Maximizing a Diet's Nutritional Value Given Certain Foods

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1 Introduction

According to a report by the BC Alliance for Healthy Living, one in six (15.6%) children in British Columbia under the age of 18 lived in households experiencing some level of food insecurity ("Ripe for Change – Food Insecurity in BC"). Additionally, over one in ten (11.8%) BC households experienced some level of food insecurity and about 3% of households experienced severe food insecurity. That's about 485,500 and 91,100 British Columbians, respectively, experience difficulty in acquiring food ("Household Food Insecurity in BC").

Food insecurity is recognized as a key public health issue in BC. It is defined as the inability to purchase healthy and safe food due to a lack of financial means ("Household Food Insecurity in Canada"). It not only increases the risk of chronic conditions, like asthma and diabetes, but also the likelihood of depression, social isolation, and distress ("Household Food Insecurity in BC", Laraia). The primary goal of food security is to improve the overall health and wellness of BC residents. It provides access to nutritious, safe, and personally acceptable food by increasing the availability of healthy food at a sustainable cost.

Household food insecurity is the inability for a household to purchase healthy and personally acceptable food. In this report, we aim to provide a solution to tackle household food insecurity by minimizing the seasonal average cost for each member of the household based on their gender.

2 Aim

This paper attempts to provide a solution to food insecurity in BC by formulating a linear programming problem to maximize the nutrition of common food items and minimize their cost. Considering the fact that the price of food products changes according to the season, we aim to find the optimal diet in each season. Furthermore, to add complexity we aim to find the optimal diet for adults with different dietary preferences. For the purpose of this report, we will only consider two dietary groups: vegetarians and omnivores.

Table 1 contains the daily nutrient requirements for each gender in order to maintain a healthy diet (Dietary Reference Intakes). This information is used in constraint formulation.

Gender	Calories (kCal/day)	Protein (g/day)	Carbohydrates (g/day)	Fiber(g/day)
Women	2104	46	237	30
Men	2720	56	306	38

Table 1: Average Nutrient Requirements

3 Data

Sourced from Statistics Canada, we used monthly average retail prices for food and other selected products' data sets. This data set contains the monthly price of 52 common products for the year 2021. For the purpose of this report, we will only be considering 15 food items in order to calculate the seasonal cost of a balanced diet.

The nutrition value of each food item is taken from a Kaggle data set. Data for each food item is further classified by the number of calories, protein, carbohydrates, fiber, and fats it contains. For simplicity, we only looked at the value of calories, proteins, carbohydrates, and fiber for the select food items selected. Figure 1 displays the cleaned data set we worked with.

Food	Grams	Calories	Protein	Fiber	Carbs	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Apples	130	70	0	1.0	18	4.53	4.49	4.57	4.55	4.64	4.66	4.79	4.97	4.90	4.61	4.44	4.76
Bacon	16	95	4	0.0	1	7.32	7.44	7.29	7.69	7.89	7.79	7.99	8.24	8.27	8.29	8.47	8.66
Bananas	150	85	1	0.9	23	1.56	1.57	1.56	1.56	1.56	1.56	1.56	1.60	1.60	1.60	1.60	1.61
Bread	35	86	3	2.4	16	2.79	2.87	2.82	2.85	2.88	2.82	2.80	2.82	2.81	2.84	2.89	2.98
Canned salmon	85	120	17	0.0	0	6.67	6.60	6.48	6.41	6.23	6.53	6.64	6.53	6.41	6.66	6.53	6.53
Canned tomatoes	240	50	2	1.0	9	1.45	1.48	1.49	1.60	1.60	1.63	1.58	1.74	1.70	1.65	1.57	1.71
Carrots	150	45	1	0.9	10	2.25	2.34	2.33	2.43	2.49	2.52	2.64	2.37	2.40	2.32	2.36	2.43
Chicken	85	185	23	0.0	0	7.35	7.00	7.73	7.87	7.76	7.72	8.07	8.13	8.30	8.51	8.18	7.87
Eggs	100	150	12	0.0	0	3.64	3.62	3.77	3.80	3.74	3.75	3.74	3.91	3.82	3.87	3.87	3.82
Ground beef	85	245	23	0.0	0	11.55	11.06	11.22	11.53	11.28	11.00	11.31	11.56	11.86	12.31	12.46	12.06
Onions	210	80	2	1.6	18	2.29	2.32	2.31	2.25	2.35	2.38	2.45	2.44	2.31	2.14	2.29	2.25
Oranges	180	60	2	1.0	16	3.59	3.77	3.61	3.66	3.90	4.04	3.97	3.88	3.90	4.02	4.12	4.11
Partly skimmed milk	258	129	9	0.0	12	5.30	5.48	5.49	5.50	5.51	5.48	5.51	5.49	5.49	5.52	5.50	5.52
Potatoes	100	100	2	0.5	22	10.34	10.09	10.31	10.23	10.58	10.50	10.72	10.67	10.19	10.17	9.89	10.36
Processed cheese slices	21	78	5	0.0	0	2.69	2.79	2.84	2.77	2.83	2.81	2.72	2.77	2.61	2.66	2.63	2.63

Figure 1: Data

In Figure 2, Figure 3, Figure 4, and Figure 5, we look at the distribution of nutritional values over grams for the food items. From the distributions, wee see that there is no food item that has low calories and moderate levels of carbohydrates, protein, and fiber. So, this tells us that diversity will be important when we create our model to ensure that nutrient requirements are met, and so that our model doesn't only use a few foods to meet the requirements.

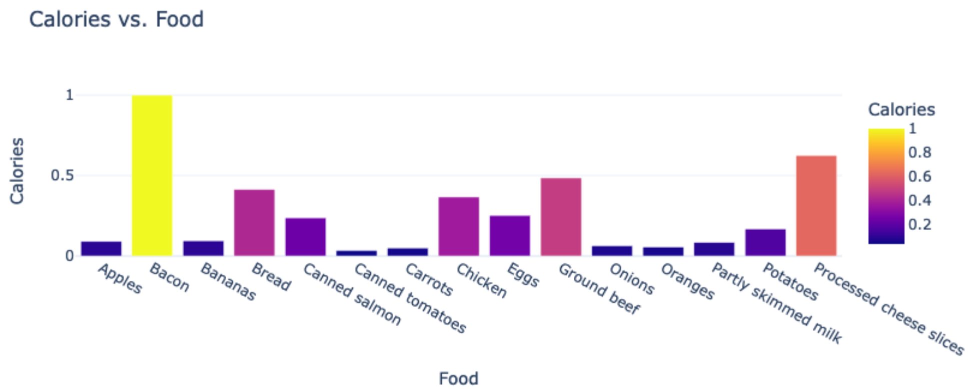

Figure 2: Calorie Distribution

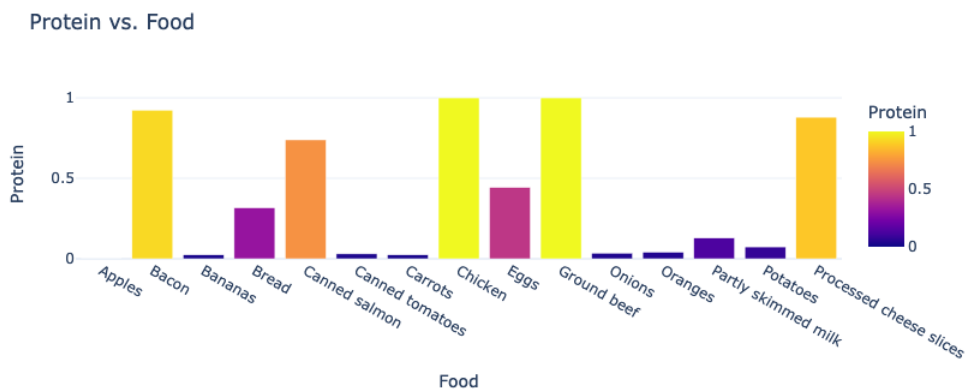

Figure 3: Protein Distribution

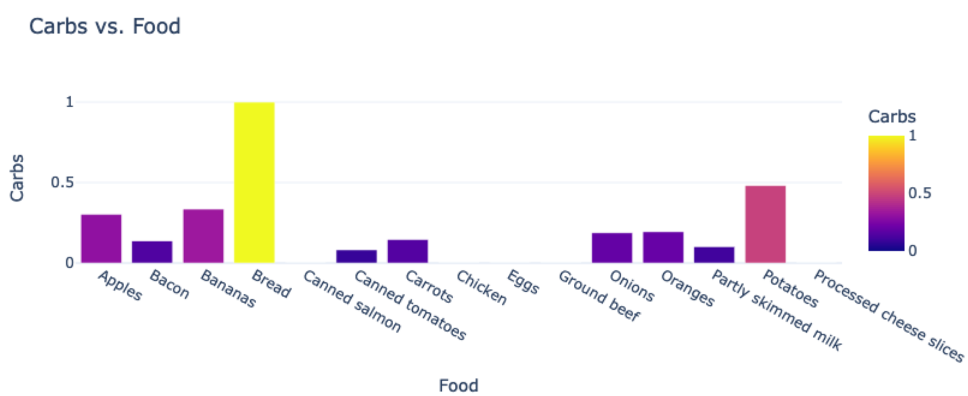

Figure 4: Carbohydrate Distribution

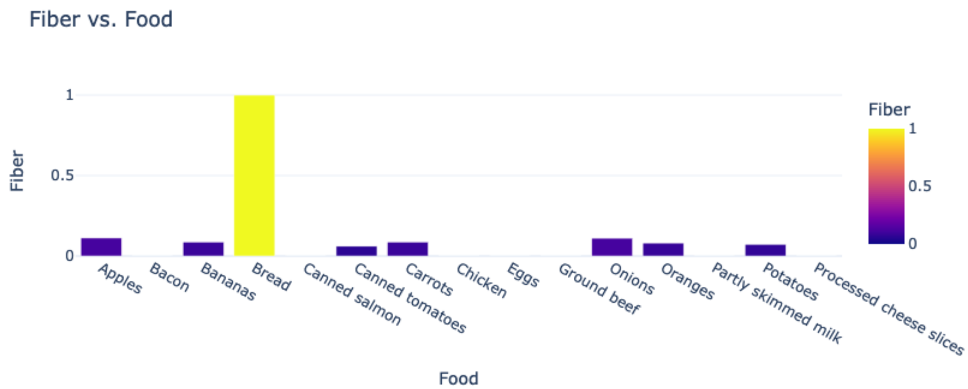

Figure 5: Fiber Distribution

4 Methods

4.1 Decision Variables

The decision variables are the quantity of food for each month. For example, $x_{1,4}$ is the decision variable for apples in April. So, for 15 foods and 12 months, we have 180 decision variables.

4.2 Objective Function

We formulated the objective function through the following steps:

1. To ensure the uniformity of units, the value of each nutrient (calories, carbohydrates, protein, and fiber) is divided by the weight of that particular food item in grams. This gives us the nutrient per gram for each food item. For example, considering only calories for apples it can be written as:

$$\text{calories per gram for apples} = \frac{\text{value of calories}}{\text{weight of apples}} = \frac{70}{130} = 0.538$$

2. Furthermore, ensure that all nutritional values can be compared, we take the value of a particular nutrient per gram over the maximum value of that nutrient amongst all food items. For example the nutritional value of carbohydrates in apples is:

$$\text{nutritional value of carbohydrates in apples} = \frac{\text{calories per gram for apples}}{\text{maximum value of calories for all foods}} = 0.090688$$

3. Then, we get the coefficient of the objective function by dividing the total nutrition value of each food item by the item's price each month. Figure 6 shows the coefficients for each variable. For example, the coefficient for the apples in April is given by:

$$c_{1,4} = \frac{\text{total nutritional value for apples}}{\text{price of apples in April}}$$

where the total nutritional value for apples is:

$$\begin{aligned}\text{total nutritional value of apples} = {} & \text{value of carbohydrates in apples} + \\ & \text{value of calories in apples} + \\ & \text{value of proteins in apples} + \\ & \text{value of fiber in apples}\end{aligned}$$

Food	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Apples	0.112	0.113	0.111	0.111	0.109	0.109	0.106	0.102	0.103	0.110	0.114	0.106
Bacon	0.282	0.277	0.283	0.268	0.261	0.265	0.258	0.250	0.249	0.249	0.243	0.238
Bananas	0.348	0.346	0.348	0.348	0.348	0.348	0.348	0.339	0.339	0.339	0.339	0.337
Bread	0.979	0.951	0.968	0.958	0.948	0.968	0.975	0.968	0.972	0.961	0.945	0.916
Canned salmon	0.146	0.148	0.151	0.152	0.157	0.150	0.147	0.150	0.152	0.147	0.150	0.150
Canned tomatoes	0.144	0.141	0.140	0.130	0.130	0.128	0.132	0.120	0.123	0.126	0.133	0.122
Carrots	0.137	0.132	0.132	0.127	0.124	0.122	0.117	0.130	0.129	0.133	0.131	0.127
Chicken	0.186	0.195	0.177	0.174	0.176	0.177	0.169	0.168	0.165	0.161	0.167	0.174
Eggs	0.191	0.192	0.185	0.183	0.186	0.186	0.186	0.178	0.182	0.180	0.180	0.182
Ground beef	0.129	0.134	0.132	0.129	0.132	0.135	0.131	0.128	0.125	0.121	0.119	0.123
Onions	0.174	0.172	0.172	0.177	0.169	0.167	0.162	0.163	0.172	0.186	0.174	0.177
Oranges	0.104	0.099	0.103	0.102	0.096	0.092	0.094	0.096	0.096	0.093	0.090	0.091
Partly skimmed milk	0.059	0.057	0.057	0.057	0.057	0.057	0.057	0.057	0.057	0.057	0.057	0.057
Potatoes	0.077	0.079	0.077	0.078	0.075	0.076	0.074	0.075	0.078	0.078	0.081	0.077
Processed cheese slices	0.560	0.540	0.530	0.543	0.532	0.536	0.553	0.543	0.577	0.566	0.572	0.572

Figure 6: Coefficients

4. Finally, we add the product of each coefficient and its corresponding decision variable to create the objective function where we want to maximize the the nutritional value of the diet while keeping costs low.

$$\max c_{1,1} \cdot x_{1,1} + c_{1,2} \cdot x_{1,2} + \cdots + c_{180,12} \cdot x_{180,12}$$

4.3 Constraints

In order to form the first constraints for this linear problem we have separated them on the basis of months, so that each month's nutrient values are independent of each other. Furthermore we have created a separate list of animal based protein items to ensure that the model selects at least two sources of protein each month.

We know that there are different dietary requirements depending on if a person is a man or woman. To create the second constraint, we use Equation 1 and Equation 2, with the corresponding values in Table 2, to find the upper bound of the calorie requirement per day (Dietary Reference Intakes).

$$\text{EER Women} = 354 - (6.91 \times \text{age}) + \Big(\text{PA} \times \big((9.36 \times \text{weight}) + (726 \times \text{height})\big)\Big) \tag{1}$$

$$\text{EER Men} = 662 - (9.53 \times \text{age}) + \Big(\text{PA} \times \big((15.91 \times \text{weight}) + (539.6 \times \text{height})\big)\Big) \tag{2}$$

Gender	Age	Weight (kg)	Height (m)	PA value
Women	25	57	1.63	1.12
Men	25	70	1.77	1.11

Table 2: Calorie Calculation Values

EER stands for estimated energy requirement/total energy expenditure, and PA stands for physical activity. For the purpose of our analysis, we have considered the age of both genders to be 25 years with low active PA values, which means that an individual does typical daily living activities plus 30 to 60 minutes of daily moderate activity ("Dietary Reference Intakes Tables"). Table 1 shows the upper bound of the respective nutrient value per day. Therefore, we get the following constraints:

- (calories of selected foods) ≤ 2104 for women
- (calories of selected foods) ≤ 2720 for men
- (protein of selected foods) ≤ 46 for women
- (protein of selected foods) ≤ 56 for men
- (carbohydrates of selected foods) ≤ 237 for women
- (carbohydrates of selected foods) ≤ 306 for men
- (fiber of selected foods) ≤ 30 for women
- (fiber of selected foods) ≤ 38 for men

The following additional constraints are added to the model to give a realistic output and a more balanced, realistic diet:

- Quantity of Apple, Banana, Tomato, Onion, Carrot, Orange, Potato ≥ 1
- Quantity of Bacon ≤ 2
- Quantity of Bread ≤ 5
- Quantity of Processed cheese slices ≤ 2
- Quantity of Bacon + Salmon + Chicken + Beef + Eggs ≥ 1
- All food quantities must be ≥ 0

And the following constraint is account for vegetarians:

- Quantity of Bacon, Salmon, Beef, Chicken $= 0$

5 Results

To get the monthly results, we take the daily results and multiply all values by 30.

We thought that fluctuations in price would change the quantity of foods eaten every month, but our model shows otherwise. That is, the quantity and types of food do not change every month. With that being said, our model is taking advantage of low priced, high nutrient items like eggs and bread to satisfy the carbohydrate and protein requirement.

Table 3 shows the ideal quantity, in grams, of each food item an individual should consume every month in order to minimize their food cost in British Columbia. The table also shows the optimal z value, which is the cost of that gender's diet for the selected amount of foods.

We can see eggs are the only values that change across gender and diet, with men needing more if they are vegetarians. Looking at the z values for each gender and diet, we see that vegetarian diets are cheaper compared to omnivorous diets, which is not a surprise considering meat products are expensive. So, that is why the model takes advantage of the low price of eggs and suggests it as the main protein source.

Notably, the difference between men and women's omnivore diets is about 410 dollars. Considering the difference in calories between women and men is 18,477.90 calories per month (or 615.93 calories per day), this comes as a surprise. However, since the model favors eggs and bread, it's not shocking that the low price of these items influences the number of eggs and bread we eat as we increase the protein and carbohydrate requirement.

Gender	Women		Men	
Diet	Omnivore	Vegetarian	Omnivore	Vegetarian
Apples	30	30	30	30
Bacon	60	0	60	0
Bananas	80.87	83.478	170.87	173.478
Bread	150	150	150	150
Canned salmon	0	0	0	0
Canned tomatoes	30	30	30	30
Carrots	30	30	30	30
Chicken	0	0	0	0
Eggs	17.261	23.043	20.761	40.543
Ground beef	0	0	0	0
Onions	30	30	30	30
Oranges	30	30	30	30
Partly skimmed milk	0	0	0	0
Potatoes	30	30	30	30
Processed cheese slices	60	60	60	60
Optimal z	2909.381	2776.596	3319.695	3186.910

Table 3: Summary of Low Active Results

The more concerning issue is the price for each of the diets. The lowest price is \$2909.38 per month, or \$727.35 per week, which is an extraordinary amount for most people. And keeping in mind that this is the price for one person, the price only multiplies as more people are considered. With that being said, when shopping, there can be sales for certain items; however, that still means that people will still be spending more than $\$2,500$ a month on food if they want to reach their nutritional requirements.

6 Sensitivity Analysis

Now, let's change the PA level to active, meaning an individual does their typical daily living activities and at least 60 minutes of moderate activity, to see how this affects the price ("Dietary Reference Intakes Tables"). So, the PA value for women changes from 1.12 to 1.27 and 1.11 to 1.25 for men. We use Equation 1 and Equation 2 again to calculate the new calorie requirements. This, in turn, changes the recommended amount of carbohydrates, protein, and fiber. Table 4 shows the new amounts in comparison with the old amounts.

Gender	Women		Men	
PA value	Low Active	Active	Low Active	Active
Calories	2104	2361	2720	3010
Carbohydrates	237	266	306	339
Protein	46	63	56	77
Fiber	30	33	38	42

Table 4: Comparison of Values

The difference now is that protein is now calculated by multiplying body weight by 1.1g/kg instead of 0.8g/kg, carbohydrates is recalculated by taking 45% of the calories and dividing that by 4g/kCal, and fiber is recalculated by taking the calories and multiplying by 14g/1000kCal ("The Minimum Carbohydrate Requirement for Adults"; "Are you getting too much protein?"; "Dietary Reference Intakes Tables").

Again, the amount of food does not vary depending on the month. So, we can summarize the results in Table 5.

Gender	Women		Men	
Diet	Omnivore	Vegetarian	Omnivore	Vegetarian
Apples	30	30	30	30
Bacon	60	0	60	0
Bananas	118.295	120.904	213.386	215.994
Bread	150	150	150	150
Canned salmon	0	0	0	0
Canned tomatoes	30	30	30	30
Carrots	30	30	30	30
Chicken	0	0	0	0
Eggs	41.892	61.675	69.718	89.5
Ground beef	0	0	0	0
Onions	30	30	30	30
Oranges	30	30	30	30
Partly skimmed milk	0	0	0	0
Potatoes	30	30	30	30
Processed cheese slices	60	60	60	60
Optimal z	3149.343	3274.508	4033.099	3900.314

Table 5: Summary of Active Results

When comparing the z values from Table 3 to those in Table 5, we see that there is a significant difference in price. The lowest difference is \$239.96 and the greatest difference is \$713.40 between active levels. It makes sense that the change in activity levels increases the price since more food and nutrients must be consumed. However, the \$713.40 increase is quite significant when increasing PA values. This can be attributed to the increase in protein since, as previously mentioned, the calculation for it has changed.

Despite only increasing the calorie limit by approximately 254 and 290 kCal for women and men respectively, the changes in carbohydrate, protein, and fiber requirements cause the quantity of food to change and price to change by quite a wide range. So, it is quite surprising that such a small calorie increase would cause such a large price jump. But, considering the change in nutrient requirements, it is expected that the prices would increase to satisfy each gender's dietary requirements.

7 Conclusion

In conclusion, the overall quantity and types of food remain the same irrespective of price changes. Bread and eggs seem to be a staple in people's diets since they are comparatively cheap. The main difference in diet depending on gender is the number of eggs consumed. For low-active people, men spend \$410 more than women on their food needs. On average people need to spend about \$2909.38 per month, or \$727.35 per week, to meet nutrient requirements. This result shows that it is impossible for a 25-year-old to be able to afford high-quality nutritious foods such as ground beef, salmon, and milk in order to maintain a healthy balanced diet. Food in BC is so overpriced that it barely meets the dietary necessities for someone earning a minimum wage making this province unlivable for students and young professionals.

This report is on individual food insecurity in BC and does not account for households, but all the findings in this paper will be valuable for supporting the health authorities and other sectors in food insecurity planning. Given the health implications of food insecurity, it is important that BC monitors individual as well as household food insecurity on a regular and consistent basis through surveillance opportunities from the Canadian Community Health Survey.

The purpose of this report is to present food insecurity data and recommend specific food items depending on gender to address food insecurity; however, it is broadly recognized that reducing household food insecurity at a population level will require policy changes that improve a household's financial circumstances as well as make food more affordable for the general public.

In future studies on this topic, we should look into household food insecurity in BC accounting for dependents and children. Furthermore, we can tackle this growing problem by looking into changes in food affordability for households in different income brackets after accounting for tax. While this area of research is not near complete, it is a valuable starting point for future studies to help fight this pressing matter in BC.

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