OUR DAILY BREAD

Psalms 27: 10
"When my father and mother forsake me, then the Lord will take me up."

MATH PROFICIENCY
(Arithmetic and Algebra)

SETS: collections of certain objects, usually number. All of the objects/ numbers within a set are called the MEMBER/ ELEMTS of the set.

- **Ex:** The set of integers is {… -3, -2, -1, 0, 1,…}

Subsets – If every element of a set B is also a member of a set A, then we say B is a subject of A; the symbol $\subset$ means "is a subject of" and the symbol $\not\subset$ means "is not a subject of."

- **Example:**
  1. A = {1, 3, 5}, B = {1, 2, 3, 4, 5}
     So, A $\subset$ B because every element in A is also in B.
  2. X = {1, 3, 5}, Y = {2, 3, 4, 5, 6}
     So, X $\not\subset$ Y because 1 is in X but not in Y

- Every set is a subset of itself (Ex: for any set A, A $\subset$ A)
- The empty set is a subset of any set A ($\varnothing \subset$ A)
- For any two sets A and B, if A $\subset$ B and B $\subset$ A then A = B.

Number of subsets = $2^{n(A)}$; where n(A) = number of elements in the finite set A

- **Example:** Q = {x, y, z}. How many subsets will Q have?
  Solution: n(Q) = 3
  Number of subsets = 2^3 = 8

Universal Set – the set containing all the elements of the other sets

- **Example:** A = {1, 2, 3, 4}, B = {1, 3, 5, 7}, and C = {7, 9, 3} and the universal set U = {1, 2, 3, 4, 5, 6, 7, 8, 9}

Set of Operations:

2. **Union (U)** – combining the elements of two (or more) sets; an element of A U B is required to EITHER A or B
   - **Example:** If A = {1, 2, 3} and B = {4, 5} then A U B = {1, 2, 3, 4, 5}
3. **Intersection ($\cap$) –** looking for the elements that two (or more) sets have in common; An element of A $\cap$ B should belong to both A and B.
   - **Example:** If A = {1, 2, 3} and B {1, 2, 4, 5} then A $\cap$ B = {1, 2}
4. **Difference –** the difference of set A and set B denoted by (A-B) is determined by removing all common elements of set A and B from set A.
   - **Example:** If A = {1, 2, 3} and B = {1, 2, 4, 5} then A – B = {3}

- **Null Set $\varnothing$ -** the set containing no elements
  Example: If A {1, 2, 3} and B = {4, 5} then A $\cap$ B = {$\varnothing$}
- **Complement –** the difference of the Universal set and a certain set A; denoted by A' or A`
  Example: Universal Set U = {1, 2, 3, 4, 5, 6, 7, 8, 8} and A = {1, 2, 3, 4}, A' = {5, 6, 7, 8, 9}

SET OF REAL NUMBERS

Natural Numbers (N) – counting numbers {1, 2, 3,…} (positive integers) or the whole numbers {0, 1, 2, 3, …} (the non-negative integers).

Integer (Z) – integers are the natural numbers and their negatives {…-3, -2, -1, 0, 1, 2, 3…}

Rational (Q) – ratios of integers that can be expressed as fractions, such as ½ = 0.5; also includes decimal values that are infinitely continuous (non terminating) but repeating, such as 1/3 = 0.333…

Real Algebraic (Ag) – irrational numbers that are algebraic; IRRATIONAL – has decimal values that are infinitely continuous (non terminating) and non repeating

Real Numbers (R) – all the numbers on the continuous number line; Real numbers may be rational or irrational, and algebraic or non-algebraic (transcendental); TRANSCENDENTAL – represented as letter symbols which denote a value

Imaginary/ Unreal Numbers (I) – cannot be found in the number line; cannot exist numerically or has no value; includes square root or negative numbers and fractions with zero as denominator

OPERATIONS ON INTEGERS

1. When multiplying or dividing two integers: if the signs are the same, the result is positive.
   - negative x positive = negative
     positive / positive = positive
     negative x negative = positive
     negative / negative = positive

2. When multiplying or dividing two integers, if the signs are different, the result is negative.
   - positive x negative = negative
   - positive / negative = negative

3. When adding two integers with the same sign, the sum has the same sign as the addends.
   - positive + positive = positive
   - negative + negative = negative

4. When adding integers of different signs, follow this two-step process:
   a) Subtract the absolute values of the numbers. Be sure to subtract the lesser absolute value from the greater absolute value.
   b) Apply the sign of the larger number.

5. When subtracting integers, change all subtraction to addition and change the sign of the number being subtracted to its opposite. Then proceed with addition.

FACTORS AND MULTIPLES:

Factors: whole numbers that, when divided into the original number, result in a quotient that is a whole number

- Example: The factors of 18 are 1, 2, 3, 6, 9 and 18 because these are the only whole numbers that divide evenly into 18.

- A prime number has only itself and the number 1 as factors: 2, 3, 5, 7, 11, 13, 17, 19, 23, …
- A composite number is a number that has more than two factors: 3, 6, 8, 9, 10, 12, 14, 15, 16, …

Common Factors: the factors that the numbers have in common; the greatest common factors (GCF) of two or two or more numbers is the largest of all the common factors.

Multiples: number that can be obtained by multiplying a number x by a whole number is called a multiple of x

Common multiples: multiples that the numbers have in common; the least common multiple (LCM) of two or more numbers is the smallest of all the common multiples.

Prime factorization: process of breaking down factors into prime numbers

LAWS OF EXPONENT:

1. Any base raised to zero is always equal to 1.
   - $(13x)^0 = 1$
   - $70^0 = 1$
2. When multiplying identical bases, copy the base and add the exponents.
   - $X^n(X^m) = X^{n+m}$
   - $3^3(3^4) = 3^7$
3. When dividing identical bases, copy the same base and subtract the exponents.
   - $X^n/X^m = X^{n-m}$
   - $3^7/3^4 = 3^3$
4. If an exponent appears outside the parentheses, multiply the exponents outside the parenthesis by the exponents inside.
   - $(X^m)^n = X^{mn}$
   - $(5^4)^3 = 5^{12}$
5. A base raised to a negative exponent (-n) is equal to the multiplicative inverse of that number raise to (n)
   - $X^{-n} = 1/n$
   - 3^{-4}
   - $= 1/3^4$

PROPERTIES OF AQAURE ROOTS

- **Square of a number** – the product of a number and itself; a x a = a^2
  Example: the number 36 is the square of the number 6 since 6 x 6 = 36; while 6 is called as the Square root of 36.
- **Perfect Square –** a whole number whose square root is also a whole number.

1. When adding or subtracting radicals with the same radicand, add or subtract only the coefficients. Keep the radicand the same.
   - $a\sqrt{b} + c\sqrt{b} = (a+c)\sqrt{b}$
   - $4\sqrt{7} + 6\sqrt{7} = (4+6)\sqrt{7} = 10\sqrt{7}$

2. You cannot combine radicals with different radicands using the addition or subtraction.
   - $\sqrt{a} + \sqrt{b} \neq \sqrt{a+b}$
   - $\sqrt{2} + \sqrt{3} \neq \sqrt{5}$

3. The product of the square roots of two numbers is the same as the square root of their product.
   - $\sqrt{a} \times \sqrt{b} = \sqrt{a \times b}$
   - $\sqrt{7} \times \sqrt{3} = \sqrt{21}$

4. The quotient of the square roots of two numbers is the square root of the quotient of the two numbers.

- $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$, where b ≠ 0
- $\frac{\sqrt{24}}{\sqrt{8}} = \sqrt{\frac{24}{8}} = \sqrt{3}$

5. The square of a square root radical is the radicand.
   - $(\sqrt{N})^2$ = N
   - $(\sqrt{4})^2 = \sqrt{4} \times \sqrt{4} = \sqrt{16} = 4$

6. To simplify a square root radical, write the radicand as the product of two factors, with one number being the largest perfect square factor. Then write the radical over each factor and simplify.
   - $\sqrt{8}) = \sqrt{4} \times \sqrt{2} = 2 \times \sqrt{2}$

DECIMALS

1. **Adding and Subtracting:** line up the decimals points then add or subtract like adding or subtracting whole numbers.
   Ex: 2.0 + 0.03 +12.3 = ?

$$\begin{array}{r} 2.0 \\ 0.03 \\ +\ \underline{12.30} \\ 14.33 \end{array}$$

2. **Multiplying decimals:** multiply them like whole numbers; then count the total number of digits to the right of all decimal points. Place your decimal point in your answer so there is the same number of digits to the right of the decimal point.
   Ex: 3.12 x 1.1 x 1.0 = ?
   3. 12 – two decimal places
   1.1 – one decimal place
   x <u>1.0 – one decimal place</u>
   3.4320 – four decimal places (2+1+1)

3. **Dividing decimals:** divide them like whole numbers; if the divisor has a decimal, move it to the right as many places as necessary until it is a whole number; then move the decimal point in the dividend the same number of places; add zeros to the dividend as necessary.
   Ex: 26/0.002 = ?
   0.002⟌26

   13000.
   0.002⟌26. = 2⟌26000.00

FRACTIONS

1. **Adding and Subtracting:** in fractions with like denominators, add or subtract the numerators and leave the denominator as it is.
   $\frac{1}{6} + \frac{4}{6} = \frac{1+4}{6} = \frac{5}{6}$ $\qquad$ $\frac{5}{7} - \frac{3}{7} = \frac{5-3}{7} = \frac{2}{7}$

- **To add or subtract fractions with unlike denominators:**
  1. Find the least common denominator (LCD).
  2. Convert the unlike denominators into the LCD. The LCD is the smallest number divisible by each of the denominators. For example, the LCD of 1/6 and ¼ is ½ because ½ is the least multiple shared by 4 and 6.
  3. Convert each fraction by multiplying both the numerator and denominator by the needed factor to get the LCS
  4. Ass or subtract the new numerator

$\frac{1}{8} + \frac{1}{12}$ LCD is 24 because 8 x 3 = 24 and 12 x 2 is 24

$\frac{1}{8} = 1 \times \frac{3}{8} \times 3 = \frac{3}{24}$ Convert the fraction

$\frac{1}{12} = 1 \times \frac{2}{12} \times 2 = \frac{2}{24}$ Convert the fraction

$\frac{3}{24} + \frac{2}{24} = \frac{5}{24}$ Add numerators only

2. **Multiplying:** multiply the numerators and the denominators
$$\frac{5}{8} \times \frac{3}{7} = \frac{5 \times 3}{8 \times 7} = \frac{15}{56}$$

3. **Dividing:** the same as multiplying first fraction by the reciprocal of the second fraction; to get the reciprocal of a fraction just swap its numerator and denominator
$$\frac{3}{4} \div \frac{2}{5} = \frac{3}{4} \times \frac{5}{2} = \frac{15}{8}$$

FUNCTIONS and EQUATIONS

Function: a set of ordered pairs in which no two ordered pairs have the same first element.
Operations of functions: substitute numerical values to the variables and perform the operation

Example: $f(x) = 2x^3 + x - 3$ and $g(x) = x^2 - 2x + 1$
Find: f(2) + g(1)
Solution: $[2(2)^2 + 2 - 3] + [(1)^2 - 2(1) + 1]$
$[2(4) + 2 - 3] + [1 - 2(1) + 1]$
$[8 + 2 - 3] + [1 - 2 + 1]$
$7 + 0 = 7$

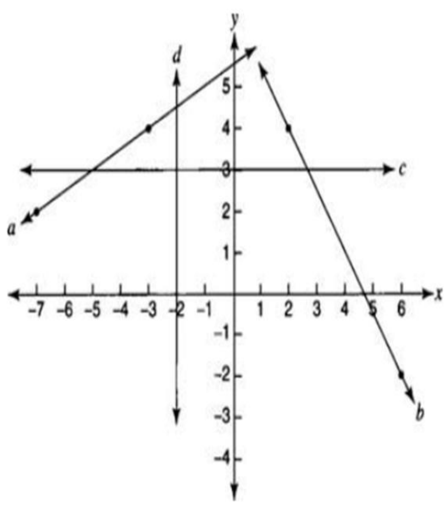
Composite Function: solve first the inner function, then substitute the result in place of the inner function and solve.

Example: $f(x) = 2x^2 + x - 2$ and $g(x) = x^2 - 3x + 3$
Find: f(g(2))
Solution: $g(2) = (2)^2 - 3(2) + 3 = 1$
$f(1) = 2(1)^2 + (1) - 2 = 1$

GRAPHS OF LINEAR FUNCTIONS

- **Standard form:** Ax + By = C
- **Slope-intercept form:** y = mx + b, where m is the slope of the line and b is the y intercept of the line (value of y if x = 0)

- **Slope of a line:** $m = \frac{y_2 - y_1}{x_2 - x_1}$, where $x_2 - x_1 = 0$
- **Midpoint formula**
  $x = \frac{x_1 + x_2}{2}$ $y = \frac{y_1 + y_2}{2}$
- **Concurrent lines:** lines on top of each other; have infinite solution set for their x and y
- **Point –slope form:** $y - y_1 = m(x - x_1)$
- **2-point form:** $\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$
- **Distance Formula:**
  $d = \sqrt{(x_1 - x_2)}^2 + (y_1 - y_2)^2$
- **Parallel Lines:** lines with equal slopes; parallel lines do not have a solution set for their x and y values
- **Perpendicular Lines:** lines whose sloes are negative reciprocals
- **Intersecting Lines:** lines meeting at a single point; have one solution set value for x and y



a has a rising slope = +

b has a falling slope = -

c is horizontal = 0

d is vertical = undefined

FACTORING EQUATIONS

- An equation in the form $ax^2 + bx + c = 0$, where a, b and c are numbers and $a \neq 0$; could be achieved by multiplying 2 binomials; binomial – equation with two terms separated by addition or subtraction.

- **FOIL METHOD:** used when multiplying binomials. FOIL represents the order used to multiply terms. First, Outer, Inner and Last. To multiply binomials, you multiply according to the FOIL order and then add the products.

- **QUADRATIC FORMULA** $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
  Where x are the roots of a quadratic formula

Standard form: $y = ax^2 + bx + c$

a) if $b^2 - 4ac = 0$ then the roots are real and equal
b) if $b^2 - 4ac < 0$ then no real roots exist
c) if $b^2 - 4ac > 0$ then the roots are real and not equal

FACTORING: expressing equations as products of simpler equations

1. Common Monomial Factor

- **Ex:** Factor $12x^3 - 16x^2 + 8x$
  x is a common factor for the first three terms. Also, the numbers 12, -16 and 8 have the greatest common factor of 4. Thus, divide all terms with 4x to get the factors.
  $12x^3/4x - 16x^2/4x + 8x/4x = 4x\,(3x^2 - 4x + 2)$

2. Difference of two squares: $(a+b)(a-b) = a^2 - b^2$

- **Ex:** Factor $64x^4 - 9y^2z^2$
  $64x^4 = (8x^2)^2$ and $9y^2z^2 = (3yz)^2$
  $64x^4 - 9y^2z^2 = (8x^2)^2 - (3yz)^2$
  $= (8x^2 + 3yz)\,(8x^2 - 3yz)$

3. Sum and Difference of two cubes

- Sum of cubes: $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
- Difference of cubes: $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

4. Trinomials of the form: $x^2 + bx + c$

- **Ex:** $x^2 - 6x + 8$
  Start by looking at the last term in the trinomial = 8, and look for two integers when multiplied together will have the product positive 8: 1x8; -1x-8; 2x4; -2x-4; Then look at the middle term of the trinomial = 6x and choose two factors from the above list that will add up to -6: -2 and -4; Now write the factors using -2 and -4: (x-2)(x-4). Use the FOIL method to double check your answer: $(x-2)(x-4) = x^2 - 6x + 8$

5. Square Trinomial

- If a trinomial is of the form $(x)^2 + 2(x)(y) + (y)^2$, it can be factored into the square of a binomial: $(x+y)^2 = (x)^2 + 2(x)(y) + (y)^2$