Effective Field Theory

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2017

Effective Field Theory Basics

Quantum Chromodynamics (QCD) in the u/d sector has approximate chiral symmetry but this symmetry is broken in two ways:

- Explicitly broken, because the u and d quark masses are not exactly zero;
- Spontaneously broken

$$SU(2)_L \times SU(2)_R \approx SU(2)_V \times SU(2)_A \to SU(2)_V,$$

that is, in the QCD ground state axial symmetry is broken while isospin symmetry is intact. We obtain 3 Goldstone bosons: the pion family!

Chiral Lagrangian

The chiral effective Lagrangian is given by an infinite series of terms with increasing number of derivatives and/or nucleon fields, with the dependence of each term on the pion field prescribed by the rules of broken chiral symmetry. Applying this Lagrangian to NN scattering generates an unlimited number of Feynman diagrams, which may suggest again an untractable problem. However, Weinberg showed that a systematic expansion of the nuclear amplitude exists in terms of $(Q/\Lambda_\chi)^\nu$, where Q denotes a momentum or pion mass, $\Lambda_\chi \approx 1$ GeV is the chiral symmetry breaking scale, and $\nu \geq 0$. For a given order ν, the number of contributing terms is finite and calculable; these terms are uniquely defined and the prediction at each order is model-independent. By going to higher orders, the amplitude can be calculated to any desired accuracy.

Chiral Lagrangian Scheme

The scheme just outlined has become known as chiral perturbation theory (χPT). Therefore, we want to describe the low-energy scenario of QCD by an Effective Field Theory (EFT). The steps to take:

- Write down the most general Lagrangian including all terms consistent with the assumed symmetries, particularly, spontaneously broken chiral symmetry.
- Calculate Feynman diagrams. Note: There will be infinitely many diagrams.
- Find a scheme for assessing the importance of the various diagrams, because we cannot calculate infinitely many diagrams.

Effective Chiral Lagrangian

The starting point for the derivation of the NN interaction is an effective chiral Lagrangian

$$\mathcal{L} = \mathcal{L}_{\pi N} + \mathcal{L}_{\pi\pi} + \mathcal{L}_{NN},$$

which is given by a series of terms of increasing chiral dimension,

$$\mathcal{L}_{\pi N} = \mathcal{L}^{(1)}_{\pi N} + \mathcal{L}^{(2)}_{\pi N} + \mathcal{L}^{(3)}_{\pi N} + \ldots,$$

$$\mathcal{L}_{\pi\pi} = \mathcal{L}^{(2)}_{\pi\pi} + \ldots,$$

$$\mathcal{L}_{NN} = \mathcal{L}^{(0)}_{NN} + \mathcal{L}^{(2)}_{NN} + \mathcal{L}^{(4)}_{NN} + \ldots,$$

where the superscript refers to the number of derivatives or pion mass insertions (chiral dimension). Good review: Epelbaum, Prog. Part. Nucl. Phys. **57**, 654 (2006).

Heavy Baryons

Common to apply the heavy baryon (HB) formulation of chiral perturbation theory in which the relativistic Lagrangian is subjected to an expansion in terms of powers of $1/M_N$ (kind of a nonrelativistic expansion), the lowest order of which is

$$\begin{aligned}\widehat{\mathcal{L}}^{(1)}_{\pi N} &= \bar{N}\left(iD_0 - \frac{g_A}{2}\vec{\sigma}\cdot\vec{u}\right)N \\ &\approx \bar{N}\left[i\partial_0 - \frac{1}{4f_\pi^2}\boldsymbol{\tau}\cdot(\boldsymbol{\pi}\times\partial_0\boldsymbol{\pi}) - \frac{g_A}{2f_\pi}\boldsymbol{\tau}\cdot(\vec{\sigma}\cdot\vec{\nabla})\boldsymbol{\pi}\right]N + \ldots\end{aligned}$$

For the parameters that occur in the leading order Lagrangian, we apply $M_N = 938.919$ MeV, $m_\pi = 138.04$ MeV, $f_\pi = 92.4$ MeV, and $g_A = g_{\pi NN}\, f_\pi/M_N = 1.29$, which is equivalent to $g^2_{\pi NN}/4\pi = 13.67$.

Heavy Baryons Lagrangian

The chiral NN force has the general form

$$V_{2\mathrm{N}} = V_\pi + V_{\mathrm{cont}},$$

where V_{cont} denotes the short-range terms represented by NN contact interactions and V_π corresponds to the long-range part associated with the pion-exchange contributions Both V_π and V_{cont} are determined within the low-momentum expansion.

Notice that the nucleon kinetic energy contributes to $\mathcal{L}^{(2)}$. The above terms determine the nuclear potential up to N^{2}LO (with the exception of the NN contact terms at NLO) in the limit of exact isospin symmetry.

Heavy Baryons Lagrangian

Consider now pion-exchange contributions to the potential

$$V_\pi = V_{1\pi} + V_{2\pi} + V_{3\pi} + \dots\,,$$

where one-, two- and three-pion exchange (3PE) contributions $V_{1\pi}$, $V_{2\pi}$ and $V_{3\pi}$ can be written in the low-momentum expansion as

$$\begin{aligned}
V_{1\pi} &= V_{1\pi}^{(0)} + V_{1\pi}^{(2)} + V_{1\pi}^{(3)} + V_{1\pi}^{(4)} + \dots \\
V_{2\pi} &= V_{2\pi}^{(2)} + V_{2\pi}^{(3)} + V_{2\pi}^{(4)} + \dots \\
V_{3\pi} &= V_{3\pi}^{(4)} + \dots\,.
\end{aligned}$$

Here, the superscripts denote the corresponding chiral order and the ellipses refer to $(Q/\Lambda)^5$- and higher order terms. Contributions due to the exchange of four- and more pions are further suppressed: n–pion exchange diagrams start to contribute at the order $(Q/\Lambda)^{2n-2}$. Notice further that in addition to isopin-invariant contributions there are isospin–breaking corrections.

The static 1PE potential at N^{3}LO has the form

$$V_{1\pi}^{(0)} + V_{1\pi}^{(2)} + V_{1\pi}^{(3)} + V_{1\pi}^{(4)} = -\left(\frac{g_A}{2F_\pi}\right)^2 (1+\delta)^2\, \tau_1 \cdot \tau_2\, \frac{\vec{\sigma}_1 \cdot \vec{q}\, \vec{\sigma}_2 \cdot \vec{q}}{\vec{q}^{\,2} + M_\pi^2}\,.$$

The 2PE contributions are convenient to express as $V_{2\pi}$ in the form:

$$\begin{aligned}
V_{2\pi} &= V_C + \tau_1\cdot\tau_2\, W_C + [V_S + \tau_1\cdot\tau_2\, W_S]\; \vec{\sigma}_1\cdot\vec{\sigma}_2 + [V_T + \tau_1\cdot\tau_2\, W_T]\; \vec{\sigma}_1\cdot\vec{q}\,\vec{\sigma}_2\cdot\vec{q} \\
&+ \qquad [V_{LS} + \tau_1\cdot\tau_2\, W_{LS}]\; i(\vec{\sigma}_1 \cdot \\
&+ \qquad [V_{\sigma L} + \tau_1\cdot\tau_2\, W_{\sigma L}]\; \vec{\sigma}_1\cdot(\vec{q}\times
\end{aligned}$$

where the superscripts C, S, T, LS and σL of the scalar functions V_C, ..., $W_{\sigma L}$ refer to central, spin–spin, tensor, spin–orbit and quadratic spin-orbit components, respectively.

Three-body forces

The first non–vanishing 3NF contribution appears at order $\nu = 3$, i.e. at N^{2}LO. The contribution from graph (a)

$$V_{2\pi}^{(3)} = \sum_{i \neq j \neq k} \frac{1}{2} \left(\frac{g_A}{2F_\pi} \right)^2 \frac{(\vec{\sigma}_i \cdot \vec{q}_i)(\vec{\sigma}_j \cdot \vec{q}_j)}{(\vec{q}_i^{\;2} + M_\pi^2)(\vec{q}_j^{\;2} + M_\pi^2)} F_{ijk}^{\alpha\beta} \tau_i^\alpha \tau_j^\beta \,,$$

where $\vec{q}_i \equiv \vec{p}_i{}' - \vec{p}_i$; $\vec{p}_i$ ($\vec{p}_i{}'$) is the initial (final) momentum of the nucleon i and

$$F_{ijk}^{\alpha\beta} = \delta^{\alpha\beta} \left[-\frac{4c_1 M_\pi^2}{F_\pi^2} + \frac{2c_3}{F_\pi^2} \vec{q}_i \cdot \vec{q}_j \right] + \sum_\gamma \frac{c_4}{F_\pi^2} \epsilon^{\alpha\beta\gamma} \tau_k^\gamma \vec{\sigma}_k \cdot [\vec{q}_i \times \vec{q}_j] \,.$$

Further three-body contributions

The contributions from the remaining graphs (b) and (c) take the form

$$V_{1\pi,\,\mathrm{cont}}^{(3)} = -\sum_{i \neq j \neq k} \frac{g_A}{8F_\pi^2} D \, \frac{\vec{\sigma}_j \cdot \vec{q}_j}{\vec{q}_j^{\,2} + M_\pi^2} \, (\tau_i \cdot \tau_j) \, (\vec{\sigma}_i \cdot \vec{q}_j), \qquad V_{\mathrm{cont}}^{(3)} = \frac{1}{2} \sum_{j \neq k} E \, (\tau_j \cdot \tau_k),$$

where D and E are the corresponding low-energy constants from the Lagrangian of order $\nu = 1$.

Chiral order	2N force	3N force	4N force
$\nu = 0$	$V_{1\pi} + V_{\mathrm{cont}}$	–	–
$\nu = 1$	–	–	–
$\nu = 2$	$V_{1\pi} + V_{2\pi} + V_{\mathrm{cont}}$	–	–
$\nu = 3$	$V_{1\pi} + V_{2\pi}$	$V_{2\pi} + V_{1\pi,\,\mathrm{cont}} + V_{\mathrm{cont}}$	–
$\nu = 4$	$V_{1\pi} + V_{2\pi} + V_{3\pi} + V_{\mathrm{cont}}$	work in progress	work in progress