Lecture Fys4480/9489, November 30 and December 1, 2023

$$G = \frac{1}{E - H_0 - H_1 + i\eta}$$

$$\frac{1}{A-B} = \frac{1}{A} + \frac{1}{A} B \frac{1}{A-B}$$

$$[A, B] \neq 0$$

$$A = E - H_0 + i\eta \qquad B = H_1$$

$$G = G^{(0)} + G^{(0)} H_I G$$

$$G^{(0)} = \frac{1}{E - H_0 + i\eta}$$

$$G = G^{(0)} + G^{(0)} T G^{(0)}$$

T-matrix $\mathcal{T} = H_I + H_I G^{(0)} H_I + \cdots$

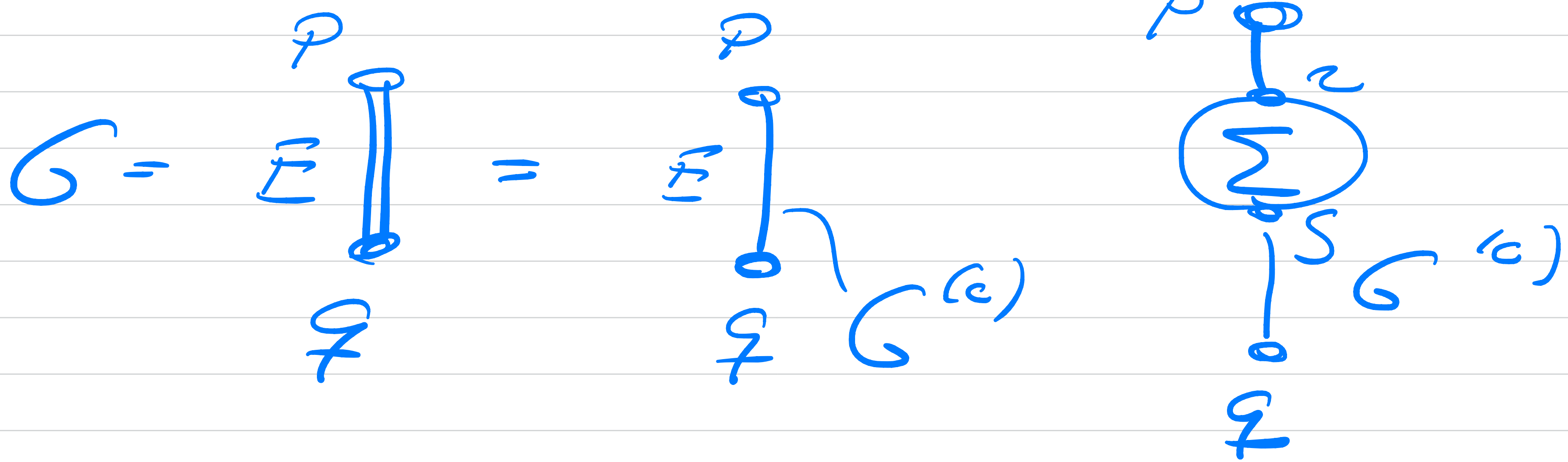
For single-particle Green's function

$$\mathcal{T} \Rightarrow \Sigma(p, q; E)$$

$$G = G(p, q; E) = G^{(0)}(p, q; E)$$
$$+ G^{(0)}(p, q; E)\, \Sigma(p, q; E)\, G^{(0)}(p, q; E)$$

$$= G^{(0)}(p, q; E) + G^{(0)}(p, q; E)$$
$$\Sigma^{*}(p, q; E)\, G(p, q; E)$$

Graphical representation



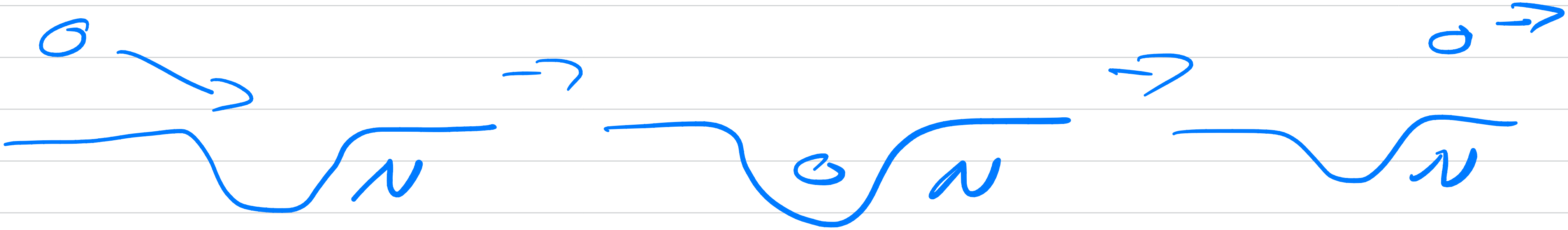
Dyson's equation. Link to MBPT

- Example single-particle case

$$\underbrace{a_a^{+}|\Phi_c\rangle}_{N+1 \text{ system}} \quad \text{— N-particle system}$$

$$\langle \Phi_0 | a_a H a_a^{+} | \Phi_0 \rangle$$

$$\underbrace{p + {}^{16}_{8}O_8}_{17} \longrightarrow \underbrace{{}^{17}_{9}F_8}_{17} \longrightarrow \underbrace{p' + {}^{16}_{8}O_8'}_{17}$$



$$E = E_0^{ref} + \Delta E^{(1)} + \Delta E^{(2)} + \ldots$$

$$\Delta E^{(1)} = \langle \Phi_0 | a_a \{ F_N + V_N \} a_a^{+} | \Phi_0 \rangle$$

$$\langle \Phi^a | F_N | \Phi^a \rangle = \langle \Phi_0 | a_a \sum_{pq} \langle p|f|q \rangle \{a_p^\dagger a_q\} a_a^\dagger |\Phi_0\rangle$$

$$\left(|\Phi^a\rangle = a_a^\dagger |\Phi_0\rangle \right)$$

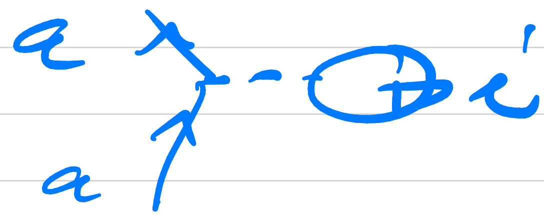
$$a_a a_p^\dagger a_q a_a^\dagger = \delta_{ap}\delta_{aq}$$

$$\langle \Phi^a | F_N | \Phi^a \rangle = \langle a|f|a\rangle$$

$$= \underbrace{\langle a|h|a\rangle}_{\text{aaa to}} + \sum_{i\le F} \langle ai|v|ai\rangle_A$$

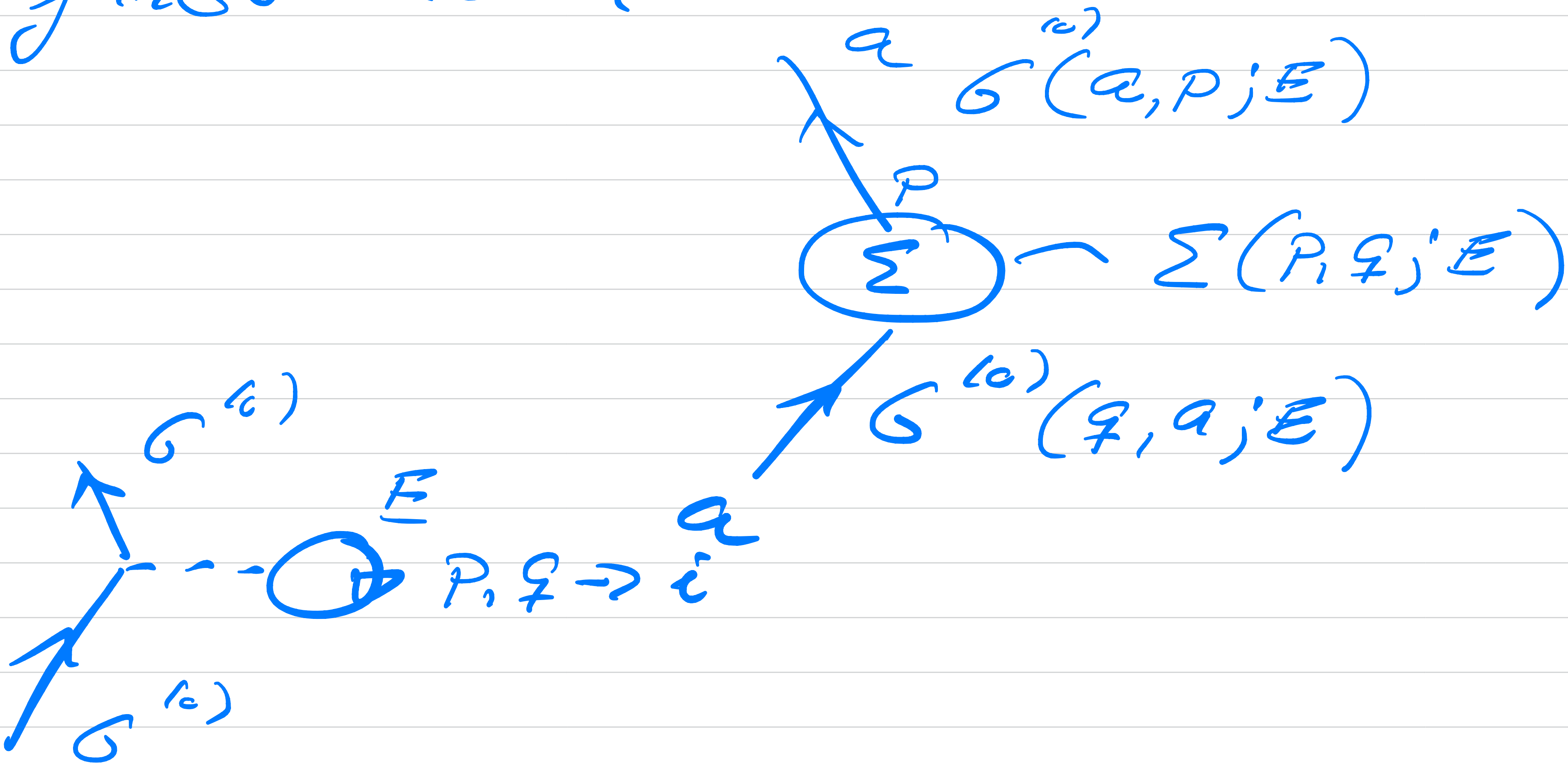
$$E_0^{ref}$$

$$E_0^{ref} + \varepsilon_a$$

$=$ 

Define self-energy

$\Sigma(p,q;E) \Rightarrow \Sigma(a,a;E)$ to first order



To second-order

$$\Delta E^{(2)} = \sum_{M} \langle \Phi^{a} | \{ F_N + V_N \} | \Phi_M \rangle \times \langle \Phi_M | \{ F_N + V_N \} | \Phi^{a} \rangle \over E_0^{a} - E_M$$

$$\Big\{ \langle \Phi_0 | a_a \sum_{pq} a_p^{+} a_q a_s a_r a_a^{+} | \Phi_0 \rangle$$

(i) $|\Phi_M\rangle = a_b |\Phi_0\rangle$ 1p state

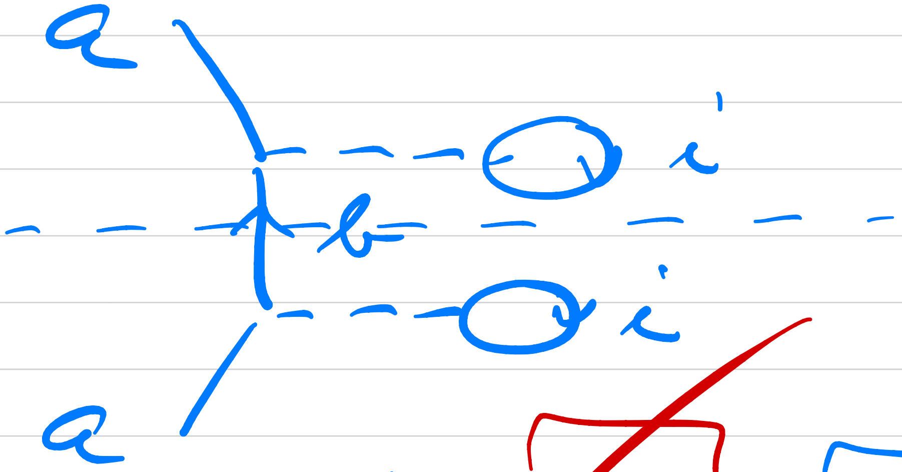
(ii) $|\Phi_M\rangle = a_a^\dagger a_b^\dagger a_i |\Phi_0\rangle$ 2p1h state

(iii) $|\Phi_M\rangle = a_b^\dagger a_j a_i |\Phi_0\rangle$ 2h1p state

$$\langle \Phi_0 | a_a a_p^\dagger a_q a_b^\dagger | \Phi_0 \rangle$$

$$\to \langle a | f | b \rangle = \sum_{i \leq F} \langle ai | v | bi \rangle_{AS}$$

$$\Rightarrow \quad \Delta E^{(2)} = \sum_{bi} \frac{|\langle ai|v|bi\rangle|^2}{\varepsilon_a - \varepsilon_b}$$

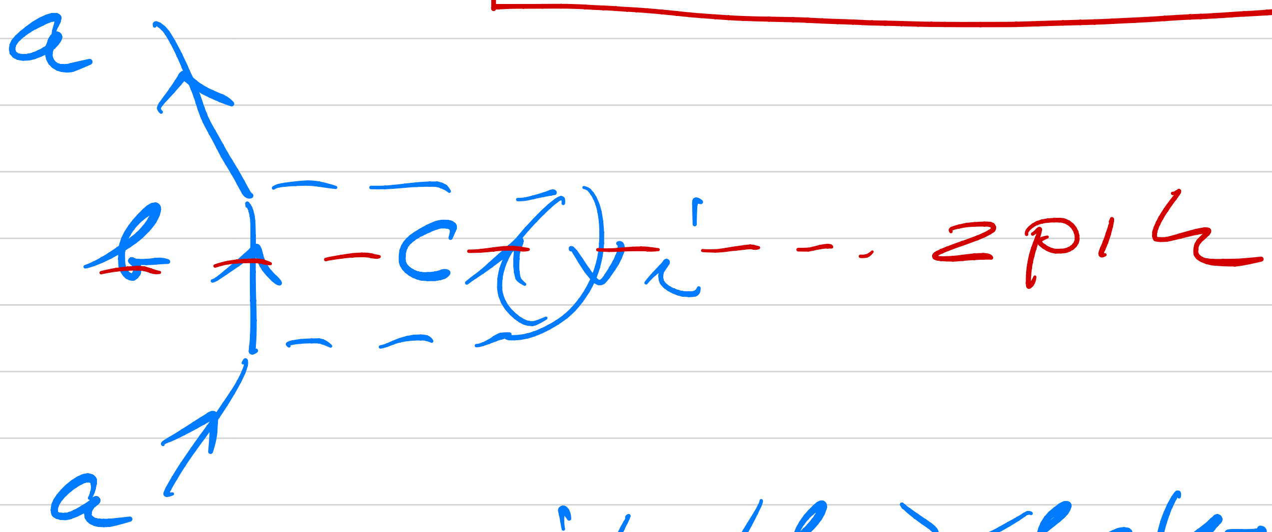


$$\langle \Phi_0 | a_a\, a_p^\dagger\, a_q^\dagger\, a_s\, a_r\, a_b^\dagger | \Phi_0 \rangle$$

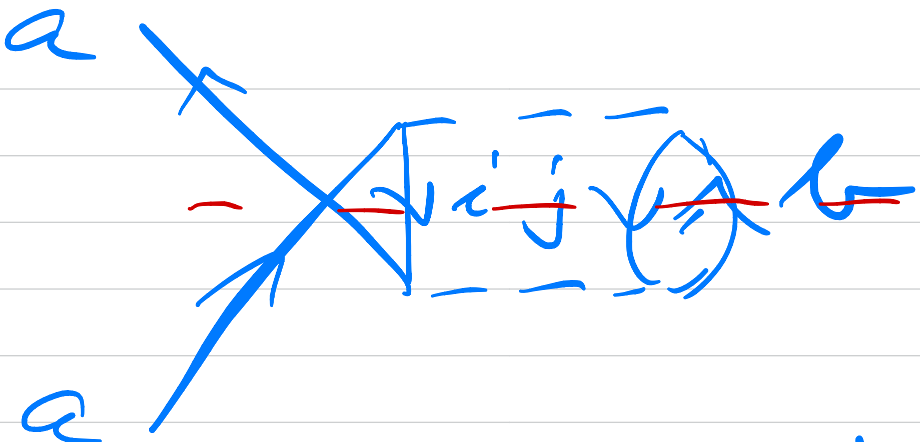
$$= 0$$

$$\langle \Phi_0 | a_a \, a_p^\dagger a_q \, \underbrace{a_b^\dagger a_c^\dagger a_i}_{2p1h} | \Phi_0 \rangle$$

$$\langle \Phi_0 | a_a \, a_p^\dagger a_q^\dagger a_s a_r a_b^\dagger a_c^\dagger a_i | \Phi_0 \rangle$$

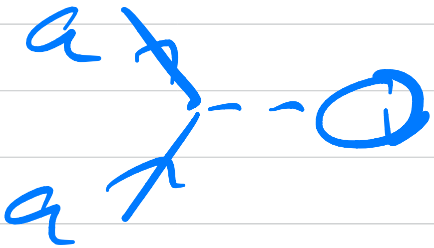
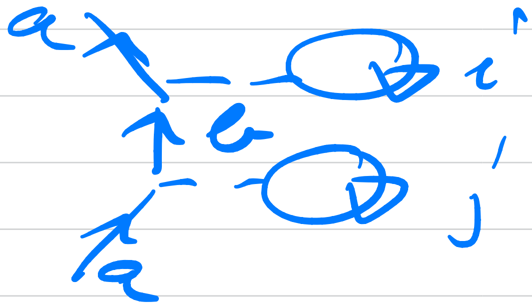


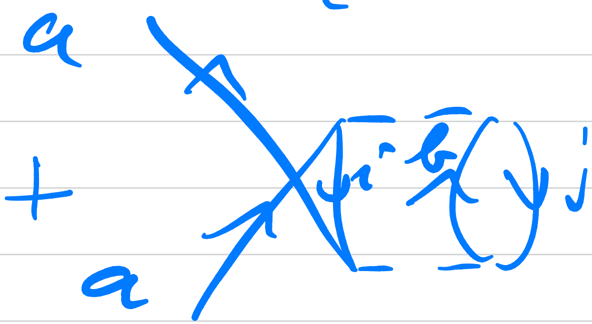
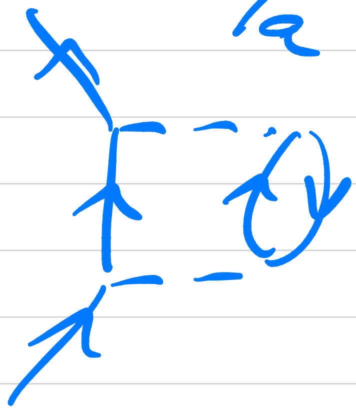
$$\frac{1}{2} \sum_{bci} \frac{\langle ai | v | bc \rangle \langle bc | v | ai \rangle}{\varepsilon_a + \varepsilon_i - \varepsilon_b - \varepsilon_c}$$



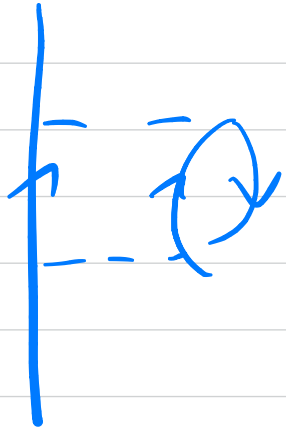
– 2h1P

$$\frac{1}{2}\sum_{ijb}\frac{\langle ij|v|ab\rangle\langle ab|v|ij\rangle}{\varepsilon_i+\varepsilon_j-\varepsilon_b-\varepsilon_a}$$

$\Sigma^{(2)} =$  $+$ 

$+$  $+$ 

TDA/RPA



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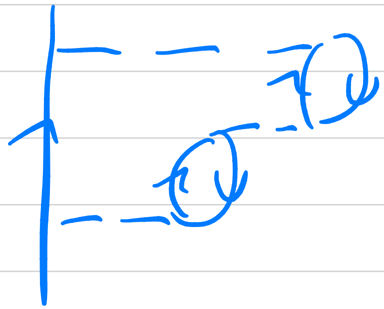

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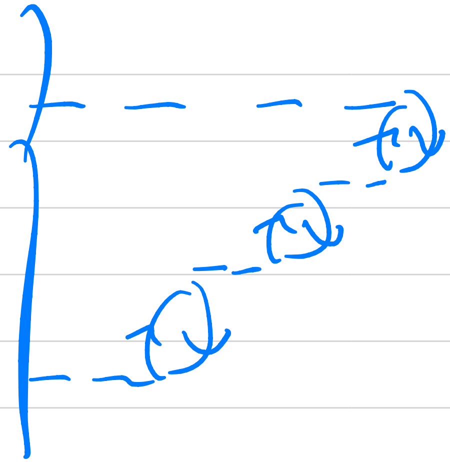

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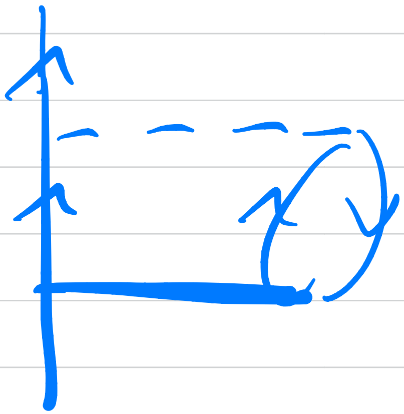
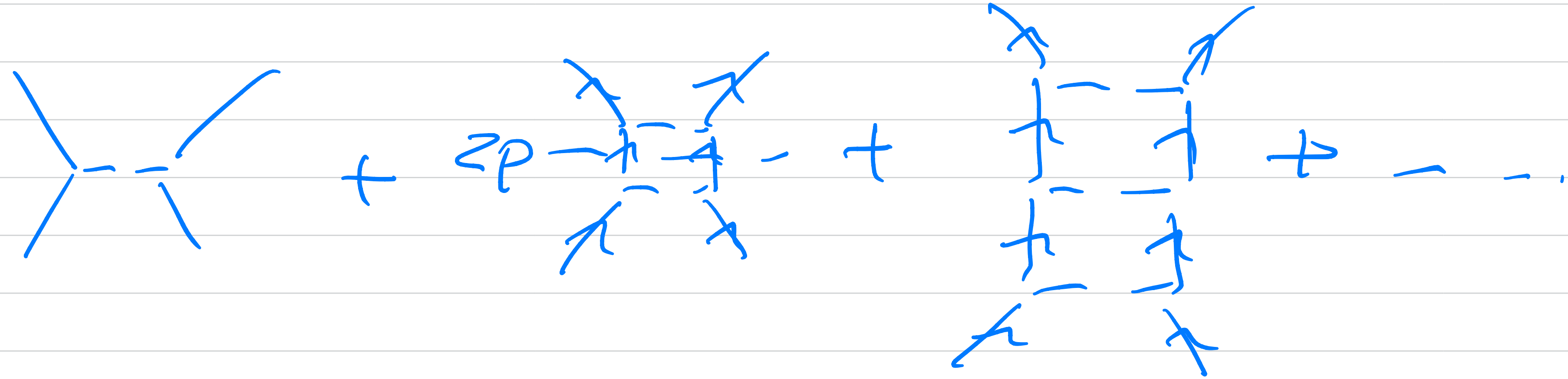
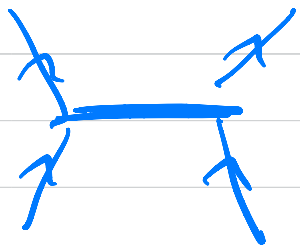
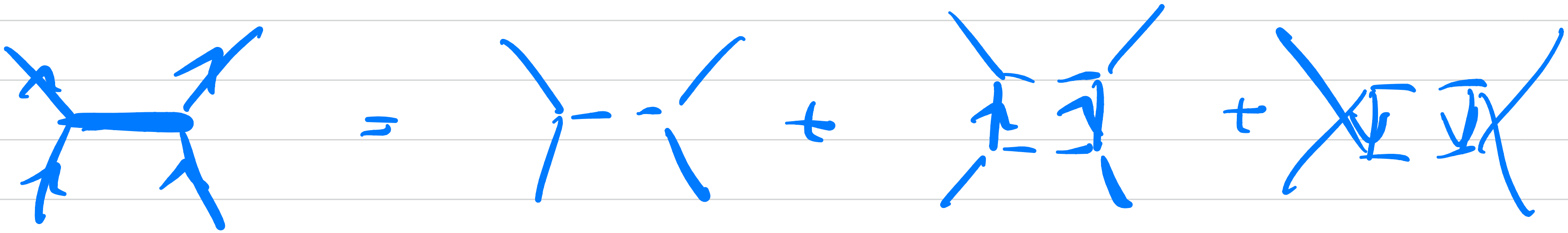


2p-diagram + 2h diagram



= 



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