Brief Overview of QCD

* Quark + gluon DoF are not optimal to describe the strong force dynamics of nuclei.

* Nevertheless, the low energy Effective Field Theories (EFT) that underpin our modern understanding of inter-nucleon forces are closely related to the underlying symmetries of QCD

* Moreover, as we'll see later in the course, remarkable progress is being made in lattice QCD so that "direct" calculations of few-nucleon systems + nuclear forces is becoming closer to reality.

* Therefore, even though > 95% of our course will be concerned with a description in terms of nucleons (+ pions), it is useful to give a birdseye view of QCD.

* Don't worry if you have no QFT background or previous experience w/ QCD. Our presentation here is necessarily impressionistic + meant only to remind you of what's governing things at a fundamental level.

QCD Lagrangian

Theory of quark ($S=\frac{1}{2}$) fields + gluon ($S=1$) fields

$$\mathcal{L}_{QCD} = \bar{\Psi}_{i,\alpha}\left[(i\gamma^{\mu}D_{\mu})_{\alpha\beta} - m_i\,\delta_{\alpha\beta}\right]\Psi_{i,\beta} - \frac{1}{4}F^{a}_{\mu\nu}F^{\mu\nu a}$$

i = "flavor" label (6 types of quarks, up, down, strange, charm, top, bottom)

α, β = "color" charge label (internal quantum # acts like analog of electric charge)

$$\boxed{(D_\mu)_{\alpha\beta} = \partial_\mu \delta_{\alpha\beta} - g A^a_\mu [T^a]_{\alpha\beta}}$$ "Gauge Covariant Derivative"

gluon field $A^a_\mu [T^a]_{\alpha\beta} \equiv [A_\mu]_{\alpha\beta}$

$a = 1, \dots 8$ "Color charge" label of the gluon fields

$[T^a]_{\alpha\beta}$ = 3X3 matrices that generate the Lie algebra of SU(3) [Gauge group of QCD]

* and the gluon Kinetic term

$$\boxed{F^a_{\mu\nu} \equiv \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g \underbrace{f^{abc}}_{\text{Structure constants of SU(3)}} A^b_\mu A^c_\nu}$$

$[T^a, T^b] = i f^{abc} T^c$

<u>Notice 2 remarkable facts</u>

(1) QCD can fit on 1 line!

(2) tiny # of input parameters

* 6 masses m_i $\quad i = u, d, s, t, b, c$

* Coupling "constant" g

Yet somehow, "all" of nuclear physics follows (in principle) from this!

Side remark:

* It doesn't really matter for us here, but the γ^μ are the usual 4x4 Dirac matrices obeying

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$$

* also, the ψ are 4-component spin $\frac{1}{2}$ Dirac Spinor fields (the dirac indices have been suppressed)

* and $\bar{\psi} = \psi^\dagger \gamma^0$

Some basic properties of quarks

flavor	charge	mass
u (up)	+2/3	~ 2-4 MeV
d (down)	−1/3	~ 4-8 MeV
c (charm)	+2/3	~ 1.3 GeV
s (strange)	−1/3	~ 100 MeV
t (top)	+2/3	~ 170 GeV
b (bottom)	−1/3	~ 4 GeV

Most of what we're concerned with in Nuclear Physics driven by the 2 light quark flavors.

Comment: (1.) 3 "generations" with repeating charge quantum #'s (each generation distinguished by behavior under Weak interactions)

(2) Nobody knows why there are 3 generations

(3.) The quark masses are actually not observable quantities since quarks cannot be isolated (more below!)

Comparison with QED (e.g., e^+, e^- + photons)

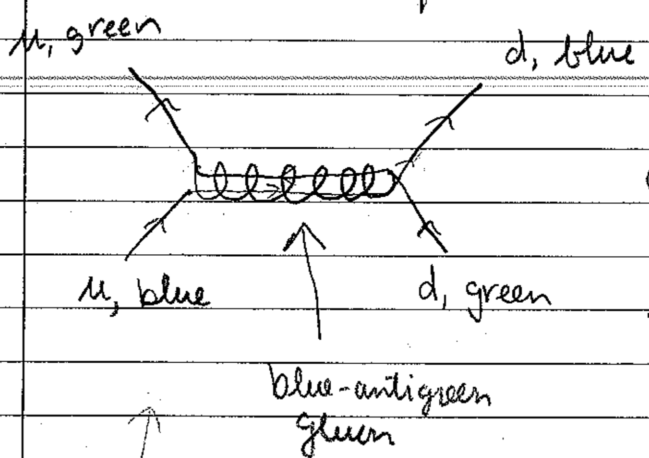
$$\mathcal{L}_{QED} = \bar{\Psi}(i\gamma^\mu D_\mu - m)\Psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

$$D_\mu = \partial_\mu - ieA_\mu$$

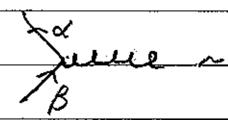
$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

QED		QCD
e^+, e^- electric charge e	$\longrightarrow$	quarks color charge g
1 photon ($S=1$, $m_\gamma=0$, no charge)	$\longrightarrow$	8 gluons ($S=1$, $m=0$, carry color charge $\Rightarrow$ self-interact!)

Forces between quarks via gluon exchange

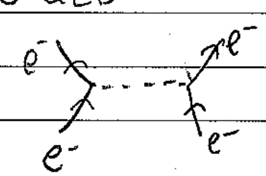


comes from the gluon part of $D_\mu = \partial_\mu - igA_\mu$

$\bar{\Psi}_\alpha g\gamma^\mu [A_\mu]_{\alpha\beta}\Psi_\beta$ ~  ~ g

i.e., this amplitude ~ $\left(\bar{\Psi} g A_\mu \Psi\right)^2 \sim \frac{g^2}{4\pi} = \alpha_s$

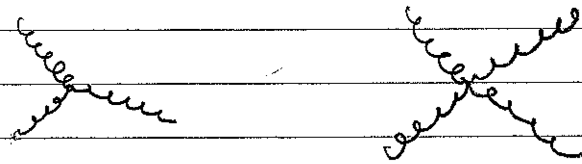
(analogy to QED

where  $\sim \frac{e^2}{4\pi} = \alpha \sim \frac{1}{137}$)

* Nonlinear gluon Self-interaction terms

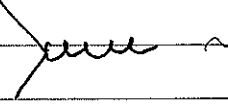
Recall gluon part of $\mathcal{L}_{QCD} \sim \left(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu\right)^2$

$\Rightarrow$



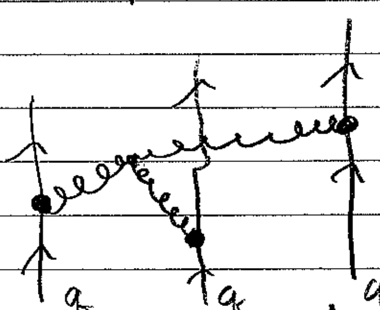
3 gluon interaction $\sim g$ 4 gluon interaction $\sim g^2$

$\therefore$ Sewing these non-linear 3- + 4-gluon vertices together with our quark-gluon vertex

 $\sim g$

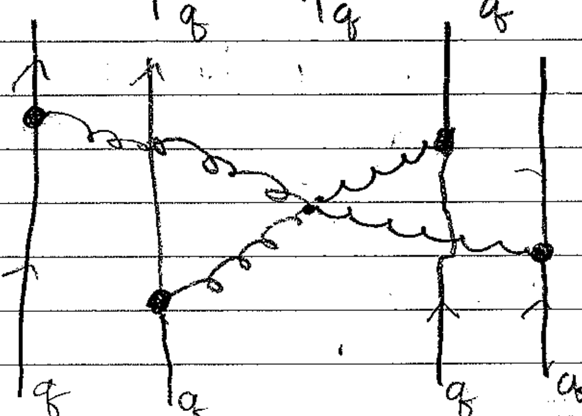
gives novel 3- + 4-quark interactions
that aren't present in a theory like QED without self-interactions

e.g.



$\sim g^4 \sim \alpha_s^2$

e.g.



$\sim g^6 \sim \alpha_s^3$

$\Rightarrow$ already at a fundamental level, we see the presence of many-body forces!

Question: What is α_s in nuclei?

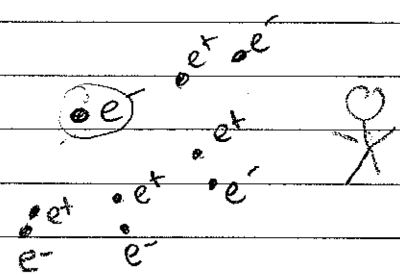
$\Downarrow$

Running Coupling

* It turns out that coupling constants in QFT are not actually constant. They depend on the momentum scale Q of the process under consideration.

* We say the coupling constants "run" as we change the energy/momentum scale.

QED: dielectric screening of the vacuum by virtual e^+e^- pairs increases the effective charge when you probe at short distances (i.e., high Q processes)



QCD: anti-screening by virtual pairs/gluons decreases the effective color charge as you probe shorter distances (higher Q processes) (unfortunately there's no qualitative picture I can draw!)

2004 Nobel
Gross, Politzer, Wilczek

$$\boxed{\frac{1}{\alpha_s(Q)} = \frac{33 - 2N_f}{6\pi} \log \frac{Q}{\Lambda_{QCD}} \qquad N_f = \#\text{ of flavors } (6 \text{ for our world})}$$

* Λ_{QCD} is the scale of QCD $\sim$ 200-400 MeV

* $m_{quarks} + \Lambda_{QCD}$ (instead of g) only inputs needed for QCD

* Chiral limit ($m_{\substack{light\\quarks}} \to 0$, $m_{\substack{heavy\\quarks}} \to \infty$), Λ_{QCD} is the only parameter.

$$\frac{1}{\alpha_s(Q)} = \frac{33-2N_f}{6\pi} \log \frac{Q}{\Lambda_{QCD}}$$

* $\alpha_s(Q)$ gets weaker as Q increases for $N_f \leq 16$

"Asymptotic freedom"

* Conversely, $\alpha_s(Q)$ gets big $\mathcal{O}(1)$ at energies relevant to nuclei => Non-perturbative

leads to 1) Confinement → DoF at low E are hadrons (composite systems of quarks) w/no net color

2) generation of mass ⟷ chiral symmetry breaking

Confinement + the Masses of Hadrons + Symmetries of QCD

bosons: "Mesons" $\pi, \rho, \ldots$ ($q\bar{q}$) (here we focus on hadrons made of $u, d, \bar{u}, \bar{d}$) like the π, ρ, N, Δ particles

fermions: "Baryons" $N, \Delta, \ldots$ (qqq)

* Show lattice QCD figure of Hadron masses + notice

1) $M_{hadrons} \sim 1$ GeV except for light π, K mesons

2) $\sim \Lambda_{QCD} \gg m_u, m_d$ (can think of Λ_{QCD} as the "standard kilogram" of QCD)

Discussion Question: What would the mass spectrum of π, ρ, N, Δ look like if zero quark masses $m_u = m_d = 0$?

Claim: $\pi \to 0$ but the others would still be $\mathcal{O}(1\,\text{GeV})$. I.e., most of our mass comes from non-perturbative QCD binding rather than the Higgs field (which is what gives $m_u + m_d$).

Degeneracy in the Hadronic Spectrum $\longleftrightarrow$ Isospin Symmetry

$m_u \approx m_d \longrightarrow$ "Isospin Symmetry" $\quad |u\rangle = |\text{isospin} \uparrow\rangle = |\tfrac{1}{2}\tfrac{1}{2}\rangle$

$|d\rangle = |\text{isospin} \downarrow\rangle = |\tfrac{1}{2}\,{-\tfrac{1}{2}}\rangle$

Isospin operator $\vec{T} = \frac{\vec{\tau}}{2}$ $\quad \tau_i$ = Pauli matrices

* This is an approximate symmetry (else $m_u = m_d$) that is clearly seen in the form of new degeneracies in the Hadron spectrum

e.g. Baryons: Nucleon $N(J^\pi = \frac{1}{2}^+)$ $\quad |n\rangle = |T=\tfrac{1}{2}, m_T = -\tfrac{1}{2}\rangle = |\underbrace{(ud)}_{S=0}d\rangle$

$m_N \sim 940$ MeV $\quad |p\rangle = |T=\tfrac{1}{2}, m_T = +\tfrac{1}{2}\rangle = |\underbrace{(ud)}_{S=0}u\rangle$

$T = \frac{1}{2}$ multiplet

Delta $\Delta(J^\pi = \frac{3}{2}^+)$ Isobars

$m_\Delta \sim 1232$ MeV

$T = \frac{3}{2}$ multiplet

	T	m_T	
$\lvert\Delta^-\rangle =$	$\lvert\tfrac{3}{2},$	$-\tfrac{3}{2}\rangle$	$= \lvert ddd\rangle$
$\lvert\Delta^0\rangle =$	$\lvert\tfrac{3}{2},$	$-\tfrac{1}{2}\rangle$	$= \lvert \underbrace{(ud)}_{S=1}d\rangle$
$\lvert\Delta^+\rangle =$	$\lvert\tfrac{3}{2},$	$\tfrac{1}{2}\rangle$	$= \lvert u\underbrace{(ud)}_{S=1}\rangle$
$\lvert\Delta^{++}\rangle =$	$\lvert\tfrac{3}{2},$	$\tfrac{3}{2}\rangle$	$= \lvert uuu\rangle$

Mesons: pseudoscalar $\pi(J^\pi = 0^-)$ pions

$m_\pi \sim 140$ MeV

$|\pi^+\rangle = |1,+1\rangle = |u\bar{d}\rangle$

$|\pi^0\rangle = |1,0\rangle = (|u\bar{u}\rangle - |d\bar{d}\rangle)/\sqrt{2}$

$|\pi^-\rangle = |1,-1\rangle = |d\bar{u}\rangle$

Vector $\rho(1^-)$ rho-meson

$m_\rho \sim 770$ MeV

$|\rho^-\rangle = |1,-1\rangle = |d\bar{u}\rangle$

$|\rho^0\rangle = |1,0\rangle = |u\bar{u}\rangle - |d\bar{d}\rangle$

$|\rho^+\rangle = |1,+1\rangle = |u\bar{d}\rangle$

later, we'll see the forces between nucleons mediated by the exchange of mesons in the effective description

Why is the Pion So light? (Chiral Symmetry)

* Let's go back to the observation that $m_u, m_d \ll \Lambda_{QCD}$
* If a quark has $m=0$, then the spin can either be in the direction of motion ("Right-handed" quark) or in the opposite direction ("Left-handed" quark)

* Claim we can decompose a Dirac spinor as

$$\psi = \psi_L + \psi_R \qquad \psi_L = \tfrac{1}{2}(1-\gamma_5)\psi$$
$$\psi_R = \tfrac{1}{2}(1+\gamma_5)\psi$$

Dirac Matrix $\gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3$

* Let's look at the quark Lagrangian for u,d quarks w/ $m_{u,d}=0$

$$\Rightarrow \mathcal{L}_q = \bar{u}(i\gamma^\mu D_\mu)u + \bar{d}(i\gamma^\mu D_\mu)d$$
$$= \bar{u}_L(i\not{D})u_L + \bar{u}_R(i\not{D})u_R + \bar{d}_L(i\not{D})d_L + \bar{d}_R(i\not{D})d_R$$

Feynman slash shorthand
$\not{D} = \gamma^\mu D_\mu$

* L,R components don't mix when $m=0$.

* $\mathcal{L}_q$ invariant under <u>independent</u> rotations of L,R component of the isospinor $\psi = \begin{pmatrix} u \\ d \end{pmatrix}$

$$\psi_L' \equiv \begin{pmatrix} u_L' \\ d_L' \end{pmatrix} = U_L \begin{pmatrix} u_L \\ d_L \end{pmatrix} \qquad U_{L,R} = 2\times 2 \text{ unitary matrices}$$

$$\psi_R' \equiv \begin{pmatrix} u_R' \\ d_R' \end{pmatrix} = U_R \begin{pmatrix} u_R \\ d_R \end{pmatrix}$$

$U(2) \sim SU(2) \times U(1)$ (any 2x2 Unitary matrix decomposed as a 2x2 Special unitary matrix times a phase)

$\therefore$ We say $\mathcal{L}_q(m_{u,d}=0)$ is symmetric under

$$SU(2)_L \times SU(2)_R \times U_L(1) \times U_R(1)$$

$$= \underbrace{SU(2)_{L+R}}_{\substack{\text{vector} \\ = \\ \text{isospin symmetry}}} \times \underbrace{SU(2)_{L-R}}_{\substack{\text{axial} \\ = \\ \text{chiral symmetry}}} \times \underbrace{U_V(1)}_{\text{baryon \# symmetry}} \times \underbrace{U_A(1)}_{\text{broken by quantum effects (anomaly)}}$$

① $SU(2)_{\text{isospin}}$ is present in Hadron spectrum

② $SU(2)_{\text{chiral}}$ implies degenerate parity partners
(axial rotations change parity)

e.g. for the nucleon + it's odd parity excitation, we'd expect them to be degenerate $N(\frac{1}{2}^+)$ and $N^*(\frac{1}{2}^-)$

But $N(\frac{1}{2}^+) = 940\,\text{MeV}$, $N^*(\frac{1}{2}^-) \cong 1500\,\text{MeV}$ } Not even close to being degenerate!

We say the full $SU_L(2) \times SU_R(2)$ symmetry is spontaneously broken to the isospin subgroup

$$SU(2)_L \times SU(2)_R \longrightarrow \underset{\text{(isospin)}}{SU(2)_{L+R}}$$

Spontaneous Symmetry Breaking

* general phenomena that pervades all areas of physics

* When the Hamiltonian (or Lagrangian) is symmetric under some symmetry group, but the ground state is not.

e.g. a magnet below $T < T_{curie}$

$$[H, R(\theta)] = 0$$ (H is invariant wrt rotating the spins that live on the atomic lattice)

Yet, for $T < T_c$ the spins collectively point in some direction, so the ground state is no longer invariant under arbitrary rotations since a direction $\vec{M} = \langle \vec{S} \rangle$ is picked out in space.

Goldstone theorem: When a continuous symmetry is broken, $J=0$ massless particles ($E \sim K$) emerge "Goldstone bosons"

* We will say much more about SSB + Goldstone Bosons later.

* For now, we claim the broken $SU(2)_A$ symmetry implies the existence of 3 massless goldstone bosons which are ... the π^0, π^+, π^- !!

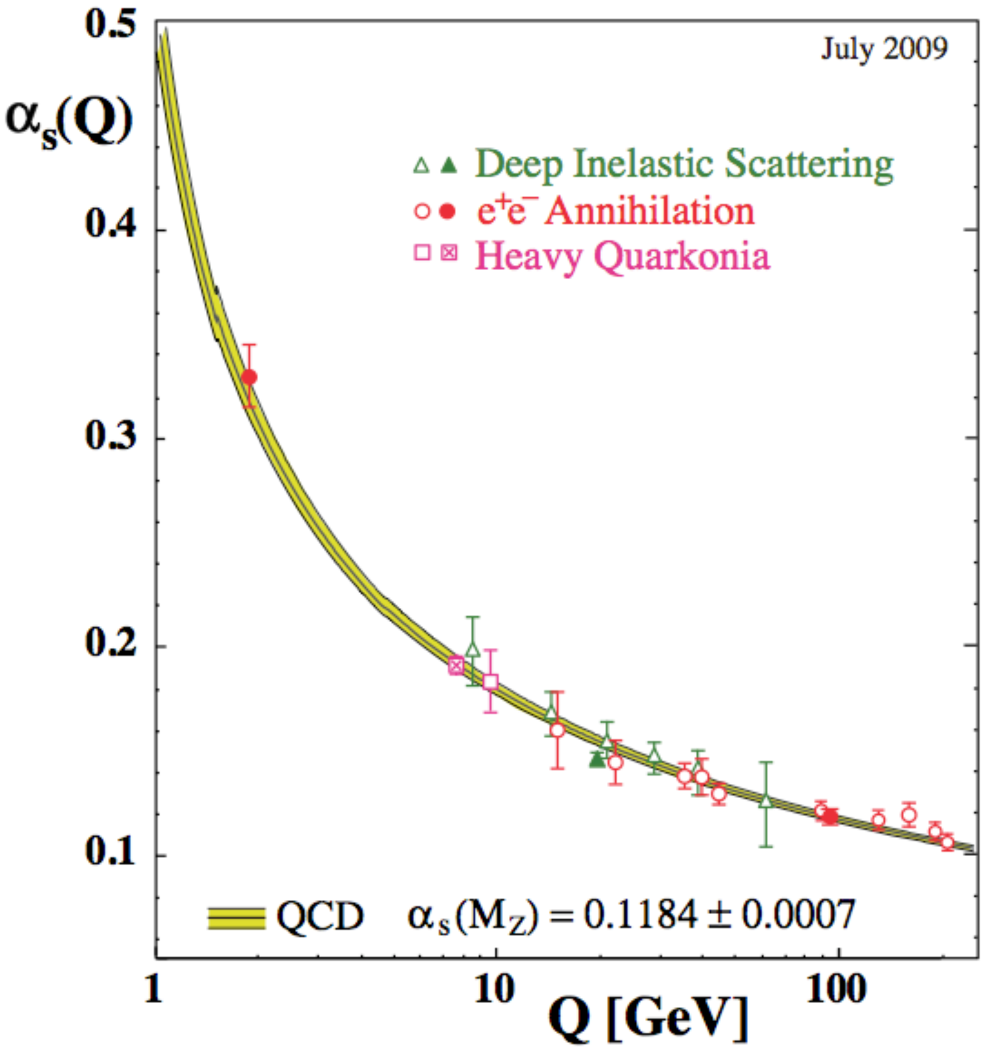
* SSB/GT also implies the interactions w/ π's are weak at low K, even though the coupling constants themselves are not small

* later, we'll see how this connects to the existence of 3, 4, ... A-body forces amongst nucleons, as well as the (fortunate!) hierarchy

$$V_{2N} > V_{3N} > V_{4N} \cdots$$

Figures from: Review of Particle Physics http://pdg.lbl.gov
Kronfeld and Quigg, Resource Letter QCD, arXiv:1002.5032.

Strong coupling



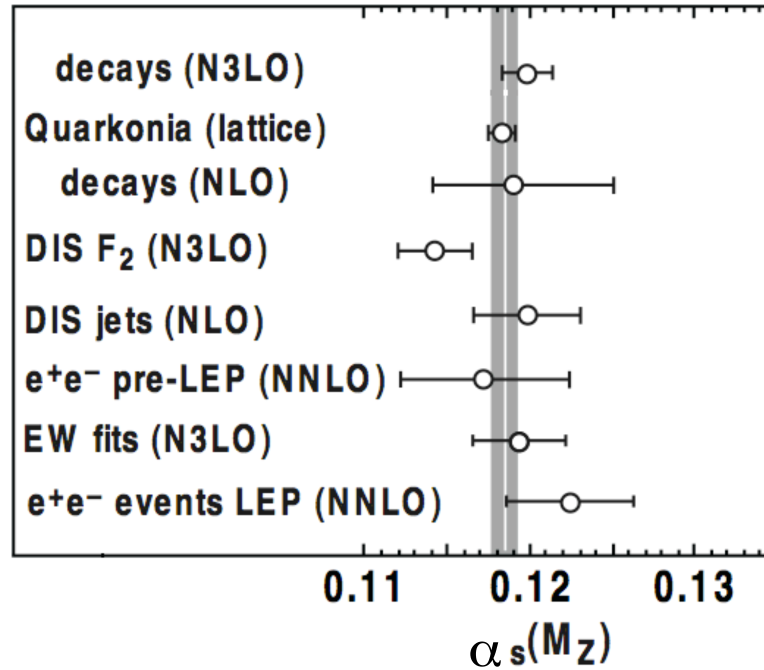


FIG. 4: Determinations of $\alpha_s(M_Z)$ from several processes. In most cases, the value measured at a scale μ has been evolved to $\mu = M_Z$. Error bars include the theoretical uncertainties. Adapted from Ref. [284].

Meson and baryon masses from lattice QCD BMW collaboration (2010)

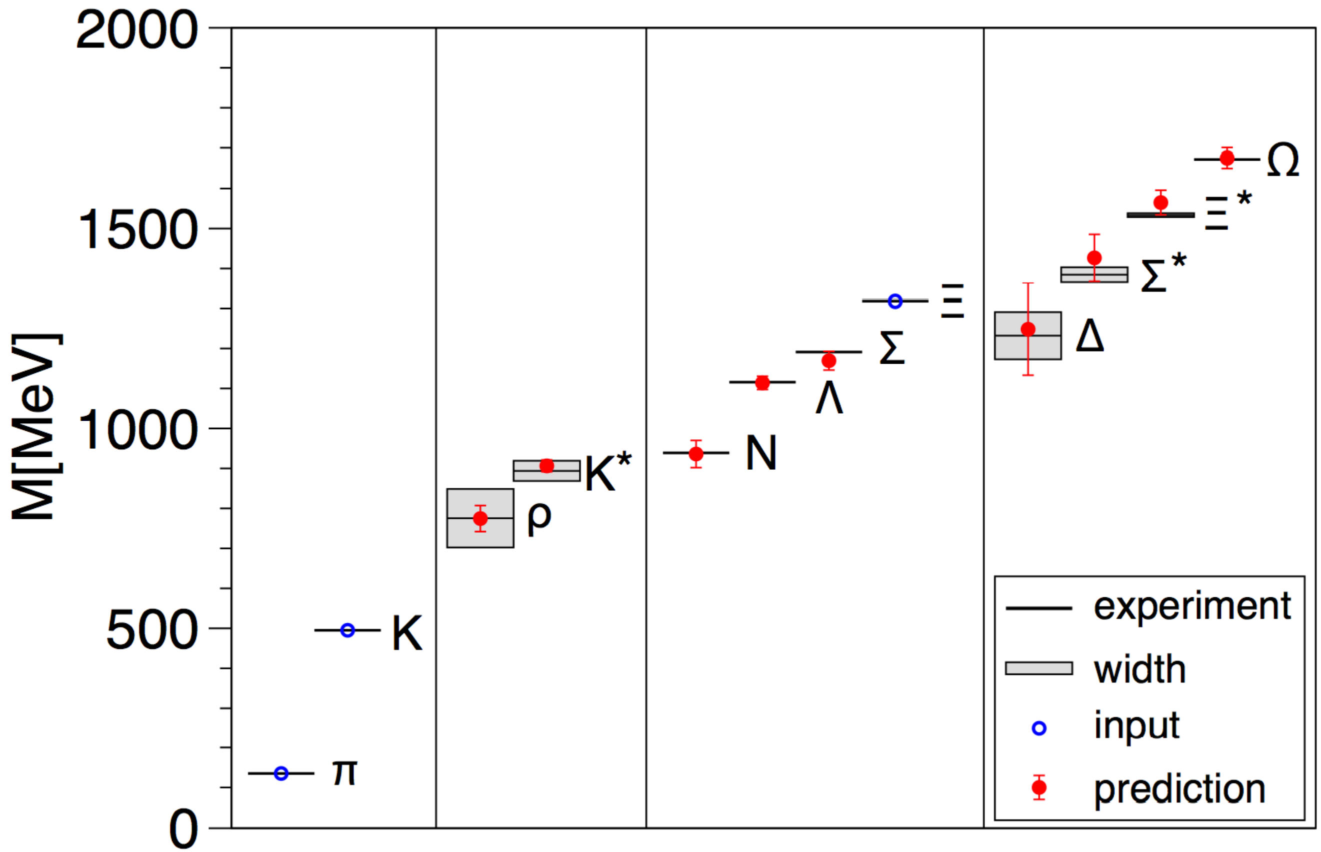


FIG. 22 Prediction of the light hadron spectrum in full $N_f = 2 + 1$ QCD according to (Durr *et al.*, 2008). Open circles are input quantities while filled circles are predictions. Experimental masses of hadrons that are stable in QCD are given with a vertical bar while for resonant states the box indicates the decay width. Experimental numbers are from (Amsler *et al.*, 2008).