FYS4480/9480, lecture November 6, 2025

FYS4480/9480 November 6

$$\Delta E_0^{(2)} = \sum_{\lambda} \frac{\langle \Phi_0 | \hat{H}_I | \Phi_\lambda \rangle \langle \Phi_\lambda | \hat{H}_1 | \Phi_0 \rangle}{\varepsilon_0 - \varepsilon_\lambda}$$

1p1h:

$$\sum_{\substack{ai \\ jk}} \frac{\langle ij | v | aj \rangle_{AS} \langle ak | v | ik \rangle_{AS}}{\varepsilon_i - \varepsilon_a}$$

2p2h:

$$\frac{1}{4} \sum_{\substack{ab \\ ij}} \frac{\langle ij | v | ab \rangle_{AS} \langle ab | v | ij \rangle_{AS}}{\varepsilon_i + \varepsilon_j - \varepsilon_a - \varepsilon_b}$$

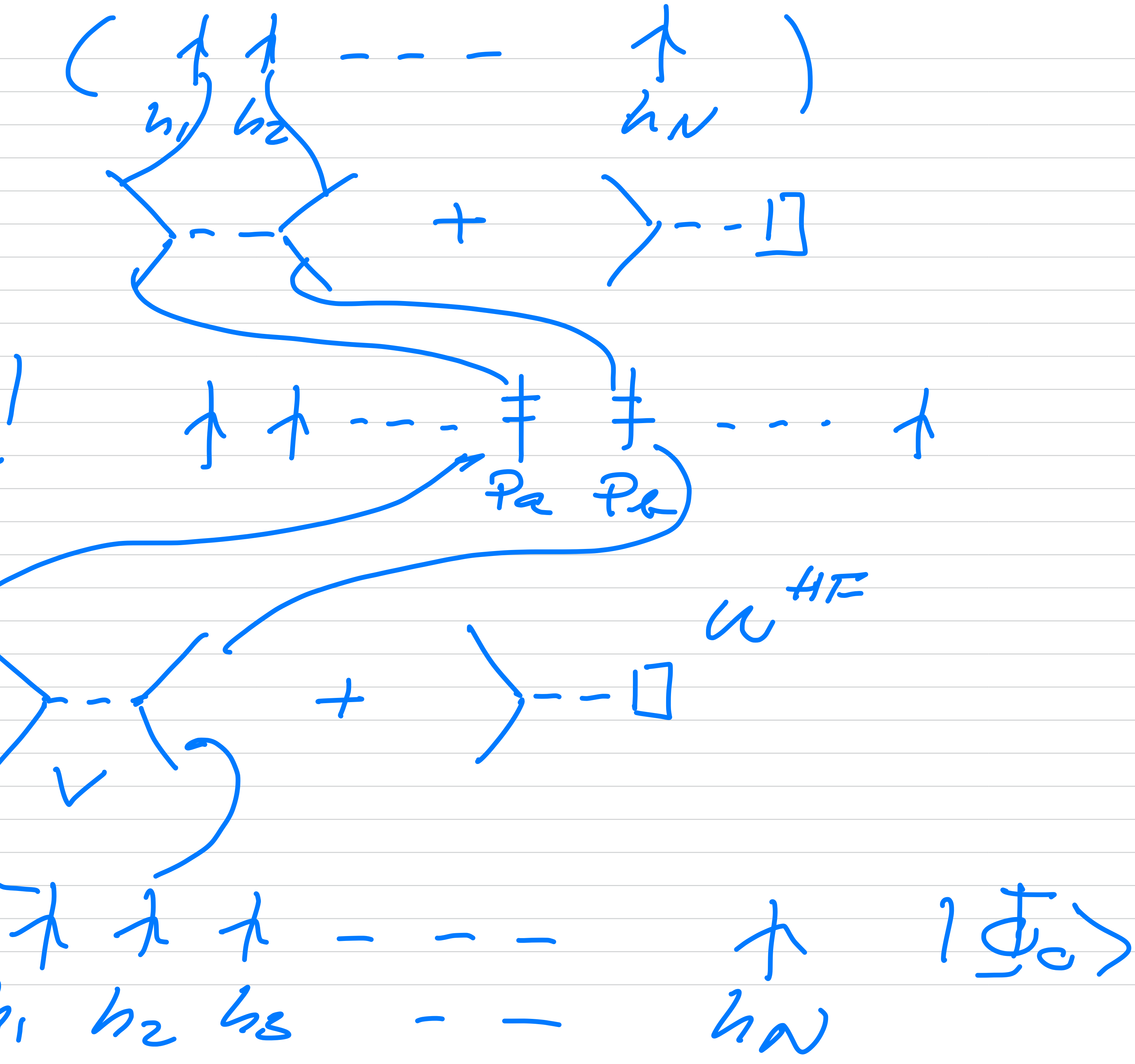
$\langle \Phi_0 |$

$\mathcal{H}_1 \rightarrow$

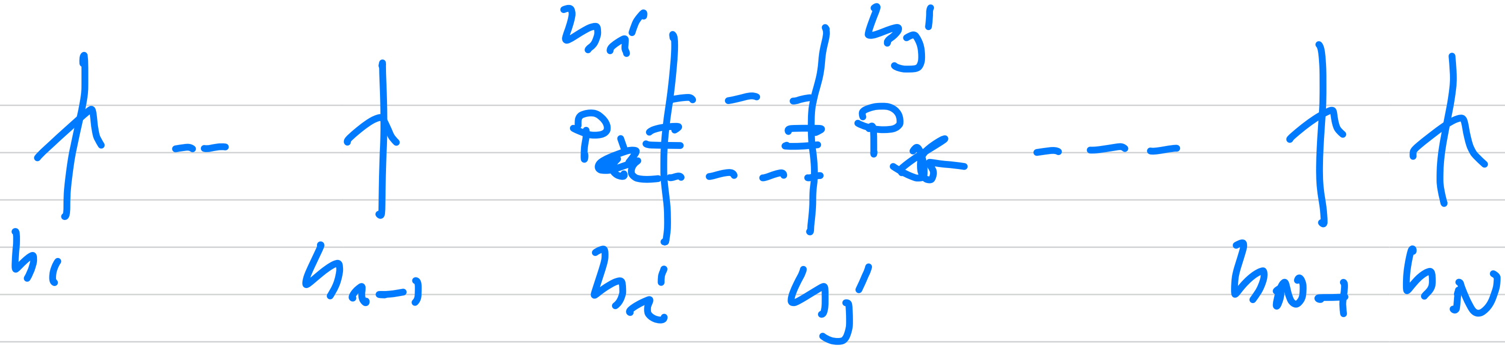
$$\frac{|\Phi_x\rangle\langle\Phi_x|}{\mathcal{E}_0 - \mathcal{E}_x}$$

$\mathcal{H}_1 \rightarrow$

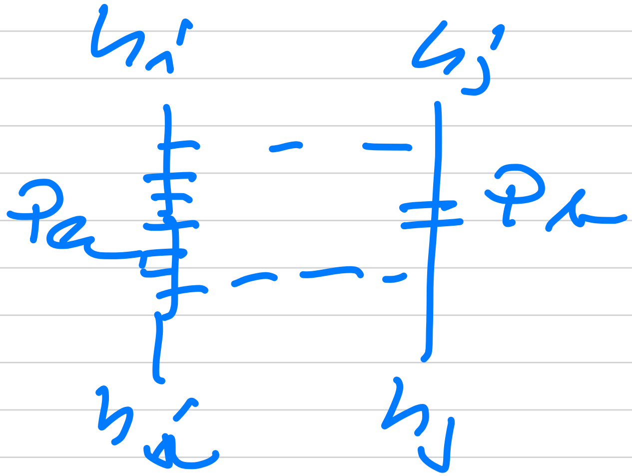
$$\prod_{i=1}^{N} a^{+}_{h_i} |0\rangle$$

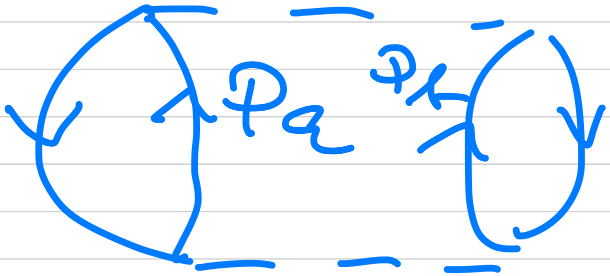


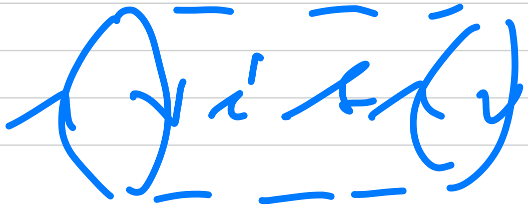
$$\sum_{\substack{s_i' s_j' \\ p_i' p_j'}}$$



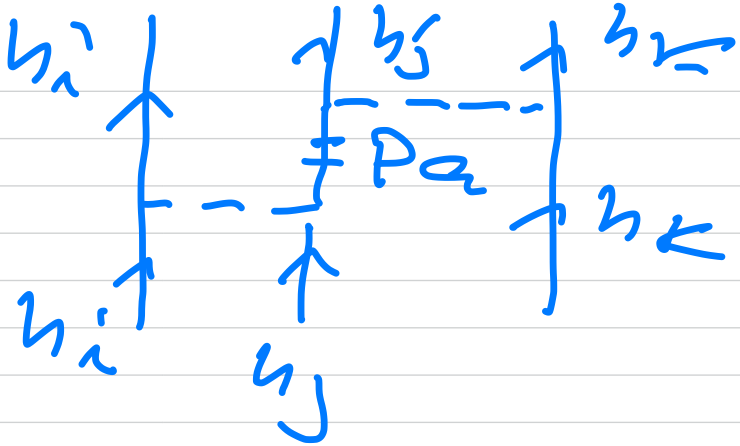
$$= \sum_{\substack{s_i' s_j' \\ p_i' p_j'}}$$



$=$ s_i  s_j' $\longrightarrow$

a  j'

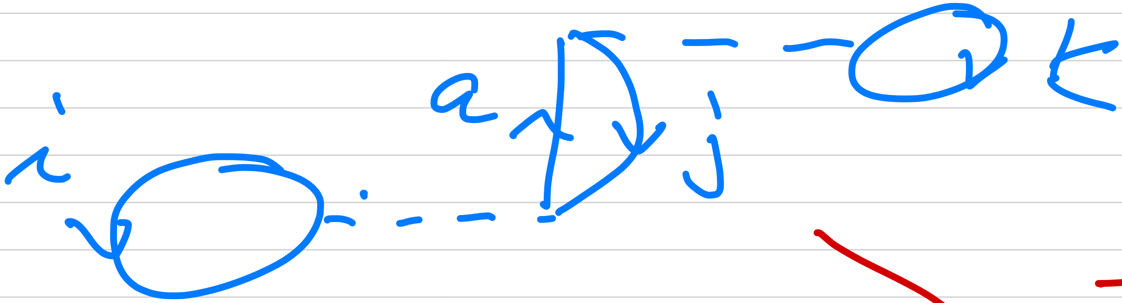
$$\sum_{n_i' n_j' n_k \atop p_a}$$



$$\langle jk|v|ak\rangle_{AS} = \langle kj|v|ka\rangle_{AS}$$

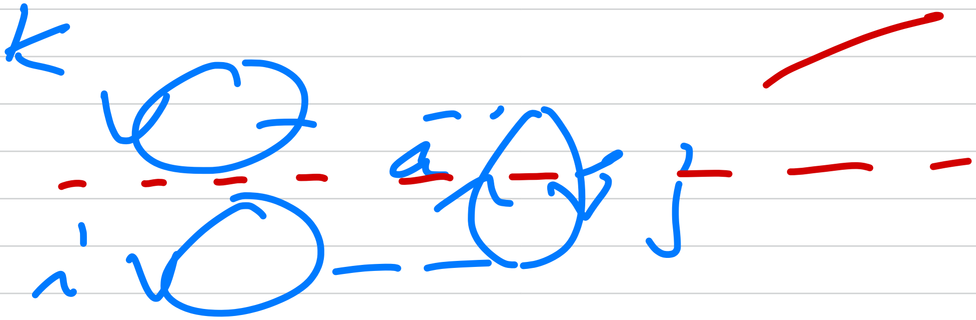
$$= -\langle jk|v|ka\rangle_{AS}$$

$=$

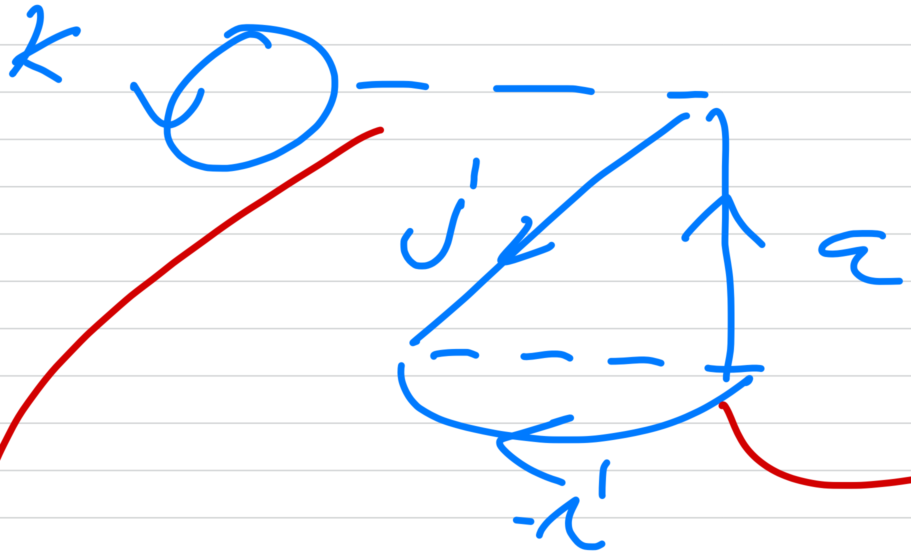


Topological equivalent

$=$

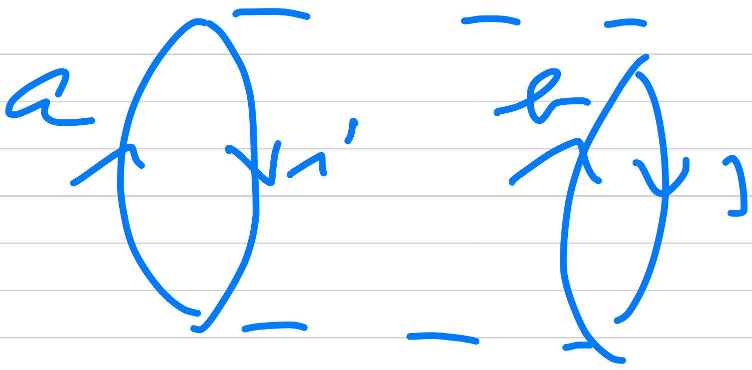


$$\sum_{aijk} \frac{\langle kj|v|ka\rangle_{AS}\langle ia|v|ij\rangle_{AS}}{\varepsilon_j - \varepsilon_a}$$



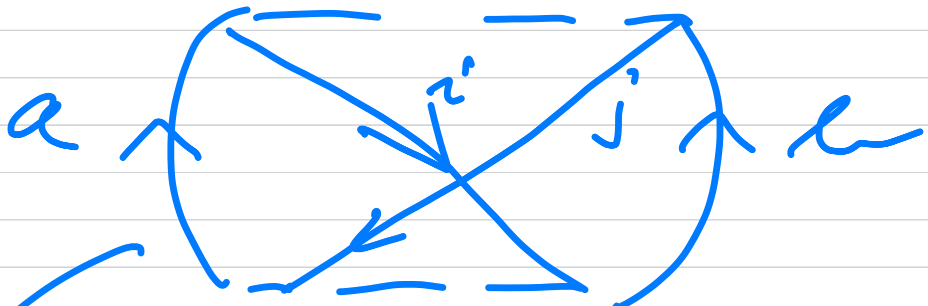
$$\langle kj'|v|ka\rangle_{AS}\langle ia|v|j'i'\rangle_{AS}$$

$$= -\sum_{\substack{ijk\\a}} \frac{\langle kj'|v|ka\rangle_{AS}\langle ia|v|i'j'\rangle_{AS}}{\varepsilon_j - \varepsilon_a} -$$



$$\langle ij|v|ab\rangle\langle ab|v|ij\rangle$$

$$\frac{1}{4}\sum_{\substack{ab\\ i'j'}}\frac{\langle i'j'|v|ab\rangle\langle ab|v|i'j'\rangle}{\varepsilon_{i'}+\varepsilon_{j'}-\varepsilon_a-\varepsilon_b}$$



$$\langle i'j'|v|ab\rangle\langle ab|v|j'i'\rangle$$

$$= -\langle i'j'|v|ab\rangle\langle ab|v|i'j'\rangle$$

$$a_i^+ \cdots a_j^+$$

$$-\frac{1}{4}\sum_{\substack{ab\\ i'j'}}\frac{\langle i'j'|v|ab\rangle\langle ab|v|i'j'\rangle}{\varepsilon_{i'}+\varepsilon_{j'}-\varepsilon_a-\varepsilon_b}$$

$$(-1)^{n_h+n_e}$$

Diagram rules

(i) Draw all distinct diagrams to a given order by linking up particle and hole lines with different interaction vertices under the restrictions that

a) the ordering of the interactions vertices is not altered.

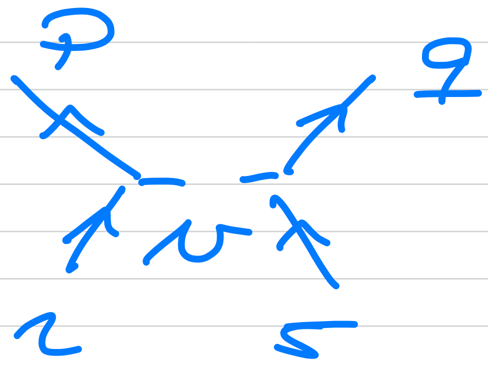
b) Particle and hole states remain particle and hole states

c) The ordering of the interactions at the end and beginning is unchanged.

(ii) For each distinct diagram

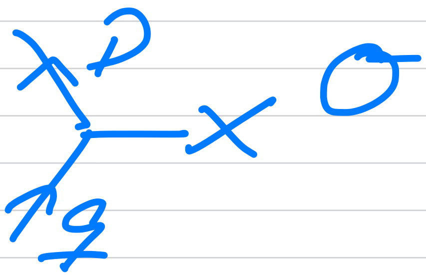
a) set up interaction vertex (>--<   >--x)

b) each of these vertices are assigned labels



$\langle pq | v | rs \rangle$

or



$\langle p | \sigma | q \rangle$

c) There is a phase factor

$$(-1)^{n_h + n_l}$$

hole lines

closed loops

d) For each interval between successive interactions we have an energy denominator

$$\frac{1}{\sum_{i \le F} \varepsilon_i - \sum_{a > F} \varepsilon_a}$$

e) There is a factor

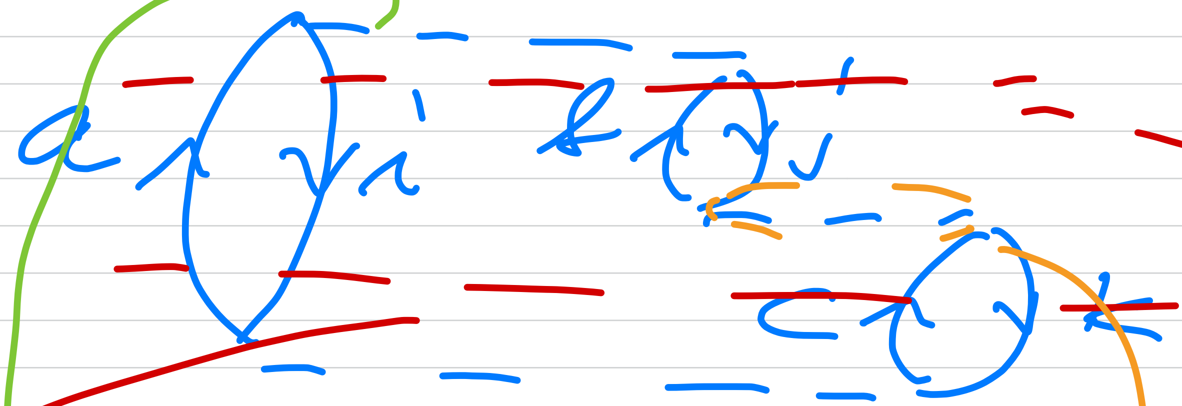
$$\left(\frac{1}{2}\right)^{n_{ep}}$$

n_{ep} = # equivalent pairs

of lines which start and end
at the same interaction vertex

f) sum freely over all inter-
mediate states

g) all lines should have a
label.



$n_h = 3$ $\qquad$ $n_\ell = 3$

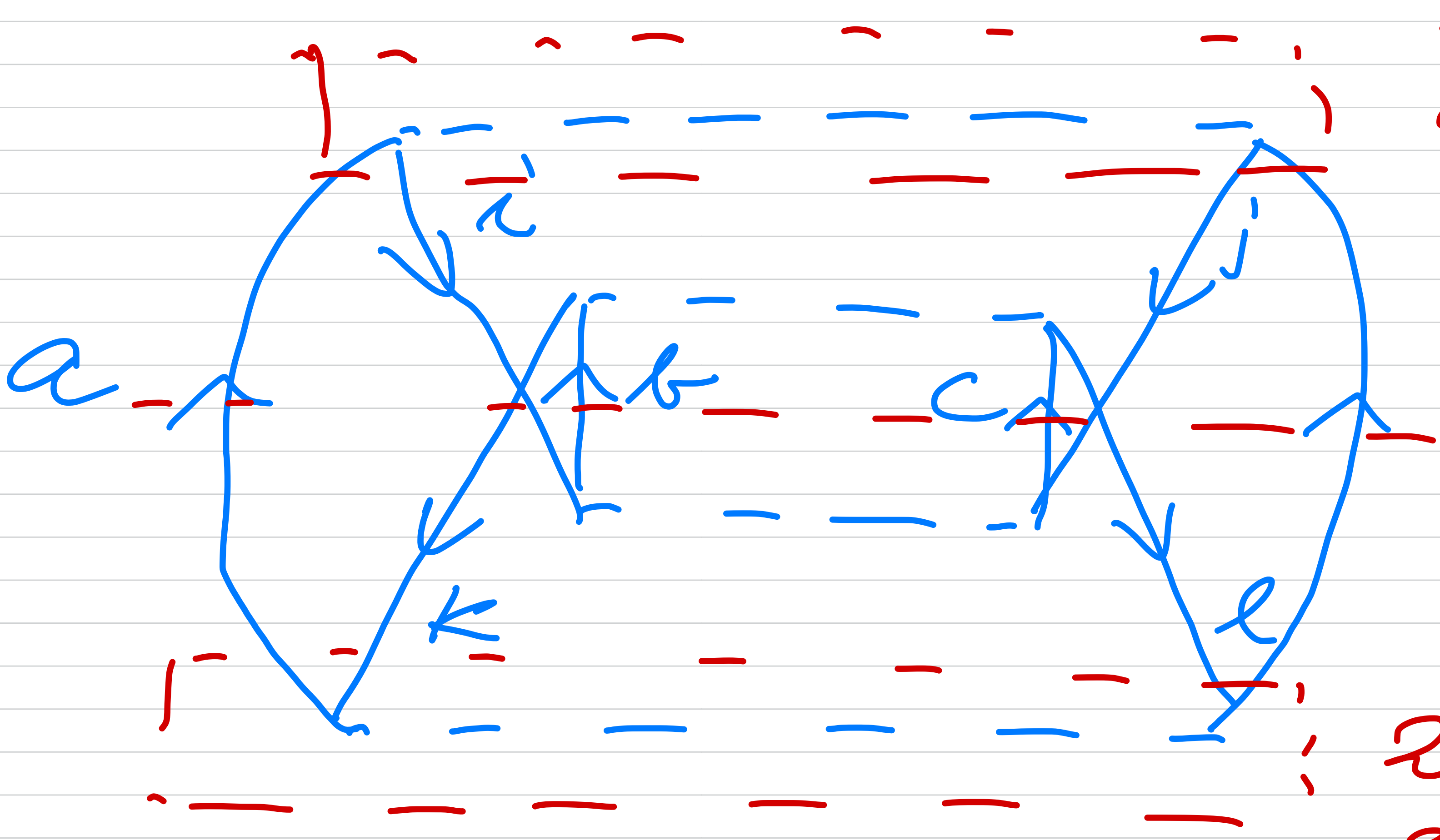
$n_{ep} = 0$

energy denominator

$$\frac{1}{(\varepsilon_i+\varepsilon_k-\varepsilon_a-\varepsilon_c)(\varepsilon_i+\varepsilon_j-\varepsilon_a-\varepsilon_b)}$$

$$\sum_{\substack{abc\\ ijk}} \frac{\langle ij|v|ab\rangle\langle bk|v|jc\rangle\langle ac|v|ik\rangle}{(\varepsilon_i+\varepsilon_k-\varepsilon_a-\varepsilon_c)(\varepsilon_i+\varepsilon_j-\varepsilon_a-\varepsilon_b)}$$

4th-order



2p2h deexcitation

4p4h

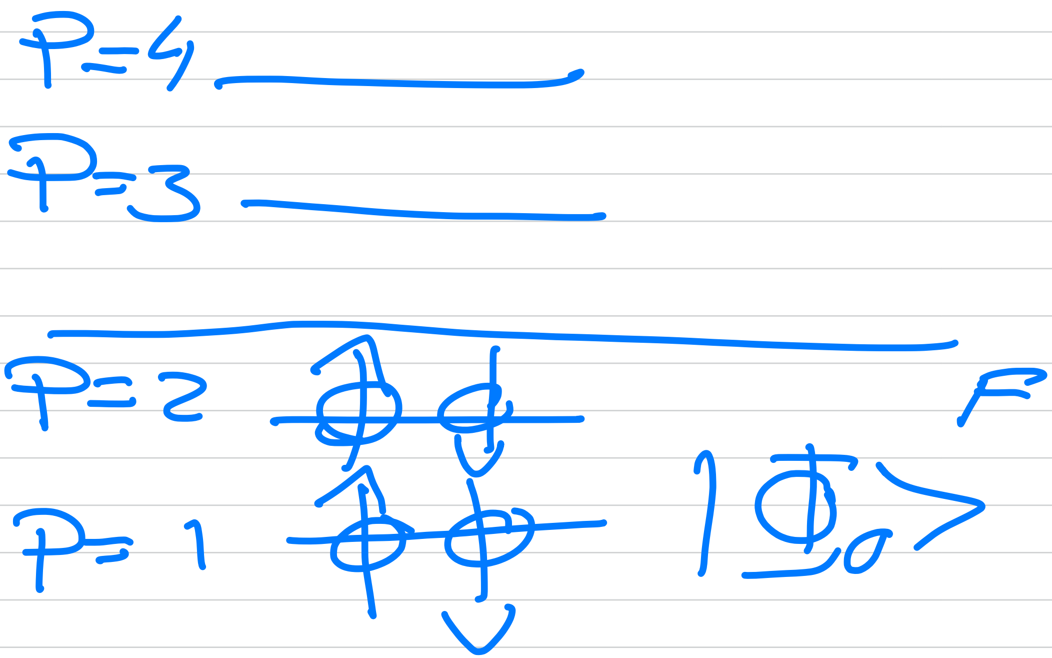
2p2h excitation

$\langle \Phi_{ij}^{ab} | \hat{H}_I | \Phi_0 \rangle$

$n_h = 4$

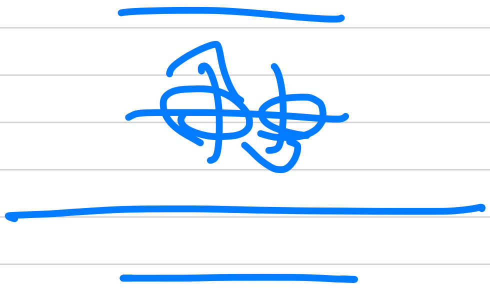
$n_l = 2$

$n_{ep} = 4 \Rightarrow \left(\frac{1}{2}\right)^4 = \frac{1}{16}$



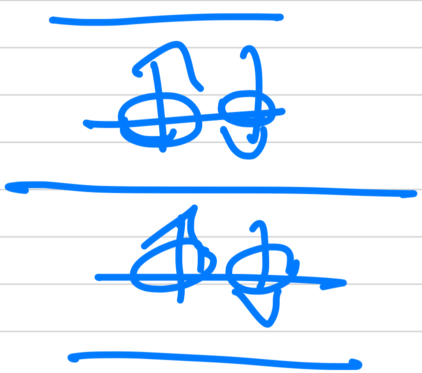
$|\Phi_0\rangle$

2p2h



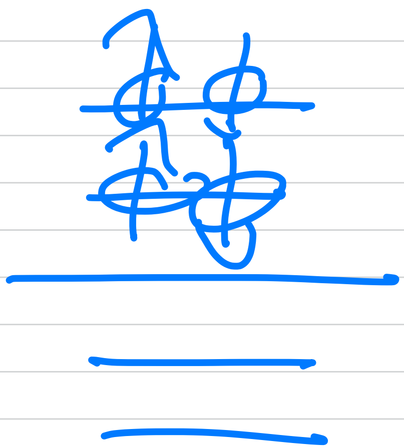
$|\Phi_1\rangle$

2p2h



$|\Phi_2\rangle$ . . . ,

4p4h



$|\Phi_i\rangle$

$$+\frac{1}{16}\sum_{\substack{abcd\\ijkl}} \frac{\langle ij|v|ad\rangle\langle kl|v|bc\rangle \times \langle bc|v|ij\rangle\langle ad|v|kl\rangle}{(\varepsilon_i+\varepsilon_j-\varepsilon_a-\varepsilon_d)(\varepsilon_i+\varepsilon_j+\varepsilon_k+\varepsilon_l-\varepsilon_a-\varepsilon_b-\varepsilon_c-\varepsilon_d)} \times \frac{1}{\varepsilon_k+\varepsilon_l-\varepsilon_a-\varepsilon_d}$$

Examples