FYS4480/9480 lecture, November 16, 2023

Diagram rules

① For a diagram with -n- H_I interactions, draw -n- vertices at times $t_0 > t_1 > t_2 > \dots > t_n$

Each vertex can be either an antisymmetric or not interaction indicated by

● or o - - - o (two-body)

or a onebody interaction

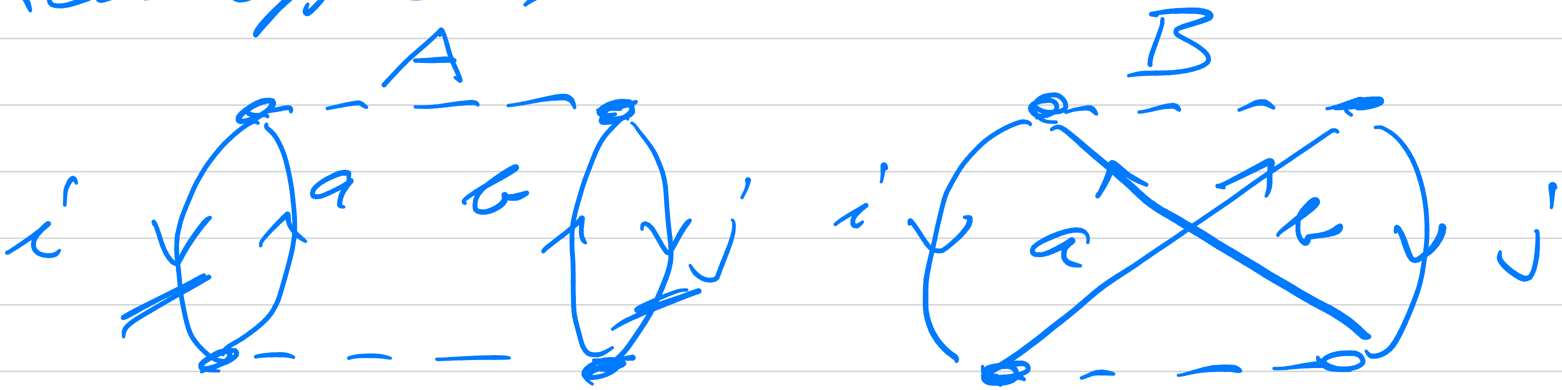
o - - - X or o - - - ⊠

② Draw all topologically distinct diagrams by linking up particle and hole lines with various vertices. Two diagrams can be made topologically equivalent by deformation of fermion lines under the restriction that the ordering of diagrams is kept.

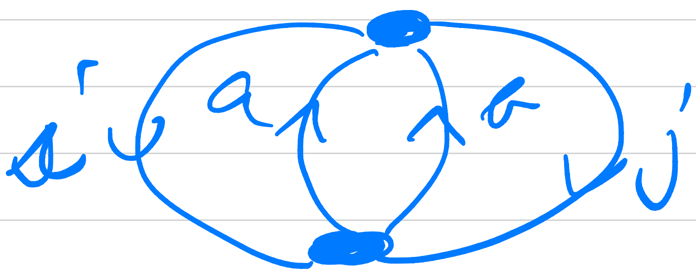
Particle lines stay as they are

hole lines are kept,
The order at to and tm
are not changed

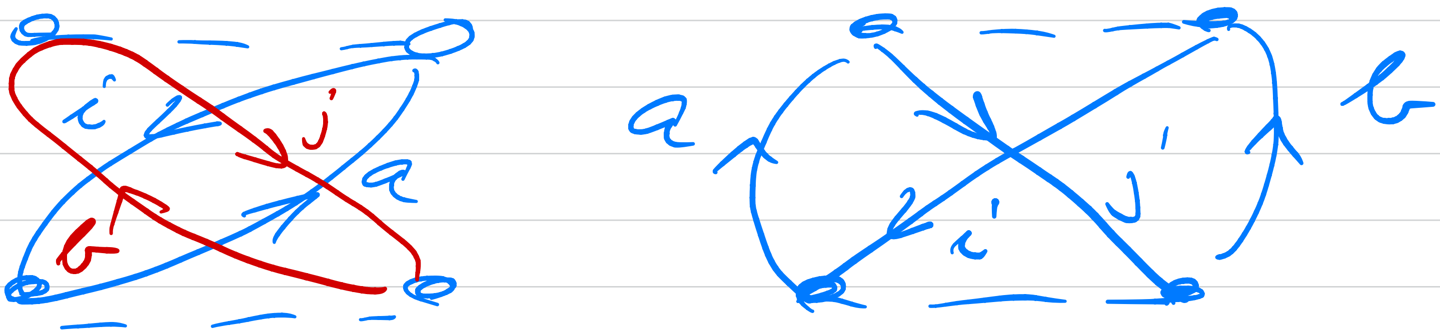
Example:



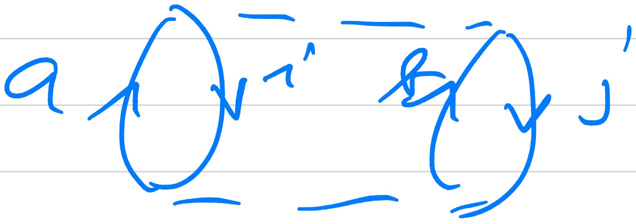
using antisymmetric
elements we can rewrite
both



A



B



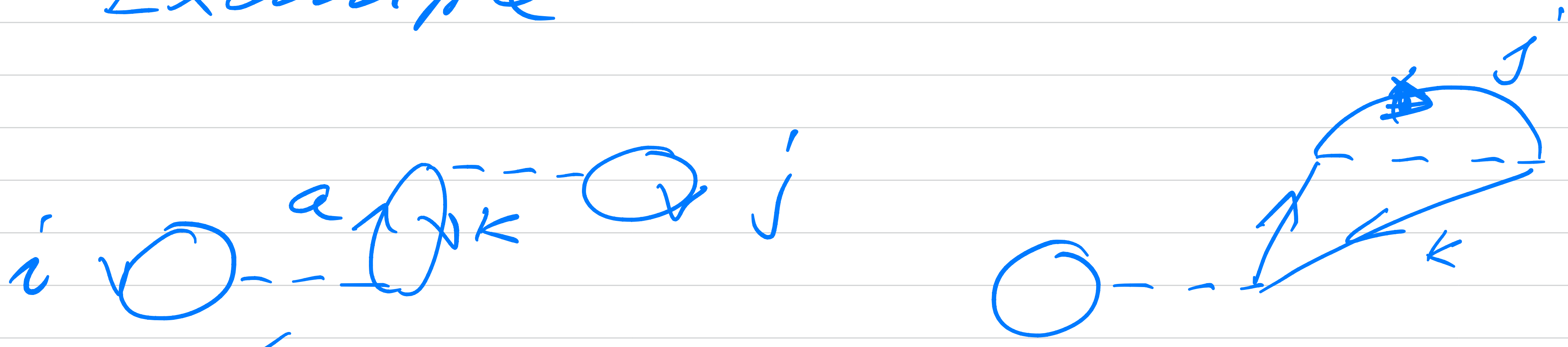
A : $\sum_{\substack{ab\\ij}} \langle ji|v|ba\rangle\langle ba|v|ji\rangle$

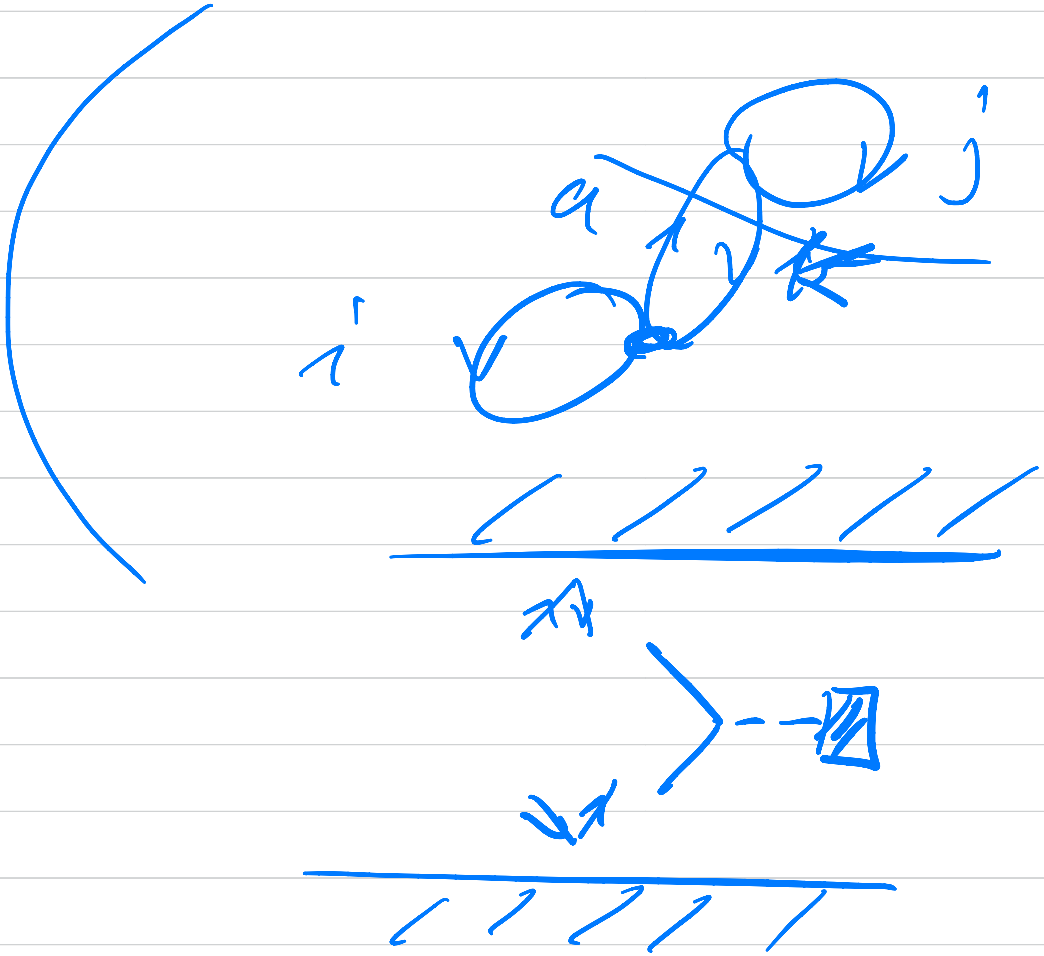
$= \langle ij|v|ab\rangle \langle ab|v|ij\rangle$

$=$

B : $\sum_{\substack{ab\\ij}} \langle ab|v|ij\rangle\langle ij|v|ab\rangle$

Example





$|p\,k$

$\langle \Phi_0 |$

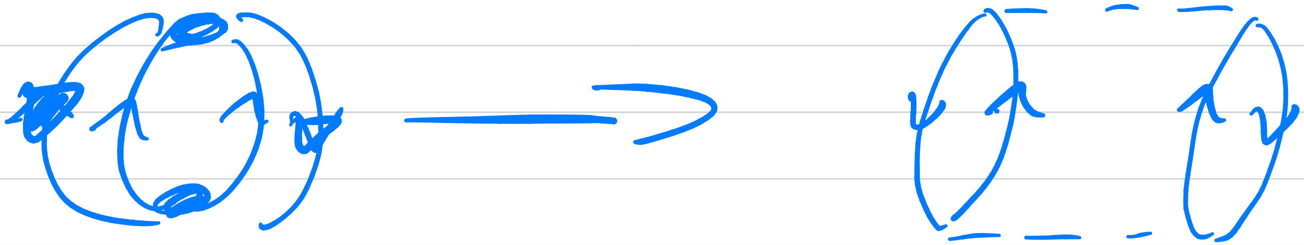
$| \Phi_0 \rangle$

$$\Delta E^{(2)} = \sum \frac{\langle \Phi_0 | H_1 | \Phi_M \rangle \langle \Phi_M | H_1 | \Phi_0 \rangle}{W_0 - W_M}$$

$$\langle i | f | a \rangle = 0$$

③ Explicit evaluation

ⓐ pull open



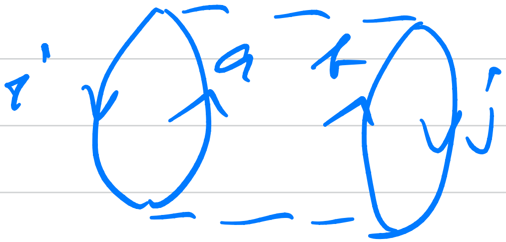
④ Each vertex has a contribution

(c) There is a factor

$$(-1)^{n_h} \times (-1)^{n_e}$$

n_h = # hole lines

n_e = # closed loops



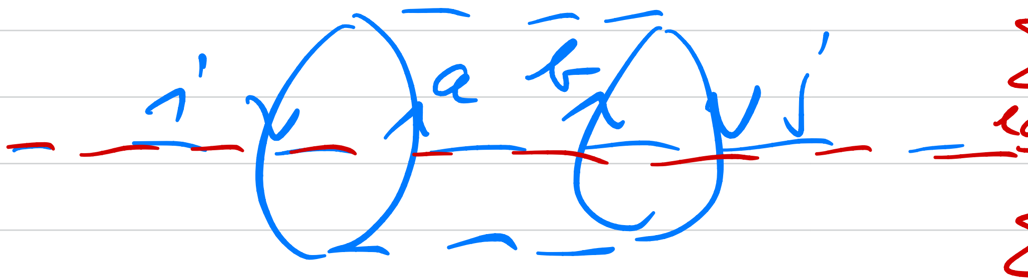
$n_h = 2$

$n_e = 2$

(d) For each interval between two vertices (with at least one state above F)

There is a factor

$$\frac{1}{\sum_{i\le F} \varepsilon_i - \sum_{a>F} \varepsilon_a}$$



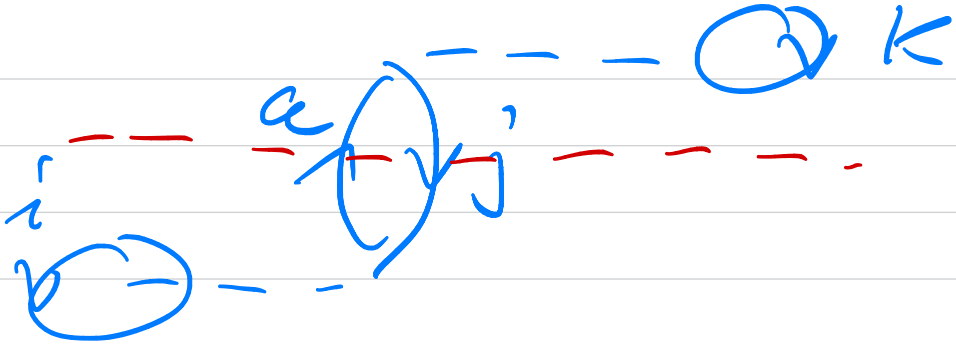
$$\sum_{i\le F} \to \varepsilon_i + \varepsilon_j$$

$$\sum_a \to \varepsilon_a + \varepsilon_b$$

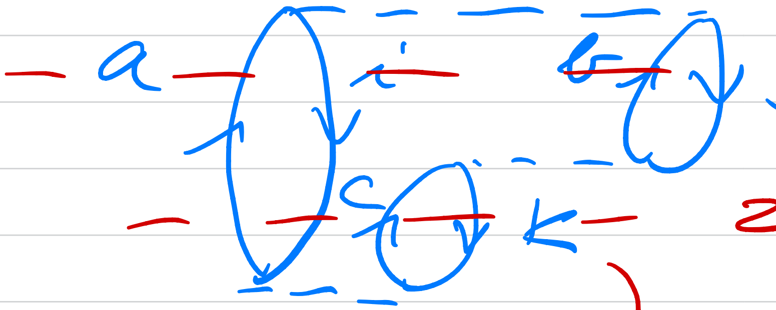
$$\frac{1}{\varepsilon_i + \varepsilon_j - \varepsilon_a - \varepsilon_b} = \frac{1}{W_0 - W_M}$$

$$|\Phi_M\rangle = a_a^+ a_b^+ a_j a_i |\Phi_0\rangle$$

$$W_M = \varepsilon_a + \varepsilon_b - \varepsilon_i - \varepsilon_j + W_0$$



$$\frac{1}{\varepsilon_j - \varepsilon_a}$$



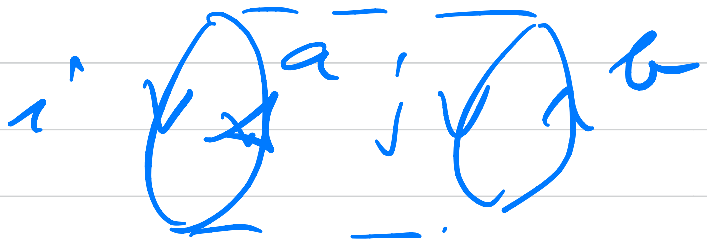
2p2h

2p2h

$$\frac{1}{\varepsilon_k + \varepsilon_i - \varepsilon_a - \varepsilon_c} \qquad \frac{1}{\varepsilon_a + \varepsilon_j - \varepsilon_a - \varepsilon_b}$$

(e) There is a factor $\left(\frac{1}{2}\right)^{n_{ep}}$

n_{ep} = # equivalent pairs

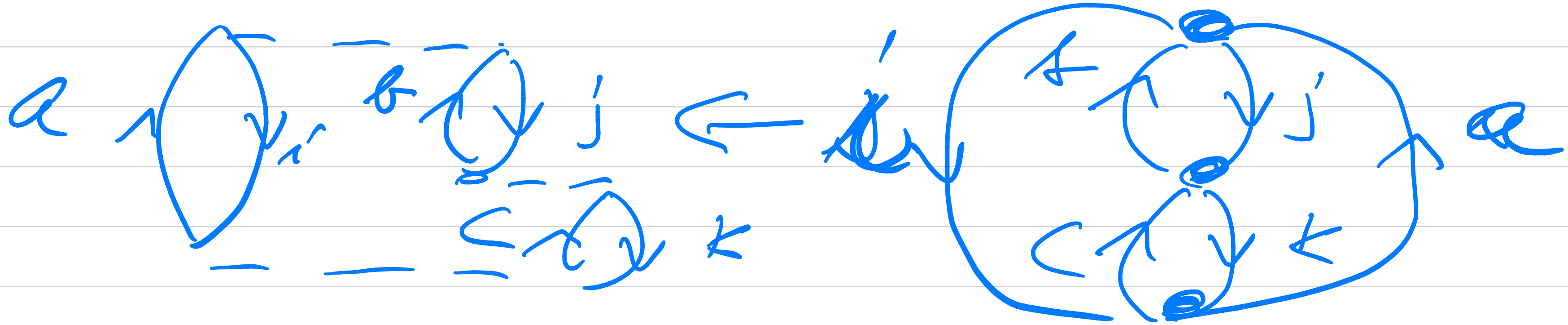


$n_{ep} = 2$

3 sum freely over all intermediate states

$$\left(\frac{1}{2}\right)^{n_{ep}} (-1)^2 (-1)^2 \sum_{\substack{ab\\ij}} \frac{\langle ij|v|ab\rangle\langle ab|v|ij\rangle}{\varepsilon_i+\varepsilon_j-\varepsilon_a-\varepsilon_b}$$

$$= \frac{1}{4} \sum_{\substack{ab\\ij}} \frac{|\langle ij|v|ab\rangle|^2}{\varepsilon_i+\varepsilon_j-\varepsilon_a-\varepsilon_b}$$



$$n_{ep} = 0 \qquad n_h = 3 \qquad n_e = 3$$

$$\sum_{\substack{abc\\ijk}} \frac{\langle ij|v|ab\rangle\langle bk|v|jc\rangle\langle ac|v|ik\rangle}{(\varepsilon_i+\varepsilon_j-\varepsilon_a-\varepsilon_b)(\varepsilon_i+\varepsilon_k-\varepsilon_a-\varepsilon_c)}$$

Example

ε_2 ———— $|\Phi_2\rangle$

ε_1 ———— $|\Phi_1\rangle$

$$H = \underbrace{\sum_{i=1}^{2} a_i^{\dagger} a_i \varepsilon_i}_{H_0} + \lambda \sum_{i\neq j=1}^{2} a_i^{\dagger} a_j$$

$$H_0 |\phi_i\rangle = \varepsilon_i |\phi_i\rangle$$

$$H_1 = \lambda (a_1^+ a_2 + a_2^+ a_1)$$

$$\langle \phi_i | H_1 | \phi_i \rangle = 0$$

$$\langle \phi_1 | \lambda (a_1^+ a_2 + a_2^+ a_1) | \phi_1 \rangle$$

$$|\phi_1\rangle = a_1^+ |0\rangle$$

$$|\phi_2\rangle = a_2^+ |0\rangle$$

$$\lambda \left\{ \langle 0 | a_1 (a_1^+ a_2 + a_2^+ a_1) a_1^+ | 0 \rangle \right\} = 0$$

$$H = \begin{bmatrix} \varepsilon_1 & \langle \phi_1 | H_1 | \Phi_2 \rangle \\ \langle \Phi_2 | H_1 | \phi_1 \rangle & \varepsilon_2 \end{bmatrix}$$

$$\langle \phi_1 | H_1 | \Phi_2 \rangle = \lambda$$

$$\lambda \{ \langle c | a_1 (a_2^\dagger a_1 + \overbrace{a_1^\dagger a_2) a_2^\dagger}^{=0} | 0 \rangle$$

$$= \lambda$$

$$\det(H - E) = 0$$

$$\det \begin{bmatrix} \varepsilon_1 - E & \lambda \\ \lambda & \varepsilon_2 - E \end{bmatrix} = 0$$

Let us look at E_1 $(< E_2)$

$$E_1 = \frac{1}{2}\left[\varepsilon_1+\varepsilon_2 - \sqrt{(\varepsilon_1-\varepsilon_2)^2+4\lambda^2}\right]$$

$$= \frac{1}{2}\left[\varepsilon_1+\varepsilon_2-(\varepsilon_2-\varepsilon_1)\times \sqrt{1+4\left(\frac{\lambda}{\varepsilon_2-\varepsilon_1}\right)^2}\,\right]$$

Taylor expand

$$E_1 = \frac{1}{2}\Big(\varepsilon_1+\varepsilon_2 - (\varepsilon_2-\varepsilon_1)\Big\{1 + \frac{4\lambda^2}{2(\varepsilon_2-\varepsilon_1)^2} - \frac{2\lambda^4}{(\varepsilon_2-\varepsilon_1)^4} + \dots\Big\}$$

$$= \varepsilon_1 - \frac{\chi^2}{\varepsilon_2 - \varepsilon_1} + \frac{\chi^4}{(\varepsilon_2 - \varepsilon_1)^3} - + \ldots$$

Link this with BW-pert theory and RS-pert theory.

BW-pert theory

$$\Delta E = \underbrace{\langle \phi_1 | H_I | \phi_1 \rangle}_{=0} + \underbrace{\frac{\langle \phi_1 | H_1 | \phi_2 \rangle \langle \phi_2 | H_1 | \phi_1 \rangle}{E_1 - \varepsilon_2}}_{=0}$$

$$+ \frac{\langle \phi_1 | H_1 | \phi_2 \rangle \langle \phi_2 | H_1 | \phi_2 \rangle \langle \phi_2 | H_1 | \phi_1 \rangle}{(E_1 - \varepsilon_2)(E_1 - \varepsilon_2)}$$

$$+ \ldots$$

$$\Delta E = \frac{\lambda^2}{E_1 - \varepsilon_2}$$

$$E_1 = \varepsilon_1 + \Delta E = \varepsilon_1 + \frac{\lambda^2}{E_1 - \varepsilon_2}$$

$$(E_1 - \varepsilon_1)(E_1 - \varepsilon_2) = \lambda^2 \Rightarrow$$

$$(E_1 - \varepsilon_1)(E_1 - \varepsilon_2) - \lambda^2 = 0 =$$

$$\det \begin{bmatrix} \varepsilon_1 - E_1 & \lambda \\ \lambda & \varepsilon_2 - E_1 \end{bmatrix} = 0$$