

# FYS4480/9480, lecture

## September 26

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$$(\mathcal{H} - E_0) \sum_{pH} c_H^p |\Phi_H^p\rangle$$

$$|\Phi_0^0\rangle = |\Phi_0\rangle$$

open

$$\langle \Phi_0 | (\mathcal{H} - E_0) | (1 + \sum_{a \neq i} c_a^a |\Phi_a^a\rangle + \sum_{a,b} c_{ij}^{ab} |\Phi_{ij}^{ab}\rangle + \dots) | \Phi_0 \rangle$$

1p1h

$$\Delta E_0 = E_0 - E_0^{\text{REF}} = \sum_{a \neq i} \langle i | f | a \rangle c_a^a + \sum_{i,j} \langle ij | v | ab \rangle c_{ij}^{ab}$$

# Hamiltonian matrix

	$0p_{0L}$	$1p_{1L}$	$2p_{2L}$	$3p_{3L}$	-	-	$Np_{NL}$
$0p_{0L}$	$E_0^{ref}$	$\langle \Phi_0   H   \Phi_0 \rangle$	$\langle \Phi_0   H   \Phi_0 \rangle$	$\langle \Phi_0   H   \Phi_0 \rangle$	0	0	0
$1p_{1L}$		X	X	X	0	-	-
$2p_{2L}$		X	X	X			
$3p_{3L}$	0	X	X	X			
1	0	0	X	X		X	
1	1	0	0	X			
1	1	1	1	0			
$Np_{NL}$	0	0	0	1	0		

$$\langle \Phi_0 | \hat{H} | \Phi_i^a \rangle = \langle i | \hat{f} | a \rangle = \langle i | \hat{h}_0 | a \rangle + \sum_{J \leq F} \langle ij | \hat{v} | aJ \rangle_{AS}$$

Hartree-Fock algorithm

$$\langle i | \hat{f} | a \rangle = 0$$

$$\left. \begin{array}{l} \hat{h}_0 | q \rangle \\ = \epsilon_q | q \rangle \end{array} \right| \begin{array}{l} \langle p | \hat{f} | q \rangle = \delta_{pq} \epsilon_p^{HF} \\ \hat{f} | q \rangle \Rightarrow \hat{f} | q \rangle_{HF} = \epsilon_q^{HF} | q \rangle_{HF} \\ \langle i | \hat{f} | i \rangle = \epsilon_i^{HF} \\ \langle a | \hat{f} | a \rangle = \epsilon_a^{HF} \end{array}$$

$$Hc = \lambda c$$

$$u u^+ = u^+ u = \underline{1} \quad u^+ = u^{-1}$$

$$u H c = \lambda u c$$

$$\uparrow$$

$$\underline{1}$$

$$\underbrace{u H u^+}_D \underbrace{u c}_y = \lambda \underbrace{u c}_y$$

$$D = [\lambda_1, \lambda_2, \dots, \lambda_n]$$

$$U = U_p U_{p-1} \dots U_1$$

$$U_p U_{p-1} \dots U_1 \text{ or } U_1^+ \dots U_p^+$$

$$U, \text{ or } U^+$$

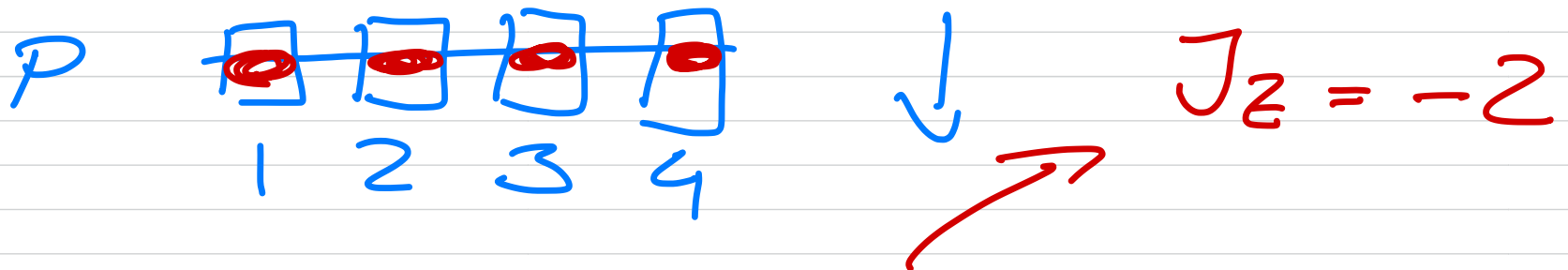
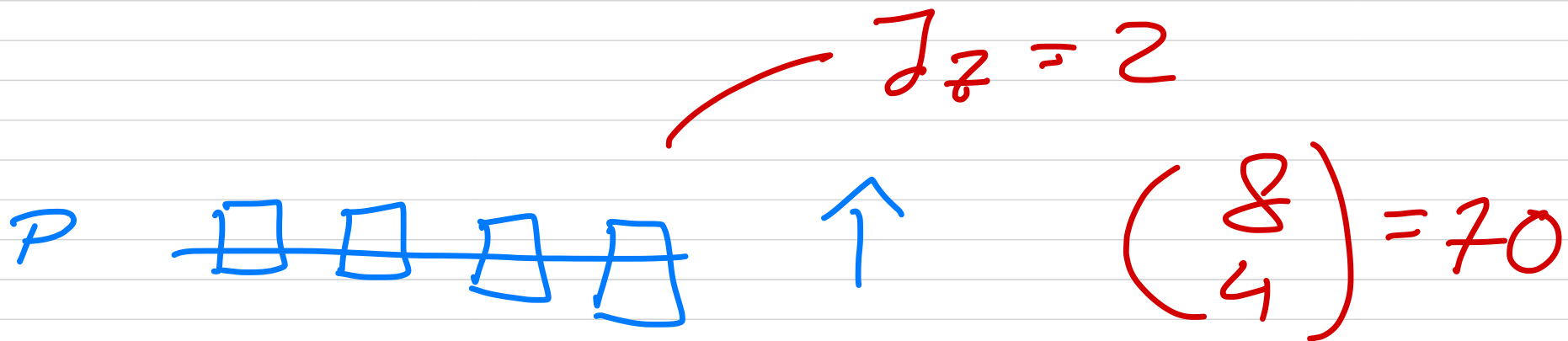
$$\begin{bmatrix} \begin{matrix} \times & \times & \times & 0 & 0 & 0 & 0 \\ \times & \times & \times & - & - & - & - \\ \times & \times & \times & - & - & - & - \\ 0 & \times & \times & - & - & - & - \\ 0 & - & \times & - & - & - & - \\ 0 & - & - & - & - & - & - \end{matrix} & \begin{matrix} \text{IP} \\ \text{IP} \\ \text{IP} \\ \text{IP} \\ \text{IP} \\ \text{IP} \end{matrix} & \begin{matrix} \times & \times & \times & 0 & 0 & 0 & 0 \\ \times & \times & \times & - & - & - & - \\ \times & \times & \times & - & - & - & - \\ \times & \times & \times & - & - & - & - \\ \times & \times & \times & - & - & - & - \\ \times & \times & \times & - & - & - & - \end{matrix} \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} x_2 & 0 & x_2 & 0 & 0 & - & - & 0 \\ 0 & x_2 & x_2 & x_2 & 0 & - & - & 1 \\ 0 & x_2 & , & , & 0 & - & - & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix}$$

$$\langle \Phi_0 | H | \Phi_n^a \rangle = \langle i | f | a \rangle$$

$$= 0$$

# Lipkin-model



$$H_0 |\Phi_0\rangle = \varepsilon_0 |\Phi_0\rangle$$

$$J = 2$$

$$J_z = -2, -1, 0, +1, +2$$



Diagrammatic representation

$$\Delta E_0 = E_0 - E_0^{\text{Ref}}$$

$$= \sum_{ai} \langle i | \hat{f} | a \rangle C_i^a + \sum_{ab} \sum_{ij'} \langle ij' | v | ab \rangle \times C_{ij}^{ab}$$

$$E_0^{\text{Ref}} = \sum_{i \in F} \langle i | \vec{h}_0 | i \rangle$$

$$+ \frac{1}{2} \sum_{i'j' \in F} \left\{ \langle i'j' | v | i'j' \rangle - \langle i'j' | v | j'i' \rangle \right\}$$

$$\langle i | \vec{h}_0 | i \rangle \rightarrow \begin{array}{c} \text{---} i' \\ \text{---} i' \\ \text{---} i' \end{array} \quad \boxed{h_0}$$

Now, hole states are written as downgoing arrows

$$\left( \begin{array}{c} i' \\ \downarrow \\ i' \end{array} \rightarrow \begin{array}{c} i' \\ \uparrow \\ i' \end{array} \right)$$



$$\sum_{i \leq R} \langle i | h_0^2 | i \rangle \rightarrow \sum_{i' \leq F} \langle i' | h_0^2 | i' \rangle = \text{Diagram of a circle with an arrow labeled } i' \text{ and a dashed box}$$

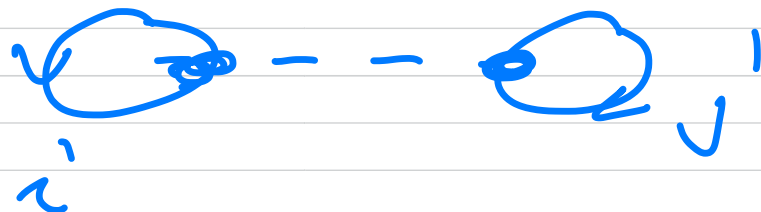
$n_h + n_e$  —  $n_e = \text{number of closed loops}$   
 (—)  $\downarrow$   
 $n_h = \text{number of holes}$

$$\sum_{i'j' \in F} \langle i'j' | \psi | i'j \rangle \rightarrow$$

$$\sum_{i'j' \in F} \begin{array}{c} \begin{array}{c} \nearrow i' \\ \nwarrow i' \end{array} \text{---} \begin{array}{c} \nwarrow j' \\ \nearrow j' \end{array} \end{array} \xrightarrow{\text{Ph-formulas}}$$

$$\sum_{i'j' \in F} \begin{array}{c} \begin{array}{c} \nwarrow i' \\ \nearrow i' \end{array} \text{---} \begin{array}{c} \nearrow j \\ \nwarrow j \end{array} \end{array}$$

=



$$n_h = 2$$

$$n_e = 2$$

$$E_0^{Ref} = \text{[Diagram: A circle with a self-loop labeled } i' \text{ and a dashed line to a crossed-out square.]}$$

$$+ \text{[Diagram: Two circles connected by a dashed line. The left circle has a self-loop labeled } i' \text{ and the right circle has a self-loop labeled } j'. \text{]}$$

$$+ \text{[Diagram: An oval with a dashed horizontal line. The top arc has an arrow pointing right labeled } i' \text{ and the bottom arc has an arrow pointing left labeled } j'. \text{]}$$

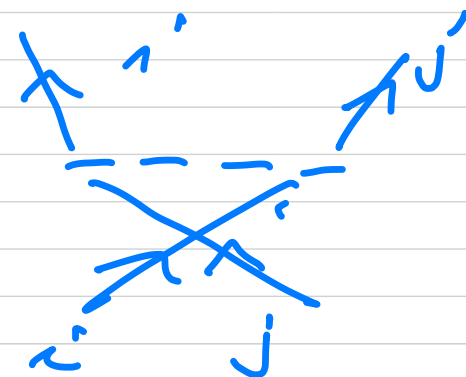
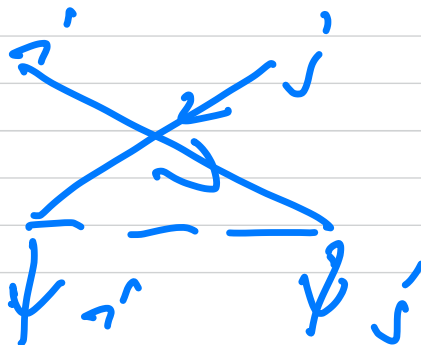
$$n_h = 2$$

$$n_e = \underline{1}$$

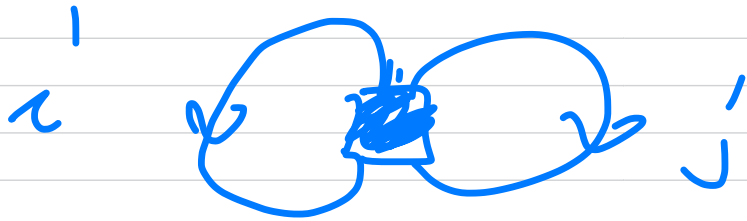
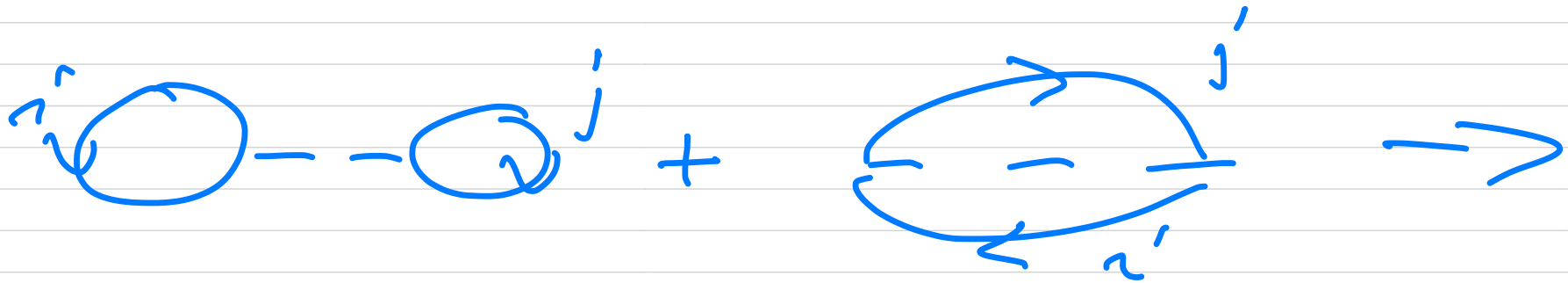
$$- \sum_{i'j' \leq F} \langle i'j' / \sigma(i'i) \rangle \Rightarrow \sum_{i'j' \leq F}$$

PH  
→

$$\sum_{i'j' \leq F}$$



Hagenholz (antisym)

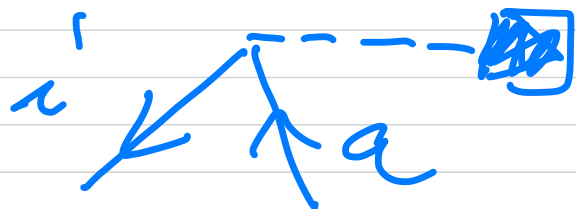


$$\langle i | \hat{f} | a \rangle = \langle \Phi_i | \hat{f} | \Phi_a \rangle$$

$$= \langle i | \hat{h}_0 | a \rangle + \sum_{J \neq i} \langle i | \hat{v} | a \rangle_{AJ}$$

$$\langle i | \hat{v} | a \rangle = \langle i | \hat{v} | a \rangle$$

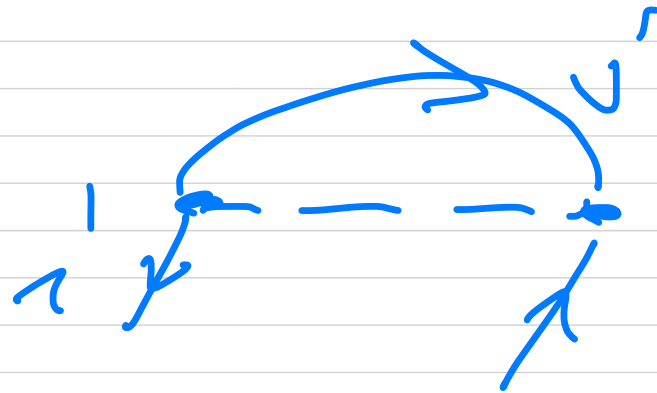
$$\hat{v} = \hat{v}$$



$$\sum_{J \neq i} \langle i | \hat{v} | a \rangle_{AJ} = \langle i | \hat{v} | a \rangle_{AJ}$$

$$- \sum_{j \in F} \langle ij | v | ja \rangle$$

$$\rightarrow \sum_{j \in F} \begin{array}{c} \downarrow i \\ \nearrow j' \\ \text{---} a \text{---} \nearrow j' \\ \downarrow j' \end{array}$$



$$\langle \Phi_0 | H | \Phi_{ij}^{ab} \rangle = \langle ij | v | ab \rangle$$

$$2p2h \quad \begin{array}{c} i' \nearrow \text{---} \text{---} \nearrow j' \\ \text{---} a \text{---} \text{---} \text{---} b \end{array} + \begin{array}{c} i' \nearrow \text{---} \text{---} \nearrow j' \\ \text{---} a \text{---} \text{---} \text{---} b \end{array}$$



$$\langle \Phi_{ij}^a | \mathcal{H} | \Phi_c \rangle = \langle ab | v | ij \rangle$$

