Lecture FYS4480/9480, September 12, 2024

FYS4480/9480 September 12

Wick's theorem, with examples

$$\langle 0|xyz\cdots w|0\rangle = N[xyz\cdots w]$$

$$+ \sum_{(1)} N[xyz\cdots w]$$

$$+ \sum_{(2)} N[xyz\cdots w]$$

$$+ \cdots \sum_{\left(\frac{M}{2}\right)} N[xyz\cdots w]$$

$$\langle 0 | a_\alpha a_\beta^\dagger | 0 \rangle = \delta_{\alpha\beta} - \langle 0 | a_\beta^\dagger a_\alpha | 0 \rangle$$

$$= \overbracket{a_\alpha a_\beta^\dagger} + N[a_\alpha a_\beta^\dagger]$$

$$\overbracket{a_\alpha a_\beta^\dagger} = \delta_{\alpha\beta}$$

$$\overbracket{a_\beta^\dagger a_\alpha} = 0 = \overbracket{a_\alpha^\dagger a_\beta^\dagger} = \overbracket{a_\alpha a_\beta}$$

Energy

$$\hat{H}_0 = \sum_{\alpha\beta} \langle \alpha | h_0 | \beta \rangle a_\alpha^\dagger a_\beta$$

$$\hat{H}_I = \frac{1}{2} \sum_{\alpha\beta\gamma\delta} \langle \alpha\beta | v | \gamma\delta \rangle a_\alpha^\dagger a_\beta^\dagger a_\delta a_\gamma$$

$$= \frac{1}{4} \sum_{\alpha\beta\gamma\delta} \langle \alpha\beta | v | \gamma\delta \rangle_{AS} \times a_\alpha^\dagger a_\beta^\dagger a_\delta a_\gamma$$

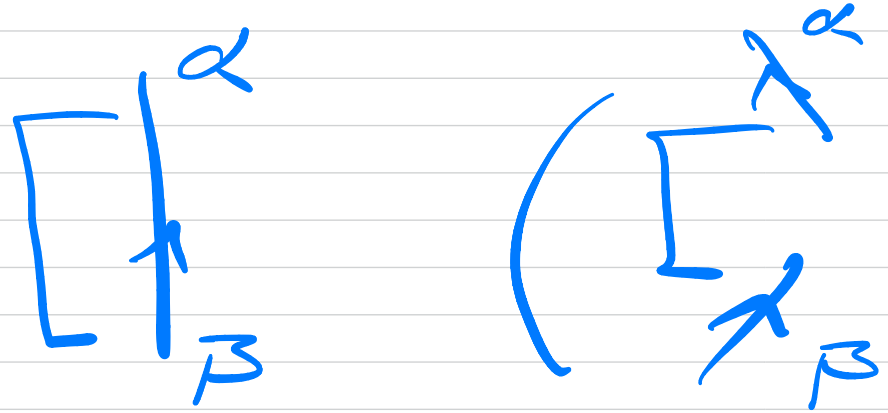
$$\langle \alpha\beta | v | \gamma\delta \rangle_{AS} = \langle \alpha\beta | v | \gamma\delta \rangle - \langle \alpha\beta | v | \delta\gamma \rangle$$

$$\hat{O}^{(1)} = \sum_{\alpha\beta} \langle\alpha|\hat{o}^{(1)}|\beta\rangle\, a_\alpha^\dagger a_\beta$$

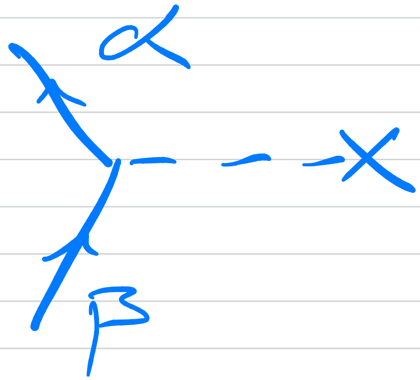
$$\hat{O}^{(2)} = \frac{1}{4}\sum_{\alpha\beta\gamma\delta} \langle\alpha\beta|\hat{o}^{(2)}|\gamma\delta\rangle_{AS} \times a_\alpha^\dagger a_\beta^\dagger a_\delta a_\gamma$$

Diagrammatic representation

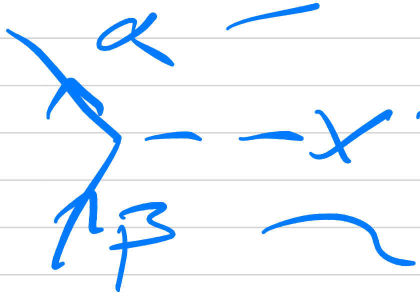
$$\overline{a_\alpha\, a_\beta^\dagger} =$$



$a_\alpha a_\beta^\dagger =$ 

$\hat{H}_0 = \sum_{\alpha\beta}$ 

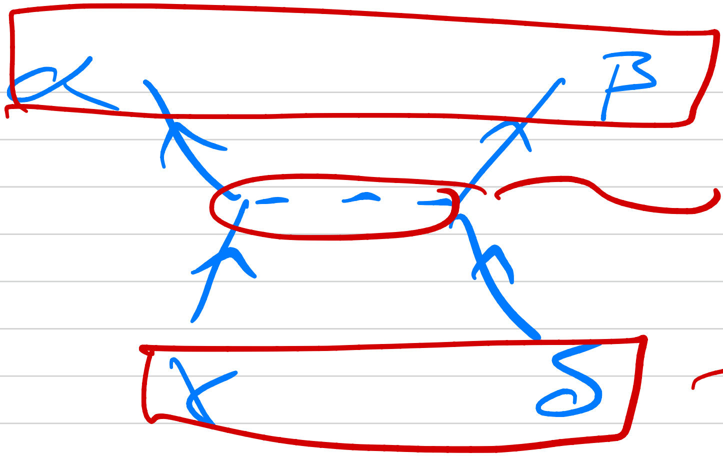
$\sum_{\alpha\beta} \langle\alpha|h_0|\beta\rangle \times a_\alpha^\dagger a_\beta$

$=$ 

$\langle\alpha|$

$\hat{h}_0$

$|\beta\rangle$

$H_I =$ 

$\sim \langle \alpha\beta|$

$\sim v$

$\sim |\gamma\delta\rangle$

$$\Big(\frac{1}{2} \sum_{\alpha\beta\gamma\delta} \langle \alpha\beta | v | \gamma\delta \rangle a_\alpha^\dagger a_\beta^\dagger \times a_\delta a_\gamma$$

$$\langle \alpha_1 \alpha_2 | H_I | \alpha_1 \alpha_2 \rangle =$$

$$\langle \alpha_1 \alpha_2 | v | \alpha_1 \alpha_2 \rangle - \langle \alpha_1 \alpha_2 | v | \alpha_2 \alpha_1 \rangle$$

$$|\alpha_1 \alpha_2\rangle = a_{\alpha_1}^\dagger a_{\alpha_2}^\dagger |0\rangle$$

$$\langle \alpha_1 \alpha_2 | H_I | \alpha_1 \alpha_2 \rangle =$$

$$\frac{1}{2} \sum_{\alpha\beta\gamma\delta} \langle \alpha\beta | v | \gamma\delta \rangle$$

$$\times \langle 0 | \underbrace{\boxed{a_{\alpha_2} a_{\alpha_1}} \boxed{a_\alpha^\dagger a_\beta^\dagger a_\delta a_\gamma} \boxed{a_{\alpha_1}^\dagger a_{\alpha_2}^\dagger}} | 0 \rangle$$

$$a_{\alpha_2} a_{\alpha_1} a_\alpha^\dagger a_\beta^\dagger a_\delta a_\gamma a_{\alpha_1}^\dagger a_{\alpha_2}^\dagger$$

$$\langle \alpha_1 \alpha_2 | v | \alpha_1 \alpha_2 \rangle$$

$$a_{\alpha_2} a_{\alpha_1} a_\alpha^\dagger a_\beta^\dagger \quad a_\delta a_\gamma a_{\alpha_1}^\dagger a_{\alpha_2}^\dagger$$

$$-\langle \alpha_1 \alpha_2 | v | \alpha_2 \alpha_1 \rangle$$

$$a_{\alpha_2} a_{\alpha_1} a^\dagger_\alpha a^\dagger_\beta a_\delta a_\gamma a^\dagger_{\alpha_1} a^\dagger_{\alpha_2}$$

$$= \langle \alpha_2 \alpha_1 | v | \alpha_2 \alpha_1 \rangle$$

$$= \langle \alpha_1 \alpha_2 | v | \alpha_1 \alpha_2 \rangle$$

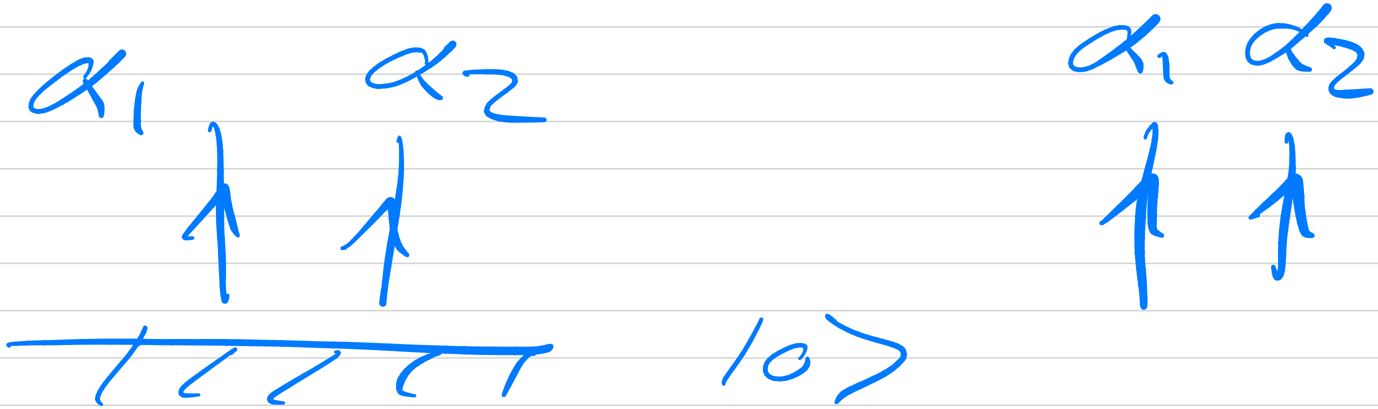
$$a_{\alpha_2} a_{\alpha_1} a^\dagger_\alpha a^\dagger_\beta a_\delta a_\gamma a^\dagger_{\alpha_1} a^\dagger_{\alpha_2}$$

$$-\langle \alpha_2 \alpha_1 | v | \alpha_1 \alpha_2 \rangle = -\langle \alpha_1 \alpha_2 | v | \alpha_2 \alpha_1 \rangle$$

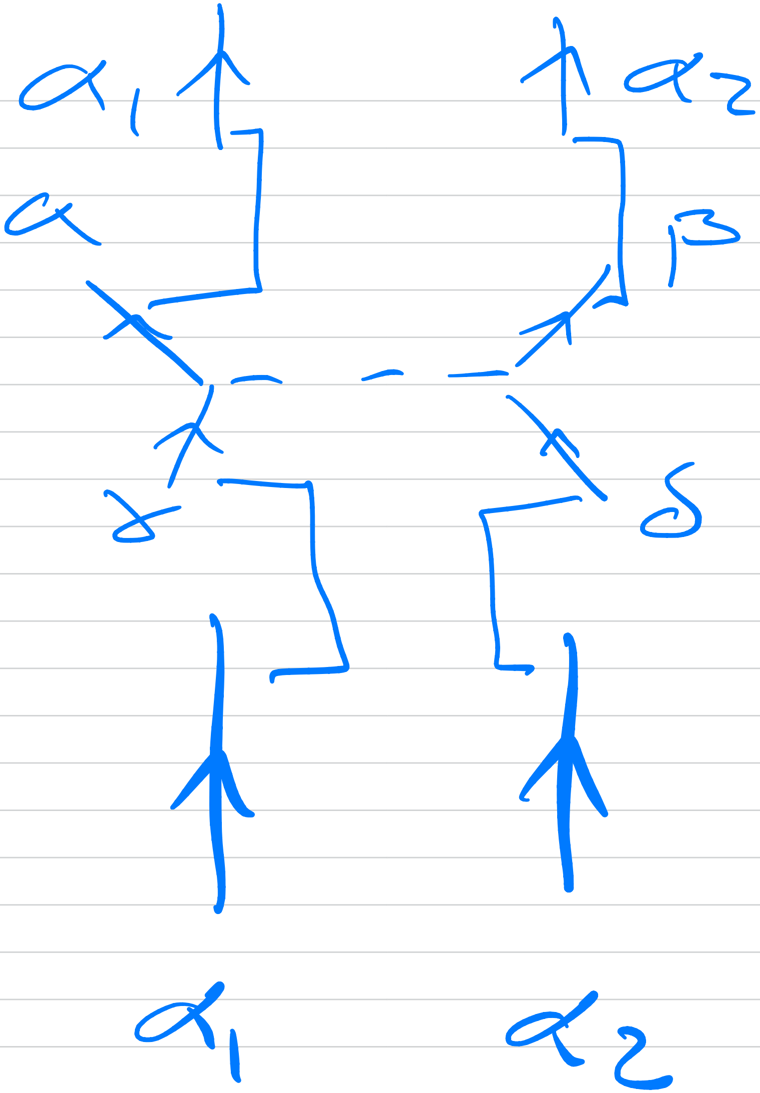
$$\langle \alpha_1 \alpha_2 | v | \alpha_1 \alpha_2 \rangle - \langle \alpha_1 \alpha_2 | v | \alpha_2 \alpha_1 \rangle$$

$$= \langle \alpha_1 \alpha_2 | v | \alpha_1 \alpha_2 \rangle_{AS}$$

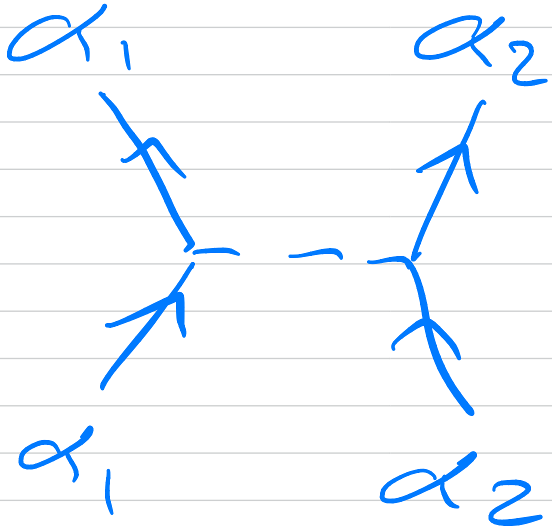
$$|\alpha_1 \alpha_2 \rangle = a^\dagger_{\alpha_1} a^\dagger_{\alpha_2} |0\rangle$$



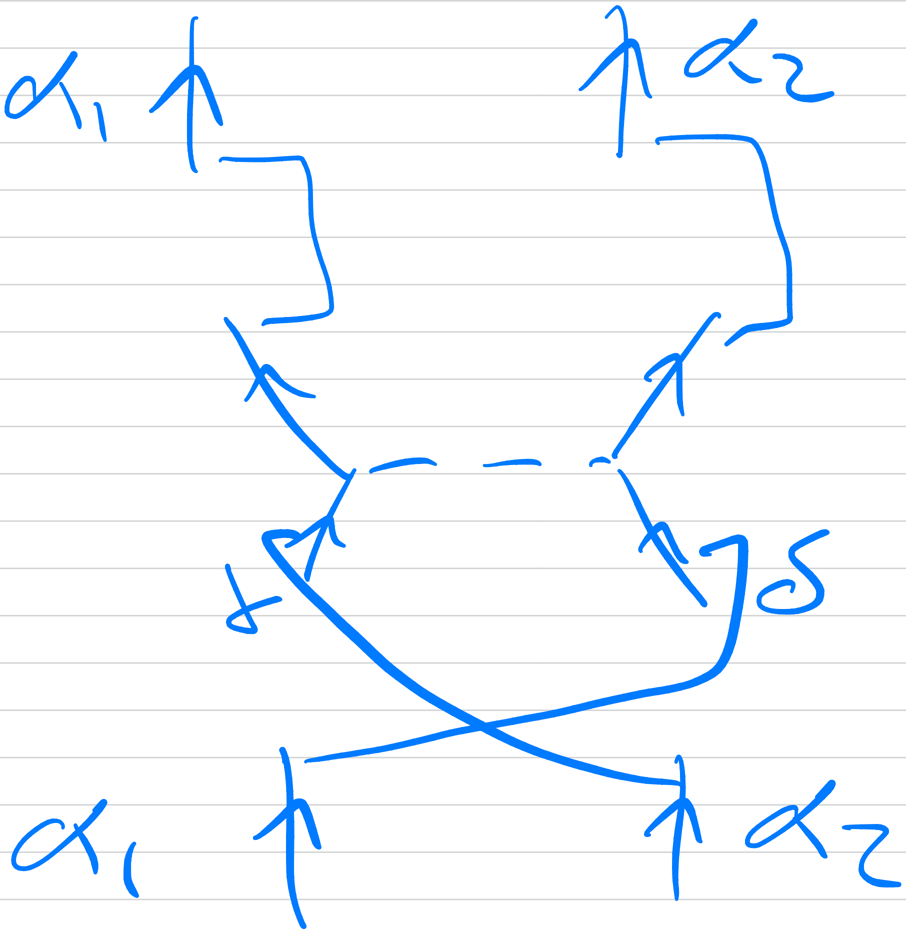
$$\sum_{\alpha\beta\gamma\delta}$$



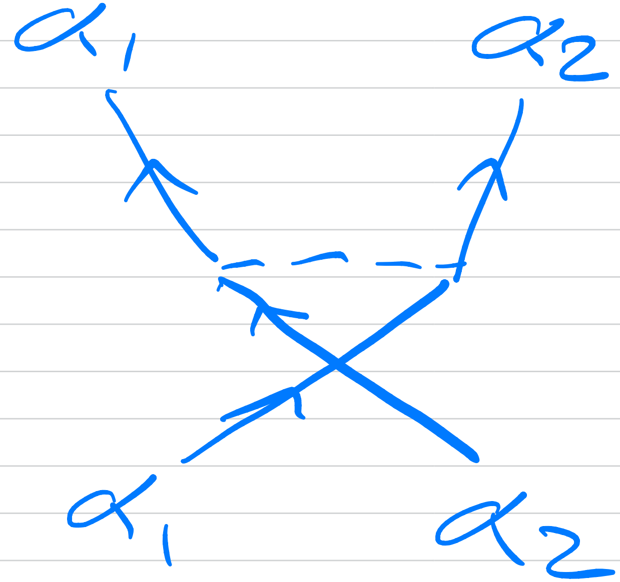
$\longrightarrow$



$\langle \alpha_1 \alpha_2 | v | \alpha_1 \alpha_2 \rangle$



$=$

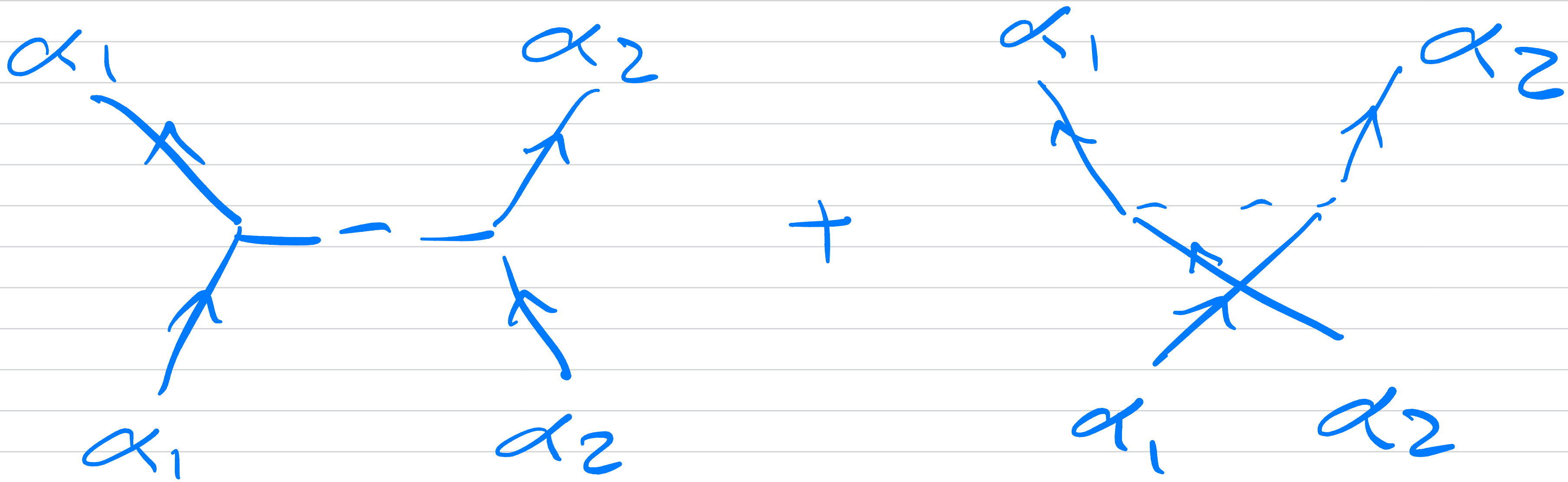


$$- \langle \alpha_1 \alpha_2 | v | \alpha_2 \alpha_1 \rangle$$

$$a_\delta a_\gamma a^\dagger_{\alpha_1} a^\dagger_{\alpha_2}$$

$$\langle \alpha_1 \alpha_2 | v | \alpha_1 \alpha_2 \rangle_{AS} =$$

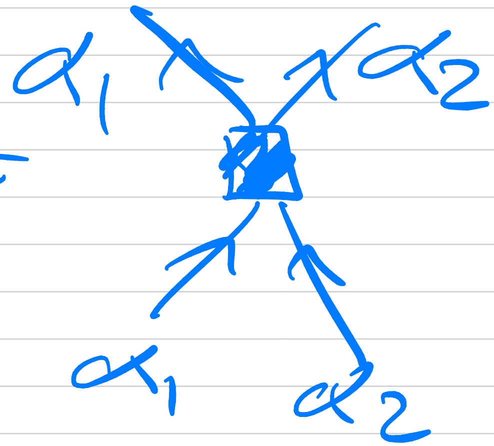
$$\langle \alpha_1 \alpha_2 | v | \alpha_1 \alpha_2 \rangle - \langle \alpha_1 \alpha_2 | v | \alpha_2 \alpha_1 \rangle$$



Feynman – Goldstone diagrams

Hugenholtz notation

$$\langle \alpha_1 \alpha_2 | v | \alpha_1 \alpha_2 \rangle_{AS} =$$



Exercises week 37

Lipkin - model

$$\hat{H} = \hat{H}_0 + \underbrace{\hat{H}_1 + \hat{H}_2}_{H_I}$$

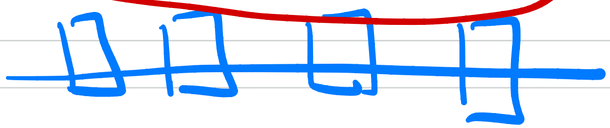
$$H_0 = \frac{1}{2}\varepsilon \sum_{p\sigma} \sigma\, a^{\dagger}_{p\sigma} a_{p\sigma}$$

$$\sigma = \{-1, +1\}$$

$N = 4$ 1   2   3   4

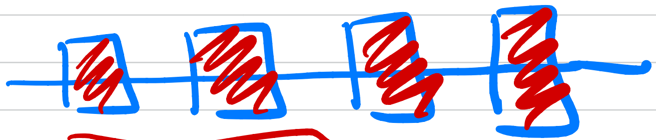
5   6   7   8

p



$\sigma = +1$

$N=4$



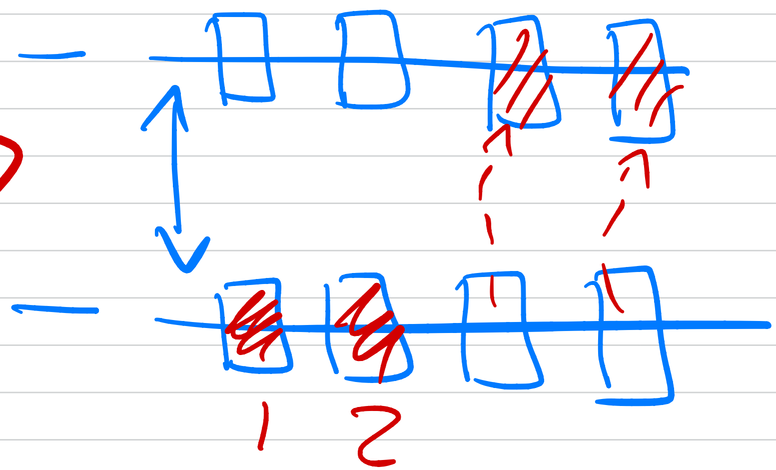
$\sigma = -1 \; (\downarrow)$

$p = 1 \;\; 2 \;\; 3 \;\; 4$

$$|\Phi_0\rangle \quad M_S = -2$$

$$M_S = +2$$

$|\Phi_1\rangle$

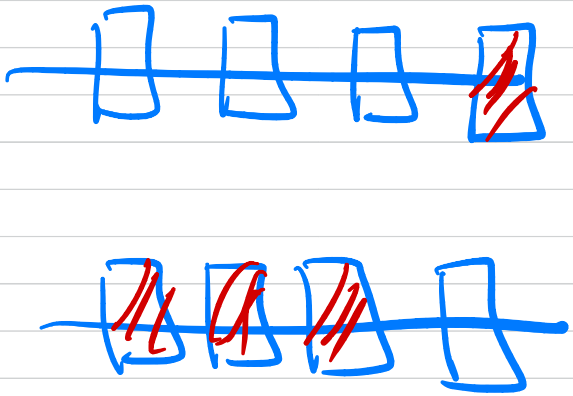


$\sigma = +1$

$\sigma = -1$

$\langle \Phi_1 | H | \Phi_0 \rangle$

$|\phi_2\rangle$



\# configurations

$$= \binom{8}{4} = \frac{8!}{4!\,4!} = 70$$

Total spin $= S = 2$

$M_S = -2, -1, 0, +1, 2$

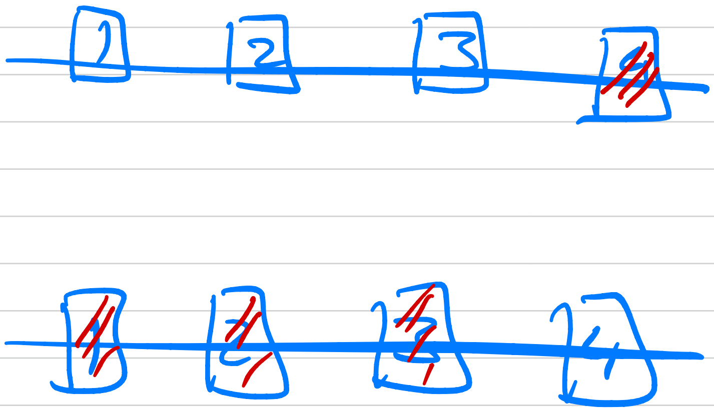
$$\hat{H}_1 = \frac{1}{2} V \sum_{\sigma p p'} a^{+}_{\sigma p} a^{+}_{\sigma p'} a_{-\sigma p'} a_{\sigma p}$$

2p2h-term

$$\hat{H}_2 = \frac{1}{2} w \sum_{\sigma p p'} a^\dagger_{\sigma p} a^\dagger_{-\sigma p'} a_{-\sigma p'} a_{\sigma p}$$

1p1h - term

p



$\sigma = 1$

$\sigma = -1$

$$\frac{1}{2} \sum_{\alpha\beta\gamma\delta} \langle \cdots \rangle a^\dagger_\alpha a^\dagger_\beta a_\delta a_\gamma$$

commutation relations for angular momentum

$$A_+ = \frac{1}{\sqrt{2}}(A_x + iA_y)$$

$$A_- = \frac{1}{\sqrt{2}}(A_x - iA_y)$$

A_z $\qquad [A_\pm, J_z] = \mp A_\pm$

$\hbar = 1$

$[A_\pm, J_\pm] = 0 \qquad [A_z, J_\pm] = \pm A_\pm$

$[A_\pm, J_\mp] = \pm 2A_z, \; [A_z, J_z] = 0$

$$J_+ = \sum_p a^\dagger_{p+} a_{p-} \qquad \sigma = \{+1, -1\}$$

$$J_- = \sum_p a^\dagger_{p-} a_{p+}$$

$$J_z = \frac{1}{2} \sum_{p\sigma} \sigma a^\dagger_{p\sigma} a_{p\sigma}$$

$$[J_+, J_-] = 2J_z$$

$$\sum_{pp'} [a^\dagger_{p+} a_{p-}, a^\dagger_{p'-} a_{p'+}] =$$

$$= \sum_{pp'} \left\{ a^\dagger_{p+} a_{p-} a^\dagger_{p'-} a_{p'+} - a^\dagger_{p'-} a_{p'+} \times a^\dagger_{p+} a_{p-} \right\}$$

$$= \sum_{p} \left\{ a^\dagger_{p+} a_{p-} - a^\dagger_{p-} a_{p-} \right\}$$

$$+ \sum_{pp'} \left\{ - a^\dagger_{p+} a^\dagger_{p'-} a_{p-} a_{p'+} + a^\dagger_{p'-} a^\dagger_{p+} a_{p'+} a_{p-} \right\}$$

$$= \sum_{p} \left\{ a^\dagger_{p+} a_{p+} - a^\dagger_{p-} a_{p-} \right\}$$

$$= \sum_{p\sigma} \sigma a^\dagger_{p\sigma} a_{p\sigma} = 2 J_z$$

$$[J_-, J_+] = -[J_+, J_-] = -2J_z$$

$$[J_z, J_z] = 0 \quad ,$$

$$[J_\pm, J_\pm] = 0$$

$$\hat{H}_0 = \varepsilon \hat{J}_z \qquad H_0 |M_S\rangle = \text{const} |M_S\rangle$$

$$\hat{H}_1 = \frac{1}{2} V (\hat{J}_+^2 + \hat{J}_-^2)$$

$$\hat{H}_2 = \frac{W}{2} (\hat{J}_+ \hat{J}_- + \hat{J}_- \hat{J}_+ - \hat{N})$$

$$M_S = -2$$

$$J_\pm |M_S\rangle = C^{\pm}_{M_S} |M_S \pm 1\rangle$$

$$= \sqrt{S(S+1) - M_S(M_S \pm 1)}\, |M_S \pm 1\rangle$$

$$\langle M_S' | H | M_S \rangle \cancel{\propto} \delta_{M_S M_S'}$$