Maximizing a Diet's Nutritional Value Given Certain Foods

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1 Abstract

This formal report addresses the pressing issue of food insecurity in British Columbia. The primary objective is to propose a viable solution to household food insecurity by optimizing the seasonal average cost per person, considering factors such as gender, dietary restrictions, and activity levels.

To achieve this goal, the report utilizes linear programming in Python, employing the pulp library to maximize the nutritional value of common food items while simultaneously minimizing their cost. Given the fluctuating prices of food products over the months, the study aims to identify the most optimal diets for both vegetarians and omnivores by evaluating 15 food items to calculate the seasonal cost of a balanced diet.

The theoretical framework introduces essential linear programming concepts, including the Simplex Algorithm and the Duality theorem. This comprehensive approach seeks to contribute to the development of effective strategies to combat food insecurity in BC.

The report delves into sensitivity analysis, uncovering significant differences in nutritional requirements and costs between low-active and active scenarios. Notably, even a slight increase in calorie limits results in considerable price variations, emphasizing the importance of nutrient adjustments.

Furthermore, the report introduces the concept of duality in linear programming, formulating a dual problem to minimize the cost of meeting constraints while maximizing shadow prices associated with each constraint. The dual solution provides insights into the value or cost per unit of each nutrient in the diet.

In conclusion, the report underscores the challenges of food insecurity in British Columbia and advocates for policy changes make nutritious food more affordable. The findings provide valuable insights for health authorities and policymakers to address individual food insecurity. The report encourages future studies to explore household food insecurity comprehensively, considering brackets, household size, location, etc., to effectively tackle this important issue in BC.

2 Introduction

According to a report by the BC Alliance for Healthy Living, one in six (15.6%) children in British Columbia under the age of 18 lived in households experiencing some level of food insecurity ("Ripe for Change – Food Insecurity in BC"). Additionally, over one in ten (11.8%) BC households experienced some level of food insecurity and about 3% of households experienced severe food insecurity. That's about 485,500 and 91,100 British Columbians, respectively, experience difficulty in acquiring food ("Household Food Insecurity in BC").

Food insecurity is recognized as a key public health issue in BC. It is defined as the inability to purchase healthy and safe food due to a lack of financial means ("Household Food Insecurity in Canada").

It not only increases the risk of chronic conditions, like asthma and diabetes, but also the likelihood of depression, social isolation, and distress (”Household Food Insecurity in BC”, Laraia). The primary goal of food security is to improve the overall health and wellness of BC residents. It provides access to nutritious, safe, and personally acceptable food by increasing the availability of healthy food at a sustainable cost.

Household food insecurity is the inability for a household to purchase healthy and personally acceptable food. In this report, we aim to provide a solution to tackle household food insecurity by minimizing the seasonal average cost for each member of the household based on their gender.

3 Aim

This paper attempts to provide a solution to food insecurity in BC by formulating a linear programming problem to maximize the nutrition of common food items and minimize their cost. Considering the fact that the price of food products changes according to the season, we aim to find the optimal diet in each season. Furthermore, to add complexity we aim to find the optimal diet for adults with different dietary preferences. For the purpose of this report, we will only consider two dietary groups: vegetarians and omnivores.

Table 1 contains the daily nutrient requirements for each gender in order to maintain a healthy diet (Dietary Reference Intakes). This information is used in constraint formulation.

Gender	Calories (kCal/day)	Protein (g/day)	Carbohydrates (g/day)	Fiber(g/day)
Women	2104	46	237	30
Men	2720	56	306	38

Table 1: Average Nutrient Requirements

4 Data

Sourced from Statistics Canada, we used monthly average retail prices for food and other selected products' data sets. This data set contains the monthly price of 52 common products for the year 2021. For the purpose of this report, we will only be considering 15 food items in order to calculate the seasonal cost of a balanced diet.

The nutrition value of each food item is taken from a Kaggle data set. Data for each food item is further classified by the number of calories, protein, carbohydrates, fiber, and fats it contains. For simplicity, we only looked at the value of calories, proteins, carbohydrates, and fiber for the select food items selected. Figure 1 displays the cleaned data set we worked with.

In Figure 2, Figure 3, Figure 4, and Figure 5, we look at the distribution of nutritional values over grams for the food items. From the distributions, wee see that there is no food item that has low calories and moderate levels of carbohydrates, protein, and fiber. So, this tells us that diversity will be important when we create our model to ensure that nutrient requirements are met, and so that our model doesn't only use a few foods to meet the requirements.

Food	Grams	Calories	Protein	Fiber	Carbs	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Apples	130	70	0	1.0	18	4.53	4.49	4.57	4.55	4.64	4.66	4.79	4.97	4.90	4.61	4.44	4.76
Bacon	16	95	4	0.0	1	7.32	7.44	7.29	7.69	7.89	7.79	7.99	8.24	8.27	8.29	8.47	8.66
Bananas	150	85	1	0.9	23	1.56	1.57	1.56	1.56	1.56	1.56	1.56	1.60	1.60	1.60	1.60	1.61
Bread	35	86	3	2.4	16	2.79	2.87	2.82	2.85	2.88	2.82	2.80	2.82	2.81	2.84	2.89	2.98
Canned salmon	85	120	17	0.0	0	6.67	6.60	6.48	6.41	6.23	6.53	6.64	6.53	6.41	6.66	6.53	6.53
Canned tomatoes	240	50	2	1.0	9	1.45	1.48	1.49	1.60	1.60	1.63	1.58	1.74	1.70	1.65	1.57	1.71
Carrots	150	45	1	0.9	10	2.25	2.34	2.33	2.43	2.49	2.52	2.64	2.37	2.40	2.32	2.36	2.43
Chicken	85	185	23	0.0	0	7.35	7.00	7.73	7.87	7.76	7.72	8.07	8.13	8.30	8.51	8.18	7.87
Eggs	100	150	12	0.0	0	3.64	3.62	3.77	3.80	3.74	3.75	3.74	3.91	3.82	3.87	3.87	3.82
Ground beef	85	245	23	0.0	0	11.55	11.06	11.22	11.53	11.28	11.00	11.31	11.56	11.86	12.31	12.46	12.06
Onions	210	80	2	1.6	18	2.29	2.32	2.31	2.25	2.35	2.38	2.45	2.44	2.31	2.14	2.29	2.25
Oranges	180	60	2	1.0	16	3.59	3.77	3.61	3.66	3.90	4.04	3.97	3.88	3.90	4.02	4.12	4.11
Partly skimmed milk	258	129	9	0.0	12	5.30	5.48	5.49	5.50	5.51	5.48	5.51	5.49	5.49	5.52	5.50	5.52
Potatoes	100	100	2	0.5	22	10.34	10.09	10.31	10.23	10.58	10.50	10.72	10.67	10.19	10.17	9.89	10.36
Processed cheese slices	21	78	5	0.0	0	2.69	2.79	2.84	2.77	2.83	2.81	2.72	2.77	2.61	2.66	2.63	2.63

Figure 1: Data

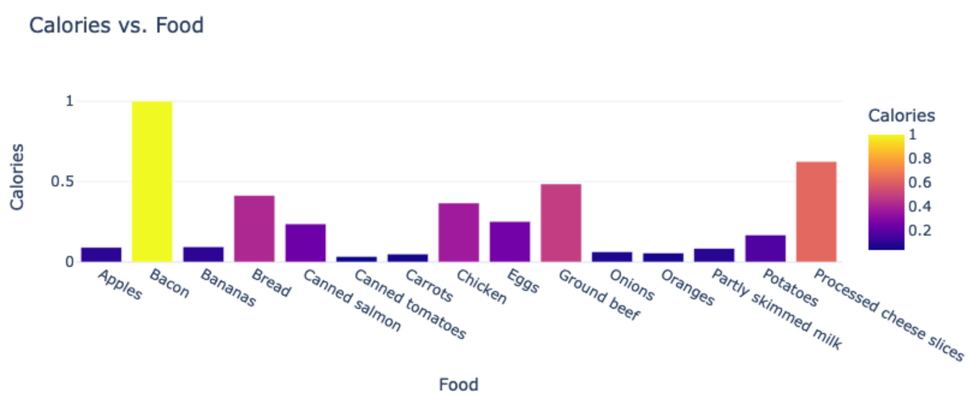

Figure 2: Calorie Distribution

5 Theory

In this section, an insight will be given about the techniques and algorithms used in this report.

5.1 Linear Programming Problem

5.1.1 Basic concepts of Linear Programming

1. A Linear Programming Problem is a mathematical model with an objective function to be maximized or minimized. The model is subject to a set of linear constraints to make the problem bounded with a definitive optimal.

2. We call the the set of all possible points that satisfy all the constraints of the LPP, the feasible region. These points represent feasible all solutions.

3. Vertices of the feasible Region are points at which the constraint lines intersect. This can be easily visualized for LPP with two decision variables but more difficult if we increase the number of decision

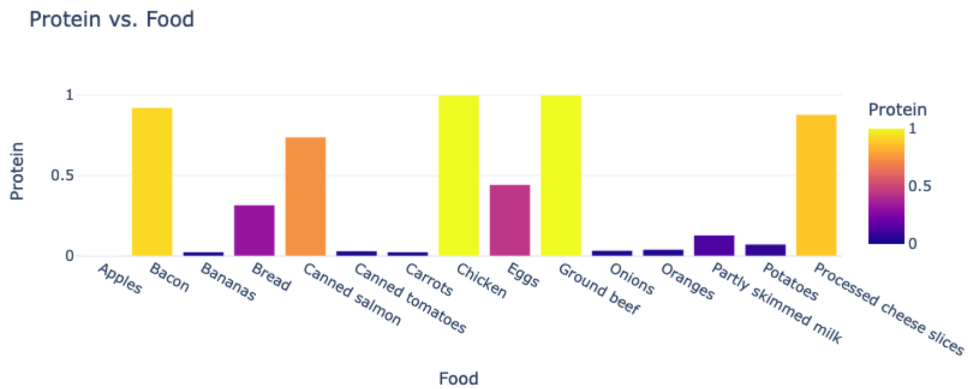

Figure 3: Protein Distribution

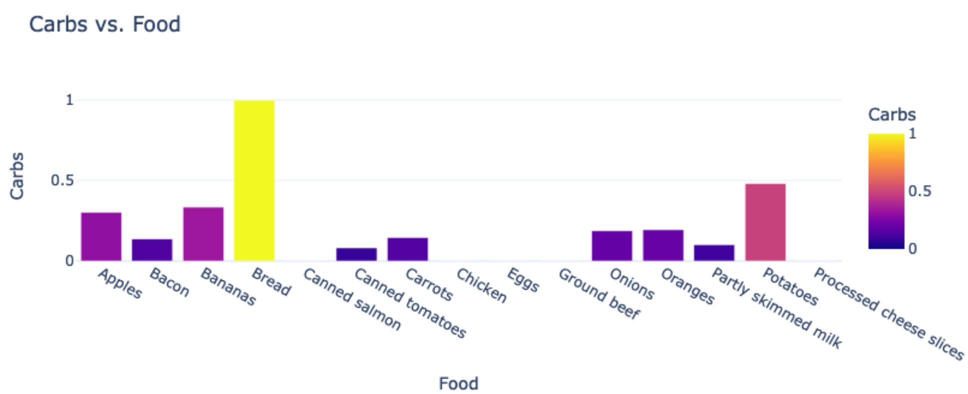

Figure 4: Carbohydrate Distribution

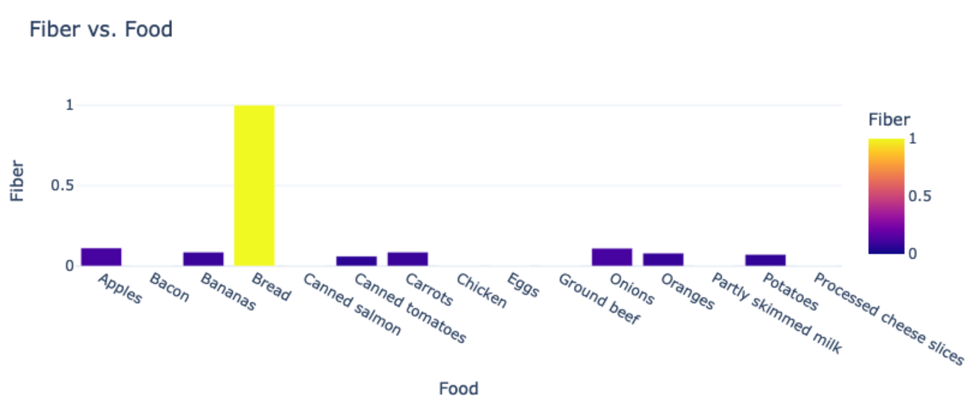

Figure 5: Fiber Distribution

variables. The Simplex Algorithm operates on the principle that the optimal solution of a linear programming problem, if it exists, will be found at one of these vertices. To do so, we require an efficient algorithm that allows us to reach that point with fewest steps possible.

5.2 The Simplex Algorithm

The Simplex Algorithm is a popular method used in linear programming for solving optimization problems. It's designed to find the best outcome in a mathematical model whose requirements are represented by linear relationships.

The Simplex Algorithm is used in various fields like business, economics, engineering, and the military, wherever there is a need to maximize efficiency or minimize cost under a set of constraints.

Here are some basic concepts of the Linear Programming:

5.2.1 Theory of the Simplex Algorithm

1. The Simplex Algorithm works with systems of linear equations. Inequalities in constraints are usually converted to equations with the addition of slack, surplus, or artificial variables.

2. The algorithm starts at a feasible vertex (corner point) of the feasible region, it usually is the origin point. If the starting point is infeasible, the 'Two-Phase' method is used to find a starting point. Where Phase I introduces an auxiliary problem that takes the current infeasible point to a feasible one

3. From the starting vertex, the algorithm detects most vertex point that minimizes or maximizes the objective function value the most and moves to that adjacent vertex. In other words, the movement is in the direction that increases (for maximization problems) or decreases (for minimization problems) the value of the objective function.

4. At each movement iteration the algorithm checks for an optimality condition. If this condition is met, the algorithm stops, and the current solution is declared optimal. Otherwise, if the optimality condition is not met, the algorithm performs a pivot operation. This involves choosing which edge of the feasible region to follow to the next vertex.

5. The algorithm terminates when it reaches a point where no adjacent vertex provides a better solution.

While the Simplex Algorithm is very efficient for most practical problems, it's worth noting that it can, in the worst case, require a number of steps exponential in the size of the problem. However, this is rare in practice.

5.3 Duality theorem

The Duality Theorem is another concept in linear programming, closely related to the idea of optimizing a problem. It states that every linear programming problem (referred to as the "Primal Problem") is associated with another linear programming problem, known as the "Dual Problem". The relationship between the primal and the dual problem has some interesting and useful properties.

Here are some basic concepts about the dual problem:

5.3.1 Basic concepts

1. The Primal problem is the original linear programming problem. It could be a maximization or minimization problem with a set of constraints.

2. Each Primal problem has a related Dual problem. If the primal is a maximization problem, the dual will be a minimization problem, and vice versa.

5.3.2 Duality theory

The Duality Theorem establishes the following key relationships between the primal and the dual problem:

1. If the primal problem has an optimal solution, then so does the dual problem, and their optimal values are equal. This means if you solve the primal problem, the solution to the dual problem is also implicitly solved, and vice versa.

2. The value of the objective function of the dual at any feasible solution is always a bound on the value of the objective function of the primal problem. Specifically, for any feasible solution of the dual problem, the objective function value is an upper bound for the primal maximization problem and a lower bound for the primal minimization problem.

3. If both the primal and the dual have feasible solutions, then they both have optimal solutions, and the optimal values of their objective functions are equal. (Strong duality)

4. Complementary slackness provides a way to check if a given feasible solution is optimal. It states that for each pair of primal and dual constraints, either the primal constraint is binding (at its equality), or the corresponding dual variable is zero, and vice versa.

All in all, the duality concept offers deep insights into the nature of the solution of the linear programming problem. It helps in understanding the underlying economics of the problem. For example, in a resource allocation problem, the dual variables can represent the shadow prices or the value of one additional unit of resources.

5.4 Sensitivity Analysis

Sensitivity Analysis in linear programming, both for the primal and dual problems, involves studying how changes in the parameters of the model affect the optimal solution and the value of the objective function. This is crucial in real-world scenarios, where data and conditions often change. Let's explore how sensitivity analysis applies to both the primal and dual problems.

5.4.1 Sensitivity Analysis in the Primal Problem

In the primal problem, sensitivity analysis often focuses on three main aspects:

- Changes in Coefficients of the Objective Function: This analysis determines how changes in the costs or profits (coefficients in the objective function) affect the optimal solution. It identifies the range within which the coefficients can vary without changing the optimal solution's structure (basis).

- Changes in the Right-Hand Side Constants: These are the constants in the constraint equations. Sensitivity analysis here examines how changes in resource availability or requirements (the constants in the constraints) influence the optimal solution. It identifies the allowable increase or decrease in these constants before the current optimal solution becomes infeasible or non-optimal.

- Adding or Removing Constraints:** This involves understanding the impact of new constraints on the existing optimal solution, or how the removal of certain constraints would affect the solution.

5.4.2 Sensitivity Analysis in the Dual Problem

In the dual problem, sensitivity analysis also focuses on similar aspects but from a different perspective:

- Changes in Dual Prices (Shadow Prices): This examines how changes in the value of resources (reflected in the dual prices or shadow prices) affect the optimal allocation of resources. Shadow prices give insights into the marginal worth of resources.

- Changes in Coefficients in the Dual Objective Function:** This is analogous to changes in the right-hand side constants of the primal problem. It looks at how changes in the values that the primal constraints must achieve (the objectives of the dual problem) affect the optimal values of the dual variables.

- Changes in the Dual Constraints: These are related to changes in the coefficients of the primal objective function. The analysis here focuses on how these changes impact the optimal resource allocation and the value of the dual objective function.

6 Methods

6.1 Decision Variables

The decision variables are the quantity of food for each month. For example, $x_{1,4}$ is the decision variable for apples in April. So, for 15 foods and 12 months, we have 180 decision variables.

```
prob = pulp.LpProblem("diet_prob", LpMaximize)
food_vars = [...]
month_vars = [...]
vars = LpVariable.dicts("var", ((food, month) for food in food_vars for month in month_vars),
lowBound=0)
b = list(vars.values())
l = []
a = 0;
for i in range(0,15):
    for j in range(1,13):
        l.append(c.iloc[i,j]*b[a])
        a += 1
```

6.2 Objective Function

We formulated the objective function through the following steps:

1. To ensure the uniformity of units, the value of each nutrient (calories, carbohydrates, protein, and fiber) is divided by the weight of that particular food item in grams. This gives us the nutrient per gram for each food item. For example, considering only calories for apples it can be written as:

$$\text{calories per gram for apples} = \frac{\text{value of calories}}{\text{weight of apples}} = \frac{70}{130} = 0.538$$

2. Furthermore, ensure that all nutritional values can be compared, we take the value of a particular nutrient per gram over the maximum value of that nutrient amongst all food items. For example the nutritional value of carbohydrates in apples is:

$$\text{nutritional value of carbohydrates in apples} = \frac{\text{calories per gram for apples}}{\text{maximum value of calories for all foods}} = 0.090688$$

3. Then, we get the coefficient of the objective function by dividing the total nutrition value of each food item by the item's price each month. Figure 6 shows the coefficients for each variable. For example, the coefficient for the apples in April is given by:

$$c_{1,4} = \frac{\text{total nutritional value for apples}}{\text{price of apples in April}}$$

where the total nutritional value for apples is:

$$\begin{aligned}\text{total nutritional value of apples} = {} & \text{value of carbohydrates in apples} + \\ & \text{value of calories in apples} + \\ & \text{value of proteins in apples} + \\ & \text{value of fiber in apples}\end{aligned}$$

Food	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Apples	0.112	0.113	0.111	0.111	0.109	0.109	0.106	0.102	0.103	0.110	0.114	0.106
Bacon	0.282	0.277	0.283	0.268	0.261	0.265	0.258	0.250	0.249	0.249	0.243	0.238
Bananas	0.348	0.346	0.348	0.348	0.348	0.348	0.348	0.339	0.339	0.339	0.339	0.337
Bread	0.979	0.951	0.968	0.958	0.948	0.968	0.975	0.968	0.972	0.961	0.945	0.916
Canned salmon	0.146	0.148	0.151	0.152	0.157	0.150	0.147	0.150	0.152	0.147	0.150	0.150
Canned tomatoes	0.144	0.141	0.140	0.130	0.130	0.128	0.132	0.120	0.123	0.126	0.133	0.122
Carrots	0.137	0.132	0.132	0.127	0.124	0.122	0.117	0.130	0.129	0.133	0.131	0.127
Chicken	0.186	0.195	0.177	0.174	0.176	0.177	0.169	0.168	0.165	0.161	0.167	0.174
Eggs	0.191	0.192	0.185	0.183	0.186	0.186	0.186	0.178	0.182	0.180	0.180	0.182
Ground beef	0.129	0.134	0.132	0.129	0.132	0.135	0.131	0.128	0.125	0.121	0.119	0.123
Onions	0.174	0.172	0.172	0.177	0.169	0.167	0.162	0.163	0.172	0.186	0.174	0.177
Oranges	0.104	0.099	0.103	0.102	0.096	0.092	0.094	0.096	0.096	0.093	0.090	0.091
Partly skimmed milk	0.059	0.057	0.057	0.057	0.057	0.057	0.057	0.057	0.057	0.057	0.057	0.057
Potatoes	0.077	0.079	0.077	0.078	0.075	0.076	0.074	0.075	0.078	0.078	0.081	0.077
Processed cheese slices	0.560	0.540	0.530	0.543	0.532	0.536	0.553	0.543	0.577	0.566	0.572	0.572

Figure 6: Coefficients

4. Finally, we add the product of each coefficient and its corresponding decision variable to create the objective function where we want to maximize the the nutritional value of the diet while keeping costs low.

$$\max c_{1,1} \cdot x_{1,1} + c_{1,2} \cdot x_{1,2} + \cdots + c_{180,12} \cdot x_{180,12}$$

6.3 Constraints

In order to form the first constraints for this linear problem we have separated them on the basis of months, so that each month's nutrient values are independent of each other. Furthermore we have created a separate list of animal based protein items to ensure that the model selects at least two sources of protein each month.

We know that there are different dietary requirements depending on if a person is a man or woman. To create the second constraint, we use Equation 1 and Equation 2, with the corresponding values in Table 2, to find the upper bound of the calorie requirement per day (Dietary Reference Intakes).

$$\text{EER Women} = 354 - (6.91 \times \text{age}) + \Big(\text{PA} \times \big((9.36 \times \text{weight}) + (726 \times \text{height})\big)\Big) \tag{1}$$

$$\text{EER Men} = 662 - (9.53 \times \text{age}) + \Big(\text{PA} \times \big((15.91 \times \text{weight}) + (539.6 \times \text{height})\big)\Big) \tag{2}$$

Gender	Age	Weight (kg)	Height (m)	PA value
Women	25	57	1.63	1.12
Men	25	70	1.77	1.11

Table 2: Calorie Calculation Values

EER stands for estimated energy requirement/total energy expenditure, and PA stands for physical activity. For the purpose of our analysis, we have considered the age of both genders to be 25 years with low active PA values, which means that an individual does typical daily living activities plus 30 to 60 minutes of daily moderate activity ("Dietary Reference Intakes Tables"). Table 1 shows the upper bound of the respective nutrient value per day. Therefore, we get the following constraints:

- (calories of selected foods) ≤ 2104 for women
- (calories of selected foods) ≤ 2720 for men
- (protein of selected foods) ≤ 46 for women
- (protein of selected foods) ≤ 56 for men
- (carbohydrates of selected foods) ≤ 237 for women
- (carbohydrates of selected foods) ≤ 306 for men
- (fiber of selected foods) ≤ 30 for women
- (fiber of selected foods) ≤ 38 for men

The following additional constraints are added to the model to give a realistic output and a more balanced, realistic diet:

- Quantity of Apple, Banana, Tomato, Onion, Carrot, Orange, Potato ≥ 1
- Quantity of Bacon ≤ 2
- Quantity of Bread ≤ 5
- Quantity of Processed cheese slices ≤ 2
- Quantity of Bacon + Salmon + Chicken + Beef + Eggs ≥ 1
- All food quantities must be ≥ 0

And the following constraint is account for vegetarians:

- Quantity of Bacon, Salmon, Beef, Chicken $= 0$

7 Code Breakdown

7.1 Data Preparation and Cleaning

The code initiates by loading a dataset containing information about various foods, including nutritional values (calories, protein, fiber, carbs) and grams per serving. Unnecessary columns are dropped, and new datasets, such as 'nutrients', are created by concatenating existing ones.

```
nutrients = pd.concat([names, vals], axis=1, join='inner')
```

7.2 Normalization of Nutrient Values

Nutrient values undergo normalization by dividing each value by the corresponding grams. Further scaling is applied by dividing them by their respective maximum values (explained in Section 6.2).

```
cal_grams = np.divide(calories_df, grams)
protein_grams = np.divide(protein_df, grams)
fiber_grams = np.divide(fiber_df, grams)
carbs_grams = np.divide(carbs_df, grams)

cal_grams_max = cal_grams/cal_grams.max()
fiber_grams_max = fiber_grams/fiber_grams.max()
carbs_grams_max = carbs_grams/carbs_grams.max()
protein_grams_max = protein_grams/protein_grams.max()
```

7.3 Calculation of Total Nutritional Value

Total nutritional values are computed by summing up the normalized nutritional values for each food item. The result is stored in a data frame with additional columns for total nutritional values.

```
nutrients_df = pd.concat([names, cal_grams_max, protein_grams_max, fiber_grams_max, carbs_grams_max ], a
```

7.4 Optimization Problem Setup using PuLP

The code formulates a linear programming problem using PuLP to maximize overall nutritional value within set constraints. Decision variables (explained in Section 6.1) represent the monthly consumption of each food item. The linear programming problem setup is finalized by adding the objective function (explained in Section 6.2), which is the sum of coefficients calculated from nutritional values in the DataFrame c.

```
cals = []
pros = []
carbs = []
fibr = []

for l in range(0, 15):
    for k in range(12 * l, 12 * (l + 1)):
        cals.append(nutrients.iloc[l, 1] * b[k])
        pros.append(nutrients.iloc[l, 2] * b[k])
        fibr.append(nutrients.iloc[l, 3] * b[k])
        carbs.append(nutrients.iloc[l, 4] * b[k])

# Monthly Breakdown
jan_cals = []
jan_pros = []
jan_carbs = []
jan_fibr = []


for i in range(0, 180, 12):
    jan_cals.append(cals[i])
    jan_carbs.append(carbs[i])
    jan_pros.append(pros[i])
    jan_fibr.append(fibr[i])


protein_items = []
# bacon
for r in range(12, 24):
    protein_items.append(b[r])


# Monthly Protein Breakdown
jan_protein_items = []


for r in range(0, 60, 12):
    jan_protein_items.append(protein_items[r])
```

```
# Repeat for other months
```

7.5 Solving the Optimization Problem and Results Extraction

The linear programming problem is solved to obtain optimized values for decision variables, representing the optimal monthly consumption of each food item.

```
results = []
for a in prob.variables():
    results.append(a.varValue)
results_df = pd.DataFrame(columns=["January", "Feburary", "March", "April", "May", "June", "July",
"August", "September", "October", "November", "December"])
results_df = pd.concat([names, results_df])
```

7.6 Post-processing of Results

Two summary DataFrames are created: one for daily consumption and another for monthly consumption. Monthly consumption values are calculated by multiplying daily values by 30, assuming 30 days in a month.

8 Hypothesis

This report hypothesizes that by applying linear programming techniques to analyze the fluctuating retail prices and nutritional content of 15 selected food items, it is possible to construct a cost-effective and nutritionally adequate diet plan tailored for each season in British Columbia. The hypothesis is specific to two dietary groups: vegetarians and omnivores, and anticipates distinct optimal diet plans for each group, accounting for gender-specific nutritional requirements as outlined in Dietary Reference Intakes. The study expects that seasonal price variations will significantly influence the composition of these diet plans, potentially leading to consistent preferences for certain low-cost, high-nutrient foods across different seasons. Additionally, the hypothesis includes the expectation that the optimal diets will adhere to the daily calorie, protein, carbohydrate, and fiber needs for both men and women, while also being financially viable for the average consumer in British Columbia. The research anticipates revealing substantial differences in the monthly food expenditure required to meet nutritional needs between vegetarians and omnivores, and between men and women, reflecting the varying cost implications of different dietary choices and gender-specific nutritional requirements.

9 Results from Primal Problem

To get the monthly results, we take the daily results and multiply all values by 30.

We thought that fluctuations in price would change the quantity of foods eaten every month, but our model shows otherwise. That is, the quantity and types of food do not change every month. With that being said, our model is taking advantage of low priced, high nutrient items like eggs and bread to satisfy the carbohydrate and protein requirement.

Table 3 shows the ideal quantity, in grams, of each food item an individual should consume every month in order to minimize their food cost in British Columbia. The table also shows the optimal z value, which is the cost of that gender's diet for the selected amount of foods.

We can see eggs are the only values that change across gender and diet, with men needing more if they are vegetarians. Looking at the z values for each gender and diet, we see that vegetarian diets are cheaper compared to omnivorous diets, which is not a surprise considering meat products are expensive. So, that is why the model takes advantage of the low price of eggs and suggests it as the main protein source.

Gender	Women		Men	
Diet	Omnivore	Vegetarian	Omnivore	Vegetarian
Apples	30	30	30	30
Bacon	60	0	60	0
Bananas	80.87	83.478	170.87	173.478
Bread	150	150	150	150
Canned salmon	0	0	0	0
Canned tomatoes	30	30	30	30
Carrots	30	30	30	30
Chicken	0	0	0	0
Eggs	17.261	23.043	20.761	40.543
Ground beef	0	0	0	0
Onions	30	30	30	30
Oranges	30	30	30	30
Partly skimmed milk	0	0	0	0
Potatoes	30	30	30	30
Processed cheese slices	60	60	60	60
Optimal z	2909.381	2776.596	3319.695	3186.910

Table 3: Summary of Low Active Results

Notably, the difference between men and women's omnivore diets is about 410 dollars. Considering the difference in calories between women and men is 18,477.90 calories per month (or 615.93 calories per day), this comes as a surprise. However, since the model favors eggs and bread, it's not shocking that the low price of these items influences the number of eggs and bread we eat as we increase the protein and carbohydrate requirement.

The more concerning issue is the price for each of the diets. The lowest price is \$2909.38 per month, or \$727.35 per week, which is an extraordinary amount for most people. And keeping in mind that this is the price for one person, the price only multiplies as more people are considered. With that being said, when shopping, there can be sales for certain items; however, that still means that people will still be spending more than $\$2,500$ a month on food if they want to reach their nutritional requirements.

10 Sensitivity Analysis

Now, let's change the PA level to active, meaning an individual does their typical daily living activities and at least 60 minutes of moderate activity, to see how this affects the price ("Dietary Reference Intakes Tables"). So, the PA value for women changes from 1.12 to 1.27 and 1.11 to 1.25 for men. We use Equation 1 and Equation 2 again to calculate the new calorie requirements. This, in turn, changes the recommended amount of carbohydrates, protein, and fiber. Table 4 shows the new amounts in comparison with the old amounts.

Gender	Women		Men	
PA value	Low Active	Active	Low Active	Active
Calories	2104	2361	2720	3010
Carbohydrates	237	266	306	339
Protein	46	63	56	77
Fiber	30	33	38	42

Table 4: Comparison of Values

The difference now is that protein is now calculated by multiplying body weight by 1.1g/kg instead of 0.8g/kg, carbohydrates is recalculated by taking 45% of the calories and dividing that by 4g/kCal, and fiber is recalculated by taking the calories and multiplying by 14g/1000kCal ("The Minimum Carbohydrate Requirement for Adults"; "Are you getting too much protein?"; "Dietary Reference Intakes Tables").

Again, the amount of food does not vary depending on the month. So, we can summarize the results in Table 5.

Gender	Women		Men	
Diet	Omnivore	Vegetarian	Omnivore	Vegetarian
Apples	30	30	30	30
Bacon	60	0	60	0
Bananas	118.295	120.904	213.386	215.994
Bread	150	150	150	150
Canned salmon	0	0	0	0
Canned tomatoes	30	30	30	30
Carrots	30	30	30	30
Chicken	0	0	0	0
Eggs	41.892	61.675	69.718	89.5
Ground beef	0	0	0	0
Onions	30	30	30	30
Oranges	30	30	30	30
Partly skimmed milk	0	0	0	0
Potatoes	30	30	30	30
Processed cheese slices	60	60	60	60
Optimal z	3149.343	3274.508	4033.099	3900.314

Table 5: Summary of Active Results

When comparing the z values from Table 3 to those in Table 5, we see that there is a significant difference in price. The lowest difference is \$239.96 and the greatest difference is \$713.40 between active levels. It makes sense that the change in activity levels increases the price since more food and nutrients must be consumed. However, the \$713.40 increase is quite significant when increasing PA values. This can be attributed to the increase in protein since, as previously mentioned, the calculation for it has changed.

Despite only increasing the calorie limit by approximately 254 and 290 kCal for women and men respectively, the changes in carbohydrate, protein, and fiber requirements cause the quantity of food to change and price to change by quite a wide range. So, it is quite surprising that such a small calorie increase would cause such a large price jump. But, considering the change in nutrient requirements, it is expected that the prices would increase to satisfy each gender's dietary requirements.

11 Duality

In the dual problem, you aim to minimize the total cost of meeting the constraints while maximizing the shadow prices associated with each constraint. The objective function is focused on minimizing costs while considering the impact of constraint relaxations.

In the context of this nutrition optimization problem, suppose a positive dual variable is associated with the constraint on calories for women. This means that if you could increase the daily calorie limit for women by a small amount, you'd be willing to pay the positive shadow price associated with it to achieve a higher nutritional value. Conversely, a negative shadow price associated with a constraint might indicate that you'd require compensation to reduce a constraint, such as moving from a vegetarian diet to one that includes meat.

To implement the dual problem for the given optimization problem, we used the following steps:

11.1 Objective Function of Dual

Minimizing the price whist meeting all the minimum nutritional requirement can be given:

$$\text{Minimize: } 2104y_{cal,w} + 2720y_{cal,m} + 46y_{pro,w} + 56y_{pro,m} + 237y_{carb,w} + 306y_{carb,m} + 30y_{fib,w} + 38y_{fib,m}$$

Let:

- $y_{cal,w}$, $y_{cal,m}$, for the calorie constraints for women and men, respectively.
- $y_{pro,w}$, $y_{pro,m}$, for the protein constraints for women and men, respectively.
- $y_{carb,w}$, $y_{carb,m}$, for the carbohydrate constraints for women and men, respectively
- $y_{fib,w}$, $y_{fib,m}$, for the fiber constraints for women and men, respectively

11.2 Dual Constraints

Each decision variable in the primal problem corresponds to a constraint in the dual problem. These constraints ensure that the cost assigned to each nutrient does not exceed the nutritional value coefficient divided by the price of the food item.

For each food item i in each month j, you have a constraint in the dual. Let's denote the coefficient for food i in month j as,$c_{i,j}$ (from your Figure 6). The dual constraints will look like:

There will be one such constraint for each of the 180 decision variables (foods across months).

Also, for the sake of simplicity, we will ignore the additional constraints.

$$y_{cal,w} + y_{cal,m} + y_{pro,w} + y_{pro,m} + y_{carb,w} + y_{carb,m} + y_{fib,w} + y_{fib,m} \geq c_{i,j}$$

For example, if we take the coefficient for apples in January which is 0.112 (from figure 6), the constraint in the dual problem would be:

$$y_{cal,w} + y_{cal,m} + y_{pro,w} + y_{pro,m} + y_{carb,w} + y_{carb,m} + y_{fib,w} + y_{fib,m} \geq 0.112$$

Or, if we take the coefficient for eggs in March, which is 0.185 (from figure 6), the constraint in the dual problem would be:

$$y_{cal,w} + y_{cal,m} + y_{pro,w} + y_{pro,m} + y_{carb,w} + y_{carb,m} + y_{fib,w} + y_{fib,m} \geq 0.185$$

Given that we have 15 foods and 12 months, the primal problem will have 180 constraints (one for each coefficient), and hence the dual problem will have 180 variables.

12 Results from Dual Problem

As mentioned before in the theory section of this report, we can apply the complementary slackness theorem and avoid running the complex simplex algorithm on the Dual problem again.

However, due to limited documentation available for the PuLP library, coupled with the author's limited experience in coding, finding the optimal slack values for the Primal problem proved to be challenging.

13 Conclusion

In conclusion, the overall quantity and types of food remain the same irrespective of price changes. Bread and eggs seem to be a staple in people's diets since they are comparatively cheap. The main difference in diet depending on gender is the number of eggs consumed. For low-active people, men spend \$410 more than women on their food needs. On average people need to spend about \$2909.38 per month, or \$727.35 per week, to meet nutrient requirements. This result shows that it is impossible for a 25-year-old to be able to afford high-quality nutritious foods such as ground beef, salmon, and milk in order to maintain a healthy balanced diet. Food in BC is so overpriced that it barely meets the dietary necessities for someone earning a minimum wage making this province unlivable for students and young professionals.

This report is on individual food insecurity in BC and does not account for households, but all the findings in this paper will be valuable for supporting the health authorities and other sectors in food insecurity planning. Given the health implications of food insecurity, it is important that BC monitors individual as well as household food insecurity on a regular and consistent basis through surveillance opportunities from the Canadian Community Health Survey.

The purpose of this report is to present food insecurity data and recommend specific food items depending on gender to address food insecurity; however, it is broadly recognized that reducing household food insecurity at a population level will require policy changes that improve a household's financial circumstances as well as make food more affordable for the general public.

One of the primary challenges we encountered was the limited documentation and support for the PuLP library. This was particularly evident in our inability to identify the coefficients of the slack variables in the primal problem, which are essential for determining the optimal point of the dual problem using the principle of complementary slackness. Additionally, the author's limited experience with Python and coding in general posed another significant limitation in effectively navigating and resolving these issues.

In future studies on this topic, we should look into household food insecurity in BC accounting for dependents and children. Furthermore, we can tackle this growing problem by looking into changes in food affordability for households in different income brackets after accounting for tax. While this area of research is not near complete, it is a valuable starting point for future studies to help fight this pressing matter in BC.

14 References

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14.1 Data sets:

Data sets used for this report can be found on the following links

1. Monthly average costs: https://github.com/ManyaNarwal/MATH340Project/blob/main/Raw_Data/selected_monthly_costs.csv
2. Nutrient of each product: https://github.com/ManyaNarwal/MATH340Project/blob/main/Raw_Data/nutrient_df.csv
3. Final cleaned data: https://github.com/ManyaNarwal/MATH340Project/blob/main/Raw_Data/cleaned_data.csv