Lecture FYS4480/9480, September 19, 2024

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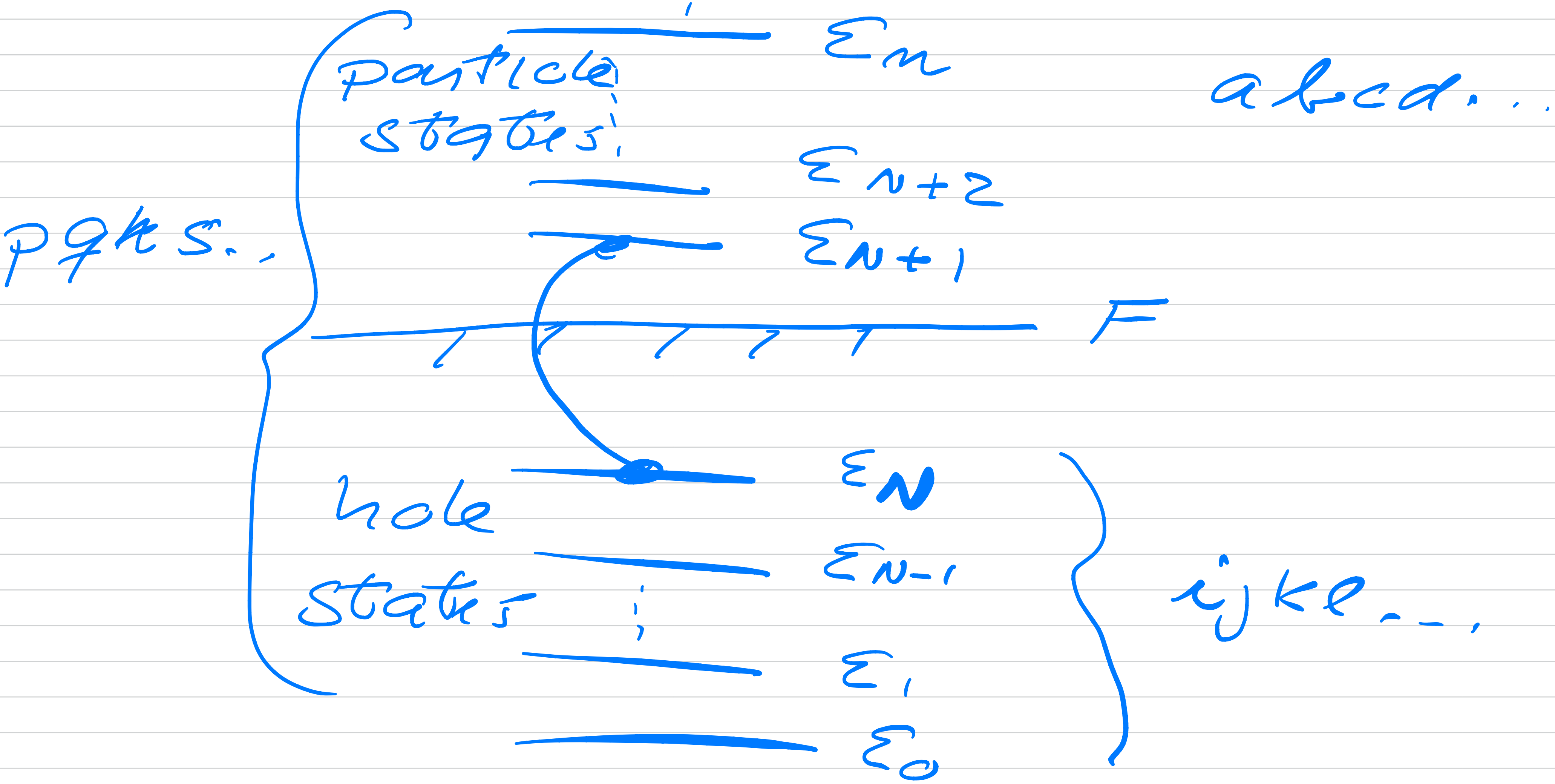
Labeling



$$\overline{a_p a_q^{\dagger}} = \delta_{pq} \quad \text{if } p,q > F$$

$$\overline{a_a a_b^{\dagger}} = \delta_{ab}$$

$$\overline{a_a^{\dagger} a_b} = 0$$

$$\overline{a_p^{\dagger} a_q} = \delta_{pq} \quad \text{if } pq \leq F$$

$$\overline{a_i^{\dagger} a_j} = \delta_{ij}$$

$|\Phi_0\rangle$ reference vacuum

$$a_a |\Phi_0\rangle = 0 \qquad a \notin |\Phi_0\rangle$$

$$a_a^\dagger |\Phi_0\rangle = |\Phi^a\rangle =$$

$$|\Phi_0\rangle = a_1^\dagger a_2^\dagger \cdots a_N^\dagger |0\rangle$$

$$|\Phi^a\rangle = a_a^\dagger a_1^\dagger a_2^\dagger \cdots a_N^\dagger |0\rangle$$

$$= (-)^N a_1^\dagger a_2^\dagger \cdots a_N^\dagger a_a^\dagger |0\rangle$$

N+1 STATE

$$a_i |\Phi_0\rangle = (-)^{i-1} a_1^\dagger a_2^\dagger \cdots a_{i-1}^\dagger a_{i+1}^\dagger \cdots a_N^\dagger |0\rangle$$

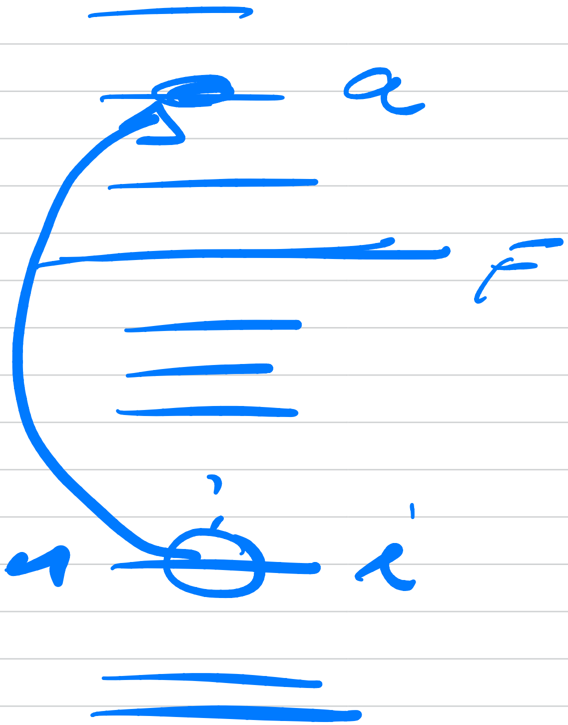
$i \in |\Phi_0\rangle$ $\qquad$ $N-1$ state

$$a_i^\dagger |\Phi_0\rangle = 0$$

$$a_a^\dagger a_i |\Phi_0\rangle = |\Phi_i^a\rangle$$

1p1h

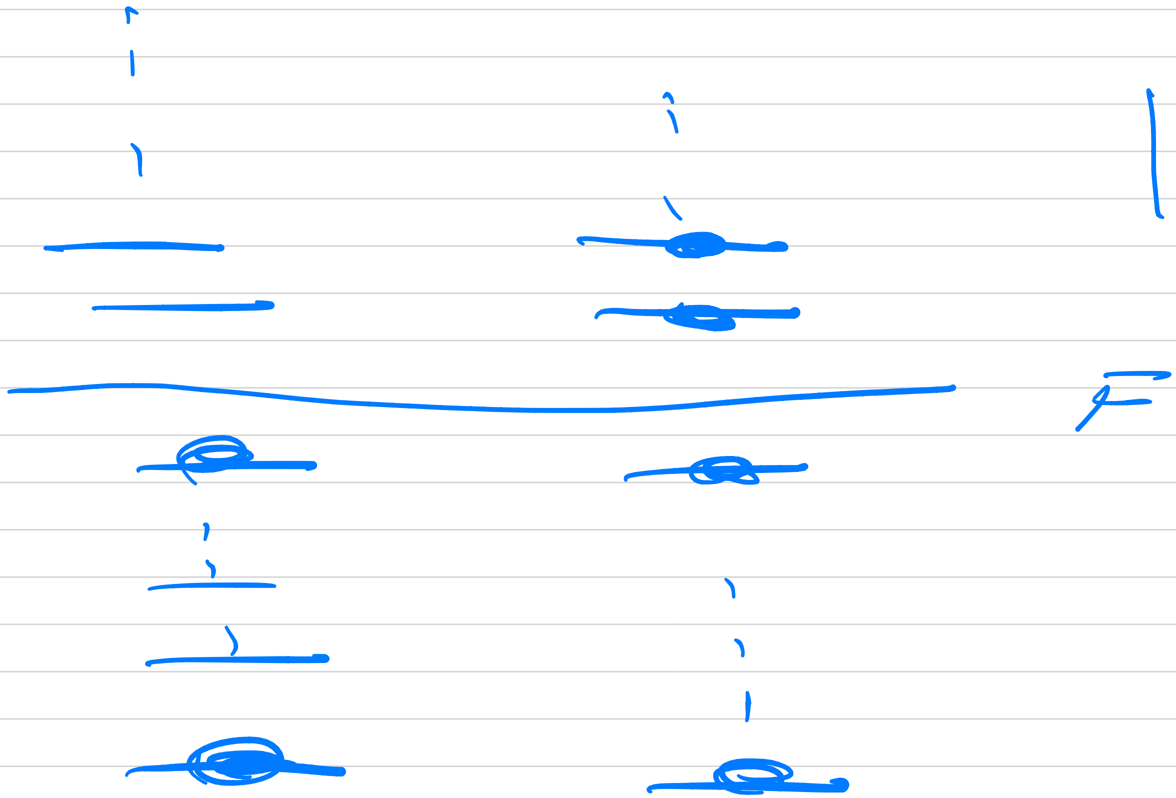
excitation



$^{18}O = {}^{18}_{8}O_{10} \quad ({}^{16}O + 2n)$

$|{}^{18}O\rangle = a_a^\dagger a_e^\dagger |\Phi_0\rangle$

$|\Phi_0\rangle \; ({}^{16}O)$



protons (8)   neutrons (8)

$$\langle \Phi_0 | \overbrace{a_q a_p^\dagger} | \Phi_0 \rangle = \delta_{pq} \text{ if } pq > F$$

$$\langle \Phi_0 | \overbrace{a_p^\dagger a_q} | \Phi_0 \rangle = \delta_{pq} \text{ if } pq \leq F$$

$$\sum_{pq} \langle p | h_0 | q \rangle a_p^\dagger a_q =$$

$$\sum_{pq} \langle p | h_e | q \rangle \{ a_p^\dagger a_q \} \quad \Bigg| \begin{array}{l} a_p^\dagger a_q \\ + a_q a_p^\dagger \\ = \delta_{pq} \end{array}$$

$$+ \sum_{pq} \langle p | h_0 | q \rangle \delta_{pq} \in i$$

$$= \sum_{pq} \langle p | h_0 | q \rangle \{ a_p^\dagger a_q \} + \underbrace{\sum_{i \le F} \langle i | h_0 | i \rangle}_{\sum_i}$$

$$\langle \Phi_0 | H_0 | \Phi_0 \rangle$$

$$H_I = \frac{1}{4} \sum_{pqrs} \langle pq \| rs \rangle_{AS}\, a_p^\dagger a_q^\dagger a_s a_r$$

$\Rightarrow$

$$a_p^\dagger a_q^\dagger a_s a_r = \underbrace{\{ a_p^\dagger a_q^\dagger a_s a_r \}}_{\text{Normal-ordering wrt new reference state } |\Phi_c\rangle}$$

$$+ \{ a_p^\dagger \overbrace{a_q^\dagger a_s}^{\delta_{qs\in i}} a_r \} + \{ \overbrace{a_p^\dagger a_q^\dagger a_s}^{-\delta_{ps\in i}} a_r \}$$

$$+ \left\{ a_p^{\dagger} a_q^{\dagger} a_s a_r \right\} + \left\{ a_p^{\dagger} a_q^{\dagger} a_s a_r \right\}$$

$$+ \left\{ a_p^{\dagger} a_q^{\dagger} a_s a_r \right\} + \left\{ a_p^{\dagger} a_q^{\dagger} a_s a_r \right\}$$

$$\Rightarrow$$

$$H_I = \frac{1}{4} \sum_{pqrs} \langle pq | v | rs \rangle \left\{ a_p^{\dagger} a_q^{\dagger} a_s a_r \right\}$$

$$+ \frac{1}{4} \sum_{pri} \langle pi | v | ri \rangle \left\{ a_p^{\dagger} a_r \right\}$$

$$\div \frac{1}{4} \sum_{qri} \underbrace{\langle iq | v | ri \rangle}_{-\langle qi | v | ri \rangle} \{ a_q^\dagger a_r \}$$

$$+ \ldots$$

$$= \frac{1}{4} \sum_{pqrs} \langle pq | v | rs \rangle_{AS} \{ a_p^\dagger a_q^\dagger a_s a_r \}$$

$$+ \sum_{pqi} \langle pi | v | qi \rangle_{AS} \{ a_p^\dagger a_q \}$$

$$+ \frac{1}{2} \sum_{ij} \langle ij | v | ij \rangle_{AS}$$

$$\hat{H} = \underbrace{E_0^{ref}} + \hat{F}_N + \hat{V}_N$$

$$\langle \Phi_c | H | \Phi_c \rangle$$

$$\hat{F}_N = \sum_{pq} \langle p | f | q \rangle \{ a_p^\dagger a_q \}$$

$$\langle p | f | q \rangle = \langle p | h_0 | q \rangle + \sum_i \langle p i | v | \underline{q} i \rangle_{AS}$$

(Mean-Field)

$$V_N = \frac{1}{4} \sum_{pqrs} \langle pq \| rs \rangle_{AS} \{ a_p^\dagger a_q^\dagger a_s a_r \}$$

Examples

$$\langle \Phi_0 | H | \Phi_i^a \rangle$$

$$|\Phi_i^a\rangle = a_a^\dagger a_i |\Phi_0\rangle$$

$$\langle \Phi_0 | E_0^{Ref} | \Phi_i^a \rangle =$$

$$E_0^{Ref} \langle \Phi_0 | \Phi_i^a \rangle = 0$$

$$\langle \Phi_0 | \Phi_0 \rangle = 1$$

$$\langle \Phi_0 | \hat{F}_N | \Phi_i^a \rangle \; ;$$

$$\langle \Phi_0 | \boxed{a_p^\dagger a_q} \boxed{a_a^\dagger a_i} | \Phi_0 \rangle$$

$$\delta_{pi} \delta_{qa}$$

$$\langle \Phi_0 | \hat{F}_N | \Phi_i^a \rangle$$

$$= \langle i | f | a \rangle = \langle i | h_0 | a \rangle$$

$$+ \sum_j \langle ij | v | aj \rangle_{AS}$$

$$\hat{F}_N | \Phi_0 \rangle$$

$$\langle \underline{\Phi}_0 | F_N | \Phi_i^a \rangle = | 1p1h \rangle$$

0p0h $\qquad$ 1p1h $\qquad$ $\times$ const

$$\langle 1p1h | 1p1h \rangle$$

$$\langle \Phi_0 | \hat{V}_N | \Phi_i^a \rangle$$

$$= \frac{1}{4} \sum_{pqrs} \langle pq | v | rs \rangle$$

$$\times \underbrace{\langle \Phi_0 | a_p^\dagger a_q^\dagger a_s a_r}\, a_a^\dagger a_i | \Phi_0 \rangle$$

$$\underbrace{a_p^\dagger a_q^\dagger a_s a_r | \Phi_0 \rangle}_{\propto\ |2p2h\rangle \text{ or } |\Phi_0\rangle} = 0$$

$$\langle \Phi_0 | H | \Phi_i^a \rangle =$$

$$\langle i | f | a \rangle = \langle i | h_0 | a \rangle$$

$$+ \sum_j \langle ij | v | aj \rangle$$

$$\langle \Phi_0 | H | \Phi_{ij}^{ab} \rangle :$$

$$E_0^{ref} \langle \Phi_0 | \Phi_{ij}^{ab} \rangle = 0$$

$$\langle \Phi_0 | F_N | \Phi_{ij}^{ab} \rangle$$

$$F_N | \Phi_0 \rangle \propto a_p^\dagger a_q | \Phi_0 \rangle$$

$$= \begin{cases} | \Phi_i^a \rangle \\ | \Phi_0 \rangle \end{cases}$$

$$\propto \text{constant} \langle \Phi_0 | \underset{2p2h}{\Phi_{ij}^{ab}} \rangle$$

$$\text{or} \quad \text{constant} \langle \underset{1p1h}{\Phi_k^c} | \underset{2p2h}{\Phi_{ij}^{ab}} \rangle$$

$$\langle \Phi_0 | \hat{V}_N | \Phi_{ij}^{ab} \rangle$$

$$\hat{V}_N |\Phi_c\rangle = \begin{cases} \text{const} \; |\Phi_c\rangle & \text{0p0h} \\ \text{const} \; |\Phi_{ke}^{cd}\rangle & \text{2p2h} \end{cases}$$

$$\langle \Phi_0 | \boxed{a_p^\dagger a_q^\dagger} \, \boxed{a_s a_r} \, \boxed{a_a^\dagger a_b^\dagger} \, \boxed{a_j a_i} | \Phi_c \rangle$$

$\delta_{pi} \delta_{qj}$

$\delta_{ra} \delta_{sb}$ $\langle ij | v | ab \rangle_{AS}$

$$\langle \Phi_0 | \hat{V}_N | \Phi_{ij}^{ab} \rangle = \langle ij || ab \rangle_{AS}$$

$$\langle \Phi_0 | H | \Phi_{ijk}^{abc} \rangle = 0$$

epoh ↑ 3p34

at most 2-body

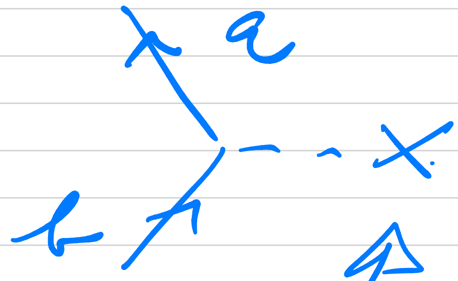
$$\langle \Phi_0 | H | \Phi_{nh}^{np} \rangle = 0$$

if $n > 2$

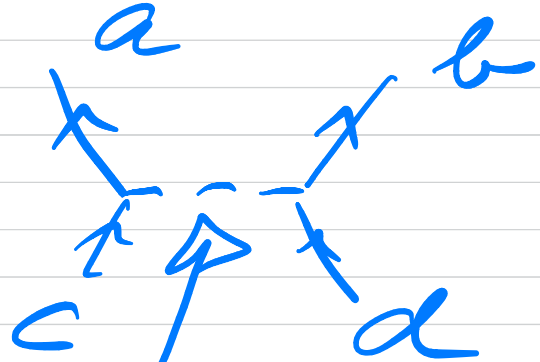
Diagrammatic representation

particle state a

$\langle a | h_0 | b \rangle$

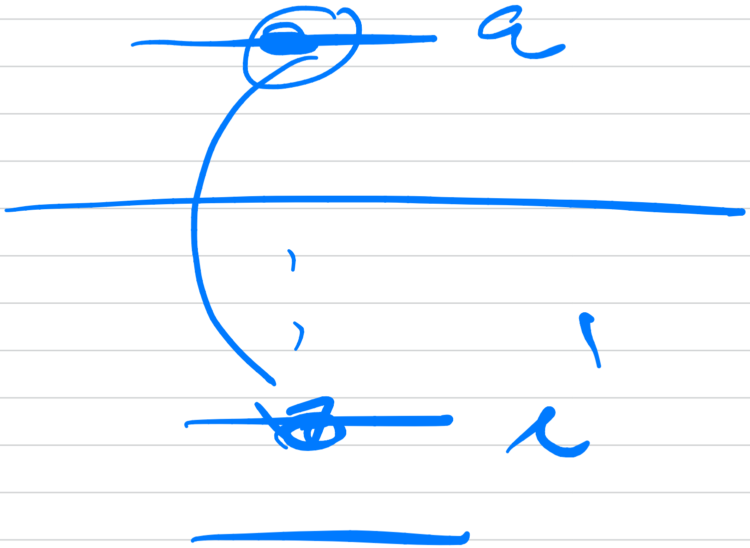


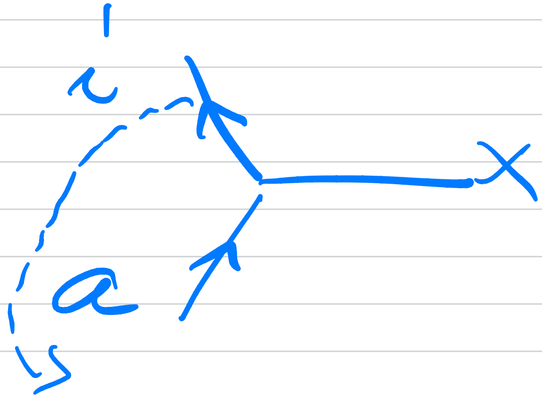
$\langle ab | v | cd \rangle$

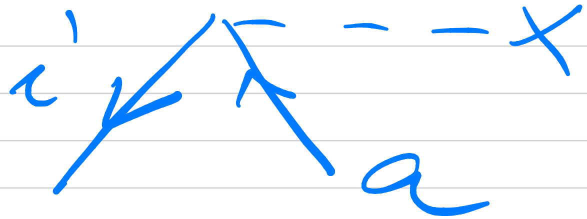


hole state i ϕ_i

$\langle i | h_0 | a \rangle$



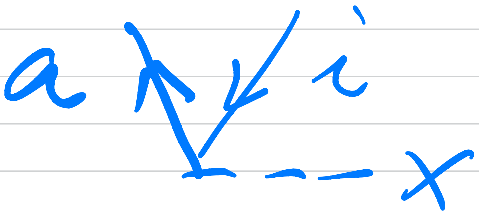




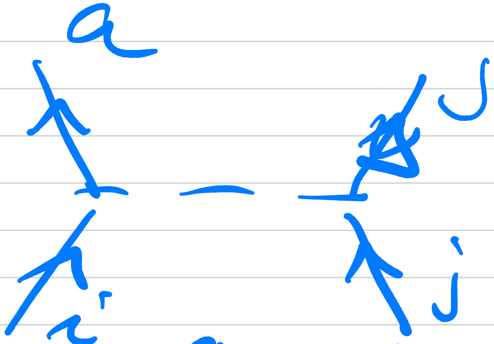
1p1h

$$\langle a|f|i\rangle = \sum_j \Big(\langle aj|v|ij\rangle - \langle aj|v|ji\rangle \Big) + \langle a|h_0|i\rangle$$

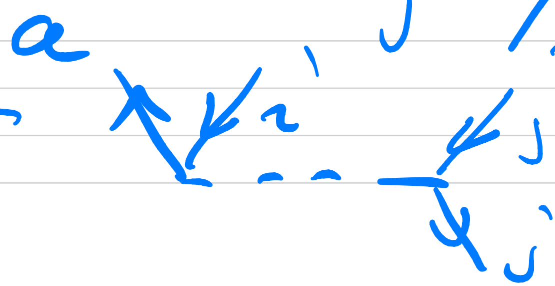
$|\Phi| h$



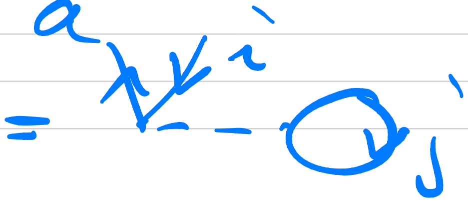
$$\sum_j \langle aj|v|ij\rangle = \sum_j$$



$$= \sum_j$$

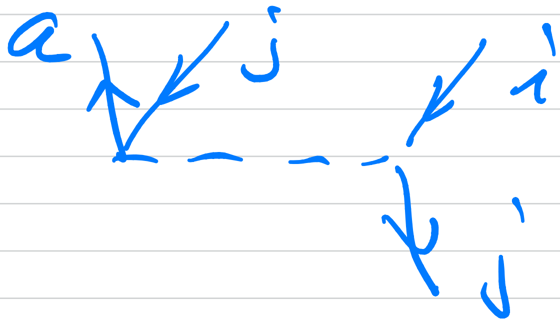


$$=$$



$$-\sum_{j} \langle a j | v | j i \rangle$$

$$= + \sum_{j}$$ 

$$= + \sum_{j}$$ 

$$=$$