

STAT 4705: Homework 1

1. An engineering firm is hired to determine if certain waterways in Virginia are safe for fishing. Samples are taken from three rivers.
 - (a) List the elements of a sample space S , using the letters F for “safe to fish” and N for “not safe to fish”.
 - (b) List the elements of S corresponding to event E that at least two of the rivers are safe for fishing.
 - (c) Define an event that has its elements the points

$$\{FFF, NFF, FFN, NFN\}$$

2. Consider the sample space $S = \{\text{copper, sodium, nitrogen, potassium, uranium, oxygen, zinc}\}$, and the events

$$A = \{\text{copper, sodium, zinc}\}$$

$$B = \{\text{sodium, nitrogen}\}$$

$$C = \{\text{oxygen}\}$$

List the elements of the sets corresponding to the following events:

- (a) A^c ;
- (b) $A \cup C$;
- (c) $(A \cap B^c) \cup C^c$;
- (d) $B^c \cap C^c$;
- (e) $A \cap B \cap C$;
- (f) $(A^c \cup B^c) \cup (A^c \cap C)$

3. (1) How many distinct permutations can be made from the letters of the word *columns*?
(2) How many of these permutations starts with the letter *m*?

4. Evaluate each of these expressions:

(1) $9!$ (2) $6!$

(3) $\binom{8}{5}$

5. The basic storage unit of a digital computer is a “bit”. A bit is a storage position that can be designated as either on (1) or off (0) at any given time. In converting picture images to a form that can be transmitted electronically, a picture element, called a *pixel* is used. Each pixel is quantized into gray levels and coded using a binary code. For example, a pixel with four gray levels can be coded using two bits by designating the gray levels by 00, 01, 10 and 11.
- (1) How many gray levels can be quantized using a four-bit code?
 - (2) How many bits are necessary to code a pixel quantized to 32 gray levels?

6. Fifteen telephones have just been received at an authorized service center. Five of these telephones are cellular, five are cordless and the other five are corded phones. Suppose that these components are randomly allocated the numbers 1, 2, ..., 15 to establish the order in which they will be serviced.
- (1) What is the probability that all the cordless phones are among the first ten to be serviced?
 - (2) What is the probability that after servicing ten of these phones, phones of only two of the three types remain to be serviced?
 - (3) What is the probability that two phones of each type are among the first six to be serviced?

7. Consider a regular deck of cards with 52 cards in total.
- (1) How many ways can we have a poker hand of 5 cards?
 - (2) Four of a kind is a poker hand that contains all four cards of one rank and any other (unmatched) card. For example, $9\clubsuit 9\spadesuit 9\diamond 9\heartsuit J\spadesuit$ is a “four of a kind”. How many ways can we have “four of a kind” ?
 - (3) In a poker hand of 5 cards, what’s the probability of getting a “four of a kind”?
 - (4) A full house is a hand that contains three matching cards of one rank and two matching cards of another rank. For example, $3\clubsuit 3\spadesuit 3\diamond 6\heartsuit 6\spadesuit$ is a “full house”. In a poker hand of 5 cards, what’s the probability of getting a “full house”?
 - (5) In a poker hand, are getting a “four of a kind” and getting a “full house” mutually exclusive events? Justify your answer.

8. Assume that in a particular military exercise involving two units, Red and Blue, there is a 60 % chance that the Red unit will successfully meet its objectives and a 70 % chance that the Blue unit will do so. There is an 18 % chance that only the Red unit will be successful. What is the probability that both units will meet their objectives? What is the probability that one or the other but not both of the units will be successful?

9. A computer center has three printers, A, B, and C, which print at different speeds. Programs are routed to the first available printer. The probability that a program is routed to printers A, B, and C are 0.6, 0.3, and 0.1, respectively. Occasionally a printer will jam and destroy a printout. The probability that printers A, B, and C will jam are 0.01, 0.05 and 0.04, respectively. Your program is destroyed when a printer jams. What is the probability that printer A is involved? Printer B is involved? Printer C is involved?

10. In an experiment to study the dependence of hypertension on smoking habits, the following data were collected on 180 individuals:

Table 1: Problem 10

	Non-smokers	Moderate Smokers	Heavy Smokers
Hypertension	21	36	30
No hypertension	48	26	19

If one of these individuals is selected at random, find the probability that the person is

- (a) experiencing hypertension, given that the person is a heavy smoker;
- (b) a nonsmoker, given that the person is experiencing no hypertension