

# Effective Field Theory

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## Effective Field Theory Basics

Quantum Chromodynamics (QCD) in the  $u/d$  sector has approximate chiral symmetry but this symmetry is broken in two ways:

- Explicitly broken, because the  $u$  and  $d$  quark masses are not exactly zero;
- Spontaneously broken

$$SU(2)_L \times SU(2)_R \approx SU(2)_V \times SU(2)_A \rightarrow SU(2)_V,$$

that is, in the QCD ground state axial symmetry is broken while isospin symmetry is intact. We obtain 3 Goldstone bosons: the pion family!

## Chiral Lagrangian

The chiral effective Lagrangian is given by an infinite series of terms with increasing number of derivatives and/or nucleon fields, with the dependence of each term on the pion field prescribed by the rules of broken chiral symmetry. Applying this Lagrangian to  $NN$  scattering generates an unlimited number of Feynman diagrams, which may suggest again an untractable problem. However, Weinberg showed that a systematic expansion of the nuclear amplitude exists in terms of  $(Q/\Lambda_\chi)^\nu$ , where  $Q$  denotes a momentum or pion mass,  $\Lambda_\chi \approx 1$  GeV is the chiral symmetry breaking scale, and  $\nu \geq 0$ . For a given order  $\nu$ , the number of contributing terms is finite and calculable; these terms are uniquely defined and the prediction at each order is model-independent. By going to higher orders, the amplitude can be calculated to any desired accuracy.

## Chiral Lagrangian Scheme

The scheme just outlined has become known as chiral perturbation theory. Therefore, we want to describe the low-energy scenario of QCD by an Effective Field Theory (EFT). The steps to take:

- Write down the most general Lagrangian including all terms consistent with the assumed symmetries, particularly, spontaneously broken chiral symmetry.
- Calculate Feynman diagrams. Note: There will be infinitely many diagrams.
- Find a scheme for assessing the importance of the various diagrams, because we cannot calculate infinitely many diagrams.

## Effective Chiral Lagrangian

The starting point for the derivation of the  $NN$  interaction is an effective chiral Lagrangian

$$\mathcal{L} = \mathcal{L}_{\pi N} + \mathcal{L}_{\pi\pi} + \mathcal{L}_{NN},$$

which is given by a series of terms of increasing chiral dimension,

$$\mathcal{L}_{\pi N} = \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\pi N}^{(2)} + \mathcal{L}_{\pi N}^{(3)} + \dots,$$

$$\mathcal{L}_{\pi\pi} = \mathcal{L}_{\pi\pi}^{(2)} + \dots,$$

$$\mathcal{L}_{NN} = \mathcal{L}_{NN}^{(0)} + \mathcal{L}_{NN}^{(2)} + \mathcal{L}_{NN}^{(4)} + \dots,$$

where the superscript refers to the number of derivatives or pion mass insertions (chiral dimension). [Good review: Epelbaum, Prog. Part. Nucl. Phys. 57, 654 \(2006\).](#)

## Heavy Baryons

Common to apply the heavy baryon (HB) formulation of chiral perturbation theory in which the relativistic Lagrangian is subjected to an expansion in terms of powers of  $1/M_N$  (kind of a nonrelativistic expansion), the lowest order of which is

$$\begin{aligned} \hat{\mathcal{L}}_{\pi N}^{(1)} &= \bar{N} \left( iD_0 - \frac{g_A}{2} \boldsymbol{\sigma} \cdot \mathbf{u} \right) N \\ &\approx \bar{N} \left[ i\partial_0 - \frac{1}{4f_\pi^2} \boldsymbol{\tau} \cdot (\boldsymbol{\pi} \times \partial_0 \boldsymbol{\pi}) - \frac{g_A}{2f_\pi} \boldsymbol{\tau} \cdot (\boldsymbol{\sigma} \cdot \boldsymbol{\nabla}) \boldsymbol{\pi} \right] N + \dots \end{aligned}$$

For the parameters that occur in the leading order Lagrangian, we apply  $M_N = 938.919$  MeV,  $m_\pi = 138.04$  MeV,  $f_\pi = 92.4$  MeV, and  $g_A = g_{\pi NN} f_\pi / M_N = 1.29$ , which is equivalent to  $g_{\pi NN}^2 / 4\pi = 13.67$ .

## Heavy Baryons Lagrangian

The chiral NN force has the general form

$$V_{2N} = V_\pi + V_{\text{cont}},$$

where  $V_{\text{cont}}$  denotes the short-range terms represented by  $NN$  contact interactions and  $V_\pi$  corresponds to the long-range part associated with the pion-exchange contributions. Both  $V_\pi$  and  $V_{\text{cont}}$  are determined within the low-momentum expansion.

Notice that the nucleon kinetic energy contributes to  $\mathcal{L}^{(2)}$ . The above terms determine the nuclear potential up to N2LO (with the exception of the NN contact terms at NLO) in the limit of exact isospin symmetry.

## Heavy Baryons Lagrangian

Consider now pion-exchange contributions to the potential

$$V_\pi = V_{1\pi} + V_{2\pi} + V_{3\pi} + \dots,$$

where one-, two- and three-pion exchange (3PE) contributions  $V_{1\pi}$ ,  $V_{2\pi}$  and  $V_{3\pi}$  can be written in the low-momentum expansion as

$$\begin{aligned} V_{1\pi} &= V_{1\pi}^{(0)} + V_{1\pi}^{(2)} + V_{1\pi}^{(3)} + V_{1\pi}^{(4)} + \dots \\ V_{2\pi} &= V_{2\pi}^{(2)} + V_{2\pi}^{(3)} + V_{2\pi}^{(4)} + \dots \\ V_{3\pi} &= V_{3\pi}^{(4)} + \dots \end{aligned}$$

Here, the superscripts denote the corresponding chiral order and the ellipses refer to  $(Q/\Lambda)^5$ - and higher order terms. Contributions due to the exchange of four- and more pions are further suppressed:  $n$ -pion exchange diagrams start to contribute at the order  $(Q/\Lambda)^{2n-2}$ . Notice further that in addition to isospin-invariant contributions there are isospin-breaking corrections.

The static 1PE potential at N3LO has the form

$$V_{1\pi}^{(0)} + V_{1\pi}^{(2)} + V_{1\pi}^{(3)} + V_{1\pi}^{(4)} = - \left( \frac{g_A}{2F_\pi} \right)^2 (1 + \delta)^2 \tau_1 \cdot \tau_2 \frac{\vec{\sigma}_1 \cdot \vec{q} \vec{\sigma}_2 \cdot \vec{q}}{\vec{q}^2 + M_\pi^2}.$$

The 2PE contributions are convenient to express as  $V_{2\pi}$  in the form:

$$\begin{aligned} V_{2\pi} &= V_C + \tau_1 \cdot \tau_2 W_C + [V_S + \tau_1 \cdot \tau_2 W_S] \vec{\sigma}_1 \cdot \vec{\sigma}_2 + [V_T + \tau_1 \cdot \tau_2 W_T] \vec{\sigma}_1 \cdot \vec{q} \vec{\sigma}_2 \cdot \vec{q} \\ &\quad + [V_{LS} + \tau_1 \cdot \tau_2 W_{LS}] i(\vec{\sigma}_1 + \vec{\sigma}_2) \cdot (\vec{q} \times \vec{k}) \\ &\quad + [V_{\sigma L} + \tau_1 \cdot \tau_2 W_{\sigma L}] \vec{\sigma}_1 \cdot (\vec{q} \times \vec{k}) \vec{\sigma}_2 \cdot (\vec{q} \times \vec{k}), \end{aligned}$$

where the superscripts  $C$ ,  $S$ ,  $T$ ,  $LS$  and  $\sigma L$  of the scalar functions  $V_C, \dots, W_{\sigma L}$  refer to central, spin-spin, tensor, spin-orbit and quadratic spin-orbit components, respectively.

### Three-body forces

The first non-vanishing 3NF contribution appears at order  $\nu = 3$ , i.e. at N2LO. The contribution from graph (a)

$$V_{2\pi}^{(3)} = \sum_{i \neq j \neq k} \frac{1}{2} \left( \frac{g_A}{2F_\pi} \right)^2 \frac{(\vec{\sigma}_i \cdot \vec{q}_i)(\vec{\sigma}_j \cdot \vec{q}_j)}{(\vec{q}_i^2 + M_\pi^2)(\vec{q}_j^2 + M_\pi^2)} F_{ijk}^{\alpha\beta} \tau_i^\alpha \tau_j^\beta,$$

where  $\vec{q}_i \equiv \vec{p}_i' - \vec{p}_i$ ;  $\vec{p}_i$  ( $\vec{p}_i'$ ) is the initial (final) momentum of the nucleon  $i$  and

$$F_{ijk}^{\alpha\beta} = \delta^{\alpha\beta} \left[ -\frac{4c_1 M_\pi^2}{F_\pi^2} + \frac{2c_3}{F_\pi^2} \vec{q}_i \cdot \vec{q}_j \right] + \sum_\gamma \frac{c_4}{F_\pi^2} \epsilon^{\alpha\beta\gamma} \tau_k^\gamma \vec{\sigma}_k \cdot [\vec{q}_i \times \vec{q}_j].$$

### Further three-body contributions

The contributions from the remaining graphs (b) and (c) take the form

$$V_{1\pi, \text{cont}}^{(3)} = - \sum_{i \neq j \neq k} \frac{g_A}{8F_\pi^2} D \frac{\vec{\sigma}_j \cdot \vec{q}_j}{\vec{q}_j^2 + M_\pi^2} (\tau_i \cdot \tau_j) (\vec{\sigma}_i \cdot \vec{q}_i), \quad V_{\text{cont}}^{(3)} = \frac{1}{2} \sum_{j \neq k} E (\tau_j \cdot \tau_k),$$

where  $D$  and  $E$  are the corresponding low-energy constants from the Lagrangian of order  $\nu = 1$ .

Chiral order	2N force	3N force	4N force
$\nu = 0$	$V_{1\pi} + V_{\text{cont}}$	—	—
$\nu = 1$	—	—	—
$\nu = 2$	$V_{1\pi} + V_{2\pi} + V_{\text{cont}}$	—	—
$\nu = 3$	$V_{1\pi} + V_{2\pi}$	$V_{2\pi} + V_{1\pi, \text{cont}} + V_{\text{cont}}$	—
$\nu = 4$	$V_{1\pi} + V_{2\pi} + V_{3\pi} + V_{\text{cont}}$	work in progress	work in progress