Q.1) $T(n) = 3T(n/2) + n^2$

A.1) $a = 3$

$b = 2$

as $a > 1, b > 1$

$\Rightarrow c = \log_b a = \log_2 3 = 1.58$

$\Rightarrow n^c = n^{1.58}$

as $f(n) > n^c$

$T(n) = \Theta f(n)$

$= \Theta(n^2)$. Ans

Q.2) $T(n) = 4T(n/2) + n^2$

A.2) $a = 4$

$b = 2.$

as $a > 1, b \geq 1$

$\Rightarrow c = \log_b a = \log_2 4 = 2. \Rightarrow n^c = n^2$

so as $f(n) = n^c$

$\Rightarrow T(n) = \Theta(n^2 \log n)$ Ans

Q.3) $T(n) = T(n/2) + 2^n$

$a = 1$

$b = 2.$

∵ the following $f()$ is exponential & not polynomial it can't be solved.

Q.4) $T(n) = 2^n T(n/2) + n^n$

– Master theorem does not apply here because a is not constant.

Q5) $T(n) = 16T(n/4) + n$

$a = 16$

$b = 4$

$\Rightarrow c = \log_b a = \log_4 16 = \frac{1.204}{0.60} = 2$

so $n^c = n^2$

as $n^c > f(n)$

$\Rightarrow T(n) = \Theta(n^2)$

Q6) $T(n) = 2T(n/2) + n\log n$

$a = 2$

$b = 2$

$a > 1, b > 1$ $\quad \Rightarrow c = \log_b a = \log_2 2 = 1. \Rightarrow n^c = 1.$

$\Rightarrow n^c = 1$.

assuming $f(n)$ i.e $n\log(n) > -1$.

$\Rightarrow T(n) = \Theta(n^{\log_b a} \log^{P+1} n)$

$= \Theta(n^1 \log^{1+1} n)$

$= \Theta(n \log^2 n)$ Ans

Q7) $T(n) = 2T(n/2) + n/\log n$

$\because$ Master theoram applies to form that are polynomial, $n/\log n$ is not polynomial so master theoram does not apply.

Q.8) $T(n) = 2T(n/4) + n^{0.51}$

A.8) $a = 2$

$b = 4.$

$\Rightarrow c = \log_b a = \log_4 2 = \frac{\log(2)}{\log(4)} = \frac{0.30}{0.60} = 0.5.$

as $f(n) > n^c$

$\Rightarrow T(n) = \theta f(n) = \theta(n^{0.51})$ Ans

Q.9.) $T(n) = 0.5T(n/2) + 1/n$

— Does not apply ∵ $a < 1$.

Q.10.) $T(n) = 16T(n/4) + n!$

$a = 16$

$b = 4$

$\Rightarrow c = \log_b a = \log_4 16 = \frac{\log(2)^4}{\log 4} = \frac{4\log 2}{\log 4}$

$= \frac{4 \times 0.30}{0.60} = 2.0$

as $n^c = n^2$

but $f(n) > n^c$

$\Rightarrow T(n) = \theta f(n)$

$= \theta f(n!)$ Ans

Q.11.) $T(n) = \sqrt{2}T(n/2) + \log n$

$a = \sqrt{2}$
$b = 2.$

$c = \log_b a = \log_2(\sqrt{2}) = \frac{\frac{1}{2}\log 2}{\log 2} = \frac{1}{2} = 0.5.$

so $n^c = n^{0.5}.$

$\Rightarrow f(n) = \log n$

as $f(n) < n^c$

$\Rightarrow T(n) = \Theta(n^c) = \Theta(n^{0.5})$ Ans

Q.12) $T(n) = \sqrt{n}\, T(n/2) + \log n$

$\because$ a is not a constant, following form cannot be solved.

Q.13) $T(n) = 3T(n/2) + n$

$a = 3$
$b = 2$

$\Rightarrow c = \log_b a = \log_2 3 = \frac{0.69}{0.30} = 2.3.$

as $n^c > f(n)$

$\Rightarrow T(n) = \Theta(n^c) = \Theta(n^{2.3})$ Ans

Q.14) $T(n) = 3T\left(\frac{n}{3}\right) + (\sqrt{n})$

$a = 3$
$b = 3$

$\Rightarrow c = \log_b a = \log_3 3 = 1 \Rightarrow n^c = n.$

as $n^c < f(n)$ i.e $n^1 > n^{1/2}$

$$\Rightarrow T(n) = \Theta(f(n))$$
$$= \Theta(n) \text{ Ans}$$

Q.15) $T(n) = 4T(n/2) + cn$

A.15) $a = 4 \quad b = 2$

$C = \log_2 4 = 2.$

$n^c = n^2 > f(n)$

$T(n) = \Theta(n^2).$

Q.16) $T(n) = 3T(n/4) + n\log n$

$a = 3,\ b = 4$

$C = \log_4 3 = 0.79$

$n^c = n^{0.79} < f(n)$

$T(n) = \Theta(n\log(n))$

Q.17) $T(n) = 3T(n/3) + n/2$

$a = 3 \quad b = 3$

$c = \log_3 3 = 1$

$n^c = n > f(n)$

$T(n) = \Theta(n)$

Q.18) $T(n) = 6T(\frac{n}{3}) + n^2\log n$

$a = 6,\ b = 3$

$c = \log_3 6 = 1.63$

$n^c = n^{1.63} < f(n)$

$T(n) = \Theta(n^2 \log n)$.

Q.19) $T(n) = 4T\left(\frac{n}{2}\right) + n/\log n$

A.19) $a = 4,\ b = 2$

$c = \log_2 4 = 2$

$n^c = n^2 > f(n)$

$T(n) = \Theta(n^2)$

Q.20.) $T(n) = 64T\left(\frac{n}{8}\right) - n^2 \log n$

$a = 64\ \ b = 8$

$c = \log_8 64 = 2$

$n^c = n^2 < f(n)$

$T(n) = \Theta(n^2 \log {}^1/n)$

Q.21) $T(n) = 7T(n/3) + n^2$

$a = 7,\ b = 3$

$c = \log_3 7 = 1.77$

$n^c = n^{1.77} < f(n)$

$T(n) = \Theta(n^2)$

Q.22) $T(n) = T(n/2) + n(2\cos n)$

$a = 1,\ b = 2,\ c = \log_2 1 = 0.$

$n^c = n^0 = 1 < f(n)$

$T(n) = \Theta(n(2 - \cos n))$