Lecture Fys4480/9480, November 17, 2023

Rayleigh-Schrödinger

$$\Delta E = \sum_{n=0}^{\infty} \langle \phi_1 | H_1 \left\{ \frac{Q}{E_1 - H_0} (H_1 - \Delta E) \right\}^n | \phi_1 \rangle$$

$$\Delta E^{(4)} = \langle \phi_1 | H_1 | \phi_1 \rangle + \langle \phi_1 | H_1 \frac{Q}{E_1 - H_0} H_1 | \phi_1 \rangle$$

$$+ \langle \phi_1 | H_1 \frac{Q}{E_1 - H_0} H_1 \frac{Q}{E_1 - H_0} H_1 | \phi_1 \rangle$$

$$- \langle \phi_1 | H_1 | \phi_1 \rangle \times \langle \phi_1 | H_1 \frac{Q}{(E_1 - H_0)^2} H_1 | \phi_1 \rangle$$

$$+$$

$$\langle H_I \rangle = \langle \phi_1 | H_1 | \phi_1 \rangle = 0$$

$$+ \left\langle H_I \frac{Q}{\varepsilon_1 - H_0} H_1 \frac{Q}{\varepsilon_1 - H_0} H_1 \frac{Q}{\varepsilon_1 - H_0} H_1 \right\rangle$$

$$- \left\langle H_1 \frac{Q}{\varepsilon_1 - H_0} \langle H_1 \rangle \frac{Q}{\varepsilon_1 - H_0} H_1 \frac{Q}{\varepsilon_1 - H_0} H_1 \right\rangle$$

$$- \left\langle H_1 \frac{Q}{\varepsilon_1 - H_0} H_1 \frac{Q}{\varepsilon_1 - H_0} \langle H_1 \rangle \frac{Q}{\varepsilon_1 - H_0} H_1 \right\rangle$$

$$+ \left\langle H_1 \frac{Q}{\varepsilon_1 - H_0} \langle H_1 \rangle \frac{Q}{\varepsilon_1 - H_0} \langle H_1 \rangle \frac{Q}{\varepsilon_1 - H_0} H_1 \right\rangle$$

$$- \left\langle H_1 \frac{Q}{\varepsilon_1 - H_0} \left\langle H_1 \frac{Q}{\varepsilon_1 - H_0} H_1 \right\rangle \frac{Q}{\varepsilon_1 - H_0} H_1 \right\rangle$$

$$\langle \Phi_1 | H_I \frac{Q}{\varepsilon_1 - H_0} H_1 | \Phi_1 \rangle$$

$$= \sum_M \frac{\langle \Phi_1 | H_1 | \Phi_M \rangle \langle \Phi_M | H_1 | \Phi_1 \rangle}{\varepsilon_1 - \varepsilon_M}$$

$$\left\{ \begin{array}{l} M = \text{second state} \\ |\Phi_M\rangle = |\Phi_2\rangle \\ \langle \Phi_1 | H_I | \Phi_2 \rangle = \lambda \end{array} \right.$$

$$= \frac{\lambda^2}{\varepsilon_1 - \varepsilon_2}$$

$$\langle \Phi_1 | H_1 \frac{G}{\varepsilon_1 - H_0} H_1 \frac{G}{\varepsilon_1 - H_0} H_1 | \Phi_1 \rangle$$

$$G = \sum_M |\Phi_M\rangle\langle\Phi_M| = |\Phi_2\rangle\langle\Phi_2|$$

$$= \frac{\langle \Phi_1 | H_I | \Phi_2\rangle\langle\Phi_2| H_I |\Phi_2\rangle\langle\Phi_2|H_1|\Phi_1\rangle}{(\varepsilon_1 - \varepsilon_2)^2}$$

$$- \frac{\langle \Phi_1 | H_1 | \Phi_2\rangle\langle\Phi_2| H_1 |\Phi_1\rangle}{(\varepsilon_1 - \varepsilon_2)^2} \times$$

$$\times \frac{\langle \Phi_1 | H_I | \Phi_2 \rangle \langle \Phi_2 | H_I | \Phi_1 \rangle}{\varepsilon_1 - \varepsilon_2}$$

$$= - \frac{\lambda^4}{(\varepsilon_1 - \varepsilon_2)^3} \qquad \Bigg| \qquad \frac{\lambda}{\varepsilon_1 - \varepsilon_2} < 1$$

From Taylor expansion of

$$E_1 = \varepsilon_1 + \frac{\lambda^2}{\varepsilon_1 - \varepsilon_2} - \frac{\lambda^4}{(\varepsilon_1 - \varepsilon_2)^3} + \dots$$

(also exact BW result)

BW

$$E_1 = \varepsilon_1 + \frac{\lambda^2}{E_1 - \varepsilon_2} \Rightarrow$$

$$(E_1 - \varepsilon_1)(E_1 - \varepsilon_2) - \lambda^2 = 0$$

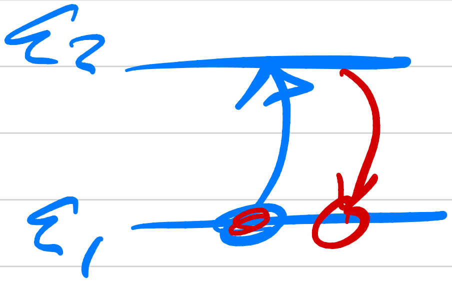
$$\frac{\lambda^2}{\varepsilon_1 - \varepsilon_2} = \frac{\langle \Phi_1 | H_1 | \Phi_2 \rangle \langle \Phi_2 | H_1 | \Phi_1 \rangle}{\varepsilon_1 - \varepsilon_2}$$

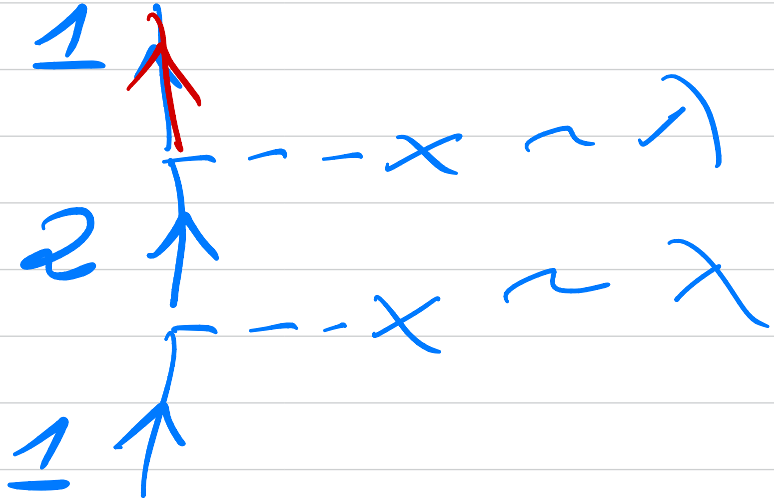
all single-particle states

$|\Phi_1\rangle = a_1^+|0\rangle$ and $|\Phi_2\rangle = a_2^+|0\rangle$

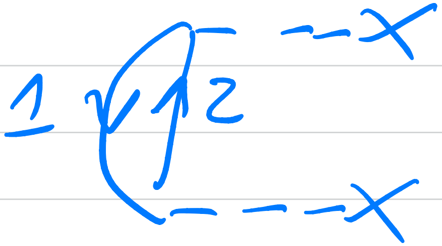
defined wrt $|0\rangle$

$$\frac{\langle 0|a_1 H_1 a_2^\dagger|0\rangle\langle 0|a_2 H_1|a_1^\dagger|0\rangle}{\varepsilon_1-\varepsilon_2}$$





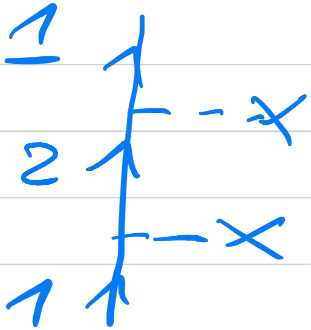
with new reference vacuum $|\Phi_1\rangle$, then 1 is hole state

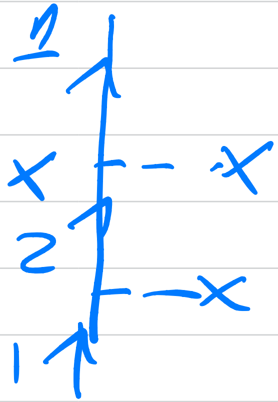


Fourth-order term in particle basis first

$$\frac{\langle \Phi_1 | H_I | \Phi_2 \rangle \langle \Phi_2 | H_1 | \Phi_1 \rangle}{(\varepsilon_1 - \varepsilon_2)^2}$$

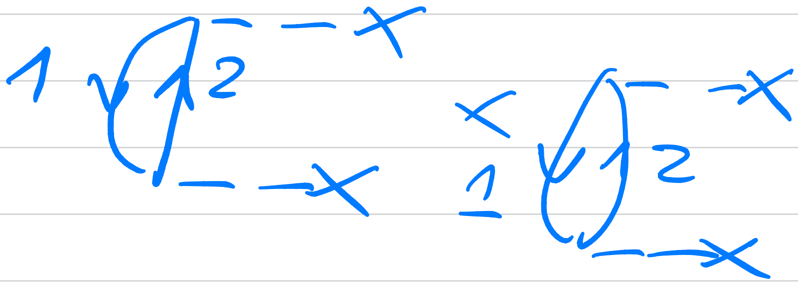
$$\times \quad \frac{\langle \Phi_1 | H_1 | \Phi_2 \rangle \langle \Phi_2 | H_1 | \Phi_1 \rangle}{(\varepsilon_1 - \varepsilon_2)}$$





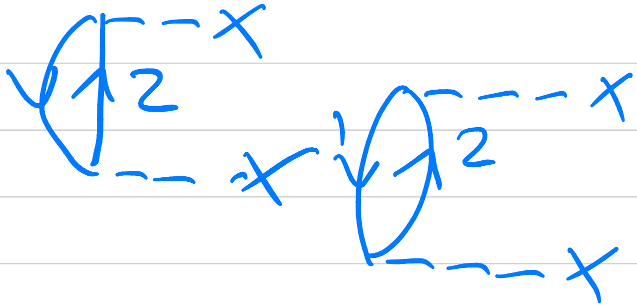
unlinked diagram

in particle-hole formalism



$n_e = 1$

$n_h = 2$



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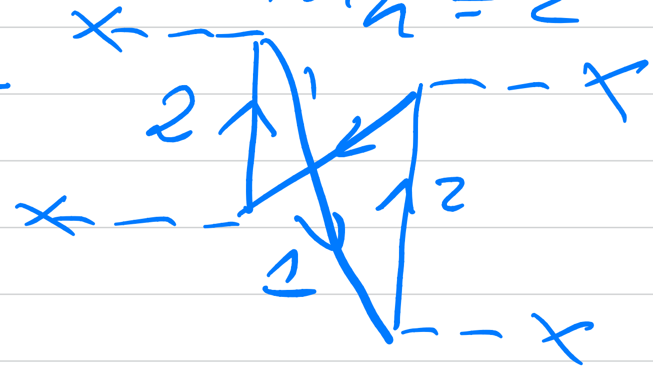
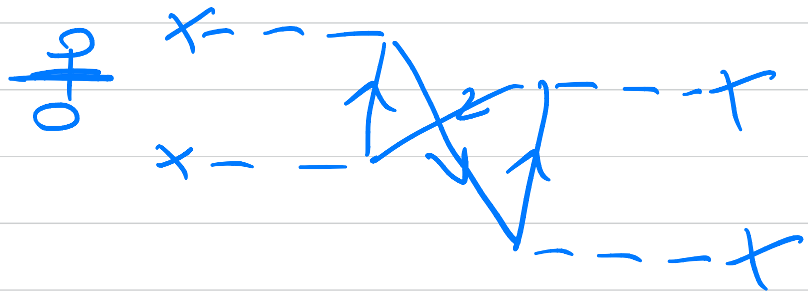


$n_h = 2$

$n_e = 2$



$$H|\psi\rangle = \lambda|\psi\rangle \qquad U = e^{-i t/\hbar \hat{H}_0}$$

$$U^\dagger U = U U^\dagger = \mathbb{1}$$

$$U H U^\dagger = D$$

$$U \underset{\mathbb{1}}{H} |\psi\rangle = \lambda U|\psi\rangle$$

$$\underbrace{U H U^\dagger}\{U|\psi\rangle\} = \underline{\lambda}\,\underbrace{\{U|\psi\rangle\}}_{|\varphi\rangle}$$

$|\varphi\rangle$

is conserved