

Exercises FYS4480/9480 for week 44, October 27-31, 2025

Exercise 1

Let $H = H_0 + V$ and $|\phi_n\rangle$ be the eigenstates of H_0 and that $|\psi_n\rangle$ are the corresponding ones for H . Assume that the ground states $|\phi_0\rangle$ and $|\psi_0\rangle$ are not degenerate. Show that

$$E_0 - \varepsilon_0 = \frac{\langle \phi_0 | V | \psi_0 \rangle}{\langle \phi_0 | \psi_0 \rangle},$$

with $H |\psi_0\rangle = E |\psi_0\rangle$ and $H_0 |\phi_0\rangle = \varepsilon_0 |\phi_0\rangle$.

- a) Define the new operators $P = |\phi_0\rangle \langle \phi_0|$ and $Q = 1 - P$. Show that these operators are idempotent.
- b) Show that for any z we have

$$|\psi_0\rangle = \langle \phi_0 | \psi_0 \rangle \sum_{n=0}^{\infty} \left(\frac{Q}{z - H_0} (z - E_0 + V) \right)^n |\phi_0\rangle,$$

and

$$E_0 = \varepsilon_0 + \sum_{n=0}^{\infty} \langle \phi_0 | V \left(\frac{Q}{z - H_0} (z - E_0 + V) \right)^n |\phi_0\rangle.$$

- c) Discuss these results for $z = E_0$ (Brillouin-Wigner perturbation theory) and $z = \varepsilon_0$ (Rayleigh-Schrödinger perturbation theory). Compare the first few terms in these expansions.

Exercise 2

Consider a system of two fermions spin $s = 1/2$ in the pair-orbitals $|m_0\rangle$ and $|-m_0\rangle$ in a single shell $j = l + s$ with $2j + 1 > 2$, where l is the orbital momentum with $l = 0, 1, 2, \dots$. Assume that the matrix elements for the interaction between the particles takes the form

$$\langle m, -m | v | m', -m' \rangle = -G.$$

- a) Show that the Brillouin-Wigner expansion from the previous exercise can be used to give

$$E_0 = -(j + 1/2)G.$$

- b) Show thereafter by direct diagonalization of the Hamiltonian matrix that this is the exact energy. Use thereafter Rayleigh-Schrödinger perturbation theory and discuss the differences.