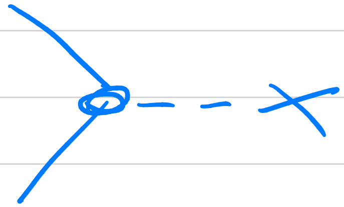
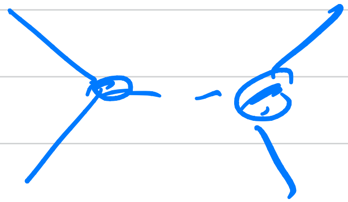
# Lecture FYS4480, September 15, 2023

# Diagrams

operator

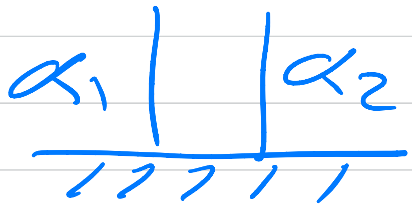


1 Body



2 Body

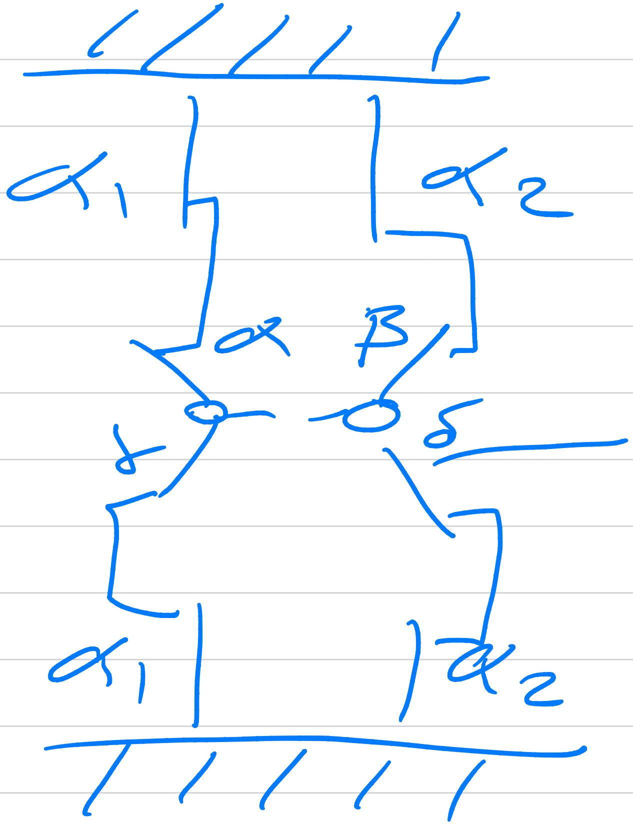
state $|\alpha_1, \alpha_2\rangle$




$|0\rangle$

$$\langle \Phi_c | H_1 | \Phi_d \rangle = \langle \alpha_1, \alpha_2 | H_1 | \alpha_1, \alpha_2 \rangle$$

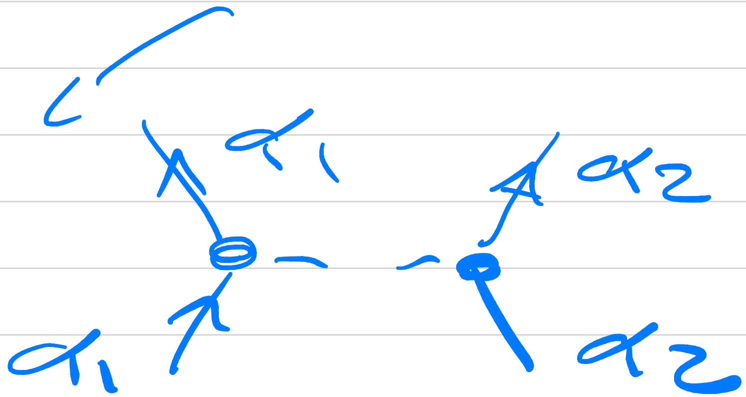
$\langle \alpha_1, \alpha_2 |$




$$a_\delta a_\gamma a^\dagger_{\alpha_1} a^\dagger_{\alpha_2}$$

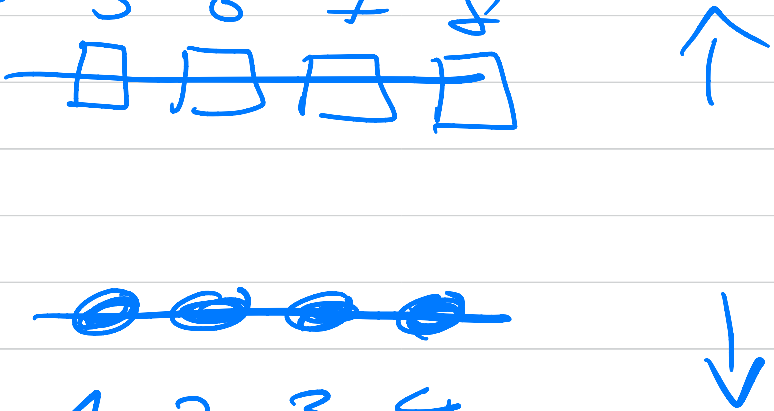
$|\alpha_1, \alpha_2\rangle$

$$\langle \alpha_1, \alpha_2 | v | \alpha_1 \alpha_2 \rangle$$



# Example: Lipkin modell

$p = 5 \quad 6 \quad 7 \quad 8$



$N = 4$

$p = 1 \quad 2 \quad 3 \quad 4$

$$|\Phi_0\rangle = a_1^+ a_2^+ a_3^+ a_4^+ |0\rangle$$

$$= |1234\rangle$$

$$\langle \Phi_0 | H_I | \Phi_0 \rangle$$

$(1,2)$, $(1,3)$, $(1,4)$
$(2,3)$, $(2,4)$, $(3,4)$

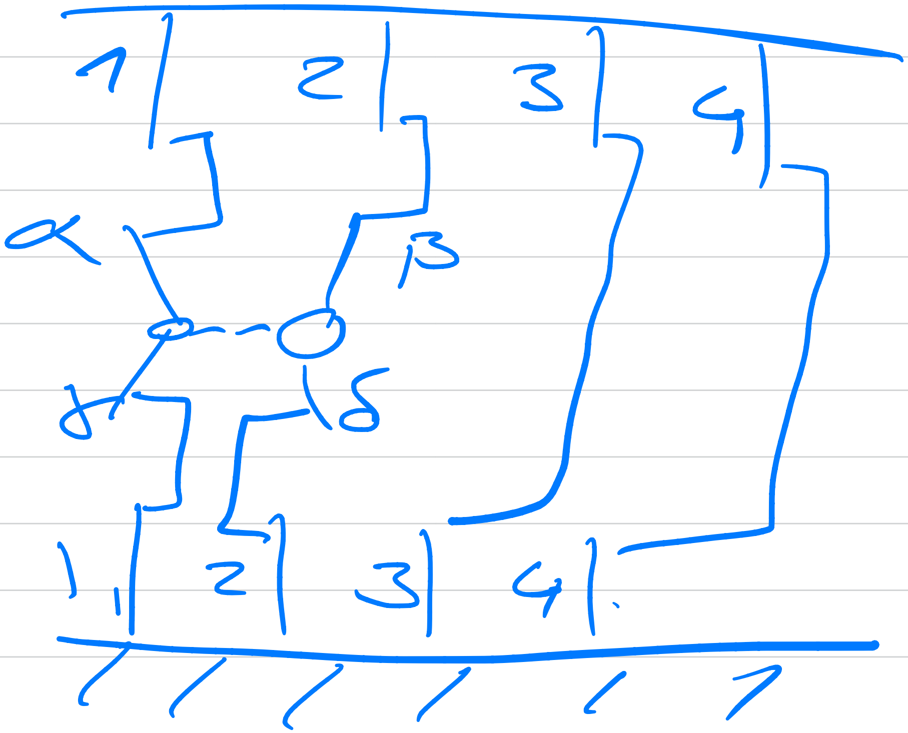
$$a_4 a_3 a_2 a_1 \; a_\alpha^+ a_\beta^+ a_\delta a_\gamma \; a_1^+ a_2^+ a_3^+ a_4^+$$

$$a_4 a_3 a_2 a_1 a_\alpha^\dagger a_\beta^\dagger a_\delta a_\gamma a_1^\dagger a_2^\dagger a_3^\dagger a_4^\dagger$$

$(1,2)$

$$\delta_{\gamma 1}\;\delta_{\alpha 1}\;\delta_{\delta 2}\;\delta_{\beta 2}\;\delta_{33}\;\delta_{44}$$

$$\langle 12|v|12\rangle$$



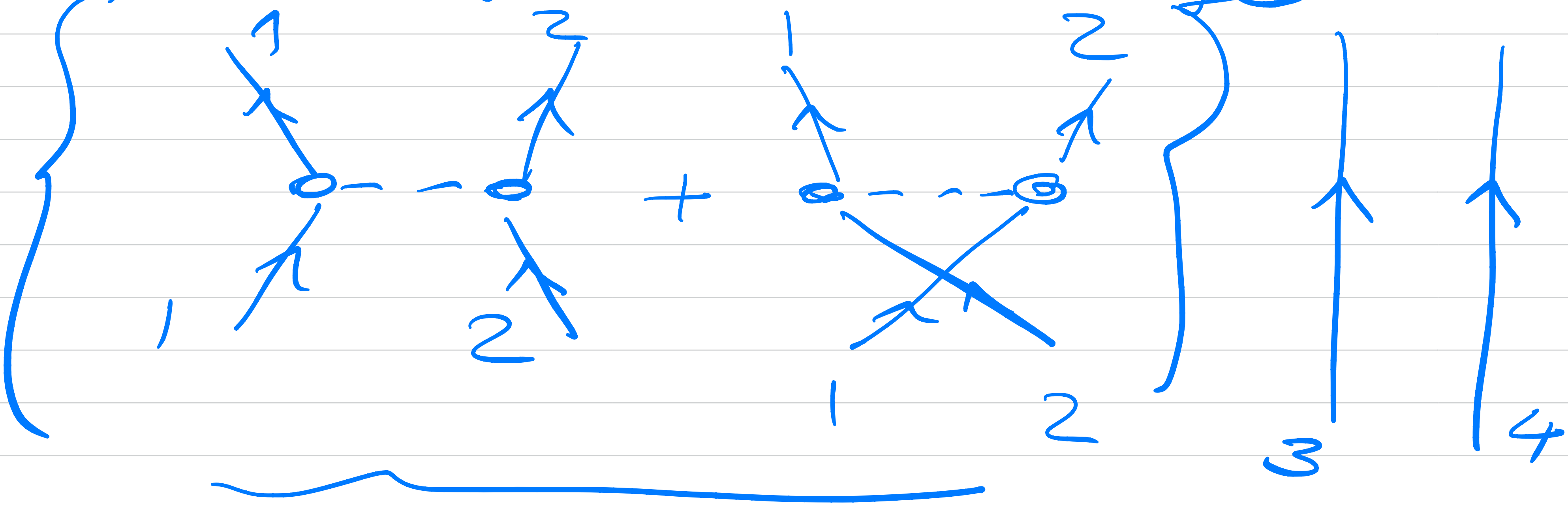

$(1\,2)$

$$\langle 12|v|12\rangle$$

$$-\langle 12|v|21\rangle$$

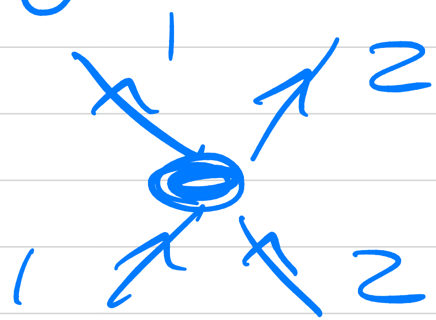
$$=\langle 12|v|12\rangle_{AS}$$

Direct exchange




Feynman-Goldstone

Hugenholtz diagram




$\langle 12 | v | 12 \rangle_{AS}$

$(1,3)$

$$\left(\langle 13|v|13\rangle - \langle 13|v|31\rangle\right)\delta_{22}\delta_{44}$$

$(1,4)$

$$\left(\langle 14|v|14\rangle - \langle 14|v|41\rangle\right)\delta_{22}$$
$$\times\,\delta_{33}$$

$(2,3)$

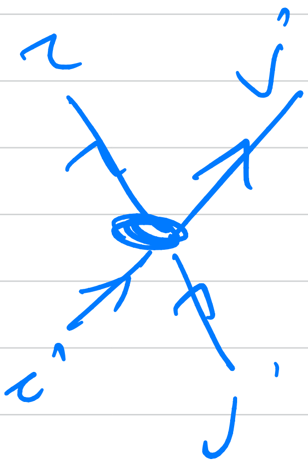
$$\left(\langle 23|v|23\rangle - \langle 23|v|32\rangle\right)\delta_{11}$$
$$\delta_{44}$$

$(2,4) \;+\; (3,4)$

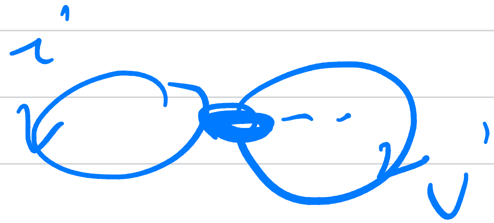
$$\langle \Phi_0 | H_1 | \Phi_0 \rangle = \sum_{i=1}^{4} \sum_{j=1}^{4} \langle ij | v | ij \rangle_{AS}$$

$(i<j)$

$$= \sum_{i<j}$$




$=$




$i, j \le F$

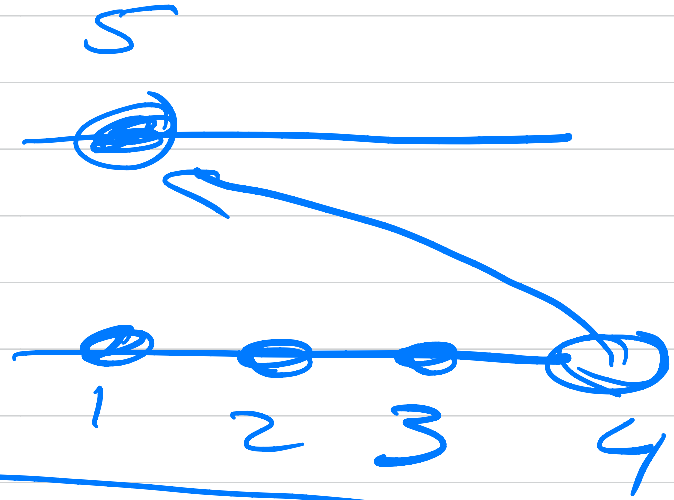
instance of the Condon-Slater rule

$$\langle \Phi_1 | H_1 | \Phi_0 \rangle = ?$$

$$|\Phi_1\rangle = a_1^\dagger a_2^\dagger a_3^\dagger a_5^\dagger |0\rangle$$

$$|\Phi_0\rangle = a_1^\dagger a_2^\dagger a_3^\dagger a_4^\dagger |0\rangle$$

$|\Phi_1\rangle =$




$$a_5 a_3 a_2 a_1 a_\alpha^\dagger a_\beta^\dagger a_\delta a_\gamma a_1^\dagger a_2^\dagger a_3^\dagger a_4^\dagger$$

$(1,2) \to (1,2) = 0$ $\qquad \delta_{45}$

$(1,3) \to (1,3) = 0$ $\qquad \delta_{45}$

$(1,4) = \langle 15|v|14\rangle - \langle 15|v|41\rangle$

$$a_5 a_3 a_2 a_1 a_4^\dagger a_5^\dagger a_5 a_4 a_1^\dagger a_2^\dagger a_3^\dagger a_4^\dagger$$

$$(2,3) \to (2,3) = 0$$

$$(2,4) \to (2,5)$$

$$\langle 25|v|24\rangle - \langle 25|v|42\rangle$$

$$(3,4) \to (3,5)$$

$$\langle 35|v|34\rangle - \langle 35|v|43\rangle$$

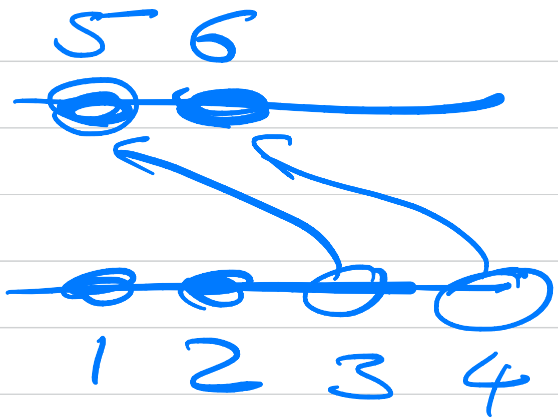
replace 4 and 5 with labels k and e

$$\langle \Phi_1 | H_1 | \Phi_0 \rangle = \sum_{i \neq \{ke\}} \langle ie|v|ik\rangle_{AS}$$

$$|\phi_2\rangle = a_1^+ a_2^+ a_5^+ a_6^+ |0\rangle$$

$|\Phi_0\rangle$ as before

$$|\phi_{12}\rangle =$$





$$\langle\phi_2|H_1|\Phi_0\rangle = \langle 56|v|34\rangle_{AS} \times \delta_{11}\,\delta_{22}$$

2 different states

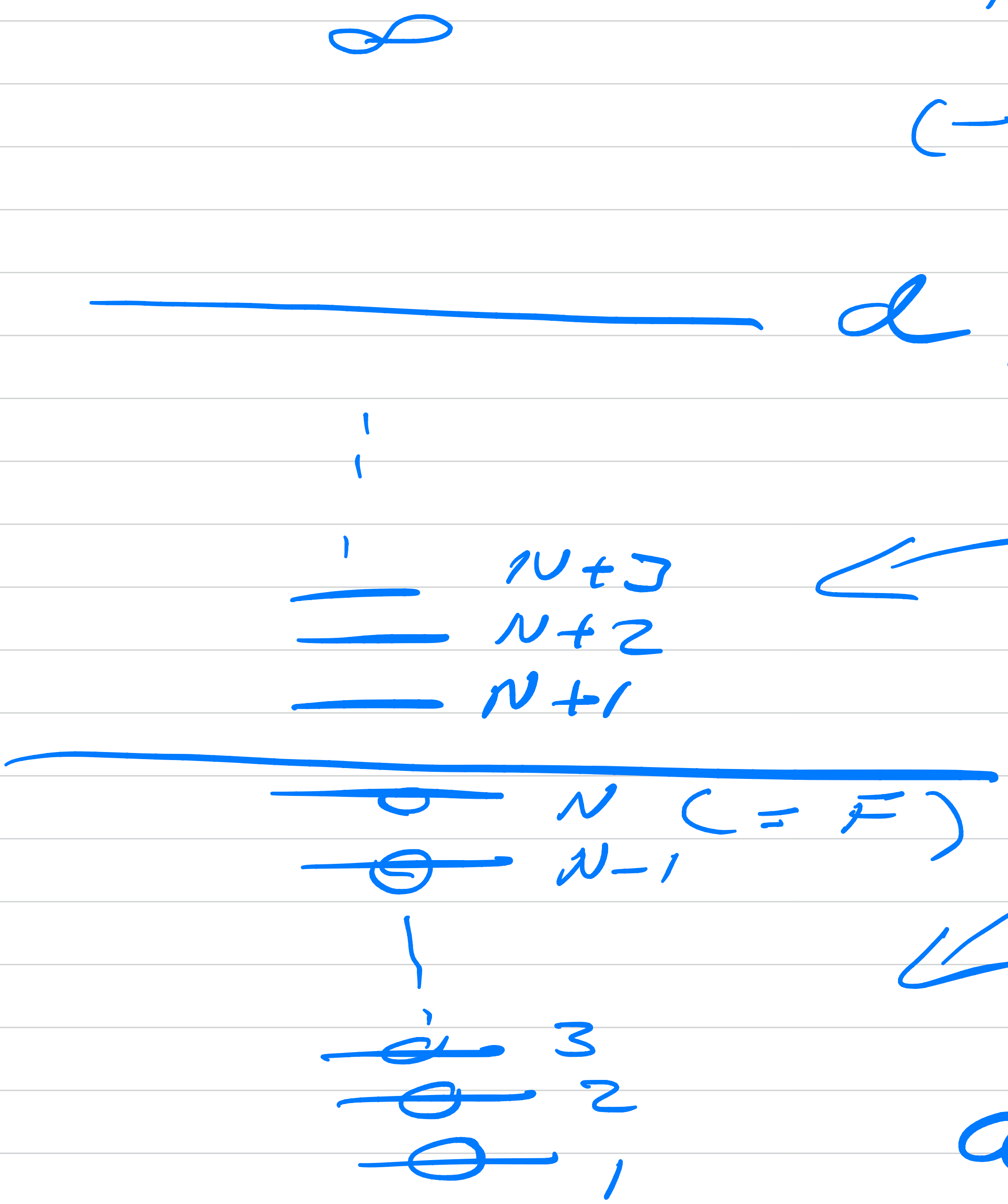
$$|\Phi(kl)\rangle \qquad |\Phi(ij)\rangle$$

$$\langle \Phi(kl) | H_I | \Phi(ij) \rangle$$

$$= \langle kl | v | ij \rangle_A$$

Particle-hole formalism to simplify.

$$|\Phi_c\rangle = a_1^\dagger a_2^\dagger a_3^\dagger \cdots a_N^\dagger |0\rangle$$




d/u

$$a^{+}_{N+1}\,\underbrace{|123\ldots N\rangle}_{N \text{ state}}$$

$$(-)^{N}\,|123\ldots N\,N{+}1\rangle$$

N+1 state

particle states

F

hole states

$$|\Phi_0\rangle$$

$$a_\alpha\,|123\ldots N\rangle$$

$$\alpha \in \{1,2,3,\ldots N\}$$

$$a_\alpha \,|123\ldots\alpha-1\;\alpha\;\alpha+1\ldots N\rangle$$

$$= (-)^{\alpha-1}\,|123\ldots\alpha-1\;\alpha+1\ldots N\rangle$$

$N-1$ particle state,

$$|\Phi_c\rangle \;(= |c\rangle) = \prod_{i=1}^{N} a_i^{+}|0\rangle$$

$$\hat{H}_0|\Phi_c\rangle = E_0|\Phi_c\rangle$$

$$E_0 = \sum_{i=1}^{N} \varepsilon_i$$

$$|\Phi_c\rangle = \prod_{i\leq F} a_i^{+}|0\rangle$$

if $a_\alpha |\Phi_0\rangle = 0$ if $\alpha \notin \{1,2,3 \dots N\}$

F

else $a_\alpha |\Phi_0\rangle \neq 0$

New operator

$$b_\alpha \,,\; b_\alpha^{+}$$

$$\{b_\alpha, b_\beta\} = \{b_\alpha^{+}, b_\beta^{+}\} = 0$$

$$\overbrace{b_\alpha b_\beta^{+}} = \delta_{\alpha\beta} \qquad \overbrace{b_\alpha^{+} b_\beta} = 0$$

$$b_\alpha^+ |\Phi_0\rangle = \begin{cases} a_\alpha^+ |\Phi_0\rangle = |\alpha\rangle & \alpha > F \\ a_\alpha |\Phi_0\rangle = |\alpha^{-1}\rangle & \alpha \leq F \end{cases}$$

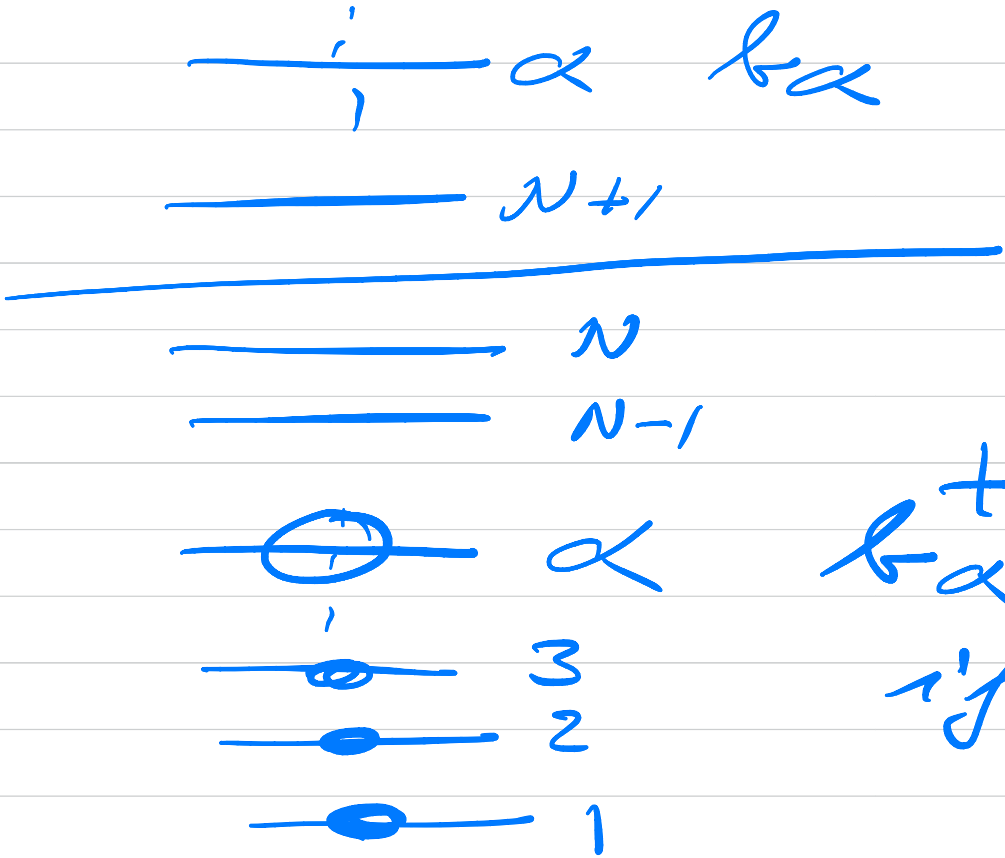
$$a_\alpha^+ |\Phi_0\rangle = |\alpha\, 1\, 2\, 3 \,\ldots\, N\rangle$$

$$= (-)^N |1\, 2\, 3 \,\ldots\, N\alpha\rangle$$

$$a_\alpha |\Phi_0\rangle = \underbrace{a_\alpha a_1^+ a_2^+ \ldots a_\alpha^+}\ldots a_N^+ |0\rangle$$

$$b_\alpha^+ = \begin{cases} a_\alpha^+ & \alpha > F \\ a_\alpha & \alpha \leq F \end{cases}$$

$$b_\alpha = \begin{cases} a_\alpha & \alpha > F \\ a_\alpha^\dagger & \alpha \le F \end{cases}$$




$b_\alpha$ destroys a particle in state $\alpha$ if $\alpha > F$

$b_\alpha^\dagger$ creates a hole if $\alpha \le F$ $\rightarrow$ $a_\alpha$

$$\hat{N} = \sum_{\alpha=1}^{\infty} a_\alpha^\dagger a_\alpha \simeq \sum_{\alpha=1}^{n} a_\alpha^\dagger a_\alpha$$

$$= \sum_{\alpha > F} a_\alpha^\dagger a_\alpha + \sum_{\alpha \le F} a_\alpha^\dagger a_\alpha$$

$$\sum_{\alpha=F+1}^{n}$$

$$a_p^\dagger a_q = \delta_{pq} \quad \text{if } p q \le F$$

$$a_i^\dagger a_j$$

$$= \sum_{\alpha > F} b_\alpha^\dagger b_\alpha + \sum_{\alpha \le F} b_\alpha b_\alpha^\dagger$$

$$b_\alpha b_\alpha^\dagger + b_\alpha^\dagger b_\alpha = \delta_{\alpha\alpha}$$

$$= \sum_{\alpha > F} b_\alpha^\dagger b_\alpha - \sum_{\alpha \le F} b_\alpha^\dagger b_\alpha$$

$$+ \underbrace{\sum_{\alpha \le F} \delta_{\alpha\alpha}}_{N} \qquad \sum_{\alpha \le F} = \sum_{\alpha = 1}^{N}$$

$$\langle \Phi_c | \hat{N} | \Phi_c \rangle = N \qquad \underbrace{\langle \Phi_c | \Phi_c \rangle}_{=1}$$

$$\langle \Phi_c | \overbrace{b_\alpha^\dagger b_\alpha}^{} | \Phi_0 \rangle = 0$$

$$N[a_\alpha^\dagger a_\alpha] = 0 = \langle c | a_\alpha^\dagger a_\alpha | c \rangle$$

Define new states

$$b_\alpha^\dagger b_\beta |\Phi_0\rangle$$

$\alpha > F \qquad \beta \leq F$


$$\binom{n}{N} = \frac{n!}{(n-N)!\,N!}$$

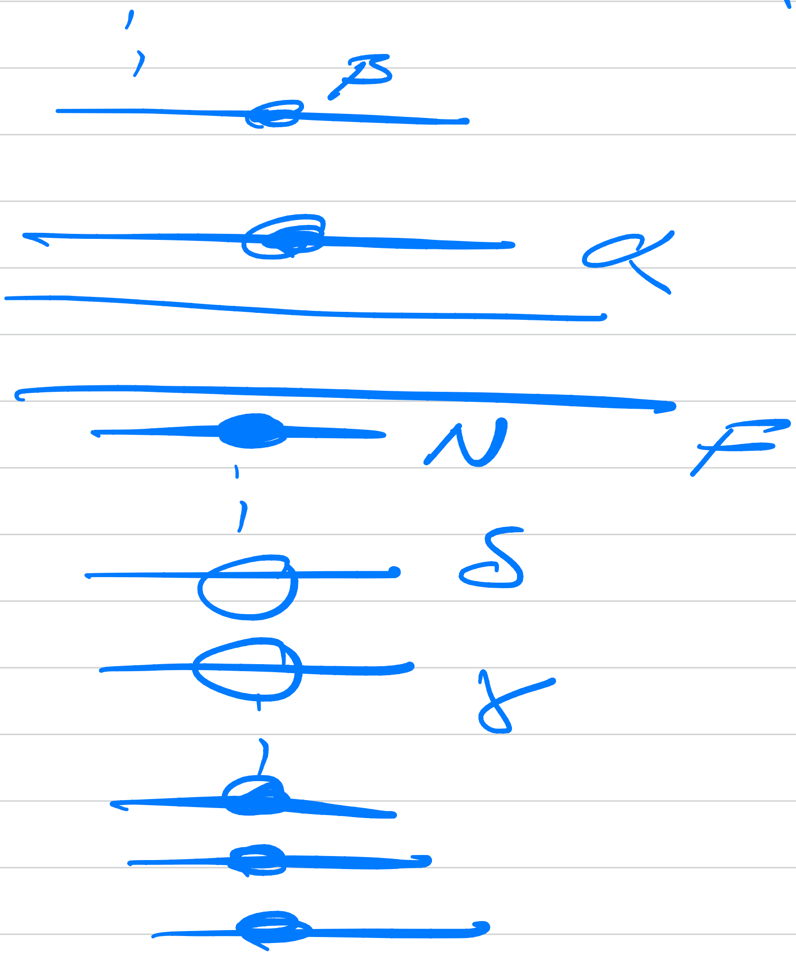
$n \geq N$

1p1h state

$$|\Phi_\beta^\alpha\rangle$$

$b^\dagger_\alpha b^\dagger_\beta b_\gamma b_\delta$

$\alpha, \beta > F \qquad \gamma, \delta \leq F$




2p2h

$|\Phi^{\alpha\beta}_{\gamma\delta}\rangle$

$|\Phi^{\alpha}\rangle = 1p$ state

$|\Phi_{\alpha}\rangle = 1h$

$$H_0 = \sum_{\alpha\beta} \langle\alpha|h_0|\beta\rangle a_\alpha^\dagger a_\beta$$

$$= \sum_{\alpha\beta > F} \langle\alpha|h_0|\beta\rangle b_\alpha^\dagger b_\beta$$

$$+ \sum_{\substack{\alpha > F \\ \beta \leq F}} \langle\alpha|h_0|\beta\rangle b_\alpha^\dagger b_\beta^\dagger$$

$$+ \sum_{\substack{\alpha \leq F \\ \beta > F}} \langle\alpha|h_0|\beta\rangle b_\alpha b_\beta$$

$$\alpha \leftrightarrow \beta$$

$$+ \sum_{\alpha\beta \leq F} \langle\alpha|h_0|\beta\rangle b_\alpha b_\beta^\dagger =$$

$$b_\alpha^+ b_\beta = b_\alpha b_\beta = b_\alpha^+ b_\beta^+ = 0$$

$$= \sum_{\alpha\beta > F} \langle \alpha | h_0 | \beta \rangle \, b_\alpha^+ b_\beta \qquad a_i a_j^+$$

$$+ \sum_{\substack{\alpha > F \\ \beta \le F}} \Big[ \langle \alpha | h_0 | \beta \rangle \, b_\alpha^+ b_\beta^+ + \langle \beta | h_0 | \alpha \rangle \, b_\beta b_\alpha \Big]$$

$$+ \sum_{\alpha \le F} \langle \alpha | h_0 | \alpha \rangle - \sum_{\alpha\beta \le} \langle \alpha | h_0 | \beta \rangle \, b_\beta^+ b_\alpha$$

$$b_\alpha b_\beta^+ = \delta_{\alpha\beta} - b_\beta^+ b_\alpha$$

$$\langle \Phi_0 | H_0 | \Phi_0 \rangle = \sum_{\alpha \le F} \langle \alpha | h_0 | \alpha \rangle = \sum_{\alpha \le F} \varepsilon_\alpha$$

$$\langle \Phi_\gamma^\delta | H_0 | \Phi_\gamma^\delta \rangle =$$

$$\varepsilon_\delta - \varepsilon_\gamma + \sum_{\alpha \le F} \varepsilon_\alpha$$

$$H_0 |\gamma\rangle = \varepsilon_\gamma |\gamma\rangle$$