

Risk Management, Volatility, and Variance Threshold Strategies in Pairs Trading

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1 Risk Management in Trading

Risk management in trading refers to the process of identifying, analyzing, and mitigating potential risks that could result in significant losses. Effective risk management strategies help ensure long-term profitability and minimize the impact of adverse market movements. Below are key components of risk management in pairs trading.

1.1 Stop-Loss Orders

A stop-loss order is an instruction to automatically sell an asset once its price drops below a specified threshold. This is a key risk management technique, as it helps to limit the loss on a trade and protect capital. In pairs trading, a stop-loss can be applied based on the spread between two assets, exiting the position if the spread deviates beyond a certain level.

Stop-Loss Condition: Exit Position if Spread > Threshold

The threshold is typically chosen based on historical price movements, volatility, or risk tolerance. For example, if the spread between two cointegrated assets exceeds a predefined threshold, the trade is exited to prevent further losses.

1.2 Position Sizing

Position sizing refers to determining the amount of capital to allocate to a particular trade. Proper position sizing is essential to risk management, as it controls the exposure to individual trades and ensures that no single position has the potential to wipe out a significant portion of the capital.

Position sizing is commonly determined based on the volatility of an asset or spread, as higher volatility increases the risk associated with a trade. The formula for position sizing is as follows:

$$\text{Position Size} = \frac{\text{Capital} \times \text{Risk Factor}}{\text{Volatility of Asset}}$$

Where:

- *Capital*: The total amount of capital available for trading.
- *Risk Factor*: A predetermined factor representing the trader's risk tolerance (e.g., 1).
- *Volatility of Asset*: A measure of how much the asset price fluctuates, often calculated as the standard deviation of past price returns.

Position sizing can be adjusted dynamically based on market conditions, particularly when volatility increases, requiring smaller position sizes to reduce exposure.

1.3 Portfolio Diversification

Portfolio diversification is a technique used to spread investments across different assets or strategies to reduce overall risk. In the context of pairs trading, diversification can involve trading multiple pairs of cointegrated assets. By doing so, a trader can ensure that the risk is not concentrated in a single asset pair, thus minimizing the potential for large losses.

The portfolio's total risk is measured by the variance of the overall portfolio, which depends on the individual asset volatilities and the correlation between the assets. The formula for portfolio variance is:

$$\text{Portfolio Variance} = \sum_{i=1}^n w_i^2 \sigma_i^2 + \sum_{i \neq j} w_i w_j \text{Cov}(i, j)$$

Where:

- w_i is the weight of the i -th asset in the portfolio.
- σ_i^2 is the variance of the i -th asset.
- $\text{Cov}(i, j)$ is the covariance between the returns of the i -th and j -th assets.

By adjusting the weights of different assets in the portfolio, traders can balance the risks and optimize the risk-return trade-off. Diversification reduces the impact of adverse movements in a single asset or pair on the overall portfolio.

1.4 Risk-Adjusted Return Measures

Risk-adjusted return metrics are used to evaluate the performance of a trading strategy while accounting for the risk involved. Two common measures include:

1.4.1 Sharpe Ratio

The Sharpe ratio is a measure of the excess return per unit of risk, where the risk is measured by the standard deviation of returns. It is used to evaluate the risk-adjusted performance of an investment or trading strategy.

$$\text{Sharpe Ratio} = \frac{R_p - R_f}{\sigma_p}$$

Where:

- R_p is the return of the portfolio.
- R_f is the risk-free rate (e.g., the return on government bonds).
- σ_p is the standard deviation (volatility) of the portfolio's returns.

A higher Sharpe ratio indicates better risk-adjusted performance.

1.4.2 Sortino Ratio

The Sortino ratio is similar to the Sharpe ratio, but it only considers downside risk (negative volatility). It is useful in cases where negative returns are of greater concern than positive fluctuations.

$$\text{Sortino Ratio} = \frac{R_p - R_f}{\sigma_d}$$

Where σ_d is the standard deviation of the negative returns.

2 Volatility and Variance in Trading

Volatility is a key measure of risk in trading, and understanding its behavior is crucial for making informed trading decisions. Volatility can be quantified in terms of the standard deviation of returns or the variance of an asset or spread.

2.1 Volatility

Volatility is a statistical measure of the dispersion of returns for a given asset or market index. Higher volatility indicates that the asset's price is fluctuating widely, increasing the potential for both higher returns and higher losses.

The standard deviation (σ) of returns is the most commonly used measure of volatility:

$$\sigma = \sqrt{\frac{1}{T} \sum_{t=1}^T (r_t - \bar{r})^2}$$

Where:

- r_t is the return at time t .
- \bar{r} is the average return over the time period.
- T is the number of observations.

2.1.1 Historical Volatility

Historical volatility is calculated based on past price data and reflects how much the asset's price has varied over a given period. It can be used to assess the risk of an asset or a spread in pairs trading. A higher historical volatility means greater risk, which may require more conservative position sizing or the implementation of risk management techniques.

2.1.2 Implied Volatility

Implied volatility refers to the market's expectation of future volatility, often inferred from the prices of options. It represents the level of uncertainty about an asset's future price movements, and higher implied volatility typically corresponds to higher options prices.

2.2 Variance Threshold Strategy

A variance threshold strategy is a risk management approach where trading decisions are based on the volatility (or variance) of an asset's price or spread. The idea is to enter or exit positions based on changes in variance, allowing the strategy to adapt to different market conditions.

2.2.1 Setting Variance Thresholds

To apply a variance threshold strategy, the trader sets a predefined threshold for the variance of the spread. If the variance of the spread exceeds this threshold, a trade is initiated or closed. This allows the strategy to take advantage of periods of high volatility and avoid trading during periods of low volatility.

The variance threshold can be set as:

$$\text{Threshold} = \mu + \lambda\sigma$$

Where:

- μ is the mean variance of the asset's spread over a historical period.
- λ is a scaling factor that determines how sensitive the threshold is to changes in volatility.
- σ is the standard deviation of the asset's spread.

A higher value of λ results in a more conservative strategy, where trades are only taken when volatility exceeds a significant level.

2.3 Rolling Volatility

Rolling volatility is calculated over a moving window of time. This technique provides a dynamic measure of volatility, which adapts to changing market conditions. Rolling volatility is useful for determining when market conditions are conducive to entering or exiting trades.

$$\sigma_{\text{rolling}}(t) = \sqrt{\frac{1}{N} \sum_{i=t-N+1}^t (r_i - \bar{r})^2}$$

Where:

- N is the window size (e.g., 30 days).
- r_i is the return at time i .

- \bar{r} is the average return over the window.

Rolling volatility can help identify periods of high volatility and provide signals for risk-adjusted decision making.

3 Conclusion

Risk management and volatility are two critical aspects of a successful trading strategy. By utilizing concepts such as stop-loss orders, position sizing, diversification, and volatility measures, traders can mitigate risk and improve their chances of success. Volatility, variance thresholds, and rolling volatility provide valuable tools for managing risk and adapting to changing market conditions. The use of risk-adjusted return metrics such as the Sharpe and Sortino ratios further enhances the evaluation of a strategy's performance in relation to its risk exposure. Ultimately, effective risk management and volatility analysis are key to maintaining consistent profitability in pairs trading.