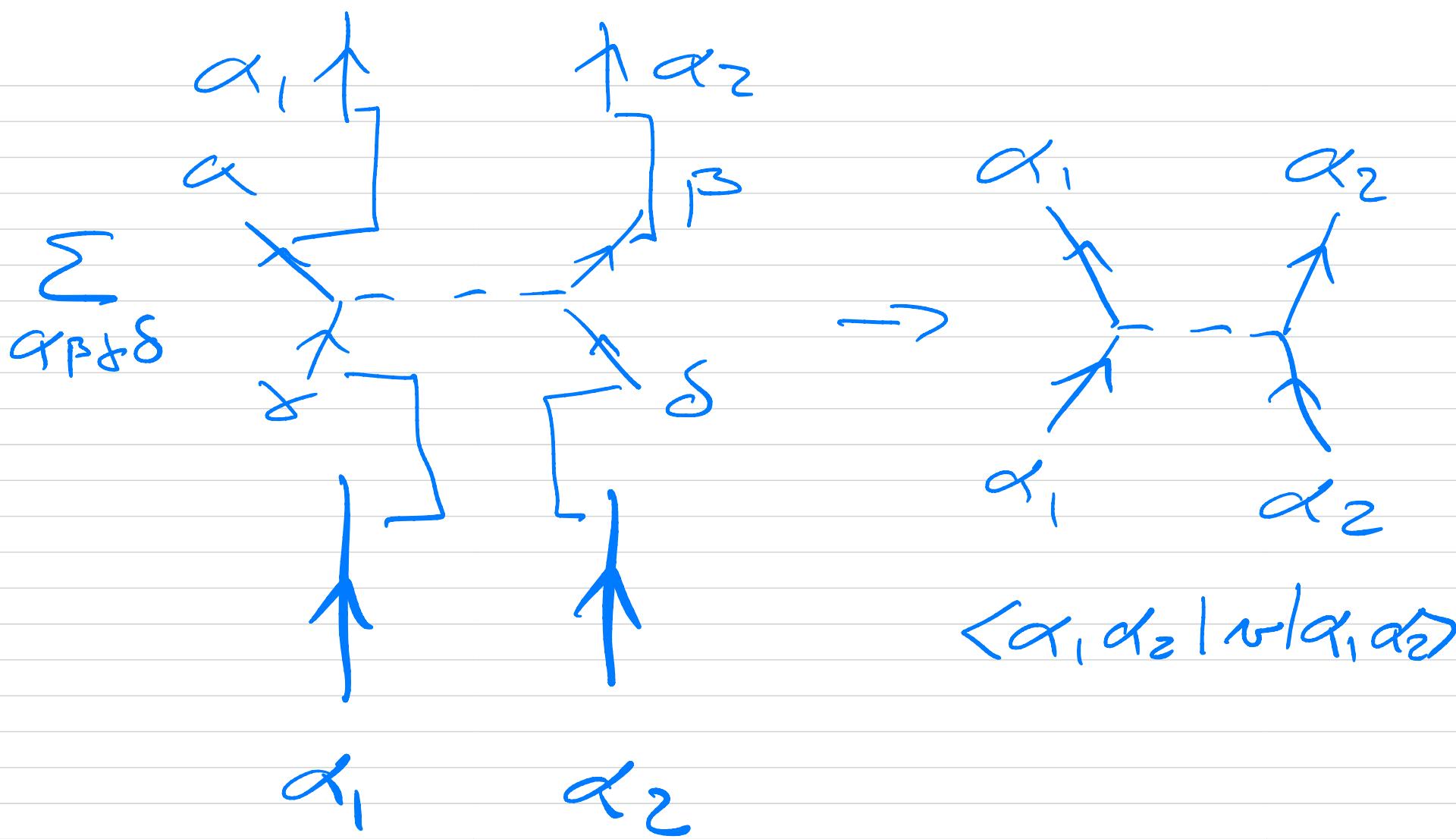
$$d_2 q_{\alpha_1} d_{\alpha}^+ q_{\beta}^+ q_S q_T q_{\alpha_1}^+ d_{\alpha_2}^+$$


$$-\langle d_2 d_1 | v | d_1 d_2 \rangle = -\langle d_1 d_2 | v | d_2 d_1 \rangle$$

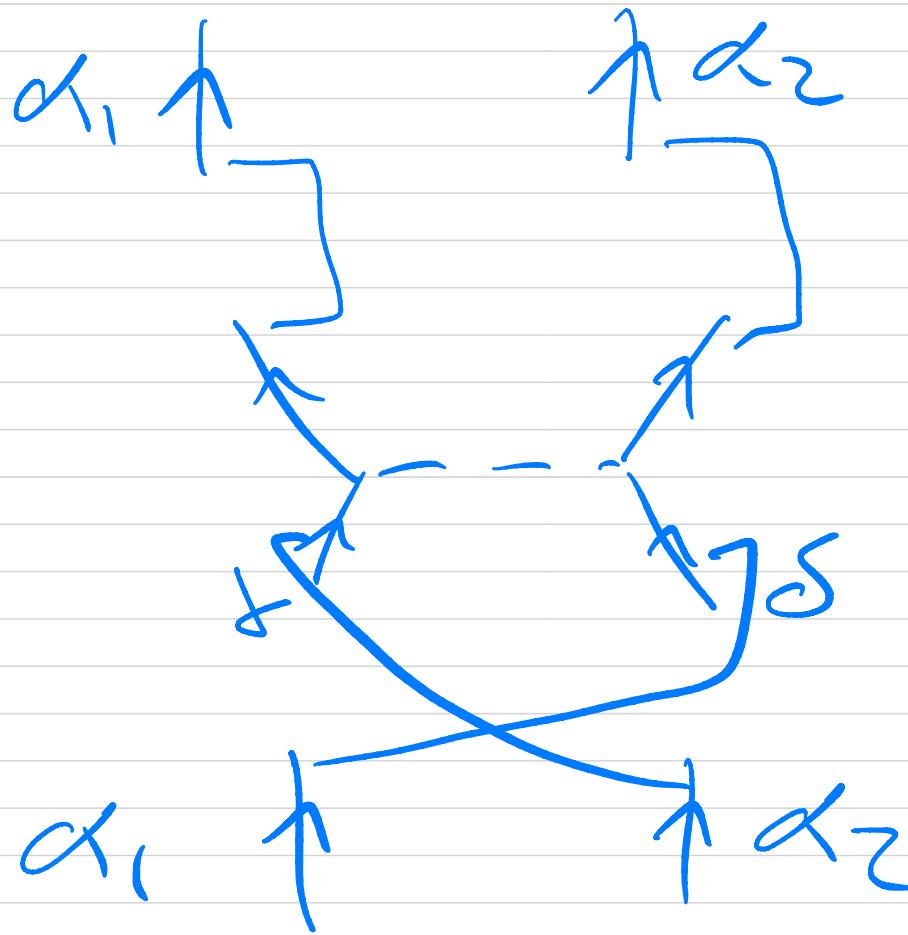
$$\langle d_1 d_2 | \nu | d_1 d_2 \rangle - \langle d_1 d_2 | \nu | d_2 d_1 \rangle \\ = \langle d_1 d_2 | \nu | d_1 d_2 \rangle_{AS}$$

$$|d_1 d_2\rangle = a_{d_1}^+ a_{d_2}^+ |0\rangle$$





$\langle d_1, d_2 | \nu | \alpha_1, \alpha_2 \rangle$

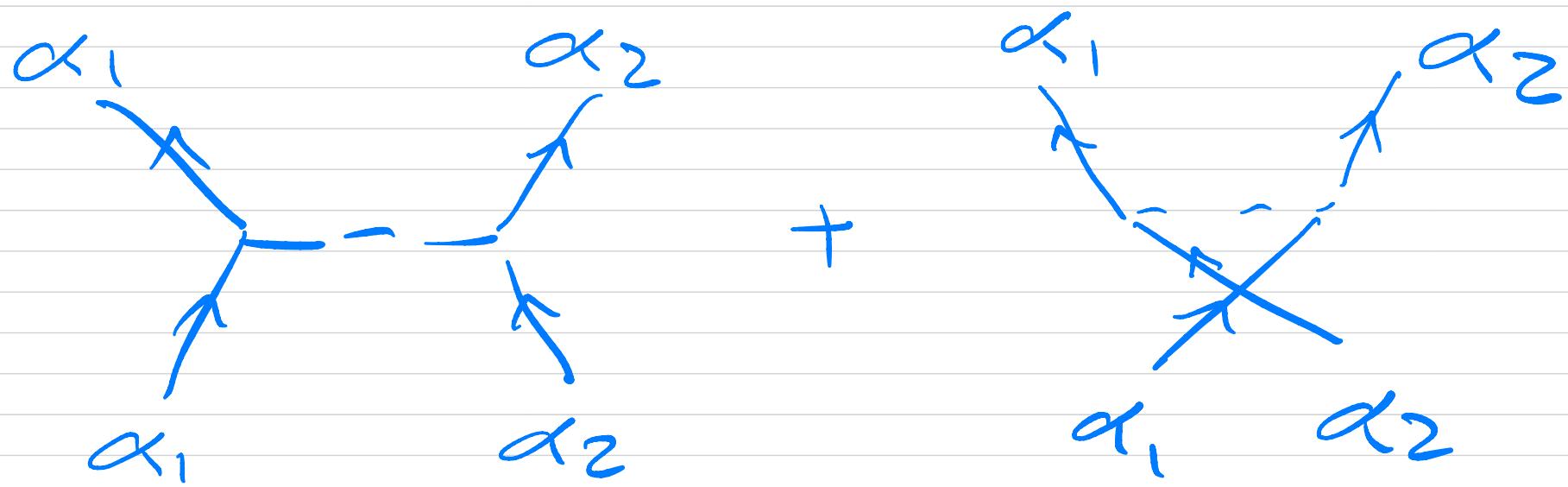


$$\begin{aligned}
 &= \text{Diagram on the left} \\
 &- \langle \alpha_1 \alpha_2 | \psi | \alpha_2 \alpha_1 \rangle
 \end{aligned}$$

$$\underbrace{\alpha_2 \alpha_1}_{\text{gluon}} \underbrace{\alpha_1 \alpha_2}_{\text{gluon}} + \underbrace{\alpha_2 \alpha_2}_{\text{fermion}} \underbrace{\alpha_1 \alpha_1}_{\text{fermion}}$$

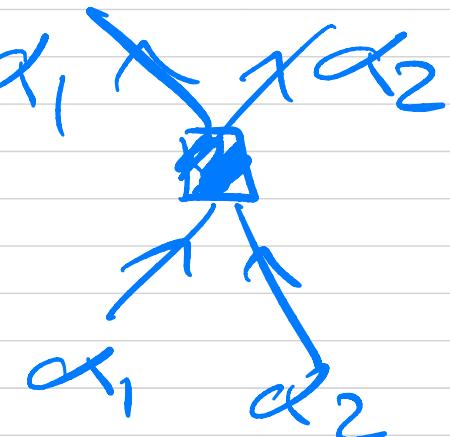
$$\langle \alpha_1 \alpha_2 | v | \alpha_1 \alpha_2 \rangle_{AS} =$$

$$\langle \alpha_1 \alpha_2 | v | \alpha_1 \alpha_2 \rangle - \langle \alpha_1 \alpha_2 | v | \alpha_2 \alpha_1 \rangle$$



Feynman - Goldstone
diagrams

Hugenholz notation

$$\langle d_1 d_2 | \omega | d_1 d_2 \rangle_{AS} =$$


The diagram shows two dipoles, d_1 and d_2 , represented by arrows originating from a central point. d_1 is a horizontal arrow pointing right, and d_2 is a vertical arrow pointing down. A horizontal line labeled x connects the tips of the two dipoles.

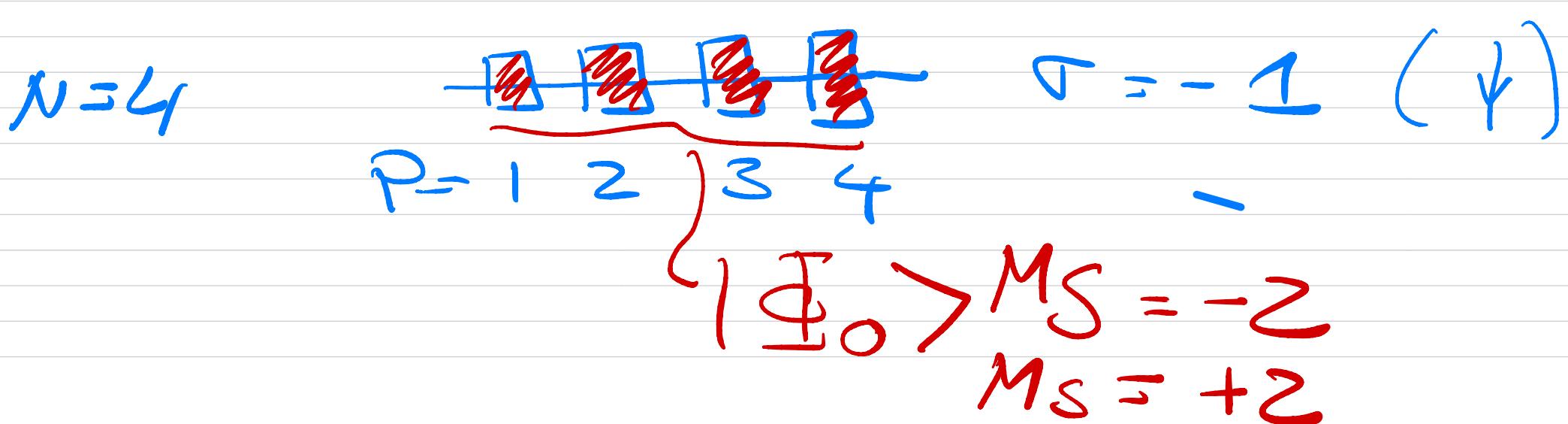
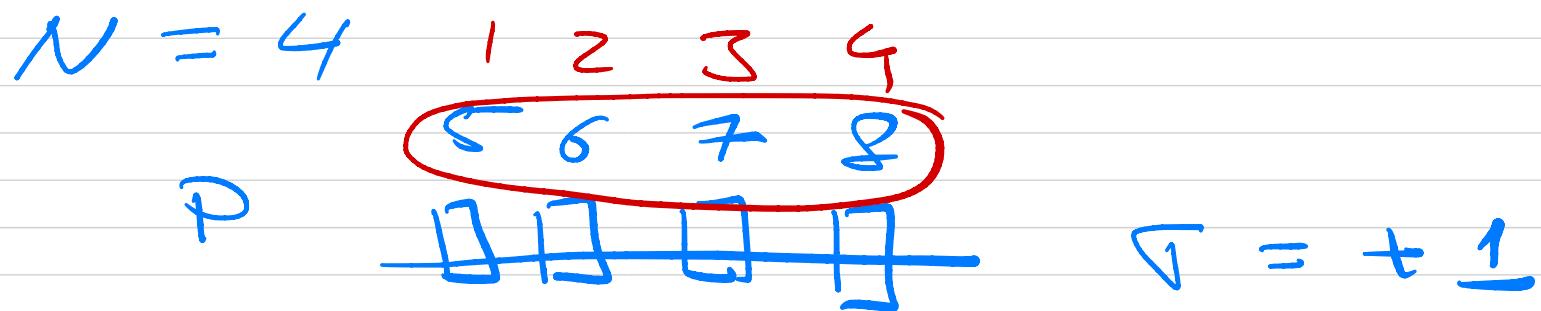
Exercises week 37

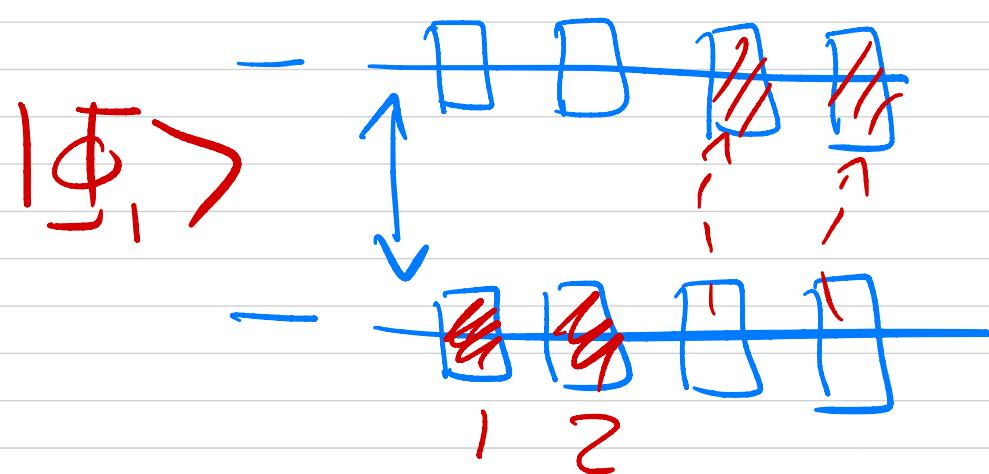
Lipkin - model

$$\hat{H} = \hat{H}_0 + \underbrace{\hat{H}_1 + \hat{H}_2}_{\hat{H}_I}$$

$$H_0 = \frac{1}{2} \varepsilon \sum_{P\Gamma} \Gamma a_{P\Gamma}^+ a_{P\Gamma}$$

$$\Gamma = \{-1, +1\}$$





$$G = +1$$

$$T = -1$$

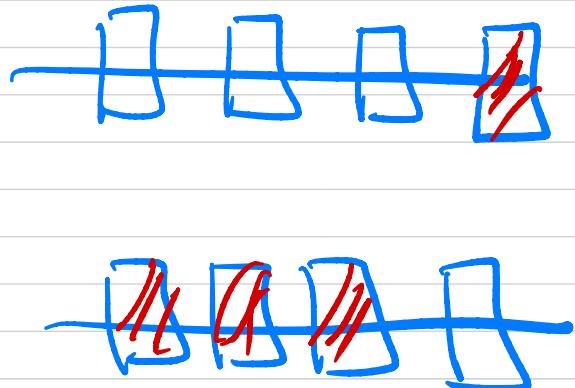
configuration
number

$$= \binom{8}{4}$$

$$= \frac{8!}{4!4!}$$

$$= 70$$

$|\Phi_2\rangle$

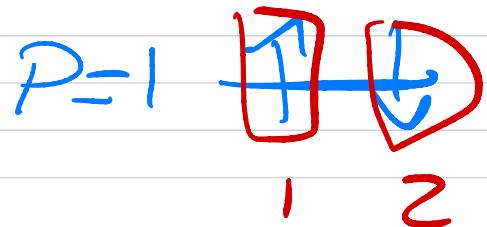


Total spin = $S = 2$

$M_S = -2, -1, 0, +1, 2$

$$\hat{H}_I = \frac{1}{2} V \sum_{\Gamma P P'} a_{\Gamma P}^+ a_{\Gamma P'}^+ a_{-\Gamma P'}^\dagger a_{\Gamma P}^\dagger$$

zpz \hbar -term



$$\hat{H}_2 = \frac{1}{2} w \sum_{\sigma p p'} a_{\sigma p}^+ a_{-\sigma p'}^\dagger a_{\sigma p'}^+ a_{-\sigma p}^\dagger$$

1p1h - term



$$\frac{1}{2} \sum_{\alpha p \delta} \langle \dots \rangle a_\alpha^\dagger q_p^\dagger q_{\delta p} q_\delta$$

commutation relations for
angular momentum

$$A_+ = \frac{1}{\sqrt{2}}(A_x + i A_y)$$

$$A_- = \frac{1}{\sqrt{2}}(A_x - i A_y)$$

$$A_z \quad [A_{\pm}, J_z] = \mp A_{\pm}$$

$$\hbar = 1$$

$$[A_{\pm}, J_{\pm}] = 0$$

$$[A_z, J_{\pm}] = \mp A_{\pm}$$

$$[A_{\pm}, J_{\mp}] = \pm 2 A_z, [A_z, J_{\pm}] = 0$$

$$J_+ = \sum_p a_{p+}^+ a_{p-}^- \quad \Gamma = \{+, -\}$$

$$J_- = \sum_p a_{p-}^+ a_{p+}^-$$

$$J_z = \frac{1}{2} \sum_{p\Gamma} (a_{p\Gamma}^+ a_{p\Gamma}^-)$$

$$[J_+, J_-] = 2J_z$$

$$\sum_{pp'} [a_{p+}^+ a_{p-}^- | a_{p'-}^+ a_{p'+}^-] =$$

$$= \sum_{pp'} \left\{ a_p^+ + q_{p'}^- q_{p'}^+ - q_{p'}^+ - a_{p'}^+ - a_{p'}^- \right. \\ \left. \times a_p^+ + q_p^- \right\}$$

$$= \sum_p \left\{ a_p^+ + q_p^- - a_p^+ - a_p^- \right\}$$

$$+ \sum_{pp'} \left\{ - a_p^+ + q_{p'}^+ - a_p^- - q_{p'}^+ + \right. \\ \left. + a_{p'}^+ - a_{p'}^- q_{p'}^+ + q_{p'}^- \right\}$$

$$= \sum_p \left\{ a_p^+ + q_p^- - a_p^+ - q_p^- \right\} \\ = \sum_p \sigma q_{p\sigma}^+ q_{p\sigma}^- = 2J_3$$

$$[\bar{J}_-, \bar{J}_+] = - [\bar{J}_+, \bar{J}_-] = -2\bar{J}_z$$

$$[\bar{J}_z, \bar{J}_z] = 0,$$

$$[\bar{J}_{\pm}, \bar{J}_{\pm}] = 0$$

$$\hat{H}_0 = \epsilon \bar{J}_z$$

$$\begin{aligned} H_0/M_S &= \\ \text{const } M_S &\rangle \end{aligned}$$

$$\hat{H}_1 = \frac{1}{2} V (\hat{J}_+^2 + \hat{J}_-^2)$$

$$\hat{H}_2 = \frac{w}{2} (\hat{J}_+ \hat{J}_- + \hat{J}_- \hat{J}_+ - \hat{N})$$

$$M_S = -2$$

$$J \pm |M_S\rangle = C_{M_S}^{\pm} |M_S \pm 1\rangle$$

$$= \sqrt{S(S+1) - M_S(M_S \pm 1)} |M_S \pm 1\rangle$$

$$\langle M_S' | H | M_S \rangle \propto S_{M_S} S_{M_S'}$$